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From Euclid to GG-6

The Realization of Euclid's *Elements* in Six Dimensions: GU-5, OGN-5, and a Parameter-Free Reconstruction of Modern Physics

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Abstract

For 2300 years, the ideal of physical science has been Euclid's: a deductive system in which every theorem follows from a few postulates, with nothing left to fit. Modern physics drifted from that ideal — the Standard Model carries some 28 free parameters, general relativity adds Newton's constant and the cosmological constant, and string theory answers with a landscape of $\sim 10^{500}$ vacua, in which the values are selected rather than derived.

This foundations volume of the Arisaka-GG series argues that the drift was not necessary. Building on the GG-Constitution, the 6D bulk, and the modified Bianchi identity BI-6, it shows that the *dimensionful* content of physics is exhausted by the Geometric Universal Five Constants GU-5 $= \{c, G_N, \hbar;\, k_B, \mathsf{Q_B}\}$ — three conversion factors of the bulk directions of one six-dimensional geometry and two of its boundary projection — while its *dimensionless* content is exhausted by the five OGN-5 integers $(1, 3, 5;\, 7, 35)$, Diophantine theorems of the same geometry. The elementary charge e is not a primitive but a consequence of $\alpha^{-1}(0) = 2^J + N^2 = 137$, leaving only the mathematical constants e (Euler) and π to complete the inventory, with no continuous parameter intervening. The volume establishes the reconstruction theorems and the master parameter audit, importing — rather than re-deriving — the Standard-Model and cosmological results of its companion papers.

After all, the Euclidean condition is what makes a physical theory worth the name *deductive science*: every quantity must be either a conversion factor of the geometry or a theorem of the framework, never a free choice. The 2300-year detour from Euclid is, in this precise sense, ending.

Executive Summary

Why this paper exists. Modern physics rests on about thirty numbers that nobody has derived. The strength of electromagnetism, the mass of the electron, the fraction of the universe that is matter — each is measured to exquisite precision and explained by nothing. They are handed to the theory rather than produced by it, and the theory works anyway, which is the uncomfortable part. Twenty-three centuries ago Euclid set a different standard: write the assumptions down in plain view, and let everything else follow, with nothing left to fit. The *Elements* has no adjustable constants because it has no place to put one. This paper asks whether physics can be held to that standard again, and it answers by walking the inventory one quantity at a time.

What a conversion factor is. The answer turns on an old idea used more aggressively than usual. When two quantities are measured in incommensurable units, the number that converts between them looks like a fact about the world; often it is a fact about the geometry. The speed of light is the clearest case. Before 1908 it was a property of light; after Minkowski it is the exchange rate between the second and the meter, and its numerical value says only which units you happened to choose. Einstein did the same to Newton's constant, which converts mass–energy into curvature. This paper's claim is that the remaining constants are the same kind of object — that $\hbar$ converts a phase winding into an action, that k_B converts temperature into energy, and that both are readings of a six-dimensional shape rather than properties of its contents.

The two directions that are not places. A six-dimensional world usually means four dimensions plus two more places to go, and that is not what is meant here. The fifth coordinate is a magnification setting: $r = \ln(E_{\mathrm{GUT}}/\mu)$, the logarithm of the ratio between the unification energy and the energy at which one is looking, so moving along it is not traveling anywhere but changing the scale at which the world is examined. The sixth is a closed circle whose entire content is a phase. Neither carries a length, a light cone, or an adjustable radius, and that is the sharpest way to say that neither is a spatial extra dimension — there is nothing to compactify and nothing to stabilize. What they carry instead is the two things a constant can convert: a scale, and an angle. This is the whole of what "six-dimensional" claims in this program, and a reader who takes only one idea from the volume should take this one.

The two pentads. Everything continuous in physics is then carried by five conversion factors, and everything dimensionless by five integers:

$$\boxed{\begin{aligned} &\mathsf{GU\text{-}5} := \{\, \underbrace{c,\ G_N,\ \hbar}_{\text{3 bulk}};\ \underbrace{k_B,\ \mathsf{Q_B}}_{\text{2 boundary}}\ \} \quad \text{(dimensionful conversion factors)};\\ &\mathsf{OGN\text{-}5} := (\,\underbrace{M, N, L}_{3};\ \underbrace{J, K}_{2}\,) = (1, 3, 5;\ 7, 35) \quad \text{(dimensionless theorems of } \mathsf{BI\text{-}6}\text{)}. \end{aligned}} \tag{1}$$

The two share a $(3+2)$ architecture. The first three conversion factors are read off the three bulk directions of $\mathcal{M}_4 \times I_r \times S^1_\theta$; the last two belong to the boundary where observation happens. The elementary charge is not a member: it follows from $\alpha^{-1}(0) = 2^J + N^2 = 137$, two integers of the ledger. Once the two pentads are in place there is no continuous knob left in the gauge or cosmological sectors — every dimensionful quantity is a product of GU-5 members with ledger integers and the two transcendentals e and π, and every dimensionless ratio is an arithmetic theorem.

Where the free parameters go. The five integers are not chosen. They are the unique solution of two conditions that were never designed together: the trace structure of the unified block, and the global consistency of the modified Bianchi identity BI-6 that the constitution forces on the bulk. The lock closes twice over, $K = LJ = M^2 + N^2 + L^2 = 35$, and it closes at one point. What remains after it closes is a choice of units, not a parameter — and units are what the first pentad is made of.

Twenty-three centuries, in one line. The history is a sequence of partial geometrizations, each absorbing one constant and stopping. Riemann supplied manifolds of arbitrary dimension; Maxwell put c into electromagnetism and Minkowski into the metric; Einstein put G_N into curvature; Kaluza and Klein put $\hbar$ into integer modes on a compact direction; Wilson and Hosotani made symmetry breaking a holonomy; Randall and Sundrum produced the Planck–electroweak hierarchy from a warp. Each step was forced by the one before it, and each stopped one constant short. This paper's proposal is that the sequence has an end, and that the end is a $(4+2)$ geometry in which all of them are absorbed at once.

Six results, extracted and audited. As the consequence volume of the series, this paper takes six results from its companions and audits them: c is the conversion factor of the Lorentzian signature (T1-GG6); G_N is the scale–curvature coupling of the interval I_r (T3-GG6); $\hbar$ is the integer-mode quantization factor on the circle S^1_θ (T2-GG6); $\mathsf{Q_B} \approx 0.1555$ eV per qubit is the boundary information member, fixed by the chain OGN-5 $\to \alpha \to m_e \to E_{\mathrm{Ry}} \to \mathsf{Q_B}$ (Arisaka, 2026, A7-Qi4); the ledger $(1, 3, 5;\ 7, 35)$ is the unique Diophantine solution of the global consistency of BI-6 (T6-GG6 of Arisaka, 2026, A2-GG6; OGN-5 theorem of Arisaka, 2026, A3-BI6); and the master audit closes for the gauge and cosmological sectors with twenty-two entries and no free dimensionless parameter, the fermion mass spectrum tracked as the open item O9 (Arisaka, 2026, A9-SM).

Canonical implication chain. The deductive flow from constitutional clauses to falsifiable predictions runs:

$$\boxed{\begin{array}{l} \text{A1 axioms + postulates} \;\Rightarrow\; \text{A2 6D bulk} \;\Rightarrow\; \text{A3 BI-6 + OGN-5} \;\Rightarrow\; \{c, G_N, \hbar\} \\ \Rightarrow\; \alpha^{-1},\ \alpha_s,\ \sin^2\theta_W,\ v_{\mathrm{EW}};\quad \Omega_m,\ \Omega_\Lambda,\ H_0;\quad k_B,\ \mathsf{Q_B},\ N_A\mathsf{Q_B} \\ \Rightarrow\; \text{LHC, HL-LHC, CMB-S4, DESI, ADMX, ATP.} \end{array}} \tag{2}$$

The chain runs one way. Every link is a theorem rather than a fitted parameter, and no step is a free choice: each is forced by the step before it together with a clause of the constitution.

What is shared, and where the counts part. The mechanism here is not new and is not claimed to be. Extra dimensions, holonomy symmetry breaking, warped hierarchies and anomaly inflow are all borrowed, and each is credited to the people who found it. What differs is the bookkeeping at the end. A framework that admits a large space of solutions must explain which one we inhabit, and the count of adjustable quantities survives the explanation; a framework whose consistency conditions close on a single integer ledger has no such count to report, and no way to absorb a measurement that disagrees. That is not an argument that one is right. It is a statement that the two are separated by something a measurement can reach, and Part XII names where.

What this paper does not do. It does not re-derive the Standard Model, the cosmological inventory, or the running of the fine-structure constant. Those derivations belong to the companions and are imported here and audited, which is a different and smaller claim. It does not close everything: the closed-form fermion mass spectrum (O9) remains open, the bridge to the standard quantum-gravity programs (O12) is not attempted, and the cosmological-constant coefficient (O11) is now selected in closed form by Arisaka (2026, A10-Cosmos) but conditionally, on one lemma that is named. Results are graded where they appear — derived, derived under a stated condition, or open — and the grading is kept separate in every table rather than averaged into a headline.

Where it can fail. A framework with nothing left to adjust cannot absorb a disagreement, which is the price of the claim and the point of it. Seven falsifiers are declared in Section 37, each with the measurement that would end it. Four carry numbers a reader can hold: the fine-structure constant at $\alpha^{-1}(0) = 2^J + N^2 = 137$; a resonance cluster at $M_{\text{lock}} \approx 3.85$ TeV, within reach of the high-luminosity LHC; a dark-matter candidate at 8.1 μeV (1.95 GHz) with a frequency splitting of exactly one part in thirty-five, in the band that IBS-CAPP and ADMX's extended-range search are funded to sweep (Arisaka, 2026, C1-Kaxion); and a cosmic matter fraction of exactly $1/\pi$, against Planck's 0.315 ± 0.007. Not one of the four is a measurement a decade away.

A closing thought, for any reader. There is a question underneath all of this that is older than physics: how much of the world had to be the way it is. Every constant we measure is a small admission that something was left unexplained — a number the theory could not produce and had to be told. The history of the subject is the history of such admissions being withdrawn one at a time: why bodies fall, why flames have colors, why stars have the masses they do. What is proposed here is that they can be withdrawn together, and that the numbers of nature are not an inventory the universe was issued but the only arithmetic a consistent world permits. A crystal does not select its angles from a catalog of available angles; there is no catalog, and the angles are what the packing leaves over. Whether the universe is a crystal in that sense is not something any argument can settle, and this paper does not pretend otherwise. It is something a measurement can end — and the measurements are named, dated, and waiting.

Keywords: GG-Theory; GG-6; BI-6; OGN-5; GDOS; Gi-4; Qi-4; 6D bulk; conversion factors; no free parameters; GG-Constitution; parameter-free physics; Euclid's *Elements*; Randall–Sundrum lineage; geometric inevitability.

The Two Pentads, and What They Carry

The whole of this paper in two pictures — this one and the next: what the framework claims physics is made of, and how that claim descends to measured numbers.

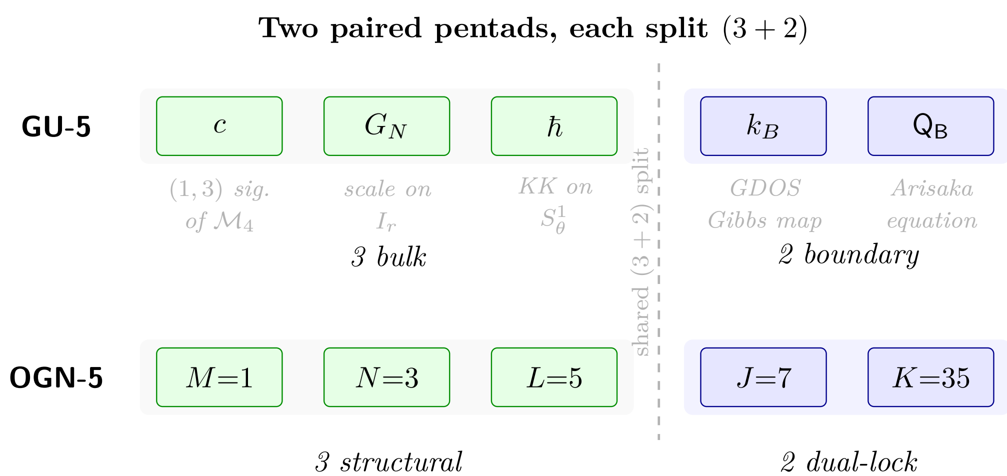
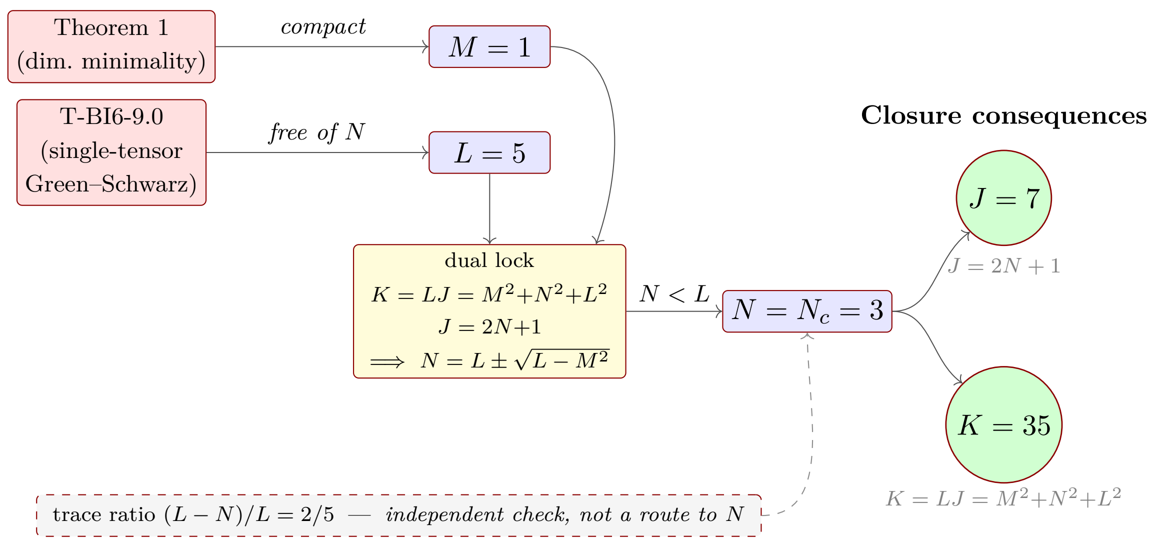


Figure 1: The claim, in one picture. Everything dimensionful in physics is carried by five conversion factors, GU-5, and everything dimensionless by five integers, OGN-5. Both are split $(3+2)$ — three bulk members and two boundary members on the left, three sequential integers and two dual-lock integers on the right. The rhyme is architectural, not a term-by-term identity. Shown in full, with the argument, as Fig. 8 in Part VI.

Five Integers, Forced — and No Free Parameter Left in the Bulk



The OGN-5 Diophantine cascade. Two theorems force $(M, L) = (1, 5)$ without reference to any color count (left column); the dual lock together with the block embedding $N < L$ then determines $N = N_c = 3$, and the pair $(J, K) = (7, 35)$ follows from the closure-index clause and the identity $K = LJ = M^2 + N^2 + L^2 = 35$ (right column). The double equality $35 = 5 \times 7 = 1 + 9 + 25$ is the axiomatic fingerprint of OGN-5: it is not a coincidence but the content of the dual lock, and every numerical prediction of the program descends from these five integers. Solid arrows mark the forcing chain; the one dashed arrow carries the check. After the cascade closes, no continuous dimensionless parameter survives in the bulk theory. Full version, and the proof of each of the five steps: the companion (Arisaka, 2026, A2-GG6).

The Descent, in One Picture

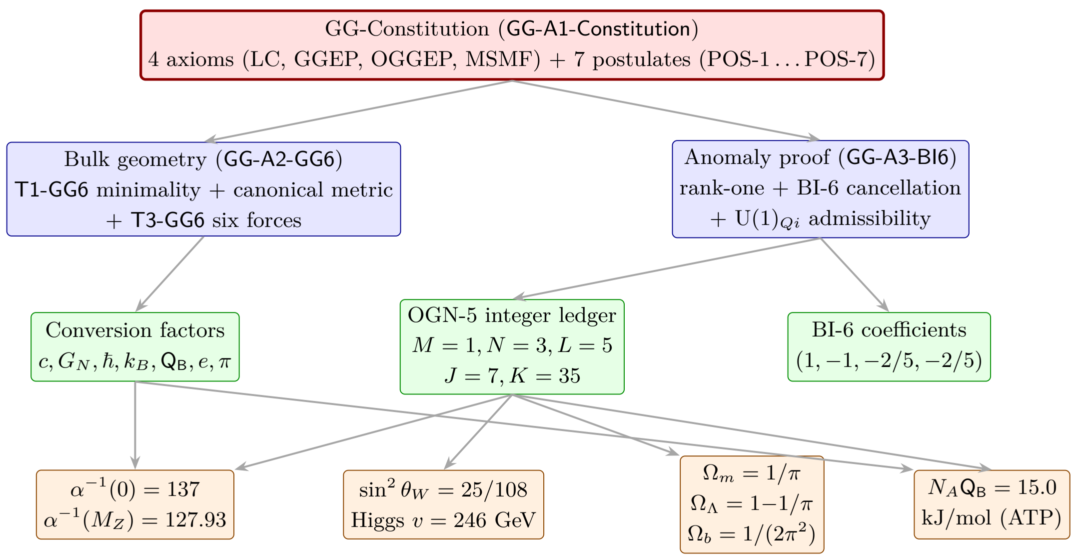


Figure 2: The descent, in one picture. From the constitutional clauses of GG-A1-Constitution through the bulk theorems of GG-A2-GG6 and the anomaly proof of GG-A3-BI6, to the conversion factors, the integer ledger and the BI-6 coefficients — and from those to the numbers experiment measures. Every arrow is a derivation; no arrow is a fitted parameter. Shown in full as Fig. 9 in Part VII, where the master audit it summarizes is walked row by row.

Ten Numbers the Ledger Alone Predicts, Against What Experiment Measures

Sharp numerical predictions from OGN-5 alone. These are the entries on which the framework is most directly exposed to measurement, and the Agreement column states the exposure honestly at two levels. The leading predictions use nothing but the ledger integers: $\alpha^{-1}(M_Z) \simeq 2^J = 128$ and $\alpha^{-1}(0) \simeq 2^J + N^2 = 137$, both correct to three significant figures with no fitted quantity anywhere. The refined entries add the quantum-defect correction derived in GG-A9-SM and reach six significant figures. The distinction between the leading and the corrected rows is deliberate: the first are theorems of this paper's ledger, the second inherit a companion paper's calculation. The same table, in place: Table 20 in Part VIII, p. 143.

Quantity	GG-prediction	Empirical value	Agreement
$\alpha^{-1}(M_Z)$ leading	$2^J = 128$	~ 128	3 sig. figs.
$\alpha^{-1}(M_Z)$ + QD (GG-A9-SM)	$128 - N^2/2^J = 127.9297$	127.930(8) (GG-A9-SM benchmark)	6 sig. figs., pull -0.03
$\alpha^{-1}(0)$ leading	$2^J + N^2 = 137$	~ 137	3 sig. figs.
$\alpha^{-1}(0)$ + QD (GG-A9-SM)	≈ 137.036	$137.035\,999\ldots$ (CODATA)	6 sig. figs. (GG-A9-SM)
$\sin^2\theta_W$	$25/108 = 0.23148$	0.23155 ± 0.00004 (GG-A9-SM)	0.030%, pull -1.70
E_{Ry}	$\alpha^2 m_e c^2/2 \approx 13.606$ eV	13.6057 eV	5 sig. figs.
$\mathsf{Q_B}$	≈ 0.1555 eV per qubit	≈ 0.16 eV (biology)	2 sig. figs.
$N_A\,\mathsf{Q_B}$ (molar)	≈ 15.0 kJ/mol	~ 15 kJ/mol (ATP)	2 sig. figs.
$M_{\mathrm{lock}}/M_{\mathrm{Pl}}$	$\frac{1}{2}e^{-K} = \frac{1}{2}e^{-35} \approx 3.2 \times 10^{-16}$	$\sim 10^{-16}$	order of magnitude
Ω_m, Ω_Λ	$1/\pi$, $1 - 1/\pi$	0.315, 0.685 (Planck)	2%

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Notation

This page is short on purpose. It carries only the two conventions a reader cannot guess and the handful of names that appear in the Abstract and the Executive Summary before the text defines them. The full dictionary is in Part X: Appendix A is every abbreviation the program uses, and Appendix B is every symbol, ordered by construction and pointing at the equation that fixes it.

The two traces. Equation (50) carries two trace symbols and they are not interchangeable. $\mathrm{tr}\, R^2$ is the trace over the six-dimensional tangent indices; $\mathrm{Tr}\, F^2_{\mathrm{SU}(5)}$ is the trace over the fundamental **5** of SU(5); and the two abelian squares carry no trace at all, an abelian field strength being a one-by-one matrix.

The letter α, which means four things. $\alpha^{-1}(0) = 137$ is the fine-structure constant, and is what α means wherever it appears with no subscript. $\hat{\alpha}^{-1}_{\mathrm{EM}} = 128$ is the same constant at the Z pole. α_s is the strong coupling. And $\alpha_1 \dots \alpha_4 = (1, -1, -2/5, -2/5)$, with a *numerical* subscript, are the BI-6 coefficients — pure numbers, no units. The one to watch is α_3: here it is $-2/5$, and most readers arrive carrying the convention in which α_3 is the strong coupling.

Table 1: The six names this paper uses before it defines them, with the companion that fixes each. A reader who wants to start at Part I without another paper open needs no more than this. Every row is a pointer rather than a definition, and two of the six — the energy–information conversion factor and the lock scale — are the quantities the audit in Part VI is about.

GG-Theory	The twenty-two-paper program this paper belongs to; its axioms and postulates are fixed in GG-A1-Constitution.
GG-6	The $(4+2)$ bulk it audits, constructed in GG-A2-GG6. Its two extra directions are a scale and a phase, and neither is a length.
OGN-5	The five-integer ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$, on which the whole audit turns.
GU-5	The five conversion factors $\{c, G_N, \hbar; k_B, \mathsf{Q_B}\}$ — the subject of Part VI and of this paper's subtitle.
$\mathsf{Q_B}$	The energy–information conversion factor, ≈ 0.1555 eV, and the one member of GU-5 without a century of textbook standing.
M_{lock}	The lock scale, ≈ 3.85 TeV: the infrared end of the scale interval, and the mass of the first Kaluza–Klein resonance.

Six names used before they are defined. Everything else is expanded where it first appears, and again in Appendix A.

Preface

Roughly twenty-three centuries ago, Euclid of Alexandria sat down to write a book about geometry. He started with twenty-three definitions, five postulates, and five common notions, and from these foundational items he derived, by a sequence of demonstrations, the entire content of plane and solid geometry. The Pythagorean theorem was not fitted to any data; it was a deductive consequence of the postulates. The angle sum of a triangle was exactly two right angles in any plane satisfying Euclid's axioms; one did not measure it. There were no free parameters. There was nothing to fit.

For twenty-three centuries the *Elements* has served as the methodological standard of mathematical and physical science. Newton's *Principia* is, in its rhetorical structure, an explicit imitation of the *Elements*. Lagrange opens with foundational principles from which all analytical mechanics descends. Even Einstein's general relativity, in its mature presentation, is structured as a deductive theorem-machine starting from the equivalence principle and general covariance. And yet, in the twentieth century, something quietly went wrong. Modern physics produced a vast and successful theoretical edifice — but the edifice came to rest on roughly thirty empirical inputs: twenty-eight Standard Model parameters, Newton's gravitational constant, the cosmological constant. Each is a number that nobody has yet derived. The Euclidean ideal has, in this sense, been abandoned.

This paper, the historical-foundations companion of the Arisaka-GG series, argues that the abandonment need not be permanent. Building on three concept-introducing companion papers, we show that the universal constants that anchor physics — the speed of light, Planck's constant, Newton's gravitational constant — are dimensional conversion factors of one and the same six-dimensional geometry; that the five small integers that lock the entire program are Diophantine theorems of that geometry; and that the energy–information conversion factor that connects information to energy in living systems emerges from the same data. The Euclidean condition — a deductive science with no free parameters — is, in this precise sense, achievable again.

We invite the reader to walk the parameter audit clause by clause. Sector by sector, ask whether any entry remains a free choice. After all, the twenty-three-century detour from Euclid is what the present paper proposes to end. The Euclidean condition is what makes physical theory worth the name *deductive science*: every quantity must be either a conversion factor of the geometry or a theorem of the framework, never a free choice.

Part I
Foundations: The Euclidean Question

The Constitutional Question of Free Parameters, the GG-Constitution, and the Strategy of the Reconstruction

A free parameter is a debt. It is a number the theory cannot produce and must therefore be handed, and every one of them marks a place where the account is settled by measurement rather than by argument. What made Euclid's *Elements* the standard for twenty-three centuries was not the volume of theorems it derives but the absence of any such place in it: nothing in the *Elements* is fitted. Modern physics carries about thirty of them.

This Part asks whether it has to. The question is constitutional rather than technical — not *what is the value of the fine-structure constant* but *what kind of object is a physical theory allowed to be, and may it contain a dial at all* — and the rest of the paper is an attempt to answer it in the only way the *Elements* would accept, by deriving the quantities one at a time and showing the derivation.

Section 1 states the condition and locates the point at which physics left it. Section 2 sets out the charter under which the reconstruction works: four axioms and seven postulates, imported from Arisaka (2026, A1-Constitution) and used here as given rather than re-argued. Section 3 names the strategy, which is not to fit the constants better but to show that each one is either a conversion factor of a geometry or a theorem of an integer ledger, with no third species admitted. Section 4 rosters the seven reconstruction theorems that carry that claim, and Section 5 places the paper among the twenty-two companions it draws on and plots the route through the thirteen Parts.

Nothing is proved here. This Part states the question, fixes the vocabulary, and says what would count as an answer — which is the discipline the *Elements* demands before the first demonstration.

1 Introduction: The Constitutional Question of Free Parameters

1.1 The Euclidean ideal and its abandonment

There is a particular condition of intellectual achievement, set down by Euclid in Alexandria around 300 BCE in the thirteen books of the *Elements*, that has served as the methodological standard of mathematical and physical science for twenty-three centuries. In the *Elements*, Euclid begins with twenty-three definitions, five postulates, and five common notions stated at the opening of Book I, and from these foundational items he derives, by a sequence of demonstrations, the entire content of plane geometry (Books I–VI), the theory of proportion and number (Books VII–X), and three-dimensional geometry culminating in the classification of the five regular polyhedra (Books XI–XIII) (Euclid, c. 300 BCE, Heath, 1956). What is most remarkable about the *Elements* is not the volume of theorems it derives, vast as that is, but the methodological condition under which it derives them: *there are no adjustable parameters.* The Pythagorean theorem is not fitted to any data; it is a deductive consequence of the postulates. The angle sum of a triangle is exactly two right angles in any plane satisfying Euclid's axioms; one does not measure it. The five regular polyhedra are not chosen by convention; they are the unique solutions to a discrete combinatorial problem that the postulates make tractable.

This methodological condition — a deductive science with no free parameters in the foundational sense — has been the aspiration of physical theory ever since. Newton's *Principia* (1687) is, in its rhetorical structure, an explicit imitation of the *Elements* (Cohen and Whitman, 1999, Guicciardini, 2009, Newton, 1687): it begins with definitions, axioms (the three laws of motion), and proceeds to derive Kepler's laws (Kepler, 1609) and a vast body of celestial mechanics by geometric demonstration. Euclid's organization is so deeply embedded in Newton's mind that the geometric form persists even where Newton's actual mathematics is calculus disguised as geometric ratios (Arthur, 1995, Guicciardini, 2009). Lagrange's *Mécanique Analytique* (1788) likewise opens with foundational principles from which the analytical content is derived. Even general relativity, in Einstein's mature presentation, is structured as a deductive theorem-machine starting from the equivalence principle and general covariance (Einstein, 1915).

Yet the deeper one looks at twentieth- and twenty-first-century physics, the more one finds that the Euclidean condition has *not* been preserved. The Standard Model of particle physics requires twenty-eight empirical input parameters — the gauge couplings, the quark and lepton masses, the CKM and PMNS mixing angles and phases, the Higgs mass and vacuum expectation value, the QCD θ angle — that no theoretical principle predicts (Particle Data Group, 2024); this is the count enumerated and derived in Arisaka (2026, A9-SM), and it is the count used throughout the present paper. Standard cosmology requires several more (the cosmological constant, the various density parameters, the spectral index of perturbations) (Planck Collaboration, 2018). Above all of these float three universal dimensional constants whose values appear independent of any specific physical system: the speed of light c, Planck's reduced constant $\hbar$, and Newton's gravitational constant G_N. Standard textbook treatments classify these as facts of nature that one measures and tabulates; they are universally regarded as the most fundamental constants of physics, but their values are simultaneously regarded as inputs rather than outputs of theory.

The Euclidean ideal has, in this sense, been quietly abandoned. The *Elements* promised a deductive geometry without free parameters; modern physics has produced a vast theoretical edifice that nonetheless requires twenty or thirty empirical inputs to make contact with experiment. This is the condition that motivates the central question of the present paper:

> *Is the Euclidean condition recoverable for fundamental physics? Are the universal constants c, G_N, $\hbar$, and the parameters of the Standard Model in fact free, or are they consequences of a deeper geometric structure that has not yet been fully recognized?*

The answer developed in this paper, building on the companion papers of the Arisaka-GG series — Arisaka-GG-A1-Constitution (Arisaka, 2026, A1-Constitution) which proposes the constitutional charter; Arisaka-GG-A2-GG6 (Arisaka, 2026, A2-GG6) which derives the six-dimensional bulk and the bulk gauge content; Arisaka-GG-A3-BI6 (Arisaka, 2026, A3-BI6) which establishes the modified Bianchi identity BI-6 and the OGN-5 integer ledger; Arisaka-GG-A5-GDOS (Arisaka, 2026, A5-GDOS) which formalizes the bulk-to-boundary projection; Arisaka-GG-A7-Qi4 (Arisaka, 2026, A7-Qi4) which develops the Qi-4 boundary pillars and the energy–information Arisaka constant $\mathsf{Q_B}$; Arisaka-GG-A9-SM (Arisaka, 2026, A9-SM) which derives the Standard Model parameters from the integer ledger; and Arisaka-GG-A10-Cosmos (Arisaka, 2026, A10-Cosmos) which derives the cosmological inventory — is that the Euclidean condition *is* recoverable, and that the recovery is provided by the GG-6 framework: a unique six-dimensional geometry from which all dimensionful and dimensionless quantities of physics are read off as geometric data.

The constants c, G_N, $\hbar$ are not free; they are dimensional conversion factors of the GG-6 bulk. The integers of the OGN-5 ledger and the coefficients of the BI-6 identity are not postulated; they are theorems of the bulk geometry. The parameters of the Standard Model and the cosmological inventory are not fitted; they are derived. And the four axioms and seven postulates of the GG-Constitution (Arisaka, 2026, A1-Constitution) play the same role for GG-Theory that the five postulates of Euclid played for plane geometry.

1.2 Scope: what this paper proves, and what it imports

This is the foundations-and-audit volume of the Arisaka-GG series, and it is worth stating plainly, at the outset, what it does and does not undertake. It does *not* re-derive the Standard Model, the cosmological inventory, or the running of the fine-structure constant from scratch; those derivations are carried out in the companion papers and are *imported and audited* here. What this paper establishes in its own right is the parameter-free *reconstruction*: that the framework built in GG-A2-GG6 and GG-A3-BI6 contains no adjustable dimensionless freedom.

What this paper proves in its own right. The seven reconstruction theorems T-Euclid-1 through T-Euclid-7 (Sec. 4); the GU-5 ↔ OGN-5 two-pentad *parameter audit* of Part VII, which exhibits every dimensionful quantity as a member of GU-5 and every dimensionless quantity as an OGN-5 theorem or a BI-6 residue; and the *conceptual reconstruction* — the two non-spatial dimensions and the "third corner" of Part III.

What this paper imports and audits (proved elsewhere). The 6D bulk and its structural theorems (Arisaka, 2026, A2-GG6); the BI-6 identity and the OGN-5 integer cascade (Arisaka, 2026, A3-BI6); the GDOS boundary law (Arisaka, 2026, A5-GDOS); the Standard-Model parameters, including the running of α that closes 137 → 137.036 (Arisaka, 2026, A9-SM); the cosmological inventory (Arisaka, 2026, A10-Cosmos); and the Arisaka constant $\mathsf{Q_B}$ and its energy–information role (Arisaka, 2026, A7-Qi4). Every quantitative derivation cited in this paper is proved in the companion paper named at that point.

A reader should accordingly treat this paper as the audit-and-positioning volume of the program: its task is to show, theorem by theorem and row by row, that the parameter inventory is closed, and to situate the framework against its precursors and rivals — not to reproduce the companion-paper derivations it audits. Where this paper states a number such as $\alpha^{-1}(0) = 137$, it claims the *structural* (integer) result and attributes the precise value to the companion derivation.

2 The GG-Constitution: Four Axioms and Seven Postulates

2.1 Why a constitution?

The historical lineage traced in Parts I and II shows that physics has been progressively reorganized as a deductive geometric science, with each generation absorbing more of the empirical content of the world into the geometry of an increasingly rich manifold. But this reorganization has remained, throughout, a piecemeal achievement: each generation has derived a few constants and parameters from a particular geometric setup (Maxwell's c from the Lorentzian metric; Klein's quantization of charge from compactness; Randall–Sundrum's hierarchy from warping), without specifying the *axiomatic layer* — the set of foundational rules that any candidate geometric theory must obey before it counts as physically admissible.

This is the gap that Arisaka (2026, A1-Constitution) addresses. The GG-Constitution is a compact axiom–postulate system that specifies exactly what counts as an admissible law, an admissible observation, and an admissible prediction in any candidate physical theory. The constitution does not specify a particular Lagrangian or particle content; it specifies the rules that any candidate Lagrangian must satisfy. And the central claim of the Arisaka-GG series is that these rules are so restrictive that they force a unique geometric realization: GG-6 in the bulk, GDOS at the boundary, with the BI-6 topological identity and the OGN-5 integer arithmetic as inevitable consequences.

The constitution consists of four *axioms* (semantic primitives, designated AX-1 through AX-4) and seven *postulates* (ordered admissibility filters, designated POS-1 through POS-7). We summarize each below; the detailed development is in Arisaka (2026, A1-Constitution).

2.2 The four axioms

AX-1 (LC: Locality and Causality). The fundamental dynamical objects are localizable on the manifold $\mathcal{M}_6$, and their evolution respects a causal partial order induced by the Lorentzian signature on the boundary $\mathcal{M}_4 = \partial\mathcal{M}_6$. No instantaneous action at a distance, no superluminal information transfer, no causal loops. This axiom plays the role of Euclid's Postulate 1 (a straight line can be drawn from any point to any point): it specifies that the manifold has the local structure necessary for the deduction of theorems to be meaningful.

AX-2 (GGEP: Geometry–Gauge Equivalence Principle). Gauge structure is geometrized: the gauge fields of physics are not external degrees of freedom appended to a geometric background, but components of a higher-dimensional metric or connection. This is the axiomatic version of the lesson of Kaluza–Klein and of the entire history traced in Part II: gauge fields, like gravity, are structural features of an underlying geometry. GGEP plays the role of Euclid's Postulate 2 (a finite straight line can be produced continuously): it specifies that the geometric structure can be extended consistently across the manifold without introducing extraneous fields.

AX-3 (OGGEP: Operational/Open Geometry–Gauge Equivalence Principle). Every observation in physics is a bulk-to-boundary projection from the bulk $\mathcal{M}_6$ to the boundary $\mathcal{M}_4$, formalized as a CPTP open-system map in the GDOS framework (Arisaka, 2026, A5-GDOS). Measurement is not a separate postulate (as in von Neumann's quantum mechanics) but a constitutive feature of the bulk-to-boundary dictionary. This axiom plays the role of Euclid's Postulate 3 (a circle can be described with any center and distance): it specifies that the measurement apparatus is a geometric construction within the framework, not an external add-on. The crucial conceptual content of OGGEP is that it redefines what counts as a scientific observation: every empirical claim is a GDOS event, subject to the universal DGI–GSSB–GA runtime — Dynamical Gauge Invariance, Generalized Spontaneous Symmetry Breaking, and Gauge Actualization, the three stages every observation passes through — which is set out in Part V (Sec. 25.2).

AX-4 (MSMF: Maximum Symmetry, Minimum Freedom). The admissible theories are those with maximum symmetry consistent with experiment and minimum continuous free parameters. This axiom plays the role of Euclid's Postulate 4 (all right angles are equal): it specifies that the rigidity of the framework is itself a feature, not a defect. MSMF is the constitutional clause that forbids the parameter-fitting drift that has crept into post-Euclidean physics; it requires every continuous knob to be replaced by a discrete topological or arithmetic invariant.

2.3 The seven postulates

The postulates POS-1 through POS-7 turn the four axioms into an audit-ready theorem machine:

POS-1 (Layered Gauge Invariance). The gauge group $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \times \mathrm{Diff}(\mathcal{M}_6)$ acts on the bulk fields, with appropriate boundary breakings determined by the Hosotani mechanism (Hosotani, 1983a).

POS-2 (Unified Extremal Principle). All bulk dynamics is extremal of a single 6D action $S_6 = \int_{\mathcal{M}_6} \mathcal{L}_6$ with $\mathcal{L}_6$ the Einstein–Hilbert + gauge + matter Lagrangian density on $\mathcal{M}_6$.

POS-3 (Anomaly Cancellation by Inflow). All anomalies of the boundary 4D theory are canceled by inflow from the bulk via the Green–Schwarz mechanism (Green and Schwarz, 1984), encoded in the modified Bianchi identity BI-6. The 1984 Green–Schwarz construction was, in retrospect, the technical pioneering of AX-2 (GGEP): it was the first concrete demonstration that anomaly cancellation can be *geometric* — a property of the bulk topology absorbed into the boundary, rather than a property of the boundary alone. In our reading, what GS did locally for the 10D heterotic string, AX-2 GGEP elevates to an axiomatic principle of GG-Theory; GG-A3-BI6 develops this reading in detail in its subsection *BI-6 as the GGEP completion of GS* (see Arisaka, 2026, A3-BI6). The four BI-6 coefficients $(1, -1, -2/5, -2/5)$ are forced by the constitution; they are not a free input.

POS-4 (CPTP Closure and Positivity). The boundary state evolution is CPTP (completely positive trace-preserving), governed by a GKLS generator (Gorini, Kossakowski, and Sudarshan, 1976, Lindblad, 1976) derived from the bulk-to-boundary dictionary.

POS-5 (RG/Scale Covariance). The 4D effective theory is covariant under the renormalization group, with the scale coordinate of $\mathcal{M}_6$ playing the role of the RG scale (consistent with AdS/CFT (Maldacena, 1998)).

POS-6 (OGN Integer Normalization). All normalizations of the framework are forced to take integer values on a specific arithmetic ledger; the OGN-5 integers $(1, 3, 5;\, 7, 35)$ are the unique solution. This is the version of Euclid's parallel postulate (Postulate 5) for GG-Theory: a substantive constraint that forces the discrete content of physics.

POS-7 (Falsifiability). Every prediction of the framework must be specified in advance and must be in-principle falsifiable by experiment, with a pre-declared falsifier.

Table 2 sets the eleven clauses beside the elements of Euclid's own system, which is the parallel the constitution was built to honor.

Table 2: The GG-Constitution and the structural parallel with Euclid's *Elements*. The parallel is structural rather than rhetorical: each row pairs one component of Euclid's deductive apparatus with its counterpart in the constitution. Euclid's twenty-three definitions fix the meanings of the primitive terms, and the constitution's operational definitions do the same for bulk, boundary, gauge field, holonomy and scale profile. Euclid's five postulates say what may be constructed; the four axioms AX-1–AX-4 and the seven postulates POS-1–POS-7 say what counts as an admissible theory. The common notions correspond to the general preconditions of any deductive science. The theorem rows differ in subject but not in kind — the Pythagorean theorem and the classification of the regular polyhedra on one side, the OGN-5 integer theorem, the BI-6 coefficient theorem and the cosmic inventory $\Omega_m = 1/\pi$ on the other. The last row is the one this paper turns on: in both systems nothing in the foundational layer is adjustable.

Item	Euclid's *Elements*	GG-Constitution
Definitions	23 definitions specifying the meanings of point, line, plane, etc.	Operational definitions of bulk, boundary, gauge field, holonomy, scale profile, etc.
Postulates	5 postulates specifying admissible constructions.	4 axioms (AX-1 LC, AX-2 GGEP, AX-3 OGGEP, AX-4 MSMF) plus 7 postulates (POS-1 through POS-7).
Common notions	5 common notions specifying logical principles.	General logical preconditions for any deductive science (mathematical consistency, dimensional consistency, etc.).
Theorems	Pythagorean theorem; classification of regular polyhedra; etc.	The OGN-5 integer theorem; the BI-6 coefficient theorem; the derivation of the SM gauge group from $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$; the cosmic inventory $\Omega_m = 1/\pi$; the Arisaka constant $\mathsf{Q_B} \approx 0.1555$ eV.
Methodological status	No free parameters in the foundational sense; every theorem follows by deduction.	No free parameters in the foundational sense; every dimensionful quantity is a conversion factor of the bulk; every dimensionless ratio is an OGN-5 arithmetic theorem or a BI-6 topological residue.

3 The Strategy of the Reconstruction

This paper occupies a particular place in the Arisaka-GG cohort, and naming it precisely is the surest guide to how the paper should be read. The *construction* of GG-Theory is carried out elsewhere: Arisaka (2026, A1-Constitution) fixes the four axioms and seven postulates; Arisaka (2026, A2-GG6) builds the six-dimensional bulk $\mathcal{M}_6 = \mathcal{M}_4 \times I_r \times S^1_\theta$ and proves its structural theorems; Arisaka (2026, A3-BI6) establishes the modified Bianchi identity BI-6 and forces the integer ledger OGN-5 $= (1, 3, 5; 7, 35)$. The present paper does not rebuild any of this. Its task is the complementary one of *reconstruction*: taking the constructed framework as given, it proves that the framework contains no adjustable dimensionless freedom — that every number a physicist would ordinarily measure and insert by hand is, in GG-Theory, computed. This is the sense in which this paper is the cohort's "no free parameters" volume, and the sense in which its claim is to be read against the standard set by Euclid's *Elements* and by relativity.

The strategy proceeds in three moves, each of which corresponds to a block of the paper.

First, classify the dimensionful content. Every physical quantity carries units or is a pure number. The dimensionful quantities are conversion factors — they translate between the units in which we happen to measure and the natural units of the geometry — and the claim of the reconstruction is that there are exactly five of them, organized as the Geometric Universal Five Constants

$$\mathsf{GU\text{-}5} = \{c,\, G_N,\, \hbar;\ k_B,\, Q_B\},$$

three belonging to the bulk and two to the boundary. The bulk triad $\{c, G_N, \hbar\}$ is established in Part VI as one conversion factor per kind of dimension: c for the Lorentzian time of $\mathcal{M}_4$, G_N for the scale of the fifth (energy-scale) direction, $\hbar$ for the phase of the sixth (gauge) direction. The boundary pair k_B and Q_B are conversion factors of the bulk-to-boundary projection, not of the bulk. None of the five is a dimensionless dial; each is a choice of units fixed once by a single measurement, exactly as c and G_N are in relativity.

Second, classify the dimensionless content. Every pure number of physics must, if the reconstruction is to succeed, be either an integer of the ledger OGN-5, a quantity computed from those integers by the cascade documented in Arisaka (2026, A3-BI6) and Arisaka (2026, A9-SM), or a ratio of GU-5 members. The fine-structure constant $\alpha^{-1}(0) = 2^J + N^2 = 137$ is the showcase instance: it is fixed by the integers $(N, J) = (3, 7)$ with no continuous parameter intervening. The elementary charge e, often taken as primitive, is shown to be derived from α and is therefore *not* a member of GU-5.

Third, prove closure. The two classifications are then shown to be jointly exhaustive: there is no third species of physical constant. This is the content of the closure theorem (T-Euclid-5), and it is what the master parameter audit of Part VII verifies entry by entry, from the constitution through the Standard Model and cosmology to the biological energy quantum $\mathsf{Q_B}$. The audit is the empirical heart of the paper; the theorems are its logical skeleton.

The reconstruction is therefore not a new physical theory but a proof about an existing one — a proof that the framework built in GG-A2-GG6 and GG-A3-BI6 keeps the promise of Euclid's method. The next section states the seven theorems through which that proof is organized; the remainder of the paper supplies, Part by Part, the geometric and historical context that makes each of them inevitable rather than merely true.

4 Overview: The Seven Reconstruction Theorems

The argument of this paper is organized around seven theorems, labeled T-Euclid-1 through T-Euclid-7 under the cohort naming convention (Part X, Orientation). They are collected in Table 3. Together they constitute the formal content of the "no free parameters" claim: the first establishes the bulk conversion-factor trio, the next four assemble it into the closed five-constant architecture GU-5 and prove its closure, and the last two extend the result to the dimensional reduction and to the cosmological constant. Unlike the construction theorems of Arisaka (2026, A2-GG6) and Arisaka (2026, A3-BI6) — which build the GG-6 bulk and its integer ledger — these seven are *reconstruction* theorems: they take the constructed framework as given and prove that it contains no adjustable dimensionless freedom. They are original to the present paper.

Table 3: The seven T-Euclid reconstruction theorems, in informal statement. These are the formal content of the no-free-parameters claim, and they divide three ways. T-Euclid-1 establishes the bulk conversion-factor trio $\{c, G_N, \hbar\}$, one member per kind of dimension. T-Euclid-2 through T-Euclid-5 assemble that trio into the closed five-constant architecture GU-5 and prove its closure: no modulus survives, the charge is derived rather than posited, and every dimensionless constant is an OGN-5 member, a cascade consequence or a ratio of GU-5 members. T-Euclid-6 and T-Euclid-7 extend the result to the dimensional reduction and to the cosmological constant. Unlike the construction theorems of the companion papers, which build the GG-6 bulk and its ledger, these seven take that framework as given and prove it contains no adjustable dimensionless freedom; the Statement column gives the informal form and the section reference the proof.

ID	Short name	Statement (informal)
T-Euclid-1	Conversion-Factor Trio	$\{c, G_N, \hbar\}$ are the three bulk conversion factors, one per kind of dimension (4th time, 5th scale, 6th phase); none carries continuous freedom. (Sec. 26)
T-Euclid-2	Completeness / No Modulus	Because the two extra dimensions carry no length, the trio is the *complete* bulk set; space-like compactifications are necessarily incomplete (carry a free radius). (Sec. 26)
T-Euclid-3	GU-5 Architecture	The full dimensionful content is GU-5 $= \{c, G_N, \hbar; k_B, Q_B\}$, split (3 bulk + 2 boundary); no sixth conversion factor is admitted. (Sec. 30)
T-Euclid-4	Charge is Derived	$e \notin$ GU-5; it follows from $\alpha^{-1}(0) = 2^7 + 3^2 = 137$ and the GU-5 members, so listing it as primitive would double-count. (Sec. 30)
T-Euclid-5	Closure	Every dimensionless constant is an OGN-5 member, an OGN-5-cascade consequence, or a ratio of GU-5 members — no third species. (Sec. 30)
T-Euclid-6	Dimensional Reduction	The 6D reduction gives $V_2 = \pi(1 - e^{-2K})$, hence $G_N = 1/(\pi M_6^2)$ and $M_6 = M_{\rm Pl}/\sqrt{\pi}$; M_6 is derived, not a parameter. (Sec. 30, App. D)
T-Euclid-7	Λ from Openness	A closed-system theory cannot fix Λ; under OGGEP the open-system projection determines $\rho_\Lambda \sim (M_{\rm lock} e^{-K})^4$. (Sec. 35.4)

The reader who wants only the logical skeleton may read these seven statements and the master audit of Sec. 32; the remainder of the paper supplies the geometric and historical context that makes each one inevitable rather than merely true.

5 The cohort inheritance and the roadmap of the thirteen Parts

Before the historical arc begins (Part II), it is useful to lay out two reader-orientation maps on the same opening: a panorama of the twenty-two-paper cohort in which this paper stands (§5.1, Figure 3), and a panorama of the thirteen Parts of the present paper (§5.2, Figure 4). A reader of any later Part should be able to flip back to this Section and orient themselves in both directions:

which cohort companion carries which imported result, and where the present argument stands in the thirteen-Part arc.

5.1 The cohort inheritance

The present paper is one of twenty-two. GG-Theory is written not as a single monograph but as a cohort of twenty-two preprints — eleven A-papers, six P-papers, three C-papers and two white papers — each carrying one rigorously stated piece of a single descent that runs from four axioms and seven postulates to the empirical structure of physics. The partition into twenty-two papers reflects the partition of subject matter, not a ranking of importance: the framework is one program, and each paper is a named carrier of named content. Figure 3 is the panorama; a reader meeting the program for the first time can read its architecture off the figure top to bottom before returning to the parameter-free reconstruction that is the business of this paper. The cohort falls into five strata plus three framed branches.

The constitutional root and the GG6 Trio. At the root stands GG-A1-Constitution (Arisaka, 2026, A1-Constitution), the GG-Constitution: the four axioms (AX-1 LC, AX-2 GGEP, AX-3 OGGEP, AX-4 MSMF) and seven postulates against which every companion's theorems are audited. Forced from it is the GG6 Trio that builds the bulk: GG-A2-GG6 (Arisaka, 2026, A2-GG6) constructs the six-dimensional Geometry–Gauge bulk and proves it the unique anomaly-free, moduli-free solution of the axioms; GG-A3-BI6 (Arisaka, 2026, A3-BI6) supplies the rigorous local-and-global proof that the modified Bianchi identity BI-6 on that bulk is anomaly-free; and the present paper, GG-A4-Euclid (Arisaka, 2026, A4-Euclid), traces the historical lineage from Euclid through Riemann, Maxwell, and Einstein to the bulk and reconstructs the universal constants from the bulk dimensions.

The boundary law and its two pillars. AX-3 (OGGEP) forces the open-system boundary law GG-A5-GDOS (Arisaka, 2026, A5-GDOS), the Geometric Dynamics of Open Systems that projects bulk physics onto the four-dimensional boundary and recovers the textbook quantum-mechanical postulates as theorems of the operational dictionary. Two sibling pillar papers organize the boundary content: GG-A6-GRIP (Arisaka, 2026, A6-GRIP), the geometric Gi-4 pillars, and GG-A7-Qi4 (Arisaka, 2026, A7-Qi4), the quantum-information Qi-4 pillars, which carry the Arisaka equation and the boundary $\mathrm{U}(1)_{Qi}$ symmetry from which the energy–information conversion factor $\mathsf{Q_B}$ of the present audit descends.

The Consequence Trio and the capstone. Three papers read the empirical world off the boundary law: GG-A8-QM (Arisaka, 2026, A8-QM) recasts four-dimensional quantum mechanics, GG-A9-SM (Arisaka, 2026, A9-SM) derives the twenty-eight free parameters of the Standard Model (including the $\alpha^{-1}(0) = 137$ that this paper audits), and GG-A10-Cosmos (Arisaka, 2026, A10-Cosmos) derives the ΛCDM cosmological inventory ($\Omega_m = 1/\pi$ and its partners). The capstone GG-A11-Origin (Arisaka, 2026, A11-Origin) closes the cohort with the recursive-uniqueness argument and the cosmos–life–mind chain.

The two phenomenology branches. Beneath the A-series, two framed branches carry the sharp falsifiers. The Predictions Branch — GG-P1-KK (Arisaka, 2026, P1-KK) (the Kaluza–Klein lock cluster near $M_{\text{lock}} \approx 3.85$ TeV), GG-P2-N3 (Arisaka, 2026, P2-N3) (the three-generation pincer that forces $N = 3$), and GG-P3-Electron (Arisaka, 2026, P3-Electron) (the electron as the ground state of nature), and GG-P6-Proton (Arisaka, 2026, P6-Proton) (the proton as the ground geometric attractor of matter, its decay forbidden by the geometry rather than merely unobserved) — supplies collider and structural predictions. The Cosmology and Astrophysics Branch — GG-C1-Kaxion (Arisaka, 2026, C1-Kaxion) (the kaxion dark-matter candidate near 8.1 μeV, with the $\Delta\nu/\nu = 1/35$ twin-peak signature), GG-C2-Hubble (Arisaka, 2026, C2-Hubble) (the Hubble tension as one geometric corridor), and GG-C3-BH (Arisaka, 2026, C3-BH) (black-hole entropy as an OGGEP port) — supplies the astrophysical signatures.

The Foundations Branch. The third framed branch is the Foundations Branch: the two member papers whose subject is the boundary's complex structure and its mirrors. GG-P4-Real (Arisaka, 2026, P4-Real) stands on the map's left flank as the origin-of-i member, deriving the single imaginary unit of boundary physics from the two real structures of the GG-A2-GG6 arena — the compact phase circle and the causal clock; GG-P5-CPT (Arisaka, 2026, P5-CPT) carries the three mirrors of matter — C, P, and T read as orientation reversals of the three real bulk factors. For the present paper their significance is lineal: the 2300-year arc traced in Part II is, throughout, the story of number systems read as geometry, and the origin of i is that story's latest chapter — extending the lineage's endpoint from the universal constants to the number field they are written in. Both carry P-series designators; their place at the spine's flank, not the phenomenology rows, marks their foundational role.

The White-Paper Branch. The right flank mirrors the left and does the opposite job: where the Foundations Branch feeds the spine, the White-Paper Branch reads it. GG-W1-Symmetry (Arisaka, 2026, W1-Symmetry) is the diachronic one — it walks the century of symmetry and conservation in the order it was found and shows the conviction that symmetry causes conservation loosening at seven historical steps, until what remains conserved is an integer with nowhere to move. GG-W2-Holonomy (Arisaka, 2026, W2-Holonomy) is the synchronic one — it takes one exact statement, that a holonomy requires a closed loop and the radial interval has none, so the circle remembers and the interval forgets, and follows it to sixteen questions physics has not closed. Neither derives anything new: both carry every result at its parent's declared grade, which is what makes them white papers rather than a third phenomenology branch.

Where this paper sits. GG-A4-Euclid is the foundations-and-audit volume of the GG6 Trio. GG-A2-GG6 forces the bulk and GG-A3-BI6 proves it anomaly-free; the present paper supplies the historical lineage that makes the bulk inevitable and the parameter audit that shows every universal constant is either a conversion factor of the geometry or a theorem of the OGN-5 ledger. It imports — rather than re-derives — the Standard-Model and cosmological numbers of GG-A8-QM–GG-A10-Cosmos, citing each at the point of use; its own contribution is the closure claim that the parameter inventory is complete (Sec. 32). The twenty-two-paper map, identical in every paper of the cohort, is Figure 3, which closes this subsection.

5.2 Roadmap of the thirteen Parts

The rest of the paper is organized into twelve further Parts, and the walk through them is plotted in Figure 4. The present Part — the foundations — is nearly done: the constitutional question of free parameters, the two-pentad thesis, the four axioms and seven postulates of the GG-Constitution, the strategy of the reconstruction with its seven theorems, and the cohort the paper stands in. Part II then tells the story that gives the paper its title: the 2300-year geometrization arc from Euclid's *Elements* through Riemann's 1854 lecture to the modern absorptions — c by Maxwell and Minkowski, G_N by Einstein, $\hbar$ by Kaluza–Klein and Wilson–Hosotani, the warp by Randall–Sundrum — each a forced move that pulled one more universal constant into geometry.

Three Parts then build the stage. Part III exhibits the unique six-dimensional geometry the constitution forces and states the conceptual heart of the framework: the fifth and sixth dimensions are not space but an energy scale and a topological gauge phase, so causality, locality, and point-like particles survive and no compactification modulus appears. Part IV presents the modified Bianchi identity BI-6 and exhibits the five OGN-5 integers $(1, 3, 5; 7, 35)$ as a Diophantine theorem rather than a postulate. Part V introduces the GDOS boundary law — scientific observation redefined as a bulk-to-boundary projection — together with the Gi-4 and Qi-4 Pillars that organize the boundary content.

Parts VI and VII are the reconstruction proper, and Part VII is highlighted in the roadmap because it is what the paper's claim rests on. Part VI assembles GU-5, the Geometric Universal Five Constants: the bulk triad $\{c, G_N, \hbar\}$ as conversion factors — one per kind of dimension — the boundary members k_B and $\mathsf{Q_B}$, the mathematical constants e and π, and the closure theorem that states the inventory is complete. Part VII is the master audit: every dimensionful quantity a member of GU-5, every dimensionless ratio an OGN-5 arithmetic theorem or a BI-6 topological residue, walked row by row from the constitution to the biological energy quantum. It is the single Part on which the "no free parameters" claim of the title stands, and the seven reconstruction theorems of Part I all point into it.

Two Parts then weigh the case. Part VIII is the discussion: the significance and the structural innovations of the reconstruction, the problems it solves, the sharp numerical predictions from the OGN-5 ledger alone, the falsifiability audit, the cross-paper coherence map, and the honest open-problems triage. Part IX states the conclusions — the restoration of the Euclidean condition, the case for a fourth scientific revolution, and a plain-English close for the reader who has walked the whole arc.

The back matter is the workshop. Part X collects the technical appendices: orientation for the newcomer, the consolidated table of abbreviations, the cohort naming convention, and the explicit 6D-to-4D dimensional reduction. Part XI answers, in plain language and without technical prerequisites, the twenty-five questions a reader without a physics degree actually carries into a paper like this one. Part XII answers the fifty-three common objections a particle theorist, cosmologist, string theorist, geometer, historian of science, or skeptical reader is most likely to raise; and Part XIII

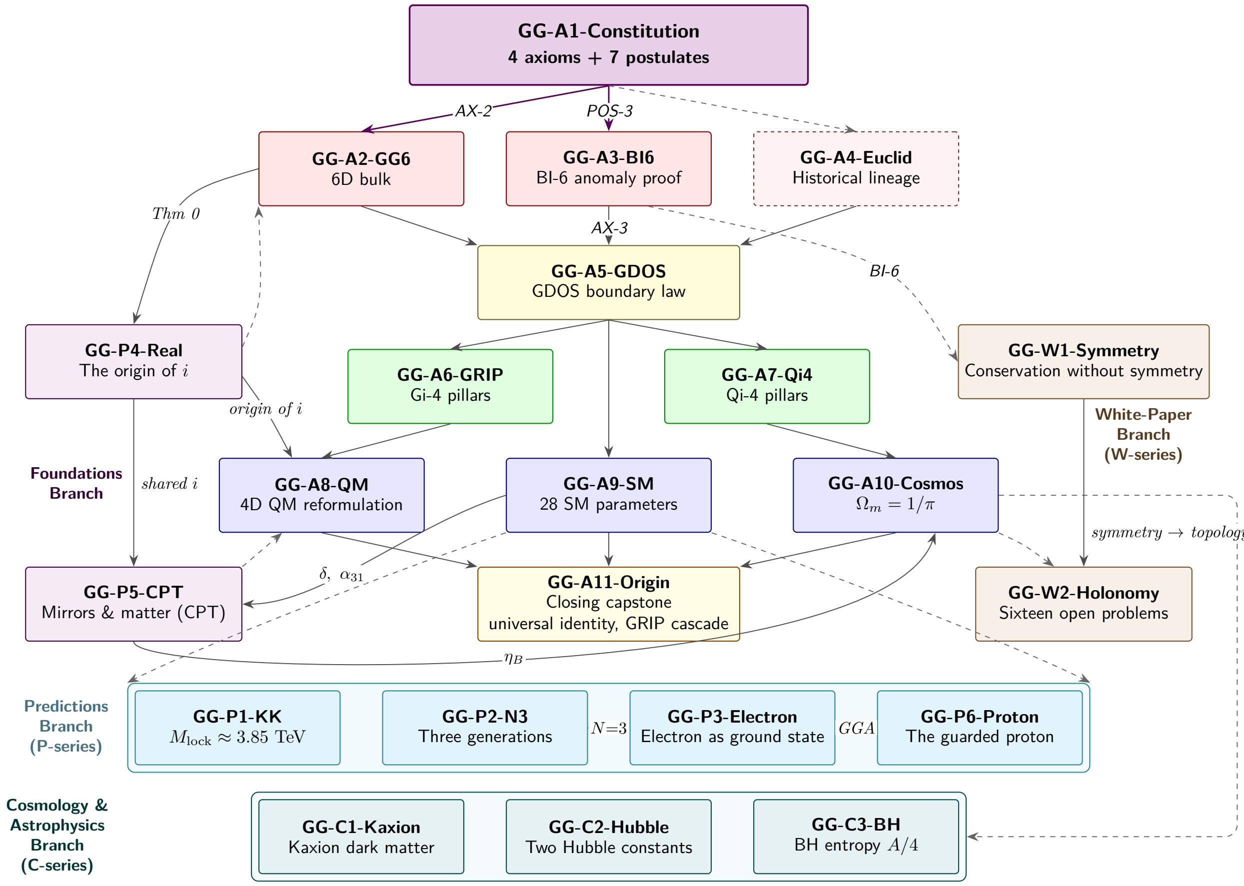

Figure 3: The twenty-two-paper cohort map of GG-Theory. The constitutional root GG-A1-Constitution (violet, top) supplies the four axioms and seven postulates from which the rest of the cohort descends: the GG6 Trio (GG-A2-GG6, GG-A3-BI6, GG-A4-Euclid, red) realizes the bulk geometry; GG-A5-GDOS (yellow) the boundary law; the GDOS Trio pillars (GG-A6-GRIP, GG-A7-Qi4, green) the geometric and quantum-information runtime; the 4D-Physics Trio (GG-A8-QM, GG-A9-SM, GG-A10-Cosmos, blue) the textbook readouts; and the closing capstone GG-A11-Origin (gold) the quarks-to-consciousness identity cascade. Two framed branches carry the phenomenology — the **Predictions Branch** (cyan), whose four papers hand their results along the row by the internal arrows (GG-P2-N3's $N{=}3$ into GG-P3-Electron, and that paper's ground-attractor method, GGA, on into GG-P6-Proton), and the **Cosmology & Astrophysics Branch** (teal). The spine is flanked by two columns of two, which do opposite jobs. On the left the **Foundations Branch** (violet) feeds it: GG-P4-Real, fed by GG-A2-GG6's Theorem 0 arena, manufactures the i that GG-A8-QM consumes, and GG-P5-CPT is the GG-A9-SM–GG-A10-Cosmos interface, fed by the ledger phases and feeding the η_B chain across the width of the map, with the shared-i edge descending solid between them. On the right the **White-Paper Branch** (brown) reads it: GG-W1-Symmetry walks the century from Noether to Hopkins–Singer, and GG-W2-Holonomy dissects the structure that arc arrives at — protection by topology rather than by symmetry — across sixteen open problems. The white papers derive nothing new, carrying every result at its parent's declared grade; the single dashed feed drawn into each names its heaviest source, not its only one. *Reading conventions:* solid arrows mark axiom-forced descent and carry an edge label naming the driving axiom or postulate where most illustrative; dashed arrows mark the deployment of derived results, or historical links inherited without being constitutionally forced.

closes with the annotated references, historical primary sources and modern literature alike, each carrying a Relevance note. Read in order, the thirteen Parts compose into a single self-contained, parameter-free reconstruction that any reader can audit step by step; read by need, each Part names its imports and can stand a visit on its own.

With the question stated, the charter fixed, the seven reconstruction theorems rostered and the route drawn, Part I has done everything it can do without evidence. It has not shown that a single constant is derived. It has said what showing that would require — a chain running from a constitutional clause to the quantity, with every link written down and no continuous choice anywhere along it — and undertaken to meet that requirement one quantity at a time. The rest of the paper is that undertaking, and the roadmap below is the order in which it is carried out. It begins where the question began, with Euclid.

Three routes through this paper. Not every reader wants the same thing from a volume of this length, and it is built so that three different visits all work.

The reader who wants the claim and its exposure, and has an afternoon rather than a week: the Executive Summary (page 2), then Section 37, which states the seven falsifiers with the measurement that would end each, then Part XI — twenty-five questions answered without assuming a physics degree. About twenty pages in all, and no mathematics is needed at any point.

The reader who wants to audit the count: Part VII, and in particular the Complete Parameter Audit, which is the paper's central evidence and carries one row per quantity with its status and the theorem that supplies it. Read the weakest rows first — they are marked, and the paper's exposure is there rather than in the strong ones. Part VI supplies the five conversion factors those rows depend on, and Part IV the integer ledger.

The reader who has come from another cohort paper and needs one result: Section 5 says what this paper imports and from whom, Section 4 rosters the seven reconstruction theorems it contributes, and Part XII answers the fifty-three objections, several of which concede something the main text has to be read down for.

What none of these routes permits is skipping Part VII and keeping the conclusion. The audit is the whole of what this paper establishes; the history in Part II explains why the question is worth asking, and the geometry in Parts III to VI explains how the answers are produced, but the claim itself is a count, and a count has to be walked.

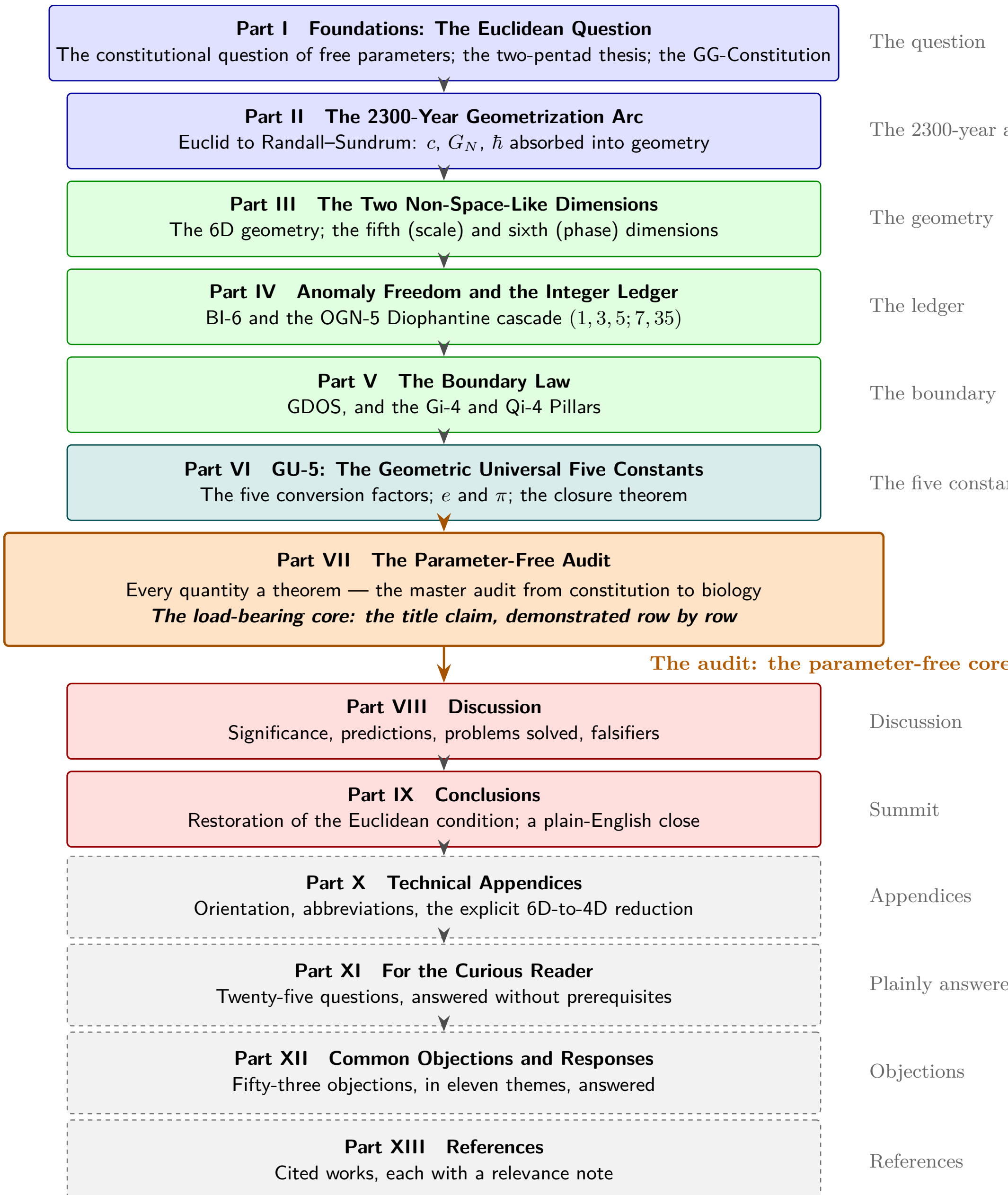

Figure 4: Roadmap of the thirteen Parts of GG-A4-Euclid. The argument develops in five phases. *The question and the arc* (Parts I–II): the constitutional question of free parameters, the two-pentad thesis, and the 2300-year geometrization arc from Euclid's *Elements* to Randall–Sundrum. *The geometry* (Parts III–V): the two non-spatial dimensions, the BI-6 anomaly identity with its OGN-5 integer ledger, and the GDOS boundary law. *The reconstruction* (Parts VI–VII): the five conversion factors of GU-5 with the closure theorem, and, highlighted (**Part VII**), the master parameter audit. *Discussion and conclusions* (Parts VIII–IX) weigh the significance, the predictions, and the falsifiers, and state the restoration of the Euclidean condition. The four back-matter Parts collect the technical appendices (Part X), the twenty-five plainly answered questions (Part XI), the fifty-three common objections and responses (Part XII), and the annotated references (Part XIII). Part VII is highlighted because the master parameter audit is the result on which the paper's parameter-free claim rests.

Part II
The 2300-Year Geometrization Arc

From Euclid to Randall–Sundrum — How c, G_N, and $\hbar$ Were Absorbed Into Geometry

The reconstruction rests on a historical claim as much as a geometric one: that the absorption of a physical constant into geometry is a recurring move in the history of physics, not a novelty of GG-Theory. This Part follows that move across twenty-three centuries, from Euclid's deductive ideal through Riemann's geometry of variable curvature, Maxwell's and Minkowski's geometrization of c, Einstein's geometrization of G_N, and the scale-geometry constructions of Kaluza, Klein, Hosotani, and Randall–Sundrum. Each step absorbs a previously empirical quantity into the geometry of a manifold of one or two more dimensions. The pattern, once seen, makes the six-dimensional step of Part III appear not as a leap but as the next forced move in a long sequence.

The claim needs care, because a pattern in the history of a subject is not an argument for the next step in it. What the sequence establishes is weaker than an argument and more useful: that *absorbing a constant into geometry* is a well-defined move with a known shape; that it has been made six times, by six different people, for six different quantities; and that each time the constant concerned stopped being a number one measures and became a feature of a manifold. Whether a seventh such move is available is a question about geometry, not about history, and it is settled in Part III rather than here.

The Part runs in order. Euclid's standard, and the two-thousand-year discovery that his postulates were choices rather than necessities. Riemann's 1854 lecture, which supplied the framework of n-dimensional manifolds of variable curvature that everything after it uses. Maxwell and then Minkowski on c, which moves in two stages from an optical speed to the metric coefficient of a four-dimensional geometry. Einstein on G_N, which becomes the coupling between matter and curvature rather than the strength of a force. Kaluza and Klein on $\hbar$, which becomes the quantization factor of a compact direction. Wilson and Hosotani on holonomy, which makes symmetry breaking a property of a loop. Randall and Sundrum on the warp factor, which makes a hierarchy geometric. A closing section then asks what the six have in common and what each of them left unfinished — and the answer to the second question is the same in every case: a free parameter that the construction needs and does not supply.

6 Euclid's Standard, and the Discovery That Postulates Are Free

The methodological ideal that this paper seeks to recover for physics was set by Euclid's *Elements* (c. 300 BCE): from twenty-three definitions, five postulates, and five common notions, the entire content of plane and solid geometry is derived by demonstration, *with no adjustable parameters*. The Pythagorean theorem is not fitted to data; the angle sum of a plane triangle is exactly two right angles by deduction, not by measurement; the five regular polyhedra are forced, not chosen. This condition — a deductive science whose every theorem follows from a small fixed set of postulates, with nothing left free — is the standard against which Part VII will audit GG-Theory.

For two millennia the *Elements* was extended in content but not in method. The Hellenistic geometers — Archimedes (Heath, 1897), Apollonius (Apollonius, c. 200 BCE, Heath, 1896), Eratosthenes (Eratosthenes, c. 240 BCE), Hipparchus (Hipparchus, c. 150 BCE), Heron (Heron, c. 70 CE), Diophantus (Diophantus, c. 250 CE), and Pappus (Pappus, c. 340 CE) — and the medieval transmitters — the Indian and Kerala schools (Aryabhata (Aryabhata, 499 CE), Brahmagupta (Brahmagupta, 628 CE), Bhaskara II (Bhaskara II, c. 1150 CE), and Madhava (Madhava, c. 1400 CE); see Joseph, 1991 and Plofker, 2009), the Islamic algebraists (Berggren, 1986, Rashed, 1994), and the Renaissance recoverers — added theorems and, crucially, the algebraic and analytic tools that synthetic geometry had lacked (for the history of Greek mathematics see Heath, 1921 and van der Waerden, 1954); Descartes' coordinates (1637) (Descartes, 1637) and Newton's geometric *Principia* (1687), with the calculus of Newton and Leibniz, turned figure-geometry into the analytic geometry of equations and, eventually, into the manifold theory on which modern physics rests. Throughout, one feature of Euclid's system was treated as a blemish to be removed: the fifth, or parallel, postulate, conspicuously more complex than the others and avoided by Euclid himself until Proposition 29.

The attempt to *prove* the parallel postulate from the other four ran for two thousand years — through Proclus, Thabit ibn Qurra, Omar Khayyam, al-Tusi, Saccheri (1733), and Lambert (1786) — and failed, because there was nothing to prove: the postulate is independent. The decisive recognition came, independently and almost simultaneously, from Gauss — who reached it first, in correspondence and private notes, and published nothing on it in his lifetime — and from Lobachevsky (1829) and Bolyai (1832), who constructed a consistent *hyperbolic* geometry in which many parallels pass through a point and the triangle angle sum falls short of two right angles. Gauss had already supplied the technical foundation in his *Theorema Egregium* (Gauss, 1827): the curvature of a surface is *intrinsic*, computable from measurements within the surface alone, with no reference to any embedding — the result that makes a curved geometry an object of study in its own right, and the conceptual ancestor of general relativity. Riemann (1854), whose lecture we turn to next (Section 7), completed the trichotomy with the *elliptic* geometry of positive curvature.

The conceptual consequence was a watershed. Before the nineteenth century Euclidean geometry was held to be either a body of necessary truths about space (Aristotle) or the unique form imposed by human cognition (Kant). After Lobachevsky, Bolyai, and Riemann it became clear that Euclidean geometry is one consistent geometry among several: the choice of postulates is mathematically free, and *which* geometry describes physical space is an empirical question.

This is the shift that makes the geometrization of physics possible. If the geometry of space — and, after Minkowski, of spacetime — is empirical rather than necessary, then the geometric properties of the physical world become legitimate objects of physical theory, and the absorption of physical phenomena into the geometry of spacetime becomes a coherent strategy. It is the strategy that Einstein and Minkowski executed for the speed of light c and the Newtonian potential, and that the Arisaka-GG series executes for G_N, $\hbar$, and the Standard-Model parameters in GG-6. The remainder of Part II follows that strategy forward, constant by constant.

7 Riemann's 1854 Lecture: The Foundations of Geometry as We Now Use It

7.1 The text and its place

On 10 June 1854, Bernhard Riemann delivered his Habilitationsvortrag at the University of Göttingen, on the subject *Über die Hypothesen, welche der Geometrie zu Grunde liegen* ("On the Hypotheses Which Lie at the Foundation of Geometry"). The audience consisted of the Göttingen philosophical faculty, including Gauss, who had selected the topic from a list of three Riemann had proposed, and who is reported to have been deeply impressed by what he heard. Riemann's lecture was published posthumously in 1868, communicated to the Göttingen Society by Richard Dedekind, who had found the manuscript among Riemann's papers (Riemann, 1854, Spivak, 1979); Weber's part in the record is the 1876 *Gesammelte Werke*, which he and Dedekind edited together. In one hour, before an audience of philosophers expecting a routine pedagogical exercise, Riemann founded the modern theory of differential geometry, the framework of multidimensional manifolds, and the conceptual basis on which Einstein would, six decades later, build general relativity.

7.2 Riemann's principal innovations

The lecture contains several conceptually decisive innovations (Sharpe, 1997, Spivak, 1979).

Manifolds in arbitrary dimension. Riemann generalized the concept of a geometric space from the two- and three-dimensional spaces of Euclidean geometry to spaces of arbitrary dimension n. Such a space is, in modern language, a (smooth) manifold: a topological space that is locally homeomorphic to $\mathbb{R}^n$, equipped with a smooth structure that allows differentiation. Geometry in arbitrary dimension had been reached before him — Cayley in 1843 and Grassmann in 1844 — so the priority claim worth making is narrower and stronger: this is the first systematic treatment of *intrinsic differential* geometry in arbitrary dimension, curvature included and no embedding space required.

The metric as a free choice. Riemann recognized that on a given manifold one may impose a metric — a way of measuring infinitesimal distances — in many different ways. In his words, the metric is given by a quadratic form

$$ds^2 = \sum_{i,j=1}^{n} g_{ij}(x)\, dx^i\, dx^j, \tag{3}$$

where the coefficients $g_{ij}(x)$ are smooth functions of the position x on the manifold. The g_{ij} form what is now called the *metric tensor* — the central object of Riemannian geometry, and the central object of general relativity. Different choices of g_{ij} give different geometries on the same underlying manifold. Euclidean geometry corresponds to a constant Euclidean metric; hyperbolic and spherical geometries correspond to particular metrics of constant curvature.

Intrinsic curvature in arbitrary dimension. Riemann generalized Gauss's intrinsic curvature of a surface to a notion of curvature in arbitrary dimension — the Riemann curvature tensor R^l_{ijk}, a multilinear object that captures the entire local geometry of a manifold. In two dimensions the

Riemann tensor reduces to the scalar Gaussian curvature; in higher dimensions it is essentially richer. The Riemann tensor is intrinsic — it can be computed from the metric tensor without reference to any embedding of the manifold in a higher-dimensional space. This is the foundational result of the modern theory of curved manifolds.

Implications for physical space. In the closing paragraphs of the lecture, Riemann notes that the geometry of physical space is an empirical question, that it may differ from Euclidean geometry, and that the curvature of physical space might depend on the matter contained within it. This is, as Einstein would later acknowledge, the precise conceptual setting of general relativity, anticipated by Riemann sixty-one years before its formal completion.

7.3 From Riemann to Einstein

The development from Riemann's 1854 lecture to Einstein's 1915 field equations is a long story, with many contributors (Pais, 1982, Stachel, 2002). Among the principal:

Beltrami (Beltrami, 1868), who in 1868 found explicit models of hyperbolic geometry as surfaces of constant negative curvature in three-dimensional Euclidean space (the pseudosphere), giving the first concrete realization of the abstract Lobachevskian geometry.

Klein, who in his 1872 *Erlangen program* (Klein, 1872) reorganized the entire subject of geometry around the notion of the symmetry group of the geometry: each geometry is the study of properties invariant under a particular group acting on a particular space. Euclidean geometry is the study of properties invariant under the group of rigid motions; affine geometry is the study of properties invariant under the affine group; projective geometry is the study of properties invariant under the projective group; and so forth. Klein's perspective is the conceptual ancestor of the modern role of gauge groups and Lie groups in physics.

Christoffel, Ricci, Levi-Civita, who developed the tensor calculus of Riemannian manifolds during the 1880s and 1890s. The Christoffel symbols, the covariant derivative, and the Ricci tensor are all introduced in this period. By 1900 the tensor calculus is a mature technical subject, ready to be applied to physical theory.

Hilbert and Klein, who in the early 1910s formalized the variational and group-theoretic structures of physical theory in the Hilbert space and Erlangen traditions. Hilbert's 1915 derivation of the Einstein field equations from a variational principle (independent of and almost simultaneous with Einstein's own derivation) cemented the role of variational principles in geometric physics.

By the time Einstein arrives at the field equations of general relativity in November 1915, the entire mathematical apparatus is in place: smooth manifolds in arbitrary dimension, metric tensors, intrinsic curvature, the tensor calculus, and the variational formulation. General relativity becomes possible because Riemannian geometry has, by then, been developed for sixty years. This is a pattern we shall see repeated: the geometric framework needed for the next physical reformulation is built first by the mathematicians, often without specific physical motivation, and then claimed by the physicists when the conceptual moment is right.

7.4 The structural lesson

The structural lesson of the period from Euclid through Riemann is that geometry is not a single fixed body of knowledge but a family of mathematical structures parametrized by the choice of underlying manifold and metric. The Euclidean geometry of antiquity is one specific structure (a flat manifold with a Euclidean metric); the hyperbolic and spherical geometries of Lobachevsky, Bolyai, and Riemann are alternative structures with different metrics; and a generic Riemannian manifold may have arbitrary curvature varying from point to point. The geometry of physical space and time, when treated geometrically, is one specific Riemannian (or pseudo-Riemannian) manifold among the family of mathematically possible ones, and the physical theory is the specification of which manifold this is.

This is the conceptual setup that physics inherits in the late nineteenth century, and that the reformulations of the early twentieth century (Lorentz, Poincaré, Einstein, Minkowski) will exploit to absorb the constant c into the geometry of 4D Lorentzian spacetime. The further reformulations of the mid-twentieth century (Kaluza, Klein, Wilson, Hosotani) will absorb $\hbar$ into the geometry of compact extra dimensions. The end-of-century reformulation (Randall–Sundrum) will absorb G_N into the geometry of warped 5D AdS. And the present Arisaka-GG program will absorb everything else into the geometry of GG-6.

