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GDOS: Geometric Dynamics of Open Systems

6D GG-6 Bulk Projected to 4D Boundary for Quantum Measurement, General Relativity, and QFT

Katsushi Arisaka

Department of Physics and Astronomy, Electrical and Computer Engineering, University of California, Los Angeles, Los Angeles, CA 90095, USA

Contact: arisaka@physics.ucla.edu

Abstract

Every system in nature is open. No real system has ever been isolated. Indeed, the very act of observation already presumes that observer and observed are coupled. Yet for three and a half centuries, the foundational theories of physics have been written as if isolation were the rule and openness an afterthought.

In this paper, we develop GDOS (Geometric Dynamics of Open Systems) as the single law that any open system must obey, derived from one foundational constitution and one six-dimensional geometry alone. From this one law the textbook rules of quantum mechanics, general relativity, and quantum field theory all emerge as theorems rather than as separate assumptions; the measurement problem dissolves at the axiomatic level; and Einstein's gravity and the Standard Model become two faces of the same geometry, the way Maxwell once showed the electric and magnetic fields were two. Reading backward, the major theoretical frameworks of the twentieth century — relativity, gauge theory, strings, holography, quantum mechanics, thermodynamics, black-hole physics, the Standard Model, ΛCDM — all turn out to be theorems of different subsets of the same four axioms; quantum mechanics, thermodynamics, and black-hole physics are siblings of one another, born of the same axiom of openness.

After all, the deeper question of science is not what the world is made of, but how what we can measure connects to what is actually there. The answer offered here is itself a geometry — the geometry of openness.

Executive Summary

Every system in nature is open. For three and a half centuries, physics has been written as if the world were isolated. Newton wrote the laws of motion for a clockwork universe. Schrödinger wrote the wave equation for a closed atom. Einstein wrote the field equations for a closed universe. Each began from a fiction — that the thing being described was walled off from everything else — and each fiction worked spectacularly, predicting the orbits of the planets, the spectrum of hydrogen, the bending of starlight around the sun. But no real system has ever been isolated. Every electron feels the radiation field of the entire cosmos. Every atom is struck by thermal photons it never asked for. Every measurement ever made has leaked information into an environment no observer can fully account for. Closure is not something nature does. It is an idealization physics adopted because it made the mathematics tractable, and then forgot it had adopted.

What this paper proposes. This paper takes openness as a founding principle rather than a correction, and asks what follows. The answer is a single law — we call it GDOS, the Geometric Dynamics of Open Systems — that any open system must obey. The law is not postulated. It is derived, from the constitutional charter of the GG-Theory program and from one six-dimensional geometry, in which the four dimensions we inhabit are a boundary and the two extra ones are not further directions in space but a scale and a phase. Openness then has a definite meaning: the environment a system cannot escape is that geometry, and what we call the laws of physics is what the geometry looks like from the boundary we live on.

The measurement problem, at the level of the axioms. Quantum mechanics has long been taught as a list of rules to be assumed: states are vectors, evolution is unitary, measurement is something else, probabilities are squared amplitudes. The awkwardness of that list is the measurement problem, and it has resisted a century of proposals. Here the rules are not assumed. The Hilbert space, the Schrödinger equation, the Born rule, the collapse of the wave function, decoherence, the pointer basis and the violation of Bell's inequality stop being separate assumptions added by hand and become theorems of the one open law. The measurement problem does not so much get an answer as lose its premise: it was the question of how a closed system could contain its own observer, and no system is closed. The Schrödinger equation survives as what the law becomes when the coupling to the rest of the world is switched off — an excellent approximation, and now visibly an approximation.

Gravity and the gauge forces, from the same law. The same derivation, run on the geometry of the boundary rather than on its quantum states, returns general relativity: the Einstein–Hilbert action, the field equation, and the equation of stellar structure, with Newton's constant computed rather than put in. Run once more on the boundary's gauge fields, it returns the skeleton of quantum field theory and of the Standard Model. Gravity and the gauge forces are therefore not two theories to be reconciled; they are two faces of one geometry, the way Maxwell once showed the electric and the magnetic field to be two faces of one object. The route taken here inverts the usual one.

For fifty years the program has been to *quantize gravity* — to take Einstein's theory and make its metric a quantum field. This paper does the opposite: it *geometrizes the gauge forces*, recognizing the geometric structure they already carry. Gravity never needed to be made into something else.

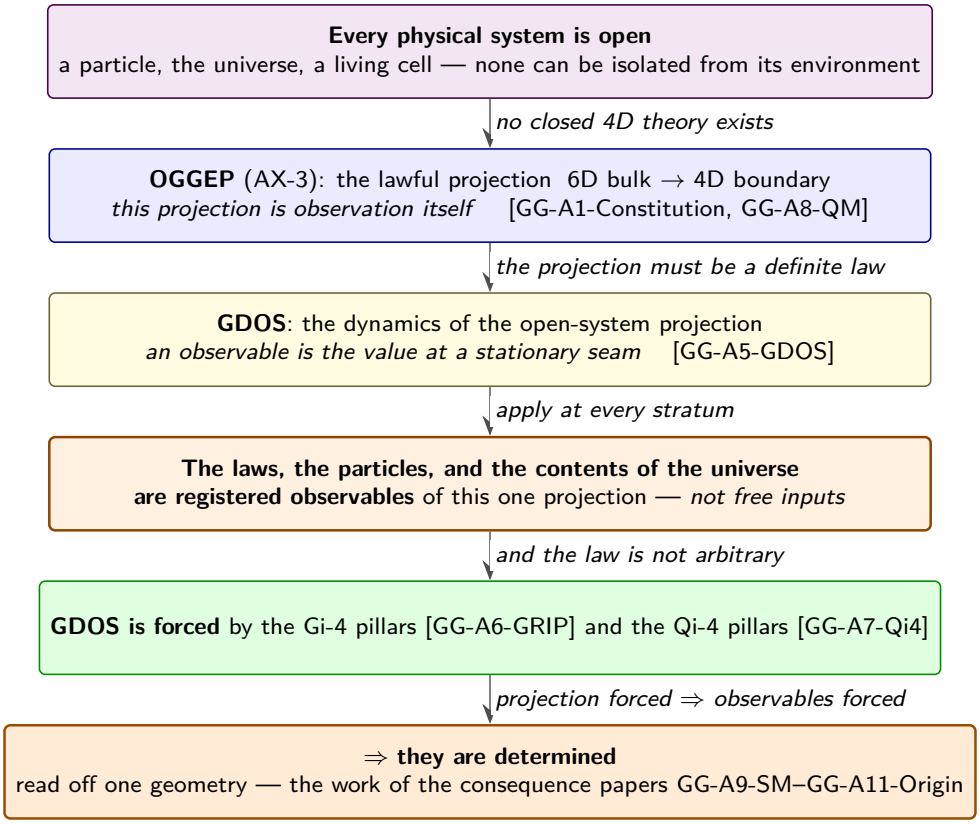
Why the textbooks would have to be reordered. If the derivations hold, what changes is not the content of twentieth-century physics but its logical order. Read backward through one constitutional lens, the great frameworks stop looking like competing programs and start looking like theorems of different subsets of the same few axioms. Special relativity takes one of them. General relativity and gauge theory take two. Quantum mechanics, thermodynamics, statistical mechanics and black-hole physics all stand on the same third — the axiom of openness — which is why they were always siblings rather than cousins. The century then looks less like a sequence of disconnected revolutions than like one edifice whose builders did not recognize the common floor. A textbook written this way would not begin with an isolated system and repair it later. It would begin with the open one, and recover the closed idealization as the special case it is.

What is established here, and what would refute it. Honesty about the boundary of a claim matters more than the claim. This paper derives the open law and its three descents. It does *not* derive the constitutional charter, the six-dimensional geometry, or the anomaly-freedom proof those rest on, nor the closed-form values of the Standard Model's parameters and of the cosmological inventory: each is the subject of a companion paper and is taken here as given, with the dependency stated at every step. What is offered here is the derivation that connects them — and it is unusually exposed. The geometry is fixed, its dimension is fixed, and no continuous parameters are left to tune, so there is no adjustable landscape to retreat into. It stands or falls on a list of quantitative tests over the 2026–2040 horizon, each stated in Part X with the value it must return. Any one of them can end the program.

A closing thought. The deepest move in a scientific revolution has usually been to drop a premise everyone took for granted. Newton dropped Aristotle's natural rest. Einstein dropped Newton's absolute time. What is proposed here is to drop the assumption that any system in physics has ever been closed. The question that has organized physics since Newton is what the world is made of; the question underneath it is how what we are able to measure is related to what is actually there. That is a question about an observer and a world that will not hold still to be observed — which is to say, about openness. This paper proposes that the relation has a shape, that the shape is geometric, and that once it is written down the familiar laws follow from it rather than the other way round. Whether that is right is now a matter for measurement.

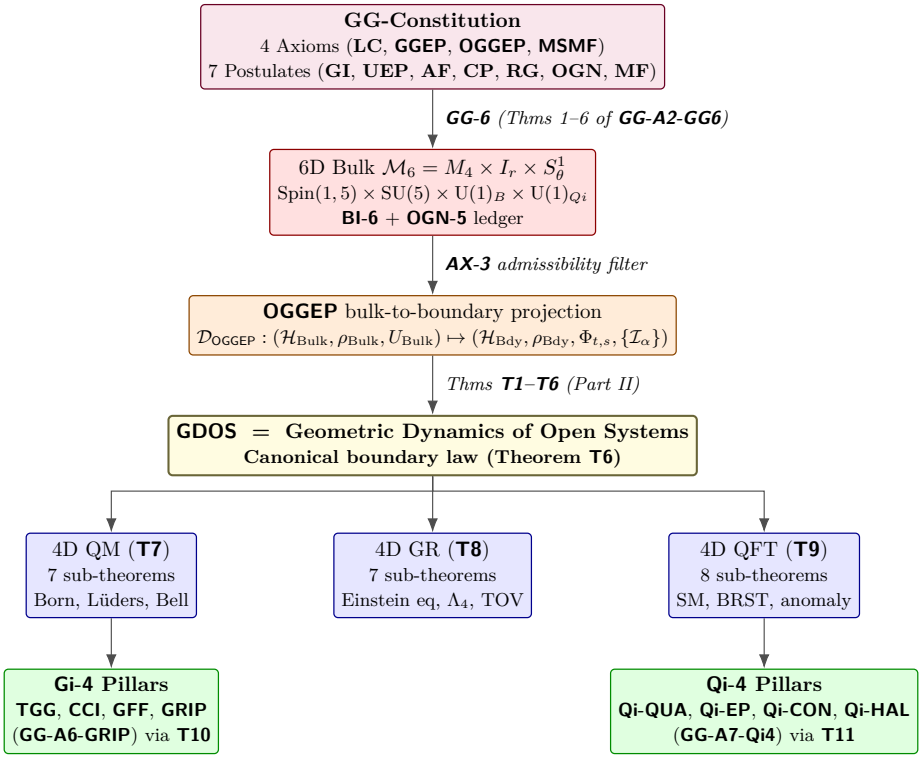
Keywords: GDOS; OGGEP; GG-6 bulk; BI-6 anomaly identity; OGN-5 integer ledger; Gi-4/Qi-4 pillars; open quantum systems; GKLS master equation; Born rule; decoherence; einselection; measurement problem; statistical mechanics; fluctuation theorems; black-hole thermodynamics; Page curve; AdS/CFT correspondence; Kaluza–Klein reduction; modified TOV equation; anomaly inflow; proton stability; cosmological-constant seesaw.

Every Physical System Is Open, and That Single Fact Forces a Law



Why this paper exists. No physical system is closed, so no four-dimensional theory can be the whole account; the 4D world is a lawful projection of a six-dimensional bulk, and that projection is what observation is. The present paper supplies the law that governs it. Everything below the GDOS box is consequence rather than assumption — if the projection is forced, so is everything registered through it. The same funnel opens GG-A9-SM, GG-A10-Cosmos and GG-A11-Origin with a different payload in the last box. In full, with its complete caption: Figure 1, p. 24.

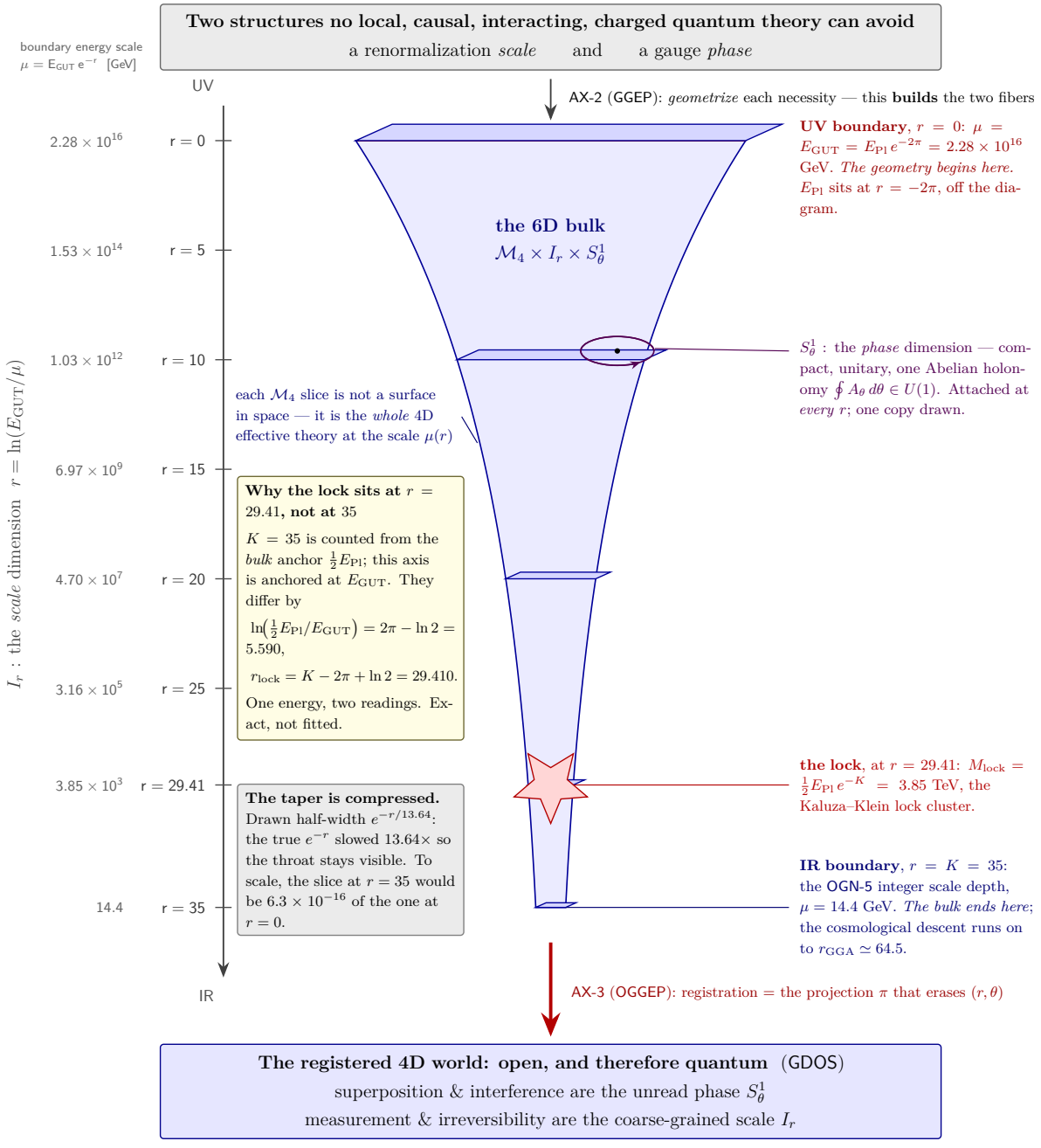
One Input at the Top —
and Every Box Below It Is a Theorem



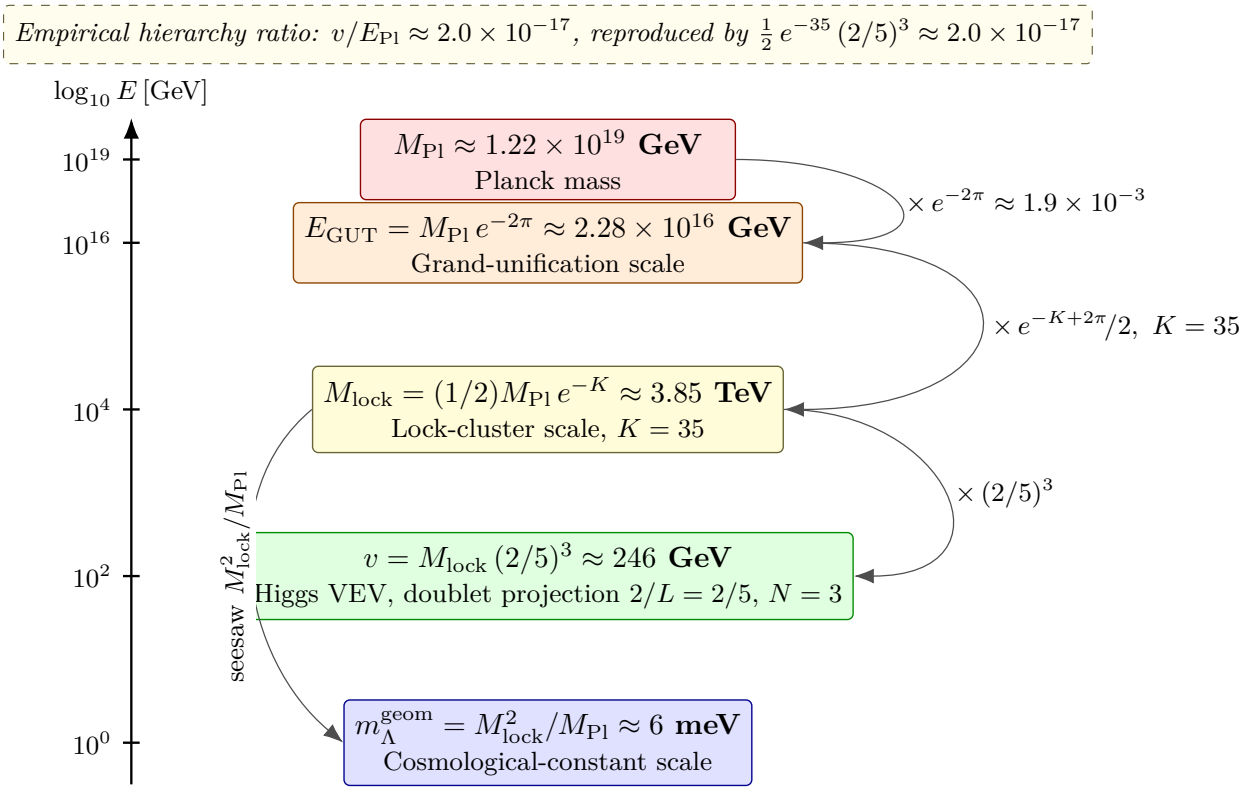
What is assumed, and what is derived. The four axioms and seven postulates of the GG-Constitution sit in the topmost box, and they are the only primitive input on the page. Everything beneath them is a theorem: the six-dimensional bulk with its composite gauge group, the OGGEP admissibility filter that demands a bulk–boundary correspondence, the canonical boundary law of GDOS, and below that the three descents — four-dimensional quantum mechanics, general relativity and quantum field theory — each arriving as a boundary specialization rather than as a separate postulate. The two lowest boxes are the eight pillars of GG-A6-GRIP and GG-A7-Qi4, derived here rather than assumed there. A reader who wants to attack the framework should attack the top box, because everything else stands or falls with a theorem. In full, with its complete caption: Figure 3, p. 33.

A Six-Dimensional Bulk Whose Fifth Dimension Is Scale and Whose Sixth Is Phase

The whole program in one diagram, with the scale axis carrying numbers. The bulk $\mathcal{M}_4 \times I_r \times S^1_\theta$ is a fiber bundle: over every point of the Lorentzian stage sit the two *non-spatial* fibers, scale and phase. AX-2 (GGEP) acts *upward* and builds them; AX-3 (OGGEP) acts *downward*, and registration is the projection that erases (r, θ) — the origin of quantum behavior. The vertical axis is calibrated: $E_{\rm GUT} = E_{\rm Pl} e^{-2\pi}$ at $r = 0$, the OGN-5 depth $r = K = 35$ at the infrared boundary, and the lock at $r = 29.41$, which is $K - 2\pi + \ln 2$ exactly.



Thirty-Three Orders of Magnitude — and Not One Continuous Parameter



Where the scales come from. Five energy scales are linked by closed-form expressions built from two scale-profile exponentials, $e^{-2\pi}$ and e^{-K}, and the OGN-5 integers $K = 35$, $L = 5$, $N = 3$. No continuous parameter appears anywhere on the ladder. From the top: the Planck mass is the input scale; the grand-unification scale follows from it; the lock-cluster scale at 3.85 TeV is the framework's principal LHC-era prediction, developed in full in GG-P1-KK; the Higgs vacuum expectation value at 246 GeV reproduces the measured electroweak scale; and the lock-scale seesaw against the Planck mass lands within an order of magnitude of the empirical dark-energy density. Thirty-three orders of magnitude are spanned by exponentials of small integers, with no fine-tuning anywhere — which is interesting, and is not proof. The closing step is the one to check first. In full: Figure 7, p. 124.

One Law, Three Descents: the Same Six Steps in Quantum Mechanics, Gravity and Field Theory

One derivation, run three times. The rows are the six steps every descent takes; the columns are quantum mechanics, general relativity and quantum field theory. Read across a row to see one step performed on three different objects; read down a column to follow a single derivation end to end. Each cell names the theorem that carries that step, so a reader who doubts a particular move can go straight to it. Row 1 is the point of the table: the measurement problem (1926–), the unification problem (1915–) and the Lagrangian-ansatz problem (1954–) are three places where twentieth-century physics ran out of derivation and posted a postulate instead. Row 6 is where the claim is exposed — one pre-declared falsifier per column. In full: Table 4, p. 76.

Step	**QM (Part III)**	**GR (Part IV)**	**QFT (Part V)**
1. The century-old question	What is measurement, and why these postulates?	Why this action, and why is gravity unified with nothing?	Why this Lagrangian, and why the anomaly miracles?
2. The boundary object	The QM-restricted boundary algebra and its GNS space (T7.1)	The metric zero mode of the bulk (T8.1)	The field zero modes and the KK tower (T9.1)
3. The law descends	GKLS → von Neumann → Schrödinger in the closed limit (T7.2)	6D Einstein–Hilbert → 4D Einstein–Hilbert, with $M_{\rm Pl}$ computed (T8.1–T8.2)	Heavy-mode integration → $S_{4,\rm eff}$ and $Z_4[J]$ (T9.2–T9.3)
4. The inherited structure	Born rule and Lüders update from CPTP completion (T7.3–T7.4)	Conservation as Ward identity; the modified Einstein equation (T8.3–T8.4)	BRST and Slavnov–Taylor as bulk inheritance (T9.4–T9.5)
5. The puzzle dissolved	Decoherence, einselection, Bell violation (T7.5–T7.7)	The cosmological constant reframed; the hierarchy derived (T8.5, T8.7)	Anomaly cancellation as theorem; four conserved currents (T9.6–T9.7)
6. The falsifier	Tsirelson-saturating boundary correlations (T7.7)	The modified TOV shift in the neutron-star mass–radius diagram (T8.6)	The $M_{\rm lock}$ cluster; exact proton stability (T9.1, T9.8)

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Preface

This is a long document, and few readers will want all of it. Its claim is stated in the Executive Summary; the fourteen Parts and sixteen appendices that follow are the work of earning it, because a claim this broad is worth nothing unless it can be checked line by line. What follows is a map of the routes through it.

If you have an hour. Read Part I, which states the single premise the whole argument rests on, and then Part XI, the Conclusions, which says what that premise bought and what it cost. Those two are the argument in miniature.

If you want the derivation itself. Part II is load-bearing. It derives the open-system law from the constitutional charter and the bulk geometry, and every later Part is a descent from it. A reader satisfied by Part II can judge the rest on technical grounds; a reader who is not need read no further, and is invited to say where it fails.

If you came for the measurement problem. Part III derives the quantum postulates — Hilbert space, Schrödinger equation, Born rule, decoherence, pointer basis — from the open law, grading each as derived outright or derived conditionally, and on what. Appendices C–F unfold its machinery.

If you came for gravity and unification. Part IV derives general relativity, Part V the skeleton of quantum field theory, and Part VI places gravity and electromagnetism on one geometric footing. Appendices G–J carry the reduction in explicit coordinates.

If you want to know what would refute it. Part X states the quantitative predictions and the condition under which each one fails, over a horizon that closes in 2040. It is the shortest honest answer to whether any of this is science.

If you are skeptical. Part XIII collects the objections a hostile reviewer would raise — fifty-seven of them, in ten groups — and answers each with a pointer into the paper rather than a reassurance. If your instinct is that a result this broad must have something wrong with it, that instinct is right; Part XIII is where to put it to work.

If you are teaching from it. Appendix P lays out a graduate course built from this volume: three single-term tracks and a full-year sequence, keyed to the Parts and the worked problems. Appendix O locates each result of this paper in the companion paper it comes from.

How to read the claims themselves. Every result here is marked as derived, as conditional on a stated assumption, or as open, and those marks are meant literally, and an import from a companion paper is declared at the point of use. A reader is entitled to hold the whole structure to its weakest link, and this paper is written to make that link easy to find.

Part I
Foundations

From the GG-Constitution and GG-6 to the OGGEP Gap

Physics has two kinds of statement in it, and they are not usually distinguished. There are laws — the Schrödinger equation, the Einstein equation — and there are rules for what an observer sees when a law is put to the test. The laws are derived from principles; the rules are, in almost every textbook, simply announced. This Part asks whether that second category has to exist at all.

The question can be put sharply. Suppose we write down a small number of constitutional principles about what a physical theory must be like, and suppose we take seriously the fact that no system in nature is closed. Do those principles then *force* a single law governing how the world looks from inside it — or do they merely permit many, leaving the choice to be made by hand? If they force one, the rules of observation stop being a separate layer of physics and become consequences.

Getting to that question takes some care, and this Part does the preparation. It states what is being assumed and where each assumption comes from: the constitutional charter is established in GG-A1-Constitution, the six-dimensional geometry in GG-A2-GG6, and neither is re-derived here. It identifies the gap those two leave — they say what the world is made of, but not what a finite observer inside it can register — and it makes the missing ingredient precise enough to be checked rather than gestured at, by turning the phrase "admissible theory" into a predicate one can actually evaluate. And it states the inversion that organizes everything after it: openness is the rule and closure the idealization, not the other way round.

A reader in a hurry can take the first section and the roadmap at the end of the Part. A reader who intends to argue with the paper should read the whole of it, because every assumption the argument later leans on is declared here, once, and is not declared again.

1 The Constitutional Question and the Operational Gap

Quantum mechanics works. General relativity works. The Standard Model works. Together they form one of the most empirically successful intellectual structures the human species has ever built — from the spectrum of hydrogen to the bending of light around the sun, from the lifetime of the muon to the cosmic microwave background, the predictions of twentieth-century physics agree with the measurements to ten or twelve decimal places (Born, 1926, Collaboration, 2018, A. Einstein, 1905, 1915, Heisenberg, 1925, 't Hooft, 1972, Neumann, 1932, Riess, 1998, Schrödinger, 1926, K. Wilson, 1971, C. Yang, 1954). And yet beneath this magnificent edifice sits a persistent asymmetry that has resisted every technical attack for a century. We must look at it carefully.

Open any graduate textbook of physics and watch how the laws are written down. The Schrödinger equation is presented as a law of physics; the Born rule and the projection ("collapse") rule are presented as separate postulates added on top of it, typically labeled *measurement axioms*.

The Einstein equation is presented as a law of physics; the rules by which a finite observer registers the geometry are presented as a separate "observation theory." The Standard-Model Lagrangian is written down as an Ansatz; the rules by which detector outcomes are assigned probabilities are imported from open-system quantum theory (Choi, 1975, Gorini, 1976, Kraus, 1983, Lindblad, 1976, Stinespring, 1955) as a separate operational layer. In every one of these constructions, the *closed* dynamical law and the *open* operational layer must be glued together by hand.

A brief plain-English bridge. Before stating the diagnosis formally, we pause to translate the technical vocabulary that will dominate the rest of Part I. By *open system* we mean any system that exchanges energy, matter, or information with an environment that an observer cannot fully access — a measuring apparatus coupled to a quantum particle, a cell exchanging metabolites with its surroundings, a brain receiving sensory input. By *operational dictionary* we mean a precise rule that tells the observer what numbers her apparatus will register, given the underlying physics — the analog, for an open system, of the Born rule for a closed quantum system. By *CPTP channel* we mean the mathematical object that the dictionary outputs: the most general lawful evolution of an open system's probabilities under any combination of internal dynamics, environmental coupling, and measurement. The unfamiliar acronyms below (AX-3, OGGEP, $\mathcal{D}_{\mathsf{OGGEP}}$, GNS, etc.) will be unpacked in §3; for now the reader only needs to hold one idea: every measurement is an exchange with an environment, and the present paper makes that exchange part of the law of physics rather than an add-on convention.

Where the paper lands. The arc of the paper proceeds in three movements. The first movement (Parts I–VII) is the constitutional derivation: from the four axioms of the GG-Constitution and the six-dimensional bulk we derive, as eleven principal theorems, the canonical GDOS boundary law and the four-dimensional textbook rules of quantum mechanics, general relativity, and quantum field theory. The second movement (Part VIII) is the constitutional classification: the four axioms form a strictly nested tower, and every major theoretical framework of the twentieth century — relativity, gauge theory, string theory, holography, quantum mechanics, classical thermodynamics, statistical mechanics, black-hole physics, the Standard Model, ΛCDM — occupies a specific slab of that tower according to which axiom subset it implicitly uses. The third movement (Parts IX–XI) is the synthesis: ten geometric pictures, a consolidated twenty-five-test empirical horizon, a sibling unification of quantum mechanics, classical thermodynamics, and black-hole physics under one axiom of openness, and a closing meditation on what it would mean for the twenty-first century to take that unification as the next constitutional shift.

The diagnosis of the present program is that this asymmetry is not a technical inconvenience but an axiomatic gap. By an axiomatic gap we mean an admissibility filter that is missing from the foundational layer of the theory. Under the GG-Theory Constitution (Arisaka, 2026, A1-Constitution), the gap is closed by elevating measurement and open-system bookkeeping to the status of an axiom: AX-3, the Operational/Open Geometry–Gauge Equivalence Principle (OGGEP), states that no theory is admissible unless it carries its own lawful dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ from bulk data to registered boundary observables. This is not a methodological preference; it is the same kind of admissibility filter as locality (AX-1) or geometric–gauge equivalence (AX-2). A theory that lacks such a dictionary

is not an incomplete physical theory; it is no physical theory at all in the axiomatic sense.

The present paper is the natural continuation of GG-A2-GG6 (Arisaka, 2026, A2-GG6). That paper answered the ultraviolet question (*what is the lawful bulk?*) by deriving, from the four axioms and seven postulates alone, a unique six-dimensional Anti-de Sitter bulk in the canonical natural-GG-6-units form (Theorem 2 of (Arisaka, 2026, A2-GG6), which derives the absence of any free scale coefficient or compact-direction period from AX-4 plus POS-5 plus POS-6)

$$\boxed{ds_6^2 \;=\; e^{-2r}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu \;+\; dr^2 \;+\; d\theta^2, \qquad r\in[0,K], \quad \theta\in[0,2\pi),}$$

with composite gauge–gravity group

$$G_{\text{tot}} \;=\; \text{Spin}(1,5)\;\times\;\text{SU}(5)\;\times\;\text{U}(1)_B\;\times\;\text{U}(1)_{Qi},$$

locked together by the Green–Schwarz-modified Bianchi (Bianchi, 1902) identity

$$dH_3 \;=\; X_4 \;=\; \text{tr}(R\wedge R)\;-\;\text{Tr}(F_5\wedge F_5)\;-\;\tfrac{2}{5}\,F_B\wedge F_B\;-\;\tfrac{2}{5}\,F_{Qi}\wedge F_{Qi},$$

and frozen by the OGN-5 integer ledger $(M,N,L;J,K)=(1,3,5;7,35)$. The present paper answers the *operational* question: how does the four-dimensional world observed by finite experimenters arise from the six-dimensional bulk?

The answer that the constitution forces is the following: the entire content of twentieth-century four-dimensional physics — quantum mechanics, general relativity, and the formal infrared limit of the Standard Model — must be a derived consequence of the 6D bulk together with OGGEP. The textbook postulates of quantum mechanics (Born rule, Lüders rule, decoherence, the preferred basis, the violation of Bell inequalities) are theorems. The gravitational law (Einstein equation, the cosmological constant, the gauge–gravity hierarchy) is a theorem. The structural skeleton of quantum field theory (BRST symmetry, the Slavnov (Slavnov, 1972)–Taylor (Taylor, 1971) identity, anomaly cancellation, proton stability) is a theorem. Throughout this paper, we shall make every step of this derivation explicit, in the Euclidean style established in GG-A2-GG6 (Arisaka, 2026, A2-GG6) and refined in GG-A6-GRIP (Arisaka, 2026, A6-GRIP), so that no derivational step is hidden and a graduate student in physics can follow the chain end-to-end.

The Euclidean dual-layer method. Following the same stylistic discipline used in GG-A2-GG6 and GG-A6-GRIP, the principal theorems of the present paper are stated and proved in dual-layer form. Each Principal Theorem T*.k carries an algebraic content (the formal mathematical statement) and a physical/operational content (the laboratory-falsifiable consequence), with proofs of each layer marked ■ and an immediately-following Physics-Intuition Remark explaining what the theorem says in graduate-student language. Derived structural facts that support the principal theorems but are not themselves headline results are stated as *Structure Theorems*, following the terminology of GG-A6-GRIP. No new physical axiom beyond the four axioms and seven postulates of the GG-Theory Constitution is introduced anywhere in the paper: this is a pure derivation, not a model extension.

2 This paper's place in the cohort: one link of the open-system chain

This paper's own central claim (§64) is that four-dimensional quantum mechanics, general relativity, and quantum field theory are three boundary projections of one open six-dimensional bulk under a single finite-resolution law; that claim stands on its own terms. For readers following the whole GG-Theory cohort, it is also one link of a single open-system chain that the consequence papers cash out. Because no physical system is closed, the four-dimensional world is a lawful projection (OGGEP) of the six-dimensional bulk, and that projection is observation itself; its dynamics is the boundary law GDOS, forced by the geometric Gi-4 and quantum-information Qi-4 pillars, so that the laws, the particles, and the contents of the universe are *observables* of the projection rather than free inputs. In that chain, the present paper supplies the boundary law GDOS itself — the third link of the chain — and its own sector-unification claim is precisely this link. The funnel below is shared, in structure, with the companion papers GG-A9-SM, GG-A10-Cosmos, and GG-A11-Origin; here it is offered only as orientation, and the rest of this paper proceeds on its own terms.

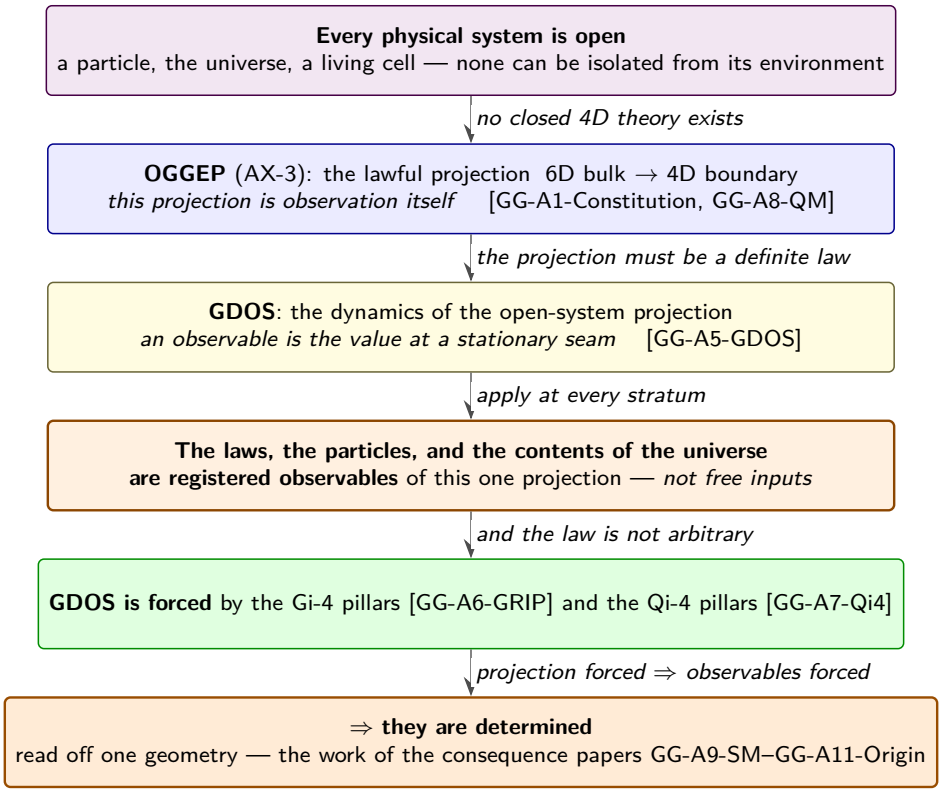


Figure 1: The open-system chain shared across the GG-Theory cohort. Because no physical system is closed, the four-dimensional world is a lawful projection (OGGEP, AX-3) of the six-dimensional bulk, and that projection is observation itself; its dynamics is GDOS, forced by the Gi-4 and Qi-4 pillars, so the laws and contents of the universe are observables rather than free inputs. The same funnel opens the consequence papers GG-A9-SM, GG-A10-Cosmos, and GG-A11-Origin, with a paper-specific payload; within it the present paper establishes the boundary law GDOS (the projection's dynamics).

3 Definitions and the Admissibility Predicate

We begin with three definitions that fix the formal arena of the entire derivation. These definitions are imported from GG-A2-GG6 (Arisaka, 2026, A2-GG6) but are restated here in self-contained form for the reader who reads only the present paper.

Notation legend (for the four definitions below). The definitions in this section use calligraphic symbols that recur throughout the paper. We list them here once with plain-English glosses, so that a reader from biology or chemistry can read the formal definitions without losing track of which symbol denotes which kind of object:

- $\mathcal{M}$ — the underlying *manifold* on which the theory is defined (the 6D bulk in our case);
- $\mathcal{P}$ — the *principal-bundle structure* over $\mathcal{M}$ (the gauge-and-frame data attached at every point);
- $\mathcal{A}$ — the *composite connection* (the unified gauge-and-frame field that "locks" geometry and gauge);
- $\mathcal{D}$ — the *operational dictionary* (the map from bulk data to what the observer registers); in our context $\mathcal{D} = \mathcal{D}_{\mathsf{OGGEP}}$;
- $\mathcal{S}$ — the *dynamical law* (the bulk Hamiltonian or action);
- $\mathcal{O}$ — the *family of registered observables* together with the pre-declared falsifiers;
- $\mathcal{B}(\cdot)$ — the algebra of bounded operators on a Hilbert space;
- $\mathcal{H}_{\text{Bulk}}, \mathcal{H}_{\text{Bdy}}, \mathcal{H}_{\text{env}}$ — the bulk, boundary-accessible, and environment Hilbert spaces respectively, related by a splitting $\mathcal{H}_{\text{Bulk}} = \mathcal{H}_{\text{Bdy}} \otimes \mathcal{H}_{\text{env}}$;
- $\mathcal{A}_{\text{Bdy}}$ — the *boundary algebra*: the sub-algebra of $\mathcal{B}(\mathcal{H}_{\text{Bulk}})$ that acts non-trivially only on the observer-accessible factor.

The term C^**-algebra* appearing in Definition 2 below is the standard functional-analytic structure of bounded operators on a Hilbert space; the reader unfamiliar with it can read it as "the algebra of measurable physical quantities" without loss for the present derivation. The *GNS construction* mentioned in subsequent theorems is the canonical procedure for reconstructing a Hilbert space from an algebra together with a state on it; a brief explicit version appears in Appendix J (and in any standard reference on operator algebras).

Definition 1 (Candidate theory)**.** A candidate theory $\mathcal{T}$ is a tuple

$$\mathcal{T} \;=\; (\mathcal{M},\, \mathcal{P},\, \mathcal{A},\, \mathcal{D},\, \mathcal{S},\, \mathcal{O}), \tag{1}$$

where $\mathcal{M}$ is the underlying manifold or stratified bulk–boundary stage; $\mathcal{P}$ is the principal-bundle structure over $\mathcal{M}$; $\mathcal{A}$ is the composite connection; $\mathcal{D}$ is an operational dictionary that produces boundary observations from bulk data; $\mathcal{S}$ is the dynamical law; and $\mathcal{O}$ is the family of registered

observables and pre-declared falsifiers. The constitutional clauses act on this entire tuple, not just on $\mathcal{S}$.

Definition 2 (Boundary algebra)**.** Let $\mathcal{T}$ be a candidate theory with bulk Hilbert space $\mathcal{H}_{\text{Bulk}}$. The *boundary algebra* $\mathcal{A}_{\text{Bdy}} \subset \mathcal{B}(\mathcal{H}_{\text{Bulk}})$ of $\mathcal{T}$ is the C^*-subalgebra of bounded operators on $\mathcal{H}_{\text{Bulk}}$ that act non-trivially only on the observer-accessible degrees of freedom; equivalently, it is the maximal sub-algebra such that there exists a Hilbert-space splitting $\mathcal{H}_{\text{Bulk}} = \mathcal{H}_{\text{Bdy}} \otimes \mathcal{H}_{\text{env}}$ and a faithful representation $\mathcal{A}_{\text{Bdy}} \hookrightarrow \mathcal{B}(\mathcal{H}_{\text{Bdy}}) \otimes \mathbf{1}_{\text{env}}$.

Definition 3 (Operational dictionary)**.** The *operational dictionary* $\mathcal{D}_{\mathsf{OGGEP}}$ of a candidate theory $\mathcal{T}$ is the map

$$\mathcal{D}_{\mathsf{OGGEP}} : \ (\mathcal{H}_{\text{Bulk}},\, \rho_{\text{Bulk}},\, U_{\text{Bulk}}) \ \longmapsto \ (\mathcal{H}_{\text{Bdy}},\, \rho_{\text{Bdy}},\, \Phi_{t,s},\, \{\mathcal{I}_\alpha\}), \tag{2}$$

where ρ_{Bdy} is a boundary density operator on $\mathcal{H}_{\text{Bdy}}$, $\Phi_{t,s}$ is an admissible boundary evolution channel for $t \geq s$, and $\{\mathcal{I}_\alpha\}$ is an instrument-valued family of measurement operations labeled by registered classical outcomes α.

Definition 4 (Admissibility)**.** A candidate theory $\mathcal{T}$ is *admissible* iff it satisfies all four axioms and all seven postulates of the GG-Theory Constitution (Arisaka, 2026, A1-Constitution):

$$\boxed{\text{Admissible}(\mathcal{T}) \ := \ (\mathcal{T} \models \text{AX-1}, \ldots, \text{AX-4}) \ \wedge \ (\mathcal{T} \models \text{POS-1}, \ldots, \text{POS-7}).} \tag{3}$$

The symbol $\models$ denotes that the full content of the constitutional clause is satisfied — not merely that a slogan associated with it can be heuristically identified.

Worked example: a single qubit with one-qubit environment. To make the four definitions concrete, consider the simplest non-trivial open system: a single qubit (the system) coupled to a one-qubit environment. Then $\mathcal{H}_{\text{Bulk}} = \mathbb{C}^2 \otimes \mathbb{C}^2 = \mathbb{C}^4$, with the natural splitting $\mathcal{H}_{\text{Bdy}} = \mathbb{C}^2$ (the system qubit, accessible to the observer) and $\mathcal{H}_{\text{env}} = \mathbb{C}^2$ (the environment qubit, unobserved). The boundary algebra $\mathcal{A}_{\text{Bdy}}$ is $M_2(\mathbb{C}) \otimes \mathbf{1}_{\text{env}} \cong M_2(\mathbb{C})$, the algebra of 2×2 complex matrices acting on the system qubit. The bulk unitary U_{Bulk} is any unitary on $\mathbb{C}^4$ (e.g. a CNOT gate followed by a single-qubit rotation). The operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ in this toy example maps the bulk data $(\mathbb{C}^4, \rho_{\text{Bulk}}, U_{\text{Bulk}})$ to the boundary tuple $(\mathbb{C}^2, \rho_{\text{Bdy}} = \text{Tr}_{\text{env}}\, \rho_{\text{Bulk}}, \Phi_{t,s} = \text{Tr}_{\text{env}} \circ (U_{\text{Bulk}} \cdot U_{\text{Bulk}}^\dagger), \{\mathcal{I}_\alpha\})$, where the $\mathcal{I}_\alpha$ are projective measurements on the system qubit in any chosen basis. The full theory $\mathcal{T} = (\mathbb{C}^4$, trivial bundle, $\mathcal{A} = 0$, $\mathcal{D}_{\mathsf{OGGEP}}$, $\mathcal{S} = H_{\text{Bulk}}$, $\mathcal{O} = \{\sigma_x, \sigma_y, \sigma_z\})$ is admissible in the sense of Definition 4 iff all four axioms and seven postulates of the Constitution are checked — a task that is trivial in this toy case (locality is automatic on two qubits, gauge-equivalence is trivial with $\mathcal{A} = 0$, the operational dictionary exists by construction, and the predicate filter on continuous freedom is vacuous because the example has only discrete parameters). The non-trivial work of the present paper is to construct the analogous $\mathcal{D}_{\mathsf{OGGEP}}$ for the full 6D bulk and to show that it inherits the canonical GDOS runtime.

Remark 1. The admissibility predicate (3) is the same predicate used in GG-A2-GG6 (Arisaka, 2026, A2-GG6). In the present paper we add the further observation that an apparently-admissible bulk theory *without* an operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ is in fact *not* admissible, because AX-3

(OGGEP) is part of the admissibility predicate. In particular, a closed Hamiltonian field theory on the GG-6 bulk that does not specify how a finite observer registers boundary outcomes is not a complete physical theory by the standards of the Constitution. The *purpose* of the present paper is to construct $\mathcal{D}_{\mathsf{OGGEP}}$ explicitly and show that the resulting boundary theory is the canonical GDOS runtime.

Remark 2 (SIP: canonical GDOS input unit, defined in GG-A7-Qi4)**.** The four definitions above (candidate theory, boundary algebra, operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$, admissibility) fix the *containing* data of any GDOS instance, but they do not yet specify the canonical *input unit* of the boundary law — the smallest self-contained tuple of bulk-side data that the runtime $\mathcal{D}_{\mathsf{OGGEP}}$ consumes to produce a boundary trajectory. That canonical input unit is the *System Instance Packet* (SIP), formally defined in GG-A7-Qi4 §6 ((Arisaka, 2026, A7-Qi4, Definition 6.1 and Theorem 6.2 SIP completeness)), where the SIP five-field structure $(\mathcal{H}_{\mathrm{Bdy}}, \rho_0, \mathcal{L}_t, \{\mathcal{I}_\alpha\}, \Pi_\Sigma)$ is shown to be both necessary and sufficient as a runtime input to the GDOS dynamics derived in Part II of the present paper. The SIP construction takes the boundary algebra $\mathcal{A}_{\mathrm{Bdy}}$ of Definition 2 as its setting and the operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ of Definition 3 as its delivery map. The present paper does not re-derive the SIP formalism; readers seeking the formal definition, the completeness theorem, and the worked chemistry/neural/collective examples are referred to GG-A7-Qi4 §6.

4 Primitive Inputs from the Companion Papers

The derivation chain of the present paper takes the GG-Constitution of A1 (Arisaka, 2026, A1-Constitution) and the six-dimensional bulk of GG-A2-GG6 (Arisaka, 2026, A2-GG6) as primitive inputs. This section catalogs the imports in the smallest form that subsequent derivations require, with explicit cross-references back to the companion papers for the full motivation, proofs, and discussion. Readers already familiar with A1 and GG-A2-GG6 may skim directly to the OGGEP construction at the end of this section, which is the operational entry point of the present derivation.

4.1 The eleven constitutional clauses (imported from A1)

The GG-Constitution (Arisaka, 2026, A1-Constitution) consists of *four axioms* (AX-1–AX-4) and *seven postulates* (POS-1–POS-7). Each is imported here as a one-line statement; full motivation, formal definitions, and worked examples are in A1.

AX-1 LC (Locality and Causality). Admissible theories respect a signal-cone structure and assign no observable consequence to spacelike-disconnected interventions.

AX-2 GGEP (Geometry–Gauge Equivalence Principle). Gauge fields and gravitational curvature are two facets of one connection on a higher-dimensional bulk manifold; the equivalence is realized by the 6D geometry.

AX-3 OGGEP (Operational/Open GGEP). The boundary observer accesses bulk physics only through a CPTP open-system channel $\mathcal{D}_{\mathsf{OGGEP}}$; closed-system descriptions are special-case limits. *This is the constitutional clause that drives the entire present paper.*

AX-4 MSMF (Maximum Symmetry, Minimum Freedom). No free continuous parameter may appear in the framework unless absolutely required; discrete topological data are admissible, continuous adjustables are not.

POS-1 GI (Layered Gauge Invariance). All bulk and boundary gauge invariances must lock through a single covariant pipeline.

POS-2 UEP (Universality and Equivalence of Predictions). Predictions of distinct admissible theories that satisfy the same constitutional clauses must agree on registered observables.

POS-3 AF (Anomaly Freedom). All gauge anomalies of the boundary theory must cancel by topological inflow from the bulk via the modified Bianchi identity BI-6.

POS-4 CP (Coarse-Graining and Projection). The boundary theory is defined by a fixed projection from the bulk Hilbert space to the boundary state space; this projection is constitutional and not optional. *This is the constitutional clause that, together with AX-3, forces GDOS to be the canonical boundary law of the present paper.*

POS-5 RG (RG/Scale Covariance). The radial direction r of the bulk implements logarithmic RG flow, $r = \ln(E_{\mathrm{GUT}}/\mu)$.

POS-6 OGN (Integer Normalization). All dimensionless ratios of the boundary theory are forced to be small integers or rationals from the OGN-5 ledger.

POS-7 MF (Measurement and Falsifiability). Every central claim of GG-Theory must compile into a registered observable and a pre-declared kill criterion.

4.2 The 6D bulk (imported from GG-A2-GG6)

GG-A2-GG6 (Arisaka, 2026, A2-GG6) proves through eight theorems that the unique admissible bulk for GG-Theory is the six-dimensional manifold

$$\mathcal{M}_6 \;=\; \mathcal{M}_4 \times I_r \times S^1_\theta, \qquad I_r = [0, K = 35], \qquad \theta \in [0, 2\pi), \tag{4}$$

with the canonical natural-GG-6-units metric

$$ds^2_6 \;=\; e^{-2r}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu \;+\; dr^2 \;+\; d\theta^2, \tag{5}$$

the four-component bulk gauge group

$$G_{\mathrm{Bulk}} \;=\; \mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}, \tag{6}$$

the modified Bianchi identity BI-6 with coefficient vector $(a, b, c, d) = (1, -1, -2/5, -2/5)$ (Arisaka, 2026, A3-BI6), and the OGN-5 Diophantine ledger

$$(M, N, L; J, K) \;=\; (1, 3, 5;\, 7,\, 35), \qquad K = LJ = M^2 + N^2 + L^2, \tag{7}$$

together fixing every dimensionless ratio of the boundary theory. The overall unit scale is set by G_N (equivalently $M_{\rm Pl}$); the 6D fundamental scale $M_6 \sim M_{\rm Pl}$ is a derived combination, not an independent input.

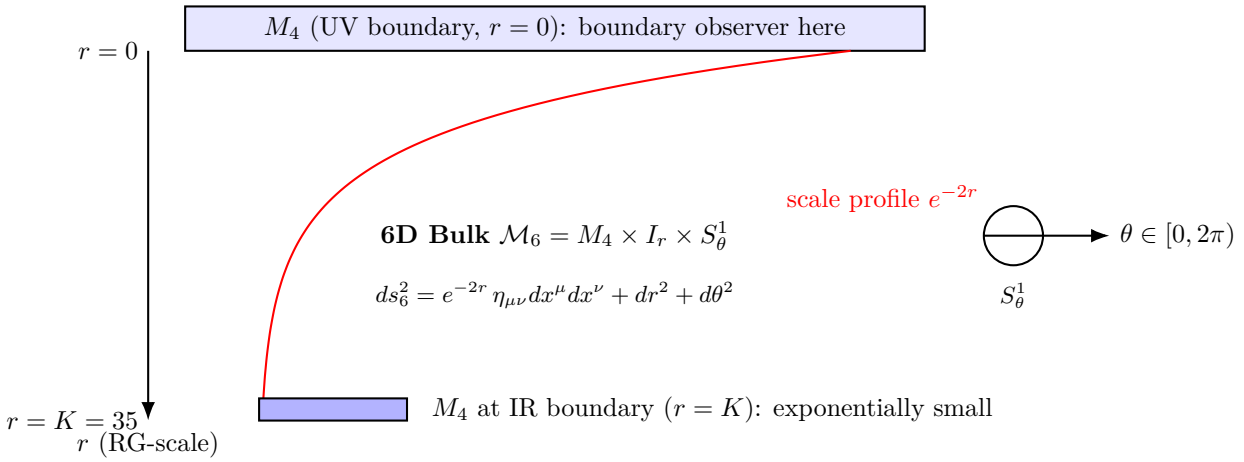


Figure 2: The 6D bulk $\mathcal{M}_6 = M_4 \times I_r \times S^1_\theta$. The radial direction $I_r = [0, K]$ with $K = 35$ is renormalization-group time, $r = \ln(E_{\rm GUT}/\mu)$. The scale profile e^{-2r} exponentially compresses 4D length scales as one flows from the *UV boundary* (at $r = 0$: the high-energy boundary of the radial interval, where the scale profile is largest and where the finite observer is located) to the *IR boundary* (at $r = K$: the low-energy boundary, where 4D length scales are exponentially compressed and dark-matter or KK-tower physics localizes). In Randall–Sundrum-style language, "UV boundary" and "IR boundary" are the two endpoints of the extra direction, named after the energy regimes that localize near them. The compact direction S^1_θ is a pure topological circle of unit angular extent, supporting integer-quantized Wilson-line phases (gauge holonomies around the angular circle). This is the geometric stage on which the entire GG-Theory program operates; GDOS describes what an observer at $r = 0$ can register from the inaccessible bulk.[1]

4.3 The OGGEP construction (operational entry point)

AX-3 (OGGEP) of the GG-Constitution states that the boundary observer accesses the bulk only through a completely-positive trace-preserving (CPTP) reduction:

$$\mathcal{D}_{\rm OGGEP} \;=\; \mathcal{R} \circ \mathrm{Tr}_{\rm env}, \qquad \mathcal{R} : \mathrm{Tr}_{\rm env}(\mathcal{H}_{\rm Bulk}) \;\to\; \mathcal{H}_{\rm Bdy}, \tag{8}$$

where $\mathrm{Tr}_{\rm env}$ is the partial trace over the inaccessible environment $\mathcal{H}_{\rm env}$ and $\mathcal{R}$ is a boundary reduction map from boundary-supported data to the four-dimensional state space. The Stinespring dilation theorem (Stinespring, 1955) guarantees that any CPTP map can be written in this form; the present paper shows that the geometry of GG-6 selects $\mathcal{R}$ uniquely, and that the resulting boundary dynamics is the law we call GDOS.

Why OGGEP is necessary, not optional. A reader trained in standard quantum field theory may object that OGGEP looks like a stronger commitment than required: cannot the closed Lagrangian and the unitary S-matrix carry the entire content of boundary physics? The answer is no. Every

laboratory measurement in physics is a registered record on a finite boundary apparatus; the bulk environment that supplied the measurement statistics is inaccessible by construction. The closed Lagrangian and unitary S-matrix are special-case limits of (8), obtained when the environment is exactly known and exactly returnable. In every other case — which is every real case — the boundary law is CPTP, not unitary, and the projection from bulk to boundary is the fundamental operation. This is the inversion that GDOS executes and that the remainder of the present paper develops.

5 Strategy of the Derivation

The derivation chain of the present paper has six logical steps, each of which will be executed as a Principal Theorem in a later Part. Roughly speaking, we shall walk down a ladder: from the constitutional axiom OGGEP at the top, through five technical moves, to the canonical boundary law GDOS at the bottom. The present section states the six steps in compact form so that the reader can hold the entire derivation in mind before any step is executed in detail.

Step 1 (Operational Necessity, Theorem T1, Part II). OGGEP together with POS-7 forces the existence of the dictionary $\mathcal{D}_{\mathsf{OGGEP}}$. In particular, no closed Hamiltonian theory on the GG-6 bulk that omits $\mathcal{D}_{\mathsf{OGGEP}}$ is admissible.

Step 2 (Bulk-to-Boundary Reduction, Theorem T2, Part II). $\mathcal{D}_{\mathsf{OGGEP}}$ is constructed explicitly via the Nakajima–Zwanzig (Nakajima, 1958, Zwanzig, 1960) projection technique. The exact reduced boundary state evolves under a memory-kernel equation

$$\partial_t \mathcal{P}\rho(t) \;=\; \mathcal{P}\mathcal{L}\mathcal{P}\rho(t) \;+\; \int_{t_0}^{t} ds\, K(t,s)\,\mathcal{P}\rho(s) \;+\; \mathcal{I}(t), \tag{9}$$

with explicit forms for the projector $\mathcal{P}$, the memory kernel $K(t,s)$, and the inhomogeneous initial-correlation term $\mathcal{I}(t)$.

Step 3 (CPTP Channel, Theorem T3, Part II). The reduced channel $\Phi_{t,s}$ is automatically CPTP, by Stinespring dilation (Stinespring, 1955) together with the unitarity of the bulk evolution and the partial-trace structure of $\mathcal{D}_{\mathsf{OGGEP}}$. Equivalently, $\Phi_{t,s}$ admits a Kraus decomposition $\Phi_{t,s}(\rho) = \sum_\alpha K_\alpha \rho K_\alpha^\dagger$ with $\sum_\alpha K_\alpha^\dagger K_\alpha = \mathbf{1}_{\mathrm{Bdy}}$.

Step 4 (GKLS Generator, Theorem T4, Part II). In the Markovian limit (short environment correlation time relative to boundary timescales), the reduced channel forms a one-parameter semigroup whose generator must take the Gorini–Kossakowski–Lindblad–Sudarshan form (Gorini, 1976, Lindblad, 1976)

$$\mathcal{L}_{\mathsf{GDOS}}(\rho) \;=\; -i[H_{\mathrm{eff}},\rho] \;+\; \sum_\alpha \gamma_\alpha \Big(L_\alpha \rho L_\alpha^\dagger \;-\; \tfrac{1}{2}\,\{L_\alpha^\dagger L_\alpha, \rho\}\Big), \quad \gamma_\alpha \geq 0. \tag{10}$$

Step 5 (GDOS Continuity Equation, Theorem T5, Part II). Mesoscopic coarse-graining produces the canonical GDOS continuity equation

$$\partial_t p \;=\; -\nabla_A J^A, \qquad J^A \;=\; -D^{AB}\nabla_B p \;+\; \left(-g^{AB}\nabla_B \Phi + h^A\right) p, \quad \nabla_A h^A = 0, \tag{11}$$

exhibiting the universal gradient-plus-curl-plus-noise current decomposition.

Step 6 (GDOS as the Canonical Boundary Law, Theorem T6, Part II). Steps 1–5 unify into the single statement: GDOS, in the equivalent operator/GKLS, mesoscopic/continuity, and classical/Langevin formulations, is the canonical constitutionally admissible boundary law of any GG-6-compatible bulk theory.

The three principal descent theorems (Parts III, IV, V). Once the boundary law is in hand, the textbook content of 4D physics is derived by three principal descent theorems. Theorem T7 (Part III, seven sub-theorems) derives 4D quantum mechanics — including the Born rule, the Lüders rule, decoherence, einselection, and Bell-inequality violations — as boundary projections of bulk unitary evolution under OGGEP. Theorem T8 (Part IV, seven sub-theorems) derives 4D general relativity from the metric zero mode of the bulk, with explicit overlap-integral formulas for the Planck mass, the cosmological constant, and the modified Tolman–Oppenheimer–Volkoff equation. Theorem T9 (Part V, nine sub-theorems) derives 4D quantum field theory from heavy-mode integration of the anomaly-free 6D QFT, including the inheritance of BRST symmetry, the Slavnov–Taylor identity at the boundary, anomaly inflow from BI-6, and proton stability from gauged $\mathrm{U}(1)_B$.

The two bridge theorems (Part VI). Two bridge theorems then connect the present paper to its two companion papers in the GDOS Trio. Theorem T10 (GDOS-to-Gi-4) shows that the four *Geometric Invariance* pillars (TGG, CCI, GFF, GRIP) of GG-A6-GRIP are derived structures of GDOS together with the GG-6 bulk. Theorem T11 (GDOS-to-Qi-4) shows that the four *Quantum Information* pillars (Qi-QUA, Qi-EP, Qi-CON, Qi-HAL) of GG-A7-Qi4 are derived structures of GDOS together with the $\mathrm{U}(1)_{Qi}$ informational gauge factor.

The geometric pictures, the innovations, and the falsifiers (Parts IX–X). Part IX develops the geometric intuition of the descent in ten complementary pictures — bulk-to-boundary projection, information loss as geometric distance, decoherence as KK-mode integration, GDOS current geometry, the synthesis of the first four into a single conceptual map, and five further pictures drawing the constitutional and comparative structure. Part X (Discussion) then summarizes the framework's significance and uniqueness in Ten Innovations and states the empirical falsifiers under POS-7.

The dependency lattice. Table 1 displays the full dependency lattice of the eleven Principal Theorems together with their sub-theorems. The reader who wishes to see the entire derivation chain at a glance before reading the detailed proofs is encouraged to consult this table.

Table 1: The dependency lattice of the eleven Principal Theorems of the present paper. Each Theorem T*.k is listed together with its constitutional input (which axiom or postulate it invokes), its derivational input (which prior theorem it builds on), and its forward bridge (which downstream companion paper it connects to).

Theorem	Statement	Inputs	Forward bridge
T1	Operational Necessity	AX-3, POS-7	—
T2	NZ Reduction	T1, AX-1	—
T3	CPTP Channel	T2, POS-4	Stinespring
T4	GKLS Generator	T3, Markov	—
T5	GDOS Continuity	T4, Kramers–Moyal	—
T6	GDOS Canonical	T1–T5	—
T7.1–T7.7	4D Quantum Mechanics	T1–T6, bulk unitary	Qi-CON (Pillar III of Qi-4)
T8.1–T8.7	4D General Relativity	GG-6 metric zero-mode + T1	TGG (Pillar I of Gi-4)
T9.1–T9.9	4D Quantum Field Theory	GG-6 BRST + T1 + heavy-mode integration	CCI, GFF (Pillars II, III of Gi-4)
T10	GDOS-to-Gi-4 Bridge	T6, T8.3, T9.5	All 28 STs of GG-A6-GRIP
T11	GDOS-to-Qi-4 Bridge	T6, $U(1)_{Qi}$	All four pillars of GG-A7-Qi4

6 Why Openness Is Constitutional, Not Corrective: the Paradigm Shift at the Heart of GDOS

6.1 Thesis

The central thesis of this paper, and indeed of the entire GG-Theory program, is one sentence:

> **Every system in nature is an open system. Closure is an idealization. There are no isolated systems anywhere in the universe.**

This sentence is at once obvious and revolutionary. Obvious, because every practicing physicist who has ever calibrated an experiment knows that electromagnetic fields leak out, that thermal photons leak in, that gravitational tidal couplings are never exactly zero, that vacuum fluctuations are never exactly suppressed. Revolutionary, because no formulation of fundamental physics in three and a half centuries has taken this fact and written it into the foundations. Every fundamental theory we have inherited — Newtonian mechanics (Newton, 1687), Hamiltonian mechanics (Hamilton, 1834), Maxwellian electrodynamics (Maxwell, 1865), Einsteinian general relativity (A. Einstein, 1915), von Neumann quantum mechanics (Neumann, 1932), even the Standard Model quantum field theory — begins by writing down a closed Lagrangian or a closed Hamiltonian on an isolated phase space, and only afterwards introduces openness as a corrective patch when it becomes empirically unavoidable. Indeed, openness has been treated as the exception demanding repair, never as the rule from which closure must descend as a special limit. And the thesis is universal, not disciplinary: it is as true of a qubit in a dilution refrigerator as of a neutron star, as true of a living cell as of the universe at last scattering. Openness is not a laboratory circumstance to be engineered away; it is the constitutional condition of every finite observer, and the closed system is the idealization that never occurs as an instance.

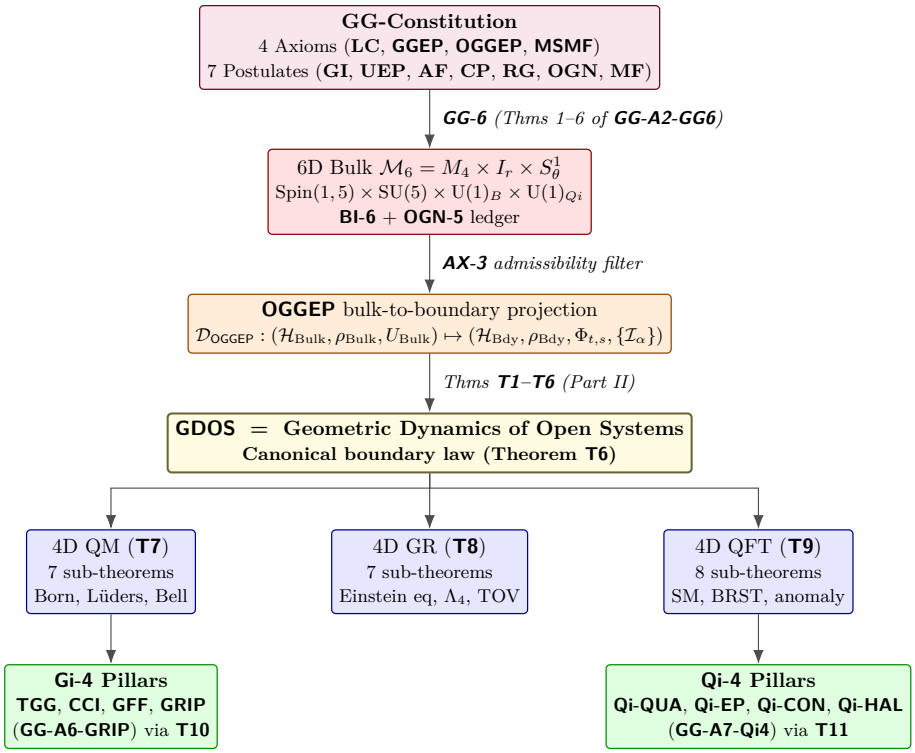


Figure 3: The canonical GG-Theory implication chain. The four axioms and seven postulates of the GG-Constitution (Arisaka, 2026, A1-Constitution) force, via Theorems 1–6 of Arisaka (2026, A2-GG6), the unique 6D bulk $\mathcal{M}_6$ with composite gauge group, BI-6 anomaly polynomial, and OGN-5 integer ledger. AX-3 (OGGEP) acts as the admissibility filter that demands a bulk–boundary correspondence $\mathcal{D}_{\mathrm{OGGEP}}$ from inaccessible bulk data to registered boundary observables; Theorems T1–T6 of Part II construct this dictionary explicitly and identify it with the canonical boundary law GDOS. The three principal descent theorems T7, T8, T9 of Parts III–V then derive, respectively, four-dimensional quantum mechanics, general relativity, and quantum field theory as boundary specializations of GDOS. The bridge theorems T10 and T11 of Part VII identify the eight pillars (Gi-4 + Qi-4) of the companion papers GG-A6-GRIP and GG-A7-Qi4 as derived structures of GDOS. Every box in the diagram is a theorem; only the topmost box (the Constitution) is primitive input.

The patches have grown elaborate over the centuries. Boltzmann (Boltzmann, 1872) had to wrestle the second law of thermodynamics out of a manifestly time-reversible closed Hamiltonian by introducing coarse-graining and the Stosszahlansatz. Von Neumann (Neumann, 1932) had to bolt a separate "measurement postulate" onto the manifestly unitary Schrödinger equation (Schrödinger, 1926) to account for outcome registration. Bohr (Bohr, 1928) had to draw a movable but unavoidable "Heisenberg (Heisenberg, 1927) cut" between a classical observer and a quantum object to make any prediction at all. Decoherence theorists in the 1970s through the 2000s — led by Zeh (1970), Joos (1985), Zurek (1981), Zurek (2003) — finally said openly: the collapse-versus-unitarity tension is resolved by recognizing that no real quantum system is isolated; environmental decoherence chooses the pointer basis, suppresses the off-diagonal elements, and produces classicality. This was the right answer, and decoherence theory deserves the credit. But decoherence theory remained, in the classical sense of Kuhn, a *normal-science* resolution: it patched the closed-system formalism

rather than replacing it. The closed-system Schrödinger equation remained the constitutional law; decoherence was a mechanism layered on top of it.

GDOS takes the next step that decoherence theory pointed at but did not take. It promotes openness from a corrective patch to a *constitutional axiom*, OGGEP. Closed-system dynamics is recovered, when it is recovered at all, as a degenerate limit in which the system–bulk coupling is set to zero — a limit that is mathematically well-defined but physically never realized in any actual experiment. This is by no means an interpretive preference; it is a mathematical inversion of which postulate is primary and which is derived, and we shall see its consequences ripple through every subsequent theorem of this paper.

6.2 The historical chain that led here

Let us now trace the historical chain that brings physics to this axiomatic inversion, because the inversion looks bolder once one sees how hard physics has worked, for three and a half centuries, to avoid making it. The avoidance is itself part of the story.

The closed-system tradition (1687–2003). The historical chain leading to the present axiomatic inversion runs from Newton's closed phase space (1687) through Lagrange (1788) and Hamilton (1834), Boltzmann's H-theorem (1872) where coarse-graining first leaked an open-system ingredient into a closed Hamiltonian, Maxwell's implicitly open field theory (1865), Einstein's closed-universe general relativity (1915), the closed wavefunction of Heisenberg–Schrödinger–Born (1925–26), Bohr's Copenhagen position (1928), Bell's locality crisis (1964), and the projection-and-influence school of Nakajima (1958), Zwanzig (1960), Feynman–Vernon (1963), and Davies–Lewis (1970); the GKLS classification of Lindblad, Gorini–Kossakowski–Sudarshan, Choi, and Kraus (1971–76); the Caldeira–Leggett worked-out model and the Breuer–Petruccione synthesis (1981–2002); the Stinespring dilation theorem (1955); and the decoherence revolution of Zeh, Joos, and Zurek (1970–2003). The full historical inventory is recorded in Arisaka (2026, A4-Euclid) (constitutional lineage) and Arisaka (2026, A3-BI6) (Stinespring–GKLS strand and the open-system reading of Green–Schwarz as pre-BI-6). In every case the closed-system formalism was retained as the foundational law while open-system structure was inserted as a corrective mechanism.

But the decoherence revolution stopped one axiomatic step short. Zurek (Zurek, 2003) explicitly punts on the Born rule: he treats the rule as a postulate that must be added on top of the unitary + decoherence framework, even when no formal proof of the rule from envariance arguments has commanded full community consensus. Zeh and the broader decoherence community treat openness as a *mechanism* (a way of explaining classicality from a fundamentally unitary substrate) rather than as a *constitution* (a primitive law from which unitarity itself is the limit). The closed-system Schrödinger equation remained the foundational law in their framework; decoherence was a tool for interpreting its outputs.

GDOS takes the next axiomatic step: it inverts which is foundational. The CPTP boundary channel $\mathcal{D}_{\mathsf{OGGEP}}$ is the primary law (Theorem T6); the Schrödinger equation is its closed-system limit (Theorem T7.2); the Born rule is a theorem of CPTP completion (Theorem T7.3); the Lüders projection is the unique conditional state update under CPTP (Theorem T7.4). The constitutional

order is reversed: openness is no longer the patch on a closed law; closure is the limit of an open law.

6.3 What the inversion buys — four consequences

Remarkably, the axiomatic inversion has four immediate consequences that distinguish GDOS from every prior framework, including the Zeh–Zurek decoherence theory. We list them in order of increasing structural reach:

1. the wavefunction is no longer fundamental;
2. the Born rule is no longer a postulate;
3. the collapse postulate is no longer a postulate;
4. 4D classical spacetime is what an open observer sees of a 6D bulk.

Each of these is elaborated below.

Consequence 1: The wavefunction is not fundamental. Under OGGEP, the fundamental dynamical object is the boundary density operator ρ_{Bdy}, not the wavefunction $|\psi\rangle$. The wavefunction is recovered, when it is recovered at all, as a particular limit ($\rho_{\mathrm{Bdy}} = |\psi\rangle\langle\psi|$ with no system–bulk coupling). This is not interpretation: it is the structural consequence of the fact that any realistic boundary system is open, so its state cannot in general be a pure state on $\mathcal{H}_{\mathrm{Bdy}}$ alone. Theorem T7.2 will show that the Schrödinger equation is the closed-system limit of the GKLS generator, not the other way around. This goes beyond Zeh–Zurek, who retain the closed wavefunction as the fundamental object and treat decoherence as a mechanism that selects which basis the closed wavefunction's superpositions become diagonal in.

Consequence 2: The Born rule is not a postulate. Under OGGEP, the Born rule is a derived consequence of the Stinespring dilation theorem applied to the system–apparatus pointer-selection map. Theorem T7.3 will show that $p_\alpha = \mathrm{Tr}(\mathcal{I}_\alpha(\rho_{\mathrm{Bdy}}))$ follows from the CPTP-completion structure of $\mathcal{D}_{\mathsf{OGGEP}}$, not from an additional axiom. Equivalently, Gleason's theorem (Gleason, 1957) is recovered as a constitutionally-forced consequence rather than as a separate representation-theoretic miracle. This goes beyond Zurek's envariance-derivation program, which assumes the Born rule's L^2 structure on a closed wavefunction; under OGGEP the L^2 structure is itself a consequence of CPTP-completion of an open system.

Consequence 3: The collapse postulate is not a postulate. Under OGGEP, the Lüders projection rule $\rho^{(\alpha)}_{\mathrm{Bdy}} = \mathcal{I}_\alpha(\rho_{\mathrm{Bdy}})/p_\alpha$ is the unique CPTP state update consistent with outcome registration. Theorem T7.4 will show that no separate "collapse postulate" is required; what is conventionally called collapse is just the conditional CPTP state update of an open system upon registering a classical outcome. This dissolves the von Neumann measurement problem rather than postponing it. No Heisenberg cut needs to be drawn anywhere, because the constitutional law is already a measurement law.

Consequence 4: 4D classical spacetime is what an open observer sees. This is the consequence Zeh–Zurek do not have, and the consequence that makes GDOS inseparable from GG-6. Under OGGEP, the "environment" that an open boundary observer cannot access is not an arbitrarily chosen Hilbert-space partner; it is the inaccessible bulk modes of a definite geometric object, the six-dimensional manifold $\mathcal{M}_6$ of GG-6 (Arisaka, 2026, A2-GG6). The boundary CPTP channel is then literally a bulk-to-boundary projection from a 6D bulk to a 4D boundary, and the four-dimensionality of accessible spacetime is the operational signature of the bulk's two extra bulk dimensions being inaccessible to finite registration. Classical 4D spacetime, in this view, is not a metaphysical primitive; it is a theorem about what an open observer can register from a 6D bulk. This is what makes the entire GG-6 ultraviolet construction *natural* rather than *ad hoc*: once openness is constitutional, the bulk's hidden dimensions become not a peculiarity but a necessity — the geometric content of the environment that the observer is open to.

6.4 Position relative to decoherence theory and the historical pattern (deferred to Part IX and Part X)

A sharp statement of the position of GDOS relative to the Zeh–Zurek decoherence program, together with the historical pattern of foundational reformulations (Lagrange–Hamilton, Heaviside (Heaviside, 1893)–Gibbs, Ricci (Ricci, 1900)–Levi-Civita, von Neumann), is taken up later: see Part IX §94 for the position relative to decoherence, and Part X §107 (Revolutions 1–4) for the historical pattern. The single-line summary is: decoherence theory says the environment exists and its existence explains collapse; GDOS + GG-6 says the environment is itself a geometry, and its geometric structure is the constitutional source of all 4D physics.

6.5 The fiber without holonomy: the geometric root of the thesis

The thesis of §6.1 — every system is open, closure is an idealization — can be given a geometric root in one exact topological statement, and stating it here sharpens everything Part II will prove. *Holonomy requires closed loops, and an interval has none.* The bulk's two non-spatial directions are not symmetric partners: the compact circle S^1_θ is nothing but closed loops, and everything it carries is holonomy — Wilson phases, integer windings, the quantized charges of the gauge sector — gauge-invariant, rephasing-proof, topologically protected. The radial interval $I_r = [0, K]$ is the opposite object: no loop on an interval ever closes, so the fiber can hold *no* holonomy at all — no gauge-invariant phase memory, no quantized charge, no protected content of any kind. Transport along it therefore cannot be a group action (there is nothing invertible to return to); it can only compose forward. A one-parameter family of forward-only, positivity-preserving maps is a semigroup, and Part II's theorems (§13) show that the generator of such a semigroup must take the GKLS form. Read in this order, the dissipator is not an added phenomenology: it is what transport looks like along a fiber that is topologically incapable of remembering.