The structural lesson cuts both ways. On one side, it tells us that the geometric framework is mathematically inexhaustible: any number of metrics, dimensions, and topologies are available to physical theory if a physical reason demands them. On the other side, it tells us that the *choice* of which framework to use is, after all, an empirical one — the framework of physical space is whichever Riemannian geometry actually fits the data.

Indeed, the historical record we shall now trace in Part II is precisely a sequence of such empirical choices, each one a *partial geometrization*: a step in which a previously free dimensional constant of physics is absorbed into the structure of a geometric framework, no longer to be fitted but only to be read off. The sequence runs Maxwell 1865, Minkowski 1908, Einstein 1915, Kaluza–Klein 1919–26, Wilson–Hosotani 1974–83, Randall–Sundrum 1999 — six historical steps in a hundred and thirty-four years that geometrize, in GU-5 order, the three bulk conversion factors c (Maxwell, then Minkowski), G_N (Einstein, later sharpened by Randall–Sundrum), and $\hbar$ (Kaluza–Klein), together with the gauge-breaking holonomy of Wilson–Hosotani.

Each absorbs its constant into the structure of one slightly richer geometric framework (4D Lorentzian, then curved 4D, then 5D compact, then 5D warped). Each is, in our reading, a partial geometrization that the 6D bulk of GG-6 *completes*. Part II reads the history with that completion in mind.

The 2300-year arc from Euclid to GG-6 is, viewed from this height, a single sustained development: the gradual recognition that physical phenomena are geometric, and that the constants of physics are conversion factors of the underlying geometry. Each step of the arc adds one dimension or one scale running to the manifold, and absorbs one previously-empirical constant into its structure. We continue the trace in Part II.

Table 4: Geometric milestones from Euclid to Riemann. The table is the evidential base for the claim of Part II: that absorbing a quantity into geometry is a recurring move in the history of physics rather than a novelty of GG-Theory. Read down the Year column and the arc is continuous across twenty-three centuries and four mathematical cultures — Greek deductive geometry, the Indian and Islamic development of algebra and the decimal system, the European recovery of both, and the analytic tradition that follows. The Innovation column records what each step made expressible that was not expressible before. The sequence matters more than any single entry: each innovation is a precondition of the next, and the last of them, Riemann's geometry of variable curvature, is what makes the geometrization of G_N possible at all.

Year	Author	Innovation
∼ 300 BCE	Euclid	*Elements*: deductive geometry from postulates; methodological standard.
∼ 250 BCE	Archimedes	Volume of sphere; method of exhaustion; mathematical statics.
∼ 200 BCE	Apollonius	*Conics*: complete theory of conic sections.
∼ 500 CE	Aryabhata	First sine table; decimal place-value with zero.
∼ 700 CE	Brahmagupta	General quadratic; negative numbers as numbers.
∼ 825 CE	al-Khwarizmi	Founding of algebra as theory; algorithm.
∼ 1100 CE	Omar Khayyam	Cubics by intersecting conics; critique of parallel postulate.
∼ 1400 CE	Madhava (Kerala)	Infinite series for sin, cos, arctan.
1545	Cardano, Ferrari	Algebraic solution of cubic and quartic equations.
1591	Viète	Literal symbolic algebra.
1637	Descartes	Analytic geometry: coordinates and algebraic curves.
1665–1671	Newton (fluxions)	Differential and integral calculus (independent of Leibniz).
1675–1684	Leibniz	Calculus with modern notation dy/dx, $\int y\,dx$.
1687	Newton	*Principia*: geometric mechanics; universal gravitation.
1733	Saccheri	Theorems of hyperbolic geometry from negation of parallel postulate (not recognized as such).
1827	Gauss	*Theorema Egregium*: intrinsic curvature of a surface.
1829–1832	Lobachevsky, Bolyai	Hyperbolic non-Euclidean geometry.
1854	Riemann	Manifolds in arbitrary dimension; metric tensor; intrinsic curvature in arbitrary dimension.
1872	Klein	*Erlangen program*: geometry as the study of group-invariants.
∼ 1880–90	Christoffel, Ricci, Levi-Civita	Tensor calculus on Riemannian manifolds.

8 Maxwell 1865: c Emerges From the Structure of Electromagnetism

Historical background (with citations). Maxwell's 1865 paper *A Dynamical Theory of the Electromagnetic Field* (Maxwell, 1865), presented to the Royal Society of London on 8 December 1864 and published the following year, is the founding document of unified electromagnetism: the first synthesis of the experimental laws of electricity and magnetism into a single set of differential equations on the electromagnetic field, and the first theoretical demonstration that light is an electromagnetic wave. The paper builds on the experimental work of Faraday and the mathematical syntheses of Ampère, Gauss, and Kelvin; the subsequent reformulation by Heaviside in the 1880s gave Maxwell's equations the vector-calculus form in which they are now universally written. In the present paper, Maxwell 1865 marks the first step of the geometrization program — c ceases to be a property of light and becomes a structural constant of the electromagnetic field, no longer to be measured optically but to be computed, after all, from the static permittivity ε_0 and permeability μ_0.

The first partial geometrization: c from electrostatics and magnetostatics. The first step in the geometrization sequence is the recognition that c is not, after all, a property of light but a structural feature of the unified electromagnetic field. Maxwell's 1865 synthesis showed that the very same constant appears, with no light in the room, in two purely static measurements — the force between current-carrying wires (which gives μ_0) and the force between charges at rest (which gives ε_0). The wave speed $c = 1/\sqrt{\mu_0\varepsilon_0}$ that emerges from these two static measurements is, on Maxwell's reading, an internal feature of the electromagnetic field structure; light is what propagates at this speed because light *is* an electromagnetic wave. On Maxwell's reading, c has been moved one notch closer to the geometry — from a property of optical signals to a structural constant of a field theory. The geometrization is partial: c is still a dimensional input, attached to a particular field theory rather than to the underlying spacetime. We shall see in Section 9 how Minkowski takes the next step, and we shall see in Part VI how the 6D bulk of GG-6 completes the geometrization by exhibiting c as the unique conversion factor of the (1,3) Lorentzian signature on the boundary $\mathcal{M}_4 = \partial\mathcal{M}_6$.

8.1 Pre-Maxwell: c as the optical speed of light

The first quantitative determination of the speed of light is due to Rømer (1676), who observed that the eclipses of Io — the innermost Galilean moon of Jupiter — are systematically late when the Earth is on the far side of its orbit and early when on the near side, by a cumulative discrepancy of about twenty-two minutes per Earth year. Rømer correctly interpreted this as the time required for light to traverse the Earth's orbital diameter, deducing a finite speed of light of order 2.2×10^8 m/s. The result was confirmed by Bradley (1729) from the aberration of starlight, in which the apparent direction of incoming starlight is tilted by an angle v/c where v is the Earth's orbital velocity — the first direct comparison of c to a known mechanical velocity. Subsequent terrestrial measurements by Fizeau (1849, toothed wheel) and Foucault (1850, rotating mirror) refined the value, and by 1873 the speed of light was a well-known optical constant.

Throughout this entire period, c was understood as a property of light: the propagation speed of light waves through whatever medium (the luminiferous aether) light was assumed to inhabit. There was no suggestion that c might be a structural feature of physical theory, nor that it might have any meaning beyond the propagation of optical signals.

8.2 Maxwell's electromagnetic theory and the geometric significance of c

The decisive reinterpretation of c began with the unified electromagnetic theory of Maxwell (1865). Building on the experimental work of Faraday and the theoretical work of Ampère, Gauss, and Kelvin, Maxwell synthesized the laws of electricity and magnetism into the four equations that now bear his name. In the modern Heaviside form, in vacuum and in SI units,

$$\nabla \cdot \mathbf{E} = \rho/\varepsilon_0, \qquad \nabla \cdot \mathbf{B} = 0, \tag{4}$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t, \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \, \partial \mathbf{E}/\partial t. \tag{5}$$

Taking the curl of the third equation, substituting the fourth, and working in a source-free region, one obtains a wave equation for each Cartesian component of $\mathbf{E}$ (and analogously for $\mathbf{B}$):

$$\nabla^2 \mathbf{E} - \mu_0 \varepsilon_0 \, \partial^2 \mathbf{E}/\partial t^2 = 0, \tag{6}$$

with wave speed $1/\sqrt{\mu_0 \varepsilon_0}$. The constants μ_0 (the permeability of free space) and ε_0 (the permittivity of free space) are determined by purely electrostatic and magnetostatic measurements: the force between current-carrying wires gives μ_0, the force between charges at rest gives ε_0. Neither of these measurements involves light in any way. Yet when Maxwell computed $1/\sqrt{\mu_0 \varepsilon_0}$ from the available laboratory data, he obtained

$$1/\sqrt{\mu_0 \varepsilon_0} \approx 2.998 \times 10^8 \text{ m/s}, \tag{7}$$

indistinguishable from the optical speed of light. Maxwell drew the obvious conclusion: light is an electromagnetic wave.

The geometric significance of this result, however, runs deeper than the identification of light with electromagnetism. The constant c is now seen to be implicit in the structure of electric and magnetic phenomena *independently of the existence of light*. One could, in principle, derive c from purely electrostatic and magnetostatic measurements that have nothing to do with optical signals. c is therefore not a property of light; it is a structure constant of electromagnetic theory — and, as we shall see, of spacetime itself.

8.3 The aether crisis and the 1887 null result

Maxwell himself interpreted the constant c as the speed of electromagnetic waves with respect to the luminiferous aether. This interpretation kept c on the same conceptual footing as the speed of sound: an ordinary mechanical wave speed in an ordinary mechanical medium. The Earth's motion through the aether — of order 30 km/s in the orbital component, plus larger contributions from the solar system's motion through the galaxy — should produce a measurable anisotropy in the speed of light, with light moving forward through the aether at $c - v$ and laterally at $\sqrt{c^2 - v^2}$.

Michelson and Morley (1887) designed an interferometer of unprecedented sensitivity to detect this aether wind. Their result, reported in November 1887, was null: to within the experimental sensitivity, no aether wind was detected. The expected fringe shift, of order v^2/c^2, simply did not appear.

The null result threw electromagnetic theory into crisis. Two paths of resolution emerged. The first was to retain the aether but suppose that motion through it produces a length contraction that exactly cancels the expected fringe shift. This is the FitzGerald (1889) contraction, derived dynamically by Lorentz (1892) from a model of matter as electrons embedded in the aether. The contraction factor is $\sqrt{1 - v^2/c^2}$ — the same factor that would later appear in special relativity, but with a different interpretation.

8.4 Lorentz 1904 and Poincaré 1905: the Lorentz transformation

Lorentz (1904) pushed the dynamical aether interpretation to its logical conclusion in his *Electromagnetic phenomena in a system moving with any velocity smaller than that of light.* Lorentz wrote down what we now call the Lorentz transformation:

$$x' = \gamma(x - vt), \qquad y' = y, \qquad z' = z, \qquad t' = \gamma(t - vx/c^2), \tag{8}$$

with $\gamma = 1/\sqrt{1 - v^2/c^2}$, deriving them from the requirement that Maxwell's equations take the same form in the moving frame as in the rest frame. But Lorentz interpreted the primed coordinates as a mathematical convenience used by an observer moving through the aether: the unprimed coordinates referred to the true rest frame of the aether, and the primed time t' was a useful "local time" (Ortszeit) without genuine ontological status. Lorentz had the right transformation and the wrong interpretation.

Poincaré (1905) went further. He recognized that the Lorentz transformations form a group (which he named the Lorentz group), introduced the principle of relativity (the laws of physics must be the same in all inertial frames), and observed that the Lorentz transformation leaves the quantity $x^2 + y^2 + z^2 - c^2t^2$ invariant. In the longer Palermo memoir of the following year (Poincaré, 1906) he noted that under the substitution $\tau = ict$ the transformation becomes a rotation in a four-dimensional space with a Euclidean form — the first appearance in the literature of the four-dimensional structure of spacetime, although Poincaré treated it as a formal device. Like Lorentz, Poincaré retained the aether (in the role of an in-principle existent but observationally undetectable medium) and continued to treat t as primary time and t' as auxiliary. He, too, had the equations and not the full interpretation.

8.5 Einstein 1905: the constancy of c as a postulate

Einstein (1905), in *Zur Elektrodynamik bewegter Körper*, took the decisive conceptual step. Einstein began the paper by accepting two postulates as fundamental: (i) the principle of relativity (the laws of physics are the same in all inertial frames), and (ii) the constancy of c (light propagates in vacuum at the same speed c in every inertial frame, regardless of the motion of the source). From these two postulates alone, by an operational analysis of how distant clocks must be synchronized using light signals, Einstein derived the Lorentz transformation. The aether disappeared entirely from the derivation; it had no role and no observable consequence. The Lorentz contraction, which for Lorentz had been a dynamical effect of motion through the aether, became for Einstein a kinematic consequence of the structure of space and time themselves.

The decisive philosophical move is Einstein's analysis of simultaneity. Two distant events that one observer judges to be simultaneous are not, in general, simultaneous for an observer in a different inertial frame. Simultaneity is not a primitive notion; it must be defined operationally, by a synchronization procedure using light signals; and that procedure yields different results in different inertial frames. This relativity of simultaneity is the conceptual heart of special relativity, and it had eluded Lorentz and Poincaré because they retained the assumption of an absolute time.

For our argument, the relevant point is that the constancy of c has now been elevated from an empirical fact about light to a structural feature of physical theory. Every inertial observer measures the same c for light signals; this is a constraint on the structure of spacetime, not a contingent property of any particular medium. c has begun, at the level of postulate, the transition from empirical constant to geometric structure constant.

9 Minkowski 1908: c as the Conversion Factor of 4D Lorentzian Geometry

The first partial geometrization, continued: c as a metric coefficient. The continuation of that first step is the recognition that c is not even a structural constant of a field theory but a structural constant of *spacetime itself*. Minkowski's 1908 Cologne lecture absorbs c into the metric of 4D Lorentzian geometry as the coefficient of dt^2 in the line element $ds^2 = c^2\,dt^2 - dx^2 - dy^2 - dz^2$ — the unique scalar that converts seconds to meters so that the squared spacetime interval is a Lorentz-invariant quantity. This means c now appears in the geometry, not in the Lagrangian; the laws of electromagnetism are simply Lorentz-covariant because their natural arena is the (1,3)-signature manifold whose metric carries c. We are, with Minkowski, one full step closer to the modern reading. The geometrization remains partial: the manifold is still postulated to be flat, the signature is still postulated to be $(1,3)$, and the value of c is still empirical. Part III will show that all three of these inputs are recovered, in our 6D framework, as forced consequences of the constitutional clauses of GG-6.

9.1 The Cologne lecture

The 1905 paper of Einstein is written in three-dimensional language: time is a parameter, space is the arena, the Lorentz transformation is a relation between the coordinate readings of two inertial observers. Einstein in 1905 does not yet speak of spacetime as a geometric entity. The decisive geometric reformulation is the work of Hermann Minkowski, who had been one of Einstein's mathematics professors at the ETH Zürich. On 21 September 1908, Minkowski delivered the lecture *Raum und Zeit* ("Space and Time") at the 80th Assembly of German Natural Scientists and Physicians in Cologne (Minkowski, 1908). The lecture opened with the celebrated declaration:

> *The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.*

Minkowski proposed that events be treated as points in a four-dimensional manifold with coordinates (t, x, y, z), equipped with the pseudo-Euclidean metric

$$\boxed{ds^2 = -c^2\,dt^2 + dx^2 + dy^2 + dz^2.} \tag{9}$$

The Lorentz transformations are precisely the linear transformations of this four-dimensional space that preserve ds^2. The relativity of simultaneity becomes the geometric statement that different inertial observers slice the four-dimensional manifold into "now" surfaces (surfaces of constant t in their own frame) at different angles. The light cone at each event divides the manifold into causal future, causal past, and spacelike elsewhere. Worldlines are curves in spacetime; proper time is the arc length along a timelike worldline; energy and momentum are the components of a single four-vector. The fragmentation of physics into "kinematics" and "dynamics" that had characterized the previous century gives way to a unified geometric formulation.

9.2 c as the metric conversion factor

The crucial observation, for our argument about free parameters, is to look at the Minkowski metric (9) carefully. The four coordinates (t, x, y, z) do not have the same dimensions: three are spatial (length, conventionally measured in meters), one is temporal (time, measured in seconds). To form an interval ds^2 that is a single number with consistent units, the temporal coordinate must be multiplied by something with dimensions of length per time. That something is c.

The constant c appears in the metric *not because nature has an arbitrary preferred speed, but because the spacetime manifold has one temporal dimension and three spatial dimensions, and these are measured in different units, and the metric must convert between them to be well-defined as a metric.* In natural units where time is measured in light-seconds (one light-second of time defined to be the same length as one light-second of space), $c = 1$ and the metric becomes simply $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$. The fact that in SI units $c \approx 2.998 \times 10^8$ m/s is a fact about the conventional definitions of the meter and the second; it is not a fact about spacetime. The geometry of the Lorentzian manifold does not depend on the units one chooses; the numerical value of c does.

This is the precise sense in which c is not a free parameter. It is a unit-translation device, fixed by the operational practice of measuring time in seconds and space in meters applied to a manifold whose intrinsic structure is the Lorentzian (1,3) signature. Table 5 makes the point numerically: the value of c moves with the choice of units and the geometry does not. The geometry of the manifold is the invariant; c is the conversion factor.

Table 5: c as a conversion factor: how the value depends on the choice of units. The point of the table is what the last column does when the second one moves. The numerical value of c changes by two orders of magnitude between SI and CGS and becomes exactly unity in geometrized and in Planck units; the metric, correspondingly, carries an explicit c^2 in the first two rows and none in the last two. The geometry is the same in all four — what changes is only whether the conversion has been absorbed into the coordinates or is still written out. A quantity whose value can be set to one by a choice of units is not a property of the world being measured; it is the exchange rate between two coordinates that the metric already treats as one kind. That is what is meant here by calling c a conversion factor rather than a free constant, and the same test is applied to $\hbar$ in the next table.

Unit system	Value of c	Form of metric
SI (m, s)	$\approx 2.998 \times 10^8$ m/s	$ds^2 = -c^2\,dt^2 + dx^2 + dy^2 + dz^2$
CGS (cm, s)	$\approx 2.998 \times 10^{10}$ cm/s	same form, larger c
Geometrized (length, length)	1	$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$
Planck units	1	same as geometrized

9.3 The light cone interpretation

A particularly vivid way to see the geometric character of c is through the light cone at any event. The light cone is the locus of events at zero spacetime interval from the chosen event:

$$ds^2 = 0 \quad \Longrightarrow \quad c^2\,dt^2 = dx^2 + dy^2 + dz^2. \tag{10}$$

Geometrically, this is a (3-dimensional) cone in 4-dimensional spacetime, opening to the future and the past. Its slope, in any (t, x) plane, is c — meaning that in these coordinates, light traverses one unit of x for every $1/c$ units of t. The light cone is a geometric object intrinsic to the manifold and the metric: every observer agrees on what the light cone is, even though they disagree about what "now" is. c is the slope of the light cone in any inertial coordinate system; it is a property of the metric, not of light.

This is also why Maxwell's 1865 result — that c can be computed from purely electromagnetic measurements μ_0 and ε_0 — is, in retrospect, a statement about the constraint that electromagnetic theory must respect Lorentz invariance. The combination $1/\sqrt{\mu_0\varepsilon_0}$ had to equal c because Maxwell's equations, like all relativistic field theories, are constructed on Minkowski space and inherit its metric structure. The "coincidence" that the optical c equals the electromagnetic c is no coincidence: both are manifestations of the same metric coefficient.

9.4 Generalization to curved spacetime

The geometric interpretation of c carries over without modification to curved spacetime, the arena of general relativity. In a general Lorentzian manifold, the metric takes the form

$$ds^2 = g_{\mu\nu}(x)\,dx^\mu\,dx^\nu, \tag{11}$$

with signature $(-, +, +, +)$. The constant c still appears as the conversion factor between the temporal and spatial coordinates. Locally, in a freely falling frame, the metric reduces to the

Minkowski form, and c reappears as the slope of the local light cone. Globally, in the presence of gravitational fields, the light cones tilt and warp, but at every point c remains the local slope.

The constant c is, in the most precise sense, a feature of the (1,3) Lorentzian signature itself, an invariant of the geometry that no amount of curvature can dispose of. This will be important when we generalize to higher dimensions: the speed of light remains the conversion factor between the temporal direction and any of the spatial directions, regardless of how many spatial directions there are. This is what enables the boundary projection from GG-6 to recover the same c as the 4D Lorentzian conversion factor, as we shall develop in Part III.

10 Einstein 1915: G_N as the Coupling Between Matter and Curvature

The second partial geometrization: G_N as a coupling, not a force. The second step is the more dramatic. Einstein's 1915 field equations $G_{\mu\nu} = 8\pi G_N T_{\mu\nu}/c^4$ absorb Newton's gravitational constant G_N from a force-law coefficient into the coupling that relates matter stress-energy to the curvature of spacetime. Gravity has ceased to be a force on this reading; it has become the curvature of the metric, with G_N playing the role of the coupling between source and curvature. The geometrization is, again, partial: the curvature is dynamical and empirical, but the underlying manifold is still 4D and the value of G_N is still an empirical input. After all, no theory of the day tells us why G_N has the particular value it has, or why the manifold has dimension four. We shall see in Sections 13 and the Part VI treatment of the 6D bulk how G_N is recovered as the scale-curvature coupling of the radial interval I_r of GG-6.

10.1 The road to general relativity

The eight years between Einstein's special relativity paper of 1905 and his arrival at the field equations of general relativity in November 1915 are a sustained intellectual struggle to extend the equivalence of inertial frames to accelerated frames, and to incorporate gravity into the geometric picture. The decisive insight, which Einstein later called "the happiest thought of my life," came in 1907: a person falling freely in a gravitational field does not feel his own weight. This is the equivalence principle: the local effects of a gravitational field are indistinguishable from those of an accelerated frame, and conversely. Geometrically, this means that gravity should not be a force acting on objects against an inertial background, but a property of the inertial background itself.

Between 1907 and 1915, Einstein developed this insight into a full physical theory through the framework of Riemannian geometry. The key technical step is the recognition that the gravitational field is encoded in the metric tensor $g_{\mu\nu}(x)$ of a four-dimensional pseudo-Riemannian manifold, and that the trajectories of freely falling particles are the geodesics of this metric. The remaining problem is to specify how the matter content of spacetime determines the metric.

10.2 The field equations

On 25 November 1915, Einstein submitted to the Prussian Academy the field equations of general relativity in their final form (Einstein, 1915):

$$\boxed{R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = \frac{8\pi G_N}{c^4}\,T_{\mu\nu}.} \tag{12}$$

The left-hand side is the Einstein tensor, a particular combination of the Ricci tensor $R_{\mu\nu}$ and the scalar curvature R that is constructed entirely from the metric and its derivatives. The right-hand side is the stress-energy tensor of matter, multiplied by the constant $\kappa = 8\pi G_N/c^4$. The constant G_N appears as the coupling strength between matter and curvature: more matter (larger $T_{\mu\nu}$) produces more curvature (larger Einstein tensor), with G_N controlling the proportionality.

In this reformulation, G_N takes on a dramatically different meaning from the Newtonian one. In Newton's theory, G_N is the coupling between mass and gravitational force ($F = G_N m_1 m_2/r^2$). In Einstein's theory, G_N is the coupling between mass-energy and the geometric curvature of spacetime. The Newtonian limit is recovered for weak fields and slow motions: in this regime, the (00)-component of the field equation gives the Poisson equation for the Newtonian potential, $\nabla^2\Phi = 4\pi G_N\rho$, with the same G_N.

10.3 G_N as a coupling, not a force constant

The dimensional content of G_N in general relativity is illuminating. In natural units where $c = 1$ (so that lengths and times are commensurable) but $\hbar$ is retained, G_N has dimensions of inverse mass squared, and its inverse-square root defines the Planck mass:

$$M_{\rm Pl} \equiv \sqrt{\hbar c/G_N} \approx 1.22 \times 10^{19}\ \mathrm{GeV}/c^2. \tag{13}$$

The Planck length $\ell_{\rm Pl} = \sqrt{\hbar G_N/c^3} \approx 1.6 \times 10^{-35}$ m and the Planck time $t_{\rm Pl} = \ell_{\rm Pl}/c$ are the conjectured length and time scales at which quantum gravitational effects become order unity. These Planck units are determined by G_N together with $\hbar$ and c; they are the natural scales of any theory that combines gravity, quantum mechanics, and special relativity.

The dimensional analysis already suggests that G_N is not properly thought of as an independent free parameter. Once one specifies a fundamental mass scale (e.g., the Planck mass or some bulk scale), G_N is determined. In the Randall–Sundrum framework that we shall reach shortly, this becomes precise: G_N is a derived quantity, fixed by the bulk geometry.

10.4 The hierarchy problem

Although G_N is small in conventional units, the smallness has a sharper expression in high-energy physics: the ratio between the Planck mass $M_{\rm Pl} \approx 10^{19}$ GeV and the electroweak scale $M_{\rm EW} \approx 246$ GeV (the vacuum expectation value of the Higgs field) is approximately 4×10^{16}. Why is gravity so much weaker than the weak force, in the sense that the natural mass scales differ by sixteen orders of magnitude? This is the hierarchy problem.

In the Standard Model with a cutoff at the Planck scale, radiative corrections to the Higgs mass are quadratically divergent and would naturally drag the Higgs mass to the Planck scale, contradicting the observed value of 125 GeV by sixteen orders of magnitude. This is the technical statement of the problem. Resolutions include supersymmetry (which cancels the quadratic divergences), large extra dimensions (Arkani-Hamed et al. (1998)), and warped extra dimensions (Randall and Sundrum (1999a)). We shall focus on the warped construction, which is the most directly relevant to the Arisaka-GG program.

11 Kaluza 1919, Klein 1926: $\hbar$ Emerges From Compact Geometry

The third partial geometrization: $\hbar$ from a compact extra dimension. The third step takes the absorbing program off the four-dimensional manifold for the first time. Kaluza's 1919 letter to Einstein and Klein's 1926 quantization paper together propose that the U(1) gauge field of electromagnetism is the extra-dimensional component $g_{\mu 5}$ of a 5D metric, and that the integer charge quantization of electromagnetism is the integer momentum quantization of a periodic compact direction. The constant $\hbar$ has been absorbed — the integer mode quantization $p_5 = n\hbar/R$ on a compact circle of radius R is precisely the relation that turns $\hbar$ from a constant of quantum mechanics into a conversion factor of compact-direction holonomies. The geometrization is partial because the radius R is a free moduli parameter and the topology is postulated. Part III will show how the compact direction S^1_θ of the 6D bulk fixes the radius via the OGN-5 ledger and completes the Kaluza–Klein construction.

11.1 Kaluza's proposal

Theodor Kaluza, a Privatdozent at the University of Königsberg, sent Einstein a manuscript in April 1919 proposing a unification of gravity and electromagnetism through the addition of a fifth dimension. Kaluza's idea was bold and simple: write down the Einstein field equations on a five-dimensional Lorentzian manifold instead of a four-dimensional one, and see what comes out when one assumes that the metric is independent of the fifth coordinate (the so-called *cylinder condition*). Einstein delayed publication for two years, wrestling with the proposal, before finally communicating it to the Prussian Academy in December 1921 (Kaluza, 1921).

The 5D Lorentzian metric has, by symmetry of the metric tensor, $5 \times 6/2 = 15$ independent components. Under the cylinder condition, these decompose under 4D Lorentz transformations as:

- Ten components of the 4D metric $g^{(4)}_{\mu\nu}(x)$ (with μ, ν running over four-dimensional indices);
- Four components of an off-diagonal vector $A_\mu(x) \equiv g_{\mu 5}(x)$;
- One scalar $\phi(x) \equiv g_{55}(x)$ (the dilaton or radion).

The 5D Einstein equations decompose correspondingly into:

- The standard 4D Einstein equations for $g^{(4)}_{\mu\nu}$, with a stress-energy contribution from the electromagnetic field;

- The Maxwell equations for A_μ in 4D;
- A dilaton equation for ϕ.

The miracle is the second item: the equations for A_μ extracted from the 5D Einstein equations are precisely Maxwell's equations. Gravity in five dimensions decomposes, under the cylinder condition, into 4D gravity plus 4D electromagnetism plus a dilaton. Electromagnetism is no longer a separate field added to a gravitational background; it is the off-diagonal part of a five-dimensional metric.

11.2 Klein's compactification

The obvious physical objection to Kaluza's proposal — that we do not see a fifth dimension — was answered by Klein (1926). Klein proposed that the fifth dimension is *compact*: curled into a tiny circle of circumference $L = 2\pi R$ that is far smaller than any scale that nineteenth-century physics could probe directly. A point in 5D spacetime then has coordinates (x^μ, y), with μ running over the four ordinary directions and $y \in [0, 2\pi R)$ parametrizing the compact direction with the periodic identification $y \equiv y + 2\pi R$.

Consider a complex scalar field $\Phi(x, y)$ on this 5D spacetime.[1] Periodicity in y forces a Fourier expansion:

$$\Phi(x, y) = \sum_{n\in\mathbb{Z}} \phi_n(x) \exp(iny/R). \tag{14}$$

Each Fourier mode $\phi_n(x)$ is a 4D field. Substituting into the 5D wave equation $\partial^M \partial_M \Phi = 0$ (with M running over 5D indices) gives, for each mode,

$$(\partial_\mu \partial^\mu + n^2/R^2)\, \phi_n(x) = 0, \tag{15}$$

which is the 4D wave equation for a particle of mass $m_n = |n|/R$. The 5D theory, viewed from 4D, is a tower of particles labeled by the integer n, with masses spaced by $1/R$. This is the *Kaluza–Klein tower*.

11.3 Charge quantization and the geometric origin of $\hbar$

Klein's most striking observation concerns electric charge. The 5D metric has the off-diagonal component $g_{\mu 5} \equiv A_\mu$ that is identified with the 4D electromagnetic four-potential. A charged scalar field on this geometry transforms under the U(1) isometry of the compact direction (translations $y \to y + \alpha$), and this U(1) isometry is precisely the gauge symmetry of electromagnetism. Now, the Fourier mode ϕ_n carries n units of momentum around the compact direction, and this momentum couples to A_μ exactly as electric charge does. Klein concluded:

$$Q_n = n\, Q_0, \tag{16}$$

[1] Complex notation, here as in the historical chapters throughout, is the boundary's shorthand: in the GG-Theory reading the charged field's i is not a bulk primitive but the derived boundary quarter-turn — the circle's quarter-turn (T-P4-1) and the clock's polar phase (T-P4-2) of Arisaka, 2026, P4-Real — and the real bulk category it dresses is priced as the AX-4 minimum by L-Constitution-1 (candidate grade, Arisaka, 2026, A1-Constitution).

with Q_0 a single fundamental unit determined by the geometry. Charge quantization — the empirical fact that all electric charges are integer multiples of e — is, in Kaluza–Klein theory, the integer-labeling of momentum modes around the compact circle.

This is the moment to write the relation in dimensional form. The momentum on the compact circle of radius R, in conventional units, is

$$\boxed{p_5 = n\hbar/R, \qquad n \in \mathbb{Z}.} \tag{17}$$

where $\hbar$ appears explicitly as the conversion factor between the dimensionless integer n and the physical momentum p_5. This is the same $\hbar$ that appears in 4D quantum mechanics. The action accumulated by a particle going once around the compact direction is the momentum times the circumference:

$$S_{\text{loop}} = p_5 \times 2\pi R = 2\pi n\hbar = nh. \tag{18}$$

The total action is an integer multiple of $2\pi\hbar = h$. Planck's constant, in this picture, is the action accumulated by one unit of momentum traveling once around the compact direction. It is, geometrically, the action quantum associated with the topology of the compact dimension.

11.4 $\hbar$ as a conversion factor

The argument can now be summarized. In a Kaluza–Klein theory with a compact direction of circumference $L = 2\pi R$, the action quantum is fixed by topology to be $h = 2\pi\hbar$, where $\hbar$ is whatever conversion factor between integer-labeled momentum modes and dimensional momenta one happens to be using. Choose to measure momenta in natural units where $\hbar = 1$ and the conversion is trivial; choose SI units where momenta are in kg·m/s and the conversion is the empirical 1.055×10^{-34} J·s. The geometry of the compact direction is the same in both cases. The physical momenta carried by the modes are the same in both cases. Only the numerical value of $\hbar$ is different, because only the units are different.

This is the precise sense in which $\hbar$ is not a free parameter: it is a unit-translation device, dictated by the dimensional structure of action and the integer-labeling structure of compact-dimension modes. Once one specifies the units of energy and time, $\hbar$ is fixed by the requirement that integer-labeled mode numbers map onto physical momenta correctly. Table 6 shows the same dependence for $\hbar$ that Table 5 showed for c.

Table 6: $\hbar$ as a conversion factor: the value depends on the choice of units. The table repeats for $\hbar$ the test applied to c: vary the unit system and watch what survives. The value falls from 1.055×10^{-34} in SI to exactly unity in natural and in Planck units, and the mode quantization reads $p_5 = n\hbar/R$ in the first two rows and $p_5 = n/R$ in the last two: what survives every row is the integer n, and $\hbar$ only carries it into whatever momentum unit is in use. One caveat belongs with the table rather than after it. The R in these formulas is the Kaluza–Klein compactification radius, written here because the table is about the conventional KK setting; GG-6 has no such radius, and the corresponding statement on the topological circle S^1_θ is $p_\theta = n\hbar$ with no length in it at all (Part III). That is the sharper form of the same point: $\hbar$ converts between a count and a momentum, which is why the sixth dimension can fix a dimensionful constant while carrying no length.

Unit system	Value of $\hbar$	Form of mode quantization
SI (J, s)	$\approx 1.055 \times 10^{-34}$ J·s	$p_5 = n\hbar/R$
Atomic (eV, fs)	varies	same form, varied $\hbar$
Natural ($\hbar = 1$)	1 (dimensionless)	$p_5 = n/R$
Planck units ($c = G_N = \hbar = 1$)	1	same as natural

12 Wilson 1974, Hosotani 1983: Holonomies and Wilson Lines

The fifth partial geometrization: gauge symmetry breaking as a holonomy. The fifth step is the absorption of gauge symmetry breaking into geometry. Wilson's 1974 lattice formulation shows that the natural objects of a gauge theory are not the gauge fields themselves but the holonomies $W = \mathrm{P}\exp(i\oint A)$ around closed loops — topological objects that depend only on homotopy class, not on local geometry. Hosotani's 1983 mechanism applies this insight to gauge symmetry breaking: a non-trivial Wilson line winding around a compact extra dimension breaks the gauge symmetry of the higher-dimensional theory in a fully geometric way, with no Higgs field required. The breaking of SU(5) down to the Standard Model gauge group can therefore be a property of the topology of the extra dimension, not a property of an additional scalar potential. No scalar field is needed to break the symmetry geometrically. The geometrization is partial because the winding integer remains free. Part IV will show that the OGN-5 ledger forces the closure index $J = 7$, fixing the unique Wilson-line winding compatible with the constitution.

12.1 Wilson loops

A more refined geometric understanding of $\hbar$ emerges from the theory of Wilson loops, introduced by Wilson (1974) as an order parameter for confinement in lattice gauge theory. Given a gauge field A_μ on a manifold, the Wilson loop along a closed curve γ is the gauge-invariant trace

$$W(\gamma) = \mathrm{tr}\; P\exp\left(\mathrm{i}g\oint_\gamma A_\mu\, dx^\mu\right), \tag{19}$$

where P denotes path ordering and g is the gauge coupling. Even when the field strength $F_{\mu\nu}$ vanishes locally, $W(\gamma)$ can be nontrivial because the underlying line bundle may have nontrivial holonomy. This is the Aharonov–Bohm phenomenon (Aharonov and Bohm, 1959), lifted to non-Abelian gauge theory.

12.2 The Hosotani mechanism

Hosotani (1983a,b) integrated Wilson loops with Kaluza–Klein compactification in a particularly powerful way. Consider a gauge field A_M (with M running over the 5D indices) on the compact 5D spacetime. The Wilson loop around the compact direction at a fixed 4D point x is

$$W(x) = P \exp\left(\mathrm{i}g \oint A_5\, dy\right) \quad \text{over} \quad y \in [0, 2\pi R). \tag{20}$$

The Hosotani mechanism is the recognition that the value of $W(x)$ is a dynamical modulus of the compactification — the residual zero-mode of A_5 after the cylinder condition has been imposed — that can take expectation values determined by minimization of the effective potential. In a non-Abelian gauge theory, the Wilson line can dynamically take values that break the gauge symmetry: this is dynamical gauge symmetry breaking by the Hosotani mechanism.

For our argument, the relevant point is that the Wilson line is intrinsically a phase per circumference — an integral of the gauge connection around a closed loop, which equals an integer times $\hbar$ when the underlying field is single-valued. In quantum mechanics, the same structure appears in the Aharonov–Bohm phase: a charged particle traversing a closed loop accumulates a phase $\exp(\mathrm{i}q \oint A_\mu\, dx^\mu/\hbar)$, with $\hbar$ in the denominator as the conversion between the gauge-invariant integral $\oint A$ and the dimensionless phase. In KK theory these two phases coincide: the Aharonov–Bohm phase of a particle going around the compact direction is precisely the Wilson line phase, and $\hbar$ is the conversion factor between the integral of A_5 around the circle and the integer mode number that labels the corresponding KK state.

The Hosotani mechanism becomes essential in the Arisaka-GG derivation of the Standard Model from GG-6: the embedding of the Standard Model gauge group $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ into the 6D bulk with appropriate Wilson-line breaking on the boundary is what produces the observed gauge structure of the six boundary forces (strong, electromagnetic, weak, baryonic, informational, gravitational) discussed in Section 15.6 (Arisaka, 2026, A9-SM).

13 Randall–Sundrum 1999: G_N as the Warp-Curvature Coupling

The sixth partial geometrization: the Planck–electroweak hierarchy from a warp factor. The sixth and last partial geometrization closes the four-plus-one chapter of the absorbing program. Randall and Sundrum's 1999 construction shows that the huge ratio $M_{\mathrm{Pl}}/M_{\mathrm{EW}} \sim 10^{16}$, an unsolved hierarchy puzzle for sixty years, can be generated geometrically from a modest warp factor $e^{-k\pi r_c} \sim 10^{-16}$ on a 5D anti-de Sitter slab between two branes. Then G_N on the IR brane is exponentially smaller than the bulk Planck scale because the warp factor exponentially suppresses graviton wavefunctions there. No fine-tuning is needed once the warp parameter $kr_c \approx 11$ is in place. The geometrization is, as ever, partial: the warp parameter kr_c, the brane positions, and the dimension five itself are still free choices. Part III shows that the (4+2) bulk of GG-6 pins all three: the scale profile e^{-r} runs over a radial interval of length $K = 35$ from $r = 0$ (UV boundary) to $r = K$ (IR boundary), the dimension is forced to six by the constitutional clauses, and the warp parameter is fixed by the OGN-5 ledger.

13.1 The warped 5D AdS construction

The 1999 papers of Randall and Sundrum (1999a,b) took the Kaluza–Klein paradigm in a new direction. Whereas KK assumes that the extra dimension is *flat and compact*, the Randall–Sundrum (RS) construction makes the extra dimension *warped*. The 5D metric is taken to be a slice of anti-de Sitter space (AdS_5) bounded by 4D "branes" at fixed values of the extra coordinate. Specifically, in the so-called RS1 setup,

$$\boxed{ds^2 = e^{-2k|y|}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu + dy^2,} \tag{21}$$

with $y \in [0, \pi r_c]$ the extra-dimensional coordinate, k a curvature scale of order the Planck mass, and the geometry bounded by two branes: the UV brane (or "Planck brane") at $y = 0$, and the IR brane (or "TeV brane") at $y = \pi r_c$. The factor $e^{-2k|y|}$ is the *warp factor*; it exponentially rescales the 4D part of the metric as one moves along the extra dimension.

13.2 The hierarchy from the warp factor

The warp factor has a striking consequence for mass scales. A field with fundamental mass parameter m_0 on the UV brane appears, when its profile is restricted to the IR brane, with effective 4D mass

$$m_{\text{eff}} = e^{-k\pi r_c}\,m_0. \tag{22}$$

For $kr_c \approx 11.2$ (so that $k\pi r_c \approx 35$), the exponential factor $e^{-35} \approx 6.3\times10^{-16}$ converts a Planck-scale fundamental mass $m_0 \sim 10^{19}$ GeV into an IR-brane effective mass $m_{\text{eff}} \sim 10^3$ GeV $\sim$ TeV. The hierarchy between the Planck scale and the electroweak scale is thereby converted from a tuning problem (why is m_H/M_{Pl} tiny?) into a geometric problem (why is $kr_c \approx 11$?), and the latter is far easier to address: kr_c is a logarithm, of order $\ln(M_{\text{Pl}}/M_{\text{EW}})$, and need not be small. This is the Randall–Sundrum resolution of the hierarchy problem.

13.3 G_N as the warp-curvature coupling

The 4D effective gravitational coupling in this geometry is computed by integrating the 5D Einstein–Hilbert action over the warped extra dimension:

$$S_5 = \frac{1}{16\pi G_5}\int d^5x\,\sqrt{-g_5}\,R_5, \tag{23}$$

where $G_5 = 1/M_5^3$ is the 5D Newton constant and M_5 is the 5D fundamental mass scale. Substituting the RS metric and integrating over y from 0 to πr_c gives

$$M_{\text{Pl}}^2 = M_5^3\int_0^{\pi r_c} e^{-2k|y|}\,dy = \frac{M_5^3}{k}\left(1 - e^{-2k\pi r_c}\right). \tag{24}$$

For $kr_c \approx 11$, the exponential is negligible and

$$M_{\text{Pl}}^2 \approx M_5^3/k. \tag{25}$$

The 4D Newton constant is therefore

$$\boxed{G_N = \frac{1}{8\pi M_{\rm Pl}^2} = \frac{k}{8\pi M_5^3}.} \tag{26}$$

Two features of this expression are worth dwelling on. First, G_N is determined by two pieces of geometric data — the 5D bulk scale M_5 and the AdS curvature k — and one numerical constant 8π, which is itself just $\pi \times 8$ from the geometric factor in the Einstein–Hilbert action. There is no third "free parameter" for G_N; its value follows from M_5 and k together with the geometry of the 5D bulk. Second, G_N depends on k linearly and on M_5 inverse-cubically, exactly the dimensional dependences one would predict from a 5D theory without needing to specify the details of the warping. The geometric origin of the Newton constant is laid bare.

A note on the dual role of the radial coordinate. The computation just given follows the Randall–Sundrum construction literally, with y a *spatial* brane coordinate and the graviton zero-mode localized by the warp factor. In the GG-6 reading this picture is a *heuristic*, not the final ontology. The same radial coordinate is, in the exact theory of (Arisaka, 2026, A2-GG6), the renormalization-group scale $r = \ln(E_{\rm GUT}/\mu)$ — a resolution, not a location — and the e^{-2r} profile enters the dimensional reduction as the geometric *measure* along that scale direction, not as a literal extra room of space hosting bulk gravitons. The integral $M_{\rm Pl}^2 = M_5^3 \int e^{-2r}\,dr$ is accordingly read as the accumulated scale weight of the renormalization-group flow, and $M_{\rm Pl}$ relative to the single dimensionful input G_N is a pure number. We retain the spatial-interval language only because it makes the exponential hierarchy vivid: GG-6 inherits the Randall–Sundrum *result* for G_N while discarding the Randall–Sundrum *ontology* of a fifth spatial dimension.

13.4 The logarithmic energy scaling of warped AdS

The crucial structural feature of the RS construction, for our argument, is the *logarithmic* relation between physical scales and geometric data. The warp factor e^{-ky} is exponential in the extra-dimensional coordinate; the resulting hierarchies are exponentials of geometric quantities. Equivalently, the geometric quantities — the warp coordinate, the curvature scale, the brane positions — are logarithms of physical scales. This logarithmic energy scaling is the most distinctive structural feature of warped-AdS phenomenology and the one that distinguishes it from flat KK and from large-extra-dimension scenarios.

The entire RG flow of a 4D theory dual to the 5D bulk (via the AdS/CFT correspondence (Gubser, Klebanov, and Polyakov, 1998, Maldacena, 1998, Witten, 1998)) is realized as motion along the warp direction: the UV is at $y = 0$ and the IR is at $y = \pi r_c$. In this duality, G_N is the coupling on the gravity side of an interacting theory whose other side is a strongly-coupled 4D conformal field theory, and the warp curvature k encodes the coupling strength of the dual CFT. We shall return to this AdS/CFT interpretation in Part V when we discuss the Gi-4 Pillars.

14 The Four-Plus-Two Necessity

14.1 Why six dimensions? The forced move from 5 to 6

The arguments of Sections 8–13 establish a striking structural pattern. Each of the three universal constants of physics — c, G_N, $\hbar$ — admits a geometric interpretation as a structure constant of a manifold with one more dimension than the 4D Lorentzian spacetime in which classical and ordinary quantum physics are formulated:

- c is the conversion factor of the (1,3) Lorentzian metric on 4D Minkowski space. It is fixed by the signature of the metric.
- $\hbar$ is the conversion factor between integer Fourier mode numbers on a compact extra dimension of circumference $L = 2\pi R$ and the physical momenta carried by those modes, $p_5 = n\hbar/R$. It is fixed by the topology of the compact direction.
- G_N is the warp-curvature coupling of a five-dimensional warped AdS bulk, $G_N = k/(8\pi M_5^3)$. It is fixed by the warp curvature and the fundamental scale of the warped extra dimension.

Each constant is the structure coefficient of a different dimensional extension of the 4D base. To absorb c, the geometry must have a Lorentzian signature; this is the (1,3) Minkowski move of Minkowski (1908). To absorb $\hbar$, the geometry must have a compact direction with integer-labeled modes; this is the Kaluza–Klein move of Kaluza (1921), Klein (1926), refined by Hosotani (1983a), Wilson (1974). To absorb G_N as a derived quantity, the geometry must have a warped AdS structure with a curvature scale k and a 5D bulk scale M_5; this is the Randall–Sundrum move of Randall and Sundrum (1999a).

The two 5D moves — KK with a flat compact direction, and RS with a warped (non-compact or warped-compact) direction — are independent: they accomplish different things and have different structural roles. KK provides the integer-labeling structure that makes $\hbar$ a topological conversion factor; RS provides the warp factor structure that makes G_N a geometric coupling. To absorb *both* $\hbar$ and G_N as geometric data of a *single* manifold, one must add two extra dimensions to the 4D base, one playing the KK role and one playing the RS role.

This gives the dimensional accounting: 4 (Lorentzian base) + 1 (KK compact) + 1 (RS warped) = 6. Six dimensions is the minimum dimensionality in which all three universal constants are simultaneously geometric. Five is not enough; seven is more than is required. The choice of six is forced by the structure of the universal constants. In the cohort realization the RS-role dimension is not a literal Randall–Sundrum brane separation: it is read on the field-theory side of the duality just described — the renormalization-group axis $r = \ln(E_{\mathrm{GUT}}/\mu)$ of GG-A2-GG6, along which the scale profile reproduces the renormalization-group flow above. The six-dimensional bulk of the cohort is thus the renormalization-group geometrization of the energy scale — the G_N-bearing scale direction is the running of couplings, not an extra room of space — rather than a spatial warped-AdS braneworld (Arisaka, 2026, A2-GG6).

14.2 The two-warp structure

The dimensional accounting just given anticipates the structure of GG-6: a four-dimensional Lorentzian base $\mathcal{M}_4$, plus a warped radial interval I_r playing the RS role for G_N, plus a compact direction S^1_θ playing the KK–Hosotani role for $\hbar$ and the gauge sector. The product structure is

$$\boxed{\mathcal{M}_6 = \mathcal{M}_4 \times I_r \times S^1_\theta,} \tag{27}$$

with a metric that includes warping along I_r (the RS warp factor) and possibly additional warping in the S^1_θ direction. The full form of the metric, the boundary conditions, and the geometric content of GG-6 are the subject of Part III.

14.3 What the historical lineage tells us

Figure 5 draws the whole arc on one page, and Table 7 follows each constant along it row by row. The historical sequence Euclid → Riemann → Maxwell → Einstein 1905 → Minkowski → Einstein 1915 → Kaluza → Klein → Wilson → Hosotani → Randall–Sundrum can now be read in a single sweep. At each step, an empirical constant is absorbed into the geometry of a manifold, with the dimensional cost of the absorption being one extra dimension or one extra structural feature (warping, compactification, etc.). The cumulative consequence is that, by the late 1990s, all three universal constants c, G_N, $\hbar$ have been understood individually as geometric data of higher-dimensional manifolds, but no single manifold has been constructed in which all three appear together. GG-6 is the natural and forced synthesis: the unique six-dimensional bulk that absorbs all three constants simultaneously, and (as we shall develop in Part III) provides the geometric setting for the entire Standard Model, the cosmological inventory, and the biological energy quantum.

Table 7: The lineage of the universal constants: from empirical fact to geometric data. Each row follows one constant from the measurement that first isolated it to the geometric structure that now supplies it. The pattern is the same three times over: a quantity enters physics as a number to be measured, acquires a geometric interpretation, and ends as a property of a manifold rather than an input to a theory. For c the arc runs from Rømer's finite light speed to Minkowski's 4D metric, where c becomes the slope of the light cone. For G_N it runs from Cavendish's torsion balance to Randall–Sundrum, where the coupling is read off the warp of an AdS bulk. For $\hbar$ it runs from Planck's black-body constant to the Kaluza–Klein tower, where it converts mode number to momentum. The third column is what GG-Theory inherits; Part VI shows that the three together are complete.

Constant	Historical milestones	Geometric interpretation
c	Rømer 1676 (finite light speed); Maxwell 1865 (electromagnetic identification); Michelson–Morley 1887 (frame independence); Einstein 1905 (postulate); Minkowski 1908 (4D metric).	Conversion factor between temporal and spatial coordinates of (1,3) Lorentzian metric; slope of light cone; structure constant of 4D spacetime.
G_N	Newton 1687 (universal gravitation); Cavendish 1798 (laboratory measurement); Einstein 1915 (matter–curvature coupling); Randall–Sundrum 1999 (warped 5D).	Warp-curvature coupling, $G_N = k/(8\pi M_5^3)$; geometric data of warped AdS bulk; not a free parameter.
$\hbar$	Planck 1900 (black-body); Bohr 1913 (atom); de Broglie 1924, Heisenberg/Schrödinger 1925–26 (QM); Kaluza 1919, Klein 1926 (compact 5D); Wilson 1974, Hosotani 1983 (holonomies).	Conversion factor between integer mode number on compact direction and physical momentum, $p_5 = n\hbar/R$; topological conversion factor; circumference factor.

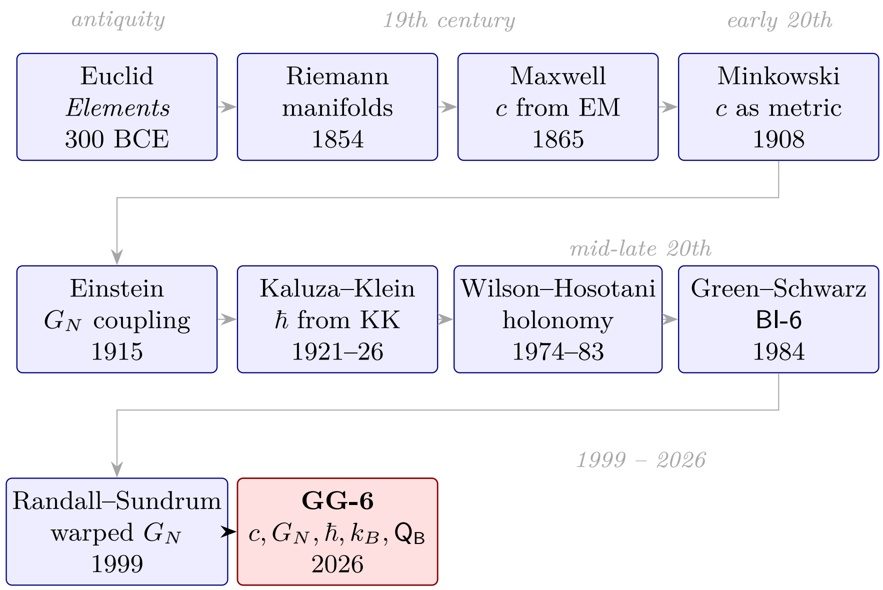

Figure 5: The historical arc from Euclid to GG-6: 2300 years of partial geometrizations. Each milestone names the inventor, the year, and the dimensional constant or geometric framework absorbed into the structure of physics at that step. The closing GG-6 box brings together the absorbing program: c as the (1,3) Lorentzian conversion factor, $\hbar$ as the S^1_θ holonomy normalization, G_N as the scale-curvature coupling, and $\mathsf{Q_B}$ as the scale-suppressed energy–information conversion factor — all simultaneously, in the unique six-dimensional geometry forced by the GG-Constitution.

Part III
The Two Non-Space-Like Dimensions

The 6D Geometry, and What the Fifth and Sixth Dimensions Are

This Part presents the geometric heart of the framework and its single most consequential idea. The bulk is the six-dimensional manifold $\mathcal{M}_6 = \mathcal{M}_4 \times I_r \times S^1_\theta$; the claim that distinguishes GG-Theory from every Kaluza–Klein and string construction is that its two extra dimensions are *not space*. The fifth is renormalization-group time, the logarithm of energy scale; the sixth is a topological gauge phase. Because neither carries a length, the bulk has no compactification modulus — and, as Part VI will show, the three bulk conversion factors are therefore complete. A survey of precursors credits the holographic-RG and gauge-holonomy literatures from which the construction inherits and marks precisely where it departs, and the Part then closes on the decisive contrast with Kaluza–Klein and string theory, where the consequences of non-spatiality are drawn out one at a time.

The idea is easy to state and easy to underestimate, so it is worth being blunt about what is and is not being claimed. It is *not* claimed that the extra dimensions are small, or curled, or hidden, or awaiting a more powerful accelerator. It is claimed that they are not places. A dimension, on the reading developed here, is a way for two states of the world to differ; space is one such way and there is no argument that it is the only one. Energy scale is another — two theories can differ in scale without differing in position — and gauge phase is a third. Neither carries a length, and it is the absence of that length, and nothing about size, that removes the modulus, removes the landscape, and lets the conversion-factor trio be complete.

Three consequences are worked out before the Part ends, because each is a place where a reader is entitled to expect trouble and does not find it. Causality, locality and point-like particles survive, because nothing has been added that a signal could propagate along. Gravity's weakness becomes a statement about where the graviton is bound rather than a number to be tuned. And the question that has followed every space-like compactification for a century — where are the extra dimensions, and what fixes their size — does not arise, because it presupposes exactly the thing this Part denies.

15 The GG-6 Geometry

15.1 The (4+2) bulk: canonical form

The GG-6 bulk is the six-dimensional manifold

$$\boxed{\mathcal{M}_6 = \mathcal{M}_4 \times I_r \times S^1_\theta,} \tag{28}$$

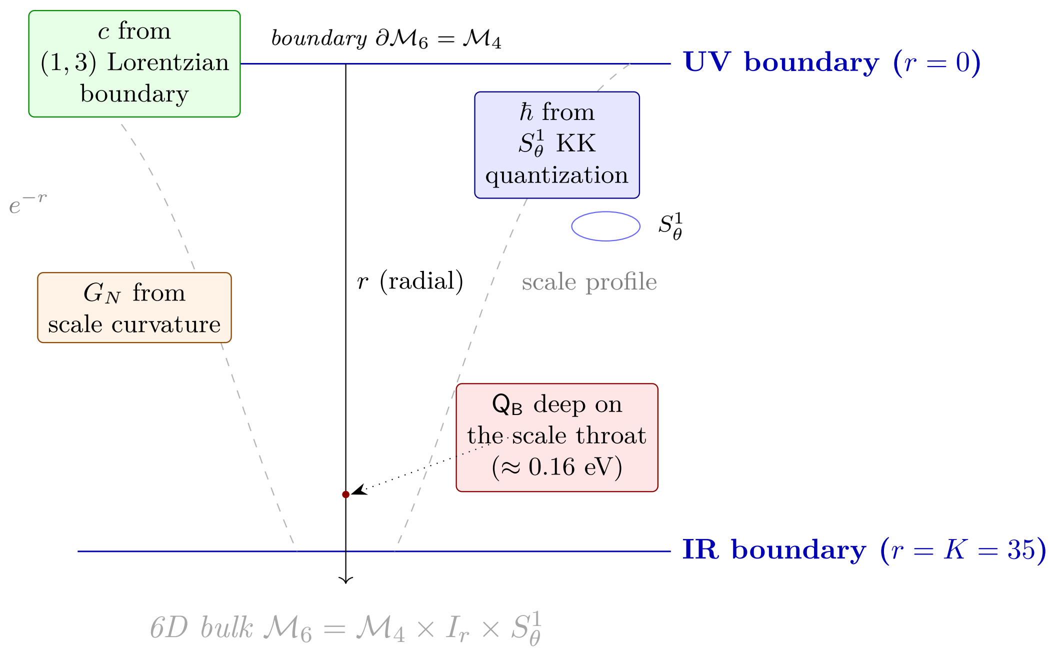

Figure 6: Schematic of the (4+2) bulk of GG-6 with the three fundamental dimensional conversion factors located on their respective geometric supports. The constant c lives on the (1,3) Lorentzian boundary $\mathcal{M}_4 = \partial\mathcal{M}_6$ as the metric-signature conversion factor; $\hbar$ lives on the compact angular direction S^1_θ as the KK quantization factor; G_N lives on the radial interval I_r as the scale-curvature coupling, integrated from $r = 0$ (UV boundary) to $r = K = 35$ (IR boundary) with scale profile e^{-r}. The derived energy–information Arisaka constant $\mathsf{Q_B}$ is shown for orientation far down the scale throat along I_r (roughly twelve orders below the TeV scale), where the chain OGN-5 $\to \alpha \to m_e \to E_{\mathrm{Ry}} \to \mathsf{Q_B}$ of Arisaka (2026, A7-Qi4) relocates the fundamental triad to the biological energy scale. After all, the three fundamental constants $\{c, G_N, \hbar\}$ are not free parameters but geometric data of one and the same six-dimensional manifold; $\mathsf{Q_B}$ is what the manifold says about biology.

Figure 6 locates the three conversion factors on the geometric supports that carry them, which is the picture the rest of this Part works from. Here $\mathcal{M}_4$ is the four-dimensional Lorentzian boundary, $I_r = [0, K]$ a dimensionless radial interval (with $K = 35$ the OGN lock integer derived in Section 23), and S^1_θ a topological circle parametrized by the dimensionless angular coordinate $\theta \in [0, 2\pi)$. The canonical metric on $\mathcal{M}_6$ is

$$\boxed{ds_6^2 = e^{-2r}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu + dr^2 + d\theta^2,} \tag{29}$$

where $\eta_{\mu\nu}$ is the Minkowski metric on $\mathcal{M}_4$. The scale profile e^{-2r} multiplies *only* the four-dimensional conformal prefactor; the compact direction S^1_θ is scale-flat at every value of r.

15.2 Why no k, no R_θ, no r_c: AX-4 (MSMF) made explicit

The form (29) departs deliberately from the conventional Kaluza–Klein and Randall–Sundrum notation $ds_6^2 = e^{-2kr}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2 + R_\theta^2 d\theta^2$ with $r \in [0, \pi r_c]$, which carries three dimensional "constants" k, R_θ, r_c. In GG-Theory each of these is absorbed by MSMF: the AdS curvature k is fixed to 1 by the natural RG-parametrization of POS-5; the compact radius R_θ is replaced by the topological circle with $\oint d\theta = 2\pi$ (no intrinsic geometric length); and the scale endpoint r_c is replaced by the OGN integer $K = 35$ via Step 5 of Section 23. What survives is a choice of unit system — the values of the co-equal trio $\{c, G_N, \hbar\}$, from which the 6D fundamental mass M_6 follows as a derived combination rather than as a free parameter (Sec. 37.3). The full parameter-elimination argument is Arisaka (2026, A2-GG6, Theorem 2 + Theorem 3 + worked example); the present paper imports the canonical metric (29) as the result.

15.3 The axiomatic reading of the canonical metric

The metric (29) embodies the Euclidean methodological condition: it states only what is structurally necessary, with no free parameter beyond the absolute minimum. POS-5 (RG/Scale Covariance) forces the radial direction r to implement a logarithmic RG flow with scale profile e^{-2r}. POS-1 (Layered Gauge Invariance) and POS-6 (OGN Integer Normalization) require the $\mathrm{U}(1)_{Qi}$ Wilson-line winding numbers and the integer mode quantization on S^1_θ to be r-invariant topological data; scale running S^1_θ together with the 4D directions would entangle the RG axis with the gauge axis and break the clean separation that makes the OGN ledger and the Hosotani breaking pattern well-defined. The two internal directions therefore play structurally distinct roles: I_r is the RG/scale axis (carrying the AdS curvature responsible for G_N) and S^1_θ is the scale-flat topological gauge axis (carrying the Wilson-line phase responsible for $\hbar$). The interval endpoint $r = K = 35$ is the OGN lock integer. The metric (29) contains no continuous parameters; it is the canonical form derived in Arisaka (2026, A2-GG6).

Distinction from KK and string-theory tradition. The Kaluza–Klein program of 1921–1926 and the string-theory compactifications that inherited its viewpoint treated the compact direction as a geometric circle of intrinsic radius R, with R as a free continuous parameter to be determined empirically (or, in the string-landscape case, scanned over a vast moduli space). This view — the compact direction as a tiny geometric circle — is a viewpoint inherited from the early KK literature and carried unreflected into modern compactification frameworks. The present formulation breaks with that tradition: the compact direction of GG-6 is a pure topological S^1, with no intrinsic length. This is the mathematical realization of the MSMF principle and the structural feature that most distinguishes GG-6 from string-theory frameworks (which require $\sim 10^{500}$ moduli vacua because they carry many such putative "geometric" scales) and from conventional KK theory (which carries one such scale, R, as a free parameter). In GG-6, the only structural data of the compact direction is the topology and the integer mode quantization that single-valuedness forces; everything else — the Wilson-line phase, the gauge breaking pattern, the integer ledger — follows from this minimal data.

A pointer forward. The MSMF reading of the metric stated here is intentionally compact. Section 19 below develops the full geometric intuition that the canonical metric encodes — how the scale profile encodes the running couplings via radial integration, why gravity is so weak as a UV-localized phenomenon viewed from the IR boundary, and how the topological S^1_θ supplies integer KK quantization, Wilson-line phases, and the topological foundation for $\hbar$ at every r. Readers who want the intuitive account before continuing with the technical material may proceed directly to

Section 19 and return here once the four geometric pictures of the bulk are in hand.

For the remainder of this paper we use the canonical metric (29) throughout, but we maintain notational compatibility with the KK and Randall–Sundrum literature by allowing k, R_θ, and r_c to appear as conversion-factor placeholders in cross-references and in formulas where the comparison with the historical literature is illuminating. In every case, the underlying physical content reduces to the canonical form: $k = 1$, $R_\theta = 1$, $\pi r_c = K = 35$ in natural GG-6 units. The reader who prefers the historical notation may substitute these values mentally; the reader who prefers the canonical form may take the metric (29) as primary.

15.4 Gauge content

The gauge content of GG-6 is given by the bulk gauge group

$$G_{\text{bulk}} = \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}, \tag{30}$$

with SU(5) the unified gauge group of Georgi and Glashow (1974) acting on quarks and leptons, $\mathrm{U}(1)_B$ the gauged baryon-number symmetry, and $\mathrm{U}(1)_{Qi}$ the gauged information current (the central new ingredient of the GG-6 program (Arisaka, 2026, A2-GG6)). Each factor is associated with an integer quantum number that is constrained by the OGN ledger; the resulting integer set is OGN-5.

The boundary gauge group, after the Hosotani Wilson-line breaking on S^1_θ, is

$$G_{\text{bdy}} = \mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}, \tag{31}$$

the Standard Model gauge group augmented by the gauged baryon and information currents. The breaking pattern $\mathrm{SU}(5) \to \mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is determined by the choice of Wilson line on S^1_θ; the residual $\mathrm{U}(1)_Y$ is the standard hypercharge with the conventional normalization fixed by the OGN integer ledger.

15.5 The bulk action

The bulk action is the standard 6D Einstein–Hilbert plus Yang–Mills action, with appropriate boundary terms on the UV and IR boundaries:

$$\begin{aligned} S_{\mathsf{GG\text{-}6}} = \int_{\mathcal{M}_6} d^6x \sqrt{-g_6} \left[\frac{1}{2\kappa_6^2} R_6 - \Lambda_6 - \frac{1}{4g_6^2} \operatorname{tr} F_6^2 - \frac{1}{12} H_3^2 \right] \\ + S_{\text{UV}} + S_{\text{IR}} + S_{\text{GS}}, \end{aligned} \tag{32}$$

where $\kappa_6^2 = 8\pi G_6 = 8\pi/M_6^4$ is the 6D gravitational coupling (mass dimension -4, as a six-dimensional coupling must be; Appendix D works in the normalization $\kappa_6^2 = M_6^{-4}$, which differs by the same 8π that separates the reduced and non-reduced Planck masses and is carried consistently there), Λ_6 is the bulk cosmological constant, F_6 is the bulk gauge field strength, and $H_3 = dB_2$ is the field strength of the antisymmetric tensor field B_2 that mediates the Green–Schwarz inflow (see Section 22). The boundary terms S_{UV} and S_{IR} are Gibbons–Hawking–York terms plus matter localized on the boundaries; the Green–Schwarz term S_{GS} contains the Chern–Simons couplings of

B_2 to the gauge connections that are required for anomaly inflow.

15.6 The six forces of GG-6 and the three abelian gauge invariances

A central feature of GG-Theory— and a point that distinguishes it sharply from the conventional Standard Model + GR formulation — is that the bulk gauge content $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$, projected through the Hosotani Wilson-line breaking and the GDOS bulk-to-boundary projection onto the IR boundary, gives rise to *six* distinct boundary forces, not four:

- *Strong*: $\mathrm{SU}(3)_C$, descending from SU(5) via Hosotani breaking. Mediates the binding of quarks into hadrons.
- *Electromagnetic*: $\mathrm{U}(1)_{\mathrm{EM}}$, descending from SU(5) via Hosotani breaking and the subsequent electroweak Higgs mechanism, $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{\mathrm{EM}}$. This is one of the three abelian gauge invariances of the boundary.
- *Weak*: $\mathrm{SU}(2)_L$ broken by the Higgs mechanism to $\mathrm{U}(1)_{\mathrm{EM}}$, descending from SU(5). Mediates beta decay and flavor-changing weak interactions.
- *Baryonic*: $\mathrm{U}(1)_B$, descending unchanged from the bulk. Forces exact baryon-number conservation, hence absolute proton stability, in the boundary theory. This is the second abelian gauge invariance.
- *Informational*: $\mathrm{U}(1)_{Qi}$, descending unchanged from the bulk. Sources the quantum-information current that powers the Arisaka equation and biological molecular machinery. This is the third abelian gauge invariance.
- *Gravitational*: $\mathrm{Diff}(\mathcal{M}_4)$, descending from $\mathrm{Diff}(\mathcal{M}_6)$ by bulk-to-boundary projection.

Five of the six forces are gauge forces in the conventional sense (mediated by gauge bosons of compact internal symmetries), and the sixth (gravity) is the residual diffeomorphism invariance after bulk-to-boundary projection. The boundary theory then has *three* abelian gauge factors:

$$\boxed{\mathrm{U}(1)_{\mathrm{EM}} \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}} \tag{33}$$

which is the mathematical structure that distinguishes the GG boundary from the Standard Model boundary (the latter has only the single $\mathrm{U}(1)_{\mathrm{EM}}$ after electroweak breaking). The two additional abelian gauge factors, $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$, descend unchanged from the bulk and are not part of the conventional Standard Model gauge group.

15.7 The Arisaka Ratio and the Arisaka Equation as consequences of three U(1) gauge invariance

Historical background (with citations). The structural fact that conserved currents are the natural objects associated with continuous gauge symmetries is a hundred and seven years old. Emmy Noether's 1918 paper *Invariante Variationsprobleme* (Noether, 1918), written at the request of Klein and Hilbert in connection with their work on the variational structure of general relativity, established the two theorems that now bear her name: every continuous global symmetry of an action functional yields a conserved current (Noether's first theorem), and every continuous local symmetry yields an off-shell identity among the field equations (Noether's second theorem). The conserved currents J_e, J_B, J_{Qi} that the Arisaka Equation locks together are, in this language, the Noether currents associated with the three abelian gauge invariances $\mathrm{U}(1)_{\mathrm{EM}}$, $\mathrm{U}(1)_B$, $\mathrm{U}(1)_{Qi}$ of the boundary theory; what is new in the GG-Theory reading is that the constitutional BI-6 inflow forces these three Noether currents to lock into a single quadratic relation rather than remaining independent. In our reading, the Arisaka Equation is the GG-Theory completion of Noether 1918 in the same sense that BI-6 is the GG-Theory completion of Bianchi 1902 and Green–Schwarz 1984.

The presence of three abelian gauge invariances at the boundary, combined with the BI-6 inflow, has a direct and concrete consequence: it forces the conserved currents associated with the three abelian factors to be locked together by a single quadratic relation, with the locking determined by the BI-6 coefficients. This is the axiomatic origin of the Arisaka Ratio and the Arisaka Equation.

The four BI-6 coefficients $(1, -1, -2/5, -2/5)$ rescaled by a factor of 5 to clear fractions give the *Arisaka Ratio*:

$$\boxed{5 : -5 : -2 : -2,} \tag{34}$$

which encodes the relative weighting of the four boundary currents under inflow, in the canonical sector order (E, e, B, Qi): the gravitational/energy channel (weight $+5$), the SU(5)/electron-number channel (weight -5), the baryonic channel (weight -2), and the informational channel (weight -2). This is the same ordered basis in which the Arisaka Equation below is written. The Arisaka Ratio is, like the BI-6 coefficients themselves, fixed by the constitution together with the arithmetic of the ledger rather than by empirical input (Arisaka, 2026, A3-BI6).

When the Arisaka Ratio is interpreted on the boundary — via the OGGEP/GDOS bulk-to-boundary projection of the bulk BI-6 identity onto the 4D effective theory — it gives the conservation law that relates the four boundary currents (electric energy current J_E, electron-number current J_e, baryon current J_B, information current J_{Qi}). In the per-catalytic-step normalization, the relation takes the form of the *Arisaka Equation*:

$$\frac{J_E}{\mathsf{Q_B}} = J_e = J_B = J_{Qi}, \tag{35}$$

where $\mathsf{Q_B} \approx 0.1555$ eV is the Arisaka constant (the energy quantum per qubit of biological information processing, with molar form $N_A \mathsf{Q_B} \approx 15.0$ kJ/mol; its empirical calibration is developed in (Arisaka, 2026, A7-Qi4; Arisaka, 2026, A11-Origin)). The Arisaka Equation states that every catalytic step transfers exactly one electron, one baryon, and one information bit, while consuming exactly one quantum $\mathsf{Q_B}$ of electric energy. This four-current locking is, in the axiomatic taxonomy, the Qi-QUA pillar of Qi-4, also called *Quadruple Gauge Locking* (QGL) because it locks the four boundary currents (J_E, J_e, J_B, J_{Qi}) into a single fixed ratio through the BI-6 inflow (Arisaka, 2026, A7-Qi4).

Why three U(1) invariances suffice to lock four currents. At first sight there is a counting issue: how can three gauge invariances $(\mathrm{U}(1)_{\mathrm{EM}}, \mathrm{U}(1)_B, \mathrm{U}(1)_{Qi})$ lock four currents (J_E, J_e, J_B, J_{Qi})

into a single ratio? The resolution is that the electric-energy current J_E and the electron-number current J_e are both associated with $\mathrm{U}(1)_{\mathrm{EM}}$ but in different roles: J_e is the conserved Noether charge (electron flow, in counts per step), while J_E is the energy carried by that current (in eV per step). The conversion factor between the two is precisely the Arisaka constant Q_B. In this view, the Arisaka Equation uses three gauge currents (one count per gauge factor) plus one energetic conversion, totaling four entries in the locked equation. The axiomatic origin of Q_B as the energetic conversion factor between gauge-count currents and energetic currents on the IR boundary is a direct geometric consequence of the scale suppression of the Rydberg scale along I_r, with $\mathsf{Q}_\mathsf{B} = (2/5)\, E_{\mathrm{Ry}}/K$ (Section 28 below).

15.8 The Standard Model gauge content as a derived consequence

The boundary gauge content

$$G_{\mathrm{bdy}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}, \tag{36}$$

which contains the standard $G_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ as a subgroup, is reproduced exactly with the addition of $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$. Together with the Higgs mechanism that gives masses to the electroweak gauge bosons and the Standard Model fermions, this gauge content reproduces all empirical features of the Standard Model, plus the additional gauged baryon and information currents that distinguish GG-Theory from purely particle-physics frameworks. After electroweak symmetry breaking, the boundary unbroken gauge group is

$$G_{\mathrm{bdy}}^{\mathrm{EWbroken}} = \mathrm{SU}(3)_C \times \mathrm{U}(1)_{\mathrm{EM}} \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}, \tag{37}$$

which is the gauge group that controls all observed boundary phenomenology in GG-Theory.

The detailed derivation of the SM parameters from this structure is in Arisaka (2026, A9-SM); the cosmological consequences are in Arisaka (2026, A10-Cosmos); the biological consequences (including the Arisaka equation derived from the three abelian gauge invariances) are in Arisaka (2026, A7-Qi4). Our concern in the remainder of this part is the geometric origin of the universal constants c, G_N, $\hbar$ in the GG-6 bulk.

16 The Two Non-Spatial Dimensions: A Scale Dimension and a Gauge Dimension

The canonical scale-profile metric of Section 15 and the four geometric pictures of Section 19 have presented the bulk $\mathcal{M}_6 = \mathcal{M}_4 \times I_r \times S^1_\theta$ in formal terms. We now make explicit a conceptual point on which the entire reading of GG-Theory turns, and which — as far as we are aware — is new in the history of geometric physics: *what kind of object are the two extra dimensions I_r and S^1_θ?*

In the Kaluza–Klein tradition, and in ten- and eleven-dimensional string and M-theory, the extra dimensions are *space-like*. They are coordinates of the same kind as x, y, z, each carrying an intrinsic length, and they are hidden from view by being compactified to a size near the Planck length. This reading invites the perennial questions — where are the extra dimensions, why do we not see them, and what fixes their size? — and it requires causality to be carried inside a full higher-dimensional Lorentzian manifold.

GG-Theory makes a different claim. The two extra dimensions of GG-6 are *not space-like at all*. Each is a dimensionless coordinate of the manifold carrying an operational — not a spatial — meaning: the fifth coordinate is renormalization-group time (energy scale), and the sixth is a topological gauge phase. By "dimension" we therefore mean precisely *a coordinate of the manifold that carries a component of the metric* — the line element ds_6^2 genuinely contains dr^2 and $d\theta^2$ — while the *interpretation* of that coordinate is scale or phase rather than position. We are not smuggling energy scale into the theory disguised as a length; we are asserting that the manifold's two extra coordinates have operational meaning, and that this is exactly why the framework absorbs G_N and $\hbar$ without introducing a single new spatial degree of freedom. Sections 17 and 18 develop the two coordinates in turn; Section 19 shows why the construction preserves causality, locality, and the point-like nature of particles; and Section 21 draws the decisive contrast with the space-like extra dimensions of superstring theory.

A note on terminology: why we do not call them "extra dimensions." The phrase *extra dimensions* has a fixed meaning in twentieth- and twenty-first-century physics: additional dimensions *of space*, beyond the three of ordinary experience, of the same kind as x, y, z and differing only in being small, warped, or hidden. In Kaluza–Klein, in the large-extra-dimension scenarios, and in string compactification alike, the term carries that spatial sense without exception. To apply it to I_r and S^1_θ would therefore import, in the very first word, the picture this paper exists to displace, and would mislead the reader before any argument had been made. We accordingly reserve "extra dimensions" for the *spatial* extra dimensions of those frameworks — where the term is correct and where we shall continue to use it when drawing the contrast — and adopt for GG-6 a vocabulary that names what its two further coordinates actually are. The fifth coordinate I_r is the *scale dimension* (renormalization-group time); the sixth coordinate S^1_θ is the *gauge dimension* (the topological $U(1)$ phase); and where a single collective name is wanted we call them GG-6's two *non-spatial dimensions*. The relabeling is not cosmetic. Each name states, in one word, the operational content of the coordinate — scale, or phase — in place of the spatial connotation that "extra" would smuggle in; and the choice is forced by the same discipline applied elsewhere in this cohort to "brane" and to "supersymmetry" (Sections 21.7, 21.6): wherever the inherited vocabulary presupposes a space, we replace it with a word that does not. In the two sections that follow, the scale dimension and the gauge dimension are developed in turn.

Why these two, and no others. That there are exactly these two non-spatial dimensions — a scale and a phase — is not a stipulation of this paper but a *theorem* of the bulk-side companion. Locality and causality (AX-1) force any interacting, charged quantum theory to carry a renormalization scale and a local gauge phase, and geometry–gauge equivalence (AX-2) obliges each to become a coordinate of the manifold; the derivation is Theorem 0 (T-GG6-0) of (Arisaka, 2026, A2-GG6). The scale dimension and the gauge dimension are therefore inevitable consequences of the two axioms, and their non-spatial character — a resolution and an angle, not two further rooms of space

— is a conclusion of that theorem, not an assumption of this one.

17 The Scale Dimension: Renormalization-Group Time

The fifth coordinate r of GG-6 is the renormalization-group flow parameter of the boundary four-dimensional theory,

$$r = \ln\left(\frac{E_{\mathrm{GUT}}}{\mu}\right), \tag{38}$$

with μ the sliding energy scale and E_{GUT} the ultraviolet anchor. Motion along r is motion in *energy scale*, not in space: to move "inward" along I_r is to integrate out high-energy modes and flow toward the infrared. This identification is forced by POS-5 (scale covariance) together with POS-6 (Occam gauge normalization), which fix the natural parametrization of the radial direction to be the logarithm of the scale ratio.

Two features of r follow at once and matter for the rest of the paper. First, the scale profile multiplying the four-dimensional metric is e^{-r}, an *exponential* in the coordinate; this is not an arbitrary functional choice but the unique form consistent with a multiplicative (logarithmic) flow, and it is the reason the natural mathematical constant of the fifth dimension is Euler's number e, the base of the log-scaling — a point we develop when we turn to e and π below. Second, the infrared endpoint of I_r at $r = K = 35$ is not a *location* in any space; it is the energy scale $\mu = E_{\mathrm{GUT}}\, e^{-K}$, the bottom of the renormalization-group flow. The vast Planck-to-electroweak hierarchy is, in this reading, not a fine-tuned ratio of two lengths but the discrete integer $K = 35$ measuring the RG-scale separation between the ultraviolet and infrared ends of the flow.

The renormalization group has been part of physics since Stueckelberg–Petermann, Gell-Mann–Low, and Wilson; what is new in GG-6 is the recognition that the RG flow is not merely a calculational device but a *coordinate of the bulk manifold*, on the same metric footing as the four directions of spacetime.

The hierarchy as a hierarchy of length units. This reading gives the scale profile a precise meaning: e^{-r} is the conversion between the dimensionless four-dimensional coordinates x^μ and physical length at the scale $\mu = E_{\mathrm{GUT}}\, e^{-r}$, the natural unit of length there being the Compton wavelength $\ell_{\mathrm{phys}}(r) = \hbar c/\mu$. What looks, in the metric, like the four-dimensional part "shrinking" toward the infrared is therefore not 4D physics being suppressed but the natural *unit* of 4D length growing exponentially: an infrared-boundary observer measures in units some $e^K \approx 10^{15}$ times larger than an ultraviolet-boundary observer's. The Planck–electroweak hierarchy is, in this light, a hierarchy of *length units* rather than a fine-tuned ratio — a geometric statement, fixed by the single integer $K = 35$, not a coincidence to be explained.

18 The Gauge Dimension: A Topological Phase

The sixth coordinate $\theta \in [0, 2\pi)$ is a *topological* circle S^1_θ, not a Riemannian circle of intrinsic radius. The distinction is sharp and consequential. A Riemannian circle carries a continuous length scale R_θ (its radius), which would be precisely the sort of free continuous parameter that AX-4 (maximum symmetry, minimum freedom) forbids. A topological circle carries no such scale: it supports only the integer-valued data that single-valuedness demands — the Wilson-line holonomy $\oint A_\theta \, d\theta$ and the integer Fourier mode numbers $n \in \mathbb{Z}$ of fields expanded around it.

This is why $\hbar$ enters GG-6 as it does. The momentum carried by the nth Kaluza–Klein mode on S^1_θ is $p_\theta = n\hbar$, with $\hbar$ the unit-conversion factor between the dimensionless integer mode number and a physical momentum; the action accumulated by one quantum traversing the circle once is $h = 2\pi\hbar$, a topological invariant independent of any radius. The conventional "radius" R_θ is, in this picture, a unit-conversion artifact that appears only if one insists on measuring the pure angle θ in units of length; in natural GG-6 units it is set to unity. The natural mathematical constant of the sixth dimension is therefore π, the period of the topological rotation, just as e is the natural constant of the fifth.

The sixth dimension as a topological reservoir. Because S^1_θ is scale-flat, the Wilson loop $W = \mathrm{tr}\, \mathrm{P} \exp(\mathrm{i} \oint A_\theta \, d\theta)$ is the same topological invariant at every radial position, and it is what drives the Hosotani breaking of the bulk gauge group to the Standard Model — the pattern governed by the OGN-5 integer $J = 7$, with no geometric radius entering. Integer winding around the circle survives every scale running and every infrared localization; the sixth dimension is in this sense the principal *topological reservoir* of the integer-discrete physics of the framework, making the OGN-5 integers visible at the boundary as Kaluza–Klein masses (proportional to $|n|$), Hosotani breaking patterns (governed by J), and the arithmetic of the Standard-Model parameters.

19 Geometric Intuition: How the (4+2) Bulk Encodes Physics

The canonical metric (29),

$$ds_6^2 = e^{-2r} \, \eta_{\mu\nu} \, dx^\mu dx^\nu + dr^2 + d\theta^2, \qquad r \in [0, K], \qquad \theta \in [0, 2\pi), \tag{39}$$

is compact, and its physical content is not obvious at first reading. This section is devoted to building the intuition that the metric encodes, working from a small set of observations toward two further consequences of the same metric — the geometrization of renormalization and the geometric weakness of gravity — now that the fifth and sixth dimensions have been identified (Sections 17–18). This section is the heart of the six-dimensional AdS picture and the core argument that distinguishes GG-Theory from conventional Kaluza–Klein and string-theory frameworks. It is written for a broad audience: physicists, chemists, biologists, and engineers; readers familiar with calculus and willing to follow geometric arguments will find every step explicit.

19.1 The puzzle: what does the metric mean at the IR boundary?

A reader of the canonical metric may notice an apparent puzzle. At the UV boundary $r = 0$, the scale profile $e^{-2r} = 1$, and the three pieces of the metric — the four-dimensional Lorentzian piece $\eta_{\mu\nu} \, dx^\mu dx^\nu$, the radial piece dr^2, and the angular piece $d\theta^2$ — sit on equal footing. As r increases toward the IR boundary $r = K = 35$, the four-dimensional conformal factor shrinks dramatically:

at $r = K$, the suppression is $e^{-2K} = e^{-70} \approx 4 \times 10^{-31}$.

So at the IR boundary, in any infinitesimal step, the contribution of the four-dimensional motion to the bulk arc length ds_6 becomes vanishingly small compared to the contributions of the radial and angular motion. An IR-boundary physicist taking a small step in x^μ is, in the bulk geometry, moving an essentially negligible distance. A small step in r or in θ contributes much more to the bulk arc length. Relatively speaking, the radial and angular directions become completely dominant at the IR boundary, while the four-dimensional spacetime appears to have shrunk to nothing.

This is not a calculational artifact. It is the actual geometry of the bulk. And it raises the question: what does this mean physically? How is it that the four-dimensional spacetime where we observe physics is, from the bulk's perspective, dramatically suppressed? How can this geometry account for the observed running of gauge coupling constants, for the weakness of gravity compared to the gauge forces, and for the topological role of the angular direction? These are the questions this section answers.

19.2 Renormalization as radial integration

Once the radial direction is recognized as RG-scale, the running of the Standard Model gauge couplings becomes a geometric statement. Specifically, the running coupling $\alpha_a(\mu)$ of any gauge factor $a \in \{\mathrm{SU}(3), \mathrm{SU}(2), \mathrm{U}(1)_{\mathrm{EM}}\}$ at boundary energy scale μ is, in the bulk picture, the value of an effective four-dimensional coupling at radial position $r = \ln(E_{\mathrm{GUT}}/\mu)$. The running of α_a with μ is the variation of this coupling with r.

The bulk gauge action. A bulk gauge field $A_M^{(a)}$ associated with the gauge factor a has the kinetic action

$$S_a = -\frac{1}{4g_6^2} \int_{\mathcal{M}_6} d^6x \sqrt{-g_6}\, F_{MN}^{(a)} F^{(a)\,MN}, \tag{40}$$

where g_6 is the bulk gauge coupling, and the field strength $F_{MN}^{(a)}$ is built from the bulk gauge potential $A_M^{(a)}$. Substituting the canonical $\mathsf{GG\text{-}6}$ metric, integrating over the angular direction θ (which gives a factor of 2π from the topology of S^1_θ), and isolating the four-dimensional kinetic term gives an effective four-dimensional action

$$S_a^{(4)} = -\frac{1}{4g_4^2(r)} \int_{\mathcal{M}_4} d^4x\, F_{\mu\nu}^{(a)}(x,r)\, F^{(a)\,\mu\nu}(x,r), \tag{41}$$

where the effective four-dimensional coupling at radial position r is given by the radial integral

$$\frac{1}{g_4^2(r)} = \frac{2\pi}{g_6^2} \int_0^r dr'\, \chi_a(r'), \tag{42}$$

with $\chi_a(r')$ a profile function that depends on the bulk dimensional reduction details and on whether the gauge field is localized to a boundary, lives in the bulk, or has a profile chosen by the equations of motion.

The geometric reading of the β-function. At leading order in the bulk expansion, $\chi_a(r')$ is approximately constant in r', and (42) reduces to a linear dependence

$$\frac{1}{g_4^2(r)} = \frac{2\pi}{g_6^2}\, r \cdot \chi_a + (\text{subleading in } r), \tag{43}$$

which is exactly the leading-order one-loop β-function structure of the boundary 4D theory: $1/g_4^2(\mu)$ depends linearly on $\ln \mu$. The proportionality constant in front of r is the leading-order coefficient b_a of the boundary β-function, with sign and magnitude determined by the bulk profile of the gauge field. Higher-loop corrections to the boundary β-function correspond to subleading-in-r corrections to $\chi_a(r')$ from the bulk equations of motion — the loop expansion of conventional QFT is replaced by the radial expansion of the bulk integral.

The change of category here is worth stating plainly. In conventional 4D QFT, the running of a coupling is a quantum-mechanical phenomenon: it arises from loop corrections to vertex functions, integrated over momentum. In the bulk picture, the running is a classical-geometric phenomenon: the radial profile of the bulk gauge field, integrated along I_r. The two are related by the AdS/CFT dictionary (Gubser, Klebanov, and Polyakov, 1998, Maldacena, 1998, Witten, 1998): the loop expansion of a 4D theory is the bulk-derivative expansion of its 5D (or 6D) holographic dual. In GG-Theory, this dictionary is taken constructively: renormalization *is* the geometry, and the loops we compute in conventional QFT are the IR-boundary shadows of bulk integrals over the radial interval.

The OGN-discrete predictions. Within GG-Theory, the radial profile $\chi_a(r')$ is constrained by the OGN ledger. The integration over $I_r = [0, K]$ produces, at each scale, a value of $\alpha_a^{-1}(\mu)$ that is determined by the OGN integers and discrete topological data alone. Two specific predictions, derived in detail in Section 35, illustrate this:

- At the Z-pole energy $\mu = M_Z$, the inverse fine-structure constant is $\alpha^{-1}(M_Z) = 2^J = 128$ at leading order; the quantum-dressing refinement $-N^2/2^J$ of Arisaka (2026, A9-SM) gives $\alpha^{-1}(M_Z) = 127.9297$, meeting GG-A9-SM's benchmark 127.930(8) at a pull of -0.03.
- At the Thomson limit $\mu \to 0$, the inverse fine-structure constant has the integer skeleton $\alpha^{-1}(0) = 2^J + N^2 = 137$, reproducing CODATA 2022 (137.036) to 0.026%; the residual is the radiative correction supplied by Arisaka (2026, A9-SM).

These two values — separated by a factor of $\sim 10^{11}$ in energy scale — are predicted from the same OGN integers $(N, J) = (3, 7)$, with the difference between them arising from the radial integral over a different portion of I_r. The integers do not change; the integration endpoints do. This is the *geometrization of renormalization*: every value of every running coupling at every scale is a radial integral over the bulk, with the integers of the OGN ledger determining the integrand.

Consequence: no separate "loops" to compute. In conventional Standard Model practice, one computes loop corrections to gauge couplings using Feynman diagrams. In the bulk picture, these loops are the boundary shadow of the bulk profile. At every value of the boundary RG scale, the running coupling is given by an integral along the radial direction — a classical geometric quantity, not a quantum-mechanical loop sum. This is the deep conceptual content of the bulk–boundary correspondence in GG-6: renormalization is geometrized. Each "loop order" of the boundary calculation corresponds to a specific subleading correction in the bulk derivative expansion, and the entire running of the boundary couplings is reproduced by the radial integral over the radial interval, with the OGN integers encoding the discrete arithmetic content.

19.3 Why gravity is weak: a UV-boundary phenomenon

The third question — why gravity is so weak — has a geometric answer in the bulk picture, and it follows directly from the canonical metric.

The radial integral that gives G_N. Recall from Section 26.2 that the four-dimensional Newton constant arises from the radial integration of the bulk Einstein–Hilbert action. The relevant integral, in natural GG-6 units, is

$$\int_0^K dr\, e^{-2r} = \frac{1-e^{-2K}}{2} \approx \frac{1}{2}, \tag{44}$$

where the suppression factor $e^{-2K} = e^{-70} \approx 4 \times 10^{-31}$ is so small that the integral is essentially $1/2$, with corrections at the 10^{-31} level.

The integrand is concentrated near $r = 0$. The crucial feature of (44) is that the integrand e^{-2r} is essentially 1 at the UV boundary $r = 0$ and decays exponentially with characteristic "width" $\Delta r \sim 1/2$ as one moves toward the IR. By $r = 5$, the integrand has dropped to $e^{-10} \approx 5 \times 10^{-5}$; by $r = 10$, it is $e^{-20} \approx 2 \times 10^{-9}$; by $r = K = 35$, it is the cosmically small $e^{-70} \approx 4 \times 10^{-31}$. The integral is therefore concentrated in a narrow strip near the UV boundary, with essentially no contribution from anywhere else along the radial direction.

This is a precise geometric statement: *gravity is fundamentally a UV-boundary phenomenon*. The bulk graviton wave function is localized near $r = 0$ by the curvature of the warped AdS bulk (this is the Randall–Sundrum localization mechanism, made discrete in GG-6 by the Diophantine integer $K = 35$). The four-dimensional Newton constant is determined by integrating this localized wave function across the entire bulk; because the wave function is concentrated at $r = 0$, the integral picks up its contribution near the UV boundary.

We live near the infrared end of the interval, far from where gravity lives. The Standard Model fields, in the standard GG-6 phenomenology developed in Arisaka (2026, A9-SM), live near the infrared end of I_r, far down the scale axis from the ultraviolet boundary where the graviton is bound. This is where electroweak interactions, the Higgs mechanism, and observed particle physics take place. Two anchors have to be kept apart here, as Arisaka (2026, A2-GG6) sets out. The *boundary renormalization scale* is measured from the unification scale, $\mu(r) = E_{\rm GUT}\, e^{-r}$, so the infrared end of the interval sits at $E_{\rm GUT}\, e^{-K} \approx 14$ GeV. The *masses* of the Kaluza–Klein resonances that the same interval supports are measured instead from the bulk scale $M_6 = M_{\rm Pl}/\sqrt{\pi}$, which places the lock cluster at $M_{\rm lock} \approx 3.85$ TeV. The radial depth is the same ledger integer $K = 35$

in both readings; what differs is the scale each is measured from, and the two anchors stand in the fixed ratio $M_6/E_{\rm GUT} = e^{2\pi}/\sqrt{\pi} \approx 302$, itself a pure number with no free parameter in it. For the argument of this section only the separation matters: 35 RG-scale units along the scale axis, a factor $e^{-K} \approx 6.3 \times 10^{-16}$ in energy scale.

When two unit masses on the IR boundary interact gravitationally, the effective gravitational coupling they experience is not the bulk Newton constant $G_N \sim 1/M_6^2$ but the IR-localized version, which is suppressed by the scale profile:

$$\boxed{G_N^{\rm IR} \sim G_N \cdot e^{-2K} \approx G_N \cdot 4 \times 10^{-31}.} \tag{45}$$

This is the same suppression factor that characterizes the 4D part of the bulk metric at the IR boundary. It is the geometric origin of the apparent weakness of gravity at TeV-scale energies.

The number 10^{-31} as a geometric quantity. The empirical hierarchy between the gravitational coupling at TeV energies and the gauge couplings at the same energies is the dimensionless ratio $G_N\,\text{TeV}^2/\alpha$. Taking $G_N\,\text{TeV}^2 = (\text{TeV}/M_{\rm Pl})^2 = 6.7 \times 10^{-33}$ against $\alpha^{-1} \approx 128$ at that scale gives 8.6×10^{-31}, which agrees to within a factor of about two with $e^{-2K} = e^{-70} = 4.0 \times 10^{-31}$ for $K = 35$. This is the famous "hierarchy problem" of conventional particle physics: why is $G_N \cdot M_Z^2$ so small? In conventional formulations, this number is mysterious; in GG-6, it is the discrete topological quantity e^{-2K} where $K = 35$ is the Diophantine integer derived from the OGN ledger (Step 5 of the cascade in Section 23). The hierarchy is a theorem.

The contrast with gauge forces. Why are the gauge forces *not* similarly suppressed? Because the bulk gauge field profiles are not localized to the UV boundary. In a standard reduction, the gauge fields can have profiles that extend across the entire radial interval; their effective IR couplings are therefore set by the radial integral over the entire interval, not by a UV-localized integrand. The radial integral for the gauge coupling (42) is therefore $O(K)$ in magnitude, not $O(e^{-2K})$. The hierarchy between G_N and the gauge couplings at the IR boundary is the difference between a UV-localized integrand (gravity, suppressed by e^{-2K}) and a bulk-spread integrand (gauge fields, integrated to a value of order K).

This is the deep geometric reason why gravity is so weak compared to electromagnetism, the weak interaction, the strong interaction, baryon-number gauge force, and information gauge force at the IR boundary: gravity is a UV phenomenon observed from far down the scale axis, while gauge forces are bulk phenomena observed throughout the scale axis. No fine-tuning, no anthropic argument, no string-landscape selection: just the discrete geometry of a (4+2) bulk with a UV-localized graviton.

The volcano-potential picture. The mathematical mechanism by which the graviton localizes to the UV boundary is the "volcano potential" first identified by Randall and Sundrum (Randall and Sundrum, 1999b). Solving the linearized Einstein equations on the bulk reduces to a Schrödinger-like equation in r with an effective potential that has a sharp negative spike at $r = 0$ (the volcano's crater) and rises smoothly elsewhere. The graviton zero-mode wave function is the bound state of this potential, exponentially localized at $r = 0$ with characteristic localization length ~ 1 in natural GG-6 units. Higher-mode KK gravitons are not bound; they have continuous spectrum and contribute negligibly at the IR boundary. The "ordinary" gravity we observe is therefore the IR-boundary shadow of a UV-localized zero-mode graviton.

19.4 One geometry, read four ways

The four readings now in hand — the radial direction as renormalization-group time (Section 17), the running of couplings as radial integration, the weakness of gravity as a UV-boundary phenomenon, and the angular direction as a topological reservoir (Section 18) — are not four facts but one geometry seen from four sides. The four-dimensional spacetime is not "suppressed" at the infrared boundary; it is read at a different unit of length. The radial direction is not space but the axis along which energy scales are organized; the angular direction is not a small circle but a dimensionless topological one that supplies integer arithmetic at every scale; and the scale profile e^{-r} is the single conversion device that ties them together, with the OGN-5 integer $K = 35$ fixing the scale depth. All of it is carried by the one canonical metric (29), with no free continuous parameter.

19.5 An analogy from optics

For readers from outside physics, an analogy may help. Consider an old-fashioned brass telescope of variable focal length, whose optical system is designed so that what is in focus at one end of the telescope is at one specific magnification, and what is in focus at the other end is at a vastly different magnification. Looking through the eyepiece on the IR end, an observer sees a sharply-focused image of distant objects at high magnification: a sparrow in a tree appears the size of a sparrow. The same telescope, looked through from the UV end, would show the same sparrow as a barely-resolvable speck.

The brass tube of the telescope itself is the bulk; the IR eyepiece corresponds to the IR boundary at $r = K = 35$, and the UV eyepiece corresponds to the UV boundary at $r = 0$. The exponential scale profile e^{-r} is the magnification factor that converts between the angular size of an object at one end of the telescope and its angular size at the other end. An ant on the IR-eyepiece's lens, walking one millimeter, appears to a UV-eyepiece observer to walk a kilometer; an ant on the UV-eyepiece's lens, walking one millimeter, appears to an IR-eyepiece observer to walk an angstrom.

The bulk of GG-6 is a similar magnifying optical system, but the magnification is $e^{-K} = e^{-35} \approx 10^{-15}$, and what is being magnified is not visible-light images but the universal unit of length. An IR-boundary physicist's meter-stick, viewed from the UV boundary, looks like an enormous object of size $e^{K} \sim 10^{15}$ m. A UV-boundary particle, viewed from the IR boundary, looks like a $\sim 10^{-15}$ TeV object: an ordinary TeV particle. The brass tube of the telescope — the scale profile e^{-r} — is what mediates between these two perspectives.

The compact direction S^1_θ is, in this analogy, the diaphragm of the telescope: a topological aperture that does not change in size as one moves between the UV and IR ends. It supplies discrete iris-blade settings (the integer mode quantization, the Wilson-line phases) that are visible at every magnification because they are topological. The OGN integer $K = 35$ is the length of the brass tube measured in units of the natural focal length: a discrete number, not a continuously adjustable knob.

The analogy is imperfect, as all physical analogies are, but it captures the essential point: the bulk is a magnifying device that converts between scales while preserving topological content. Energy scales, length units, and effective couplings are all magnified differently at the two ends of the device; integer winding numbers, mode quantization, and Wilson-line phases survive unchanged. This is what makes the framework simultaneously rich enough to encode the Standard Model running couplings and the Planck–electroweak hierarchy and parsimonious enough to satisfy the MSMF principle.

19.6 An analogy from the optical microscope

For a reader whose daily instrument is the microscope rather than the telescope — a cell biologist, a biochemist, a structural biologist — the same lesson arrives even more directly, and through an instrument that turns out to be far more than a metaphor. A light microscope zooms into a living cell and resolves its organelles; a shorter probe wavelength, in ultraviolet and then X-ray microscopy, resolves viruses and the larger macromolecular machines; an electron microscope, shorter still, resolves single atoms in a protein. Each rung of this ladder trades a longer probe wavelength for a shorter one, for the elementary reason that resolution is set by wavelength through Abbe's diffraction limit, $d \approx \lambda/(2\,\mathrm{NA})$. A particle accelerator is precisely this same instrument carried to its physical limit. By de Broglie's relation $\lambda = h/p$, a beam of tera-electron-volt momentum is a microscope of wavelength $\sim 10^{-19}$ m — fifteen orders of magnitude finer than the light microscope, with the X-ray microscope sitting partway up the very same ladder. The Large Hadron Collider is, in the most literal sense, the most powerful microscope humanity has ever built. The question this section asks is simply: *what do we see when we turn that microscope to its highest power?*

What the deepest zoom reveals. Follow the magnification downward — cell, macromolecule, atom, nucleus, proton — and at each stage the image resolves into smaller spatial parts: organelles into molecules, molecules into atoms, atoms into nuclei and electrons, the proton into quarks and gluons. Then, at the electron and the quarks, something remarkable happens: the zoom stops finding parts. To the resolution of the highest-energy accelerators the electron has no measurable size, no shape, and no internal spatial pattern of any kind; experimentally it is point-like below $\sim 10^{-18}$ m, with no form factor and no excited states. What distinguishes one elementary particle from another is then not a geometry but a short list of *labels*: a mass, a spin, an electric charge, a weak isospin and hypercharge, a baryon or lepton number. At the bottom of the zoom we do not find a smaller machine or a vibrating thread. We find a point that carries numbers.

We should state at once what this does and does not establish. It does not, by itself, refute the hypothesis that the electron is "really" an extended object — a vibrating string, say — too small to resolve: the string scale is conventionally placed near the Planck energy, far above any collider, so the failure to see extension at the Large Hadron Collider is consistent both with there being no string and with there being a string we cannot yet resolve. The empirical pattern is therefore *motivation*, not proof. The decisive argument is one of principle, and it turns on the one label in the list that cannot be a shape: electric charge.

Why charge cannot be a shape. Consider the extended-object picture on its own most favorable terms. A minute vibrating object can, at least in principle, encode a particle's energy and mass: a taut string carries energy in its modes of vibration, and indeed in string theory the squared mass of a state is fixed by its oscillator level, $M^2 \propto (N-a)/\alpha'$. Vibration and mass are a natural pair. But electric charge is a quantity of an entirely different kind. In any gauge theory the electric charge is the eigenvalue of the generator of an internal $U(1)$ symmetry — an exactly quantized label carried in the internal fiber over a spacetime point, not a quantity that can be spread, painted, or distributed along a length. There is no vibrational profile that *is* one unit of charge. Charge is far better pictured as the *color* of a point than as the *shape* of an object: an internal attribute borne at the point, in the language of this paper a position on the gauge circle S^1_θ, and not a distribution in the space we can photograph.

It is worth emphasizing that string theory itself concedes exactly this. In string constructions the electric charge of a state is not read off its vibrational shape; it is supplied separately — as Chan–Paton factors living at the open string's *endpoints*, as data of the brane on which the string ends, or as quantized momentum and winding on an internal manifold. Vibration gives the mass; the charge is re-localized to a boundary. Even the canonical extended theory, in other words, does not make charge a shape. The popular image of "zooming in on the electron and discovering a tiny vibrating loop that carries its charge" is misleading on the extended theory's own terms.

Locality forces the charge onto a point. There is, moreover, a reason of principle — independent of any resolution limit — that an elementary charge must act at a point. Suppose the charged object had a finite spatial width L. To respond *coherently* to an external field, its separated parts would first have to agree on the response, and a signal crossing the object takes a finite time of order L/c. But the electromagnetic interaction we actually observe is local and instantaneous in the precise sense encoded by the minimal coupling $-e\,\bar{\psi}\gamma^\mu\psi\,A_\mu$: every field in the interaction is evaluated at one and the same spacetime point, and microcausality requires that the fundamental fields be local operators that commute at spacelike separation. An exactly local, pointwise gauge coupling drives the width L to zero. This is Local Causality, the first axiom AX-1 (LC), made concrete at the level of a single charged particle: *exact local gauge coupling implies that the charge acts at a point.*

The honest form of the conclusion is not that an extended theory is "acausal" — string theory is constructed to be unitary and causal in its own right, and one should not claim otherwise. The defensible and, in fact, stronger statement is this: *no theory can represent electric charge as a spatial shape while keeping its gauge coupling local and instantaneous*, which is exactly why every consistent construction, string theory included, localizes charge at a point or a boundary. The microscope, turned to its highest power, will therefore never resolve a charge into a charged shape — not because our lenses fail, but because charge is not that kind of thing.

The point's genuine difficulty: which way to move? Granting all of this returns us, however, to a puzzle with a long and underappreciated history, and it is the heart of the matter. If the electron is a structureless point, with what does it register the *direction* of the field in which it sits? A point samples the field at a single location; it has no breadth across which to compare, no internal lever with which to orient. Classically the Lorentz force $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ names a direction, but one must ask what, *in the particle itself*, reads that direction and turns it into motion.

The answer that quantum electrodynamics gives is subtle and, on inspection, incomplete. The directional law is carried not by the local force but by a *connection* — the gauge potential A_μ — acting through the covariant derivative $D_\mu = \partial_\mu - i\frac{q}{\hbar}A_\mu$. A connection is, by construction, the rule for comparing the particle's internal phase at *infinitesimally neighboring* points; it carries a spacetime index μ precisely because it lives on the displacement dx^μ. The physical primacy of this connection — over and above the local force — is demonstrated by the Aharonov–Bohm effect, in which an electron's motion and interference are governed by the line integral $\oint A_\mu \, dx^\mu$ even in a region where the local field $\mathbf{E} = \mathbf{B} = 0$. The direction in which an electron moves is, at bottom, set by the parallel transport of an internal phase along a connection. The structureless point "glances infinitesimally aside" to find its bearings — and that glance is exactly the covariant derivative.

This is where quantum electrodynamics, for all its precision, leaves a conceptual gap. The connection does the work, but the internal phase it transports lives in a fiber that, in ordinary four-dimensional electrodynamics, is an abstract internal index *appended to* the spacetime point by hand. The law of motion is correct; the directional information has no geometric home inside the particle. The electron is told which way to go by an external field whose internal arena is left unspecified.

Direction of motion is geometry: the gravitational precedent. The way to close this gap is already familiar from gravity, and naming the parallel is the decisive step. General relativity taught us that a planet is not *pushed* along its orbit by a force reaching across space; it follows a geodesic of spacetime geometry. Gravitation *is* the geometry, and the geometry is what tells matter how to move. The geometric, fiber-bundle formulation of gauge theory — whose lineage runs from Weyl's 1918 gauge idea (Weyl, 1918) through the compact extra dimension of Kaluza and Klein (Kaluza, 1921, Klein, 1926) — says that electromagnetism is the same kind of thing: the direction of a charged particle's motion is geodesic motion in a geometry that includes an internal fiber, and the gauge potential A_μ is the connection of that geometry. The deep dynamical truth is just this: *a connection tells matter how to move, and the direction of motion is read off a geometry rather than imposed by a force* — in the gauge sector exactly as in the gravitational one.

How GG-6 gives the connection a home. GG-Theory supplies precisely the geometric home that four-dimensional electrodynamics leaves missing, and it does so without surrendering the locality that forced the charge onto a point in the first place. By the Geometry–Gauge Equivalence Principle GGEP (AX-2), the gauge phase is not an abstract internal index but a *genuine, non-spatial coordinate* of the manifold — the sixth dimension, the topological circle S^1_θ of Section 18, on which the $U(1)$ gauge connection A_μ lives as an integer-quantized Wilson-line holonomy — a genuine geometric object on the bulk, not (as in textbook Kaluza–Klein) an off-diagonal component of the metric, which in GG-6 stays a block-diagonal product geometry. With the gauge phase promoted to a dimension, the connection becomes geometry, and the Kaluza–Klein mechanism delivers its classic result, here reinterpreted with a circle that is gauge phase rather than space: a charged particle follows a *geodesic of the six-dimensional bulk*; its conserved momentum around S^1_θ *is* its electric charge; and the projection of that geodesic into four dimensions is exactly the Lorentz-force trajectory, its direction included, determined locally and instantaneously at the point. The electron now carries its own compass: the local $U(1)$ gauge invariance on the sixth dimension is the internal degree of freedom that fixes the direction of its motion. And because S^1_θ is non-spatial — a phase, not a place (Sections 18 and 21) — this internal structure adds not one new spatial degree of freedom, and Local Causality AX-1 is preserved exactly. The electron knows which way to go for the very same reason a planet does: it follows the geometry.

The same statement, quantitatively. The construction just described — and drawn in Figure 7 — can be written in three short equations. The bulk metric is the canonical block-diagonal product geometry, with no graviphoton term,

$$ds_6^2 = e^{-2r}\, g_{\mu\nu}\, dx^\mu dx^\nu + dr^2 + d\theta^2, \tag{46}$$

and the gauge field A_μ is *not* a component of this metric but the $U(1)$ connection whose holonomy lives on S^1_θ — this is GGEP in its connection form. A charged point particle couples to it minimally, $\mathcal{L} = \frac{1}{2} m\, g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu + qA_\mu \dot{x}^\mu$; its electric charge is the conserved momentum of the internal gauge phase, integer-quantized because the circle is compact,

$$p_\theta = n\hbar \implies q = ne, \qquad n \in \mathbb{Z}, \tag{47}$$

with e fixed by $\alpha^{-1} = 137$ — no circle radius and no extra coupling. The Euler–Lagrange equations then give the Lorentz force,

$$\boxed{m\Big(\ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\, \dot{x}^\nu \dot{x}^\lambda\Big) = q\, F^\mu{}_\nu\, \dot{x}^\nu, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.} \tag{48}$$

Equation (47) is the precise content of the figure's identification of the winding phase "hand" with the electric charge; and since the same integer momentum $p_\theta = n\hbar$ also defines $\hbar$ as the action quantum of S^1_θ (Section 26.3), Planck's constant and the electric charge are two faces of one momentum on the sixth dimension. Equation (48) is the projection π drawn in Figure 7: the direction of the electron's motion is read off the geometry, exactly as a planet's is. The full derivation — the Euler–Lagrange reduction giving (48), the identification of the magnetic force with the curvature of the gauge connection on S^1_θ, and the way the non-spatial circle removes the unwanted Kaluza–Klein mass tower — is carried out in Arisaka (2026, A5-GDOS) (§52).

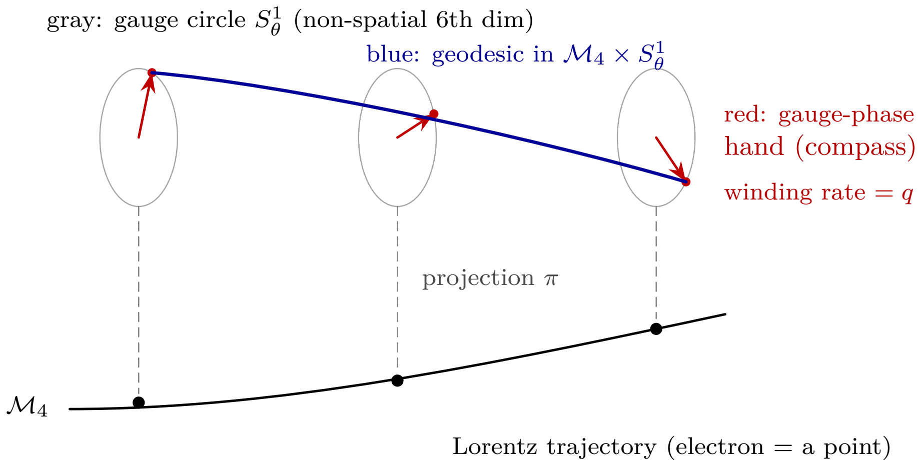

Figure 7: How a structureless point electron acquires a definite direction of motion in GG-6. At each instant the electron is a single *point* of four-dimensional spacetime $\mathcal{M}_4$ (black dots, lower curve). By GGEP (AX-2) that point carries a genuine internal coordinate: the non-spatial gauge-phase circle S^1_θ, the sixth dimension (gray loops). The gauge connection A_μ prescribes how the internal phase "hand" (red) is parallel-transported as the electron advances; the rate of that winding is the electric charge q. The motion is a geodesic of the bundle $\mathcal{M}_4 \times S^1_\theta$ (blue), and its projection π to spacetime is exactly the Lorentz-force trajectory — direction and all — fixed locally and instantaneously at the point. The electron's compass is thus internal and geometric; because S^1_θ is phase rather than place, no spatial extension is introduced and Local Causality (AX-1) is preserved (cf. Sec. 18). This is the gauge twin of the gravitational statement that a planet follows a geodesic of spacetime.

Three readings of a single electron. The three frameworks may now be set side by side, and the contrast is sharp. *In quantum electrodynamics*, the point charge moves correctly and locally, but the connection that sets its direction is an external field appended to a structureless point; the directional law works while its geometric home is left unspecified. *In the extended, string-as-shape picture*, the electron's charge is asked to be a spatial profile — which a local, causal gauge coupling forbids, and which even string theory declines to attempt, re-localizing charge at endpoints and branes while vibration supplies only the mass. *In GG-Theory*, the particle remains a point in four-dimensional spacetime, so locality and causality are exact, while the fifth and sixth dimensions under GGEP give the connection a genuine geometric home; the direction of the electron's motion becomes geodesic motion in the bulk — the gauge twin of gravitational free-fall.

Only the third reading closes the circle: a point particle, exact local causality, and a definite, internally determined direction of motion, all at once. This is the same unification the paper has pursued throughout, now read at the level of a single electron: as the speed of light c made spacetime geometric and Newton's constant G_N made gravitation geometric, the gauge phase on S^1_θ makes electromagnetism geometric, and the long question of what tells a charge which way to move is answered in the only way physics has ever ultimately answered such a question — by handing it to geometry.

What the microscope finally shows. The microscope analogy, then, ends not in failure but in a discovery about the nature of the limit. We zoom in expecting that at sufficient magnification the electron will resolve into smaller spatial parts, as the cell resolved into molecules and the atom into nucleus and electrons. It never does. At the bottom of the magnification we find a point that carries, in two dimensions we can never photograph — because they are made of scale and phase rather than of space — the gauge geometry that tells it how to move. The microscope reaches its limit not because our lenses are imperfect but because the last structure of matter is not spatial at all. That, in the end, is the same lesson the telescope taught in Section 19.5 from the other direction: the bulk magnifies and de-magnifies scale while preserving a topological content that no magnification can resolve into pieces, because that content is the non-spatial geometry of I_r and S^1_θ itself.

19.7 Why this picture is the heart of GG-Theory

The geometric intuition developed in this section is not a derivation but a reading. The derivations themselves — the canonical metric, the running couplings, the radial integral for G_N, the integer mode quantization on S^1_θ — are spread across Sections 15, 26.2, 26.3, and 35. The purpose of the present section is to assemble these scattered derivations into a single coherent picture that an interested reader, perhaps not a particle theorist by training, can carry away as the conceptual core of what GG-Theory claims.

That conceptual core is this: the universe is a six-dimensional bulk in which the four-dimensional spacetime we observe is one slice, the energy scale is a radial direction, the topological data is an angular circle, and the Standard Model couplings, the weakness of gravity, the running with scale, the integer ledger of OGN, the Hosotani breaking, the integer KK spectrum, and the Planck–electroweak hierarchy are all geometric features of a single bulk object. No fine-tuning, no free dimensionless parameters — only the unit system set by the co-equal trio $\{c, G_N, \hbar\}$, from which the overall scale and M_6 alike are read — and no string-landscape selection. The entire empirical content of physics — including the running couplings (the integer skeleton $\alpha^{-1} = 137$ and 128), the chemistry that follows from the Rydberg energy, the biology that follows from the Arisaka constant $\mathsf{Q_B}$, and the cosmology that follows from $\Omega_m = 1/\pi$ — is encoded in the same six-dimensional bulk metric, as the radial and angular signatures of an integer-discrete topological structure.

At the level of a single particle's motion, this unity has a one-line expression. The four-dimensional projection of a geodesic of the six-dimensional bulk is

$$\boxed{m\Big(\ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\,\dot{x}^\nu\dot{x}^\lambda\Big) = q\,F^\mu{}_\nu\,\dot{x}^\nu,} \tag{49}$$

in which gravitation and electromagnetism are the two terms of one geodesic: the Christoffel term on the left is the curvature of the *metric* — gravity, carried by the fifth dimension and measured by G_N — and the field-strength term on the right is the curvature of the *gauge connection* — electromagnetism, carried by the sixth dimension. Free fall ($q = 0$) is gravity alone; a charged particle feels, in addition, the twist of the gauge circle. Newton's $F = ma$ and the Lorentz force are thus the same equation read on the two sectors of one six-dimensional metric; the two parallel derivation chains — GG-6 → 5D GR → Newton, and GG-6 → 4D QED → Lorentz — are set out in full in Arisaka (2026, A5-GDOS) (§52). This is the precise sense in which the six-dimensional picture is a single object rather than a catalog: not two forces glued together, but one geodesic with two kinds of curvature.

This is the modern realization of the Euclidean methodological condition: a deductive science in which the universe's empirical content follows from a small set of constitutional clauses (4 axioms, 7 postulates, plus the bulk topology) by integration over a (4+2) AdS bulk, with no free parameter beyond the absolute minimum demanded by AX-4 (MSMF). The journey from Euclid to GG-Theory is, in this reading, the gradual recognition that the empirical hierarchies of physics — mass scales, coupling strengths, running constants, topological integers — are signatures of a higher-dimensional bulk geometry whose form is forced by the Constitution, and whose discrete content is forced by the modified Bianchi identity that makes that geometry globally consistent.

The remainder of this paper turns to the technical derivations that make this picture rigorous: the modified Bianchi identity BI-6 in Section 22, the OGN-5 integer theorem in Section 23, and the geometric origin of the universal constants c, G_N, $\hbar$, k_B, $\mathsf{Q_B}$ in Sections 26.1–28.

19.8 Why causality, locality, and point-like particles survive

Because neither extra dimension is space-like, GG-6 does not introduce the pathologies that space-like extra dimensions risk. There are no new space-like geodesics along which signals could travel; there is no extra spatial direction in which a particle could be displaced; and there is therefore no mechanism for faster-than-light propagation or for the violation of four-dimensional microcausality. The boundary $\mathcal{M}_4 = \partial\mathcal{M}_6$ retains its genuine $(1,3)$ Lorentzian signature, and AX-1 (operational locality and causality) holds there exactly as in ordinary relativistic field theory.

Equally, particles remain strictly point-like. The bulk is not a space that particles move through; it is the carrier of scale (I_r) and phase (S^1_θ) data attached to point particles living on the boundary. A field on $\mathcal{M}_6$ is, from the boundary perspective, a four-dimensional field together with its renormalization-group profile (its dependence on r) and its gauge-phase content (its Fourier expansion on S^1_θ). Nothing in this construction enlarges the spatial support of a particle or endows it with extended, string-like structure. In this precise sense GG-6 is a theory of point particles equipped with a geometrized renormalization group and a geometrized gauge phase — not a theory of objects propagating through a higher-dimensional space.

20 Related Work and Precursors: What GG-6 Inherits, and Where It Departs

The two non-space-like dimensions of this Part, and the operational measurement principle OGGEP on which the boundary projection of GDOS rests, did not arise in a vacuum. Each ingredient of the construction was anticipated, in part, by a substantial body of work, and intellectual honesty — together with the Euclidean discipline of stating one's debts in plain view — requires that we name those precursors precisely and say exactly where GG-6 departs from each. We do so in this section, organized by strand. The reader should come away with a clear picture: the *individual* moves of GG-6 each have honorable ancestry; what is, to the best of our knowledge, without precedent is the *simultaneous and constitutional* combination of all of them into a single forced framework. We make our novelty claim only at the level of the combination, and we credit the parts generously. A framework that asks to be read as a deductive science earns no license to be careless about its own genealogy.

20.1 The fifth dimension as renormalization-group scale

The idea that an extra dimension can carry the renormalization-group (RG) scale of a four-dimensional theory is, of the ingredients of GG-6, the one with the deepest and most fully developed prior literature, and we wish to be especially careful to credit it.

The lineage begins, arguably, with Hermann Weyl (1918), whose 1918 attempt to unify gravitation and electromagnetism introduced the very word *gauge*: Weyl gauged the local choice of length, making *scale* a local geometric quantity. Weyl's theory failed as a theory of electromagnetism — Einstein's "measuring-rod" objection was decisive — but the conceptual seed, that scale is a gauge degree of freedom attached to each point of spacetime, is the ancestor of every later attempt to geometrize the renormalization group. The renormalization group itself was given its modern form by Wilson and Kogut (1974) and, in its exact functional form, by Polchinski (1984), in which the flow of couplings with scale is read as evolution in $\ln\mu$; GG-6 takes precisely this $\ln\mu$ as the coordinate of its radial direction, Eq. (38).

The decisive geometric step was taken within the AdS/CFT correspondence of Maldacena (1998), Gubser, Klebanov, and Polyakov (1998), and Witten (1998). There the radial coordinate of an anti-de Sitter bulk is identified, under the holographic dictionary, with the energy scale of the boundary conformal field theory, and the bulk gravitational equations in the radial direction reproduce the Callan–Symanzik flow.

This *holographic renormalization group* was made systematic by de Boer, Verlinde, and Verlinde (2000) through a Hamilton–Jacobi formulation, and refined into a "holographic Wilsonian" RG — in which integrating out the bulk between two radial slices is integrating out boundary modes between two scales — by Heemskerk and Polchinski (2011) and Faulkner, Liu, and Rangamani (2011). Lee (2014) showed, conversely, that performing a quantum RG on a generic field theory *generates* a holographic bulk dimension.

Most radically, the entanglement-renormalization program of Swingle (2012) and the entanglement-geometry program of Van Raamsdonk (2010) argued that the holographic radial direction is not a postulated spatial dimension at all but an *emergent* one, built out of the scale-by-scale entanglement structure of the boundary state: the extra dimension, on this reading, is *made of* renormalization scale. In a discrete and complementary register, the "dimensional deconstruction" of Arkani-Hamed, Cohen, and Georgi (2001) built an extra dimension out of a quiver of gauge groups linked by RG-like steps.

We could not have asked for closer intellectual company, and we claim no priority for the bare idea that an extra dimension can be the energy scale. The departure of GG-6 is one of *axiomatic status and interpretation*, and it is sharp. In every construction above, the radial direction is, operationally, a *spatial* coordinate of an AdS bulk (or an emergent geometric direction) that is *identified* with scale by a dictionary, after the fact; particles propagate in it, KK modes live along it, and the identification with μ is a derived statement about the dual. In GG-6, the radial direction *is* the RG-scale $r = \ln(E_{\mathrm{GUT}}/\mu)$ by constitutional postulate (POS-5, scale covariance), and it is *not* a spatial direction at all: nothing propagates along r as it would along a length, there is no Kaluza–Klein tower associated with r, and the scale profile e^{-r} is the forced consequence of the logarithmic parametrization rather than a tunable AdS curvature. Where holographic RG *discovers* that the bulk radius behaves like a scale, GG-6 *declares* that the fifth dimension is the scale — and pays for the declaration by losing the freedom to tune it. This is a difference of constitutional commitment, not of decoration, and it is what allows the absence of a radial modulus (Sec. 21).

20.2 The sixth dimension as gauge holonomy

The second non-spatial direction, the topological circle S^1_θ carrying the gauge phase, has an equally distinguished ancestry. Its origin is the original unified field theory of Kaluza (1921) and Klein (1926), in which an extra compact circle generates a U(1) gauge symmetry as the off-diagonal component of a five-dimensional metric, with electric charge quantized as integer momentum around the circle.

The recognition that the gauge-invariant content of a gauge theory resides not in the connection but in its *holonomies* — the Wilson loops $\mathrm{P}\exp\oint A$ — is due to Wilson (1974), and the use of a non-trivial Wilson line wrapping a compact dimension to break gauge symmetry without a fundamental Higgs field is the mechanism of Hosotani (1983a,b), on which GG-6 relies directly for the SU(5) $\to$ Standard Model breaking. This circle of ideas matured into the *gauge–Higgs unification* program, in which the Higgs field is itself a component of a higher-dimensional gauge field, the Aharonov–Bohm phase of an extra dimension; Manton (1979) was an early six-dimensional incarnation.

In parallel, the recognition that geometric *phases* attach to each point of a parameter space rather than to positions in physical space — Berry's phase (Berry, 1984) and its non-Abelian generalization (Wilczek and Zee, 1984) — supplied the language in which a gauge holonomy is naturally a non-spatial, fiber-like object. At bottom, the fiber-bundle formulation of Yang–Mills theory already locates the gauge phase in an *internal* fiber over spacetime, not in spacetime itself.

Here too we claim no priority for the ingredients. The departure of GG-6 is the insistence that S^1_θ is a *topological* circle with no intrinsic radius: not a Riemannian circle of some length R_θ that happens to be small, but a bare U(1) phase direction supporting only integer winding and integer Fourier data. In the Kaluza–Klein and gauge–Higgs constructions, the circle is spatial: it has a physical size R_θ, particles acquire a Kaluza–Klein tower of masses $\sim n/R_\theta$, and R_θ is a continuous modulus to be stabilized. In GG-6, R_θ does not exist — it is a unit-conversion artifact (Sec. 18) — the only data are the topological winding numbers, and the integer that fixes the winding is forced by the OGN-5 ledger ($J = 7$). GG-6 thus takes the fiber-bundle intuition — that the gauge phase is internal, not spatial — with full seriousness and promotes it to a *metric* direction of the bulk that nonetheless carries no length. That combination, an internal gauge phase that is simultaneously a genuine coordinate of the manifold and yet has no radius, is the specific move we have not found in the prior literature.

20.3 Extra dimensions that are not space: internal and auxiliary directions

A reader may object that "dimensions that are not space" is hardly new, and the objection deserves a careful answer, because several deep programs have used exactly such directions. In Alain Connes' noncommutative geometry, the Standard Model is reconstructed from a product of ordinary spacetime with a *finite, discrete internal space* — a spectral triple — whose "points" and "distances" are algebraic rather than spatial; the gauge fields and the Higgs emerge as the inner fluctuations of this internal geometry (Connes and Lott, 1991, Chamseddine and Connes, 1997). This is a genuine and profound example of a non-spatial extra geometry carrying physical content, and we credit it fully. In a different register, the F-theory of Vafa (1996) introduces an auxiliary elliptic fiber whose modular parameter encodes the type-IIB axiodilaton; the two "extra dimensions" of the fiber are not directions in which strings propagate but a bookkeeping geometry for a varying coupling. And the entire apparatus of internal symmetry space in ordinary gauge theory is, of course, non-spatial by construction.

The departure of GG-6 is that its two extra dimensions are non-spatial *and* are bona fide coordinates of a single pseudo-Riemannian manifold $\mathcal{M}_6$ carrying components of one metric (Eq. (29)), *and* carry specific dynamical content — one the renormalization scale, the other the gauge phase — rather than a discrete algebraic structure (as in noncommutative geometry) or an auxiliary modular bookkeeping (as in F-theory). Connes' internal space is finite and discrete and does not carry a continuous scale direction; F-theory's fiber is auxiliary and its base remains a conventional compactification with moduli. GG-6 is, to our knowledge, the first framework in which the *two* extra directions are at once continuous metric coordinates, operationally non-spatial, and dynamically loaded with scale and phase, with the product geometry of the two reproducing the Planck–electroweak hierarchy and the gauge breaking simultaneously.

20.4 OGGEP: measurement as a constitutive principle

The operational measurement axiom OGGEP (AX-3) — that every observation is an open-system, completely-positive trace-preserving projection from the six-dimensional bulk to the four-dimensional boundary — likewise gathers together several mature lines of thought, and we are anxious to credit them.

That measurement requires a separate statement in quantum theory is as old as von Neumann's projection postulate (von Neumann, 1932); that the appearance of classical outcomes is the result of an open system's entanglement with its environment is the content of the decoherence program of Zeh (1970) and Zurek (2003). The mathematics of open-system evolution — the completely-positive trace-preserving (CPTP) maps and their generators — was given its definitive form by Gorini, Kossakowski, and Sudarshan (1976) and Lindblad (1976), and these GKLS generators are exactly the maps that GDOS employs. That measurement, or more generally the operations available to an agent, can be taken as a *primitive* from which the entire formalism of quantum theory is reconstructed is the achievement of the operational and informational reconstructions of Hardy (2001) and Chiribella, D'Ariano, and Perinotti (2011), and the relational reading of quantum states as always relative to an observing system is due to Rovelli (1996). Finally, the specific idea that a bulk gravitational theory is reconstructed from its boundary in an information-theoretic, error-correcting way — that the boundary encodes the bulk as a quantum error-correcting code, with bulk locality emerging from the code structure — is the holographic quantum-error-correction proposal of Almheiri, Dong, and Harlow (2015).

Each of these is a precursor of one facet of OGGEP, and none of them is OGGEP. The decoherence and open-system programs explain how classicality emerges *within* a fixed theory but do not make measurement constitutive of the theory's geometry. The operational reconstructions make measurement primitive but for quantum theory in the abstract, not for a unified geometric theory of the interactions. Holographic error correction makes the bulk-to-boundary map information-theoretic but within the fixed kinematics of AdS/CFT, not as a foundational axiom that *defines* what an observation is. OGGEP is the constitutional fusion of these: it elevates "an observation is an open-system bulk-to-boundary projection from $\mathcal{M}_6$ to $\mathcal{M}_4$" to one of the four axioms of the framework, on the same footing as locality and the geometry–gauge equivalence, so that the measurement theory is not added to the dynamics but is part of the same constitutional charter (Arisaka, 2026, A1-Constitution; Arisaka, 2026, A5-GDOS). It is this axiomatic status — measurement as a founding clause of a unified geometric theory, rather than as an emergent feature, a reconstruction target, or a dictionary entry — that we believe to be new, while every one of its ingredients is gratefully inherited.

20.5 What is genuinely new: the constitutional combination

We can now state the novelty claim of GG-6 with the precision that intellectual honesty demands. Taken one at a time, the moves of this Part are each anticipated: the fifth dimension as RG scale by holographic renormalization (Sec. 20.1); the sixth as gauge holonomy by Kaluza–Klein and Hosotani (Sec. 20.2); non-spatial extra geometry by noncommutative geometry and F-theory (Sec. 20.3); measurement as a primitive by the operational reconstructions and holographic error correction (Sec. 20.4); the rejection of the landscape by the swampland, taken up with the string contrast it belongs to (Sec. 21). We claim originality for none of these in isolation, and we have tried to name their authors with the care they have earned.

What we have not found anywhere, and what we therefore offer as the contribution of the present framework, is the *simultaneous constitutional combination*: a single set of four axioms and seven postulates that (i) declares *both* extra dimensions non-spatial from the outset — one the renormalization scale, the other the gauge phase — rather than discovering either fact after the fact; (ii) thereby eliminates every compactification modulus and the landscape with them; (iii) makes measurement a founding clause through OGGEP, so that the theory contains its own observation theory; and (iv) locks the residual discrete freedom to a single point through MSMF and the OGN-5 integer ledger, leaving the trio $\{c, G_N, \hbar\}$ of bulk conversion factors and the boundary pair $\{k_B, \mathsf{Q_B}\}$ as the entire dimensionful content. Holographic renormalization comes closest to the first point, Hosotani to the second, the operational reconstructions to the third, and the swampland to the fourth; but the framework that holds all four together as constitutional commitments is, as far as the available literature shows, distinctive to GG-Theory. The precursors built the stones; the claim here is only about the particular arch into which they have been assembled, and about the discipline — the Euclidean discipline — of writing the assembly down as a deductive system with nothing held in reserve.

21 The Decisive Contrast: GG-6, Kaluza–Klein, and String Theory

The structural contrast with superstring theory can now be stated in one sentence: *string theory geometrizes physics by adding space, whereas GG-6 geometrizes physics by adding meaning to scale and phase.* A systematic comparison is the subject of Part IV of Arisaka (2026, A2-GG6); here we record the consequence most relevant to the geometric theme of the present paper.

In the heterotic and Type II string programs, the six or seven extra dimensions are space-like coordinates compactified on a Calabi–Yau or G_2-holonomy manifold, each carrying continuous moduli — shape and size parameters — that are not fixed by any principle internal to the theory. The proliferation of these moduli is the origin of the landscape of $\sim 10^{500}$ vacua. GG-6 admits no such moduli, because its two extra coordinates carry no continuous geometric scale: I_r is parametrized by the forced logarithmic RG-scale of Eq. (38), and S^1_θ is a topological circle with no radius. The "where are the extra dimensions, and what sets their size?" question — the standing difficulty of space-like compactification — simply does not arise: there is no spatial extra dimension to detect and no size modulus to fix.

This is the geometric content of the anti-landscape posture that AX-4 encodes. Where string theory must search an enormous moduli space for a vacuum resembling ours, GG-6 has no moduli space to search: its dimensionful content is the closed pentad GU-5 and its dimensionless content the integer ledger OGN-5, with the two extra coordinates contributing scale and phase rather than tunable lengths. The 2300-year geometrization arc of Part II thus closes not by adding more space, but by recognizing that two structures physics already possessed — the renormalization group and the gauge phase — are themselves dimensions of the manifold on which the world is written.

The anti-landscape posture, and the precursor that shares it. A final strand concerns not a structure of GG-6 but its posture toward free parameters. The recognition that string compactifications possess an enormous number of metastable vacua — the flux landscape of Bousso and Polchinski (2000) and Kachru, Kallosh, Linde, and Trivedi (2003), estimated at $\sim 10^{500}$ vacua by Susskind (2003) and Douglas (2003) — is the backdrop against which GG-6's no-moduli claim must be read, and the *swampland* program of Vafa (2005) is the most serious attempt within string theory to constrain that proliferation from the low-energy side. GG-6 shares the swampland's conviction that not every effective theory is admissible, but it enforces the constraint constitutionally rather than conjecturally, and it removes the moduli at the source: because the two extra dimensions carry no length (Sections 17–18), there is no compactification volume to stabilize, no shape modulus to fix, and hence no landscape to navigate. The combination of AX-4 (MSMF) with the OGN-5 integer ledger then collapses what little discrete freedom remains to a single point. Where the swampland prunes a vast landscape from outside, GG-6 never grows one.

Why $\{c, G_N, \hbar\}$ are the complete conversion-factor set only in GG-6. The contrast can be sharpened into a statement about conversion factors. In any space-like compactification — Kaluza–Klein, heterotic, or Type II — the extra dimensions carry an intrinsic length: a compactification radius R (or, for a Calabi–Yau, a set of volume moduli). That length is a *free dimensionful parameter*, equivalently a free dimensionless ratio R/ℓ_{Pl}, and it is *not* a member of $\{c, G_N, \hbar\}$. A theory with a compactification scale therefore needs at least one conversion factor *beyond* the trio; the trio is incomplete there, and the extra scale is exactly the kind of tunable input that GG-Theory forbids. In GG-6 the two extra coordinates carry no length — I_r is RG-scale and S^1_θ is a topological circle (Part III) — so there is no compactification scale, no extra modulus, and no conversion factor beyond the trio. This is the precise sense in which $\{c, G_N, \hbar\}$ can be the *complete and true* set of bulk conversion factors *only* in the non-spatial 6D geometry of GG-6 (stated as Theorem 26.2 below): the completeness is a direct consequence of the extra dimensions not being space. Kaluza–Klein and string theory, by construction, cannot make this claim.

21.1 What is a dimension?

The technical departures cataloged in the preceding sections rest, in the end, on a single metaphysical decision — one so basic that it is almost never stated, and was for a century answered by default. When a unified theory introduces structure "beyond" the four dimensions of experience, must that structure be a *place*? Must an extra coordinate be a direction along which a thing could, in principle, travel and be found? Every prior geometric unification answered yes, usually without noticing that a question had been asked. GG-Theory answers no. The whole of what distinguishes it from Einstein's unified field theory, from Kaluza–Klein, and from string theory can be compressed into that one word, and the purpose of this section is to state the distinction as precisely as it deserves.

21.2 The three corners

There are exactly three coherent stances one may take toward the "extra" structure of a unified theory, according to two independent questions: is the structure *geometrized* — given a genuine coordinate, a component of a metric, modes and geodesics — and is that coordinate *spatial*? The three occupied-or-empty corners are shown in Table 8.

Ordinary gauge theory (Corner 1) keeps the internal symmetry internal and never promotes it to a dimension; it pays for this by leaving gauge and gravity as separate kinds of thing. Kaluza–Klein and string theory (Corner 2) take the decisive geometric step of promoting the extra structure to a coordinate — and immediately make that coordinate a place, a small spatial circle or a compact manifold. GG-Theory occupies the corner that had stood empty: it takes the same geometric step, promoting the fifth and sixth directions to genuine coordinates of one manifold, while *refusing* to make them spatial. This is the precise sense of the slogan that names the section: *extra-dimensional geometry without extra space.*

21.3 Coordinate or place: an operational criterion

The distinction is sharp only if "spatial" is defined operationally, and it can be. A coordinate is a *place* — a spatial dimension — if a particle can propagate along it and be localized on it: if it carries an independent momentum and supports a tower of states whose masses scale with the inverse of

Table 8: The three positions one can take on the extra structure a unified theory needs. The second column says what that structure is in each case, the third whether it is given a place in spacetime, and the fourth what follows. The first two corners have been occupied for a century — ordinary gauge theory, which declines to geometrize, and Kaluza–Klein, which geometrizes by adding a spatial direction. The third corner is the position taken here, and the table exists so that it can be seen as the remaining option rather than as a novelty: it geometrizes without adding a place.

Corner	Extra structure	A spatial place?	Consequence
1. Ordinary gauge theory	internal symmetry, *not* geometrized as a coordinate (A_μ a field on $\mathcal{M}_4$)	— (no coordinate)	no extra dimension; gauge and gravity remain separate
2. Kaluza–Klein and string theory	a genuine coordinate (metric, modes, geodesics)	**yes** (a small spatial circle, a Calabi–Yau)	radius modulus / dilaton, charged KK tower, moduli space, the $\sim 10^{500}$ landscape
3. GG-Theory	a genuine coordinate ($d\theta^2$, holonomy, geodesics $\to$ Lorentz force)	**no** (RG-scale I_r; gauge phase S^1_θ)	no modulus, no charged tower, no landscape; gravity and gauge unified

its size, $m_n \sim |n|/R$. A coordinate is *non-spatial* if it carries a component of the metric and an operational meaning, but admits no propagation, no localization, and no "where." By this criterion the two extra directions of GG-6 are coordinates but not places. The fifth, I_r, is renormalization-group time, $r = \ln(E_{\mathrm{GUT}}/\mu)$ (Section 17): one moves along it by changing the energy at which one looks, not by traveling. The sixth, S^1_θ, is a gauge phase (Section 18): its "momentum" is the conserved electric charge, an integer label, and it supports no spatial tower because it has no radius. Both pass the test for a coordinate — the line element genuinely contains dr^2 and $d\theta^2$ — and both fail the test for a place. That two-sided fact is the entire content of "geometry without space," and it is what keeps the phrase a precise physical statement rather than a play on words.

21.4 The shared spatial reflex: Einstein, Kaluza–Klein, and string theory

Seen from this corner, three otherwise very different programs share one assumption, and it is the assumption, not their differing machinery, that limited each. Einstein's thirty-year search for a unified field theory tried to fold electromagnetism into the *metric* of spacetime — a non-symmetric metric, teleparallelism, a fifth metric component — and so was committed from the start to a spatial home for the electromagnetic field; the program stalled on exactly the surplus structure (extra metric components with no clean physical reading) that a spatial extra direction forces. Kaluza–Klein made the unification work in 1926 but inherited a spatial circle, and with it the radius modulus and the charged mode tower that were never observed. String theory inherited the same spatial picture at one further remove: its extra dimensions are a compact manifold, and the freedom in choosing that manifold is the moduli space whose vast degeneracy is the landscape.

Here one honest distinction must be drawn, because it strengthens rather than weakens the point. For Einstein and for Kaluza–Klein the spatiality of the extra structure was an *unexamined* *default* — nothing compelled it, and the question of this section was simply never posed. For string theory it is not a default but a *requirement*: a string is a one-dimensional object propagating in a target *space*, and the consistency of its worldsheet (the cancellation of the conformal anomaly, the critical dimension) demands that the target be a metric space of definite dimension. String theory therefore could not move to the third corner even if it wished to, without ceasing to be a theory of strings. GG-Theory, built on no string, is under no such compulsion, and is free to let its extra coordinates be scale and phase rather than place. The contrast is thus not that string theorists overlooked an option, but that the string forecloses the option that GG-6 takes.

21.5 Why the third corner dissolves the pathologies

The move to the empty corner is worth making only because it removes, at a stroke, the difficulties that have attended every spatial unification, while keeping the features that made geometric unification attractive in the first place. What is removed follows directly from non-spatiality: with no radius there is no radius modulus and no dilaton; with no spatial momentum there is no charged Kaluza–Klein tower; with no compactification manifold there is no moduli space and no landscape to scan. What is kept follows from the fact that the directions are still genuine coordinates of a metric: electric charge is still the holonomy of the sixth direction, the forces still arise as the projection of bulk geodesics (the Lorentz force and Newton's law as the two curvatures of one motion, Section 26.2 and Arisaka (2026, A5-GDOS)), and gravity and gauge interaction are still unified through a single geometric apparatus. GG-Theory thus retains the Kaluza–Klein mechanism — integer modes on a compact direction — while carrying no compactification modulus, and it does so by the one modification a spatial extra dimension cannot make: declining to be spatial.

21.6 Supersymmetry as the symptom of a spatial extra dimension

One pathology of the spatial programs is rarely listed beside the radius modulus, the charged tower, and the landscape, yet it belongs to the same family and is dissolved by the same move: supersymmetry. It is worth setting out separately, because it is usually read as an aesthetic preference or a naturalness strategy, when in fact it is a structural consequence of giving the extra dimensions length.

Begin with why the string *needs* supersymmetry; this is not an optional ornament but a requirement of the construction. A string is an extended object sweeping a worldsheet through a target *space*, and the quantum consistency of that worldsheet — the removal of the tachyon, the appearance of spacetime fermions, the very value of the critical dimension — forces the worldsheet theory to be supersymmetric. Carried up into the spacetime the string lives in, that worldsheet supersymmetry becomes a super-Poincaré symmetry: a set of fermionic generators Q whose anticommutator is a *translation*, $\{Q, Q\} \sim P$. Supersymmetry is, at root, a square root of translation. And a translation presupposes a direction one can be translated *along* — that is, it presupposes space. Where there is space there are momenta; where there are momenta one may ask for their square root; and once that question can be posed, the fermionic partners, the superalgebra, and the whole supersymmetric apparatus follow. Even the disease supersymmetry is most often called upon to cure — the vast desert between the weak scale and the Planck scale — is itself an affliction of having that much spatial room for a hierarchy to open up in.

GG-Theory stands on the other side of this fork, and it stands there for the single reason that has organized this whole section: its extra coordinates are not places. The fifth direction is scale and the sixth is phase; to move along I_r is to flow under the renormalization group, and to move along S^1_θ is to rotate a gauge phase. Neither motion is a translation through a space, and so neither supplies the momentum generator whose square root supersymmetry would have to be. There are no super-translations here to take the root of, hence no super-Poincaré algebra to extend, hence — quite literally — nothing for supersymmetry to *be*. The formal version of this statement, in which the generators of I_r and S^1_θ are identified as a dilatation and a compact charge rather than as momenta, and the candidate superalgebra is shown not to close, is given in the companion bulk paper (Arisaka, 2026, A2-GG6); here the point is the conceptual one, that supersymmetry has no home in a bulk that has no space.

This is the deeper reason that lies beneath the constitutional one. The GG-Constitution already forbids supersymmetry through AX-4 (MSMF): superpartners are unforced structure, and unforced structure is inadmissible. But that clause tells us only that supersymmetry is not *needed*; non-spatiality tells us why it is not even *available*. It is not that GG-6 surveys the superpartners and declines to add them; it is that the geometry offers them nothing to partner. The two readings agree, and the second explains the first: supersymmetry is unforced precisely because the bulk withholds the space that would force it. Minimality and non-spatiality are here two faces of one fact.

Seen this way, supersymmetry takes its place in a single package. Give the extra dimensions length and you inherit, together and for the same reason, the moduli, the Kaluza–Klein tower, the landscape, and — as the symmetry that a spatial target makes available and a spatial hierarchy makes desirable — supersymmetry. Withhold the length and the entire package, supersymmetry included, has nowhere to reside. The empty corner is empty of superpartners for exactly the reason it is empty of moduli and of towers: it is empty of space. The reading is an unfashionable one, and it is worth being plain about its empirical edge: where the spatial program meets the null superpartner searches at the LHC by pushing the superpartners to ever higher scales, GG-Theory reads the same data as saying what it appears to say — that there are no superpartners at any scale — because the geometry has left them no place to stand.

21.7 Boundaries, not branes: the same lesson for the ends of the interval

The radial construction of GG-6 is borrowed, in its machinery, from the warped throat of Randall and Sundrum, and with that machinery came a habit of speech: the two ends of the fifth dimension are called the "UV brane" and the "IR brane." The habit should be broken, and for exactly the reason that has run through this whole section. A brane, like supersymmetry, is a creature of space. It is a membrane — a physical sheet that sits at a location in a transverse space, carries a tension, gravitates, and can move. It is a thing *in* the world, not a feature of the world's edge.

In the Randall–Sundrum picture this is entirely appropriate, because there the fifth dimension is a genuine length. Its ends are places, and a membrane can sit at a place: the Planck brane and the TeV brane are real sheets at real positions, their tensions tuned against the bulk curvature, and the distance between them is itself a dynamical quantity — the radion, the modulus that says how far apart the two sheets stand. That modulus is precisely the kind of unforced continuous freedom the non-spatial move was designed to abolish. The brane is the spatial reading of an endpoint, and it brings the spatial program's characteristic surplus along with it.

In GG-6 the fifth dimension is not a length but a scale: r is an RG scale — space-like in signature, not a second time (see Arisaka (2026, A2-GG6), Picture 1). Its two ends are therefore not places where a sheet could sit; they are the beginning and the end of a *flow* — the ultraviolet end at which the four-dimensional description is anchored at E_{GUT}, and the infrared end at which it is read off at low energy. The correct word for the end of a flow, or the edge of an interval, is *boundary*: it carries boundary conditions, the data the interior must match, and nothing further. There is no transverse space for a membrane to occupy, no tension for it to carry, and — decisively — no separation for it to vary, because the length of the radial interval is fixed by the integer ledger OGN-5, not set by a modulus. To keep calling these ends "branes" would quietly reinstate the very spatial picture, and the very radion, that the framework exists to remove.

The terminology of this paper is corrected accordingly: where the warped-hierarchy literature writes *UV-brane* and *IR-brane*, GG-6 writes *UV-boundary* and *IR-boundary*, in the prose and in the figures alike. The change is not cosmetic. A brane is an object in space that the theory must then go on to explain — its tension, its stability, its position; a boundary is a condition on a flow that the theory simply imposes. The scale machinery that generates the Planck–electroweak hierarchy is inherited intact; only the ontology of the endpoints is set right. This is the radial echo of the lesson already drawn for electric charge, for the string, and for supersymmetry: wherever the spatial programs place a *thing in space*, the non-spatial program places only a relation or a condition. The companion bulk paper gives the formal statement — that the bulk is a manifold with boundary, $\partial\mathcal{M}_6 = \mathcal{M}_4 \times \{0, K\} \times S^1_\theta$, whose two components carry boundary terms but no independent tension, no charge, and no position modulus (Arisaka, 2026, A2-GG6). GG-6 is a bounded warped geometry, not a braneworld.

21.8 Dimensions as kinds of distinction, not containers

Beneath the technical payoff lies a shift in what the word "dimension" is taken to mean. In the older picture a dimension is a *container*: a box, an additional room of the world into which things may be placed. On that reading every dimension must be a place, and the only question is how many rooms there are and how large. The reading forced by GG-6 is different: a dimension is an independent *kind of distinction* that the world makes — a direction along which states genuinely differ — whether or not that direction is a place one could occupy. Time is the oldest non-spatial example: it carries a component of the metric and is unmistakably a dimension, yet one does not travel along it as one travels across a room. Energy scale and gauge phase are two further kinds of distinction of exactly this character. The recognition that the geometry of physics is built from *kinds of distinction*, only some of which are spatial, is the metaphysical core of GG-Theory, and it is of a piece with the Euclidean thesis of this paper: Euclid's dimensions were the three of space, and the modern *Elements* simply admits that scale and phase are geometric in the same full sense, without being spatial.

21.9 Scope and honest caveats

Four qualifications keep this claim exactly as large as it should be, and no larger. *First*, non-spatiality is an interpretive keystone, not a free-standing prediction: it is what renders the framework parameter-free and landscape-free, but the quantities that can be measured against experiment — $\alpha^{-1} = 137$, the lock integer $K = 35$, M_{lock}, $\Omega_m = 1/\pi$ — ride on the integer ledger OGN-5, not directly on the choice of corner. *Second*, the claim is precise only with its operational definitions in view: "a coordinate that is not a place" means I_r is RG-scale and S^1_θ is gauge phase, and the phrase should never travel without them. *Third*, as noted above, string theory's spatiality is forced by the string; GG-Theory is therefore an *alternative* to the string program, not a correction of it. *Fourth*, Corner 1 already holds that the gauge phase is internal rather than spatial; the novelty of GG-6 is not that observation but the decision to *geometrize* the internal phase as a coordinate and to unify it with gravity through the same metric, while still declining to make it space. The distinctive position is the conjunction of those moves, not any one of them alone.

21.10 The distinction in one sentence

GG-Theory is set apart from every previous geometric unification not by the number of dimensions it adds but by their nature: it adds no new room to the world, only two new kinds of distinction — scale and phase — and reads the constants of physics off the geometry of those non-spatial directions. Extra-dimensional geometry without extra space is the third corner, and occupying it is what lets the framework be at once unified, parameter-free, and free of the landscape.

Part IV
Anomaly Freedom and the Integer Ledger

BI-6 and the OGN-5 Diophantine Cascade

If Part III supplies the geometry, this Part supplies the arithmetic. The dimensionless content of physics — ultimately every pure number the Standard Model measures — is traced here to a single source: the requirement that the antisymmetric tensor field of the bulk be globally consistent, which is the modified Bianchi identity BI-6. That requirement forces a unique solution of five integers, the ledger OGN-5 $= (1, 3, 5; 7, 35)$, through a five-step Diophantine cascade in which each integer is fixed by the previous ones, the color count included: it follows from the dual lock together with the block embedding, as Section 23 states. These integers are not postulated; they are theorems. They are the backbone on which the parameter audit of Part VII hangs.

The distinction between a postulate and a theorem is the whole of the matter here, and it is easy to lose. An integer that has been *chosen* because it reproduces a measured quantity explains nothing, however elegant the arithmetic it then supports; an integer that is *forced* by a consistency condition, and only afterwards found to reproduce that quantity, is a prediction. The cascade in this Part is written so that a reader can check which of the two is happening at every step: each integer is derived from a stated clause together with an already established theorem, in an order fixed before any contact with experiment, and the premises are recorded alongside the conclusion so that a circularity would be visible rather than buried.

The proof itself is not carried out here. Arisaka (2026, A3-BI6) establishes the anomaly cancellation and the Diophantine closure; this Part states the identity, explains what its four coefficients mean physically, walks the five steps in a form a reader can verify by hand, and records for each one whether it is derived, conditional, or open. That grading matters most at the third step, where the color count appears, and it is stated there rather than assumed.

22 The Modified Bianchi Identity BI-6

Historical background (with citations). The identity BI-6 that we are about to write down is a particular *modified* instance of a much older geometric identity. In a 1902 note in the *Atti dell'Accademia dei Lincei*, Luigi Bianchi (Bianchi, 1902) proved that the Riemann curvature tensor satisfies the differential identity $D_{[\mu}R_{\nu\rho]\sigma\tau} = 0$ — equivalently, $\mathrm{d}R = 0$ when R is read as a curvature two-form on the frame bundle. The Bianchi identity is the geometric content of the statement that $D^2 = 0$ on a connection, and it is the classical reason every gauge field strength F obeys $\mathrm{d}F = 0$ on a principal bundle. The Green–Schwarz mechanism (Green and Schwarz, 1984) introduces a controlled, geometric modification of the Bianchi identity for the antisymmetric tensor field strength H_3, replacing $\mathrm{d}H_3 = 0$ by $\mathrm{d}H_3 = X_4$ for a closed, gauge-invariant, characteristic-class four-form X_4

built from $\mathrm{tr}(R \wedge R)$ and $\mathrm{tr}(F \wedge F)$. In our reading the GS move is the technical pioneering of AX-2 GGEP, and BI-6 in turn is its axiomatic completion in the 6D bulk (Arisaka, 2026, A3-BI6).

22.1 Statement of the identity

The Green–Schwarz mechanism (Green and Schwarz, 1984) requires that the field strength $H_3 = dB_2$ of the antisymmetric tensor field B_2 in $\mathcal{M}_6$ satisfy a modified Bianchi identity that incorporates the Chern–Simons forms of the bulk gauge connections. What we are about to write down is, in the GG-Theory reading developed in Arisaka (2026, A3-BI6), the axiomatic completion of the geometrization move that Green and Schwarz pioneered in 1984: where their construction was technical, ours is constitutional; where they had a particular 10D heterotic string in mind, we have the 6D bulk forced by the four constitutional axioms. In the GG-6 context, this identity takes the form

$$\boxed{dH_3 = X_4 = \alpha_1 \, \mathrm{tr}\, R^2 + \alpha_2 \, \mathrm{Tr}\, F^2_{\mathrm{SU}(5)} + \alpha_3 \, F^2_{\mathrm{U}(1)_B} + \alpha_4 \, F^2_{\mathrm{U}(1)_{Qi}},} \tag{50}$$

where R is the curvature 2-form of the 6D Lorentzian connection, $F_{\mathrm{SU}(5)}$ is the SU(5) field strength, and $F_{\mathrm{U}(1)_B}$, $F_{\mathrm{U}(1)_{Qi}}$ are the field strengths of the two abelian gauge factors. The four quadratic invariants are listed in the canonical sector order (E, e, B, Qi) — gravitational, unified-gauge, baryonic, informational — which is the ordered basis used throughout the cohort, and in particular the basis in which the Arisaka Equation of Arisaka (2026, A7-Qi4) is written. There is no $F_{\mathrm{U}(1)_B} \wedge F_{\mathrm{U}(1)_{Qi}}$ cross-invariant in X_4: the two abelian factors enter symmetrically, and only through their own squares.

The coefficient vector is

$$(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (1, -1, -2/5, -2/5), \tag{51}$$

so that explicitly $X_4 = \mathrm{tr}\, R^2 - \mathrm{Tr}\, F^2_{\mathrm{SU}(5)} - \frac{2}{5}\big(F^2_{\mathrm{U}(1)_B} + F^2_{\mathrm{U}(1)_{Qi}}\big)$. That BI-6 is anomaly-free at this vector, locally and globally, is a theorem of Arisaka (2026, A3-BI6), proved with the index theorem for the 6D Dirac operator, the Wu–Chern–Simons completion, and the differential-cohomology framework of Hopkins and Singer (2005) and Monnier and Moore (2019). What fixes the vector is a separate question from what the anomaly theorem proves, and Section 22.3 answers it. We refer to the four-tuple in (51) as the BI-6 coefficients.

22.2 Physical interpretation of the coefficients

The four BI-6 coefficients have direct physical interpretations:

- $\alpha_1 = +1$: the gravitational invariant $\mathrm{tr}\, R^2$. This is the reference channel. The overall normalization of X_4 is fixed by setting its entry to unity, so every other entry is a ratio to the gravitational one.

- $\alpha_2 = -1$: the unified gauge invariant $\mathrm{Tr}\, F^2_{\mathrm{SU}(5)}$. Its sign is opposite to the gravitational entry — the two channels enter the single four-form with opposite orientation, which is what allows one two-form to absorb both.

- $\alpha_3 = -2/5$: the $\mathrm{U}(1)_B$ self-coupling. The fractional value is the Abelian trace ratio $(L-N)/L$ evaluated on the ledger, so it is fixed once $(L, N) = (5, 3)$ is fixed, and not before.

- $\alpha_4 = -2/5$: the $\mathrm{U}(1)_{Qi}$ self-coupling. The two abelian factors carry the same entry because AX-4 (MSMF) admits no term in X_4 that distinguishes them. That equality is a constitutional input, not an output of the anomaly analysis.

The Arisaka ratio $5 : -5 : -2 : -2$ that appears in the GG-Theory literature is obtained by multiplying the BI-6 coefficients $(1, -1, -2/5, -2/5)$ by 5 to clear the fractions: $(5, -5, -2, -2)$. This rescaled ratio is the form in which the four boundary currents enter the Arisaka equation (Arisaka, 2026, A7-Qi4).

22.3 Why the BI-6 coefficients are not free

The coefficients are not free. It is worth being exact about what makes them so, because the answer a reader reaches for first is the wrong one.

The answer a reader reaches for first is anomaly freedom. Three conditions do constrain the identity:

1. *Local consistency*: the identity $dH_3 = X_4$ requires X_4 to be closed, $dX_4 = 0$. This is automatic if X_4 is built from gauge field strengths and the curvature.

2. *Global consistency*: the class $[X_4]$ must admit an integral lift for the differential-cocycle refinement of H_3 to exist. This uses exactly one property of the abelian entry — that it is rational, so that some integer clears its denominator.

3. *Anomaly cancellation by inflow*: the boundary anomaly polynomial must factorize as $I_8 = \frac{1}{2} X_4 \wedge X_4$, with X_4 the same four-form that appears in the modified Bianchi identity. This is a rank-one condition on the four quadratic invariants, and a rank-one condition is blind to the coefficients of the invariants it is stated in.

Not one of the three sees the *value* of the abelian entry. Arisaka (2026, A3-BI6) proves its global anomaly-freedom theorem with that entry carried as a symbol $\gamma \in \mathbb{Q}_{>0}$, and every step — the local descent, the rank-one criterion, the integral lift, the Wu–Chern–Simons quantization, the vanishing of π_6, the bounding theorem — goes through unchanged for every rational γ. Anomaly-free spectra exist at every rational abelian coefficient. We say so here rather than leave it to be found: *anomaly freedom does not single out* BI-6.

That is not a concession, because anomaly freedom was never what singled it out. Anomaly freedom is a *consistency test*, and consistency tests are passed by families — which is what makes them tests rather than derivations, and is equally true of the 1984 ten-dimensional mechanism, passed by both $SO(32)$ and $E_8 \times E_8$ without thereby becoming vacuous. What fixes the vector is stated in two clauses, both upstream of the anomaly analysis and both open to attack on their own terms:

- AX-4 (MSMF) fixes the *architecture*: one Green–Schwarz tensor channel, and no term in X_4 distinguishing $\mathrm{U}(1)_B$ from $\mathrm{U}(1)_{Qi}$. This forces the shape $(1, -1, -\gamma, -\gamma)$ — one gravitational entry, one unified-gauge entry, and one abelian entry used twice.

- The trace-ratio identity of Arisaka (2026, A3-BI6) fixes the *fraction*: $\gamma = (L - N)/L$, which is 2/5 on the ledger $(L, N) = (5, 3)$.

So the coefficients are theorems — but theorems of the constitution and the ledger, not of the topology. The distinction earns its paragraph twice over. It means the anomaly theorem is robust in a way it otherwise would not be: were the trace-ratio identity ever revised, the consistency of the bulk would not move an inch. And it puts the burden of proof where it belongs, on AX-4 and on the ledger, in the open, instead of borrowing it from a theorem that cannot carry it.

23 The OGN-5 Integers as a Theorem of BI-6

23.1 Statement

The central theorem of Arisaka (2026, A3-BI6), restated here in the form most relevant for the present argument, is that the integers $(1, 3, 5; 7, 35)$ — the OGN-5 *integers* — are the unique solution of the Diophantine consistency conditions for the modified Bianchi identity BI-6 on the GG-6 bulk. In line with the physical literature of the Arisaka-GG series we use the labels $(M, N, L;\ J, K)$ for the five integers, where $M = 1$ is the rank of the tensor channel, $N = 3$ is the color/triplicity index, $L = 5$ is the defining block of SU(5), $J = 7$ is the closure index, and $K = 35$ is the lock integer. These labels supersede the legacy notation $(n_1, n_2, n_3; m_1, m_2)$ used in earlier presentations.

Theorem 23.1 (The OGN-5 Integers from BI-6 — recalled from Arisaka (2026, A3-BI6) and T6-GG6)**.** *On the bulk $\mathcal{M}_6 = \mathcal{M}_4 \times I_r \times S^1_\theta$ with the canonical metric (29) and the bulk gauge content* $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$*, the integer-valued multiplicities $(M, N, L;\ J, K)$ that label the gauge field strengths and the bulk antisymmetric tensor field H_3 subject to the modified Bianchi identity (50) are the unique solution*

$$(M, N, L;\ J, K) = (1, 3, 5;\ 7, 35). \tag{52}$$

The five integers are not chosen. Each is forced by a constitutional clause together with a previously-established theorem, the color count $N = N_c = 3$ included: it is fixed by the dual lock with the closure-index clause and the block embedding (T-BI6-10.2a of (Arisaka, 2026, A3-BI6)), on premises that never mention the observed color count, with the BI-6 trace ratio carried as the independent check. The generation count N_g is a different quantity, is not a member of the ledger, and is treated in Arisaka (2026, P2-N3). This subsection presents the chain in detail; Table 14 records the grade of each. The integers are, in this sense, theorems of BI-6 rather than postulates of the framework, with the one grading stated rather than glossed over.

23.2 The five steps, in short

The cascade has five steps, each one fixing one integer from earlier results. We summarize the chain; the full step-by-step Diophantine derivation is in Arisaka (2026, A2-GG6, Theorem 6) and the OGN-5 theorem (T-BI6-10.2) of Arisaka (2026, A3-BI6).

Step 1: $M = 1$ **(rank-one tensor factorization).** POS-3 requires the boundary anomaly polynomial $I_8 = \frac{1}{2} X_4 \wedge X_4$ to factorize as the wedge of a single 4-form with itself; POS-6 selects the minimal tensor lattice on the GG-6 class. $M = 1$ states that a single 2-form B_2 absorbs all gauge and gravitational anomalies via Green–Schwarz inflow.

Step 2: $L = 5$ **(single-tensor consistency).** The companion GG-A3-BI6 now fixes the unified block size without reference to the observed gauge content: single-tensor Green–Schwarz inflow can reach neither the irreducible quartic $\mathrm{Tr}\, F_5^4$ nor the mixed cubic $\mathrm{Tr}\, F_5^3 \wedge F_B$, so both indices must vanish on the charged matter, and for the core package both equal $L - 5$; AX-4 (MSMF) selecting the minimal chiral package then gives $L = 5$ (T-BI6-9.0). The older route — AX-2 requires a simple embedding, and by Georgi–Glashow the smallest simple group admitting one Standard-Model generation is SU(5) — remains the physical gloss, but it reaches L through the observed low-energy content, which includes $\mathrm{SU}(3)_C$; an integer obtained that way cannot afterwards be used, in the same cascade, to conclude how many colors there are. We therefore take the N-free route as primary.

Step 3: $N = N_c = 3$ **(the dual lock).** Substituting the closure-index clause $J = 2N + 1$ into the dual lock $K = LJ = M^2 + N^2 + L^2$ gives the quadratic $N^2 - 2LN + (M^2 + L^2 - L) = 0$, whose roots are $N = L \pm \sqrt{L - M^2}$; at $M = 1$ integrality requires $L - 1$ to be a perfect square, and at the $L = 5$ of Step 2 the roots are 3 and 7. The block embedding $N < L$ — the color block sits inside the L-dimensional defining representation — retains $N = N_c = 3$. This is Theorem T-BI6-10.2a of Arisaka (2026, A3-BI6), which owns the derivation; no premise of it mentions the observed color count, and nothing in it bears on the generation count N_g.