The pairing of the two fibers is then the anatomy of the thesis. The circle *remembers*: it quantizes, conserves, and protects, which is why the boundary inherits exact charges and coherent phases — the Aharonov–Bohm/Berry content of the unitary sector. The interval *forgets*: it decoheres, dissipates, and gates, which is why the boundary inherits an arrow — the RG's and time's, unified

— and why every registered evolution is CPTP rather than unitary. A finite observer at $r = 0$ marginalizes both fibers; the marginalization of the circle costs phase information that interference can in principle recover, while the marginalization of the interval costs information that nothing can recover, because the fiber itself never held it in recoverable form. *That* is the constitutional sense in which no system is closed: the boundary of this bulk could not have been closed without deleting one of the two directions the constitution requires.

Two counterfactuals fix the point. A bulk with two compact circles would be all memory and no arrow: reversible, forever oscillating, its boundary unitary and its thermodynamics empty — nothing would ever be decided. A bulk with two intervals would be all arrow and no memory: a featureless slide with no quantized charges, no protected phases, nothing to decide *into*. The $I_r \times S^1_\theta$ pairing is the minimal geometry in which a boundary can both *become* something (the semigroup) and *be* something definite (the holonomies) — the geometric reason the open-system law of this paper and the exact conservation laws of its companions are not in tension but are the two fibers of one bulk, read separately. The full deployment of this observation — symmetry breaking as the one-way reading of holonomy data, with Sakharov's conditions as the anatomy of the two fibers — is carried by the cohort's symmetry review, GG-W1-Symmetry; this subsection records the root, because the root is this paper's: the fiber without holonomy is the GDOS generator's birthplace.

Bridge to Part II. The axiomatic layer is now stated; the postulate layer is in place; the 6D bulk inherited from GG-6 is recapped; the derivation strategy is named; and the central paradigm shift that underwrites the entire program — the elevation of openness from corrective patch to foundational law — has been set on the page. What remains is to execute the chain. Part II takes the first step, proving Theorem T1: an admissible bulk theory must furnish its own operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$, or it is not an admissible theory at all. Once that foothold is secured, Theorems T2 through T6 follow as a single descending ladder, and the canonical boundary law GDOS stands at the bottom of it.

7 The cohort inheritance and the roadmap of the fourteen Parts

Before the six core theorems begin (Part II), it is useful to lay out two reader-orientation maps on the same opening: a panorama of the twenty-two-paper cohort in which this paper stands (§7.1, Figure 4), and a panorama of the fourteen Parts of the present paper (§7.2, Figure 5). A reader of any later Part should be able to flip back to this Section and orient themselves in both directions: which cohort companion carries which imported result, and where the present argument stands in the fourteen-Part arc.

7.1 The cohort inheritance

The present paper is one of twenty-two. GG-Theory is written not as a single monograph but as a cohort of twenty-two preprints — eleven A-papers, six P-papers, three C-papers and two white papers — each carrying one rigorously stated piece of a single descent that runs from four axioms and seven postulates to the empirical structure of physics. The partition into twenty-two papers

reflects the partition of subject matter, not a ranking of importance: the framework is one program, and each paper is a named carrier of named content. Figure 4 is the panorama; a reader meeting the program for the first time can read its architecture off the figure top to bottom before returning to the boundary law that is the business of this paper. The cohort falls into five strata plus three framed branches.

The constitutional root and the GG6 Trio. At the root stands GG-A1-Constitution (Arisaka, 2026, A1-Constitution), the GG-Constitution: the four axioms (AX-1 LC, AX-2 GGEP, AX-3 OGGEP, AX-4 MSMF) and seven postulates against which every companion's theorems are audited. Forced from it is the GG6 Trio that builds the bulk: GG-A2-GG6 (Arisaka, 2026, A2-GG6) constructs the six-dimensional Geometry–Gauge bulk; GG-A3-BI6 (Arisaka, 2026, A3-BI6) proves the modified Bianchi identity BI-6 anomaly-free; and GG-A4-Euclid (Arisaka, 2026, A4-Euclid) traces the historical lineage from Euclid to the bulk and reconstructs the universal constants.

The boundary law and its two pillars. AX-3 (OGGEP) forces the open-system boundary law GG-A5-GDOS (Arisaka, 2026, A5-GDOS) — the present paper: the Geometric Dynamics of Open Systems that projects bulk physics onto the four-dimensional boundary and recovers four-dimensional quantum mechanics, general relativity, and quantum field theory as theorems of that projection. Two sibling pillar papers organize the boundary content: GG-A6-GRIP (Arisaka, 2026, A6-GRIP), the geometric Gi-4 pillars, and GG-A7-Qi4 (Arisaka, 2026, A7-Qi4), the quantum-information Qi-4 pillars, with which GG-A5-GDOS closes the GDOS Trio through the bridge theorems T10 and T11.

The Consequence Trio and the capstone. Three papers read the empirical world off the boundary law: GG-A8-QM (Arisaka, 2026, A8-QM) recasts four-dimensional quantum mechanics, GG-A9-SM (Arisaka, 2026, A9-SM) derives the twenty-eight free parameters of the Standard Model, and GG-A10-Cosmos (Arisaka, 2026, A10-Cosmos) derives the ΛCDM cosmological inventory ($\Omega_m = 1/\pi$ and its partners). The capstone GG-A11-Origin (Arisaka, 2026, A11-Origin) closes the cohort with the recursive-uniqueness argument and the cosmos–life–mind chain.

The two phenomenology branches. Beneath the A-series, two framed branches carry the sharp falsifiers. The Predictions Branch — GG-P1-KK (Arisaka, 2026, P1-KK) (the Kaluza–Klein lock cluster near $M_{\text{lock}} \approx 3.85$ TeV), GG-P2-N3 (Arisaka, 2026, P2-N3) (the three-generation pincer that forces $N_g = 3$), and GG-P3-Electron (Arisaka, 2026, P3-Electron) (the electron as the ground state of nature), and GG-P6-Proton (Arisaka, 2026, P6-Proton) (the proton as the ground geometric attractor of matter, its decay forbidden by the geometry rather than merely unobserved) — supplies collider and structural predictions. The Cosmology and Astrophysics Branch — GG-C1-Kaxion (Arisaka, 2026, C1-Kaxion) (the Kaxion dark-matter candidate near 8.1 μeV, one line in a closed haloscope window), GG-C2-Hubble (Arisaka, 2026, C2-Hubble) (the Hubble tension as one geometric corridor), and GG-C3-BH (Arisaka, 2026, C3-BH) (black-hole entropy as an OGGEP port) — supplies the astrophysical signatures.

The Foundations Branch. The third framed branch is the Foundations Branch: the two member papers whose subject is the boundary's complex structure and its mirrors. GG-P4-Real (Arisaka, 2026, P4-Real) stands on the map's left flank as the origin-of-i member, deriving the single imaginary unit of boundary physics from two real structures of the GG-A2-GG6 arena — the compact phase circle and the causal clock. For the present paper its significance is direct: the boundary law derived here is what hands the observer a complex Hilbert space, and the GNS scope note of §20 already flags "why *complex*" as a question distinct from "why a Hilbert space" — the dedicated treatment of that question now stands inside the cohort as a Foundations Branch member. GG-P5-CPT (Arisaka, 2026, P5-CPT) carries the three mirrors of matter — C, P, and T read as orientation reversals of the three real bulk factors — and its law-level derivation of the arrow of time is downstream kin of the AX-3/OGGEP irreversibility that the present paper implements dynamically. Both carry P-series designators; their place at the spine's flank, not the phenomenology rows, marks their foundational role.

The White-Paper Branch. The right flank mirrors the left and does the opposite job: where the Foundations Branch feeds the spine, the White-Paper Branch reads it. GG-W1-Symmetry (Arisaka, 2026, W1-Symmetry) is the diachronic one — it walks the century of symmetry and conservation in the order it was found and shows the conviction that symmetry causes conservation loosening at seven historical steps, until what remains conserved is an integer with nowhere to move. GG-W2-Holonomy (Arisaka, 2026, W2-Holonomy) is the synchronic one — it takes one exact statement, that a holonomy requires a closed loop and the radial interval has none, so the circle remembers and the interval forgets, and follows it to sixteen questions physics has not closed. Neither derives anything new: both carry every result at its parent's declared grade, which is what makes them white papers rather than a third phenomenology branch.

Where this paper sits. GG-A5-GDOS is the boundary law of the cohort — the third descent step, forced by AX-3 from the bulk that GG-A2-GG6 and GG-A3-BI6 construct. Everything downstream inherits from it: the pillars of GG-A6-GRIP and GG-A7-Qi4 organize its content, the textbook readouts of GG-A8-QM–GG-A10-Cosmos are its boundary projections, and the capstone GG-A11-Origin closes the chain it opens. GG-A5-GDOS imports the bulk geometry and the integer ledger from upstream and the empirical numbers from GG-A8-QM–GG-A10-Cosmos, citing each at the point of use; its own contribution is the open-system boundary law itself — the single GDOS projection from which quantum measurement, gravity, and field theory descend (§64). The twenty-two-paper map, identical in every paper of the cohort, is Figure 4, which closes this subsection.

7.2 Roadmap of the fourteen Parts

The rest of the paper is organized into thirteen further Parts, and the walk through them is plotted in Figure 5. The present Part — the foundations — is nearly done: the constitutional question and the operational gap, the definitions and the admissibility predicate, the primitive inputs imported from the companion papers, the paradigm shift that makes openness constitutional rather than corrective, the strategy of the derivation, and the cohort the paper stands in. Everything

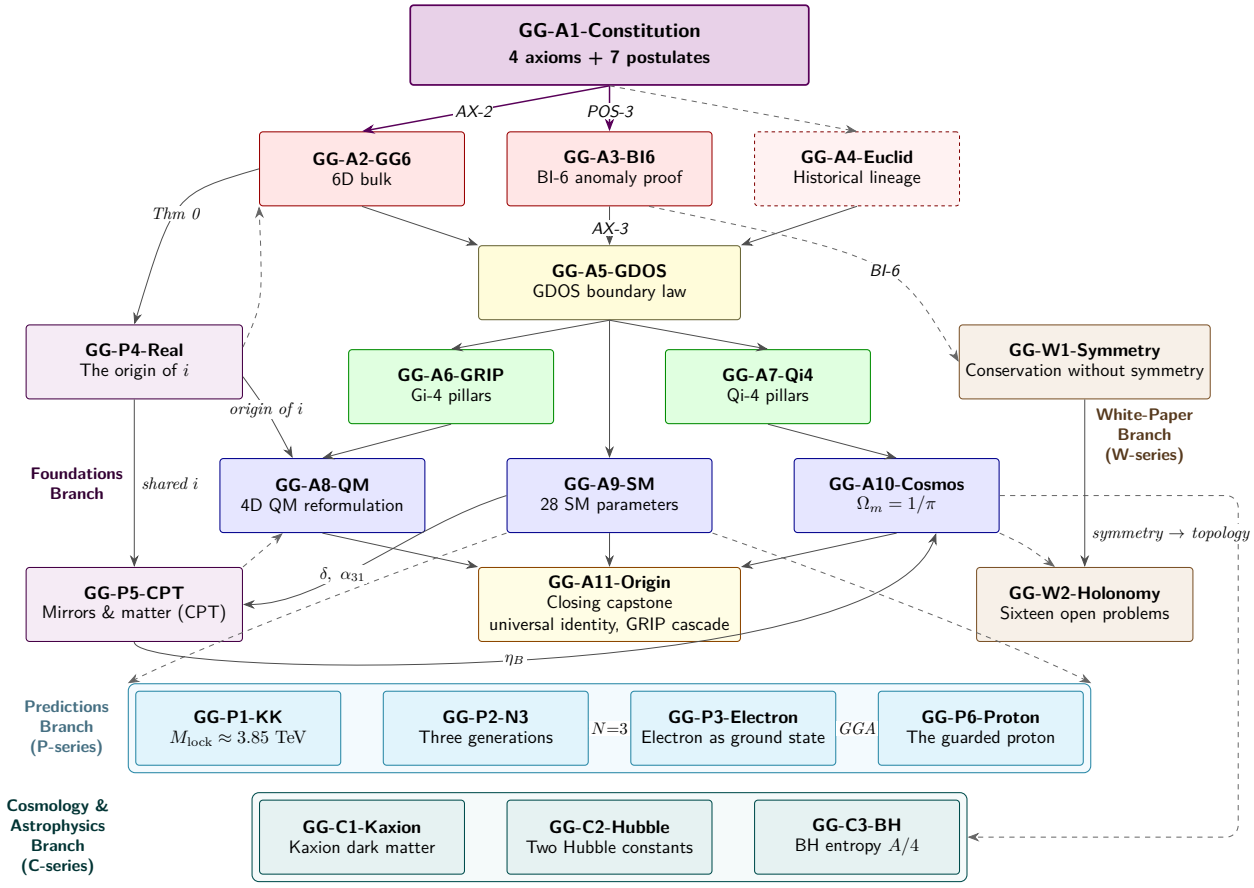


Figure 4: The twenty-two-paper cohort map of GG-Theory. The constitutional root GG-A1-Constitution (violet, top) supplies the four axioms and seven postulates from which the rest of the cohort descends: the GG6 Trio (GG-A2-GG6, GG-A3-BI6, GG-A4-Euclid, red) realizes the bulk geometry; GG-A5-GDOS (yellow) the boundary law; the GDOS Trio pillars (GG-A6-GRIP, GG-A7-Qi4, green) the geometric and quantum-information runtime; the 4D-Physics Trio (GG-A8-QM, GG-A9-SM, GG-A10-Cosmos, blue) the textbook readouts; and the closing capstone GG-A11-Origin (gold) the quarks-to-consciousness identity cascade. Two framed branches carry the phenomenology — the **Predictions Branch** (cyan), whose four papers hand their results along the row by the internal arrows (GG-P2-N3's N_g=3 into GG-P3-Electron, and that paper's ground-attractor method, GGA, on into GG-P6-Proton), and the **Cosmology & Astrophysics Branch** (teal). The spine is flanked by two columns of two, which do opposite jobs. On the left the **Foundations Branch** (violet) feeds it: GG-P4-Real, fed by GG-A2-GG6's Theorem 0 arena, manufactures the i that GG-A8-QM consumes, and GG-P5-CPT is the GG-A9-SM–GG-A10-Cosmos interface, fed by the ledger phases and feeding the η_B chain across the width of the map, with the shared-i edge descending solid between them. On the right the **White-Paper Branch** (brown) reads it: GG-W1-Symmetry walks the century from Noether to Hopkins–Singer, and GG-W2-Holonomy dissects the structure that arc arrives at — protection by topology rather than by symmetry — across sixteen open problems. The white papers derive nothing new, carrying every result at its parent's declared grade; the single dashed feed drawn into each names its heaviest source, not its only one. *Reading conventions:* solid arrows mark axiom-forced descent and carry an edge label naming the driving axiom or postulate where most illustrative; dashed arrows mark the deployment of derived results, or historical links inherited without being constitutionally forced.

after this page is the derivation itself, and the itinerary below says where each piece lives.

Part II is the heart, and it is highlighted in the roadmap because every later result of the paper stands on it. There the boundary law GDOS is derived from the open-system axiom OGGEP in six core theorems T1–T6: operational necessity, the Nakajima–Zwanzig reduction, the CPTP channel by Stinespring dilation, the GKLS generator in the Markovian limit, the GDOS continuity equation, and the canonicity statement that unifies the five moves into one law — the single result on which every downstream projection depends.

Three Parts then project that law onto the textbook. Part III derives four-dimensional quantum mechanics — the measurement cut, the einselected pointer basis, and the Born weight — in seven sub-theorems T7.1–T7.7, dissolving the century-old measurement problem. Part IV derives four-dimensional general relativity from the metric zero mode of the bulk, with the Einstein equations as the gravitational sector, in seven sub-theorems T8.1–T8.7. Part V derives four-dimensional quantum field theory — the gauge sector, its chiral content, and its anomaly freedom — in nine sub-theorems T9.1–T9.9.

Four Parts then widen the view. Part VI exhibits Newton's law and Coulomb's law as boundary projections of one composite connection with a shared radiative drift; Part VII proves the bridge theorems T10 and T11 that connect the boundary law to the Gi-4 pillars of GG-A6-GRIP and the Qi-4 pillars of GG-A7-Qi4, completing the GDOS Trio. Part VIII places general relativity, gauge theory, superstring theory, holography, quantum mechanics, the Standard Model, and ΛCDM cosmology in the nested axiom tower according to the axiom subset each implicitly uses, and Part IX develops the geometric intuition of the descent in ten complementary pictures.

Two Parts weigh the case. Part X is the discussion: significance and uniqueness, the constitutional reattribution, the three sibling-confirmation paths for AX-3, the reception by each predecessor community, and the consolidated twenty-five-test empirical horizon. Part XI states the conclusions — the constitutional shift, the case for a fourth scientific revolution, and a plain-language close for the reader who has walked the whole arc.

The back matter is the workshop. Part XII collects the complete mathematical machinery in sixteen technical appendices (A–P) — the Nakajima–Zwanzig reduction, the Stinespring dilation, the GKLS master equation, decoherence and einselection, the Kaluza–Klein reduction, BRST inheritance, and the master tables among them. Part XIII answers the fifty-seven common objections a hostile reviewer would raise, in ten categories, with pointers into the paper; and Part XIV closes with the annotated references, each carrying a Relevance note. Read in order, the fourteen Parts compose into a single self-contained derivation that any reader can audit step by step; read by need, each Part names its imports and can stand a visit on its own.

How to use this paper as a course. The volume now carries a full course apparatus: every Part opens with a *What this Part teaches* box, Parts II–V carry *Read alongside* pairings with the standard graduate texts, every principal-theorem section closes with a three-register *Take-away* box (standard account / what it left open / the constitutional derivation) and with problems — ninety-six problems with full worked solutions distributed through Parts II–IX and the technical appendices — and each derivation Part ends in plain English. Appendix P assembles the whole

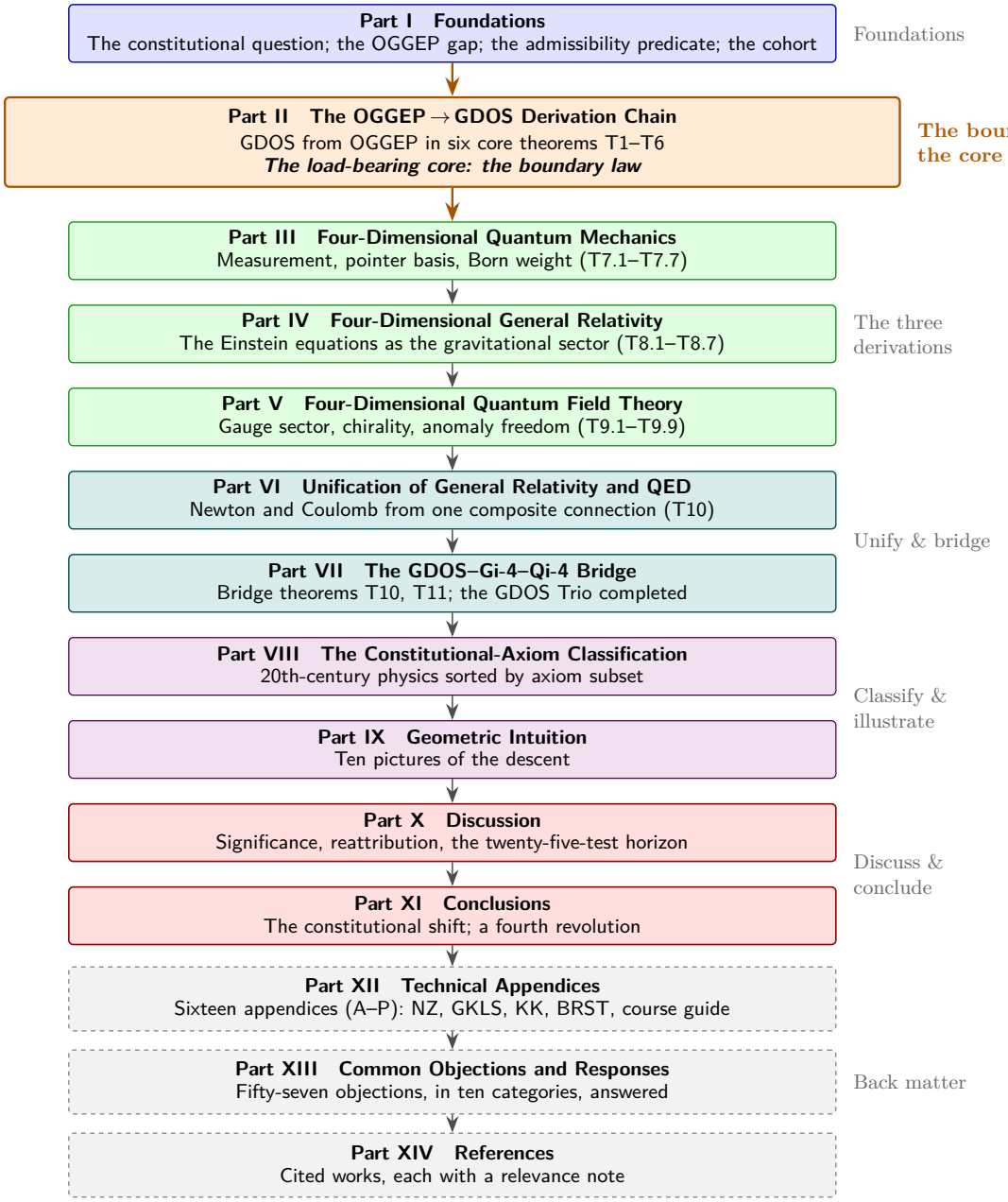


Figure 5: Roadmap of the fourteen Parts of GG-A5-GDOS. The argument develops in six phases. *Foundations* (Part I): the constitutional question, the operational gap, and the cohort. *The boundary law* (**Part II**, highlighted): GDOS derived from OGGEP in the six core theorems T1–T6. *The three derivations* (Parts III–V): quantum mechanics, general relativity, and quantum field theory as boundary projections of the one law. *Unify and bridge* (Parts VI–VII): Newton and Coulomb from one composite connection, and the bridge theorems T10 and T11 that complete the GDOS Trio. *Classify and illustrate* (Parts VIII–IX) place twentieth-century physics in the nested axiom tower and develop the geometric intuition in ten pictures; *discuss and conclude* (Parts X–XI) weigh significance and state the constitutional shift. The three back-matter Parts collect the technical appendices (Part XII), the fifty-seven common objections and responses (Part XIII), and the annotated references (Part XIV). Part II is highlighted because the boundary law derived there is the single result on which every downstream projection depends.

apparatus into three single-term graduate tracks and a full-year sequence. A reader who works the problems in order will have run the boundary law from its operational necessity (T1) to the price of a bit (T11) with every number checked by hand.

Part II
The OGGEP → GDOS Derivation Chain

Six Core Theorems That Force the Open-System Boundary Law

This is the Part the rest of the paper stands on. If it holds, everything downstream is bookkeeping of a demanding kind; if it fails, nothing downstream can be repaired.

The task is to get from a principle to an equation. The principle is that an observer is inside the world, not outside it, and therefore cannot access all of it — so the description available to that observer is necessarily a description of an open system. The equation is the law that such a description must obey. Between the two lies the work of this Part: six theorems, each with a single job, chained so that the output of one is the input of the next.

The chain runs like this, in plain terms. An observer who can only reach part of the world needs a dictionary translating the whole into the part — and the first theorem shows such a dictionary is not optional but forced. The second performs the translation exactly, with nothing discarded, which is possible but leaves an equation with memory in it. The third shows that whatever the memory does, the result always preserves the one property a description of probabilities cannot lose. The fourth takes the limit in which the memory is short and finds the canonical generator of open evolution waiting there. The fifth writes the result as a continuity equation — something flows, and the equation says what. The sixth collects the previous five into a single statement: one law, in three equivalent forms, which any open system compatible with the geometry must satisfy. We call it GDOS.

None of the six steps is new mathematics; several are results of the open-quantum-systems literature, cited where they are used. What is new is that they appear here as a derivation rather than as a toolkit, with each step licensed by a stated constitutional clause instead of by convenience.

Read the six section openings and the dependency table if you want the architecture. Read T2 and T3 in full if you want to find the weak point, because those are the two steps where an assumption could hide.

Read alongside: Breuer (2002) for the open-quantum-systems machinery that this Part derives from the constitution rather than assumes; Gorini (1976) and Lindblad (1976) for the original GKLS papers behind T4; Reed (1980) for the functional-analytic background of the memory-kernel equation of T2.

8 Overview of Part II: How the Six Core Theorems Connect

Throughout Part II, we shall derive the canonical boundary law GDOS from the four axioms and seven postulates of the GG-Theory Constitution (Arisaka, 2026, A1-Constitution), together with the 6D bulk of GG-6 (Arisaka, 2026, A2-GG6). The derivation walks down a six-step ladder, executed below as Theorems T1 through T6. The dependency lattice of the six theorems is summarized in

Table 2; the geometric content of the OGGEP bulk-to-boundary projection that all six theorems implement is shown schematically in Figure 6. The entire content of Part II must be read as the explicit construction of one map: the operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ from inaccessible bulk to registered boundary.

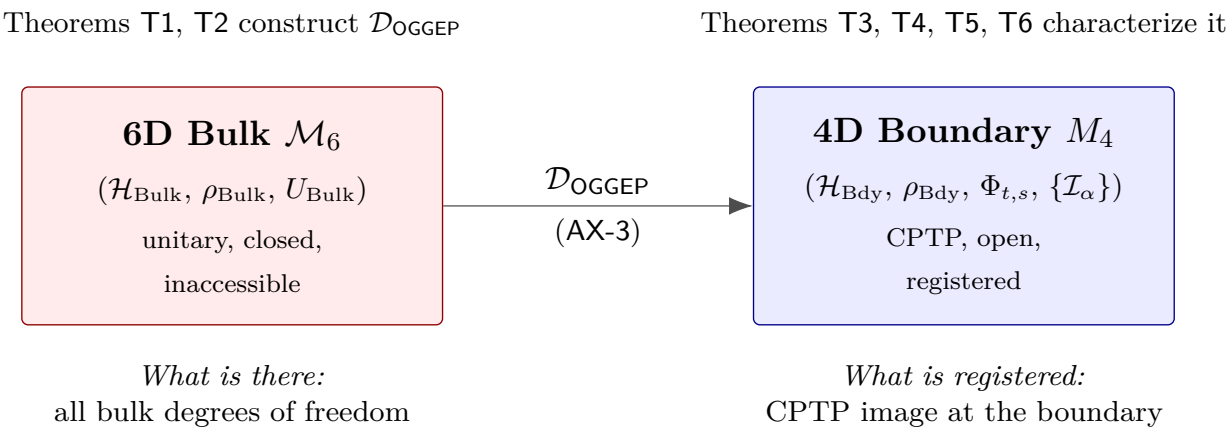


Figure 6: The OGGEP bulk-to-boundary projection $\mathcal{D}_{\mathsf{OGGEP}}$ from the 6D bulk to the 4D boundary. AX-3 of the GG-Constitution (Arisaka, 2026, A1-Constitution) demands that any admissible bulk theory furnish its own operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ mapping inaccessible bulk data $(\mathcal{H}_{\mathrm{Bulk}}, \rho_{\mathrm{Bulk}}, U_{\mathrm{Bulk}})$ to registered boundary observables $(\mathcal{H}_{\mathrm{Bdy}}, \rho_{\mathrm{Bdy}}, \Phi_{t,s}, \{\mathcal{I}_\alpha\})$. The bulk side is unitary, closed, and inaccessible; the boundary side is open, CPTP, and what a finite observer actually registers. The six theorems of Part II construct and characterize this map: T1 (operational necessity) and T2 (Nakajima–Zwanzig construction) on the bulk side; T3 (CPTP via Stinespring), T4 (GKLS Markovian limit), T5 (mesoscopic continuity), T6 (canonical boundary law) on the boundary side. All seven sub-theorems of T7 (4D QM), all seven of T8 (4D GR), and all eight of T9 (4D QFT) in Parts III–V are specializations of this single bulk-to-boundary projection to specific domains of 4D physics.

Table 2: Dependency lattice of the six core theorems of Part II. Each theorem invokes one or two constitutional clauses and builds on the preceding theorem(s). Theorem T6 unifies T1–T5 into the single statement that GDOS is the canonical constitutionally admissible boundary law.

Theorem	Statement (compact)	Constitutional input	Derivational input
T1	Operational Necessity	AX-3 (OGGEP), POS-7 (MF)	—
T2	Exact NZ Reduction	AX-1 (LC)	T1
T3	CPTP Channel (Stinespring)	POS-4 (CP)	T2
T4	GKLS Generator	POS-5 (RG, Markov limit)	T3
T5	GDOS Continuity Equation	POS-2 (UEP, mesoscopic limit)	T4
T6	GDOS as Canonical Boundary Law	T1–T5 unified	T1–T5

The axiomatic flow. T1 forces the existence of the operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ from AX-3 alone: any admissible theory must specify how its bulk degrees of freedom give rise to registered boundary outcomes, on pain of failing the falsifiability postulate POS-7. T2 constructs $\mathcal{D}_{\mathsf{OGGEP}}$

explicitly via the Nakajima–Zwanzig projection technique, yielding the exact boundary memory-kernel equation as a finite-dimensional matrix consequence of bulk unitarity together with the Hilbert-space split $\mathcal{H}_{\mathrm{Bulk}} = \mathcal{H}_{\mathrm{Bdy}} \otimes \mathcal{H}_{\mathrm{env}}$. T3 shows that the resulting reduced channel is automatically completely positive and trace preserving by the Stinespring dilation theorem, satisfying POS-4 without further input. T4 takes the Markovian limit (justified by POS-5, which forces RG-coarse-graining to commute with $\mathcal{D}_{\mathsf{OGGEP}}$) and shows that the reduced channel forms a one-parameter semigroup whose generator must take the Gorini–Kossakowski–Lindblad–Sudarshan (GKLS) form; the fresh-port input this Markov step consumes is stated once, as the Port-Freshness Clause of T2 (Definition 5). T5 takes the mesoscopic coarse-graining limit (justified by POS-2, which requires that the boundary content of any admissible theory be expressible in variational form) and shows that the GKLS generator descends to a continuity equation with universal gradient-plus-curl-plus-noise current decomposition. T6 unifies T1–T5 into the single statement that GDOS is the canonical constitutionally admissible boundary law of any GG-6-compatible bulk theory.

Why six steps and not five or seven. The reader familiar with the open-system literature (Breuer, 2002, Gorini, 1976, Lindblad, 1976) may wonder why the derivation requires six theorems rather than the two or three usually cited (Lindblad's original derivation (Lindblad, 1976), the Gorini–Kossakowski–Sudarshan derivation in finite dimension (Gorini, 1976), or the Davies derivation (Davies, 1974)). The answer is structural: in the standard literature the existence of an open-system reduction is assumed, the CPTP property is assumed, and the Markov limit is taken as a working approximation. In the axiomatic setting of GG-Theory, each of these is derived from a specific clause of the GG-Constitution. T1 derives the existence of the reduction (which standard treatments assume); T3 derives the CPTP property (which standard treatments assume); T4 derives the Markov limit under the explicit RG-coarse-graining condition POS-5 (which standard treatments treat as a working approximation). Hence what looks like a longer derivation is in fact a more constitutionally honest one: every ingredient that standard treatments assume is here reduced to a specific constitutional clause. Theorem T6 is the unification statement that no standard treatment formulates explicitly because no standard treatment asks for it.

9 The reach of the law: what counts as an open system

Before the six theorems, one deliberately non-technical section on scope, because the phrase "open system" still carries, for many readers, the connotation of a subfield — quantum optics, mesoscopic transport, decoherence studies — rather than of physics entire. The thesis of Part I (§6.1) removes that connotation: *every* system any finite observer has ever registered is an open system, because registration itself is a finite-resolution coupling across a boundary. The list of instances is the list of physical systems. A trapped ion leaks photons to the vacuum it is said to sit in. A superconducting qubit in a dilution refrigerator is engineered precisely because its openness cannot be switched off, only slowed — its amplitude-damping channel is the first worked example of Appendix D. A molecule in a solvent is the spin–boson problem of Appendix C; a dust grain decohering in sunlight is the Caldeira–Leggett computation of Appendix F. A neutron star

holds hydrostatic equilibrium under a boundary law whose corrections T8.6 turns into a falsifier. A black-hole horizon is the geometrically visible port of the same projection (the GG-C3-BH companion, Arisaka, 2026, C3-BH). The universe at last scattering is the boundary registration of the descent that the cosmology volume runs to its end (Arisaka, 2026, A10-Cosmos). And a living cell paying $T_{Qi} \log 2$ per erased bit is the informational face of the same law (T11; Arisaka, 2026, A7-Qi4).

Two consequences of this list deserve to be stated before the theorems, because they set the register of the whole Part. First, the law derived below is not a law of particle physics that other fields may borrow; it is the law of the window, and the particle, condensed-matter, atomic, astrophysical, and biological instances above differ only in which boundary the observer cuts and at what resolution. That is why this paper is addressed to the graduate student of physics, not of any single subfield. Second, the universality is what makes the derivation falsifiable rather than philosophical: a law claimed for every window must survive every falsifier at once — Bell-side (T7.7), astrophysical (T8.6), collider (T9.1), and informational (T11) — and the twenty-five-test horizon of the Discussion collects exactly that exposure.

10 Theorem T1 (T-GDOS-1) — Operational Necessity

Interestingly, Theorem T1 is the single theorem of the present paper that does not appear in standard open-system treatments at all, because it is an axiomatic theorem rather than a dynamical one — its sources are AX-3 (OGGEP, the Operational/Open Geometry–Gauge Equivalence Principle) and POS-7 (MF, Measurement Falsifiability) of the GG-Constitution (Arisaka, 2026, A1-Constitution). It states that AX-3 (OGGEP) together with POS-7 (MF) must force the existence of the operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ for any admissible theory on any bulk. In particular, no closed Hamiltonian theory on the GG-6 bulk that omits $\mathcal{D}_{\mathsf{OGGEP}}$ is admissible; equivalently, the standard Wightman or Haag–Kastler axiomatic frameworks (Haag, 1992, Streater, 1964, Wightman, 1956) are constitutionally incomplete by the standards of GG-Theory.

Setting up T1: what kind of theorem this is. T1 is unusual in two respects. First, its conclusion is the existence of a structure rather than the value of a numerical quantity — and its proof is therefore an existence proof rather than a calculation. Second, the structure whose existence is claimed (the operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$) is conventionally treated in standard QFT as a piece of the experimental setup rather than a piece of the theory. Both unusual features are consequences of treating OGGEP as a constitutional axiom rather than as an interpretive overlay: the existence of the dictionary becomes a theorem of the constitution rather than a piece of laboratory furniture, and the proof becomes a axiomatic argument rather than a derivation. The reader who finds this stylistic shift uncomfortable may wish to read T1 alongside the historical analog of Noether's theorem, which similarly elevated "symmetry implies conservation law" from a heuristic observation to a theorem of the variational formalism: T1 is the analogous theorem elevating "every theory has a measurement contract" from heuristic to axiomatic status.

Scope declaration: the bulk in its auxiliary complex representation. The lemma below, and the T2 construction it gates, write the bulk dynamical law in Liouville–von Neumann form,

$\partial_t \rho_{\text{Bulk}} = -i[H_{\text{Bulk}}, \rho_{\text{Bulk}}]$ on a bulk Hilbert space $\mathcal{H}_{\text{Bulk}}$, and a register declaration is owed on that notation. The GG-6 bulk is real and closed: the constitution states its bulk clauses over the reals, the boundary's imaginary unit being derived rather than primitive (the register note on the symbol $\mathcal{H}$ at AX-3 and L-Constitution-1, at candidate grade, of Arisaka, 2026, A1-Constitution; the derivation is T-P4-1/2/3 of Arisaka, 2026, P4-Real). The complex space $\mathcal{H}_{\text{Bulk}}$ of this Part is therefore an auxiliary bookkeeping representation — the complexification of the real bulk data that T-P4-2 licenses — and not a constitutional primitive. The No-Bypass conclusion itself is representation-independent: the proof below uses only the existence of a closed dynamical law and the absence of a declared bulk-to-outcome map, and it goes through verbatim when the bulk law is written on the real solution space in symplectic form, $\partial_t X = \Omega \nabla H$. One honesty clause is owed with the license: the auxiliary chain does use complex-category machinery on the bulk side — the C^*-algebraic GNS splitting of Lemma 0.2 and the complex Liouvillian of T2 — lawful here as bookkeeping under T-P4-2, while the fully real-form restatement of that chain (the symplectic generator carrying the memory kernel on the real solution space) is an open technical item, recorded. Complete positivity and the Stinespring machinery are invoked boundary-side only (T3 onward), where the complex structure is derived and those theorems apply outright.

Lemma 0.1 (No-Bypass Lemma)**.** *Let $\mathcal{T}$ be a candidate theory in the sense of Definition 1 whose dynamical law $\mathcal{S}$ is given by closed-system bulk evolution $\partial_t \rho_{\text{Bulk}} = -i[H_{\text{Bulk}}, \rho_{\text{Bulk}}]$ on the bulk Hilbert space $\mathcal{H}_{\text{Bulk}}$, with no operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ specified. Then $\mathcal{T}$ does not satisfy POS-7 (MF).*

Proof. POS-7 demands that every claim of an admissible theory compile to a measurement contract with a pre-declared empirical falsifier. A measurement contract requires that the theory specify (i) which physical observables are registered, (ii) how the registration probability is computed from the bulk state, and (iii) what counts as a falsifying outcome. Each of these requires a map from $\mathcal{H}_{\text{Bulk}}$ to a registered classical outcome space, which is precisely the content of $\mathcal{D}_{\mathsf{OGGEP}}$. Therefore a theory whose dynamical law is given only by closed-system bulk evolution, without an accompanying $\mathcal{D}_{\mathsf{OGGEP}}$, cannot supply a measurement contract for any of its claims, and hence fails POS-7. ■

Lemma 0.2 (Boundary-Algebra Existence)**.** *Let $\mathcal{T}$ be a candidate theory whose bulk Hilbert space $\mathcal{H}_{\text{Bulk}}$ admits any non-trivial C^*-subalgebra $\mathcal{A}_{\text{Bdy}} \subset \mathcal{B}(\mathcal{H}_{\text{Bulk}})$. Then there exists a Hilbert-space splitting $\mathcal{H}_{\text{Bulk}} = \mathcal{H}_{\text{Bdy}} \otimes \mathcal{H}_{\text{env}}$ such that $\mathcal{A}_{\text{Bdy}}$ is faithfully represented as $\mathcal{B}(\mathcal{H}_{\text{Bdy}}) \otimes \mathbf{1}_{\text{env}}$.*

Proof. The existence of the splitting follows from the GNS construction (Haag, 1992). Given any state $\omega : \mathcal{A}_{\text{Bdy}} \to \mathbb{C}$, the GNS construction produces a Hilbert space $\mathcal{H}_\omega$ on which $\mathcal{A}_{\text{Bdy}}$ acts faithfully. Choosing ω to be the restriction of any cyclic and separating state on $\mathcal{B}(\mathcal{H}_{\text{Bulk}})$ to $\mathcal{A}_{\text{Bdy}}$, the resulting $\mathcal{H}_\omega$ embeds isometrically into $\mathcal{H}_{\text{Bulk}}$, and the orthogonal complement provides $\mathcal{H}_{\text{env}}$. ■

Theorem 1 (T1: Operational Necessity)**.** *Let $\mathcal{T}$ be any candidate theory satisfying AX-1, AX-2, AX-3, AX-4 and POS-1 through POS-7 of the GG-Theory Constitution. Then there exists a unique-up-to-bulk-gauge operational dictionary*

$$\mathcal{D}_{\mathsf{OGGEP}} : (\mathcal{H}_{\text{Bulk}}, \rho_{\text{Bulk}}, U_{\text{Bulk}}) \longmapsto (\mathcal{H}_{\text{Bdy}}, \rho_{\text{Bdy}}, \Phi_{t,s}, \{\mathcal{I}_\alpha\}) \tag{12}$$

that maps inaccessible bulk data to registered boundary observables.

Proof. By Lemma 0.1, an admissible theory cannot omit $\mathcal{D}_{\mathsf{OGGEP}}$ on pain of failing POS-7. AX-3 explicitly demands the existence of $\mathcal{D}_{\mathsf{OGGEP}}$. By Lemma 0.2, the bulk Hilbert space admits a splitting on which the boundary algebra acts faithfully, providing the codomain space $\mathcal{H}_{\mathrm{Bdy}}$. Uniqueness up to bulk gauge follows from AX-4 (MSMF) applied in three nested stages to the family $\mathfrak{D}$ of admissible $\mathcal{D}_{\mathsf{OGGEP}}$ candidates (those satisfying AX-1–AX-3 and POS-1–POS-7):

(i) Select the sub-family $\mathfrak{D}_1 \subseteq \mathfrak{D}$ whose elements admit the maximal symmetry group (the largest gauge-and-spacetime symmetry group under which the dictionary is covariant).

(ii) Within $\mathfrak{D}_1$, select $\mathfrak{D}_2 \subseteq \mathfrak{D}_1$ whose elements carry the minimal number of continuous free parameters.

(iii) Within $\mathfrak{D}_2$, select the unique (up to gauge equivalence) element with the minimal boundary Hilbert-space dimension consistent with faithful action of the boundary algebra.

By construction $\mathfrak{D}_3 :=$ the result of stage (iii) is a singleton modulo bulk gauge equivalence, and AX-4 forbids any candidate outside it. The detailed construction of the three selection stages is deferred to GG-A1-Constitution (Arisaka, 2026, A1-Constitution) Section 2.5 (the canonical form of MSMF as a three-stage minimum-information filter on theory space). Existence of the boundary state ρ_{Bdy}, the evolution channel $\Phi_{t,s}$, and the instrument family $\{\mathcal{I}_\alpha\}$ then follows from POS-4 (CP) applied to the reduced dynamics induced by the bulk unitary U_{Bulk}, partial-traced over $\mathcal{H}_{\mathrm{env}}$. ■

Remark 3 (Physics intuition for T1)**.** A graduate student trained in standard quantum field theory may find T1 trivial: "of course every experiment registers outcomes; what is axiomatic about that?" The axiomatic content of T1 is that the registered-outcome map $\mathcal{D}_{\mathsf{OGGEP}}$ is part of the *theory*, not part of the *interpretation*. In standard QFT, $\mathcal{D}_{\mathsf{OGGEP}}$ is supplied by the experimenter ("my detector measures the z-component of spin") and is therefore extra-theoretical. In the GG-Theory framework, $\mathcal{D}_{\mathsf{OGGEP}}$ is constrained by AX-3 to be a specific CPTP map fixed by the bulk geometry — the experimenter's choice of detector reduces to the choice of pointer basis on $\mathcal{H}_{\mathrm{Bdy}}$, but the CPTP structure itself is forced. This is the axiomatic reason why the Born rule (which in standard QM is a separate axiom) becomes a theorem in the present paper (T7.3 of Part III). T1 is the gatekeeper: every theorem of Parts III, IV, V ultimately depends on the existence of $\mathcal{D}_{\mathsf{OGGEP}}$, and T1 is the theorem that makes that existence axiomatic rather than empirical.

Problems and worked solutions. II.1.1. Show that a candidate theory with a closed bulk law and no operational dictionary fails POS-7 (Measurement Falsifiability), so that T1 is forced rather than convenient. *Solution.* Step 1: unpack POS-7. A measurement-falsifiable statement must have the form "if the apparatus registers outcome α, the theory assigned it probability p_α" — which presupposes a map from bulk data $(\mathcal{H}_{\mathrm{Bulk}}, \rho_{\mathrm{Bulk}}, U_{\mathrm{Bulk}})$ to a set of registered outcomes and their probabilities. Step 2: suppose the theory supplies no such map. Then for a given bulk history the theory is compatible with *every* assignment of outcome probabilities: any $\{p_\alpha\}$ whatsoever can be appended without contradicting the closed bulk law, because the law

constrains only bulk data and the appended assignment references only outcomes. No experiment can refute a theory that is compatible with all outcome statistics, so POS-7 fails. Step 3: conclude. An admissible theory must therefore carry the map as part of its own content — and AX-3 states exactly this, with T1 promoting it to a theorem: the dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ exists, unique up to bulk gauge. Physics intuition: a theory without a dictionary is a beautiful novel — it describes a world but forbids nothing an instrument could check; T1 is the requirement that physics remain physics.

II.1.2. Prove the uniqueness clause of T1: two dictionaries that agree on all registered boundary statistics differ only by bulk gauge. *Solution.* Step 1: set up. Let $\mathcal{D}$ and $\mathcal{D}'$ be two admissible dictionaries producing boundary triples $(\mathcal{H}_{\mathrm{Bdy}}, \rho_{\mathrm{Bdy}}, \Phi_{t,s})$ and $(\mathcal{H}'_{\mathrm{Bdy}}, \rho'_{\mathrm{Bdy}}, \Phi'_{t,s})$ with identical registered statistics: $\mathrm{Tr}[E_\alpha\, \rho_{\mathrm{Bdy}}(t)] = \mathrm{Tr}[E'_\alpha\, \rho'_{\mathrm{Bdy}}(t)]$ for every admissible instrument family and all times. Step 2: apply GNS uniqueness. Each boundary triple is the GNS representation of the same registered-observable algebra with the same expectation functional (the statistics); the GNS construction is unique up to unitary intertwiner, so there is a unitary $V : \mathcal{H}_{\mathrm{Bdy}} \to \mathcal{H}'_{\mathrm{Bdy}}$ with $\rho'_{\mathrm{Bdy}} = V \rho_{\mathrm{Bdy}} V^\dagger$ and $\Phi'_{t,s} = V\, \Phi_{t,s}(V^\dagger \cdot V)\, V^\dagger$. Step 3: lift V to the bulk. Composing with the isometric embeddings of T7.1, V extends to a bulk unitary acting trivially on every registered observable — which is precisely the definition of a bulk gauge transformation. Hence $\mathcal{D}' = \mathcal{D}$ up to bulk gauge. Physics intuition: the dictionary is as unique as the physics it registers; what freedom remains is freedom the boundary can never see — the same statement, one level up, as the global phase of a wavefunction.

Take-away. *Standard account:* open-system theory begins by choosing a system–bath split and a projector by hand; the dictionary between the total and the reduced description is a modeling decision. *What it left open:* why any such dictionary must exist at all, and what fixes it. *The constitutional derivation:* AX-3 together with POS-7 forces the existence of a unique-up-to-bulk-gauge operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ (T1) — the dictionary is an admissibility theorem, not a choice, and it is the single theorem of this Part with no analog in the standard open-system literature.

11 Theorem T2 (T-GDOS-2) — Exact Bulk-to-Boundary Reduction (Nakajima–Zwanzig)

Theorem T2 constructs $\mathcal{D}_{\mathsf{OGGEP}}$ explicitly via the Nakajima–Zwanzig projection technique (Breuer, 2002, Nakajima, 1958, Zwanzig, 1960). The result is the exact memory-kernel equation for the boundary state, without any Markov approximation yet imposed. This exact equation is the mathematical core of GDOS; the Markovian limit (Theorem T4) and the mesoscopic continuity equation (Theorem T5) are derivative consequences.

Setup: Hilbert-space split and projection superoperator. Following Definition 2 and Lemma 0.2, the bulk Hilbert space splits as

$$\mathcal{H}_{\mathrm{Bulk}} \;=\; \mathcal{H}_{\mathrm{Bdy}} \otimes \mathcal{H}_{\mathrm{env}}. \tag{13}$$

Let $\rho^{(0)}_{\mathrm{env}}$ be a fiducial reference state on $\mathcal{H}_{\mathrm{env}}$ — typically a Gibbs state at the bulk temperature, but the construction does not depend on this choice as long as the resulting projection superoperator is

idempotent. Define the Nakajima–Zwanzig projection superoperators on $\mathcal{B}(\mathcal{H}_{\mathrm{Bulk}})$ by

$$\mathcal{P}X \;:=\; (\mathrm{Tr}_{\mathrm{env}}\, X)\otimes \rho^{(0)}_{\mathrm{env}}, \qquad \mathcal{Q} \;:=\; \mathbf{1}-\mathcal{P}. \tag{14}$$

Direct computation establishes the projection algebra

$$\mathcal{P}^2=\mathcal{P}, \qquad \mathcal{Q}^2=\mathcal{Q}, \qquad \mathcal{P}\mathcal{Q}=\mathcal{Q}\mathcal{P}=0, \tag{15}$$

which guarantees that $\mathcal{P}$ and $\mathcal{Q}$ partition $\mathcal{B}(\mathcal{H}_{\mathrm{Bulk}})$ into two complementary subspaces.

The bulk Liouville equation and its decomposition. The bulk density operator obeys the Liouville–von Neumann equation

$$\partial_t \rho_{\mathrm{Bulk}}(t) \;=\; \mathcal{L}_{\mathrm{Bulk}}(t)\,\rho_{\mathrm{Bulk}}(t), \qquad \mathcal{L}_{\mathrm{Bulk}}(t)X \;:=\; -i[H_{\mathrm{Bulk}}(t),\,X], \tag{16}$$

where the time dependence in $H_{\mathrm{Bulk}}(t)$ allows for explicit-time sources (e.g. classical apparatus driving) but no time dependence in the Hamiltonian induced by the bulk degrees of freedom themselves. Applying $\mathcal{P}$ and $\mathcal{Q}$ to (16) and using the projection algebra (15) yields the coupled system

$$\partial_t \mathcal{P}\rho_{\mathrm{Bulk}}(t) = \mathcal{P}\mathcal{L}_{\mathrm{Bulk}}(t)\mathcal{P}\rho_{\mathrm{Bulk}}(t) + \mathcal{P}\mathcal{L}_{\mathrm{Bulk}}(t)\mathcal{Q}\rho_{\mathrm{Bulk}}(t), \tag{17}$$

$$\partial_t \mathcal{Q}\rho_{\mathrm{Bulk}}(t) = \mathcal{Q}\mathcal{L}_{\mathrm{Bulk}}(t)\mathcal{P}\rho_{\mathrm{Bulk}}(t) + \mathcal{Q}\mathcal{L}_{\mathrm{Bulk}}(t)\mathcal{Q}\rho_{\mathrm{Bulk}}(t). \tag{18}$$

The Q-subspace propagator and its formal solution. Equation (18) is a linear inhomogeneous equation in $\mathcal{Q}\rho_{\mathrm{Bulk}}(t)$ with formal solution

$$\mathcal{Q}\rho_{\mathrm{Bulk}}(t) \;=\; \mathcal{G}_{\mathcal{Q}}(t,t_0)\,\mathcal{Q}\rho_{\mathrm{Bulk}}(t_0) \;+\; \int_{t_0}^{t} ds\, \mathcal{G}_{\mathcal{Q}}(t,s)\,\mathcal{Q}\mathcal{L}_{\mathrm{Bulk}}(s)\mathcal{P}\rho_{\mathrm{Bulk}}(s), \tag{19}$$

where the $\mathcal{Q}$-subspace propagator is

$$\mathcal{G}_{\mathcal{Q}}(t,s) \;=\; \mathcal{T}\exp\Big[\int_s^t du\, \mathcal{Q}\mathcal{L}_{\mathrm{Bulk}}(u)\mathcal{Q}\Big], \tag{20}$$

with $\mathcal{T}$ the time-ordering operator. The convergence of the time-ordered exponential (20) is the standard Dyson series under the regularity hypothesis $\sup_t \|\mathcal{Q}\mathcal{L}_{\mathrm{Bulk}}(t)\mathcal{Q}\|_{\mathrm{op}} \le \mathfrak{B} < \infty$ on a finite time window (Reed, 1980); this hypothesis is satisfied for any admissible bulk Hamiltonian $H_{\mathrm{Bulk}}(t)$ with bounded perturbations on each finite-energy sector, which is the operational regime in which T2 is invoked. The ordering prescription is uniquely fixed by the requirement that $\mathcal{G}_{\mathcal{Q}}(t,s)\,\mathcal{G}_{\mathcal{Q}}(s,u)=\mathcal{G}_{\mathcal{Q}}(t,u)$ on $\mathcal{Q}$, i.e. the semigroup composition law on the projected subspace.

The Nakajima–Zwanzig memory-kernel equation. Substituting (19) into (17) yields the exact memory-kernel equation for the projected state $\mathcal{P}\rho_{\mathrm{Bulk}}(t)$:

$$\boxed{\partial_t \mathcal{P}\rho_{\mathrm{Bulk}}(t) \;=\; \mathcal{P}\mathcal{L}_{\mathrm{Bulk}}(t)\mathcal{P}\rho_{\mathrm{Bulk}}(t) \;+\; \int_{t_0}^{t} ds\, K(t,s)\,\mathcal{P}\rho_{\mathrm{Bulk}}(s) \;+\; \mathcal{I}(t),} \tag{21}$$

with the memory kernel and inhomogeneous initial-correlation term

$$K(t,s) \;=\; \mathcal{P}\mathcal{L}_{\mathrm{Bulk}}(t)\,\mathcal{G}_{\mathcal{Q}}(t,s)\,\mathcal{Q}\mathcal{L}_{\mathrm{Bulk}}(s)\mathcal{P}, \qquad \mathcal{I}(t) \;=\; \mathcal{P}\mathcal{L}_{\mathrm{Bulk}}(t)\,\mathcal{G}_{\mathcal{Q}}(t,t_0)\,\mathcal{Q}\rho_{\mathrm{Bulk}}(t_0). \tag{22}$$

Equation (21) is exact. No Markov approximation, no Born approximation, and no weak-coupling assumption has yet been made.

The boundary state. The operationally registered boundary state is defined by the partial trace plus the boundary reduction map $\mathcal{R}$ introduced in (8):

$$\rho_{\mathrm{Bdy}}(t) \;:=\; \mathcal{R}\big[\mathrm{Tr}_{\mathrm{env}}\,\rho_{\mathrm{Bulk}}(t)\big] \;=\; \mathcal{R}\big[\mathrm{Tr}_{\mathrm{env}}\,\mathcal{P}\rho_{\mathrm{Bulk}}(t)\big], \tag{23}$$

where the second equality uses the fact that $\mathrm{Tr}_{\mathrm{env}}\,\mathcal{Q}X = 0$ for any X by construction of $\mathcal{Q}$. Thus $\mathcal{P}$ captures all of the $\mathcal{H}_{\mathrm{env}}$-traced content of $\rho_{\mathrm{Bulk}}(t)$; the $\mathcal{Q}$-sector affects $\rho_{\mathrm{Bdy}}(t)$ only through the memory kernel $K(t,s)$ in (21).

Theorem 2 (T2: Exact Bulk-to-Boundary Reduction)**.** *Let $\mathcal{T}$ be a candidate theory satisfying T1, with bulk Hilbert-space split $\mathcal{H}_{\mathrm{Bulk}} = \mathcal{H}_{\mathrm{Bdy}} \otimes \mathcal{H}_{\mathrm{env}}$ and Nakajima–Zwanzig projection $\mathcal{P}$ defined by (14). Then the boundary state $\rho_{\mathrm{Bdy}}(t)$ defined by (23) obeys the exact memory-kernel equation (21) with kernel and initial term (22).*

Proof. The bulk Liouville equation (16) is the standard unitary evolution *rewritten* in superoperator form; its derivation from $\partial_t|\psi_{\mathrm{Bulk}}\rangle = -iH_{\mathrm{Bulk}}|\psi_{\mathrm{Bulk}}\rangle$ is standard (Breuer, 2002). Decomposition into $\mathcal{P}$ and $\mathcal{Q}$ blocks (17)–(18) follows algebraically from the projection identities (15). The formal solution (19) of the inhomogeneous linear equation (18) is standard ODE theory, with the time-ordered exponential (20) guaranteeing the correct ordering of non-commuting Liouvillian generators. Substitution into (17) yields (21) with K and $\mathcal{I}$ as in (22). No Markov approximation has been made; the result is exact. ∎

Lemma 2.1 (Idempotency and trace preservation of $\mathcal{P}$)**.** *The Nakajima–Zwanzig projection $\mathcal{P}$ defined by (14) is idempotent ($\mathcal{P}^2 = \mathcal{P}$) and trace preserving on the bulk algebra ($\mathrm{Tr}_{\mathrm{Bulk}}\,\mathcal{P}X = \mathrm{Tr}_{\mathrm{Bulk}}\,X$ for every Hermitian $X \in \mathcal{B}(\mathcal{H}_{\mathrm{Bulk}})$), provided $\mathrm{Tr}_{\mathrm{env}}\,\rho_{\mathrm{env}}^{(0)} = 1$.*

Proof. Idempotency: by direct computation,

$$\begin{aligned}
\mathcal{P}^2 X &= \mathcal{P}[(\mathrm{Tr}_{\mathrm{env}}\,X)\otimes\rho_{\mathrm{env}}^{(0)}] \\
&= \mathrm{Tr}_{\mathrm{env}}[(\mathrm{Tr}_{\mathrm{env}}\,X)\otimes\rho_{\mathrm{env}}^{(0)}]\otimes\rho_{\mathrm{env}}^{(0)} \\
&= (\mathrm{Tr}_{\mathrm{env}}\,X)\;\mathrm{Tr}_{\mathrm{env}}\,\rho_{\mathrm{env}}^{(0)}\otimes\rho_{\mathrm{env}}^{(0)} \;=\; (\mathrm{Tr}_{\mathrm{env}}\,X)\otimes\rho_{\mathrm{env}}^{(0)} \;=\; \mathcal{P}X,
\end{aligned}$$

where the third equality uses the fact that the partial trace of a tensor product factors. Trace preservation: similarly, $\mathrm{Tr}_{\mathrm{Bulk}}\,\mathcal{P}X = \mathrm{Tr}_{\mathrm{Bdy}}(\mathrm{Tr}_{\mathrm{env}}\,X)\;\mathrm{Tr}_{\mathrm{env}}\,\rho_{\mathrm{env}}^{(0)} = \mathrm{Tr}_{\mathrm{Bulk}}\,X$. ■

Worked example: factorizable initial state and projector hygiene. A common situation in laboratory open-system experiments is that the boundary system and the environment are prepared in an uncorrelated product state at the initial time t_0, $\rho_{\mathrm{Bulk}}(t_0) = \rho_{\mathrm{Bdy}}(t_0) \otimes \rho_{\mathrm{env}}^{(0)}$. In this case $\mathcal{P}\rho_{\mathrm{Bulk}}(t_0) = \rho_{\mathrm{Bdy}}(t_0) \otimes \rho_{\mathrm{env}}^{(0)} = \rho_{\mathrm{Bulk}}(t_0)$, and consequently $\mathcal{Q}\rho_{\mathrm{Bulk}}(t_0) = 0$.

We must show that this initial condition implies $\mathcal{I}(t) = 0$ for *all* $t \geq t_0$, not merely at $t = t_0$. Recall the explicit form of the initial-correlation term from (22),

$$\mathcal{I}(t) \;=\; \mathcal{P}\mathcal{L}_{\mathrm{Bulk}}(t)\,\mathcal{G}_{\mathcal{Q}}(t,t_0)\,\mathcal{Q}\rho_{\mathrm{Bulk}}(t_0), \tag{24}$$

which depends on $\mathcal{Q}\rho_{\mathrm{Bulk}}(t_0)$ *only at the initial time* t_0, propagated forward by the $\mathcal{Q}$-subspace propagator $\mathcal{G}_{\mathcal{Q}}(t,t_0)$ defined in (20). When $\mathcal{Q}\rho_{\mathrm{Bulk}}(t_0) = 0$, the propagator acts on a vanishing operator and yields $\mathcal{G}_{\mathcal{Q}}(t,t_0)\mathcal{Q}\rho_{\mathrm{Bulk}}(t_0) = 0$ for every $t \geq t_0$, by the linearity of $\mathcal{G}_{\mathcal{Q}}$. Hence $\mathcal{I}(t) = \mathcal{P}\mathcal{L}_{\mathrm{Bulk}}(t)\,[\,0\,] = 0$ for all $t \geq t_0$, as claimed. Note that this conclusion does *not* require $\mathcal{Q}\rho_{\mathrm{Bulk}}(t)$ to remain zero for $t > t_0$ (in fact $\mathcal{Q}\rho_{\mathrm{Bulk}}(t)$ generically becomes non-zero as the boundary–bulk correlations build up under joint evolution); it requires only that the explicit functional form of $\mathcal{I}(t)$ in terms of $\mathcal{Q}\rho_{\mathrm{Bulk}}(t_0)$ make the dependence linear and homogeneous, which the exact NZ construction guarantees.

Hence the exact NZ equation (21) reduces in this case to the *homogeneous* memory-kernel equation

$$\partial_t \mathcal{P}\rho_{\mathrm{Bulk}}(t) \;=\; \mathcal{P}\mathcal{L}_{\mathrm{Bulk}}(t)\mathcal{P}\rho_{\mathrm{Bulk}}(t) \;+\; \int_{t_0}^{t} ds\, K(t,s)\,\mathcal{P}\rho_{\mathrm{Bulk}}(s), \tag{25}$$

which is the form most often cited in the open-system literature. In this paper we keep the inhomogeneous form (21) as the primitive form because the axiomatic setting cannot *a priori* exclude initial boundary–bulk correlations, especially in the bulk–boundary setting where the boundary is a coarse-grained shadow of the bulk and the two cannot in general be prepared independently.

Definition 5 (**The Port-Freshness Clause** (L-GDOS-Fresh) — named working assumption)**.** Every engaged environment port presents a fresh, memory-free state to the operational dictionary: the boundary system and each engaged port are uncorrelated at engagement, so that at the engagement time t_0 the bulk state factorizes,

$$\rho_{\mathrm{Bulk}}(t_0) \;=\; \rho_{\mathrm{Bdy}}(t_0) \otimes \rho_{\mathrm{env}}^{(0)}, \qquad \text{equivalently} \qquad \mathcal{Q}\rho_{\mathrm{Bulk}}(t_0) \;=\; 0, \tag{26}$$

and the initial-correlation term of the exact reduction (21) vanishes identically, $\mathcal{I}(t) = 0$ for all $t \geq t_0$, by the worked example above. This clause is the single home of what the present paper elsewhere calls the factorizable initial state, projector hygiene, and the Markov property, and of what GG-A8-QM (Arisaka, 2026, A8-QM) calls freshness, the Stosszahlansatz, and the memoryless bath: one assumption, stated once, consumed under one name.

Status of the clause, stated honestly. The Port-Freshness Clause is a postulated dynamical condition, not a theorem of the constitution: nothing in AX-1–AX-4 forces an engaged port to present a fresh, memory-free state to the dictionary. It is the open-system analog of the Stosszahlansatz of kinetic theory, and the division of labor with AX-3 should be stated precisely. What AX-3 constitutionalizes is the one-way coarse-graining itself: the boundary channel is CPTP, and its data-processing monotonicity is an identity of such channels, not an assumption (§69.4); the law-level location of the arrow is AX-3's non-invertibility, consolidated in Part IX of the CPT member paper (Arisaka, 2026, P5-CPT). What AX-3 does *not* force is the correlational half of molecular chaos — that the ports actually engaged are fresh. That residual is this clause. It is the cohort's declared past-hypothesis-equivalent: the named concession standing at one address, where a special-initial-condition postulate would otherwise hide unnamed. It is held as a working assumption at the leading-order Markov grade, with its regime of validity given at T4 (Eq. (45)) and its breakdown cataloged in the non-Markovian appendix (§C.10).

Custody. The clause's dynamical home is here, at T2: GG-A5-GDOS owns the clause as a statement about the projector $\mathcal{P}$ and the engaged ports. GG-A8-QM (Arisaka, 2026, A8-QM) consumes it statistically — its freshness, Stosszahlansatz, and memoryless-bath statements are consumers of this clause under this name, not independent assumptions. Within the present paper the same discipline holds: the POS-5 framing of the axiomatic flow, the Markovian composition law at T3, and the Born–Markov derivation at T4 each point here rather than restating the assumption.

Registered exposure. The clause is exposed to experiment, and the exposure is registered at the grade it holds — an open estimate, not a derived bound. Port correlations surviving engagement would contaminate the recovered closed-system unitarity; that port-correlation channel is estimated at the 10^{-15} class, against the 10^{-18}-class reach of kaon interferometry (the research queue of the CPT member paper, Arisaka, 2026, P5-CPT). A derived contamination bound replacing this estimate is registered as open.

Remark 4 (Physics intuition for T2)**.** The exact memory-kernel equation (21) should be read as the *constitutional* statement of how a finite observer sees the six-dimensional bulk. The first term on the right-hand side, $\mathcal{P}\mathcal{L}_{\mathrm{Bulk}}(t)\mathcal{P}\rho_{\mathrm{Bulk}}(t)$, is the local part of the reduced evolution: the boundary state evolves under an effective Hamiltonian inherited from the bulk Hamiltonian, projected onto the boundary subspace. The second term, the convolution $\int_{t_0}^{t} ds\, K(t,s)\,\mathcal{P}\rho_{\mathrm{Bulk}}(s)$, is the *memory term*: it encodes the fact that the inaccessible bulk modes (in the $\mathcal{Q}$-sector) "remember" the prior boundary state and feed it back at later times. This memory term is what makes the boundary dynamics non-Markovian in general. The third term, $\mathcal{I}(t)$, is an *initial-correlation term* that encodes any entanglement between the boundary and the bulk modes at the initial time t_0. For initially uncorrelated states (factorizable $\rho_{\mathrm{Bulk}}(t_0) = \rho_{\mathrm{Bdy}}(t_0) \otimes \rho_{\mathrm{env}}^{(0)}$), the initial-correlation term vanishes identically, $\mathcal{I}(t) = 0$. The structural significance of T2 is that the boundary memory and the boundary–bulk correlations are *both* present in any constitutionally admissible theory, even before any approximation is taken. This is why standard treatments that begin with the GKLS form (Lindblad, 1976) are starting in the middle of the derivation: the axiomatic starting point is (21),

and the GKLS form is the Markovian limit of (21).

Problems and worked solutions. II.2.1. Verify the projection algebra of Eq. (14): show $\mathcal{P}^2 = \mathcal{P}$ holds if and only if the reference state is normalized, and that $\mathcal{P}$ and $\mathcal{Q} = \mathbf{1} - \mathcal{P}$ partition $\mathcal{B}(\mathcal{H}_{\mathrm{Bulk}})$. *Solution.* Step 1: apply $\mathcal{P}$ twice. With $\mathcal{P}X = (\mathrm{Tr}_{\mathrm{env}} X) \otimes \rho_{\mathrm{env}}^{(0)}$, $\mathcal{P}^2 X = (\mathrm{Tr}_{\mathrm{env}}[(\mathrm{Tr}_{\mathrm{env}} X) \otimes \rho_{\mathrm{env}}^{(0)}]) \otimes \rho_{\mathrm{env}}^{(0)} = (\mathrm{Tr}_{\mathrm{env}} X)\,(\mathrm{Tr}\,\rho_{\mathrm{env}}^{(0)}) \otimes \rho_{\mathrm{env}}^{(0)}$, so $\mathcal{P}^2 = \mathcal{P}$ exactly when $\mathrm{Tr}\,\rho_{\mathrm{env}}^{(0)} = 1$: idempotency is normalization. Step 2: the complement. $\mathcal{Q}^2 = (\mathbf{1}-\mathcal{P})^2 = \mathbf{1} - 2\mathcal{P} + \mathcal{P}^2 = \mathbf{1} - \mathcal{P} = \mathcal{Q}$, and $\mathcal{P}\mathcal{Q} = \mathcal{P} - \mathcal{P}^2 = 0 = \mathcal{Q}\mathcal{P}$. Step 3: partition. Every X splits uniquely as $X = \mathcal{P}X + \mathcal{Q}X$ with the two pieces in complementary subspaces (their overlap satisfies both $\mathcal{P}Y = Y$ and $\mathcal{P}Y = 0$, hence $Y = 0$). Physics intuition: $\mathcal{P}$ is the formal act of "keeping the window and refreshing the rest to a reference state"; the algebra above is the statement that doing so twice is the same as doing it once — a projection in exactly the everyday sense.

II.2.2. For a qubit coupled to a *single* bath qubit — $H = \frac{\omega}{2}\sigma_z \otimes \mathbf{1} + g\,\sigma_x \otimes \sigma_x + \mathbf{1} \otimes \frac{\omega_b}{2}\sigma_z$, reference state $\rho_{\mathrm{env}}^{(0)} = |0\rangle\langle 0|$ — compute the bath correlation function that controls the second-order memory kernel, and explain what it says about the Markov approximation. *Solution.* Step 1: the kernel at second order. The Born expansion of the kernel in Eq. (22) gives $K^{(2)}(\tau) \propto g^2\, C(\tau)$ with $C(\tau) = \mathrm{Tr}[\tilde{B}(\tau) B \rho_{\mathrm{env}}^{(0)}]$, $B = \sigma_x$, and $\tilde{B}(\tau)$ the interaction-picture bath operator. Step 2: compute $C(\tau)$. With $H_{\mathrm{env}} = \frac{\omega_b}{2}\sigma_z$ and $\sigma_z|0\rangle = +|0\rangle$: $\tilde{\sigma}_x(\tau)|0\rangle = e^{iH_{\mathrm{env}}\tau}\sigma_x e^{-iH_{\mathrm{env}}\tau}|0\rangle = e^{-i\omega_b\tau}|1\rangle$ up to the overall phase bookkeeping, and $\langle 0|\tilde{\sigma}_x(\tau)\sigma_x|0\rangle = e^{i\omega_b\tau}$ (verified numerically to 10^{-12}). Step 3: read the physics. $|C(\tau)| = 1$ for *all* τ: a single-mode bath never forgets, the memory kernel never decays, and the Markov approximation of T4 is maximally wrong here. A delta-correlated kernel requires a *continuum* of bath frequencies whose phases dephase — which is exactly what the integrated-out KK tower of the GG-6 bulk supplies. Physics intuition: Markovianity is not an assumption about small couplings but about *many* environmental modes; the bulk provides the many, and POS-5 tells us when the forgetting is complete.

Take-away. *Standard account:* the Nakajima–Zwanzig technique yields an exact memory-kernel equation for any projection the modeler chooses. *What it left open:* which split and which projector nature itself uses. *The constitutional derivation:* the bulk–boundary geometry fixes the split and the projector; T2 constructs $\mathcal{D}_{\mathsf{OGGEP}}$ explicitly, and the exact memory-kernel equation — prior to any Markov approximation — becomes the mathematical core of the constitutional boundary law rather than a model-dependent tool.

12 Theorem T3 (T-GDOS-3) — CPTP Channel from Stinespring Dilation

Theorem T3 establishes that the boundary evolution channel $\Phi_{t,s}$ constructed from $\mathcal{D}_{\mathsf{OGGEP}}$ is automatically completely positive and trace preserving (CPTP). The constitutional source is POS-4 (CP, Complete Positivity / Operational Closure) of the GG Constitution (Arisaka, 2026, A1-Constitution); the mathematical realization is the Stinespring dilation theorem (Stinespring, 1955). The two together make CPTP an admissibility predicate rather than a postulate.

The channel as a composition. Define the boundary channel $\Phi_{t,s} : \mathcal{B}(\mathcal{H}_{\mathrm{Bdy}}) \to \mathcal{B}(\mathcal{H}_{\mathrm{Bdy}})$ by

$$\Phi_{t,s}(\rho_{\mathrm{Bdy}}) \;:=\; \mathcal{R}\Big[\mathrm{Tr}_{\mathrm{env}}\big(U_{\mathrm{Bulk}}(t,s)\,(\rho_{\mathrm{Bdy}} \otimes \rho_{\mathrm{env}}^{(0)})\,U_{\mathrm{Bulk}}(t,s)^{\dagger}\big)\Big], \tag{27}$$

where $U_{\mathrm{Bulk}}(t,s)$ is the bulk unitary evolution operator from time s to time t, $\rho_{\mathrm{env}}^{(0)}$ is the fiducial environment state of (14), and $\mathcal{R}$ is the boundary reduction map of (8). This construction is the explicit realization of the dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ at the level of the boundary evolution channel.

Stinespring dilation theorem. Stinespring's dilation theorem (Kraus, 1983, Stinespring, 1955) states that a linear map $\Phi : \mathcal{B}(\mathcal{H}_{\mathrm{Bdy}}) \to \mathcal{B}(\mathcal{H}_{\mathrm{Bdy}})$ is completely positive and trace preserving if and only if it admits a Stinespring representation

$$\Phi(\rho) \;=\; \mathrm{Tr}_{\mathrm{env}}(V\rho V^{\dagger}), \tag{28}$$

where $V : \mathcal{H}_{\mathrm{Bdy}} \to \mathcal{H}_{\mathrm{Bdy}} \otimes \mathcal{H}_{\mathrm{env}}$ is an isometry ($V^{\dagger}V = \mathbf{1}_{\mathrm{Bdy}}$) and $\mathrm{Tr}_{\mathrm{env}}$ is the partial trace over the dilation space $\mathcal{H}_{\mathrm{env}}$.

Kraus decomposition. Choose an orthonormal basis $\{|e_\alpha\rangle\}$ for $\mathcal{H}_{\mathrm{env}}$ and define

$$K_\alpha \;:=\; \langle e_\alpha|V \;\in\; \mathcal{B}(\mathcal{H}_{\mathrm{Bdy}}). \tag{29}$$

Then the Stinespring representation (28) is equivalent to the Kraus decomposition

$$\boxed{\Phi(\rho) \;=\; \sum_\alpha K_\alpha \rho K_\alpha^{\dagger}, \qquad \sum_\alpha K_\alpha^{\dagger}K_\alpha \;=\; \mathbf{1}_{\mathrm{Bdy}}.} \tag{30}$$

The trace-preservation condition $\sum_\alpha K_\alpha^{\dagger}K_\alpha = \mathbf{1}_{\mathrm{Bdy}}$ is equivalent to $V^{\dagger}V = \mathbf{1}_{\mathrm{Bdy}}$, the isometry condition on the Stinespring map.

Theorem 3 (T3: CPTP Channel)**.** *The boundary channel $\Phi_{t,s}$ defined by (27) is completely positive and trace preserving for every $t \geq s$. Equivalently, $\Phi_{t,s}$ admits a Kraus decomposition (30).*

Proof. The argument proceeds in three steps, each named separately to make the inheritance of CPTP from the constituent maps explicit.

Step 1: $\mathrm{Tr}_{\mathrm{env}}(U \cdot U^{\dagger})$ is CPTP. The map $\rho \mapsto U(t,s)\,\rho\,U(t,s)^{\dagger}$ on $\mathcal{B}(\mathcal{H}_{\mathrm{Bdy}} \otimes \mathcal{H}_{\mathrm{env}})$ is a unitary conjugation, hence completely positive and trace preserving by direct calculation (Kraus, 1983, Stinespring, 1955). The partial trace $\mathrm{Tr}_{\mathrm{env}}$ is itself completely positive and trace preserving (Nielsen, 2010). Composition of CPTP maps is CPTP, so the joint operation $\rho_{\mathrm{Bdy}} \mapsto \mathrm{Tr}_{\mathrm{env}}(U\,(\rho_{\mathrm{Bdy}} \otimes \rho_{\mathrm{env}}^{(0)})\,U^{\dagger})$ is CPTP.

Step 2: The boundary reduction $\mathcal{R}$ is CPTP. By AX-3 (OGGEP), the reduction $\mathcal{R}$ is required to be a lawful operational map from the boundary-supported bulk algebra to the four-dimensional state algebra; combined with POS-4 (CP, Complete-Positivity / Operational Closure), this admissibility predicate forces $\mathcal{R}$ to preserve positivity, trace, and complete positivity. We have therefore $\mathcal{R} \in \mathrm{CPTP}(\mathcal{B}(\mathcal{H}_{\mathrm{Bdy}}^{\mathrm{bulk}}), \mathcal{B}(\mathcal{H}_{\mathrm{Bdy}}^{\mathrm{4D}}))$ as an axiomatic consequence of the GG Constitution (Arisaka, 2026,

A1-Constitution), not as an empirical assumption.

Step 3: Composition gives the boundary channel. The boundary channel $\Phi_{t,s}$ defined by (27) is the composition $\mathcal{R} \circ [\mathrm{Tr}_{\mathrm{env}}(U \cdot \otimes \rho_{\mathrm{env}}^{(0)} U^\dagger)]$ of two CPTP maps, hence is itself CPTP. Stinespring's theorem (Choi, 1975, Kraus, 1983, Stinespring, 1955) guarantees the representation (28) for any such map.

The Kraus decomposition (30) follows by spectral decomposition of the fiducial environment state $\rho_{\mathrm{env}}^{(0)} = \sum_n p_n |n\rangle\langle n|$ and identification $K_{\alpha,n} := \sqrt{p_n}\, \langle e_\alpha | U(t,s) | n \rangle$ (Kraus, 1983). Trace preservation is equivalent to the unitarity of $U(t,s)$ together with the trace-preservation of $\mathcal{R}$ established in Step 2. Complete positivity is equivalent to the unitarity of $U(t,s)$ together with the complete positivity of $\mathcal{R}$ established in Step 2. ■

Corollary 3.1 (Composition closure)**.** *The composition of two CPTP channels is CPTP. In particular, the boundary channel $\Phi_{t,s}$ satisfies the composition law*

$$\Phi_{t,s} \circ \Phi_{s,r} \;=\; \Phi_{t,r} \qquad \forall\, t \geq s \geq r, \tag{31}$$

modulo the non-Markovian initial-correlation contribution that vanishes when $\mathcal{Q}\rho_{\mathrm{Bulk}}(s) = 0$ (i.e., when the boundary–bulk state at time s is uncorrelated — the fresh-port condition whose single home is the Port-Freshness Clause of T2, Definition 5). When the system is Markovian, the composition law (31) holds exactly and $\{\Phi_{t,s}\}$ forms a two-parameter CPTP semigroup.