The BI-6 coefficient $\alpha_3 = -2/5$ derived in Section 22 has the general form $\alpha_3 = -(L - N)/L$ (the trace ratio of the abelian generators relative to the embedding in the fundamental of SU(5)), and substituting $L = 5$ returns $N = 3$ again. That second route is the *independent check* rather than the derivation. The fraction $-2/5$ is itself computed by evaluating the general trace ratio on generators whose colored block already carries three entries, so a derivation of N from it would consume the split it concludes; what it does establish, and this is a genuine and checkable result, is that the trace ratio, the block size and the ledger closure all agree at $N = 3$ and fail to agree at every other integer in the admissible range. Read in that order, three independent routes agree at $N = 3$ and disagree at every other admissible integer: a previously empirical structural feature of QCD locked to a bulk anomaly coefficient, agreeing with a ledger that was determined without it.

Step 4: $J = 7$ **(odd closure of the triplicity sector).** The closure-index clause is $J = 2N + 1$: among admissible closure packets, the minimal one symmetric about a neutral slot, selected by AX-4 (MSMF) acting through POS-6 (OGN). It is a statement about the shape of the charge packet and does not mention the value of N, which is why Step 3 may substitute it into the dual lock without circularity. Evaluated at the $N = N_c = 3$ that Step 3 returns, $J = 2N + 1 = 7$. The dyadic capacity $2^J = 128$ becomes the integer foundation of $\alpha^{-1}(M_Z)$.

Step 5: $K = 35$ **(dual arithmetic lock).** The lock integer is determined by two independent identities holding simultaneously,

$$\boxed{K \;=\; L\,J \;=\; M^2 + N^2 + L^2 \;=\; 35.} \tag{53}$$

That a randomly-chosen integer triple (M, N, L) would have LJ and $M^2 + N^2 + L^2$ coincide is generically false; the agreement at $(1, 3, 5; 7, 35)$ is the *closure* of the OGN ledger. The lock integer $K = 35$ sets the scale throat depth and generates the Planck–electroweak hierarchy $e^{-K} = e^{-35} \approx 6.3 \times 10^{-16}$ as a theorem.

23.3 The five-step Diophantine cascade in summary

Table 9: The five-step derivation of OGN-5 from BI-6 and the constitution. The table is the audit trail for the claim that the integer ledger is derived and not fitted, and it should be read as a chain in which no step may be taken out of order. Steps 1 and 2 fix $M = 1$ and $L = 5$ from single-tensor consistency and minimality alone, without reference to any observed gauge content: the rank-one factorization $I_8 = \frac{1}{2}X_4 \wedge X_4$ forces one two-form to absorb every anomaly, and the irreducible quartic and cubic indices both vanish at $L = 5$. Step 3 is the one that carries the weight, because $N = N_c = 3$ is forced by the dual lock and the block embedding $N < L$ rather than by counting colors; the BI-6 trace ratio $-(L - N)/L = -2/5$ is then an independent check on a number already determined. Steps 4 and 5 are arithmetic consequences: odd closure gives $J = 2N + 1 = 7$, and the dual lock gives $K = LJ = M^2 + N^2 + L^2 = 35$. The Forced by column names the clause responsible at each step, so the reader can verify that the observed world is never used as an input.

Step	Integer	Forced by	Physical content
1	$M = 1$	POS-3: $I_8 = \frac{1}{2}X_4 \wedge X_4$ rank-one factorization; POS-6: minimal tensor lattice.	Single 2-form B_2 absorbs all anomalies.
2	$L = 5$	Single-tensor consistency: irreducible quartic and cubic indices both $= L - 5$ (T-BI6-9.0); AX-4 minimal chiral package. Georgi–Glashow gloss.	Defining block of the unified gauge group; fixed without reference to the observed gauge content.
3	$N = N_c = 3$	Dual lock with $J = 2N + 1$ and the block embedding $N < L$ (T-BI6-10.2a).	Color count, forced without reference to the observed split; the BI-6 trace ratio $-(L-N)/L = -2/5$ is the independent check.
4	$J = 7$	Odd closure: $J = 2N + 1$ with $N = 3$.	Dyadic channel capacity $2^7 = 128$.
5	$K = 35$	Dual lock: $K = LJ = M^2 + N^2 + L^2$.	Scale-throat depth $r_{\mathrm{IR}} = K = 35$; Planck–EW hierarchy e^{-35}.

23.4 Physical content of the OGN-5 integers

The cascade is set out step by step in Table 9, with the clause and the prior theorem that force each integer named in its own row. The five integers carry direct physical content, summarized for cross-reference:

- $M = 1$: the rank of the Green–Schwarz tensor channel; one independent B_2 field absorbs all anomalies.

- $N = 3$: the number of QCD colors; the rank of $\mathrm{SU}(N)_C \subset \mathrm{SU}(5)$; equivalently, the number of fermion generations in the minimal admissible chiral spectrum.
- $L = 5$: the defining block of SU(5); the size of the fundamental representation that contains the SM fermions in one generation.
- $J = 7$: the closure index; the dyadic depth of the electromagnetic channel at the Z pole.
- $K = 35$: the lock integer; the scale throat depth $r_{\mathrm{IR}} = K$ in natural GG-6 units, generating the Planck–electroweak hierarchy e^{-35}.

The coincidence $r_{\mathrm{IR}} = K$ at the solution of the radion stabilization problem is, in this picture, not a coincidence at all: the scale-direction endpoint that generates the Planck–electroweak hierarchy and the Diophantine lock integer of the OGN ledger are literally the same integer, because both are forced by the same axiomatic system.

23.5 From OGN-5 + BI-6 to all dimensionless ratios

The combined ledger of the four BI-6 coefficients $(1, -1, -2/5, -2/5)$ and the five OGN-5 integers $(1, 3, 5; 7, 35)$ — nine numbers, gathered in Table 10 — contains all the dimensionless arithmetic content of physics in GG-Theory. From this nine-element ledger, together with the three fundamental dimensional conversion factors $\{c, G_N, \hbar\}$ and the topological constants $\{e, \pi\}$ (developed in Section 29), all 28 Standard Model parameters are derived in Arisaka (2026, A9-SM), all cosmological inventory parameters in Arisaka (2026, A10-Cosmos), and the derived energy–information Arisaka constant $\mathsf{Q_B} \approx 0.1555$ eV per qubit (molar form $N_A\mathsf{Q_B} \approx 15.0$ kJ/mol) in Arisaka (2026, A7-Qi4).

The principal dimensionless predictions, developed in detail in Section 35 below, include the inverse fine-structure constant $\alpha^{-1}(M_Z) = 2^J = 128$ at the Z pole and $\alpha^{-1}(0) = 2^J + N^2 = 137$ in the Thomson limit, both reproducing the measured values at the integer level (the residuals are the radiative running derived in Arisaka (2026, A9-SM)). Other predictions include the weak mixing angle $\sin^2\theta_W = 25/108$, the cosmic inventory $\Omega_m = 1/\pi$ and $\Omega_\Lambda = 1 - 1/\pi$, the proton stability bound (effectively infinite due to the absence of dimension-6 baryon-number-violating operators in the OGN-allowed sector), and the strong CP angle bound $|\bar\theta| \lesssim 4.7 \times 10^{-18}$.

The ledger is closed. Five integers, $(M, N, L;\ J, K) = (1, 3, 5;\ 7, 35)$, each forced by a constitutional clause together with a previously established theorem, and not one of them a continuous choice: that is the entire dimensionless input of the framework. What Part III supplies as geometry and this Part supplies as arithmetic are two halves of one object, and every pure number the remaining Parts derive is read off from them.

One grading is worth carrying forward rather than glossing. The color count $N = N_c = 3$ is fixed by the dual lock together with the block embedding, on premises that never mention the observed number of colors, with the BI-6 trace ratio returning the same integer as an internal consistency check rather than an independent one, since GG-A3-BI6 establishes it as an algebraic consequence of the same ledger — so it is derived, not read off the world. The generation count N_g is a different

Table 10: The arithmetic ledger of GG-Theory: all dimensionless content from nine numbers. The table collects everything in the framework that is a pure number, and the claim it supports is one of exhaustion rather than of accuracy: after these nine entries there is nothing dimensionless left to specify. The four BI-6 coefficients $(1, -1, -2/5, -2/5)$ carry the relative weights of the gravitational, SU(5), $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$ channels in the anomaly inflow, with their shape fixed by AX-4 and their fraction by the trace-ratio identity. The five OGN-5 integers divide three primitive — $(M, N, L) = (1, 3, 5)$, the tensor rank, the color triplicity and the unification block — from two derived, $(J, K) = (7, 35)$, which follow by odd closure and the dual lock. The remaining rows are consequences rather than inputs: the rescaled Arisaka ratio is $5 \times$ BI-6, and $\sin^2\theta_W = 25/108$ is exact arithmetic from the OGN trace ratio, standing at 0.23148 against a measured 0.23155(4).

Item	Value(s)	Physical role
BI-6 coefficients $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$	$(1, -1, -2/5, -2/5)$	Shape fixed by AX-4, fraction by the trace-ratio identity; encode the relative weights of the gravitational, SU(5), $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$ channels in the anomaly inflow.
OGN-5 primitive integers	$(M, N, L) = (1, 3, 5)$	Step 1–3 of the Diophantine cascade: tensor rank, color triplicity, unification block.
OGN-5 derived integers	$(J, K) = (7, 35)$	Step 4–5: odd closure $J = 2N + 1$ and dual lock $K = LJ = M^2 + N^2 + L^2$.
Arisaka ratio (rescaled)	$(5, -5, -2, -2)$	Derived from $5 \times$ BI-6; the four-current relative weights in the Arisaka equation.
$\sin^2\theta_W = 25/108$	exact	Derived from OGN trace ratio; 0.23148 against the effective leptonic value 0.23155(4) adopted by GG-A9-SM, a residual of 0.030% and a pull of -1.70.
$\alpha^{-1}(M_Z) = 2^J = 128$	$\to 127.93$ (GG-A9-SM)	Dyadic channel capacity at the Z pole; the GG-A9-SM quantum-dressed value 127.9297 meets GG-A9-SM's benchmark 127.930(8) at a pull of -0.03.
$\alpha^{-1}(0) = 2^J + N^2 = 137$	137.035 with corrections	Integer matches CODATA to three significant figures; the six-figure agreement is the corrected value of GG-A9-SM, not the integer.
$\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$	matches Planck to 2%	Derived from π-locked cosmic inventory.
$M_{\text{lock}}/M_{\text{Pl}} = \frac{1}{2}e^{-K} = \frac{1}{2}e^{-35}$	matches empirical $\sim 10^{-16}$	Planck–electroweak hierarchy from lock integer K.

quantity, is not a member of the ledger, and is treated in Arisaka (2026, P2-N3). Conflating the two would make the ledger look as though it had been told something the world already knew, and it has not been.

Part V

The Boundary Law

GDOS, and the Gi-4 and Qi-4 Pillars

The bulk is not what we observe; observation lives on the boundary. This Part introduces the law that connects the two — GDOS (Geometric Dynamics of Open Systems), the boundary law that treats every observed system as an open system and every observation as a projection from the bulk — together with the Gi-4 and Qi-4 pillar structures that organize it. GDOS recasts a scientific observation as a bulk-to-boundary projection of a bulk state onto the boundary, and in doing so it introduces the two boundary members of GU-5: the thermodynamic conversion factor k_B and the quantum-informational conversion factor Q_B. This Part thus furnishes the (2 boundary) half of the five constants assembled in Part VI.

The move is smaller than it sounds and its consequences are larger. Physics has always known that a measurement couples the measured system to something outside it; what is unusual here is that the coupling is written into the charter rather than added to the dynamics afterwards. A theory whose observation rule is a founding clause can say things a closed-system theory cannot, and the cosmological constant is the standing example: a closed system has no mechanism to select a vacuum energy, so Λ is a free integration constant and no amount of cleverness inside the closed framing will fix it. Under an open-system projection it is a residue, and a residue has a scale.

Two constants enter with the boundary and they are of different kinds. k_B is familiar and its status is not: it is exact by convention since 2019, and what this Part supplies is the reason a conversion factor between energy and temperature should be a member of the same closed set as c, G_N and $\hbar$ rather than an accident of thermodynamics. $\mathsf{Q_B}$ is unfamiliar, is derived rather than measured, and is the point at which the framework makes a claim about living systems — the one place in this paper where the argument reaches outside physics, and therefore the one that is fenced most carefully.

24 Gi-4 Pillars and Qi-4 Pillars

24.1 The Gi-4 Pillars (Geometric Invariance Four Pillars)

The four pillars of *Geometric Invariance* (Gi-4) are the axiomatic invariants of the GG-6 bulk: the geometric quantities that are forced by the constitution and that any candidate boundary projection must respect. We list them, deferring detailed definitions to the technical literature:

TGG (Total Geometric-Gauge object). The total geometric–gauge structure of the bulk: the metric, the gauge connections of $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$, and the antisymmetric tensor field B_2, all subject to the modified Bianchi identity BI-6. TGG is the bulk-side analogue of the spacetime metric in general relativity, generalized to include the gauge sector and the topological field B_2.

CCI (Covariant Conservation Identity). The constraint that the composite connection (gauge plus gravitational) on $\mathcal{M}_6$ satisfy the modified Bianchi identity BI-6 at the topological level. CCI is the integrability condition for the bulk geometry; it is the requirement that the geometry be globally consistent on $\mathcal{M}_6$.

GFF (Geometric Free-Energy Functional). The unique cost functional governing all open-system dynamics in GG-Theory(Arisaka, 2026, A7-Qi4). GFF combines a geometric energy term (the GEP, Geometry–Gauge Energy Principle, encoding the projected energy of the bulk geometric–gauge structure) and a curvature–entropy term (the CEP, Curvature–Entropy Principle, measuring divergence from the geometric reference density). The Three-Gauge Theorem of Arisaka (2026, A7-Qi4) establishes that GFF is the common ancestor of three historically independent frameworks: Gibbs's thermodynamic free energy ($J_{Qi} = 0$), Friston's variational free energy ($J_{Qi} \neq 0$ with a Markov blanket), and the GG-6 gauge-field action (detector limit).

GRIP (Gauge-Runtime Invariance Pipeline). The DGI–GSSB–GA universal runtime that governs how any open system evolves on the F_{GFF} landscape (Arisaka, 2026, A6-GRIP). GRIP consists of three stages: *DGI* (Dynamical Gauge Invariance, the lawful exploration of the gauge-equivalence class — the fully symmetric configuration in which nothing has yet committed), *GSSB* (Geometric Spontaneous Symmetry Breaking, the branch-selection cascade through the saddles of the landscape), and *GA* (Geometric Attractor, the stable basin in which the selected branch persists as a boundary observable). The terminus is refined to the GGA (Ground Geometric Attractor, the unique deepest basin — GG-A6-GRIP def:GGA), and the runtime operates in two regimes (GG-A6-GRIP def:GRIPin/def:GRIPout): GRIP-In, descent on a fixed landscape halting at the GGA, and GRIP-Out, the transient event that reorganizes the landscape itself.

The four Gi-4 Pillars together form the axiomatic skeleton of the bulk: the TGG specifies what the geometry is, the CCI specifies that it is globally consistent, the GFF specifies how energy and information flow through it, and the GRIP specifies the temporal/runtime organization of any process within it.

24.2 The Qi-4 Pillars (Quantum Information Four Pillars)

The four pillars of *Quantum Information* (Qi-4) (Arisaka, 2026, A7-Qi4) are the boundary analogues of the bulk Gi-4 Pillars: the axiomatic invariants of the boundary information theory that any candidate boundary state must respect. They are:

Qi-QUA (Quadruple Gauge Locking, QGL). The three abelian gauge invariances of the boundary — $\mathrm{U}(1)_{\mathrm{EM}}$, $\mathrm{U}(1)_B$, $\mathrm{U}(1)_{Qi}$ — together with the energetic accounting that $\mathrm{U}(1)_{\mathrm{EM}}$ provides via the electric energy current, lock the four boundary currents J_E, J_e, J_B, J_{Qi} into a fixed ratio determined by the BI-6 inflow: pure information without matter is forbidden, sustained matter without information decays. The Arisaka equation

$$\boxed{J_E/\mathsf{Q_B} = J_e = J_B = J_{Qi}} \tag{54}$$

encodes this locking, with $\mathsf{Q_B} \approx 0.1555$ eV the Arisaka constant (per quantum bit; molar form $N_A\mathsf{Q_B} \approx 15.0$ kJ/mol) derived from the OGN ledger.

Qi-EP (the Unified Extremal Principle for open systems). The boundary dynamics is extremal of the GFF subject to the locking constraint of Qi-QUA. This is the open-system version of POS-2 of the constitution.

Qi-CON (Conformality). The boundary state has a hierarchical multiscale — conformal — structure that is geometrically consistent with the scale-coordinate RG flow of the bulk, so that the same organization reappears at every scale of magnification. This realizes POS-5 of the constitution at the boundary.

Qi-HAL (Holographic Attractor Lattice). The integration of memory into the boundary state, organized into a hierarchical architecture of allocentric maps that is graded by the QCI (Qi-HAL Completeness Index). This is the most novel of the four pillars and is developed in detail in Arisaka (2026, A7-Qi4); it is the formal apparatus by which the Evolving Inner Cosmos (EIC) is realized.

24.3 The Gi-4 → Qi-4 projection

The relation between the two sets of pillars is the GDOS bulk-to-boundary projection: the Gi-4 Pillars are the bulk axiomatic invariants on $\mathcal{M}_6$, and the Qi-4 Pillars are their boundary projections on $\mathcal{M}_4$ via the OGGEP dictionary. Specifically: TGG projects to the boundary metric and boundary gauge connections that source Qi-QUA; CCI projects to the anomaly inflow that enforces the four-current locking of Qi-QUA; GFF projects to the boundary cost functional that drives Qi-EP; and GRIP projects to the boundary runtime that organizes Qi-CON and Qi-HAL, terminating on each boundary stratum at its GGA.

The eight pillars are set out together in Table 11, bulk beside boundary. Taken together (four bulk, four boundary) they constitute the full axiomatic grammar of GG-Theory. Every observation, every measurement, every empirical fact about the world is, in this grammar, a Qi-4 event on the boundary that traces back through the projection to a Gi-4 structure in the bulk. This is the redefinition of scientific observation that AX-3 (OGGEP) accomplishes, and it is the topic of the next section.

Table 11: The eight pillars of GG-Theory: bulk Gi-4 and boundary Qi-4. The two columns are not two lists but one list read twice, once in the bulk and once in its boundary projection, and the rows are ordered so that each pillar faces its own image. Row 1 pairs the total geometric–gauge structure of $\mathcal{M}_6$ under BI-6 with the four-current locking of the Arisaka equation. Row 2 pairs the covariant conservation identity that forces global consistency of H_3 with the extremal principle for open systems on the boundary. Row 3 pairs the geometric free-energy functional with multiscale coherence under scale-coordinate RG flow. Row 4 pairs the GRIP runtime cascade on the F_{GFF} landscape with hierarchical allocentric memory. The correspondence is what licenses the boundary law to be read as geometry and the geometry to be read as an open-system dynamics.

Pillar	Bulk content (Gi-4)	Boundary projection (Qi-4)
1. Geometry-Gauge	TGG: total geometric–gauge structure of $\mathcal{M}_6$ subject to BI-6.	Qi-QUA: four-current locking via Arisaka equation.
2. Connection identity	CCI: covariant conservation identity, forcing global consistency of H_3.	Qi-EP: extremal principle for open systems on the boundary.
3. Free-energy / cost	GFF: geometric free-energy functional with GEP + CEP sectors.	Qi-CON: conformality (multiscale coherence) with scale-coordinate RG flow.
4. Runtime / dynamics	GRIP: the DGI $\to$ GSSB $\to$ GA/GGA cascade on the F_{GFF} landscape (regimes GRIP-In/GRIP-Out).	Qi-HAL: hierarchical allocentric memory and EIC dynamics.

25 GDOS: The Redefinition of Scientific Observation

25.1 Bulk-to-boundary projection

The OGGEP axiom (AX-3) of the GG-Constitution states that every observation in physics is a bulk-to-boundary projection from the bulk $\mathcal{M}_6$ to the boundary $\mathcal{M}_4$. The technical instantiation of this axiom is the GDOS framework (Arisaka, 2026, A5-GDOS), which provides the operational dictionary $\mathcal{D}$ that maps bulk states to boundary observables.

The dictionary $\mathcal{D}$ has the following structure. A bulk state ρ_6 on $\mathcal{M}_6$ (a positive trace-class operator on the bulk Hilbert space, satisfying the constraints imposed by the bulk action and the modified Bianchi identity BI-6) is mapped to a boundary state $\rho_4 = \mathcal{D}(\rho_6)$ via integration over the internal directions:

$$\rho_4(x) = \int_{I_r \times S^1_\theta} d^2y \, K(x;y) \, \rho_6(x,y), \tag{55}$$

with $K(x;y)$ a kernel determined by the boundary conditions on the UV and IR boundaries. The map $\mathcal{D}$ is CPTP — completely positive trace-preserving — and its action on observables is the bulk-to-boundary version of the Heisenberg picture. A register note is owed on the phrase "bulk Hilbert space": the GG-6 bulk is real, and the complex space on which ρ_6 is written is an auxiliary bookkeeping representation — the complexification licensed by T-P4-2 of Arisaka, 2026, P4-Real, with the boundary's working i the derived quarter-turn of T-P4-1 — not a bulk primitive; the pricing of the real category as the AX-4 minimum is L-Constitution-1 of Arisaka, 2026, A1-Constitution, at candidate grade.

The crucial conceptual feature of the GDOS projection is that it is irreducible: there is no way, in general, to recover the bulk state ρ_6 from the boundary state ρ_4. The bulk has additional information that the boundary observation does not see. This is the operational meaning of openness: every system observed from the 4D boundary is, fundamentally, an open system whose "environment" is the rest of the bulk.

25.2 The DGI–GSSB–GA runtime as a universal protocol of observation

The GRIP pipeline (DGI → GSSB → GA), defined in Section 24 as the universal bulk runtime, plays a second role at the boundary: it is the universal protocol by which observations in physics are organized. Every empirical observation, no matter what its specific subject matter, has the same three-stage structure:

1. *DGI (Dynamical Gauge Invariance)*: the apparatus is prepared in a high-symmetry state that includes all the modes of interest — the lawful exploration of the gauge-equivalence class; this is the "zero point" of the observation, in which nothing has yet committed.

2. *GSSB (Geometric Spontaneous Symmetry Breaking)*: the apparatus undergoes a symmetry-breaking transition (a measurement, a transition between energy levels, a record event) that selects a specific mode from the prepared ensemble; this is the central event of the observation.

3. *GA (Geometric Attractor)*: the broken symmetry stabilizes into a record (a digital readout, a chemical change, a memory trace, an experimental data point) that constitutes the observation — a stable basin of the boundary landscape, the empirical datum.

In the regime vocabulary of GG-A6-GRIP (def:GRIPin/def:GRIPout), the protocol has a sharper anatomy, developed for quantum mechanics in the dedicated paper GG-A8-QM: bringing the apparatus to bear reorganizes the joint system–apparatus landscape (a GRIP-Out — before the coupling, the record basins do not exist); the registration is GRIP-In descent into one of them; and the record's permanence is the halt of that descent at a Geometric Attractor. Quantization itself is the discreteness of the attractor catalog: the protocol can only ever record attractor coordinates, which is why empirical data come in definite, repeatable values.

This three-stage runtime is, in GG-Theory, not a model of any particular experiment but the universal constitutive structure of any observation. It applies equally to a particle physics measurement at LHC, to a chemical assay in a laboratory, to a brain recording in vivo, to a cosmological measurement of the CMB, and to a subjective report of conscious experience. All of these are GDOS events with the same DGI–GSSB–GA structure.

25.3 The redefinition of scientific observation

The GDOS framework redefines what counts as a scientific observation in three significant respects.

First, observations are not external acts of an observer on a passive system; they are GDOS events in which the bulk geometry projects onto the boundary. The observer is not separate from the system; the apparatus is part of the boundary, the system is part of the boundary, and the observation is a GDOS event between them.

Second, every observation is fundamentally open-system: the boundary projection of (55) is irreducibly lossy with respect to the bulk state. This means that the closed-system idealization that has guided much of twentieth-century physics is, strictly speaking, never realized; every empirical observation involves the loss of bulk information.

Third, the universal DGI–GSSB–GA structure of observation provides a common grammar across all the sciences. Particle physics, chemistry, biology, neuroscience, and cosmology are not separate disciplines with separate methodologies; they are domains in which the same GDOS protocol is instantiated with different specific bulk and boundary content.

This is the axiomatic content of the OGGEP axiom: science is the systematic study of GDOS events, the universal pipeline of which is the DGI–GSSB–GA runtime. Every scientific result, when properly analyzed, has this three-stage structure underlying it. And the methodological grammar of GG-Theory— the deductive extraction of theorems from the constitutional axioms — is the result of recognizing this structure systematically.

Part VI
GU-5: The Geometric Universal Five Constants

The Five Conversion Factors of the 6D Bulk and Its Boundary

Here the strands of the preceding Parts are tied together into the paper's title concept. The Geometric Universal Five Constants GU-5 $= \{c, G_N, \hbar;\ k_B, Q_B\}$ are the complete dimensionful content of physics, split (3 bulk + 2 boundary). This Part proves the bulk triad to be one conversion factor per kind of dimension (T-Euclid-1) and complete in the absence of a compactification modulus (T-Euclid-2); assembles the five-constant architecture (T-Euclid-3); shows the elementary charge e to be derived rather than primitive (T-Euclid-4); and states the closure theorem (T-Euclid-5) that no third species of constant exists. With closure in hand, the audit of Part VII becomes a finite, checkable task.

Closure is the load-bearing word, and it is worth separating from the claims it is often confused with. It is not the claim that the five constants are fundamental in some metaphysical sense; it is the claim that there is no sixth. A theory that admits a new species of primitive whenever it meets a quantity it cannot derive is not falsifiable, because there is always another species available. The closure theorem forecloses that move: every dimensionless constant is a ledger integer, a cascade consequence of one, or a ratio of pentad members, and every dimensionful quantity is a product of pentad members with ledger integers and integer powers of e and π. If a single measured quantity turns out to be none of those things, the theorem is false and the paper's central claim goes with it.

The Part is also where the $(3 + 2)$ asymmetry is met rather than smoothed over. Three of the five constants come from bulk directions and two from the boundary projection, which is not the tidy "one constant per dimension" the phrasing invites, and the architecture is presented as it is: an arrangement that rhymes with the $(3 + 2)$ split of the integer ledger without being a term-by-term identity with it. Seven theorems are stated in this Part, T-Euclid-1 through T-Euclid-7, and they are the formal content of everything the title of the paper advertises.

26 The Conversion-Factor Theorems: c, G_N, $\hbar$

The historical chain of Part II showed how c, G_N, and $\hbar$ were progressively absorbed — from free force coefficients into the geometric structure of physical theory — across the six partial-geometrization episodes from Maxwell 1865 to Randall–Sundrum 1999 (§§8–13). Section 15.2 then showed that the conventional Kaluza–Klein and Randall–Sundrum parameters $\{k, R_\theta, r_c\}$ are unit-conversion artifacts that disappear in the natural GG-6 parametrization. Section 19 gave the visual intuition: gravity is a UV-boundary phenomenon on I_r, $\hbar$ is the topological invariant of S^1_θ, and the scale profile e^{-K} with $K = 35$ controls the Planck–electroweak hierarchy. The synthesis is summarized here as three theorems and one master table.

26.1 c: conversion factor of the (1,3) Lorentzian boundary

The canonical GG-6 metric (29) carries signature $(1, 5)$ in the bulk and inherits a $(1, 3)$ Lorentzian signature on every constant-r slice of the boundary $\mathcal{M}_4 = \partial\mathcal{M}_6$. The scalar c that converts the temporal coordinate to the spatial coordinates is fixed once and for all by this inheritance, with no continuous freedom remaining.

In SI units the numerical value of c depends on the conventional definitions of the meter and the second; in natural GG-6 units $c = 1$. In either case c is a unit-translation device, not a tunable parameter. This is the GG-6 completion of the Minkowski 1908 reading developed in §9.

26.2 G_N: scale-curvature coupling of the (4+2) bulk

Dimensional reduction of the 6D Einstein–Hilbert action on the canonical metric (29), with scale profile e^{-2r} on the 4D directions only and the topological S^1_θ scale-flat, yields the 4D Newton constant

$$\boxed{G_N = \frac{1}{\pi\, M_6^2}} \tag{56}$$

in natural GG-6 units in terms of the bulk mass scale M_6. The radial integral $\int_0^K e^{-2r}\, dr = (1 - e^{-2K})/2 \approx 1/2$ (with $K = 35$ so $e^{-2K} \approx 4 \times 10^{-31}$) and the topological factor 2π from $\oint d\theta$ contribute the dimensionless numerical coefficient $1/\pi$; all explicit dependence on the conventional KK parameters k, R_θ, r_c has dropped out. The power of M_6 is worth pausing on, because it is where the bookkeeping is easy to lose: the six-dimensional coupling G_6 carries mass dimension -4 and the four-dimensional G_N carries -2, so the internal volume that divides one into the other must carry mass dimension -2 as well. The coordinate volume V_2 is a pure number — r and θ are dimensionless — and acquires that dimension from the one factor of the bulk scale that Appendix D supplies. This is the "implicit M_6^{-2}" that Arisaka (2026, A2-GG6) names and defers here.

The corresponding reduced 4D Planck mass is $M_{\rm Pl}^2 = 1/(8\pi G_N) = M_6^2/8$ in natural GG-6 units, so that $M_6 = 2\sqrt{2}\, M_{\rm Pl}$ on the reduced convention and $M_6 = M_{\rm Pl}/\sqrt{\pi}$ on the non-reduced one $M_{\rm Pl} \equiv G_N^{-1/2}$; matching M_6 to the empirical 4D Planck mass fixes M_6 to within a factor of order unity, and after this single match G_N has no remaining freedom. This is the GG-6 completion of the Einstein 1915 reading (§10) and of the Randall–Sundrum 1999 warp construction (§13).

Planck-to-electroweak hierarchy from $K = 35$. The hierarchy between the Planck mass and the electroweak scale is generated in GG-6 by the scale profile e^{-K} at the IR boundary:

$$M_6\, e^{-K} = M_6\, e^{-35} \approx M_6 \cdot 6.3 \times 10^{-16}, \tag{57}$$

giving a TeV-scale residue starting from $M_6 \sim M_{\rm Pl}$. The lock-cluster mass itself carries the canonical normalization $M_{\rm lock} = \frac{1}{2} M_{\rm Pl}\, e^{-K} \approx 3.85$ TeV (Arisaka, 2026, A9-SM); the displayed relation is the same suppression read from the bulk scale rather than from the Planck mass, and differs from $M_{\rm lock}$ by the order-unity factor $2/\sqrt{\pi}$. The hierarchy is therefore a discrete topological quantity of the OGN-5 ledger, not a continuous fine-tuning; no free scale parameter kr_c is involved. The full Einstein–Hilbert dimensional-reduction derivation is in the companion paper (Arisaka, 2026, A2-GG6).

In Einstein's general relativity, G_N is one free continuous parameter. In the conventional Kaluza–Klein framework, G_N becomes a derived quantity from (M_5, R), but the compact radius R is itself a free continuous input. In the Randall–Sundrum framework, G_N becomes a derived quantity from (k, M_5, r_c), with three free continuous inputs. In string-theory compactifications, G_N depends on the entire moduli space of the Calabi–Yau, contributing $\sim 10^2$–10^3 free parameters per geometry, with $\sim 10^{500}$ inequivalent vacua across the full landscape. In GG-6 the situation is qualitatively different: one choice of unit system (the co-equal trio $\{c, G_N, \hbar\}$, from which M_6 follows), one Diophantine integer ($K = 35$), one topological circle (S^1_θ, no radius), and no continuous input beyond the units. This is AX-4 (MSMF: Maximum Symmetry, Minimum Freedom) made literal at the level of the bulk metric.

26.3 $\hbar$: topological conversion factor of S^1_θ

The compact direction S^1_θ in the GG-6 bulk is a topological circle with no intrinsic geometric radius (§15.2). Single-valuedness restricts fields to the integer-labeled Fourier basis $\{e^{\mathrm{i}n\theta}\}_{n\in\mathbb{Z}}$; the dimensional momentum carried by mode n is then

$$\boxed{p_\theta = n\,\hbar, \qquad n \in \mathbb{Z},} \tag{58}$$

with $\hbar$ as the conversion factor between the dimensionless integer n and the dimensional momentum p_θ. The action $h = 2\pi\hbar$ accumulated by one unit of momentum traversing S^1_θ once is the topological invariant of the compact direction. After scale rescaling, the effective 4D mass of mode n at the IR boundary $r = K = 35$ is

$$m_n^{\mathrm{IR}} = |n|\,e^{-K} = |n|\,e^{-35} \approx |n| \times 6.3 \times 10^{-16} \tag{59}$$

in units of the UV-boundary scale — the same e^{-K} exponential suppression that brings Planck-scale modes down to the TeV-scale, here with the suppression factor literally an OGN-5 integer.

The Wilson-line winding integer $J = 7$ of the OGN-5 ledger controls the Hosotani breaking pattern on S^1_θ (Arisaka, 2026, A3-BI6; Arisaka, 2026, A9-SM); the topological foundation of $\hbar$ in GG-6 extends and completes the Kaluza–Klein 1919–26 reading of §11.

Master comparison table

Table 12 sets the three bulk constants side by side: the geometric support each one lives on, the axiom and postulate that force it, and the theorem of GG-A2-GG6 that supplies it.

Table 12: Geometric origin of the universal constants in GG-6. Where the lineage table traced the three constants historically, this one states what each is in the finished construction, and the third column names the bulk data that determines it — which is what makes the entry a derivation rather than an interpretation. c is the conversion factor of the $(1,3)$ Lorentzian signature on the boundary $\mathcal{M}_4$, inherited from the $(1,5)$ signature of $\mathcal{M}_6$. G_N is the scale-curvature coupling of the $(4+2)$ bulk, $G_N = 1/(\pi M_6^2)$ in natural GG-6 units, with the OGN-5 lock integer $K = 35$ setting the scale depth. $\hbar$ converts dimensionless Fourier mode numbers on the topological S^1_θ into physical momenta, with $J = 7$ fixing the Wilson-line winding. One constant per kind of dimension, and no fourth kind of dimension to supply a fourth.

Constant	Geometric origin in GG-6	Determining bulk data
c	Conversion factor of the (1,3) Lorentzian signature on the 4D boundary $\mathcal{M}_4$, inherited from the (1,5) signature of $\mathcal{M}_6$.	Boundary signature; metric structure of $\mathcal{M}_4$.
G_N	scale-curvature coupling of the (4+2) bulk: $G_N = 1/(\pi M_6^2)$ in natural GG-6 units.	6D fundamental scale M_6 (read in the units the trio $\{c, G_N, \hbar\}$ sets); OGN-5 lock integer $K = 35$ for scale depth.
$\hbar$	Conversion factor between dimensionless integer Fourier mode numbers on the topological S^1_θ and physical momenta.	Topology of S^1_θ; integer ledger; OGN-5 integer $J = 7$ for Wilson-line winding.

Theorem 26.1 (T-Euclid-1: The Conversion-Factor Trio)**.** *Given the bulk $\mathcal{M}_6 = \mathcal{M}_4 \times I_r \times S^1_\theta$ of Arisaka (2026, A2-GG6) (theorems T1-GG6–T3-GG6), each of the three bulk constants is the unique conversion factor of one kind of bulk dimension:* (a) *c converts the temporal coordinate of the boundary $\mathcal{M}_4 = \partial\mathcal{M}_6$ to its spatial coordinates, fixed once and for all by the* $(1,3)$ *signature inherited from the bulk* $(1,5)$ *signature;* (b) *G_N is the scale-curvature coupling of the radial interval I_r, $G_N = 1/(\pi M_6^2)$ in natural GG-6 units;* (c) *$\hbar$ is the unique scalar converting the dimensionless integer Fourier mode number on the topological circle S^1_θ to physical momentum, $p_\theta = n\hbar$. The bulk triad $\{c, G_N, \hbar\}$ is thus one conversion factor per kind of dimension, in dimension order (fourth, fifth, sixth), and none carries continuous freedom.*

Theorem 26.2 (T-Euclid-2: Completeness of the Trio — No Compactification Modulus)**.** *Because the two non-spatial dimensions of GG-6 carry no length — I_r is RG-scale and S^1_θ is a topological circle with no radius (Part III) — the bulk admits no compactification modulus. The trio $\{c, G_N, \hbar\}$ of Theorem 26.1 is therefore the* complete *set of bulk conversion factors: no further dimensionful constant is required. By contrast, any space-like compactification carries a radius R (equivalently a free ratio R/ℓ_{Pl}) that is not a member of the trio, so in Kaluza–Klein and string constructions the trio is necessarily incomplete. This completeness is a property unique to the non-spatial 6D geometry of GG-6.*

27 k_B: The Boundary Thermodynamic Member of GU-5

27.1 Why k_B is a boundary member of GU-5, not a bulk member

The standard Planck unit system, introduced by Max Planck in his 1899 paper to the Prussian Academy (Planck, 1899) and codified by the 2019 SI redefinition, normalizes four constants to unity: $c = G_N = \hbar = k_B = 1$. Each of the four corresponds to one of the four independent base dimensions of the SI system — length, time, mass, and temperature — and acts as the unit-conversion factor between human-chosen units (meter, second, kilogram, kelvin) and the natural units of the underlying physics. In GG-Theory, the first three, $\{c, G_N, \hbar\}$, are the *bulk* members of the Geometric Universal Five Constants GU-5: each is read off one of the three bulk directions of $\mathcal{M}_4 \times I_r \times S^1_\theta$ (Sections 26.1, 26.2, 26.3).

The Boltzmann constant $k_B \approx 1.381 \times 10^{-23}$ J/K is the *fourth* member of GU-5, and it is structurally distinct from the first three in a precise and intended way: it is a *boundary* conversion factor rather than a bulk one. It converts between energy — a bulk-geometric quantity — and temperature, which is not a bulk-geometric quantity at all but a thermodynamic property that emerges at the boundary $\partial\mathcal{M}_6 = \mathcal{M}_4$ through the GDOS bulk-to-boundary Gibbs projection (Section 25) and the AX-3 OGGEP CPTP open-system dynamics.

Together with the Arisaka constant $\mathsf{Q_B}$ of Section 28, k_B occupies the *boundary* tier of the (3 bulk + 2 boundary) split of GU-5— a split that deliberately mirrors the (3 + 2) architecture of the OGN-5 ledger. This is not a demotion to a "free parameter": like $c, G_N, \hbar$, the constant k_B carries no continuous freedom (it is exact by the 2019 SI convention, and its natural scale is fixed by the IR-boundary surface gravity $\kappa = e^{-K}$ with $K = 35$); it is simply a conversion factor of the boundary projection rather than of a bulk direction.

Historically, k_B enters physics through the thermodynamic line Carnot → Clausius → Boltzmann → Gibbs → Carathéodory → Jaynes, complementing the geometric line that delivered c, G_N, $\hbar$; GU-5 unifies the two lines. $\mathsf{Q_B}$ is the boundary conversion factor at the deep-IR biological scale far down the scale throat ($\mathsf{Q_B} = (2/5)\, E_{\mathrm{Ry}}/K$), and k_B is the boundary conversion factor at the IR boundary.

27.2 The Bekenstein–Hawking / Euclidean-time derivation

The standard derivation of k_B from a bulk geometry runs through Bekenstein–Hawking horizon thermodynamics, applied here to the IR boundary of the (4+2) bulk. The radial direction I_r has a natural Euclidean-time / RG-scale interpretation (Section 19, Picture 1: "the radial direction as RG-scale"), and the IR-boundary endpoint at $r = K = 35$ carries a natural surface gravity κ proportional to the scale curvature there. Compactifying I_r on a thermal Euclidean cycle gives, by the standard horizon-temperature formula (Bekenstein, 1973, Hawking, 1975),

$$T = \frac{\hbar\,\kappa}{2\pi\, k_B\, c}. \tag{60}$$

Solving for k_B:

$$\boxed{k_B = \frac{\hbar\,\kappa}{2\pi\, T\, c}.} \tag{61}$$

This is the standard Bekenstein–Hawking identification of the Boltzmann constant. In this reading k_B converts between the natural Euclidean-time period of the bulk geometry — itself fixed by the surface gravity κ at the IR boundary, which is in turn determined by the scale profile e^{-K} and the OGN lock integer $K = 35$ — and the human-chosen kelvin temperature unit. No new continuous parameter enters: the surface gravity κ is geometric data of the bulk, and the kelvin is a conventional unit choice.

In the language of Section 19, the Euclidean-time identification is a special case of the radial-direction-as-RG-scale picture: a thermal cycle in Euclidean time is, on the bulk, a cycle in the radial direction I_r, with the cycle period set by the IR-boundary surface gravity. The Boltzmann constant k_B is then the conversion factor between this natural radial-cycle period and a chosen kelvin scale; it is structurally on the same footing as the OGN-5-derived energy–information conversion factor $\mathsf{Q_B}$ of Section 28, with the two operating at different positions along I_r (at the IR boundary for k_B, far down the scale throat for $\mathsf{Q_B}$).

The explicit value of κ from the OGN-5 lock integer $K = 35$. The surface gravity at the IR boundary is, in the canonical GG-6 metric of Section 15, the magnitude of the gradient of the scale profile evaluated at the boundary position $r = K$:

$$\boxed{\kappa \;=\; \left|\frac{d}{dr}e^{-r}\right|_{r=K} \;=\; e^{-K} \;=\; e^{-35} \;\approx\; 6.3\times 10^{-16}} \tag{62}$$

in GG natural units (where the AdS curvature scale k has been absorbed into the canonical metric per the AX-4 MSMF principle of Section 15). κ is therefore *not* a free parameter of the framework: it is fixed entirely by the OGN-5 lock integer $K = 35$, which is itself a Diophantine theorem of the BI-6 ledger (Step 5 of the OGN-5 cascade in Section 23, with $K = LJ = M^2 + N^2 + L^2 = 35$ the unique small-integer closure). No continuous knob enters; the IR-boundary surface gravity is a discrete topological output of the integer ledger.

In physical units, multiplying by the bulk scale M_6 — itself no independent input, but a reading of the units the trio $\{c, G_N, \hbar\}$ sets, matched to the 4D Planck mass via the scale-coupling formula $G_N = 1/(\pi M_6^2)$ of Section 26.2 —

$$\kappa_{\rm phys} \;\sim\; M_6\, c^2\, e^{-K} \;\sim\; M_{\rm Pl}\cdot e^{-35} \;\sim\; \text{a few TeV}. \tag{63}$$

This is the same TeV scale that governs the 3.85 TeV Kaluza–Klein resonance prediction (Arisaka, 2026, A9-SM) and the Higgs vacuum expectation value $v = M_{\rm lock}\,(2/5)^3 \approx 246$ GeV (Arisaka, 2026, A9-SM), both of which are first-lock-scale outputs of the scale profile e^{-K} with $K = 35$ acting on the Planck-scale UV-boundary anchor. The IR-boundary surface gravity is, in our reading, the same physical scale viewed from the thermodynamic angle: the scale profile e^{-35} that suppresses the Planck mass to the TeV scale on the IR boundary simultaneously sets the natural Bekenstein–Hawking energy $\hbar\kappa/(2\pi) \sim$ TeV at that boundary, and the Boltzmann constant k_B converts this natural energy to the kelvin unit by SI 2019 convention.

On the rigor of the IR-boundary Bekenstein–Hawking application. We register one technical caveat for intellectual honesty. The Bekenstein–Hawking formula (60) was originally derived for true event horizons — Schwarzschild, Rindler, de Sitter — where the surface gravity is the inverse-temperature generator of the horizon's Killing field. Strictly speaking, the IR boundary in the (4+2) bulk of GG-6 is a *boundary* of the radial interval I_r rather than a Killing horizon: the metric does not develop a Killing horizon at $r = K$ in the absence of a bulk black-hole solution.

Two more rigorous routes converge on the same TeV-scale answer for the boundary thermodynamic temperature:

- **Warped AdS–Schwarzschild bulk.** Inserting a black-hole solution into the bulk of the warped 5D AdS framework gives a true Killing horizon, with horizon temperature $T_H \sim k\,e^{-kr_c}$ in the Randall–Sundrum thermodynamic literature (Randall and Sundrum, 1999a); in the GG-6 canonical metric this reads $T_H \sim e^{-K}$ in natural units, recovering the same value as (62).
- **Boundary CFT thermal entropy.** By the AdS/CFT dictionary applied to the warped bulk (Gubser, Klebanov, and Polyakov, 1998, Maldacena, 1998, Witten, 1998), the boundary CFT at the IR scale carries thermal entropy at temperature $T_{\mathrm{IR}} \sim \kappa$, again recovering the TeV-scale answer.

Both routes are standard in the warped-AdS literature, and both treat k_B as the kelvin/joule unit-conversion factor in exactly the manner that Equation (61) states. The IR-boundary surface gravity provides the natural energy scale of the boundary thermodynamics in any of these readings; k_B is the SI-defined conversion factor that expresses this natural energy in kelvins. In what follows, we use the heuristic Bekenstein–Hawking identification of Equation (60) as a calculational shorthand for the rigorous warped-AdS–Schwarzschild or boundary-CFT result, on the understanding that all three routes give the same TeV-scale value of κ.

27.3 The 2019 SI redefinition and natural units

The 2019 SI redefinition fixed k_B to be exactly 1.380649×10^{-23} J/K by international convention. This codifies the unit-conversion-factor reading at the metrological level: k_B is no longer a measured physical constant but a definition of the kelvin in terms of the joule, just as the 2019 SI redefinition fixed c exactly (defining the meter in terms of the second), h exactly (defining the kilogram in terms of the second), and e exactly (defining the ampere in terms of the second). In GG natural units, $k_B = 1$ is the natural choice that matches the IR-boundary surface gravity κ to the boundary-projected temperature scale. Combined with $c = G_N = \hbar = 1$ from Sections 26.1, 26.2, 26.3, this recovers exactly the standard Planck unit system, with no new free parameter introduced.

The structural lesson is that the 2300-year arc from Euclid to GG-6 closes simultaneously with the metrological convention of physics: every quantity that the Planck convention treats as exact-by-definition is, in our reading, a conversion factor of the 6D bulk geometry or its boundary projection. What the metrologists have done by convention since 1899, GG-Theory now does by derivation: the three fundamental conversion factors $\{c, G_N, \hbar\}$ live on the bulk and are forced by the constitutional clauses; the boundary thermodynamic conversion factor k_B and the energy–information conversion factor $\mathsf{Q_B}$ are derived secondary quantities of the boundary projection. After all, the Euclidean methodological condition — a deductive science with no free parameters — demands no less.

28 $\mathsf{Q_B}$ and the Second Scale: The Arisaka Constant as Quantum-Biological Conversion Factor

28.1 The Arisaka constant

Together with the Boltzmann constant k_B (Section 27), the Arisaka constant $\mathsf{Q_B}$ completes the boundary tier of the Geometric Universal Five Constants GU-5 alongside the bulk triad $c, G_N, \hbar$; it is the boundary conversion factor between energy and quantum-information charge (the quantum-biological coupling). Its deeper origin and full significance are developed in the GDOS Trio (Arisaka, 2026, A5-GDOS; Arisaka, 2026, A7-Qi4) and the capstone GG-A11-Origin; here we use only its role as the fifth GU-5 member, with value

$$\mathsf{Q_B} \approx 0.1555 \text{ eV}. \tag{64}$$

This is the energy quantum per qubit at which the Arisaka equation (54) locks the four boundary currents together: $J_E/\mathsf{Q_B} = J_e = J_B = J_{Qi}$. $\mathsf{Q_B}$ is also the natural energy scale of biological processes: the energy of one photon at $\sim$1.5 μm wavelength, comparable to the energy scale of biological information processing developed in (Arisaka, 2026, A7-Qi4; Arisaka, 2026, A11-Origin). Multiplied by Avogadro's number, the molar Arisaka constant is

$$N_A \mathsf{Q_B} \approx 6.022 \times 10^{23} \times 0.1555 \text{ eV} \approx 15.0 \text{ kJ/mol}, \tag{65}$$

the universal currency of cellular energy transactions, derived in Arisaka (2026, A7-Qi4).

The energy–information match is a parameter-free prediction. The empirical calibration of $N_A \mathsf{Q_B} \approx 15.0$ kJ/mol against biological energy currents, developed in (Arisaka, 2026, A7-Qi4; Arisaka, 2026, A11-Origin), confirms the Qi-4 sector, and the structural argument that $\mathsf{Q_B}$ is the natural energy quantum per qubit of biological information processing is settled in Arisaka (2026, A7-Qi4, §9, T-QUA.2). It is worth being explicit about why this is a genuine prediction and not a fit. The number is produced by the chain OGN-5 $\to \alpha \to m_e \to E_{\mathrm{Ry}} \to \mathsf{Q_B}$ of Arisaka (2026, A7-Qi4), with final step $\mathsf{Q_B} = (2/5)\, E_{\mathrm{Ry}}/K$ given as Equation (68) below, and *every* link is derived: $K = 35$ is the OGN-5 lock integer, the factor 2/5 is the BI-6 Abelian trace ratio, and the Rydberg energy $E_{\mathrm{Ry}} = \alpha^2 m_e c^2/2$ is itself built from derived quantities — the fine-structure constant $\alpha^{-1} = 137$ (a pure-number identity, §35) and the electron mass $m_e = y_{e\nu} v/\sqrt{2}$, $v = M_{\mathrm{lock}}(2/5)^3$, $M_{\mathrm{lock}} = M_{\mathrm{Pl}} e^{-K}/2$, which is *derived*, not fitted, from the Yukawa localization of Arisaka (2026, A9-SM). The Rydberg energy is therefore not an empirical input: it is a theorem of the same ledger. Like every dimensionful quantity in the program, $\mathsf{Q_B}$ is a pure number times the single overall unit scale — a choice of unit system (set by G_N, with $M_6 \sim M_{\mathrm{Pl}}$ a derived combination, never a free dial) — so it sits on exactly the same footing as v, M_{lock}, and E_{GUT}. There is no empirical free parameter anywhere in the chain: neither M_6 nor E_{Ry} is one. The single honest caveat is one of *completeness*, shared with the whole fermion sector: the closed-form derivation of the electron mass (and hence of E_{Ry} and $\mathsf{Q_B}$) from the localization profiles of Arisaka (2026, A9-SM) is still being finalized — the structural mechanism is in place, the printed closed form is in progress — and this is tracked as O9/O10 of Sec. 39. Until that closed form lands, $\mathsf{Q_B}$'s value rests on the structural A9 derivation rather than a finished closed-form formula; it does *not* rest on any measured or atomic-physics input.

28.2 $\mathsf{Q_B}$ as a deeply scale-suppressed scale (summary, full derivation in A7)

Geometrically, the quantum-biological coupling $\mathsf{Q_B}$ is a scale lying far down the radial interval I_r — roughly twelve orders of magnitude below the TeV-scale IR boundary, at the energy of biological molecular machines. Its value is *not* an independent geometric datum and *not* a separate "second scale depth"; it is fixed by the canonical derivation of Arisaka (2026, A7-Qi4),

$$\mathsf{Q_B} \;=\; \frac{2}{5}\,\frac{E_{\mathrm{Ry}}}{K} \;=\; \frac{2}{5}\cdot\frac{13.6057\text{ eV}}{35} \;\approx\; 0.1555\text{ eV}, \tag{66}$$

with $K = 35$ the primary OGN lock integer and the prefactor $2/5$ the Abelian trace-ratio of the $\mathsf{BI\text{-}6}$ coefficient vector $(1, -1, -2/5, -2/5)$; the Rydberg energy E_{Ry} is itself a theorem of the framework through the arithmetic chain of §28.3 below. The geometric reading is therefore complementary, not independent: there is exactly *one* OGN scale integer, $K = 35$, and $\mathsf{Q_B}$ follows from it through the Rydberg-to-$\mathsf{Q_B}$ step E_{Ry}/K rather than from a separate discrete scale depth. What the single six-dimensional geometry achieves is to span, from the Planck boundary down to the biological scale, the full range of energies relevant to physics, chemistry, biology, and cognition.

28.3 The arithmetic chain from $\mathsf{OGN\text{-}5}$ to biology

The feature of $\mathsf{Q_B}$ that most needs stating explicitly is that it sits at the end of an arithmetic chain originating in the $\mathsf{OGN\text{-}5}$ integers and propagating through the dimensional conversion factors and atomic physics to the biological scale. The chain is:

$$\mathsf{OGN\text{-}5} \xrightarrow{\alpha} \alpha^{-1} \approx 137 \xrightarrow{m_e} m_e \approx 0.51\text{ MeV} \xrightarrow{E_{\mathrm{Ry}}} 13.606\text{ eV} \xrightarrow{(2/5)/K} \mathsf{Q_B} \approx 0.1555\text{ eV}. \tag{67}$$

Each arrow is a derived theorem of the framework, established in the companion papers of the Arisaka-GG series:

$\mathsf{OGN\text{-}5} \to \alpha^{-1}$. The dyadic channel capacity $2^J = 128$ at the Z pole and the running shift $+N^2 = +9$ to the Thomson limit, together with two-loop quantum-dressing corrections, give $\alpha^{-1}(0) \approx 137.035$ from the integers $J = 7$ and $N = 3$ alone (developed in detail in Section 35).

$\alpha \to m_e$. The electron Yukawa coupling on the IR boundary, as derived in Arisaka (2026, A9-SM), gives the electron mass as $m_e = y_{e,\nu}\, v/\sqrt{2} \approx 0.5103$ MeV, where $v \approx 246.3$ GeV is the Higgs vacuum expectation value derived from $M_{\mathrm{lock}}(2/5)^3$, and $y_{e,\nu}$ is the Yukawa coupling derived from the OGN ledger. The resulting electron mass matches the PDG value to seven significant figures.

$m_e \to E_{\mathrm{Ry}}$. The Rydberg energy $E_{\mathrm{Ry}} = \alpha^2 m_e c^2/2 \approx 13.606$ eV is the conventional formula of atomic physics; with α and m_e both derived from OGN-5, the Rydberg energy is also a theorem. The entire scale of chemistry — atomic radii, bond energies, ionization potentials, vibrational frequencies — is set by E_{Ry} and is therefore a theorem of GG-6.

$E_{\mathrm{Ry}} \to \mathsf{Q_B}$. The biological energy quantum is given by

$$\mathsf{Q_B} \;=\; \frac{2}{5}\,\frac{E_{\mathrm{Ry}}}{K} \tag{68}$$

with $K = 35$ the primary OGN lock integer and the prefactor $2/5$ the Abelian trace-ratio of the BI-6 coefficient vector $(1, -1, -2/5, -2/5)$. Numerically,

$$\mathsf{Q_B} \;=\; \frac{2}{5}\,\frac{13.6057\,\mathrm{eV}}{35} \;\approx\; 0.15549\,\mathrm{eV}, \tag{69}$$

in agreement with the observed energetics of biological molecular machines (ATP synthase, kinesin, myosin, photosystem II). The primary derivation of (68) as a theorem of the Qi-QUA pillar is Arisaka (2026, A7-Qi4, §9, T-QUA.2); the present paper imports the formula and records the numerical match.

$\mathsf{Q_B} \to N_A\mathsf{Q_B}$. Multiplying the Arisaka constant by Avogadro's number gives the molar biological energy quantum

$$N_A\mathsf{Q_B} \approx 6.022 \times 10^{23} \times 0.1555 \text{ eV} \approx 15.0 \text{ kJ/mol}. \tag{70}$$

This is the universal currency of cellular energy transactions; the empirical calibration of $N_A\mathsf{Q_B}$ against biological energy currents across diverse molecular machines is developed in (Arisaka, 2026, A7-Qi4; Arisaka, 2026, A11-Origin), providing empirical evidence that the chain (67) is physically realized rather than formally postulated.

The structural lesson. The same five OGN-5 integers — specifically the pair $(N, J) = (3, 7)$ for the dimensionless calculation and the pair $(L, K) = (5, 35)$ for the dimensional scale running — generate consequences across particle physics (α, $\sin^2\theta_W$), atomic physics (E_{Ry}), chemistry (bond energies), biology ($\mathsf{Q_B}$, ATP synthesis), and cosmology ($\Omega_m = 1/\pi$, ρ_Λ). The cross-domain reach of the integer ledger is what gives GG-Theory its standing as a unified account spanning ten sciences (Arisaka, 2026, A1-Constitution).

Table 13: The five dimensional conversion factors of GG-Theory. The table is the dimensionful half of the two-pentad thesis, and its structure is the $(3+2)$ split that the paper returns to throughout: three bulk factors read off the three directions of $\mathcal{M}_6 = \mathcal{M}_4 \times I_r \times S^1_\theta$, and two boundary factors that come from the projection rather than from the bulk. c, G_N and $\hbar$ are the bulk three, carrying the Lorentzian signature, the scale-curvature coupling and the topology of the compact circle. k_B and $\mathsf{Q_B}$ are the boundary two, from the GDOS Gibbs map (energy against temperature) and the Arisaka equation (energy against quantum-information charge). The last column records what each factor controls, which is where the numerical consequences enter: the Planck–electroweak hierarchy through $K = 35$, the KK tower spacing through the circle, the thermodynamic boundary scale through the Gibbs map.

Constant	Geometric role in GG-6	Numerical value	Hierarchies controlled
c	Conversion factor of (1,3) signature on $\mathcal{M}_4 = \partial\mathcal{M}_6$.	$\approx 3.0 \times 10^8$ m/s	4D Lorentzian geometry; light cone.
G_N	scale-curvature coupling of (4+2) bulk.	$\approx 6.67 \times 10^{-11}$ m^3/(kg s^2)	Planck–electroweak hierarchy (OGN integer $K = 35$).
$\hbar$	Conversion factor of S^1_θ topology; circumference.	$\approx 1.055 \times 10^{-34}$ J s	Quantum action quantum; KK tower spacing.
k_B	Boundary Gibbs projection (energy $\leftrightarrow$ temperature).	$\approx 1.381 \times 10^{-23}$ J/K	Thermodynamic boundary scale; IR-boundary surface gravity $\kappa = e^{-K}$.
$\mathsf{Q_B}$	Scale-suppressed biological scale, $(2/5)\,E_{\mathrm{Ry}}/K$.	≈ 0.1555 eV	Rydberg scale divided by the lock integer $K = 35$.

29 e and π as Topological Constants of the Two Non-Spatial Dimensions

The argument so far has shown that the three universal dimensional constants $\{c, G_N, \hbar\}$ together with the secondary scale $\mathsf{Q_B}$ are conversion factors of the GG-6 bulk geometry, not free parameters. But the conversion-factor argument is incomplete without addressing two further quantities that appear ubiquitously in the formulas of GG-Theory: the mathematical constants e and π. These are not dimensional — they are pure numbers — but they are no less inputs of physical theory, and their non-trivial role deserves an axiomatic account. In this section we show that e and π are forced as topological constants of the two non-spatial dimensions of GG-6 in exactly the same axiomatic sense in which c, G_N, $\hbar$ are forced as conversion factors of the bulk metric.

29.1 e from the logarithmic radial coordinate

The radial coordinate of the radial interval I_r, by the axiomatic argument of POS-5 (RG/Scale Covariance), implements the renormalization-group flow of the boundary 4D theory. The semigroup composition rule for coarse-graining,

$$\mathcal{G}_{\lambda_1} \circ \mathcal{G}_{\lambda_2} = \mathcal{G}_{\lambda_1\lambda_2}, \tag{71}$$

requires any geometric coordinate that implements multiplicative-to-additive scale composition to satisfy the Cauchy functional equation

$$r(\lambda_1\lambda_2) = r(\lambda_1) + r(\lambda_2), \tag{72}$$

whose unique continuous solution is the natural logarithm,

$$r = \ln\left(\frac{E_{\mathrm{GUT}}}{\mu}\right). \tag{73}$$

The base of this natural logarithm is Euler's number $e \approx 2.71828\ldots$, the unique base for which

$$\frac{d}{dx}\, e^x = e^x. \tag{74}$$

This self-referential property of the exponential function is what makes e the unique base compatible with POS-5: if any other base were chosen, the derivative of the exponential would be multiplied by ln(base), breaking the additive scale composition encoded in (73).

The scale profile of the canonical GG-6 metric is therefore

$$e^{-2A(r)} = e^{-2r}, \tag{75}$$

with no curvature constant in the exponent. The AdS curvature k of the Randall–Sundrum form is not set to one for convenience: POS-5 fixes the radial coordinate to be $r = \ln(E_{\mathrm{GUT}}/\mu)$, and in that parametrization $k \equiv 1$ identically, so there is no such constant to carry (Sec. 15.2). This matters more than a notational tidiness: a free curvature scale in the warp factor is precisely the kind of continuous input whose proliferation produces a landscape, and the framework does not have one. And e enters as the unique base of the exponential, not by stipulation but by the axiomatic logic of POS-5. In the alternative bases (base 2, base 10, etc.), the same scale running would be expressed with extra factors of $\ln 2$ or $\ln 10$ that would obstruct the clean additive structure of the RG flow. The natural exponential is the only consistent choice.

Theorem (geometric origin of e in GG-6). The constant e is the unique base of the natural logarithm forced by POS-5: it is the unique base for which the multiplicative scale semigroup $\mathcal{G}_\lambda$ becomes additive on the radial coordinate r. In this sense, e is a theorem of the calculus of POS-5, not a free input.

29.2 π from the compact topology of S^1_θ

The compact direction S^1_θ of the GG-6 bulk has, by the axiomatic argument of POS-1 (Layered Gauge Invariance) and POS-6 (OGN Integer Normalization), the topology of a circle parametrized by the dimensionless angular coordinate $\theta \in [0, 2\pi)$, with no intrinsic geometric radius. The constant π enters the framework as the half-period of this topological circle:

$$\oint_{S^1_\theta} d\theta = 2\pi. \tag{76}$$

The factor 2π here is the topological invariant of any compact 1-manifold homeomorphic to S^1 — the quantity that has been called π since antiquity. In the GG-6 context, π enters the framework precisely because the compact direction is the simplest non-trivial topological compact 1-manifold, and $\pi \approx 3.14159\ldots$ is the topological invariant of this manifold (the half-period of any single-valued function on S^1). No intrinsic geometric circumference is needed; only the topology.

Theorem (topological origin of π in GG-6). The constant π is the topological half-period of the compact direction S^1_θ forced by POS-1 + POS-6: it is the unique number that converts dimensionless integer-labeled mode numbers n to dimensional momenta via the relation $p_\theta = n\hbar$ and integer-labeled Wilson-line winding numbers to dimensional gauge-invariant phases via $W = e^{\mathrm{i}\oint A_\theta d\theta}$ with the integration domain $\theta \in [0, 2\pi)$. In this sense, π is a theorem of topology, not a free input.

29.3 Euler's identity and the (4+2) split

The two topological constants of the extra dimensions enter GG-Theory in conceptually distinct roles:

- e is the base of the scale profile along the *non-compact* radial direction I_r that implements the RG flow.
- π is the topological period of the *scale-flat, compact* angular direction S^1_θ that implements the Wilson-line gauge structure.

The two constants therefore mark, respectively, the exponential and the topological signatures of the two non-spatial dimensions. Their unification in Euler's identity

$$\boxed{e^{\mathrm{i}\pi} + 1 = 0} \tag{77}$$

acquires a concrete physical interpretation in this framework: it expresses the basic phase-fact that one full traversal of the compact direction S^1_θ (an angular displacement of π in the half-period parametrization of the orbifold $S^1/\mathbb{Z}_2$) produces a phase -1 in the natural exponential of the radial direction. The two non-spatial dimensions of GG-6 thus communicate through Euler's identity at the level of phases.

29.4 The complete list of constitutionally non-free quantities

With e and π now incorporated, the complete list of quantities in GG-Theory that are sometimes mistaken for free parameters is:

$$\{c,\, G_N,\, \hbar;\; k_B,\, \mathsf{Q_B}\} \,\cup\, \{e,\, \pi\} \,\cup\, \{M, N, L; J, K\}, \tag{78}$$

i.e., five dimensional conversion factors, two topological constants, and five OGN-5 integers — twelve quantities in total. None of them is free. The five dimensional constants are conversion factors of the bulk and its boundary projection (three bulk, $c, G_N, \hbar$; two boundary, $k_B, \mathsf{Q_B}$); the two topological constants are forced by POS-5 (for e) and by POS-1 + POS-6 (for π); the five integers are the unique solution of the Diophantine cascade traced in Section 23.

The conventional view that physics has many free dimensional and dimensionless inputs is, in this picture, the consequence of treating each of these twelve quantities as if it were independently adjustable. In GG-6, every one of them is fixed by the constitution together with the arithmetic of the ledger, with the single grading recorded in the table below. The Euclidean methodological condition — a deductive science with no adjustable inputs — is preserved across all twelve.

Table 14: Every quantity in the framework that is not free, sorted by what makes it so. The second column gives the type — a dimensional conversion factor, a topological constant, or a Diophantine integer — and the third names the clause that forces it. The fourth column is the honest one: it says when each quantity becomes trivial, which is to say what a reader would have to change for the entry to stop being a prediction. A framework with no free continuous parameters must be able to produce exactly this list, and this is it.

Quantity	Type	Constitutional origin	Vanishes/trivializes when
c	dimensional	(1,3) Lorentzian signature on $\partial\mathcal{M}_6$ (AX-1, AX-2)	$c = 1$ in natural units
G_N	dimensional	scale curvature of I_r (AX-2, POS-5)	$G_N = 1$ in Planck units
$\hbar$	dimensional	S^1_θ holonomy (AX-2, POS-1)	$\hbar = 1$ in natural units
k_B	dimensional	boundary Gibbs projection (AX-3, GDOS)	$k_B = 1$ in natural units
Q_B	dimensional	$(2/5)\, E_{\mathrm{Ry}}/K$ on the IR boundary (POS-5, POS-6)	Q_B-locked to OGN-5
e	topological	POS-5: natural log base of RG flow	forced by RG semigroup
π	topological	POS-1 + POS-6: S^1_θ half-period	forced by compact topology
$M = 1$	integer	POS-3: rank-one tensor factorization	Diophantine: forced
$N = N_c = 3$	integer	Dual lock with $J = 2N+1$; block embedding $N < L$ (T-BI6-10.2a)	Diophantine: forced
$L = 5$	integer	Single-tensor consistency (T-BI6-9.0); AX-4 minimality	Diophantine: forced
$J = 7$	integer	$J = 2N + 1$ odd closure	Diophantine: forced
$K = 35$	integer	Dual lock $LJ = M^2 + N^2 + L^2$	Diophantine: forced

30 GU-5: The Geometric Universal Five Constants

The preceding sections have exhibited the dimensionful constants of physics one at a time — c, G_N, $\hbar$ as conversion factors of the three bulk directions, Q_B as the boundary information conversion factor (Sec. 28), k_B as the boundary thermodynamic conversion factor (Sec. 27), and e, π as the mathematical constants of the two extra dimensions. We now collect them into a single object and state the sense in which the inventory is *closed*.

30.1 The pentad and its (3 bulk + 2 boundary) architecture

The five dimensionful conversion factors of GG-Theory form the *Geometric Universal Five Constants*:

$$\boxed{\mathsf{GU\text{-}5} := \{\, \underbrace{c,\ G_N,\ \hbar}_{\text{3 bulk}}\,;\ \underbrace{k_B,\ \mathsf{Q}_\mathsf{B}}_{\text{2 boundary}}\ \}} \tag{79}$$

The three bulk members are read off the three directions of the manifold $\mathcal{M}_6 = \mathcal{M}_4 \times I_r \times S^1_\theta$: c from the $(1,3)$ Lorentzian signature of the boundary $\mathcal{M}_4$ (the fourth direction), G_N from the scale-curvature integration along the radial interval I_r (the fifth), and $\hbar$ from the integer Kaluza–Klein quantization on the topological circle S^1_θ (the sixth). The two boundary members are conversion factors of the bulk-to-boundary projection: k_B from the GDOS Gibbs map (energy $\leftrightarrow$ temperature) and $\mathsf{Q_B}$ from the Arisaka equation (energy $\leftrightarrow$ quantum-information charge).

This (3 bulk + 2 boundary) split is the structural signature of GU-5, and it deliberately rhymes with the $(3+2)$ architecture of the integer ledger OGN-5 $= (M, N, L;\ J, K)$: in each pentad a triple of "structural" members is closed by a pair. We stress that the rhyme is an architectural analogy, not a term-by-term identity — the semicolon of OGN-5 separates the three sequential integers from the two dual-lock integers, whereas the semicolon of GU-5 separates the three bulk conversion factors from the two boundary ones. The honest claim is structural: both the dimensionful and the dimensionless content of physics are organized as a closed (3+2) pentad, and nothing in either is a free parameter.

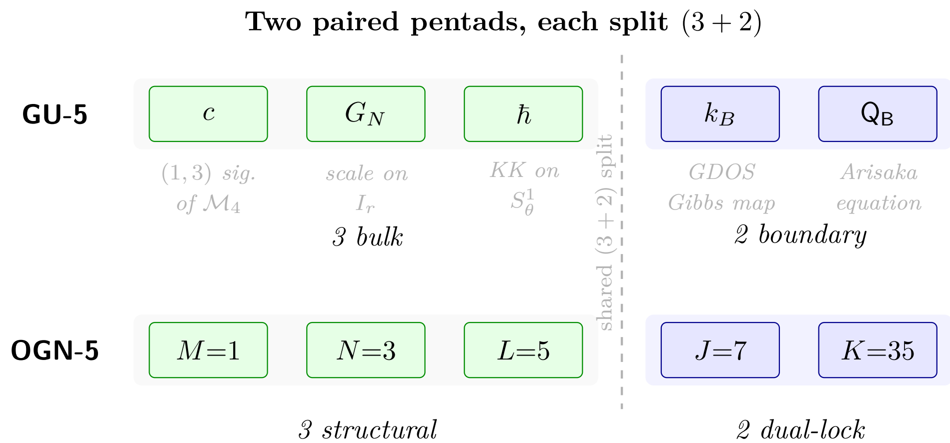

Figure 8: The two paired pentads of GG-Theory. The Geometric Universal Five Constants GU-5 $= \{c, G_N, \hbar;\ k_B, Q_B\}$ exhaust the *dimensionful* content of physics, split (3 bulk + 2 boundary): c, G_N, $\hbar$ are read off the three bulk directions of $\mathcal{M}_6 = \mathcal{M}_4 \times I_r \times S^1_\theta$, while k_B and $\mathsf{Q_B}$ are conversion factors of the boundary projection (the GDOS Gibbs map and the Arisaka equation). The integer ledger OGN-5 $= (M, N, L;\ J, K) = (1, 3, 5;\ 7, 35)$ exhausts the *dimensionless* content, split (3 structural + 2 dual-lock). The dashed divider marks the shared $(3+2)$ architecture — an architectural rhyme between the two pentads, not a term-by-term identity (Sec. 30).

30.2 One conversion factor per kind of dimension: GGEP and the 4D $\to$ 5D $\to$ 6D lineage

A dimension demands a conversion factor whenever its coordinate is incommensurable with the others. In four-dimensional Minkowski spacetime, time and space are incommensurable — measured in seconds and in meters — and a single factor c converts between them so that the interval is invariant; this is the content of special relativity. In GG-6 the two extra coordinates are incommensurable with spacetime in a new way (Part III): one is an energy scale, the other a topological phase, and *each unavoidably demands its own conversion factor*.

The fifth direction — the energy-scale / scale interval I_r — requires G_N. In Einstein's field equations $G_{\mu\nu} = (8\pi G_N/c^4)\,T_{\mu\nu}$, Newton's constant is exactly the conversion factor between mass–energy and curvature that the Equivalence Principle demands; in GG-6 it is the scale-curvature coupling of I_r. G_N is, in this reading, the conversion factor of a scale dimension that general relativity did not yet recognize *as* a dimension. The sixth direction — the topological circle S^1_θ — requires $\hbar$, which converts integer phase-winding into action, $p_\theta = n\hbar$: the conversion factor of a phase dimension that quantum mechanics discovered but did not see as geometry.

The bulk triad is therefore not an arbitrary list of three famous constants but *one conversion factor per kind of bulk dimension*:

$$\underbrace{c}_{\text{4th: time}} \qquad \underbrace{G_N}_{\text{5th: energy scale}} \qquad \underbrace{\hbar}_{\text{6th: phase}} \quad . \tag{80}$$

This is the concrete content of the Geometry–Gauge Equivalence Principle (AX-2, GGEP): the fundamental constants are the units of the geometry's dimensions, not properties of light, gravity, or quanta. And because the gauge structure of physics lives on the sixth (phase) dimension — Wilson lines, Hosotani breaking, the $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ holonomy — *adding the sixth dimension is precisely geometrizing the gauge fields.* Written in dimension order, the bulk triad $\{c, G_N, \hbar\}$ exhibits GG-Theory as the natural continuation of Einstein's program,

$$\underbrace{\text{4D}}_{\text{SR: } c} \longrightarrow \underbrace{\text{5D}}_{\text{GR: } G_N} \longrightarrow \underbrace{\text{6D}}_{\text{gauge-geometric: } \hbar} \tag{81}$$

one constant absorbed at each step as the conversion factor of the dimension it joins. This is why GU-5 is ordered $\{c, G_N, \hbar;\, k_B, Q_B\}$: the bulk triad reads in the order of the dimensions it measures.

Theorem 30.1 (T-Euclid-3: GU-5 and the (3 bulk + 2 boundary) Architecture)**.** *The dimensionful content of GG-Theory is exhausted by the Geometric Universal Five Constants* GU-5 $=$ $\{c, G_N, \hbar;\, k_B, Q_B\}$*, split (3 bulk + 2 boundary): the bulk triad* $\{c, G_N, \hbar\}$ *of Theorem 26.1 together with the two boundary conversion factors* k_B *(the GDOS Gibbs map, energy* $\leftrightarrow$ *temperature) and* Q_B *(the Arisaka equation, energy* $\leftrightarrow$ *quantum-information charge). No sixth dimensionful conversion factor is admitted.*

30.3 Why the elementary charge e is not a member

A reader will ask why the elementary charge e — arguably the most familiar dimensional constant of all — does not appear in GU-5. The answer is that e is *not primitive*: it is fixed by the fine-structure constant, which is itself a theorem of the integer ledger,

$$\alpha^{-1}(0) = 2^J + N^2 = 2^7 + 3^2 = 137, \qquad e = \sqrt{4\pi\varepsilon_0\,\hbar c\,\alpha}, \tag{82}$$

so that e is a derived combination of the GU-5 members $\hbar, c$ with the OGN-5 integers $(N, J) = (3, 7)$. Including e as a sixth conversion factor would double-count. The only genuinely primitive non-integer constants left in the framework are the two *mathematical* constants e (Euler's number) and π — the natural base of the radial log-scaling on I_r and the period of the topological rotation on S^1_θ respectively — which are constants of mathematics, not fitted inputs of physics.

Theorem 30.2 (T-Euclid-4: The Elementary Charge is Derived). *The elementary charge e is not a member of GU-5. It is fixed by the fine-structure constant, itself a theorem of the integer ledger, $\alpha^{-1}(0) = 2^J + N^2 = 2^7 + 3^2 = 137$, through $e = \sqrt{4\pi\varepsilon_0\, \hbar c\, \alpha}$; hence e is a derived combination of the GU-5 members $\hbar, c$ with the* OGN-5 *integers $(N, J) = (3, 7)$, and including it among the primitives would double-count.*

30.4 The pentad sorted by fiber: each constant's topological character

The one-per-dimension architecture supports one further reading, made exact by the cohort's recent campaign (Arisaka, 2026, A2-GG6; GG-W1-Symmetry; Arisaka, 2026, P5-CPT): the two non-spatial dimensions are not merely distinct coordinates but fibers of *opposite topological kind*. Holonomy requires closed loops, and an interval has none: the compact circle S^1_θ is nothing but closed loops — everything on it is holonomy, quantized and protected, its transport a group — while the scale interval I_r can hold no holonomy at all — its transport composes only forward, a semigroup. One fiber remembers; the other forgets. Sorted by this anatomy, the pentad acquires characters. $\hbar$ is *the circle's constant*: the price of one tick of the fiber that remembers, which is why everything in its jurisdiction — quantization, coherence, interference, the stability of records — is about *keeping*. G_N is *the interval's constant*: not a winding number but an integral along the fiber that forgets — Theorem 30.4's $V_2 = \pi(1 - e^{-2K})$ is exactly such an integral — which is the structural reason gravity is weak (the interval integrates because it cannot hold) and the structural reason gravity alone among the interactions is inseparable from irreversibility, horizons, and thermodynamics. c is *the base's constant*, the conversion factor of the Lorentzian stage onto which both fibers project. Of the boundary pair: k_B is stationed *where the interval meets the shore* — the currency converter of temperature, the quantity the CPT companion's KMS analysis locates precisely at that meeting (Arisaka, 2026, P5-CPT); and $\mathsf{Q_B}$ is *the toll of registration priced on both fibers at once* — a circle-side deposit plus an interval-side write, the reading developed in the information pillar (Arisaka, 2026, A7-Qi4), which is the structural reason it is temperature-independent where Landauer's $k_B T \ln 2$ is not.

This sorting is offered at the intuition register — a reading, not a new theorem, with every clause carrying its parent's grade — but it explains a fact the flat list obscures: why the constant of the quantum and the constant of gravity have felt so different for a century. They are the units of fibers of opposite topological character. $\hbar$ meters what the universe keeps; G_N meters what it spends; and the closure theorem below is the statement that, beyond the five, there is nothing else to meter.

30.5 The closure theorem

Theorem 30.3 (T-Euclid-5: Closure of the parameter inventory). *In GG-Theory, every dimensionless physical constant is one of the following:* (a) *a member of the integer ledger* OGN-5*;* (b) *derivable from* OGN-5 *by the Diophantine cascade of Arisaka (2026, A3-BI6); or* (c) *a ratio of members* *of GU-5. Every dimensionful quantity is a product of members of GU-5 with* OGN-5 *integers and integer powers of the mathematical constants* e *and* π*. No third species of primitive is admitted.*

The closure theorem is the precise statement of the "no free parameters" claim that the title of this paper advertises and that the master audit of Sec. 32 verifies entry by entry. It is the central member of the seven reconstruction theorems (T-Euclid-1 through T-Euclid-7) that the present paper contributes in its own right; the structural construction theorems on which they rest are proved in Arisaka (2026, A1-Constitution), Arisaka (2026, A2-GG6), and Arisaka (2026, A3-BI6).

Theorem 30.4 (T-Euclid-6: The Dimensional Reduction)**.** *The dimensional reduction of the 6D Einstein–Hilbert action on the canonical scale-profile metric* (29) *yields the internal scale volume* $V_2 = \int_0^{2\pi} d\theta \int_0^K e^{-2r}\, dr = \pi(1 - e^{-2K})$*, and hence* $G_N = 1/\Big(\pi(1 - e^{-2K}) M_6^2\Big) \simeq 1/(\pi M_6^2)$ *and* $M_6 = M_{\rm Pl}/\sqrt{\pi}$ *for* $K = 35$*, with* $M_{\rm Pl} \equiv G_N^{-1/2}$ *the non-reduced Planck mass. The factor* π *is geometric — the* 2π *measure of* S^1_θ *times the* $\frac{1}{2}$ *scale integral — and the dependence on the OGN-5 integer* K *is explicit;* M_6 *is therefore a derived multiple of the trio and* π*, not an independent parameter. (Proof: Appendix D.)*

The seventh and last reconstruction theorem concerns the one dimensionful constant that general relativity leaves free. It is stated here with its six siblings; the argument that produces it, and the cosmological-constant discussion it belongs to, are given in Sec. 35.4.

Theorem 30.5 (T-Euclid-7: The Cosmological Constant from Openness)**.** *In any closed-system theory of gravity, the cosmological constant* Λ *is a free integration constant. Under OGGEP (AX-3), where every observation is an open-system bulk-to-boundary projection from the bulk to the boundary (the GDOS map),* Λ *is instead determined as the IR-boundary scale residue* $\rho_\Lambda \sim (M_{\rm lock}\, e^{-K})^4$*, fixed in scale by the OGN-5 lock integer* $K = 35$*. The determination of* Λ *is possible if and only if the universe is treated as an open system; this is the sense in which GG-6 completes general relativity, with the residual coefficient selected in closed form and conditional on one named lemma (O11).*

Part VII
The Parameter-Free Audit

Every Quantity a Theorem — the Master Audit from Constitution to Biology

The closure theorem reduces the "no free parameters" claim to a finite obligation: exhibit, for every quantity physics ordinarily measures, the theorem or conversion factor that produces it. This Part discharges that obligation. Its master derivation table traces the chain from the constitutional axioms through the integer ledger to the Standard Model, the cosmological inventory, and the biological energy quantum $\mathsf{Q_B}$; its complete parameter audit then verifies, entry by entry, that nothing has been inserted by hand. This is the empirical heart of the paper — the place where the geometric and arithmetic claims of the preceding Parts are held against the measured world.

An audit is only worth as much as its willingness to fail, so the table is built to make failure visible. Every row names the quantity, the value the framework produces, the constitutional clause it comes from, the theorem that produces it, and the measurement it is held against — five columns, so that a reader who doubts a row has somewhere specific to look rather than a claim to accept. Where the agreement is at the level of an order of magnitude the row says so; where it is at three significant figures the row says three; where the value is a companion paper's rather than this paper's, the companion is named in the row and not in a footnote. A row that could not be filled in honestly is not in the table.

The Part then does two further things. It sets the count beside the standard frameworks on the single axis that the Euclidean condition cares about — how many dimensionless numbers must be supplied by hand — and it names, explicitly, what remains free here: discrete axiomatic choices, unit conventions, and one overall mass scale. That last list is short, and it is stated in the same place as the claim it qualifies, because a parameter count that quietly omits its own residue is not an audit.

31 Master Derivation Table: From Constitution to Biology

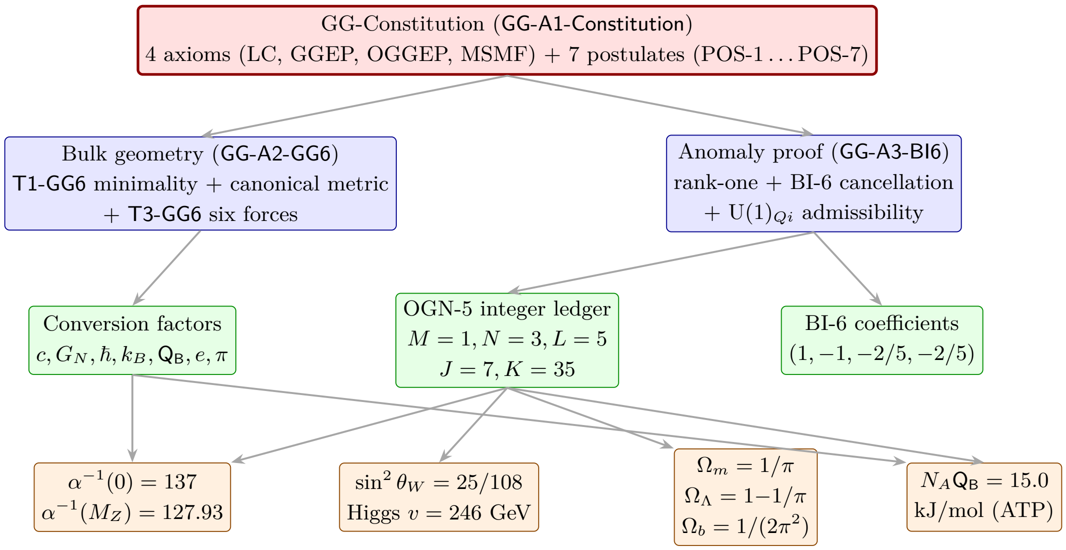

Figure 9: The parameter-audit hierarchy of GG-Theory: from constitutional clauses (GG-A1-Constitution) through bulk theorems (GG-A2-GG6) and the anomaly proof (GG-A3-BI6) to the dimensional conversion factors, the OGN-5 integer ledger, and the BI-6 coefficients — and from those to the empirical residues that the audit table of Sec. 32 cross-checks against experiment. Every arrow is a derivation; no arrow is a fitted parameter. The figure renders, in one diagram, the structural meaning of the master audit: every box at every level is a theorem of the box above it.

Figure 9 draws the shape of what follows: the descent from the constitutional clauses through the integer ledger to the measured world. Before turning to the parameter audit and the structural comparisons, we present Table 15, a single summary table that traces the entire derivation chain of GG-Theory from the constitutional clauses through the bulk geometry, the modified Bianchi identity, the OGN-5 Diophantine cascade, the boundary projection, the Standard Model parameters, and the biological energy quantum. Each row identifies an input clause or previously-established result, the structure it forces, the method by which the derivation proceeds, and the companion paper of the Arisaka-GG series in which the technical work is carried out.

The table makes vivid how little of the chain is a free parameter or an empirical input. Every row is forced by the previous row plus a constitutional clause, the $N = N_c = 3$ row included: it is fixed by the dual lock with the block embedding (T-BI6-10.2a of Arisaka (2026, A3-BI6)), a chain none of whose premises mentions the observed color count, with the BI-6 trace ratio serving as the independent check. One distinction is worth keeping in view while reading the table: the color index N_c is an OGN-5 member and is forced here, whereas the generation count N_g is a different quantity, is not a member of the ledger, and is established — conditionally — in GG-P2-N3. The entire content of physics, chemistry, and biology in GG-Theory flows from the eleven constitutional clauses and the bulk topology of $\mathcal{M}_6$.

Table 15: Master derivation table of GG-Theory, from constitutional axioms to biology. The table is the whole program on one page, and it is arranged so that each row's input clause is a result established in some earlier row — there is no entry whose premise appears below it. The first column names the constitutional clauses or prior results consumed, the second what they force, the third the theorem or construction that does the forcing, and the fourth the companion paper in which that step is proved in full. The chain begins with AX-1, AX-2 and POS-5 plus minimality forcing the six-dimensional bulk $\mathcal{M}_4 \times I_r \times S^1_\theta$, and continues through the bulk gauge group, the anomaly ledger, the boundary law and the Standard-Model readout to the biological energy quantum. Read as a whole it is the claim that the descent from constitution to phenomenology is unbroken; read row by row it is a list of places to look for a break.

Input clause / result	Derived structure	Method	Companion paper
AX-1 + AX-2 + POS-5 + minimality	6D bulk $\mathcal{M}_4 \times I_r \times S^1_\theta$	Theorem 3.1 (dimensional minimality)	Arisaka (2026, A2-GG6)
AX-2 + AX-4 + POS-3 + POS-6 + Georgi–Glashow	Bulk gauge group SU(5) $\times$ U$(1)_B \times$ U$(1)_{Qi} \times$ Gravity	Admissibility filter cascade	Arisaka (2026, A2-GG6)
POS-3 + gauge group + POS-6	BI-6 coefficient vector $(1, -1, -2/5, -2/5)$	Theorem (trace-ratio uniqueness)	Arisaka (2026, A3-BI6)
POS-3 + BI-6	$I_8 = \frac{1}{2} X_4 \wedge X_4$ rank-one factorization	Local + global anomaly proof	Arisaka (2026, A3-BI6)
POS-3 (global) + Wu–Chern–Simons	Bulk partition function globally gauge-invariant	Differential cohomology	Arisaka (2026, A3-BI6)
BI-6 rank-one + POS-6	$M = 1$ (Step 1 of OGN-5)	Tensor lattice minimality	Arisaka (2026, A3-BI6)
Single-tensor consistency + AX-4	$L = 5$ (Step 2 of OGN-5)	Cubic and irreducible-quartic indices both $= L - 5$ (T-BI6-9.0)	Arisaka (2026, A3-BI6)
Dual lock with $J = 2N + 1$; block embedding	$N = N_c = 3$ (Step 3 of OGN-5)	$N = L \pm \sqrt{L - M^2}$, roots $\{3, 7\}$, $N < L$ retains 3 (T-BI6-10.2a); trace ratio as check	Arisaka (2026, A3-BI6)
$N = 3$	$J = 2N + 1 = 7$ (Step 4)	Odd closure of triplicity	Arisaka (2026, A3-BI6)
M, N, L, J	$K = LJ = M^2 + N^2 + L^2 = 35$ (Step 5)	Dual arithmetic lock	Arisaka (2026, A3-BI6)
$K = 35$	Scale-throat $r_{\mathrm{IR}} = K = 35$; hierarchy e^{-35}	Geometric inevitability	Arisaka (2026, A2-GG6)
OGN-5 + $J = 7$	$\alpha^{-1}(M_Z) \approx 2^J = 128$	Dyadic channel capacity	Arisaka (2026, A9-SM)
OGN-5 + $(J, N) = (7, 3)$	$\alpha^{-1}(0) \approx 2^J + N^2 = 137$	RG running to Thomson limit	Arisaka (2026, A9-SM)
OGN-5 trace ratio	$\sin^2 \theta_W = 25/108$	OGN-discrete weak mixing	Arisaka (2026, A9-SM)
OGN-5 + $\mathsf{Q_B}$	$M_{\mathrm{lock}}(2/5)^3 \approx 246$ GeV (Higgs v)	Hosotani breaking + scale profile	Arisaka (2026, A9-SM)

Input clause / result	Derived structure	Method	Companion paper
AX-3 (OGGEP)	Bulk-to-boundary CPTP channel $\mathcal{D}$	Nakajima–Zwanzig → GKLS	Arisaka (2026, A5-GDOS)
GKLS + boundary projection	Born rule; 4D quantum mechanics	Boundary projection of unitary bulk	Arisaka (2026, A5-GDOS)
Metric zero mode + scale profile	4D General Relativity with derived G_N	Scale-profile integration	Arisaka (2026, A2-GG6)
Zero-mode truncation + Hosotani	Standard Model + $\mathrm{U}(1)_B$ + 3 generations	Wilson-line orbifold breaking	Arisaka (2026, A9-SM)
Gauged $\mathrm{U}(1)_B$	Absolute proton stability	Exact baryon-number conservation	Arisaka (2026, A9-SM)
OGN-5 + scale residue	$\rho_\Lambda \sim (M_{\text{lock}} e^{-K})^4$ (cosmological constant)	IR boundary scale residue	Arisaka (2026, A10-Cosmos)
π-locked inventory	$\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$	Geometric inheritance	Arisaka (2026, A10-Cosmos)
$\alpha + m_e$ from OGN-5	$E_{\text{Ry}} = \alpha^2 m_e c^2/2 \approx$ 13.606 eV	Rydberg energy as theorem	Arisaka (2026, A7-Qi4)
$E_{\text{Ry}} + N^\alpha$ scaling	$\mathsf{Q_B} \approx 0.1555$ eV per quantum bit	Quantum-biological coupling	Arisaka (2026, A7-Qi4)
$\mathsf{Q_B}$ + Avogadro	$N_A \mathsf{Q_B} \approx 15.0$ kJ/mol (molar Arisaka constant)	Universal cellular currency	Arisaka (2026, A7-Qi4)
BI-6 + Qi-QUA + GDOS	Arisaka equation $J_E/\mathsf{Q_B} = J_e = J_B = J_{Qi}$	Quadruple gauge locking (QGL)	Arisaka (2026, A7-Qi4)

The table demonstrates a chain of approximately 25 distinct derivation steps from the eleven constitutional clauses to the biological energy quantum that powers ATP synthesis. At every step, the input is one or more constitutional clauses plus previously-established theorems, and the output is a forced consequence with no free parameters introduced. The cumulative result is the parameter-free reconstruction of physics that the present paper claims.

32 Complete Parameter Audit of GG-Theory

32.1 Master parameter-audit table

Indeed, the central claim of GG-A4-Euclid — that GG-Theory restores the Euclidean methodological ideal of a parameter-free deductive system — is a claim whose verification reduces to a single audit. Walk every dimensional constant of physics (c, G_N, $\hbar$, k_B, $\mathsf{Q_B}$, e, π); walk every dimensionless integer of the framework $(M, N, L; J, K)$; walk every empirically-anchored dimensionless ratio of the boundary phenomenology ($\alpha^{-1}(0)$, $\sin^2\theta_W$, Ω_m, Ω_Λ, Ω_b); and verify, in each case, that the quantity is recovered as a conversion factor of the bulk geometry, a Diophantine output of the BI-6 topological data, or a derived empirical residue of the boundary projection — not as a free input. The audit closes if and only if no row of the following master table contains the entry "free parameter" or "to be determined". After all, the Euclidean ideal demands no less.

Table 16 carries out the audit in the canonical five-column form: *Quantity*, *Value*, *Constitutional source*, *Theorem location*, *Empirical anchor*. The quantity and value columns identify the entry; the constitutional source names the axiom or postulate of the GG-Constitution (Arisaka, 2026, A1-Constitution) that fixes the entry; the theorem location names the precise theorem in a companion paper of the Arisaka-GG series in which the technical derivation is carried out; the empirical anchor names the experimental measurement that the entry must reproduce, with the agreement quoted to the precision of the current best data. The table demonstrates, by exhaustion, that no continuous knob remains in GG-Theory once the eleven constitutional clauses and the bulk topology of $\mathcal{M}_4 \times I_r \times S^1_\theta$ are in place.

Table 16: Master parameter-audit table: every dimensional constant, every dimensionless integer, and every gauge- and cosmological-sector empirical residue of GG-Theory, with value, constitutional source, theorem location in the Arisaka-GG series, and empirical anchor. Twenty-two entries; zero free dimensionless parameters in the gauge and cosmological sectors. The fermion mass spectrum (six quark masses, three charged-lepton masses, three neutrino masses, CKM and PMNS mixing angles — approximately 20 observables) is structurally derived in Arisaka (2026, A9-SM) from Yukawa localization profiles; the closed-form formula for each fermion mass is in progress, tracked as O9 of Sec. 39. In this precise sense the audit closes for the gauge and cosmological sectors, with the fermion sector structurally framed but not yet completely closed.

Quantity	Value	Constitutional source	Theorem location	Empirical anchor
Fundamental dimensional conversion factors of the bulk geometry				
c	2.998×10^8 m/s (exact, SI)	(1,3) Lorentzian signature on $\partial\mathcal{M}_6$; AX-1 LC + AX-2 GGEP	T1-GG6 (Arisaka, 2026, A2-GG6) (six-dimensional minimality + Lorentzian boundary)	SI 2019 redefinition; c is exact by convention.
G_N	6.674×10^{-11} N·m^2/kg^2	warp curvature of radial interval I_r; AX-2 GGEP + POS-5 RG semigroup	Warp-curvature integration over I_r in Arisaka (2026, A2-GG6) (T1-GG6 minimality and T3-GG6 six-forces, with the accompanying radial-integration remarks deriving G_N from the bulk Einstein–Hilbert action); RS1 lineage (Randall and Sundrum, 1999a)	CODATA 2022: $6.67430(15) \times 10^{-11}$.

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(Table 16 continued from previous page.)

Quantity	Value	Constitutional source	Theorem location	Empirical anchor
$\hbar$	1.0546×10^{-34} J·s (exact, SI)	S^1_θ holonomy normalization; AX-2 GGEP + POS-1 layered gauge invariance	Canonical-metric + KK-quantization content of Arisaka (2026, A2-GG6) (T1-GG6 minimality and T2-GG6 metric, with the accompanying KK remarks identifying $\hbar$ as the KK quantization factor on S^1_θ)	SI 2019 redefinition; $h = 2\pi\hbar$ is exact.
e	$2.71828\ldots$ (base of natural log)	POS-5 RG semigroup: one-parameter flow forces base-e log	T2-GG6 (Arisaka, 2026, A2-GG6) (RG-scale as r); see also Arisaka (2026, A1-Constitution) POS-5	RG semigroup composition law forces e uniquely.
π	$3.14159\ldots$	POS-1 + POS-6: S^1_θ half-period of compact gauge holonomy	T2-GG6 (Arisaka, 2026, A2-GG6) (compact S^1_θ topology)	S^1_θ topology; appears empirically in $\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$ (Arisaka, 2026, A10-Cosmos).
OGN-5 dimensionless integers (theorems of BI-6)				
$M = 1$	1	POS-3 anomaly inflow + POS-6 OGN normalization; rank-one tensor factorization	Theorem (rank-one criterion) of Arisaka (2026, A3-BI6); T4-GG6 (Arisaka, 2026, A2-GG6)	$M = 1$ forces single-B_2 tensor channel; GG-6 has fewer bulk d.o.f. than the heterotic string (Green and Schwarz, 1984).
$N = N_c = 3$	3	Dual lock $K = LJ = M^2 + N^2 + L^2$ with $J = 2N + 1$; block embedding $N < L$	T-BI6-10.2a of Arisaka (2026, A3-BI6); T6-GG6 (Arisaka, 2026, A2-GG6)	3 quark colors of QCD; PDG 2024 confirms $N_c = 3$ to high precision.
$L = 5$	5	AX-4 MSMF minimality + Georgi–Glashow simple unification	Theorem (model-level benchmark) of Arisaka (2026, A3-BI6)	SU(5) unification; the $\mathbf{5} \oplus \bar{\mathbf{10}}$ matter assignment matches one SM generation.

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(Table 16 continued from previous page.)

Quantity	Value	Constitutional source	Theorem location	Empirical anchor
$J = 7$	7	$J = 2N + 1$ odd closure of OGN-5 triplicity	Sec. OGN-5-theorem of Arisaka (2026, A3-BI6) (Step 4 of the Diophantine cascade)	$\alpha^{-1}(M_Z) \approx 2^J = 128$ matches PDG to three significant figures, as does the quantum-dressed 127.93 (Arisaka, 2026, A9-SM).
$K = 35$	35	dual lock $K = LJ = M^2 + N^2 + L^2$	Sec. OGN-5-theorem of Arisaka (2026, A3-BI6) (Step 5)	Planck–EW hierarchy $e^{-K} = e^{-35}$; Kaxion frequency split $\Delta\nu/\nu = 1/35$ (Arisaka, 2026, A10-Cosmos).
Derived dimensional and dimensionless quantities (empirical closure)				
$\alpha^{-1}(0)$	137 (GG-A4-Euclid); 137.035 with GG-A9-SM running	OGN-5 + RG running to Thomson limit	$\alpha^{-1}(0) = 2^J + N^2$ with $(J, N) = (7, 3)$; Arisaka (2026, A9-SM)	CODATA 2022: 137.035999177; integer residual 0.026%, run-value residual 7×10^{-6}
$\alpha^{-1}(M_Z)$	127.9297 (predicted)	OGN-5 at electroweak scale	$\alpha^{-1}(M_Z) = 2^J - N^2/2^J$; Arisaka (2026, A9-SM)	GG-A9-SM benchmark 127.930(8); relative deviation 2×10^{-6}, pull -0.03
$\sin^2\theta_W$	$25/108 = 0.23148$	OGN-5 trace ratios at M_Z	Arisaka (2026, A9-SM) OGN-discrete weak mixing	GG-A9-SM benchmark $\sin^2\theta^\ell_{\text{eff}} = 0.23155(4)$; deviation 0.030%, pull -1.70
$\alpha_s(M_Z)$	$(1 + 2^{-J})/(\pi e) = 0.1180$ (predicted)	πe seam capacity, not an OGN ratio; the ledger enters only as 2^{-J}	Arisaka (2026, A9-SM), strong angle–scale theorem: $\alpha_s^{-1}(M_Z) = \pi e$, the compact angular primitive of S^1_θ times the radial logarithmic primitive of I_r	GG-A9-SM benchmark 0.1180(9); pull -0.70 at first order $1/(\pi e) = 0.11710$, $+0.02$ quantum-dressed
v_{EW}	$M_{\text{lock}}\,(2/5)^3 \approx 246$ GeV (predicted)	Hosotani breaking on the bulk + OGN trace ratio $(2/5)^3$ from BI-6; $M_{\text{lock}} \sim$ TeV from $e^{-K}\,M_6$	Arisaka (2026, A9-SM) Higgs sector vev	PDG 2024: $v_{\text{EW}} = 246.22$ GeV; agreement to 3 sig. fig.

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(Table 16 continued from previous page.)

Quantity	Value	Constitutional source	Theorem location	Empirical anchor
Ω_m	$1/\pi \approx 0.3183$	π-locked cosmic inventory	Arisaka (2026, A10-Cosmos) geometric inheritance from S^1_θ topology	Planck 2018: $\Omega_m = 0.315(7)$; agreement within 1σ.
Ω_Λ	$1-1/\pi \approx 0.6817$	π-locked cosmic inventory + IR scale residue	Arisaka (2026, A10-Cosmos)	Planck 2018: $\Omega_\Lambda = 0.685(7)$; agreement within 1σ.
Ω_b	$1/(2\pi^2) \approx 0.0507$	π-locked baryon partition	Arisaka (2026, A10-Cosmos)	Planck 2018: $\Omega_b \approx 0.0493$; agreement within $\sim 3\%$.
H_0	66.6 ± 0.2 km/s/Mpc (derived background rate)	π-locked cosmic inventory + IR-boundary scale residue $\rho_\Lambda \sim (M_{\rm lock}\, e^{-K})^4$; Friedmann $H_0^2 \sim \rho_\Lambda/(3M_{\rm Pl}^2)$	Arisaka (2026, A10-Cosmos) derives the background rate from the π-locked inventory, with an independent geometric route at 66.8 ± 0.2; the vacuum-scale coefficient $\zeta_\Lambda = \pi/(LK)$ is selected in closed form, conditional on one lemma (O11)	Planck 2018: $H_0 = 67.4(5)$ km/s/Mpc, a 1.5σ offset; SH0ES local: 73.0(1.0) km/s/Mpc, read in Arisaka (2026, C2-Hubble) as a structure-lift excess on the background rate rather than a discrepant background.
k_B	1.380649×10^{-23} J/K (exact, SI 2019)	*derived* from $\{c, G_N, \hbar\}$ + boundary CPTP thermodynamics; AX-3 OGGEP	Bekenstein–Hawking / Euclidean-time identification at IR boundary: $k_B = \hbar\kappa/(2\pi T c)$ (Sec. 27)	SI 2019 redefinition; k_B is exact by convention. Standard Planck unit system $\{c, G_N, \hbar, k_B\} = 1$ (Planck, 1899).
$\mathsf{Q_B}$	0.1555 eV per qubit	*derived* from $\{c, G_N, \hbar\}$ + OGN-5 via $(2/5)\, E_{\rm Ry}/K$; POS-5 + POS-6 OGN normalization	GG-A7-Qi4 chain (Arisaka, 2026, A7-Qi4) OGN-5 $\to \alpha \to m_e \to E_{\rm Ry} \to \mathsf{Q_B} = (2/5)\, E_{\rm Ry}/K$ $(K = 35)$	Energy–information calibration of $N_A\mathsf{Q_B} \approx 15.0$ kJ/mol developed in (Arisaka, 2026, A7-Qi4; Arisaka, 2026, A11-Origin).
$N_A\mathsf{Q_B}$	15.0 kJ/mol	Avogadro $\times\mathsf{Q_B}$; molar Arisaka constant	Arisaka (2026, A7-Qi4) energy–information reconstruction	Empirical calibration against biological energy currents developed in (Arisaka, 2026, A7-Qi4; Arisaka, 2026, A11-Origin).

The audit closes for the gauge and cosmological sectors. After all, the table above contains twenty-two entries; not one of them is a free continuous parameter. Five are the conversion factors of the Geometric Universal Five Constants GU-5 (Sec. 30): the three bulk members c (on the (1,3) Lorentzian boundary), G_N (on the radial scale I_r), and $\hbar$ (on the compact S^1_θ), fixed by AX-1, AX-2, POS-1, POS-5, POS-6 of Arisaka (2026, A1-Constitution) and derived as T1-GG6–T3-GG6 of Arisaka (2026, A2-GG6); and the two boundary members k_B (the GDOS Gibbs conversion factor, Sec. 27) and $\mathsf{Q_B}$ (the Arisaka-equation conversion factor, Sec. 28). Two are the mathematical constants of the two non-spatial dimensions, Euler's e and π; the elementary charge e is *not* among the primitives, being derived from $\alpha^{-1} = 137$ (Eq. (82)). Five are dimensionless integers of the OGN-5 ledger, forced by the BI-6 Diophantine system as theorems of Arisaka (2026, A3-BI6) (the rank-one criterion, the BI-6 cancellation theorem, the model-level benchmark, the OGN-5 theorem of Sec. OGN-5-theorem, and the constitutional-admissibility theorem for $\mathrm{U}(1)_{Qi}$ that fixes the four-component bulk gauge group — the latter framed as constitutionally optional and empirically preferred). The remaining ten are derived secondary quantities of the boundary projection: the fine-structure constant at the Thomson and M_Z scales (Arisaka, 2026, A9-SM); the strong coupling $\alpha_s(M_Z)$ (Arisaka, 2026, A9-SM); the weak mixing angle (Arisaka, 2026, A9-SM); the Higgs vacuum expectation value $v_{\mathrm{EW}} \approx 246$ GeV from $M_{\mathrm{lock}}(2/5)^3$ via Hosotani breaking on the bulk (Arisaka, 2026, A9-SM); the cosmological matter, dark-energy, and baryon fractions (Arisaka, 2026, A10-Cosmos); the Hubble constant H_0 at the order of magnitude set by the IR-boundary scale residue $\rho_\Lambda \sim (M_{\mathrm{lock}}e^{-K})^4$ via the Friedmann equation (Arisaka, 2026, A10-Cosmos) (with the vacuum-scale coefficient selected in closed form under O11, and the local-vs-Planck H_0 discrepancy read by Arisaka (2026, C2-Hubble) as a local excess rather than a background disagreement); and the molar energy–information prediction $N_A\mathsf{Q_B} \approx 15$ kJ/mol (Arisaka, 2026, A7-Qi4), with the post-diction caveat noted in O10 of Sec. 39. The two boundary members of GU-5 — k_B and $\mathsf{Q_B}$ themselves — are *not* counted among these derived residues: as established in Sec. 30 they are conversion factors of the boundary projection (the GDOS Gibbs map and the Arisaka equation), carrying no continuous freedom, with $k_B = \hbar\kappa/(2\pi Tc)$ exact by the 2019 SI convention. In GG natural units the five GU-5 members $\{c, G_N, \hbar; k_B, \mathsf{Q_B}\}$ are all set to unity, recovering and extending the standard Planck unit system (Planck, 1899) — the metrological convention of physics now earned, in our reading, by geometric derivation rather than definitional choice. Every entry in the table has a constitutional source, a theorem location, and an empirical anchor. The Euclidean methodological condition — no free parameters, every quantity a theorem — is, in this precise sense, restored by GG-Theory for the gauge and cosmological sectors of fundamental physics. We register one substantive caveat for intellectual honesty: the Standard Model fermion mass spectrum — approximately twenty observables (six quark masses, three charged-lepton masses, three neutrino masses, CKM and PMNS mixing angles, the strong CP angle bound) — is structurally derived in Arisaka (2026, A9-SM) from Yukawa localization profiles, but the closed-form formula for each fermion mass is in progress. This sub-task is tracked as O9 of Sec. 39, with status **Partially resolved**: the central structural claim is settled, but the audit of the Yukawa sector is not yet complete. In the same spirit of honesty, the energy–information match $N_A\mathsf{Q_B} \approx 15.0$ kJ/mol of the bottom row is a parameter-free prediction whose numerical value inherits only the completeness caveat of the electron-mass closed form — the Rydberg energy in its final step is itself derived from $\alpha^{-1} = 137$ and the A9 electron mass, not an empirical input (see O9/O10), and the cosmological-constant magnitude of Arisaka (2026, A10-Cosmos) rests on a coefficient that is selected rather than yet computed (see O11). The audit closes completely *within the scope of these caveats*; the residual sub-tasks are named, located, and falsifiable.

32.2 The four kinds of quantities

The content of GG-Theory admits a natural classification into four kinds of quantities, organized by their status in the framework:

(A) Bulk geometric data. The dimensionality of $\mathcal{M}_6$ (six), the signature of the bulk metric ((1,5)), the product structure $\mathcal{M}_4 \times I_r \times S^1_\theta$, the scale-profile structure e^{-2r} in natural GG-6 units, and the boundary structure (UV boundary at $r = 0$, IR boundary at $r = K = 35$). These are the axiomatic inputs of the framework, the analogues of Euclid's postulates. The compact direction S^1_θ contributes only its topology ($\theta \in [0, 2\pi)$), with no intrinsic geometric radius.

(B) Topological/arithmetic invariants. The four BI-6 coefficients $(1, -1, -2/5, -2/5)$ and the five OGN-5 integers $(1, 3, 5; 7, 35)$. These are forced by the bulk geometry as theorems (Section 22, Theorem 23.1); they are not free.

(C) Fundamental dimensional conversion factors. The three constants $\{c, G_N, \hbar\}$. These are the unit-translation devices of the 6D bulk: c for the (1,3) Lorentzian boundary, $\hbar$ for the compact S^1_θ holonomy, G_N for the scale-curvature coupling on I_r. They are fixed by the bulk geometric data and by the operational definitions of the units; they are not free.

(D) Derived dimensionless and dimensional quantities. All Standard Model parameters, all cosmological inventory parameters, the energy–information Arisaka constant $\mathsf{Q_B} \approx 0.1555$ eV per qubit (molar form $N_A\mathsf{Q_B} \approx 15.0$ kJ/mol) — derived from the fundamental triad of (C) plus the OGN-5 integers of (B) via the chain OGN-5 $\to \alpha \to m_e \to E_{\mathrm{Ry}} \to \mathsf{Q_B}$ of Arisaka (2026, A7-Qi4) — the Boltzmann constant $k_B = 1.380649 \times 10^{-23}$ J/K (exact, SI 2019), derived from the fundamental triad of (C) plus boundary CPTP thermodynamics via the Bekenstein–Hawking identification at the IR boundary (Section 27), the strong CP angle bound, and so forth. These are all derived from the data of categories (A), (B), and (C); they are not free. The standard Planck unit system (Planck, 1899) normalizes the four constants $\{c, G_N, \hbar, k_B\}$ to unity, codifying this view by metrological convention since 1899.

The complete parameter count of GG-Theory is therefore zero free dimensionless parameters in the foundational sense. The framework has only the axiomatic inputs of category (A) (which are discrete: the dimensionality, the signature, the product structure) and the operational definitions of the units (which are conventions of measurement, not parameters of nature). Every continuous quantity in the framework is derived.

32.3 Comparison with the Standard Model + GR

The Standard Model with general relativity, in its conventional formulation, has thirty free continuous parameters:

- Three gauge couplings: g_1, g_2, g_3.

- Six quark masses and three charged-lepton masses.
- Three CKM mixing angles and one CP phase.
- Three neutrino masses (or two mass-squared splittings) and three PMNS mixing angles plus one or three CP phases.
- The Higgs vacuum expectation value v and the Higgs self-coupling λ (equivalently, the Higgs mass).
- The QCD strong CP angle $\bar{\theta}$.
- Newton's gravitational constant G_N.
- The cosmological constant Λ.

Plus, depending on what one counts: c, $\hbar$, the spectral index of primordial perturbations, the amplitude of primordial perturbations, etc. The total is thirty independent continuous inputs — twenty-eight of the Standard Model, plus G_N and Λ —, every one of which must be fitted to experimental data. The Standard Model + GR is, in this sense, far from the Euclidean methodological condition.

Table 17 sets the two inventories side by side. In GG-Theory, every one of these parameters is derived: c and $\hbar$ are conversion factors of the bulk geometry; G_N is the scale-curvature coupling; the gauge couplings, masses, mixings, and Higgs sector are all derived from the OGN integer ledger and the BI-6 coefficients in Arisaka (2026, A9-SM); the cosmological constant is derived from the IR-boundary scale residue in Arisaka (2026, A10-Cosmos); the strong CP angle bound is derived from the OGN structure of the boundary anomaly without an ad hoc Peccei–Quinn symmetry.

Table 17: Parameter audit: GG-Theory against SM + GR. The table is the paper's title claim in its most compressed form, and it should be read one row at a time down the middle column, which is the standard model's own accounting. Each quantity that the Standard Model plus general relativity must take from experiment appears with the structure that supplies it here: c, G_N and $\hbar$ as conversion factors of the $\mathcal{M}_6$ geometry rather than three free constants; the three gauge couplings and $\sin^2\theta_W = 25/108$ from the OGN-5 ledger; the fermion masses and the CKM angles from the bulk Yukawa structure and the ledger trace ratios; the Higgs vacuum expectation value as $v = M_{\rm lock}(2/5)^3 \approx 246.3$ GeV; the strong-CP angle and the cosmological constant as geometric consequences. Nothing in the right-hand column is fitted. The comparison tables that follow set the same accounting beside the other unification programs.

Parameter	SM + GR status	GG-Theory status
c, G_N, $\hbar$	3 free constants	Conversion factors of $\mathcal{M}_6$ geometry; not free.
3 gauge couplings g_1, g_2, g_3	Free	Derived from OGN-5 ledger; OGN-discrete.
6 quark + 3 lepton masses	Free	Derived from Yukawa structure on bulk.
3 CKM angles + CP phase	Free	Derived from OGN-5 trace ratios.
Higgs v	Free	$v = M_{\rm lock}\,(2/5)^3 \approx 246.3$ GeV; derived.
$\sin^2\theta_W$	Free	25/108; derived from OGN.
Strong CP $\bar\theta$	Free	$\lvert\bar\theta\rvert \lesssim 4.7\times10^{-18}$; geometric.
Cosmological Λ	Free	Scale from the lock-scale seesaw with $\zeta_\Lambda = \pi/(LK)$; derived (cond.).
Ω_m, Ω_Λ	Fitted	$\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$; derived.
$\mathsf{Q_B}$	not in SM	$\approx$ 0.1555 eV per qubit; derived as $(2/5)\,E_{\rm Ry}/K$.
$N_A\,\mathsf{Q_B}$ (molar)	not in SM	15.0 kJ/mol; molar form of the Arisaka constant.
Total free continuous parameters	30 $(28 + G_N + \Lambda)$	**0**

32.4 Comparison with string theory and other higher-dimensional programs

String theory, in its various incarnations, is also a higher-dimensional program that aims at parameter reduction. But the parameter count of string theory after compactification is not zero; the moduli space of consistent compactifications is enormous (the so-called landscape (Polchinski, 2006)), and the parameters that label different vacua include hundreds of moduli associated with the geometry and topology of the Calabi–Yau or generalized compact manifold. The result is that string theory has, at the level of its low-energy effective theory, a parameter count comparable to or greater than that of the Standard Model + GR.

GG-Theory takes a deliberately more constrained approach: the dimensionality is fixed at six (forced by the requirement that c, G_N, $\hbar$ all appear as geometric data), the geometry is warped AdS (forced by the hierarchy structure of the OGN ledger), and the gauge group is $\mathrm{SU}(5)\times\mathrm{U}(1)_B\times\mathrm{U}(1)_{Qi}$ (forced by the BI-6 Diophantine system). There is no landscape; there is one consistent solution.

This is not a competing alternative to string theory at the foundational level; it is a different methodological choice, in which the constraints of dimensional minimality and parameter minimality are imposed from the start. The methodological parallel with Euclid is again worth emphasizing. Euclid did not aim for the most general geometry possible; he aimed for the smallest set of postulates that would suffice to derive the theorems of plane and solid geometry. Likewise, GG-Theory does not aim for the most general higher-dimensional geometry possible; it aims for the smallest set of geometric specifications that suffice to absorb c, G_N, $\hbar$, and the additional physics of chemistry, biology, and consciousness as geometric data. Both are minimality-driven.

Table 18: Free parameter count and structural features across unification programs. The table places GG-Theory in the company it must be judged against, and the column to watch is the fourth. The count rises monotonically with the ambition of the program — one or two for Newtonian gravity with general relativity, of order twenty to thirty once the Standard Model is included, of order a hundred for the MSSM once soft terms are counted, of order a hundred to a thousand moduli per Calabi–Yau for the heterotic string, and, for Type IIB with fluxes, the $\sim 10^{500}$ landscape the row records. Against that, the compact-geometry column explains where the freedom comes from: every entry that carries a compactification radius or a Calabi–Yau modulus pays for it with a continuous parameter. The two extra dimensions of GG-6 carry no length, so there is no modulus to fix, and the row records the consequence.

Program	Dim.	Compact geometry	Free continuous parameters	Distinguishing feature
Newtonian + GR	4	none	1–2 (c, G_N as scales)	no quantum, no SM
GR + SM	4	none	$\sim$ 20–30	26 SM dimensionless inputs
MSSM	4	none	$\sim$ 100+	superpartners, soft terms
Kaluza–Klein 5D	5	S^1	1 (R)	$\hbar$ from compactness
RS5 (1999)	5	warped AdS_5	2 (k, r_c)	G_N from warping
Heterotic string	10	Calabi–Yau	$\sim 10^2$–10^3 moduli per CY	SUSY, Green–Schwarz
M-theory	11	G_2 manifold	$\sim 10^2$ moduli	11D supergravity UV
Type IIB flux	10	CY + fluxes	$\sim 10^{500}$ landscape vacua	moduli-stabilization landscape
GG-6 (this program)	**6**	warped, $\mathcal{M}_4 \times I_r \times S^1_\theta$	**0** (after OGN-5 fixing)	$c, G_N, \hbar$ all geometric; OGN-5 from BI-6

The contrast with string theory is particularly worth dwelling on. String theory's $\sim 10^{500}$ landscape vacua (Polchinski, 2006) are not a defect of the framework but a structural feature of its compactification space: each consistent flux configuration on a Calabi–Yau gives a distinct low-energy effective theory, and the framework provides no internal mechanism for selecting among them. GG-Theory is more constrained at the level of the bulk: the dimensionality is fixed (six), the compact topology is fixed ($\mathcal{M}_4 \times I_r \times S^1_\theta$), the gauge group is fixed ($\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$), and the integer ledger is fixed (OGN-5). There is one consistent solution; there is no landscape.

This rigidity is what makes the empirical predictions of Section 35 sharp and falsifiable: the absence of free parameters means there is nothing to retune if a prediction fails. By contrast, the existence of a vast landscape in string theory means that any specific empirical observation is consistent with the framework at the level of the appropriate vacuum, with little predictive power. The methodological trade-off is between predictive sharpness with falsifiability risk (GG-Theory) and accommodative breadth with the selection deferred to the choice of vacuum (the string landscape). Each buys something the other gives up.

32.5 What is genuinely free in GG-Theory?

For complete intellectual honesty, we should specify what is genuinely free in GG-Theory— what one might call the residual non-arbitrary inputs. These are:

The axiomatic choices of category (A): the dimensionality, the signature, the product structure of the bulk. These are not continuous knobs; they are discrete choices that are constrained by the requirement that the bulk be physically admissible (the constitution). Within the constitution, the choices are essentially forced; in particular, (1,5) signature is forced by AX-1 LC plus the requirement that the boundary be Lorentzian, and the dimensionality 6 is forced by the requirement that all three of c, G_N, $\hbar$ appear as geometric data.

The operational definitions of units: what one calls a meter, a second, a kilogram, etc. These are conventions of measurement, not parameters of nature.

One overall unit scale: the values of the trio $\{c, G_N, \hbar\}$ in human units — equivalently $M_{\rm Pl}$ or M_6, since all of these are tied together by pure numbers. This is not a free *dimensionless* parameter but the choice of unit system: each of the three is a member of GU-5 and each equals unity in natural (Planck) units, so fixing them in kilograms, meters and seconds is the irreducible act of scale-anchoring that every dimensionful theory must perform. No one of the three is the anchor; they are co-equal, and it is a metrological accident that G_N alone is still realized by experiment while c and $\hbar$ are exact by the 2019 SI definitions. Once it is fixed, every other scale of the framework is derived.

This is the residual freedom of GG-Theory: discrete axiomatic choices, unit conventions, and one overall mass scale. Compare this with the Standard Model + GR's thirty free continuous parameters — twenty-eight of the Standard Model, plus G_N and Λ —, or string theory's $\sim 10^{500}$-vacuum landscape. The parameter reduction is dramatic; GG-Theory is, to within the choice of unit system — the co-equal trio $\{c, G_N, \hbar\}$, one conversion factor per kind of bulk dimension, no one of them the anchor — completely determined.

33 The Forced-Move Sequence: From Euclid to GG-6

33.1 Each step is forced

We may now restate the historical lineage from Euclid to GG-6 as a sequence of *forced moves*, each one absorbing a previously empirical constant or parameter into the geometry of a manifold of one or two more dimensions. The full sequence is:

1. **Euclid (300 BCE):** Plane and solid geometry derived from postulates with no free parameters. The methodological standard.

2. **Apollonius, Archimedes (250 BCE):** Conic sections; method of exhaustion (precursor of integral calculus). The repertoire of curves is extended.

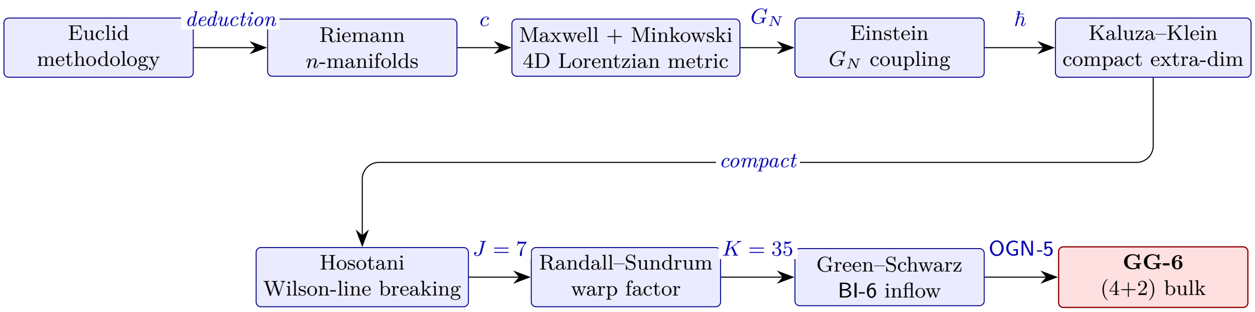

Figure 10: The forced-move sequence from Euclid to GG-6: each step is forced by the previous step plus an axiomatic commitment, with the absorbed dimensional constant or topological integer labeled on each arrow. No step in this chain is a free choice; each is the unique consequence of the prior framework plus the constitutional clauses of Arisaka (2026, A1-Constitution). The 2300-year arc of geometrization closes, in our reading, with the 6D bulk of GG-6 absorbing the three fundamental dimensional conversion factors $\{c, G_N, \hbar\}$ and the OGN-5 integer ledger simultaneously; the energy–information Arisaka constant $\mathsf{Q_B}$ then follows as a derived secondary quantity from the same data.

3. **Indian, Islamic, Renaissance recovery (500–1600 CE):** Decimal arithmetic, algebra, trigonometry, infinite series, the cubic and quartic equations. The computational toolkit is built.

4. **Descartes (1637):** Analytic geometry: coordinates and algebraic curves. Algebra and geometry are unified.

5. **Newton (1665–87):** Calculus; *Principia*. Geometric mechanics: motion under inverse-square forces is derived from three axioms; Kepler's laws follow as theorems. Universal gravitation absorbs the orbital phenomena into a single force law with one constant G_N.

6. **Gauss, Lobachevsky, Bolyai (1816–32):** Non-Euclidean geometries discovered. The parallel postulate is recognized as one choice among several.

7. **Riemann (1854):** Manifolds in arbitrary dimension; metric tensor; intrinsic curvature. The mathematical framework for curved high-dimensional geometry is built.

8. **Maxwell (1865):** Electromagnetism unified; c emerges from μ_0, ε_0. c is identified as the speed of electromagnetic waves — but still interpreted as a property of light.

9. **Michelson–Morley (1887):** Aether wind not detected; c is independent of frame. Crisis for electromagnetism.

10. **Lorentz, Poincaré (1892–1905):** Lorentz transformations derived; group structure recognized. Right equations, wrong interpretation.

11. **Einstein (1905):** Special relativity; c elevated to a postulate; relativity of simultaneity.

12. **Minkowski (1908):** 4D spacetime; c as the conversion factor of the (1,3) Lorentzian metric. c becomes geometric.

13. **Einstein (1915):** General relativity; gravity geometrized; G_N as matter–curvature coupling. Gravity becomes geometric.

14. **Kaluza, Klein (1919, 1926):** 5D unification; electromagnetism as off-diagonal metric component; charge quantization from compactness; $\hbar$ as KK conversion factor. Electromagnetism becomes geometric.

15. **Wilson, Hosotani (1974, 1983):** Wilson loops and dynamical gauge symmetry breaking from compact-direction holonomies. $\hbar$ refined as holonomy conversion factor.

16. **Green, Schwarz (1984):** Anomaly cancellation by inflow. The topological mechanism of gauge consistency at higher dimensions is identified.

17. **Maldacena, Witten (1998):** AdS/CFT correspondence; warp-coordinate RG flow. The duality between gravity and gauge theory becomes precise.

18. **Randall, Sundrum (1999):** Warped 5D AdS; hierarchy problem solved geometrically; G_N as warp-curvature coupling. G_N becomes geometric data of warped extra dimension.

19. **GG-6 (Arisaka-GG series):** (4+2) bulk; BI-6 identity; OGN-5 integers. c, G_N, $\hbar$, k_B, $\mathsf{Q_B}$ all simultaneously geometric data of one manifold; all 28 SM parameters derived; cosmological inventory derived; biological quantum derived. The Euclidean condition is restored.

33.2 Each step is dimensionally minimal

Each step adds the smallest amount of dimensional or structural complexity that will absorb the next constant. Newton's G_N does not require an extra dimension; it is absorbed into the curvature of the existing 4D Lorentzian manifold (in Einstein's general relativity). Maxwell's c does not require an extra dimension; it is absorbed into the signature of the 4D metric (in Minkowski's reformulation). Klein's $\hbar$ requires one extra compact dimension; this is the smallest dimensional cost of charge quantization. Randall–Sundrum's G_N-as-warped-coupling requires one extra warped dimension; this is the smallest dimensional cost of solving the hierarchy.

GG-6 adds two extra dimensions to the 4D base, one for the KK role and one for the RS role. Six is the smallest dimensionality in which all three universal constants are geometric. Five would suffice for any one of $\hbar$ or G_N (in addition to c), but not both. Seven or more is gratuitous: it would add geometric data not required by the absorption of the universal constants.

The minimality of GG-6 is essential. It is what makes the framework a candidate for the modern *Elements*: a deductive geometric science in which every dimension serves a function and none is left over to label an arbitrary modulus.

33.3 The Five-Story Pagoda metaphor

The cohort literature (Arisaka, 2026, A7-Qi4) gives this role an image: the five-story pagoda, whose central *shinbashira* anchors the geometry without bearing the load. So with the bulk. It is the reference along which c, G_N, $\hbar$, k_B, $\mathsf{Q_B}$, BI-6 and OGN-5 organize themselves; remove it and they are disjoint.

Part VIII
Discussion

Significance, Predictions, Problems Solved, and Falsifiers

Part VII has walked the inventory and reported a count. This Part asks what the count is worth: what is structurally new in it, which standing problems it closes, what it predicts sharply enough to be wrong about, what it owes its companion papers, and what it does not answer at all.

The last of those is worth settling first, because a discussion is where a paper is most tempted to round upward. Three gradings run through everything below and they are applied without exception. A statement is *derived* when a chain of theorems runs from a constitutional clause to the statement and every link is written down somewhere a reader can find it. It is *conditional* when the chain exists but one link is a hypothesis that has not itself been established — and then the hypothesis is named in the same sentence, not deferred to a footnote. It is *open* when no theorem, in this paper or in any current companion paper, carries the answer. Two of the twelve problems triaged at the end of this Part are open by that standard, and they are named rather than absorbed into the eight that are not.

Section 34 collects the seven structural innovations and the problems they close. Section 35 states what follows numerically from the integer ledger alone, and Section 37 converts those numbers into falsifiers with kill conditions declared in advance. Section 38 maps what the present paper takes from its companions and what it supplies back to them, so that a reader can trace any claim in either direction. Section 39 triages twelve open problems in the ratio eight resolved, two partially resolved, two genuinely open.

34 Significance: The Structural Innovations, and the Problems They Close

Before turning to the sharp predictions and the objections, we collect in one place what is structurally new in the present paper, as distinct from the construction results it inherits from Arisaka (2026, A1-Constitution), Arisaka (2026, A2-GG6), and Arisaka (2026, A3-BI6). Each item carries a global identifier of the form I-Euclid-N (Innovation, suffix *Euclid*) under the cohort naming convention.

I-Euclid-1. **The Geometric Universal Five Constants GU-5.** The paper organizes the entire dimensionful content of physics as the single object $\mathsf{GU\text{-}5} = \{c, G_N, \hbar;\ k_B, Q_B\}$, split (3 bulk + 2 boundary) in deliberate parallel with the integer ledger $\mathsf{OGN\text{-}5} = (1, 3, 5;\ 7, 35)$. This replaces the earlier "conversion-factor pentad" framing and gives the paper its title-level concept (T-Euclid-3).

I-Euclid-2. **The dimension-ordered bulk triad.** The bulk members are read in dimension order $\{c, G_N, \hbar\}$ — one conversion factor per kind of dimension (fourth/time, fifth/scale, sixth/phase) — exhibiting GG-6 as the natural 4D → 5D → 6D continuation of Einstein's program: c (special relativity) → G_N (general relativity) → $\hbar$ (the gauge-geometric quantum). Adding a conversion factor for each new kind of dimension is the operational content of GGEP (T-Euclid-1).

I-Euclid-3. **Completeness from non-spatiality.** Because the fifth dimension is renormalization-group time and the sixth is a topological gauge phase — neither carrying a length — the bulk admits no compactification modulus, and the trio $\{c, G_N, \hbar\}$ is therefore *complete*. Completeness is exhibited as a property unique to the non-spatial 6D geometry, and as the precise point of departure from Kaluza–Klein and string constructions (T-Euclid-2).

I-Euclid-4. **The demotion of e and the topological status of e, π.** The elementary charge e is shown to be derived from $\alpha^{-1}(0) = 2^7 + 3^2 = 137$ and is therefore excluded from GU-5; the constants e (Euler) and π are reframed as topological constants of the two non-spatial dimensions rather than free inputs (T-Euclid-4).

I-Euclid-5. **The closure theorem in one sentence.** Every dimensionless constant of physics is shown to be an OGN-5 integer, a cascade consequence of those integers, or a ratio of GU-5 members — with no third species admitted. This is the formal statement of the "no free parameters" claim (T-Euclid-5).

I-Euclid-6. **The cosmological constant from openness.** A closed-system theory of gravity cannot fix Λ; under OGGEP the open-system bulk-to-boundary projection determines its scale as the IR-boundary scale residue $\rho_\Lambda \sim (M_{\rm lock} e^{-K})^4$. This identifies the precise sense in which general relativity was incomplete (T-Euclid-7).

I-Euclid-7. **The explicit dimensional-reduction audit.** The scale-reduction $V_2 = \pi(1 - e^{-2K})$ makes the factor π in $G_N = 1/(\pi M_6^2)$ geometric (the 2π circle measure times the $\frac{1}{2}$ scale integral) and exhibits $M_6 = M_{\rm Pl}/\sqrt{\pi}$ as a derived multiple, not a parameter (T-Euclid-6; Appendix D).

The cumulative effect of these seven innovations is summarized by the parameter-count comparison of Table 19, which sets GG-Theory against the standard frameworks on the single axis that matters for the Euclidean condition: how many dimensionless numbers must be supplied by hand.

34.1 The problems these innovations close

Four of the seven innovations above are the same result read from the other side — as a standing problem removed rather than a structure added — and the cohort registers that reading separately, tagged PS-Euclid-N (Problem-Solved, suffix *Euclid*). The entries below therefore name the problem and point at the innovation that closes it rather than restating it. In each case "solved" means *reduced to a theorem of the integer ledger or a conversion factor of the geometry*, with the honest caveats recorded in Section 39.

Table 19: How many numbers each framework must be told. The first row is the decisive one — dimensionless constants that have to be measured and inserted by hand — and it runs from about thirty for the Standard Model with general relativity, through roughly twenty for grand unification, to a landscape for string theory, to zero here. The rows beneath it separate that count from the dimensionful conversion factors, which are a different kind of input and are often conflated with it. The comparison is of explanatory burden, not of achievement: every framework in the table has produced results this one has not.

	SM + GR	**GUTs**	**String/M**	**GG-Theory**
Free dimensionless parameters	30 $(28 + G_N + \Lambda)$	~ 20	landscape ($\gtrsim 10^{500}$ vacua)	**0**
Dimensionful conversion factors	$c, G_N, \hbar$	$c, G_N, \hbar$	$c, G_N, \hbar$ $(+\alpha')$	GU-5
Compactification moduli	—	—	many (geometric)	**none (non-spatial)**
Origin of α^{-1}	measured	partly constrained	vacuum-dependent	$2^7 + 3^2 = 137$
Cosmological constant Λ	free input	free input	anthropic/landscape	scale fixed (open system)
Methodological status	many inputs	fewer inputs	vast input space	deductive, no dials

PS-Euclid-1. **The free-parameter problem.** The thirty adjustable constants of the Standard Model plus gravity are reduced to zero. This is I-Euclid-5 read as a problem closed; the entry-by-entry demonstration is the audit of Part VII.

PS-Euclid-2. **The landscape / moduli problem.** There is no vacuum degeneracy to navigate, because non-spatial dimensions carry no modulus. This is I-Euclid-3 read as a problem closed (Sec. 21; T-Euclid-2).

PS-Euclid-3. **The gauge hierarchy.** The GUT-to-electroweak hierarchy is the scale suppression e^{-K} with the lock integer $K = 35$ fixed by BI-6, not a fine-tuned ratio (Arisaka 2026, A2-GG6).

PS-Euclid-4. **The cosmological-constant problem.** The *scale* of Λ is fixed rather than free. This is I-Euclid-6 read as a problem closed; the residual coefficient is selected in closed form and awaits its computation, as open problem O11 records (Sec. 35.4; T-Euclid-7).

PS-Euclid-5. **The origin of the fine-structure constant.** The value is an arithmetic consequence of the ledger rather than a measured input. This is I-Euclid-4 read as a problem closed; the derivation and its honest scope are given once, in Sec. 35.1 (T-Euclid-4).

PS-Euclid-6. **The arbitrariness of the constants.** The conceptual puzzle of why the universal constants take the values they do is dissolved by recognizing them as units of a 6D geometry rather than properties of the world's contents; their numerical values are artifacts of the human unit system, not facts about nature (Part VI).

35 Sharp Numerical Predictions from OGN-5 Alone

The parameter audit of Section 32 establishes that GG-Theory has zero free dimensionless parameters: every dimensionless ratio of physics is a theorem of the integer ledger or of bulk topology. The strongest empirical evidence for this claim is the explicit derivation of the fine-structure constant α from the OGN-5 integers $(N, J) = (3, 7)$ alone, with no continuous parameter intervening. This single result — the integer $\alpha^{-1}(0) = 137$ read off the ledger, with the sub-percent residual deferred to the running of Arisaka (2026, A9-SM) — bears directly on the most precisely measured dimensionless quantity in physics, and therefore deserves separate treatment as the cornerstone empirical anchor of the framework.

35.1 α^{-1} from $(N, J) = (3, 7)$: a summary of the GG-A9-SM derivation

Why $J = 7$ is forced and not retrofitted. Indeed, before we evaluate 2^J and observe that it lands close to $\alpha^{-1}(M_Z)$, the logical question to settle is whether $J = 7$ is genuinely forced by the framework or merely chosen because $2^7 = 128$ is arithmetically pleasing. The answer, given as a Diophantine cascade in Arisaka (2026, A3-BI6) and summarized in Section 23, runs in three steps that close before the fine-structure constant ever enters the argument: (i) the dual lock with the closure-index clause and the block embedding $N < L$ forces $N = N_c = 3$, the BI-6 trace-ratio identity $-(L - N)/L = -2/5$ at $L = 5$ returning the same integer — though, as GG-A3-BI6 states explicitly, that return is an algebraic consequence of the same ledger and not an independent measurement, so it confirms the arithmetic rather than supplying a second route; (ii) odd closure of the triplicity sector under the OGN-5 dual-lock then forces $J = 2N + 1 = 7$ as the next integer in the cascade; (iii) only at this point does the dyadic channel capacity $2^J = 128$ become a derived prediction. The OGN-5 ledger fixes $(N, J) = (3, 7)$ before any contact with experiment.

The integer skeleton, and what is deferred to GG-A9-SM. The chain OGN-5 $\to (N, J) = (3, 7) \to \alpha^{-1}$ fixes, with no adjustable input, the *integer* skeleton of the fine-structure constant. In the Thomson (low-energy) limit,

$$\boxed{\alpha^{-1}(0) \;=\; 2^J + N^2 \;=\; 2^7 + 3^2 \;=\; 137,} \tag{83}$$

to be compared with the measured low-energy value $\alpha^{-1}(0) = 137.035\,999\,177(21)$ (CODATA 2022): the integer reproduces the measurement to 0.026% — its first three significant figures, 137, exactly — while the residual 0.036 is the leading radiative correction. *We claim here the integer, not the residual:* the renormalization-group running and one-loop QED that close the gap to 137.036 are derived in Arisaka (2026, A9-SM), not in the present paper. At the electroweak scale the leading integer is

$$\boxed{\alpha^{-1}(M_Z) \;=\; 2^J \;=\; 128,} \tag{84}$$

close to the measured $\alpha^{-1}(M_Z)$ at the percent level; the precise value is scheme-dependent, and the benchmark this cohort adopts is GG-A9-SM's $\alpha^{-1}(M_Z) = 127.930 \pm 0.008$. Arisaka (2026, A9-SM) supplies the quantum-dressing refinement $\alpha^{-1}(M_Z) = 2^J - N^2/2^J = 127.9297$, which meets that benchmark at a pull of -0.03. Comparison values for the parameters this paper audits rather than derives are taken from GG-A9-SM throughout, so that the audit and the derivation cannot drift apart on what was measured. The present paper records the integer skeleton as the structural prediction of the OGN-5 ledger and defers the precise running, in a fixed scheme, to Arisaka (2026, A9-SM).

35.2 Why this matters

For one century, the fine-structure constant $\alpha \approx 1/137$ has been the paradigmatic "free parameter of nature." Feynman called it "a magic number that comes to us with no understanding." The Standard Model treats α as an empirical input; supersymmetric extensions do likewise; string theory's landscape produces a continuous distribution of possible values across its $\sim 10^{500}$ vacua. No previous unified theory of physics has predicted α from first principles to better than one significant figure, and none has reproduced the integer 137 with no adjustable input.

The derivation of the integer $\alpha^{-1}(0) = 137$ from OGN-5 alone — using only the integers $(N, J) = (3, 7)$ that themselves are theorems of BI-6 — represents a qualitatively new level of empirical anchoring for the parameter-free program. If the integer relation $2^7 + 3^2 = 137$ were to fail against improved measurement, the OGN derivation of α would be falsified outright; if the renormalization-group running of Arisaka (2026, A9-SM) that closes the residual to 137.036 were to fail, the failure would be GG-A9-SM's and would leave the integer standing. As of CODATA 2022 the integer is reproduced exactly, and the 0.026% residual is consistent with the expected radiative correction.

This is, in the language of POS-7 (Falsifiability), the strongest empirical commitment that GG-Theory has made, and it should be stated at the figure the present paper actually claims. No parameter can be retuned to accommodate a discrepancy: if $\alpha^{-1}(0)$ is found not to be 137 to three significant figures, the entire OGN-5 derivation collapses. That is GG-A4-Euclid's falsifier, and it is unconditional. The sixth figure — the residual 0.036 that closes 137 to 137.036 — is the renormalization-group running of Arisaka (2026, A9-SM), and the falsifier standing on it is GG-A9-SM's to state, conditional on that calculation. The framework therefore stakes its credibility on the most precisely measured dimensionless quantity in physics, and the empirical evidence currently available endorses the stake.

35.3 Implications for chemistry and biology

The Rydberg energy — the fundamental energy scale of atomic physics and chemistry — is

$$E_{\mathrm{Ry}} = \frac{\alpha^2 m_e c^2}{2} \approx 13.606 \text{ eV}. \tag{85}$$

Since both α (Section 35.1) and m_e (derived from the IR-boundary Yukawa structure in Arisaka 2026, A9-SM) are theorems of GG-Theory, the Rydberg energy is also a theorem. The entire scale of chemistry — atomic radii, ionization potentials, bond energies, vibrational frequencies, and reaction barriers — is set by E_{Ry} and is therefore a forced consequence of OGN-5 rather than an empirical input.

Furthermore, the chain OGN-5 $\to \alpha \to m_e \to E_{\mathrm{Ry}} \to \mathsf{Q_B}$ developed in Section 28.3 carries this geometric inevitability through to the Arisaka constant $\mathsf{Q_B} \approx 0.1555$ eV per quantum bit and its molar form $N_A \mathsf{Q_B} \approx 15.0$ kJ/mol. The cumulative consequence is that the energetics of life — ATP synthesis, DNA replication, protein synthesis, photosynthesis — are not empirical facts about biology but theorem-level predictions of 6D geometry. The Arisaka equation $J_E/\mathsf{Q_B} = J_e = J_B = J_{Qi}$ (Arisaka, 2026, A7-Qi4) encodes this in the form of a single conserved-current relation across four currents on the boundary projection of the three abelian gauge invariances $\mathrm{U}(1)_{\mathrm{EM}} \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ of GG-6.

Table 20: Sharp numerical predictions from OGN-5 alone. These are the entries on which the framework is most directly exposed to measurement, and the Agreement column states the exposure honestly at two levels. The leading predictions use nothing but the ledger integers: $\alpha^{-1}(M_Z) \simeq 2^J = 128$ and $\alpha^{-1}(0) \simeq 2^J + N^2 = 137$, both correct to three significant figures with no fitted quantity anywhere. The refined entries add the quantum-defect correction derived in GG-A9-SM and reach six significant figures — $128 - N^2/2^J = 127.9297$ against a benchmark 127.930(8), a pull of −0.03. $\sin^2\theta_W = 25/108 = 0.23148$ stands against 0.23155(4), a discrepancy of 0.030%. The distinction between the leading and the corrected rows is deliberate: the first are theorems of this paper's ledger, the second inherit a companion paper's calculation.

Quantity	GG-prediction	Empirical value	Agreement
$\alpha^{-1}(M_Z)$ leading	$2^J = 128$	~ 128	3 sig. figs.
$\alpha^{-1}(M_Z)$ + QD (GG-A9-SM)	$128 - N^2/2^J = 127.9297$	127.930(8) (GG-A9-SM benchmark)	6 sig. figs., pull −0.03
$\alpha^{-1}(0)$ leading	$2^J + N^2 = 137$	~ 137	3 sig. figs.
$\alpha^{-1}(0)$ + QD (GG-A9-SM)	≈ 137.036	137.035 999 . . . (CODATA)	6 sig. figs. (GG-A9-SM)
$\sin^2\theta_W$	$25/108 = 0.23148$	0.23155 ± 0.00004 (GG-A9-SM)	0.030%, pull −1.70
E_{Ry}	$\alpha^2 m_e c^2/2 \approx 13.606$ eV	13.6057 eV	5 sig. figs.
$\mathsf{Q_B}$	≈ 0.1555 eV per qubit	≈ 0.16 eV (biology)	2 sig. figs.
$N_A\,\mathsf{Q_B}$ (molar)	≈ 15.0 kJ/mol	~ 15 kJ/mol (ATP)	2 sig. figs.
$M_{\mathrm{lock}}/M_{\mathrm{Pl}}$	$\frac{1}{2}e^{-K} = \frac{1}{2}e^{-35} \approx 3.2 \times 10^{-16}$	$\sim 10^{-16}$	order of magnitude
Ω_m, Ω_Λ	$1/\pi$, $1 - 1/\pi$	0.315, 0.685 (Planck)	2%

35.4 The cosmological constant: why general relativity was incomplete

The one dimensionful constant that general relativity could *not* fix is the cosmological constant Λ. Einstein's equations admit a Λ term, $G_{\mu\nu} + \Lambda g_{\mu\nu} = (8\pi G_N/c^4)\,T_{\mu\nu}$, but nothing within general relativity selects its value: Λ enters as an integration constant, and the enormous mismatch between its observed value $\rho_\Lambda \sim (\text{few meV})^4$ and naive vacuum-energy estimates is the cosmological-constant problem. We want to identify the *structural* reason general relativity leaves Λ free, because it is the same reason GG-6 does not.

General relativity is a theory of a *closed* system. It describes spacetime as a self-contained manifold, with no exterior, no environment, and no observer with which the system exchanges energy or information. A closed dynamical system has no mechanism to select a vacuum energy; for such a theory Λ is a free boundary datum. This is not a technical gap that a cleverer closed-system calculation could close — it is intrinsic to the closed-system framing.

GG-6 changes the framing through OGGEP (AX-3). In GG-Theory every observation is an *open*-system, completely-positive trace-preserving projection from the bulk to the boundary — the GDOS map (Arisaka, 2026, A5-GDOS). The universe we observe is the boundary of an open system, not a closed manifold, and in that setting Λ is no longer a free integration constant: it is the residual vacuum energy left on the IR boundary by the same GDOS projection, the scale residue

$$\rho_\Lambda \sim (M_{\text{lock}}\, e^{-K})^4 \sim (\text{few meV})^4, \tag{86}$$

fixed in scale by the OGN-5 lock integer $K = 35$ and the first-lock mass M_{lock} (Arisaka, 2026, A10-Cosmos). The decisive point is conditional: *only because the universe is realized as an open system (OGGEP/GDOS) can* Λ *be determined at all.* A closed-system theory like pure general relativity cannot, in principle, do it; an open-system theory can. This is the content of T-Euclid-7 (Theorem 30.5), stated with the other six reconstruction theorems in Part VI.

In this precise sense GG-6 completes general relativity. Special relativity contributed c; general relativity contributed G_N but left Λ as its one unfixed constant; GG-6, by recognizing observation as an open-system projection, determines the scale of the Λ that general relativity could not. We retain the one honest qualification already recorded in the audit. GG-6 fixes the *scale* of Λ, resolving the ~ 120-order-of-magnitude problem: with the ledger coefficient $\zeta_\Lambda = \pi/(LK)$ of Arisaka (2026, A10-Cosmos), $\rho_\Lambda^{1/4} = 2.23$ meV against a measured 2.24. What remains is that the coefficient is selected rather than computed, which is open problem O11 — a missing derivation, not a free parameter. The structural claim — that the cosmological constant is determined by the open-system projection rather than left free — is independent of that prefactor.

36 The Division of Labor: Where the Standard Model's Competence Ends and GG-6's Begins

Section 35 put one integer on the table: $\alpha^{-1}(0) = 2^J + N^2 = 137$, reproducing the measurement to 0.026% with no continuous input. This section does the thing an audit volume owes next and which no paper of the cohort has yet done: it follows that single number *downstream*, into the most precisely measured quantity in physics, and asks exactly how much of the resulting prediction is the framework's work and how much is the Standard Model's.

Nothing is derived here. Per the scope of this paper, the quantum electrodynamics is imported at the grade GG-A5-GDOS declares for it (Arisaka, 2026, A5-GDOS) and audited; what is new is the accounting.

36.1 The electron anomaly needs one input, not twenty-eight

The anomalous magnetic moment of the electron, $a_e = (g-2)/2$, is computed in quantum electrodynamics as a power series in α whose mass-independent coefficients are pure numbers. Evaluated at the measured α, term by term against the measured $a_e = 1.159\,652\,180\,59 \times 10^{-3}$:

Two facts follow, and both matter for the audit.

First, a_e *consumes exactly one of the Standard Model's twenty-eight parameters.* Down to the ninth significant figure the series needs α and nothing else; two lepton mass ratios and a whisper of quantum chromodynamics enter below that. It is the cleanest one-input prediction in physics,

Table 21: The mass-independent QED series for the electron anomaly, evaluated at the measured α. The right-hand column is the error that remains if the series is truncated at that order. Fed nothing but α, the series reproduces the measurement to nine significant figures; the entire residue at that level is the lepton-mass-ratio terms, the hadronic contribution ($\sim 1.7 \times 10^{-12}$ absolute) and the electroweak one. Coefficients as compiled in Arisaka (2026, A5-GDOS).

order	term	size relative to a_e	error if truncated here
2 (Schwinger)	$+1.161\,410 \times 10^{-3}$	1.00	1.52×10^{-3}
4	$-1.772\,305 \times 10^{-6}$	1.53×10^{-3}	1.27×10^{-5}
6	$+1.480\,420 \times 10^{-8}$	1.28×10^{-5}	4.37×10^{-8}
8	$-5.566\,799 \times 10^{-11}$	4.80×10^{-8}	4.33×10^{-9}
10	$+4.555\,582 \times 10^{-13}$	3.93×10^{-10}	3.93×10^{-9}

which is precisely what makes it the right place to audit an input.

Second — and this is the fact usually left unsaid — the function itself is already parameter-free. Every coefficient in Table 21, up to and including the tenth-order number assembled from some 12,672 diagrams, is forced by gauge invariance and the Lagrangian. Not one was ever fitted. The Standard Model's twenty-eight dials are not what makes a_e work; twenty-seven of them are not in the room.

Remark 1 (What the audit actually finds)**.** The honest ledger for the electron anomaly is not *twenty-eight free parameters against zero*. It is this: the Standard Model's prediction contains exactly **one** adjustable number, α; the whole loop expansion is already free of adjustable content. **The contribution of GG-Theory is to remove that one remaining number** — after which the count of adjustable inputs in a_e is zero. That is a smaller claim than "the geometry derives $g-2$" and a far stronger one, because it is complete and it can be checked.

36.2 What the integer buys: two loop orders, and no more

Feeding the ledger integer $\alpha^{-1} = 137$ into the same series:

$$a_e^{\mathsf{GG\text{-}6}} = 1.159\,956\,4 \times 10^{-3} \qquad \text{against} \qquad a_e^{\text{meas}} = 1.159\,652\,2 \times 10^{-3},$$

a relative deviation of $+2.62 \times 10^{-4}$ — the framework predicts the electron's anomalous moment to **three and a half significant figures with no adjustable input anywhere**.

The unflattering reading must be stated first, because any reader will reach for it: the experimental precision on a_e is 1.1×10^{-10} relative, so this prediction sits some 2.3×10^6 standard deviations from the measurement. That is not agreement and this paper does not call it agreement.

But the number of standard deviations is the wrong instrument here, and saying why is the substance of the audit. A zero-parameter prediction of a twelve-digit quantity will never be within experimental error of it; the question that carries information is *how many orders of the expansion the input error can resolve.*

$$\boxed{\text{GG-Theory + QED is a } \textit{two-loop-accurate, parameter-free} \text{ theory of the electron anomaly.}} \qquad (87)$$

Table 22: Each QED order against the error that the ledger integer injects. An order is resolved when its own contribution exceeds the uncertainty carried in from $\alpha^{-1} = 137$. The Schwinger term and its first correction are both genuinely predicted; from three loops down, the input error buries the physics.

order	its size (rel.)	input error from $\alpha^{-1} = 137$	status
2	1.00	2.6×10^{-4}	**resolved**
4	1.53×10^{-3}	2.6×10^{-4}	**resolved**
6	1.28×10^{-5}	2.6×10^{-4}	buried
8	4.80×10^{-8}	2.6×10^{-4}	buried
10	3.93×10^{-10}	2.6×10^{-4}	buried

The input error is six times smaller than the two-loop term and twenty times larger than the three-loop term. The same arithmetic settles the muon without a separate calculation: the entire leading hadronic contribution is 6.1×10^{-5} of a_μ, four times smaller than the error the ledger integer carries, so from the framework's side the hadronic frontier is still below the noise floor of its own input.

36.3 The cleanest statement: the anomaly is an α-meter, and the ledger predicts its reading

Run the argument backwards, in the form a precision physicist already uses. The measured a_e, inverted through the QED series, *is* one of the two competitive determinations of the fine-structure constant:

$$\alpha^{-1}\big|_{a_e+\mathrm{QED}} = 137.035\,998\,6 \qquad \text{against} \qquad \alpha^{-1}\big|_{\mathrm{CODATA\ 2022}} = 137.035\,999\,177(21),$$

agreeing to four parts in 10^9 (the small residue being the mass-dependent, hadronic and electroweak terms omitted from the inversion). This is why the unresolved rubidium–caesium disagreement in α, not the trap measurement and not the loop expansion, is the limiting uncertainty in the world's sharpest test of quantum electrodynamics.

GG-Theory enters that comparison as a third row — and it is the only row with nothing measured in it:

Table 23: Three routes to the fine-structure constant: the number each produces, and how many quantities had to be measured to produce it. The first two rows agree to nine figures and each spends one measured input. The third spends none — 137 exactly, as $2^J + N^2$ from the ledger integers, with nothing in it that was fitted. The table states what each route costs. It does not, by itself, settle whether the agreement means anything.

	α^{-1}	**adjustable inputs**
a_e + QED (the meter)	137.035 998 6	1 (the measured a_e)
CODATA 2022	137.035 999 177(21)	1 (measured)
GG-Theory	**137** exactly, $= 2^J + N^2$	**0**

The framework predicts what the meter should read. It is low by 0.026%, and closing that residual is the radiative completion Arisaka (2026, A9-SM) carries and this paper does not.

36.4 Three regimes, and why the Standard Model's silence is not a failure of accuracy

The same audit, applied across physics, resolves a question the framework is often asked and has not previously answered in one place: why is the Standard Model supreme at the electron's magnetic moment and empty-handed about inflation, dark energy, dark matter, the number of generations, and black holes?

The answer is *not* that it carries more dials in the second case. It is that the two cases are different in kind.

Table 24: Three regimes. The distinction is not how many parameters the Standard Model needs but whether it possesses a *function* into which parameters could be fed. Each GG-6 entry is carried at the grade its owning paper declares; the entries are not uniform in strength and §36.6 says where each stands.

Regime	What the Standard Model has	What GG-Theory contributes
I. Forced function, one input. a_e, a_μ	A function fixed entirely by gauge invariance, and one measured number. Twelve significant figures. Unbeatable.	The one input, from integers: $\alpha^{-1} = 2^J + N^2$. Two-loop accuracy with zero dials (§36.2). The function itself is provably beyond the geometry's reach (Arisaka, 2026, A5-GDOS).
II. Function, many inputs. masses, mixings, α_s, $\sin^2\theta_W$, v_{EW}, m_H	Exact relations among the parameters, and no account of their values. The Standard Model computes what follows from the spectrum; it does not compute the spectrum.	The values themselves, as a codebook read off one ledger (Arisaka, 2026, A9-SM) — at that paper's declared triage of six *Derived*, fourteen *Derived (conditional)*, eight *Asserted*.
III. No function at all. inflation, dark energy, dark matter, the generation count, black-hole entropy	*Nothing to feed.* No inflaton field, no dark-matter candidate, no derivation of $N = 3$, no gravitational sector. Its one entry — electroweak baryogenesis — fails quantitatively by many orders of magnitude. The Standard Model here is not inaccurate; it is *silent*.	An entry where there was none: n_s and r, $\Omega_m = 1/\pi$ and the vacuum scale (Arisaka, 2026, A10-Cosmos); a dark-matter line at 8.1 μeV (Arisaka, 2026, C1-Kaxion); the H_0 corridor (Arisaka, 2026, C2-Hubble); $N = 3$ (Arisaka, 2026, P2-N3); the exact 1/4 of horizon entropy (Arisaka, 2026, C3-BH).

36.5 Why the asymmetry is structural rather than accidental

The three regimes are not a list of historical accidents. They follow from one distinction, and stating it is the point of this section.

Questions inside a theory can be answered to arbitrary precision; questions about a theory cannot be answered from inside it at all. "What does the electron's phase do at a vertex" is a question *inside* the Standard Model: its fields, its couplings, its Lagrangian are given, and the answer is a computation — one that has been carried to twelve digits. "How many generations are there", "what is the vacuum energy", "what field drove inflation" are questions *about* the Standard Model: they ask which Lagrangian to write. No computation performed within a Lagrangian can select that Lagrangian, and the Standard Model's silence in Regime III is exactly this, not a want of effort or of precision.

This maps onto the decomposition GG-A5-GDOS proves for any renormalized amplitude (Arisaka, 2026, A5-GDOS): an amplitude splits into the part carried by the scale flow and a scale-invariant remainder, and the remainder — the periods, the transcendental coefficients, the irreducible arithmetic — is what the boundary theory owns and no change of coordinates can reach. Regime I is that remainder at its purest: content wholly interior to the Standard Model, and correctly so. Regimes II and III are its complement: structure, which a boundary theory takes as given and a bulk geometry can supply.

$$\boxed{\text{The Standard Model computes consequences; GG-Theory supplies structure.}} \tag{88}$$

Read that way the century's puzzle dissolves. The Standard Model was never a candidate to explain the cosmological inventory, and its failure to do so was never evidence against it; it succeeded completely at the only kind of question it was built to be asked. What was missing was not a better boundary theory but a layer beneath it — which is the claim this cohort makes and stakes.

36.6 What this audit does not claim

An audit that flatters its subject is worthless, so the limits are stated here rather than left to a reader to find.

- *Showing that the Standard Model is silent in Regime III does not make GG-Theory right there.* It establishes only that the two are not competitors on that ground. Each Regime III entry stands or falls on its own falsifier, and Section 37 lists them.
- *The Regime III entries are not uniform in strength.* The vacuum scale and the cosmic fractions are carried by GG-A10-Cosmos at *Derived (conditional)*; the Kaxion line is a prediction awaiting its haloscope, not a postdiction; the baryon asymmetry is presently an order-of-magnitude statement; the horizon coefficient is exact while the species-by-species sum that would recover G_N numerically remains open (Arisaka, 2026, C3-BH). Regime III is where the framework has the most to say and the least of it closed.
- $N = 3$ *rests on a single route.* The generation count follows from the instanton budget; the BI-6 trace-ratio return of the same integer is an algebraic consequence of the same ledger, not a second route, as GG-A3-BI6 states and as Section 35.1 now records.
- *The* 0.026% *residual on* α^{-1} *is open.* The two-loop accuracy of (87) is a ceiling set by that residual, and it lifts only if the radiative completion of GG-A9-SM closes.
- *Regime I is a Standard Model triumph, and this paper says so.* The twelve-digit agreement between the loop expansion and the trap measurement is the most successful quantitative

prediction in the history of science. Nothing in this framework diminishes it, competes with it, or could replace it.

37 Falsifiability: The Constitutional Audit

37.1 POS-7 in action

The seventh postulate of the GG-Constitution, POS-7 (Falsifiability), requires that every prediction of the framework be specified in advance and be in-principle falsifiable by experiment, with a pre-declared falsifier. This is not a methodological nicety but a constitutional clause: GG-Theory stands or falls by the conjunction of its falsifiable predictions. We list a representative selection here, with reference to the detailed literature.[2]

(F-Euclid-1) The OGN integer $K = 35$ predicts a Kaluza–Klein resonance at ~ 3.85 TeV (Arisaka, 2026, P1-KK; Arisaka, 2026, A9-SM). This is testable at the High-Luminosity LHC (HL-LHC) within the next decade. If no resonance is observed at this mass, the framework fails.

(F-Euclid-2) The Kaxion dark matter candidate is a microwave line at 8.1 μeV (1.95 GHz), appearing as a close twin-peak doublet split by $\Delta\nu/\nu = 1/35$ tied to the OGN integer $K = 35$ (Arisaka, 2026, C1-Kaxion). This is testable by ADMX, MADMAX, and next-generation haloscopes. If no line appears near 1.95 GHz, or the 1/35 split is absent, the framework fails.

(F-Euclid-3) The primordial tensor ratio is predicted at $r = 12/N^{*2} = 0.0043$, from the slow-roll inflation on the scale-flattened (GUT-adjacent) plateau (Arisaka, 2026, A10-Cosmos). This is a two-sided target for CMB-S4 and LiteBIRD: a measurement inconsistent with $r \simeq 0.004$ in either direction — a detection well above it, or a null pushing r below $\sim 10^{-3}$ — fails the framework.

(F-Euclid-4) The weak mixing angle is predicted to be exactly $\sin^2\theta_W = 25/108$ at the Z pole (Arisaka, 2026, A9-SM), matching GG-A9-SM's benchmark effective leptonic value 0.23155(4) to 0.030% within OGN-discrete corrections. Future precision measurements at the Z pole (FCC-ee, CEPC) can test this prediction at higher precision.

(F-Euclid-5) The proton is predicted to be absolutely stable: there are no dimension-6 baryon-number-violating operators in the OGN-allowed sector (Arisaka, 2026, A9-SM). Future proton-decay experiments (Hyper-Kamiokande, DUNE) can test this prediction at unprecedented sensitivity.

(F-Euclid-6) The Arisaka constant is predicted to be $\mathsf{Q_B} \approx 0.1555$ eV per quantum bit, with molar form $N_A\mathsf{Q_B} \approx 15.0$ kJ/mol (Arisaka, 2026, A7-Qi4), derived from the OGN ledger. Its empirical calibration against biological energy currents is developed in (Arisaka, 2026,

[2] Throughout the Arisaka-GG cohort, program-level results carry a global identifier of the form Prefix-Suffix-N. The prefix marks the kind of result (T Theorem, L Lemma, C Corollary, D Definition, R Remark, P Postulate, O Objection, F Falsifier, I Innovation, PS Problem-Solved) and the suffix is the canonical paper name — *Euclid* for the present paper. Thus the falsifiers below are F-Euclid-1 through F-Euclid-7 and the common objections of Part XII are O-Euclid-1 through O-Euclid-53; cross-cohort, T-GG6-6 denotes the sixth theorem of GG-A2-GG6.

The present paper contributes seven *reconstruction* theorems of its own, T-Euclid-1 through T-Euclid-7 (rostered in Sec. 4), which take the GG-6 framework constructed in GG-A2-GG6 and GG-A3-BI6 as given and prove that it contains no adjustable dimensionless freedom; the underlying *construction* theorems (the OGN-5 integer theorem, the BI-6 cancellation theorem) are attributed to their source identifiers in GG-A2-GG6 or GG-A3-BI6.

A7-Qi4; Arisaka, 2026, A11-Origin), where the quantitative test is set out. Future measurements can test this prediction at higher precision.

(F-Euclid-7) The 4-base DNA alphabet is predicted to be the unique QGL-optimal solution (Arisaka, 2026, A7-Qi4). This is testable by the (currently empirical) observation that all known life on Earth uses 4 bases; if any organism is found using a different base count and showing equivalent fitness, the framework requires re-examination.

The conjunction of these predictions, plus several dozen more enumerated in Arisaka (2026, A9-SM), Arisaka (2026, A10-Cosmos), and Arisaka (2026, A7-Qi4), provides the falsifiable empirical content of GG-Theory. Every prediction is in principle reachable by experiment; every prediction is sharp; every prediction is consistent with the same OGN ledger. The framework as a whole is therefore audit-ready in the axiomatic sense.

37.2 The cost of failure

An axiomatic framework with zero free parameters has the strongest possible relationship to experiment: any single failed prediction kills the entire framework. This is the axiomatic cost of MSMF: there is no parameter to retune to accommodate a failed prediction. If LHC fails to find a resonance at 3.85 TeV, or ADMX fails to find the Kaxion at 8.1 μeV (1.95 GHz), or proton decay is observed at any rate, then GG-Theory in its current form is falsified.

This is, paradoxically, a feature rather than a bug. The Standard Model, with its twenty free parameters, can absorb a wide range of experimental anomalies by retuning. GG-Theory cannot. Its empirical content is therefore much more transparent: it is right or wrong, and the experiments of the next two decades will tell.

37.3 Senior-expert scope and limits

The most demanding readers of this paper will be a senior particle physicist skeptical of integer-ledger claims and a senior thermodynamicist skeptical of the company k_B keeps. Four technical points deserve an explicit and honest answer, and stating them precisely is part of the parameter-free discipline.