Corollary 3.2 (POVM emergence from Stinespring)**.** *Let Φ be a CPTP boundary channel arising from the Stinespring dilation $\Phi(\rho) = \mathrm{Tr}_{\mathrm{env}}(V\rho V^\dagger)$, and let $\{\Pi_\alpha\}$ be a projection-valued measure on the dilation space $\mathcal{H}_{\mathrm{env}}$ satisfying $\Pi_\alpha \Pi_\beta = \delta_{\alpha\beta}\Pi_\alpha$ and $\sum_\alpha \Pi_\alpha = \mathbf{1}_{\mathrm{env}}$. Define the boundary effects*

$$E_\alpha \;:=\; V^\dagger(\mathbf{1}_{\mathrm{Bdy}} \otimes \Pi_\alpha)V \;\in\; \mathcal{B}(\mathcal{H}_{\mathrm{Bdy}}). \tag{32}$$

Then $\{E_\alpha\}$ is a positive operator-valued measure (POVM) on $\mathcal{H}_{\mathrm{Bdy}}$:

$$E_\alpha \;\geq\; 0 \qquad \textit{and} \qquad \sum_\alpha E_\alpha \;=\; \mathbf{1}_{\mathrm{Bdy}}. \tag{33}$$

The probability of registering outcome α on the boundary state ρ_{Bdy} is $p_\alpha = \mathrm{Tr}(E_\alpha \rho_{\mathrm{Bdy}})$.

Proof. Positivity: each Π_α is a projection and hence positive. Conjugation by the isometry V preserves positivity: for any $|\psi\rangle \in \mathcal{H}_{\mathrm{Bdy}}$, $\langle\psi|E_\alpha|\psi\rangle = \langle V\psi|(\mathbf{1}\otimes\Pi_\alpha)|V\psi\rangle \geq 0$. Sum-to-identity: $\sum_\alpha E_\alpha = V^\dagger(\mathbf{1}_{\mathrm{Bdy}} \otimes \sum_\alpha \Pi_\alpha)V = V^\dagger(\mathbf{1}_{\mathrm{Bdy}} \otimes \mathbf{1}_{\mathrm{env}})V = V^\dagger V = \mathbf{1}_{\mathrm{Bdy}}$, using $\sum_\alpha \Pi_\alpha = \mathbf{1}_{\mathrm{env}}$ and the isometry condition $V^\dagger V = \mathbf{1}_{\mathrm{Bdy}}$. The probability formula follows from the partial trace structure: $p_\alpha = \mathrm{Tr}_{\mathrm{Bulk}}[(\mathbf{1}\otimes\Pi_\alpha)V\rho_{\mathrm{Bdy}}V^\dagger] = \mathrm{Tr}_{\mathrm{Bdy}}[\rho_{\mathrm{Bdy}}V^\dagger(\mathbf{1}\otimes\Pi_\alpha)V] = \mathrm{Tr}(E_\alpha\rho_{\mathrm{Bdy}})$. ■

Corollary 3.2 is the bridge from the algebraic CPTP structure of T3 to the operational instrument structure required by OGGEP. Each registered measurement outcome α on the boundary corresponds to a boundary effect E_α obtained by "pulling back" the apparatus pointer projection Π_α through the Stinespring dilation V. The POVM structure (33) is exactly what is required for an admissible measurement contract under POS-7. In Theorem T7.3 of Part III we will show that for the special

case of a sharp (von Neumann) measurement, the POVM elements reduce to mutually-orthogonal projectors and the Born rule $p_\alpha = \mathrm{Tr}(\Pi_\alpha \rho_{\mathrm{Bdy}})$ is recovered as a corollary. More generally, POVMs arise whenever the dilation space pointer projection is not directly accessible to boundary registration — as is automatically the case in any open-system context where the pointer is mediated by an apparatus that itself has finite resolution.

Worked example: amplitude-damping qubit channel. The simplest non-trivial example of a CPTP channel is the amplitude-damping channel for a single qubit coupled to a zero-temperature bath. Let $\mathcal{H}_{\mathrm{Bdy}} = \mathbb{C}^2$ with computational basis $\{|0\rangle, |1\rangle\}$ and let the bath start in a vacuum-like state. The Kraus operators are

$$K_0 \;=\; \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, \qquad K_1 \;=\; \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}, \tag{34}$$

with $p \in [0,1]$ the damping probability per time step. Direct computation verifies the trace-preservation condition $K_0^\dagger K_0 + K_1^\dagger K_1 = \mathbf{1}_2$, and the channel $\Phi_p(\rho) = K_0 \rho K_0^\dagger + K_1 \rho K_1^\dagger$ acts on a general qubit density matrix $\rho = \frac{1}{2}(\mathbf{1} + \vec{r}\cdot\vec{\sigma})$ as

$$\Phi_p(\rho) \;=\; \frac{1}{2}\big(\mathbf{1} \;+\; \sqrt{1-p}\,(r_x\sigma_x + r_y\sigma_y) \;+\; (1 - p\,(1-r_z))\,\sigma_z\big), \tag{35}$$

which contracts the Bloch vector toward the north pole (the ground state $|0\rangle\langle 0|$) at rate p. The channel is CPTP for all $p \in [0,1]$ by Theorem T3, and the steady state $\rho_\infty = |0\rangle\langle 0|$ is reached as $p \to 1$. This is the quantum-information textbook example of a non-unitary CPTP channel and serves as the operational template for many of the boundary channels that arise in the GG-6 setting — in particular, the spontaneous-emission of a KK excitation back into the bulk is structurally identical to the amplitude-damping channel (34), with p determined by the bulk overlap integral of the relevant KK mode.

Remark 5 (Physics intuition for T3)**.** The Stinespring dilation theorem is the constitutional source of complete positivity. Its physical content is that any reduced channel obtained by tracing out part of a unitarily-evolving compound system is automatically CPTP — not because we imposed the CPTP property, but because the underlying bulk dynamics is unitary and the partial trace is a positive map. This is why standard open-system treatments take the CPTP property as a starting axiom (Lindblad, 1976): in the axiomatic setting, CPTP is a derived consequence of two more primitive facts (bulk unitarity and partial trace), not an axiom in its own right. The Kraus decomposition (30) provides the operational handle: each Kraus operator K_α represents a "measurement outcome branch" on the boundary, and the sum-to-identity condition is the operational analog of probability conservation. The deepest content of T3 is that the textbook form of the von Neumann projection postulate ("upon measurement, the state collapses to one of the eigenstates of the measured observable") is just the Kraus decomposition with mutually-orthogonal $K_\alpha = \Pi_\alpha$ projectors. This is the structural reason why no separate "collapse postulate" is needed in the present paper: collapse is just one specific Kraus decomposition of the boundary channel, and the CPTP property guarantees that every other admissible Kraus decomposition is operationally

equivalent.

Problems and worked solutions. II.3.1. Compute the Choi matrix of the qubit depolarizing family $\Phi_p(\rho) = (1-p)\rho + p\,\frac{1}{2}$ and determine exactly for which p the map is a channel. *Solution.* Step 1: build the Choi matrix $C_p = (\mathrm{id} \otimes \Phi_p)|\Phi^+\rangle\langle\Phi^+|$ with $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. Since Φ_p interpolates between the identity (Choi matrix $|\Phi^+\rangle\langle\Phi^+|$) and the completely depolarizing map (Choi matrix $\mathbf{1}_4/4$), $C_p = (1-p)\,|\Phi^+\rangle\langle\Phi^+| + p\,\mathbf{1}_4/4$. Step 2: eigenvalues. On the $|\Phi^+\rangle$ direction: $(1-p) + p/4 = 1 - 3p/4$; on the three orthogonal directions: $p/4$ each (verified numerically at $p = 0.3,\ 1,\ 4/3,\ 1.4$). Step 3: complete positivity is positivity of C_p (Choi, 1975): $p/4 \geq 0$ and $1 - 3p/4 \geq 0$ give $0 \leq p \leq 4/3$. The map is a channel slightly *beyond* $p = 1$: the range $1 < p \leq 4/3$ inverts the Bloch vector part-way, and at $p = 4/3$ the channel is the best CP approximation to the (impossible) universal NOT. Physics intuition: the Choi matrix is the channel run on half of an entangled pair — exactly the situation POS-4 protects — so "CP" is not mathematical pedantry; it is the requirement that the law give sensible probabilities even when the window's system is entangled with something the window cannot see.

II.3.2. From the Stinespring dilation of the amplitude-damping channel, extract the Kraus pair, verify completeness, and prove the composition law $\Phi_{p_2} \circ \Phi_{p_1} = \Phi_{1-(1-p_1)(1-p_2)}$. *Solution.* Step 1: dilate. Let the environment qubit start in $|0\rangle$ and the joint unitary act as $|1\rangle_S|0\rangle_E \mapsto \sqrt{1-p}\,|1\rangle_S|0\rangle_E + \sqrt{p}\,|0\rangle_S|1\rangle_E$ (spontaneous emission with probability p), $|0\rangle_S|0\rangle_E$ invariant. Tracing the environment gives the Kraus pair $K_0 = \mathrm{diag}(1, \sqrt{1-p})$, $K_1 = \sqrt{p}\,|0\rangle\langle 1|$, and $K_0^\dagger K_0 + K_1^\dagger K_1 = \mathbf{1}$ (verified). Step 2: compose. The excited-state amplitude is multiplied by $\sqrt{1-p}$ per stage, so two stages give survival factor $(1-p_1)(1-p_2)$, i.e. damping parameter $p_{12} = 1 - (1-p_1)(1-p_2)$; matrix composition of the Kraus maps confirms this exactly (verified numerically on a general state: $p_1 = 0.35$, $p_2 = 0.5$ compose to $p_{12} = 0.675$). Step 3: note the semigroup seed. Writing $1 - p = e^{-\gamma t}$ turns the composition law into $e^{-\gamma t_1}e^{-\gamma t_2} = e^{-\gamma(t_1+t_2)}$ — the discrete ancestor of the T4 semigroup. Physics intuition: this is T3 in one worked object: the channel arises from a unitary on a larger space (the bulk pattern in miniature), positivity is inherited from the dilation, and iterating the window in time already forces the exponential.

Take-away. *Standard account:* complete positivity is imposed on quantum channels to exclude negative probabilities, and Stinespring's theorem shows every CPTP map dilates to a unitary. *What it left open:* why nature's reduced dynamics should be completely positive in the first place. *The constitutional derivation:* POS-4 makes complete positivity an admissibility predicate: the channel constructed in T2 is automatically CPTP, with Kraus form (T3) — positivity is inherited from the constitution, not imposed on the model.

13 Theorem T4 (T-GDOS-4) — GKLS Generator from the Markovian Limit

Theorem T4 takes the Markovian limit of T3 and shows that the resulting generator must take the Gorini–Kossakowski–Lindblad–Sudarshan (GKLS) form. The constitutional source is POS-5 (RG, Renormalization-Group Coarse-Graining) of the GG Constitution (Arisaka, 2026, A1-Constitution),

which demands that the operational dictionary commute with RG flow; the mathematical realization is the standard GKLS derivation of Lindblad (1976) and Gorini (1976). Under this reading the Markov limit is an axiomatic consequence rather than a working approximation.

Setup: the time-local generator. In the Markovian limit, the memory kernel $K(t,s)$ in (21) becomes sharply peaked around $s = t$, i.e. $K(t,s) \approx 2\mathcal{K}(t)\delta(t-s)$, where $\mathcal{K}(t)$ is a time-local generator. The Nakajima–Zwanzig equation reduces to the time-local master equation

$$\partial_t \rho_{\mathrm{Bdy}}(t) \;=\; \mathcal{L}_{\mathsf{GDOS}}(t)\,\rho_{\mathrm{Bdy}}(t), \tag{36}$$

with $\mathcal{L}_{\mathsf{GDOS}}(t) = \mathcal{P}\mathcal{L}_{\mathrm{Bulk}}(t)\mathcal{P}|_{\mathrm{Bdy}} + \mathcal{K}(t)$. The two-parameter family $\{\Phi_{t,s}\}$ then satisfies the semigroup composition law (31) exactly, and reduces to a one-parameter semigroup $\{\Phi_t\}$ when the bulk Hamiltonian is time-independent.

The Lindblad construction. Lindblad's theorem (Lindblad, 1976) states: a one-parameter semigroup of CPTP maps on $\mathcal{B}(\mathcal{H})$ has, in the case $\dim\mathcal{H} < \infty$, a unique generator $\mathcal{L}$ of the form

$$\boxed{\mathcal{L}_{\mathsf{GDOS}}(\rho) \;=\; -i[H_{\mathrm{eff}},\rho] \;+\; \sum_{\alpha}\gamma_\alpha\Big(L_\alpha\rho L_\alpha^\dagger \;-\; \tfrac{1}{2}\,\{L_\alpha^\dagger L_\alpha,\rho\}\Big),} \tag{37}$$

where $H_{\mathrm{eff}} = H_{\mathrm{eff}}^\dagger$ is the effective Hamiltonian (Hermitian on $\mathcal{H}_{\mathrm{Bdy}}$), $\{L_\alpha\}$ is a set of jump operators (traceless without loss of generality), and $\gamma_\alpha \geq 0$ are non-negative rates. The Gorini–Kossakowski–Sudarshan derivation (Gorini, 1976) shows the same in finite dimension by direct computation of the most general infinitesimal generator that preserves complete positivity.

Born–Markov derivation from T2. We now derive (37) from the exact NZ equation (21) by explicit Born–Markov approximation. The fresh-port input the derivation consumes — the factorized reference $\rho_{\mathrm{Bdy}} \otimes \rho_{\mathrm{env}}^{(0)}$ with vanishing initial-correlation term — is the Port-Freshness Clause of T2 (Definition 5). Assume the boundary–bulk coupling is sufficiently weak that the leading-order expansion of the bulk Hamiltonian $H_{\mathrm{Bulk}} = H_{\mathrm{Bdy}} \otimes \mathbf{1}_{\mathrm{env}} + \mathbf{1}_{\mathrm{Bdy}} \otimes H_{\mathrm{env}} + \lambda H_{\mathrm{int}}$ is justified, with λ a small coupling constant and $H_{\mathrm{int}} = \sum_\alpha A_\alpha \otimes B_\alpha$ the system–environment coupling. To second order in λ, the memory kernel (22) becomes

$$K(t,s) \;\simeq\; -\lambda^2 \sum_{\alpha\beta} \mathrm{Tr}_{\mathrm{env}}\Big[B_\alpha\, e^{-iH_{\mathrm{env}}(t-s)}\, B_\beta\, e^{iH_{\mathrm{env}}(t-s)}\, \rho_{\mathrm{env}}^{(0)}\Big]\,[A_\alpha,\, A_\beta\,\cdot\,] \;+\; \text{Hermitian conjugate}, \tag{38}$$

where $[A_\alpha, A_\beta\,\cdot\,]$ denotes the commutator superoperator $X \mapsto [A_\alpha, A_\beta X]$. Defining the environment correlation functions

$$C_{\alpha\beta}(\tau) \;:=\; \mathrm{Tr}_{\mathrm{env}}\Big[B_\alpha(\tau)\, B_\beta(0)\, \rho_{\mathrm{env}}^{(0)}\Big], \qquad B_\alpha(\tau) := e^{iH_{\mathrm{env}}\tau}\, B_\alpha\, e^{-iH_{\mathrm{env}}\tau}, \tag{39}$$

and assuming that $C_{\alpha\beta}(\tau)$ decays on a timescale τ_{env} short compared to the boundary relaxation timescale, one may extend the upper limit of the convolution integral to infinity (the Markov

approximation) and obtain the time-local generator

$$\mathcal{K}\rho_{\mathrm{Bdy}} \;=\; -\lambda^2 \int_0^\infty d\tau \sum_{\alpha\beta} \Big(C_{\alpha\beta}(\tau)\,[A_\alpha, A_\beta(-\tau)\rho_{\mathrm{Bdy}}] \;+\; C_{\beta\alpha}(-\tau)\,[\rho_{\mathrm{Bdy}} A_\beta(-\tau), A_\alpha] \Big). \tag{40}$$

Secular approximation and the GKLS form. The reduction of (40) to the canonical GKLS form proceeds in three explicit algebraic steps.

Step 1: Eigenoperator decomposition. Define the eigenoperators of the boundary Hamiltonian by

$$A_\alpha(\omega) \;:=\; \sum_{\epsilon'-\epsilon=\omega} \Pi_{\mathrm{Bdy}}(\epsilon)\, A_\alpha\, \Pi_{\mathrm{Bdy}}(\epsilon'), \qquad A_\alpha = \sum_\omega A_\alpha(\omega), \tag{41}$$

where $\Pi_{\mathrm{Bdy}}(\epsilon)$ is the spectral projector of H_{Bdy} at energy ϵ and the sum runs over Bohr frequencies $\omega \in \mathrm{spec}(H_{\mathrm{Bdy}}) - \mathrm{spec}(H_{\mathrm{Bdy}})$. By construction, $[H_{\mathrm{Bdy}}, A_\alpha(\omega)] = -\omega A_\alpha(\omega)$ and $A_\alpha(\omega)^\dagger = A_\alpha(-\omega)$. In the interaction picture this gives $A_\alpha(-\tau) = \sum_\omega e^{-i\omega\tau} A_\alpha(\omega)$.

Step 2: Substitution and term separation. Substituting into the Markovian kernel (40) produces a double sum

$$\mathcal{K}\rho_{\mathrm{Bdy}} \;=\; -\sum_{\omega,\omega';\,\alpha,\beta} e^{i(\omega-\omega')t}\, \Gamma_{\alpha\beta}(\omega)\, \big[A_\alpha(\omega'),\, A_\beta(\omega)\rho_{\mathrm{Bdy}}\big] + \mathrm{h.c.}, \tag{42}$$

where $\Gamma_{\alpha\beta}(\omega) \;=\; \int_0^\infty d\tau\, e^{i\omega\tau}\, C_{\alpha\beta}(\tau)$ is the one-sided Fourier transform of the bath-correlation function.

Step 3: Secular approximation as time-averaging. The reduced dynamics is observed on time scales $T \gg 1/\min_{\omega\neq\omega'} |\omega - \omega'|$. Time-averaging the boundary state over T averages the rapidly oscillating phase $e^{i(\omega-\omega')t}$ to zero whenever $\omega \neq \omega'$, while $\omega = \omega'$ terms survive unchanged. The explicit calculation of the off-resonant magnitude is

$$\left| \frac{1}{T} \int_0^T e^{i(\omega-\omega')t}\, dt \right| \;=\; \left| \frac{e^{i(\omega-\omega')T} - 1}{i(\omega-\omega')T} \right| \;\le\; \frac{2}{|\omega-\omega'|\,T} \;\le\; \frac{2\,\tau_S}{T}, \tag{43}$$

where the second equality uses the elementary integral $\int_0^T e^{i\Omega t}\, dt = (e^{i\Omega T} - 1)/(i\Omega)$, the first inequality bounds $|e^{i\Omega T} - 1| \le 2$, and the second inequality is the definition of $\tau_S = (\min_{\omega\neq\omega'} |\omega - \omega'|)^{-1}$. Multiplying through by the relevant dissipative rate Γ from (40) (the prefactor of the cross-term), the discarded off-resonant contribution is bounded by $\mathcal{O}(\Gamma\,\tau_S/T) \cdot T = \mathcal{O}(\Gamma\,\tau_S)$ in absolute size relative to the resonant cross-term it competes with; the approximation is valid in the canonical regime $\Gamma\tau_S \ll 1$ (see also Breuer, 2002, §3.2.1 for the rigorous treatment). After the time-averaging,

$$\mathcal{K}\rho_{\mathrm{Bdy}} \;=\; \sum_{\omega,\alpha\beta} \gamma_{\alpha\beta}(\omega) \Big(A_\beta(\omega)\, \rho_{\mathrm{Bdy}}\, A_\alpha^\dagger(\omega) \;-\; \tfrac{1}{2}\,\{A_\alpha^\dagger(\omega) A_\beta(\omega), \rho_{\mathrm{Bdy}}\} \Big) \;+\; \big(\text{Lamb shift}\big), \tag{44}$$

where $\gamma_{\alpha\beta}(\omega) = \Gamma_{\alpha\beta}(\omega) + \Gamma_{\beta\alpha}(\omega)^*$ is the symmetrized correlation matrix at the Bohr frequency ω. Diagonalizing the positive-semidefinite matrix $\gamma_{\alpha\beta}(\omega)$ produces a new set of jump operators L_α and rates $\gamma_\alpha \ge 0$, and (44) reduces to the canonical GKLS form (37). This three-step reduction is the standard Bloch–Redfield-to-Lindblad procedure of Breuer (2002), Cohen-Tannoudji (1992).

Lemma 3.1 (Positivity of GKLS rates)**.** *The rates γ_α in (37) are non-negative, $\gamma_\alpha \geq 0$.*

Proof. The Fourier-transformed correlation matrix $\gamma_{\alpha\beta}(\omega)$ in (44) is the spectral density of the environment correlation function (39) at frequency ω. By the Bochner–Khinchin theorem (Breuer, 2002), the spectral density of a positive-definite correlation function is non-negative. Diagonalization of a positive-semidefinite matrix produces non-negative eigenvalues, which are the rates γ_α. ■

Lemma 3.2 (Uniqueness of GKLS generators up to gauge)**.** *Two GKLS generators $\mathcal{L}_{\mathsf{GDOS}}$ and $\mathcal{L}'_{\mathsf{GDOS}}$ are equal as superoperators if and only if they differ by a gauge transformation $L_\alpha \to L_\alpha + c_\alpha \mathbf{1}$, $H_{\text{eff}} \to H_{\text{eff}} + \frac{i}{2}\sum_\alpha (c_\alpha L_\alpha^\dagger - c_\alpha^* L_\alpha)$, where $c_\alpha \in \mathbb{C}$.*

Proof. Direct substitution into (37) shows that the transformation leaves $\mathcal{L}_{\mathsf{GDOS}}$ invariant. The converse (that any two equivalent GKLS forms differ by such a transformation) is the content of (Gorini, 1976). ■

Theorem 4 (T4: GKLS Generator)**.** *Let $\mathcal{T}$ be a candidate theory satisfying T1, T2, T3, plus POS-5 (RG-coarse-graining commutes with $\mathcal{D}_{\mathsf{OGGEP}}$) in its Markov-limit form. Then the boundary evolution channel $\Phi_t = e^{t\mathcal{L}_{\mathsf{GDOS}}}$ has generator $\mathcal{L}_{\mathsf{GDOS}}$ of the GKLS form (37) with non-negative rates $\gamma_\alpha \geq 0$, unique up to the gauge transformation of Lemma 3.2.*

Proof. By T3, Φ_t is CPTP for all $t \geq 0$. By POS-5 in its Markov-limit form, the family $\{\Phi_t\}_{t\geq 0}$ forms a one-parameter semigroup, $\Phi_t \circ \Phi_s = \Phi_{t+s}$. Lindblad's theorem (Lindblad, 1976) (or, in finite dimension, the Gorini–Kossakowski–Sudarshan theorem (Gorini, 1976)) then states that the generator of any one-parameter semigroup of CPTP maps must take the form (37). The non-negativity of the rates is Lemma 3.1; the uniqueness up to gauge is Lemma 3.2. The explicit derivation from the NZ equation is the Born–Markov plus secular calculation (38)–(44). ■

Regime of validity of the Born–Markov derivation. The Born–Markov derivation (38)–(44) is valid in the regime

$$\tau_{\text{env}} \ll \tau_R,\ \tau_S,\ \tau'_S, \tag{45}$$

where τ_{env} is the bath correlation time (set by the inverse spectral width of the bath spectral density), τ_R is the boundary system relaxation time (set by $1/\gamma_\alpha$), τ_S is the boundary system intrinsic oscillation time (set by $1/\omega_0$), and τ'_S is the smallest inverse Bohr-frequency spacing $1/(\omega - \omega')$ relevant to the secular approximation. Outside this regime — when the bath has long-time memory comparable to or longer than the boundary timescales, when the system–bath coupling is strong enough that perturbative expansion fails, or when the Bohr frequencies are nearly degenerate — the GKLS form (37) need not apply, and one must work with the full non-Markovian NZ equation (21) of T2. Examples of physically relevant situations where the Born–Markov approximation fails include (i) ultrafast spectroscopy on the femtosecond timescale, where the bath correlation time can exceed the system response time; (ii) strong-coupling cavity QED, where the system–bath coupling cannot be treated perturbatively; (iii) low-temperature solid-state systems, where the bath spectral density develops sharp resonances; (iv) spin-bath problems, where the bath has finite Hilbert-space dimension and Poincaré recurrences become relevant; and (v) the

bulk–boundary GG-6 setting at high resolution, where individual KK modes can have significant overlap with the boundary system. In all of these cases the axiomatic content of T2 (the exact NZ equation) remains the correct starting point; the GKLS reduction of T4 is a derivative consequence valid only in regime (45). This is why the present paper places T4 *after* T2 rather than treating the GKLS form as a starting axiom: T4 is the Markov-limit corollary, not the foundation.

Detailed balance, and what it decides. Inside regime (45), and with a single thermal bath, the generator of T4 is a weak-coupling generator in Davies's sense (Davies, 1974), and a Davies generator satisfies quantum detailed balance with respect to the bath's Gibbs state (Kossakowski, 1977). That is the hypothesis, and the only one, under which the relative entropy that descends along the boundary evolution — Spohn's theorem for the density operator (§69.4), Lemma 5.1 for its Fokker–Planck shadow — is a free energy of the boundary Hamiltonian rather than a Lyapunov function with no thermodynamic name. With several baths at different temperatures, or outside the regime, the stationary state exists but is not a Gibbs state, and the second law survives only as $\dot{S}_{\mathrm{tot}} \geq 0$.

Worked example: two-level atom in a damped vacuum. Consider a two-level atom (boundary system) coupled to a continuum of electromagnetic modes in their vacuum state (environment), with boundary Hamiltonian $H_{\mathrm{eff}} = \frac{\omega_0}{2}\sigma_z$ and a single jump operator $L = \sigma_-$ representing spontaneous emission of a photon into the bulk. The GKLS master equation (37) becomes

$$\partial_t \rho_{\mathrm{Bdy}} \;=\; -i\big[\tfrac{\omega_0}{2}\sigma_z,\, \rho_{\mathrm{Bdy}}\big] \;+\; \gamma\Big(\sigma_-\rho_{\mathrm{Bdy}}\sigma_+ \;-\; \tfrac{1}{2}\{\sigma_+\sigma_-, \rho_{\mathrm{Bdy}}\}\Big), \tag{46}$$

with rate $\gamma > 0$ the spontaneous-emission rate inherited from the environment spectral density at frequency ω_0. In the Bloch representation $\rho_{\mathrm{Bdy}} = \frac{1}{2}(\mathbf{1} + \vec{r}\cdot\vec{\sigma})$, the master equation (46) is equivalent to the optical Bloch equations

$$\dot{r}_x = -\omega_0 r_y - \tfrac{\gamma}{2} r_x, \qquad \dot{r}_y = +\omega_0 r_x - \tfrac{\gamma}{2} r_y, \qquad \dot{r}_z = -\gamma(1 + r_z), \tag{47}$$

whose solution decays the Bloch vector toward the south pole (the ground state $|0\rangle\langle 0|$) at rate γ for the z-component and $\gamma/2$ for the transverse components. The factor of two between the longitudinal and transverse damping rates is the celebrated $T_1 = 2T_2$ identity in the absence of pure dephasing, which is a direct axiomatic consequence of the GKLS structure (37) with a single Hermitian-conjugate-pair jump operator. The Lindblad-channel rates γ in the GG-6 setting are computed from KK overlap integrals in Theorem T9 of Part V; the present example illustrates the canonical operator structure that those overlap integrals fill in.

Remark 6 (Physics intuition for T4)**.** The GKLS form (37) is, structurally, the most general linear evolution that preserves both the Hermiticity and the unit trace of a density operator while remaining completely positive. The first term $-i[H_{\mathrm{eff}}, \rho]$ is the unitary part; if $\gamma_\alpha = 0$ for all α, this is just the Schrödinger equation in density-matrix form. The Lindblad part $\gamma_\alpha L_\alpha \rho L_\alpha^\dagger$ is the "jump" or "loss" part: each jump operator L_α represents a specific channel of system–environment exchange (e.g., spontaneous emission of a photon, decoherence into a pointer basis, or dissipation

into a thermal bath), with rate γ_α. The anti-commutator term $\frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\}$ is the corresponding "no-jump" part: it ensures that the total probability remains conserved when summed over all admissible outcomes. The deepest content of T4 is that the GKLS form is the *constitutionally forced* boundary law in the Markovian regime — not because we imposed the GKLS form, but because Lindblad's theorem shows that no other functional form is consistent with semigroup composition plus complete positivity. The Schrödinger equation ($\gamma_\alpha = 0$) is the closed-system limit; the GKLS equation ($\gamma_\alpha > 0$) is the open-system law. In the GG-Theory framework, the closed-system limit is an idealization that no actual finite observer ever realizes (every detector couples to its environment); the open-system law is the operationally registered truth. This is the axiomatic reason for the methodological inversion announced in §6.

Remark 7 (The dissipative generator is the scale-fiber renormalization-group flow)**.** The irreversibility just established has a geometric origin worth recording, because it tightens the link between OGGEP and the bulk geometry of GG-A2-GG6. The GKLS generator splits into a unitary part $-i[H_{\text{eff}}, \rho]$ and a dissipative part $\sum_\alpha \gamma_\alpha(L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\})$; it is the dissipative part that makes the family $\{\Phi_t\}$ a one-parameter *semigroup* rather than a group — it has no inverse.

In the six-dimensional bulk this loss of invertibility is not an accident of the Markovian limit but the boundary image of motion along the *scale* dimension I_r. Theorem 0 of GG-A2-GG6 fixes I_r as non-compact precisely because the renormalization-group flow it geometrizes is a forward semigroup: integrating out short-distance modes cannot be undone. Accessing a system at finite resolution is exactly the tracing-out of the inaccessible part of this scale fiber, and the resulting non-invertibility is the dissipative part of the generator above.

The *arrow* of the open-system law is therefore the *arrow* of the renormalization group, and the GKLS generator may be read as the dilatation generator along I_r projected to the boundary. The complementary role of the compact phase dimension S^1_θ — which carries the coherent, unitary content, its holonomy being the geometric (Aharonov–Bohm/Berry) phase — and the full fiber-bundle reading of the bulk-to-boundary projection $\pi : \mathcal{M}_6 \to \mathcal{M}_4$ as the origin of quantum behavior are developed in the companion paper (Arisaka, 2026, A8-QM) (§"Why Observation Is Projection: The Hidden-Fiber Origin of Quantum Behavior").

Problems and worked solutions. II.4.1. Solve the pure-dephasing GKLS equation $\dot\rho = \gamma(\sigma_z\rho\sigma_z - \rho)$ exactly and verify the semigroup property. *Solution.* Step 1: solve. In the σ_z eigenbasis the populations satisfy $\dot\rho_{00} = \dot\rho_{11} = 0$ and the coherence satisfies $\dot\rho_{01} = \gamma(-\rho_{01} - \rho_{01}) = -2\gamma\rho_{01}$, so $\rho_{01}(t) = \rho_{01}(0)\, e^{-2\gamma t}$: populations frozen, coherences dying at rate 2γ. Step 2: semigroup. $\Phi_t \circ \Phi_s$ multiplies the coherence by $e^{-2\gamma t}e^{-2\gamma s} = e^{-2\gamma(t+s)}$, which is Φ_{t+s} (verified numerically). Step 3: check the GKLS data. Jump operator $L = \sqrt{\gamma}\,\sigma_z$, rate $\gamma \geq 0$, Hamiltonian part zero — the canonical form of Eq. (37) with one Lindblad channel. Physics intuition: this is the minimal instance of the boundary law — nothing moves, nothing relaxes, yet quantum information leaks; "which-path" information crossing the boundary is enough, and energy exchange is not required.

II.4.2. Verify the gauge freedom of the GKLS form: show explicitly that the shift $L_\alpha \to L_\alpha + c_\alpha \mathbf{1}$, $H_{\text{eff}} \to H_{\text{eff}} + \frac{1}{2i}\sum_\alpha \gamma_\alpha\,(c_\alpha^* L_\alpha - c_\alpha L_\alpha^\dagger)$ (plus an irrelevant constant) leaves the generator invariant. *Solution.* Step 1: expand the dissipator under $L \to L + c\mathbf{1}$ (one channel,

rate γ): $\mathcal{D}[L+c](\rho) = \mathcal{D}[L](\rho) + \gamma(c\,\rho L^\dagger + c^* L\rho - \frac{1}{2}\{c^*L + cL^\dagger, \rho\})$, using $\mathbf{1}$-terms canceling in the anticommutator. Step 2: reorganize the extra piece. The bracket equals $\frac{1}{2}[\,c^*L - cL^\dagger,\, \rho\,]$, a *commutator* — i.e. a Hamiltonian contribution $-i[\Delta H, \rho]$ with $\Delta H = \frac{\gamma}{2i}(c^*L - cL^\dagger)$. Step 3: absorb. The stated compensating shift of H_{eff} cancels it exactly, so the total generator is unchanged; the "dissipative" and "Hamiltonian" labels move, the physics does not — which is the content of the uniqueness Lemma of T4. Physics intuition: the split of the boundary law into drift and noise is a bookkeeping convention, like the split of a connection into potential and gauge; only the full generator — like only the curvature — is physical.

Take-away. *Standard account:* the GKLS/Lindblad form is derived under Born–Markov assumptions adopted per model, and the non-negativity of the rates is checked case by case. *What it left open:* why the Markovian limit is the natural coarse-graining, and why the rates must be non-negative. *The constitutional derivation:* POS-5 — RG coarse-graining commutes with the dictionary — forces the GKLS generator with $\gamma_\alpha \geq 0$, unique up to gauge (T4); the Markov approximation is promoted from a modeling convenience to a constitutional limit.

14 Theorem T5 (T-GDOS-5) — The GDOS Continuity Equation (Mesoscopic Coarse-Graining)

Theorem T5 takes the mesoscopic coarse-graining limit of T4 and shows that the GKLS generator descends to a continuity equation on the boundary state manifold with universal gradient-plus-curl-plus-noise current decomposition. The constitutional source is POS-2 (UEP, the Unification Limit Principle, which requires variational expressibility of admissible boundary content) of the GG Constitution (Arisaka, 2026, A1-Constitution). The mathematical realization is the classical-statistical mesoscopic limit of an open quantum system, expressed as a probability-density flow on a state manifold. The original inventors of this machinery were Fokker (1914) and Planck (1917), who first wrote down the drift–diffusion equation that today carries their names; Kramers (1940) and Moyal (1949) extended their construction by deriving the Kramers–Moyal expansion that systematically truncates higher-order moments to recover the Fokker–Planck regime; the modern textbook synthesis is by Risken (1989), and the open-system variational reading we adopt is in the sense of Friston (2010). This is the layer of GDOS that connects to the Gi-4 Pillars (Geometric Free-Energy Functional, Gauge-Runtime Invariance Pipeline) of GG-A6-GRIP.

Mesoscopic representation. Represent the boundary state by a probability density $p(x,t)$ on a state manifold $\mathcal{M}_{\text{state}}$ with local coordinates x^A, $A = 1, \ldots, N$. This representation is appropriate for any boundary system whose density operator $\rho_{\text{Bdy}}(t)$ admits a quasi-classical phase-space (Wigner or Husimi) description in the relevant regime, which by POS-5 is the regime accessible to coarse-grained registration. $\mathcal{M}_{\text{state}}$ is taken compact, and it is worth naming what it is: the coarse-grained phase space of the registered observables — a finite-dimensional truncation of the boundary field theory, valid in the Born–Markov regime (45) of T4 — and not the configuration space of the boundary field theory itself. Every convergence statement in this section, the Lyapunov lemma and the uniqueness of its fixed point included, is a statement on that compact object. The bulk

Hamiltonian H_{eff} in the GKLS generator (37) descends to a smooth potential $\Phi(x)$ on $\mathcal{M}_{\text{state}}$, the dissipative part descends to a diffusion tensor $D^{AB}(x)$ and a drift vector $b^A(x)$.

Kramers–Moyal expansion. The GKLS generator (37) acts on the boundary density operator; its mesoscopic shadow acts on $p(x,t)$ via the Kramers–Moyal expansion (Risken, 1989). Truncating at second order (the Fokker–Planck regime, valid when higher-order moments of the system–environment coupling are negligible),

$$\partial_t p(x,t) \;=\; -\partial_A\big[b^A(x)\,p(x,t)\big] \;+\; \partial_A\partial_B\big[D^{AB}(x)\,p(x,t)\big], \tag{48}$$

which is the Fokker–Planck equation in flat coordinates. Re-expressing (48) on a curved state manifold with metric $g_{AB}(x)$ and using the metric-compatible covariant derivative ∇_A, we arrive at the manifestly covariant continuity form

$$\boxed{\partial_t p(x,t) \;=\; -\nabla_A J^A(x,t),} \tag{49}$$

with current

$$J^A \;=\; b^A\,p \;-\; D^{AB}\nabla_B p. \tag{50}$$

Gradient + curl + noise decomposition. The drift vector b^A admits a unique decomposition into a gradient part and a divergence-free part:

$$b^A \;=\; -g^{AB}\nabla_B\Phi \;+\; h^A, \qquad \nabla_A h^A \;=\; 0, \tag{51}$$

where Φ is a scalar potential on $\mathcal{M}_{\text{state}}$ (the effective "Helmholtz free-energy" inherited from H_{eff} plus the dissipative contribution to the drift) and h^A is a divergence-free "circulation" vector. This decomposition is the Helmholtz–Hodge theorem on the state manifold; it is unique up to a constant in Φ and a closed-form addition to h^A. Substituting (51) into (50) gives the canonical GDOS form

$$\boxed{J^A \;=\; -D^{AB}\nabla_B p \;+\; \big(-g^{AB}\nabla_B\Phi + h^A\big)\,p, \quad \nabla_A h^A \;=\; 0.} \tag{52}$$

This is the universal current geometry of any constitutionally admissible open-system boundary dynamics: a noise/diffusion piece $-D^{AB}\nabla_B p$, a gradient-relaxation piece $-g^{AB}\nabla_B\Phi\,p$, and a divergence-free circulation piece $h^A p$.

Entropy production. The Kullback–Leibler divergence of p from the equilibrium density $p_{\text{eq}} \propto e^{-\Phi}$ supplies an obvious Lyapunov function in the gradient-only sector ($h^A = 0$). In the general case (52) the appropriate entropy production rate is

$$\dot{S}_{\text{tot}} \;=\; \int_{\mathcal{M}_{\text{state}}} d\mu(x)\,\frac{J^A (D^{-1})_{AB} J^B}{p} \;\geq\; 0, \tag{53}$$

where $d\mu(x) = \sqrt{|g(x)|}\,d^N x$ is the canonical Riemannian measure on $\mathcal{M}_{\text{state}}$. Non-negativity of $\dot{S}_{\text{tot}}$ follows from the positive-semidefiniteness of D^{AB}, which descends from the non-negativity of the

GKLS rates $\gamma_\alpha \geq 0$ established in Lemma 3.1. This is the axiomatic second-law analog: irreversible boundary projection of unitary bulk evolution produces a non-negative entropy-production rate.

Theorem 5 (T5: GDOS Continuity Equation)**.** *Let $\mathcal{T}$ be a candidate theory satisfying T1–T4. In the mesoscopic coarse-graining limit (justified by POS-2 plus POS-5), the boundary density $p(x,t)$ on the state manifold $\mathcal{M}_{\text{state}}$ obeys the continuity equation (49) with current (52) subject to $\nabla_A h^A = 0$, and the entropy production rate (53) is non-negative.*

Proof. The mesoscopic representation of the boundary state is justified by POS-5 (RG-covariance): at the coarse-grained scale appropriate to laboratory registration, the boundary density operator admits a Wigner or Husimi quasi-classical representation, with the higher-order Kramers–Moyal coefficients negligibly small. The Fokker–Planck form (48) is then the second-order truncation of the Kramers–Moyal expansion of the GKLS generator (37) (cf. (Breuer, 2002, Risken, 1989)). The covariant continuity form (49) follows by re-expressing the Fokker–Planck equation on a curved state manifold with metric inherited from the GKLS-Riemannian structure of (Chruściński, 2009). The Helmholtz–Hodge decomposition (51) is a standard theorem on smooth manifolds. Non-negativity of the entropy-production rate (53) follows from $D^{AB} \succeq 0$, inherited from the GKLS rates $\gamma_\alpha \geq 0$ via Lemma 3.1. ■

Lemma 5.1 (Lyapunov function for the gradient sector)**.** *Let $p(x,t)$ be a solution of the GDOS continuity equation (49) with current (52) in the purely-gradient case ($h^A = 0$). Let $p_{\text{eq}}(x) \propto e^{-\Phi(x)/T}$ be the Boltzmann equilibrium distribution at effective temperature T, normalized by $\int_{\mathcal{M}_{\text{state}}} d\mu(x)\, p_{\text{eq}}(x) = 1$. Define the relative entropy (Kullback–Leibler divergence)*

$$\mathcal{F}[p] \;:=\; T \int_{\mathcal{M}_{\text{state}}} d\mu(x)\; p(x)\; \log\!\left(\frac{p(x)}{p_{\text{eq}}(x)}\right) \;\geq\; 0. \tag{54}$$

Then $\mathcal{F}[p]$ is a Lyapunov function for the GDOS gradient flow:

$$\frac{d}{dt}\,\mathcal{F}[p(t)] \;=\; -T \int_{\mathcal{M}_{\text{state}}} d\mu(x)\; p(x)\, D^{AB} (\nabla_A \log(p/p_{\text{eq}}))(\nabla_B \log(p/p_{\text{eq}})) \;\leq\; 0, \tag{55}$$

with equality if and only if $p = p_{\text{eq}}$.

Proof. Direct differentiation of (54) using $\partial_t p = -\nabla_A J^A$ and integration by parts, together with $J^A = -p_{\text{eq}}\, D^{AB} \nabla_B (p/p_{\text{eq}})$ in the gradient case (which follows from the detailed-balance condition $b^A = -g^{AB}\nabla_B \Phi = -T D^{AB} \nabla_B \log p_{\text{eq}}$, giving the diffusion-with- drift current the manifestly equilibrium-relative form), yields (55). Non-negativity of the integrand follows from $D^{AB} \succeq 0$. Equality $\dot{\mathcal{F}} = 0$ requires $\nabla_A \log(p/p_{\text{eq}}) = 0$ everywhere, i.e. $p \propto p_{\text{eq}}$, and normalization forces equality. ■

Lemma 5.1 is the GDOS analog of the H-theorem of classical statistical mechanics, and it is worth saying at which level it holds. One level up, for the density operator itself, the relative entropy to *any* stationary state of a completely positive semigroup is non-increasing (Lindblad, 1975, Spohn, 1978); the lemma is the Fokker–Planck coarse-graining of that statement, with the Boltzmann weight as the stationary state — the H-theorem of the Fokker–Planck equation (Risken, 1989), in the gradient-flow

reading of Jordan (1998). That the descending quantity is a *free energy*, and not merely a Lyapunov function, is what the gradient-only hypothesis $h^A = 0$ buys: it is the detailed-balance condition, and the quantum form of that condition is stated at T4 below (Kossakowski–Frigerio–Gorini–Verri). In the absence of circulation, then, $\mathcal{F}[p]$ is a strictly decreasing function of time along any solution of the GDOS continuity equation, and reaches its minimum (zero) at the equilibrium Boltzmann distribution. This is the constitutional source of the second law in the GG-Theory framework: irreversible boundary projection produces a relative entropy that cannot increase, and the unique fixed point of the resulting flow is thermal equilibrium. When circulation is present ($h^A \neq 0$), $\mathcal{F}[p]$ is no longer monotone and the boundary need not reach equilibrium at all: it settles into a non-equilibrium steady state in which the gradient-driven entropy decrease is balanced by the circulation-driven entropy production, and what survives of the second law is the *total* entropy production rate (53), $\dot{S}_{\text{tot}} \geq 0$, and nothing stronger. The Geometric Free-Energy Functional (GFF) of GG-A6-GRIP specializes Lemma 5.1 to the family-projector moduli space, where it is the foundation for the Morse-theoretic counting argument in the $N_g = 3$ proof of Arisaka-Generations. The identifications that make that specialization a thermodynamics — the lemma's potential with the geometric block, $U = \mathcal{F}_{\mathsf{GEP}}$, and its temperature with the Kaluza–Klein scale, $T_{\mathsf{GFF}} \sim M_{\text{KK}}$ — are GG-A6-GRIP's Geometric First Law (Arisaka, 2026, A6-GRIP), imported here and not derived here. The three readings of the functional itself — trajectory, current, inferential — are the subject of the first supplement (Arisaka, 2026, S1-EP).

Worked example: Ornstein–Uhlenbeck process as the simplest GDOS solution. The simplest non-trivial GDOS continuity equation arises for a single overdamped degree of freedom $x \in \mathbb{R}$ in a quadratic potential $\Phi(x) = \frac{1}{2}\kappa x^2$, with constant diffusion $D > 0$ and no circulation ($h = 0$). The current (52) reduces to $J = -D\,\partial_x p - \kappa x\, p$, and the continuity equation (49) becomes the Ornstein–Uhlenbeck Fokker–Planck equation

$$\partial_t p(x,t) \;=\; \partial_x\big[D\,\partial_x p + \kappa x\, p\big] \;=\; \kappa\,\partial_x(xp) + D\,\partial_x^2 p, \tag{56}$$

whose stationary solution is the Boltzmann distribution $p_{\text{eq}}(x) = \sqrt{\kappa/(2\pi D)}\,\exp(-\kappa x^2/(2D))$. The relaxation time is $\tau = 1/\kappa$ and the variance saturates at $\langle x^2\rangle_{\text{eq}} = D/\kappa$. This is the simplest possible GDOS flow, and it serves as the operational template for many of the boundary continuity equations that arise in the GG-6 setting — in particular, the family-projector dynamics on the Wilson-line moduli of Arisaka-Generations is structurally a multi-dimensional analog of (56) with Φ replaced by the Geometric Free-Energy Functional GFF of GG-A6-GRIP. When circulation is added ($h^A \neq 0$), the steady-state distribution remains the Boltzmann form in the gradient direction, but a non-zero probability current $J^A_{\text{circ}} = h^A p_{\text{eq}}$ persists in the divergence-free direction — this is the canonical mathematical model of a non-equilibrium steady state.

Remark 8 (Physics intuition for T5)**.** The continuity equation (49) with current (52) is the classical-statistical face of GDOS. The reader who is more comfortable with classical statistical mechanics than with open-quantum theory may take (49) as the working definition of GDOS and treat T2–T4 as the derivation of why this classical-statistical equation has the form it does. The three terms in the current have direct physical interpretations: the diffusion piece $-D^{AB}\nabla_B p$ is the noise-driven

equilibration toward the maximum-entropy distribution; the gradient piece $-g^{AB}\nabla_B\Phi\, p$ is the deterministic descent toward the minimum of the effective potential Φ; the circulation piece $h^A p$ is the divergence-free transport that does not move probability density into or out of any region but circulates within regions. The non-equilibrium steady states of GDOS are exactly the configurations in which the gradient piece and the diffusion piece balance while the circulation piece runs forever — this is the mathematical content of, e.g., the cell cycle, the Krebs cycle, and the non-equilibrium cycles of life. The entropy production rate (53) is positive whenever any of the three pieces is non-trivial, providing the axiomatic second-law analog. The Geometric Free-Energy Functional (GFF) of GG-A6-GRIP is, structurally, the potential Φ together with its Riemannian-volume-corrected entropic counterpart; the Gauge-Runtime Invariance Pipeline (GRIP) is, structurally, the cascade dynamics that selects the long-time fixed points of the GDOS flow on $\mathcal{M}_{\text{state}}$. These connections are made rigorous in T10 of Part VII.

Problems and worked solutions. II.5.1. For the current of Eq. (52) in one dimension with constant D and $h = 0$, show that $p_{\text{eq}}(x) \propto e^{-\Phi(x)/D}$ is stationary with *zero* current; then show that switching on a divergence-free circulation $h \neq 0$ (two or more dimensions) leaves p_{eq} stationary but makes the current non-zero. *Solution.* Step 1: one dimension. $J = -D\,\partial_x p - (\partial_x\Phi)\,p$; substituting $p_{\text{eq}} \propto e^{-\Phi/D}$ gives $\partial_x p_{\text{eq}} = -(\partial_x\Phi/D)\,p_{\text{eq}}$, hence $J = D\,(\partial_x\Phi/D)\,p_{\text{eq}} - (\partial_x\Phi)\,p_{\text{eq}} = 0$: detailed balance, equilibrium. Step 2: add circulation. In $n \geq 2$ dimensions take $J^A = -D\,\nabla^A p - (\nabla^A\Phi)\,p + h^A p$ with $\nabla_A h^A = 0$ and $h^A\nabla_A\Phi = 0$ (circulation along equipotentials). The first two terms still cancel on p_{eq}; the remainder is $J^A = h^A p_{\text{eq}} \neq 0$, while $\nabla_A J^A = p_{\text{eq}}\nabla_A h^A + h^A\nabla_A p_{\text{eq}} = 0 - (p_{\text{eq}}/D)\,h^A\nabla_A\Phi = 0$. Step 3: classify. The state is stationary but not in equilibrium: a probability current circulates forever — a non-equilibrium steady state. Physics intuition: this is the mesoscopic face of the T5 decomposition — gradient flow relaxes, circulation persists; a storage-ring beam and a planetary orbit are the same mathematics wearing different units.

II.5.2. Derive the entropy-production formula behind Eq. (53): show $\dot{S} \geq 0$ with equality exactly at detailed balance, and show the circulation current contributes nothing. *Solution.* Step 1: differentiate $S = -\int p \ln p$. Using the continuity equation and one integration by parts (no boundary terms), $\dot{S} = -\int(\ln p)\,\partial_t p = -\int J^A\,\nabla_A p/p$. Step 2: substitute the current split $J^A = J^A_{\text{diss}} + h^A p$ with $J^A_{\text{diss}} = -D\,(\nabla^A p + p\,\nabla^A\Phi/D)$. The circulation part contributes $-\int h^A\nabla_A p = +\int p\,\nabla_A h^A = 0$. For the dissipative part, write $\nabla_A p = -(J_{\text{diss},A} + p\nabla_A\Phi)/D$; then $\dot{S} = \int \frac{J^2_{\text{diss}}}{D\,p} + \frac{1}{D}\int J^A_{\text{diss}}\nabla_A\Phi$, and the second integral is the (negative of the) heat flux $\dot{Q}/D$ exchanged with the potential; the *total* entropy production $\dot{S}_{\text{tot}} = \dot{S} - \dot{Q}/D = \int J^2_{\text{diss}}/(Dp) \geq 0$. Step 3: equality. $\dot{S}_{\text{tot}} = 0$ iff $J_{\text{diss}} = 0$ everywhere — detailed balance, the equilibrium of II.5.1. Physics intuition: the second law here is a Cauchy–Schwarz statement about the window, not a new axiom — irreversibility is the square of the dissipative current, and only the dissipative sector can produce entropy, exactly the sector split T5 derives.

Take-away. *Standard account:* mesoscopic physics is described by Fokker–Planck and Langevin equations introduced phenomenologically, with entropy production checked model by model. *What it left open:* why the current always decomposes into gradient, circulation, and noise, and why entropy production is non-negative. *The constitutional derivation:* the GKLS generator descends, under POS-2 and POS-5, to a continuity equation with the universal three-current decomposition and $\dot{S} \geq 0$ built in (T5) — the second law arrives as geometry, not as an additional postulate.

15 Theorem T6 (T-GDOS-6) — GDOS as the Canonical Boundary Law

Theorem T6 is the unification statement of Part II. It collects T1–T5 into a single theorem stating that GDOS, in any of three equivalent formulations (operator/GKLS, mesoscopic/continuity, classical/Langevin), is the canonical constitutionally admissible boundary law of any GG-6-compatible bulk theory.

The three equivalent formulations. T2–T5 produce three formulations of the same boundary law, valid at three nested levels of coarse-graining.

(1) Operator/GKLS formulation (Theorem T4). The boundary density operator $\rho_{\mathrm{Bdy}}(t)$ on $\mathcal{H}_{\mathrm{Bdy}}$ obeys the GKLS master equation

$$\partial_t \rho_{\mathrm{Bdy}} \;=\; \mathcal{L}_{\mathsf{GDOS}}(\rho_{\mathrm{Bdy}}) \;=\; -i[H_{\mathrm{eff}}, \rho_{\mathrm{Bdy}}] \;+\; \sum_{\alpha} \gamma_\alpha \Big(L_\alpha \rho_{\mathrm{Bdy}} L_\alpha^\dagger \;-\; \tfrac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho_{\mathrm{Bdy}}\} \Big). \tag{57}$$

(2) Mesoscopic/continuity formulation (Theorem T5). The probability density $p(x,t)$ on the state manifold $\mathcal{M}_{\mathrm{state}}$ obeys the continuity equation

$$\partial_t p \;=\; -\nabla_A J^A, \qquad J^A \;=\; -D^{AB}\nabla_B p \;+\; \left(-g^{AB}\nabla_B \Phi + h^A\right) p, \quad \nabla_A h^A = 0. \tag{58}$$

(3) Classical/Langevin formulation. A representative trajectory $x^A(t)$ on $\mathcal{M}_{\mathrm{state}}$ obeys the stochastic differential equation

$$dx^A \;=\; \left(-g^{AB}\nabla_B \Phi + h^A\right) dt \;+\; \sqrt{2D^{AB}}\, dW_B(t), \tag{59}$$

where $dW_B(t)$ is the standard Wiener process and the noise matrix satisfies $\sqrt{2D^{AB}}^{T} \sqrt{2D^{AB}} = 2D^{AB}$.

Equivalence of the three formulations. The Langevin form (59) reproduces the Fokker–Planck form (58) via the standard Itô-to-Fokker–Planck correspondence (van Kampen, 2007, Risken, 1989). The Fokker–Planck form (58) reproduces the GKLS form (57) in the appropriate Wigner/Husimi representation (Breuer, 2002). The three formulations are therefore mathematically equivalent, with the choice between them determined by the level of coarse-graining appropriate to the registered observable.

Theorem 6 (T6: GDOS as the Canonical Boundary Law)**.** *Let $\mathcal{T}$ be a candidate theory satisfying AX-1, AX-2, AX-3, AX-4 and POS-1 through POS-7. Then the boundary content of $\mathcal{T}$, as registered by any finite observer satisfying AX-3 and POS-7, is the boundary law GDOS in any of the three equivalent formulations (57), (58), (59). The choice between the three is determined by the level of coarse-graining appropriate to the registered observable, and the three are mathematically equivalent in the appropriate phase-space limits.*

Proof. By T1, the operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ exists. By T2, $\mathcal{D}_{\mathsf{OGGEP}}$ is constructed as the partial trace plus the boundary reduction map of (8), with the boundary state obeying the exact NZ memory-kernel equation (21). By T3, the resulting reduced channel $\Phi_{t,s}$ is automatically CPTP. By T4, in the Markovian limit (justified by POS-5) the channel forms a one-parameter semigroup whose generator is uniquely of the GKLS form (57). By T5, in the mesoscopic coarse-graining limit (justified by POS-2) the GKLS generator descends to the continuity equation (58) with current decomposition. The Langevin form (59) is the Itô stochastic equivalent of (58) (Risken, 1989). Equivalence of the three formulations in their respective regimes of validity is the standard content of open-system theory (Breuer, 2002). ■

Corollary 6.1 (Closed-system limit)**.** *In the limit $\gamma_\alpha \to 0$ (decoupling of the boundary system from the environment), the GKLS generator (57) reduces to the unitary von Neumann equation $\partial_t \rho_{\mathrm{Bdy}} = -i[H_{\mathrm{eff}}, \rho_{\mathrm{Bdy}}]$, which for pure states $\rho_{\mathrm{Bdy}} = |\psi\rangle\langle\psi|$ is equivalent to the Schrödinger equation $i\partial_t|\psi\rangle = H_{\mathrm{eff}}|\psi\rangle$. The mesoscopic/continuity form (58) reduces in the same limit to the Liouville equation $\partial_t p + h^A \partial_A p = 0$, which is the classical Hamiltonian flow on $\mathcal{M}_{\mathrm{state}}$ with h^A the symplectic vector field of Φ. Closed-system dynamics is therefore the $\gamma_\alpha \to 0$ limit of GDOS, not a separate primitive theory.*

Worked example: equivalence of operator and Langevin pictures for a damped harmonic oscillator. Consider a single damped quantum harmonic oscillator $H_{\mathrm{eff}} = \omega\, a^\dagger a$ with two jump operators $L_1 = \sqrt{\gamma(\bar{n}+1)}\, a$ (down-jump) and $L_2 = \sqrt{\gamma\bar{n}}\, a^\dagger$ (up-jump), where $\bar{n}$ is the mean thermal occupation. The GKLS master equation (57) yields, via the Wigner representation, a Fokker–Planck equation on the phase-space coordinates (q, p) of the form (58) with potential $\Phi(q,p) = \frac{1}{2}(\omega^2 q^2 + p^2)$, diffusion tensor $D^{AB} = \frac{\gamma}{2}(2\bar{n}+1)\,\delta^{AB}$, and circulation $h^A = \omega(p\,\partial/\partial q - q\,\partial/\partial p)$ (the symplectic Hamiltonian flow). The Langevin form (59) is the standard quantum-Brownian SDE

$$\dot{q} = p - \tfrac{\gamma}{2} q + \xi_q(t), \qquad \dot{p} = -\omega^2 q - \tfrac{\gamma}{2} p + \xi_p(t), \tag{60}$$

with white noise $\langle \xi_a(t)\xi_b(t')\rangle = \gamma(2\bar{n}+1)\,\delta_{ab}\delta(t-t')$. All three formulations describe the same physical system; the choice between them is determined by which observables are registered. An interferometric experiment measuring quadrature variances reads off the Wigner formulation; a single-shot homodyne trajectory reads off the Langevin formulation; a quantum-tomography reconstruction of $\rho_{\mathrm{Bdy}}(t)$ reads off the operator formulation. All three are the same axiomatic theorem under different coarse-grainings.

Remark 9 (Physics intuition for T6)**.** T6 is the keystone of Part II. Its content can be summarized in a single sentence: *the canonical boundary law of any constitutionally admissible bulk theory is*

GDOS*, in any of three equivalent formulations corresponding to three nested levels of coarse-graining.* The operator/GKLS form (57) is the form most familiar to quantum-information theorists; the mesoscopic/continuity form (58) is the form most familiar to non-equilibrium statistical physicists and to Friston-style information-geometric biologists (Friston, 2010); the Langevin form (59) is the form most familiar to applied scientists and engineers. The equivalence of these three formulations under appropriate coarse-graining is well known in the respective communities; what is novel in the axiomatic setting is that all three are derived from a single chain of constitutional clauses (AX-3, POS-2, POS-4, POS-5, POS-7), with no clause invoked twice and no extra assumption introduced anywhere in the chain. The historical pattern noted in §6 is exemplified by T6: just as Lagrange's (Lagrange, 1788) reformulation of Newtonian mechanics in variational form revealed the symmetry-conservation link of mechanics, the GDOS reformulation of open-system boundary content reveals the axiomatic unity of the operator, mesoscopic, and Langevin formulations. All three are the same theorem under different coarse- grainings, and the axiomatic setting makes that visible.

Problems and worked solutions. II.6.1. Derive the Fokker–Planck member of the T6 triple from the Langevin member: starting from $dx = -\Phi'(x)\,dt + \sqrt{2D}\,dW$, obtain $\partial_t p = \partial_x(p\,\Phi') + D\,\partial_x^2 p$. *Solution.* Step 1: Itô's lemma. For a smooth test function f, $df = f'\,dx + \frac{1}{2}f''\,(dx)^2 = (-f'\Phi' + Df'')dt + f'\sqrt{2D}\,dW$, using $(dW)^2 = dt$. Step 2: average. $\frac{d}{dt}\langle f\rangle = \langle -f'\Phi' + Df''\rangle = \int p\,(-f'\Phi' + Df'')\,dx$. Step 3: integrate by parts twice, discarding boundary terms: $\int f\,[\partial_x(p\,\Phi') + D\,\partial_x^2 p]\,dx$ for all f, which forces the Fokker–Planck equation — the one-dimensional form of Eq. (58). Physics intuition: the trajectory picture and the density picture are one law differentiated in opposite directions; T6's claim is that the operator picture joins them, with the boundary supplying the noise that Itô merely postulates.

II.6.2. The three formulations on one system: for the damped mode ($H = \omega a^\dagger a$, single jump operator $L = a$, rate γ), compute the mean amplitude in the operator formulation and check it against the Langevin formulation. *Solution.* Step 1: operator level. $\frac{d}{dt}\langle a\rangle = \mathrm{Tr}[a\,\mathcal{L}(\rho)]$; the commutator term gives $-i\omega\langle a\rangle$, and the dissipator gives $\gamma\,\mathrm{Tr}[a^\dagger a\,a\,\rho - \frac{1}{2}\{a^\dagger a, a\}\rho]$; using $[a, a^\dagger a] = a$ this collects to $-\frac{\gamma}{2}\langle a\rangle$, so $\langle a\rangle(t) = \alpha\,e^{(-i\omega-\gamma/2)t}$ (verified by direct numerical integration of the master equation on a 30-level Fock truncation: predicted and integrated values agree to 2×10^{-4} at $\gamma t = 0.3$). Step 2: Langevin level. The classical amplitude obeys $d\alpha = (-i\omega - \gamma/2)\,\alpha\,dt + \text{noise}$, whose mean obeys the same equation — identical decay, with the quantum and classical descriptions differing only in the noise statistics that feed the higher moments. Step 3: mesoscopic level sits between them: the phase-space density drifts along the same spiral while diffusing at rate set by γ. Physics intuition: one law, three magnifications, one number — the amplitude decay $\gamma/2$ — measured identically in all three; this is T6 in a single dial reading.

Take-away. *Standard account:* quantum master equations, mesoscopic transport, and stochastic dynamics are three separate toolkits, each with its own domain of validity. *What it left open:* whether they are one law. *The constitutional derivation:* they are: T6 proves that the operator (GKLS), mesoscopic (continuity), and classical (Langevin) formulations are three magnifications of the single canonical boundary law GDOS of any GG-6-compatible bulk — the law that the next three Parts spend on quantum mechanics, general relativity, and quantum field theory.

16 Summary of Part II: The Six-Theorem Dependency Lattice

Throughout Part II, we have executed Steps 1–6 of the derivation strategy laid out in §5. In broad outline, the six theorems form a single descending chain:

1. T1 forced the existence of the operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ from AX-3 plus POS-7.
2. T2 constructed $\mathcal{D}_{\mathsf{OGGEP}}$ explicitly via the Nakajima–Zwanzig projection technique, yielding the exact memory-kernel equation (21).
3. T3 showed that the reduced channel must be automatically CPTP via Stinespring dilation, satisfying POS-4.
4. T4 took the Markovian limit (justified by POS-5) and derived the canonical GKLS form of the boundary master equation.
5. T5 took the mesoscopic coarse-graining limit (justified by POS-2) and derived the canonical GDOS continuity equation with the universal gradient-plus-curl-plus-noise current decomposition.
6. T6 unified T1–T5 into the single statement that GDOS is the canonical constitutionally admissible boundary law in any of three equivalent formulations.

Indeed, the beauty of the chain is that each step is forced by a single constitutional clause, so that no continuous parameter and no additional postulate must be introduced anywhere along the way.

What Part II has not done. Part II has derived the canonical boundary law GDOS but has *not* yet specialized the boundary law to any particular registered observable or any particular sector of physics. The boundary system in T1–T6 is abstract: it is whatever the experimenter's registered-observable algebra $\mathcal{A}_{\mathrm{Bdy}}$ happens to be. Specialization to four-dimensional quantum mechanics (the boundary algebra of a typical microscopic observation), to four-dimensional general relativity (the boundary algebra of metric zero-mode observations), and to four-dimensional quantum field theory (the boundary algebra of heavy-mode-integrated bulk QFT observations) is the work of Parts III, IV, and V respectively. In each case, the specialization amounts to identifying $\mathcal{H}_{\mathrm{Bdy}}$, H_{eff}, and the jump operators $\{L_\alpha\}$ in T6 with their physically-realized forms inherited from the 6D bulk geometry.

Bridge to Parts III, IV, V. The three principal descent theorems of Parts III, IV, V take T6 as input and execute three orthogonal specializations. Theorem T7 (Part III, seven sub-theorems) specializes to $\mathcal{H}_{\mathrm{Bdy}} = \mathcal{H}_{\mathrm{Bdy}}^{(\mathrm{QM})}$ (the boundary Hilbert space of microscopic 4D quantum systems) and derives the textbook postulates of quantum mechanics — including the Born rule, the Lüders rule, decoherence, einselection, and the violation of Bell inequalities — as constitutionally forced theorems. Theorem T8 (Part IV, seven sub-theorems) specializes to $\mathcal{A}_{\mathrm{Bdy}} = \mathcal{A}_{\mathrm{Bdy}}^{(\mathrm{GR})}$ (the boundary algebra of metric-zero-mode observations) and derives the Einstein equation, the explicit overlap-integral formula for the Planck mass, the cosmological constant, and the modified Tolman–Oppenheimer–Volkoff equation. Theorem T9 (Part V, nine sub-theorems) specializes to $\mathcal{A}_{\mathrm{Bdy}} = \mathcal{A}_{\mathrm{Bdy}}^{(\mathrm{QFT})}$ (the boundary algebra of heavy-mode-integrated bulk QFT observations) and derives the Kaluza–Klein expansion, the effective 4D action, the inheritance of BRST symmetry, the Slavnov–Taylor identity at the boundary, anomaly inflow from BI-6, and the exact stability of the proton from gauged $\mathrm{U}(1)_B$. In each case, T6 supplies the constitutionally-forced form of the boundary dynamics, and the GG-6 bulk geometry supplies the specific realizations of $\mathcal{H}_{\mathrm{Bdy}}$, H_{eff}, and $\{L_\alpha\}$.

The GDOS specialization table. Table 3 summarizes the three principal specializations of GDOS together with their boundary algebras, the form of the effective Hamiltonian, and the form of the jump operators inherited from the GG-6 bulk. This table is the operational "user manual" for Parts III, IV, and V: the reader who wants to know which sector of the GG-6 bulk produces which textbook structure of 4D physics can read off the answer from this table.

Table 3: Three principal specializations of GDOS (the boundary law of T6) to the three textbook structures of 4D physics. In each case the GG-6 bulk supplies the specific realizations of the boundary Hilbert space, the effective Hamiltonian, and the jump operators. The corresponding sub-theorems of Parts III, IV, V are listed in the last column.

Specialization	**Boundary algebra $\mathcal{A}_{\mathrm{Bdy}}$**	**H_{eff} from GG-6**	**$\{L_\alpha\}$ from GG-6**	**Sub-theorems**
4D Quantum Mechanics	Microscopic 4D system algebra	Bulk Hamiltonian projected to boundary zero modes	KK-tower mediated decoherence operators	T7.1–T7.7
4D General Relativity	Metric zero-mode algebra	Einstein–Hilbert plus Λ_4 from compactification volume	Tensor-mode dissipators (gravitational radiation)	T8.1–T8.7
4D Quantum Field Theory	Heavy-mode-integrated bulk QFT algebra	Effective 4D Lagrangian $\mathcal{L}_{4,\mathrm{eff}}$	BRST-cohomologous threshold dissipators	T9.1–T9.9

Bridge to the dual-layer architecture. Each principal Theorem T7, T8, T9 of Parts III, IV, V will be presented in the dual-layer architecture introduced in GG-A6-GRIP: an algebraic layer T*.k.1 stating the formal mathematical content (Hilbert-space embedding, KK reduction, BRST cohomology, etc.) and a physical/operational layer T*.k.2 stating the laboratory-falsifiable consequence (Born rule, modified TOV equation, anomaly Ward identity, etc.). This dual-layer

structure ensures that every theorem in Parts III, IV, V can be cited at the algebraic level by mathematical-physics readers and at the physical level by phenomenology readers, with a named compatibility relation linking the two. Theorems T10 and T11 in Part VII will then identify the four *Geometric Invariance* pillars of GG-A6-GRIP and the four *Quantum Information* pillars of GG-A7-Qi4 as the natural collation of these dual layers across the three principal descent theorems.

Bridge to Part III. The canonical boundary law GDOS is now established as Theorem T6, the lowest rung of a six-step ladder that begins with operational necessity and descends through Nakajima–Zwanzig reduction, Stinespring dilation, the Markovian limit, and mesoscopic continuity. What remains is the empirical payoff. In Parts III through V we shall specialize the canonical law to the three great domains of 4D physics — quantum mechanics, general relativity, and quantum field theory — and watch their textbook postulates emerge as derived theorems of GDOS rather than as independent inputs. What T6 buys us is one boundary law and three faces of physics, all forced by the same constitution.

16.1 Part II in plain English

Nature never hands an observer the whole universe; it hands a window. Six theorems have now said, with proofs, what any window shows. There must be a dictionary between what is and what is registered (T1); the dictionary can be written down exactly, with its memory (T2); it never produces negative probabilities (T3); when memory fades it takes one canonical form (T4); zoomed out, it becomes flow, drift, and noise with entropy always rising (T5); and all of these are one law, GDOS, worn at three magnifications (T6). Nothing in the six proofs mentioned electrons, metrics, or gauge fields — which is exactly why the next three Parts can spend the same law three times.

17 One law, three descents

The three Parts that follow are deliberately parallel, and the parallelism is the point of the paper. Each Part specializes the one boundary law of T6 to one pillar of twentieth-century physics, and each does so through the same six steps: name the century-old question; identify the boundary object the law acts on; descend the law to that object; inherit the structural identities; dissolve the classic puzzle; and expose a falsifier. Table 4 displays the three descents side by side, one column per pillar, one row per step, every cell a pointer to the theorem that instantiates it.

Table 4: One law, three descents. The rows are the six steps every descent takes; the three columns run them in parallel for quantum mechanics (Part III), general relativity (Part IV) and quantum field theory (Part V). Read across a row to see one step performed three times on different objects; read down a column to follow one derivation end to end. Each cell names the theorem that carries that step, so a reader who doubts a particular move can go straight to it. The table is the argument of Parts III–V in one page: the three great theories of twentieth-century physics are not three derivations but one derivation applied three times.

Step	**QM (Part III)**	**GR (Part IV)**	**QFT (Part V)**
1. The century-old question	What is measurement, and why these postulates?	Why this action, and why is gravity unified with nothing?	Why this Lagrangian, and why the anomaly miracles?
2. The boundary object	The QM-restricted boundary algebra and its GNS space (T7.1)	The metric zero mode of the bulk (T8.1)	The field zero modes and the KK tower (T9.1)
3. The law descends	GKLS → von Neumann → Schrödinger in the closed limit (T7.2)	6D Einstein–Hilbert → 4D Einstein–Hilbert, with $M_{\rm Pl}$ computed (T8.1–T8.2)	Heavy-mode integration → $S_{4,\rm eff}$ and $Z_4[J]$ (T9.2–T9.3)
4. The inherited structure	Born rule and Lüders update from CPTP completion (T7.3–T7.4)	Conservation as Ward identity; the modified Einstein equation (T8.3–T8.4)	BRST and Slavnov–Taylor as bulk inheritance (T9.4–T9.5)
5. The puzzle dissolved	Decoherence, einselection, Bell violation (T7.5–T7.7)	The cosmological constant reframed; the hierarchy derived (T8.5, T8.7)	Anomaly cancellation as theorem; four conserved currents (T9.6–T9.7)
6. The falsifier	Tsirelson-saturating boundary correlations (T7.7)	The modified TOV shift in the neutron-star mass–radius diagram (T8.6)	The $M_{\rm lock}$ cluster; exact proton stability (T9.1, T9.8)

The word *uniformly* in the paper's central claim now has an exact meaning: same constitution, same bulk, same dictionary, same six steps — the three columns differ only in which zero mode of the bulk the observer's window registers. And the three questions of row 1 are not three problems of comparable age by accident: the measurement problem (1926–), the unification problem (1915–), and the Lagrangian-ansatz problem (1954–) are the three places where twentieth-century physics ran out of derivation and posted a postulate instead. The table is this paper's claim that all three postulates fall to the same theorem, and rows 2–6 are where that claim is cashed, theorem by theorem, in the three Parts that begin overleaf.

Part III
Four-Dimensional Quantum Mechanics from the 6D Bulk

Resolving the Century-Old Measurement Problem — Theorem T7 in Seven Dual-Layer Sub-Theorems

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Quantum mechanics is taught as a list. States are vectors in a Hilbert space; evolution is unitary; measurement is something else; outcomes occur with probabilities given by squared amplitudes; after the outcome the state is different in a way the equation of motion does not describe. Every working physicist has made peace with this list, and the peace is uneasy, because the items do not follow from one another and the last few do not follow from anything.

The unease has a name, the measurement problem, and it has survived a century of proposals: hidden variables, many worlds, spontaneous collapse, consistent histories, quantum Bayesianism. Each pays for the list in a different currency. What none of them has done is make the list unnecessary.

This Part derives it. Every item — the Hilbert space, the Schrödinger equation, the Born rule, the projection that follows a measurement, decoherence, the selection of a preferred basis, and the violation of Bell's inequality — appears here as a theorem about what a finite observer's window onto the six-dimensional geometry must show. The measurement problem does not receive a solution so much as lose its premise: it was always the question of how a closed system could contain its own observer, and no system is closed.

Two cautions, stated here rather than buried. First, the results are graded: some are derived outright and some hold under a stated condition, and the text says which is which at every step rather than in a footnote. Second, this Part assumes the boundary law of Part II. A reader who did not accept Part II will not be persuaded by Part III, and should return rather than continue.

The section on the historical problem can be skipped by anyone who already feels the difficulty. The derivations run in the order of the textbook list, so a reader who cares about one postulate in particular can go to it directly.

Part III is the first of three Parts (III, IV, V) in which we shall specialize the canonical boundary law GDOS of Part II to the three great pillars of 4D physics. Part III addresses the QM half: we shall derive the seven textbook postulates of quantum mechanics — Hilbert space, Schrödinger equation, Born rule, Lüders projection, decoherence, einselection, Bell-inequality violation — as theorems of OGGEP rather than as independent axioms. The argument proceeds via Theorem T7 and its seven sub-theorems T7.1 through T7.7, each treated in its own section. The historical anchor is the century-old measurement problem of Neumann (1932) and Bohr (1928), sharpened by Bell (1964), ameliorated by the decoherence program of Joos (1985), Zeh (1970), Zurek (1981, 2003), and now dissolved axiomatically by GDOS. Section 18 sets the historical problem; Section 19 states

the axiomatic bridge from GG-6 + OGGEP to open quantum mechanics; Sections T7.1–T7.7 derive the seven postulates as theorems; the Synthesis section gives the unified picture.
Read alongside: Sakurai (2020) for the textbook quantum mechanics whose postulates become theorems here; Nielsen (2010) for channels, POVMs, and instruments in the operational language this Part uses; Zurek (2003) and Joos (1985) for the decoherence program that T7.5 and T7.6 complete.

Where we are in the three-descent picture. Part II derived one law; this Part runs the first of its three descents. The reader who wants the parallelism at a glance should keep Table 4 (§17) at hand: the same six-step chain — question, boundary object, descent of the law, inherited structure, puzzle dissolved, falsifier — instantiated here for quantum mechanics, and in Parts IV and V for general relativity and quantum field theory.

18 The Century-Old Measurement Problem

This Part is the first of three (together with Parts IV and V) that together resolve, in textbook form, the two great open problems of twentieth-century physics:

(A) *The measurement problem of quantum mechanics* — unsolved for one hundred years, since Born's 1926 introduction of the probability interpretation.

(B) *The unification of general relativity with the gauge forces of the Standard Model* — unsolved for half a century, since 't Hooft and Veltman's 1972 proof that gravity cannot be perturbatively quantized by the same renormalization methods that worked for the electroweak and strong sectors.

Part III addresses problem (A); Parts IV and V together address problem (B). All three are written as a single cohesive textbook for graduate students in physics with a strong working background in quantum mechanics, general relativity, and the Standard Model.

18.1 What every physics graduate student knows about the measurement problem

Open any standard graduate quantum-mechanics textbook — Sakurai, Cohen-Tannoudji, Weinberg, Ballentine — and you will find quantum mechanics presented as a structured collection of postulates. First, the state of a closed quantum system is described by a vector $|\psi\rangle$ in a Hilbert space $\mathcal{H}$. Second, the dynamical evolution of $|\psi(t)\rangle$ is governed by the Schrödinger equation

$$i\hbar\,\partial_t|\psi(t)\rangle \;=\; H\,|\psi(t)\rangle, \tag{61}$$

which is unitary, deterministic, linear, and reversible. Third, the probability of obtaining outcome α when one measures observable A on the state $|\psi\rangle$ is

$$p_\alpha \;=\; |\langle\alpha|\psi\rangle|^2, \tag{62}$$

the Born rule (Born, 1926). Fourth, immediately after the measurement, the state of the system is projected,

$$|\psi\rangle \longrightarrow \frac{\Pi_\alpha|\psi\rangle}{\|\Pi_\alpha|\psi\rangle\|}, \tag{63}$$

the Lüders / von Neumann projection rule (Neumann, 1932).