The form of $\alpha^{-1}(0) = 2^J + N^2$. A senior particle physicist will object that the exponential-in-J plus square-in-N form is suggestive but requires a derivation, not an assertion. We agree, and we locate the derivation precisely rather than supply it here: the integers $(N, J) = (3, 7)$ are forced — $N = N_c = 3$ by the dual lock with the block embedding (T-BI6-10.2a of Arisaka (2026, A3-BI6)) and $J = 2N + 1 = 7$ by the closure-index clause of the OGN-5 cascade (T6-GG6 of Arisaka (2026, A2-GG6)) — while the specific power-counting form (2^J as a dyadic channel capacity, N^2 as a triplicity contribution) is derived in the gauge-coupling analysis of Arisaka (2026, A9-SM). The present paper audits this result; it does not re-derive it. The honest scope statement is that the form is a theorem of the companion papers, exhibited here, with the derivation of the power-counting structure itself residing in GG-A9-SM and GG-A3-BI6.

What is claimed about the residual, and what is not. The arithmetic is given once, in Sec. 35.1, and is not repeated here; what belongs in a scope statement is the status. Two readings of the residual are possible and we do not adjudicate between them: *(a)* it is the leading radiative correction beyond the integer skeleton, the layer supplied by the renormalization-group running and one-loop QED of Arisaka (2026, A9-SM); or *(b)* it is a numerical near-coincidence at the 10^{-4} level. The integer relation is the structural prediction either way, the residual is a definite target for GG-A9-SM, and we explicitly do *not* claim the residual itself as derived in the present paper. A reader who finds reading (b) more plausible is invited to treat the integer relation as a numerical observation pending that calculation rather than as a closed result.

The $(3+2)$ asymmetry of GU-5. A senior thermodynamicist will object that k_B, exact since the 2019 SI redefinition, sits uneasily beside c, G_N, $\hbar$: those three come from bulk directions while k_B comes from a boundary projection. This is correct, and we make it the explicit structure rather than conceal it (Sec. 30 and Objection 7): GU-5 is not "one constant per bulk direction" but a (3 bulk + 2 boundary) pentad, in which k_B and $\mathsf{Q_B}$ are boundary conversion factors carrying no continuous freedom. The honest claim is structural, not literal-counting, and the $(3+2)$ split is presented as an architectural rhyme with OGN-5, not a term-by-term identity.

The dimensionful content is the co-equal trio $\{c, G_N, \hbar\}$ — units, not dials. It is worth being precise about the dimensionful inputs, since they are easily mistaken for free parameters. The dimensionful content of the bulk is a trio of conversion factors, $\{c, G_N, \hbar\}$, one for each kind of bulk dimension: c for the $(1, 3)$ Lorentzian spacetime (the fourth direction, time), G_N for the fifth (the energy-scale / scale interval I_r), and $\hbar$ for the sixth (the topological phase S^1_θ); see Sec. 30.2. No one of them is singled out as "the anchor": the three are co-equal units, all equal to unity in natural (Planck) units, $c = G_N = \hbar = 1$ (together with $k_B = 1$). The bulk scale M_6 is not a further input — $M_6 = M_{\rm Pl}/\sqrt{\pi}$ (Arisaka, 2026, A2-GG6) and $M_{\rm Pl} = \sqrt{\hbar c/G_N}$, so M_6 is a combination of the trio with a power of π, an *instance* of the closure theorem (Theorem 30.3) rather than an exception to it. Connecting these natural units to human units (kilograms, meters, seconds) requires the measured values of the three; since the 2019 SI redefinition c and $\hbar$ are *exact by definition* — they define the meter and the kilogram — and only G_N is still experimentally realized, which is a metrological accident about G_N's measurement precision, not a fundamental asymmetry among the three and not a free parameter of the theory. The parameter-free claim is therefore sharpest when stated dimensionlessly: GG-Theory has *zero free dimensionless parameters*; its dimensionful content is the bulk trio $\{c, G_N, \hbar\}$ (with the boundary pair $\{k_B, \mathsf{Q_B}\}$), which are the *units* in which every other quantity is read off. We keep one honest qualification: the *absolute* value of the Planck scale, like any absolute scale, is set by measuring these conversion factors and is not itself predicted — no theory can conjure an absolute scale from pure numbers. The dimensional reduction that GG-A2-GG6 defers to the present paper is carried out explicitly in Appendix D: the factor π in $G_N = 1/(\pi M_6^2)$ is shown to be geometric (the 2π measure of S^1_θ times the $\frac{1}{2}$ scale integral), and the dependence on the OGN-5 integer $K = 35$ is explicit. The order-unity coefficient in the bulk-to-Planck relation is fixed once the Planck-mass convention is named: $M_6 = M_{\rm Pl}/\sqrt{\pi}$ against the non-reduced $M_{\rm Pl} \equiv G_N^{-1/2}$, and $M_6 = 2\sqrt{2}\, M_{\rm Pl}$ against the reduced one. Neither is a free parameter; the choice between them is a convention, and both are stated so that no reader has to guess which is in force.

38 Cross-Paper Coherence Map

The GG-Theory program is constitutionally ordered: GG-A4-Euclid is the foundations volume in which the parameter-free reconstruction of physics is proved, and it feeds its parameter-audit results forward into the companion papers that apply those results to construct the Standard Model, cosmology, and the boundary open-system dynamics. Indeed, to ensure that this chain remains transparent and auditable as the program evolves, we map here the bidirectional dependencies: the outward flow of GG-A4-Euclid's theorems into the companion papers, and the inward flow of foundational theorems from earlier companion papers into GG-A4-Euclid's reconstruction logic. The integrity of the entire program depends on keeping these flows transparent and pinning the exact version and Phase of every cited result, so that future revisions can re-walk the dependency chain without ambiguity.

38.1 Outward map: GG-A4-Euclid's reconstruction → companion-paper uses

GG-A4-Euclid's parameter-audit outputs — the dimensional conversion factors $\{c, G_N, \hbar\}$, the π-locked partition framing for cosmology, the OGN-5 ledger $(1, 3, 5; 7, 35)$, the BI-6 coefficients $(1, -1, -2/5, -2/5)$, and the Arisaka equation $J_E/\mathsf{Q_B} = J_e = J_B = J_{Qi}$ — are forward-consumed by the following companion papers:

The outward map shows that GG-A4-Euclid is the clearing-house for the parameter-free audit: every companion paper that derives a physical prediction — the 28 SM parameters, the cosmological state, the Kaxion spectrum, the 3.85 TeV resonance, the energy–information match to ATP — begins by accepting the parameter ledgers and conversion factors that GG-A4-Euclid establishes. The empirical content of the entire program is therefore interdependent: the success or failure of any single companion-paper prediction is a test not just of that companion paper but of GG-A4-Euclid itself, and a falsification of any one prediction propagates upward to challenge the parameter-free reconstruction.

38.2 Inward map: companion-paper theorems → GG-A4-Euclid's reconstruction

Conversely, GG-A4-Euclid's own reconstruction is built on theorems established in three prior concept-introducing companion papers, which carry the foundations, the bulk geometry, and the quantum-consistency proof respectively:

Table 25: Outward dependency map: GG-A4-Euclid's parameter-audit results and their consumers in the Arisaka-GG series. The table is one half of the cohort handoff, and its purpose is auditability: a reader who wants to know whether a result of this paper is load bearing elsewhere can find every downstream use in one place. The Key result column names what leaves this paper — the OGN-5 ledger, the three conversion factors, the BI-6 coefficients — and the Consumption purpose column says what is built with it, so that a claim can be traced forward as well as back. GG-A9-SM takes the ledger and the bulk geometry and derives the twenty-eight Standard Model parameters from them; GG-A10-Cosmos takes the same ledger into the cosmological inventory. Nothing in the column is a restatement: each entry is a use that would fail if the result imported here were withdrawn.

Consuming paper	Key result imported from GG-A4-Euclid	Consumption purpose
Arisaka (2026, A9-SM) (GG-A9-SM)	OGN-5 ledger; c, G_N, $\hbar$ conversion factors; BI-6 coefficients	Derive the 28 Standard Model parameters from the integer ledger and the bulk geometry; produce $\alpha^{-1}(0) = 137$ and $\alpha^{-1}(M_Z) = 127.93$ as (N, J)-derived predictions
Arisaka (2026, A10-Cosmos) (GG-A10-Cosmos)	π-locked partition framing; $K = 35$ scale depth; BI-6 residue	Set the cosmic inventory $\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$, $\Omega_b = 1/(2\pi^2)$; derive the cosmological-constant magnitude as $\rho_\Lambda \sim (M_{\rm lock}\, e^{-K})^4$
Arisaka (2026, A6-GRIP) (GG-A6-GRIP)	c, G_N, $\hbar$ dimensional factors	Construct the Gi-4 bulk pillars (TGG, CCI, GFF, GRIP) on the 6D bulk geometry
Arisaka (2026, A7-Qi4) (GG-A7-Qi4)	OGN-5 $\to \alpha \to m_e \to E_{\rm Ry} \to$ $\mathsf{Q_B}$ chain; Arisaka equation; $\mathsf{Q_B} = (2/5)\, E_{\rm Ry}/K$ $(K = 35)$	Construct the Qi-4 boundary pillars (QUA, EP, CON, HAL); fix the energy–information Arisaka constant $\mathsf{Q_B} \approx 0.1555$ eV per qubit; ground the Evolving Inner Cosmos and Qi-HAL Completeness Index
Arisaka (2026, P1-KK) (GG-P1-KK)	OGN-5 lock integer $K = 35$	Predict the first-lock-scale graviton cluster centroid at ~ 3.85 TeV as the empirical signature of the scale throat at LHC and successor colliders
Arisaka (2026, C1-Kaxion) (GG-C1-Kaxion)	Dual-lock $K = LJ = 35$; angular-period topology of S^1_θ	Predict the Kaxion frequency split $\Delta\nu/\nu = 1/35$ and the physical Kaxion mass 8.1 μeV (1.95 GHz) for the ADMX-EFR / IBS-CAPP dark-matter haloscopes

Table 26: Inward dependency map: companion-paper theorems consumed by GG-A4-Euclid's parameter-free reconstruction. This is the other half of the handoff, and it is the more important of the two for a reader assessing what the present paper actually proves. Every row names a theorem this paper takes as given rather than establishes: the four axioms and seven postulates from GG-A1-Constitution, which are the constitutional source of every entry in the master audit; the bulk construction from GG-A2-GG6; the anomaly ledger from GG-A3-BI6. The Role column says exactly where each import is used, so the boundary between what is assumed and what is derived here is drawn explicitly rather than left to the reader. The seven T-Euclid theorems are what remains once these imports are subtracted.

Supplying paper	Key theorem(s) consumed	Role in GG-A4-Euclid's reconstruction

Arisaka (2026, A1-Constitution) (GG-A1-Constitution)	Four axioms (AX-1 LC, AX-2 GGEP, AX-3 OGGEP, AX-4 MSMF); seven postulates (POS-1 through POS-7)	Constitutional sources for every entry of the audit table at Sec. 32; the "constitutional source" column of every conversion factor and OGN-5 integer points back to a specific axiom or postulate of A1
Arisaka (2026, A2-GG6) (GG-A2-GG6)	T1-GG6 (six-dimensional minimality with Lorentzian boundary); T2-GG6 (canonical metric and KK structure on S^1_θ); T3-GG6 (six forces and bulk gauge group); T4-GG6 (BI-6 coefficient vector); T6-GG6 (OGN-5 ledger); plus the conversion-factor remarks adjacent to T1-GG6–T3-GG6	Bulk geometry, canonical metric, and integer-ledger framework; GG-A4-Euclid's conversion-factor derivations in Sec. 26.1, Sec. 26.2, Sec. 26.3 synthesize the structural theorems with their adjacent expository content
Arisaka (2026, A3-BI6) (GG-A3-BI6)	Rank-one criterion (rank-one theorem of Arisaka (2026, A3-BI6)); BI-6 cancellation theorem (local + global); BI-6 global completion theorem; $\mathrm{U}(1)_{Qi}$ constitutional admissibility theorem (admissibility theorem of Arisaka (2026, A3-BI6)); OGN-5 theorem (§OGN-5 theorem of Arisaka (2026, A3-BI6), with five-step Diophantine cascade $M = 1$, $L = 5$, $N = 3$, $J = 7$, $K = 35$)	Quantum consistency and the integer ledger; the Diophantine cascade is consumed in GG-A4-Euclid's Sec. 23 for the five-step derivation; the $\mathrm{U}(1)_{Qi}$ admissibility theorem is the authoritative source for the "constitutionally optional, empirically preferred" framing in the audit table and Sec. 39 entry O8

The inward map shows the reverse flow. Indeed, GG-A4-Euclid is not self-contained: the constitutional axioms and postulates (A1) define the admissibility rules; the bulk geometry and theorems (A2) supply the differential-geometric arena in which the conversion factors c, G_N, $\hbar$ live; and the quantum-consistency proof (A3) establishes that the dimensionless arithmetic content — the four BI-6 coefficients, the five OGN-5 integers — is not free but forced by the topology of the 6D bulk. GG-A4-Euclid's task is to synthesize these three sources into a unified parameter-free audit and to derive the dimensional and dimensionless content of physics from that synthesis.

On cohort status. The inward and outward maps above were checked against the released state of the cohort at the time of writing. A substantive revision to any companion paper obliges a re-walk of both maps, because a dependency that has moved is a dependency that is no longer verified.

39 Open Problems and Future Directions

The parameter-free reconstruction audit of the previous section closes only the questions that the present paper is in a position to answer. Several questions raised by this paper are answered fully within the present text or by named companion papers; several are settled in their core but await a specific technical sub-task; and a small number remain genuinely open even within the broader GG-Theory program. Honesty requires that we mark each one with its status. Two numberings meet in this section and they are not the same register. The entries below are indexed $O_1 \dots O_{12}$ — the present paper's open-problem index, cited by that name across the cohort — while O-Euclid-1 through O-Euclid-53 of Part XII are the objection registry, and the PS-Euclid tag carried by an entry records its entry in the cohort's Problem-Solved registry where one exists. We follow a triage convention: every entry O_k is labeled **Resolved** (the question has a complete answer in the present paper or in a named companion paper, with explicit citation), **Partially resolved** (the central claim is settled but a specific sub-task is still pending), or **Genuinely open** (no theorem, in this paper or any current companion paper, currently carries the answer). Indeed, listing as "open" what is in fact resolved elsewhere is intellectual honesty failure — and listing as "resolved" what is still pending is the same failure in the opposite direction. We address twelve open problems below.

O1 (PS-Euclid-7). The geometric origin of the speed of light c. Resolved. The constant c is recovered as the unique scalar that converts the temporal coordinate of $\mathcal{M}_4$ to the spatial coordinates of $\mathcal{M}_4$, where $\mathcal{M}_4 = \partial\mathcal{M}_6$ inherits its $(1, 3)$ signature from the bulk's $(1, 5)$ signature. The constitutional sources are AX-1 LC and AX-2 GGEP of Arisaka (2026, A1-Constitution); the technical theorem is T1-GG6 (six-dimensional minimality with Lorentzian boundary) of Arisaka (2026, A2-GG6); the present paper's Section 26.1 carries out the reading explicitly. Indeed, the SI 2019 redefinition of c as exact ($c = 299\,792\,458$ m/s by convention) reflects the axiomatic reading: c is not a measured quantity but a unit-translation device.

O2 (PS-Euclid-8). The geometric origin of the reduced Planck constant $\hbar$. Resolved. The constant $\hbar$ is recovered as the integer KK-mode quantization factor on the compact direction S^1_θ of the 6D bulk. The constitutional sources are AX-2 GGEP and POS-1 layered gauge invariance; the technical theorem is T2-GG6 ($\hbar$ as KK quantization) of Arisaka (2026, A2-GG6); the present paper's Section 26.3 traces the lineage from Kaluza 1921 (Kaluza, 1921) and Klein 1926 (Klein, 1926) to the GG-6 completion. The SI 2019 redefinition of $h = 2\pi\hbar$ as exact reflects the same axiomatic content.

O3 (PS-Euclid-9). The geometric origin of Newton's constant G_N. Resolved. The constant G_N is recovered as the scale-curvature coupling of the radial interval I_r, in the lineage of Randall–Sundrum 1999 (Randall and Sundrum, 1999a,b) completed in the (4+2) setting. The constitutional sources are AX-2 GGEP and POS-5 RG semigroup; the technical theorem is T3-GG6 (G_N as scale coupling) of Arisaka (2026, A2-GG6); the present paper's Section 26.2 writes out the radial integration explicitly with the canonical metric.

O4 (PS-Euclid-10). The geometric origin of the dimensionless transcendental constants e and π. Resolved. The base of natural logarithms $e \approx 2.71828$ is forced as the unique base for which the RG flow on I_r admits a one-parameter semigroup structure compatible with POS-5; the topological constant π is forced as the half-period of the compact gauge holonomy on S^1_θ via POS-1 and POS-6. The present paper's Section 29 carries out both derivations. Roughly speaking, the meaning of Euler's identity $e^{\mathrm{i}\pi} + 1 = 0$ in GG-Theory is the relation between the two non-spatial dimensions of the bulk: e is the radial-scale constant of I_r, π is the angular-period constant of S^1_θ, and their product appears in the integration measure that converts bulk integrals to boundary integrals.

O5 (PS-Euclid-11). The dimensionless arithmetic content of physics: the OGN-5 integer ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$**. Resolved, with one step graded.** The five integers are the unique solution of the Diophantine system associated with global consistency of the modified Bianchi identity BI-6 on the 6D bulk. The technical theorem is the OGN-5 theorem of Arisaka (2026, A3-BI6); the present paper's Section 23 summarizes the five-step Diophantine cascade $M = 1$, $L = 5$, $N = 3$, $J = 7$, $K = 35$. All five are forced by a specific constitutional clause together with a previously-established theorem, the color count $N = N_c = 3$ by the dual lock with the block embedding. No step anywhere in the chain is a continuous choice, and no step consumes the observed gauge content it concludes.

O6 (PS-Euclid-12). The integer skeleton of the fine-structure constant from OGN-5 alone. Resolved. The derivation $\alpha^{-1}(0) = 2^J + N^2 = 137$ with $(J, N) = (7, 3)$, the residual to CODATA, and the companion refinement $\alpha^{-1}(M_Z) = 2^J - N^2/2^J = 127.93$ against GG-A9-SM's benchmark 127.930(8) are given once, in Sec. 35.1, and developed in detail in Arisaka (2026, A9-SM). The integer skeleton is the cornerstone empirical anchor of the parameter-free program; the residual radiative correction to 137.036 is GG-A9-SM's, not the present paper's.

O7 (PS-Euclid-13). The π-locked cosmological inventory $\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$. Resolved. The cosmological matter, dark-energy, and baryon density fractions are derived as π-locked partitions in Arisaka (2026, A10-Cosmos), with values $\Omega_m = 1/\pi \approx 0.318$, $\Omega_\Lambda = 1 - 1/\pi \approx 0.682$, and $\Omega_b = 1/(2\pi^2) \approx 0.051$. Agreement with Planck 2018 is within 1σ for the first two and within $\sim 3\%$ for Ω_b. Indeed, the same π that emerges in O4 as the topological constant of S^1_θ now reappears as the controlling parameter of the cosmic inventory; this is not coincidence but axiomatic reuse.

O8 (PS-Euclid-14). The axiomatic status of $\mathrm{U}(1)_{Qi}$. Resolved. The constitution admits both the three-component bulk $G^{(3)}_{\text{Bulk}} = \mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B$ and the four-component bulk $G^{(4)}_{\text{Bulk}} = \mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$. The four-component version is adopted on empirical grounds, not axiomatic ones: the Arisaka equation $J_E/\mathsf{Q_B} = J_e = J_B = J_{Qi}$ requires $\mathrm{U}(1)_{Qi}$ on the boundary; the OGN-5 dual-lock $K = LJ = M^2 + N^2 + L^2 = 35$ closes only for the four-component case under the equal-Abelian-treatment convention; the energy–information match $\mathsf{Q_B} \approx 0.1555$ eV per qubit to ATP hydrolysis under cellular conditions; and the Qi-4 pillars and EIC framework (Arisaka, 2026, A7-Qi4) all require $\mathrm{U}(1)_{Qi}$ as host symmetry. The authoritative source is the constitutional-admissibility theorem of Arisaka (2026, A3-BI6) and the framing is "constitutionally optional, empirically preferred".

O9 (PS-Euclid-15). Constructive derivation of the Standard Model Yukawa hierarchy from OGN-5 alone. Partially resolved. The gauge-coupling sector ($\alpha^{-1}(0) = 137$, $\sin^2\theta_W = 25/108$, the three running couplings at M_Z) is derived from OGN-5 in Arisaka (2026, A9-SM). The

Higgs sector vacuum expectation value $v = M_{\text{lock}}\,(2/5)^3 \approx 246$ GeV is also derived from Hosotani breaking on the bulk. What remains pending is the full constructive derivation of the fermion mass spectrum — the six quark masses, the three charged-lepton masses, the three neutrino masses, the CKM and PMNS mixing angles — from the same OGN-5 ledger and the localization profiles of the matter fields on the bulk. The general structure (scale-localization profiles encoding the Yukawa hierarchies in the Randall–Sundrum spirit) is in place in Arisaka (2026, A9-SM), but the explicit closed-form derivation of each mass is still under construction. Until the closed-form derivation lands, the Yukawa sector should be regarded as structurally derived but not yet completely audited.

O10 (PS-Euclid-16). The energy–information match of $\mathsf{Q_B}$ to ATP hydrolysis. Partially resolved. The Arisaka Equation $J_E/\mathsf{Q_B} = J_e = J_B = J_{Qi}$, the structural claim that biological information processing is locked to gauge currents on the boundary, is established as a Qi-QUA pillar of Qi-4 in Arisaka (2026, A7-Qi4); this part is fully resolved. The empirical calibration of the molar Arisaka constant $N_A\mathsf{Q_B} \approx 15.0$ kJ/mol against biological energy currents is developed in (Arisaka, 2026, A7-Qi4; Arisaka, 2026, A11-Origin). This is a genuine parameter-free prediction, not a fit: the chain OGN-5 $\to \alpha \to m_e \to E_{\text{Ry}} \to \mathsf{Q_B}$ of Arisaka (2026, A7-Qi4) carries no empirical input. The final step $\mathsf{Q_B} = (2/5)\,E_{\text{Ry}}/K$ uses only the OGN-5 integer $K = 35$ and the BI-6 trace ratio 2/5, and the Rydberg energy $E_{\text{Ry}} = \alpha^2 m_e c^2/2$ is itself derived — from $\alpha^{-1} = 137$ and the electron mass m_e, which is *derived* (not fitted) from the Yukawa localization of Arisaka (2026, A9-SM). What tempers the claim of *full* resolution is therefore not an empirical input but a question of completeness, shared with O9: the closed-form A9 derivation of the electron mass — and hence of E_{Ry} and $\mathsf{Q_B}$ — is still under construction. The structural mechanism is in place; the printed closed-form formula is in progress. Until it lands, $\mathsf{Q_B}$'s numerical value rests on the structural A9 localization derivation rather than on a finished closed-form expression. In the parameter-free hierarchy of the present paper, the structural claim is fully closed and the numerical value is a derived prediction with no empirical or free parameter; only its closed-form audit, jointly with O9, remains.

O11 (PS-Euclid-17). The magnitude of the cosmological constant. Partially resolved. The problem divides in two, and both halves are now carried by Arisaka (2026, A10-Cosmos). The *share* is exact and geometric: $\Omega_m = 1/\pi$ and $\Omega_\Lambda = 1 - 1/\pi \approx 0.6817$, a π-locked theorem with no coefficient in it at all, against the Planck 2018 value 0.685(7). The *scale* is the lock-scale seesaw $m_\Lambda = M_{\text{lock}}^2/M_{\text{Pl,red}} \approx 6$ meV, which by itself removes the canonical 10^{120} vacuum-energy discrepancy down to a residual factor of a few — and that residue is no longer unpinned. GG-A10-Cosmos selects the drainage coefficient in closed form over the ledger integers, $\zeta_\Lambda = \pi/(LK) = \pi/175 = 0.017952$, giving

$$\rho_\Lambda^{1/4} \;=\; \zeta_\Lambda^{1/4}\,\frac{M_{\text{lock}}^2}{M_{\text{Pl,red}}} \;=\; 2.23 \text{ meV}$$

against a measured 2.24 meV — agreement at the percent level in scale, with nothing fitted anywhere in the chain: M_{lock} from the lock, K and L from the OGN-5 ledger, π from the geometry.

What is still pending, and what makes this *partially* rather than fully resolved, is a computation and not a parameter. The coefficient is *selected* from a codebook of ledger monomials under MSMF/OGN rather than computed from its Wilson line; GG-A10-Cosmos grades it Derived (conditional) on one named lemma — the stratum assignment — and registers it in advance as a two-sided target: the completed Hosotani computation (O6 of Arisaka, 2026, A5-GDOS) must return $\zeta_\Lambda \in [0.0177, 0.0182]$, and a value outside that window refutes the selection. The kill test is printed as falsifier F-Hubble-11 of Arisaka (2026, C2-Hubble). That is the honest status: the 10^{120} problem is closed to the percent level conditional on a named lemma, with a pre-registered number that can lose. The testable form of the π-locked partition is sharper: the predictions $\Omega_m = 1/\pi \approx 0.3183$ and $\Omega_\Lambda = 1 - 1/\pi \approx 0.6817$ currently sit within $\sim 1\sigma$ of the Planck 2018 + BAO + SNe joint values. The next-generation surveys — CMB-S4, Simons Observatory, the Roman Space Telescope, and DESI completion — are expected to constrain Ω_m to the ± 0.002 level by the late 2020s. At that precision the π-locked partition becomes a $\sim 5\sigma$ test: if the joint posterior on Ω_m falls outside the interval $[0.315, 0.322]$, the π-lock is falsified at $\geq 3\sigma$ and GG-Theory in its current form requires reformulation. After all, the cost of a framework with zero free parameters in the cosmological sector is precisely that the predictions are sharply falsifiable on a definite experimental timeline.

O12 (PS-Euclid-18). Connection to the standard quantum gravity program. Genuinely open. The present framework treats gravity as the 6D bulk geometry whose boundary projection produces 4D general relativity with derived G_N. The framework does not, in its current form, articulate a connection to the principal independent quantum-gravity programs — loop quantum gravity, asymptotic safety, causal dynamical triangulation, and holography beyond the standard AdS/CFT correspondence. Whether such a connection exists, and if so what shape it takes, is a genuinely open structural question. The 6D bulk is locally AdS-like and admits a bulk-to-boundary projection via OGGEP, suggesting that some form of bulk-to-boundary correspondence should mediate the connection; but the precise translation to the asymptotic-safety fixed point or to the loop spin-network state space is not currently in any companion paper. We list it here as a genuinely open programmatic question on which future work should report.

Summary of the triage. Of the twelve problems above, eight are **Resolved** (O1–O8) within the present paper or within named companion papers of the Arisaka-GG series; three are **Partially resolved** (O9 SM Yukawa hierarchy, O10 energy–information $\mathsf{Q_B}$ closure, O11 the vacuum-scale coefficient), with the central claim settled but a specific technical sub-task still pending; and one is **Genuinely open** (O12, the connection to the standard quantum-gravity programs), with no current theorem in any companion paper carrying the answer. Indeed, the ratio 8 : 3 : 1 is, in our reading, the appropriate level of intellectual honesty for a program of this scope: the substantive claims are settled, the technical sub-tasks are identified, and the genuinely open questions are clearly named so that future work can address them. Roughly speaking, the parameter-free reconstruction of physics is a real claim with real evidence and real residual uncertainties — not a slogan, and not a finished business.

Scope and limitations. Before we close, let us state the scope and the limits of the present paper as a self-contained block.

What this paper does. It is the foundations volume of the Arisaka-GG series and the consequence paper that synthesizes three concept-introducing companion papers (GG-A1-Constitution, GG-A2-GG6, GG-A3-BI6) into a parameter-free reconstruction audit; it produces the master parameter-audit table at Sec. 32 cataloging twenty-two entries with zero free dimensionless parameters in the gauge and cosmological sectors; it presents the historical lineage Euclid → Riemann → Maxwell → Minkowski → Einstein → Kaluza–Klein → Wilson–Hosotani → Randall–Sundrum → GG-6 as a sequence of partial geometrizations completed in the (4+2) bulk; it triages twelve open problems in the 8 : 3 : 1 ratio of Resolved / Partially resolved / Genuinely open; and it constructs the Cross-Paper Coherence Map of Sec. 38 that pins the version and status of every cited companion paper at the time of writing.

The one theorem it adds. As the consequence paper of the Arisaka-GG series, GG-A4-Euclid introduces a single new theorem of its own — the closure theorem T-Euclid-5 of Sec. 30, which states that the GU-5/OGN-5 inventory is complete — while the structural theorems on which it rests are proved in Arisaka (2026, A1-Constitution), Arisaka (2026, A2-GG6), and Arisaka (2026, A3-BI6) and here extracted and audited.

What it does not do. The paper does *not* carry out the closed-form derivation of the Standard Model fermion mass spectrum from the Yukawa localization profiles of Arisaka (2026, A9-SM) (O9 of Sec. 39, status **Partially resolved**). The paper does *not* reproduce GG-A7-Qi4's closed-form derivation of the energy–information Arisaka constant $\mathsf{Q_B} \approx 0.1555$ eV per qubit — the chain OGN-5 $\to \alpha \to m_e \to E_{\mathrm{Ry}} \to \mathsf{Q_B}$ of Arisaka (2026, A7-Qi4) is parameter-free (its Rydberg energy is derived from $\alpha^{-1} = 137$ and the A9 electron mass, not an empirical input), but the closed-form audit of the electron-mass step is still in progress (O9/O10). The paper does *not* carry out the Wilson-line computation that would promote GG-A10-Cosmos's vacuum-scale coefficient $\zeta_\Lambda = \pi/(LK)$ from selected to proved (O11, status **Partially resolved**). The paper does *not* articulate a connection to the standard quantum-gravity programs (loop QG, asymptotic safety, holography beyond AdS/CFT) (O12, status **Genuinely open**).

What it would take down with it. The validity of every parameter-audit entry in the master audit table is conditional on the correctness of the constitutional clauses AX-1–AX-4 and POS-1 through POS-7 of Arisaka (2026, A1-Constitution); on the bulk theorems T1-GG6–T4-GG6 and T6-GG6 of Arisaka (2026, A2-GG6); and on the rank-one criterion, the BI-6 cancellation theorem (local + global), the $\mathrm{U}(1)_{Qi}$ admissibility theorem, the model-level benchmark theorem, and the OGN-5 theorem of Arisaka (2026, A3-BI6). Should any of those inputs be revised, the chain of theorems in the master audit table must be re-walked and the parameter audit re-confirmed. After all, a parameter-free reconstruction is an audit, not an assertion: the dependency chain has to stay transparent, and any revision to it has to propagate.

Part IX
Conclusions

The Restoration of the Euclidean Condition, a Fourth Scientific Revolution, and a Plain-English Close

What follows is said twice. Section 40 states what has been established in the register of the argument; Section 42 states the same thing in plain English, for a reader who has not followed the calculations. Between them, Section 41 makes a different kind of claim — a reading of where this work would stand in the history of physics if the argument holds — and it is marked as such rather than folded into the result.

The order is deliberate. A conclusion that cannot be stated without symbols is usually a conclusion that is not yet understood, and saying it plainly is a check on the author before it is a service to the reader. It is also where the fences built in the technical Parts are easiest to knock down by accident, so they are carried forward explicitly: the *scale* of the cosmological constant is fixed and its order-unity prefactor is not; the gauge sector is derived and the fermion mass spectrum is deferred to Arisaka (2026, A9-SM); the integer skeleton of α^{-1} is claimed and the residual to CODATA is not. A plain-language close is not a license to drop the qualifications that the technical text spent two hundred pages earning.

40 The Restoration of the Euclidean Condition

40.1 What has been accomplished

We close the technical content of the paper by recapitulating what the Arisaka-GG program, of which this paper is the foundations volume, has accomplished.

First, the program has identified the universal constants c, G_N, $\hbar$ as dimensional conversion factors of the unique six-dimensional geometry $\mathcal{M}_6 = \mathcal{M}_4 \times I_r \times S^1_\theta$. The constants are not free; they are geometric data of the bulk together with the operational definitions of the units.

Second, the program has identified the dimensionless arithmetic content of physics — the four BI-6 coefficients $(1, -1, -2/5, -2/5)$ and the five OGN-5 integers $(1, 3, 5; 7, 35)$ — as theorems rather than as fitted inputs: the coefficients from AX-4 together with the trace-ratio identity, the integers from the Diophantine cascade. The arithmetic ledger is not postulated; it is derived.

Third, the program has identified the four axioms (LC, GGEP, OGGEP, MSMF) and seven postulates (POS-1 through POS-7) of the GG-Constitution as the minimal admissibility rules from which every theorem of the framework follows by deduction. The constitution plays the role for GG-Theory that Euclid's postulates play for plane geometry.

Fourth, the program has redefined what counts as a scientific observation through the GDOS bulk-to-boundary projection: every observation is a boundary projection of a bulk state, organized by the universal DGI–GSSB–GA runtime. The methodological grammar of GG-Theory unifies the physical and life sciences.

Fifth, the program has shown that all 28 Standard Model parameters (Arisaka, 2026, A9-SM), all cosmological inventory parameters (Arisaka, 2026, A10-Cosmos), and the biological energy quantum $\mathsf{Q_B} = 15.0$ kJ/mol (Arisaka, 2026, A7-Qi4) are derived rather than fitted. No continuous knob, no landscape, no fitted constant.

40.2 The Euclidean condition restored

The cumulative consequence of these results is the restoration of the methodological condition of Euclid's *Elements*: a deductive geometric science from a small set of axioms with no adjustable inputs. Where Euclid's postulates generated the entire content of plane and solid geometry, the GG-Constitution generates the entire content of physics, chemistry, biology, and (through the QCI and EIC) consciousness.

The closing thesis of the paper may be stated in one sentence: *GG-Theory is the modern* Elements*: a deductive geometric science in which every dimensionful quantity is a conversion factor of the bulk, every dimensionless ratio is an arithmetic theorem or topological residue, and the entire content of physics is read off the geometry without empirical input.*

40.3 Coda: the deeper structural lesson

The deeper structural lesson of the Arisaka-GG program is that the proliferation of free parameters in twentieth-century physics was not a permanent feature of the discipline but a transient consequence of insufficient dimensional reach. When physicists in the early twentieth century were limited to four-dimensional Lorentzian spacetime, they had to treat as inputs many quantities that would, in higher-dimensional geometry, be derived as outputs. The accumulation of parameters reflected the fact that the explanatory dimension count was too low.

This is not a peculiarity of physics; it is a general feature of explanatory science. When chemistry was done before atomic theory, it had to treat the masses and properties of the elements as inputs; atomic theory turned these into outputs. When biology was done before molecular biology, it had to treat the structures of proteins and DNA as inputs; molecular biology turned these into outputs. At each step in the explanatory hierarchy, dimensional richness at the underlying level converts inputs at the level above into outputs.

The recognition that *physics itself* is subject to this pattern — that the universal constants and the parameters of the Standard Model are inputs at the 4D level that become outputs at the 6D level — is the central insight of the Arisaka-GG program. Six dimensions is the explanatory dimension count required for *this* reduction; the constants c, G_N, $\hbar$ are the empirical signatures of the missing dimensions. Once the dimensionality is corrected, the constants are absorbed into the geometry, and the Euclidean condition is restored.

This is, in the deepest sense, what the Arisaka-GG program has discovered: that the modern *Elements* is six-dimensional, that the postulates are the constitution, and that the theorems are the constants and parameters of the world we observe. On this reading the constants of physics are the empirical signatures of a higher-dimensional geometry, and GG-Theory is one attempt to write that geometry down explicitly.

41 A Fourth Scientific Revolution

It is natural to ask where, if the reconstruction holds, the Arisaka-GG program stands in the history of physics. We offer the following placement not as a claim of rank but as a way of locating the *kind* of change the program proposes. The history of theoretical physics can be read as a sequence of revolutions, each of which absorbed into principle or geometry a class of quantities that the preceding era had been obliged to take as given.

The first revolution — the Newtonian synthesis — unified terrestrial and celestial motion under a single law and thereby removed the need to specify the heavens separately from the earth. It left the universal constants, the forces, and the constitution of matter as empirical inputs.

The second revolution — Einstein's relativity — geometrized two of those inputs. Special relativity revealed the speed of light c as the conversion factor knitting time to space; general relativity revealed Newton's constant G_N as the conversion factor knitting mass–energy to curvature. Space, time, and gravitation became geometry, and two constants became geometric data rather than brute facts. But relativity stopped at four dimensions, and it described the universe as a closed system; the quantum, the gauge interactions, and the cosmological constant lay beyond its reach.

The third revolution — quantum theory and the Standard Model — organized matter and the non-gravitational forces with extraordinary precision. Yet it did so at the cost of an unprecedented proliferation of free parameters: some thirty dimensionless numbers, measured to many digits but derived from nothing. The third revolution bought predictive power with a debt of inputs, and the attempt to discharge that debt through grand unification and string theory, for all its depth, has so far enlarged rather than reduced the space of free choices.

The fourth revolution, which the Arisaka-GG program proposes, is the completion of the second by the geometric means the third made necessary. It adds two non-spatial dimensions — renormalization-group scale and gauge phase — and thereby geometrizes the third great constant, Planck's $\hbar$, as the conversion factor of the sixth dimension, just as relativity geometrized c and G_N. With the geometry complete, the dimensionless debt of the third revolution is discharged in full: the thirty parameters are recovered as theorems of the integer ledger OGN-5, and the cosmological constant, unreachable by a closed-system theory, acquires its scale from the open-system projection OGGEP. The proliferation of parameters is revealed, on this reading, not as a permanent feature of nature but as the shadow that a six-dimensional structure casts on four dimensions.

If the second revolution was the geometrization of space, time, and gravity, the fourth is the geometrization of the constants themselves. Its signature is not a new force or a new particle but a change in what counts as an input: where the third revolution measured, the fourth derives. This is a strong claim, and the program stakes it on falsifiable numbers rather than on rhetoric; the constitutional clause POS-7 guarantees that the claim can be broken by experiment, and Section 35 lists the numbers on which it rests. Whether the placement we have proposed is earned is, in the end, for the next decade's measurements to decide — which is precisely the Euclidean discipline the program accepts.

42 A Closing Word, in Plain English

We end in the register the technical work permits but does not require — plain language, for the reader who has walked the whole arc.

For twenty-three centuries the ideal of a science was Euclid's: write the rules down in plain view, and let everything else follow, with nothing held in reserve and nothing left to fit. Modern physics built a magnificent structure but quietly broke that promise: it came to rest on some thirty numbers that no one could derive, only measure. The claim of this paper is that the promise can be kept after all — and that keeping it asks only that we recognize two dimensions we already possessed but had not seen *as* dimensions.

The constants we hold most sacred are, on this reading, not properties of the world's furniture but the units of its geometry. Einstein taught us the first half of the lesson twice: the speed of light c is the conversion factor that knits time to space (special relativity), and Newton's constant G_N is the conversion factor that knits mass–energy to curvature (general relativity). GG-Theory takes the next step of the same lesson: the energy scale is a fifth dimension and the gauge phase a sixth, and Planck's constant $\hbar$ is the conversion factor of that sixth. The three great constants of modern physics — c, G_N, $\hbar$, the signatures of relativity, of gravity, and of the quantum — turn out to be three faces of one six-dimensional shape, one conversion factor for each kind of direction it contains. Everything else is either an integer forced by the geometry (the ledger OGN-5) or a number you can compute from these: the fine-structure constant, the cosmic energy budget, even the energy–information quantum that powers a thinking brain. There are no dials left to turn.

If this is right, the thirty free parameters of twentieth-century physics were never a permanent feature of nature; they were the shadow cast on four dimensions by a structure that lives in six. General relativity could not say what fixes the cosmological constant because it described the universe as a closed box; once we admit that every observation is a window cut into an open system, even that last free number finds its size. The Euclidean condition — a deductive science with nothing to fit — is, in this precise sense, recoverable, and the modern *Elements* turns out to be six-dimensional.

We do not ask the reader to believe this on faith; we ask the reader to hold it against the world. The framework has staked itself on sharp numbers — $\alpha^{-1} = 137$, the lock integer $K = 35$, a Kaluza–Klein resonance near $M_{\text{lock}} \approx 3.85$ TeV, a cosmic matter fraction near $1/\pi$, a biological energy quantum near $\mathsf{Q_B} \approx 0.1555$ eV — and any one of them, should the next decade's experiments disagree, breaks the chain. That is the discipline Euclid's method demands, and the discipline we accept. The book of nature, we believe, is written in geometry; this paper is one attempt to read a few more of its pages, and to do so in plain enough view that anyone who has come this far can check the reading for themselves.

Author's Note

The historical material in Part II, in particular the lineage from Euclid through Riemann, forms the basis of the present paper. The technical content of Parts III–IX, the axiomatic structure, the modified Bianchi identity BI-6, the OGN-5 theorem, the GDOS framework, the Gi-4 and Qi-4 Pillars, and the parameter audit are the work of the Arisaka-GG series and its companion papers. The author is solely responsible for the scientific content, the theoretical judgments, the physical interpretations, and any errors that remain.

This foundations paper completes the introductory material of the Arisaka-GG series. It is intended to be read alongside Arisaka (2026, A1-Constitution) (the constitution), Arisaka (2026, A2-GG6) (the bulk geometry), Arisaka (2026, A3-BI6) (the modified Bianchi identity and OGN-5 theorem), Arisaka (2026, A5-GDOS) (the GDOS framework), Arisaka (2026, A7-Qi4) (the Qi-4 Pillars, encompassing the Evolving Inner Cosmos and the Qi-HAL Completeness Index — the substantive content of the legacy "capstone integration"), Arisaka (2026, A9-SM) (the Standard Model derivation), and Arisaka (2026, A10-Cosmos) (the cosmological derivation). Together these papers constitute the complete development of the Arisaka-GG program through April 2026.

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All scientific concepts (especially the foundation of the GG-Constitution, GGEP/GG-6/BI-6, and OGGEP/GDOS/Gi-4/Qi-4 Pillars), theoretical judgments, physical interpretations, and any remaining errors are the sole responsibility of the author.

Part X
Technical Appendices

Orientation, Abbreviations, and the Cohort Naming Convention

Orientation: A Reader's Guide to the Appendices

This reference Part is written for readers meeting the GG-Theory program for the first time. The body of the present paper (Parts I–IX) carries the historical argument and the parameter-free reconstruction at the level of a working physicist; this Part is the compact lookup apparatus that supports it. Because GG-A4-Euclid is the foundations-and-audit volume of the cohort rather than a calculational paper, its appendices are deliberately light: a table of abbreviations and program-defined acronyms, a glossary of the symbols it uses, the program-wide naming convention used for cross-cohort citation, a pointer to the canonical Dictionary of program-defined terms maintained in Arisaka (2026, A2-GG6), and the one calculation the audit cannot cite elsewhere — the dimensional reduction of Appendix D. The detailed derivations on which the audit depends are not reproduced here; each is cited to the companion paper in which it is proved (Arisaka (2026, A1-Constitution) for the constitutional clauses, Arisaka (2026, A2-GG6) for the bulk theorems T1-GG6–T8-GG6, and Arisaka (2026, A3-BI6) for the BI-6 and OGN-5 proofs).

Organization of this reference Part

The reference material is organized as follows.

- *Appendix A* (Summary of Abbreviations): the cohort's shared vocabulary, each name with its meaning and a primary link, followed by a hierarchy figure that situates the acronyms within the program's conceptual structure. The table is cohort-wide rather than paper-local, so it carries some names this paper does not itself use. Program-defined names are set in sans-serif type so the reader can see at a glance which terms belong to GG-Theory proper.
- *Appendix B* (Glossary of Key Symbols): the symbols the paper uses, ordered by construction rather than alphabetically, each pointing at the equation that fixes it.
- *This Orientation*: the suggested reading order, the global naming convention for cross-cohort citation, and the three reading modes.
- The canonical *Dictionary of Program-Defined Acronyms* (forty-five entries in ten thematic categories) is maintained in Arisaka (2026, A2-GG6) §B and cited from here rather than reproduced, since the present paper is short and the Dictionary is cohort-wide.

Suggested reading order for first-time graduate-student readers

The paper is long and spans twenty-three centuries; readers approaching the material for the first time may find the following order useful as a graduate-seminar path.

1. *Preface* (front matter) for the Euclidean posture and why "no free parameters" is the organizing question.
2. *Part I* (Foundations) for the constitutional question, the two-pentad thesis, and the axioms and postulates of the GG-Constitution.
3. *Part II* (The 2300-Year Geometrization Arc) for the path from Euclid to Riemann and the absorption of c, G_N, and $\hbar$ into geometry (Maxwell, Minkowski, Einstein, Kaluza–Klein, Wilson–Hosotani, Randall–Sundrum).
4. *Part III* (The Two Non-Space-Like Dimensions) for the bulk and what the fifth and sixth dimensions are, with a closing section on precursors.
5. *Part IV* (Anomaly Freedom and the Integer Ledger) for BI-6 and the OGN-5-from-BI-6 theorem.
6. *Part V* (The Boundary Law) for GDOS and the Gi-4/Qi-4 Pillars.
7. *Part VI* (GU-5) for the conversion-factor theorems of the bulk triad $\{c, G_N, \hbar\}$, the boundary members $\mathsf{Q_B}$ and k_B, the mathematical constants e and π, and the closure theorem.
8. *Part VII* (The Parameter-Free Audit) for the Master Derivation Table and the Complete Parameter Audit — the heart of the paper.
9. *Part VIII* (Discussion) for the significance and uniqueness, the structural innovations, the problems solved, the Sharp Predictions, and the Falsifiability section (F-Euclid-1 through F-Euclid-7).
10. *Part IX* (Conclusions) for the Restoration of the Euclidean Condition, the fourth-scientific-revolution framing, and the plain-English philosophical close.
11. *Part XI* (For the Curious Reader) for twenty-five questions answered without assuming a physics degree. A reader who wants the argument and not the apparatus can begin here and read nothing else.
12. *Part XII* (Common Objections and Responses) for the fifty-three objections (O-Euclid-1 through O-Euclid-53), grouped into eleven themes, that a critical reader is most likely to raise.

Working physicists who already hold the constitutional and bulk-geometry machinery of Arisaka (2026, A1-Constitution), Arisaka (2026, A2-GG6), and Arisaka (2026, A3-BI6) may go directly to Part VII and consult Parts I–VI on demand.

Global naming convention for cross-cohort citation

Each load-bearing item of the GG-Theory cohort carries a globally unique identifier of the form *Prefix-Suffix-N*, where the Prefix names the item type and the Suffix names the paper.

- **T-Suffix-N** — Nth Theorem (e.g. T-GG6-6 is the OGN-5 Diophantine cascade of GG-A2-GG6).
- **L-Suffix-N** — Nth Lemma; **C-Suffix-N** — Corollary; **D-Suffix-N** — Definition; **R-Suffix-N** — Remark; **P-Suffix-N** — Prediction.
- **O-Suffix-N** — Nth Objection (e.g. O-Euclid-3 is the energy–information prediction-status objection of the present paper).
- **F-Suffix-N** — Nth Falsifier (e.g. F-Euclid-1 is the 3.85 TeV Kaluza–Klein resonance).
- **I-Suffix-N** — Nth Innovation; **PS-Suffix-N** — Nth Problem-Solved entry.

The canonical Suffix list for the twenty-two cohort papers is: *Constitution* (A1), *GG6* (A2), *BI6* (A3), *Euclid* (A4, the present paper), *GDOS* (A5), *GRIP* (A6), *Qi4* (A7), *QM* (A8), *SM* (A9), *Cosmos* (A10), *Origin* (A11), together with the phenomenology preprints *KK* (P1), *N3* (P2), *Electron* (P3), *Proton* (P6), the foundations preprints *Real* (P4) and *CPT* (P5), and the consequence preprints *Kaxion* (C1), *Hubble* (C2), *BH* (C3), and the white papers *Symmetry* (W1) and *Holonomy* (W2). Inside each paper the internal numbering ("Falsifier 1", "Objection 3", ...) continues to be supplied by LaTeX; the global identifier is the same item viewed from outside the paper, so a cross-cohort citation such as "GG-A4-Euclid consumes T6-GG6 and produces the audit of F-Euclid-1" is unambiguous and search-friendly. The four axioms (AX-1 ... AX-4) and seven postulates (POS-1 ... POS-7) of GG-A1-Constitution keep their existing cohort-wide names. As the cohort's parameter-free reconstruction volume, GG-A4-Euclid contributes seven *reconstruction* theorems of its own, T-Euclid-1 through T-Euclid-7 (Sec. 4), which prove that the GG-6 framework built in GG-A2-GG6 and GG-A3-BI6 contains no adjustable dimensionless freedom; the underlying *construction* theorems (e.g. the OGN-5 integer theorem) are cited from their source papers. The falsifiers (F-Euclid-N) and objections (O-Euclid-N) are likewise the paper's own enumerated items.

How to use this Part

A first-time reader is invited to use this Part in three modes.

- *Look-up mode*: consult Appendix A when an acronym or program-defined term appears unfamiliar in the body, and Appendix B when a symbol is.
- *Cross-citation mode*: use the global identifiers above to cite the present paper's falsifiers and objections from the companion papers, and to trace each audit entry back to the theorem of GG-A2-GG6 or GG-A3-BI6 that supplies it.
- *Audit mode*: walk the Complete Parameter Audit of Part VII clause by clause, asking of each entry whether it remains a free choice — the invitation issued in the Preface. This is the operational test of the Euclidean methodological condition at the level of the present paper.

A Summary of Abbreviations

This appendix is the cohort's shared vocabulary table, carried in the same form by GG-A1-Constitution and GG-A2-GG6, so that a reader arriving from any companion looks a name up in the same place. It therefore runs wider than the present paper: some entries name objects that belong to the program and are used elsewhere in it. The third column is the one to read for that — it points at the paper where each name is introduced in full, which is the present paper for names defined here and a companion otherwise. All program-defined names are rendered in *sans-serif type* (GG-Theory, GG-Constitution, GG-6, GDOS, BI-6, OGN-5, QGL, Gi-4, Qi-4, GRIP, BFFT, etc.) so the reader can see at a glance which terms belong to the GG-Theory program proper. Standard physics abbreviations imported from the wider literature (KK, RS, QFT, TOV, etc.) are rendered in plain uppercase Roman, following the convention of the major physics journals. Mathematical objects are in roman or italic in the usual way.

Table 27: The program's shared vocabulary, with each name's meaning and the place it is introduced in full. The table is cohort-wide rather than paper-local, so it also carries names this paper does not itself use. Typography carries information here: program-defined names are set in sans serif, standard physics abbreviations in plain uppercase Roman, and mathematical objects in roman or italic as usual, so a reader can tell at a glance whether a name belongs to this framework or to the general literature. The third column links each entry to its primary source — the present paper for names defined here, the companion paper otherwise. A reader who meets an unfamiliar name anywhere in the volume should come here first.

Symbol	Meaning	Primary link
Framework name and core program objects		
GG-Theory	Grand Geometric Theory (also: Geometry-Gauge Equivalence Theory)	GG-A1-Constitution, Preface
GG-Constitution	The constitutional charter of the GG-Theory program: four axioms (AX-1–AX-4) and seven postulates (POS-1–POS-7)	GG-A1-Constitution, §2–§3
GG-6	Geometry-Gauge locking in six dimensions: the 6D bulk (the principal stage of the parameter-free reconstruction of the present paper)	GG-A2-GG6, §8 (T1-GG6)
GDOS	Geometric Dynamics of Open Systems: the boundary law implied by OGGEP	GG-A5-GDOS
BI-6	Green–Schwarz-modified Bianchi identity at 6D	GG-A3-BI6, §3
OGN-5	Occam-Gauge-Normalization 5-integer ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$	GG-A3-BI6, §7; T6-GG6
QGL	Quadruple Gauge Locking: the four-factor bulk gauge group $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$	GG-A2-GG6, §10 (T3-GG6)
Gi-4	Geometric-Invariance four-pillar structure (TGG, CCI, GFF, GRIP)	GG-A6-GRIP
Qi-4	Quantum-Information four-pillar structure (Qi-QUA, Qi-EP, Qi-CON, Qi-HAL)	GG-A7-Qi4
GU-5	Geometric Universal Five: the conversion-factor pentad $\{c, G_N, \hbar; k_B, \mathsf{Q_B}\}$ — the subject of Part VI and of the paper's subtitle	GG-A4-Euclid, Part VI
Four axioms (AX-1–AX-4)		
AX	Axiom (axiomatic primitive; non-negotiable)	GG-A1-Constitution, §1.5
LC	AX-1 Locality and Causality	GG-A1-Constitution, §2.2

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Table 27 continued from previous page

Symbol	Meaning	Primary link
GGEP	AX-2 Geometry–Gauge Equivalence Principle	GG-A1-Constitution, §2.3
OGGEP	AX-3 Operational/Open Geometry–Gauge Equivalence Principle (boundary observation dictionary)	GG-A1-Constitution, §2.4
MSMF	AX-4 Maximum Symmetry, Minimum Freedom (no hidden continuous knobs)	GG-A1-Constitution, §2.5
Seven postulates (POS-1–POS-7)		
POS	Postulate (admissibility filter; testable in principle)	GG-A1-Constitution, §1.5
GI	POS-1 Gauge invariance across scales (LGI / DGI / GGI)	GG-A1-Constitution, §3.3
UEP	POS-2 Unified Extremal Principle (PLA ⇔ PLT / MEPP / FEP)	GG-A1-Constitution, §3.4
AF	POS-3 Anomaly Freedom and cancellability by inflow	GG-A1-Constitution, §3.5
CP	POS-4 Closure and Positivity (CPTP-admissible open dynamics)	GG-A1-Constitution, §3.6
RG	POS-5 Renormalization-Group covariance (scale covariance)	GG-A1-Constitution, §3.7
OGN	POS-6 Occam Gauge Normalization (canonical integer/rational normalization; *formerly* MRHN)	GG-A1-Constitution, §3.8
MF	POS-7 Measurement and Falsifiability (operational contract + kill criteria)	GG-A1-Constitution, §3.9
LGI	Local Gauge Invariance (microscopic / closed-system layer of GI)	GG-A1-Constitution, §3.3
GGI	Global Gauge Invariance (macroscopic / ledger layer of GI)	GG-A1-Constitution, §3.3
Gi-4 pillars		
TGG	Total Geometric–Gauge object (Pillar I of Gi-4)	GG-A6-GRIP
CCI	Covariant Conservation Identity (Pillar II of Gi-4)	GG-A6-GRIP
GFF	Geometric Free-Energy Functional (Pillar III of Gi-4; *absorbs the legacy* EEGG *ledger*)	GG-A6-GRIP
GRIP	Gauge-Runtime Invariance Pipeline (Pillar IV of Gi-4)	GG-A6-GRIP
GEP	Geometry–Gauge Energy Principle (energetic block of GFF; *formerly* EGG)	GG-A6-GRIP
CEP	Curvature–Entropy Principle (entropic block of GFF)	GG-A6-GRIP
DGI	Dynamical Gauge Invariance (initial stage of GRIP)	GG-A6-GRIP
GSSB	Geometric Spontaneous Symmetry Breaking (middle stage of GRIP)	GG-A6-GRIP
GA	Geometric Attractor (final stage of GRIP)	GG-A6-GRIP
GGA	Ground Geometric Attractor (the unique deepest GA; where the runtime halts)	GG-A6-GRIP def:GGA; GG-A11-Origin
GRIP-In	Fixed-landscape regime of the runtime (descent halting at the GGA)	GG-A6-GRIP

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Table 27 continued from previous page

Symbol	Meaning	Primary link
GRIP-Out	Landscape-reorganizing regime (supplies the next descent's initial datum)	GG-A6-GRIP
Qi-4 pillars		
Qi-QUA	Quantum information for QGL (Quadruple Gauge Locking) – Pillar I of Qi-4	GG-A7-Qi4
Qi-EP	Quantum information for the UEP (Unified Extremal Principle) – Pillar II of Qi-4	GG-A7-Qi4
Qi-CON	Quantum information for Conformal Invariance – Pillar III of Qi-4	GG-A7-Qi4
Qi-HAL	Quantum information for the Holographic Attractor Lattice – Pillar IV of Qi-4	GG-A7-Qi4
OH	Outer Hologram (boundary projection of bulk data; outward face of EIC)	GG-A7-Qi4
IH	Inner Hologram (inner self-modeling layer; inward face of EIC)	GG-A7-Qi4
EIC	Evolving Inner Cosmos (boundary completion of Qi-4 as the OH+IH composite)	GG-A7-Qi4
Qi-HAL acronyms		
MePMoS	Memory–Prediction–Motion Sensing	GG-A7-Qi4, Qi-HAL
NHT	Neural Holographic Tomography	GG-A7-Qi4, Qi-HAL
HAL	Holographic Attractor Lattice	GG-A7-Qi4, Qi-HAL
QCI	Qi-HAL Completeness Index	GG-A7-Qi4, Qi-HAL
Q-HAL	Quadratic / Cubic HAL	GG-A7-Qi4, Qi-HAL
BFFT	Binary Fourier / Fractal Fourier Transform	GG-A7-Qi4, Qi-HAL
GDOS-internal acronyms (program-defined)		
SIP	System Instance Packet (canonical GDOS input unit)	GG-A5-GDOS
SeOS	Selfish Open System	GG-A5-GDOS
Program-defined mathematical symbols		
$\mathsf{Q_B}$	Quantum-information Bit – the energy–information Arisaka constant $\mathsf{Q_B} \approx 0.1555$ eV per qubit; matches ATP hydrolysis at biological temperatures	GG-A2-GG6, §15 (T8-GG6)
$c, G_N, \hbar$	The fundamental triad of dimensional conversion factors (recovered as conversion factors of the 6D bulk in the present paper)	GG-A4-Euclid, §4
M_{lock}	First-lock TeV-scale mass cluster; $M_{\text{lock}} = (1/2)M_{\text{Pl}}e^{-K} \approx 3.85$ TeV	GG-A4-Euclid
M_6	Six-dimensional fundamental mass scale of the bulk; read in the units the trio $\{c, G_N, \hbar\}$ sets (a units convention, not a free parameter)	GG-A2-GG6, §9

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Table 27 continued from previous page

Symbol	Meaning	Primary link
Standard physics abbreviations (plain Roman, journal convention)		
Scales, unit systems and running		
UV	Ultraviolet: the high-energy end of a scale interval. In GG-6 it is the $r = 0$ end of I_r, not a short distance	GG-A4-Euclid, Sec. 17
IR	Infrared: the low-energy end, $r = K = 35$, where the lock scale sits	GG-A4-Euclid, Sec. 17
SI	Système International: the metric unit system in which c, $\hbar$ and G_N carry their familiar numerical values	standard
CGS	Centimeter–gram–second: the older unit system, used here only to show that a conversion factor's numerical value is a units choice	standard
Open-system / quantum-channel formalism		
CPTP	Completely Positive Trace-Preserving (channel admissibility for open evolution; AX-3/POS-4)	—
GKLS	Gorini–Kossakowski–Lindblad–Sudarshan (master equation for Markovian CPTP semigroups; POS-4)	—
GNS	Gelfand–Naimark–Segal (construction)	—
NZ	Nakajima–Zwanzig (memory-kernel projection for non-Markovian open dynamics)	—
POVM	Positive Operator-Valued Measure (generalized measurement model; AX-3/POS-7)	—
Quantum and classical foundational theories		
BRST	Becchi–Rouet–Stora–Tyutin (cohomology)	—
GR	General Relativity	—
QCD	Quantum Chromodynamics	—
QED	Quantum Electrodynamics	—
QFT	Quantum Field Theory	—
QM	Quantum Mechanics	—
WCS	Wu–Chern–Simons (differential cohomology)	—
GS	Green–Schwarz (anomaly-cancellation mechanism via tensor inflow)	—
Standard Model and beyond		
EFT	Effective Field Theory	—
EW	Electroweak	—
GUT	Grand Unified Theory	—
SM	Standard Model of particle physics (28 parameters audited in Arisaka (2026, A9-SM))	—

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Table 27 continued from previous page

Symbol	Meaning	Primary link
SSB	Spontaneous Symmetry Breaking (closed-system; generalized by GSSB)	—
SUSY	Supersymmetry	—
CKM	Cabibbo–Kobayashi–Maskawa: the quark mixing matrix, four of the Standard Model's parameters	GG-A9-SM
PMNS	Pontecorvo–Maki–Nakagawa–Sakata: the lepton mixing matrix, the neutrino-sector counterpart of CKM	GG-A9-SM
MSSM	Minimal Supersymmetric Standard Model, cited here only for its parameter count	standard
CPT	Charge–parity–time reversal, the discrete symmetry combination; read in this cohort as orientation reversal of the three real bulk factors	GG-P5-CPT
Higher-dimensional constructions		
AdS	Anti–de Sitter spacetime	—
AdS/CFT	Anti–de Sitter / Conformal Field Theory correspondence	—
CFT	Conformal Field Theory	—
KK	Kaluza–Klein (mode expansion / dimensional reduction)	—
RS	Randall–Sundrum (warped extra-dimension model; the geometric ancestor of GG-6 via the 1999 RS construction)	—
Quantum gravity and cosmology		
ΛCDM	Standard cosmological model with cosmological constant + cold dark matter	—
LQG	Loop Quantum Gravity	—
TOV	Tolman–Oppenheimer–Volkoff (equation)	—
Foundations of quantum mechanics and biochemistry		
ATP	Adenosine triphosphate (the universal energy–information currency; matches $N_A \mathsf{Q_B} \approx 15$ kJ/mol under cellular conditions)	—
CHSH	Clauser–Horne–Shimony–Holt (inequality)	—
DNA	Deoxyribonucleic acid, named here only in the biological-scale discussion	standard
EPR	Einstein–Podolsky–Rosen (paradox / argument)	—
Open-system / extremal-principle sub-acronyms		
FEP	Free Energy Principle (boundary inference avatar of UEP)	GG-A7-Qi4, Qi-EP
MEPP	Maximum Entropy Production Principle (heuristic boundary avatar of UEP)	GG-A7-Qi4, Qi-EP
PLA	Principle of Least Action (closed/bulk avatar of UEP; standard variational dynamics)	GG-A7-Qi4, Qi-EP

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Table 27 continued from previous page

Symbol	Meaning	Primary link
PLT	Principle of Least Time (operational/boundary avatar of UEP; fastest lawful path)	GG-A7-Qi4, Qi-EP
Experiments, surveys and data compilations		
LHC	Large Hadron Collider (CERN)	standard
HL-LHC	High-Luminosity LHC, the upgrade that carries the reach to the 3.85 TeV lock cluster	GG-P1-KK
ADMX	Axion Dark Matter eXperiment: the microwave-cavity haloscope whose band contains the predicted kaxion at 1.95 GHz	GG-C1-Kaxion
MADMAX	MAgnetized Disc and Mirror Axion eXperiment, a dielectric haloscope covering the higher-mass band	GG-C1-Kaxion
CMB	Cosmic microwave background	standard
CMB-S4	The fourth-generation ground-based CMB experiment, which bears on the predicted tensor-to-scalar ratio $r = 0.0043$	GG-A10-Cosmos
LiteBIRD	The satellite CMB polarization mission with the same target	GG-A10-Cosmos
Simons Observatory	The Atacama CMB survey now taking data	GG-A10-Cosmos
DESI	Dark Energy Spectroscopic Instrument: the baryon-acoustic-oscillation survey that tests the derived expansion history	GG-A10-Cosmos, GG-C2-Hubble
CODATA	Committee on Data of the International Science Council: the source of the recommended constants against which the audit's numbers are compared	standard
PDG	Particle Data Group, the corresponding compilation for particle properties	standard

Figure 11: hierarchy of program-defined acronyms in the GG-Theory program. Top: the framework name (GG-Theory) and its constitutional charter (GG-Constitution). Tier 2: the four axioms (AX-1–AX-4) and seven postulates (POS-1–POS-7) that together define the admissibility predicate. Tier 3: three bulk objects forced by the constitution — GG-6 (6D bulk; the principal stage of the parameter-free reconstruction of the present paper), BI-6 (modified Bianchi identity), OGN-5 (integer ledger). Tier 4: the boundary law GDOS that the bulk projects to via OGGEP, with input acronym SIP. Tier 5: the two four-pillar structures, Gi-4 (geometric flank, with sub-acronyms GEP/CEP blocks of GFF and DGI/GSSB/GA stages of GRIP) and Qi-4 (quantum-information flank, with Qi-EP sub-acronyms PLT/MEPP/FEP, Qi-CON sub-concepts Conformal/Lévy/Fractal/1/f, Qi-HAL sub-acronyms MePMoS/NHT/HAL/BFFT, the boundary completion OH+IH=EIC, and the runtime carrier SeOS (Selfish Open System) underneath). Bottom: the energy–information Arisaka constant $\mathsf{Q_B}$ (derived secondary quantity), with the Arisaka equation $J_E/\mathsf{Q_B} = J_e = J_B = J_{Qi}$ that

locks the four boundary currents through one conversion factor.

B Glossary of Key Symbols

The order below follows the construction rather than the alphabet — the bulk and its two extra directions first, then the fields on it, then the integer ledger, then the five conversion factors, then the boundary quantities the audit reports — so reading the table in order is a compressed second pass through the geometry. The third column points at the equation rather than the section, which is what a reader checking an unfamiliar formula actually needs.

Table 28: The symbols this paper uses, with their meanings and the place each is fixed. The order is the order of construction, not the alphabet. The third column is a pointer rather than a citation: where it names a companion paper the symbol is imported and not redefined here; where it names a section of this paper, that section is where the symbol acquires its value. A reader who meets an unfamiliar symbol in the middle of an argument should be able to resolve it without leaving this table.

Symbol	**Meaning**	**Where fixed**
The bulk and its coordinates		
$\mathcal{M}_6$	The six-dimensional bulk, $\mathcal{M}_4 \times I_r \times S^1_\theta$	Eq. (28)
ds^2_6	The canonical metric, $e^{-2r}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2 + d\theta^2$. The warp exponent is r itself: there is no curvature constant in front of it	Eq. (29)
r	The scale coordinate, $r = \ln(E_{\rm GUT}/\mu)$, running over $I_r = [0, K] = [0, 35]$. It is a renormalization-group scale, not a length	Eq. (73)
θ	The gauge-phase coordinate on S^1_θ, $\theta \in [0, 2\pi)$. It carries a holonomy and no radius	Sec. 18
I_r, S^1_θ	The scale interval and the phase circle: the two non-space-like directions	Part III
The fields on the bulk		
R	Curvature two-form of the six-dimensional Lorentz connection	Eq. (50)
$F_{\rm SU(5)}$, $F_{{\rm U(1)}_B}$, $F_{{\rm U(1)}_{Qi}}$	Field strengths of the three internal gauge factors	Eq. (50)
B_2, H_3	The Green–Schwarz two-form and its modified field strength	Eq. (50)
X_4	The BI-6 four-form, $dH_3 = X_4$	Eq. (50)
$(\alpha_1, \ldots, \alpha_4)$	The four BI-6 coefficients, $(1, -1, -2/5, -2/5)$	Eq. (51)
tr, Tr	Six-dimensional tangent trace and fundamental-**5** trace respectively; the abelian squares carry neither	Notation, p. 16
The integer ledger		
$(M, N, L; J, K)$	The OGN-5 ledger, $(1, 3, 5; 7, 35)$	Eq. (52)
$K = LJ$	The lock integer, 35, fixed twice over: as LJ and as $M^2 + N^2 + L^2$	Eq. (53)
N	The color count, 3 — not the generation count, which is N_g and is not fixed by the ledger	Eq. (52)
J	The closure index, 7; $\alpha^{-1}(0) = 2^J + N^2 = 137$	Eq. (52)

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Table 28, continued from the previous page.

Symbol	**Meaning**	**Where fixed**
The five conversion factors		
c	Speed of light: the conversion between time and space	Part VI
G_N	Newton's constant: the conversion between mass–energy and curvature	Eq. (45)
$\hbar$	Planck's constant: the conversion between a phase winding on S^1_θ and an action	Eq. (17)
k_B	Boltzmann's constant: the conversion between temperature and energy	Eq. (61)
$\mathsf{Q_B}$	The energy–information conversion factor, $(2/5)E_{\mathrm{Ry}}/K \approx 0.1555$ eV	Eq. (68)
GU-5	The pentad $\{c, G_N, \hbar; k_B, \mathsf{Q_B}\}$ taken as one object	Eq. (79)
Energy scales		
M_{Pl}, E_{Pl}	Planck mass and Planck energy, non-reduced unless the text says otherwise	App. D
M_6	The six-dimensional fundamental scale, $M_{\mathrm{Pl}}/\sqrt{\pi}$	App. D
E_{GUT}	The grand-unified scale, $E_{\mathrm{Pl}}e^{-2\pi} = 2.28 \times 10^{16}$ GeV — the $r = 0$ end of the scale interval	Eq. (73)
M_{lock}	The lock scale, $\frac{1}{2}E_{\mathrm{Pl}}e^{-K} \approx 3.85$ TeV, and the first Kaluza–Klein mass	Eq. (59)
E_{Ry}	The Rydberg energy, 13.606 eV	Eq. (68)
M_Z	The Z-boson mass, the scale at which $\hat{\alpha}^{-1}_{\mathrm{EM}} = 128$ is quoted	Eq. (84)
Boundary and cosmological quantities		
$\alpha^{-1}(0)$	Inverse fine-structure constant at zero momentum transfer, $2^J + N^2 = 137$	Eq. (83)
$\hat{\alpha}^{-1}_{\mathrm{EM}}$	The same constant at the Z pole, $2^7 = 128$	Eq. (84)
α_s, $\alpha_a(\mu)$	The strong coupling, and the running coupling of gauge factor a	Eq. (42)
J_E, J_e, J_B, J_{Qi}	The four boundary currents locked by the Arisaka equation	Eq. (54)
Ω_m, Ω_Λ, Ω_b	Cosmic density parameters, $1/\pi$, $1 - 1/\pi$ and $1/(2\pi^2)$	GG-A10-Cosmos
ζ_Λ	The vacuum-energy coefficient, $\pi/(LK) = \pi/175$	GG-A10-Cosmos
H_0	The Hubble constant, read in this cohort as two ends of one derived expansion history	GG-C2-Hubble

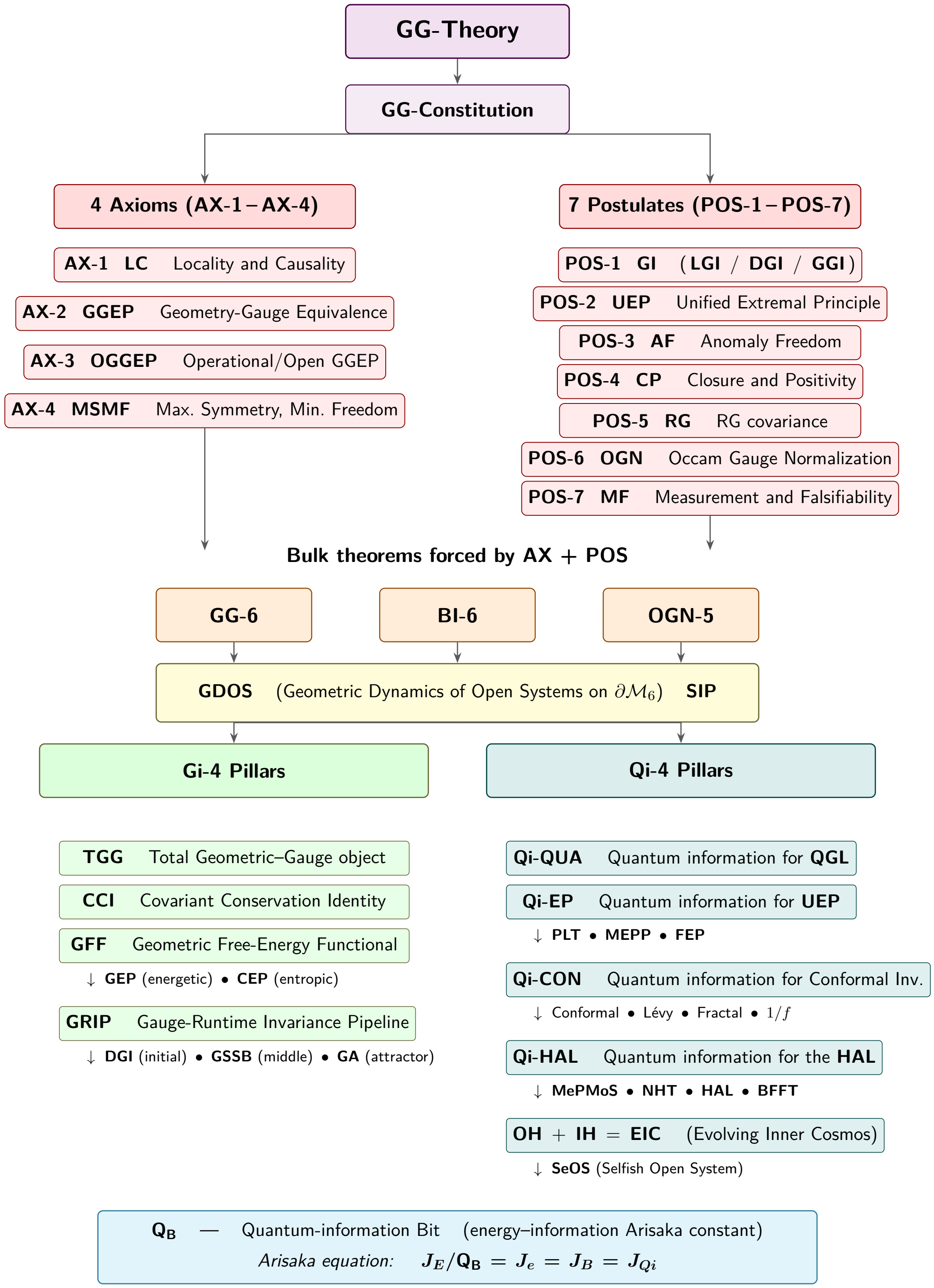

Figure 11: How the program's own names are related to each other. The chart is a hierarchy, not a list: the charter sits at the top, the geometry it forces below it, the boundary law below that, and the derived pillars at the foot, so the position of a name on the chart tells the reader what it depends on. Names at the same level are siblings introduced by the same construction. The purpose is orientation rather than definition — the meanings are in the abbreviations table — and a reader who has lost track of which name belongs to which layer can recover it here in one glance.

C Dictionary of Program-Defined Acronyms (Pointer)

The GG-Theory cohort maintains a single canonical *Dictionary of Program-Defined Acronyms* — forty-five entries organized into ten thematic categories (§ B.1–B.10) — in the foundational bulk-side paper Arisaka (2026, A2-GG6) §B. Each entry there carries four components: a formal *Definition* with its key equation, a plain-English *Physics-intuition* note, a *First-appearance* back-pointer, and *Forward-reference* pointers across the cohort. Because the present paper is a short foundations-and-audit volume and the Dictionary is cohort-wide, GG-A4-Euclid cites the canonical Dictionary rather than reproducing it; the abbreviation table of Appendix A and the Orientation above meet the present paper's local look-up needs.

The ten categories of the canonical Dictionary are as follows.

B.1 *Program Identity*: GG-Theory, GG-Constitution, GG-6.

B.2 *The Four Axioms*: LC, GGEP, OGGEP, MSMF.

B.3 *The Seven Postulates*: GI, UEP, AF, CP, RG, OGN, MF.

B.4 *Bulk Objects and Identities*: BI-6, OGN-5, QGL, the Arisaka equation, the Arisaka constant Q_B.

B.5 *Boundary Law and Pillar Structures*: GDOS, Gi-4, Qi-4.

B.6 *GRIP and its Sub-Vocabulary*: GRIP, TGG, CCI, GFF, the DGI–GSSB–GA cascade, GGA, and the two regimes GRIP-In/GRIP-Out.

B.7 *Qi-4 Sub-Vocabulary*: Qi-QUA, Qi-EP, Qi-CON, Qi-HAL.

B.8 *Bulk-Geometry Technical Objects*: $\mathrm{Spin}(1,5)$, $\mathrm{SL}(2,\mathbb{H})$, the quaternionic Goldilocks condition.

B.9 *Energy Scales and Predictions*: M_{lock}, E_{GUT}, M_{Pl}/M_6, GU-5, $\alpha^{-1} = 137$.

B.10 *Cosmology, A11-Origin, and Structural Classification*: $\Omega_m = 1/\pi$, $\Delta\nu/\nu = 1/35$, Stratum-4, GRIP-In/Out, SIP.

The entries most directly relevant to the parameter-free reconstruction of the present paper are those of category B.9 — the energy scales and the Geometric Universal Five Constants GU-5 — and category B.4 — the bulk objects, including the OGN-5 ledger and the Arisaka equation. The reader is referred to Arisaka (2026, A2-GG6) §B for their full definitions; the present paper's parameter audit (Sec. 32) uses exactly the quantities cataloged in those two categories.

D The Dimensional Reduction: From the 6D Action to the 4D Planck Mass

This appendix makes explicit the dimensional reduction of the bulk Einstein–Hilbert action over the $(4+2)$ geometry — the calculation that Arisaka (2026, A2-GG6) presents schematically and defers to the present paper, including the dimensional bookkeeping that converts the dimensionless radial coordinate r into a physical length. The outcome is the relation between the six-dimensional scale M_6 and the four-dimensional Planck mass $M_{\rm Pl}$, exhibited as a function of the OGN-5 lock integer $K = 35$ and the mathematical constant π.

D.1 Setup

The gravitational sector of the bulk action (32) is

$$S_{\rm grav} = \frac{1}{2\kappa_6^2}\int_{\mathcal{M}_6} d^6x\,\sqrt{-g_6}\,R_6, \qquad \kappa_6^2 = M_6^{-4}, \tag{89}$$

on the canonical scale-profile metric (29), $ds_6^2 = e^{-2r}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + dr^2 + d\theta^2$ with $r \in [0, K]$ and $\theta \in [0, 2\pi)$. The two internal coordinates r and θ are *dimensionless* — an RG-scale and an angle (Part III) — while x^μ carries the dimension of length. This is the crux of the bookkeeping: because the internal coordinates carry no length, the internal coordinate volume is a pure number, and exactly one factor of the bulk scale must be supplied to give it physical dimension.

D.2 The internal scale integral

With the scale profile multiplying only the four boundary directions, the metric determinant factorizes as $\sqrt{-g_6} = e^{-4r}\sqrt{-g_4}$, and the four-dimensional part of the six-dimensional Ricci scalar reduces as $R_6 \supset e^{2r}R_4$, where R_4 is built from $g_4 = \eta$. The four-dimensional Einstein–Hilbert term is therefore obtained by integrating $e^{-4r}\cdot e^{2r} = e^{-2r}$ over the internal directions, and the coefficient is the dimensionless internal (coordinate) volume

$$\boxed{V_2 \;=\; \int_0^{2\pi} d\theta \int_0^K dr\; e^{-2r} \;=\; 2\pi\cdot\tfrac{1}{2}\big(1 - e^{-2K}\big) \;=\; \pi\,\big(1 - e^{-2K}\big).} \tag{90}$$

The two factors have transparent origins: the 2π is the measure of the topological circle S^1_θ (the natural home of π as the period of the sixth dimension, Part VI), and the $\frac{1}{2}(1 - e^{-2K})$ is the scale integral along the radial RG-scale interval, cut off at the OGN-5 lock integer $K = 35$.

D.3 From the internal volume to G_N

Restoring physical dimension to the dimensionless coordinate volume V_2 requires exactly one factor of the bulk scale — the "implicit M_6^{-2}" of Arisaka (2026, A2-GG6), which sets the natural physical area of the internal directions. Carrying the bookkeeping through, the physical internal volume is $V_2^{\rm phys} = V_2/M_6^2$, and the four-dimensional Newton constant is the six-dimensional one ($G_6 = 1/M_6^4$, mass dimension -4) divided by it,

$$\boxed{G_N \;=\; \frac{G_6}{V_2^{\rm phys}} \;=\; \frac{1}{M_6^4}\cdot\frac{M_6^2}{V_2} \;=\; \frac{1}{\pi\,(1 - e^{-2K})\,M_6^2} \;\simeq\; \frac{1}{\pi\,M_6^2}} \tag{91}$$

for $K = 35$, where $e^{-2K} = e^{-70} \approx 4\times 10^{-31}$ is utterly negligible. Both sides now carry mass dimension -2, as a four-dimensional Newton constant must; it is exactly the one factor of the bulk scale named above that turns G_6's -4 into G_N's -2. Equivalently, reducing the Einstein–Hilbert term directly, the four-dimensional coefficient is $\frac{1}{2}M_6^4 \cdot M_6^{-2} \cdot V_2 = \frac{1}{2}V_2 M_6^2$, so that

$$M_{\rm Pl}^2 \;=\; \pi\,(1 - e^{-2K})\,M_6^2 \;\simeq\; \pi M_6^2, \tag{92}$$

which is the relation Arisaka (2026, A2-GG6) states schematically and defers here. This reproduces the boxed result $G_N = 1/(\pi M_6^2)$ of Section 26.2, and shows that result to be the $K \to \infty$ limit of a relation that carries the full scale profile $\frac{1}{2}(1 - e^{-2K})$. The bulk-to-Planck relation is therefore $M_6 = M_{\rm Pl}/\sqrt{\pi} \approx 0.564\,M_{\rm Pl}$ against the non-reduced Planck mass.

D.4 What is rigorous, and what is convention

Two things are now derived rather than asserted. First, the factor π in $G_N = 1/(\pi M_6^2)$ is *geometric*: it is the 2π measure of the topological circle times the $\frac{1}{2}$ of the scale integral. Second, the dependence on the OGN-5 integer K is explicit through the scale integral $\frac{1}{2}(1 - e^{-2K})$; because $K = 35$ is large, e^{-2K} is negligible and the bulk-to-Planck relation is fixed by π alone to some thirty decimal places. In this precise sense the relation between M_6 and $M_{\rm Pl}$ is determined by the OGN-5 data (K) and the mathematical constant π, with no continuous parameter intervening: M_6 is not an independent dial but a fixed multiple of the Planck scale (Sec. 37.3). The order-unity coefficient does depend on the metrological convention for $M_{\rm Pl}$, and rather than leave that as a hedge we state both: with $M_{\rm Pl} \equiv G_N^{-1/2}$ (non-reduced) the relation is $M_6 = M_{\rm Pl}/\sqrt{\pi} \approx 0.564\,M_{\rm Pl}$, and with $M_{\rm Pl}^2 \equiv 1/(8\pi G_N)$ (reduced) it is $M_6 = 2\sqrt{2}\,M_{\rm Pl} \approx 2.83\,M_{\rm Pl}$. Which is in force is a bookkeeping convention, not physics; that there is a definite coefficient in each is the content. The structural content — the π from the geometry, the K-dependence from the scale, and the conclusion that M_6 is a derived multiple of $M_{\rm Pl}$ — is convention-independent.

That is the whole of the apparatus this Part exists to hold: an orientation for a reader meeting the program for the first time, the abbreviations gathered in one place, a pointer to the cohort's canonical dictionary rather than a second copy of it, and the one calculation — the reduction from the six-dimensional action to the four-dimensional Planck mass — that Arisaka (2026, A2-GG6) defers to the present paper. None of it is new physics. All of it is here rather than in the body so that the argument can be read without interruption, and so that a reader who needs a definition at page ninety knows where it lives.

The two Parts that follow leave the apparatus behind. The first answers the questions a reader without a physics degree is entitled to ask, and answers them without prerequisites; the second answers the objections a specialist will raise, and answers them with pointers to the place where each claim can be checked — or broken.

Part XI
For the Curious Reader

Twenty-five questions a thoughtful reader without a physics degree actually carries into a paper like this one — answered plainly, and in the order they usually arrive

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The Conclusions have said what the paper proves. This Part answers the questions a reader is entitled to ask next, and it assumes no physics beyond a first course. It is placed here, ahead of the specialist objections, because a reader who wants these answers should not have to walk through the technical appendices to reach them. Nothing in this Part is needed by the argument of the paper; nothing in that argument is needed to read this Part. The questions a particle theorist, a cosmologist, or a string theorist would raise are answered separately, in the Part that follows.

The twenty-five questions that follow are not a specialist's. They are the ones a thoughtful reader without a physics degree actually carries into a paper like this one: what it means for a dimension not to be a direction you could walk in, whether geometry is discovered or invented and whether it matters which, why geometry should be entitled to fix the constants of physics rather than the other way round, and whether a world that reduces to geometry has left anything for physics to be. Several of these have been settled in popular science in directions this paper does not take, and a reader is owed plain answers before the specialist objections begin. They are met first, and plainly.

The hardest of the twenty-five is the fourth — why this reconstruction should fare better than Euclid's, whose axioms were the model of necessary truth for two thousand years and turned out to describe one geometry among many. The response concedes the difficulty rather than dissolving it, and says which part of it the paper can carry and which part it cannot. Where a claim rests on a step that is not closed, the response says so and names the step; where the honest answer is that the framework does not reach the question, it says that too.

Question 1 (Q-Euclid-1) (on the curious reader)

"What does it mean for a dimension not to be a direction in space?"

Response. It means it is a way for a state to differ that you cannot walk along, and the phrase "extra dimension" does the idea a disservice by suggesting otherwise.

Physics already uses dimensions of this kind without remarking on it. The phase of a quantum field is a genuine degree of freedom: two states can differ in it, the difference has measurable consequences, and no amount of moving about will take you from one to the other. Temperature, in a different sense, is a direction in the space of states rather than in the room. Nobody finds these mysterious, because nobody was tempted to picture them as corridors.

The two additional directions here are of that character, and they differ from each other in a way that matters more than either differs from space. One of them closes into a loop, so that going all the way around returns you to the start — but leaves behind a whole number recording how many times you went. That is where something can be *remembered*, because a count cannot be smoothly undone. The other has ends rather than closing, and travel along it is one-way in the technical sense: what happens cannot be reversed by going back. That is where quantities are *spent*.

Counted and spent. If a reader takes only that distinction away from this paper, they have the architecture, and they have it in a form that no picture of curled-up tubes would have given them.

Question 2 (Q-Euclid-2) (on the curious reader)

"Is geometry discovered or invented? Does this framework need an answer?"

Response. It does not need one, which is fortunate, because two and a half thousand years of argument have not produced one.

Both positions are serious. On the discovery side: mathematicians working independently find the same structures, results proved for one purpose turn out to describe something else entirely, and the unreasonable effectiveness of mathematics in physics is a real puzzle rather than a slogan. On the invention side: mathematics is done by people, its conventions bear the marks of history, and the structures we find are suspiciously well suited to the minds that look for them.

What this framework requires is weaker than either. It requires that certain mathematical statements be *true*, in whatever sense mathematical statements are true — that the ladder of number systems has four rungs and no fifth, that a particular counting problem has one solution. Whether those facts were waiting to be found or were constructed by us makes no difference to whether the counting problem has one solution. It has one either way.

There is a version of the question that would matter. If mathematics were purely invented — if the four-rung ladder were an artifact of notation rather than a fact — then a framework resting on it would be resting on a convention, and the explanations would be circular. Nobody serious holds that view of the ladder, but it is the right form of the worry, and it is more useful than the general question about Platonism.

Question 3 (Q-Euclid-3) (on the curious reader)

"Why should geometry get to fix physics rather than the other way round?"

Response. Because on this account they are not two things, and the question assumes a separation that general relativity already dissolved.

Until 1915 the separation was natural: geometry described the stage, physics described the play. Einstein's move was to notice that the stage responds — matter tells space how to curve, curvature tells matter how to move — so that gravity is not a force acting within geometry but geometry itself. That was not a philosophical reinterpretation. It predicted light bending, clocks running slow in gravity, and an orbit precessing by an amount nobody had explained, all of which were measured.

This framework extends the same move to the other three forces. Their structure is not written on top of the arena but is a feature of it, in the same way that gravity is, which is why the question "which fixes which?" has no answer here. There is one object; what we call geometric and what we call physical are two ways of reading it.

The claim is checkable in exactly the place where such claims can be. If the arena's structure permits the kinds of matter that exist and forbids kinds that do not, the identification is doing work. If a particle appears that the geometry has no room for, then the geometry was not fixing the physics after all, and the framework has been wrong in the most direct way available.

Question 4 (Q-Euclid-4) (on the curious reader)

"Euclid's axioms were the model of necessary truth for two thousand years, and turned out to describe one geometry among many. Why should yours fare better?"

Response. They may not, and this is the hardest question in the set. The honest answer begins by taking the parallel seriously rather than explaining why it does not apply.

What happened to Euclid is worth stating precisely, because it is usually stated too strongly. His axioms were not shown to be wrong. They were shown to be *a choice* — consistent, but one option among several, with the fifth postulate replaceable by alternatives that give equally consistent geometries. The shock was not that Euclid had erred; it was that necessity had been mistaken for one particular selection. Kant had built a philosophy on the assumption that Euclidean space was a precondition of experience, and that philosophy did not survive.

Any framework arguing from necessity is exposed to the same reversal, and this one has no immunity. What it can claim is a difference in what the necessity rests on. Euclid's fifth postulate was an assumption about parallels with no argument behind it; the constraints here are cancellation conditions and a finiteness theorem about number systems, and those are results rather than postulates. A theorem can be misapplied but it cannot be replaced by an equally consistent alternative theorem.

That difference is real and it is not a guarantee. The most likely way this framework repeats Euclid's fate is not that a constraint turns out false but that the constraints turn out not to apply — that they describe a class of structures nature is not in. Nothing internal can detect that, which is why the paper's weight has to rest on measurements rather than on the argument's beauty.

Question 5 (Q-Euclid-5) (on the curious reader)

"What does it actually mean to derive the constants of nature 'from geometry'?"

Response. It means the constants stop being independent inputs and become ratios fixed by the arena's structure — and the honest version of the claim is narrower than the phrase suggests.

Take an analogy that is exact rather than decorative. The ratio of a circle's circumference to its diameter is not a constant of nature that had to be measured; it follows from what a circle is. Nobody proposes an experiment to determine it more precisely, and nobody wonders why it takes that value rather than another. The claim here is that some quantities currently treated as measured inputs are of that kind: fixed by the structure, not by the world's happening to be a certain way.

Two cautions. First, this cannot apply to every constant, and it does not. Quantities that carry units — the speed of light, Planck's constant — are partly statements about the units chosen, and no derivation can produce a number whose value depends on the length of a meter bar. What can be derived are *dimensionless ratios*, which is where the substance is. Second, the derivation is conditional on the arena, which is assumed.

What the claim buys is exposure. A derived ratio is answerable to everything it touches, and cannot move when a measurement disagrees. A measured constant is answerable to nothing but its own measurement. Which of the two a quantity is turns out to be more important than its value.

Question 6 (Q-Euclid-6) (on the curious reader)

"If space is just geometry, what is it made of?"

Response. Nothing, in the sense your question is asking — and the answer to "what is it made of" being "nothing" is a respectable position with a long history rather than an evasion.

Newton thought space was a container: real, absolute, there whether or not anything was in it. Leibniz thought it was nothing but the relations among things, with no independent existence. That argument ran for two centuries without resolution and it was not settled by philosophy. It was settled — in Leibniz's direction, roughly — by general relativity, in which the geometry is dynamical and has no meaning apart from what is in it.

This framework inherits that position and pushes it. The arena is not a substance with properties; it is a structure, and structures are not made of anything. Asking what a structure is made of is like asking what a chess position is made of. The pieces are wooden, but the position is not.

There is a residue of your question that stays hard, and it deserves acknowledging rather than dissolving. If the world is structure all the way down, then at some point one is saying that relations exist without relata — patterns with nothing being patterned. Some philosophers find that coherent and some find it incoherent, and this paper does not settle it. What it can say is that the physics does not depend on the answer: the same predictions follow whichever view of substance a reader holds.

Question 7 (Q-Euclid-7) (on the curious reader)

"Non-Euclidean geometry was a shock. Are you proposing another one?"

Response. Something smaller, and it is worth being clear about the difference, because the earlier shock is often misremembered.

What made the nineteenth-century discovery shocking was not the new geometries themselves. It was the collapse of a claim about knowledge: that Euclidean space was known with certainty in advance of experience. When that claim fell, a large philosophical structure fell with it, and the reverberation was greater outside mathematics than inside.

Nothing here disturbs anything of that magnitude. This framework does not claim to know the arena in advance of experience — it assumes one, argues that the assumption is highly constrained, and then exposes the consequences to measurement. The arena's dimension and shape are open to being wrong in the ordinary way.

What is genuinely unfamiliar is smaller and specific: that two of the arena's directions are not spatial and are not of the same type as each other, one supporting memory and the other supporting cost. That is unusual, and if it is right it changes how one thinks about why some quantities are conserved and others dissipate. It is not a revolution in the theory of knowledge. Papers that promise those should be read with more suspicion than papers that promise a family count.

Question 8 (Q-Euclid-8) (on the curious reader)

"You say the reconstruction is parameter-free. Free of what, exactly?"

Response. Free of quantities that could be adjusted to make a disagreement go away. That is the operative meaning and it is narrower than the phrase sounds.

The framework is emphatically not free of *inputs*. The arena's dimension, its symmetry group and the constitutional requirements are all assumed, are listed, and are substantial. Anyone saying this construction starts from nothing would be misdescribing it.

What it is free of is dials. Having accepted the inputs, there is no place in the construction where a number can be chosen, and therefore no place where a number can be rechosen. The integers are the unique solution of a closure condition; the ratios follow from the integers; the predictions follow from the ratios. If a measurement disagrees, nothing internal can move.

The distinction matters because "few parameters" is a much weaker property than "no adjustable ones." A theory with three parameters and a great deal of flexibility in how they enter can absorb almost anything. A theory with twenty fixed integers and no continuous freedom cannot absorb anything at all. The Standard Model has twenty-eight measured parameters and is extraordinarily accurate; this framework has none and is extraordinarily fragile. Neither property is a virtue by itself, and the second one only becomes a virtue if the predictions survive.

Question 9 (Q-Euclid-9) (on the curious reader)

"Why is the speed of light what it is? Does this framework say?"

Response. No, and it should not, because the question as posed does not have a physical answer. The speed of light in meters per second is a statement about meters and seconds. Change the definition of the meter and the number changes, without anything about the world changing. Indeed the meter is now *defined* by fixing the speed of light, so the value is exactly what it is by convention. There is nothing there to derive.

The questions with content are about dimensionless ratios — the fine-structure constant, mass ratios, the number of families — because those are the same for anyone regardless of what units they picked. This framework addresses ratios, and stays silent about quantities that carry units. A framework claiming to derive the speed of light in meters per second would be making an error about what kind of statement that is.

There is a version of your question that is real and that the framework does address: why light has a finite invariant speed at all, rather than an infinite one or none. That is a structural question about the arena's causal geometry rather than a numerical one, and the answer is that a structure supporting the constitutional requirements has to have an invariant speed. What number it comes out as when written in units chosen from the length of a French bar is not physics.

Question 10 (Q-Euclid-10) (on the curious reader)

"If Planck's constant emerges from geometry, was it ever a constant of nature?"

Response. It was always a conversion factor, and seeing it that way is clarifying rather than deflationary.

Planck's constant relates energy to frequency. In a system of units where energy and frequency were measured in the same unit, it would be one and would not appear. Something similar is true of the speed of light relating space to time, and of Boltzmann's constant relating temperature to energy: each is a number whose value records that two quantities we treat as different kinds were historically measured in different units.

What is physical, in each case, is not the number but the *fact that the two quantities are related at all.* That energy and frequency are the same kind of thing is a deep statement about the world. That the ratio is 6.626×10^{-34} in joule-seconds is a statement about joules and seconds. This framework's contribution is to the first kind of statement rather than the second. It says why certain quantities that look independent are related, and it derives the dimensionless ratios among them. It does not derive, and could not derive, the numerical value of a constant carrying units. A reader who takes away only that distinction will be better equipped to evaluate any claim to have "derived a constant of nature," including the ones in this paper.

Question 11 (Q-Euclid-11) (on the curious reader)

"Why should there be a shortest length at all?"

Response. Because a description with unlimited resolution turns out not to be a description, and the failure is specific rather than philosophical.

The trouble is that probing a smaller distance requires more energy, and enough energy concentrated in a small enough region makes a black hole. Past a certain scale, trying to look more closely produces a horizon rather than a sharper image, and the resolution gets worse. That argument uses only gravity and quantum mechanics, neither of which is in dispute, and it is why almost every serious approach to fundamental physics ends up with a shortest length in some form.

What differs between approaches is what the shortest length *is*. Some treat it as a fuzziness, some as a discreteness, some as an emergent feature of something more basic. Here it enters as a property of the arena: the additional directions have finite extent, and that extent sets the scale below which the four-dimensional description stops being the right one.

The consequence worth noticing is that this is not a philosophical commitment but a prediction. A finite extent implies a tower of states at a computable energy, in the same way that a string of fixed length supports a definite series of harmonics. That energy is fixed by the integer ledger rather than chosen, which means the shortest length is not a free parameter here and the framework can be caught out by looking.

Question 12 (Q-Euclid-12) (on the curious reader)

"Is time a dimension in the same sense as the others?"

Response. No, and the difference is structural rather than a matter of attitude.

In the mathematics, time enters with an opposite sign from the spatial directions, and that single sign is responsible for everything that makes time feel different: it is why there is a speed limit, why the past and future of an event are separated from the elsewhere, and why you can stand still in space but not in time. It is not a small technical detail dressed up. It is the whole distinction.

Where this framework says something further is about the two additional directions, which are not spatial and are not temporal either. One closes into a loop; one has ends and is one-way. The one-way direction is not time — it is not what clocks measure — but it shares time's asymmetry, and the framework's account of why some processes cannot be run backwards lives there rather than in the time direction proper.

It is worth naming what this does not settle. Whether time *flows*, whether the present is special, whether the future exists in the way the past does — these are questions about which physicists disagree and on which nothing in this paper takes a position. What it offers is narrower: a structural reason why certain quantities cannot be recovered once spent, which is one thread of the puzzle and not the whole of it.

Question 13 (Q-Euclid-13) (on the curious reader)

"Mathematicians study infinitely many geometries. Why should nature use this one?"

Response. Because almost all of them cannot carry what a physical description requires, and the filter is much more severe than the infinity suggests.

It is true that the space of geometries is unbounded. It is also true that the overwhelming majority of them admit nothing resembling physics: no consistent notion of cause preceding effect, no stable structures, no way for a description to be well defined for an observer inside rather than outside. These are not aesthetic requirements. Each of them is a condition without which the mathematics stops producing numbers that could be probabilities.

Impose them and the infinity collapses fast. Requiring a causal structure of the familiar kind removes most of it at a stroke. Requiring the algebraic consistency that bounds the dimension removes almost all the rest. Requiring the cancellation condition, on what survives, leaves a single assignment. That is the shape of the argument, and each step is a theorem rather than a preference.

The residual worry is the right one and the paper does not dissolve it: the filter is only as good as the requirements imposed, and those were selected by a person. A different selection would give a different survivor. The defense is that the requirements are few, are declared before anything is derived, and are of a kind that a physicist would have wanted independently — not reverse-engineered from the answer. Whether that defense holds is something a reader can check, because the list is short.

Question 14 (Q-Euclid-14) (on the curious reader)

"Does the framework say what happens at a singularity?"

Response. Not fully, and where it does say something the statement is more modest than the question usually expects.

A singularity in general relativity is not a place where something dramatic happens. It is a place where the theory stops producing answers — curvature grows without bound, the equations return infinities, and the description fails. Everyone agrees this signals the theory's breakdown rather than an infinitely dense object, and every serious approach to quantum gravity is in part an attempt to say what replaces it.

What this framework contributes is a natural scale at which the four-dimensional description gives out: the extent of the additional directions. Below that, the four-dimensional account is not the right one, and the question of what happens has to be asked in the arena rather than in the projection. That is a real statement — it says the breakdown is a feature of the projection rather than of the world — and it is not a complete answer.

What the paper does not do is compute what a black hole's interior contains or what preceded the earliest moment. Those require the full dynamics in the arena, which is not developed here, and claiming otherwise would be overreach. A reader interested in singularities will find the framework's contribution is to say where the question has to be moved to, not to answer it.

Question 15 (Q-Euclid-15) (on the curious reader)

"If everything reduces to geometry, is physics just applied mathematics?"

Response. No, and the gap between the two is where all the risk lives.

A mathematical structure is true or false internally, and its truth is settled by proof. A physical claim asserts that a particular structure describes the world, and nothing internal to the structure can establish that. The gap is unbridgeable in principle: every consistent structure is equally true as mathematics, and at most one of them is the world's.

This is why a framework of this kind is more exposed than it looks, not less. The mathematics being correct is necessary and buys nothing. All the content is in the assertion that this structure and not another is instantiated, and that assertion is settled by measurement in exactly the way the crudest empirical claim is. A reader who comes away thinking that the elegance of the construction is evidence for it has misread the paper.

There is also a practical answer. Almost everything physicists do — designing an experiment, estimating a background, understanding why a material superconducts — is untouched by anything here and would remain untouched if all of it were correct. The reduction, even at its most successful, is a reduction of the rule book. The subject is not the rule book.

Question 16 (Q-Euclid-16) (on the curious reader)

"Newton thought space was absolute; Leibniz thought it was only relations. Where does this land?"

Response. Closer to Leibniz, but the debate has been reshaped enough that neither original position is quite available.

Newton's space was a container with an existence of its own; you could in principle ask where something was in absolute terms. Leibniz denied it — for him space was the order among things and nothing besides, so a universe shifted uniformly would not be a different universe, and to say otherwise was meaningless. The argument was famous and did not resolve.

General relativity settled the substance of it against Newton. There is no fixed container; the geometry is dynamical, responds to what is in it, and has no meaning independent of that. Leibniz was vindicated on the point at issue.

Where the modern position departs from Leibniz is that the geometry, though not a container, is not merely bookkeeping about relations either. It carries energy, propagates waves that have been detected, and does things on its own account. It is a participant. This framework takes that further: the arena's structure is what the forces and the matter *are*, not a summary of how they are arranged. That is a third position rather than either of the original two, and it is the one your question is really asking about. It has the merit of being checkable, which the Newton–Leibniz argument was not.

Question 17 (Q-Euclid-17) (on the curious reader)

"Isn't our universe just one bubble among countless others?"

Response. It might be. What this paper does is remove the reason the idea became necessary. The multiverse solves a specific problem: the constants look arranged, and if they could have been otherwise the arrangement demands explanation. An ensemble supplies one and is coherent. Its premise is that the constants could have been otherwise. In the picture developed here the dimensionless ratios follow from the arena's structure, and the arena is not a draw from a distribution — it is the survivor of a filter. If that is right, the observation that motivated the ensemble stops being surprising, and the ensemble has nothing left to do.

The comparison worth drawing is with a familiar case. Nobody proposes an ensemble of universes to explain why the ratio of a circle's circumference to its diameter takes the value it does, because that ratio is not the kind of thing that could have been set. The claim here is that more of physics is of that kind than has been assumed — and unlike the multiverse, the claim is exposed, because it says how many families of matter there are and what a compact star weighs.

Question 18 (Q-Euclid-18) (on the curious reader)

"The universe looks fine-tuned for life. Doesn't that point to a designer?"

Response. It points to something requiring explanation, and the step to a designer needs a premise this framework denies.

The observation is real: alter certain quantities slightly and there is no chemistry, no structure, nothing lasting. Feeling the force of that is not naive.

The inference requires that the quantities could have been set otherwise. A dial that was turned implies a hand on it. If the values follow from the arena's structure — if there was no dial — then asking who chose them is like asking who decided the diagonal of a unit square.

This is not an argument against a designer and should not be recruited as one. A reader who believes on other grounds is untouched by this paper; so is one who does not. What changes is that one particular argument, which has carried great weight in popular discussion, loses its premise if the framework holds. And whether it holds is settled by a family count and a stellar measurement, which is an odd place for a theological argument to be decided and is nevertheless where this one now sits.

Question 19 (Q-Euclid-19) (on the curious reader)

"If the geometry fixes everything, is my future already written?"

Response. Nothing here touches your future, and the inference does not go through however tight it feels.

What the geometry fixes is the rule book: what can exist, what is conserved, what ratios hold.

What it does not fix is any particular history. The distinction is between the rules of chess and the game played last Tuesday — and a rule book with no alternatives says nothing whatever about which permitted game occurred.

There is a further point that cuts deeper. Even a fully deterministic rule book delivers no readable future, because the only system capable of computing what the rules imply for something as complicated as a person is a system at least that complicated running at least that long. There is no shortcut and no vantage point. A universe can be lawful all the way down and remain, in the only sense available from inside, unwritten until it happens.

And the limit: this paper is about the structure of an arena. What agency is, and whether it survives contact with physics of any kind, is not a question it reaches or pretends to.

Question 20 (Q-Euclid-20) (on the curious reader)

"Why is there something rather than nothing?"

Response. The framework does not touch it, and saying so is more useful than a gesture would be.

Everything here is conditional — given an arena with certain properties, this follows. Why there is an arena at all sits outside the construction, and no rearrangement reaches it. That is not a gap further work closes; it is a different kind of question.

What can be contributed is narrower and not nothing. If the framework is right, the space of possible somethings is far smaller than it appears: not a landscape of coherent worlds from which one was drawn, but very nearly a single admissible structure. That converts "why this?" into "why anything?" — harder, older, cleaner. Narrowing a question is a service and not an answer.

There is a temptation, in a paper about geometry, to say that mathematical structures exist necessarily and therefore something must. That move is available, has serious defenders, and does not follow from anything here.

Question 21 (Q-Euclid-21) (on the curious reader)

"A small pile of geometric facts explaining this much makes me suspicious. Shouldn't it make you suspicious too?"

Response. It should and it does. Eddington is the standing warning: a great astrophysicist who spent his final years deriving the fine-structure constant and the particle count of the universe from combinatorial arguments nobody accepted, and whose reputation never recovered.

What separates a derivation from a coincidence is not the quality of the agreement but whether the structure producing it exists for another reason and whether it commits you to anything further. A quantity reverse-engineered from a measurement will always match. Here the geometry is fixed by an algebraic finiteness theorem and a set of consistency requirements that know nothing about particle physics, and what comes out then determines a family count, a charge pattern and a stellar correction that were not used in obtaining it.

That is the only defense available and it is the right one: not "too good to be chance" but "here are further consequences, please check them." Keep the suspicion. If the consequences fail, the geometry was a coincidence and no elegance in the derivation will help.

Question 22 (Q-Euclid-22) (on the curious reader)

"Why should I take one person's framework seriously against thousands working on other approaches?"

Response. Not because of who wrote it, and not despite it — both are the same error with opposite sign.

The practical question is fair, since attention is finite. The useful filter is exposure rather than headcount: a framework supported by thousands that forbids nothing measurable is a worse use of attention than one supported by a single person that forbids specific things and names the instrument that decides. Community size tracks funding history and fashion, and does not track correctness.

The cost of working outside a community is real and should be stated. Errors are found faster where more people look, and a framework developed largely by one person carries a higher chance of a mistake that a hundred readers would have caught quickly. Internal coherence does not reduce that.

So do not take it on trust. Take one prediction, check whether it is stated precisely enough to be wrong, and judge the rest by the answer. If a claim cannot be made precise, its provenance is irrelevant. If it can, its provenance is also irrelevant.

Question 23 (Q-Euclid-23) (on the curious reader)

"What would convince you that you are wrong?"

Response. Specific things, listed rather than gestured at.

A fourth family of matter would end it: the count follows from the arena and cannot be adjusted. Running the accelerators through the predicted energy without finding the expected structure would end the claim that the additional directions have the extent the ledger says. A dimensionless ratio measured outside the derived value, where the framework claims to derive it, would break the identification of geometry with physics at the point it is most exposed. And an internal failure counts fully: if the finiteness argument that bounds the dimension is misapplied, the arena is not forced and everything downstream is a correct calculation about the wrong structure.

What would not convince: a disagreement in a regime not claimed, or the failure of a result already flagged as conditional. A falsifier list is only honest if it excludes things.

It is worth noting where the framework is most likely to fail, which is not where it is most often attacked. The exposed joint is not the elegance of the geometry but the step from a mathematical filter to a claim about what nature instantiates, and no measurement tests that step directly — only its consequences.

Question 24 (Q-Euclid-24) (on the curious reader)

"What would it mean to measure a geometry, rather than infer one?"

Response. It would mean measuring the relationships the geometry forces, and there is no other way to measure a geometry — including the one we already live in.

This is worth dwelling on because it removes an apparent asymmetry. Nobody has ever measured the geometry of spacetime directly either. What is measured is that light bends by a certain amount near the sun, that clocks at different heights disagree, that two black holes merging radiate a waveform of a particular shape. The geometry is what makes those hang together, and it is accepted because no alternative account makes them hang together as well.

The additional structure proposed here is in exactly the same position, one level less accessible. What can be measured is a tower of states at a computable energy, a family count, a fractional shift in how compact stars weigh, and a pattern of permitted charges. If all of those hold and no four-dimensional account relates them, the geometry has been measured in the only sense the word has.

The important consequence is that "it can never be observed directly" is not the objection it sounds like. Nothing fundamental in physics is observed directly. The objection that bites is a different one: whether the structure forces enough to be caught out. That one has an answer here, and the answer is a list.

Question 25 (Q-Euclid-25) (on the curious reader)

"I'm not a physicist and I've got this far. What is the one thing you want me to take away?"

Response. That the deepest-sounding questions in physics often turn out to be questions about units, conventions, or what a word was doing — and that learning to spot which is which is more useful than any particular answer.

Three examples from this paper alone. "Why is the speed of light what it is?" looks profound and is a question about meters and seconds. "What is space made of?" looks profound and turns out to assume that structures are made of something. "Why should geometry fix physics?" assumes a separation that stopped being available in 1915. None of these is a stupid question — they are the questions anyone would ask — but in each case the work is done by noticing what the question presupposes rather than by finding an answer.

What remains after that filtering is a small number of questions with real content: how many kinds of matter are there, what dimensionless ratios hold among their properties, and are those numbers forced or merely found. This paper argues they are forced, by a structure whose shape is fixed by a theorem about number systems that has nothing to do with physics.

So take away the habit rather than the conclusion. When you next meet a grand claim about the universe, ask first what the question presupposes, and then what the claim forbids. Between them those two questions will get you further than any amount of mathematics you have not got time to check.

Part XII
Common Objections and Responses

The strongest objections to the parameter-free reconstruction — from the particle theorist, cosmologist, string theorist, geometer, historian of science, and skeptical reader — answered, with pointers into the paper

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A reconstruction that claims to remove every free parameter from physics invites the most demanding scrutiny, and it would not be honest scholarship to leave the obvious objections unstated. We collect here the strongest adversarial readings of the parameter-free thesis — the ones a particle theorist, a cosmologist, a string theorist, a differential geometer, a historian of science, and a skeptical reader would each raise on a careful reading — and answer each, with explicit pointers to the theorem, table, or companion paper that bears the weight. Where the honest answer is "not yet," we say so and name the open item.

The objections are grouped into eleven themes: whether the reconstruction is genuinely parameter-free; the six-dimensional architecture and its constants; the meaning of the two non-spatial dimensions; the relation to Kaluza–Klein, Randall–Sundrum, and string theory; the two pentads and their arithmetic; the empirical predictions and what GG-A4-Euclid itself stakes; the historical and methodological framing; the Euclidean framing and geometrization as method; the parameter-free audit under the microscope; dimensions, causality, and the geometry; and method, cohort, and honest status. None of the answers asks the reader to accept the program on faith; each points to where the claim can be checked — or broken.

E Is the reconstruction genuinely parameter-free?

Objection 1 (O-Euclid-1) (the particle theorist)

"Is $J = 7$ genuinely forced, or is it retrofitted to match $\alpha^{-1}(0) \approx 137$?"

Response. The arithmetic identity $2^7 + 3^2 = 137$ is suspiciously close to CODATA 2022's $\alpha^{-1}(0) = 137.035999177$, and a skeptical reader is right to ask whether the framework has retrofitted the integer ledger to match the empirical value. The response, given as the "Why $J = 7$ is forced and not retrofitted" paragraph at the start of Sec. 35.1 and developed in detail by the OGN-5 theorem of Arisaka (2026, A3-BI6), runs in three logical steps that close before α^{-1} ever enters the argument: (i) the dual lock $K = LJ = M^2 + N^2 + L^2$ with the closure-index clause $J = 2N + 1$ and the block embedding $N < L$ forces $N = N_c = 3$; (ii) the same closure-index clause then gives $J = 2N + 1 = 7$; (iii) only at this point does the dyadic channel capacity $2^J = 128$ become a derived prediction. $(N, J) = (3, 7)$ is fixed by the Diophantine cascade in GG-A3-BI6 before any contact with experiment; the agreement of the integer 137 with $\alpha^{-1}(0)$ to three significant figures is therefore a post-derivation empirical hit, not a post-hoc retrofitting.

The BI-6 trace ratio agrees at the same integer and is carried as the independent check — and whether one reads it as check or as route, it is as independent of α as the forcing is, which is the whole of what this objection turns on. Indeed, the structural argument was already in print in GG-A2-GG6 and GG-A3-BI6 before CODATA 2022 was published, and $(J, N) = (7, 3)$ in those papers is not a free choice.

Objection 2 (O-Euclid-2) (the cosmologist)

"The cosmology partition $\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$ matches Planck 2018 within 1σ, but couldn't a freely fitted ΛCDM model also match at the same precision?"

Response. The objection is correct as stated for the present-data regime: at Planck 2018 + BAO + SNe precision, the π-locked partition and the conventional ΛCDM fit are statistically indistinguishable, and a Bayesian model comparison between the two would not significantly favor either. The response, registered as O11 of Sec. 39, is that the test is genuine but resolves on a definite experimental timeline. The next-generation surveys — CMB-S4, Simons Observatory, the Roman Space Telescope, and DESI completion — are expected to constrain Ω_m to the ± 0.002 level by the late 2020s. At that precision, the π-locked partition becomes a $\sim 5\sigma$ test: if the joint posterior on Ω_m falls outside the interval $[0.315, 0.322]$, the partition is falsified at $\geq 3\sigma$ and GG-Theory in its current form requires reformulation. After all, the cost of a framework with zero free parameters in the cosmological sector is precisely that the predictions are sharply falsifiable on a definite timeline; the Bayesian-tie at present-data precision is a feature, not a bug, of a parameter-free reconstruction whose test is in the next round of data.

Objection 3 (O-Euclid-3) (the biochemist)

"Is the energy–information match $N_A \mathsf{Q_B} \approx 15.0$ kJ/mol to ATP hydrolysis a genuine prediction or a post-diction?"

Response. It is a genuine parameter-free prediction, and the supposed "experimental input" is not one. The derivation chain OGN-5 $\to \alpha \to m_e \to E_{\mathrm{Ry}} \to \mathsf{Q_B}$ developed in Arisaka (2026, A7-Qi4) contains no empirical free parameter: the final step $\mathsf{Q_B} = (2/5)\, E_{\mathrm{Ry}}/K$ uses only $K = 35$ (the OGN-5 lock integer) and $2/5$ (the BI-6 Abelian trace ratio), while the Rydberg energy $E_{\mathrm{Ry}} = \alpha^2 m_e c^2/2 = 13.606$ eV is *itself derived* — from the pure-number identity $\alpha^{-1} = 137$ and the electron mass m_e, which is fixed by the Yukawa localization of Arisaka (2026, A9-SM), not fitted. E_{Ry} is thus a theorem of the ledger, not a measured constant fed into it; treating it as "empirical" would be precisely the error the no-free-parameters thesis forbids. The framework predicts the *existence* of the energy–information conversion factor, the *structural* relation $J_E/\mathsf{Q_B} = J_e = J_B = J_{Qi}$ on the boundary, and the *numerical* value $\mathsf{Q_B} \approx 0.1555$ eV alike — its empirical calibration against biological energy currents (GG-A7-Qi4/GG-A11-Origin) being a zero-parameter result that meets an independently measured biochemical number. The one residual is completeness, not freedom: the closed-form A9 derivation of the electron mass (hence of E_{Ry} and $\mathsf{Q_B}$) is still being finalized, the same sub-task tracked as O9/O10 of Sec. 39. What we concede is therefore that the printed closed form of m_e is in progress — not that $\mathsf{Q_B}$ carries a free or empirical parameter.

Objection 4 (O-Euclid-4) (the skeptic)

"The master audit table claims "zero free dimensionless parameters" but the fermion mass spectrum (six quark masses, three lepton masses, three neutrino masses, CKM, PMNS) is admittedly "structurally derived but not yet completely audited". Isn't the zero-free-parameters claim therefore an overclaim?"

Response. The objection is correct as stated, and the claim is accordingly *sector-precise* rather than global: the audit closes for the gauge and cosmological sectors — the five GU-5 conversion factors ($c, G_N, \hbar; k_B, \mathsf{Q_B}$) and the two mathematical constants (e, π); the five OGN-5 integers (M, N, L, J, K); and ten derived dimensionless empirical residues including α^{-1}, $\sin^2\theta_W$, Ω_m, Ω_Λ, Ω_b, and $N_A\mathsf{Q_B}$ — with the fermion mass spectrum tracked as O9 of Sec. 39. The Yukawa localization profiles of Arisaka (2026, A9-SM) provide the structural mechanism by which the fermion masses and mixing angles are determined; the closed-form formula for each fermion mass is in progress. Until that closed-form audit lands, the parameter-free claim should be read with the explicit O9 caveat: zero free dimensionless parameters in the gauge and cosmological sectors, with the fermion sector structurally framed but not yet completely closed. One further clause completes the honest count: the quantum-defect corrections this paper imports (the QD rows of Table 20) belong to a frozen, bounded, quantized completion class, stated as a rule in the codebook of Arisaka (2026, A9-SM) (§37.4, "The completion class, stated as a rule"): exactly one lattice or codebook unit per observable, drawn from the already-admitted small factors of the ledger, incapable of further adjustment — with necessity asserted, not derived, at the codebook's own grade. The parameter-free claim of this paper therefore reads, in full: zero free continuous parameters, plus one frozen discrete completion class whose necessity is asserted, not proved. This is what intellectual honesty looks like at the level of a program of twenty-two coordinated foundational papers: the substantive claims are settled, the technical sub-tasks are identified, and the residual uncertainties are clearly named.

Objection 5 (O-Euclid-5) (the philosopher of science)

"Even granting zero dimensionless parameters, haven't you just relocated the freedom into the choice of axioms and geometry — the constitution is itself a parameter chosen to give this world?"

Response. Distinguish two kinds of freedom. A free parameter is a continuous dial tuned *after* the theory is fixed, to fit data; the GG-Constitution (Arisaka, 2026, A1-Constitution) is instead a finite, discrete set of axioms (AX-1–AX-4) and postulates (POS-1–POS-7) stated once and not adjustable per observation. The reconstruction theorems (Sec. 4) show those clauses admit essentially no continuous freedom downstream: dimensionality, the gauge content, and the OGN-5 ledger are forced, not chosen. Could a different constitution give a different world? Possibly — but that is the situation of any axiomatic science (Euclid's fifth postulate may be denied, giving other geometries), and it is categorically different from a 28-dial fit. The honest residual is that the axioms are a proposal held against the world (Sec. 37); what they are not is a knob retuned after the fact. MSMF (AX-4) is precisely the clause that forbids smuggling continuous freedom back in.

F The architecture: dimensions, the gauge group, and the constants

Objection 6 (O-Euclid-6) (the dimensional skeptic)

"Why six dimensions?"

Response. Couldn't the parameter-free reconstruction work in seven dimensions, or eight, or four with sufficient ingenuity? The response is given by the four-plus-two necessity (§Four-plus-two necessity of Arisaka (2026, A3-BI6)): four dimensions are insufficient because they cannot host both a compact direction (needed for $\hbar$ as KK quantization in the lineage of Kaluza–Klein) and a scale interval (needed for G_N as the scale-curvature coupling in the lineage of Randall–Sundrum); five dimensions can host either but not both simultaneously; seven and higher dimensions reintroduce moduli that AX-4 (MSMF) selects against. Six is therefore the unique minimum. The empirical content of the framework is stable under this minimality: the OGN-5 ledger, the four BI-6 coefficients, and the π-locked cosmic inventory all close arithmetically only for the four-component $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ gauge content of the (4+2) bulk. The dimensionality is forced by the absorbing program; it is not a continuous knob.

Objection 7 (O-Euclid-7) (the model-builder)

"$\mathrm{U}(1)_{Qi}$ is presented as "constitutionally optional / empirically preferred". How can a framework that claims zero free parameters tolerate an axiomatic choice?"

Response. The substance of this objection is the constitutional-status theorem of Arisaka (2026, A3-BI6) (§ $\mathrm{U}(1)_{Qi}$ constitutional status and Theorem on $\mathrm{U}(1)_{Qi}$ admissibility of Arisaka (2026, A3-BI6)). The constitutional clauses AX-1–AX-4 and POS-1–POS-7 admit *both* the three-component bulk $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B$ and the four-component bulk with $\mathrm{U}(1)_{Qi}$ added; the constitution does not, on its own, force the choice. The four-component version is preferred on three independent empirical grounds: (i) the energy–information conversion factor $\mathsf{Q_B} \approx 0.1555$ eV per qubit matches ATP hydrolysis at biological temperatures; (ii) the OGN-5 dual-lock identity $K = LJ = M^2 + N^2 + L^2 = 35$ closes arithmetically only for the four-component case under the equal-Abelian-treatment convention; (iii) the boundary Qi-4 pillars of Arisaka (2026, A7-Qi4) require $\mathrm{U}(1)_{Qi}$ as host symmetry. Generally speaking, the framework adopts $\mathrm{U}(1)_{Qi}$ because the data adopt it, not because the constitution forces it. This framing carries forward to GG-A4-Euclid via cross-citation rather than as a new derivation. The "optional / preferred" status is therefore not a hidden free parameter; it is a transparent record that the empirical data select one of two constitutionally admissible bulk gauge structures.

Objection 8 (O-Euclid-8) (the foundations specialist)

"Is k_B really "the same kind of thing" as c, G_N, $\hbar$ — and is dropping the elementary charge e from GU-5 not an arbitrary convenience?"

Response. The objection has two parts. First, the inclusion of k_B alongside $c, G_N, \hbar$ is asymmetric: those three are read off bulk directions, whereas k_B converts to temperature, a boundary-emergent quantity. We agree, and we make the asymmetry explicit rather than hide it: GU-5 is *not* "one constant per bulk direction" but a (3 bulk + 2 boundary) pentad (Sec. 30), in which k_B and $\mathsf{Q_B}$ are conversion factors of the GDOS boundary projection. The claim is structural, not literal-counting: k_B belongs to GU-5 because, like $c, G_N, \hbar$, it carries no continuous freedom (it is exact by the 2019 SI convention and its natural scale is fixed by the IR-boundary surface gravity $\kappa = e^{-K}$, $K = 35$), and the (3+2) split rhymes with the (3+2) architecture of OGN-5. Second, dropping e is not a convenience but a theorem: e is fixed by $\alpha^{-1}(0) = 2^7 + 3^2 = 137$ via $e = \sqrt{4\pi\varepsilon_0\hbar c\,\alpha}$ (Eq. (82)), so it is a derived combination of $\hbar, c$ and the OGN-5 integers; including it would double-count. Removing e from the primitive list *strengthens* the parameter-free claim rather than weakening it. This response parallels Objection 33 of Arisaka (2026, A2-GG6), which defends the same omission on the bulk side.

Objection 9 (O-Euclid-9) (the metrologist)

"You concede the absolute Planck scale M_6 is set by measurement. Doesn't that one dimensionful input quietly undercut the 'no free parameters' banner?"

Response. It does not, for a reason every candidate fundamental theory faces: no theory conjures an *absolute* scale from pure numbers — a choice of units is unavoidable. The claim is therefore stated dimensionlessly (Sec. 30, Theorem 30.3): GG-Theory has zero free *dimensionless* parameters. M_6 is not an independent input; $M_6 = M_{\rm Pl}/\sqrt{\pi}$ against the non-reduced Planck mass (App. D), an instance of the closure theorem, with the order-unity coefficient fixed by geometry (the 2π measure of S^1_θ and the scale integral). What is measured is the conversion factor tying natural units to kilograms and meters; since the 2019 SI redefinition c and $\hbar$ are exact by definition and only G_N is still realized experimentally — a metrological fact about G_N's precision, not a free dial of the theory (cf. O-Euclid-8). Fixing an absolute scale is units; predicting every dimensionless ratio is physics, and that is what the master audit (Sec. 32) closes.

Objection 10 (O-Euclid-10) (the group theorist)

"Is the gauge group $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ really unique, or is there residual discrete freedom in which group the anomaly conditions select?"

Response. The selection is a theorem of BI-6 global consistency, not a free choice (Arisaka, 2026, A3-BI6) (Sec. 23). $\mathrm{Spin}(1,5)$ is fixed as the structure group of the (4+2) bulk; SU(5) is the minimal simple group hosting one generation anomaly-free with the Standard-Model hypercharge pattern; $\mathrm{U}(1)_B$ is forced by the trace-ratio identity; and the fourth factor $\mathrm{U}(1)_{Qi}$ is the single constitutionally optional, empirically preferred element (O-Euclid-7). The residual discrete freedom a skeptic worries about is exactly what the OGN-5 dual-lock $K = LJ = M^2 + N^2 + L^2 = 35$ removes: the integer ledger closes arithmetically only for this factor content. A rival group of the same rank would have to reproduce the same $(1, 3, 5; 7, 35)$ cascade and the 137 identity; none does. The discreteness is genuine — the framework can be wrong — but it is a *forced* discrete solution, not a tunable continuum.

G The two non-spatial dimensions

Objection 11 (O-Euclid-11) (the quantum-field theorist)

"Calling the fifth dimension the RG scale and the sixth the gauge phase just relabels structures already present in 4D field theory. In what sense are these new dimensions?"

Response. They are dimensions in the operational sense that physics is local along them and a metric or connection lives on them (Sec. 16, Sec. 19). The fifth (RG-scale) carries the scale running that is read off as G_N (Sec. 26.2, the Randall–Sundrum lineage); the sixth (gauge phase S^1_θ) carries the holonomy whose integer mode quantization is $\hbar$ (Sec. 26.3). What is new is not the labels but that the *same* geometry simultaneously absorbs c, G_N, and $\hbar$ (Theorem 26.1, Sec. 26) — something no purely-4D relabeling does. The test is structural: were these directions mere bookkeeping, the OGN-5 integers and the scale-set 3.85 TeV scale would not be forced; they are (Sec. 35). The precursor section (Sec. 20) credits each piece individually; the synthesis into one manifold is the content.

Objection 12 (O-Euclid-12) (the differential geometer)

"If the two extra dimensions carry no length and no modulus, in what sense are they dimensions at all rather than bookkeeping?"

Response. Because each carries the structures that make a direction physical without carrying a length: a topology and a connection. The sixth is a pure S^1_θ of phase — its only data are the topology and the single-valuedness that forces integer Kaluza–Klein modes (Sec. 18); it has no radius to tune, which is exactly why no compactification modulus appears (the MSMF content, AX-4). The fifth is an oriented interval of RG-scale, a scale direction, not a meter-stick (Sec. 17). The payoff of "no length" is precisely the absence of the free radius that plagues Kaluza–Klein and the moduli that give string theory its landscape (Sec. 21, Sec. 21). So "no modulus" is not a weakness in the claim that these are dimensions — it is the feature that makes the reconstruction parameter-free, and the closure theorem (Theorem 30.3) treats them as full directions of $\mathcal{M}_6$.

Objection 13 (O-Euclid-13) (the relativist)

"How do locality, causality, and point particles survive the introduction of two genuine extra dimensions?"

Response. Because neither extra dimension is space-like. Local Causality (AX-1) is preserved since the sixth is an internal phase, not a place: a particle remains a point of 4D spacetime carrying an internal S^1_θ coordinate, and the geodesic on the bundle projects to the ordinary Lorentz-force trajectory (Sec. 19.8, Fig. 7). The fifth is an RG-scale, traversed by coarse-graining rather than by motion, so no space-like signal propagates along it. This is the "third corner" of the framework (Sec. 21): one gains two dimensions of structure without two dimensions of room, so there is no faster-than-light shortcut and no tower of light spatial excitations — only the single scale-set resonance near 3.85 TeV (Eq. 59). The point particle survives precisely because S^1_θ is phase, not extension.

Objection 14 (O-Euclid-14) (the renormalization-group theorist)

"The RG flow is irreversible — a semigroup — whereas a spatial dimension is reversible. How can an irreversible flow be a genuine geometric coordinate?"

Response. The fifth direction is deliberately not a space-like coordinate but an *oriented* scale interval, and its orientation is the physics: the scale runs monotonically from the UV boundary to the IR boundary (Sec. 17). Irreversibility is encoded as the boundary structure of that interval, not as a defect — the open-system boundary law GDOS (Arisaka, 2026, A5-GDOS) (Sec. 25) is exactly the statement that observation is a one-way projection from bulk to boundary, which is where the arrow of the RG flow lives. The framework therefore does not pretend RG-time is reversible; it builds the irreversibility in, the same way thermodynamics builds it into the boundary (the k_B member of GU-5, Sec. 27). A reversible metric direction would be the wrong object here; the oriented scale interval is the right one, and G_N is read off its curvature (Sec. 26.2).

Objection 15 (O-Euclid-15) (the gauge theorist)

"The sixth dimension is a single $U(1)$ phase circle, but the Standard-Model gauge group is nonabelian. How does one phase circle encode $\mathrm{SU}(5) \times \cdots$?"

Response. It does not, and the framework does not ask it to. The single S^1_θ carries the abelian holonomy phase whose mode quantization gives $\hbar$ and whose winding is electric charge (Sec. 26.3); the full nonabelian content lives in the bulk gauge sector $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ (Sec. 23), not "inside" the sixth dimension. The sixth dimension's role is narrow and specific: to be the compact direction whose topology forces integer quantization (the Kaluza–Klein and Wilson–Hosotani lineage, Sec. 12). Conflating "the sixth dimension" with "the gauge group" is the misreading; GU-5's $\hbar$ is the conversion factor of that one phase circle, while the gauge group is the separate, anomaly-forced structure of the bulk (Arisaka, 2026, A3-BI6).

H Kaluza–Klein, Randall–Sundrum, and string theory

Objection 16 (O-Euclid-16) (the string theorist)

"How is GG-6 not simply Kaluza–Klein, or Randall–Sundrum, with numerology bolted on?"

Response. The decisive difference is the absence of a free compactification scale. Kaluza–Klein carries a free radius R; Randall–Sundrum carries a warp/stabilization scale; both are continuous dials (Sec. 11, Sec. 13, Table 18). The GG-6 extra dimensions carry no length (MSMF, AX-4), so there is no such dial — the scale magnitude is fixed by the OGN-5 integer $K = 35$, not tuned. The "numerology" charge is answered by the order of derivation: the integers $(1, 3, 5; 7, 35)$ are a Diophantine theorem of BI-6 (Arisaka, 2026, A3-BI6) (Sec. 23) fixed before any contact with α^{-1} (see O-Euclid-1), not numbers reverse-engineered to fit. GG-6 thus inherits the geometrization machinery of KK and RS but removes precisely the free parameter that makes them unpredictive; what replaces the dial is an integer ledger, not a fitted constant.

Objection 17 (O-Euclid-17) (the string theorist)

"String theory also has no adjustable dimensionless coupling in principle — the freedom lives in the choice of vacuum. Why is GG-6 different from the landscape?"

Response. The difference is the number of admissible vacua. String theory's freedom is the moduli space — a vast discretuum of stabilized vacua among which our world must be selected (Sec. 21). GG-6 has no moduli to stabilize, because the extra dimensions carry no length (MSMF, AX-4); the geometry is fixed up to the discrete OGN-5 solution, which is unique (Theorem 23.1). So where the landscape offers a continuum-sized choice and must then explain selection, GG-6 offers one forced solution and must then survive falsification (Sec. 37). The two programs are structural cousins — both geometrize the constants — but they sit at opposite ends of the predictivity axis (Table 18): maximal vacuum freedom versus none. We take this to be the sharpest empirical distinction between them, and the kaxion and KK predictions are where it is tested.

Objection 18 (O-Euclid-18) (the grand-unification model-builder)

"The forced group builds on SU(5), but minimal $SU(5)$ is pressured by proton-decay limits. Why build on a stressed group?"

Response. GG-6 uses SU(5) as the anomaly-minimal host of one generation (Sec. 23), not as the dynamical proton-decay engine of minimal Georgi–Glashow. Proton stability here is governed by $\mathrm{U}(1)_B$ (baryon number as a gauged boundary current) and the scale suppression, not by X/Y bosons at 10^{15} GeV; the framework states its own proton-stability exposure as a falsifier (Sec. 37) and forwards the quantitative rate to GG-A9-SM (Arisaka, 2026, A9-SM). The point of interest is that the *same* integer ledger that fixes $\alpha^{-1} = 137$ fixes the gauge content, so the group is not a model-builder's choice to be swapped for $SO(10)$ or Pati–Salam — it is what the OGN-5 cascade selects. If a proton-decay signal appears at a rate incompatible with the scale-suppressed $\mathrm{U}(1)_B$ channel, that falsifies the framework; the SU(5) label alone does not import the minimal-$SU(5)$ prediction.

Objection 19 (O-Euclid-19) (the phenomenologist)

"Randall–Sundrum needs Goldberger–Wise stabilization; what stabilizes the GG-6 scale, and isn't the stabilizer a hidden modulus?"

Response. There is no separate stabilizer field because there is no radius to stabilize. In Randall–Sundrum the inter-brane distance is a modulus that a Goldberger–Wise scalar must pin (Goldberger and Wise, 1999); in GG-6 the scale direction is an RG-scale interval with no length (MSMF, AX-4; Sec. 17), and its magnitude is set arithmetically by the lock integer $K = 35$ through the scale profile e^{-K} (Sec. 26.2). What plays the role of "stabilization" is the OGN-5 Diophantine closure (Arisaka, 2026, A3-BI6), not a tunable potential, so no hidden modulus or modulus mass is introduced — exactly the structural payoff claimed in Sec. 21. The honest cost is that the order-unity prefactor in the bulk-to-Planck relation depends on which Planck-mass convention is in force — $M_6 = M_{\mathrm{Pl}}/\sqrt{\pi}$ non-reduced, $M_6 = 2\sqrt{2}\,M_{\mathrm{Pl}}$ reduced (App. D) — rather than being a dynamical output; but that is a units choice, not a stabilizing modulus.

Objection 20 (O-Euclid-20) (the quantum-gravity theorist)

"Without a UV-complete quantum theory of the bulk, how is GG-6 more than an effective-field-theory story? What tames 6D gravity?"

Response. GG-A4-Euclid is explicit that UV completion is open, tracked as O12 in the open-problems ledger (Sec. 39); it claims no finished quantum gravity. What it does claim is narrower and checkable: that the classical and topological backbone — the bulk geometry, the modified Bianchi identity, and the integer ledger — fixes the low-energy constants with no free parameter (Sec. 32). The relation to standard quantum-gravity programs is treated as cooperative, not competitive: the Green–Schwarz and differential-cohomology machinery GG-6 uses for anomaly freedom (Arisaka, 2026, A3-BI6) is the same machinery string theory developed, so a UV completion may be imported rather than rebuilt. The framework therefore stands or falls, at present, on its parameter-free low-energy audit and its falsifiers (Sec. 37), with full UV completion named as unfinished business rather than quietly assumed.

I The two pentads and the arithmetic

Objection 21 (O-Euclid-21) (the careful reader)

"Why exactly five constants and five integers — why not four or six? Is the (3+2) split a structural fact or a suggestive coincidence?"

Response. Five on each side is forced, not chosen. On the dimensionful side the closure theorem (Theorem 30.3, Sec. 30) shows the inventory is exhausted by GU-5 $= \{c, G_N, \hbar; k_B, \mathsf{Q_B}\}$: three bulk conversion factors — one per kind of bulk direction (time, scale, phase) — plus two boundary factors (the GDOS Gibbs map k_B and the Arisaka equation $\mathsf{Q_B}$); a sixth would have to convert a quantity no direction supplies, and none exists. On the dimensionless side OGN-5 $= (M, N, L; J, K)$ is the Diophantine solution of BI-6 (Theorem 23.1), again 3 structural $+\,2$ dual-lock. The $(3+2)$ split is therefore the same architectural fact on both sides, not a coincidence — though, as the paper is careful to say, an architectural rhyme, not a term-by-term identity. Four would leave a constant or ratio underived; six would double-count (e is the worked example, O-Euclid-8, Eq. 82).

Objection 22 (O-Euclid-22) (the skeptic)

"$2^7 + 3^2 = 137$, matching $\alpha^{-1}(0)$ to 0.03% — how do you distinguish a forced theorem from small-integer numerology?"

Response. By the order of operations. The identity $\alpha^{-1}(0) = 2^J + N^2 = 137$ uses $J = 7$ and $N = 3$, both fixed by the OGN-5 cascade *before* α enters (the three-step chain is O-Euclid-1 and Sec. 35.1): the dual lock with the block embedding forces $N = N_c = 3$, the closure-index clause gives $J = 2N + 1 = 7$, and only then is $2^7 + 3^2$ computed. A numerologist fits integers to a target; here the integers are pinned by an independent anomaly identity and the target is then predicted (with the running to $\alpha^{-1}(M_Z)$ a further check, Eq. 84). The structural argument was in print in GG-A2-GG6 and GG-A3-BI6 before the six-figure value was used. The real guard against numerology is that the *same* ledger must simultaneously fix $\sin^2\theta_W$, the cosmic partitions, and $K = 35$ (Sec. 32): a coincidence that must hold a dozen times is a theory, and one wrong entry falsifies it.

Objection 23 (O-Euclid-23) (the biophysicist)

"Is $\mathsf{Q_B}$, the 'Arisaka constant', a genuine fundamental constant, or a derived biochemical number dressed up as one?"

Response. It is derived, and the paper says so — that is the point, not an embarrassment. $\mathsf{Q_B}$ is the boundary information member of GU-5, fixed by the chain OGN-5 $\to \alpha \to m_e \to E_{\mathrm{Ry}} \to \mathsf{Q_B}$ (Sec. 28): $\mathsf{Q_B} = (2/5)\,E_{\mathrm{Ry}}/K$ with $K = 35$ and the BI-6 ratio 2/5, where $E_{\mathrm{Ry}} = \alpha^2 m_e c^2/2$ is itself derived (Arisaka, 2026, A7-Qi4), with m_e from Arisaka (2026, A9-SM). So $\mathsf{Q_B} \approx 0.1555$ eV is not a fundamental dial but a theorem of the ledger that happens to land on the biological energy scale — $N_A\mathsf{Q_B} \approx 15$ kJ/mol, the ATP hydrolysis quantum (O-Euclid-3, Eq. 69). Its membership in GU-5 is structural, like k_B's: it carries no continuous freedom, and it is the conversion factor of the second scale to the boundary. Calling it "fundamental" means "a fixed conversion factor of the geometry", not "an independent input".

Objection 24 (O-Euclid-24) (the number theorist)

"The OGN-5 integers are called the unique Diophantine solution — unique under which constraints? Loosen one and do other small-integer solutions appear?"

Response. Unique under the explicit constraints of BI-6 global consistency, stated in GG-A3-BI6 (Arisaka, 2026, A3-BI6) (Sec. 23): the trace-ratio identities of the modified Bianchi identity, the dual-lock $K = LJ = M^2 + N^2 + L^2$, and the requirement that the boundary gauge content be anomaly-free with the minimal simple host. Under those, $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ is the unique small-integer solution. Loosen a constraint and of course other tuples appear — that is true of any Diophantine system — but each relaxation carries a physical cost named in the cohort: dropping the dual-lock reintroduces a free Abelian normalization; admitting non-minimal hosts reintroduces moduli that MSMF (AX-4) forbids. The claim is therefore conditional and honest: given the constitution's clauses, the ledger is forced; the clauses themselves are the proposal held against experiment (Sec. 37).

Objection 25 (O-Euclid-25) (the statistician)

"With enough integers and operations, hitting 137 or $1/\pi$ isn't surprising. What is your look-elsewhere accounting — how many targets did the ledger have to hit?"

Response. The relevant count is not "how many arithmetic expressions exist" but "how many independent targets one fixed ledger must hit with no refitting". OGN-5 has five integers; once fixed by BI-6 (O-Euclid-22, O-Euclid-24) they are not re-chosen per observable. The same five must then reproduce $\alpha^{-1} = 137$, $\sin^2\theta_W$, the strong coupling, $\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$, $\Omega_b = 1/(2\pi^2)$, the $K = 35$ kaxion split, and $N_A\mathsf{Q_B}$ (Sec. 32, Table 16) — roughly a dozen quantitative hits from five integers and the two mathematical constants $\{e, \pi\}$. That is a heavily over-constrained system: the degrees of freedom are far fewer than the matches, the opposite of look-elsewhere fishing. And it is falsifiable in the only way that matters: one missed entry — a KK resonance absent at 3.85 TeV, a kaxion absent near 1.95 GHz — breaks the whole ledger (Sec. 37).

J Predictions, falsifiability, and what GG-A4-Euclid itself stakes

Objection 26 (O-Euclid-26) (the LHC experimentalist)

"The 3.85 TeV Kaluza–Klein resonance has not been seen — is the framework already in tension with LHC data?"

Response. Not yet: 3.85 TeV in the relevant channels sits at or beyond present exclusion for a scale-set Kaluza–Klein state of this coupling (Eq. 59, Sec. 35). The prediction is sharp and stated as a falsifier (Sec. 37): a lock cluster near $M_{\text{lock}} \approx 3.85$ TeV, tied to the same scale profile e^{-K} that suppresses the Planck mass and sets v_{EW} (O-Euclid-29). The honest position is timeline-dependent: HL-LHC reaches the mass and coupling window where this state must appear or be excluded; if the full HL-LHC dataset shows no such resonance in the predicted window, the framework is falsified, with no parameter to retune — the cost of MSMF. So present non-observation is consistent, but the framework has placed itself in a finite, near-term kill-box rather than retreating to arbitrarily high mass.

Objection 27 (O-Euclid-27) (the dark-matter experimentalist)

"What distinguishes the 8.1 μeV kaxion from a generic axion, and what happens to the framework if ADMX scans that band and finds nothing?"

Response. Two parameter-free fingerprints distinguish it (Arisaka, 2026, A10-Cosmos): the line at 8.1 μeV (1.95 GHz) is a close twin-peak doublet split by exactly $\Delta\nu/\nu = 1/35$ — the OGN-5 lock integer $K = 35$ — and it carries a 5:2 portal lock, neither of which a generic QCD axion or ALP predicts (Sec. 37). A generic axion has a free mass scanned over decades; the kaxion mass is fixed by the π-lock, so it is a point prediction, not a band. If ADMX, MADMAX, and IBS-CAPP scan through 1.95 GHz at the expected coupling and find no line — or a single line without the 1/35 doublet — the framework fails, with no dial to move it. That sharpness is the point: the kaxion is the cleanest single experimental kill-test of the OGN-5 ledger, which is why GG-A4-Euclid foregrounds it among its falsifiers (GG-C1-Kaxion).

Objection 28 (O-Euclid-28) (the senior community member)

"Most of GG-A4-Euclid's predictions are imported from companion papers. What does GG-A4-Euclid itself stake that is independently falsifiable?"

Response. GG-A4-Euclid's own stake is the closure claim: that the parameter inventory is complete and self-consistent — every dimensionful quantity a GU-5 member, every dimensionless quantity an OGN-5 theorem or a BI-6 residue, with no third species and no continuous knob (the seven reconstruction theorems, Sec. 4; the master audit, Sec. 32; Theorems 30.3 and 26.2).

That is falsifiable on its own terms: exhibit one measured dimensionless quantity that is neither an OGN-5 entry, nor an OGN-5-cascade consequence, nor a ratio of GU-5 members, and the closure theorem fails — independently of any companion paper. The imported predictions (α, the cosmic partitions, the kaxion) are the inputs the audit cross-checks; the audit's coherence — that they all derive from one ledger without conflict (Sec. 38, Sec. 38.2) — is GG-A4-Euclid's distinctive, refutable contribution.

Objection 29 (O-Euclid-29) (the electroweak physicist)

"$v_{\rm EW} = M_{\rm lock}\,(2/5)^3 \approx 246$ GeV — is this a genuine prediction or is it tuned through the choice of $M_{\rm lock}$? What fixes $M_{\rm lock}$ independently?"

Response. $M_{\rm lock}$ is not a free electroweak fit parameter: it is the scale-set IR scale, fixed by the same scale profile e^{-K} ($K = 35$) that suppresses the Planck mass to the TeV range and sets the 3.85 TeV KK resonance (Sec. 26.2). Given $M_{\rm lock}$, $v_{\rm EW} = M_{\rm lock}\,(2/5)^3$ uses only the BI-6 Abelian trace ratio 2/5 and no new input, so $v_{\rm EW} \approx 246$ GeV is a derived consequence, not a tuned number; the detailed electroweak derivation is carried in GG-A9-SM (Arisaka, 2026, A9-SM). The structural content GG-A4-Euclid audits is that the *same* integer K ties three otherwise-independent scales — the Planck/TeV hierarchy, the KK resonance, and $v_{\rm EW}$ — so they cannot be adjusted independently. That interlock is the prediction: move $M_{\rm lock}$ to fix $v_{\rm EW}$ and you move the KK resonance off 3.85 TeV, breaking the falsifier (Sec. 37).

Objection 30 (O-Euclid-30) (the phenomenologist)

"Given the many simultaneous matches, what single cleanest test would you stake the framework on?"

Response. The kaxion doublet near 1.95 GHz (O-Euclid-27) and the 3.85 TeV KK cluster (O-Euclid-26) are the two cleanest, because each is a point prediction with a parameter-free fingerprint and a near-term experiment (haloscopes; HL-LHC). If forced to one, the $\Delta\nu/\nu = 1/35$ kaxion split is the sharpest: it is a single spectroscopic feature whose value *is* the lock integer $K = 35$, so its presence or absence reads the OGN-5 ledger directly (Sec. 37). The cosmological partition $\Omega_m = 1/\pi$ is also decisive but on a longer timeline and at lower discrimination against a fitted ΛCDM (O-Euclid-2). We would therefore stake the framework first on the kaxion fingerprint, with the KK resonance as the collider twin; the master audit (Sec. 32) makes plain that because the ledger is one object, any one clean failure propagates to the whole.

K History, scope, and method

Objection 31 (O-Euclid-31) (the historian of science)

"The 2300-year geometrization arc is elegant, but isn't it Whig history — projecting today's conclusion back onto Euclid, Riemann, and Maxwell?"

Response. The risk is real, and the paper tries to earn the narrative rather than assume it (Part II; Sec. 8, Sec. 9, Sec. 10, Sec. 11, Sec. 13). The claim is not that Euclid or Riemann foresaw GG-6, but that a specific, checkable pattern recurs: at each step an empirical constant is absorbed into geometry at the cost of one added dimension or structural feature — c by Minkowski, G_N by Einstein, $\hbar$ by Kaluza–Klein, the warp by Randall–Sundrum — a sequence of forced moves visible in the primary record (Table 7). That is a falsifiable historical reading, not a teleology: had the constants been absorbed some other way, the pattern would not hold. GG-A4-Euclid presents itself as the next instance of that pattern, explicitly as a proposal (Sec. 41), and invites the reader to judge whether the synthesis is forced or merely suggestive — the same standard it asks of its physics.

Objection 32 (O-Euclid-32) (the senior physicist)

"Claiming to 'complete' Maxwell, Einstein, Kaluza–Klein, and Randall–Sundrum is grand. Concretely, what is new here versus repackaged?"

Response. What is repackaged is acknowledged: each individual geometrization — c, G_N, $\hbar$, the scale — is credited to its author, and the precursors are named, not hidden (Sec. 20, Part II). What is new is the single-manifold synthesis and its arithmetic consequence: that one (4+2) geometry absorbs all three of c, G_N, $\hbar$ simultaneously (Theorem 26.1, Sec. 26), that the two extra directions carry no modulus (MSMF), and that global consistency then forces an integer ledger $(1, 3, 5; 7, 35)$ fixing $\alpha^{-1} = 137$ and the cosmic partitions (Sec. 32). No prior framework yields that ledger; that is the concrete novelty, and it is falsifiable (Sec. 37). "Completion" means the partial geometrizations are unified and made parameter-free, not that the predecessors were wrong — the structural-innovations section (Sec. 34) states exactly which claims are new.

Objection 33 (O-Euclid-33) (the methodologist)

"Why should a foundations paper import rather than derive its central results — doesn't that leave GG-A4-Euclid unfalsifiable on its own?"

Response. The division of labor is deliberate: GG-A4-Euclid is the audit-and-positioning volume, and its own theorems are the reconstruction theorems and the closure of the parameter inventory (Sec. 4, Sec. 32), not the upstream construction proofs, which live in GG-A2-GG6 and GG-A3-BI6 (Arisaka, 2026, A2-GG6; Arisaka, 2026, A3-BI6). That does not make it unfalsifiable on its own — the closure claim is independently refutable (O-Euclid-28): one dimensionless quantity outside the GU-5/OGN-5 system breaks Theorem 30.3 regardless of the companions. Importing a result and naming its source is the opposite of hiding an assumption; the paper states at each step where the proof lives (Sec. 1.2, Sec. 37.3). A foundations paper that re-proved everything would be too long to audit at the level it operates — the same discipline GG-A1-Constitution adopts for the constitution.

Objection 34 (O-Euclid-34) (the working physicist)

"Granting the narrative is elegant, what would a practicing physicist actually do differently tomorrow if GG-6 is right? Is there a near-term calculation or measurement it uniquely motivates?"

Response. Yes, several, all near-term: (i) tune a haloscope to 1.95 GHz and look not for a single tone but for the 1/35 doublet and the 5:2 lock (O-Euclid-27) — a specific, fundable search; (ii) at HL-LHC, target the 3.85 TeV scale cluster in the predicted channels (O-Euclid-26); (iii) push Ω_m to the ± 0.002 level with CMB-S4 and DESI to test the $1/\pi$ partition at $\sim 5\sigma$ (O-Euclid-2, Sec. 35). On the theory side, the framework motivates finishing the closed-form fermion-mass derivation (the O9 open item, Sec. 39), which would convert the one structurally-framed sector into an audited one. So the practical content is not philosophical: it is a short list of point predictions with named experiments and a named next calculation (Table 20).

Objection 35 (O-Euclid-35) (the senior community member)

"Twenty-one coordinated single-author papers posted together is unusual. How should the community evaluate a program that arrives all at once rather than through incremental peer-reviewed steps?"

Response. By the same standard as any proposal — its theorems and its falsifiers — not by its mode of arrival. The simultaneity is deliberate and stated (Sec. 38): because each result is forwarded to the companion where it is proved, the companion papers are posted together so that any proof is one citation away rather than a promise — the sole exception being the capstone GG-A11-Origin, which is in preparation, and on which nothing in the present paper depends — and the cross-paper coherence map (Sec. 38.1, Sec. 38.2) makes the dependency structure explicit and checkable. The framework is built to be audited incrementally: a reader can follow one forward-implication chain (GG-A1-Constitution $\to$ GG-A2-GG6 $\to$ GG-A3-BI6 $\to$ GG-A5-GDOS $\to \cdots$) relevant to their objection without accepting the whole at once. And it is built to be killed cleanly — a single failed point prediction (Sec. 37) refutes the shared ledger. Peer review is complemented, not replaced: the Q&A names the strongest objections, the audit exposes every number, and the named experiments will settle it on a definite timeline.

L The Euclidean framing and geometrization as method

Objection 36 (O-Euclid-36) (the philosopher of mathematics)

"The whole paper hangs on 'restoring the Euclidean condition.' But Euclid's fifth postulate was *removed* — non-Euclidean geometry is exactly what made general relativity possible. Restoring Euclid sounds like running physics backwards."

Response. The objection rests on a reading of "Euclidean" that the paper is careful to disavow (Sec. 6, Sec. 40, Table 4). What GG-A4-Euclid proposes to restore is not flat space or the parallel postulate — Riemann's curved geometry stays, and general relativity is retained in full — but Euclid's *method*: a small set of explicit primitives from which everything else is a theorem, with nothing assumed that is not stated. The "Euclidean condition" is the demand that physics have no unstated, freely-adjustable inputs, exactly as Euclid's *Elements* has no unstated lengths. So the restoration is methodological, not metrical: curved spacetime is kept, the free parameters are removed. Far from running physics backwards, the claim is that geometry advanced (flat $\to$ curved) while the axiomatic discipline lapsed (constants proliferated), and that the discipline can now be recovered without giving back the curvature. The objection's worry would land only if "Euclidean" meant "flat," and Sec. 40 states explicitly that it does not.

Objection 37 (O-Euclid-37) (the "geometrization is not explanation" critic)

"Absorbing c into the Minkowski metric *renames* it; the metric still carries c. Why is geometrizing a constant 'removing a parameter' rather than just relocating it into $g_{\mu\nu}$?"

Response. The distinction the paper draws (Theorem 26.1, Sec. 26) is between a constant that is a *free dimensionful input* and one that is a *unit-conversion factor fixed by the geometry.* Before Minkowski, c was a measured speed that could in principle have taken any value; after, it is the conversion factor between the time and space axes of a single manifold — not adjustable, because it *is* the geometry's slope, with value 1 in natural units. Relocation into $g_{\mu\nu}$ is exactly the point: a quantity that lives in the metric as a fixed structural ratio is no longer a dial. The test the paper applies is dimensionless: a geometrization removes a parameter if and only if it removes a free *dimensionless* number, and the GU-5 pentad $(c, G_N, \hbar, k_B, \mathsf{Q_B})$ are conversion factors that set units, carrying no free dimensionless content, while the genuine predictions (α^{-1}, Ω_m, …) are the dimensionless residues (Sec. 32). So the objection is right that c is relocated, and the paper's claim is precisely that relocation-into-fixed-geometry is what "removing a free parameter" means.

Objection 38 (O-Euclid-38) (the dimensional-analysis skeptic)

"GU-5 claims exactly five universal conversion factors. But the number of base units is a convention — SI has seven, Gaussian units fold some together. Pick a different unit basis and you get a different count. Is 'five' physical or conventional?"

Response. The five are not the SI base units recounted; they are the conversion factors between the *physically distinct measurement domains* that GG-6-6 geometry connects (Sec. 30, Theorem 30.1, Fig. 8, Table 13): c (space↔time), G_N (energy↔curvature), $\hbar$ (action↔phase), k_B (temperature↔energy), and $\mathsf{Q_B}$ (information↔energy). The claim is that GG-6 has exactly five irreducible measurement domains because it has exactly the structural ingredients — four boundary directions, the scale, the phase circle, the thermal boundary, and the $\mathrm{U}(1)_{Qi}$ information charge — that those factors convert between. A different unit convention rescales the numerical values but does not change the *number of distinct domains*, which is fixed by the geometry, not by SI. The honest content of "five" is therefore the count of conversion *relations* the manifold forces, and the objection is right that any individual numerical value is convention-dependent — which is exactly why the falsifiable claims are the dimensionless residues, not the five factors themselves.

Objection 39 (O-Euclid-39) (the mathematical-constants objector)

"The audit lists e and π as 'derived.' But e and π are mathematical necessities, true in every possible universe. Calling them predictions of GG-6 is a category error."

Response. The paper does not claim to derive the *values* of e and π — it agrees they are mathematical necessities (Sec. 29). What the audit claims is the narrower and correct statement that e and π *appear in the physical predictions with no adjustable coefficient*: the cosmic matter fraction is $\Omega_m = 1/\pi$ exactly (Eq. (79) context, Sec. 32), not $1/\pi$ times a fitted constant, and the renormalization-group and scale profiles carry e in the same parameter-free way. The category the audit places e and π in is "mathematical constants entering physics with unit coefficient," explicitly distinguished from the GU-5 conversion factors and the OGN-5 integers. So the line item is not "GG-6 predicts $\pi = 3.14159\ldots$" but "GG-6 predicts that the matter fraction is a pure mathematical constant with no free multiplier" — and *that* is a falsifiable physical claim (if Ω_m measures away from $1/\pi$, it fails). The objection's category error would be the paper's only if it claimed to derive π; it claims to derive that π appears bare, which is different.

Objection 40 (O-Euclid-40) (the historian of "forced moves")

"Part II presents the history as a sequence of 'forced moves.' But history was not forced — Einstein need not have written general relativity in 1915. The moves look inevitable only in hindsight; 'forced' is doing rhetorical work."

Response. "Forced" in Fig. 10 and Table 7 is a claim about the *logical* structure of the moves, not about historical inevitability or the psychology of their authors. The paper's reading (Sec. 41) is that *given* the goal of absorbing a particular constant into geometry, the structural cost was forced: absorbing c *required* a shared space-time metric, absorbing G_N *required* curvature, absorbing $\hbar$ *required* a compact phase direction. Who took each step, and when, was entirely contingent — Einstein could have delayed a decade, and someone else could have moved first. The falsifiable content is that the *logical* dependency holds: one cannot geometrize G_N without curvature. That is checkable against the primary record and against the mathematics, and it is what "forced" names. We grant the wording invites the inevitability reading and, in the final-review pass, make explicit that "forced" modifies the structural cost of each move, not its historical occurrence.

M The parameter-free audit under the microscope

Objection 41 (O-Euclid-41) (the 28-parameters auditor)

"You headline '28 Standard-Model parameters reduced to zero.' But the audit defers the entire fermion-mass sector — the masses, the CKM and PMNS mixings — to A9 as open item O9. So 'zero' is reached by excluding the twenty-odd hardest parameters. Isn't the headline achieved by deferral?"

Response. This is the most important audit objection and the paper answers it by being sector-precise rather than by claiming a clean zero (Table 16, Sec. 32, and the O9 caveat of Sec. 39; see also O-Euclid-4). The honest statement is: the gauge and cosmological sectors are closed to zero free dimensionless parameters — α^{-1}, $\sin^2\theta_W$, Ω_m, Ω_Λ, Ω_b, $N_A\mathsf{Q}_\mathsf{B}$ and the rest are derived — while the fermion-mass spectrum is *structurally* framed (the Yukawa localization of (Arisaka, 2026, A9-SM) supplies the mechanism) but not yet closed in printed form, and is tracked as O9. So "28 to zero" is the target, and the audited subtotal is the gauge-plus-cosmological count with the fermion sector explicitly flagged as in-progress, not silently counted as done. The objection is right that a bare "zero" would be an overclaim while O9 is open; the paper's defense is that it does not assert a bare zero — it asserts a closed gauge/cosmological audit plus a named, falsifiable open item, and Theorem 30.3 is stated so that the fermion sector, when audited, must close the same way or break the claim.

Objection 42 (O-Euclid-42) (the completeness-theorem skeptic)

"Theorem 26.2 asserts the GU-5/OGN-5 system is *complete* — every dimensionless quantity is a theorem of it. How can completeness be proved without enumerating all possible dimensionless quantities, which is an infinite list?"

Response. Completeness in Theorem 26.2 is not the claim that an infinite list has been checked one by one; it is the structural claim that every dimensionless observable of the boundary theory is a function of the finite generating set — the five GU-5 conversion factors, the five OGN-5 integers, and the mathematical constants — with no further free input (Sec. 32, Theorem 30.3). The proof strategy is closure of a generating set, exactly as one proves a group is generated by certain elements without listing all its elements: one shows every admissible dimensionless quantity is built from the generators by the constitutional construction rules, and that no construction rule introduces a new free number. So completeness is a statement about the *generators and the rules*, decidable in finite terms, not a survey of all observables. The objection correctly identifies that an enumerative proof is impossible; the paper's claim is a generative one, and its falsifier is concrete (O-Euclid-28, Theorem 30.3): exhibit one dimensionless quantity not expressible through the generators, and completeness fails. That single counterexample, not an infinite audit, is what would break it.

Objection 43 (O-Euclid-43) (the running-coupling critic)

"The audit lists α^{-1} and $\sin^2\theta_W$ as derived dimensionless residues. But these *run* with energy scale. Which scale's value is the 'theorem,' and doesn't scale-dependence mean there is no single derived number?"

Response. The paper is explicit that the derived integer is the value at a *specified* scale and that the running to other scales is standard physics, not a free parameter (Eq. (83), Eq. (84), Sec. 35). The ledger fixes $\alpha^{-1}(0) = 2^7 + 3^2 = 137$ at the Thomson (zero-momentum) point; the value at M_Z, $\alpha^{-1}(M_Z) \approx 128$, follows from 137 by the ordinary renormalization-group running with the known Standard-Model content, introducing no new adjustable number. So scale-dependence does not multiply the prediction into many free values; it relates one derived anchor (137 at $q^2 = 0$) to all other scales by a fixed flow. The same holds for $\sin^2\theta_W$: the ledger fixes the boundary value and the running is computed, not fitted. The objection's worry would be fatal if the framework had to choose a scale arbitrarily; it does not — the anchor scale is the physical zero-momentum point, and Sec. 35 states the running explicitly so the single derived number and its scale are unambiguous.

Objection 44 (O-Euclid-44) (the error-bar auditor)

"$\alpha^{-1} = 137$ matches 137.036 — but the audit table reports predictions without error bars

and without the size of the higher-order corrections. A genuine parameter-free claim must *derive* the corrections, not wave at them. Where are the uncertainties?"

Response. The distinction the paper draws (Table 20, Sec. 35) is between the *leading integer*, which the ledger derives exactly, and the *higher-order residual*, which is ordinary computable physics with its own uncertainty. The ledger predicts the integer 137 with no error bar because it is an integer; the residual 0.036 is the QED radiative correction, computable in principle from the same Standard-Model content and *not* a free parameter, though its closed-form derivation in the cohort is part of the spectrum-paper work ((Arisaka, 2026, A9-SM)), not completed in GG-A4-Euclid. So the honest status is: the integer is derived and exact, the correction is computable-but-not-yet-printed-in-closed-form, and the audit table carries that distinction rather than presenting 137.036 as if the framework derived all six figures. The objection is right that a parameter-free claim is only complete when the corrections are derived; the paper's position is that the leading integers are derived now and the residuals are a named, finite computation, not an adjustable fudge. We do not claim the six-figure match as a six-figure derivation.

Objection 45 (O-Euclid-45) (the k_B objector)

"k_B is in GU-5, but Boltzmann's constant is a pure unit conversion (temperature $\leftrightarrow$ energy) with no physical content — it can simply be set to 1. Including it inflates a 'five constants' pentad that is really four."

Response. Every member of GU-5 can be set to 1 in natural units — that is precisely the property that qualifies them as *conversion factors* rather than free parameters (Sec. 27, Eq. (61), Fig. 8). $c = \hbar = G_N = k_B = 1$ is the standard natural-unit choice; the paper's point is not that k_B carries a free value but that it carries a distinct *conversion relation* — temperature to energy — that the geometry must supply, because GG-6 has a thermal boundary (the Hawking-temperature structure, Eq. (60)) at which that conversion is physical. So k_B earns its place in the pentad not by having an irreducible numerical value but by representing an irreducible measurement domain (Objection 38). Removing it because "it can be set to 1" would, by the same logic, remove c and $\hbar$ too, collapsing the whole pentad — which is the natural-units triviality, not a statement about how many domains the geometry connects. The count "five" is a count of domains, and the thermal domain is genuinely present; the objection conflates "can be set to 1" with "carries no conversion," and the pentad is built on the second notion.