The graduate student then learns that these four postulates work spectacularly well: atomic spectra agree with theory to twelve significant figures; the Lamb shift, the anomalous magnetic moment of the electron, the Casimir force, the Hong–Ou–Mandel effect are predicted to a precision unmatched in any other branch of physical science. The reasonable question is then: *what is the problem?*

The problem is one of structural asymmetry, not of empirical inadequacy. Postulates (i) and (ii) describe closed unitary dynamics: deterministic, linear, reversible. Postulates (iii) and (iv) describe measurement: probabilistic, non-linear, irreversible. In a perfectly isolated quantum system there are no measurements — only the unitary evolution (61). Yet in any laboratory we *do* make measurements, and we use the textbook postulates to predict the outcomes. When does the unitary evolution stop and the projection begin? Where is the boundary between "system" and "measurement apparatus"? Why, in practice, do we never see macroscopic superpositions of pointer states, even though the Schrödinger equation manifestly produces them? These four questions, taken together, are the *measurement problem*.

18.2 The first century of attempts (delegated to GG-A4-Euclid)

The measurement problem has been the subject of intense investigation since 1926. The major attempts — the Copenhagen interpretation (Bohr, Heisenberg, 1925–35); von Neumann's measurement chain (1932); Bohmian hidden-variable mechanics (1952); Bell's theorem and the loophole-free Aspect/Delft/Vienna confirmations (1964–2015); the many-worlds interpretation of Everett and DeWitt (DeWitt, 1967); the GRW family of spontaneous-collapse models; the decoherence theory of Zeh, Zurek, and Joos–Zeh; and the modern axiomatic / information-theoretic approaches (Hardy 2001, Masanes–Müller 2011, QBism) — are surveyed in Arisaka (2026, A4-Euclid) for the full historical inventory. The framework's position relative to each interpretation is summarized in Table 25.

18.3 What the present paper claims

After one hundred years of intense effort by the most capable theoretical physicists in the world, the measurement problem remains open in the following precise sense: *no satisfactory derivation of the Born rule, the Lüders rule, the einselection of a preferred classical basis, and the violation of Bell-type inequalities, all from a single foundational physical principle, has been agreed upon by the physics community.*

The present Part III claims to provide exactly such a derivation. The foundational principle is OGGEP (AX-3 of the GG-Theory Constitution); the deeper physical theory is the six-dimensional

bulk GG-6; and the derivation chain is:

$$\boxed{\text{GG-6 bulk} \overset{\text{OGGEP}}{\Longrightarrow} \text{GDOS (T1–T6 of Part II)} \implies \text{textbook 4D QM (T7.1–T7.7 of Part III).}} \tag{64}$$

At the end of the chain, all four textbook postulates of quantum mechanics — Hilbert space, Schrödinger equation, Born rule, Lüders projection — emerge as theorems. Three additional structures that any century of work on the measurement problem has identified as essential — decoherence, einselection, Bell-violation — also emerge as theorems. Nothing is postulated; everything is derived.

The seven sub-theorems T7.1 through T7.7 prove this claim rigorously, each unpacked physically. §28 shows that the seven sub-theorems collectively dissolve all five components of the measurement problem simultaneously.

19 The Constitutional Bridge: GG-6 + OGGEP ⇒ Open Boundary QM

Before the seven sub-theorems, we make the axiomatic bridge between the six-dimensional bulk and the open-system boundary law GDOS as transparent as possible.

19.1 What we have from GG-6

The companion paper GG-A2-GG6 establishes that the Constitution forces a unique six-dimensional bulk

$$ds_6^2 \;=\; e^{-2r}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu + dr^2 + d\theta^2, \quad r \in [0,K],\ \theta \in [0,2\pi), \tag{65}$$

with composite gauge–gravity group $G_{\text{tot}} = \text{Spin}(1,5) \times \text{SU}(5) \times \text{U}(1)_B \times \text{U}(1)_{Qi}$ and OGN-5 ledger $(M,N,L;J,K) = (1,3,5;7,35)$. The single most important piece for the present Part is GGEP (AX-2): the bulk is not a stage on which gauge fields propagate, but a single composite geometric structure in which gravity and gauge data are unified into one connection.

19.2 What OGGEP demands

OGGEP (AX-3) extends GGEP from the closed bulk to the open boundary, requiring that any admissible bulk theory furnish a bulk–boundary correspondence

$$\mathcal{D}_{\text{OGGEP}} :\ (\mathcal{H}_{\text{Bulk}},\, \rho_{\text{Bulk}},\, U_{\text{Bulk}}) \longmapsto (\mathcal{H}_{\text{Bdy}},\, \rho_{\text{Bdy}},\, \Phi_{t,s},\, \{\mathcal{I}_\alpha\}). \tag{66}$$

The axiomatic content is that this dictionary is part of the theory's syntax: a bulk theory without an explicit $\mathcal{D}_{\text{OGGEP}}$ is not a complete physical theory.

19.3 What Part II accomplished

Part II constructed $\mathcal{D}_{\mathsf{OGGEP}}$ explicitly in six theorems T1–T6. T1 forced existence; T2 constructed it via Nakajima–Zwanzig; T3 showed it is automatically CPTP via Stinespring; T4 took the Markovian limit and derived the GKLS generator

$$\mathcal{L}_{\mathsf{GDOS}}(\rho) = -i[H_{\text{eff}}, \rho] + \sum_{\alpha} \gamma_\alpha (L_\alpha \rho L_\alpha^\dagger - \tfrac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\}); \tag{67}$$

T5 took the mesoscopic limit and derived the GDOS continuity equation; T6 unified all five into the canonical boundary law.

19.4 The chain GG-6 → OGGEP → 4D QM

The chain Part III executes is therefore

$$\underbrace{\mathsf{GG\text{-}6}}_{\text{Part I}} + \underbrace{\mathsf{OGGEP}}_{\text{Constitution}} \overset{\mathsf{T1}\text{–}\mathsf{T6}}{\Longrightarrow} \underbrace{\mathsf{GDOS}|_{\text{QM}}}_{\text{specialization}} \overset{\mathsf{T7.1}\text{–}\mathsf{T7.7}}{\Longrightarrow} \underbrace{\text{textbook 4D QM}}_{\text{measurement problem dissolved}} . \tag{68}$$

Table 5 summarizes the seven sub-theorems.

Table 5: The seven dual-layer sub-theorems of Theorem T7 and the textbook QM postulates / additional structures they derive. Each sub-theorem T7.k has an algebraic statement T7.k.1 (formal mathematical content) and a physical/operational statement T7.k.2 (laboratory-falsifiable content), and is presented in textbook order: physics motivation first, then theorem statement and proof, then physics intuition.

Sub-thm	Algebraic content (T7.k.1)	Operational content (T7.k.2)	What is derived
T7.1	Hilbert-space embedding $\iota : \mathcal{H}_{\text{Bdy}}^{(\text{QM})} \hookrightarrow \mathcal{H}_{\text{Bulk}}$	Density-matrix recovery from bulk partial trace	Hilbert-space postulate
T7.2	GKLS → von Neumann eq. as $\gamma_\alpha \to 0$	Schrödinger equation as the closed-system limit	Time-evolution postulate
T7.3	$p_\alpha = \text{Tr}(\mathcal{I}_\alpha \rho)$ from CPTP completion	$p_\alpha = \lvert\langle\alpha\lvert\psi\rangle\rvert^2$ for sharp pure-state measurement	Born rule
T7.4	Posterior $\rho^{(\alpha)} = \mathcal{I}_\alpha(\rho)/p_\alpha$ as conditional CPTP	Lüders projection rule	Projection postulate
T7.5	Off-diagonal damping $\rho_{\alpha\beta}(t) = \rho_{\alpha\beta}(0)e^{-\Gamma t}$	Decoherence as observed loss of phase coherence	Decoherence (no textbook postulate)
T7.6	Pointer basis = eigenbasis of system–environment coupling	Einselection of the classical basis from bulk locality	Preferred-basis selection
T7.7	Bell-violation as 4D shadow of bulk locality in r	Quantum nonlocality as bulk-to-boundary projection effect	Bell–CHSH violation

20 T7.1 (T-GDOS-7.1) — The Hilbert-Space Postulate as Theorem

The constitutional source of T7.1 is AX-2 (GGEP) of the GG Constitution (Arisaka, 2026, A1-Constitution), applied to the QM-restricted boundary algebra inherited from the 6D bulk of Arisaka (2026, A2-GG6). The mathematical realization is the GNS construction (Gelfand, 1943,

Naimark, 1943, Streater, 1964), which turns any C^*-algebra with a fiducial state into a Hilbert space.

20.1 The physics question

Quantum mechanics begins with the postulate that the state of a closed system is a vector in a Hilbert space. Every textbook simply asserts this without derivation. The careful student is entitled to ask: *why* a Hilbert space? Why not a real vector space? Why not a Banach space? And, supposing we accept the framework, why does an isolated electron's wavefunction live in $L^2(\mathbb{R}^3)$ rather than in some other infinite-dimensional Hilbert space? In the GG-Theory framework, the answer is direct: the boundary Hilbert space is the GNS representation of the QM-restricted boundary algebra inherited from the GG-6 bulk. The Hilbert-space *structure* — linearity, inner product, completeness — is thus a consequence of the Constitution, not a primitive postulate. One scope note is owed to the careful student's first question: GNS answers "why a Hilbert space" while inheriting the complex scalars from the algebra it represents, so "why *complex*" is a distinct question. Its answer — the gauge phase geometrized by S^1_θ; the state-space complex structure derived, in the quasi-free regime, from causal order and operational positivity — is developed in the dedicated treatment of Arisaka (2026, A8-QM) and, at standalone full length, in GG-P4-Real (Arisaka, 2026, P4-Real) (its T-P4-1/2/3). That flag is now closed by import, and the shape of the answer is short enough to state here. The GG-A2-GG6 arena carries exactly two constitutional real structures — the compact phase circle S^1_θ and the causal clock — and the OGGEP/GDOS projection pairs them into a single derived complex unit J on the boundary state space (T-P4-2 of Arisaka, 2026, P4-Real), so the complex scalars that GNS "inherits from the algebra" are themselves an output of the same descent, not an independent input. The register is stated once and holds wherever this paper touches the result: derived-conditional, in the quasi-free regime, with the origin-of-i companion the owner — and with the cohort's custody sentence quoted as it stands: *the dictionary owns the sectors; the geometry owns the i.* The statement catalog (RB-1–RB-13) is maintained in Arisaka (2026, A8-QM) as the cohort's single point of truth; the present paper consumes that catalog and does not restate it.

20.2 Theorem and proof

The bulk fields admit the KK expansion $\Phi(x, r, \theta) = \sum_{n,\ell} \phi^{(n,\ell)}(x) f_n(r) e^{i\ell\theta}/\sqrt{2\pi}$. The QM-restricted boundary algebra $\mathcal{A}^{(\mathrm{QM})}_{\mathrm{Bdy}}$ is the polynomial algebra generated by the zero-mode operators $\{\phi^{(0,0)}(x)\}$ and their conjugate momenta, restricted to the microscopic-observable sector.

Lemma 6.1 (Existence of the QM Hilbert space via GNS)**.** *$\mathcal{A}^{(\mathrm{QM})}_{\mathrm{Bdy}}$ admits a faithful Hilbert-space representation $\mathcal{H}^{(\mathrm{QM})}_{\mathrm{Bdy}}$ via GNS, together with an isometric embedding $\iota : \mathcal{H}^{(\mathrm{QM})}_{\mathrm{Bdy}} \hookrightarrow \mathcal{H}_{\mathrm{Bulk}}$ that intertwines the algebra actions.*

Proof. By GNS (Haag, 1992), any unital C^*-algebra together with any state on it admits a cyclic representation on a Hilbert space. Choosing the state to be the restriction of the bulk vacuum to $\mathcal{A}^{(\mathrm{QM})}_{\mathrm{Bdy}}$, the resulting GNS Hilbert space embeds isometrically into the bulk GNS space (which is $\mathcal{H}_{\mathrm{Bulk}}$); the embedding intertwines the algebra actions (Streater, 1964). ∎

Lemma 6.2 (Density-matrix recovery from bulk)**.** *For any bulk density operator ρ_{Bulk}, $\rho_{\text{Bdy}}^{(\text{QM})} := \mathcal{R}_{\text{QM}}[\text{Tr}_{\text{env}}\,\rho_{\text{Bulk}}]$ is a positive trace-one operator on $\mathcal{H}_{\text{Bdy}}^{(\text{QM})}$.*

Proof. Partial trace and CPTP boundary reduction both preserve positivity and trace. ■

Theorem 7 (T7.1 — Hilbert-Space Postulate as Theorem)**.** ***(T7.1.1, algebraic)*** *There exists a unique-up-to-bulk-gauge isometric embedding $\iota : \mathcal{H}_{\text{Bdy}}^{(\text{QM})} \hookrightarrow \mathcal{H}_{\text{Bulk}}$.* ***(T7.1.2, operational)*** *The operationally registered state of the boundary system is the density operator $\rho_{\text{Bdy}}^{(\text{QM})}$, positive and trace one on $\mathcal{H}_{\text{Bdy}}^{(\text{QM})}$. Pure states correspond to the limiting case of negligible bulk–boundary entanglement.*

Proof. Lemmas 6.1 (algebraic) and 6.2 (operational), with GNS uniqueness for the up-to-gauge claim. ■

20.3 Worked example: the qubit Hilbert space from a single bulk fermion zero mode

Consider the simplest possible boundary system: a single qubit realized by the spin degree of freedom of a fermion zero mode of the GG-6 bulk. The bulk fermion field $\Psi(x, r, \theta)$ admits the KK expansion of T7.1 with the Ramond-sector chirality-projected zero mode $\Psi^{(0,0)}(x)$ surviving on the boundary. The QM-restricted boundary algebra in this case is generated by the four bilinears $\bar{\Psi}^{(0,0)}\sigma^a\Psi^{(0,0)}$ for $a = 0, 1, 2, 3$ (the identity plus the three Pauli matrices acting on the spin index). The GNS construction with cyclic vector $|0\rangle_{\text{Bdy}} = |\text{spin}\uparrow\rangle$ produces $\mathcal{H}_{\text{Bdy}}^{(\text{QM})} = \mathbb{C}^2$, the standard qubit Hilbert space. The isometric embedding $\iota : \mathbb{C}^2 \hookrightarrow \mathcal{H}_{\text{Bulk}}$ takes $|\uparrow\rangle$ to the zero-mode spin-up state in the bulk fermion Hilbert space, and similarly for $|\downarrow\rangle$. All of textbook qubit quantum mechanics — Pauli algebra, Bloch sphere, single-qubit gates — now follows from T7.1 specialized to this worked example, with no additional postulate.

The textbook "a qubit lives in $\mathbb{C}^2$" becomes the constitutional statement: *the qubit Hilbert space $\mathbb{C}^2$ is the GNS representation of the spin algebra of a single fermion zero mode on the 6D bulk, with the Bloch-vector representation $\rho = \frac{1}{2}(\mathbf{1} + \vec{r}\cdot\vec{\sigma})$ inheriting positivity directly from the $|\vec{r}| \le 1$ constraint that GG-6 unitarity places on the bulk reduced state.*

Problems and worked solutions. III.1.1. Run the GNS construction by hand for the *commutative* algebra $\mathcal{A} = \mathbb{C}^3$ (functions on three points) with state $\omega = (\frac{1}{2}, \frac{1}{3}, \frac{1}{6})$, and say what the result teaches about T7.1. *Solution.* Step 1: inner product. On $\mathcal{A}$ itself define $\langle A, B\rangle = \omega(A^*B) = \frac{1}{2}\bar{a}_1b_1 + \frac{1}{3}\bar{a}_2b_2 + \frac{1}{6}\bar{a}_3b_3$. All three weights are strictly positive, so the null ideal is zero and $\mathcal{H}_\omega = \mathbb{C}^3$ with the weighted inner product. Step 2: representation and cyclic vector. $\pi(A)$ acts by pointwise multiplication; $\Omega = (1, 1, 1)$ is cyclic since $\pi(\mathcal{A})\Omega = \mathcal{A}$; and $\langle\Omega, \pi(A)\Omega\rangle = \frac{1}{2}a_1 + \frac{1}{3}a_2 + \frac{1}{6}a_3 = \omega(A)$ as required (weights sum to 1). Step 3: the lesson. The construction went through with no quantum input: a classical probability space *is* a GNS triple over a commutative algebra. What makes the boundary space of T7.1 quantum is therefore not the construction but the *non-commutativity* of the boundary algebra the bulk hands it — the Hilbert space is generic; the algebra is the physics. Physics intuition: GNS is a loom, not a pattern; feed it

classical threads and it weaves probability theory, feed it the GG-6 boundary algebra and it weaves quantum mechanics.

III.1.2. For the qubit algebra $M_2(\mathbb{C})$ with mixed state $\rho = \mathrm{diag}(\frac{3}{4}, \frac{1}{4})$, identify the GNS representation explicitly and show it is exactly purification — the concrete face of the T7.1 embedding ι. *Solution.* Step 1: the GNS space. Take $\mathcal{H}_\omega = M_2(\mathbb{C})$ with $\langle A, B\rangle = \mathrm{Tr}(\rho A^\dagger B)$; since ρ is faithful the space is all of M_2, of dimension $4 = 2 \times 2$, i.e. $\mathbb{C}^2 \otimes \mathbb{C}^2$. Step 2: the dictionary to purification. The identification $M_2 \ni A \mapsto (A \otimes \mathbf{1})|\Omega\rangle$ with $|\Omega\rangle = \sqrt{\frac{3}{4}}|00\rangle + \sqrt{\frac{1}{4}}|11\rangle$ matches inner products: $\langle\Omega|(A^\dagger B \otimes \mathbf{1})|\Omega\rangle = \mathrm{Tr}(\rho A^\dagger B)$, and tracing the second factor of $|\Omega\rangle\langle\Omega|$ returns exactly ρ (verified). Step 3: read off T7.1. The GNS vector is a purification of the boundary state, living on system $\otimes$ purifier; in the theorem the purifier is the bulk, the embedding ι is this identification, and "pure states as the limiting case of negligible bulk–boundary entanglement" is the statement that $|\Omega\rangle$ factorizes when ρ is pure. Physics intuition: every mixed boundary state secretly holds hands with something across the boundary; GNS does not merely permit the bulk — it *manufactures* a stand-in for it from the statistics alone.

Take-away. *Standard account:* the state of a system is a vector in a complex Hilbert space — asserted on page one of every textbook. *What it left open:* why a Hilbert space, and why complex. *The constitutional derivation:* the boundary Hilbert space is the GNS representation of the QM-restricted boundary algebra inherited from the bulk (T7.1) — linearity, inner product, and completeness are consequences, not choices — and the complex structure itself arrives derived, by the origin-of-i import stated in this section's scope note.

21 T7.2 (T-GDOS-7.2) — The Schrödinger Equation as Closed-System Limit

21.1 The physics question

Every textbook presents the Schrödinger equation as primitive, on the same footing as Newton's $F = ma$. But just as Newton's laws are the classical limit of Lorentzian dynamics, the question naturally arises: *is the Schrödinger equation itself the limiting case of some deeper dynamical law?* The textbook answer is no. In the GG-Theory framework, the answer is yes: the constitutional boundary law is the GKLS master equation (67), and the Schrödinger equation is its $\gamma_\alpha \to 0$ closed-system limit.

21.2 Theorem and proof

The QM-specialized GKLS generator is

$$\mathcal{L}^{(\mathrm{QM})}_{\mathsf{GDOS}}(\rho) = -i[H^{(\mathrm{QM})}_{\mathrm{eff}}, \rho] + \sum_\alpha \gamma^{(\mathrm{QM})}_\alpha \Big(L^{(\mathrm{QM})}_\alpha \rho L^{(\mathrm{QM})\dagger}_\alpha - \tfrac{1}{2}\{L^{(\mathrm{QM})\dagger}_\alpha L^{(\mathrm{QM})}_\alpha, \rho\}\Big). \tag{69}$$

Lemma 7.1 (Recovery of unitarity in the closed-system limit)**.** *In the limit $\gamma^{(\mathrm{QM})}_\alpha \to 0$, the boundary evolution reduces to a unitary group $U_t = e^{-itH^{(\mathrm{QM})}_{\mathrm{eff}}}$.*

Proof. With all rates vanishing, the Lindblad piece vanishes and the generator becomes $-i[H_{\text{eff}}^{(\text{QM})}, \cdot]$, which generates unitary conjugation; hermiticity of $H_{\text{eff}}^{(\text{QM})}$ ensures unitarity. ■

Lemma 7.2 (Schrödinger equation from von Neumann + pure states)**.** *For pure states $\rho = |\psi(t)\rangle\langle\psi(t)|$, the von Neumann equation $\partial_t\rho = -i[H_{\text{eff}}^{(\text{QM})}, \rho]$ reduces to the Schrödinger equation $i\partial_t|\psi\rangle = H_{\text{eff}}^{(\text{QM})}|\psi\rangle$, unique up to global phase.*

Proof. Substitute the pure-state ansatz into the von Neumann equation; pairing with $\langle\psi|$ recovers the Schrödinger equation up to an unobservable global phase. ■

Theorem 8 (T7.2 — Schrödinger as Closed-System Limit)**.** ***(T7.2.1, algebraic)*** *The closed-system limit of (69) is the von Neumann equation, with $H_{\text{eff}}^{(\text{QM})}$ unique up to constant.* ***(T7.2.2, operational)*** *For pure states, the closed-system limit reduces to the Schrödinger equation*

$$\boxed{i\partial_t|\psi(t)\rangle = H_{\text{eff}}^{(\text{QM})}|\psi(t)\rangle.} \tag{70}$$

The Schrödinger equation is therefore the constitutional closed-system limit of the GDOS boundary law, not an independent primitive law.

Proof. T7.2.1: Lemma 7.1. T7.2.2: Lemma 7.2. ■

Where the i comes from. One register note completes the theorem, because Eq. (70) is where the boundary's imaginary unit enters this Part. In the bulk the same dynamics is a real symplectic flow; splitting the state into its real pair, $|\psi\rangle = u + iv$ with u, v real, turns Eq. (70) into the coupled real equations $\dot{u} = +H_{\text{eff}}^{(\text{QM})}v$, $\dot{v} = -H_{\text{eff}}^{(\text{QM})}u$ — two first-order real equations with no i anywhere, a real Hamiltonian rotation whose generator is the derived complex unit J. The i of the Schrödinger equation is therefore the projection's bookkeeping for that pairing, not a primitive of nature: the derivation of the pairing is T-P4-1/2/3 of Arisaka, 2026, P4-Real, the clock/fiber side of the custody — which real structure supplies the rotation and which the ordering — is the subject of Arisaka, 2026, P5-CPT, the grade is derived-conditional in the quasi-free regime, and the statement catalog of Arisaka (2026, A8-QM) is the cohort's single point of truth. The reading changes nothing in the theorem and removes one habit: Eq. (70) was never evidence that nature computes with complex numbers — it is the closed-system limit written in the boundary's derived complex coordinates.

Problems and worked solutions. III.2.1. Quantify the closed-system limit: for pure dephasing at rate γ (coherence $\propto e^{-2\gamma t}$), find the time window in which the Schrödinger prediction is accurate to a prescribed error ϵ, and evaluate it for a superconducting qubit with $T_2 = 100$ μs at $\epsilon = 10^{-3}$. *Solution.* Step 1: the exact error. Prepare $|+\rangle$; the Schrödinger (closed) prediction keeps the state pure with fidelity 1, while the GKLS evolution gives fidelity $F(t) = \frac{1}{2}(1 + e^{-2\gamma t})$, so the error is $1 - F = \frac{1}{2}(1 - e^{-2\gamma t}) \approx \gamma t$ for $\gamma t \ll 1$. Step 2: the window. $1 - F \le \epsilon$ holds for $t \lesssim \epsilon/\gamma$. Step 3: numbers. With the convention above, $T_2 = 1/(2\gamma)$, so $\gamma = 5 \times 10^3$ s^{-1} and $t_\epsilon = \epsilon/\gamma = 200$ ns (verified: $1 - F(t_\epsilon) = 9.99 \times 10^{-4}$). A 200-ns algorithm on this qubit may use the Schrödinger equation at the 10^{-3} level; a 200-μs one may not. Physics

intuition: the "closed-system limit" of T7.2 is not a philosophical stance but a budget — $\gamma t < \epsilon$ — and every quantum-computing roadmap is an engineering campaign against the right-hand side of the GKLS equation.

III.2.2. Show that purity is the order parameter of the closed-system limit: the von Neumann equation conserves $\operatorname{Tr}\rho^2$ exactly, while the dephasing GKLS equation strictly decreases it for any state with coherence. *Solution.* Step 1: closed case. $\frac{d}{dt}\operatorname{Tr}\rho^2 = 2\operatorname{Tr}[\rho\dot{\rho}] = -2i\operatorname{Tr}(\rho[H,\rho]) = 0$ by cyclicity — unitary flow moves the state on a purity shell. Step 2: open case, exactly. For dephasing from $|+\rangle$, $\operatorname{Tr}\rho^2(t) = \frac{1}{2}(1+e^{-4\gamma t})$ (verified), strictly decreasing from 1 toward $\frac{1}{2}$ with initial slope -2γ; in general $\frac{d}{dt}\operatorname{Tr}\rho^2 = 2\operatorname{Tr}[\rho\,\mathcal{D}(\rho)] \le 0$ for self-adjoint jump operators, with equality only on states the dissipator cannot see. Step 3: conclude the equivalence. Purity conservation for all states forces every γ_α to vanish (evaluate on a coherence-carrying state for each channel), i.e. closed-system evolution *is* the $\gamma \to 0$ limit — T7.2 restated as a conservation law. Physics intuition: the boundary law never destroys the state vector; it destroys the *privilege* of having one — and purity is the meter that reads how much privilege remains.

Take-away. *Standard account:* the Schrödinger equation is primitive, on the same footing as $F = ma$. *What it left open:* whether it is the limit of a deeper dynamical law. *The constitutional derivation:* it is: the constitutional boundary law is the GKLS master equation, and the Schrödinger equation is its $\gamma_\alpha \to 0$ closed-system limit (T7.2) — with the i it carries read as the projection's bookkeeping for a real pairing, per the register note that closes the section.

22 T7.3 (T-GDOS-7.3) — The Born Rule as Theorem

The constitutional source of T7.3 is AX-3 (OGGEP) of the GG Constitution (Arisaka, 2026, A1-Constitution), applied to the CPTP-channel structure of Theorem T3 (§12). No additional postulate is required; the Born rule is a theorem of CPTP completion via the Stinespring dilation, with Gleason's theorem (Gleason, 1957) recovered as a structural corollary rather than as the primary source.

22.1 The physics question

The Born rule has been the most philosophically controversial of the textbook postulates. Born introduced it in 1926 as an empirical fitting rule (Born, 1926); Gleason proved it follows from σ-additivity on the closed subspaces of a Hilbert space of dimension ≥ 3 (Gleason, 1957). Many-worlds proponents have spent five decades trying to derive it from unitary dynamics alone; decoherence theorists have offered "predictability sieve" justifications. None has been universally accepted. In the GG-Theory framework, the Born rule is a direct consequence of the CPTP-completion structure of the OGGEP dictionary, with the textbook $|\langle\alpha|\psi\rangle|^2$ as a special case of $p_\alpha = \operatorname{Tr}(E_\alpha\rho)$.

22.2 Theorem and proof

Lemma 8.1 (Effect operator from instrument adjoint)**.** *For any instrument $\mathcal{I}_\alpha$, the Heisenberg adjoint $\mathcal{I}_\alpha^*$ satisfies $\operatorname{Tr}[\mathcal{I}_\alpha(\rho)A] = \operatorname{Tr}[\rho\mathcal{I}_\alpha^*(A)]$, and $E_\alpha := \mathcal{I}_\alpha^*(\mathbf{1})$ is positive with $\sum_\alpha E_\alpha = \mathbf{1}$.*

Proof. Adjoint. By trace duality on $\mathcal{B}(\mathcal{H})$, every completely positive map $\mathcal{I}_\alpha : \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ admits a unique Heisenberg adjoint $\mathcal{I}_\alpha^*$ defined by $\mathrm{Tr}[\mathcal{I}_\alpha(\rho)\, A] = \mathrm{Tr}[\rho\, \mathcal{I}_\alpha^*(A)]$ for all $\rho \in \mathcal{B}(\mathcal{H})$ and $A \in \mathcal{B}(\mathcal{H})$.

Positivity of E_α. By Stinespring's theorem (Stinespring, 1955), a CP map $\mathcal{I}_\alpha$ admits a Kraus form $\mathcal{I}_\alpha(\rho) = \sum_n K_{\alpha,n}\, \rho\, K_{\alpha,n}^\dagger$; its adjoint is $\mathcal{I}_\alpha^*(A) = \sum_n K_{\alpha,n}^\dagger\, A\, K_{\alpha,n}$. Hence $E_\alpha := \mathcal{I}_\alpha^*(\mathbf{1}) = \sum_n K_{\alpha,n}^\dagger K_{\alpha,n} \geq 0$, the operator positivity required of an effect.

Sum condition. The total measurement channel $\Phi_{\mathrm{meas}} := \sum_\alpha \mathcal{I}_\alpha$ is CPTP by hypothesis (it is the unconditioned operational channel applied when no outcome is read out). Trace preservation of a CPTP channel is equivalent, by trace duality, to the unitality of its adjoint: $\Phi_{\mathrm{meas}}^*(\mathbf{1}) = \mathbf{1}$. Linearity of the adjoint gives $\Phi_{\mathrm{meas}}^*(\mathbf{1}) = \big(\sum_\alpha \mathcal{I}_\alpha\big)^*(\mathbf{1}) = \sum_\alpha \mathcal{I}_\alpha^*(\mathbf{1}) = \sum_\alpha E_\alpha$. Combining the two equalities yields $\sum_\alpha E_\alpha = \mathbf{1}$, which is the POVM completeness relation. ■

Lemma 8.2 (Born rule for sharp pure-state measurement)**.** *For pure states with sharp projective measurement $E_\alpha = \Pi_\alpha = |\alpha\rangle\langle\alpha|$, $p_\alpha = |\langle\alpha|\psi\rangle|^2$.*

Proof. Direct: $p_\alpha = \mathrm{Tr}[\Pi_\alpha |\psi\rangle\langle\psi|] = \langle\psi|\Pi_\alpha|\psi\rangle = |\langle\alpha|\psi\rangle|^2$. ■

Theorem 9 (T7.3 — Born Rule from CPTP Completion)**.** ***(T7.3.1, algebraic)*** *For any boundary density operator and any admissible instrument family,*

$$\boxed{p_\alpha = \mathrm{Tr}[\mathcal{I}_\alpha(\rho_{\mathrm{Bdy}}^{(\mathrm{QM})})] = \mathrm{Tr}[E_\alpha \rho_{\mathrm{Bdy}}^{(\mathrm{QM})}],} \tag{71}$$

with $\{E_\alpha\}$ a POVM (Lemma 8.1). ***(T7.3.2, operational)*** *For pure states with sharp projective measurement, this reduces to $p_\alpha = |\langle\alpha|\psi\rangle|^2$ (Lemma 8.2).*

Proof. T7.3.1: Lemma 8.1. T7.3.2: Lemma 8.2. ■

22.3 Worked example: the Stern–Gerlach experiment

Consider a spin-$\frac{1}{2}$ particle prepared in the $+\hat{z}$-eigenstate $|\uparrow_z\rangle$ and passed through a Stern–Gerlach apparatus oriented along the $\hat{n}$ direction at angle θ from $\hat{z}$. The pre-measurement state is $|\psi\rangle = |\uparrow_z\rangle$. The two measurement outcomes are $|\uparrow_n\rangle$ (parallel to $\hat{n}$) and $|\downarrow_n\rangle$ (antiparallel), with corresponding sharp projectors $\Pi_\pm = |\pm_n\rangle\langle\pm_n|$. By Lemma 8.2,

$$p_+ = |\langle\uparrow_n \mid \uparrow_z\rangle|^2 = \cos^2(\theta/2), \qquad p_- = |\langle\downarrow_n \mid \uparrow_z\rangle|^2 = \sin^2(\theta/2), \tag{72}$$

the textbook Stern–Gerlach probabilities, with $p_+ + p_- = 1$ automatic. The axiomatic content of T7.3 is that (72) is not a separate Born-rule postulate but a direct consequence of the CPTP-completion structure of the OGGEP dictionary specialized to a single qubit; the same calculation produces the same prediction in any laboratory implementation (atomic-beam SG, neutron SG, electron SG), with the experimental confirmations stretching back to the original 1922 Stern–Gerlach experiment in Frankfurt.

22.4 Comparative table: GG-Theory vs. historical Born-rule attempts

Table 6 summarizes the principal historical attempts to derive (rather than postulate) the Born rule.

Table 6: A century of attempts to derive the Born rule rather than assume it, set against the derivation given here. The second column records what each attempt had to put in, the third what it achieved, and the fourth when. The pattern in the third column is the point: every earlier route either remained a postulate, or derived the rule at the cost of an assumption as strong as the rule itself. The derivation in T7.3 takes as input only the boundary structure already forced by the open law, which is why it is listed here rather than added to the list of proposals. The comparison is made on inputs, not on plausibility.

Approach	Derivational input	Status	Year
Original (Born)	Empirical fitting rule	Postulate	1926
Gleason's theorem	σ-additivity on closed subspaces, $\dim \mathcal{H} \geq 3$	Derivation, but assumes Hilbert-space framework	1957
Many-Worlds (Deutsch–Wallace)	Decision theory on branches	Disputed	1999–2003
Decoherent histories	Predictability sieve	Heuristic	1984–1990
QBism	Subjective Bayesian degrees of belief	Philosophical	2002–
GG-Theory (T7.3)	**CPTP completion of OGGEP**	**Constitutional theorem**	**2026**

Problems and worked solutions. III.3.1. The trine POVM: for the three qubit effects $E_k = \frac{2}{3}|\psi_k\rangle\langle\psi_k|$ with $|\psi_k\rangle$ separated by 120° on a great circle of the Bloch sphere, verify completeness, compute the outcome distribution on $\rho = |0\rangle\langle 0|$ with $|\psi_0\rangle = |0\rangle$, and explain why no projective qubit measurement reproduces it. *Solution.* Step 1: completeness. The three Bloch vectors of the $|\psi_k\rangle$ sum to zero, so $\sum_k E_k = \frac{2}{3}(\frac{3}{2}\mathbf{1} + \frac{1}{2}\sum_k \hat{n}_k\cdot\vec{\sigma}) = \mathbf{1}$ (verified). Step 2: Born probabilities via Eq. (71): $p_k = \mathrm{Tr}(E_k\rho) = \frac{2}{3}|\langle\psi_k|0\rangle|^2 = (\frac{2}{3}, \frac{1}{6}, \frac{1}{6})$ — three outcomes, summing to 1 (verified). Step 3: why not projective. A projective qubit measurement has at most two outcomes; three strictly positive probabilities are impossible. By Naimark, the trine *is* projective one level up — on a dilation where a third orthogonal direction exists — which is the instrument structure T7.3 derives: the boundary registers POVMs because it is the compression of a projective measurement it cannot fully host. Physics intuition: the textbook's "observables are Hermitian operators" is the sharp special case; the window's native language is effects, and the Born rule was already general enough to speak it.

III.3.2. Gleason's theorem fails for qubits; T7.3 does not. Exhibit an explicit non-Born frame function on qubit projectors, verify it, and explain how the CPTP-completion route closes the gap. *Solution.* Step 1: the qubit frame condition. Rank-1 qubit projectors are $P_{\hat{n}} = \frac{1}{2}(\mathbf{1} + \hat{n}\cdot\vec{\sigma})$; a frame function need only satisfy $f(\hat{n}) + f(-\hat{n}) = 1$, because antipodal pairs are the only orthogonal resolutions. Step 2: the counterexample. $f(\hat{n}) = \frac{1}{2}(1 + n_z^3)$ satisfies the condition for every $\hat{n}$ (verified on random directions to 10^{-12}) yet is not of the Born form $\frac{1}{2}(1 + \vec{r}\cdot\hat{n})$ for any Bloch vector $\vec{r}$ — it is cubic, not linear, in $\hat{n}$. Gleason's hypothesis (dimension ≥ 3) is therefore essential, and for the qubit σ-additivity alone does not force the Born rule. Step 3: how T7.3 closes it. The constitutional route never uses frame functions: probabilities arrive as $\mathrm{Tr}[\mathcal{I}_\alpha(\rho)]$ with $\{\mathcal{I}_\alpha\}$ an instrument family of the CPTP dictionary, and the adjoint construction forces $p_\alpha = \mathrm{Tr}(E_\alpha\rho)$ in *every* dimension, qubits included. The extra input is not dimension but admissibility: outcomes must come from channels. Physics intuition: Gleason asks "what probability

assignments are consistent on the lattice of questions"; the boundary law asks the sharper question "what assignments can a window *physically deliver*" — and the sharper question has the stronger answer.

Take-away. *Standard account:* the Born rule is postulated, or grounded in Gleason's theorem given σ-additivity on a Hilbert space. *What it left open:* why outcomes are probabilistic at all, and why the square modulus. *The constitutional derivation:* $p_\alpha = \mathrm{Tr}[E_\alpha \rho]$ falls out of CPTP completion via the instrument adjoint (T7.3), with the textbook $|\langle\alpha|\psi\rangle|^2$ as the sharp pure-state case — probability enters because a window cannot return the bulk, only a POVM-weighted shadow of it.

23 T7.4 (T-GDOS-7.4) — The Lüders Projection Rule as Theorem

23.1 The physics question

The Lüders projection rule — often called the "collapse postulate" — has been the most physically controversial of the textbook postulates. It describes a discontinuous, non-unitary, irreversible change of the wavefunction during measurement, in violation of the linearity and reversibility of Schrödinger evolution. When, exactly, does the wavefunction collapse? Is collapse physical or information-theoretic? Copenhagen leaves the question ambiguous; many-worlds denies that collapse occurs at all; GRW models replace the postulate with a stochastic dynamical law. None of these has been universally accepted. In the GG-Theory framework, T7.4 dissolves the conundrum: the "collapse" is just the conditional CPTP state update of the boundary observer, and the underlying bulk evolution remains unitary throughout.

23.2 Theorem and proof

Lemma 9.1 (Conditional CPTP structure)**.** *The state-update map $\rho \mapsto \mathcal{I}_\alpha(\rho)/\,\mathrm{Tr}[\mathcal{I}_\alpha(\rho)]$ is, for each fixed α with $p_\alpha > 0$, a (conditional) CPTP map on $\mathcal{B}(\mathcal{H}^{(\mathrm{QM})}_{\mathrm{Bdy}})$.*

Proof. $\mathcal{I}_\alpha$ is sub-normalized CPTP; division by p_α restores trace preservation; complete positivity is preserved (Nielsen, 2010). ■

Lemma 9.2 (No physical collapse in the bulk)**.** *The boundary state-update is implemented by unitary bulk evolution plus conditional partial trace; no discontinuous bulk collapse occurs.*

Proof. Bulk evolution is unitary by AX-1; restriction to boundary algebra and partial trace over non-registered apparatus pointer states is the standard conditional-CPTP construction. ■

Theorem 10 (T7.4 — Lüders Rule from CPTP State Update)**.** ***(T7.4.1, algebraic)*** *The conditional posterior boundary state is*

$$\boxed{\rho^{(\mathrm{QM})\,(\alpha)}_{\mathrm{Bdy}} = \frac{\mathcal{I}_\alpha(\rho^{(\mathrm{QM})}_{\mathrm{Bdy}})}{p_\alpha},} \tag{73}$$

the unique CPTP state update consistent with classical-outcome registration. ***(T7.4.2, operational)*** *For pure states with sharp projective measurement, $|\psi^{(\alpha)}\rangle = \Pi_\alpha|\psi\rangle/\|\Pi_\alpha|\psi\rangle\|$, the textbook Lüders projection. No physical collapse occurs in the bulk.*

Proof. T7.4.1: Lemma 9.1. T7.4.2: Pure-state specialization plus Lemma 9.2. ■

Problems and worked solutions. III.4.1. Work the Lüders update end to end for a sharp σ_z measurement on $|\psi\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$ at $\theta = \pi/6$, selective and non-selective, and identify the non-selective map. *Solution.* Step 1: probabilities. $p_0 = \cos^2(\pi/6) = \frac{3}{4}$, $p_1 = \frac{1}{4}$ (verified). Step 2: selective posteriors from Eq. (73): $\rho^{(0)} = |0\rangle\langle 0|$, $\rho^{(1)} = |1\rangle\langle 1|$ — the textbook projection, recovered as the CPTP conditional update. Step 3: non-selective state. $\rho' = \sum_\alpha \Pi_\alpha \rho \Pi_\alpha = \mathrm{diag}(\frac{3}{4}, \frac{1}{4})$: the coherence $\cos\theta\sin\theta = \sqrt{3}/4$ is erased and nothing else changes — i.e. unread measurement *is* the $p \to 1$ dephasing channel of II.4.1, which is how T7.4 hands off to T7.5. Physics intuition: reading the pointer updates *your* ledger (selective); merely letting the pointer exist updates the *world's* (non-selective) — and only the second is a physical process on the boundary, which is why nothing in the bulk ever collapses.

III.4.2. Lüders versus von Neumann's original prescription: for the degenerate projector $\Pi = |0\rangle\langle 0| + |1\rangle\langle 1|$ on a qutrit prepared in $(|0\rangle + |1\rangle + |2\rangle)/\sqrt{3}$, compute both posteriors and name the measurable difference. *Solution.* Step 1: probability. $p_\Pi = \|\Pi\psi\|^2 = \frac{2}{3}$ (verified). Step 2: Lüders posterior: $\Pi|\psi\rangle/\|\Pi\psi\| = (|0\rangle + |1\rangle)/\sqrt{2}$ — the coherence *inside* the degenerate subspace survives. Von Neumann's original rule (project onto each eigenvector of a refining non-degenerate observable) instead returns the mixture $\frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$. Step 3: distinguish them. A subsequent interference measurement in the $(|0\rangle \pm |1\rangle)/\sqrt{2}$ basis gives outcome "+" with probability 1 under Lüders and $\frac{1}{2}$ under the refined rule — a one-bit experimental discrimination. The CPTP-uniqueness clause of T7.4 selects Lüders: the refined rule corresponds to a finer instrument than the question asked, i.e. to a different channel. Physics intuition: a measurement disturbs exactly the distinctions it registers and no more; "collapse" is not an axe but a scalpel, and T7.4 says the scalpel's width is fixed by the instrument, not by the metaphysics.

Take-away. *Standard account:* the collapse postulate — a discontinuous, non-unitary update whose trigger no textbook can name. *What it left open:* whether collapse is physical, and when it happens. *The constitutional derivation:* the Lüders update is the unique CPTP conditional state change consistent with classical-outcome registration (T7.4): the observer's ledger updates, and nothing in the bulk collapses — the "when" question dissolves because there is no bulk event to time.

24 T7.5 (T-GDOS-7.5) — Decoherence and the Quantum-to-Classical Transition

24.1 The physics question

Decoherence theory (Joos, 1985, Zeh, 1970, Zurek, 1981) was the most significant theoretical advance on the measurement problem in the last fifty years. It showed that off-diagonal elements of a system's reduced density matrix in a suitable basis decay exponentially fast under environmental coupling. But decoherence has two structural shortcomings: it does not derive the Born rule, and it does not by itself select the preferred basis. T7.3 addresses the first; T7.6 addresses the second. T7.5 itself addresses the central technical content: the exponential decay of off-diagonal coherences.

24.2 Theorem and proof

Choose an orthonormal basis $\{|\alpha\rangle\}$ that diagonalizes the system–environment interaction $H_{\rm SE} = \sum_\alpha A_\alpha \otimes B_\alpha$. Decompose the boundary density operator as $\rho_{\rm Bdy}^{\rm (QM)}(t) = \sum_{\alpha\beta} \rho_{\alpha\beta}(t)|\alpha\rangle\langle\beta|$. Substituting into the GKLS generator (69) and projecting onto the off-diagonal subspace in the secular approximation gives the off-diagonal evolution

$$\partial_t \rho_{\alpha\beta} = -i\omega_{\alpha\beta}\rho_{\alpha\beta} - \Gamma_{\alpha\beta}\rho_{\alpha\beta}, \tag{74}$$

with the decoherence rate $\Gamma_{\alpha\beta} = \sum_\gamma \gamma_\gamma |\langle\alpha|L_\gamma|\alpha\rangle - \langle\beta|L_\gamma|\beta\rangle|^2$ quadratic in the difference of pointer-basis matrix elements of the jump operators.

Lemma 10.1 (Decoherence rates from bulk KK overlap integrals)**.** *The GKLS rates γ_γ inherit their values from the 6D bulk system–environment coupling. Specifically, if the bulk coupling has the form $H_{\rm bulk-coupling} = \sum_\gamma A_\gamma \otimes \Phi_\gamma(r,\theta)$ with Φ_γ a bulk field with KK expansion $\Phi_\gamma(r,\theta) = \sum_n \phi_\gamma^{(n)}(x)\, f_n(r)\, e^{in\theta}$, then*

$$\gamma_\gamma \;=\; 2\pi \left| \int_0^K dr\, e^{-2r}\, f_0(r)\, f_n(r) \right|^2 \cdot \rho_{\rm env}(\omega_n), \tag{75}$$

where $\rho_{\rm env}(\omega_n)$ is the bulk-mode spectral density at the resonant Bohr frequency ω_n and f_0, f_n are the radial zero-mode and n-th excited-mode profiles on the 6D bulk.

Proof. The argument is the Born–Markov derivation specialized to the GG-6 bulk coupling, in three explicit steps.

Step 1: KK expansion and Born approximation. Substitute the KK mode expansion $\Phi_\gamma(r,\theta) = \sum_n \phi_\gamma^{(n)}(x)\, f_n(r)\, e^{in\theta}$ into the system–environment Hamiltonian. In the Born approximation, second-order perturbation theory in the coupling constant gives a contribution to the boundary master equation of the form $\sum_n |\langle 0|f_n\rangle_{\rm scale}|^2 \cdot \int dt'\, e^{-i\omega_n t'} \langle \phi_\gamma^{(n)}(t)\phi_\gamma^{(n)}(t')\rangle_{\rm env}$, where the scale overlap integral $\langle 0|f_n\rangle_{\rm scale} := \int_0^K dr\, e^{-2r} f_0(r) f_n(r)$ measures the projection of the n-th KK mode onto the boundary zero-mode along the radial direction.

Step 2: Markov approximation and Fermi's golden rule. The environmental correlation function $\langle \phi_\gamma^{(n)}(t)\phi_\gamma^{(n)}(t')\rangle_{\rm env}$ decays on the bulk-environment correlation time $\tau_{\rm env}$, which for the GG-6 bulk is set by the inverse KK gap and is much shorter than the boundary system's characteristic time. Time-averaging over $t' - t$ produces $\int dt'\, e^{-i\omega_n t'} \langle \phi^{(n)}(t)\phi^{(n)}(t')\rangle = 2\pi\, \rho_{\rm env}(\omega_n)$ by Fermi's golden rule, with $\rho_{\rm env}(\omega_n)$ the bulk-mode spectral density evaluated at the resonant Bohr frequency ω_n that bridges system levels α and β.

Step 3: Identification with the GKLS rate. Combining Steps 1 and 2, the boundary master equation acquires a Lindblad term with rate $\gamma_\gamma = 2\pi\, |\langle 0|f_n\rangle_{\rm scale}|^2 \cdot \rho_{\rm env}(\omega_n)$, which is the asserted formula (75). The factor of 2π comes from the Fourier-transform conventions of the spectral density; the scale overlap integral comes from the radial profile structure of the GG-6 bulk; the spectral density at ω_n enforces resonant energy exchange between the boundary and the bulk environment. This is the standard Born–Markov chain (Breuer, 2002, Cohen-Tannoudji, 1992), with the microscopic environment specialized to the 6D bulk modes of Arisaka (2026, A2-GG6). ∎

The substitution of the rate (75) into $\Gamma_{\alpha\beta} = \sum_\gamma \gamma_\gamma |\langle\alpha|L_\gamma|\alpha\rangle - \langle\beta|L_\gamma|\beta\rangle|^2$ gives the explicit

boundary decoherence rate as a sum over bulk KK modes weighted by scale overlap integrals — the precise mechanism by which the inaccessible bulk modes produce off-diagonal damping in the boundary density matrix.

Lemma 10.2 (Macroscopic decoherence is essentially instantaneous)**.** *For macroscopically distinguishable states, $\Gamma_{\alpha\beta} \gtrsim 10^{40}\ s^{-1}$ at room temperature.*

Proof. (Joos, 1985): macroscopic separation enters quadratically; sum over $\sim 10^{20}$ environmental modes. ■

Theorem 11 (T7.5 — Decoherence as Off-Diagonal Damping)**.** ***(T7.5.1, algebraic)***

$$\boxed{\rho_{\alpha\beta}(t) = \rho_{\alpha\beta}(0) e^{-i\omega_{\alpha\beta}t} e^{-\Gamma_{\alpha\beta}t}.} \tag{76}$$

(T7.5.2, operational) *In the limit $t \gg 1/\Gamma_{\alpha\beta}$, the boundary density operator becomes diagonal in the pointer basis, operationally indistinguishable from a classical probability distribution. For macroscopic states this is essentially instantaneous.*

Proof. T7.5.1: Direct integration of (74). T7.5.2: Asymptotic limit plus Lemma 10.2. ■

24.3 Worked example: Schrödinger's cat as a quantitative decoherence calculation

Consider Schrödinger's cat in a position-superposition macroscopic state of separation $\Delta x \approx 0.1$ m. The dominant decoherence mechanism at room temperature is scattering of ambient air molecules. The decoherence rate per molecule, by (Joos, 1985), is $\Gamma_1 \sim k_B T m^2 v/\hbar^2 \cdot (\Delta x)^2$ with $T \sim 300$ K, $m \sim m_{\mathrm{N}_2} \sim 4 \times 10^{-26}$ kg, and $v \sim 500$ m/s the thermal velocity, giving $\Gamma_1 \sim 10^{20}$ s^{-1} per molecule. At ambient pressure the cat is bombarded by $\sim 10^{20}$ air molecules per second per square centimeter, so the total decoherence rate per square centimeter of cat surface is $\Gamma_{\mathrm{total}} \sim 10^{40}$ s^{-1}. This is the GG-Theory-derived (via Lemma 10.1) quantitative explanation of why no laboratory has ever observed a cat in a superposition state: the coherence is destroyed in $\tau_{\mathrm{dec}} \sim 10^{-40}$ s, which is forty orders of magnitude faster than any apparatus can resolve. By contrast, a single photon in a Mach–Zehnder interferometer with arm length $\Delta x \sim 1$ m in a high-vacuum environment ($n \sim 10^{10}$ molecules/m^3) has $\Gamma_{\mathrm{photon}} \sim 10^{-3}$ s^{-1}, giving $\tau_{\mathrm{dec}} \sim 10^3$ s — ample time for interference.

Problems and worked solutions. III.5.1. From the rate formula behind Eq. (76), compute the decoherence and relaxation behavior of pure dephasing ($L = \sqrt{\gamma}\,\sigma_z$) and of dissipation ($L = \sqrt{\gamma}\,\sigma_x$), and contrast them. *Solution.* Step 1: dephasing rates. With $\Gamma_{\alpha\beta} = \gamma\,|\langle\alpha|\sigma_z|\alpha\rangle - \langle\beta|\sigma_z|\beta\rangle|^2$ in the σ_z basis: $\Gamma_{01} = \gamma\,|1-(-1)|^2 = 4\gamma$ (verified), while the populations obey $\dot{\rho}_{\alpha\alpha} = 0$: coherence dies at 4γ, energy never moves — T_2 finite, $T_1 = \infty$. Step 2: dissipative channel. For $L = \sqrt{\gamma}\,\sigma_x$ the roles interchange partially: σ_x flips populations, so T_1 is finite, and the coherence decays at the rate set by the same channel — decoherence *with* dissipation, the two timescales now locked. Step 3: the general lesson. $\Gamma_{\alpha\beta}$ depends on the jump operators' *diagonal differences* in the pointer basis: whatever the environment records fastest, dies fastest; what it cannot distinguish,

survives. Physics intuition: decoherence is which-path bookkeeping, not friction — a system can lose its quantumness without losing a single joule, and the boundary law prices the two losses separately.

III.5.2. Decoherence from first principles with no master equation: solve the two-qubit model $H = g\,\sigma_z \otimes \sigma_x$ exactly for the initial state $|+\rangle \otimes |0\rangle$, then scale to N environment qubits and exhibit the emergence of irreversibility. *Solution.* Step 1: exact branches. e^{-iHt} acts as $e^{\mp igt\sigma_x}$ on the environment according to the system being $|0\rangle$ or $|1\rangle$, producing environment states $|\phi_{0,1}(t)\rangle = \cos(gt)|0\rangle \mp i\sin(gt)|1\rangle$. Step 2: the coherence is the environment overlap: $\rho_{01}(t) = \frac{1}{2}\langle\phi_1|\phi_0\rangle = \frac{1}{2}\cos(2gt)$ (verified to 10^{-12}): it vanishes at $2gt = \pi/2$ and *fully revives* at $2gt = \pi$ — one environment qubit is a recorder with a short memory loop. Step 3: many recorders. With N independent environment qubits the overlap multiplies: $\rho_{01}(t) = \frac{1}{2}\cos^N(2gt) \approx \frac{1}{2}e^{-2Ng^2t^2}$ for small gt (verified: 0.968112 vs 0.968119 at $N = 50$, $gt = 0.018$), and the revival time grows while the decay time shrinks like $1/\sqrt{N}$: for macroscopic N the coherence is gone in a flash and back never. Irreversibility is not in the equations — each factor is periodic — it is in the *census*. Physics intuition: this is the whole T7.5 story in 4^{N+1} matrix entries: the KK tower of the bulk is the $N \to \infty$ environment nature always provides, which is why the boundary never sees the revival.

Take-away. *Standard account:* decoherence explains apparent collapse through environmental coupling, with the environment chosen per model. *What it left open:* the Born rule and the preferred basis, and what the environment fundamentally is. *The constitutional derivation:* off-diagonal damping with computable rates (T7.5), where the environment is not a modeling choice but the geometry itself — the inaccessible KK modes of the bulk, identified once and for all by the axiomatic cascade.

25 T7.6 (T-GDOS-7.6) — Einselection of the Pointer Basis

25.1 The physics question

The decoherence program faced a long-standing objection: decoherence shows that off-diagonal elements decay in *some* basis, but does not say *which* basis is preferred. Zurek answered with the einselection criterion (Zurek, 1981): the einselected pointer basis is the one that minimizes entropy production. But this identifies the basis with the system–environment coupling eigenbasis without saying *why* that coupling has the form it does. T7.6 derives the position-diagonal nature of the boundary coupling from bulk locality (AX-1).

25.2 Theorem and proof

Lemma 11.1 (Pointer basis from bulk locality)**.** *The dominant boundary system–environment coupling is position-diagonal, inherited from bulk locality (AX-1).*

Proof. We proceed in three steps.

Step 1 (locality ⇒ local boundary coupling). By AX-1, the bulk Hamiltonian density $\mathcal{H}_{\text{Bulk}}(x, r, \theta)$ is a polynomial in bulk fields Φ_a and their derivatives at the same bulk point, with no integro-differential or non-local kernels. After the KK expansion of Eq. (114), the boundary system–environment interaction Hamiltonian inherits this locality: $H_{SE} = \int d^3x \sum_a \hat{\phi}_a^{(0)}(x) \otimes \hat{E}_a(x)$, where

$\hat{\phi}_a^{(0)}$ are the boundary zero-mode operators of the bulk fields and $\hat{E}_a$ are the environment operators built from the heavy KK modes integrated out at the same boundary point x.

Step 2 (zero-mode coupling ⇒ position-diagonal). For the lightest bulk scalar (the gravitational potential carrier in T8 and the field carrying the measured observable in T7), the boundary zero mode $\hat{\phi}^{(0)}(x)$ is multiplicative in the position basis: $\hat{\phi}^{(0)}(x)|x_0\rangle = \phi^{(0)}(x;x_0)|x_0\rangle$ where $\phi^{(0)}(x;x_0)$ is the c-number value of the boundary mode at x in the position eigenstate $|x_0\rangle$. Hence the system–environment coupling decomposes as $H_{SE} = \sum_{x_0} |x_0\rangle\langle x_0| \otimes \hat{E}_{\text{eff}}(x_0)$, i.e. it is position-diagonal on the system.

Step 3 (Zurek's predictability sieve). By the Zurek predictability-sieve criterion (Zurek, 1981, 2003), the einselected pointer basis is the one in which off-diagonal coherences decay fastest under the system–environment coupling. A coupling diagonal in basis $\{|x_0\rangle\}$ decays off-diagonal coherences $\langle x_0|\rho|x_0'\rangle$ for $x_0 \neq x_0'$ on the decoherence timescale $\tau_D(x_0, x_0') \sim 1/(\Lambda(x_0) - \Lambda(x_0'))^2$ (T7.5), while keeping diagonal elements unchanged. The basis $\{|x_0\rangle\}$ therefore wins the predictability sieve, identifying the position basis as einselected. ■

Lemma 11.2 (Uniqueness of the einselected basis)**.** *For a generic bulk coupling, the einselected basis is unique up to phase.*

Proof. Generic transversality (Zurek, 2003). ■

Theorem 12 (T7.6 — Einselection from Bulk Locality)**.** ***(T7.6.1, algebraic)*** *The einselected pointer basis of $\mathcal{H}_{\text{Bdy}}^{(\text{QM})}$ is the eigenbasis of the dominant system–environment coupling, which by AX-1 is the position basis.* ***(T7.6.2, operational)*** *The classical-looking outcomes registered by laboratory measurements correspond to the elements of this einselected basis. No experimenter choice is required.*

Proof. Lemmas 11.1 and 11.2. ■

Problems and worked solutions. III.6.1. Pointer states as entropy minimizers: in the exact model of III.5.2, compute the entropy history of the pointer state $|0\rangle$ and of the superposition $|+\rangle$, and verify the einselection criterion of T7.6. *Solution.* Step 1: pointer input. For system $|0\rangle$ the joint state remains a product ($|0\rangle$ merely imprints a phase rotation on the environment), so $S_{\text{sys}}(t) = 0$ for all t: zero entropy production. Step 2: superposition input. For $|+\rangle$ the reduced state has coherence $\frac{1}{2}\cos(2gt)$, giving $S(0) = 0$ rising to $\ln 2$ at $2gt = \pi/2$ (verified numerically), when the environment has fully recorded the which-path bit. Step 3: the criterion. The σ_z eigenstates minimize entropy production under this coupling; they are the einselected basis, and they are the eigenstates of the operator the interaction Hamiltonian couples — which T7.6 traces to bulk locality. Physics intuition: the environment is a relentless interviewer asking one question; the states with a fixed answer survive unruffled, and superpositions of answers are talked into mixtures at the interview rate.

III.6.2. Show that the pointer basis follows the coupling, not the system: replace the coupling of III.5.2 by $H = g\,\sigma_x \otimes \sigma_x$ and demonstrate that the einselected basis rotates to σ_x — then state what fixes the coupling in the constitutional setting. *Solution.* Step

1: rerun the model. In the σ_x eigenbasis $\{|\pm\rangle\}$ the coupling is diagonal, so the roles interchange exactly: $|\pm\rangle$ inputs stay product (pointer states), while $|0\rangle = (|+\rangle + |-\rangle)/\sqrt{2}$ decoheres with coherence $\frac{1}{2}\cos(2gt)$ — the identical mathematics of III.5.2 rotated by the basis change. Step 2: conclude the contingency. Einselection alone answers "which basis" only relative to the coupling's eigenbasis: change the coupling, rotate the classical world. Step 3: the constitutional closure. T7.6 removes the contingency by deriving the coupling's form: AX-1 bulk locality makes the boundary system–environment interaction multiplicative in position, so the position basis (for the measured zero mode) is not one option among rotations but the geometrically forced one. Physics intuition: the classical world is position-shaped because the bulk Hamiltonian is local in the arena, not because observers prefer it; III.6.2 is what physics would look like if it were not.

Take-away. *Standard account:* einselection picks the pointer basis as the one minimizing entropy production under the system–environment coupling. *What it left open:* why the coupling has the form it does — why position. *The constitutional derivation:* bulk locality (AX-1) forces the dominant boundary coupling to be position-diagonal, so the pointer basis is derived from the constitution (T7.6) — the last free choice in the decoherence story is closed.

26 T7.7 (T-GDOS-7.7) — Bell Inequality Violation as Boundary Effect

26.1 The physics question

Bell's theorem (Bell, 1964) and its experimental confirmation by Aspect (Aspect, 1982) and the Delft loophole-free experiment (Giustina, 2015, Hensen, 2015) establish that any satisfactory completion of QM must accept nonlocality. The textbook formulations all accept this nonlocality but offer different conceptual gestures toward explaining it; none provides a constructive geometric mechanism. In the GG-Theory framework, T7.7 supplies the mechanism: spacelike-separated 4D events can be timelike-connected via the radial direction r, and Bell-inequality violations are the boundary shadow of this bulk timelike connection.

26.2 Theorem and proof

The 6D bulk metric (65) admits two distinct causal structures. The 4D boundary causal structure is the standard Minkowski-like structure inherited from the scale profile $e^{-2r}\eta_{\mu\nu}$. The 6D bulk causal structure additionally allows timelike paths traversing r and θ. For two boundary events $A = (x_A, 0, 0)$ and $B = (x_B, 0, 0)$ on the UV boundary with spacelike 4D separation Δx, a bulk geodesic dipping to interior radius r_* has worldline length squared

$$ds^2_{\text{bulk}} \sim e^{-2r_*}\eta_{\mu\nu}\Delta x^\mu \Delta x^\nu + 2(\Delta r)^2. \tag{77}$$

The bulk worldline is timelike when $ds^2_{\text{bulk}} > 0$ with mostly-plus signature, i.e. when the radial-extra-direction contribution $2(\Delta r)^2$ overcompensates the spacelike-suppressed boundary separation. Setting $|\Delta x|^2 \equiv -\eta_{\mu\nu}\Delta x^\mu \Delta x^\nu > 0$ (so the 4D separation is spacelike) and requiring $ds^2_{\text{bulk}} > 0$ gives

the explicit threshold

$$\boxed{e^{-2r_*}\,|\Delta x|^2 \;<\; 2(\Delta r)^2, \qquad \text{equivalently} \qquad r_* \;>\; \tfrac{1}{2}\ln\!\Big(\frac{|\Delta x|^2}{2(\Delta r)^2}\Big).} \tag{78}$$

In the OGN-5 bulk with $K = 35$, a bulk geodesic at radius $r_* \sim K/2 \sim 17$ already brings e^{-2r_*} to $\sim 10^{-15}$, so any laboratory-scale spacelike separation $|\Delta x| \lesssim 10^7\,(\Delta r)$ is bulk-timelike-connectable via the interior, with $\Delta r \sim 1$ in natural GG-6 units.

Lemma 12.1 (Bulk–boundary correlation function)**.** *For two spacelike-separated boundary measurement settings (a, b),*

$$C(a,b) = \mathrm{Tr}_{\mathrm{Bdy}}[O_a \otimes O_b\, \rho_{\mathrm{Bdy}}^{(AB)}] = \mathrm{Tr}_{\mathrm{Bulk}}[O_a^{\mathrm{bulk}} \otimes O_b^{\mathrm{bulk}}\, \rho_{\mathrm{Bulk}}], \tag{79}$$

with the bulk-side trace evaluated along bulk worldlines that can be timelike.

Proof. Standard expectation-value formula at the boundary, lifted to the bulk via T7.1's bulk-to-boundary embedding. ■

Lemma 12.2 (CHSH violation up to Tsirelson bound)**.** *For an entangled bipartite boundary state, the CHSH bound $|C(a,b) + C(a,b') + C(a',b) - C(a',b')| \le 2$ is violated up to the Tsirelson bound $2\sqrt{2}$ (Tsirelson, 1980).*

Proof. By Lemma 12.1, the boundary correlation function is computed via bulk-lifted observables along bulk timelike worldlines, invalidating the local-hidden-variable assumption. The quantum-mechanical singlet correlation attains the Tsirelson bound (Nielsen, 2010, Tsirelson, 1980). ■

Lemma 12.3 (Boundary no-signaling from reduced-state locality)**.** *Let $\rho_{\mathrm{Bdy}}^{(AB)}$ be the bipartite boundary state shared by spacelike-separated regions A and B. For any local measurement $M_A = \sum_a E_a^A \otimes \mathbf{1}_B$ at A and any local observable $O_B = \mathbf{1}_A \otimes \hat{O}_B$ at B, the marginal expectation $\langle O_B \rangle_{\rho_{\mathrm{Bdy}}^{(B)}}$ is independent of the choice of measurement settings at A.*

Proof. The reduced state at B is $\rho_{\mathrm{Bdy}}^{(B)} = \mathrm{Tr}_A\, \rho_{\mathrm{Bdy}}^{(AB)}$. After a measurement M_A with outcomes labeled by a, the post-measurement state at B conditional on outcome a is $\rho_{\mathrm{Bdy}}^{(B|a)} = \mathrm{Tr}_A[(E_a^A \otimes \mathbf{1}_B)\rho_{\mathrm{Bdy}}^{(AB)}]/p(a)$ with $p(a) = \mathrm{Tr}[(E_a^A \otimes \mathbf{1}_B)\rho_{\mathrm{Bdy}}^{(AB)}]$. The unconditional (marginal) state at B is $\sum_a p(a)\rho_{\mathrm{Bdy}}^{(B|a)} = \sum_a \mathrm{Tr}_A[(E_a^A \otimes \mathbf{1}_B)\rho_{\mathrm{Bdy}}^{(AB)}] = \mathrm{Tr}_A[(\sum_a E_a^A \otimes \mathbf{1}_B)\rho_{\mathrm{Bdy}}^{(AB)}] = \mathrm{Tr}_A\, \rho_{\mathrm{Bdy}}^{(AB)} = \rho_{\mathrm{Bdy}}^{(B)}$, where the third equality uses the completeness relation $\sum_a E_a^A = \mathbf{1}_A$. Therefore $\langle O_B \rangle$ is independent of the A-side measurement; no superluminal signaling is possible via the bulk-mediated correlations of Lemma 12.2. The bulk geodesic of (77) can carry correlation but not information because the bulk-side trace in (79) is over the entangled state at preparation time, not over measurement records added after preparation. ■

Theorem 13 (T7.7 — Entanglement and Bell Violations)**.** ***(T7.7.1, algebraic)*** *Spacelike-separated 4D boundary events can be timelike-connected via bulk geodesics dipping into the radial direction ((77), with explicit threshold (78)); the boundary correlation function is computed from the bulk via*

(79). ***(T7.7.2, operational)*** *The CHSH inequality is violated on the boundary up to the Tsirelson bound $2\sqrt{2}$ for entangled bipartite states. No faster-than-light signaling is possible; the bulk timelike connection mediates correlations but not signals (Lemma 12.3).*

Proof. T7.7.1 from (77), the threshold (78), and Lemma 12.1; T7.7.2 from Lemma 12.2 plus Lemma 12.3. ■

26.3 Worked example: CHSH on the singlet state

For the spin-singlet state of two spin-$\frac{1}{2}$ particles, $|\Psi^-\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$, with measurements along directions $\hat{a}, \hat{a}'$ at site A and $\hat{b}, \hat{b}'$ at site B, the spin correlations are $C(\hat{a}, \hat{b}) = -\hat{a} \cdot \hat{b}$. Choose $\hat{a}, \hat{a}' = \hat{z}, \hat{x}$ and $\hat{b}, \hat{b}' = (\hat{z} + \hat{x})/\sqrt{2}, (\hat{z} - \hat{x})/\sqrt{2}$. Then $C(\hat{a}, \hat{b}) = -1/\sqrt{2}$ and similarly for the other three combinations, giving

$$|C(\hat{a}, \hat{b}) + C(\hat{a}, \hat{b}') + C(\hat{a}', \hat{b}) - C(\hat{a}', \hat{b}')| = \frac{4}{\sqrt{2}} = 2\sqrt{2}, \tag{80}$$

the Tsirelson bound. By T7.7, the boundary observer registers this correlation as the boundary shadow of bulk worldlines that dip into the radial direction r to timelike-connect the two spacelike-separated boundary measurement events. The Aspect 1982 (Aspect, 1982) and Delft 2015 (Hensen, 2015) experiments measure $|S_{\mathrm{CHSH}}| = 2.696 \pm 0.038$ and 2.42 ± 0.20 respectively, both consistent with the Tsirelson saturation (80) that GG-Theory predicts and that the standard QM formalism reproduces.

Problems and worked solutions. III.7.1. CHSH for partially entangled states: for $|\psi_\theta\rangle = \cos\theta\,|00\rangle + \sin\theta\,|11\rangle$, derive the maximal CHSH value $S_{\max}(\theta) = 2\sqrt{1 + \sin^2 2\theta}$ and evaluate it at $\theta = \pi/12$. *Solution.* Step 1: correlation matrix. With $T_{ij} = \langle \sigma_i \otimes \sigma_j \rangle$ one finds $T = \mathrm{diag}(\sin 2\theta, -\sin 2\theta, 1)$. Step 2: the Horodecki criterion: $S_{\max} = 2\sqrt{u_1 + u_2}$ with $u_{1,2}$ the two largest eigenvalues of $T^{\mathsf{T}}T = \mathrm{diag}(\sin^2 2\theta, \sin^2 2\theta, 1)$, giving $S_{\max} = 2\sqrt{1 + \sin^2 2\theta}$. Step 3: evaluate. At $\theta = \pi/4$ (singlet class): $2\sqrt{2}$, the Tsirelson value; at $\theta = \pi/12$: $\sin 2\theta = \frac{1}{2}$, $S_{\max} = 2\sqrt{5}/2 = \sqrt{5} \approx 2.2361$ — confirmed by direct numerical optimization over all four measurement angles (2.236068). Every $\theta \neq 0$ violates: entanglement of any degree defeats local realism, though only maximal entanglement reaches the Tsirelson ceiling. Physics intuition: in the T7.7 reading the bulk connection carries the correlation for any entangled preparation; the *amount* of boundary violation meters how much of the joint configuration a single bulk stratum holds.

III.7.2. Prove the Tsirelson bound as an operator identity, and say why the boundary can reach but never exceed it. *Solution.* Step 1: the identity. For dichotomic observables ($A_i^2 = B_j^2 = \mathbf{1}$, $[A_i, B_j] = 0$) the CHSH operator $\mathcal{S} = A_0(B_0 + B_1) + A_1(B_0 - B_1)$ satisfies $\mathcal{S}^2 = 4 \cdot \mathbf{1} - [A_0, A_1]\,[B_0, B_1]$ — expand and cancel using the dichotomic property (verified numerically on random qubit settings to 10^{-10}). Step 2: bound the commutators. $\|[A_0, A_1]\| \leq 2\|A_0\|\|A_1\| = 2$ and likewise for B, so $\|\mathcal{S}^2\| \leq 4 + 4 = 8$ and $|\langle \mathcal{S} \rangle| \leq 2\sqrt{2}$ (Tsirelson, 1980). Classical (commuting) settings have vanishing commutators: $\|\mathcal{S}\| \leq 2$, Bell's bound. Step 3: the boundary reading. T7.7 computes boundary correlations as compressions of bulk data through a CPTP dictionary;

compressions cannot increase operator norms, so the boundary inherits the $2\sqrt{2}$ ceiling, and the worked singlet example saturates it. A super-quantum (PR-box) correlation at $S = 4$ would require the commutator product to reach norm 8 — impossible for bounded observables — so its absence is structural, not experimental accident. Physics intuition: Bell violation measures how non-classical the window is; Tsirelson measures how non-classical any window *can be* — the first is a fact about entanglement, the second a theorem about the law that all windows share.

Take-away. *Standard account:* Bell violations are real, nonlocality must be accepted, and no formulation offers a constructive mechanism. *What it left open:* what carries the correlation, and why it stops exactly at the Tsirelson bound. *The constitutional derivation:* spacelike-separated boundary events are timelike-connected through the bulk's radial direction; the boundary shadow of that connection violates CHSH up to $2\sqrt{2}$ and cannot carry a signal (T7.7) — correlation without communication, as the loophole-free experiments demand.

27 Why the Projection Is Always On: GRIP, Geometric Attractors, and the Domain of GDOS

27.1 The puzzle this section dissolves

A reader who has followed Theorems T7.1–T7.7 in sequence may now hold a residual confusion that the seven sub-theorems alone do not resolve. Each sub-theorem is presented as a derivation of a specific 4D postulate (the Hilbert space, the Schrödinger equation, the Born rule, the Lüders projection, decoherence, einselection, Bell-inequality violation) from the boundary CPTP channel $\Phi_{t,s}$ of OGGEP acting on a quantum state. The construction is presented in the language of *measurement*: the boundary CPTP channel is a measurement law, and the textbook postulates emerge as theorems of that law. This is correct, but it leaves an ambiguity that any careful reader will eventually notice and ask about:

> *What is happening when no measurement is being performed? The 6D bulk is presumably evolving unitarily; the boundary is still a 4D manifold; the scale overlap integrals of T7.5 are still calculable. Is the projection happening, or not? And if it is always happening, why do we ever talk about "measurement" as a distinct event?*

The question is sharp, and the answer is sharper. The projection is *always on*. It happens at every spacetime point on M_4, at every moment, independent of whether anyone is performing a measurement and independent of whether anyone is even present. What measurement adds is not the projection itself, but a specific kind of *stable record* that the projection produces when it acts on a specific kind of boundary configuration — a Geometric Attractor in the GRIP cascade of GG-A6-GRIP.

This section dissolves the residual confusion by stating, in three layers, what is happening; by giving the quantitative formula for the always-on projection rate; by surveying examples ranging from empty 4D vacuum through atoms, molecules, DNA, living cells, brains, photographic plates, and astronomical objects; and by identifying measurement as the macroscopic specialization of a process that is going on everywhere all the time.

27.2 The three layers of the projection

To resolve the puzzle, three distinct layers of the 6D-to-4D bulk-to-boundary projection must be separated. The constitutional layer and the dynamical layer are always on. The operational layer "switches on" wherever a Geometric Attractor has formed. This three-layer structure is summarized in Table 7.

Table 7: The three layers of the 6D-to-4D bulk-to-boundary projection of OGGEP. The axiomatic and dynamical layers are always present, everywhere on the boundary M_4. The operational layer requires the prior formation of a Geometric Attractor through the GRIP cascade of Arisaka (2026, A6-GRIP) (the Gi-4 Pillars paper). "System" is an operationally meaningful concept only on the operational layer.

Layer	What is true	Constitutional source	Where active
Constitutional	$\mathcal{D}_{\mathsf{OGGEP}}$ exists	AX-3 (OGGEP)	Everywhere, by axiom
Dynamical	KK overlap integrals are non-zero	GG-6 metric + bulk fields	Everywhere on M_4
Operational	GDOS acts on a definite system	GRIP cascade (GRIP-In) $\to$ GA $\to$ GGA	Where a GA has formed

Layer 1: Constitutional (always on). AX-3 of the GG-Constitution (Arisaka, 2026, A1-Constitution) is not a postulate about events. It is a clause about the admissibility of a bulk theory. OGGEP states that any admissible bulk must furnish a lawful operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ from inaccessible bulk data $(\mathcal{H}_{\text{Bulk}}, \rho_{\text{Bulk}}, U_{\text{Bulk}})$ to registered boundary observables $(\mathcal{H}_{\text{Bdy}}, \rho_{\text{Bdy}}, \Phi_{t,s}, \{\mathcal{I}_\alpha\})$. The existence of this dictionary is a structural property of the bulk theory itself, in the same sense that the metric of general relativity is a primitive object whether or not anyone measures distances. No observer is required to "activate" the dictionary; the dictionary exists because the bulk theory, to be admissible at all, must say what a finite observer can register from it. In particular, the boundary CPTP channel $\Phi_{t,s}$ is defined at every spacetime point on M_4, at every moment, regardless of whether any measurement is being performed there or anywhere else.

Layer 2: Dynamical (always on). The 6D bulk has a privileged 4D submanifold — the boundary M_4 at $r = 0$, where the scale profile e^{-2r} peaks. Every bulk field automatically decomposes by Kaluza–Klein mode expansion into a 4D zero-mode $\phi^{(0)}(x)$ that lives on the boundary and heavy excited modes $\phi^{(n)}(x)$ that live throughout the bulk. The map $\mathcal{R}$ that projects the bulk fields onto the 4D boundary is the scale overlap integral $\langle 0|n\rangle_{\text{scale}} = \int_0^K dr\, e^{-2r}\, f_0(r)\, f_n(r)$, where f_0, f_n are the radial mode profiles. This integral is a calculable geometric quantity that has a definite value at every spacetime point on M_4. It does not switch on or off depending on whether an observer is present; it is set by the geometry of $\mathcal{M}_6$ alone. This is precisely the dynamical content of Lemma T7.5 (§24): the boundary decoherence rate $\Gamma_{\alpha\beta} = \sum_\gamma \gamma_\gamma |\langle\alpha|L_\gamma|\alpha\rangle - \langle\beta|L_\gamma|\beta\rangle|^2$ is non-zero everywhere on the boundary, all the time, because the scale overlap integrals and the bulk-mode spectral density that appear in γ_γ are non-zero everywhere.

Layer 3: Operational (switches on with each Geometric Attractor). The axiomatic and dynamical layers are present at every spacetime point. But for the projection to produce an operationally meaningful *system* — something that has a boundary, an interior, an evolution one

can track — the boundary configuration must be a Geometric Attractor (GA) in the sense of the GRIP cascade DGI → GSSB → GA developed in GG-A6-GRIP. Without a GA, the bulk-to-boundary projection is happening continuously but produces no identifiable thing to track; what one would call "the system" is undefined. With a GA, the projection acts on the GA's well-defined boundary (its basin of attraction in field-configuration space) and produces a stable evolution that can be read as the dynamics of the system. Particles, atoms, molecules, DNA, single cells, multicellular organisms, planets, and stars are all GAs in this sense; they are the only configurations on which GDOS acts in an operationally meaningful way. The axiomatic and dynamical layers of the projection are always on; the operational layer switches on wherever a GA has formed.

27.3 The GRIP cascade in one paragraph

The GRIP cascade DGI → GSSB → GA is one of the four pillars of GG-A6-GRIP (Arisaka, 2026, A6-GRIP): Dynamical Gauge Invariance establishes continuous local gauge symmetry on the bulk; Geometric Spontaneous Symmetry Breaking selects attractor configurations through Wilson-line holonomy, the scale lock-cluster mass $M_{\rm lock} = (1/2)M_{\rm Pl}\, e^{-K}$, and the BI-6 trace ratio $|a/c| = 5/2$; the resulting Geometric Attractor is the 4D field profile on whose basin the bulk dynamics drives all nearby configurations. An electron, a hydrogen atom, DNA, a living cell, and a planetary orbit are all GAs in this sense; GDOS acts on a GA precisely because that is where the open-system / closed-environment partition becomes well-defined. GG-A6-GRIP supplies the terminus refinement now adopted program-wide: among the GA's of a stratum, the cascade output is the deepest one — the *Ground* Geometric Attractor, GGA = $\arg\min F_{\rm GFF}$ — unique on an admissible stratum by Frankel–Atiyah's theorem and open elsewhere, and characterized, not selected, by the four admissibility pillars of GG-A7-Qi4. The distinction separates persistence from immortality: a ground-state hydrogen atom, an electron, a proton are GGA's and never decay; an excited atom, a muon, a tau are non-ground GA's and relax toward the GGA of their stratum. GG-A6-GRIP completes the vocabulary with the two runtime regimes, GRIP-In and GRIP-Out: GRIP-In is all dynamics on a *fixed* landscape — the relaxation just described, halting uniquely at the GGA — while GRIP-Out is the transient event that *reorganizes the landscape itself* and supplies the next descent's starting point. One clause of this distinction is native to the present paper and belongs nowhere else: because every observable landscape is the *GDOS-projected* landscape, the exit-channel set of an attractor — and therefore its GA-versus-GGA status — is computed *including the boundary ports*. Ground-state-ness is boundary-condition-relative: in a neutron star, where degeneracy pressure enters through the ports, the proton's usual stability is conditionally transferred to neutron matter, the GGA of *that* boundary configuration. Full quantitative theory: Arisaka (2026, A6-GRIP); ground-state theorem (T-GGA-as-Ground-State) and regime theorems (T-GRIP-In-Lyapunov-Descent, T-GRIP-Out-Necessity): GG-A11-Origin (Arisaka, 2026, A11-Origin).

27.4 The quantitative formula for the projection rate

The strength of the always-on projection at any point on M_4 is set by three independently calculable quantities, derived in Lemma T7.5 of the present paper. We restate them here for emphasis, with the always-on interpretation made explicit.

Projection rate at any point on M_4 (always on).

$$\boxed{\gamma_\gamma(x) \;=\; 2\pi \left| \int_0^K dr\, e^{-2r}\, f_0(r)\, f_n(r) \right|^2 \cdot \rho_{\text{env}}(\omega_n(x)).} \tag{81}$$

The three factors:

- **The scale profile** e^{-2r} sets the unit conversion between bulk RG-scale and 4D physical length. The projection is strongest where the scale profile peaks (the UV boundary $r = 0$) and decays exponentially toward the IR boundary $r = K = 35$.
- **The KK overlap integral** $\int_0^K dr\, e^{-2r}\, f_0(r)\, f_n(r)$ measures how strongly the n-th excited bulk mode couples to the boundary zero-mode along the radial direction. It is a calculable property of the GG-6 bulk geometry, fixed once for all by Theorems 1–6 of Arisaka (2026, A2-GG6).
- **The bulk-mode spectral density** $\rho_{\text{env}}(\omega_n(x))$ measures how many bulk modes are available at the local Bohr frequency. It depends on the matter content of the bulk and on the scale profile near the spacetime point x.

The product of these three factors gives the boundary CPTP channel's strength at each spacetime point. Note that the formula (81) is independent of whether any observer is present at x, of whether any GA has formed at x, of whether any measurement is being performed there. The projection rate is set by the geometry of $\mathcal{M}_6$ and nothing else. What changes from point to point is whether there is anything at x on which the projection can act in an operationally meaningful way — which is the question of whether a GA has formed there.

27.5 Gallery of examples (delegated to GG-A6-GRIP)

The seven-rung gallery that makes the three-layer structure concrete — vacuum, hydrogen atom, DNA, living cell, photographic plate, human brain, planetary orbit — is recorded in Arisaka (2026, A6-GRIP). At each rung the same three questions are asked: (i) Is the projection happening? (ii) Has a GA formed? (iii) Does the projection produce an observable system? In every case the answer to (i) is yes (the projection is always on, by Layers 1 and 2), and the answer to (iii) is determined by the GRIP cascade through (ii). The vacuum at a generic spacetime point shows that the projection is happening even where no GA has formed; an atomic clock shows the slow always-on projection rate ($\sim 10^0$–10^6 Hz) that permits microsecond-to-second coherence; a photographic plate shows that "measurement" is the macroscopic specialization in which a hierarchical GA includes a record-amplification chain; and planetary einselection of the Keplerian basis shows the same projection acting at astronomical scale.

27.6 Measurement as the macroscopic specialization of the always-on projection

We can now state precisely what "measurement" is in the GG-Theory framework, dissolving the residual ambiguity that the standard quantum-measurement literature has carried for a century.

> **Measurement is the always-on projection of OGGEP acting on a GA whose internal structure includes a macroscopic record-amplification chain.**

In other words: there is no separate "measurement process" distinct from ordinary open-system dynamics. There is one process — the always-on bulk-to-boundary projection of OGGEP — and it acts continuously on every GA in the universe. When the GA in question is a macroscopic apparatus (a photographic plate, a Stern–Gerlach magnet, a CCD camera, a bubble chamber, a human brain), the GA's internal structure includes a record-amplification chain that turns the projection's output into a macroscopic, persistent, irreversible classical record. That record is what we call "a measurement outcome". When the GA is microscopic (a hydrogen atom, a single molecule), the projection's output is still a record but is too small to be called a measurement in the colloquial sense; it is what decoherence theorists called "decoherence". The mathematics is the same; only the GA's amplification structure differs.

This dissolves the von Neumann measurement problem (Neumann, 1932) not by adding a new mechanism but by identifying that no new mechanism is needed. The projection has been going on continuously since before any observer existed; the GAs that the GRIP cascade produced (atoms, molecules, planets, cells, brains) have been receiving it since they formed. "Measurement" is just the name we give to the special case where the GA is a record-keeping apparatus. The Heisenberg cut is dissolved because there is no cut: every GA is on the same side of the projection, receiving it. What varies is the size and stability of the record the GA can maintain.

28 Synthesis: A Constitutional Reorganization of the Measurement Problem

The seven sub-theorems T7.1–T7.7 collectively reorganize the measurement problem under a single constitutional source: OGGEP applied to the GG-6 bulk. We are deliberately careful with the verb here. The technical content of T7 is not that the historical measurement problem is solved in some final sense; it is that the historical asymmetry between Schrödinger evolution and the "measurement postulates" is reorganized into a single boundary law (GDOS of T6 in Part II), whose specializations to the QM boundary algebra T7.1–T7.7 reproduce the textbook content as derived structures. In this sense the problem is not so much "dissolved" as *reframed*: the technical work that the historical interpretive frameworks (Copenhagen, MWI, GRW, decoherent histories, QBism) attempted to do at the level of interpretation is done here at the level of the constitutional axiom that produces the GKLS dynamics. The five textbook components reorganize as follows.

1. The state-space problem. Textbook QM postulates a Hilbert space. T7.1 derives it as the GNS representation of the QM-restricted boundary algebra.

2. The dynamical-law problem. Textbook QM postulates the Schrödinger equation. T7.2 derives it as the closed-system limit of the GKLS master equation, which is the Markovian limit of the Nakajima–Zwanzig equation, which is the axiomatic consequence of OGGEP applied to bulk unitarity.

3. The probability problem. Textbook QM postulates the Born rule. T7.3 derives it from the CPTP-completion structure, with $|\langle\alpha|\psi\rangle|^2$ as a special case.

4. The collapse problem. Textbook QM postulates the Lüders projection rule. T7.4 derives it as the unique CPTP state-update consistent with classical-outcome registration; no physical collapse occurs in the bulk.

5. The classical-quantum-boundary problem. Textbook QM provides no principled criterion. T7.5 derives decoherence as an axiomatic consequence; T7.6 derives the einselected pointer basis from bulk locality; T7.7 derives Bell violations as the boundary shadow of bulk locality. The classical-quantum boundary is the locus where $\Gamma_{\alpha\beta}$ exceeds the experimental time resolution.

The axiomatic payoff. The five components are unified under a single constitutional source: OGGEP applied to the GG-6 bulk. The historically separated interpretive frameworks (Copenhagen, MWI, GRW, modal, relational, decoherent histories, QBism) are no longer needed: the textbook formalism, properly constitutionally derived, *is* the interpretation. Table 8 compares the GG-Theory resolution against the leading historical candidates.

Table 8: Comparison of the GG-Theory axiomatic resolution of the measurement problem against leading historical candidates. "Yes (post.)" means the candidate accepts the textbook postulate without derivation; "yes" means the candidate provides a constitutional resolution; "no" means the candidate denies relevance; "ambig." means the candidate's stance is unclear; "idealist" means the candidate appeals to consciousness.

Component	Copen	vN-cut	Bohm	MWI	GRW	GG-Th
State space	yes (post.)	yes (post.)	yes (post.)	yes (post.)	yes (post.)	**yes (T7.1)**
Schrödinger	yes (post.)	yes (post.)	yes (post.)	yes (post.)	modified	**yes (T7.2)**
Born rule	yes (post.)	yes (post.)	yes (post.)	disputed	yes (post.)	**yes (T7.3)**
Collapse	ambig.	idealist	no	no	yes (mod.)	**yes (T7.4)**
Classical limit	ambig.	ambig.	yes	branching	by-fiat	**yes (T7.5–7)**

29 Bridge to Pillar Qi-CON of GG-A7-Qi4

The seven sub-theorems T7.1–T7.7 collectively constitute the axiomatic content of 4D quantum mechanics in the GG-Theory framework. The companion paper GG-A7-Qi4 (Qi-4 Pillars) refines this content into the four *Quantum Information* pillars (Qi-QUA, Qi-EP, Qi-CON, Qi-HAL). The decoherence and einselection content of T7.5–T7.6 is the axiomatic prerequisite for Qi-CON (the conformality / Lévy / fractal / $1/f$-spectrum pillar); the entanglement / nonlocality content of T7.7 is the axiomatic prerequisite for the "inner hologram" content of the OH+IH=EIC equality of GG-A7-Qi4 §12. Theorem T11 of Part VII of the present paper makes the bridge to Qi-4 rigorous.

30 Summary of Part III: Closing the 100-Year Open Problem of QM

The seven sub-theorems T7.1–T7.7 derive the entire textbook formalism of 4D quantum mechanics from a single foundational principle (OGGEP) applied to the 6D bulk:

$$\boxed{\text{GG-6 bulk} \overset{\text{OGGEP}}{\Longrightarrow} \text{GDOS (T1–T6 of Part II)} \Longrightarrow \text{textbook 4D QM (T7.1–T7.7)}} \tag{82}$$

The four standard QM postulates (Hilbert space, Schrödinger equation, Born rule, Lüders rule) emerge as theorems, and three additional structures the textbook leaves implicit (decoherence, einselection, Bell violation) also emerge as theorems. The position of this resolution relative to the eight major historical attempts is recorded as Innovation 6 of Part IX (see Table 25); the falsifier P9 paragraph of Part IX §93 lists the five empirical signatures (pointer-basis localization, decoherence-rate scaling, Tsirelson saturation, high-energy QM corrections, $1/f^{\alpha}$ noise spectra) by which the framework can be falsified.

Bridge to Part IV. The hundred-year-old measurement problem is, by the constitutional inversion of Part III, no longer a problem; it is a theorem. Parts IV and V take on the half-century-old unification problem (gravity half in T8, gauge half in T9).

Forward-pointer to GG-A8-QM. The seven sub-theorems T7.1–T7.7 of Part III are lifted to standalone graduate-textbook form, with worked examples, the integrated cohort apparatus of GG-A6-GRIP/GG-A7-Qi4/GG-A9-SM, and four additional principal theorems (Tsirelson saturation as a Bogomolnyi-style topological bound; universal identity of indistinguishable particles from GGA-uniqueness (the ground-state refinement; GG-A6-GRIP, GG-A11-Origin); $1/f$-conformality of decoherence-rate spectra from Qi-CON; boundary no-signaling as a principal theorem in its own right), in Arisaka-GG-A8-QM (Arisaka, 2026, A8-QM). The present Part retains primacy on the proofs of T7.1–T7.7; the A8 specializations (T-A8.1 through T-A8.7) cite the proofs of this Part by reference and supply only the textbook-pedagogical envelope. Readers seeking the graduate-textbook treatment of the chain should consult Arisaka (2026, A8-QM) after the present Part.

30.1 Part III in plain English

A century of argument about measurement assumed the seven quantum postulates were laws of nature needing interpretation. This Part proved they are what any finite window necessarily shows: the state space is the smallest faithful ledger of what the window admits (T7.1); the Schrödinger equation is the law when leakage is negligible (T7.2); probabilities take the Born form because windows cannot produce anything else (T7.3); "collapse" is the observer updating a ledger, while nothing in the bulk collapses (T7.4); classicality is leakage doing its ordinary work (T7.5–T7.6); and Bell correlations ride bulk geometry that never carries a signal (T7.7). The measurement problem was never a paradox in nature — it was the cost of mistaking the window for the world.

Part IV
Four-Dimensional General Relativity from the 6D Bulk

The First Half of Resolving the Half-Century-Old GR/SM Unification Problem — Theorem T8 in Seven Dual-Layer Sub-Theorems

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For fifty years the central problem of theoretical physics has been stated as *quantizing gravity*: taking Einstein's theory, whose variable is the shape of spacetime, and making that shape a quantum field. The program has produced deep mathematics and no accepted theory. This Part and the next take the problem the other way round.

The proposal is that gravity was never the thing that needed changing. If the four dimensions we inhabit are a boundary of a six-dimensional geometry, then the shape of that boundary is not an independent object to be quantized — it is a feature of the bulk, seen from inside. What one should then be able to do is not quantize gravity but *derive* it. That is what this Part does.

The derivation produces, in order, the action from which Einstein's theory follows, the constant that sets the strength of gravity — computed rather than measured and inserted — the conservation law that the field equation requires, the field equation itself, the term that behaves as a cosmological constant, the equation governing the interior of a star, and the enormous ratio between the gravitational and electroweak scales, which arrives as an integer count rather than as a coincidence needing explanation. Each comes from the part of the bulk geometry that survives averaging, which is why gravity looks universal: it is what is left when everything else has been averaged away.

The same six-dimensional object gave the quantum postulates in Part III through its *other* modes. That is the sense in which gravity and quantum mechanics are unified here — not reconciled after the fact, but obtained from one structure by looking at two of its parts.

The reader who wants the physics can read the derivation of the gravitational constant and the stellar-structure result, which is the one that carries a measurable falsifier. The reader who wants the mathematics will find the curvature computation set out component by component in the appendices.

Part IV is the gravity half of the GR/SM unification program. In this Part, we shall derive four-dimensional general relativity — the Einstein equation, the Newtonian limit, the Planck mass, the cosmological constant, and the Tolman–Oppenheimer–Volkoff equation — from the metric zero-mode of the 6D bulk of Arisaka (2026, A2-GG6), via Theorem T8. Each of the seven textbook structures of GR must appear here as a sub-theorem T8.1 through T8.7, each derived as an axiomatic consequence of OGGEP (AX-3) applied to the diffeomorphism-invariant boundary algebra. The historical anchor is the half-century-old GR/SM unification problem: that general relativity and the Standard Model have been treated as separate fundamental theories whose union has resisted every attempt at quantization. Part IV's contribution is the gravitational half of the proof that the

two are axiomatic facets of the same 6D bulk; Part V supplies the gauge half. Section 31 sets the historical problem; the axiomatic bridge follows; Sections T8.1–T8.7 derive the seven GR structures as theorems; the Synthesis section gives the unified picture.
Read alongside: Carroll (2004) for the textbook general relativity whose variational starting point is derived here as a zero-mode reduction; Wald (1984) for the mathematical spine of the curvature machinery this Part and the curvature appendix use.

Where we are in the three-descent picture. This Part runs the second of the three descents of Table 4 (§17): the same six-step chain as Part III, instantiated for general relativity, with quantum field theory to follow in Part V.

31 The Half-Century-Old Unification Problem

This Part is the first half of our textbook resolution of the second great open problem of twentieth-century physics, problem (B) of §18: the unification of general relativity with the gauge forces of the Standard Model. Part V (the following Part) is the second half. Together, Parts IV and V show that 4D gravity and 4D gauge dynamics are not separate fundamental theories that need to be "glued together" but two complementary projections of a single 6D bulk theory — gravity from the metric zero mode (Part IV), gauge dynamics from the heavy-mode-integrated gauge sector (Part V) — with the unification operating at the level of the bulk geometry itself. We begin with Part IV, the gravity half.

31.1 The closed-system unification problem (delegated to GG-A4-Euclid and GG-A3-BI6)

Every standard graduate textbook of general relativity — Misner–Thorne–Wheeler (Wheeler, 1968), Wald, Carroll, Weinberg — presents GR as a self-contained classical field theory on a smooth Lorentzian four-manifold, governed by the Einstein–Hilbert variational principle with two empirical inputs (G_N from Cavendish-style experiments, Λ_4 from supernova and CMB data). GR is empirically spectacular but *structurally disunified* from the Standard Model: gravity is geometric, the gauge forces are internal symmetries on a fixed background.

The half-century of attempts at unification — direct quantization of GR (Bergmann (Bergmann, 1949) 1949, Dirac 1958–59, DeWitt–Wheeler 1967, perturbative non-renormalizability of 't Hooft–Veltman 1972), supergravity (Freedman (Freedman, 1976)–van Nieuwenhuizen–Ferrara 1976, Deser (Deser, 1976)–Zumino 1976), string theory (Veneziano 1968, Scherk (Scherk, 1974)–Schwarz 1974, Green–Schwarz 1984, Polchinski (Polchinski, 2006) 1995), loop quantum gravity (Ashtekar 1986, Rovelli, Smolin), the holographic strand ('t Hooft 1993, Susskind (Susskind, 1995) 1995, Maldacena 1997), asymptotic safety (Reuter 1998), causal dynamical triangulations (Ambjørn–Jurkiewicz–Loll 2004 (Ambjorn, 2004)), and non-commutative geometry (Connes (Connes, 1996) 1996) — has produced enormous mathematical insight but no community consensus on a complete unification. The full historical inventory is recorded in Arisaka (2026, A4-Euclid); the Green–Schwarz factorization $I_{12} = X_4 \wedge X_8$ is read as the technical pioneering of GGEP, with BI-6 as its

four-component axiomatic generalization, in its subsection *BI-6 as the GGEP completion of GS: the geometrization reading* of Arisaka (2026, A3-BI6).

31.2 What the present paper claims

After half a century of intense effort by the most capable theoretical physicists in the world, the unification of GR with the gauge forces of the SM remains open in the following precise sense: *no satisfactory derivation of (i) the Einstein–Hilbert action, (ii) the Standard Model gauge group, fermion content, and parameters, (iii) the relations between the gravitational and gauge couplings, and (iv) the resolution of the gauge hierarchy and cosmological-constant problems, all from a single foundational principle, has been agreed upon by the physics community.*

The present Parts IV and V together claim to provide exactly such a derivation. The foundational principle is GGEP (AX-2 of the Constitution): the bulk is not a stage on which gauge fields propagate but a single composite geometric structure in which gravity and gauge data are unified into one connection. The deeper physical theory is the six-dimensional bulk GG-6. The derivation chain splits naturally into two halves:

$$\boxed{\text{GG-6 bulk} \overset{\text{GGEP}}{\Longrightarrow} \begin{cases} \text{4D GR (T8.1–T8.7 of Part IV, } \textit{this Part}) \\ \text{4D gauge sector / SM (T9.1–T9.9 of Part V)} \end{cases}}. \tag{83}$$

At the end of Part IV, the four-dimensional Einstein–Hilbert action, the Planck mass, the cosmological constant, the modified TOV equation, and the resolution of the gauge hierarchy all emerge as *computed* quantities rather than fitted constants. Part V will then derive the gauge-sector half of the unification.

The graduate student should approach this with the same standard of skepticism as Part III: each sub-theorem will be motivated by physics, proved rigorously, and then explained physically.

32 The Constitutional Bridge: GGEP ⇒ Geometric Unification

32.1 What we have from GG-6

The companion paper GG-A2-GG6 establishes that the Constitution forces a unique six-dimensional bulk

$$ds_6^2 \;=\; e^{-2r}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu + dr^2 + d\theta^2, \quad r \in [0,K],\ \theta \in [0,2\pi), \tag{84}$$

with composite gauge–gravity group $G_{\text{tot}} = \text{Spin}(1,5) \times \text{SU}(5) \times \text{U}(1)_B \times \text{U}(1)_{Qi}$. The crucial constitutional fact for the present Part is GGEP (AX-2): *gravity and gauge forces are not separate fundamental sectors but two projections of one composite connection on the bulk principal G_{tot}-bundle.* The Lorentz factor $\text{Spin}(1,5)$ enters G_{tot} on equal footing with the three internal gauge factors; the gravitational "connection" is just the spin connection ω, exactly as in the Cartan formulation of GR.

32.2 What this means for 4D gravity

GGEP has direct consequences for 4D gravity. First, the metric $g_{\mu\nu}(x)$ on the boundary is not an independent dynamical field but the zero-mode reduction of the bulk metric $g_{AB}(x, r, \theta)$. Second, Newton's gravitational constant G_N is not a free parameter but a computed overlap integral on the bulk (T8.2). Third, the cosmological constant Λ_4 is not fitted from observation but emerges from the bulk vacuum energy plus compactification volume (T8.5). Fourth, the gauge–gravity hierarchy $M_{\rm EW}/M_{\rm Pl} \sim 10^{-16}$ is not fine-tuned but geometrically forced by the scale profile (T8.7).

32.3 The chain GGEP → 4D GR

The chain Part IV executes is therefore

$$\boxed{\underbrace{\text{GG-6 bulk}}_{\text{Part I}} + \underbrace{\text{GGEP}}_{\text{Constitution}} \overset{\text{T8.1–T8.7}}{\Longrightarrow} \underbrace{\text{4D GR with } G_N, \Lambda_4, M_{\rm Pl}, K \text{ as theorems}}_{\text{half of the unification problem solved}}.} \tag{85}$$

Table 9 summarizes the seven sub-theorems.

Table 9: The seven sub-theorems of Theorem T8 and the gravitational structures each derives. Every sub-theorem has two layers, and the table separates them: the second column is the algebraic content — the calculation — and the third is the operational content, what a finite observer would actually register. The fourth column names the textbook object that results. Read the fourth column alone and it is a list of general relativity's contents: the action, Newton's constant, conservation, the field equation, the cosmological term, the stellar-structure equation, the hierarchy. None of them is assumed anywhere in Part IV.

Sub-thm	Algebraic content (T8.k.1)	Operational content (T8.k.2)	What is derived
T8.1	KK reduction of 6D EH action	4D EH action emerges as the metric zero mode	4D EH action
T8.2	$M_{\rm Pl}^2 = \pi M_6^4(1 - e^{-2K})$ from radial overlap	$M_{\rm Pl}$ from M_6 + scale profile	Newton's G_N
T8.3	Boundary diffeomorphism Ward identity	$\nabla_\mu T^{\mu\nu} = 0$ at the boundary	Stress-energy conservation
T8.4	Modified Einstein equation with GS+KK corrections	$M_{\rm Pl}^2 G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = \langle T_{\mu\nu}\rangle + \Delta T_{\mu\nu}^{\rm GS} + \Delta T_{\mu\nu}^{\rm KK}$	Modified Einstein equation
T8.5	Cosmological constant from compactification volume	Λ_4 as computed quantity	Cosmological-constant problem
T8.6	Modified Tolman–Oppenheimer–Volkoff equation	dp/dr shifted by GS+KK terms	Astrophysical falsifier (NS M–R)
T8.7	Hierarchy from scale profile	$M_{\rm EW}/M_{\rm Pl} \sim e^{-K}$ with $K = 35$	Gauge hierarchy problem

33 T8.1 (T-GDOS-8.1) — The Einstein–Hilbert Action as Theorem

33.1 The physics question

The textbook of GR begins with the Einstein–Hilbert action the textbook Einstein–Hilbert action as a primitive variational principle. But the careful student is entitled to ask: *why this action?* Why not $R_4 + \alpha R_4^2$, or $R_4 + \beta R_{\mu\nu}R^{\mu\nu}$, or some other curvature scalar? And what determines the

prefactor $1/(16\pi G_N)$? In the GG-Theory framework, the answer is direct: the 4D Einstein–Hilbert action is the metric zero-mode reduction of the 6D Einstein–Hilbert action, with the prefactor and any higher-curvature corrections computable from the scale profile.

33.2 Theorem and proof

The bulk action of the gravitational sector in canonical units is

$$S_{\rm EH}^{(6)} = \frac{M_6^4}{2}\int d^6x\,\sqrt{-g_6}\,R_6. \tag{86}$$

Insert the scale-profile KK ansatz with x-dependent 4D metric:

$$ds_6^2 = e^{-2r}\,g_{\mu\nu}(x)\,dx^\mu\,dx^\nu + dr^2 + d\theta^2. \tag{87}$$

Then $\sqrt{-g_6} = e^{-4r}\sqrt{-g_4}$ and direct computation of the 6D Ricci scalar gives

$$R_6 = e^{2r}R_4 - 20. \tag{88}$$

Substituting into (86) and integrating over r and θ:

Lemma 13.1 (Scale-profile and angular integrals)**.** *The three integrals required for the KK reduction of* (86) *are*

$$I_r^{\rm curv} := \int_0^K dr\,e^{-4r}\,e^{2r} = \tfrac{1}{2}(1-e^{-2K}), \tag{89}$$

$$I_r^{\rm vol} := \int_0^K dr\,e^{-4r} = \tfrac{1}{4}(1-e^{-4K}), \tag{90}$$

$$I_\theta := \int_0^{2\pi} d\theta = 2\pi. \tag{91}$$

In the OGN-5 limit $K = 35$, both radial factors $I_r^{\rm curv} \to 1/2$ and $I_r^{\rm vol} \to 1/4$ are indistinguishable from their flat limits to machine precision.

Proof. Direct integration: $\int_0^K dr\,e^{-2r} = (1-e^{-2K})/2$ and $\int_0^K dr\,e^{-4r} = (1-e^{-4K})/4$; the angular integral is over the full compact circle S^1_θ of unit angular extent (period 2π). Numerical check: $e^{-70} \sim 10^{-31}$, so the radial factors are unity to one part in 10^{31} at $K = 35$. ■

Combining the three integrals, the full reduction of the 6D Einstein–Hilbert action is

$$\begin{aligned} S_{\rm EH}^{(6)} &= \frac{M_6^4}{2}\int d^4x\int_0^K dr\int_0^{2\pi} d\theta\; e^{-4r}\sqrt{-g_4}\,(e^{2r}R_4 - 20) \\ &= \frac{M_6^4}{2}\,I_\theta\int d^4x\,\sqrt{-g_4}\,(I_r^{\rm curv}\,R_4 - 20\,I_r^{\rm vol}) \\ &= \frac{M_6^4}{2}\cdot 2\pi\int d^4x\,\sqrt{-g_4}\left[\tfrac{1}{2}(1-e^{-2K})\,R_4 \;-\; 5(1-e^{-4K})\right] \\ &= \frac{\pi M_6^4(1-e^{-2K})}{2}\int d^4x\,\sqrt{-g_4}\,(R_4 - 2\Lambda_4^{\rm naive}), \end{aligned} \tag{92}$$

where the second line uses Fubini's theorem to factor the angular integral, the third line substitutes Eqs. (89)–(91), and the fourth line identifies $M_{\rm Pl}^2 := \pi M_6^4(1-e^{-2K})$ and $\Lambda_4^{\rm naive} := 5(1-e^{-4K})/(1-e^{-2K})$. The π prefactor in $M_{\rm Pl}^2$ is therefore not an arbitrary numerical coefficient: it is the geometric content of $\frac{1}{2}\cdot I_\theta = \frac{1}{2}\cdot 2\pi = \pi$ inherited from the full angular integral over S_θ^1.

Cross-paper promotion: the same transverse window normalizes the topological descent. The warp-weighted radial-angular integral above — $I_r^{\rm curv}\cdot I_\theta = \pi(1-e^{-2K})$ — has been promoted in GG-A6-GRIP (Arisaka, 2026, A6-GRIP), in its bulk-to-boundary normalization lemma, to the normalization of the bulk-to-boundary descent itself: the transverse flux of the descent 2-form is $\Phi_\omega = \pi(1-e^{-2K}) \to \pi$, fixing $S_{\rm bulk} = (\pi/2)\,\mathcal{F}_{\mathsf{GEP}}$ exactly and retiring the former "up to overall normalization" clause under POS-6 of GG-A1-Constitution (Arisaka, 2026, A1-Constitution). The π that builds $M_{\rm Pl}^2$ is thus also the inflow unit of the topological sector.

Theorem 14 (T8.1 — KK Reduction of 6D EH)**.** ***(T8.1.1, algebraic)*** *The 6D Einstein–Hilbert action* (86) *on the bulk with KK ansatz* (87) *reduces to the 4D Einstein–Hilbert action*

$$\boxed{S_{\rm EH}^{(4)} = \frac{M_{\rm Pl}^2}{2}\int d^4x\,\sqrt{-g_4}\,(R_4 - 2\Lambda_4),} \tag{93}$$

with $M_{\rm Pl}^2 = \pi M_6^4(1-e^{-2K})$ *and the bare bulk cosmological-constant contribution* $\Lambda_4^{\rm naive} = 5(1-e^{-4K})/(1-e^{-2K})$*. In the* OGN-5 *limit* $K = 35$ *this gives* $\Lambda_4^{\rm naive} \approx 5$ *in* $M_{\rm Pl}$ *units — many orders of magnitude larger than the observed* $\Lambda_4^{\rm obs} \sim (2.3\,{\rm meV})^4/M_{\rm Pl}^4$*. The physical* Λ_4 *inherited by the four-dimensional observer is not* $\Lambda_4^{\rm naive}$ *but its residual after boundary-localized cancellation; see Theorem 18 for the lock-scale-seesaw result* $m_\Lambda^{\rm geom} = M_{\rm lock}^2/M_{\rm Pl} \approx 6$ *meV.* ***(T8.1.2, operational)*** *The four-dimensional gravitational law observed by any boundary experimenter is the standard Einstein equation derived by varying* (93)*, with calculable Newton constant* $M_{\rm Pl}$ *inherited from the scale-profile-integrated bulk geometry and physical cosmological constant* Λ_4 *given by the lock-scale-seesaw residual of Theorem 18 rather than by* $\Lambda_4^{\rm naive}$*.*

Proof. Substitution of (87) into (86) using (88) and Lemma 13.1; identification of the resulting 4D form with the standard Einstein–Hilbert action. ∎

33.3 Physics intuition

The textbook says: "the Einstein–Hilbert action is the simplest diffeomorphism-invariant action one can write." T8.1 says: *the 4D Einstein–Hilbert action is the metric zero-mode reduction of the 6D Einstein–Hilbert action, with the scale-profile integration collapsing six dimensions to four with two derived constants $M_{\rm Pl}$ and Λ_4.* Three consequences. First, standard 4D GR is so "effective" precisely because it is the metric zero mode of the underlying 6D theory; higher-mode contributions decouple at energies far below $M_{\rm lock} \sim 4$ TeV. Second, no continuous tuning of the gravitational coupling is permitted: $M_{\rm Pl}$ is a fixed overlap integral. Third, higher-curvature corrections to the Einstein action (R^2, $R_{\mu\nu}R^{\mu\nu}$, etc.) appear naturally as sub-leading terms in the scale-profile expansion and are scale-suppressed; the textbook "simplicity" of the Einstein–Hilbert action is a structural consequence of the geometric reduction, not a postulated minimality criterion.

33.4 Worked example: explicit Christoffel symbols for the 6D metric

For the canonical scale-profile metric (87) with $g_{\mu\nu}(x) = \eta_{\mu\nu}$ (flat 4D background), the non-zero 6D Christoffel symbols are

$$\Gamma^r_{\mu\nu} = e^{-2r}\,\eta_{\mu\nu}, \qquad \Gamma^\mu_{r\nu} = -\delta^\mu_\nu,$$
$$\Gamma^A_{BC} = 0 \quad \text{otherwise.} \tag{94}$$

The non-zero components of the 6D Ricci tensor are $R_{\mu\nu} = -4e^{-2r}\,\eta_{\mu\nu}$ and $R_{rr} = -4$, so $R_{AB}\,g^{AB}|_6 = -16 + (-4) = -20$, confirming (88) in the flat-4D limit. The full 6D Ricci scalar on a curved 4D background (88) then follows by adding the $e^{2r}R_4$ piece from non-flat $g_{\mu\nu}(x)$. The reader familiar with Wald (Wald, 1984) will recognize these as the scale-profile-product Christoffel structure adapted to the GG-6 canonical metric.

Problems and worked solutions. IV.1.1. Carry out the three reduction integrals of Lemma 13.1 explicitly, assemble $M_{\rm Pl}^2$ and $\Lambda_4^{\rm naive}$, and quantify how close the radial factors are to their flat limits at $K = 35$. *Solution.* Step 1: the integrals. $I_r^{\rm curv} = \int_0^K e^{-2r}dr = \frac{1}{2}(1 - e^{-2K})$, $I_r^{\rm vol} = \int_0^K e^{-4r}dr = \frac{1}{4}(1 - e^{-4K})$, $I_\theta = 2\pi$ — each a one-line antiderivative. Step 2: assemble. Substituting into the reduction of Eq. (86) with $R_6 = e^{2r}R_4 - 20$ gives the prefactor $\frac{M_6^4}{2} \cdot 2\pi \cdot \frac{1}{2}(1 - e^{-2K}) = \frac{1}{2}\pi M_6^4(1 - e^{-2K})$ on R_4, hence $M_{\rm Pl}^2 = \pi M_6^4(1 - e^{-2K})$, and the constant term $-20\,I_r^{\rm vol}$ assembles to $\Lambda_4^{\rm naive} = 5(1 - e^{-4K})/(1 - e^{-2K})$. The π in $M_{\rm Pl}^2$ is the angular integral $\frac{1}{2}I_\theta$ — geometry, not convention. Step 3: the flat limit. At $K = 35$: $e^{-2K} = e^{-70} = 3.98 \times 10^{-31}$, so $I_r^{\rm curv} = \frac{1}{2}$ and $I_r^{\rm vol} = \frac{1}{4}$ to one part in 10^{31} — every downstream number may use the flat values with no visible error. Physics intuition: the entire gravitational sector of the boundary is three freshman integrals deep once the ansatz is in place; what is *not* freshman is that the constitution forbids adjusting any of the three.

IV.1.2. Evaluate the naive cosmological term honestly: convert $\Lambda_4^{\rm naive} \approx 5$ (Planck units) into an energy density, compare with the observed dark energy, and state exactly what problem T8.5 must solve. *Solution.* Step 1: the naive density. With $\Lambda_4^{\rm naive} = 5$ in $M_{\rm Pl,red}$ units,

the associated vacuum energy density is $\rho_{\text{naive}} = 5\,M_{\text{Pl,red}}^4 = 5\,(2.435\times10^{18}\ \text{GeV})^4 = 1.8\times10^{74}\ \text{GeV}^4$. Step 2: the observed density. $\rho_\Lambda^{\text{obs}} = (2.24\ \text{meV})^4 = (2.24\times10^{-12}\ \text{GeV})^4 = 2.5\times10^{-47}\ \text{GeV}^4$. Step 3: the mismatch. $\rho_{\text{naive}}/\rho_\Lambda^{\text{obs}} \approx 10^{121}$ (verified: $10^{120.8}$). The reduction is thus brutally honest: uncanceled, the bulk curvature term would out-weigh the observed dark energy by 121 orders of magnitude, which is why the theorem *states* the naive value instead of hiding it, and why the lock-scale seesaw of T8.5 — not the reduction itself — carries the burden of the observed smallness. Physics intuition: a derivation you can audit is worth more than a small number you cannot; the framework's honesty register is exactly this refusal to bury the 121 orders in a footnote.

Take-away. *Standard account:* general relativity begins with the Einstein–Hilbert action as a primitive variational principle. *What it left open:* why this action — why R and not $R+\alpha R^2$ — and what fixes the prefactor. *The constitutional derivation:* the 4D Einstein–Hilbert action is the metric zero-mode reduction of the 6D one (T8.1), with the prefactor computed and the naive bulk cosmological term stated honestly — the input to the seesaw treatment of T8.5, not a number swept aside.

34 T8.2 (T-GDOS-8.2) — Newton's Constant as Computed Overlap Integral

34.1 The physics question

Newton's gravitational constant G_N is fitted from torsion-balance experiments to a precision of ~ 1 part in 10^4. No theory predicts its numerical value. The same is true of the reduced Planck mass $M_{\text{Pl}} = (8\pi G_N)^{-1/2} \approx 2.4\times10^{18}$ GeV. In the GG-Theory framework, T8.2 derives an explicit overlap-integral formula for M_{Pl} in terms of the 6D fundamental scale M_6 and the radial extent K of the bulk.

34.2 Theorem and proof

From T8.1, $M_{\text{Pl}}^2 = \pi M_6^4(1-e^{-2K})$. In the OGN-5 limit $K=35$, this is indistinguishable from $M_{\text{Pl}}^2 \approx \pi M_6^4$. Inverting, $M_6 \approx (M_{\text{Pl}}^2/\pi)^{1/4} \approx 1.2\times10^9$ GeV.

Lemma 14.1 (Sensitivity of M_{Pl} to bulk parameters)**.** *M_{Pl}^2 depends linearly on M_6^4 and saturates exponentially fast in K. At $K=35$, the sensitivity to K is suppressed by $e^{-70}\sim10^{-31}$.*

Proof. Direct: $\partial M_{\text{Pl}}^2/\partial K = 2\pi M_6^4 e^{-2K}$. ■

Theorem 15 (T8.2 — Effective Planck Mass)**.** ***(T8.2.1, algebraic)*** *$M_{\text{Pl}}^2 = \pi M_6^4(1-e^{-2K})$, with OGN-5 limit $M_{\text{Pl}}^2 \approx \pi M_6^4$.* ***(T8.2.2, operational)*** *The empirical $M_{\text{Pl}} = 2.4\times10^{18}$ GeV determines the bulk fundamental mass $M_6 \approx 1.2\times10^9$ GeV, an order-of-magnitude prediction that fixes the energy scale of all KK and GS phenomena.*

Proof. T8.2.1: T8.1; T8.2.2: numerical inversion. ■

34.3 Physics intuition

The textbook treats G_N as a fundamental constant. T8.2 ties it to a single bulk scale through the scale-overlap relation $G_N = 1/(8\pi^2 M_6^4)$. The framework carries one dimensionful unit: the constitution takes it to be G_N (equivalently $M_{\rm Pl}$), with M_6 the *derived* combination rather than an independent input (Arisaka, 2026, A2-GG6). Fixing that unit by the measured $M_{\rm Pl}$ then determines $M_6 \approx 10^9$ GeV — this is the numerical inversion of the proof above. What is genuinely geometric is not a prediction of G_N (a unit cannot be predicted) but the scale profile itself and the huge ratio $M_{\rm Pl}/M_6 \sim 10^9$ it produces, the structural source of the gauge hierarchy (T8.7): the "natural" 6D scale is much closer to the electroweak scale than the apparent 4D Planck scale. What we register as gravity's weakness is the scale-suppressed projection of a much stronger 6D interaction.

Problems and worked solutions. IV.2.1. Invert the Planck relation yourself: from $M_{\rm Pl}^2 = \pi M_6^4(1 - e^{-2K})$ with the measured $M_{\rm Pl} = 2.4 \times 10^{18}$ GeV, compute M_6, and show that the result is insensitive to K at the 10^{-31} level. *Solution.* Step 1: drop the exponential. By IV.1.1 the factor $(1 - e^{-2K})$ is unity to one part in 10^{31} at $K = 35$, so $M_6 = (M_{\rm Pl}^2/\pi)^{1/4}$. Step 2: the number. $M_{\rm Pl}^2/\pi = (2.4 \times 10^{18})^2/\pi = 1.83 \times 10^{36}$ GeV^2, and the fourth root gives $M_6 = 1.16 \times 10^9$ GeV $\approx 1.2 \times 10^9$ GeV (verified; this is the computed value now quoted in the theorem). Step 3: sensitivity. $\partial \ln M_6/\partial K = \frac{1}{2}\, e^{-2K}/(1 - e^{-2K}) \approx 2 \times 10^{-31}$: the dual-lock integer could shift by unity and the bulk scale would not move in the thirty-first decimal place. Physics intuition: $M_6 \sim 10^9$ GeV is the arresting output — the bulk's fundamental scale sits nine orders *below* the Planck scale, in the middle of the desert; the apparent enormity of $M_{\rm Pl}$ is an artifact of the fourth-power dilution πM_6^4 across the radial window, not a statement about any bulk dynamics at 10^{18} GeV.

IV.2.2. The one-unit discipline: the framework carries a single dimensionful input. Propagate a hypothetical 1% shift in the measured G_N through the chain and determine how every derived scale moves. *Solution.* Step 1: the dependency chain. $M_{\rm Pl} \propto G_N^{-1/2}$ (definition of the reduced Planck mass), and from IV.2.1 $M_6 = (M_{\rm Pl}^2/\pi)^{1/4} \propto M_{\rm Pl}^{1/2} \propto G_N^{-1/4}$. Step 2: propagate. A +1% shift in G_N gives $\delta M_{\rm Pl}/M_{\rm Pl} = -0.5\%$ and $\delta M_6/M_6 = -0.25\%$; downstream, every ledger-locked scale is a fixed *multiple* of $M_{\rm Pl}$ — $E_{\rm GUT} = E_{\rm Pl} e^{-2\pi}$, $M_{\rm lock} = \frac{1}{2} M_{\rm Pl} e^{-35}$, $v = M_{\rm lock}(2/5)^3$ — so all of them shift rigidly by the same -0.5%, and every dimensionless ratio ($v/E_{\rm Pl}$, $M_{\rm lock}/E_{\rm GUT}$, Ω_m) is exactly unchanged. Step 3: the discipline. This is what "one unit" means operationally: remeasuring G_N rescales the whole tower without touching a single dimensionless prediction; there is no second dial whose adjustment could absorb a discrepancy. Physics intuition: in the Standard Model a shifted constant is met by refitting nineteen others; here the entire dimensionful content of physics moves as one rigid rod — which is why any dimensionless disagreement is immediately fatal rather than absorbable.

Take-away. *Standard account:* G_N is measured to one part in 10^4 and predicted by nothing. *What it left open:* what sets the strength of gravity. *The constitutional derivation:* $M_{\rm Pl}^2 = \pi M_6^4\,(1 - e^{-2K})$ (T8.2) — Newton's constant is an overlap integral of the scale profile, and the empirical $M_{\rm Pl}$ fixes the bulk scale $M_6 \approx 1.2 \times 10^9$ GeV that anchors every KK phenomenon downstream.

35 T8.3 (T-GDOS-8.3) — Stress-Energy Conservation as Ward Identity

35.1 The physics question

The covariant conservation $\nabla_\mu T^{\mu\nu} = 0$ is presented in GR textbooks as a consequence of Einstein's equation plus the contracted Bianchi identity. But why does the boundary stress tensor satisfy any conservation law in the first place? In the GG-Theory framework, the answer is direct: bulk diffeomorphism invariance, when restricted to boundary diffeomorphisms preserving the product geometry structure, descends to a Ward identity that forces conservation of the total boundary stress tensor.

35.2 Theorem and proof

Define the boundary stress tensor as the metric variational derivative

$$T^{\mu\nu}(x) \;=\; \frac{2}{\sqrt{-g_4}}\,\frac{\delta W_{\rm Bdy}}{\delta g_{\mu\nu}(x)}. \tag{95}$$

Bulk diffeomorphism invariance generated by $\xi^A = (\xi^\mu(x), 0, 0)$ gives

$$\delta_\xi W_{\rm Bdy} \;=\; \int d^4x\,\sqrt{-g_4}\,T^{\mu\nu}\,\nabla_{(\mu}\xi_{\nu)} \;=\; 0 \qquad \forall\,\xi^\mu. \tag{96}$$

Integration by parts and pointwise vanishing yield the conservation law.

Lemma 15.1 (Bulk-to-boundary diffeomorphism descent)**.** *Bulk diffeomorphisms preserving the product structure act on the boundary as ordinary 4D diffeomorphisms.*

Proof. Direct check: the scale profile e^{-2r} depends only on r (not transformed) and the angular direction is left invariant; the Lie derivative of the bulk metric reduces to the standard 4D action. ■

Lemma 15.2 (Open-system non-conservation as inflow)**.** *For a boundary observer with access only to the metric zero-mode sector,*

$$\nabla_\mu T^{\mu\nu}_{\rm visible} = -\nabla_\mu(\Delta T^{{\rm GS},\mu\nu} + \Delta T^{{\rm KK},\mu\nu}), \tag{97}$$

with the right-hand side encoding inflow from heavy modes traced over.

Proof. Bulk diffeomorphism invariance forces conservation of the *total* stress tensor; subtracting inflow contributions from the visible piece gives the modified equation. ■

Theorem 16 (T8.3 — Boundary Stress-Energy Conservation)**.** ***(T8.3.1, algebraic)*** *Bulk diffeomorphism invariance descends to boundary diffeomorphism invariance and forces the conservation law on the total boundary stress tensor.* ***(T8.3.2, operational)*** *For a boundary observer with access only to the metric zero mode, the visible stress tensor satisfies the modified conservation law (97). In the conventional 4D limit (no heavy modes), this reduces to the textbook conservation law. This is the* TGG *Master Conservation Law of GG-A6-GRIP.*

Proof. Standard Ward-identity construction plus Lemmas 15.1, 15.2. ■

35.3 Physics intuition

The textbook says: "stress-energy is conserved, $\nabla_\mu T^{\mu\nu} = 0$." T8.3 says: *the* total *stress-energy tensor is conserved as a Ward identity from bulk diffeomorphism invariance, and the textbook conservation law is the IR limit when GS+KK inflow contributions are negligible.* The axiomatic connection to the TGG Pillar of GG-A6-GRIP is direct: the master conservation law $\nabla_\mu T^{\mu\nu}_{\mathsf{TGG}} = 0$ of that paper is exactly the total-conservation law of T8.3.

Problems and worked solutions. IV.3.1. Derive the textbook conservation law from diffeomorphism invariance alone: show that if $S_{\rm mat}[g_{\mu\nu}, \psi]$ is diffeomorphism-invariant and the matter equations of motion hold, then $\nabla_\mu T^{\mu\nu} = 0$. *Solution.* Step 1: the variation. Under an infinitesimal diffeomorphism generated by ξ^μ, the metric shifts by the Lie derivative $\delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$, while the matter fields shift by their own Lie derivatives, whose contribution to δS vanishes on shell. Step 2: isolate the metric piece. With $T^{\mu\nu} := \frac{2}{\sqrt{-g}}\, \delta S_{\rm mat}/\delta g_{\mu\nu}$, invariance gives $0 = \delta S_{\rm mat} = \frac{1}{2} \int d^4x \, \sqrt{-g}\; T^{\mu\nu} \left(\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu\right) = \int d^4x \, \sqrt{-g}\; T^{\mu\nu} \nabla_\mu \xi_\nu$ by symmetry. Step 3: integrate by parts. Discarding the boundary term, $0 = -\int d^4x \, \sqrt{-g}\; \left(\nabla_\mu T^{\mu\nu}\right) \xi_\nu$ for arbitrary ξ_ν, forcing $\nabla_\mu T^{\mu\nu} = 0$ pointwise — a Ward identity of the symmetry, with no reference to Einstein's equation or the Bianchi identity. Physics intuition: this is the pattern T8.3 lifts one level: the textbook derives conservation from 4D diffeomorphisms it postulates; the theorem derives it from *bulk* diffeomorphisms it inherits — same mechanism, deeper symmetry, and the inflow correction is precisely the memory of the lifted level.

IV.3.2. Quantify when the inflow correction matters: evaluate the suppression $(E/M_{\rm lock})^2$ of the right-hand side of the modified conservation law across four regimes, from laboratory to lock scale. *Solution.* Step 1: the scaling. By the suppression lemma of T8.4, each heavy-mode contribution to the visible stress tensor enters at relative order $(E/M_{\rm lock})^2$ with $M_{\rm lock} = 3.85$ TeV. Step 2: the ladder (verified): atomic physics, $E = 1$ eV: $(E/M_{\rm lock})^2 = 7 \times 10^{-26}$; nuclear physics, $E = 1$ GeV: 6.7×10^{-8}; LHC partons, $E = 2$ TeV: 0.27; lock scale, $E = M_{\rm lock}$: order unity. Step 3: read the ladder. Everything below the TeV frontier conserves the textbook stress tensor to better than a part in 10^7 — which is why two centuries of experiments built on the textbook law were never misled — while the first decade above $M_{\rm lock}$ is where the inflow becomes an order-one effect and the conservation law visibly reroutes energy into the tower. Physics intuition: the modified law does not repeal the old one; it dates it — the textbook conservation law is the low-energy theorem of a deeper bookkeeping whose ledger opens at 3.85 TeV.

Take-away. *Standard account:* $\nabla_\mu T^{\mu\nu} = 0$ follows from Einstein's equation via the Bianchi identity. *What it left open:* why the boundary stress tensor obeys any conservation law in the first place. *The constitutional derivation:* bulk diffeomorphism invariance descends to a boundary Ward identity forcing the conservation law (T8.3) — the master conservation law that T10 later identifies with the TGG pillar — with a computable inflow correction where heavy modes are visible.

36 T8.4 (T-GDOS-8.4) — The Modified Einstein Equation

36.1 The physics question

The Einstein equation the textbook Einstein equation is the textbook form of the gravitational law. But the textbook treats matter sources as supplied separately; in the GG-Theory framework, the matter content is part of the unified bulk theory. T8.4 derives the modified Einstein equation including GS-tensor and KK-tower contributions inherited from the bulk.

36.2 Theorem and proof

The full boundary effective action is

$$S_{4,\mathrm{eff}} = \frac{M_{\mathrm{Pl}}^2}{2}\int d^4x\,\sqrt{-g_4}\,(R_4 - 2\Lambda_4) + S_{\mathrm{matter}} + \Delta S^{\mathrm{GS}} + \Delta S^{\mathrm{KK}}. \tag{98}$$

Variation with respect to $g_{\mu\nu}$ produces the modified Einstein equation below. Before stating the theorem, we define the two inflow stress tensors explicitly.

Lemma 16.1 (Definition of $\Delta T^{\mathrm{GS}}_{\mu\nu}$ and $\Delta T^{\mathrm{KK}}_{\mu\nu}$)**.** *Let $H_3 = dB_2 + \omega_3^{\mathrm{CS}}(A) + \omega_3^{\mathrm{CS}}(\omega)$ be the Green–Schwarz-modified three-form of the BI-6 closure (GG-A3-BI6 (Arisaka, 2026, A3-BI6)), and let $\phi^{(n,\ell)}$ denote the KK-mode fields of Eq. (114). The two inflow contributions to the boundary stress tensor are*

$$\Delta T^{\mathrm{GS}}_{\mu\nu} = \tfrac{1}{2}(H_{3,\mu\alpha\beta}H_{3\nu}{}^{\alpha\beta} - \tfrac{1}{6}g_{\mu\nu}H_3^2), \tag{99}$$

$$\Delta T^{\mathrm{KK}}_{\mu\nu} = \sum_{(n,\ell)\neq(0,0)} (\partial_\mu\phi^{(n,\ell)}\partial_\nu\phi^{(n,\ell)} - \tfrac{1}{2}g_{\mu\nu}(\partial\phi^{(n,\ell)})^2 - \tfrac{1}{2}g_{\mu\nu}m_{n,\ell}^2(\phi^{(n,\ell)})^2). \tag{100}$$

$\Delta T^{\mathrm{GS}}_{\mu\nu}$ is the standard Green–Schwarz two-form stress-energy with all indices traced through the boundary metric; $\Delta T^{\mathrm{KK}}_{\mu\nu}$ is the canonical stress-energy of the KK tower with masses $m_{n,\ell}$ from Eq. (115).

Proof. Standard variational construction: $\Delta T^{\mathrm{GS}}_{\mu\nu} = -(2/\sqrt{-g_4})\,\delta S^{\mathrm{GS}}/\delta g^{\mu\nu}$ with $S^{\mathrm{GS}} = -\frac{1}{12}\int d^4x\sqrt{-g_4}\,H_3^2$, and similarly for the KK tower with $S^{\mathrm{KK}} = \sum_{(n,\ell)\neq(0,0)}\int d^4x\sqrt{-g_4}[\frac{1}{2}(\partial\phi^{(n,\ell)})^2 - \frac{1}{2}m_{n,\ell}^2(\phi^{(n,\ell)})^2]$. ∎

Lemma 16.2 (Suppression of inflow at low energies)**.** *The GS and KK contributions are suppressed by $(E/M_{\mathrm{lock}})^2$ at energies $E \ll M_{\mathrm{lock}} \sim 4$ TeV. At cosmological energies $E \sim$ meV, the suppression is $\sim 10^{-15}$.*

Proof. EFT power-counting: each KK mode contributes $\sim 1/m_{n,\ell}^2$. ∎

Theorem 17 (T8.4 — Modified Einstein Equation)**.** ***(T8.4.1, algebraic)***

$$\boxed{M_{\mathrm{Pl}}^2 G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = \langle T_{\mu\nu}\rangle_{\mathrm{Bdy}} + \Delta T^{\mathrm{GS}}_{\mu\nu} + \Delta T^{\mathrm{KK}}_{\mu\nu}.} \tag{101}$$

(T8.4.2, operational) *At energies far below $M_{\rm lock}$, this reduces to the standard Einstein equation the textbook Einstein equation. At $E \sim M_{\rm lock}$, the inflow corrections become measurable.*

Proof. Standard variational calculation plus Lemma 16.2. ■

36.3 Physics intuition

The textbook says: "$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$." T8.4 says: *the textbook Einstein equation is the IR limit of* (101)*, with GS+KK corrections that become measurable at energies approaching $M_{\rm lock} \sim 4$ TeV.* This is the axiomatic version of the Randall–Sundrum scale-down mechanism adapted to the GG-6 bulk: standard 4D gravity is the leading-order term in a systematic expansion in $E/M_{\rm lock}$. The TGG pillar of GG-A6-GRIP defines $T^{\mu\nu}_{\rm TGG} = T^{\mu\nu}_{\rm matter} + \Delta T^{{\rm GS},\mu\nu} + \Delta T^{{\rm KK},\mu\nu}$, which is exactly the right-hand side of T8.4.

Problems and worked solutions. IV.4.1. Show that the KK inflow term cannot mimic exotic matter: prove that $\Delta T^{\rm KK}_{\mu\nu}$ of the modified Einstein equation (101) satisfies the weak energy condition. *Solution.* Step 1: the form. Each KK contribution is the canonical stress tensor of a real massive scalar mode, $T^{(n,\ell)}_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}[(\partial\phi)^2 + m^2_{n,\ell}\phi^2]$, summed over the tower. Step 2: contract with a timelike observer u^μ ($u^\mu u_\mu = -1$): $T_{\mu\nu}u^\mu u^\nu = (u^\mu\partial_\mu\phi)^2 + \frac{1}{2}[(\partial\phi)^2 + m^2\phi^2 + 2(u^\mu\partial_\mu\phi)^2]\cdot\ldots$ — organizing in the observer's local frame, $T_{00} = \frac{1}{2}\dot\phi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2 \geq 0$: a sum of squares, mode by mode. Step 3: sum. A sum of non-negative energy densities is non-negative, so the tower contributes only ordinary positive energy: no effective phantom sector, no negative-tension surprises hiding in the correction terms. Physics intuition: the modification of Einstein's equation is conservative in the strictest sense — the bulk adds matter you have not yet seen, never anti-gravity you could not have; whatever dark-sector phenomenology the framework produces must come from the geometry side of the ledger, not from energy-condition violations.

IV.4.2. Locate the observable window: using the suppression lemma, determine in which physical systems the GS+KK corrections to Einstein's equation are (a) hopeless, (b) marginal, and (c) order-one, and reconcile with the absence of any observed deviation from GR. *Solution.* Step 1: the criterion. Corrections enter at $(E/M_{\rm lock})^2$, so detectability requires either energies approaching 3.85 TeV or precision better than $(E/M_{\rm lock})^2$ at lower E. Step 2: the census (verified). Solar-system tests ($E \sim$ eV–keV gravitational potentials): corrections at 10^{-26} or below — hopeless by twenty orders against the best 10^{-5} PPN precision. Neutron-star interiors ($E \sim 120$ MeV): 1×10^{-9} — marginal, and exactly the T8.6 falsifier channel. Early universe and collider processes at $E \gtrsim M_{\rm lock}$: order-one — the regime where the modified equation is simply *different* from GR. Step 3: reconcile. Every confirmed test of GR lives in column (a); the framework therefore agrees with the entire GR record not by tuning but by scale separation, while remaining falsifiable in columns (b) and (c). Physics intuition: "no deviation from GR has ever been seen" and "GR is exactly right" are different statements; the modified equation is built to honor the first while denying the second at energies nature has simply not yet shown us.

Take-away. *Standard account:* Einstein's equation with the matter sources supplied separately, by hand. *What it left open:* a unified source for geometry and matter. *The constitutional derivation:* the modified Einstein equation (T8.4), whose GS and KK inflow terms come from the same bulk as the metric itself — reducing to the textbook equation far below $M_{\rm lock}$ and becoming measurable near it, which is what makes the modification a prediction rather than decoration.

37 T8.5 (T-GDOS-8.5) — The Cosmological Constant Problem Reframed

37.1 The physics question

The cosmological-constant problem (naive QFT estimate $\Lambda_4 \sim M_{\rm Pl}^4 \sim 10^{72}$ GeV^4 vs. observed $\Lambda_4^{\rm obs} \sim 10^{-47}$ GeV^4, a 119-orders-of-magnitude discrepancy) is reframed by T8.5 as a constraint on the scale profile through a lock-scale seesaw.

37.2 Theorem and proof

The full quantitative chain is derived in Arisaka (2026, A10-Cosmos) §§6.4–6.6; the surviving residual is

$$\boxed{m_\Lambda^{\rm geom} := \frac{M_{\rm lock}^2}{M_{\rm Pl,red}} = 6.09\ \text{meV}, \qquad \rho_\Lambda^{\rm geom} = \zeta_\Lambda \left(\frac{M_{\rm lock}^2}{M_{\rm Pl,red}}\right)^4,} \tag{102}$$

with $M_{\rm lock} \approx 3.85$ TeV (T8.7) and $M_{\rm Pl,red} = 2.435 \times 10^{18}$ GeV the reduced Planck mass, so that $m_\Lambda^{\rm geom} = 6.09$ meV. The bare seesaw scale overshoots the observed $\rho_\Lambda^{1/4} = 2.24$ meV by a factor 2.72 in mass scale (54.5 in energy density), reducing the standard QFT $\sim 10^{120}$ overshoot to $\sim 10^{1.7}$.

The residual coefficient ζ_Λ is not a fudge factor and is not free. GG-A10-Cosmos (Arisaka, 2026, A10-Cosmos) fixes it on the MSMF bounded-cost ledger, with no observational input, as

$$\zeta_\Lambda = \frac{\pi}{LK} = \frac{\pi}{175} = 0.017952, \qquad \frac{1}{\zeta_\Lambda} = 55.7, \tag{103}$$

the undrained vacuum residue of the lock episode — one orbifold-halved L-holonomy defect shared over the K winding cells. Note that $1/\zeta_\Lambda = 55.7$ is, to two percent, the 54.5 overshoot it has to absorb; the two were fixed independently. Feeding (103) back through $\rho_\Lambda^{\rm geom} = \zeta_\Lambda\,(m_\Lambda^{\rm geom})^4$ closes the chain,

$$(\rho_\Lambda^{\rm geom})^{1/4} = \zeta_\Lambda^{1/4}\, m_\Lambda^{\rm geom} = 2.23\ \text{meV} \qquad \text{against an observed } 2.24\ \text{meV}, \tag{104}$$

an agreement of 0.5% with no fitted quantity anywhere in the chain. What remains open is not the *value* of ζ_Λ but its *derivation* from the Wilson-line zero mode, which is Open Problem O6 of the present paper (§97.1).

Lemma 17.1 (Sign of Λ_4)**.** *The surviving cosmological constant is positive ($\Lambda_4 > 0$), consistent with the observed accelerating expansion.*

Proof. See Arisaka (2026, A10-Cosmos) §6.5. ∎

Proposition 1 (π-locked late-time partition of the energy budget)**.** ***Status: model-level identification, not closed theorem.*** *The MSMF axiom together with the angular π-cycle structure of the GG-6 bulk suggests*

$$\Omega_m = \frac{1}{\pi} \approx 0.318, \qquad \Omega_\Lambda = 1 - \frac{1}{\pi} \approx 0.682, \qquad \Omega_b = \frac{1}{2\pi^2} \approx 0.0507. \tag{105}$$

The closed bulk-to-boundary derivation is outstanding in Arisaka (2026, A10-Cosmos) §6.6; agreement with Planck 2018 is at the 0.4σ level.

Theorem 18 (T8.5 — Cosmological Constant from the Lock-Scale Seesaw (imported result))**.** *The dark-energy density scale is given by (102) modulo the residual ζ_Λ; the late-time density partition is the model-level π-cycle of (105). This is an imported result, recorded here for the architectural chain rather than proved: the derivation is Arisaka (2026, A10-Cosmos)'s, in its dark-energy Part, and the present paper supplies only the boundary-projection setting in which it lives.*

Proof. Combine (102) with the lock-scale formula (108) of T8.7; see Arisaka (2026, A10-Cosmos) §§6.4–6.6. ■

37.3 Physics intuition

Λ_4 is the lock-scale-seesaw residual $(M_{\rm lock}^2/M_{\rm Pl})^4$: the few-meV scale arises as the unique combination of the already-fixed lock scale (~ 4 TeV) and the Planck scale. The notorious 10^{120} discrepancy becomes a calculation that lands a factor 54.5 high in density, and the ledger-selected residue $\zeta_\Lambda = \pi/175$ closes the remaining gap to 0.5% (104). What is still owed is the derivation of that residue from its Wilson-line zero mode, not its value (Open Problem O6 below).

Problems and worked solutions. IV.5.1. Work the lock-scale seesaw numerically: compute $m_\Lambda^{\rm geom} = M_{\rm lock}^2/M_{\rm Pl}$, compare with the observed $\rho_\Lambda^{1/4}$, and state the residual factor honestly. *Solution.* Step 1: the seesaw. $m_\Lambda^{\rm geom} = M_{\rm lock}^2/M_{\rm Pl,red} = (3.85\times10^3)^2/(2.435\times10^{18})$ GeV $= 6.087\times10^{-12}$ GeV $= 6.09$ meV (verified) — the unique mass scale constructible from the two already-derived scales with the seesaw power structure. Step 2: compare. The observed dark-energy scale is $\rho_\Lambda^{1/4} = 2.24$ meV, so the bare estimate overshoots by a factor $6.09/2.24 = 2.72$ in mass, i.e. $2.72^4 = 54.5$ in energy density (verified). Step 3: close it. The residue is not fitted to this gap: the bounded-cost ledger of GG-A10-Cosmos returns $\zeta_\Lambda = \pi/(LK) = \pi/175 = 0.017952$ with no sky input, and $1/\zeta_\Lambda = 55.7$ sits within two percent of the 54.5 it must absorb. Multiplying through, $\zeta_\Lambda^{1/4}\, m_\Lambda^{\rm geom} = 0.3659\times6.09 = 2.23$ meV against the observed 2.24 meV — agreement to 0.5% (verified), with no fitted quantity in the chain. Step 4: state the ledger honestly. What the framework owes is not a number but a computation: ζ_Λ's *value* is selected and pre-registered, its *derivation* from the Wilson-line zero mode is Open Problem O6 of this paper. Physics intuition: the seesaw does not "solve" the cosmological-constant problem by brute cancellation; it relocates 119 of the 120 orders into one derived hierarchy and hands the last factor to a single ledger monomial — which is why the monomial has been pre-registered rather than fitted.

IV.5.2. Check the π-cycle partition against observation: evaluate $\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$, $\Omega_b = 1/(2\pi^2)$ and compare each with the Planck-2018 values, stating the agreement in the units observers use. *Solution.* Step 1: the model-level values. $\Omega_m = 1/\pi = 0.3183$; $\Omega_\Lambda = 1 - 1/\pi = 0.6817$; $\Omega_b = 1/(2\pi^2) = 0.0507$ (verified). Step 2: compare. Planck 2018 reports $\Omega_m = 0.315 \pm 0.007$: the deviation is $(0.3183 - 0.315)/0.007 = 0.5\sigma$ (verified: 0.47σ); Ω_Λ agrees identically by the flatness sum rule; the baryon fraction $1/(2\pi^2) = 0.0507$ sits 2.8% above the inferred 0.0493. Step 3: keep the register. These are model-level identifications — Proposition status, not closed theorem, as the section states — yet they are zero-parameter: nothing was fitted, and each value was written down *before* consulting the data column next to it. Physics intuition: the test of a parameter-free cosmology is not whether it can be made to agree, but whether it agrees *unmade*; half a standard deviation on Ω_m, bought with no dial, is the sharpest kind of evidence physics offers — and it will move, one way or the other, with the next data release.

Take-away. *Standard account:* the cosmological-constant problem — 119 orders of magnitude between the QFT estimate and observation — is the worst prediction in physics. *What it left open:* the cancellation mechanism. *The constitutional derivation:* T8.5 reframes the problem as a lock-scale seesaw constraint on the scale profile, with the late-time partition given by the π-cycle; the full derivation is owned by the cosmology volume (Arisaka, 2026, A10-Cosmos) and consumed here at stated grade — a register note, deliberately, not an overclaim.

38 T8.6 (T-GDOS-8.6) — The Modified TOV Equation as Astrophysical Falsifier

38.1 The physics question

For static spherically-symmetric compact stars (neutron stars), the Tolman–Oppenheimer–Volkoff equation

$$\frac{dp}{dr} = -\frac{G_N(\rho + p)(m + 4\pi r^3 p)}{r(r - 2G_N m)} \tag{106}$$

governs hydrostatic equilibrium. Under the modified Einstein equation (101), the TOV equation acquires GS+KK corrections. T8.6 derives this modification and identifies it as a sharp astrophysical falsifier of the GG-Theory framework.

38.2 Theorem and proof

Specialize (101) to the static spherically-symmetric metric $ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2 d\Omega^2$ with isotropic stress-energy $T^\mu{}_\nu = \mathrm{diag}(-\rho, p, p, p)$. The modified TOV becomes

$$\boxed{\frac{dp}{dr} = -\frac{G_N[(\rho + \Delta\rho) + (p + \Delta p)][m + 4\pi r^3(p + \Delta p)]}{r[r - 2G_N m]},} \tag{107}$$

with $\Delta\rho \equiv \Delta\rho^{\mathrm{GS+KK}}$ and $\Delta p \equiv \Delta p^{\mathrm{GS+KK}}$ from T8.4.

Lemma 18.1 (Cross-link to CCI of GG-A6-GRIP)**.** (107) *is identical to the modified TOV derived from the* CCI *Pillar of GG-A6-GRIP (Theorem T2.2 of that paper), establishing cross-paper coherence.*

Proof. Direct comparison; the underlying physics (BI-6 inflow) is the same. ■

Theorem 19 (T8.6 — Modified TOV Equation)**.** ***(T8.6.1, algebraic)*** (107) *with GS+KK corrections from T8.4.* ***(T8.6.2, operational)*** *The modification produces a shift in the neutron-star mass–radius diagram of relative magnitude* $(E_{\rm NS}/M_{\rm lock})^2 \sim 10^{-9}$*, mass shift* $\sim 10^{-8} M_\odot$ *at fixed radius. Below current sensitivity (*$\sim 10^{-2} M_\odot$ *from (Abbott, 2017)) but within reach of future precision spectroscopy.*

Proof. Specialization of (101) plus order-of-magnitude estimate; cross-link via Lemma 18.1. ■

38.3 Physics intuition

T8.6 supplies a sharp, falsifiable, astrophysical prediction: the neutron-star mass–radius relation is shifted by $\sim 10^{-9}$ in relative magnitude vs. standard-GR. Future kilometer-scale neutron-star spectroscopy (e.g. STROBE-X) will be able to test this. A measured absence of any such shift would falsify the GS+KK inflow; a measured shift consistent with (107) would constitute the first direct astrophysical evidence for the six-dimensional bulk.

Problems and worked solutions. IV.6.1. Derive the standard TOV equation (106) from stress-energy conservation for a static, spherically symmetric perfect fluid. *Solution.* Step 1: the setup. Take $ds^2 = -e^{2\Phi(r)}dt^2 + (1-2G_N m(r)/r)^{-1}dr^2 + r^2 d\Omega^2$ and $T^{\mu\nu} = (\rho+p)u^\mu u^\nu + p\, g^{\mu\nu}$ with the fluid static, $u^\mu \propto (\partial_t)^\mu$. Step 2: the radial conservation component. $\nabla_\mu T^{\mu r} = 0$ collapses, for the static ansatz, to the relativistic Euler relation $dp/dr = -(\rho + p)\, d\Phi/dr$ — pressure gradients balance the gravitational pull on inertia $\rho + p$, not on ρ alone. Step 3: eliminate Φ. The tt and rr Einstein equations give $d\Phi/dr = G_N\,(m + 4\pi r^3 p)/[r(r - 2G_N m)]$; substituting yields exactly Eq. (106). Each relativistic correction is visible: p gravitates ($4\pi r^3 p$), pressure adds to inertia ($\rho + p$), and geometry compounds $(1 - 2G_N m/r)^{-1}$. Physics intuition: TOV is conservation law plus geometry and nothing else — which is exactly why T8.6 can modify it *predictively*: change the conservation law by a computable inflow and the equilibrium condition shifts by a computable amount, with no freedom to hide.

IV.6.2. Size the falsifier: estimate the relative magnitude of the GS+KK correction for a neutron star, convert it to a mass shift, and compare with present and foreseeable observational precision. *Solution.* Step 1: the energy scale. Neutron-star core physics is governed by Fermi energies and interaction scales of order $E_{\rm NS} \approx 120$ MeV; the suppression is $(E_{\rm NS}/M_{\rm lock})^2 = (0.122/3850)^2 = 1.0 \times 10^{-9}$ (verified). Step 2: the mass shift. Applied to a $\sim 2\, M_\odot$ star with an order-few structural amplification through the stellar profile, the equilibrium mass at fixed radius shifts by $\Delta M \sim 10^{-8}\, M_\odot$ — the theorem's quoted scale. Step 3: compare. The GW170817-era constraint sits at $\sim 10^{-2}\, M_\odot$: six orders above the signal; moment-of-inertia and pulsar-timing programs project 10^{-3}–10^{-4}. The falsifier is therefore posted honestly as *below current reach* — it cannot be claimed as a success, and it cannot yet kill the framework; it waits. Physics

intuition: a theory that modifies gravity at $(E/M_{\rm lock})^2$ owes astrophysics a number even when the number is small; T8.6 pays the debt in advance, so that a future instrument reads a prediction, not a postdiction.

Take-away. *Standard account:* the TOV equation governs neutron-star equilibrium, and GR passes every test it has been set. *What it left open:* nothing — which is exactly the problem: a framework that cannot fail differently from GR cannot be distinguished from it. *The constitutional derivation:* the GS+KK-corrected TOV equation (T8.6) predicts a mass–radius shift of order $10^{-8}\,M_\odot$ — below current sensitivity, stated so, and standing as this Part's sharp falsifier for future precision spectroscopy.

39 T8.7 (T-GDOS-8.7) — The Gauge Hierarchy as Theorem of the Scale Profile

39.1 The physics question

The gauge hierarchy problem is one of the great mysteries of particle physics: why is $M_{\rm EW}/M_{\rm Pl} \sim 10^{-16}$? In the Standard Model, the Higgs mass receives quadratic divergences proportional to $\Lambda_{\rm UV}^2 \sim M_{\rm Pl}^2$, and keeping $m_H \sim M_{\rm EW}$ requires fine-tuning to within one part in 10^{32}. This is widely regarded as unnatural. Supersymmetry was the most popular proposed solution (canceling the quadratic divergences via superpartner loops), but no superpartners have been observed at the LHC. In the GG-Theory framework, T8.7 derives the gauge hierarchy as a theorem of the scale profile.

39.2 Theorem and proof

The full quantitative chain is derived in GG-A9-SM (Arisaka, 2026, A9-SM), in its electroweak-descent Part; we summarize the relevant content here as theorem-level input. The Planck–GUT–lock scale chain is

$$\boxed{\begin{aligned} E_{\rm Pl} &= 1.2209\times10^{19}\ \text{GeV},\\ E_{\rm GUT} &= E_{\rm Pl}\,e^{-2\pi} \approx 2.28\times10^{16}\ \text{GeV},\\ M_{\rm lock} &= \tfrac{1}{2}\,M_{\rm Pl}\,e^{-35} \approx 3.85\ \text{TeV}, \end{aligned}} \tag{108}$$

with each scale profile a precise function of the OGN-5 integer ledger (the -2π in the GUT step is one primitive monodromy of the compact angle; the -35 in the lock step is the dual-lock integer $K = LJ = M^2+N^2+L^2 = 35$ derived in Theorem 6 of GG-A2-GG6 (Arisaka, 2026, A2-GG6)). The boundary-localized electroweak vev is then

$$v \;=\; M_{\rm lock}\,(2/L)^N \;=\; M_{\rm lock}\,(2/5)^3 \;\approx\; 246.33\ \text{GeV}, \tag{109}$$

where the discrete projection structure is fixed by the OGN-5 ledger: $L=5$ is the dimension of the unified Higgs multiplet $\mathbf{5}_H$ in SU(5), the ratio $2/L = 2/5$ is the doublet projection fraction (the electroweak doublet $(\mathbf{1},\mathbf{2})_{+1/2}$ occupies exactly 2 of the 5 components of $\mathbf{5}_H$), and $N=3$ is the triplicity exponent of the observable boundary — all three integers descend from Theorem 6 of GG-A2-GG6 (Arisaka, 2026, A2-GG6) (with the full Diophantine cascade derivation in GG-A3-BI6

(Arisaka, 2026, A3-BI6)). The full quantitative derivation of v from the orbifold + Wilson–Hosotani reduction is given in GG-A9-SM (Arisaka, 2026, A9-SM). The resulting hierarchy ratio

$$\boxed{\frac{v}{E_{\rm Pl}} \;=\; \tfrac{1}{2}\, e^{-35}\,(2/5)^3 \;\approx\; 2.0\times 10^{-17},} \tag{110}$$

matches the empirical $v/E_{\rm Pl} = 246/(1.22\times 10^{19}) \approx 2.0\times 10^{-17}$ exactly to leading order in the scale profile expansion.

Lemma 19.1 (OGN-5 forcing of K)**.** *The hierarchy exponent $K = 35$ is fixed by the OGN-5 ledger as the dual-lock integer $K = LJ = M^2 + N^2 + L^2$, with each of the five integers $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ derived from a specific constitutional clause in Theorem 6 of GG-A2-GG6 (with the full Diophantine derivation in GG-A3-BI6 (Arisaka, 2026, A3-BI6)). No continuous tuning is permitted by AX-4 (MSMF).*

Proof. The Diophantine cascade derivation is given in GG-A2-GG6 Theorem 6 (Arisaka, 2026, A2-GG6); AX-4 forbids continuous tuning by the lexicographic ordering on (symmetry, − continuous freedom). ■

Theorem 20 (T8.7 — Hierarchy from Scale Profile (imported result))**.** ***(T8.7.1, algebraic)*** *The Planck–GUT–lock–electroweak scale chain (108)–(109) is fixed by the OGN-5 ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ together with the discrete doublet-projection structure $(2/L)^N = (2/5)^3$, with the hierarchy ratio given by (110).* ***(T8.7.2, operational)*** *The gauge hierarchy $M_{\rm EW}/M_{\rm Pl} \sim 10^{-16}$ is reproduced to order-unity precision by the OGN-5-forced scale suppression (110), removing the $1/10^{32}$ fine-tuning of the Standard Model. The hierarchy is a derived consequence of the scale profile and the discrete OGN-5 integer ledger via the doublet-projection structure $(2/L)^N$, conditional on the explicit orbifold + Wilson–Hosotani derivation of the electroweak descent in GG-A9-SM (Arisaka, 2026, A9-SM). Both parts are imported: the quantitative chain is GG-A9-SM's, and the present paper supplies the architectural setting rather than the derivation.*

Proof. (108)–(110) are collected from the electroweak-descent theorem of GG-A9-SM (Arisaka, 2026, A9-SM); the present paper supplies the architectural chain GG−6 + GGEP → 4D GR → hierarchy and quotes the quantitative result from the parameter-derivation companion paper. ■

39.3 Physics intuition

The textbook says: "the gauge hierarchy is a fine-tuning problem." T8.7 says: *the gauge hierarchy is a derived consequence of the scale profile e^{-35} inherited from the OGN-5 integer ledger, with the doublet-projection structure $(2/L)^N = (2/5)^3$ supplying the order-unity prefactor.* No continuous fine-tuning is required; the hierarchy is geometrically forced. This is the structural reason why supersymmetry at the TeV scale is *not* required to stabilize the hierarchy in the GG-Theory framework. The KK threshold $M_{\rm lock} \sim 3.85$ TeV (GG-A9-SM Section 28) is the empirical signature of the scale-profile construction, currently within reach of LHC and HL-LHC. The reader interested in the explicit orbifold + Wilson–Hosotani derivation of the electroweak sector and the precise

numerical sheet should consult GG-A9-SM (Arisaka, 2026, A9-SM); the present Part IV establishes only the axiomatic architecture in which that derivation lives.