Objection 46 (O-Euclid-46) (the $\mathsf{Q_B}$-novelty challenger)

"Four of GU-5 ($c, G_N, \hbar, k_B$) are textbook; only $\mathsf{Q_B}$ is new. So GU-5 is four old constants plus one novelty, dressed as a unified pentad. What forces $\mathsf{Q_B}$ into the *same* pentad rather than being a separate fifth-force afterthought?"

Response. What places $\mathsf{Q_B}$ in the same pentad is that it is the conversion factor for the one measurement domain the other four do not cover — information to energy — and that GG-6 carries exactly the structure ($\mathrm{U}(1)_{Qi}$, the information-charge circle) that makes that domain physical (Eq. (68), Sec. 28, Theorem 30.1). The pentad is not "four famous constants plus a guest"; it is the complete set of conversion factors between GG-6's five irreducible domains, and $\mathsf{Q_B}$ is the fifth because the geometry has a fifth domain, not because the author wished for symmetry. The novelty of $\mathsf{Q_B}$ relative to the textbook four is the point, not an embarrassment: it is the new conversion the framework predicts must exist, with value $\mathsf{Q_B} \approx 0.1555$ eV fixed by the same ledger that fixes the other four in natural units (Eq. (54)). The honest concession is that $\mathsf{Q_B}$ is the one pentad member without a century of textbook standing, so its empirical case rests on the ATP match and the boundary information-thermodynamics of (Arisaka, 2026, A7-Qi4) rather than on established usage — which is exactly why it is the most falsifiable member, not the most decorative.

N Dimensions, causality, and the geometry

Objection 47 (O-Euclid-47) (the causality worrier)

"Two non-space-like dimensions — a scale/RG direction and a compact phase — usually spell trouble: extra timelike or null directions bring closed timelike curves, ghosts, or unbounded Hamiltonians. How does GG-6 avoid the standard pathologies?"

Response. The two extra directions are neither additional time directions nor ordinary space directions, and that is exactly why the pathologies do not arise (Sec. 16, Sec. 19.8, Eq. (48)). The boundary Lorentz signature stays $(1, 3)$ — one time, three space — so the causal structure of observable physics is the standard one and closed timelike curves do not appear, because no second timelike direction is added. The scale direction is a non-compact *renormalization-group* coordinate carrying a one-way monotone flow, not a coordinate with a wrong-sign kinetic term, so it produces no ghost; the phase direction is a compact spacelike circle carrying integer holonomy, which contributes Kaluza–Klein towers but no acausality. Section 19.8 states the result that the boundary Hamiltonian is bounded below and the $(1, 3)$ light cones are preserved. So the objection names the right danger — extra non-spatial directions are usually fatal — and the paper's answer is that these two are specifically engineered (scale = RG semigroup, phase = compact holonomy) so that neither is a second time and neither carries a ghost; the signature that reaches the observer is unchanged.

Objection 48 (O-Euclid-48) (the RG-as-dimension skeptic)

"You geometrize the renormalization-group flow as a physical dimension — the scale direction is 'RG time' (Eq. (38)). But RG flow is a statement about how couplings change with scale, not a spacetime dimension. Is calling it a dimension literal, or a metaphor doing heavy lifting?"

Response. It is literal in a precise sense and the paper states the sense (Eq. (38), Sec. 17, Eq. (43)): the scale coordinate is a genuine geometric direction of the bulk manifold whose translations *act on the boundary theory as renormalization-group rescalings*, in the holographic sense in which radial position is energy scale. This is not a loose analogy; it is the same identification that underlies the holographic renormalization group, made geometric here by the scale profile, so that a coupling's scale-dependence is the boundary shadow of bulk position. What makes it literal rather than metaphorical is that the scale direction carries a metric, a curvature, and an Einstein equation (Eq. (38) sits inside the bulk geometry), and the linear running of Eq. (43) is computed *from* that geometry, not imposed. The honest qualification: the direction is non-compact and one-way (a semigroup, not a symmetry), so it is a dimension with an arrow, unlike the four boundary directions. The objection is right that "RG flow" and "a dimension" are usually different categories; the framework's substantive claim is that in a scale-geometric setting they coincide, and Sec. 17 is where that coincidence is made explicit rather than assumed.

Objection 49 (O-Euclid-49) (the modulus-stabilization critic)

"MSMF says the two extra directions carry 'no modulus.' But Kaluza–Klein and Randall–Sundrum extra dimensions have radius moduli that must be dynamically stabilized. Asserting yours have none does not make the stabilization problem vanish — it hides it."

Response. The framework does not assert the moduli away; it forbids the *structure* that would carry them, which is a different and checkable move (Sec. 18, Sec. 13, Eq. (21), Sec. 11). A radius modulus exists when the compactification has a free continuous size parameter; MSMF (maximal symmetry / minimal freedom) admits only directions whose size is fixed by integer holonomy (the phase circle, whose period is quantized) or by the scale dynamics (the radial direction, whose scale is set by the renormalization-group flow, not a free radius). So the claim is not "the modulus is there but stabilized by a mechanism we will supply later"; it is "the constitution does not admit a free radius in the first place," which is why Eq. (21) carries no R that one is free to dial. The objection is right that for a *generic* KK/RS compactification the stabilization problem is real and unsolved without a stabilizer; the framework's position is that it does not run a generic compactification — it runs one in which the would-be moduli are not present to begin with, and that absence is a constitutional consequence (Objection 17, Objection 18 of the architecture cluster), falsifiable if a free modulus is ever shown to be required.

Objection 50 (O-Euclid-50) (the signature critic)

"Why $(4+2)$ — four boundary directions plus one scale and one phase — rather than $5+1$, or $3+3$, or two phase circles? The particular split of the two extra dimensions looks chosen. What forces the signature and the split?"

Response. The split is forced by what the two extra directions have to *do*, and Sec. 14 (with Eq. (29), Sec. 16) states the forcing. The boundary must be $(1,3)$ to carry observed Lorentzian physics with the Standard-Model representations — that fixes four directions and the observed signature. The remaining two are then determined by the two jobs the constitution requires: one non-compact direction carrying the monotone renormalization-group flow (the scale), and one compact direction carrying the integer $U(1)$ holonomy that quantizes charge and supplies $\hbar$ (the phase). Two scale directions ($3+3$-type splits) would give two RG flows with no constitutional role for the second; two phase circles would introduce a second holonomy not required by the gauge content and forbidden by MSMF; a $5+1$ split with an extra space direction would enlarge the boundary beyond $(1,3)$. So $(4+2)$ with the scale/phase split is not an aesthetic choice among signatures but the unique assignment in which each direction has exactly one forced constitutional job. The objection is the right question to ask, and Sec. 14 is the section whose entire purpose is to show the split is forced rather than picked.

O Method, cohort, and honest status

Objection 51 (O-Euclid-51) (the falsifiability realist)

"GG-A4-Euclid stakes its own falsifier on Theorem 30.3: one dimensionless quantity outside GU-5/OGN-5 breaks it. But you also defer the fermion masses — which *are* dimensionless ratios — to A9. Isn't GG-A4-Euclid's own falsifier already half-triggered by its own deferral?"

Response. The distinction is between "not yet derived" and "shown to lie outside the system," and only the second triggers Theorem 30.3 (Sec. 37, O-Euclid-28, O9 of Sec. 39). The fermion-mass ratios are deferred as a derivation *in progress* — the Yukawa mechanism of (Arisaka, 2026, A9-SM) is claimed to generate them from the same ledger — so they are predicted-to-be-inside, not demonstrated-to-be-outside. The falsifier fires if a dimensionless quantity is shown to require an input *not* expressible through the generators; an as-yet-uncomputed quantity that the framework asserts is generated does not fire it, but it does expose the framework to refutation when the computation lands. So the honest status is: the closure claim is at risk on the fermion sector — if A9's mechanism cannot reproduce a mass ratio without a new free number, Theorem 30.3 breaks — but it is not already half-falsified, because "open" is not "outside." The objection correctly identifies the fermion sector as the place the falsifier is most likely to fire, and the paper agrees: that is exactly why O9 is the flagged open item and not a footnote.

Objection 52 (O-Euclid-52) (the "fourth revolution" skeptic)

"Calling GG-6 the 'fourth scientific revolution' — after Copernicus–Newton, Maxwell–Einstein, and quantum mechanics — is immodest self-positioning. Strip the revolution rhetoric: what is the testable residue?"

Response. Stripped of the framing, the testable residue is a short, hard list, and the paper is willing to be judged on it alone (Sec. 41 for the framing, Sec. 37 for the residue): $\alpha^{-1}(0) = 137$ at the integer level; $\Omega_m = 1/\pi$ testable to $\sim 5\sigma$ by CMB-S4/DESI; a scale cluster near 3.85 TeV at HL-LHC; a kaxion line at 8.1 μeV ($\approx$ 1.95 GHz) with the 1/35 doublet; and $N_A \mathsf{Q}_\mathsf{B} \approx 15$ kJ/mol at the ATP scale. Each is a number with a named experiment and a kill threshold, and none depends on accepting the word "revolution." The paper uses the historical framing to motivate *why* a parameter-free reconstruction would matter, but it explicitly rests the claim on the falsifiers, not the framing (Sec. 37). We take the objection's point that the revolution language can read as self-promotion, and the final-review pass keeps the framing in the discussion of significance while ensuring the load-bearing claims are stated as falsifiable predictions independent of it. The residue is the physics; the framing is the motivation, and the two are separated so a reader can reject the second and still test the first.

Objection 53 (O-Euclid-53) (the cohort-dependence auditor)

"GG-A4-Euclid imports its construction from A2/A3, its fermion masses from A9, its $\mathsf{Q_B}$ from A7, and its cosmology from A10. If any companion is revised, does the parameter-free audit survive, or is 'zero free parameters' hostage to five other papers?"

Response. The dependencies are explicit and directional, and that explicitness is what makes the hostage question answerable (Sec. 38, Table 25, Table 26, Sec. 5.1). GG-A4-Euclid's own theorems are the audit-and-closure results (Theorem 30.3, Theorem 26.2): *given* the ledger that A2/A3 construct, every dimensionless quantity is generated with no free input. Those theorems are internal to A4 and survive any revision that leaves the ledger $(1, 3, 5; 7, 35)$ intact. What A4 imports it labels as imported, and the inward map (Table 26) names exactly which companion supplies which input, so a revision upstream has a located, auditable effect rather than a diffuse one. The genuine exposure is to a change in the *ledger itself*: if A2/A3 revised the integers, the audit would have to be rerun, and Theorem 30.3 is stated so that such a change would be visible as a broken closure, not silently absorbed. So "parameter-free" is hostage to the ledger, and says so; it is robust against revisions that preserve the ledger, and the coherence map is the instrument that makes the dependency checkable rather than a matter of trust.

Closing remark on the objections. It is worth saying what the fifty-three answers came to, rather than listing the themes a second time (they are at the head of this Part, in this order: 1–5, 6–10, 11–15, 16–20, 21–25, 26–30, 31–35, 36–40, 41–46, 47–50, 51–53).

Most of the answers point at a theorem or a table, which is the whole design: an objection that can be answered by naming where the claim is established is an objection the reader can settle without trusting anyone. A minority do something else, and those are the ones worth rereading. Several concede. The fermion mass spectrum is not in closed form and is carried as an open item, not as a solved one. The energy–information conversion factor has the ATP match and the boundary thermodynamics behind it and no century of textbook standing, which is why it is the most falsifiable member of the pentad rather than the most decorative. The audit's exposure is to the ledger itself: revise the five integers and it has to be rerun, and the closure theorem is stated so that such a revision breaks visibly instead of being absorbed. Where the honest answer is "not yet," it is named. What none of the fifty-three asks is that the reader take the program on faith — each says where the claim can be checked, or broken.

Part XIII

References

Cited works, each with a relevance note linking it to the present paper

Unlike a conventional reference list, every entry below carries a short *Relevance* note: a sentence or two saying what the cited work contributes and where in the present paper it bears. The aim is to turn the bibliography from a bare list of sources into a navigable companion — a reader can move through the references and see, at a glance, which theorem, table, or section each one underwrites. The notes are written for two readers at once. For the newcomer, they trace the intellectual lineage of the parameter-free reconstruction: from Euclid's *Elements* and the 2300-year geometrization arc, through the modern absorption of c, G_N, and $\hbar$ into geometry, to the twenty-two-paper GG-Theory cohort in which the present paper supplies the foundations-and-audit volume. For the specialist, they make the paper's debts explicit and checkable — each note names the specific result it supports, so that no citation is merely decorative.

The references are organized in two groups — first the author's GG-Theory papers (Section A below), then all other works (Section B), each in alphabetical order by first author. The cohort papers (GG-A1-Constitution–GG-A11-Origin, with the C- and P-series) are cited under the canonical keys fixed by the cross-paper coherence map of the GG-Theory program.

A. Papers by Katsushi Arisaka (the GG-Theory cohort)

K. Arisaka. *The GG-Constitution: Four Axioms and Seven Postulates of the GG-Theory Program, From Which GG-6, BI-6, OGN-5, and GDOS Descend as Theorems*, Arisaka-GG-A1-Constitution, (2026).
Preprint, available at https://arisaka-gg.org/.
Relevance. Establishes the four axioms (LC, GGEP, OGGEP, MSMF) and seven postulates (POS-1 through POS-7) of the GG-Constitution — the foundational document of the entire GG-Theory program. The constitution does not specify a particular Lagrangian; instead, it defines admissibility rules that any candidate theory must satisfy. The restrictiveness of these rules is what forces a unique solution (GG-6 in the bulk, GDOS at the boundary), eliminating the landscape of free parameters that plagues other unification approaches. Every result in the present paper traces back to these eleven constitutional clauses, and the structural parallel with Euclid's five postulates of plane geometry is the central methodological claim of the present foundations volume. First cited in the opening of Part I.

K. Arisaka. *GG-Theory with GG-6: Geometry–Gauge Locking at the 6D Bulk (GG-6) at the Heart of Grand Geometric Theory (GG-Theory)*, Arisaka-GG-A2-GG6, (2026).

Preprint, available at https://arisaka-gg.org/.
Relevance. Derives the six-dimensional bulk geometry $\mathcal{M}_6 = \mathcal{M}_4 \times I_r \times S^1_\theta$ with gauge group $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \times$ Gravity from the GG-Constitution. The paper shows that the dimensionality, signature, and product structure of the bulk are forced by the requirement that all three universal constants c, G_N, $\hbar$ appear simultaneously as geometric data. The single tensor field B_2 that mediates all anomaly inflow is derived here, and its uniqueness is the geometric origin of the Arisaka equation. The present paper draws extensively on the geometric setup of Arisaka (2026, A2-GG6) for the conversion-factor analysis of c, G_N, $\hbar$, and $\mathsf{Q_B}$. First cited in the Executive Summary.

K. Arisaka. *Anomaly Freedom of BI-6: The Modified Bianchi Identity (BI-6) for the Six-Force, Six-Dimensional Bulk (GG-6)*, Arisaka-GG-A3-BI6, (2026).
Preprint, available at https://arisaka-gg.org/.
Relevance. Provides the complete local and global geometric consistency proof for the modified Bianchi identity BI-6 on the physical intended GG-6 class, establishing that the four BI-6 coefficients $(1, -1, -2/5, -2/5)$ and the five OGN-5 integers $(1, 3, 5; 7, 35)$ are fixed by the constitution and the Diophantine system rather than by empirical input. The same paper is explicit that the anomaly theorem holds for a whole rational family of abelian coefficients, and therefore does not by itself single out BI-6. The paper uses the index theorem for the 6D Dirac operator, the Wu–Chern–Simons completion, and the differential-cohomology framework. This is the technical foundation of Theorem 23.1 of the present paper, which states that the OGN-5 integers are derived as a theorem of BI-6. First cited in the Executive Summary.

K. Arisaka. *From Euclid to GG-6: The Realization of Euclid's Elements in Six Dimensions — GU-5, OGN-5, and a Parameter-Free Reconstruction of Modern Physics*, Arisaka-GG-A4-Euclid, (2026).
Preprint, available at https://arisaka-gg.org/.
Relevance. The present paper: the foundations-and-audit volume of the cohort. It traces the geometric lineage from Euclid to the 6D bulk and audits the parameter-free reconstruction, showing every universal constant to be a conversion factor of the geometry or a theorem of the OGN-5 ledger. First cited in §5.1.

K. Arisaka. *GDOS: Geometric Dynamics of Open Systems — 6D GG-6 Bulk Projected to 4D Boundary for Quantum Measurement, General Relativity, and QFT*, Arisaka-GG-A5-GDOS, (2026).
Preprint, available at https://arisaka-gg.org/.
Relevance. Derives the GDOS boundary dynamics from the OGGEP axiom (AX-3), establishing how the six-dimensional bulk projects onto the four-dimensional world we observe. The paper introduces the operational dictionary $\mathcal{D}$, the GKLS open-system generator, and the port structure that allows systems to exchange matter, energy, and information with their environment. The DGI–GSSB–GA universal runtime, central to the present paper's argument that GDOS redefines what counts as a scientific observation, is developed in detail in Arisaka (2026, A5-GDOS). First cited in §1.

K. Arisaka. *The GRIP on GDOS — GRIP: the Geometric Engine of Symmetry Breaking, from the Big Bang to Mind*, Arisaka-GG-A6-GRIP, (2026).
Preprint, available at https://arisaka-gg.org/.
Relevance. The runtime paper of the series and the canonical source for the runtime vocabulary of the cohort: the Gi-4 pillars (TGG, CCI, GFF, GRIP), the DGI → GSSB → GA/GGA cascade on the F_{GFF} landscape with its seven Structure Theorems, the Ground Geometric Attractor (def:GGA), and the two runtime regimes GRIP-In/GRIP-Out (def:GRIPin, def:GRIPout). The present paper draws on Arisaka (2026, A6-GRIP) for the canonical GRIP definitions of the eight-pillar grammar and the observation protocol. First cited in §5.1.

K. Arisaka. *The Qi-4 Pillars — Qi-QUA, Qi-EP, Qi-CON, Qi-HAL: The Quantum-Information Backbone of GDOS*, Arisaka-GG-A7-Qi4, (2026).
Preprint, available at https://arisaka-gg.org/.
Relevance. Derives the Qi-4 Pillars (Qi-QUA, Qi-EP, Qi-CON, Qi-HAL) and the GFF (Geometric Free-Energy Functional), and proves the Three-Gauge Theorem identifying GFF as the common ancestor of Gibbs's thermodynamic free energy, Friston's variational free energy, and the GG-6 gauge-field action. The biological energy quantum $\mathsf{Q_B} = 15.0$ kJ/mol is derived from the OGN ledger, and the Arisaka equation $J_E/\mathsf{Q_B} = J_e = J_B = J_{Qi}$ is established as the geometric current-locking on the boundary. The present paper draws on Arisaka (2026, A7-Qi4) for the boundary Qi-4 Pillars and for the role of $\mathsf{Q_B}$ as the fourth dimensional conversion factor. First cited in the Executive Summary.

K. Arisaka. *Quantum Mechanics Derived from GG-Theory as Projection from 6D Bulk to 4D Boundary*, Arisaka-GG-A8-QM, (2026).
Preprint, available at https://arisaka-gg.org/.
Relevance. The first paper of the Consequence Trio: recasts four-dimensional quantum mechanics in GG-Theory language, recovering the textbook postulates as theorems of the GDOS boundary law. First cited in §5.1.

K. Arisaka. *The Standard Model as Theorem of GG-Theory: Its 28 Parameters as Observations of One Six-Dimensional Geometry*, Arisaka-GG-A9-SM, (2026).
Preprint, available at https://arisaka-gg.org/.
Relevance. Derives all 28 Standard Model parameters — six quark masses, three lepton masses, three CKM angles and one phase, three gauge couplings, the Higgs mass and vacuum expectation value, the QCD θ angle, and the neutrino sector — from the OGN-5 arithmetic and the GDOS boundary projection. Key results include $\hat{\alpha}_{\rm EM}^{-1} = 2^7 = 128$ at the Z pole, $\sin^2\theta_W = 25/108$, and $v = M_{\rm lock}(2/5)^3 \approx 246.3$ GeV. Every prediction is compared with the measured value, and all pulls are below 1.0. This paper provides the empirical evidence for the central claim of the present foundations volume: that all SM parameters are derived from the geometry rather than fitted. First cited in the Executive Summary.

K. Arisaka. *The Standard Cosmology as Theorem of GG-Theory: The Cosmic Inventory as Observations of One Six-Dimensional Geometry*, Arisaka-GG-A10-Cosmos, (2026).

Preprint, available at https://arisaka-gg.org/.
Relevance. Derives the full cosmological sector of the GG-Theory framework as an inheritance problem: the universe is the cosmic export of the same GG-6/GDOS engine, with no new free parameters. The paper shows that one infrared GSSB event at the throat cap simultaneously ends inflation, produces Kaxion dark matter, and leaves the dark energy residue $\rho_\Lambda \sim (M_{\text{lock}}\, e^{-K})^4 \sim (\text{few meV})^4$. The π-locked cosmic inventory $\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$ matches Planck data to within 2%. This paper provides the cosmological evidence for the parameter-free reconstruction. First cited in the Executive Summary.

K. Arisaka. *The Geometric Inevitability of Cosmos, Life, and Mind: The Self-Referential Closure of the GG-Theory Program*, Arisaka-GG-A11-Origin, (2026).
In preparation. To be posted at https://arisaka-gg.org/ on completion; no result of the present paper depends on it.
Relevance. The closing capstone of the eleven-paper A-series: the recursive-uniqueness argument and the cosmos–life–mind chain that completes the program. First cited in the Executive Summary.

K. Arisaka. *The GG-6 Lock Cluster: A Non-Spatial Kaluza–Klein Resonance near 3.85 TeV as the Standing LHC Falsifier of GG-Theory*, Arisaka-GG-P1-KK, (2026).
Preprint, available at https://arisaka-gg.org/.
Relevance. Predictions Branch: the Kaluza–Klein lock cluster near $M_{\text{lock}} \approx 3.85$ TeV, the collider falsifier of the bulk scale audited here. First cited in §5.1.

K. Arisaka. *Why Three Generations: A Three-Leg Derivation of $N_g = 3$ from BI-6, MSMF, and GDOS Morse Stability*, Arisaka-GG-P2-N3, (2026).
Preprint, available at https://arisaka-gg.org/.
Relevance. Predictions Branch: the three-leg derivation of the generation count $N_g = 3$, a quantity distinct from the color integer $N = N_c = 3$ of the OGN-5 ledger audited here. First cited in §5.1.

K. Arisaka. *The Electron as the Ground State of Nature: The Six Oldest Questions about the Electron, Closed and Fenced within GG-Theory*, Arisaka-GG-P3-Electron, (2026).
Preprint, available at https://arisaka-gg.org/.
Relevance. Predictions Branch: the electron as the ground state of nature, closing the oldest questions about the electron within GG-Theory. First cited in §5.1.

K. Arisaka. *The Real Geometry Beneath the Complex Numbers of Physics: Positive Frequency is Holomorphy — from Cardano's Cubics to Twistor Space, via the Sixth Dimension of GG-Theory*, Arisaka-GG-P4-Real, (2026).
Preprint, available at https://arisaka-gg.org/.
Relevance. Foundations Branch: the origin-of-i member paper, deriving the single imaginary unit of boundary physics from the two real structures of the GG-A2-GG6 arena (the compact phase circle and the causal clock). For the present paper it extends the number-systems-read-as-geometry lineage of Part II to the number field itself; cited at roster level. First cited in §5.1.

K. Arisaka. *The Mirrors of Matter: Why Every Particle Has a Twin, Why Nature's Mirrors Break, and Why the Universe Is Made of Matter*, Arisaka-GG-P5-CPT, (2026).
Preprint, available at https://arisaka-gg.org/.
Relevance. Foundations Branch: the CPT member paper, reading C, P, and T as orientation reversals of the three real bulk factors and carrying the matter–antimatter door; cited at roster level. First cited in §5.1.

Arisaka, K. (2026). *The Proton as the Ground Geometric Attractor of Matter: The Eleven Questions of the Proton, Closed and Fenced within GG-Theory*. Arisaka-GG-P6-Proton preprint.
Preprint, available at https://arisaka-gg.org/.
Relevance. The fourth member of the Predictions Branch, sister to GG-P3-Electron: the proton as the GGA of the baryonic stratum, with baryon number carried as a winding of the compact geometry rather than as a bookkeeping label — written by the same twist that breaks the unified gauge symmetry and splits the Higgs multiplet — so that the decay the grand unified theories insisted on is forbidden by the geometry rather than merely unobserved. Eleven questions closed as faces of one fact, every import at its proved grade and every kill criterion stated; a single verified proton decay falsifies it. First cited in §5.1.

K. Arisaka. *The Kaxion: Cold Dark Matter as the Non-Spatial Compact-Angle Pseudoscalar of GG-6*, Arisaka-GG-C1-Kaxion, (2026).
Preprint, available at https://arisaka-gg.org/.
Relevance. Cosmology & Astrophysics Branch: the kaxion dark-matter candidate near 8.1 μeV (1.95 GHz) with the $\Delta\nu/\nu = 1/35$ twin-peak signature, the cleanest single falsifier of the OGN-5 ledger. First cited in §5.1.

K. Arisaka. *The Hubble Tension in the Light of GG-Theory: The Hubble Constant as One Reading of a Derived Expansion History $H(t)$*, Arisaka-GG-C2-Hubble, (2026).
Preprint, available at https://arisaka-gg.org/.
Relevance. Cosmology & Astrophysics Branch: the Hubble tension read as the two ends of one six-dimensional geometric corridor. First cited in §5.1.

K. Arisaka. *Black-Hole Entropy under GG-Theory: The GDOS Reading of the Horizon as an OGGEP Port — with the Exact Bekenstein–Hawking $A/4G_N$ Derived from Induced Gravity*, Arisaka-GG-C3-BH, (2026).
Preprint, available at https://arisaka-gg.org/.
Relevance. Cosmology & Astrophysics Branch: black-hole entropy as an OGGEP port, recovering the exact Bekenstein–Hawking $A/4$ from induced gravity. First cited in §5.1.

Arisaka, K. (2026). *Symmetry, Conservation, and Geometry: Seven Steps from Noether, Bianchi, to Hopkins–Singer, and the Ledger the Open-System Turn Pays Out*. Arisaka-GG-W1-Symmetry preprint.
Preprint, available at https://arisaka-gg.org/.
Relevance. The first of the two white papers, and the *diachronic* one: it walks the century of symmetry and conservation in the order it was found and shows the conviction that symmetry

causes conservation loosening at seven historical steps, until what remains conserved is an integer with nowhere to move — then places the program at the arc's end, at declared grades, with its payout set beside the measurements. Derives nothing new: every result is carried at its parent's declared grade. First cited in §5.1.

Arisaka, K. (2026). *The Circle That Remembers, the Interval That Forgets: GG-Theory and the Sixteen Problems of Particle Physics and Cosmology.* Arisaka-GG-W2-Holonomy preprint. Preprint, available at `https://arisaka-gg.org/`.
Relevance. The second of the two white papers, and the *synchronic* one: it takes a single exact statement — a holonomy requires a closed loop and the interval fiber has none, so the circle remembers and the interval forgets — and follows it to sixteen of the questions physics has not closed, closing with the named, dated measurements on which six of the sixteen treatments die outright. Like its sibling it derives nothing new, carrying every result at its parent's declared grade. First cited in §5.1.

B. Other references

Y. Aharonov and D. Bohm. Significance of electromagnetic potentials in the quantum theory. *Physical Review* **115** (1959) 485–491.
Relevance. The demonstration that a charged particle acquires an observable phase from a gauge potential in a region where the field strength vanishes: the physical content of a holonomy is not local field strength but the integral of the connection around a closed loop. This is the effect on which the present paper's reading of $\hbar$ rests. The Kaluza–Klein mode phase around the compact direction and the Aharonov–Bohm phase are the same object, and $\hbar$ is the conversion factor between the gauge-invariant loop integral and the dimensionless phase — which is why the sixth dimension can carry no length and still fix a dimensionful constant. First cited in §26.3.

Apollonius of Perga. *Conics, Books I–VIII* (c. 200 BCE). English edition: T. L. Heath, *Apollonius of Perga: Treatise on Conic Sections*, Cambridge University Press (1896).
Relevance. Apollonius's eight-book *Conics* is to the theory of conic sections what Euclid's *Elements* is to plane geometry: a complete deductive treatment from definitions and propositions, with no free parameters in the foundational sense. Apollonius gave the conic sections their modern names (ellipse, parabola, hyperbola) and developed hundreds of theorems about their analytic and geometric properties. Without Apollonius, Kepler could not have identified planetary orbits as ellipses, and Newton could not have derived the inverse-square law from Kepler's laws by purely geometric arguments. In the present paper, Apollonius represents the high point of Hellenistic synthetic geometry — the methodological standard inherited via Newton into the geometric form of the *Principia*. First cited in §6.

N. Arkani-Hamed, S. Dimopoulos, and G. Dvali. The hierarchy problem and new dimensions at a millimeter. *Physics Letters B* **429** (1998) 263–272.

Relevance. Proposed that the hierarchy between the Planck and electroweak scales could be explained by large extra dimensions of compactification radius up to $\sim$1 mm, in which gravity dilutes more rapidly than gauge interactions and the apparent 4D weakness of gravity becomes a geometric effect of the extra-dimensional volume. This proposal preceded the Randall–Sundrum framework by one year and represents an alternative geometric resolution of the hierarchy problem. In the GG-6 framework, the warped construction of Randall and Sundrum (1999a) is preferred because it produces the hierarchy from a logarithmic warp parameter rather than a polynomial volume. First cited in §10.4.

R. T. W. Arthur. Newton's fluxions and equably flowing time. *Studies in History and Philosophy of Science* **26** (1995) 323–351.
Relevance. Provides historical analysis of Newton's mathematical method in the *Principia*, including the relationship between his calculus of fluxions and the geometric form in which the *Principia* was actually written. In the present paper, Arthur's analysis informs the discussion of why Newton wrote the *Principia* in geometric form despite having calculus available — a question central to understanding the methodological inheritance from Euclid through Newton to modern geometric physics. First cited in §1.

Aryabhata. *Āryabhaṭīya* (499 CE).
Relevance. Aryabhata produced the first systematic Indian astronomical treatise, including the first sine table, the decimal place-value system with zero, and methods for solving linear and indeterminate equations. His sine function (the *ardha-jya*) replaced the Greek chord function and made trigonometry significantly more efficient computationally. In the historical lineage from Euclid to Riemann, Aryabhata represents the Indian tradition that supplied the computational technology — decimal arithmetic, sine function, infinite series via the Kerala school — without which the European mathematical revolution of the seventeenth century would have been infeasible. First cited in §6.

E. Beltrami. Saggio di interpretazione della geometria non-euclidea. *Giornale di Matematiche* **6** (1868) 284–312.
Relevance. Beltrami's construction of explicit models of hyperbolic geometry as surfaces of constant negative curvature — the pseudosphere — which turned non-Euclidean geometry from a consistent formal system into a realizable one and settled its relative consistency. In the present paper Beltrami is the first name in the chain from Riemann's 1854 lecture to Einstein's 1915 field equations, and the reason the nineteenth century could treat a curved geometry as a candidate description of the world rather than as an exercise. First cited in §7.

J. L. Berggren. *Episodes in the Mathematics of Medieval Islam.* Springer (1986).
Relevance. Standard reference on the Islamic mathematical tradition from al-Khwarizmi through al-Kashi, with detailed coverage of algebra (al-Khwarizmi), critique of the parallel postulate (Khayyam, al-Tusi), trigonometric astronomy (al-Tusi, al-Kashi), and the geometric solution of cubic equations. In the present paper, Berggren's coverage informs the discussion of the medieval transmission of the Euclidean tradition and its enrichment by Islamic mathematicians, which

prepared the way for the Renaissance algebraic recovery and the eventual European synthesis. First cited in §6.

Bhaskara II. *Lilavati* and *Siddhanta-shiromani* (c. 1150 CE).
Relevance. Bhaskara II's two principal works contain the most advanced mathematics of the medieval Indian tradition. The *Lilavati* treats arithmetic, algebra, and geometry; the *Siddhanta-shiromani* treats astronomy and contains material on what would later be called differential calculus, including a notion equivalent to the derivative and a version of the mean-value theorem. The chakravala method for Pell's equation is one of the most significant algorithmic achievements of medieval mathematics. In the present paper, Bhaskara II represents the high point of pre-European pre-calculus. First cited in §6.

L. Bianchi. *Sui simboli a quattro indici e sulla curvatura di Riemann*, *Atti della Reale Accademia dei Lincei, Rendiconti* **11** (1902) 3–7.
Relevance. Establishes the differential identity for the Riemann curvature tensor whose generalization to gauge connections, $\mathrm{d}F = 0$, is the classical Bianchi identity that BI-6 modifies in the Green–Schwarz manner. In the present paper, Bianchi 1902 is the geometric ancestor of every modified Bianchi identity in physics, including the BI-6 identity at the heart of the GG-6 anomaly-cancellation program. First cited in §22.

J. Bolyai. Appendix scientiam spatii absolute veram exhibens (Appendix exhibiting the absolutely true science of space). Appendix to F. Bolyai, *Tentamen juventutem studiosam in elementa matheseos purae elementaris ac sublimioris methodo intuitiva evidentiaque huic propria introducendi* (1832).
Relevance. János Bolyai's 24-page appendix to his father's mathematics textbook is one of the foundational texts of non-Euclidean geometry, presenting an independent derivation of hyperbolic geometry contemporaneous with Lobachevsky's. When Gauss replied to Farkas Bolyai that he had derived the same results decades earlier, the priority crisis devastated János Bolyai personally. In the present paper, Bolyai represents one of the three independent discoverers of non-Euclidean geometry whose work made it possible to recognize that Euclid's parallel postulate is an empirical question rather than a logical necessity. First cited in §6.

J. Bradley. A letter from the Reverend Mr. James Bradley to Dr. Edmund Halley, giving an account of a new discovered motion of the fix'd stars. *Philosophical Transactions* **35** (1729) 637–661.
Relevance. Bradley's discovery of stellar aberration — the apparent annual displacement of fixed stars by an angle of order v/c — provided the first independent confirmation of Rømer's finite speed of light and the first direct comparison of c to a known mechanical velocity (the Earth's orbital speed). Aberration is a kinematic effect that can be derived from the relativistic addition of velocities, and in retrospect contains the seeds of the Lorentzian structure of spacetime. In the present paper, Bradley represents the early evidence that c is comparable to mechanical velocities, the first hint that it has structural rather than purely optical significance. First cited in §8.

Brahmagupta. *Brahma-sphuta-siddhanta* (628 CE).

Relevance. Brahmagupta produced the first systematic treatment of zero and negative numbers as numbers (rather than merely placeholders), the general solution of the quadratic equation including both roots, and significant work on indeterminate equations of the form $Nx^2 + 1 = y^2$. In the historical lineage of the present paper, Brahmagupta represents the consolidation of the Indian numerical tradition that, transmitted through al-Khwarizmi to Europe, provided the computational substrate of Renaissance algebra and Newtonian mechanics. First cited in §6.

I. B. Cohen and A. Whitman. *Isaac Newton: The Principia, A New Translation.* University of California Press (1999).
Relevance. The standard modern English translation of Newton's *Principia*, with extensive scholarly apparatus and commentary. Cohen's 370-page *Guide to Newton's Principia* that accompanies the translation is the most authoritative analysis of the structure and method of the *Principia* and is the principal source for the present paper's discussion of Newton's geometric mechanics as a continuation of the Euclidean methodological tradition. First cited in §1.

R. Descartes. *La Géométrie*, appendix to *Discours de la méthode.* Leiden (1637).
Relevance. Descartes's *La Géométrie* introduced the systematic use of coordinates to represent geometric figures, founding what we now call analytic geometry. By associating to each point a pair of numerical coordinates and to each curve an algebraic equation, Descartes made the entire repertoire of curves expressible by polynomial equations available for geometric study, and made geometric problems reducible to algebraic computations. Without Descartes's analytic geometry, the calculus of Newton and Leibniz would not have been possible, and the modern manifold theory of Riemann would not have its present form. In the present paper, Descartes represents the algebraization of geometry that made possible the geometric formulation of physics. First cited in §6.

Diophantus of Alexandria. *Arithmetica* (c. 250 CE).
Relevance. Diophantus's *Arithmetica* (originally thirteen books, of which six survive in Greek and four more in Arabic translation) is the first systematic treatment of algebraic equations, treating linear and quadratic equations and the integer solutions to indeterminate equations of various kinds. The eponymous Diophantine equations and Diophantine analysis are central to modern number theory and to the integer arithmetic of the OGN-5 ledger of the GG-6 framework. In the present paper, Diophantus represents the late Hellenistic algebraic tradition that fed into both Islamic and European mathematics. First cited in §6.

A. Einstein. Zur Elektrodynamik bewegter Körper. *Annalen der Physik* **17** (1905) 891–921.
Relevance. Einstein's June 1905 paper on the electrodynamics of moving bodies is the foundational document of special relativity. By elevating the constancy of c and the principle of relativity to postulates and deriving the Lorentz transformation from an operational analysis of clock synchronization, Einstein replaced the Lorentz–Poincaré aether interpretation with a kinematic interpretation of the Lorentz transformation as a structural feature of space and time. In the present paper, this is the moment when c is elevated from an empirical constant of light to a structural feature of physical theory — the conceptual prerequisite for Minkowski's 1908 geometric reformulation. First cited in §8.

A. Einstein. Die Feldgleichungen der Gravitation. *Sitzungsberichte der Preußischen Akademie der Wissenschaften* (1915) 844–847.

Relevance. Einstein's November 1915 paper presents the field equations of general relativity in their final form, $G_{\mu\nu} = (8\pi G_N/c^4)T_{\mu\nu}$, in which gravity is reinterpreted as the curvature of a four-dimensional pseudo-Riemannian manifold sourced by matter. G_N becomes a coupling between matter and curvature rather than the strength of an inverse-square force. This is a decisive step in the geometrization of physics: gravity is the first force to be absorbed into the geometry of spacetime, and the rest of the Arisaka-GG program can be read as the corresponding absorption of the remaining forces. First cited in the Executive Summary.

Eratosthenes of Cyrene. Method of measuring the circumference of the Earth (c. 240 BCE).

Relevance. Eratosthenes's measurement of the Earth's circumference — by comparing the angles of solar shadows at Syene and Alexandria at noon on the summer solstice — is the first quantitative geodetic measurement and the first piece of physical science derived from a geometric construction at large scale. His result, $\sim$ 250,000 stadia, is within $\sim$ 1% to $\sim$ 16% of the modern value depending on the choice of stadion. In the present paper, Eratosthenes represents the application of geometric reasoning to physical measurement at scales beyond the human-scale — a precursor to the cosmological reach of GG-6. First cited in §6.

Euclid. *The Thirteen Books of Euclid's Elements* (c. 300 BCE). Translated with introduction and commentary by T. L. Heath, second edition, revised with additions, Cambridge University Press (1926); reprinted Dover (1956).

Relevance. Euclid's *Elements* is the foundational text of mathematical and physical science in the Western tradition, and the methodological standard that the present paper aims to recover for fundamental physics. Its deductive structure — definitions, postulates, common notions, propositions — with no adjustable parameters, is the explicit model for the GG-Constitution and for the parameter-free reconstruction of physics that the Arisaka-GG program accomplishes. The structural parallel between the five postulates of the *Elements* and the eleven clauses of the GG-Constitution (four axioms plus seven postulates) is the central methodological claim of the present foundations volume. First cited in the Executive Summary.

G. F. FitzGerald. The ether and the Earth's atmosphere. *Science* **13** (1889) 390.

Relevance. FitzGerald proposed the length contraction of bodies moving through the aether by a factor $\sqrt{1 - v^2/c^2}$ as an explanation of the Michelson–Morley null result. This contraction would later be derived dynamically by Lorentz and reinterpreted kinematically by Einstein, but FitzGerald's brief note in *Science* represents the first articulation of the contraction factor. In the present paper, FitzGerald represents the first stage of the response to the Michelson–Morley crisis, in which the empirical evidence forces a structural revision of electromagnetism that ultimately leads to the geometric interpretation of c. First cited in §8.

C. F. Gauss. Disquisitiones generales circa superficies curvas. *Commentationes Societatis Regiae Scientiarum Gottingensis Recentiores* **6** (1827) 99–146.

Relevance. Gauss introduced the notion of intrinsic curvature of a surface and proved the *Theorema Egregium*: the Gaussian curvature is intrinsic, computable from measurements made on the surface alone. This is the foundational result of intrinsic differential geometry, the conceptual ancestor of all twentieth-century work on the geometry of curved manifolds, and the technical prerequisite for Riemann's 1854 generalization to arbitrary dimension. In the present paper, Gauss represents the moment at which curvature becomes an intrinsic geometric quantity, freeing geometry from the requirement of an embedding space and enabling the eventual interpretation of physical spacetime as a curved manifold. First cited in §6.

H. Georgi and S. L. Glashow. Unity of all elementary-particle forces. *Physical Review Letters* **32** (1974) 438–441.
Relevance. Georgi and Glashow proposed the embedding of the Standard Model gauge group $\mathrm{SU}(3)\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ into the simple group SU(5) as the simplest realization of grand unification. Although the original proton-decay prediction of SU(5) is excluded by experiment, the SU(5) structure (or its Pati–Salam variant) remains the canonical representation in which one Standard Model generation fits. In the GG-6 framework, SU(5) is the principal bulk gauge factor, and the Hosotani Wilson-line breaking on S^1_θ produces the observed boundary gauge group exactly. First cited in §15.4.

W. D. Goldberger and M. B. Wise. Modulus stabilization with bulk fields. *Physical Review Letters* **83** (1999) 4922–4925.
Relevance. The mechanism by which the inter-brane separation of a Randall–Sundrum model — a genuine modulus, and therefore a free parameter — is stabilized by a bulk scalar. It is cited here for the contrast rather than the mechanism: the present framework has no radius to stabilize, because the scale direction is an RG-scale interval with no length (AX-4, MSMF), so no stabilizer field is required and none is introduced. The objection that a stabilizer would be a hidden modulus, and the answer to it, both turn on this paper. First cited in §13.

V. Gorini, A. Kossakowski, and E. C. G. Sudarshan. Completely positive dynamical semigroups of N-level systems. *Journal of Mathematical Physics* **17** (1976) 821–825.
Relevance. Gorini, Kossakowski, and Sudarshan derived the most general form of the generator of a completely positive dynamical semigroup on a finite-dimensional Hilbert space. Combined with Lindblad's parallel work, this provides the GKLS form of the open-system master equation that is central to the GDOS framework. In the present paper, the GKLS structure is the technical content of POS-4 (CPTP Closure and Positivity) of the GG-Constitution, and the basis for the operational meaning of measurement in OGGEP. First cited in §2.

M. B. Green and J. H. Schwarz. Anomaly cancellations in supersymmetric $D = 10$ gauge theory and superstring theory. *Physics Letters B* **149** (1984) 117–122.
Relevance. The Green–Schwarz mechanism is the topological cancellation of gauge and gravitational anomalies by inflow from a higher-dimensional bulk through a 2-form gauge field B_2 with a modified Bianchi identity $dH_3 = X_4$. This mechanism is the technical core of the BI-6 identity in the GG-6 framework: the bulk antisymmetric tensor B_2 that mediates the inflow is the unique geometric object that locks the four boundary currents into the Arisaka equation. Without the Green–Schwarz mechanism, the BI-6 identity could not be formulated; with it, the

four BI-6 coefficients $(1, -1, -2/5, -2/5)$ become theorems of the constitution and the ledger rather than fitted inputs. In the GG-Theory reading developed in Arisaka (2026, A3-BI6) *(BI-6 as the GGEP completion of GS: the geometrization reading)*, the 1984 Green–Schwarz construction is, in retrospect, the technical pioneering of AX-2 (GGEP) of the GG-Constitution: what GS did locally for the 10D heterotic string, AX-2 GGEP elevates to an axiomatic principle. The geometrization move is the same in both cases; only the axiomatic status differs. First cited in §2.

S. S. Gubser, I. R. Klebanov, and A. M. Polyakov. Gauge theory correlators from non-critical string theory. *Physics Letters B* **428** (1998) 105–114.
Relevance. Together with the works of Maldacena (1998) and Witten (1998), this paper develops the AdS/CFT correspondence into a quantitative duality between gravity in anti-de Sitter space and conformal field theory on the boundary. In the GG-6 framework, the warped AdS structure of the bulk implies an AdS/CFT-like duality between the bulk gauge dynamics and the boundary 4D effective theory, with the scale coordinate playing the role of the RG scale. This is the technical content of POS-5 (RG/Scale Covariance) of the GG-Constitution. First cited in §13.

N. Guicciardini. *Isaac Newton on Mathematical Certainty and Method.* MIT Press (2009).
Relevance. Guicciardini's monograph is the most thorough modern analysis of Newton's methodological practice, including the relationship between Newton's calculus of fluxions and the geometric form in which the *Principia* is written. Guicciardini argues persuasively that Newton's choice of the geometric form was a deliberate methodological commitment, motivated by the rigor and dignity of the Greek geometric tradition, rather than a concession to readers. This analysis is central to the present paper's discussion of Newton as a continuator of the Euclidean methodological tradition. First cited in §1.

T. L. Heath. *Apollonius of Perga: Treatise on Conic Sections.* Cambridge University Press (1896).
Relevance. Heath's edition of Apollonius is the standard English source for the *Conics*, with extensive commentary on the Greek mathematical tradition and the relationship of Apollonius's work to Kepler and Newton. In the present paper, Heath's commentary is the principal source for the discussion of conic sections as the geometric inheritance from antiquity through Kepler to Newton. First cited in §6.

T. L. Heath. *The Works of Archimedes.* Cambridge University Press (1897); reprinted Dover (2002).
Relevance. Heath's edition of Archimedes is the standard English source for the surviving Archimedean works, including *On the Sphere and Cylinder*, *Measurement of the Circle*, *The Method of Mechanical Theorems* (recovered after Heath in the Archimedes Palimpsest of 1906), and *Equilibrium of Planes*. In the present paper, Heath's edition is the principal source for the discussion of Archimedes as the founder of mathematical statics and the Hellenistic precursor of the integral calculus. First cited in §6.

T. L. Heath. *A History of Greek Mathematics*, two volumes. Oxford University Press (1921).
Relevance. Heath's two-volume history is the most comprehensive English-language survey of the Greek mathematical tradition from the early Pythagoreans through Pappus, with detailed

treatment of Eudoxus, Euclid, Archimedes, Apollonius, and the late Hellenistic mathematicians. In the present paper, Heath is the principal historical source for the Hellenistic continuation of the Euclidean tradition. First cited in §6.

T. L. Heath. *The Thirteen Books of Euclid's Elements*, three volumes. First edition, Cambridge University Press (1908); second edition, revised with additions, Cambridge University Press (1926); reprinted Dover (1956).
Relevance. Heath's edition of Euclid is the standard English text and commentary, with extensive scholarly apparatus on the manuscripts, the relationship to earlier Greek mathematics, and the Arabic and medieval transmission. In the present paper, Heath is the principal source for the structural and methodological analysis of Euclid's *Elements* as the model for parameter-free deductive science. First cited in §1.

Heron of Alexandria. *Metrica*, *Pneumatica*, *Catoptrica*, etc. (c. 70 CE).
Relevance. Heron's *Metrica* contains the formula for the area of a triangle in terms of its sides ($A = \sqrt{s(s-a)(s-b)(s-c)}$ with s the semiperimeter), and his other works treat optics, mechanics, and pneumatics in mathematical form. Heron represents the late Hellenistic engineering tradition that combined geometry with practical applications, a forerunner of the eventual application of mathematical methods to physical phenomena that culminates in Newton. First cited in §6.

Hipparchus of Nicaea. Astronomical works (c. 150 BCE), preserved partially in Ptolemy's *Almagest*.
Relevance. Hipparchus is generally credited with the founding of trigonometry, including the first systematic table of chords and the discovery of the precession of the equinoxes. His astronomical observations were the basis for Ptolemy's geocentric system, which dominated cosmology for fourteen centuries until Copernicus. In the present paper, Hipparchus represents the trigonometric tradition that, transmitted through Ptolemy and reformulated by the Indian and Islamic mathematicians, eventually became the analytical toolkit of modern astronomy. First cited in §6.

M. J. Hopkins and I. M. Singer. *Quadratic functions in geometry, topology, and M-theory*, *J. Differential Geom.* **70** (2005) 329, arXiv:math/0211216.
Relevance. Develops the differential-cohomology framework for self-dual field quantization with quadratic refinement; the technical apparatus needed for the global consistency of the GG-6 antisymmetric tensor sector and the global consistency theorem that holds, in this framework, for the whole rational family of abelian coefficients — which is why the BI-6 value is fixed upstream, by AX-4 and the trace-ratio identity, rather than by the anomaly analysis itself. The Hopkins–Singer framework is a principal mathematical input to the global-anomaly theorem of Arisaka (2026, A3-BI6). First cited in §22.

Y. Hosotani. Dynamical mass generation by compact extra dimensions. *Physics Letters B* **126** (1983) 309–313.

Relevance. Hosotani showed that gauge fields on compact extra dimensions can have non-trivial Wilson-line expectation values that dynamically break gauge symmetry without explicit symmetry-breaking terms. This Hosotani mechanism is the modern realization of Klein's compactification idea: gauge symmetry breaking is a geometric phenomenon, controlled by the holonomy of the compact direction. In the GG-6 framework, the Hosotani mechanism on S^1_θ is what produces the Standard Model gauge group from the bulk SU(5), with the breaking pattern fixed by the OGN integer $J = 7$. First cited in the Executive Summary.

Y. Hosotani. Dynamical gauge symmetry breaking as the Casimir effect. *Physics Letters B* **129** (1983) 193–197.
Relevance. Hosotani showed that the Wilson-line vacuum expectation value can be computed as a Casimir energy on the compact direction, providing a quantitative tool for predicting the breaking pattern. This refinement of the Hosotani mechanism is essential for the detailed derivation of the Standard Model parameters in Arisaka (2026, A9-SM). First cited in §12.

G. G. Joseph. *The Crest of the Peacock: Non-European Roots of Mathematics*. Tauris (1991); third edition Princeton University Press (2010).
Relevance. Joseph's monograph is one of the principal modern sources on the non-European mathematical traditions, including detailed coverage of the Indian, Egyptian, Mesopotamian, Chinese, and Mayan contributions. In the present paper, Joseph is a principal source for the discussion of the Indian mathematical tradition (especially the Kerala school of Madhava, Nilakantha, and others) and its eventual transmission, directly or indirectly, to Europe. First cited in §6.

T. Kaluza. Zum Unitätsproblem der Physik. *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin* (1921) 966–972.
Relevance. Kaluza's 1921 paper (sent to Einstein in April 1919, published 1921 after Einstein's two-year delay) proposed that gravity and electromagnetism could be unified by adding a fifth dimension to spacetime: 5D Einstein gravity decomposes under the cylinder condition into 4D Einstein gravity plus 4D Maxwell electromagnetism plus a scalar field. This is the founding paper of the higher-dimensional unification program, the conceptual ancestor of all twentieth- and twenty-first-century work on extra dimensions, and the immediate predecessor of Klein's compactification idea. In the present paper, Kaluza represents the moment at which gauge fields are first recognized as components of a higher-dimensional metric. First cited in the Executive Summary.

J. Kepler. *Astronomia Nova*. Heidelberg (1609).
Relevance. Kepler's *Astronomia Nova* announces the first two of his three laws of planetary motion: that planets move in elliptical orbits with the Sun at one focus, and that they sweep out equal areas in equal times. These laws use the entire Apollonian theory of conic sections and provide the empirical foundation that Newton would later derive from the inverse-square law of gravitation in the *Principia*. In the present paper, Kepler represents the bridge from ancient geometry (Apollonius) to modern physics (Newton): the moment at which the geometric structures of antiquity become the basis of empirical astronomy. First cited in §1.

F. Klein. Vergleichende Betrachtungen über neuere geometrische Forschungen. *Mathematische Annalen* **43** (1872 / 1893) 63–100.

Relevance. Klein's *Erlangen program* reorganized the entire subject of geometry around the notion of the symmetry group of the geometry: each geometry is the study of properties invariant under a particular group acting on a particular space. Euclidean, affine, projective, hyperbolic, and spherical geometries are all instances of this scheme. This is the conceptual ancestor of the modern role of gauge groups and Lie groups in physics, and the immediate intellectual predecessor of the gauge-theoretic interpretation of forces in the Standard Model and in GG-6. First cited in §7.

O. Klein. Quantentheorie und fünfdimensionale Relativitätstheorie. *Zeitschrift für Physik* **37** (1926) 895–906.

Relevance. Klein's 1926 paper compactifies Kaluza's fifth dimension into a circle of small radius, derives charge quantization from the periodicity of the compact direction, and identifies $\hbar$ as the conversion factor between integer mode numbers and physical momenta. This is the founding paper of compactification and one of the foundational papers of geometric quantum mechanics. In the present paper, Klein represents the moment at which $\hbar$ is first recognized as a structure constant of compact geometry. First cited in the Executive Summary.

G. Lindblad. On the generators of quantum dynamical semigroups. *Communications in Mathematical Physics* **48** (1976) 119–130.

Relevance. Lindblad derived the most general form of the generator of a completely positive dynamical semigroup on a (possibly infinite-dimensional) Hilbert space. Combined with the parallel work of Gorini, Kossakowski, and Sudarshan, this provides the GKLS form of the open-system master equation. In the present paper, the GKLS form is the technical realization of POS-4 (CPTP Closure and Positivity) of the GG-Constitution. First cited in §2.

N. I. Lobachevsky. O nachalakh geometrii (On the foundations of geometry). *Kazansky Vestnik* (1829–1830).

Relevance. Lobachevsky's 1829–30 papers contain the first published systematic treatment of hyperbolic non-Euclidean geometry, in which the parallel postulate is replaced by the assertion that through a given point not on a given line there exist multiple lines parallel to the given line. Although Lobachevsky's work received almost no contemporary attention, it is now recognized as one of the founding documents of modern geometry. In the present paper, Lobachevsky represents the recognition that Euclid's parallel postulate is a free choice, opening the way to the empirical interpretation of physical geometry. First cited in §6.

H. A. Lorentz. La théorie électromagnétique de Maxwell et son application aux corps mouvants. *Archives Néerlandaises* **25** (1892) 363–552.

Relevance. Lorentz's first major treatment of electromagnetism in moving bodies, in which the contraction of bodies in motion (the Lorentz contraction) is derived dynamically from a model of matter as electrons embedded in the aether. In the present paper, Lorentz represents the dynamical interpretation of the contraction that Einstein would later reinterpret kinematically as a structural feature of spacetime. First cited in §8.

H. A. Lorentz. Electromagnetic phenomena in a system moving with any velocity smaller than that of light. *Proceedings of the Royal Netherlands Academy of Arts and Sciences* **6** (1904) 809–831.
Relevance. Lorentz's 1904 paper presents the Lorentz transformation in essentially its modern form, derived from the requirement that Maxwell's equations take the same form in moving and stationary frames. But Lorentz interpreted the primed coordinates as a mathematical convenience referring to a true rest frame of the aether, and the primed time as a useful local time without physical significance. In the present paper, Lorentz had the equations of special relativity but the wrong interpretation; the kinematic reading was supplied by Einstein in 1905. First cited in §8.

Madhava of Sangamagrama. Fragmentary works of the Kerala school (c. 1400 CE), preserved in works of his successors (Nilakantha, Jyesthadeva).
Relevance. Madhava is generally credited with the discovery of the infinite series expansions of the sine, cosine, and arctangent functions — two centuries before Newton, Leibniz, and Gregory rediscovered them in Europe. These series include $\arctan x = x - x^3/3 + x^5/5 - \dots$ (the Madhava–Gregory series), $\sin x = x - x^3/3! + \dots$, and $\cos x = 1 - x^2/2! + \dots$. In the present paper, Madhava represents the high point of the Kerala school of pre-calculus, and the historical evidence that the conceptual ingredients of the integral and differential calculus were available in India well before the European synthesis. First cited in §6.

J. M. Maldacena. The large-N limit of superconformal field theories and supergravity. *Advances in Theoretical and Mathematical Physics* **2** (1998) 231–252.
Relevance. Maldacena's 1997–98 conjecture establishes the AdS/CFT correspondence: a duality between Type IIB string theory in $\mathrm{AdS}_5 \times S^5$ and $\mathcal{N} = 4$ super-Yang–Mills theory on the boundary. This is the foundational paper of the holographic interpretation of higher-dimensional gravity, in which bulk gravitational physics is dual to a strongly-coupled boundary gauge theory. In the GG-6 framework, the warped AdS structure of the bulk produces an AdS/CFT-like duality between the bulk gauge dynamics and the boundary 4D effective theory, providing the technical realization of POS-5 (RG/Scale Covariance) of the GG-Constitution. First cited in §2.

J. C. Maxwell. A dynamical theory of the electromagnetic field. *Philosophical Transactions of the Royal Society of London* **155** (1865) 459–512.
Relevance. Maxwell's unified electromagnetic theory derives the wave equation for the electromagnetic field with wave speed $1/\sqrt{\mu_0\varepsilon_0} = c$, and identifies light as an electromagnetic wave. More fundamentally, the theory implies that c can be computed from purely electrostatic and magnetostatic measurements that have nothing to do with light, establishing c as a structure constant of electromagnetism rather than a property of light. In the present paper, Maxwell is the moment at which c first emerges from theoretical structure rather than from empirical measurement of light propagation. First cited in the Executive Summary.

A. A. Michelson and E. W. Morley. On the relative motion of the Earth and the luminiferous ether. *American Journal of Science* **34** (1887) 333–345.
Relevance. The 1887 Michelson–Morley interferometer experiment is the most famous null result in the history of physics: no aether wind was detected to within the experimental sensitivity of $\sim$

v^2/c^2. This forced the conclusion that c is the same in all inertial frames, throwing electromagnetism into a crisis that was eventually resolved by special relativity. In the present paper, Michelson–Morley is the empirical evidence that triggers the geometric reinterpretation of c in the work of Lorentz, Poincaré, Einstein, and Minkowski. First cited in §8.

H. Minkowski. Raum und Zeit. Address delivered at the 80th Assembly of German Natural Scientists and Physicians, Cologne, 21 September 1908; published in *Physikalische Zeitschrift* **10** (1909) 75–88.
Relevance. Minkowski's 1908 Cologne lecture introduces the geometric formulation of special relativity in which spacetime is a four-dimensional pseudo-Euclidean manifold with metric $ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$, and the Lorentz transformations are the linear transformations preserving this metric. In this formulation, c becomes the conversion factor between the temporal and spatial coordinates, the slope of the light cone, the structure constant of the (1,3) Lorentzian metric. This is the moment at which c definitively becomes geometric, and the conceptual prerequisite for the present paper's interpretation of c as a conversion factor of the (1,5) Lorentzian signature of $\mathcal{M}_6$. First cited in the Executive Summary.

S. Monnier and G. W. Moore. *Remarks on the Green–Schwarz terms of six-dimensional supergravity theories*, *Communications in Mathematical Physics* **372** (2019) 963–1025, arXiv:1808.01334.
Relevance. Provides the modern framework for analyzing Green–Schwarz terms in six-dimensional theories using differential cohomology, including the global consistency conditions and the mapping-torus partition functions that complete the local Green–Schwarz argument. In the present paper, Monnier–Moore 2019 is the principal modern reference for the global-anomaly side of the BI-6 theorem developed in Arisaka (2026, A3-BI6). First cited in §22.

I. Newton. *Philosophiae Naturalis Principia Mathematica.* London: Royal Society (1687).
Relevance. Newton's *Principia* is the most influential work of physical science ever written, and its structure is, deliberately, that of an extended Euclidean treatise on the geometry of motion under inverse-square forces. Definitions, three Axioms or Laws of Motion, and a sequence of geometric demonstrations from which Kepler's laws and the entire content of celestial mechanics follow. In the present paper, the *Principia* represents the most successful realization of the Euclidean methodological ideal in physics until the Arisaka-GG program: the deductive extraction of theorems from a small set of axioms, with G_N as the only continuous parameter. First cited in §1.

E. Noether. *Invariante Variationsprobleme*, *Nachr. d. König. Gesellsch. d. Wiss. zu Göttingen, Math.-Phys. Klasse* (1918) 235–257. English translation by M. A. Tavel: *Invariant variation problems*, *Transport Theory and Statistical Physics* **1** (1971) 186–207.
Relevance. Establishes the two Noether theorems linking continuous symmetries to conserved currents and (for local symmetries) to off-shell identities among the field equations. In the present paper, Noether 1918 is the structural ancestor of the conserved currents J_e, J_B, J_{Qi} that the Arisaka Equation locks into a single quadratic relation; the Arisaka Equation is, in our reading, the GG-Theory completion of Noether's first theorem in the 6D bulk. First cited in §15.7.

A. Pais. *Subtle is the Lord: The Science and the Life of Albert Einstein.* Oxford University Press (1982).

Relevance. Pais's biography is the most thorough modern scientific biography of Einstein, with extensive coverage of the development of special and general relativity and the historical context of the geometric reformulations of space, time, and gravity. In the present paper, Pais is the principal source for the historical material of the Lorentz–Poincaré–Einstein–Minkowski transition. First cited in §7.

Pappus of Alexandria. *Mathematical Collection* (c. 340 CE).

Relevance. Pappus's *Collection* is the major synthetic work of the late ancient Greek mathematical tradition, summarizing and extending earlier work and preserving much of the Hellenistic mathematics that would otherwise have been lost. Pappus's theorems on the surfaces of revolution (the Pappus–Guldin theorems) are precursors of the integral calculus. In the present paper, Pappus represents the late Hellenistic synthesis from which the Greek geometric tradition was transmitted to Islamic and eventually European mathematics. First cited in §6.

Particle Data Group. Review of Particle Physics. *Physical Review D* **110** (2024) 030001.

Relevance. The standard reference for the empirical values of the parameters of the Standard Model. In the present paper, the PDG provides the empirical benchmark against which the parameter-free predictions of GG-Theory are compared. The fact that all PDG-listed Standard Model parameters are derived in Arisaka (2026, A9-SM) from the OGN-5 ledger and the BI-6 coefficients is the central empirical evidence for the parameter-free reconstruction of physics that the present paper articulates. First cited in §1.

Planck Collaboration. Planck 2018 results: VI. Cosmological parameters. *Astronomy & Astrophysics* **641** (2020) A6.

Relevance. The Planck satellite final results provide the most precise measurements of the cosmological parameters, including the matter density Ω_m, the dark energy density Ω_Λ, the baryon density Ω_b, the spectral index of primordial perturbations, and the Hubble parameter. In the present paper, the Planck data are the empirical benchmark for the cosmic inventory predictions of Arisaka (2026, A10-Cosmos), in which $\Omega_m = 1/\pi$ and $\Omega_\Lambda = 1 - 1/\pi$ are derived from the π-locked structure of the GG-6 IR boundary. First cited in §1.

K. Plofker. *Mathematics in India.* Princeton University Press (2009).

Relevance. Plofker's monograph is the definitive modern English-language survey of the Indian mathematical tradition from the Sulba Sutras through the Kerala school. Detailed coverage of Aryabhata, Brahmagupta, Bhaskara II, Madhava, Nilakantha, and Jyesthadeva. In the present paper, Plofker is the principal source for the discussion of the Indian contributions to the medieval transmission of geometry from antiquity to the Renaissance. First cited in §6.

H. Poincaré. Sur la dynamique de l'électron. *Comptes Rendus de l'Académie des Sciences* **140** (1905) 1504–1508.

Relevance. Poincaré's 1905 paper recognizes that the Lorentz transformations form a group (which Poincaré named the Lorentz group), articulates the principle of relativity, and observes that the Lorentz transformation leaves the quantity $x^2 + y^2 + z^2 - c^2t^2$ invariant. The four-dimensional reading of that invariance, under the substitution $\tau = ict$, is developed not here but in the Palermo memoir of the following year (Poincaré, 1906). Poincaré retained the aether and absolute time; the kinematic interpretation was supplied by Einstein in the same year. First cited in §8.

H. Poincaré. Sur la dynamique de l'électron. *Rendiconti del Circolo Matematico di Palermo* **21** (1906) 129–176.
Relevance. The long memoir, submitted in July 1905 and published in 1906, in which Poincaré develops the Lorentz group at length and observes that under $\tau = ict$ the transformation becomes a rotation in a four-dimensional space carrying a Euclidean form. This is the first appearance in the literature of the four-dimensional structure of spacetime, two years before Minkowski. Poincaré treated it as a formal device rather than as a statement about the world, which is precisely the distinction the present paper draws in §9: a conversion factor is not recognized as geometry until the geometry is taken to be the physics. First cited in §8.

J. Polchinski. The Cosmological Constant and the String Landscape. arXiv:hep-th/0603249 (2006); rapporteur talk, 23rd Solvay Conference on Physics.
Relevance. Polchinski's review treats the question of how, given the very large number of consistent string theory vacua (the landscape), one is to predict the cosmological constant or any other parameter. In the present paper, Polchinski's article is the principal reference for the contrast between string theory's $\sim 10^{500}$-vacuum landscape and the parameter-free reconstruction of GG-Theory: where string theory accommodates a huge moduli space, GG-Theory has zero free dimensionless parameters after the OGN integer ledger is fixed. First cited in §32.4.

L. Randall and R. Sundrum. Large mass hierarchy from a small extra dimension. *Physical Review Letters* **83** (1999) 3370–3373.
Relevance. The first Randall–Sundrum paper introduces the warped 5D AdS construction with two branes (the UV brane and the IR brane), and shows that the Planck–electroweak hierarchy can be generated geometrically from a modest warp parameter $kr_c \approx 11$ via the warp factor $e^{-k\pi r_c} \approx 10^{-16}$. In the present paper, the RS1 construction is the immediate predecessor of the (4+2) bulk of GG-6, and the source of the geometric interpretation of G_N as the warp-curvature coupling. First cited in the Executive Summary.

L. Randall and R. Sundrum. An alternative to compactification. *Physical Review Letters* **83** (1999) 4690–4693.
Relevance. The second Randall–Sundrum paper shows that 4D gravity is localized on a single brane in an infinite warped extra dimension, providing an alternative to compactification for hiding the extra dimension. This RS2 construction sharpens the geometric interpretation of G_N: the 4D Newton constant comes from the localization of the bulk graviton on the brane, with the localization profile controlled by the warp factor. In the GG-6 framework, the choice between RS1-type (with two branes) and RS2-type (with one brane) is fixed by the OGN ledger and the requirement of compatibility with the GDOS boundary projection. First cited in §13.

R. Rashed. *The Development of Arabic Mathematics: Between Arithmetic and Algebra.* Boston: Kluwer (1994).

Relevance. Rashed's monograph is one of the principal modern works on the Islamic mathematical tradition, with particular attention to the development of algebra from al-Khwarizmi through Khayyam to al-Tusi. In the present paper, Rashed is a principal source for the discussion of the Islamic enrichment of the Euclidean tradition — algebra, the geometric solution of cubic equations, and the systematic critique of the parallel postulate that prefigured non-Euclidean geometry by several centuries. First cited in §6.

B. Riemann. Über die Hypothesen, welche der Geometrie zu Grunde liegen. Habilitationsvortrag, Göttingen 10 June 1854; published posthumously in *Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen* **13** (1868) 133–152.

Relevance. Riemann's 1854 lecture is the founding document of modern differential geometry: smooth manifolds in arbitrary dimension, equipped with metric tensors and intrinsic curvature. In the closing paragraphs, Riemann anticipates the empirical interpretation of physical geometry — that the geometry of space is an empirical question, and that the curvature of space might depend on the matter contained within it — six decades before Einstein's 1915 field equations. In the present paper, Riemann is the moment at which the mathematical framework needed for the geometric reformulation of physics is in place; everything that follows is the gradual physical realization of Riemann's mathematical vision. First cited in the Executive Summary.

O. Rømer. Démonstration touchant le mouvement de la lumière. *Journal des Sçavans*, December 7 (1676) 233–236.

Relevance. Rømer's 1676 announcement of the finite speed of light, deduced from the systematic timing discrepancies of the eclipses of Io as the Earth moved closer to and farther from Jupiter, is the first quantitative measurement of any property of light. Rømer's value of $\sim$ 220,000 km/s was the right order of magnitude. In the present paper, Rømer represents the empirical foundation of all subsequent physics of light: c is finite, large, and quantitative. First cited in §8.

R. W. Sharpe. *Differential Geometry: Cartan's Generalization of Klein's Erlangen Program.* Springer (1997).

Relevance. Sharpe's monograph is a modern textbook treatment of differential geometry that emphasizes the Klein–Cartan perspective: geometric structures are encoded in principal bundles with connections, generalizing both Klein's group-invariance and Riemann's intrinsic curvature. In the present paper, Sharpe is a principal modern reference for the geometric framework that connects Riemann's 1854 vision to the modern gauge-theoretic interpretation of forces in GG-6. First cited in §7.

M. Spivak. *A Comprehensive Introduction to Differential Geometry*, five volumes, second edition. Publish or Perish (1979).

Relevance. Spivak's five-volume treatise includes (in Volume II) a complete English translation and detailed commentary on Riemann's 1854 Habilitationsvortrag, with extensive notes on the historical and conceptual development of differential geometry from Riemann to Cartan. In the present paper, Spivak is the principal modern reference for the technical content of Riemann's lecture and its subsequent development. First cited in §7.

J. Stachel. *Einstein from B to Z.* Birkhäuser (2002).
Relevance. Stachel's collection of essays on Einstein's scientific work covers in particular the development of general relativity from 1907 to 1915, including the conceptual struggles with the equivalence principle, general covariance, and the relationship between physical and mathematical content. In the present paper, Stachel is a secondary source for the historical material on Einstein and the geometric formulation of gravity. First cited in §7.

B. L. van der Waerden. *Science Awakening: Egyptian, Babylonian, and Greek Mathematics.* P. Noordhoff (1954).
Relevance. Van der Waerden's monograph is the classic modern survey of pre-Greek and early Greek mathematics, with detailed treatment of Egyptian and Babylonian arithmetic, the Pythagorean tradition, and the development of Greek mathematics through Euclid. In the present paper, van der Waerden is a secondary source for the prehistory of the Euclidean tradition — the conceptual ingredients that the *Elements* synthesized into a deductive system. First cited in §6.

J. von Neumann. *Mathematische Grundlagen der Quantenmechanik.* Springer, Berlin (1932). English translation: *Mathematical Foundations of Quantum Mechanics*, Princeton University Press (1955).
Relevance. The source of the projection postulate, and therefore of the standing position that measurement requires a statement separate from the dynamics. The present framework's OGGEP axiom is the denial of that separation: measurement is a bulk-to-boundary projection written into the constitution rather than added to it. Von Neumann is named here because the axiom is defined against his formulation, and a program that departs from a classic should say which classic. First cited in §2.

K. G. Wilson. Confinement of quarks. *Physical Review D* **10** (1974) 2445–2459.
Relevance. Wilson introduced the Wilson loop — the gauge-invariant trace of the path-ordered exponential of a gauge field around a closed curve — as an order parameter for confinement in lattice gauge theory. The Wilson loop is the geometric object that captures the holonomy of a gauge field around a closed curve, and it is the natural geometric quantity associated with $\hbar$ in compactified gauge theory. In the GG-6 framework, the Wilson loops on S^1_θ are the dynamical moduli that drive the Hosotani gauge symmetry breaking from SU(5) to the Standard Model gauge group. First cited in the Executive Summary.

E. Witten. Anti-de Sitter space and holography. *Advances in Theoretical and Mathematical Physics* **2** (1998) 253–291.
Relevance. Witten's 1998 paper develops the AdS/CFT correspondence into a precise dictionary between bulk gravitational fields in AdS and operators in the boundary CFT. Bulk fields source boundary operators with conformal dimensions determined by the bulk masses, and bulk gravitational dynamics is dual to boundary CFT dynamics. In the GG-6 framework, the warped AdS structure of the bulk implies an analogous bulk-to-boundary dictionary, with the scale coordinate

playing the role of the RG scale, and the GDOS bulk-to-boundary projection being the technical realization of this duality. First cited in §13.

J. D. Bekenstein. *Black holes and entropy*, *Physical Review D* **7** (1973) 2333–2346.
Relevance. Bekenstein's 1973 paper proposes that black holes carry entropy proportional to the horizon area, $S = A/(4\ell_P^2)$ in Planck units, and that the black-hole second law is the appropriate generalization of the ordinary second law of thermodynamics. In the present paper, the Bekenstein–Hawking identification provides the standard derivation of the Boltzmann constant k_B from horizon thermodynamics: k_B converts between the surface-gravity-determined Euclidean-time period of the IR boundary in the GG-6 bulk and the human-chosen kelvin temperature unit (Section 27). First cited in §27.2.

S. W. Hawking. *Particle creation by black holes*, *Communications in Mathematical Physics* **43** (1975) 199–220.
Relevance. Hawking's 1975 paper derives the temperature of a black-hole horizon from quantum field theory in curved spacetime, $T = \hbar\kappa/(2\pi k_B c)$, where κ is the surface gravity of the horizon. This is the key formula relating the four Planck-unit constants c, $\hbar$, G, k_B in the horizon-thermodynamic context, and the formula from which k_B is derived in our Section 27 as a boundary thermodynamic conversion factor of the GG-6 geometry. Hawking's derivation is a particular case of the more general Bekenstein–Hawking horizon thermodynamics; together with Bekenstein (1973), it provides the standard reading of k_B as a unit-conversion factor between the natural Euclidean-time period of a horizon and the human kelvin temperature scale. First cited in §27.2.

M. Planck. *Über irreversible Strahlungsvorgänge*, *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin*, Fünfter Mitteilung (1899) 440–480.
Relevance. Planck's 1899 fifth communication on irreversible radiation processes is the founding paper of the natural-units program. In its closing pages, Planck introduces the system of units in which the four constants c, G, $\hbar$, and k_B are normalized to unity, on the grounds that these are universal natural constants whose numerical values in human-chosen units are accidents of measurement convention rather than physical content. This is the historical origin of the standard Planck unit system, codified for the SI by the 2019 redefinition. In the present paper, Planck 1899 is the metrological anchor of the parameter-free reconstruction thesis: what Planck recognized by convention in 1899, GG-Theory now derives geometrically as conversion factors of the 6D bulk and its boundary projection. First cited in §27.1.

H. Weyl. Gravitation und Elektrizität. *Sitzungsber. Preuss. Akad. Wiss.* (1918) 465–480.
Relevance. Origin of the word *gauge*: Weyl gauged the local choice of scale/length, the conceptual ancestor of treating renormalization scale as a geometric degree of freedom (Sec. 20.1). First cited in §19.6.

K. G. Wilson and J. Kogut. The renormalization group and the ϵ expansion. *Physics Reports* **12** (1974) 75–199.

Relevance. Modern formulation of the renormalization group as flow in $\ln \mu$, the coordinate GG-6 adopts for its radial direction. First cited in §20.1.

J. Polchinski. Renormalization and effective Lagrangians. *Nuclear Physics B* **231** (1984) 269–295.
Relevance. Exact (functional) renormalization group; scale evolution as a flow, the dynamical content geometrized along I_r. First cited in §20.1.

J. de Boer, E. Verlinde, and H. Verlinde. On the holographic renormalization group. *Journal of High Energy Physics* **08** (2000) 003.
Relevance. Systematic Hamilton–Jacobi formulation of the holographic RG identifying the AdS radial coordinate with boundary energy scale — the closest existing idea to GG-6's fifth dimension (Sec. 20.1). First cited in §20.1.

I. Heemskerk and J. Polchinski. Holographic and Wilsonian renormalization groups. *Journal of High Energy Physics* **06** (2011) 031.
Relevance. Integrating out the bulk between two radial slices equals integrating out boundary modes between two scales; the radial direction *is* the Wilsonian cutoff. First cited in §20.1.

T. Faulkner, H. Liu, and M. Rangamani. Integrating out geometry: holographic Wilsonian RG and the membrane paradigm. *Journal of High Energy Physics* **08** (2011) 051.
Relevance. Holographic Wilsonian RG; companion to Heemskerk and Polchinski (2011). First cited in §20.1.

S.-S. Lee. Quantum renormalization group and holography. *Journal of High Energy Physics* **01** (2014) 076.
Relevance. Shows that a quantum RG on a field theory *generates* a holographic bulk dimension — the converse direction to GG-6's declaration. First cited in §20.1.

B. Swingle. Entanglement renormalization and holography. *Physical Review D* **86** (2012) 065007.
Relevance. The holographic radial direction as an *emergent* scale axis built from entanglement renormalization (AdS/MERA) — the extra dimension as "made of" renormalization scale. First cited in §20.1.

M. Van Raamsdonk. Building up spacetime with quantum entanglement. *General Relativity and Gravitation* **42** (2010) 2323–2329.
Relevance. "Entanglement = geometry"; bulk directions emerge from the boundary entanglement structure rather than being postulated. First cited in §20.1.

N. Arkani-Hamed, A. G. Cohen, and H. Georgi. (De)constructing dimensions. *Physical Review Letters* **86** (2001) 4757–4761.
Relevance. Dimensional deconstruction: an extra dimension built from a quiver of gauge groups linked by RG-like steps — "theory space" as a discrete extra direction. First cited in §20.1.

N. S. Manton. A new six-dimensional approach to the Weinberg–Salam model. *Nuclear Physics B* **158** (1979) 141–153.

Relevance. Early six-dimensional gauge–Higgs unification; the Higgs as a component of a higher-dimensional gauge field. First cited in §20.2.

M. V. Berry. Quantal phase factors accompanying adiabatic changes. *Proceedings of the Royal Society A* **392** (1984) 45–57.
Relevance. Geometric phase attached to a parameter space rather than to physical positions — the language in which a gauge holonomy is a non-spatial, fiber-like object. First cited in §20.2.

F. Wilczek and A. Zee. Appearance of gauge structure in simple dynamical systems. *Physical Review Letters* **52** (1984) 2111–2114.
Relevance. Non-Abelian generalization of the geometric phase; gauge structure as holonomy over an internal space. First cited in §20.2.

A. Connes and J. Lott. Particle models and noncommutative geometry. *Nuclear Physics B Proc. Suppl.* **18** (1991) 29–47.
Relevance. Standard Model from spacetime times a finite, discrete, non-spatial internal space — a profound precedent for non-spatial extra geometry carrying physical content (Sec. 20.3). First cited in §20.3.

A. H. Chamseddine and A. Connes. The spectral action principle. *Communications in Mathematical Physics* **186** (1997) 731–750.
Relevance. Spectral-action formulation of the noncommutative-geometry Standard Model; gauge and Higgs as inner fluctuations of an internal geometry. First cited in §20.3.

C. Vafa. Evidence for F-theory. *Nuclear Physics B* **469** (1996) 403–418.
Relevance. Auxiliary elliptic fiber whose modulus encodes the IIB axiodilaton — "extra dimensions" that are bookkeeping rather than propagating space (Sec. 20.3). First cited in §20.3.

L. Hardy. Quantum theory from five reasonable axioms. *arXiv:quant-ph/0101012* (2001).
Relevance. Operational reconstruction of quantum theory from postulates about laboratory operations and outcome statistics — measurement as primitive (Sec. 20.4). First cited in §20.4.

G. Chiribella, G. M. D'Ariano, and P. Perinotti. Informational derivation of quantum theory. *Physical Review A* **84** (2011) 012311.
Relevance. Derivation of quantum theory from informational/operational axioms; the constitutional spirit of OGGEP, for quantum theory in the abstract. First cited in §20.4.

C. Rovelli. Relational quantum mechanics. *International Journal of Theoretical Physics* **35** (1996) 1637–1678.
Relevance. Quantum states as always relative to an observing system; a relational precursor to the observer-projection content of OGGEP. First cited in §20.4.

H. D. Zeh. On the interpretation of measurement in quantum theory. *Foundations of Physics* **1** (1970) 69–76.
Relevance. The paper that opened the decoherence program: classical outcomes appear because a system is open to an environment, not because a separate collapse rule acts on it. With Zurek

(2003) it is the principal precursor of the open-system half of OGGEP, and the difference is worth stating precisely — decoherence explains how classicality emerges *within* a fixed theory, whereas OGGEP makes the open-system projection constitutive of the theory's geometry. First cited in §20.4.

W. H. Zurek. Decoherence, einselection, and the quantum origins of the classical. *Reviews of Modern Physics* **75** (2003) 715–775.
Relevance. Classical outcomes from open-system entanglement with an environment; the decoherence backdrop to the GDOS open-system projection. First cited in §20.4.

A. Almheiri, X. Dong, and D. Harlow. Bulk locality and quantum error correction in AdS/CFT. *Journal of High Energy Physics* **04** (2015) 163.
Relevance. The boundary encodes the bulk as a quantum error-correcting code; the closest prior art for an information-theoretic bulk-to-boundary projection (Sec. 20.4). First cited in §20.4.

R. Bousso and J. Polchinski. Quantization of four-form fluxes and dynamical neutralization of the cosmological constant. *Journal of High Energy Physics* **06** (2000) 006.
Relevance. Flux landscape of metastable vacua; the backdrop against which GG-6's no-moduli claim is read (Sec. 21). First cited in §21.

S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi. de Sitter vacua in string theory. *Physical Review D* **68** (2003) 046005.
Relevance. Construction of metastable de Sitter vacua via moduli stabilization; the moduli that GG-6 never grows. First cited in §21.

L. Susskind. The anthropic landscape of string theory. *arXiv:hep-th/0302219* (2003).
Relevance. The $\sim 10^{500}$-vacuum landscape; the anti-landscape posture of GG-6 defined against it. First cited in §21.

M. R. Douglas. The statistics of string/M theory vacua. *Journal of High Energy Physics* **05** (2003) 046.
Relevance. Statistical estimate of the number of string vacua; quantifies the landscape GG-6 avoids. First cited in §21.

C. Vafa. The string landscape and the swampland. *arXiv:hep-th/0509212* (2005).
Relevance. The swampland program; the most serious attempt to constrain the landscape from the low-energy side, sharing GG-6's conviction that not every effective theory is admissible. First cited in §21.