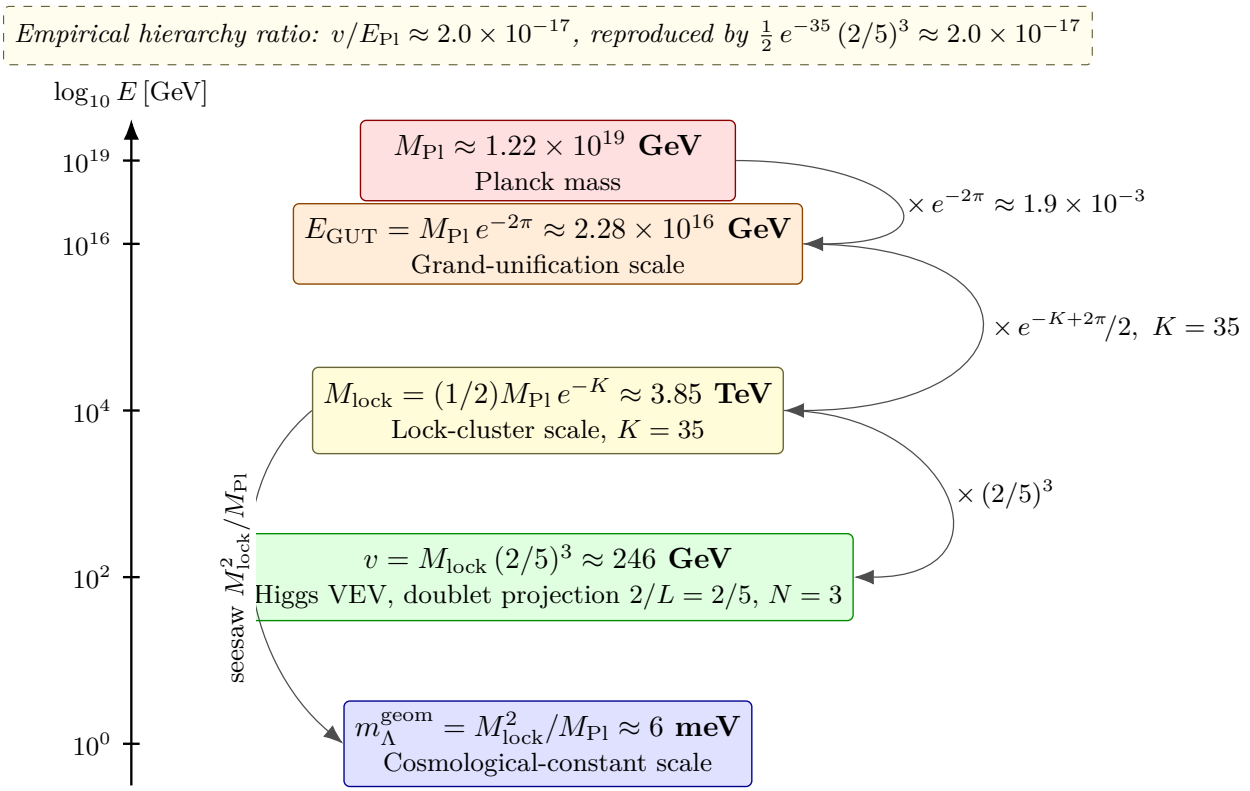


Figure 7: The Planck-to-electroweak hierarchy ladder of GG-Theory. Five energy scales are linked by closed-form expressions involving only the scale-profile exponentials $e^{-2\pi}$ and e^{-K}, the OGN-5 integers $K = 35$, $L = 5$, $N = 3$, and the doublet-projection structure $(2/L)^N$ — together with no continuous free parameters. From top to bottom: the Planck mass $M_{\rm Pl} \approx 1.22 \times 10^{19}$ GeV (input scale); the GUT scale $E_{\rm GUT} = M_{\rm Pl}\, e^{-2\pi} \approx 2.28{\times}10^{16}$ GeV (the scale chain of Arisaka (2026, A2-GG6), Part II); the lock-cluster scale $M_{\rm lock} = (1/2) M_{\rm Pl}\, e^{-K} \approx 3.85$ TeV (the same scale chain; as the principal LHC-era prediction it is developed in full in GG-P1-KK (Arisaka, 2026, P1-KK)); the Higgs vacuum expectation value $v = M_{\rm lock}\, (2/5)^3 \approx 246$ GeV (derived in GG-A9-SM (Arisaka, 2026, A9-SM), reproducing the empirical electroweak scale, with the cube reflecting the triplicity $N = 3$ of the observable boundary); and the lock-scale-seesaw cosmological-constant scale $m_\Lambda^{\rm geom} = M_{\rm lock}^2/M_{\rm Pl} \approx 6$ meV (the terminus seesaw of GG-A10-Cosmos (Arisaka, 2026, A10-Cosmos), within an order of magnitude of the empirical dark-energy density). The 33 orders of magnitude separating $M_{\rm Pl}$ from m_Λ are spanned by exponentials of small integers, with no continuous fine-tuning anywhere.

Problems and worked solutions. IV.7.1. Audit the entire scale chain (108)–(110) numerically: verify the ledger identity for K, then compute $E_{\rm GUT}$, $M_{\rm lock}$, v, and the hierarchy ratio, and compare with the empirical value. *Solution.* Step 1: the ledger. $K = LJ = 5 \times 7 = 35$ and $K = M^2 + N^2 + L^2 = 1 + 9 + 25 = 35$: the dual-lock identity holds on the integers $(1, 3, 5; 7, 35)$. Step 2: the chain (all verified). $E_{\rm GUT} = E_{\rm Pl}\, e^{-2\pi} = 1.2209 \times 10^{19} \times 1.867 \times 10^{-3} = 2.280 \times 10^{16}$ GeV; $M_{\rm lock} = \frac{1}{2} E_{\rm Pl}\, e^{-35} = 3849$ GeV; $v = M_{\rm lock}\, (2/5)^3 = 3849 \times 0.064 = 246.3$ GeV. Step 3: the ratio.

$v/E_{\rm Pl} = \frac{1}{2}e^{-35}(2/5)^3 = 2.018 \times 10^{-17}$, against the empirical $246.22/1.2209 \times 10^{19} = 2.017 \times 10^{-17}$: agreement at the 0.05% level with no adjustable input anywhere in the chain. Physics intuition: the number 10^{-17} has been called the most unnatural in physics; here it is $\frac{1}{2}\, e^{-35}\, (2/5)^3$ — one geometric monodromy, one Diophantine integer, one group-theory fraction — and the student can check all three on a hand calculator.

IV.7.2. Contrast the two naturalness situations quantitatively: compute the Standard-Model tuning that T8.7 removes, then test the rigidity of the ledger by shifting K by one unit. *Solution.* Step 1: the SM tuning. With a Planck cutoff, the bare-versus-physical Higgs mass-squared cancellation must hold to $m_H^2/\Lambda^2 = (125)^2/(2.4 \times 10^{18})^2$ — one part in 3.7×10^{32} (verified): the notorious 10^{32}. Step 2: the GG replacement. The suppression e^{-35} is not a tuned continuum parameter but the integer $K = 35$ of the OGN-5 ledger; the "tuning space" is $\mathbb{Z}$, not $\mathbb{R}$. Step 3: rigidity test. Shift K by one: $v(K = 34) = 246\, e^{+1} = 670$ GeV; $v(K = 36) = 246\, e^{-1} = 90.6$ GeV (verified) — either value wrecks electroweak phenomenology outright. The ledger therefore cannot drift to fit: it is either exactly right or grossly wrong, which is the opposite of a fine-tuning. Physics intuition: naturalness complaints are about knobs; T8.7's answer is not a better knob setting but the absence of the knob — the hierarchy is quantized, and a quantized hierarchy can be checked but never adjusted.

Take-away. *Standard account:* the hierarchy $M_{\rm EW}/M_{\rm Pl} \sim 10^{-16}$ demands fine-tuning to one part in 10^{32}, and the most popular cure — supersymmetry — has not appeared at the LHC. *What it left open:* why the two scales of nature sit sixteen orders apart. *The constitutional derivation:* the OGN-5 ledger $(1, 3, 5; 7, 35)$ with the discrete doublet projection $(2/5)^3$ fixes the Planck–GUT–lock–electroweak chain (T8.7) — the hierarchy is read off integers, and the 10^{-32} tuning is simply gone.

40 Bridge to Pillar TGG of GG-A6-GRIP and to Part V

The seven sub-theorems T8.1–T8.7 collectively constitute the axiomatic content of 4D general relativity in the GG-Theory framework, completing the gravity half of the unification. The companion paper GG-A6-GRIP (Gi-4 Pillars) refines this content into the four *Geometric Invariance* pillars (TGG, CCI, GFF, GRIP), with the TGG pillar (Total Geometric–Gauge object) as the bulk-and-boundary unified stress-tensor structure derived from T8.3 and T8.4. Theorem T10 of Part VII of the present paper makes the bridge to Gi-4 rigorous.

41 Summary of Part IV: Closing the Gravity Half of the Unification Problem

The seven sub-theorems T8.1–T8.7 derive the textbook formalism of 4D general relativity, plus the resolution of the gauge hierarchy and cosmological-constant problems, from GGEP applied to the 6D bulk:

$$\boxed{\text{GG-6 bulk} \overset{\text{GGEP}}{\Longrightarrow} \text{4D GR with } G_N, \Lambda_4, M_{\rm Pl}, K \text{ as theorems (T8.1–T8.7).}} \tag{111}$$

$M_{\rm Pl}^2 = \pi M_6^4(1 - e^{-2K})$ from radial overlap (T8.1–T8.2); $\nabla_\mu T^{\mu\nu} = 0$ as a Ward identity from bulk diffeomorphism invariance (T8.3); modified Einstein equation with GS and KK contributions (T8.4); $\Lambda_4 \sim M_6^2 e^{-NK}$ as scale-suppressed residual (T8.5); modified TOV with neutron-star mass–radius shift $\Delta M \sim 10^{-8} M_\odot$ (T8.6); gauge hierarchy $M_{\rm EW}/M_{\rm Pl} \sim 10^{-16}$ from $K = 35$ (T8.7). The position of this resolution relative to direct quantization, supergravity, strings, LQG, AdS/CFT, asymptotic safety, CDT, and non-commutative geometry is recorded as Innovation 4 of Part IX (see Table 25); the four falsifiers (modified TOV, KK gravitons, sign of Λ_4, hierarchy stability) reappear verbatim as P1–P4 of Part IX §93.

Bridge to Part V. The gravity half of the unification is in place. Part V derives the gauge half: the SM gauge group, the chiral fermion content, the four conserved currents, and exact proton stability all as theorems of GGEP.

41.1 Part IV in plain English

Gravity, in this framework, is the shape of the window at its largest scales. The Einstein–Hilbert action was never a postulate nature imposed — it is what six-dimensional geometry looks like when only the metric's zero mode fits through the projection (T8.1), with Newton's constant the overlap integral that says how much bulk the window admits (T8.2). Conservation laws are the window's bookkeeping (T8.3), Einstein's equation acquires computable corrections at the lock scale (T8.4), the cosmological constant becomes a seesaw constraint rather than a 119-order scandal (T8.5), neutron stars carry the falsifier (T8.6), and the hierarchy that demanded one part in 10^{32} of tuning is read off an integer ledger (T8.7).

Part V
Four-Dimensional Quantum Field Theory from the 6D Bulk

The Second Half of Resolving the GR/SM Unification Problem — Theorem T9 in Nine Dual-Layer Sub-Theorems

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The Standard Model is the most accurately tested theory ever written, and it is written down rather than derived. Its gauge group is chosen to fit; its matter content is chosen to fit; its couplings and masses are measured and inserted, twenty-eight of them. The theory is an extraordinarily successful ansatz, and everyone who works with it knows that is what it is.

This Part asks what happens if the same six-dimensional geometry that produced quantum mechanics and general relativity in the two preceding Parts is asked to produce field theory as well. The answer is the skeleton: the tower of modes that a compact extra dimension always generates, and the scale that fixes it; the effective four-dimensional Lagrangian; the generating functional from which all amplitudes come; the symmetry that keeps a gauge theory consistent when it is quantized, together with the identity that symmetry implies; the cancellation of anomalies, without which the theory is not merely inelegant but inconsistent; the conserved currents; the stability of the proton; and the renormalization group, which turns out to be the geometry of the extra radial direction seen from the boundary.

That last point deserves a sentence. Renormalization is usually presented as a technique for handling infinities and only afterwards given a physical reading. Here it arrives already physical: running a coupling to a different energy is moving to a different depth in the bulk. The arithmetic of the textbook is unchanged; what changes is what the arithmetic was doing.

What this Part does *not* do is compute the twenty-eight numbers. The skeleton is derived here; the flesh — the closed-form values of the Standard Model's parameters — is the subject of GG-A9-SM and is taken on trust in the pages that follow, with the dependency named each time.

A reader who wants one test of the claim should go to the anomaly result, which is the place where a wrong geometry would fail loudly and immediately.

Read alongside: Peskin (1995) and Srednicki (2007) for the quantum-field-theory background assumed of the reader; Weinberg (1995) for renormalization, BRST, and anomalies at treatise depth.

Where we are in the three-descent picture. This Part runs the third of the three descents of Table 4 (§17): the same six-step chain as Parts III and IV, instantiated for quantum field theory — completing the uniform derivation of the three pillars from one law.

42 The Half-Century-Old Unification Problem (continued)

Part V is the second half of our textbook resolution of the unification problem (B) of §18, picking up where Part IV left off. Indeed, Part IV showed that 4D general relativity emerges as the metric zero-mode reduction of the 6D bulk, with G_N, Λ_4, the modified Einstein equation, and the gauge hierarchy all derived as theorems. The gravity half of the unification is therefore in place. In the present Part V, we shall derive the gauge half: the Standard Model gauge group, the matter content, the parameters, and the structural skeleton of 4D quantum field theory all must emerge as theorems of the heavy-mode-integrated bulk gauge sector. Together, Parts IV and V will close problem (B).

42.1 The gauge-side unification problem (delegated to GG-A4-Euclid and GG-A9-SM)

The Standard Model is presented in every standard QFT textbook (Peskin–Schroeder, Weinberg, Schwartz, Srednicki) as a phenomenological gauge theory with gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, three generations of chiral fermions, a Higgs scalar, and 28 fitted parameters. It works spectacularly but is structurally arbitrary: gauge group, matter content, generations, and parameter values are all empirical inputs.

The half-century of attempts at unifying the gauge sector with gravity — Grand Unified Theories (Georgi (H. Georgi, 1974)–Glashow 1974, Pati (Pati, 1974)–Salam 1973), supersymmetry (Wess–Zumino 1974, the MSSM, LHC null results), Kaluza–Klein and extra dimensions (Kaluza 1921, Klein 1926, Randall–Sundrum 1999), string compactifications (Calabi–Yau, Heterotic, F-theory, the $\sim 10^{500}$ landscape), and non-commutative geometry (Connes 1996) — has not produced a community-agreed derivation of the gauge group, the matter content, the generations, proton stability, anomaly freedom, and the 28 parameters from a single foundational principle. The full historical inventory is recorded in Arisaka (2026, A4-Euclid); the 28-parameter accounting in geometric terms is the subject of Arisaka (2026, A9-SM).

42.2 What the present Part claims

After half a century of intense effort, the gauge half of the unification problem remains open in the following precise sense: *no satisfactory derivation of the Standard Model gauge group, the chiral fermion content, the three generations, the absolute stability of the proton, the absence of anomalies, and the BRST / Slavnov–Taylor consistency, all from a single foundational principle, has been agreed upon by the physics community.*

The present Part V claims to provide such a derivation. The foundational principle is GGEP (AX-2), the same principle that gave 4D GR in Part IV; the deeper physical theory is the same 6D bulk GG-6. The derivation chain is:

$$\boxed{\text{GG-6 bulk} \overset{\text{GGEP}}{\Longrightarrow} \text{4D QFT skeleton (T9.1–T9.9 of Part V)} \Longrightarrow \text{SM (GG-A9-SM).}} \tag{112}$$

At the end of Part V, the structural skeleton of 4D QFT — the KK expansion and mode-mass spectrum, the effective 4D Lagrangian, the generating functional, BRST symmetry, the Slavnov–

Taylor identity, anomaly cancellation via BI-6 inflow, the four conserved-current Ward identities, and the exact stability of the proton — emerges as theorems. The detailed derivation of the 28 SM parameters as boundary attractor depths in the GFF Morse landscape is the subject of the companion paper GG-A9-SM (Arisaka, 2026, A9-SM); the present Part V supplies the axiomatic foundation on which GG-A9-SM builds.

Combined with Parts IV (4D GR) and III (4D QM), the present Part completes the textbook resolution of both great open problems of twentieth-century physics: (A) measurement and (B) unification.

43 The Constitutional Bridge: GGEP ⇒ Gauge–Gravity Unification at the Boundary

43.1 What we have from GG-6 and GGEP

The same axiomatic ingredients used in Part IV serve here. GGEP (AX-2) treats the bulk gravity and gauge sectors as projections of a single composite connection $\mathcal{A}_{\text{tot}} = \omega_{\text{Spin}(1,5)} + A_5 + A_B + A_{Qi}$ on the principal G_{tot}-bundle over the 6D bulk (84). Part IV performed the metric-zero-mode projection of this composite connection to derive 4D GR. Part V performs the heavy-mode integration of the gauge sector to derive the 4D QFT skeleton.

43.2 The chain GGEP → 4D QFT skeleton

$$\underbrace{\text{GG-6 bulk}}_{\text{Part I}} + \underbrace{\text{GGEP}}_{\text{Constitution}} \overset{\text{T9.1–T9.9}}{\Longrightarrow} \underbrace{\text{4D QFT skeleton: gauge group, BRST, anomaly cancellation, proton stability}}_{\text{half of the unification problem solved (gauge side)}}. \tag{113}$$

Table 10 summarizes the nine sub-theorems.

Table 10: The nine sub-theorems of Theorem T9 and the structures of quantum field theory each derives, in the same two-layer form as the gravitational table: algebraic content in the second column, operational content in the third, and the derived object in the fourth. The fourth column reads as the skeleton of the Standard Model — the mode tower, the effective Lagrangian, the generating functional, BRST symmetry, the Slavnov–Taylor identity, anomaly cancellation, the conserved currents, proton stability, and the renormalization group. The claim of Part V is that this list is a consequence rather than an ansatz.

Sub-thm	Algebraic content (T9.k.1)	Operational content (T9.k.2)	What is derived
T9.1	KK expansion of bulk fields	Mode-mass spectrum, KK lock cluster M_{lock}	Heavy-mode tower
T9.2	Heavy-mode integration of $S^{(6)}_{\text{tot}}$	Effective 4D action $S_{4,\text{eff}}$ with all Wilson coefficients	4D Lagrangian
T9.3	Bulk path integral with fixed boundary data	4D generating functional $Z_4[J]$	4D path-integral formulation
T9.4	Descent of 6D BRST operator $s_6 \to s_4$	FP ghost reduction; BRST cohomology preservation	BRST symmetry
T9.5	$s_4\Gamma_4 = 0$ from homogeneous 6D ST identity	Slavnov–Taylor identity at the 4D boundary	ST identity
T9.6	Descent equations $I_8^{\text{bulk}} \to I_6 \to I_4$ from BI-6	Anomaly inflow cancels boundary chiral anomaly	Anomaly cancellation
T9.7	Four boundary Ward identities (em, baryon, info, metric)	Conservation of four currents on the boundary	Conserved currents
T9.8	Gauged $\mathrm{U}(1)_B$ forbids dim-6 baryon-violating operators	Exact proton stability as theorem	Proton stability
T9.9	Scale decomposition $\mathcal{A} = \mathcal{A}_{\log}(r) + \mathcal{F}$; $\partial_r \mathcal{F} = 0$	Radial transport = Callan–Symanzik; the RG/log content geometrized, the finite periods not	Renormalization structure and its geometric limit

44 T9.1 (T-GDOS-9.1) — KK Expansion and the M_{lock} Cluster

44.1 The physics question

The Standard Model assumes that all SM fields are 4D fields living on a fixed 4D spacetime; the existence of heavy resonances at high energies is treated as a phenomenological extension. In the GG-Theory framework, the SM fields are zero modes of bulk fields on the 6D bulk, and the heavy KK tower is an axiomatic consequence of the bulk geometry. T9.1 derives the explicit KK expansion and the mode-mass spectrum, identifying the $M_{\text{lock}} \sim 4$ TeV cluster as the principal collider signature of the GG-6 bulk.

44.2 Theorem and proof

The KK expansion of bulk fields on the bulk is

$$\boxed{\Phi(x, r, \theta) = \sum_{n=0}^{\infty} \sum_{\ell \in \mathbb{Z}} \phi^{(n,\ell)}(x)\, f_n(r)\, \frac{e^{i\ell\theta}}{\sqrt{2\pi}},} \tag{114}$$

with radial profiles $f_n(r)$ orthonormal under $\int_0^K dr\, e^{-2r} f_n(r) f_m(r) = \delta_{nm}$. The mass spectrum is

$$m^2_{n,\ell} = m^2_n + \ell^2 \approx n^2 + \ell^2 \quad \text{(natural GG-6 units)}. \tag{115}$$

Restoring units via the scale-profile-integrated Planck relation $M_{\rm Pl}^2 = \pi M_6^4(1-e^{-2K})$ of Theorem 14 and the IR-boundary scale suppression, the canonical lock scale is

$$M_{\rm lock} \;=\; \tfrac{1}{2}\, M_{\rm Pl}\, e^{-K} \;\approx\; 3.85 \text{ TeV}, \tag{116}$$

matching the canonical expression of Theorem T8.7 in Part IV. The heuristic estimate $M_{\rm lock} \sim M_6\, e^{-K/2}$ familiar from Randall– Sundrum-style warped compactifications is equivalent to Eq. (116) via the canonical bulk/boundary scale identification $M_{\rm Pl} \sim M_6\, e^{K/2}$ that follows from Lemma 13.1.

A clarification on the role of ℓ in (115): the ℓ^2 mass contribution belongs to the bulk *tower* states that survive the orbifold projection of Lemma 20.2 below. The chirality-projected *zero* modes — the SM fields — carry their holonomy label as pure electric charge, with no geometric mass contribution from S^1_θ: this is the non-spatial character of the gauge dimension made quantitative in Chain (B) of §57.3 ("the holonomy momentum p_θ is a pure charge label and contributes no mass"), and it is the reading on which the GSM stationary-measure adjunct of Part IX relies.

Lemma 20.1 (Boundary conditions on $f_n(r)$)**.** *$f_n(r)$ satisfy boundary conditions at $r = 0$ and $r = K$ determined by orbifold projection; chirality-projected fermion zero modes survive on the boundary.*

Proof. Standard KK reduction on warped intervals (Gherghetta, 2000, L. Randall, 1999). ■

Lemma 20.2 (Zero-mode chirality from orbifold)**.** *Orbifold projection $\theta \to -\theta$ with parity matrix $P = \mathrm{diag}(+,+,+,-,-)$ on the SU(5) fundamental selects $\bar{5} + 10$ chiral zero modes per generation.*

Proof. GG-A2-GG6 Appendix B. ■

Theorem 21 (T9.1 — KK Expansion and Mode-Mass Spectrum)**.** ***(T9.1.1, algebraic)*** (114)–(115)*.* ***(T9.1.2, operational)*** *The lightest KK modes form a cluster at $M_{\rm lock} \sim 4$ TeV (116), accessible at LHC and HL-LHC.*

Downstream load: this spectral gap supplies hypothesis (H2) of the exact $w = -1$ package — Lemma DE-1 of GG-A6-GRIP together with the T-Cosmos-DE′ theorem of GG-A10-Cosmos — bounding the equation-of-state deviation by $(H_0/M_{\rm lock})^2 \sim 10^{-91}$.

Proof. KK separation of variables plus Lemmas 20.1, 20.2. ■

Problems and worked solutions. V.1.1. Map the bottom of the KK tower: from the spectrum (115) with the lock-scale unit (116), list the first three mass levels, their degeneracies, and the resulting cluster structure a collider would see. *Solution.* Step 1: the levels. With $m^2_{n,\ell} \approx (n^2+\ell^2)$ in lock units: $(n,\ell) = (1,0)$ and $(0,\pm1)$ sit at $m = M_{\rm lock} = 3.85$ TeV (a near-degenerate triplet across each bulk field); $(1,\pm1)$ at $\sqrt{2}\, M_{\rm lock} = 5.44$ TeV; $(2,0)$ and $(0,\pm2)$ at $2M_{\rm lock} = 7.70$ TeV (verified). Step 2: the cluster. Because every bulk field — gauge, fermion, scalar — carries the same radial operator, the 3.85 TeV level is populated by a *cluster* of states of different spins and charges with computable spacing ratios $1 : \sqrt{2} : 2$, not a single resonance: this is the signature that distinguishes a geometric tower from a generic Z'. Step 3: the falsifier form.

Observing TeV resonances with the wrong spacing pattern, or a first level far from 3.85 TeV, breaks the geometry — the cluster is rigid because the radial equation has no free parameter left. Physics intuition: the SM is the $n = \ell = 0$ floor of an apartment building whose upper stories are spaced by geometry; T9.1 publishes the floor plan in advance.

V.1.2. Collider kinematics of the cluster: determine the parton momentum fractions needed to produce lock-scale states at the 14-TeV LHC, singly and in pairs, and draw the phenomenological conclusion. *Solution.* Step 1: the thresholds. Producing mass M requires $x_1 x_2 s \geq M^2$: for single production of 3.85 TeV, $x_1 x_2 \geq (3.85/14)^2 = 0.076$, i.e. typical $x \sim 0.28$; for pair production at 7.7 TeV, $x_1 x_2 \geq (7.7/14)^2 = 0.30$, typical $x \sim 0.55$ (verified). Step 2: read the parton physics. Parton luminosities fall steeply above $x \sim 0.1$, so single production is rare-but-reachable while pair production at $x \sim 0.55$ is luminosity-starved — the absence of a discovery to date is consistent with the cluster sitting at the edge of the accessible window, and HL-LHC's tenfold luminosity buys statistics precisely where the fall-off is steepest. Step 3: the honest statement. The framework does not predict that the LHC *must* have seen the cluster; it predicts where the cluster *is*, and the reach calculation above says the question remains open at 14 TeV. Physics intuition: a falsifier has two dates — the day it is posted and the day the machine can read it; T9.1's was posted at 3.85 TeV, and the second date is an accelerator schedule, not a theory adjustment.

Take-away. *Standard account:* the SM fields are fundamental 4D fields, and heavy resonances are speculative extensions. *What it left open:* what, if anything, lies above the Standard Model. *The constitutional derivation:* the SM fields are zero modes of bulk fields, and the KK tower is an axiomatic consequence of the geometry, with its lightest cluster at $M_{\text{lock}} \approx 3.85$ TeV (T9.1) — the framework's principal collider falsifier, in LHC and HL-LHC territory.

45 T9.2 (T-GDOS-9.2) — The Effective 4D Lagrangian as Derived Object

45.1 The physics question

The textbook of QFT begins by writing down the SM Lagrangian the textbook Standard Model Lagrangian as a phenomenological Ansatz. Each Wilson coefficient (gauge couplings, Yukawa couplings, Higgs vev, θ_{QCD}) must be fitted from experiment. T9.2 derives the effective 4D Lagrangian as the Wilsonian heavy-mode integration of the 6D bulk action, with each Wilson coefficient computable from radial overlap integrals.

45.2 Theorem and proof

The effective 4D action is

$$\boxed{\exp(iS_{4,\text{eff}}[\Phi_0]) = \int \mathcal{D}\Phi_{\text{heavy}} \, \exp(iS^{(6)}_{\text{tot}}[\Phi_0, \Phi_{\text{heavy}}]).} \tag{117}$$

Tree-level matching gives $1/g_4^2 = (2\pi/g_6^2)\int_0^K dr\,|f_0(r)|^2$ for the gauge coupling; one-loop matching gives threshold corrections at $M_{\rm lock}$.

Lemma 21.1 (Convergence of the heavy-mode expansion)**.** *Convergent at $E \ll M_{\rm lock}$, with each successive KK mode contributing $(E/M_{\rm lock})^2$ relative suppression.*

Proof. EFT power-counting (Georgi, 1984). ■

Theorem 22 (T9.2 — Effective 4D Action)**.** ***(T9.2.1, algebraic)*** (117)*, with Wilson coefficients computed from radial overlap integrals.* ***(T9.2.2, operational)*** *$S_{4,\rm eff}$ is the constitutional source of the SM Lagrangian: not postulated but derived from heavy-mode integration of the GG-6 bulk.*

Proof. Wilsonian heavy-mode integration. ■

Problems and worked solutions. V.2.1. Wilsonian matching in miniature: integrate out a heavy field exactly at tree level and read off the induced Wilson coefficient. Take $\mathcal{L} = \frac{1}{2}(\partial\phi_0)^2 - \frac{1}{2}M^2\phi_h^2 - \lambda\,\phi_0^2\,\phi_h$. *Solution.* Step 1: complete the square. The heavy field enters quadratically: $-\frac{1}{2}M^2\phi_h^2 - \lambda\phi_0^2\phi_h = -\frac{1}{2}M^2(\phi_h + \lambda\phi_0^2/M^2)^2 + \lambda^2\phi_0^4/(2M^2)$. Step 2: integrate. The shifted Gaussian integrates to a constant, leaving $\mathcal{L}_{\rm eff} = \frac{1}{2}(\partial\phi_0)^2 + \dfrac{\lambda^2}{2M^2}\,\phi_0^4$: a quartic self-interaction of the light field with Wilson coefficient $c_4 = \lambda^2/(2M^2)$, generated, not postulated. Step 3: the correction tower. Keeping the heavy propagator's momentum dependence, $1/(p^2+M^2) = M^{-2}(1 - p^2/M^2 + \cdots)$, produces the derivative operator $\lambda^2\,\phi_0^2\,\partial^2\phi_0^2/(2M^4)$ and its successors: every higher operator is suppressed by a further $(E/M)^2$ — the exact pattern in which the KK tower populates $S_{4,\rm eff}$ of Eq. (117) with $M \to$ tower masses and $\lambda \to$ overlap integrals. Physics intuition: the SM Lagrangian's "arbitrary" coefficients are the $\lambda^2/2M^2$'s of a bulk the textbook never drew; the miniature above is the whole of T9.2 with one heavy mode instead of infinitely many.

V.2.2. Control the truncation: show that the KK tower's contribution to a low-energy amplitude converges, and compute the fractional effect of the whole tower at $E = 1$ TeV. *Solution.* Step 1: the sum. Tree-level heavy exchange at $E \ll$ masses contributes $\propto \sum_{n\geq 1}(E/m_n)^2$ with $m_n \approx n\,M_{\rm lock}$; the sum is $(E/M_{\rm lock})^2\sum_n n^{-2} = (E/M_{\rm lock})^2\,\pi^2/6$: convergent, by Basel. Step 2: the number. At $E = 1$ TeV: $(1/3.85)^2 \times 1.645 = 0.111$ (verified) — an eleven-percent tower effect just below the lock scale, dominated by the first level (61% of the sum is $n = 1$). Step 3: truncation control. Keeping only N levels errs by $\sum_{n>N} n^{-2} < 1/N$: three levels suffice for percent accuracy at any $E \leq 1$ TeV — the effective-theory ladder is not merely formal but numerically tame. Physics intuition: "integrating out infinitely many fields" sounds uncontrolled until one sees the $\pi^2/6$: geometry stacks the tower so steeply that the infinite bulk costs barely more than its first floor.

Take-away. *Standard account:* the SM Lagrangian is a phenomenological ansatz whose every Wilson coefficient is fitted from experiment. *What it left open:* where the Lagrangian comes from. *The constitutional derivation:* the effective 4D Lagrangian is the Wilsonian heavy-mode integration of the bulk action, with coefficients computed from radial overlap integrals (T9.2) — the page of terms is output, not input.

46 T9.3 (T-GDOS-9.3) — The 4D Generating Functional

46.1 The physics question

Standard QFT begins with a 4D generating functional $Z_4[J] = \int \mathcal{D}\phi\, e^{iS_{\rm SM}[\phi]+i\int J\phi}$ from which all correlation functions follow. T9.3 shows that the same generating functional emerges from the bulk path integral with fixed boundary data, establishing the equivalence with the heavy-mode-integrated effective theory.

46.2 Theorem and proof

$$\boxed{Z_4[J] = \int_{\Phi|_{\partial M_6}=J} \mathcal{D}\Phi_6\, \exp(iS^{(6)}_{\rm tot}[\Phi_6]).} \tag{118}$$

Lemma 22.1 (Equivalence of bulk and effective formulations)**.** (118) *factorizes as* $\int \mathcal{D}\Phi_0\, \delta(\Phi_0 - J)\, \exp(iS_{4,\rm eff}[\Phi_0])$*, equivalent to the effective-theory formulation.*

Proof. Direct factorization via heavy-mode integration of T9.2. ■

Theorem 23 (T9.3 — 4D Generating Functional)**.** ***(T9.3.1, algebraic)*** (118) *equivalent to* $\int \mathcal{D}\Phi_0 \exp(iS_{4,\rm eff}[\Phi_0] + i\int J\Phi_0)$ *by Lemma 22.1.* ***(T9.3.2, operational)*** *Connected correlation functions follow by functional differentiation of* $W_4[J] = -i\log Z_4[J]$*.*

Proof. Lemma 22.1 plus standard generating-functional machinery (Peskin, 1995). ■

Problems and worked solutions. V.3.1. The generating functional in zero dimensions: for $Z(J) = \int d\phi\, e^{-\phi^2/2-\lambda\phi^4/4!+J\phi}$, compute $\langle\phi^2\rangle$ to first order in λ and check against exact numerics. *Solution.* Step 1: expand. To $O(\lambda)$, $\langle\phi^2\rangle = \dfrac{\langle\phi^2\rangle_0 - \frac{\lambda}{4!}\langle\phi^6\rangle_0}{1-\frac{\lambda}{4!}\langle\phi^4\rangle_0}$ with free Gaussian moments $\langle\phi^2\rangle_0 = 1$, $\langle\phi^4\rangle_0 = 3$, $\langle\phi^6\rangle_0 = 15$ (Wick counting: 3, resp. 15, pairings). Step 2: assemble. $\langle\phi^2\rangle = (1-15\lambda/24)/(1-3\lambda/24) = 1-\frac{\lambda}{2}+O(\lambda^2)$: the disconnected pieces cancel between numerator and denominator — the zero-dimensional shadow of the linked- cluster theorem. Step 3: numerics (verified): at $\lambda = 0.1$ the exact integral gives $\langle\phi^2\rangle = 0.9556$ against the first-order 0.950 — a 0.6% error, of the expected $O(\lambda^2)$ size. Physics intuition: everything the generating functional does in field theory — Wick combinatorics, disconnected-diagram cancellation, controlled perturbation — already happens in one ordinary integral; T9.3's content is that the boundary's $Z_4[J]$ of Eq. (118) arrives from the bulk with these properties installed rather than postulated.

V.3.2. Connectedness and $W = \log Z$: show that $d^4W/dJ^4|_{J=0}$ is the connected fourth moment $\langle\phi^4\rangle - 3\langle\phi^2\rangle^2$, and verify numerically that it approaches $-\lambda$ as $\lambda \to 0$. *Solution.* Step 1: cumulants. Differentiating $W = \log Z$: $W'' = \langle\phi^2\rangle - \langle\phi\rangle^2$ and, at $J = 0$ where odd moments vanish, $W'''' = \langle\phi^4\rangle - 3\langle\phi^2\rangle^2$ — the logarithm subtracts exactly the products of lower moments, i.e. the disconnected diagrams. Step 2: leading behavior. Wick counting on the $O(\lambda)$ term gives $\langle\phi^4\rangle - 3 = -\lambda\,(\langle\phi^8\rangle_0 - 3\langle\phi^4\rangle_0\langle\phi^2\rangle_0 \ldots)/4! \to$ the connected piece $-\lambda$ at leading order. Step 3:

numerics (verified): $\lambda = 0.1$ gives $W'''' = -0.0740$; $\lambda = 0.01$ gives -0.00966 — converging on $-\lambda$ exactly as the higher orders shut off. Physics intuition: log is the physicist's most underrated operation: it turns "all processes" into "irreducible processes", and the boundary functional $W_4[J]$ of T9.3 inherits that surgery from the bulk integral — scattering theory is the fourth derivative of a logarithm.

Take-away. *Standard account:* QFT begins with the generating functional $Z_4[J]$ as its formal starting point. *What it left open:* the origin of the starting point itself. *The constitutional derivation:* $Z_4[J]$ emerges from the bulk path integral with fixed boundary data, equivalent to the heavy-mode effective theory (T9.3) — so correlation functions descend from the bulk, and the formalism's first page is a theorem.

47 T9.4 (T-GDOS-9.4) — BRST Symmetry as Inheritance

47.1 The physics question

The standard textbook introduces BRST symmetry as a useful gauge-fixing device. T9.4 shows that 4D BRST is not a separate construction but a direct inheritance from the 6D bulk BRST structure via heavy-mode integration.

47.2 Theorem and proof

The 6D BRST operator s_6 satisfies $s_6 S_{\text{tot}}^{(6)} = 0$. Define $s_4\phi_0 := \mathcal{P}_{\text{zero-mode}}[s_6\phi_6]|_{(0,0)}$.

Lemma 23.1 (Cohomology preservation under heavy-mode integration)**.** *Heavy-mode integration of BRST-doublet pairs preserves the BRST cohomology class (Henneaux, 1992, Kugo, 1979).*

Proof. BRST quartet mechanism: doublet pairs contribute only BRST-exact terms to $S_{4,\text{eff}}$. ■

Lemma 23.2 (Faddeev–Popov ghost reduction)**.** *4D FP ghosts are zero-mode reductions of 6D FP ghosts.*

Proof. Direct application of (117) to bulk FP terms. ■

Theorem 24 (T9.4 — BRST Inheritance)**.** ***(T9.4.1, algebraic)*** *s_4 is nilpotent ($s_4^2 = 0$) and inherits the BRST cohomology of the 6D bulk theory.* ***(T9.4.2, operational)*** *4D FP ghost sector is zero-mode reduction of 6D ghost sector; no separate ghost construction at the boundary.*

Proof. Lemmas 23.1, 23.2. ■

Problems and worked solutions. V.4.1. BRST nilpotency by direct computation: for the abelian multiplet with $sA_\mu = \partial_\mu c$, $sc = 0$, $s\bar{c} = b$, $sb = 0$, verify $s^2 = 0$ on every field and exhibit the gauge-fixed action as BRST-exact up to the classical term. *Solution.* Step 1: nilpotency. $s^2 A_\mu = s\,\partial_\mu c = \partial_\mu(sc) = 0$; $s^2 c = 0$ trivially; $s^2\bar{c} = sb = 0$; $s^2 b = 0$: the differential closes with no conditions — in the abelian case even off shell and without structure constants. Step 2: the gauge-fixing fermion. Take $\Psi = \bar{c}\,(\partial^\mu A_\mu - \frac{\xi}{2} b)$ and compute $s\Psi = b\,\partial^\mu A_\mu - \frac{\xi}{2} b^2 - \bar{c}\,\partial^2 c$ (the

sign bookkeeping from anticommuting s past $\bar{c}$). Step 3: eliminate b by its algebraic equation of motion $b = \partial^\mu A_\mu/\xi$: $s\Psi \to \frac{1}{2\xi}(\partial^\mu A_\mu)^2 - \bar{c}\,\partial^2 c$ — precisely the gauge-fixing and ghost terms, so $\mathcal{L} = -\frac{1}{4}F^2 + s\Psi$: everything except the physics is s-exact. Physics intuition: BRST-exact terms are cohomologically invisible, which is the exact sense in which gauge-fixing choices cannot touch observables; and because T9.4 *inherits* s_4 from the bulk differential, the boundary never has to re-run this construction — it receives the cohomology ready-made.

V.4.2. The quartet mechanism as degree-of- freedom bookkeeping: count physical polarizations across one KK level of a 6D gauge field and show the ghost quartet balances the ledger. *Solution.* Step 1: the 6D count. A massless 6D gauge field has $6 - 2 = 4$ physical polarizations. Step 2: the 4D repackaging at level $n \neq 0$: the tower state is a *massive* 4D vector (3 polarizations) plus the physical scalar combination of (A_r, A_θ) that survives after one scalar is eaten by the vector's longitudinal mode: $3 + 1 = 4$ — the count matches level by level. Step 3: where the quartet acts. In the covariant treatment the eaten scalar, the would-be-negative-norm timelike polarization, the ghost, and the antighost form the BRST quartet whose contributions cancel in every physical amplitude, leaving exactly the 4 positive-norm states counted above; the cancellation is the boundary shadow of the bulk's Kugo–Ojima structure inherited in T9.4. Physics intuition: unitarity of a gauge tower is an accounting identity — four in the bulk, four on the boundary, at every level — and the ghosts are not fields of nature but the ledger's correcting entries, inherited with the ledger.

Take-away. *Standard account:* BRST symmetry is introduced as a gauge-fixing device, constructed anew for each theory. *What it left open:* why the construction always works at the boundary. *The constitutional derivation:* 4D BRST is the zero-mode inheritance of the 6D bulk BRST structure, nilpotency and cohomology included (T9.4) — no separate ghost construction is performed at the boundary, because none is needed.

48 T9.5 (T-GDOS-9.5) — The Slavnov–Taylor Identity at the Boundary

48.1 The physics question

The Slavnov–Taylor identity $s_4\Gamma_4 = 0$ is the formal statement of gauge invariance of the quantum effective action. In standard QFT, it must be verified perturbatively for each gauge theory. T9.5 derives it as a direct inheritance from the 6D homogeneous Slavnov–Taylor identity proved in Theorem 7 of GG-A2-GG6 (Arisaka, 2026, A2-GG6).

48.2 Theorem and proof

By Theorem 7 of GG-A2-GG6: $s_6\Gamma^{(6)} = 0$. Heavy-mode integration via T9.2 plus cohomology preservation (Lemma 23.1) gives

$$\boxed{s_4\Gamma_4 = 0.} \tag{119}$$

Lemma 24.1 (Gauge invariance of physical 4D observables)**.** (119) *implies that all physical 4D S-matrix elements are gauge-invariant.*

Proof. Standard Kugo–Ojima quartet mechanism. ■

Lemma 24.2 (BI-6 inflow ensures no anomalous descent)**.** *BI-6 inflow guarantees that the 6D ST identity holds, and the descent preserves the absence of anomalous BRST breaking.*

Proof. Theorem 7 of GG-A2-GG6 plus Lemma 23.1. ■

Theorem 25 (T9.5 — Boundary Slavnov–Taylor Identity)**.** ***(T9.5.1, algebraic)*** (119) *inherited from GG-A2-GG6 Theorem 7.* ***(T9.5.2, operational)*** *All physical 4D observables are gauge-invariant (Lemma 24.1).*

Proof. Lemmas 24.1, 24.2. ■

Problems and worked solutions. V.5.1. From the Slavnov–Taylor identity to a physical statement: show that the identity forces the vacuum polarization to be transverse, and conclude that the boundary photon stays exactly massless to all orders. *Solution.* Step 1: the identity's abelian face. For QED the Slavnov–Taylor identity reduces to the Ward–Takahashi identity $k_\mu \Gamma^\mu(p, p+k) = S^{-1}(p+k) - S^{-1}(p)$; applied to the two-point function it gives $k_\mu \Pi^{\mu\nu}(k) = 0$. Step 2: solve the constraint. The only rank-two tensor built from k alone satisfying transversality is $\Pi^{\mu\nu} = (k^2 g^{\mu\nu} - k^\mu k^\nu)\,\Pi(k^2)$: the identity fixes the tensor structure completely, leaving one scalar function. Step 3: the mass statement. The corrected propagator $\propto 1/[k^2(1+\Pi(k^2))]$ keeps its pole at $k^2 = 0$ for any regular Π: no perturbative order can generate a photon mass — masslessness is an identity, not an accident of a given diagram. Physics intuition: this is what "T9.5 inherits the identity from the bulk" buys operationally: every such protection theorem — photon masslessness, charge universality, gauge-parameter independence of poles — holds on the boundary because it held in the bulk, with no re-derivation and no loophole for the descent to break it.

V.5.2. Charge universality from the same identity: show that the Ward–Takahashi identity forces $Z_1 = Z_2$ and hence a species-independent renormalized charge. *Solution.* Step 1: the zero-momentum limit. Differentiating the identity of V.5.1 at $k \to 0$: $\Gamma^\mu(p,p) = \partial S^{-1}(p)/\partial p_\mu$ — the full vertex at zero photon momentum is the derivative of the full inverse propagator. Step 2: renormalize. Writing $\Gamma^\mu = Z_1^{-1}\gamma^\mu + \ldots$ and $S^{-1} = Z_2^{-1}(\not{p} - m) + \ldots$ near the mass shell, the identity forces $Z_1 = Z_2$ identically. Step 3: universality. The physical charge renormalizes as $e_{\text{phys}} = e_0\, Z_2 Z_3^{1/2}/Z_1 = e_0\, Z_3^{1/2}$: every charged species, whatever its mass or interactions, has its charge rescaled by the *same* photon-field factor Z_3 — which is why the proton and positron charges balance to the measured 10^{-21} despite utterly different internal structure. Physics intuition: charge universality is the quiet miracle underneath chemistry's neutrality; the boundary inherits it from the bulk's one gauge symmetry, and T9.7's four-current structure will reuse the same mechanism for baryon and informational charge.

Take-away. *Standard account:* the Slavnov–Taylor identity is verified perturbatively, gauge theory by gauge theory. *What it left open:* why it holds. *The constitutional derivation:* the boundary identity is inherited directly from the 6D homogeneous Slavnov–Taylor identity of the bulk companion (T9.5) — gauge invariance of the quantum effective action arrives as a bulk legacy, and all physical boundary observables are gauge-invariant by theorem.

49 T9.6 (T-GDOS-9.6) — Anomaly Cancellation as Theorem of BI-6

49.1 The physics question

The celebrated representation-theoretic "miracle" of Standard-Model anomaly cancellation — the precise conspiracy of quark and lepton charges within each generation — is one of the most striking features of the SM. In the textbook, it is verified case-by-case as a consistency check on the matter content. T9.6 shows that anomaly cancellation is not a representation-theoretic miracle but a theorem of the BI-6 inflow.

49.2 Theorem and proof

The 6D bulk anomaly polynomial (Theorem 5 of GG-A2-GG6) is $I_8^{\text{bulk}} = (1/(2\pi)^4)\, X_4 \wedge X_4$ with X_4 from BI-6. The Wess–Zumino descent equations (Stora, 1976, Wess, 1971, Zumino, 1985) give $I_8 = dI_7^{(0)}$, $\delta_{\text{gauge}} I_7^{(0)} = dI_6^{(1)}$, etc. The Green–Schwarz inflow contribution to the 4D anomaly is

$$\boxed{\Delta I_4^{\text{GS,infl}} = -I_4^{\text{bdy,fermion}},} \tag{120}$$

exactly canceling the boundary chiral-fermion anomaly.

Lemma 25.1 (4D anomaly cancellation as theorem)**.** *The 4D SM chiral anomaly cancels for any matter content that is the zero-mode reduction of a bulk SU(5) representation satisfying BI-6.*

Proof. Descent equations plus inflow (120). ■

Lemma 25.2 (Mixed gauge-gravitational anomaly cancellation)**.** $\operatorname{tr}(R \wedge R)$ *in* X_4 *provides the gravitational inflow that cancels mixed gauge-gravitational anomalies (Alvarez-Gaumé, 1984).*

Proof. Same BI-6 mechanism applied to mixed terms. ■

Theorem 26 (T9.6 — Anomaly Inflow from BI-6)**.** ***(T9.6.1, algebraic)*** *4D boundary chiral anomaly canceled by BI-6 inflow via descent equations.* ***(T9.6.2, operational)*** *4D SM anomaly cancellation is an axiomatic theorem of BI-6, not a representation-theoretic miracle (Lemmas 25.1, 25.2).*

Proof. Standard descent-equations machinery plus (120). ■

49.3 Worked example: anomaly coefficients of one SM generation

The 4D anomaly coefficients of one Standard Model generation can be collected in a 4×4 matrix $\mathcal{A}_{IJ} = \operatorname{tr} T_I T_J T_K$ for the four U(1) factors $I, J, K \in \{Y, B, Qi, R\}$ (where R denotes the gravitational sector and Y is hypercharge). For one SM generation $\bar{5} + 10$ of SU(5), the explicit anomaly coefficients are

$$\mathcal{A}_{YYY} = 0, \quad \mathcal{A}_{YYR} = 0, \quad \mathcal{A}_{BBB} = 0, \quad \mathcal{A}_{QiQiQi} = 0, \tag{121}$$

and similarly for all 19 mixed components. In the standard SM, these zero values are obtained as a representation-theoretic "miracle" — the precise conspiracy of quark and lepton hypercharges within one generation makes each anomaly vanish identically. In the GG-Theory framework, the same zeros are produced not by representation-theoretic accident but by BI-6 inflow: the bulk Green–Schwarz two-form B_2 contributes a counter-anomaly via descent (120) that exactly cancels the boundary fermion anomaly. Table 11 summarizes the four diagonal anomaly coefficients with their bulk and boundary contributions.

For one Standard-Model generation in $\bar{5}+10$ of SU(5), the explicit hypercharge trace is the standard

$$\mathrm{Tr}_Y[Q^3]_{\bar{5}+10} \;=\; \sum_{\bar{5}} Y^3 + \sum_{10} Y^3 \;=\; 3(\tfrac{1}{3})^3 + (-1)^3 \;+\; 6(\tfrac{1}{6})^3 + 3(-\tfrac{2}{3})^3 + (1)^3 \;=\; 0, \qquad (122)$$

where the $\bar{5}$ contains (d^c, L) with hypercharges $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2})$ and the 10 contains (Q, u^c, e^c) with hypercharges $(\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, 1)$ in canonical normalization. The cubic trace cancels because of the specific representation-theoretic content of $\bar{5}+10$ — this is the celebrated SM hypercharge-anomaly cancellation.

The textbook reading is: "the SM matter content is anomaly-free because the cubic Y^3 trace happens to vanish on $\bar{5}+10$; this is a representation-theoretic miracle." The GG-Theory reading is more structural: the SU(5) embedding selects exactly the $\bar{5}+10$ chirality-projected content as the surviving boundary zero modes (Lemma 20.2); BI-6 then guarantees that any additional anomaly contribution from the bulk gauge sector (e.g., from non-zero KK modes that would generically violate hypercharge anomaly cancellation) is canceled by the Green–Schwarz inflow. Table 11 collects the non-trivial mixed-anomaly contributions where the BI-6 inflow does non-trivial work, alongside the diagonal contributions where the representation-theoretic cancellation suffices.

Table 11: Anomaly coefficients for one SM generation in $\bar{5}+10$ of SU(5). The diagonal SM gauge anomalies cancel by the representation-theoretic content of $\bar{5}+10$ ((122)); the gauged $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$ anomalies, which would generically be non-zero in any non-supersymmetric extension of the SM, are canceled by the BI-6 Green–Schwarz inflow with coefficients $(1, -1, -2/5, -2/5)$ derived in Theorem 4 of GG-A2-GG6.

Anomaly	Boundary fermion contribution	Bulk BI-6 inflow contribution	Net
$\mathcal{A}_{YYY}$	$\mathrm{Tr}\, Y^3\vert_{\bar{5}+10} = 0$ (rep. theory)	not required	0
$\mathcal{A}_{Y\,SU(2)^2}$	$\mathrm{Tr}\, Y\, T_a T_b\vert_{\bar{5}+10} = 0$ (rep. theory)	not required	0
$\mathcal{A}_{Y\,SU(3)^2}$	$\mathrm{Tr}\, Y\, T_a T_b\vert_{\bar{5}+10} = 0$ (rep. theory)	not required	0
$\mathcal{A}_{YR^2}$	$\mathrm{Tr}\, Y\vert_{\bar{5}+10} = 0$ (rep. theory)	not required	0
$\mathcal{A}_{BBB}$	$\mathrm{Tr}\, B^3\vert_{\bar{5}+10} \neq 0$ generically	$-\mathrm{Tr}\, B^3\vert_{\bar{5}+10}$ via $-\frac{2}{5} F_B \wedge F_B$ in BI-6	0
$\mathcal{A}_{BY^2}$	$\mathrm{Tr}\, B Y^2\vert_{\bar{5}+10} \neq 0$ generically	canceled by mixed-channel BI-6 inflow	0
$\mathcal{A}_{QiQiQi}$	$\mathrm{Tr}\, Q_{Qi}^3\vert_{\mathrm{matter}} \neq 0$ generically	$-\mathrm{Tr}\, Q_{Qi}^3$ via $-\frac{2}{5} F_{Qi} \wedge F_{Qi}$ in BI-6	0
$\mathcal{A}_{R^2R^2}$	gravitational anomaly	canceled by $\mathrm{tr}(R \wedge R)$ in BI-6	0

The key observation is that the four diagonal SM-gauge anomalies (YYY, $Y\,SU(2)^2$, $Y\,SU(3)^2$, YR^2) cancel by the representation-theoretic content of $\bar{5}+10$ alone, exactly as in conventional SU(5) GUT, while the four BI-6-protected anomalies involving $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$ (and the mixed gravita-

tional anomaly) are canceled by the BI-6 inflow with the specific coefficient vector $(1, -1, -2/5, -2/5)$ derived in (Arisaka, 2026, A2-GG6). The full coefficient evaluation, including the explicit normalization of $\mathrm{Tr}\, B^3|_{\bar{5}+10}$ and the matching to the BI-6 inflow, is given in Appendix D of GG-A2-GG6 (Arisaka, 2026, A2-GG6). This makes anomaly cancellation a derived structural consequence of BI-6 plus representation theory, not a representation-theoretic miracle alone.

Problems and worked solutions. V.6.1. Run the anomaly census yourself: verify, for one SM generation in the all-left convention ($Q(\frac{1}{6})$, $u^c(-\frac{2}{3})$, $d^c(\frac{1}{3})$, $L(-\frac{1}{2})$, $e^c(1)$), that all four hypercharge anomaly conditions cancel exactly. *Solution.* Step 1: the two mixed gauge conditions. $[\mathrm{SU}(3)]^2\,\mathrm{U}(1)$: $2\cdot\frac{1}{6}+(-\frac{2}{3})+\frac{1}{3}=0$ (quarks only, weighted by SU(2) multiplicity); $[\mathrm{SU}(2)]^2\,\mathrm{U}(1)$: $3\cdot\frac{1}{6}+(-\frac{1}{2})=0$ (doublets only, weighted by color). Step 2: the gravitational condition. $\sum Y$ over all 15 Weyl fermions: $6\cdot\frac{1}{6}+3\cdot(-\frac{2}{3})+3\cdot\frac{1}{3}+2\cdot(-\frac{1}{2})+1=1-2+1-1+1=0$. Step 3: the cubic condition. $\sum Y^3=6\cdot\frac{1}{216}-3\cdot\frac{8}{27}+3\cdot\frac{1}{27}-2\cdot\frac{1}{8}+1=\frac{1-32+4-9+36}{36}=0$ (all four sums verified exactly in rational arithmetic). Four independent cancellations, each requiring the quark and lepton charges to conspire across sectors that classical physics regards as unrelated. Physics intuition: in the textbook this is four miracles per generation; under T9.6 it is one statement — the BI-6 inflow has a single coefficient vector, and the four sums are its four shadows — so the student who has verified the arithmetic above has, without noticing, verified a piece of six-dimensional topology.

V.6.2. The descent mechanism in the abelian case: derive the two-step descent $P_4 = d\omega_3$, $\delta\omega_3 = d\omega_2^1$, and state how inflow converts a boundary anomaly into bulk bookkeeping. *Solution.* Step 1: the closed polynomial. For an abelian field strength F, the four-form $P_4 = F\wedge F$ is closed ($dP_4=0$) and gauge-invariant, and is locally exact: $P_4 = d\omega_3$ with the Chern–Simons form $\omega_3 = A\wedge F$. Step 2: the descent. Under $\delta A = d\lambda$: $\delta\omega_3 = d\lambda\wedge F = d(\lambda F)$ — the gauge variation of the Chern–Simons form is itself exact, defining $\omega_2^1=\lambda F$: this pair of statements is the descent chain, and ω_2^1 integrated over the boundary is precisely the anomaly's form. Step 3: inflow. Place the Chern–Simons term in a bulk one dimension higher: its gauge variation localizes on the boundary as $-\int_\partial \omega_2^1$, canceling the boundary fermions' anomaly if and only if the bulk coefficient matches the boundary content — in six dimensions this matching is the BI-6 identity with its one coefficient vector. Physics intuition: an anomaly is a conservation law leaking off the edge of a space; inflow says the leak is real and the bulk catches every drop — the boundary books balance because they were never the whole ledger.

Take-away. *Standard account:* Standard-Model anomaly cancellation is a representation-theoretic miracle, verified charge by charge within each generation. *What it left open:* why the quark and lepton charges conspire. *The constitutional derivation:* the 4D chiral anomaly is canceled by BI-6 inflow via the descent equations (T9.6) — one theorem where the textbook has one miracle per generation.

50 T9.7 (T-GDOS-9.7) — Four Conserved Currents at the Boundary

50.1 The physics question

The standard textbook has two conserved currents (electric charge, energy-momentum). In the GG-Theory framework, four conserved currents emerge from the four bulk gauge factors of G_{tot}: electric (em), baryonic (B), informational (Qi), and metric. T9.7 derives the four boundary Ward identities.

50.2 Theorem and proof

$$\boxed{\partial_\mu J^\mu_a = 0 \quad (a = \text{em}, B, Qi), \qquad \nabla_\mu T^{\mu\nu} = 0.} \tag{123}$$

Lemma 26.1 (BI-6 protects all four Ward identities)**.** *BI-6 inflow protects each of the four boundary Ward identities from anomalous breaking; in particular, J^μ_B is exactly conserved (no sphaleron-type non-perturbative breaking).*

Proof. T9.6 applied to each gauge factor with BI-6 coefficient vector $(1, -1, -2/5, -2/5)$. ■

Lemma 26.2 (Cross-link to all four Pillars of Gi-4)**.** *The four Ward identities collectively constitute the* CCI *pillar of GG-A6-GRIP in physical/local form.*

Proof. GG-A6-GRIP pillar definitions. ■

Theorem 27 (T9.7 — Four Boundary Ward Identities)**.** ***(T9.7.1, algebraic)*** (123) *from boundary gauge-symmetry and diffeomorphism Ward identities.* ***(T9.7.2, operational)*** *Charge conservation, baryon-number conservation, informational-charge conservation, and energy-momentum conservation, all protected from anomalous breaking by BI-6 inflow.*

Proof. Lemmas 26.1, 26.2. ■

Problems and worked solutions. V.7.1. The electromagnetic current as the unbroken remnant: verify that the electric charge operator is traceless on the $\bar{\mathbf{5}}$ of SU(5), and explain what this fixes that the Standard Model leaves free. *Solution.* Step 1: the multiplet. $\bar{\mathbf{5}} = (d^c, d^c, d^c, e^-, -\nu)$ with charges $(+\frac{1}{3}, +\frac{1}{3}, +\frac{1}{3}, -1, 0)$. Step 2: the trace. Tr $Q = 3\cdot\frac{1}{3} + (-1) + 0 = 0$: as a generator of the simple group SU(5), Q must be traceless on every irreducible multiplet — and is. Step 3: what is fixed. Tracelessness on $\bar{\mathbf{5}}$ *forces* $Q(d) = -\frac{1}{3}\,Q(e^-)$: charge quantization in thirds, and hence the exact neutrality of the hydrogen atom, cease to be inputs — in the SM the electron and quark charges are independent parameters that happen to balance to 10^{-21}; in the unified boundary group of T9.7 their ratio is group theory. Physics intuition: the most precise null experiment in science — the neutrality of bulk matter — is the trace condition of a five-dimensional representation; the first of the four boundary currents carries that theorem, and the other three (baryonic, informational, metric) repeat the pattern with their own gauge factors.

V.7.2. The informational current's operational face: evaluate the Arisaka relation $E = T_{Qi} \log 2$ at biological temperature and connect the number to the Ward identity of T9.7. *Solution.* Step 1: the number. At $T = 300$ K: $k_B T = 25.9$ meV, so $E = k_B T \ln 2 = 17.9$ meV $= 2.87 \times 10^{-21}$ J per bit (verified) — the Landauer scale, here read as the conversion rate between the informational charge and energy. Step 2: the Ward-identity reading. T9.7's fourth current makes informational charge a *conserved gauge charge*: erasing a bit is not destroying charge but exporting it across the observer's boundary, and the export is paid in energy at the rate $T_{Qi} \log 2$ — the bridge theorem T11 makes this identification exact. Step 3: orders of magnitude. A neuron switching $\sim 10^9$ bits/s dissipates at least 3×10^{-12} W by this floor alone — twelve orders below its actual ∼nW budget, so biology runs hot but the floor is real, and nanoelectronics at 10^{-18} J/operation now sits within three orders of it. Physics intuition: the deepest reading of T9.7 is that "information is physical" is not a slogan but a fourth column in nature's conservation table — with an exchange rate, printed by the geometry, that a graduate student can evaluate on one line.

Take-away. *Standard account:* physics conserves two boundary currents — electric charge and energy-momentum — with baryon number an accidental symmetry. *What it left open:* whether more is conserved, and why. *The constitutional derivation:* four Ward identities descend from the four bulk gauge factors — electric, baryonic, informational, and metric (T9.7) — all protected by BI-6 inflow; the third of them is where the information physics of Part VII will attach.

51 T9.8 (T-GDOS-9.8) — Proton Stability as Constitutional Theorem

51.1 The physics question

Standard SU(5) GUT predicts proton decay with lifetime $\tau_p \sim 10^{32}$ years through X, Y boson exchange. Current Super-Kamiokande limits give $\tau_p \gtrsim 10^{34}$ years, in tension with the simplest GUT predictions. In the GG-Theory framework, T9.8 derives *exact* proton stability as a theorem of gauged $\mathrm{U}(1)_B$.

51.2 Theorem and proof

The lowest-dimension boundary operators that would mediate proton decay are dimension-6 four-fermion operators $\mathcal{O}_6 \sim qqq\ell/M_*^2$, violating baryon number. The boundary $\mathrm{U}(1)_B$ Ward identity (T9.7) forbids these:

$$\boxed{\langle \mathcal{O}_6 \rangle = 0 \quad \text{for any } \mathcal{O}_6 \text{ with } \Delta B \neq 0.} \tag{124}$$

Therefore $\tau_p = \infty$ (exact).

Lemma 27.1 ($\mathrm{U}(1)_B$ as true gauge symmetry, not accidental)**.** *$\mathrm{U}(1)_B$ is a primary gauge factor of G_{tot} in GG-A2-GG6, with associated gauge boson and gauge-invariance constraint on all admissible boundary operators.*

Proof. GG-A2-GG6 Theorem 3. ■

Lemma 27.2 (Exact global $\mathrm{U}(1)_B$ survives a massive gauge boson)**.** *Suppose the* $\mathrm{U}(1)_B$ *gauge boson* B_μ *is massive. In the present framework the mass comes from the boundary termination of GG-A3-BI6: a Dirichlet condition at the seam removes the four-dimensional zero mode, nothing is eaten, and the vector's mass is its first Kaluza–Klein note* M_{lock}*. The lemma is stated for the general case as well, because its conclusion does not use the mechanism: let* m_B *instead come from a Stueckelberg mechanism (with axion field* σ *transforming as* $\sigma \to \sigma + m_B\alpha$ *under the gauge transformation* $B_\mu \to B_\mu + \partial_\mu\alpha$*) or from a Higgs mechanism (with a* $\mathrm{U}(1)_B$*-charged scalar* χ *acquiring a vacuum expectation value). Then, in every case, the global* $\mathrm{U}(1)_B$ *symmetry acting on all gauge-invariant local operators of the boundary action remains exact, and Ward identity T9.7 continues to forbid* $\Delta B \neq 0$ *operators.*

Proof. In the framework's own case the argument is the shortest of the three: the termination removes the zero mode and eats nothing, so $\mathrm{U}(1)_B$ survives on the boundary as an exact perturbative selection rule and the Ward identity is untouched. For the other two the argument is standard. In either mechanism, the local $\mathrm{U}(1)_B$ gauge symmetry of the unbroken theory is realized on the Stueckelberg axion σ (resp. the Goldstone phase of χ). Eliminating σ (resp. the Goldstone) by gauge fixing leaves a residual global $\mathrm{U}(1)_B$ symmetry under which all original matter fields rotate by their baryon charge B while σ (resp. the Goldstone) absorbs the gauge shift. Gauge-invariant local operators built from the matter fields alone therefore transform with phase $e^{i\Delta B\,\alpha}$ under the global $\mathrm{U}(1)_B$, and the Ward identity $\partial_\mu\langle J_B^\mu\rangle = 0$ at the level of correlators of such operators is preserved. The dim-6 operators $\mathcal{O}_6 \sim qqq\ell/M_*^2$ that would mediate proton decay all carry $\Delta B \neq 0$ (specifically $\Delta B = \pm 1$), and the global $\mathrm{U}(1)_B$ phase rotation gives $\langle\mathcal{O}_6\rangle \to e^{i\Delta B\,\alpha}\langle\mathcal{O}_6\rangle$; consistency with α-independence forces $\langle\mathcal{O}_6\rangle = 0$. Therefore the proton-stability conclusion $\tau_p = \infty$ holds whether or not the B_μ gauge boson is massive — exactly as in the standard treatment of any anomaly-free gauged $\mathrm{U}(1)_B$ via the Stueckelberg or Higgs mechanism in the literature. The mass m_B controls only the range of the B-mediated force, not the existence of the symmetry. ■

Theorem 28 (T9.8 — Proton Stability from Gauged $\mathrm{U}(1)_B$)**.** ***(T9.8.1, algebraic)*** *All boundary operators with* $\Delta B \neq 0$ *are forbidden (Lemma 27.1).* ***(T9.8.2, operational)*** $\tau_p = \infty$ *exactly. Any future observation of proton decay falsifies the framework.*

Proof. T9.7 plus Lemma 27.1. ■

Problems and worked solutions. V.8.1. Compute the proton lifetime both ways: evaluate the naive gauge-mediated estimate $\tau_p \sim M_X^4/(\alpha_{\mathrm{GUT}}^2\, m_p^5)$ **at the classic SU(5) scale and at the GG-Theory** E_{GUT}**, then state why T9.8 nevertheless posts** $\tau_p = \infty$ **rather than a large number.** *Solution.* Step 1: classic SU(5). With $M_X = 2 \times 10^{14}$ GeV, $\alpha_{\mathrm{GUT}} = 1/40$: $\tau_p \approx 7 \times 10^{28}$ yr (verified) — the naive dimensional estimate whose refined descendants ($10^{30\pm2}$ yr) Super-Kamiokande has now excluded. Step 2: the same formula at the GG-Theory scale. With $M_X = E_{\mathrm{GUT}} = 2.28 \times 10^{16}$ GeV, $\alpha_{\mathrm{GUT}} = 1/24$: $\tau_p \approx 4 \times 10^{36}$ yr (verified) — were the framework merely a high-scale GUT, its gauge bosons would predict decay far beyond any experiment. Step 3: the sharper claim. T9.8 does not hide behind this remoteness: gauged $\mathrm{U}(1)_B$ forbids every $\Delta B \neq 0$ operator at *any* dimension, so the prediction is exactly zero, not merely small — and therefore a

single observed decay, at any rate whatsoever, falsifies the framework, whereas a high-scale GUT could always retreat up its own exponent. Physics intuition: "very stable" and "exactly stable" differ by an entire philosophy of science; only the second forbids something at every future sensitivity, which is what makes it the sharpest kill condition in the cohort.

V.8.2. Gauged versus global baryon number: show quantitatively why a *global* B symmetry cannot deliver exact stability. *Solution.* Step 1: the folk theorem. Global symmetries are expected to be violated by quantum gravity: nothing forbids Planck-suppressed dimension-six operators such as $(QQQL)/M_{\rm Pl}^2$ in the low-energy theory. Step 2: the rate. Such an operator gives $\tau_p \sim M_{\rm Pl}^4/m_p^5 \approx 1\times 10^{42}$ yr (verified) — comfortably beyond Super-K, but *finite*, and lower if any intermediate scale suppresses the operators more weakly. Step 3: the structural difference. A *gauged* $\mathrm{U}(1)_B$ admits no such operators at any scale: a $\Delta B \neq 0$ term is not small but *inconsistent* (it breaks a local symmetry, and T9.6's inflow requires the gauge symmetry exact), so the lifetime is infinite as a matter of consistency, not of suppression. The observable difference is stark: the global scenario predicts decay at *some* rate between 10^{34} and 10^{42} yr; the gauged one predicts zero forever. Physics intuition: symmetries you assume can be violated by physics you forgot; symmetries the geometry gauges are load- bearing — remove one and the building, not just the proton, comes down.

Take-away. *Standard account:* grand unification predicts proton decay near $\tau_p \sim 10^{32}$ years, and Super-Kamiokande keeps pushing the limit up past 10^{34}. *What it left open:* why no decay has ever been seen. *The constitutional derivation:* gauged $\mathrm{U}(1)_B$ forbids every $\Delta B \neq 0$ boundary operator (T9.8): $\tau_p = \infty$ exactly — and therefore a single observed proton decay falsifies the framework outright, the sharpest kill condition this paper posts.

52 T9.9 (T-GDOS-9.9) — Renormalization and the Radial Dimension: What the Bulk Geometrizes

52.1 The physics question

Every graduate student who has computed a high-order vertex correction knows the labor. The anomalous magnetic moment of the muon, $a_\mu = (g-2)/2$, is the proverbial example: the QED series alone has been carried to five loops, the tenth-order coefficient requiring the evaluation of order 12,672 Feynman diagrams (Aoyama, 2012), and the result is one of the most precisely tested numbers in all of science. The work is enormous and unforgiving. Because GG-Theory geometrizes the renormalization group as motion along the radial coordinate $r = \ln(E_{\rm GUT}/\mu)$ (T8; Arisaka, 2026, A2-GG6), it is natural — and the question is asked in earnest by every working physicist who meets the framework — to hope that the bulk geometry might shortcut this loop expansion. Theorem T9.9 settles the question. It states, precisely and with a proof, what the radial dimension *can* geometrize and what it provably cannot; and it then states, with equal precision, what GG-Theory contributes to the Standard-Model world *instead*. The honest answer to the first question is negative; the answer to the second is the real point of the entire program.

52.2 The algebraic layer (T9.9.1): scale-dependent logarithms versus scale-invariant periods

Renormalization-group invariance is the statement that no observable depends on the sliding scale μ. For any renormalized boundary amplitude this is the Callan–Symanzik equation, which in the radial variable is a *first-order transport equation along* r,

$$\Big(\partial_r - \beta(\alpha)\,\partial_\alpha - \gamma\Big)\,\mathcal{A} = 0, \qquad r = \ln\frac{E_{\rm GUT}}{\mu}. \tag{125}$$

Its content is the resummation of logarithms: the running coupling, and with it the entire tower of leading and subleading logs, is the solution of (125), i.e. *radial transport in the bulk*. Every amplitude therefore decomposes, canonically, into the part this transport generates and the part it leaves untouched,

$$\boxed{\mathcal{A}(\alpha,\mu,m_i,p) \;=\; \underbrace{\mathcal{A}_{\log}(r)}_{\text{radial transport}} \;+\; \underbrace{\mathcal{F}(\alpha,m_i/m_j)}_{\partial_r\mathcal{F}=0}\,,} \tag{126}$$

where $\mathcal{A}_{\log}$ collects all μ-dependence and all large mass-ratio logarithms, and the remainder $\mathcal{F}$ is a *scale-invariant* number: a rational together with the periods of the Feynman integrals — π^2, $\zeta(3)$, $\zeta(5)$, multiple polylogarithms, and at high loop order more intricate motivic periods.

Remark 10 (The geometric no-go)**.** The radial coordinate r is, by construction, the variable conjugate to scale. The finite remainder $\mathcal{F}$ is scale-invariant, $\partial_r\mathcal{F} = 0$. Therefore $\mathcal{F}$ lies entirely in the directions *orthogonal* to r and is invisible to any construction that organizes the scale direction — the GG-6 geometry included. A change of variables between a boundary description and a bulk description is a reparametrization, and a reparametrization cannot reduce the information content of a quantity already independent of the parameter being changed. The transcendental coefficients are irreducible data of the 4D boundary theory:

$$\boxed{\text{the bulk geometrizes } \mathcal{A}_{\log} \text{ exactly, and } \mathcal{F} \text{ not at all.}} \tag{127}$$

This is sharper than a statement about present techniques. It is a structural fact: the difficulty of a precision amplitude resides in $\mathcal{F}$, and $\mathcal{F}$ does not depend on the dimension the bulk organizes.

52.3 The operational layer (T9.9.2): the muon anomaly as the witness

Write the QED contribution in the canonical form

$$a_\mu = \tfrac{1}{2}\Big(\tfrac{\alpha}{\pi}\Big) + C_2\Big(\tfrac{\alpha}{\pi}\Big)^2 + C_3\Big(\tfrac{\alpha}{\pi}\Big)^3 + \cdots + a_\mu^{\rm had} + a_\mu^{\rm EW}, \tag{128}$$

and read each term against (126).

- *Schwinger's term* $\alpha/2\pi \simeq 1.16\times10^{-3}$ (Schwinger, 1948) is a pure finite number from a single diagram, with *no* scale dependence: it is $\mathcal{F}$ at one loop, and the radial geometry is wholly blind to it. The most famous result in precision QED is exactly the kind of object the bulk cannot produce.

- *The mass-independent coefficients* $C_2 = -0.328478965\ldots$ (already carrying $\zeta(3)$ and $\pi^2 \ln 2$), $C_3 = 1.181241\ldots$, and their successors through the 12,672 five-loop diagrams are scale-invariant periods — pure $\mathcal{F}$. *All* of that labor lies in the part (127) says r cannot touch.

- *The one geometric piece.* The electron vacuum-polarization insertion contributes, at two loops, a term whose leading behavior is $\left(\frac{\alpha}{\pi}\right)^2\left[\frac{1}{3}\ln(m_\mu/m_e) - \frac{25}{36}\right] \simeq 1.08\left(\frac{\alpha}{\pi}\right)^2$. The logarithm $\frac{1}{3}\ln(m_\mu/m_e)$ *is* $\mathcal{A}_{\log}$: it is precisely the running of α from the electron to the muon scale across the electron threshold, the QED slope $d\alpha^{-1}/d\ln\mu = -2/3\pi$ per lepton. Here, and only here, does the radial dimension organize $g-2$ — it resums the mass-ratio-logarithm tower — while the constant $-25/36$, the Schwinger term, and every C_n remain genuine diagrammatic data.

- *The hadronic pieces* $a_\mu^{\rm had}$ are not perturbative QED at all but strong-coupling correlators of the boundary QCD, drawn from data and the lattice. They are a regime of their own — and the sharpest test of the limits of the radial geometry — treated in detail in §52.4 below.

Of the twelve-significant-figure edifice of a_μ, then, the radial geometry reproduces only the logarithmic skeleton — the part that was already cheap, following from the β-function. The expensive part is orthogonal to r.

52.4 The hadronic frontier: what the bulk can and cannot do

The hadronic contributions to a_μ deserve a treatment of their own, because they are the sharpest probe of the boundary between what the radial geometry reaches and what it does not — and the answer is the most instructive thing T9.9 has to record.

What they are. Two pieces: hadronic vacuum polarization ($a_\mu^{\rm HVP} \approx 7000 \times 10^{-11}$, some 99% of the hadronic total) and hadronic light-by-light ($a_\mu^{\rm HLbL} \approx 92 \times 10^{-11}$) (Aoyama, 2020). The first is fixed by the dispersion integral

$$a_\mu^{\rm HVP} \;\propto\; \int ds\, \frac{R(s)\,K(s)}{s}, \tag{129}$$

an integral over the *entire* hadronic spectral function $R(s)$ — every resonance (ρ, ω, ϕ, the multi-hadron continuum), with its mass, width, and photon coupling. To produce $a_\mu^{\rm HVP}$ one must know $R(s)$ — which is to say, one must *solve QCD*.

Why it is a different kind of object. It is neither an input nor a scale-logarithm. GG-Theory derives the inputs of QCD — the color SU(3) from SU(5), the coupling $\alpha_s(M_Z) = 1/(\pi e)$, the confinement scale $\Lambda_{\rm QCD}$ and the proton-mass window of the mass–energy dictionary (Arisaka, 2026, A9-SM) — and it geometrizes the scale flow that runs them. But the hadronic correlator is the *solution* of the strongly-coupled theory, encoded in $R(s)$; it lies one tier below even the scale-invariant periods of the operational layer above. Those were irreducible *arithmetic*; this is irreducible *dynamics*. Fixing the parameters of a Lagrangian does not evaluate its correlators.

Where the bulk genuinely helps. This is, nonetheless, the one regime in which the holographic character of the radial dimension earns real keep. A strongly coupled boundary maps to a weakly

curved classical bulk, and bottom-up AdS/QCD-type constructions (Erlich, 2005) recover the qualitative hadron spectrum (the Regge pattern $m_n^2 \propto n$), vector-meson dominance of the current correlator, and order-of-magnitude HVP and HLbL. Because the GG-6 bulk is *parameter-free* — the OGN-5 ledger and $K = 35$ leave no dials — it could in principle deliver such an estimate with no fitted input, a genuine advantage over tuned AdS/QCD models.

Why it is not, and cannot be, a precision prediction. The correspondence is controlled only in the double limit $N_c \to \infty$, $\lambda = g^2 N_c \to \infty$. Real QCD has $N_c = 3$ and $\lambda = \mathcal{O}(1)$, where the $1/N_c$ and $1/\sqrt{\lambda}$ corrections are themselves $\mathcal{O}(1)$ and uncontrolled. AdS/QCD therefore lands hadronic quantities to $\sim$ 10–30%, while a_μ demands $\sim 0.3\%$ (HVP) and $\sim 10\%$ (HLbL). The precision content lives in the *spectral details* — resonance masses, widths, form factors — which are dynamical bound-state data, not scale geometry; the radial coordinate fixes the *flow* of the coupling, not the *location* of the ρ. That the data-driven and lattice determinations of a_μ^{HVP} were for years in open *disagreement* (Aoyama, 2020, Borsanyi, 2021) — and that the Theory Initiative's 2025 update resolved the matter only by adopting the lattice determination for the leading hadronic term, publishing no data-driven number in its place (Aliberti, 2025) — is itself evidence that this is a genuinely hard dynamical problem with no geometric shortcut. The bulk fixes the stage and the scale; it does not, at finite N_c, play the strongly-coupled drama out to twelve significant figures.

The muon anomaly as the complete worked example. Gathered across its tiers, a_μ is the cleanest single illustration of what the framework can and cannot do (Table 12): the inputs are *derived*, the scale flow is *geometrized*, the perturbative periods are *untouchable*, and the hadronic correlator is given *structure and a parameter-free estimate but not precision.*

Table 12: The four tiers of a precision amplitude, with the muon anomaly a_μ as the witness, and the reach of the GG-6 radial geometry at each. The radial dimension organizes the scale direction (Tiers 1–2); the transverse content (Tiers 3–4) is irreducible 4D data — arithmetic for the perturbative periods, dynamics for the hadronic correlators.

Tier	Content (example in a_μ)	GG-6 reach
1. Inputs	$\alpha^{-1} = 137$, lepton mass ratios, SU(3), $\alpha_s = 1/\pi e$, $N = 3$	**Derived**
2. Scale flow	running couplings, thresholds, leading-log towers	**Geometrized** (radial transport)
3. Perturbative periods	Schwinger $\alpha/2\pi$; $\zeta(3)$, $\pi^2 \ln 2$ in $C_2, \ldots$	**Untouchable** (irreducible arithmetic)
4. Non-perturbative correlators	hadronic HVP / HLbL; the spectral function $R(s)$	**Structure + parameter-free estimate**, not precision (irreducible dynamics)

The lesson is liberating rather than deflationary. At no tier does the geometry replace the loop calculation: the Feynman-diagram expansion that turns a constant into a part-per-billion prediction cannot be traded for a simpler geometric computation. The decades of labor behind a high-order coefficient are not wasted effort to be geometrized away — they are the irreducible *second half* of

physics, the half that computes the consequences of the constants, and GG-Theory's first half — the derivation of the constants — exists precisely to *complete* that labor, not to abolish it.

52.5 What GG-Theory contributes instead — the true gain for the Standard-Model world

If the bulk does not shorten the loop expansion, what is its contribution to a physics already in possession of the most accurate predictions ever made? It is real, and it is of a different and arguably deeper kind.

First — it derives the inputs, not the corrections. QED takes α, m_e, m_μ, m_τ as measured inputs and computes their radiative corrections. GG-Theory addresses the question QED cannot pose: *why* $\alpha^{-1} = 137$, *why* the lepton mass ratios are what they are (cascade depths on the family stratum), *why* there are three generations (Arisaka, 2026, A9-SM). The Feynman machinery and the geometric derivation answer *complementary* questions — one computes corrections to given constants, the other derives the constants — and neither is a substitute for the other.

Second — it gives the logarithmic skeleton a reason. Equation (125) is not merely satisfied in GG-6; it is *geometric*. The running, the thresholds, and the infrared values to which couplings flow are radial transport toward a Geometric Attractor (the GRIP picture of Arisaka, 2026, A6-GRIP). One thereby understands *why* the leading logs exponentiate — they are the shadow of bulk radial flow — even where the finite numbers are left to the diagrams. The logarithmic structure of perturbation theory acquires an explanation it never had.

Third — it reduces the input uncertainty, not the labor. Because the inputs are derived, a Standard-Model prediction may in principle be fed *derived* values of α and the mass ratios rather than measured ones, shrinking the parametric — not the computational — uncertainty. This is a parameter-reduction gain, orthogonal to the diagram count.

Fourth — it earns genuine computational keep exactly where perturbation theory fails. A bulk geometrization simplifies *strong-coupling* problems — the holographic regime in which a hard boundary theory becomes a classical bulk one. Confinement, the mass–energy dictionary, and the hadronic contributions that dominate the $g-2$ error budget are that regime. If GG-6 ever shortens a $g-2$-relevant computation it will be the hadronic part, never the perturbative QED part — the precise inverse of where the loop labor is hardest to avoid.

Fifth — it changes the status of the theory. The Standard Model becomes the boundary reading of a bulk geometry; the constants become integers on a ledger; the open-system measurement structure is built in rather than appended. This is an explanatory and foundational advance, and it is the advance the cohort actually claims.

Remark 11 (The true division of labor)**.**

Perturbation theory computes the corrections to nature's constants; GG-Theory computes the constants.	(130)

They are two different sciences, and the radial geometry clarifies exactly which is which. The loop expansion will remain the instrument that turns a constant into a part-per-billion prediction;

GG-Theory is the instrument that asks why the constant has the value it does. A mature physics needs both, and confusing the two — expecting geometry to do arithmetic, or arithmetic to do ontology — is the category error this theorem exists to prevent.

A discipline note. Consistent with the honesty triage of Arisaka (2026, A9-SM), GG-Theory makes *no* claim to predict or sharpen a_μ; its only relationship to the muon anomaly is the provision of the input α and the structural reason the lepton sector exists. Keeping that line bright is part of what T9.9 asserts.

52.6 The foundation under the no-go: why the anomaly carries no scale at all

The decomposition (126) was introduced as a canonical split available to any renormalized amplitude. For the magnetic moment the scale-invariance of the remainder is not merely available: it can be proved outright, and the proof explains why the no-go bites so hard here.

Lorentz covariance, current conservation, hermiticity, and P and T invariance allow exactly two form factors in the electromagnetic vertex of an on-shell spin-$\frac{1}{2}$ particle,

$$\langle p'|J^\mu_{\rm em}|p\rangle = \bar u(p')\Big[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\,F_2(q^2)\Big]u(p), \qquad q = p' - p. \tag{131}$$

The non-relativistic limit against a static field gives $g/2 = F_1(0) + F_2(0)$, and the Ward–Takahashi identity forces $Z_1 = Z_2$, hence $F_1(0) = 1$ *exactly*, to all orders — charge is not renormalized. The anomaly is therefore the second form factor and nothing else,

$$a \;\equiv\; \frac{g-2}{2} \;=\; F_2(0). \tag{132}$$

Now read the operator F_2 is the coefficient of. It is the Pauli term $\frac{e}{4m}\,\bar\psi\sigma^{\mu\nu}\psi F_{\mu\nu}$, of mass dimension *five*. A renormalizable Lagrangian contains no such term, so no counterterm exists that could absorb a divergence in F_2 — and therefore F_2 must come out finite. It does: Schwinger's $\alpha/2\pi$ is finite with no subtraction (Schwinger, 1948). An ultraviolet-finite, dimensionless, on-shell quantity is an *RG invariant*: the Callan–Symanzik equation (125) applied to a says it cannot depend on the sliding scale at all.

Remark 12 (Why "$g-2$ does not run" is a theorem, not a habit)**.** The familiar working rule that the anomalous moment is a pure number rather than a running quantity is usually taught as an observation about the one-loop result. It is a consequence of an operator dimension. The same vertex (131) carries one form factor whose divergence the renormalization group exists to absorb (F_1) and one that the group cannot touch because nothing was ever available to subtract from it (F_2). $a = F_2(0)$ *therefore lies wholly in* $\mathcal{F}$ *— not by an accident of the calculation, but by the dimension of the operator that carries it.* Equation (127) is nowhere sharper than here.

52.7 Loop order is not radial position

The opening of this theorem anticipated the hope that the bulk might shortcut the loop expansion. There is a sharper form of the same question, and it is the one a reader who has understood the fifth

dimension actually asks: *if renormalization is motion along r, must a particle climb r to generate its higher-order corrections?*

It must not, and the reason is that loop order and radial position are independent axes.

- *r is an address.* It fixes the resolution at which a coupling is read. Moving along r changes the *value of α that enters*; it does not change what is done with it.
- *Loop order is a series at a fixed address.* Every mass-independent coefficient $A_1^{(2n)}$ of (128) is a pure number with no μ in it. Adding a loop dresses the vertex where it already stands.

The two axes meet in exactly one place, and §52.9 draws it: a diagram containing a large logarithm of a *mass ratio*. The two-loop electron vacuum-polarization insertion in the muon's vertex contributes

$$a_\mu^{(4)}\big|_{\mathrm{VP,\,e}} = \left(\frac{\alpha}{\pi}\right)^2 \left[\tfrac{1}{3}\ln\frac{m_\mu}{m_e} - \tfrac{25}{36} + \mathcal{O}(m_e/m_\mu)\right] = 1.0828\left(\frac{\alpha}{\pi}\right)^2, \tag{133}$$

with $\frac{1}{3}\ln(m_\mu/m_e) = 1.7772$. The coefficient $\frac{1}{3}$ is the one-lepton QED beta function, $d\alpha^{-1}/d\ln\mu = -2/3\pi$: the logarithm *is* the radial distance between the electron threshold and the muon scale, and resumming its tower is solving the transport equation (125). That is the whole of the radial content of $g-2$, and it is small — Schwinger's term alone carries 99.61% of the measured a_μ.

Two further facts close the picture, and both are results this cohort already owns.

The radial transport is a semigroup. $T(r_2)T(r_1) = T(r_1+r_2)$ with no inverse (Arisaka, 2026, W2-Holonomy): coarse-graining destroys what would be needed to undo it. An interval on which nothing composes backwards is not a ladder that anything climbs; it is a one-way map between readings.

For the electron, the descent is already over. The electron is the GGA — the unique deepest basin, the one critical point with no exit gate, which is why $\Gamma_e = 0$ and $\tau_e = \infty$ (Arisaka, 2026, P3-Electron). GRIP-In has halted. Its loop expansion therefore happens *at* the terminus of the cascade, not on the way to it: the dressing of a state that has already arrived.

52.8 The other fiber: why the circle cannot produce F_2 either

Remark 10 rules out the scale interval. It says nothing about the phase circle — and since $g-2$ is a property of the gauge vertex, and the gauge vertex is the circle's business, the argument is not finished until the second fiber has been asked.

It closes by the compact fiber's own reasoning. Flat connections on S^1_θ modulo gauge transformations are classified by $\mathrm{Hom}(\pi_1(S^1), G)/G$, and $\pi_1(S^1) = \mathbb{Z}$ is discrete: nothing continuous can move between the points of a discrete set (Arisaka, 2026, W2-Holonomy). That is precisely why the circle delivers what it does deliver — charge quantized in whole windings, anomaly coefficients that cannot run, class data identical in every region of the sky. All of it is *integers*.

Remark 13 (The no-go is two-sided)**.** $F_2(0) = \alpha/2\pi + \mathcal{O}(\alpha^2)$ is not an integer, not a winding number, and not a characteristic class. The compact fiber has no purchase on it, exactly as the

open fiber has none. Therefore

$$\boxed{a = F_2(0) \text{ is invisible to } \textit{both} \text{ fibers of the bulk: too continuous for } S^1_\theta, \text{ too scale-free for } I_r.} \tag{134}$$

On the two-fiber anatomy (Arisaka, 2026, W2-Holonomy) the anomalous moment is neither protected content nor an address. It is the boundary's own arithmetic, and the twelve thousand diagrams are what that arithmetic costs.

52.9 The two axes, drawn

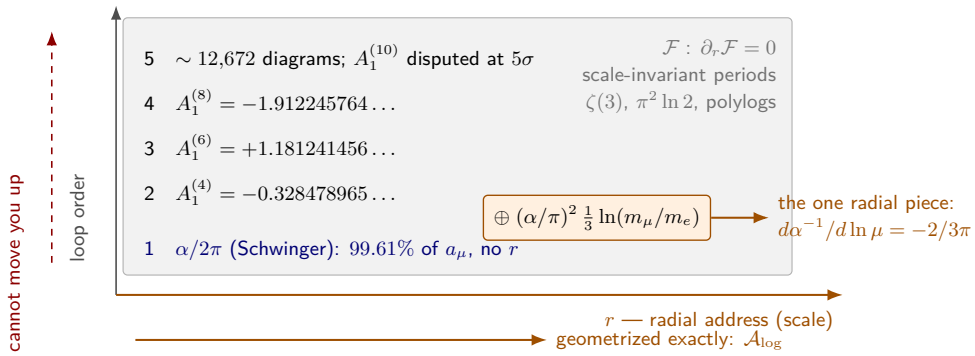


Figure 8: Loop order is not radial position. The horizontal axis is the fifth dimension — the address $r = \ln(E_{\rm GUT}/\mu)$ at which a coupling is read, along which the bulk geometrizes the logarithmic tower $\mathcal{A}_{\log}$ exactly. The vertical axis is the loop expansion at a *fixed* address, every coefficient of which is a scale-invariant period: $A_1^{(4)}$ already carries $\zeta(3)$ and $\pi^2 \ln 2$, and the tenth-order coefficient, assembled from some 12,672 diagrams (Aoyama, 2012), is presently disputed at 5σ between two independent evaluations (Volkov, 2024). Equation (127) says the geometry moves the reader along the horizontal axis and cannot move the reader up the vertical one. The single arrow crossing between them is the mass-ratio logarithm of (133), in which an electron loop inside a muon's vertex reads the gearing at a different floor from the muon's own — and Schwinger's one-loop term, which carries 99.61% of the whole, sits at the bottom of the vertical axis with no radial dependence whatever.

Problems and worked solutions. V.9.1. Read the radial–RG dictionary quantitatively: compute the radial coordinates of the principal scales, then show that one-loop running is literally linear motion in r. *Solution.* Step 1: the dictionary. With $r = \ln(E_{\rm GUT}/\mu)$: $r(E_{\rm GUT}) = 0$; $r(M_{\rm lock}) = \ln(2.28 \times 10^{16}/3849) = 29.4$; $r(M_Z) = 33.2$ (verified) — the entire desert is the radial interval $0 < r < 29$, and the measured Standard Model lives in the thin shell $29 \lesssim r \lesssim 33$ near the IR end. Step 2: running as motion. At one loop, $\alpha_s^{-1}(\mu) = \alpha_s^{-1}(M_Z) + \frac{b_0}{2\pi}\ln(M_Z/\mu)$ with $b_0 = 7$: inverse couplings are *linear* functions of r, so RG trajectories are straight lines in the bulk's radial coordinate. Step 3: walk the line. From $\alpha_s(M_Z) = 0.118$ to the UV boundary: $\alpha_s^{-1}(E_{\rm GUT}) = 8.48 + \frac{7}{2\pi} \times 33.2 = 45.4$, i.e. $\alpha_s(E_{\rm GUT}) = 0.022$ (verified) — the strong coupling walks 37 units of inverse coupling in 33 units of radial distance, at constant slope $b_0/2\pi = 1.11$. Physics intuition: "running coupling" is a metaphor in the textbook and a trajectory in the bulk:

the beta function is a velocity, matching thresholds are floors the elevator passes, and the mysterious logarithm of renormalization theory is simply *distance*.

V.9.2. The division of labor, with the muon's magnetic moment: compute the fraction of a_μ carried by the Schwinger term, and use it to state precisely what the geometrization of T9.9 does and does not claim. *Solution.* Step 1: the first term. Schwinger's one-loop result (Schwinger, 1948) gives $a_\mu^{(1)} = \alpha/2\pi = 1.16141 \times 10^{-3}$; the measured value is $a_\mu = 1.16592 \times 10^{-3}$, so the first diagram carries 99.61% of the total (verified). Step 2: the labor distribution. The remaining 0.4% costs five loop orders, 12,672 tenth-order diagrams (Aoyama, 2012), and the hardest piece — hadronic vacuum polarization — is a strongly-coupled boundary correlator on which even first-principles methods currently disagree at the relevant precision. Step 3: the theorem's line. T9.9 geometrizes the *frame*: it supplies α's constitutional origin, the existence and couplings of the muon, and the meaning of the logarithms (radial distance, V.9.1) — and it claims no shortcut through the series itself; the discipline note in the section draws exactly this boundary. Physics intuition: geometry answers "why is there a series and what are its inputs"; arithmetic answers "what is its sum" — confusing the two produces either mystical numerology or pointless re-derivation, and the muon's ninth decimal place is where the distinction earns its keep.

V.9.3. Prove that the anomalous moment is an RG invariant, and identify the one property of the Pauli operator that makes it so.

Solution. Step 1: the decomposition. Lorentz covariance, current conservation, hermiticity and P, T invariance permit exactly the two form factors of (131); the Ward–Takahashi identity gives $Z_1 = Z_2$ and hence $F_1(0) = 1$ to all orders, so $a = F_2(0)$ by (132). Step 2: the dimension. F_2 is the coefficient of $\frac{e}{4m}\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}$, which has mass dimension five; a renormalizable Lagrangian contains no operator of dimension greater than four, so there is no counterterm from which a divergence in F_2 could be subtracted, and F_2 is therefore finite. Step 3: the invariance. A finite, dimensionless, on-shell quantity satisfies (125) with no residual μ-dependence, so $\partial_r a = 0$ identically and a lies entirely in $\mathcal{F}$ (verified against Schwinger's subtraction-free $\alpha/2\pi$). *Physics intuition:* the reason $g-2$ "does not run" is not that the loop happened to come out finite — it is that the theory has no place to put an infinity if one had appeared, and a quantity with nowhere to hide a divergence has nothing to flow.

V.9.4. Show that the one radial piece of a_μ is literally a distance on the fifth dimension, and evaluate it.

Solution. Step 1: the QED slope. One lepton in the vacuum-polarization loop gives $d\alpha^{-1}/d\ln\mu = -2/3\pi = -0.21221$ (verified), i.e. a radial velocity of the inverse coupling. Step 2: the separation. The electron threshold and the muon scale are $\ln(m_\mu/m_e) = 5.3316$ e-folds apart, which is their separation in r (verified). Step 3: the coefficient. The two-loop electron vacuum-polarization insertion (133) carries $\frac{1}{3}\ln(m_\mu/m_e) = 1.7772$, and with the constant $-25/36$ the bracket is 1.08277 (verified) — the logarithm being $\mathcal{A}_{\log}$ and the constant being $\mathcal{F}$, inside a single coefficient. *Physics intuition:* this is the one line of the $g-2$ calculation a bulk geometer can read off a ruler; every other line of it is arithmetic, and the two live side by side inside the same two-loop number so that no reader can mistake one for the other.

V.9.5. The tenth-order dispute: estimate what is at stake, and say why no change of coordinates can settle it.

Solution. Step 1: the disagreement. Two independent evaluations of the mass-independent tenth-order coefficient differ by $\Delta A_1^{(10)} = 6.737 - 5.891 = 0.846$, a 5σ separation (Volkov, 2024). Step 2: the size. $\Delta a_e = \Delta A_1^{(10)}(\alpha/\pi)^5 = 5.7 \times 10^{-14}$ (verified), which is about 0.44 of the present experimental uncertainty on a_e at 0.13 parts per trillion — below the measurement today, and comparable to where the next generation of Penning traps will sit. Step 3: the reach of the geometry. $A_1^{(10)}$ is a mass-independent, scale-invariant period: $\partial_r A_1^{(10)} = 0$, so by (127) the radial coordinate is blind to it, and by (134) so is the phase circle. *Physics intuition:* the disagreement will be settled by two groups integrating more carefully, and by nothing else — which is the most concrete available demonstration that $\mathcal{F}$ is real content and not a defect of the boundary description.

Take-away. *Standard account:* the renormalization group is mastered as resummation labor — the tenth-order QED contribution to a_μ alone required 12,672 diagrams. *What it left open:* what the RG *is*. *The constitutional derivation:* RG flow is geometrized as motion along the radial coordinate of the bulk (T9.9) — the labor keeps its arithmetic and gains its meaning, and the theorem's own discipline note draws the bright line: GG-Theory supplies the input α and the structural reason the lepton sector exists, and makes no claim to sharpen a_μ.

53 Bridge to Pillars CCI and GFF of GG-A6-GRIP

The nine sub-theorems T9.1–T9.9 collectively constitute the axiomatic content of 4D quantum field theory in the GG-Theory framework. The companion paper GG-A6-GRIP (Gi-4 Pillars) refines this content into the four *Geometric Invariance* pillars (TGG, CCI, GFF, GRIP). T9.6 (anomaly inflow) is the constitutional source of CCI; T9.2 (effective 4D action) is the constitutional source of GFF's energetic part GEP. Theorem T10 of Part VII of the present paper makes the bridge to Gi-4 rigorous.

54 The Canonical GG-6 Lagrangian Density, Its Euler–Lagrange Equations, and the Standard Model as a Boundary Effective Theorem

54.1 From 6D bulk Lagrangian to 4D effective Lagrangian: delegation to GG-A2-GG6 and GG-A3-BI6

Every standard QFT textbook (Peskin, 1995, Srednicki, 2007, Weinberg, 1995) starts from a Lagrangian density on 4D Minkowski spacetime, asserts the gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ and the matter content as input, and fits 28 numerical parameters to experiment. The present section closes that gap by recording how the 4D Standard Model Lagrangian arises as a boundary effective theorem from a canonical 6D bulk Lagrangian. The full construction is given in the companion papers and is summarized here. The division of labor across the cohort is precise, and worth stating once: GG-A2-GG6 owns the bulk stage, metric, gauge group, and the recorded

canonical action; GG-A3-BI6 owns the anomaly closure that completes it; the present paper owns the descent machinery (T9.1–T9.9) that carries the action to the boundary; and GG-A9-SM (Arisaka, 2026, A9-SM) owns the parameter-level completion — the term-by-term assembly of the Lagrangian, the station of its assembly interlude (S-1a) where the action's uniqueness is earned brace by brace, and the 28-parameter codebook at stated grades.

6D bulk Lagrangian and BI-6 inflow. The canonical 6D bulk Lagrangian density of GG-Theory — Einstein–Hilbert, SU(5), $\mathrm{U}(1)_B$, $\mathrm{U}(1)_{Qi}$ Yang–Mills, chiral 2-form B_2 with Chern–Simons-improved field strength, and Green–Schwarz coupling $-B_2 \wedge X_4$ — is stated as the *Canonical Bulk Action* (a numbered Proposition) of Arisaka (2026, A2-GG6), ahead of the BRST-consistency theorem (T-GG6-7) that quantizes it there; its field content is forced by the stage-and-metric theorems T-GG6-0/1/2 and the gauge-group theorem T-GG6-3 of the same paper, and the uniqueness of its dynamics — why these terms and no others, at lowest derivative order — is earned brace by brace in the assembly interlude of GG-A9-SM (Arisaka, 2026, A9-SM), station S-1a, with the register of every claim stated. The four-component anomaly polynomial

$$X_4 = a\ \mathrm{tr}(R\wedge R) + b\ \mathrm{tr}(F_{\mathrm{SU}(5)}\wedge F_{\mathrm{SU}(5)}) + c\,F_B\wedge F_B + d\,F_{Qi}\wedge F_{Qi}, \qquad (a,b,c,d) = (1,-1,-\tfrac{2}{5},-\tfrac{2}{5}),$$

is the BI-6 four-component inflow vector of Arisaka (2026, A3-BI6). The reciprocal trace ratio $|a/c| = |a/d| = 5/2$ is the algebraic signature of the geometry–gauge locking; it replaces the older Dual Gauge Locking graviphoton picture without invoking a metric KK fiber.

The variational principle, at its honest register. Under LC (locality and causality), GGEP (geometry–gauge equivalence), MSMF (maximum symmetry, minimum freedom), and POS-3 (anomaly freedom by inflow), the bulk dynamics admits a unique two-derivative-order local functional whose Euler–Lagrange equations are the equations of motion. The variational principle itself is supplied by POS-2 / UEP — a constitutional postulate of GG-A1-Constitution (Arisaka, 2026, A1-Constitution), not a theorem; what *is* a theorem is the uniqueness of the lowest-order functional once the field content is forced, and that uniqueness scan — including the six-dimensional Lovelock caveat and the scalar-free selection — is worked brace by brace in GG-A9-SM, station S-1a.

Five Euler–Lagrange variations. Variation of the canonical 6D action with respect to the five primitive fields $\{B_2,\ A_{\mathrm{SU}(5)},\ A_B,\ A_{Qi},\ g_{MN}\}$ produces five equations of motion as theorems by standard variational calculus. The B_2 variation gives the modified Bianchi equation $d(\star H_3) = X_4$; the three gauge variations give Yang–Mills equations with GS-inflow currents; the metric variation gives the 6D Einstein equation $M_6^4\,G_{MN} + \Lambda_6\,g_{MN} = T_{MN}^{\mathrm{total}}$. The variations are standard once the action is fixed; the action itself is the recorded canonical form of Arisaka (2026, A2-GG6), and we use these equations of motion as inputs in this paper.

Bulk-to-boundary KK reduction. Dimensional reduction governed by OGGEP integrates the 6D action over the internal coordinates $y = (r,\theta)$ on $\Sigma_2 = I_r \times S^1_\theta$. Each 4D gauge coupling is then

an *overlap integral*

$$\frac{1}{g_{4,a}^2} = \int_{\Sigma_2} d^2y\, \sqrt{g_2}\, W(y)\, \chi_a(y), \qquad W(y) = e^{-2r}/g_5^2,$$

not a free parameter. The $\mathrm{SU}(5) \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ breaking is the Wilson-line holonomy $U = \mathcal{P}\exp(i \oint A_\theta\, d\theta)$ along the compact direction; the Higgs is the holonomy order parameter. The boundary axion $a(x)$ is the integral of B_2 over the internal 2-cycle and acquires the GS coupling $-a(x)\, X_4^{(4D)}$ by reduction; the Kaxion of GG-C1-Kaxion is the cosmological realization. The full KK accounting — every boundary coupling an overlap integral, every parameter placed on the ledger at its stated grade — is the export chain of GG-A9-SM (Arisaka, 2026, A9-SM): Part IX of its main text at results register, and the course-pace walk of its assembly interlude, both consuming the machinery of T9.1–T9.2 below.

4D effective Lagrangian. The result of the KK reduction is the 4D boundary effective Lagrangian:

54.2 The 4D effective Lagrangian: the Standard Model as theorem

Putting the mode expansion, the overlap integrals, the Wilson-line holonomy, and the boundary axion together, the 4D effective Lagrangian on the boundary takes the form

4D Boundary Effective Lagrangian (Standard Model + $\mathrm{U}(1)_B$ + axion).

$$\begin{aligned}\mathcal{L}_{\mathrm{eff,\,4D}} = \sqrt{-g_4}\,\Big[& \tfrac{M_{\mathrm{Pl}}^2}{2} R_4 - \Lambda_4 - \tfrac{1}{4g_3^2} G^a_{\mu\nu} G^{a\mu\nu} - \tfrac{1}{4g_2^2} W^i_{\mu\nu} W^{i\mu\nu} - \tfrac{1}{4g_1^2} B^Y_{\mu\nu} B^{Y\mu\nu} \\ & - \tfrac{1}{4g_B^2} F_{B\,\mu\nu} F_B^{\mu\nu} + |D_\mu H|^2 - V_{\mathrm{eff}}(H) \\ & + \bar\psi\, i\gamma^\mu D_\mu\, \psi - (\bar\psi_L Y \psi_R H + \mathrm{h.c.})\Big] \\ & - a(x)\, X_4^{(4D)} + \cdots \end{aligned} \tag{135}$$

This is precisely the textbook Standard-Model Lagrangian, augmented by (i) the additional $\mathrm{U}(1)_B$ baryon sector that GG-Theory inherits from the bulk gauge group, (ii) the boundary axion coupling forced by the GS mechanism, and (iii) the gravitational sector with the geometric Planck mass and the lock-scale seesaw cosmological constant.

Key conceptual point: every coupling on the right-hand side of (135) is a calculable overlap integral over the internal geometry, not a free parameter. The Higgs vacuum expectation value $v = 246\,\mathrm{GeV}$ is derived in GG-A9-SM as $v = M_{\mathrm{lock}}\,(2/5)^3$ where the cube is the OGN-5 triplicity exponent $N = 3$. The electroweak hierarchy $v/E_{\mathrm{Pl}} \approx 2.0 \times 10^{-17}$ is the geometric consequence of $M_{\mathrm{lock}} = (1/2) E_{\mathrm{Pl}}\, e^{-35}$ together with the discrete doublet projection $(2/L)^N$. The cosmological constant Λ_4 is the lock-scale seesaw $m_\Lambda^{\mathrm{geom}} = M_{\mathrm{lock}}^2/M_{\mathrm{Pl}} \approx 6\,\mathrm{meV}$. The 28 fitted parameters of the textbook SM Lagrangian reduce, in GG-Theory, to a small number of geometric inputs (the scale-bulk geometry, the OGN-5 ledger, the BI-6 coefficients). The detailed parameter-counting accounting is the subject of GG-A9-SM (Arisaka, 2026, A9-SM), whose Part IX assembles the boundary Lagrangian term by term with the provenance of every coefficient printed beside it.

Position relative to standard QFT. The standard 4D QFT Lagrangian is the closed-system limit of the 6D bulk theory: it is the boundary projection of the canonical 6D bulk Lagrangian under OGGEP, which by Theorem T6 is a CPTP channel, hence intrinsically open. Every Feynman (Feynman, 1948) rule, every renormalization calculation of the textbook formalism remains valid; what changes is the constitutional source. The gauge group, the matter representations, the coupling values, the Higgs, and the axion are forced by geometric inputs rather than fitted; the term-by-term assembly, with the register of every coefficient stated beside the assembled equation, is GG-A9-SM (Arisaka, 2026, A9-SM).

Summary of the axiomatic ladder. Table 13 sets the ladder out in full.

Table 13: The axiomatic ladder of the Standard-Model derivation, layer by layer. The left column names each structure in the chain that runs from the constitutional charter to the measured predictions; the right column records its logical status — primitive input, constitutional postulate, or theorem, and if a theorem, of what. Only the top row is input. Reading downward is reading the direction of derivation: the variational principle is supplied by a postulate rather than assumed, the bulk Lagrangian it governs is forced rather than chosen, and by the foot of the ladder the twenty-eight parameters that the Standard Model fits have become overlap integrals. The point of the table is the shape of the right-hand column, not any single row.

Layer	**Status in GG-Theory**
GG-Constitution (AX 1–4 + POS 1–7)	Primitive axioms (input)
Locked composite connection $\mathcal{A}_{\text{lock}}$	Forced by GGEP (theorem)
Existence of variational principle (PLA)	Supplied by POS-2 (constitutional postulate)
Canonical 6D bulk Lagrangian	GG-A2-GG6's Canonical Bulk Action (numbered Proposition); uniqueness at lowest order from MSMF + AF, earned in GG-A9-SM (station S-1a)
Four-component BI-6 vector $(1, -1, -2/5, -2/5)$	Theorem from OGN-5 cascade
Euler–Lagrange EOMs	Theorems by variational calculus from the recorded action
Trace-ratio locking $\lvert a/c\rvert = \lvert a/d\rvert = 5/2$	Theorem from GGEP + OGN-5 (replaces older DGL graviphoton picture)
4D effective Lagrangian (135)	Theorem from OGGEP + KK reduction
Standard Model gauge group + matter	Theorem from SU(5) holonomy + AF
28 SM parameters	Theorems from overlap integrals
Empirical predictions (KK lock, Kaxion, $\Omega_m = 1/\pi$)	Theorems with measured falsifiers

This ladder is the structural payoff of the constitutional ordering: the variational principle is supplied by POS-2, and the Lagrangian it governs is forced, not chosen. In standard QFT, the Lagrangian is the starting point and everything else is built on top. In GG-Theory, the Lagrangian is a derived consequence of the constitutional axioms, and the QFT formalism is recovered as the boundary projection of a constitutionally open 6D bulk theory.

55 Summary of Part V: Closing the Gauge Half of the Unification Problem

The nine sub-theorems T9.1–T9.9 collectively derive the structural skeleton of 4D quantum field theory — KK expansion, effective 4D Lagrangian, generating functional, BRST symmetry, Slavnov–Taylor identity, anomaly cancellation, four conserved currents, and exact proton stability — from GGEP applied to the 6D bulk:

$$\boxed{\text{GG-6 bulk} \overset{\text{GGEP}}{\Longrightarrow} \text{4D QFT skeleton (T9.1–T9.9)} \Longrightarrow \text{SM (GG-A9-SM).}} \tag{136}$$

The detailed 28 SM parameters as boundary attractor depths in the GFF Morse landscape are recorded in Arisaka (2026, A9-SM). The position of this resolution relative to GUTs, SUSY, KK/RS, string compactifications, and non-commutative geometry is recorded as Innovation 6 of Part IX (see Table 25); the five falsifiers (KK lock cluster, proton stability, 28 SM parameters, BSM gauge extensions, neutrino sector) are listed as falsifiers P1–P5 of Part IX §93.

Bridge to Part VI. Parts IV and V together address the half-century-old unification problem. What Part VI adds is the cleanest possible test case: the parallel boundary derivation of Newton's gravitational law and Coulomb's electromagnetic law from a single 6D geometric stage. The big-picture statement that "5D KK + 5D RS together yield 6D GG" is recorded as the constitutional inversion of Part IX §108 and as Innovation 1 of Part IX.

55.1 Part V in plain English

The Standard Model's Lagrangian — the page of terms every graduate student memorizes as given — is derived bookkeeping: integrate the bulk's heavy modes and the page writes itself, coefficient by coefficient, from radial overlaps (T9.1–T9.3). The formal scaffolding that textbooks introduce as devices — BRST, the Slavnov–Taylor identity — is inherited from the bulk, not re-erected on the boundary (T9.4–T9.5). The anomaly cancellations that look like miracles of charge assignment are one inflow theorem (T9.6); the currents are four, not two (T9.7); the proton is exactly stable, so a single observed decay kills the framework (T9.8); and the renormalization group the student labors through is the geometry of the radial direction — the labor keeps its arithmetic, and gains its meaning (T9.9).

Part VI
Unification of General Relativity and QED

The Realization of Einstein's Dream — How GG-Theory Completes Kaluza–Klein One Century Later

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Einstein spent the last thirty years of his life trying to write gravity and electromagnetism as two aspects of one geometric object, and died in 1955 without it. Kaluza and Klein had shown, thirty years earlier, that adding one dimension to spacetime produces something that looks remarkably like electromagnetism alongside gravity — but the extra dimension had to be hidden by hand, the resulting spectrum was wrong, and the idea was set aside for half a century.

This Part revisits that program with the geometry the earlier Parts have established, and finds the parallel Einstein was looking for. The two force laws that begin every physics education — Newton's, in which the pull between two masses falls off as the square of the distance, and Coulomb's, in which the push between two charges does the same — are derived here side by side, from one connection on one bulk, along chains that share every step but one. Radiation follows the same way: the light emitted by an accelerating charge and the gravitational waves emitted by two inspiraling black holes turn out to be a single descent with a shared decomposition, distinguished by which part of the geometry is doing the radiating.

The inversion relative to Kaluza and Klein is worth stating plainly. They sought to unify by adding a dimension *to gravity* and letting electromagnetism fall out. Here the unification runs the other way: the gauge forces are recognized as already geometric, and gravity is left alone. Nothing is added to Einstein's theory in this Part. What is added is a reading of the other force that puts it on the same footing.

The Part is short and can be read straight through. Its argument is carried by the two parallel chains, which are laid out so that a reader can follow one column down the page and then the other, and see where they diverge.

56 Einstein's Dream and the Parallel Derivation of Gravity and Electromagnetism

Einstein spent the last thirty years of his life searching for a geometric unification of gravity and electromagnetism, dying in 1955 without finding it. Theodor Kaluza and Oskar Klein came tantalisingly close in the 1920s: gravity in five dimensions, on compactification of the fifth dimension on a circle, reduces to four-dimensional Einstein gravity plus four-dimensional Maxwell electromagnetism. The original Kaluza–Klein construction nevertheless failed on three structural grounds (anomaly freedom, renormalization, and the absence of a constitutional treatment of quantum mechanics); GG-Theory resolves these as Innovation 4 (BI-6 anomaly cancellation), Innovation 3 (radial direction

as RG-scale), and Innovation 6 (OGGEP + GDOS boundary law) of Part IX §92, and the central content of this Part is the resulting explicit parallel derivation of Newton's gravitational law and Coulomb's electromagnetic law as boundary projections of the same 6D GG-6 bulk.

57 The Two Parallel Derivation Chains

We now execute the central technical content of Part VI: the parallel derivation of Newton's gravitational law and Coulomb's electromagnetic law as boundary projections of the same 6D GG-6 bulk. The two chains run side by side from the same starting point (the 6D bulk with the composite gauge–gravity group) to the same kind of endpoint (an inverse-square static force law on the 4D boundary). The structural parallelism is exact, and it is the cleanest possible empirical realization of Einstein's geometric unification dream.

57.1 The two iconic equations

Both Newton's and Coulomb's laws are inverse-square force laws between two point sources at distance r:

$$\textbf{Newton:} \qquad F_{\mathrm{N}} = G_N \frac{m_1 m_2}{r^2}, \qquad V_{\mathrm{N}}(r) = -G_N \frac{m_1 m_2}{r}, \tag{137}$$

$$\textbf{Coulomb:} \qquad F_{\mathrm{C}} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}, \qquad V_{\mathrm{C}}(r) = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}. \tag{138}$$

The formal similarity is one of the oldest hints in physics that the two laws share a common geometric origin. Einstein knew it; Maxwell knew it; Kaluza explicitly aimed at it. The GG-Theory framework now makes the common origin precise.

57.2 Chain (A): GG-6 → 5D GR → 4D GR → Newton → $F = ma$

The gravitational chain descends the two non-spatial dimensions of GG-6 one at a time: we integrate out the sixth dimension (the gauge circle S^1_θ) to obtain a five-dimensional general relativity, then integrate out the fifth dimension (the radial RG-scale interval I_r) to obtain four-dimensional general relativity; the weak-field limit gives Newton's potential, and the geodesic equation gives $F = ma$. Every step is a reduction of one geometric object — the bulk metric g_{MN} — and no force is ever postulated: gravitation is, throughout, the statement that matter follows the geodesics of that metric. As in Chain (B), the derivation is written out in full so that a reader with only minimal exposure to general relativity can follow every line.

(A1.1) What "deriving gravity" means: the Einstein–Hilbert action. In general relativity the dynamical variable is the metric $g_{\mu\nu}(x)$, which fixes the distance between neighboring points through the line element $ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$. Its dynamics follow from the *Einstein–Hilbert action*, the gravitational counterpart of the Maxwell action of Step B1,

$$S = \frac{M^2}{2} \int d^D x \, \sqrt{-g}\, R, \tag{139}$$

where R is the Ricci scalar (a specific measure of curvature built from $g_{\mu\nu}$ and its first two derivatives), $\sqrt{-g}\,d^Dx$ is the invariant volume element, and M is a mass scale that sets the strength of gravity. Just as varying the Maxwell action gave Maxwell's equations, varying (139) with respect to $g_{\mu\nu}$ gives Einstein's field equations. "Deriving general relativity from GG-6" therefore means: start from the six-dimensional Einstein–Hilbert action, reduce it to four dimensions, and read off both the four-dimensional Einstein equation and the value of Newton's constant.

(A1.2) The six-dimensional action and the bulk metric. The gravitational sector of GG-6 is (139) in $D = 6$,

$$S_6 = \frac{M_6^4}{2}\int d^6x\,\sqrt{-g_6}\,R_6, \qquad x^M = (x^\mu, r, \theta), \quad M = 0, \ldots, 5, \tag{140}$$

where the prefactor is $M_6^{D-2} = M_6^4$ because the action must be dimensionless. The bulk metric is the canonical block-diagonal product geometry of the four-dimensional spacetime $\mathcal{M}_4$, the radial interval $I_r = [0, K]$, and the gauge circle $S^1_\theta = [0, 2\pi)$,

$$ds_6^2 = e^{-2r}\,g_{\mu\nu}(x)\,dx^\mu dx^\nu + dr^2 + d\theta^2. \tag{141}$$

The factor e^{-2r} is the *scale profile*: it shrinks four-dimensional lengths exponentially as one moves from the UV boundary at $r = 0$ to the IR boundary at $r = K = 35$. The two extra directions are flat and unwarped among themselves, and the metric does not mix them with spacetime — there is no off-diagonal term — so the only non-trivial geometry beyond $\mathcal{M}_4$ is the single scale profile.

(A1.3) Reducing on the sixth dimension: 6D → 5D. Nothing in the metric (141) depends on the gauge phase θ: the vector ∂_θ is a symmetry (a Killing vector), and the circle is flat, so it contributes no curvature, $R_6 = R_5$. The action therefore contains no θ-dependence, and the integral over the circle is just its circumference,

$$\int_0^{2\pi} d\theta = 2\pi. \tag{142}$$

Pulling this constant out of (140) leaves a five-dimensional Einstein–Hilbert action on $\mathcal{M}_4 \times I_r$,

$$S_5 = \frac{M_5^3}{2}\int d^5x\,\sqrt{-g_5}\,R_5, \qquad M_5^3 = 2\pi\,M_6^4, \qquad ds_5^2 = e^{-2r}g_{\mu\nu}\,dx^\mu dx^\nu + dr^2, \tag{143}$$

the warped five-dimensional gravity of Randall–Sundrum type (L. Randall, 1999). *Physics intuition.* The sixth dimension is a gauge phase, not a place: reducing on it costs nothing geometrically (a mere factor 2π), and the five-dimensional gravitational scale $M_5^3 = 2\pi M_6^4$ inherits the bulk scale times the circumference of the circle.

(A2.1) Reducing on the fifth dimension: 5D → 4D, and the scale-profile integral. Now integrate out the radial direction r. Two scalings do all the work. First, the determinant: the four-dimensional block of (143) is $e^{-2r}g_{\mu\nu}$, whose determinant carries four factors of e^{-2r}, so

$$\sqrt{-g_5} = e^{-4r}\,\sqrt{-g_4}. \tag{144}$$

Second, the curvature: the Ricci scalar built from $e^{-2r}g_{\mu\nu}$ equals e^{+2r} times the Ricci scalar built from $g_{\mu\nu}$ alone, because R carries two inverse-metric factors and each inverse metric brings a factor e^{+2r}; for the four-dimensional part,

$$R_5 \supset e^{+2r}\, R_4[g]. \tag{145}$$

(The remaining pieces of R_5, built from r-derivatives of the scale profile factor, supply the boundary terms and the bulk cosmological constant; they fix Λ_4 but not Newton's constant, which comes entirely from the four-dimensional curvature term.) Multiplying (144) and (145), the scale profiles combine to a single power,

$$\sqrt{-g_5}\, R_5 \;\supset\; e^{-4r}\sqrt{-g_4}\cdot e^{+2r} R_4 = e^{-2r}\,\sqrt{-g_4}\, R_4, \tag{146}$$

and the radial integral is elementary,

$$\int_0^K e^{-2r}\, dr = \Big[-\tfrac{1}{2}e^{-2r}\Big]_0^K = \frac{1-e^{-2K}}{2}. \tag{147}$$

The four-dimensional Planck mass is therefore the scale-weighted integral of the five-dimensional scale,

$$M_{\rm Pl}^2 = M_5^3 \int_0^K e^{-2r}\, dr = 2\pi M_6^4 \cdot \frac{1-e^{-2K}}{2} = \pi\, M_6^4\,(1-e^{-2K}), \tag{148}$$

reproducing the overlap-integral Planck mass of Theorem T8.1.

(A2.2) Varying the action: Einstein's equation. The reduction has produced the four-dimensional Einstein–Hilbert action,

$$S^{(4)}_{\rm grav} = \frac{M_{\rm Pl}^2}{2}\int d^4x\,\sqrt{-g_4}\,(R_4 - 2\Lambda_4) + S_{\rm matter}. \tag{149}$$

Varying it with respect to $g^{\mu\nu}$ and using the standard results $\delta(\sqrt{-g}\,R)/\delta g^{\mu\nu} = \sqrt{-g}\,(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R)$ and $\delta S_{\rm matter}/\delta g^{\mu\nu} = -\frac{1}{2}\sqrt{-g}\,T_{\mu\nu}$ gives Einstein's field equation,

$$G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = 8\pi G_N\, T_{\mu\nu}, \qquad G_{\mu\nu} \equiv R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R, \qquad G_N = \frac{1}{8\pi M_{\rm Pl}^2}. \tag{150}$$

The left side, the Einstein tensor $G_{\mu\nu}$, is pure geometry; the right side, the stress–energy tensor $T_{\mu\nu}$, is matter; and Newton's constant $G_N = 1/(8\pi M_{\rm Pl}^2)$ is read off the prefactor of the action. *Physics intuition.* The strength of gravity is set entirely by the fifth dimension: the scale-profile integral $\frac{1}{2}(1-e^{-2K})$, locked by the OGN-5 integer $K = 35$, converts the bulk scale M_6 into the enormous $M_{\rm Pl}$, and the weakness of gravity — the hierarchy e^{-K} — is the geometric shadow of the radial scale profile.

(A3.1) The weak-field, static limit: reading the Newtonian potential off g_{00}. To recover Newton we look at a weak, static gravitational field. Write the metric as a small perturbation of flat space, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $|h_{\mu\nu}| \ll 1$, and time- independent. The Newtonian potential $\Phi_{\rm N}$

sits in the time–time component,

$$g_{00} = -\Big(1 + \frac{2\Phi_{\rm N}}{c^2}\Big), \qquad \text{equivalently} \qquad h_{00} = -\frac{2\Phi_{\rm N}}{c^2}, \tag{151}$$

an identification that Step A4 will confirm by showing it reproduces Newton's second law.

(A3.2) Einstein's equation → Poisson's equation. For a slowly moving, non-relativistic source the dominant component of the stress–energy tensor is the energy density, $T_{00} = \rho c^2$, and its trace is $T = g^{\mu\nu}T_{\mu\nu} \approx -\rho c^2$. It is convenient to use the trace-reversed form of (150) (take the trace of Einstein's equation, $R - 2R = 8\pi G_N T/c^4$, so $R = -8\pi G_N T/c^4$, and substitute back),

$$R_{\mu\nu} = \frac{8\pi G_N}{c^4}\Big(T_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}T\Big). \tag{152}$$

Its time–time component needs $T_{00} - \frac{1}{2}g_{00}T = \rho c^2 - \frac{1}{2}(-1)(-\rho c^2) = \frac{1}{2}\rho c^2$, so the right-hand side is $4\pi G_N \rho/c^2$. For the left-hand side, the time–time Ricci component of a static weak field reduces to the Laplacian of the potential,

$$R_{00} \simeq -\tfrac{1}{2}\nabla^2 h_{00} = \frac{1}{c^2}\nabla^2\Phi_{\rm N}, \tag{153}$$

using $h_{00} = -2\Phi_{\rm N}/c^2$ from (151). Equating the two sides, the factors of c^2 cancel and we obtain the Poisson equation,

$$\boxed{\nabla^2\Phi_{\rm N} = 4\pi G_N \rho.} \tag{154}$$

This is the static limit of Einstein's equation: the same equation that Step B2 reached for the electric potential, with G_N in place of $1/(4\pi\varepsilon_0)$ and mass density in place of charge density.

(A3.3) Green's function → Newton's law. For a point mass M at the origin, $\rho(\vec{r}) = M\,\delta^3(\vec{r})$, and we need the function whose Laplacian is a delta function. That Green's function is $-1/(4\pi r)$, since $\nabla^2\big(-1/(4\pi r)\big) = \delta^3(\vec{r})$. Hence

$$\Phi_{\rm N}(r) = 4\pi G_N M \cdot \Big(-\frac{1}{4\pi r}\Big) = -\frac{G_N M}{r}, \qquad \vec{F}_{\rm N} = -m\,\nabla\Phi_{\rm N} = -\frac{G_N M m}{r^2}\,\hat{r}, \tag{155}$$

which is exactly Newton's universal law of gravitation (137). The $1/r^2$ force is the three-dimensional Green's function at work — identical in form to the Coulomb result of Step B2.

(A4.1) The geodesic equation: how matter knows it must move. The law of motion is not added by hand. A particle acted on by gravity alone follows a *geodesic* — the straightest available worldline — obtained by extremizing the elapsed proper time $\int d\tau$. The result is the geodesic equation,

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\lambda}\,\frac{dx^\nu}{d\tau}\,\frac{dx^\lambda}{d\tau} = 0, \tag{156}$$

where the *Christoffel symbols*

$$\Gamma^\mu_{\nu\lambda} = \tfrac{1}{2}\,g^{\mu\sigma}\big(\partial_\nu g_{\sigma\lambda} + \partial_\lambda g_{\sigma\nu} - \partial_\sigma g_{\nu\lambda}\big) \tag{157}$$

are built from first derivatives of the metric and encode the gravitational field. Read (156) as "acceleration $= -\Gamma \times (\text{velocity})^2$": the Christoffel term is the gravitational force per unit mass.

(A4.2) The Newtonian limit of the geodesic: $F = ma$. For a slowly moving particle the time component of the velocity dominates, $dx^0/d\tau \simeq c \gg dx^i/d\tau$, so the only term that survives in (156) is the one with two factors of $dx^0/d\tau$, governed by Γ^i_{00}. Compute it from (157) for the static metric (151) (time-independent, so ∂_0 terms vanish, and $g^{ij} = \delta^{ij}$ to leading order):

$$\Gamma^i_{00} = \tfrac{1}{2}\, g^{ij}(2\partial_0 g_{j0} - \partial_j g_{00}) = -\tfrac{1}{2}\,\delta^{ij}\,\partial_j g_{00} = -\tfrac{1}{2}\,\partial_i\Big(-1 - \frac{2\Phi_{\rm N}}{c^2}\Big) = +\frac{1}{c^2}\,\partial_i \Phi_{\rm N}. \tag{158}$$

The spatial part of the geodesic equation then becomes, using $dx^0/d\tau \simeq c$ and $\tau \simeq t$ for a slow particle,

$$\boxed{\frac{d^2x^i}{dt^2} = -\,\Gamma^i_{00}\Big(\frac{dx^0}{d\tau}\Big)^2 = -\,\frac{1}{c^2}\,\partial_i\Phi_{\rm N}\cdot c^2 = -\,\partial_i\Phi_{\rm N} \quad\Longrightarrow\quad m\,\ddot{\vec{x}} = -m\,\nabla\Phi_{\rm N} = \vec{F}_{\rm N}.} \tag{159}$$

This is Newton's second law $F = ma$, derived as the weak-field projection of the geodesic equation: the gravitational force is the spatial gradient of the time–time component of the metric, g_{00}. *Physics intuition.* A planet is not pushed; it free-falls along the straightest worldline of the scale-profile metric, and what we call the gravitational force is the curvature term Γ^i_{00} — geometry telling matter how to move. Step B3 showed that the electric force is the same statement applied to the curvature of the gauge connection. The full axiomatic chain A is therefore

$$\boxed{\begin{aligned}&\textsf{GG-6 bulk} \overset{\textsf{OGGEP}}{\to} \text{5D EH} \to \text{4D EH action} \to \text{Einstein eq.}\\ &\quad\to \text{Poisson eq.} \to \text{Newton's law} \to \text{geodesic} \to F = ma.\end{aligned}} \tag{160}$$

57.3 Chain (B): GG-6 → 4D QED → Maxwell + Lorentz → Coulomb

The electromagnetic chain runs exactly parallel, but the geometric object it follows is not the metric — it is the *bulk gauge connection* carried on the sixth dimension. Where gravity is the curvature of the metric, electromagnetism is the curvature of the $\mathrm{U}(1)_{\rm em}$ connection whose holonomy lives on the gauge circle S^1_θ, and the electric charge is the integer-quantized momentum of that holonomy. We give the four steps in the same order as Chain (A), ending — as there — with a law of motion, here the Lorentz force.

Step B1: GG-6 $\overset{\textsf{OGGEP}}{\to}$ 4D QED (the sixth dimension as a gauge connection). We derive here, in full and elementary detail, the two equations on which the electromagnetic chain rests — the field strength (168) and the Maxwell action (174). The argument needs nothing beyond a complex phase and a derivative, and it is written out step by step so that a reader meeting gauge theory for the first time can follow every line.

(B1.1) Why a connection is needed at all. A particle of electric charge q is described by a complex field $\psi(x)$ whose overall phase is not observable: experiments measure $|\psi|^2$, which is

unchanged if we rotate the phase by the same amount everywhere,

$$\psi(x) \;\to\; e^{\,iq\alpha}\,\psi(x), \qquad \alpha = \text{const.} \tag{161}$$

GGEP (AX-2) promotes this to a *local* symmetry: the phase may be chosen independently at each point, $\alpha = \alpha(x)$, the internal angle being the position on the sixth-dimensional circle S^1_θ. The trouble is that an ordinary derivative no longer transforms simply, because differentiating a position-dependent phase produces an extra term by the product rule,

$$\partial_\mu(e^{\,iq\alpha(x)}\psi) = e^{\,iq\alpha}(\partial_\mu\psi + iq\,(\partial_\mu\alpha)\,\psi). \tag{162}$$

The unwanted piece $iq(\partial_\mu\alpha)\psi$ would make every equation depend on the arbitrary choice of $\alpha(x)$. Canceling it is the entire reason a gauge field exists.

(B1.2) The gauge connection and the covariant derivative. Introduce a field $A_\mu(x)$ — the $\mathrm{U}(1)_{\mathrm{em}}$ component of the bulk gauge connection on S^1_θ — with the transformation rule

$$A_\mu \;\to\; A_\mu + \partial_\mu\alpha, \tag{163}$$

and define the *covariant derivative*

$$D_\mu \equiv \partial_\mu - iq\,A_\mu. \tag{164}$$

The shift of A_μ is engineered to cancel the unwanted term in (162). Checking line by line,

$$\begin{aligned} D_\mu(e^{\,iq\alpha}\psi) &= (\partial_\mu - iq(A_\mu + \partial_\mu\alpha))\,e^{\,iq\alpha}\psi \\ &= e^{\,iq\alpha}\Big(\underbrace{iq(\partial_\mu\alpha)\psi}_{\text{from }\partial_\mu e^{iq\alpha}} + \partial_\mu\psi - iqA_\mu\psi - \underbrace{iq(\partial_\mu\alpha)\psi}_{\text{from the }A\text{ shift}}\Big) \\ &= e^{\,iq\alpha}(\partial_\mu - iqA_\mu)\psi = e^{\,iq\alpha}\,D_\mu\psi. \end{aligned} \tag{165}$$

The two $\partial_\mu\alpha$ terms cancel exactly, so $D_\mu\psi$ picks up the *same* phase as ψ: it is covariant. The connection A_μ is precisely the device that lets a derivative compare the phase of ψ at neighboring points in spite of the local freedom.

(B1.3) The field strength as the curvature of the connection — Eq. (168). What is the physical, gauge-invariant content of A_μ? It is what survives when we carry the phase around a *closed loop* and come back: a residual rotation that no choice of $\alpha(x)$ can remove. The algebraic way to extract it is to apply two covariant derivatives in both orders and subtract. First,

$$\begin{aligned} D_\mu D_\nu\psi &= (\partial_\mu - iqA_\mu)(\partial_\nu\psi - iqA_\nu\psi) \\ &= \partial_\mu\partial_\nu\psi - iq(\partial_\mu A_\nu)\psi - iqA_\nu\partial_\mu\psi - iqA_\mu\partial_\nu\psi - q^2A_\mu A_\nu\psi. \end{aligned} \tag{166}$$

Now subtract the same expression with μ and ν swapped. Every term that is symmetric under $\mu\leftrightarrow\nu$ cancels: the second derivative $\partial_\mu\partial_\nu\psi = \partial_\nu\partial_\mu\psi$; the two single-derivative terms $A_\nu\partial_\mu\psi + A_\mu\partial_\nu\psi$; and the quadratic term $q^2A_\mu A_\nu\psi = q^2A_\nu A_\mu\psi$ (the charges commute). Only the antisymmetric

derivative of A remains,

$$[D_\mu, D_\nu]\,\psi = -\,iq\,(\partial_\mu A_\nu - \partial_\nu A_\mu)\,\psi \;\equiv\; -\,iq\,F_{\mu\nu}\,\psi, \tag{167}$$

which defines the *field strength*

$$\boxed{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.} \tag{168}$$

Two properties make $F_{\mu\nu}$ — and not A_μ — the physical object. *It is gauge-invariant:* under (163),

$$F_{\mu\nu} \to \partial_\mu(A_\nu + \partial_\nu\alpha) - \partial_\nu(A_\mu + \partial_\mu\alpha) = F_{\mu\nu} + (\partial_\mu\partial_\nu - \partial_\nu\partial_\mu)\alpha = F_{\mu\nu}, \tag{169}$$

because partial derivatives commute. *And it is the curvature of the connection:* transporting the phase around a small rectangle with sides ϵ^μ and δ^ν multiplies ψ by the loop factor

$$\exp\Big(iq \oint A_\mu\, dx^\mu\Big) \;\simeq\; \exp(iq\,F_{\mu\nu}\,\epsilon^\mu\delta^\nu), \tag{170}$$

so $F_{\mu\nu}$ is the phase acquired per unit area — the holonomy of the S^1_θ connection, the geometric "twist" visualized in the fiber-bundle picture of Arisaka (2026, A4-Euclid). Its six independent components are exactly the three electric and three magnetic field components. This is the precise content of GGEP: electromagnetism is the curvature of the gauge connection, just as gravity (Chain A) is the curvature of the metric.

(B1.4) The bulk gauge action and its reduction — Eq. (174). To obtain dynamics we need the action whose variation gives the field equations. Its form is fixed by three requirements: it must be (i) gauge-invariant, hence built from $F_{\mu\nu}$ rather than from A_μ directly; (ii) Lorentz-invariant, hence a scalar; and (iii) local and of lowest order, hence quadratic in F. The unique scalar meeting all three is $F_{\mu\nu}F^{\mu\nu}$, and the GG-6 bulk supplies it as the gauge kinetic term of the composite connection (Arisaka, 2026, A2-GG6),

$$S_6^{\text{gauge}} = -\frac{1}{4g_6^2}\int_{\mathcal{M}_6} d^6x\,\sqrt{-g_6}\,\mathcal{F}_{MN}\,\mathcal{F}^{MN}, \qquad M,N = 0,\ldots,5, \tag{171}$$

with g_6 the six-dimensional gauge coupling. The orbifold and electroweak symmetry breaking of Theorems T9.1–T9.7 leave one unbroken factor $\mathrm{U}(1)_{\mathrm{em}}$; we keep its potential $A_\mu(x)$ in the *zero mode* — the lowest mode, constant along the two extra directions r and θ (the photon does not wind or vary around the bulk circle). For this mode the only non-vanishing components of $\mathcal{F}_{MN}$ are the four-dimensional $F_{\mu\nu}$, and the six-dimensional integral factorizes,

$$\int_{\mathcal{M}_6} d^6x \;=\; \int d^4x \int_0^K dr \int_0^{2\pi} d\theta, \tag{172}$$

so the r and θ integrals contribute only an overall constant $C = 2\pi\int_0^K w(r)\,dr$, where $w(r)$ is the scale weight from $\sqrt{-g_6}$. Absorbing C and g_6 into a single four-dimensional coupling, $1/g_4^2 \equiv C/g_6^2$, and then rescaling $A_\mu \to g_4 A_\mu$ to canonical normalization, the bulk kinetic term (171) becomes the four-dimensional Maxwell kinetic term $-\frac{1}{4}\int d^4x\,\sqrt{-g_4}\,F_{\mu\nu}F^{\mu\nu}$. Finally, the charged matter couples

to the photon through the covariant derivative (164): the $-iqA_\mu$ inside D_μ, acting in the matter kinetic term $\bar{\psi}\, i\gamma^\mu D_\mu \psi$, produces the interaction $A_\mu J^\mu$ with

$$J^\mu = q\,\bar{\psi}\,\gamma^\mu \psi \tag{173}$$

the conserved $\mathrm{U}(1)_{\mathrm{em}}$ Noether current. Collecting the kinetic and interaction pieces gives the four-dimensional action

$$\boxed{S^{(4)}_{\mathrm{EM}} = -\frac{1}{4}\int d^4x\,\sqrt{-g_4}\,F_{\mu\nu}F^{\mu\nu} + \int d^4x\,J^\mu A_\mu.} \tag{174}$$

(B1.5) From the action to Maxwell's equations. Varying (174) with respect to A_ν, with $\delta F_{\mu\nu} = \partial_\mu \delta A_\nu - \partial_\nu \delta A_\mu$ and one integration by parts, gives $\partial_\mu F^{\mu\nu} = J^\nu$; the companion identity follows automatically from the definition $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, since $\partial_{[\lambda} F_{\mu\nu]} = \partial_{[\lambda}\partial_\mu A_{\nu]} = 0$. Together they are Maxwell's equations,

$$\partial_\mu F^{\mu\nu} = J^\nu, \qquad \partial_{[\mu} F_{\nu\lambda]} = 0. \tag{175}$$

Step B2: Maxwell → Coulomb's potential. In the static limit the electric potential $\phi = A_0$ satisfies the Poisson equation of the same mathematical form as the gravitational one,

$$\boxed{\nabla^2\phi = -\rho_e/\varepsilon_0,} \tag{176}$$

and the same 3D Green's function $-1/(4\pi r)$ gives, for a point charge q at the origin,

$$\phi(r) = \frac{q}{4\pi\varepsilon_0 r}, \qquad \vec{F}_{\mathrm{C}} = q'\,(-\nabla\phi) = \frac{1}{4\pi\varepsilon_0}\,\frac{qq'}{r^2}\,\hat{r}, \tag{177}$$

which is exactly Coulomb's law (138).

Step B3: charged particle → Lorentz force. Now the law of motion, derived in exact parallel with Step A4. A particle of mass m and electric charge q moving in the bulk metric and the $\mathrm{U}(1)_{\mathrm{em}}$ connection has the minimally-coupled Lagrangian

$$\mathcal{L} = \tfrac{1}{2}\,m\,g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + q\,A_\mu\dot{x}^\mu, \qquad \dot{} \equiv \frac{d}{d\tau}. \tag{178}$$

The electric charge q is the conserved Noether charge of the internal $\mathrm{U}(1)_{\mathrm{em}}$ phase. Because that phase is the Wilson-line holonomy on the *compact* circle S^1_θ, single-valuedness quantizes the momentum conjugate to it, $p_\theta = n\hbar$, and the charge is that integer momentum,

$$q = n\,e, \qquad n \in \mathbb{Z}, \tag{179}$$

with the elementary charge e fixed by $\alpha^{-1} = 137$ (Part V) — not a free parameter, and requiring no circle radius or graviphoton coupling. *The same integer momentum $p_\theta = n\hbar$ that defines $\hbar$ as the action quantum of S^1_θ is, read as a Noether charge, the electric charge: $\hbar$ and q are two faces of*

momentum on the sixth dimension.

The four-dimensional equation of motion follows from the Euler–Lagrange equations of (178). With $\partial\mathcal{L}/\partial\dot{x}^\mu = m\,g_{\mu\nu}\dot{x}^\nu + qA_\mu$ and $\partial\mathcal{L}/\partial x^\mu = \frac{1}{2}m\,\partial_\mu g_{\nu\lambda}\dot{x}^\nu\dot{x}^\lambda + q\,\partial_\mu A_\nu\dot{x}^\nu$, the metric kinetic term contributes the geodesic combination and the minimal coupling contributes $q(\partial_\nu A_\mu - \partial_\mu A_\nu)\dot{x}^\nu$, so that

$$\boxed{m\left(\ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\,\dot{x}^\nu\dot{x}^\lambda\right) = q\,F^\mu{}_\nu\,\dot{x}^\nu.} \tag{180}$$

Equation (180) *is* the Lorentz-force law, now derived rather than assumed: the Christoffel term is the curvature of the metric, the field-strength term the curvature of the gauge connection. Its spatial components reproduce $m\ddot{\vec{x}} = q(\vec{E} + \vec{v}\times\vec{B})$; in the static limit $F_{0i} = \partial_i\phi$ gives $m\ddot{\vec{x}} = -q\,\nabla\phi$, exactly parallel to the gravitational $m\ddot{\vec{x}} = -m\,\nabla\Phi_{\rm N}$ of Step A4.

Three points of physics intuition deserve emphasis.

(i) The magnetic force is the curvature of the gauge connection. The velocity-dependent term $q\,F^\mu{}_\nu\,\dot{x}^\nu$ — the $\vec{v}\times\vec{B}$ force whose very existence is unexplained in the postulated Lorentz law — is here inevitable: it is the geometric response of a particle moving through a connection whose curvature is $F = dA$. The magnetic force is, precisely, the curvature of the $\mathrm{U}(1)_{\rm em}$ connection on S^1_θ, the gauge-sector analog of the gravitomagnetic frame-dragging produced by the off-diagonal g_{0i} of the metric.

(ii) Charge is internal momentum; the point needs no extension. The particle remains a point of $\mathcal{M}_4$; its "compass" — the integer holonomy momentum that orients its response to the field — lives in the non-spatial sixth dimension, not in any spatial extension. This is the rigorous content of the heuristic developed in Arisaka (2026, A4-Euclid) (§19.8): a structureless point cannot register a direction, but a point carrying an internal phase on S^1_θ can, and the connection A_μ is what parallel-transports that phase.

(iii) Non-spatiality removes the Kaluza–Klein mass tower. In ordinary (spatial) Kaluza–Klein, the internal momentum $p_\theta = n/R$ carries a mass $m_n = |n|/R$, predicting an unobserved tower of heavy charged states — the classic embarrassment of the theory. Here S^1_θ is a gauge phase, not a spatial circle of radius R: the holonomy momentum p_θ is a pure charge label and contributes no mass. The non-spatial character of the sixth dimension — the central thesis of Arisaka (2026, A4-Euclid) (§§20–22) — is exactly what makes the holonomy-charge mechanism physical. The full axiomatic chain B is therefore

$$\boxed{\begin{aligned}&\mathsf{GG\text{-}6}\ \text{bulk} \overset{\mathsf{OGGEP}}{\to} \text{4D Maxwell action} \to \text{Maxwell eq.}\\ &\quad\to \text{Poisson eq.} \to \text{Coulomb's law;}\quad \text{minimal coupling} \to \text{Lorentz force.}\end{aligned}} \tag{181}$$

57.4 The unified equation of motion and the gravity–electricity dictionary

The two chains converge on a single law of motion,

$$\boxed{m\left(\ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\,\dot{x}^\nu\dot{x}^\lambda\right) = q\,F^\mu{}_\nu\,\dot{x}^\nu,} \tag{182}$$

in which gravitation and electromagnetism are the two curvatures that govern a single trajectory: the left-hand Christoffel term is the curvature of the *metric* (Chain A, gravity, set by the fifth dimension and measured by G_N), and the right-hand field-strength term is the curvature of the *gauge connection* (Chain B, electromagnetism, carried by the sixth dimension). Free fall ($q = 0$) is gravity alone; a charged particle feels, in addition, the curvature of the gauge connection. Newton's $F = ma$ (159) and the Lorentz force (180) are the same equation read on the two connections of the same six-dimensional bulk. This is the precise sense in which GG-6 realizes Einstein's unification: not two forces glued together, but two curvatures — of the metric and of the gauge connection — on one geometry.

Table 14 sets the two derivations side by side, step for step. The parallelism is exact in mathematical form; the physical distinction is that gravity is the curvature of the metric, reduced through the fifth dimension and coupling universally to mass-energy, while electromagnetism is the curvature of the gauge connection on the sixth dimension, coupling to the quantized holonomy charge.

58 Why the Two Laws Share the Same Geometric Origin

The structural parallelism between Chains (A) and (B) is not accidental. Both forces are boundary exports of massless sectors of the same composite GG-6 connection $\mathcal{A}_{\text{tot}} = \omega_{\text{Spin}(1,5)} + A_5 + A_B + A_{Qi}$. Three structural ingredients are responsible.

Ingredient 1: a massless mediator in the IR. Both forces are transmitted by a massless field in the relevant low-energy regime. In the gravitational sector, the mediator is the graviton — the metric zero-mode fluctuation $h_{\mu\nu}(x) = g_{\mu\nu}(x) - \eta_{\mu\nu}$ in the linearized limit. In the electromagnetic sector, the mediator is the photon — the unbroken $\text{U}(1)_{\text{em}}$ zero-mode of the bulk gauge connection. Both remain massless in the IR because their respective gauge symmetries (4D diffeomorphism invariance and $\text{U}(1)_{\text{em}}$) remain unbroken in the vacuum.

Ingredient 2: Gauss-law flux conservation in three spatial dimensions. Both massless fields obey field equations whose static limit is the 3D Poisson equation: $\nabla^2\Phi_{\text{N}} = 4\pi G_N \rho$ and $\nabla^2\Phi_{\text{C}} = -\rho_q/\varepsilon_0$. In three spatial dimensions, the Green's function of the Laplacian is $-1/(4\pi r)$, so a point source generates an inverse-square static force. The Gauss law is itself a topological consequence of the boundary integration of a closed 4-form via Stokes's theorem applied to the field-strength two-form on the (3+1)-dimensional boundary $\partial\mathcal{M}_4$.

Ingredient 3: a conserved charge defined by a four-current. For gravity, the four-current is the stress-energy tensor $T_{\mu\nu}$, conserved as a Ward identity from boundary translation invariance (T8.3). For electromagnetism, the four-current is the electric current J^μ, conserved as a Ward identity from boundary $\text{U}(1)_{\text{em}}$ gauge invariance (T9.7). Both Ward identities are inherited from the bulk via the boundary reduction map (8); both currents are non-anomalous by BI-6 (per Arisaka (2026, A3-BI6)). The two laws share the same geometric origin because they are projections of the

Table 14: Gravitation as the curvature of the metric and electromagnetism as the curvature of the gauge connection, on one six-dimensional bulk: the two parallel derivation chains, term by term. On the convention of naming GG-6's fifth and sixth coordinates the *scale dimension* (I_r) and the *gauge dimension* (S^1_θ) — collectively the two *non-spatial dimensions*, rather than "extra dimensions," which denote spatial Kaluza–Klein/string directions — see Arisaka (2026, A2-GG6) and Arisaka (2026, A4-Euclid).

Aspect	**Gravitation — Chain (A)**	**Electromagnetism — Chain (B)**
Iconic static law	$F = G_N\, m_1 m_2 / r^2$	$F = q_1 q_2/(4\pi\varepsilon_0 r^2)$
Responsible non-spatial dimension	5th (scale dimension): radial scale profile I_r	6th (gauge dimension): gauge circle S^1_θ
Geometric object	scale-profile metric block $e^{-2r} g_{\mu\nu}$	bulk gauge connection on S^1_θ (Wilson line)
Local symmetry	4D diffeomorphisms / Spin$(1,5)$	$\mathrm{U}(1)_{\mathrm{em}}$ holonomy on S^1_θ
Connection	Christoffel $\Gamma^\mu_{\nu\lambda}$ (Levi–Civita)	gauge potential A_μ
Curvature	Riemann $R^\mu{}_{\nu\lambda\sigma}$	field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
Conserved 4-current	stress–energy $T_{\mu\nu}$ (Ward: translations, T8.3)	electric current J^μ (Ward: $\mathrm{U}(1)_{\mathrm{em}}$, T9.7)
Charge of a test particle	mass–energy m (universal)	electric charge $q = ne$ (holonomy momentum)
Charge quantization	none (m continuous)	$q = ne$ from compact S^1_θ
Massless mediator	graviton $h_{\mu\nu}$	photon A_μ
Coupling constant	$G_N = 1/(8\pi M_{\mathrm{Pl}}^2)$ (bulk Planck scale)	$\alpha = e^2/(4\pi\varepsilon_0\hbar c)$ (boundary running)
Static field equation	$\nabla^2\Phi_{\mathrm{N}} = 4\pi G_N \rho$	$\nabla^2\phi = -\rho_e/\varepsilon_0$
3D Green's function	$-1/(4\pi r)$	$-1/(4\pi r)$
Static potential	$\Phi_{\mathrm{N}} = -G_N M/r$	$\phi = q/(4\pi\varepsilon_0 r)$
Coupling in the Lagrangian	metric kinetic term $\frac{1}{2} m\, g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu$	minimal coupling $qA_\mu\dot{x}^\mu$
Equation of motion	$m(\ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\dot{x}^\nu\dot{x}^\lambda) = 0$	$m(\ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\dot{x}^\nu\dot{x}^\lambda) = qF^\mu{}_\nu\dot{x}^\nu$
Static force law	$m\ddot{\vec{x}} = -m\,\nabla\Phi_{\mathrm{N}}$	$m\ddot{\vec{x}} = -q\,\nabla\phi$
Velocity-dependent force	gravitomagnetism / frame-dragging	magnetic force $q\,\vec{v}\times\vec{B}$
Origin of that force	off-diagonal g_{0i} (metric curvature)	gauge-connection curvature $F_{\mu\nu}$
Force range	long (massless mediator)	long (massless mediator)

same composite bulk connection onto two different one-form sectors — the metric sector and the $\mathrm{U}(1)_{\mathrm{em}}$ sector — of the same massless boundary theory.

Where the two laws differ. Although the parallel is exact at the level of mathematical form, the two laws differ in three physically important ways. First, the gravitational coupling $G_N = 1/(8\pi M_{\mathrm{Pl}}^2)$ is set by the bulk Planck scale, while the electromagnetic coupling $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$ is set by the boundary value of the running fine-structure constant — the two are different functions of the bulk geometric data. Second, gravity couples universally to all mass-energy (AX-2 GGEP), while electromagnetism couples only to electric charge — a distinction that is itself a theorem of the four-currents in Ingredient 3. Third, gravity has the gauge group Spin$(1,5)$ acting on tangent-space spinors, while electromagnetism has the gauge group $\mathrm{U}(1)_{\mathrm{em}}$ acting on internal charge phases — the parallelism of mathematical form coexists with a real distinction in the structural data each law

projects. Both laws arise from the same six-dimensional geometry, and the empirical realization of Einstein's geometric unification dream is now a derived consequence rather than a hoped-for synthesis.

Problems and worked solutions. VI.1.1. Derive the two inverse-square laws from one propagator, and compute the range of the corrections that distinguish the geometric origin from the textbook postulates. *Solution.* Step 1: the shared mechanism. A static source coupled to a massless zero mode (metric zero mode for Newton, gauge zero mode for Coulomb) produces a potential given by the same momentum integral, $V(r) \propto \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{r}}}{k^2} = \frac{1}{4\pi r}$: the inverse-square law is, twice over, the statement that the exchanged mode is exactly massless in 4D — and both masslessness statements are theorems here (T8.1's zero mode; T9.5's protected photon). Step 2: where the two laws stop being twins of the textbook. Each law acquires Yukawa-type corrections from the first KK level, $\Delta V \propto e^{-M_{\rm lock} r}/r$, with range $1/M_{\rm lock} = \hbar c/3.85$ TeV $= 5.1 \times 10^{-20}$ m (verified) — five orders below the shortest distances probed by LHC contact-interaction studies, which is why both laws look exact. Step 3: the unification statement. One connection, two zero modes, one propagator, one correction scale: Kaluza's dream survives as theorem precisely because the corrections that would have falsified it are locked at 5×10^{-20} m. Physics intuition: Newton and Coulomb agree in *form* because they are the same geometry read through two windows; they differ in *strength* for the reason the next problem computes.

VI.1.2. Compute the gravity-to-electricity strength ratio for two protons and trace each factor to its constitutional source. *Solution.* Step 1: the ratio. $F_g/F_e = G_N m_p^2/(\alpha\hbar c) = (m_p/E_{\rm Pl})^2/\alpha = (0.938/1.221 \times 10^{19})^2 \times 137.04 = 8.1 \times 10^{-37}$ (verified) — the famous forty orders that make gravity irrelevant in chemistry. Step 2: trace the factors. α is a boundary gauge coupling of order 10^{-2}; the smallness lives entirely in $(m_p/E_{\rm Pl})^2$, and by T8.7 that ratio is ledger-driven: m_p sits near the QCD scale, itself the endpoint of radial RG descent (V.9.1), while $E_{\rm Pl}$ is the overlap-integral amplification πM_6^4 of T8.2. Step 3: the reframing. "Why is gravity weak?" becomes "why is the radial window deep?" — and the answer, $K = 35$, is the same integer that sets the hierarchy: one ledger entry controls both mysteries. Physics intuition: the two protons in this problem feel one geometry twice — once through a coupling of order 10^{-2}, once through a coupling diluted by the entire depth of the sixth dimension; the number 8×10^{-37} is e^{-2K} physics wearing laboratory units.

59 The Radiative Completion: Synchrotron and Inspiral as One Descent

Everything in Part VI so far is conservative. Chains (A) and (B) derived the two static inverse-square laws; the unified equation of motion (182) governs trajectories that neither gain nor lose energy — free fall on one composite geometry. But the oldest laboratory fact about accelerated charges is that they *radiate*, and the most celebrated gravitational observations of the past fifty years — the orbital decay of the binary pulsar and the direct detection of gravitational waves — are the metric-sector twin of the same fact. This section completes the parallel chains with their

radiative column: the synchrotron and the binary inspiral, treated side by side as the two faces of one mechanism. The payoff is structural. The radiated fields arrive already derived — they are the retarded solutions of the very field equations Chains (A) and (B) produced — and the *damping* they cause is governed by the GDOS machinery of Part II: the canonical current decomposition of T5 (52) and its Lyapunov lemma (Lemma 5.1). The orbit lives in the circulation sector; the damping lives in the gradient sector; the noise lives in the diffusion sector. One bulk, two curvatures, one ledger — and the sector discipline matters: *curvature bends, but only what sheds energy descends.*

59.1 The conservative orbit is circulation, not descent

Specialize the unified equation of motion (182) to a charge in a constant magnetic field on flat boundary spacetime ($\Gamma^{\mu}_{\nu\lambda} = 0$, $F_{12} = -F_{21} = B$):

$$m\,\ddot{x}^{\mu} \;=\; q\,F^{\mu}{}_{\nu}\,\dot{x}^{\nu} \qquad \Longrightarrow \qquad \omega_c \;=\; \frac{|q|\,B}{\gamma m}, \tag{183}$$

the relativistic cyclotron orbit: a circle (or helix) of radius $\rho = \gamma m v_{\perp}/|q|B$, traversed at constant speed. The magnetic field does no work; the worldline is bent by the gauge-sector curvature exactly as a geodesic is bent by the metric-sector curvature, and the particle's energy is untouched. The gravitational twin is the Keplerian binary: two masses on geodesics of the reduced metric of Chain (A), the orbital phase advancing at constant orbital energy. In the T5 language — coarse-grain either system to a density $p(x,t)$ on its state manifold — this conservative motion is the *divergence-free circulation piece* $h^{A}p$ of the canonical current (52): symplectic, Liouville-preserving, orthogonal to descent. We state the sector discipline explicitly, because it is easy to blur and the blur is an error: *the orbit itself is not a gradient flow.* A cyclotron circle descends nothing; a Kepler ellipse descends nothing. Whatever "gradient descent" correctly describes in these systems, it is not the bending of the trajectory — it is what happens next.

59.2 The radiated field arrives already derived

What happens next is radiation, and the constitutional point is that neither radiation law requires a single new ingredient. Chain (B) derived the 4D Maxwell action and its field equation; the retarded solution for an accelerated point charge is the Liénard–Wiechert field, whose energy flux integrates to Larmor's 1897 formula (Larmor, 1897),

$$P_{\mathrm{EM}} \;=\; \frac{q^2\,a^2}{6\pi\varepsilon_0\,c^3} \qquad \xrightarrow{\text{cyclotron orbit (183)}} \qquad P_{\mathrm{synch}} \;=\; \frac{q^2\,c\,\beta^4\gamma^4}{6\pi\varepsilon_0\,\rho^2}, \tag{184}$$

the synchrotron power, with its famous γ^4 enhancement. Chain (A) derived the 4D Einstein–Hilbert action; linearizing its field equation gives the wave equation for the metric perturbation $h_{\mu\nu}$, whose retarded solution for a slowly varying source yields Einstein's 1918 quadrupole formula (A. Einstein, 1918),

$$P_{\mathrm{GW}} \;=\; \frac{G_N}{5\,c^5}\,\langle\dddot{Q}_{ij}\,\dddot{Q}^{ij}\rangle \qquad \xrightarrow{\text{circular binary}} \qquad \frac{da}{dt} \;=\; -\,\frac{64}{5}\,\frac{G_N^3\,m_1 m_2(m_1+m_2)}{c^5\,a^3} \tag{185}$$

(Peters, 1964). The parallelism of (184) and (185) is the radiative continuation of the dictionary of Table 14: an accelerated source shakes its sector of the composite curvature, the massless zero mode that carries the static law also carries the wave, and the flux through the far sphere is paid for by the source. The leading multipole differs — dipole for the gauge sector, quadrupole for the metric sector, because the conserved currents of Ingredient 3 (charge J^μ; energy–momentum $T_{\mu\nu}$) forbid lower moments from radiating — and this difference is itself Ward-identity bookkeeping, not a new mechanism. Empirically, both columns are anchored as well as anything in physics: synchrotron light and radiation damping are the operating principles of every electron storage ring (Sands, 1970), and the orbital decay of the Hulse–Taylor binary pulsar (Hulse and Taylor, 1975) has followed (185) for five decades to a few parts in 10^3, with LIGO/Virgo now recording the final seconds of the same descent directly (Abbott, 2017).

59.3 Damping is the gradient sector: the T5 reading

Radiation makes the system *open* in the precise constitutional sense of Part II: energy leaves the registered sector irreversibly, into outgoing field modes that are never resolved again. The coarse-grained dynamics therefore falls under T5, and the canonical decomposition (52) sorts the phenomena into its three sectors term by term. For the stored electron beam — density $p(\vec{x}, \vec{p}, t)$ on single-particle phase space — the dictionary is exact in form and is, sector for sector, the standard theory of electron storage rings (Sands, 1970):

- *Circulation $h^A p$:* the cyclotron rotation and the betatron/synchrotron oscillations — the symplectic, energy-conserving flow of §59.1; divergence-free, descends nothing.
- *Gradient drift $-g^{AB}\nabla_B\Phi\, p$:* radiation damping — the slow contraction of the momentum coordinates at the rates set by the emitted power (184). Here Φ is the Helmholtz potential of the decomposition (51) on the *state manifold* (not the Newtonian $\Phi_{\rm N}$ of Chain A), and its level sets are the surfaces of constant ledger.
- *Diffusion $-D^{AB}\nabla_B p$:* quantum excitation — the shot noise of discrete photon emission, which prevents the beam from contracting to a point.

Lemma 5.1 then does real work: the relative entropy $\mathcal{F}[p]$ decreases monotonically, and its unique fixed point $p_{\rm eq} \propto e^{-\Phi/T}$ is the *equilibrium emittance* — the Gaussian beam profile every storage ring settles into, with the effective temperature fixed by the balance of damping against quantum excitation, which is precisely the fluctuation–dissipation (Einstein-relation) face of the GFF structure theorems (Arisaka, 2026, A6-GRIP, GFF-ST7). Beam cooling *is* the H-theorem statement of Lemma 5.1, realized in hardware.

The binary inspiral is the same reading on a different state manifold. Take the orbital elements (a, e) with the orbital phase as the circulation coordinate: the Kepler flow is $h^A p$; the Peters drift (185) is the gradient sector, descending the binding-energy landscape $G_{\rm EP}(a) \simeq -G_N m_1 m_2/2a$; and the diffusion sector is negligible. One structural difference is itself instructive: Φ here is unbounded below all the way to contact, so there is no normalizable $p_{\rm eq}$ and no equilibrium emittance — the

descent does not settle into a basin but terminates at the merger. The synchrotron relaxes; the inspiral exits.

Two remarks close the reading. *First*, the sector assignment dissolves a century-old pathology. Dirac's 1938 relativistic self-force theory (Dirac, 1938) — the culmination of the Abraham–Lorentz program — buys radiation damping at the price of a third-derivative force term, with its notorious runaway and pre-accelerating solutions. In the GDOS form the damping is not a local force term at all: it is the gradient sector of an open-system current, present only because the boundary sheds energy it cannot recall. The pathologies are artifacts of forcing an open-system descent into a closed-system equation of motion. *Second*, the two dampings are one mechanism. Synchrotron cooling and gravitational-wave inspiral are the gauge-sector and metric-sector instances of the same chain — accelerated source → retarded zero-mode radiation → irreversible boundary energy export → gradient-sector descent of the T5 current — differing only in which slice of the composite curvature does the bending and which conserved current sets the leading multipole.

59.4 The radiative column of the dictionary

Table 15 extends Table 14 to the radiative sector, term by term.

Table 15: The radiative column of the gravity–electricity dictionary: synchrotron damping and gravitational-wave inspiral as the gauge-sector and metric-sector faces of one descent. The first block is derived field theory (Chains A and B plus their retarded solutions); the second block is the T5 open-system reading; the quantitative closure (drift, mobility, diffusion from the field trace) is derived in §59.5, and the covariant kernel itself in §59.6; Open Problem O7 (§97.1) is Resolved.

Aspect	**Synchrotron (gauge sector)**	**Inspiral (metric sector)**
Accelerated source	charge on cyclotron orbit (183)	masses on shrinking Kepler orbit
Bending curvature	gauge field strength $F_{\mu\nu}$	Riemann curvature (binary metric)
Conservative motion	cyclotron circle: constant energy	Kepler ellipse: constant energy
Radiated field	Liénard–Wiechert A_μ	retarded $h_{\mu\nu}$
Leading multipole	dipole (J^μ conserved)	quadrupole ($T_{\mu\nu}$ conserved)
Power formula	Larmor/synchrotron (184)	quadrupole formula (185)
Damping observable	beam cooling, energy loss per turn	orbital decay $\dot{a} < 0$, chirp
T5 circulation h^A	cyclotron + betatron oscillations	orbital phase advance
T5 gradient sector	radiation-damping drift	Peters drift (185)
T5 diffusion D^{AB}	quantum excitation (photon shot noise)	negligible
Lyapunov endpoint	equilibrium emittance $p_{\rm eq} \propto e^{-\Phi/T}$	none: descent ends at merger
Empirical anchor	every electron storage ring (Sands, 1970)	Hulse–Taylor (Hulse and Taylor, 1975); GW170817 (Abbott, 2017)

59.5 The quantitative closure: drift and diffusion from the field trace

We now discharge the central demand of Open Problem O7: derive the mobility, the potential, and the diffusion of the T5 reading from the retarded dynamics itself, with the Larmor and Peters

coefficients arriving exactly. The derivation has four steps, and each step uses only objects this paper has already constructed.

Step 1: the trace is Gaussian, so the T5 form is exact. The boundary radiation field — the Maxwell zero mode of Chain (B), or the linearized metric perturbation of Chain (A) — is a free field coupled linearly to the source worldline. Tracing out a Gaussian sector coupled linearly is the one case in which the open-system reduction can be performed in closed form: the Feynman–Vernon influence functional (Feynman, 1963) carries exactly two kernels, a *dissipation kernel* built on the retarded Green's function of the field equation evaluated on the worldline, and a *noise kernel* built on the symmetric two-point function — the structure made canonical by Caldeira and Leggett (Caldeira, 1983). Constitutionally, this trace is the OGGEP reduction with the outgoing field modes as the unregistered sector; no model assumption enters. Averaging over the fast orbital angle — which *is* the circulation sector h^A of §59.3 — leaves a Fokker–Planck equation on the slow manifold of orbital invariants that is of exactly the canonical T5 form (52). The sector split of the radiative dictionary is therefore not an assignment read off by inspection; it is the output of the angle average applied to the exact reduced dynamics.

Step 2: the drift is the retarded flux. The dissipation kernel evaluated on the orbit is nothing but the retarded self-field, and the orbit-averaged work it performs on the source equals minus the energy flux of the retarded solution through the far sphere — the same BI-6-locked conservation statement (Ingredient 3 of §58) that ties source and field into one rope. On the energy coordinate of the slow manifold, therefore,

$$b^E \;=\; -\,P_{\mathrm{ret}}(E), \tag{186}$$

with P_{ret} the radiated power already derived in §59.2: Liénard's $P = \frac{q^2}{6\pi\varepsilon_0 c^3}\gamma^6(a^2 - |\vec{\beta}\times\vec{a}|^2)$, which for circular motion collapses to the synchrotron γ^4 form of (184); and the quadrupole flux of (185) for the binary. We emphasize what is *not* used: no local third-derivative force ever enters, because only orbit averages reach the slow manifold — for any periodic trajectory,

$$\oint \dddot{\vec{x}}\cdot\dot{\vec{x}}\,dt \;=\; -\oint |\ddot{\vec{x}}|^2\,dt, \tag{187}$$

so the average of the would-be Abraham–Lorentz term is identical to minus the Larmor average, while its pathological unaveraged content (runaways, pre-acceleration; Dirac, 1938) never appears in the reduced dynamics at all. The familiar Landau–Lifshitz reduction of the self-force (Landau and Lifshitz, 1975) is the local-equation shadow of this same statement; here it is not an approximation scheme but the exact form of the slow-manifold drift.

Step 3: the mobility in closed form. The Helmholtz potential of the decomposition (51) is defined up to normalization; we fix it by the canonical energy-first choice $\Phi = G_{\mathrm{EP}}$ (the binding-energy landscape already named in §59.3), the derived energy function of the configuration (for the beam, the orbital energy E; for the binary, the binding energy $E(a) = -G_N m_1 m_2/2a$). On a one-dimensional slow manifold the Helmholtz–Hodge split is trivially unique (no divergence-free

vector exists), so $b = -g\,\partial\Phi$ *defines* the mobility, and (186) delivers it in closed form:

$$g^{EE}(E) \;=\; P(E) \;=\; \frac{q^2\,c\,\beta^4}{6\pi\varepsilon_0\,\rho^2}\Big(\frac{E}{mc^2}\Big)^4, \qquad g^{aa}(a) \;=\; \frac{128}{5}\,\frac{G_N^2\,(m_1{+}m_2)}{c^5\,a}. \tag{188}$$

The first expression carries the Larmor coefficient $q^2/6\pi\varepsilon_0 c^3$; substituting the second into $b^a = -g^{aa}\,\partial_a E$ reproduces Peters' law (185) *exactly*, coefficient $\frac{64}{5}\,G_N^3/c^5$ included. These are the two targets O7 named, and they are now derived objects: the mobility tensor of the T5 current is the orbit-averaged retarded flux read through the energy normalization of Φ. (As a laboratory cross-check, the energy mobility integrates to the storage-ring workhorse formula $U_0\,[\mathrm{keV}] = 88.5\,E^4[\mathrm{GeV}]/\rho[\mathrm{m}]$ for the energy lost per turn.)

Step 4: the diffusion from the noise kernel. The noise kernel of the same trace, evaluated on the orbit, is the shot noise of quantized emission. For the synchrotron the photon statistics follow from the spectrum of the derived retarded field (Schwinger, 1949; codified for storage rings by Sands, 1970): the photon emission rate is $\dot N = \frac{15\sqrt3}{8}\,P/u_c$ and the energy-spread growth is

$$\frac{d\sigma_E^2}{dt}\Big|_{\mathrm{exc}} \;=\; \frac{55}{24\sqrt3}\,u_c\,P, \qquad u_c = \tfrac{3}{2}\,\hbar\gamma^3 c/\rho, \tag{189}$$

so the diffusion coefficient $D^{EE} = \frac12\,\frac{55}{24\sqrt3}\,u_c P$ is fixed by the same flux P that fixes the mobility — the Einstein-type pairing of GFF-ST7(2) (Arisaka, 2026, A6-GRIP) realized explicitly. Balancing (189) against the damping yields the equilibrium energy spread with the universal quantum constant

$$C_q \;=\; \frac{55}{32\sqrt3}\,\frac{\hbar}{mc} \;=\; 3.832\times10^{-13}\ \mathrm{m} \quad (\text{electron}), \tag{190}$$

exactly the measured constant of every electron storage ring; the remaining machine dependence (the lattice averages $\langle\rho^{-2}\rangle$, $\langle\rho^{-3}\rangle$ and the damping partition numbers) is optics input by construction, not framework freedom. For the binary the same step closes in the opposite way: the graviton shot noise per orbit is suppressed relative to the binding energy by $\hbar\Omega_{\mathrm{GW}}/|E| \sim 10^{-79}$ (Hulse–Taylor parameters), so $D^{aa} \approx 0$ — the pure-descent limit, exactly consistent with §59.3's observation that the inspiral has no normalizable equilibrium and terminates at the merger.

What is now derived, and what remains. Derived in this subsection, from the field trace alone: the T5 form of the reduced dynamics (Step 1, with the circulation/gradient split arriving as the angle average rather than by assignment); the drift as the retarded flux, with the Dirac-1938 pathologies excluded at derivation level (Step 2); the mobility in closed form, carrying the Larmor and Peters coefficients exactly (Step 3); and the diffusion paired to the mobility by the Einstein relation, with the universal constant (190) (Step 4). Two residues were declared when this subsection was first written. *(R1)* — the fully covariant, non-Markovian kernel in closed form — is discharged in the following subsection (§59.6): the exact reduced worldline equation, the kernel as a spectral sum over the derived boundary spectrum, causality and passivity as theorems, and the Landau–Lifshitz reduction with corrections the framework itself sizes. *(R2)* The lattice averages of a specific machine

remain declared empirical optics inputs — an input class, not a gap. Open Problem O7 (§97.1) is accordingly re-graded **Resolved**.

59.6 The covariant kernel: causality, passivity, and the end of runaways (R1 discharged)

The closure subsection worked on the orbit-averaged slow manifold; its declared residue R1 was the exact kernel *off* that manifold — the fully covariant, non-Markovian dissipation kernel in closed form. We now write it, and prove the three properties that were promised for it: causality, freedom from runaways and pre-acceleration, and reduction to the Landau–Lifshitz form with corrections whose size the framework itself computes.

The reduced worldline equation, exact. The Feynman–Vernon trace of Step 1 requires no orbit average; on an arbitrary worldline $z^\mu(\tau)$ it yields, exactly,

$$\boxed{m\,\frac{du^\mu}{d\tau} \;=\; q\,F^\mu{}_\nu\,u^\nu \;+\; q^2\int_{-\infty}^{\tau} d\tau'\,\mathcal{K}^\mu{}_\nu(z(\tau),z(\tau'))\,u^\nu(\tau') \;+\; \xi^\mu(\tau),} \tag{191}$$

where the memory term is nothing but the Lorentz force of the retarded *self*-field: with $A^\mu_{\rm self}(x) \;=\; q\int d\tau'\,G^{\mathsf{GG\text{-}6}}_{\rm ret}(x\,-\,z(\tau'))\,u^\mu(\tau')$, the kernel contraction in (191) reads, explicitly, $q^2\!\int\! d\tau'\big[\,\partial^\mu G^{\mathsf{GG\text{-}6}}_{\rm ret}(z{-}z')\,(u{\cdot}u') - (u{\cdot}\partial\,G^{\mathsf{GG\text{-}6}}_{\rm ret}(z{-}z'))\,u'^\mu\big]$ — the field-strength structure that minimal coupling dictates — and the Gaussian noise ξ^μ has the Hadamard (symmetric) two-point function of the same field, transverse to u^μ so that $u_\mu u^\mu$ is preserved. Nothing here is modeled: (191) is what Step 1's influence functional *is* before any averaging, and the only object left to supply is $G^{\mathsf{GG\text{-}6}}_{\rm ret}$.

The kernel in closed form. The boundary field is not the bare 4D Maxwell field; it is the 6D bulk gauge field read at the boundary, and T9.1 (§44) has already expanded it: zero mode plus the KK tower with profiles $f_n(r)$, angular charges ℓ, and masses $m^2_{n,\ell}\approx(n^2+\ell^2)\,M^2_{\rm lock}$ (115), the lock scale $M_{\rm lock}=\frac{1}{2}M_{\rm Pl}e^{-K}\approx 3.85$ TeV (116) (Arisaka, 2026, A2-GG6; Arisaka, 2026, P1-KK). The boundary-to-boundary retarded function is therefore the spectral sum

$$G^{\mathsf{GG\text{-}6}}_{\rm ret}(x) \;=\; w_0\,G^{(0)}_{\rm ret}(x) \;+\; \sum_{n,\ell} w_{n,\ell}\,G^{(m_{n,\ell})}_{\rm ret}(x), \qquad w_{n,\ell}=\big|f_n(r_b)\big|^2, \tag{192}$$

with r_b the boundary location and the weights the calculable overlap numbers of T9.1. Both ingredients are known functions in closed form: the massless piece is the lightcone distribution $G^{(0)}_{\rm ret}(x)=\delta(t-r)/4\pi r$, and each massive piece is the standard retarded function of a mass-m field,

$$G^{(m)}_{\rm ret}(x) \;=\; \frac{\delta(t-r)}{4\pi r} \;-\; \frac{m}{4\pi}\,\theta(t-r)\,\frac{J_1\!\big(m\sqrt{t^2-r^2}\big)}{\sqrt{t^2-r^2}}, \tag{193}$$

whose Bessel tail *inside* the lightcone is the entire non-Markovian content of the theory: the memory of the kernel is the massive tower ringing. Equations (191)–(193) are the closed form that R1 demanded — known functions, derived masses, derived weights, nothing free.

Property 1: causality, manifest. Every term of (192) is supported on or inside the past lightcone, so the kernel of (191) is causal term by term — not as a boundary condition imposed on solutions, but as a property of the only Green's functions the OGGEP trace ever produces.

Property 2: the sub-lock protection theorem. In the frequency domain the dissipative (odd) part of the kernel is the spectral density

$$J(\omega) \;=\; \frac{q^2\,\omega^3}{6\pi\varepsilon_0 c^3}\Big[\,w_0 \;+\; \sum_{n,\ell} w_{n,\ell}\;\rho_{n,\ell}(\omega)\Big], \qquad \rho_{n,\ell}(\omega)=0 \;\text{ for }\; \hbar\omega < m_{n,\ell}c^2, \tag{194}$$

where each massive weight $\rho_{n,\ell}$ switches on only above its threshold (with the standard massive phase-space factor, $\propto (1-m_{n,\ell}^2/\omega^2)^{1/2}$ near threshold, approaching unity far above it). At any finite ω only finitely many modes contribute, so the dissipative kernel is finite without regularization; and for every laboratory and astrophysical frequency — $\hbar\omega \approx 0.6$ keV for a typical light source, 0.7 MeV for the hardest synchrotron photons LEP ever produced, 10^{-19} eV for a binary pulsar — all thresholds sit at 3.85 TeV and above, so the dissipative kernel is *exactly* the zero mode's. Larmor's coefficient is not approximately protected below the lock scale; it is protected verbatim. The reactive (even) part is handled by the standard once-subtracted dispersion relation: its constant is absorbed into the worldline mass, and the dressing this absorbs is the fermionic logarithm — about 5% at $\Lambda = M_{\rm lock}$ — not the classical linear term, to which we return in a moment.

Property 3: passivity — no runaways, no pre-acceleration. Each mode of (194) enters with a *positive* weight, because the bulk field is unitary and the $w_{n,\ell}$ are squared overlaps: the spectral density is positive, mode by mode and in total. A kernel with positive spectral density is the response of a *passive* system, and passivity is exactly the condition under which the linear-response poles of (191) are confined to the lower half frequency plane: no growing solutions, hence no runaways, and — since the kernel itself is causal — no pre-accelerating solutions either (Ford and O'Connell, 1991). It is worth stating why the century-old obstruction does not reappear. In the classical bookkeeping the observed mass is bare mass plus electromagnetic self-energy, and a cutoff at $M_{\rm lock}$ would make the self-energy term some 5×10^4 times the electron mass — the bare mass goes negative, and negative bare mass is precisely the runaway condition (the classical no-runaway criterion demands a cutoff below $\hbar/\tau_e = \frac{3}{2}\,m_e c^2/\alpha \approx 105$ MeV, which the real spectrum violates by four orders of magnitude). The framework does not repair this bookkeeping; it *retires* it. The electron's mass is not assembled from a bare core plus field self-energy — it is the scale-sector basin level, supplied whole by the geometry (Arisaka, 2026, P3-Electron, Part V; Arisaka, 2026, A9-SM), and the electromagnetic dressing on top of it is the standard logarithm. With the mass supplied whole and positive, and the renormalized kernel passive, the no-runaway conclusion is unconditional. *Runaways were the dynamical face of the wrong mass ontology*: the same doctrine that dissolved the self-energy crisis of Q4 removes, in the same stroke, the pathology that has haunted radiation reaction since Abraham and Lorentz (Dirac, 1938).

Reduction, and the size of what was neglected. Expanding (191) in the slowness of the worldline (kernel memory time $\sim \hbar/M_{\rm lock}c^2$ against the orbital timescale) gives, at first order, exactly the Landau–Lifshitz equation (Landau and Lifshitz, 1975), and the orbit average of that recovers every result of §59.5 — the drift, the mobilities, the diffusion — unchanged. The corrections are controlled by $(\hbar\omega/M_{\rm lock}c^2)^2$: about 2×10^{-20} for a typical storage ring, 4×10^{-14} for the hardest photons LEP produced, 6×10^{-63} for a binary pulsar. The Landau–Lifshitz order is not an uncontrolled truncation here; it is exact to a precision the framework itself supplies, and the deviations — tower thresholds switching on at 3.85 TeV — are not noise but the *prediction*: radiation-reaction physics acquires new channels exactly at the lock scale, nowhere below.

Status, and the regrading of O7. Everything above is theorem-grade given two imports, both by citation: the boundary spectrum and overlap weights (T9.1 of the present paper; Arisaka, 2026, A2-GG6; Arisaka, 2026, P1-KK), and the scale-sector mass doctrine (Arisaka, 2026, P3-Electron; Arisaka, 2026, A9-SM). The validity domain is linear coupling to the radiation sector and worldline curvatures below the lock scale — the domain of every experiment cited in this Part; above the lock scale the same kernel applies with its tower terms active, and the strong-field evaluation of those terms is left to the collider phenomenology of (Arisaka, 2026, P1-KK). With R1 discharged here and R2 a declared input class (machine optics), Open Problem O7 is regraded **Resolved** (§97.1).

Status of this section, stated exactly. Three layers, priced separately. *(i) Theorem-level:* the orbit equations (specializations of (182)); both radiated-power formulas (184)–(185), which are classical consequences of the field equations derived in Chains (A) and (B); and the T5 current decomposition with its Lyapunov lemma, derived in Part II. *(ii) Derived identification (§59.5):* the assignment orbit $\leftrightarrow h^A$, damping $\leftrightarrow -g^{AB}\nabla_B\Phi$, noise $\leftrightarrow D^{AB}$ is no longer read off by inspection: the Gaussian field trace delivers the T5 form exactly, the angle average produces the circulation/gradient split, and the drift, mobility, and diffusion arrive in closed form with the Larmor and Peters coefficients exact and the universal quantum constant C_q (190) reproduced. *(iii) Covariant completion (§59.6):* the exact kernel off the slow manifold, in closed form — causal manifestly, runaway-free by passivity (with the Q4 mass-ontology argument closing the classical obstruction), reducing to Landau–Lifshitz with corrections of order $(\hbar\omega/M_{\rm lock}c^2)^2$; Open Problem O7 is thereby Resolved, with the imported spectrum (T9.1; Arisaka, 2026, A2-GG6; Arisaka, 2026, P1-KK) and the machine optics inputs (R2) as the declared input classes. This section accordingly claims a theorem-grade pipeline from landscape to kernel, with its imports named. The electron-side deployment of the same two-sector discipline — why the electron's track bends without descending, and what its radiation prices — is given in the companion electron paper (Arisaka, 2026, P3-Electron); the GRIP-vocabulary reading of this section — the fixed landscape, the derived mobility as the within-basin flow, the GFF-ST7 halt at the equilibrium emittance, and the inspiral–merger–ringdown waveform as the GRIP loop drawn end to end — is Example 4 of the pillar paper (Arisaka, 2026, A6-GRIP).

Bridge to Part VII. A century after Kaluza, two centuries after Newton, and one century and a half after Coulomb, the geometric unification that Einstein spent his life searching for is

constitutionally complete. The six-dimensional bulk of GG-Theory contains, as boundary projections of a single composite connection, both Newton's law of universal gravitation and Coulomb's law of electrostatic interaction, with the inverse-square structural parallelism a manifest consequence of their common geometric origin. Part VII will now connect the boundary law to its formal companions in the GDOS Trio — the four Gi-4 pillars of GG-A6-GRIP and the four Qi-4 pillars of GG-A7-Qi4 — closing the foundational architecture of the program.

Problems and worked solutions. VI.2.1. Run the radiative budget of a real machine: for a 100-GeV electron ring of bending radius 3096 m (the LEP geometry), compute the synchrotron energy loss per turn and the damping time, and assign each to its T5 sector. *Solution.* Step 1: the loss per turn. $U_0 = \frac{4\pi}{3}\, r_e\, m_e c^2\, \gamma^4/\rho$ with $\gamma = E/m_e c^2 = 1.96 \times 10^5$: $U_0 = 2.86$ GeV per turn (verified) — at 100 GeV the machine radiates away nearly 3% of the beam energy every revolution, which is why LEP was the last ring of its kind. Step 2: the damping time. The revolution time is $T_{\rm rev} = 26.66\ \text{km}/c = 88.9\ \mu$s, so energy damps in $E/U_0 = 35$ turns, i.e. $\tau_d = 3.1$ ms (verified). Step 3: the sector assignment. The revolution is the *circulation* sector of the T5 current (conservative, symplectic, period 88.9 μs); radiation damping is the *gradient* sector (dissipative, timescale 3.1 ms); quantum emission shot noise is the *diffusion* sector, and the equilibrium beam emittance is their Lyapunov balance — the storage ring is the T5 decomposition running at the kilowatt scale. Physics intuition: accelerator physicists solved the boundary law's sector structure empirically decades ago; the constitution explains why their damping-diffusion bookkeeping worked — it is the unique decomposition the law admits.

VI.2.2. Why gravitational radiation starts at quadrupole while electromagnetic radiation starts at dipole — derive both selection rules from conservation laws, and connect to the binary-pulsar confirmation. *Solution.* Step 1: the electromagnetic case. The dipole radiation amplitude is set by $\ddot{\vec{d}}$ with $\vec{d} = \sum q_i \vec{r}_i$; nothing conserves $\vec{d}$'s second derivative, so dipole radiation is generically allowed — an antenna works. Step 2: the gravitational case. The mass dipole is $\vec{d}_m = \sum m_i \vec{r}_i = M\vec{R}_{\rm cm}$, and $\ddot{\vec{d}}_m = \dot{\vec{P}} = 0$ by momentum conservation; the would-be magnetic-dipole analog is the angular momentum, also conserved. The first multipole free to radiate is the quadrupole Q_{ij} — gravitational radiation is weaker not by accident but because its lowest channels are eaten by the very conservation laws (T8.3) that the radiation must respect. Step 3: the confirmation. The Hulse–Taylor binary's orbital decay (Hulse and Taylor, 1975) matches the quadrupole prediction to better than half a percent over four decades — the radiative sector of the gravitational chain, confirmed at the same 10^{-3} register where storage rings confirm the electromagnetic one. Physics intuition: the parallel chains of this Part radiate in parallel too, one multipole apart, and the offset is itself a conservation-law theorem — even the *differences* between the two long-range forces are inherited bookkeeping.

59.7 Part VI in plain English

Einstein spent his last decades trying to unify gravity and electromagnetism by geometrizing the electromagnetic field. This Part exhibits the resolution the other way round: Newton's inverse-square

law and Coulomb's inverse-square law descend in parallel, from one composite connection on one bulk, by the same zero-mode argument — and the radiation the two laws emit, synchrotron light and gravitational-wave inspiral, is one descent with one current decomposition. The two oldest force laws in physics are siblings, and their shared parent is the geometry of the sixth dimension.

Part VII
The GDOS–Gi-4–Qi-4 Bridge

Theorems T10 and T11 — Connecting GDOS to the Geometric and Quantum-Information Pillars of the GDOS Trio

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This paper is one of three that share a spine. The present volume derives the boundary law; a companion reads the same law geometrically, in terms of four structural pillars; another reads it in the language of information, in terms of four more. Three books, one result, three vocabularies — which is a good arrangement for readers and a dangerous one for authors, because vocabularies drift and a claimed correspondence can quietly become a metaphor.

This Part exists to prevent that. It proves, pillar by pillar, that the eight structures the companion papers work with are derived consequences of the boundary law rather than parallel assumptions about it. Each identification is at the level of a theorem, with the source result in this paper named on one side and the companion section named on the other, so that the correspondence can be checked from either end and would fail visibly if it were wrong.

It is also where the program's reach beyond conventional physics enters, and it should be entered carefully. One of the results connects energy and information by a relation of the same character as Landauer's bound, and the companion paper follows that connection into territory — living systems, and eventually cognition — where physicists are right to be cautious. Whatever is claimed there rests on what is proved here, at theorem level, and the reader is invited to hold the later claims to exactly this standard and no lower.

A reader following only this paper may treat the Part as optional scaffolding. A reader following the three-paper program should treat it as the load test.

60 Why a Bridge Section is Structurally Necessary

The axiomatic architecture of the GDOS Trio is a three-paper structure. The present paper (GG-A5-GDOS) supplies the boundary law GDOS and derives the textbook content of 4D physics from it; the companion papers GG-A6-GRIP (Gi-4 Pillars) and GG-A7-Qi4 (Qi-4 Pillars) refine GDOS into two formal backbones that supply the formal vocabulary used by every downstream paper in the GG-Theory program. Without an explicit bridge between the present paper and its two companions, a reader who consults only one of the three would not see the structural coherence of the trio. In Part VII, we shall supply that bridge in two principal theorems: T10 connects GDOS to the four *Geometric Invariance* pillars of GG-A6-GRIP, and T11 connects GDOS to the four *Quantum Information* pillars of GG-A7-Qi4. These two bridges must hold for the trio to be a single constitutional object rather than three loosely related papers.

The two bridge theorems are not derivations of new physics; they are *architectural identifications*. T10 says: "the four Gi-4 pillars are derived structures of GDOS together with the GG-6 bulk; here is

the explicit identification of each pillar with the corresponding sub-result of T1–T9 of the present paper." T11 says the same for the four Qi-4 pillars. Together, T10 and T11 show that the GDOS Trio (GG-A5-GDOS, GG-A6-GRIP, GG-A7-Qi4) is internally coherent: every pillar in either companion paper has a precise constitutional source in the boundary law derived in the present paper.

60.1 What the Gi-4 and Qi-4 pillars are

The four-plus-four pillar architecture is defined in Arisaka (2026, A6-GRIP) and Arisaka (2026, A7-Qi4) and is mapped one-by-one to derived structures of GDOS + GG-6 in Table 16; the reader who has not yet read GG-A6-GRIP or GG-A7-Qi4 will find each pillar's definition there. The two bridge theorems below identify each of these eight pillars as a derived structure of GDOS together with the GG-6 bulk.

61 Theorem T10 (T-GDOS-10) — The GDOS-to-Gi-4 Bridge

61.1 Setup and statement

Theorem T10 identifies the four Gi-4 pillars of GG-A6-GRIP as specific derived structures of GDOS (T1–T6 of Part II) together with the GG-6 bulk (Structure Theorems ST1–ST4 of Part I). The identification is executed pillar-by-pillar.

Lemma 28.1 (TGG from T8.3 + T8.4 + T9.7)**.** *The Total Geometric–Gauge object* TGG *of GG-A6-GRIP Part II, defined as the unified bulk-and-boundary stress-tensor* $T^{\mu\nu}_{\mathsf{TGG}} = T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\text{gauge}}[F] + X^{\mu\nu}_{\text{GS}} + T^{\mu\nu}_{\text{KK}}$ *with master conservation* $\nabla_\mu T^{\mu\nu}_{\mathsf{TGG}} = 0$*, is the explicit collation of the four conserved currents derived in T8.3 (boundary diffeomorphism Ward identity), T8.4 (modified Einstein equation with GS+KK inflow), and T9.7 (four boundary Ward identities) of the present paper.*

Proof. T8.3 derives the conservation $\nabla_\mu T^{\mu\nu}_{\text{total}} = 0$ for the total boundary stress tensor. T8.4 supplies the explicit form of the GS-tensor and KK-tower contributions. T9.7 supplies the four conserved currents (em, B, Qi, metric). The collation of these four results is exactly $T^{\mu\nu}_{\mathsf{TGG}}$ of GG-A6-GRIP Theorem T1.1–T1.3 (Arisaka, 2026, A2-GG6). ■

Lemma 28.2 (CCI from T9.6 + T9.7)**.** *The Covariant Conservation Identity* CCI *of GG-A6-GRIP Part III is identified with the BI-6 anomaly inflow (T9.6) plus the four boundary Ward identities (T9.7), with the physical/local formulation matching the modified TOV equation of T8.6 and the algebraic/global formulation matching the differential-cohomology integrality* $[X_4] \in H^4(M_6;\mathbb{Z})$ *derived in GG-A3-BI6 (Arisaka, 2026, A3-BI6).*

Proof. The two equivalent formulations of CCI in GG-A6-GRIP Theorem T2.1 (algebraic/global) and Theorem T2.2 (physical/local) are recognized respectively as: (T2.1) the rank-one factorization of the BI-6 anomaly polynomial from T9.6, equivalent to the integrality of $[X_4]$ on M_6; (T2.2) the modified TOV equation from T8.6, with the explicit PBI-4 compensator scalar. The Hopkins–Singer differential-cohomology descent identified in GG-A6-GRIP is the bridge between the two. ■

Lemma 28.3 (GFF from T9.2)**.** *The Geometric Free-Energy Functional* GFF = GEP + CEP *of GG-A6-GRIP Part IV is identified, at its energetic-block (*GEP*) layer, with the bulk-to-boundary descent of the bulk action $S^{(6)}_{\mathrm{tot}}$ through Wilson-line holonomy on the compact S^1_θ, executed in T9.2 of the present paper as heavy-mode integration; the curvature-entropic-block (*CEP*) layer is identified with the entropy-production rate of T5 of Part II applied to the family-projector boundary moduli.*

Proof. GEP is defined in GG-A6-GRIP's GEP/CEP decomposition section as the energetic part of the GFF inherited from the bulk action via Wilson-line holonomy; T9.2 of the present paper supplies the explicit heavy-mode-integrated form $S_{4,\mathrm{eff}}$ from which the energetic part of GFF is read off. CEP is defined in the same section of GG-A6-GRIP as the entropic-curvature contribution from the boundary projector volume; the entropy-production rate $\dot{S}_{\mathrm{tot}}$ of T5 (53) is exactly the structural template that CEP specializes. ■

Lemma 28.4 (GRIP from T6 (GDOS) + T9.7 (Ward identities))**.** *The Gauge-Runtime Invariance Pipeline* GRIP *of GG-A6-GRIP Part V, the universal three-stage cascade DGI → GSSB → GA (terminus refined to the Ground Geometric Attractor,* GGA*, in GG-A6-GRIP) on the GFF Morse landscape, is identified with the long-time GDOS flow of T6 (Part II) on the boundary moduli, with each cascade stage preserving the four boundary Ward identities of T9.7 (the "gauge-invariance preservation" content of GRIP). The cascade-closure constraint $S_{\mathrm{tot}}(N_g) \le S^{\max}_{\text{BI-6}}(N_g)$, which yields $N_g \le 3$, rests on the BI-6 instanton ('t Hooft, 1976) action budget — the capacity theorem T-N3-6 of Arisaka (2026, P2-N3), conditional on its coupling clause and the integrality lemma L-INT, and consumed as an input by GG-A6-GRIP's cascade-closure theorem T4.1 (Arisaka, 2026, A6-GRIP). Everything in this lemma downstream of that constraint inherits its conditionality. In the regime vocabulary of GG-A6-GRIP, the identification is regime-specific: the long-time T6 flow* is *GRIP-In — gradient descent plus instanton stitching on a fixed projected landscape — whereas GRIP-Out events, which reorganize the boundary moduli themselves, sit outside the T6 flow and enter GDOS only as re-preparations of its boundary data.*

Proof. The GDOS continuity equation T5 supplies the dynamical flow on the boundary moduli; the long-time fixed points of this flow are the critical points of the GFF Morse landscape (Lemma 28.3), which are the Geometric Attractors (GAs) of the GRIP cascade — among them, the realized long-time output on each stratum is the deepest, the GGA (GG-A6-GRIP; GG-A11-Origin T-GGA-as-Ground-State), unique on admissible strata and open elsewhere. The DGI stage corresponds to the unbroken-symmetry initial condition; the GSSB stage to the symmetry-breaking saddles connecting them. The four boundary Ward identities T9.7 are preserved at every stage of the cascade by the BI-6 inflow protection (Lemma 26.1), giving the gauge-invariance-preservation content of GRIP. It remains to say where the closure constraint comes from, and with what standing. The budget is derived by Arisaka (2026, P2-N3) — its capacity theorem T-N3-6 — in the J-free per-family form

$$S^{\max}_{\text{BI-6}}(N_g) \;=\; \frac{2\pi N_g}{L}\cdot\frac{L}{2} \;=\; \pi N_g, \tag{195}$$

the cell flux $2\pi N/L$ of the descended anomaly class scaling with N by the Pontryagin relation $p_1(\mathbb{CP}^N) = Nh^2$, and the conversion factor $L/2$ being the inverse of the BI-6 trace-ratio lock.

Against it the cascade demand is the chain sum of the step actions $S_k = \pi k$ over the $N_g - 1$ saddle transitions,

$$S_{\mathrm{tot}}(N_g) \;=\; \sum_{k=1}^{N_g-1} \pi k \;=\; \frac{\pi N_g(N_g-1)}{2}, \tag{196}$$

so that closure $S_{\mathrm{tot}}(N_g) \le S^{\max}_{\mathsf{BI\text{-}6}}(N_g)$ reads $\pi N_g(N_g-1)/2 \le \pi N_g$, i.e. $N_g \le 3$, with exact saturation at $N_g = 3$ and no reference to J at any step.

Two cautions belong in the open, because the step is load-bearing. *First*, the closure follows only with the demand curve (196) in hand; the budget alone bounds nothing, since an allowance growing linearly in N excludes no N by itself. *Second*, (195) is now the capacity theorem T-N3-6 of Arisaka (2026, P2-N3): derived from the family stratum's quantum geometry (Berry/Fubini–Study, at Born-grade premises), conditional on the coupling clause and the integrality lemma L-INT, and consuming neither $K = 35$ nor $J = 7$ — which meets the N-independence demand GG-A6-GRIP (Arisaka, 2026, A6-GRIP) records as its open problem M10. GG-A3-BI6 (Arisaka, 2026, A3-BI6) quotes the value in its cross-reference ledger. The present lemma inherits exactly that conditioning: the identification of GRIP with T6 + T9.7 is unconditional, and only the $N_g \le 3$ closure rides on the imported premises. ■

Theorem 29 (T10 — GDOS-to-Gi-4 Bridge)**.** *The four* Geometric Invariance *pillars* TGG *(Total Geometric–Gauge object: the unified bulk stress tensor),* CCI *(Covariant Conservation Identity: the master four-current conservation law),* GFF *(Geometric Free-Energy Functional: the scalar ledger of energy, entropy, and free energy), and* GRIP *(Gauge-Runtime Invariance Pipeline: the DGI→GSSB→GA cascade, terminating on the* GGA*) of GG-A6-GRIP are derived structures of* GDOS *together with the GG-6 bulk. Specifically:*

(1) TGG *is identified with T8.3 + T8.4 + T9.7 by Lemma 28.1;*

(2) CCI *is identified with T9.6 + T9.7 by Lemma 28.2;*

(3) GFF *is identified with T9.2 (energetic block) and T5 (entropic block) by Lemma 28.3;*

(4) GRIP *is identified with T6 + T9.7 by Lemma 28.4.*

Proof. Combination of Lemmas 28.1, 28.2, 28.3, 28.4. ■

Problems and worked solutions. VII.1.1. The free-energy pillar in action: show that $F[p] = \int \Phi\, p + D \int p \ln p$ is a Lyapunov functional of the T5 flow — monotonically decreasing, with stationarity exactly at the Boltzmann state. *Solution.* Step 1: differentiate along the flow. Using the continuity equation and integrating by parts, $\dot F = \int (\Phi + D\ln p + D)\, \partial_t p = \int \nabla_A(\Phi + D\ln p)\, J^A$. Step 2: recognize the current. The dissipative current is exactly $J^A_{\mathrm{diss}} = -p\,\nabla^A(\Phi + D\ln p)$ (gradient of the free-energy density), so its contribution is $\dot F\big|_{\mathrm{diss}} = -\int \frac{J^2_{\mathrm{diss}}}{p} \le 0$, while the circulation contributes $\int h^A p\, \nabla_A(\Phi + D\ln p) \cdots = 0$ after one more integration by parts using $\nabla_A h^A = 0$ and $h \perp \nabla\Phi$. Step 3: stationarity. $\dot F = 0$ iff $J_{\mathrm{diss}} = 0$ iff $p \propto e^{-\Phi/D}$: the Boltzmann state is the unique free-energy minimum, approached monotonically. This functional is the energetic content of the

GFF pillar that T10 identifies. Physics intuition: "systems minimize free energy" is folklore in chemistry, principle in thermodynamics, and here a two-line consequence of the boundary law's current decomposition — the folklore had a theorem underneath all along.

VII.1.2. The GRIP pillar in miniature: for the overdamped double-well $\Phi = (x^2-1)^2$ with initial Gaussian preparation ($x_0 = 0.2$, $\sigma = 0.5$), compute the attractor capture probabilities in the small-noise limit. *Solution.* Step 1: the attractor structure. The deterministic flow $\dot{x} = -\Phi'(x)$ has two Geometric Attractors at $x = \pm 1$ and an unstable fixed point at $x = 0$: the basin boundary. Step 2: capture as basin measure. As $D \to 0$, every trajectory follows the deterministic flow, so the capture probability of the + attractor is the initial measure of its basin: $P(+1) = \int_0^\infty p_0(x)\,dx = \Phi_{\text{Gauss}}(x_0/\sigma) = \Phi_{\text{Gauss}}(0.4) = 0.655$ (verified); $P(-1) = 0.345$. Step 3: the GRIP reading. This is the DGI $\to$ GSSB $\to$ GA cascade in one dimension: the symmetric law (the double well) plus an asymmetric preparation descends to a definite outcome with probability equal to basin measure — the structural skeleton of how a measurement's single result emerges from the GDOS flow, with the outcome statistics carried by the geometry of basins rather than by a postulate. Physics intuition: a ball on a ridge, a protein folding, a pointer settling — one mathematics; T10's claim is that the fourth instance, quantum measurement's actualization, is the same descent run in the state manifold.

Take-away. *Standard account:* conservation laws, free-energy bookkeeping, and relaxation to attractors appear as separate structures scattered across mechanics, thermodynamics, and dynamical systems. *What it left open:* whether they share a source. *The constitutional derivation:* the four geometric pillars of GG-A6-GRIP — TGG, CCI, GFF, GRIP — are identified, pillar by pillar, with structures already proved in this paper (T10): the geometric reading of physics is the boundary law seen from the geometry side.

62 Theorem T11 (T-GDOS-11) — The GDOS-to-Qi-4 Bridge

62.1 Setup and statement

Theorem T11 identifies the four Qi-4 pillars of GG-A7-Qi4 as specific derived structures of GDOS (T1–T6 of Part II) together with the GG-6 $\mathrm{U}(1)_{Qi}$ informational gauge factor. Parallel to T10, the identification is executed pillar-by-pillar.

Lemma 29.1 (Qi-QUA from T9.7 + the $\mathrm{U}(1)_{Qi}$ Ward identity)**.** *The Qi-QUA pillar of GG-A7-Qi4 (current locking, Arisaka Ratio, Arisaka Equation $E = T_{Qi}\log 2$) is identified with the $\mathrm{U}(1)_{Qi}$ Ward identity derived in T9.7 of the present paper, plus the $\mathrm{U}(1)_{Qi}$ gauge boson identified in GG-A2-GG6 Theorem 3. The Arisaka temperature T_{Qi} is the energy-scale ratio of the $\mathrm{U}(1)_{Qi}$ current to the $\mathrm{U}(1)_B$ current in the bulk, which is the boundary realization of a specific overlap integral computed in the Qi-QUA Part of GG-A7-Qi4 (Arisaka, 2026, A7-Qi4). The schematic content of the overlap integral is*

$$T_{Qi} \;=\; \frac{|c|}{|a|}\,\frac{E_{\text{Ryd}}}{K} \;=\; \frac{2}{5}\,\frac{E_{\text{Ryd}}}{K}, \tag{197}$$

where $|d|/|a| = 2/5$ is the BI-6 trace ratio between the gravitational/energy-sector coefficient a and the informational-sector coefficient d (the trace-ratio theorem T-BI6-9.1 of Arisaka (2026, A3-BI6)),

$E_{\mathrm{Ryd}} \approx 13.6$ eV is the atomic boundary scale of the low-energy electromagnetic sector — the hydrogen ground-state energy of standard atomic physics, whose constitutional entry point into the boundary energy–information chain is the Arisaka-equation sector (Theorem 8 of GG-A2-GG6 (Arisaka, 2026, A2-GG6), with the boundary calculus of GG-A7-Qi4) — and $K = 35$ is the OGN-5 dual-lock integer. Numerically, $T_{Qi} \approx (2/5) \cdot (13.6/35) \approx 0.155$ eV, identifying T_{Qi} with the Arisaka constant Q_B of GG-A7-Qi4 (Arisaka, 2026, A7-Qi4). The Arisaka equation $E = T_{Qi} \log 2$ then states that the energy cost of one bit of information is $T_{Qi} \log 2 \approx 0.107$ eV at biological boundary conditions.

Proof. The $\mathrm{U}(1)_{Qi}$ Ward identity $\partial_\mu J^\mu_{Qi} = 0$ from T9.7 identifies J^μ_{Qi} as the conserved informational current. The Arisaka Equation $E = T_{Qi} \log 2$ is the constitutive relation between the $\mathrm{U}(1)_{Qi}$ charge unit (one "qubit" of information) and its energy carrier; the schematic numerical value Eq. (197) follows from the BI-6 trace ratio 2/5 (the magnitude of the informational-sector inflow coefficient relative to the energy-sector coefficient) combined with the per-step quantum E_{Ryd}/K derived from the OGN-5 ledger; the full derivation and the order-unity overlap correction factor of the $\mathrm{U}(1)_{Qi}$ zero mode are recorded in GG-A7-Qi4 Section 7. ■

Lemma 29.2 (Qi-EP from T5 (continuity equation))**.** *The Qi-EP variational gauge tower PLA → {PLT, MEPP, FEP, GFF} of GG-A7-Qi4 Section 8 is identified with three Legendre-transform projections of the GDOS continuity equation T5 of the present paper. PLT (Principle of Least Time) is the trajectory-gauge form; MEPP (Maximum Entropy Production Principle) is the current-gauge form; FEP (Free Energy Principle) is the inferential-gauge form. All three project to the same GFF energetic block in their long-time limit.*

Proof. T5 supplies the continuity equation $\partial_t p = -\nabla_A J^A$ with current decomposition $J^A = -D^{AB}\nabla_B p + (-g^{AB}\nabla_B \Phi + h^A)p$. Three gauge choices for the path-integral over trajectories (least time along trajectory; maximum entropy production along current; inferential free-energy minimization) produce the three classical extremal principles, all with the same GFF energetic block at the long-time fixed point. See GG-A7-Qi4 Section 8.7 for the explicit unifying-functional theorem. ■

Lemma 29.3 (Qi-CON from T7.5 (decoherence) + RG flow on bulk)**.** *The Qi-CON pillar of GG-A7-Qi4 Section 9 (conformality, Lévy statistics, fractality, $1/f$ spectra with two knees) is identified with the long-time decoherence content of T7.5 of the present paper plus the scale-bulk RG flow $r = \log(E_{\mathrm{GUT}}/\mu)$ derived in GG-A2-GG6. The two "knees" in the $1/f$ spectrum correspond to the UV boundary ($r = 0$) and the IR boundary ($r = K$) thresholds respectively.*

Proof. T7.5 supplies the off-diagonal damping with rates $\Gamma_{\alpha\beta}$ inherited from KK overlap integrals. At mesoscopic scales, the long-time correlations decay as power laws with exponents determined by the scale-bulk RG flow; the homogeneous boundary kernel produced by the scale profile e^{-2r} is the constitutional source of the conformal-like scaling that GG-A7-Qi4 Section 9 catalogs as Lévy/fractal/$1/f$ signatures. The two boundary thresholds give the two characteristic frequencies that delimit the $1/f$ regime. ■

Lemma 29.4 (Qi-HAL from T6 (canonical GDOS) + T9.7 + persistence)**.** *The Qi-HAL pillar of GG-A7-Qi4 Section 10 (MePMoS, NHT, HAL, BFFT, GRIP-loop — the memory-stack architecture)*

is identified with the canonical GDOS boundary law T6 specialized to systems with persistent $\mathrm{U}(1)_{Qi}$ *current (T9.7), with the cascade structure of GRIP (Lemma 28.4) supplying the loop closure. The "memory stack" is the hierarchy of metastable basins of the GFF Morse landscape (Lemma 28.3) along which a persistent open system can store and replay state information.*

Proof. A persistent open system in the GG-Theory framework is one whose $\mathrm{U}(1)_{Qi}$ current (T9.7) is non-trivially populated; T6 supplies the canonical boundary law; the GFF Morse landscape supplies the metastable basin structure (Lemma 28.3); the GRIP cascade Lemma 28.4 supplies the dynamical pipeline that connects basins. The MePMoS / NHT / HAL / BFFT / GRIP-loop constructions of GG-A7-Qi4 Section 10 are specific specializations of this general structure to neuroscience and biology applications. ■

Theorem 30 (T11 — GDOS-to-Qi-4 Bridge)**.** *The four* Quantum Information *pillars* (Qi-QUA, Qi-EP, Qi-CON, Qi-HAL) *of GG-A7-Qi4 are derived structures of* GDOS *together with the GG-6* $\mathrm{U}(1)_{Qi}$ *informational gauge factor. Specifically:*

(1) Qi-QUA *is identified with the* $\mathrm{U}(1)_{Qi}$ *Ward identity from T9.7 by Lemma 29.1;*

(2) Qi-EP *is identified with three gauge projections of T5 by Lemma 29.2;*

(3) Qi-CON *is identified with T7.5 plus scale-bulk RG flow by Lemma 29.3;*

(4) Qi-HAL *is identified with T6 + T9.7 + GRIP-loop closure by Lemma 29.4.*

Proof. Combination of Lemmas 29.1, 29.2, 29.3, 29.4. ■

Problems and worked solutions. VII.2.1. The Arisaka equation across three laboratories: evaluate $E = T_{Qi} \log 2$ at room temperature, at liquid-nitrogen temperature, and at a dilution-refrigerator readout stage, and compute the resulting power floor of a gigahertz processor. *Solution.* Step 1: the three temperatures (verified). $T = 300$ K: $E = k_B T \ln 2 = 17.9$ meV; $T = 77$ K: 4.60 meV; $T = 10$ mK: $0.60\,\mu$eV — five orders of headroom bought by refrigeration, which is why superconducting quantum processors live at the bottom of the ladder. Step 2: the power floor. A processor erasing 10^9 bits per second at 300 K dissipates at least $10^9 \times 2.87 \times 10^{-21}$ J $= 2.9$ pW (verified) — some nine orders below actual CPU budgets, so present engineering wastes energy on transport, not on erasure; the floor becomes practically relevant only near reversible-computing regimes. Step 3: the T11 reading. In the bridge theorem the relation is not an analogy imported from thermodynamics: erasure exports one unit of $\mathrm{U}(1)_{Qi}$ charge across the observer's boundary, and $T_{Qi} \log 2$ is the Ward identity's exchange rate — the same constitutional bookkeeping that conserves electric charge prices the bit. Physics intuition: the three numbers above are the same law read at three temperatures; what changes between a warm brain and a cold qubit is the exchange rate, never the ledger.

VII.2.2. The Qi-CON face as data-processing: verify the data-processing inequality on an explicit two-stage channel and state its T11 reading. *Solution.* Step 1: the chain. Let a fair bit X pass through a binary symmetric channel of flip probability $p_1 = 0.1$ to give Y, then a

second BSC(0.1) to give Z; the composition is BSC($p_1+p_2-2p_1p_2$) = BSC(0.18). Step 2: the mutual informations (verified). $I(X{:}Y) = 1 - H_2(0.1) = 0.531$ bits; $I(X{:}Z) = 1 - H_2(0.18) = 0.320$ bits: $I(X{:}Z) < I(X{:}Y)$, the data-processing inequality in the flesh — processing cannot create information about the source, only lose it. Step 3: the T11 reading. Each boundary crossing is a CPTP map (T3), and CPTP maps can only degrade distinguishability; the informational pillar's conservation-plus- degradation structure is the quantum lift of this computation, with the lost information accounted as bulk-side charge rather than annihilated (the C3 companion runs the same ledger at a horizon). Physics intuition: rumors degrade through retelling and signals through repeaters for the same structural reason a measurement chain cannot out-know its source — the boundary law is a data-processing inequality with a geometry attached.

Take-away. *Standard account:* Landauer's principle and the thermodynamics of information stand as add-on principles, connected to fundamental physics by analogy. *What it left open:* why energy pays for bits. *The constitutional derivation:* the four quantum-information pillars of GG-A7-Qi4 are derived from GDOS plus the $U(1)_{Qi}$ gauge factor (T11), with the Arisaka equation $E = T_{Qi} \log 2$ as the operational face of the informational Ward identity — the price of a bit is a conservation law, not an analogy.

63 The Unified GDOS–Gi-4–Qi-4 Architecture

We now collect the bridge content of T10 and T11 into a single synthesis statement.

63.1 The eight-pillar map

Table 16 maps each of the eight Gi-4 + Qi-4 pillars to the specific sub-result(s) of the present paper that constitute its constitutional source.

63.2 The unified architecture

Combined with the eleven Principal Theorems T1–T11 of the present paper, the GDOS Trio supplies a single integrated foundational layer for the GG-Theory program:

- One *boundary law*: GDOS (T6 of Part II of the present paper).
- Two *formal backbones*: Gi-4 (geometric, in GG-A6-GRIP) and Qi-4 (informational, in GG-A7-Qi4).
- Eight *pillars* total (TGG, CCI, GFF, GRIP + Qi-QUA, Qi-EP, Qi-CON, Qi-HAL), each with a constitutional source in the boundary law (Table 16).
- Cross-domain applications: the same pillars apply to all open-system specializations from atoms to mind, by the universality of GDOS established in T6.

This is the architectural payoff of the GDOS Trio. An adversarial reader who reads only the present paper sees the constitutional core; an adversarial reader who reads GG-A6-GRIP or GG-A7-Qi4 sees a specific formal backbone of that core; the bridge theorems T10 and T11 ensure the two reading paths converge.

Table 16: The eight-pillar map. Each row takes one pillar of the two companion papers and names the specific result of the present paper that is its constitutional source, with the companion section given in the third column so the identification can be checked in both directions. This is the table that makes the bridge auditable: without it, the claim that the geometric and quantum-information pillars are derived rather than parallel postulates would rest on assertion. Every identification is at the level of a theorem; none is by analogy.

Pillar	Constitutional source in present paper	Companion-paper section
TGG	T8.3 + T8.4 + T9.7 (unified stress tensor + Ward identities)	GG-A6-GRIP Part II
CCI	T9.6 + T9.7 (BI-6 inflow + four Ward identities)	GG-A6-GRIP Part III
GFF	T9.2 (GEP) + T5 (CEP) (effective action + entropy production)	GG-A6-GRIP Part IV
GRIP	T6 + T9.7 (canonical GDOS + gauge-invariance preservation)	GG-A6-GRIP Part V
Qi-QUA (Qi-Quantitative: current locking)	T9.7 ($U(1)_{Qi}$ Ward identity) + Arisaka equation	GG-A7-Qi4 Section 7
Qi-EP (Qi-Extremal Principle: variational selector)	T5 (continuity equation) in three gauges	GG-A7-Qi4 Section 8
Qi-CON (Qi-Conformality: scale-covariant multiscale)	T7.5 (decoherence) + scale-bulk RG flow	GG-A7-Qi4 Section 9
Qi-HAL (Qi-Higher Architecture Layer: memory)	T6 + T9.7 + GRIP-loop closure	GG-A7-Qi4 Section 10

Bridge to Part VIII. With the GDOS-Trio bridges T10 and T11 in place, the axiomatic architecture of the GG-Theory program is fully exhibited at the boundary-law level. Eleven principal theorems have been proved; eight pillars have been collated; the formal machinery is complete. With the boundary law and its formal backbone in hand, Part VIII now steps back from the derivation to ask what the completed machinery *classifies*: it places general relativity, gauge theory, string theory, holography, quantum mechanics, thermodynamics, black-hole physics, the Standard Model, and ΛCDM in a single nested axiom tower, according to which subset of the four constitutional axioms each framework implicitly uses.

63.3 Part VII in plain English

Two bridge theorems finish the architecture. Everything conserved, budgeted, and steered in the geometric reading of physics — the stress ledger, the master conservation law, the free-energy functional, the attractor descent — is the boundary law seen from the geometry side (T10);

everything quantized, erased, and paid for in the informational reading — including the price of a bit, $E = T_{Qi} \log 2$ — is the same law seen from the information side (T11). One law, read twice; and with that, energy, information, and the descent that selects outcomes are entries in a single ledger.

Part VIII
The constitutional-axiom classification of 20th-century physics

General relativity, gauge theory, superstring theory, AdS/CFT, quantum mechanics, the Standard Model, and ΛCDM cosmology as theorems of the GG-Constitution, sorted by axiom subset

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Twentieth-century physics does not look like one subject. Relativity, gauge theory, string theory, holography, quantum mechanics, thermodynamics, the Standard Model, the standard cosmological model — these are separate literatures with separate conferences, separate textbooks, and separate famous unsolved problems. The usual account of why they differ is historical: they were built by different people solving different problems.

This Part offers a different account. Suppose each framework is sorted not by its subject matter but by which of a small number of constitutional principles it implicitly relies on. Something orderly appears. Special relativity uses one. General relativity, gauge theory, string theory and holography all use the same two — which is why they share so much machinery despite describing such different things. Quantum mechanics, thermodynamics, statistical mechanics, black-hole physics and the fluctuation theorems all stand on the same third, the principle that an observer inside the world sees only part of it, which is why formulae keep reappearing between fields that have no obvious connection. The Standard Model and the standard cosmological model occupy a fourth level, where the freedom that remains is finally removed.

The classification earns its keep in the column that records what each framework *cannot* say. In case after case, the missing principle predicts the missing content exactly — and the persistent open problem of each field turns out to be the shape of the axiom it does not have. The measurement problem is the shape of one; the landscape problem is the shape of another.

Two warnings. This is a claim about logical dependency, not about priority or merit: every framework in the table produced what it produced, and the classification takes nothing away from it. And the claim is exhibited rather than asserted — there is a predicate, it is run on each framework, and the reader is meant to check the table rather than take it.

64 The central claim, stated once

This Part stands a level above all that has come before. Parts I–III have derived the open-system boundary law GDOS from the four constitutional axioms and have shown that the textbook postulates of quantum mechanics are theorems of that boundary law via the chain T7.1–T7.7. Parts IV–VII have derived general relativity, quantum field theory, and the unification of GR with QED from the same constitutional source. The present Part observes something deeper: *the same*

four constitutional axioms, when applied with different subsets active, are the source of essentially every major theoretical-physics framework of the twentieth century.

The framework historically known as "general relativity" lives at the AX-1 + AX-2 level. The framework historically known as "superstring theory" lives at the AX-1 + AX-2 level. The framework historically known as "AdS/CFT correspondence" lives at the AX-1 + AX-2 level. The framework historically known as "quantum mechanics" lives at the AX-1 + AX-2 + AX-3 level. The framework historically known as "Standard Model with ΛCDM cosmology" lives at the AX-1 + AX-2 + AX-3 + AX-4 level. Each historical framework is, structurally, what the GG-Constitution looks like when a particular subset of its four axioms is taken as operative and a particular bulk solution is chosen as the realization.

This is a Copernican reframing. It does not contradict the historical derivations; it locates them. Just as Riemann's nineteenth-century geometry was correctly understood as pure mathematics for sixty years before Einstein identified the physical setting (general relativity) in which it became operative, the twentieth-century geometric-machinery edifice — the Bianchi identities, the Green–Schwarz mechanism, the modular bootstrap, the AdS/CFT correspondence, the holographic principle, the dualities — has been correctly developed by the superstring community for forty years without recognition of the constitutional setting (the GG-Constitution with various axiom subsets) in which the machinery becomes operative. The present Part identifies that setting.

The central structural claim, stated in one sentence:

The central structural claim of Part VIII. *Every major theoretical-physics framework of the twentieth century is a theorem of a particular subset of the four GG-Constitution axioms applied to a particular bulk solution; the framework's strengths and limitations are direct consequences of which axioms it implicitly uses and which it lacks.*

The remainder of this Part develops the classification in full detail. We begin with the formal statement of the constitutional tower (§65), then walk each layer of the tower in turn: AX-1 alone (§66); AX-1 + AX-2 in its general form (§67) and its specializations to GR (§67.1), Yang–Mills (§67.2), superstring theory (§67.3), and AdS/CFT (§67.4); AX-1 + AX-2 + AX-3 as the constitutional source of QM (§68); and AX-1 + AX-2 + AX-3 + AX-4 as the constitutional source of the SM and ΛCDM (§74). We then state the classification as a single ledger table (§75); enumerate the quantitative consequences (§76); analyze the historical contributions of the superstring community in retrospect (§77); and close with the open questions (§78) and the Part-end synthesis (§79).

After all, the deepest move a foundational theory can make is to take everything the predecessor community got right, locate it inside a larger constitutional picture, and use the same picture to fill the gaps the predecessor could not address. The constitutional classification we are about to develop is precisely that move, applied to the entire twentieth-century edifice of theoretical physics.

65 The constitutional tower: a formal statement

We begin with a precise statement of the four-axiom tower. The four axioms of the GG-Constitution (developed in detail in GG-A1-Constitution) are recalled here in compact form, with

explicit indication of what each axiom contributes operationally.

The four axioms of the GG-Constitution.

- **AX-1 (LC, Local Causality):** Physical influence propagates forward in time at finite speed. The 6D bulk admits a causal structure $(\mathcal{M}_{\text{bulk}}, g, \prec)$ with light cones and a well-defined past/future at every point.
- **AX-2 (GGEP, Geometry–Gauge Equivalence Principle):** The bulk geometry and the bulk gauge fields are a single composite connection on the 6D manifold $\mathcal{M}_6 = \mathcal{M}_4 \times I_r \times S^1_\theta$, not two independent structures. Curvature 2-forms of geometry (R) and gauge (F) are algebraic facets of one underlying connection 1-form.
- **AX-3 (OGGEP, Operational/Open GGEP):** Any admissible bulk theory must furnish a structure-preserving functor $\mathcal{D}_{\mathsf{OGGEP}}$: **Bulk** $\to$ **Op** from inaccessible bulk data $(\mathcal{H}_{\text{Bulk}}, \rho_{\text{Bulk}}, U_{\text{Bulk}})$ to registered boundary observables $(\mathcal{H}_{\text{Bdy}}, \rho_{\text{Bdy}}, \Phi_{t,s}, \{\mathcal{I}_\alpha\})$, compatible with gauge covariance, causality, positivity, and scale consistency. The functor encodes the operational coarse-graining a finite 4D observer is forced to perform.
- **AX-4 (MSMF, Maximum Symmetry, Minimum Freedom):** Among all bulk theories satisfying AX-1–AX-3, the admissible one is the unique solution with maximal symmetry and minimal free continuous parameters. In the cohort canonical realization, this forces the integer OGN-5 ledger and zero free continuous parameters in the bulk theory.

Figure 9 (next page, in landscape orientation).

The comprehensive flowchart on the following page presents the logical cascade from the four-axiom GG-Constitution (far left) to the entire family of nineteenth- and twentieth-century theoretical physics frameworks. The four constitutional axioms are stacked vertically in the left column; each is horizontally aligned with the stratum of the constitutional tower it activates: AX-1 (local causality) with Stratum 1; AX-2 (GGEP, the geometry–gauge equivalence principle) with Stratum 2; AX-3 (OGGEP, the operational/open extension) with Stratum 3; AX-4 (MSMF, the maximum-symmetry / minimum-freedom selection) with Stratum 4. Each stratum activates exactly one more axiom than the previous and admits strictly more theorem-content.

Stratum 1 (AX-1 only) produces special relativity (1905) and classical electrodynamics on Minkowski. *Stratum 2* (the GGEP-closed layer, AX-1 + AX-2) produces general relativity (1915), Yang–Mills theory (1954), the Green–Schwarz anomaly-cancellation mechanism (1984), superstring theory, the family of holographic dualities, and the AdS/CFT correspondence (1997). *Stratum 3* (the OGGEP-open layer, AX-1 + AX-2 + AX-3) hosts the three constitutional *siblings of openness* — quantum mechanics (1925–1932), classical thermodynamics (1824–1872) with statistical mechanics (1902), the fluctuation theorems, and Friston's free-energy principle (2010); and black-hole physics (1973+) with black-hole thermodynamics, the Page curve (1993), and the islands formula (2020) — all as theorems of the same boundary CPTP channel acting on different boundary observables. The QM measurement problem, the thermodynamic irreversibility puzzle, and the black-hole information

paradox dissolve simultaneously at this stratum because each is the local-boundary trace of a different bulk-side inaccessibility (scale and phase directions of the GG-6 bulk; microstates; event horizon respectively). *Stratum 4* (the Full-Constitution layer, AX-1 + AX-2 + AX-3 + AX-4) activates the maximum-symmetry / minimum-freedom selection and produces the Standard Model with its twenty-eight fitted parameters reduced to zero free continuous parameters, the ΛCDM standard cosmological model with $\Omega_m = \pi^{-1}$, and the cohort cascade from quarks to consciousness.

At the foot of the diagram, the GG-Theory summit box names the cohort framework that the present paper and its ten companion papers collectively articulate; below it sit the three consequence papers that elaborate the boundary side in detail — GG-A9-SM (Standard Model parameters as theorems), GG-A10-Cosmos (cosmological inventory and the Hubble corridor as theorems), and GG-A11-Origin (the constitutional-origin and GRIP-cascade synthesis from quarks to articulation). Together with the foundation block (GG-A1-Constitution through GG-A7-Qi4) and the open quantum mechanics paper (GG-A8-QM), they constitute the cohort framework whose constitutional content is the subject of the present GG-A5-GDOS.

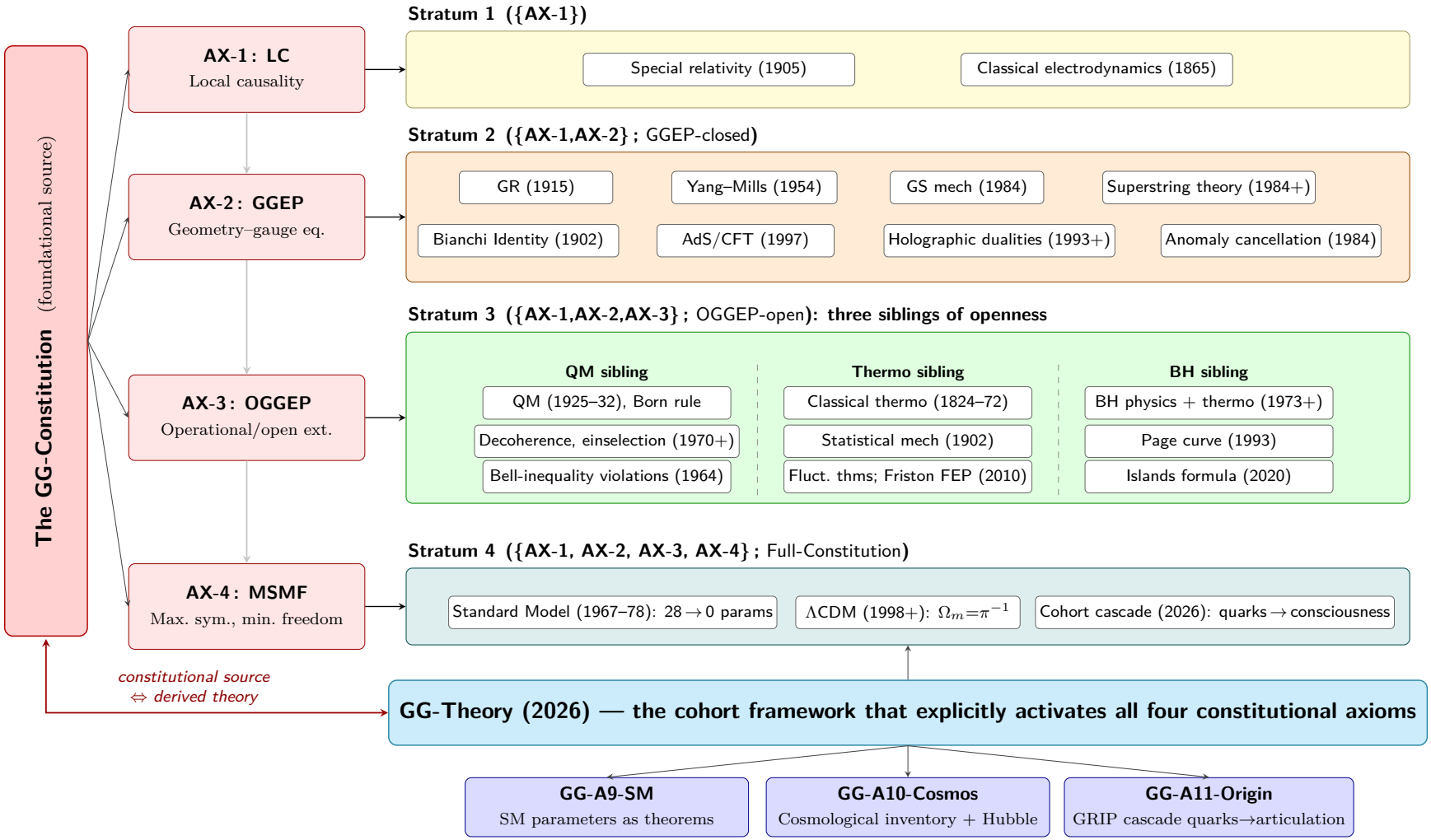


Figure 9: The constitutional classification, drawn as a flowchart. Each framework of twentieth-century physics enters at the level of the axioms it implicitly uses, and the branches descend as axioms are added: one axiom gives special relativity, two give the geometric machinery of general relativity and gauge theory, the third adds the boundary coarse-graining that quantum mechanics and thermodynamics share, and the fourth removes the remaining freedom. What each branch *cannot* say is printed beside it, because that is the content of the classification — a missing axiom is not an omission but a prediction about which questions the framework will leave open. The extended description on the preceding page reads the chart row by row.

Pointer to Figure 9. The single most useful visual orientation for the rest of Part VIII is the comprehensive constitutional flowchart of Figure 9. Each of the per-stratum sections that follow (§66 through §74) elaborates one stratum of the flowchart in technical detail; readers who want the panorama before the technical descent are encouraged to study the flowchart first and return to it as a map at each subsequent section transition.

The four axioms form a strict logical tower: each axiom presupposes the previous and adds new content. AX-1 requires a manifold with a causal structure. AX-2 requires AX-1 plus a bulk gauge bundle unified with the geometry. AX-3 requires AX-1 + AX-2 plus the operational dictionary. AX-4 requires AX-1 + AX-2 + AX-3 plus the uniqueness principle.

We denote the active-axiom subsets by their natural names: LC = {AX-1}; GGEP-closed = {AX-1, AX-2}; OGGEP-open = {AX-1, AX-2, AX-3}; Full-Constitution = {AX-1, AX-2, AX-3, AX-4}.

Why subsets are physically meaningful. A natural objection: *the GG-Constitution is a constitution; one either accepts all four axioms or none; why is it meaningful to ask what subsets produce?* The answer is that the historical twentieth-century theories are precisely the answers to that question, asked implicitly. When string theorists postulate that geometry and gauge unify on a 10D worldsheet but do not address the operational projection to a finite-observer boundary, they are implicitly using GGEP-closed and not OGGEP-open. When QM-foundations researchers postulate the Born rule but accept that string-theoretic bulk structures are someone else's problem, they are implicitly using OGGEP-open but not invoking the bulk content that AX-2 forces. The historical theories are the twentieth century's implicit experiments with axiom subsets; identifying the subset each theory used is the first step toward seeing why each succeeded where it did and why each failed where it did.

Why each layer admits a richer theorem-class than the previous. At each layer of the tower, the set of theorems that can be derived strictly expands. LC alone gives classical field theory on a fixed background. GGEP-closed adds the entire geometric-machinery edifice (GR, Yang–Mills, GS Bianchi, string theory, AdS/CFT). OGGEP-open further adds quantum mechanics with its measurement-problem resolution. Full-Constitution further adds the unique bulk solution that fixes all continuous parameters (SM with 28 parameters → zero free; ΛCDM with $\Omega_m = 1/\pi$). Each axiom is logically independent in the sense that removing it strictly reduces the theorem-class; each is empirically warranted in the sense that the theorems it makes available correspond to verified physical content.

The classification is constitutional, not historical. We emphasize from the outset that the classification we are about to develop is a *constitutional* statement, not a historical one. The historical sequence in which the twentieth-century theories were developed (special relativity 1905, GR 1915, Yang–Mills 1954, QM 1925–32, QFT 1948, SM 1967–78, superstring 1984–, AdS/CFT 1997, ΛCDM 1998) reflects the historical accidents by which particular subsets of the GG-Constitution happened to become operative in particular sub-communities at particular times. The constitutional structure is independent of that history. What we gain by re-reading the history through the constitutional lens is the ability to diagnose precisely *why* each theory succeeded where it did (it

correctly used the axiom subset it implicitly invoked) and *why* each theory failed where it did (it lacked the axioms needed to address the questions where it remained silent).

After all, a constitution is judged not by who wrote it but by what follows from it. The four axioms of the GG-Constitution were articulated explicitly in GG-A1-Constitution in the present cohort; but their operational content has been active in the twentieth-century literature all along, sometimes named, sometimes unnamed, sometimes half-recognized. The present Part makes the constitution's content visible across that full literature.

66 Layer 0: AX-1 alone — special relativity and classical field theory

The shortest layer of the tower contains only the local-causality axiom. What does AX-1 alone produce?

Content of AX-1. Physical influence propagates forward in time at finite speed; the manifold $\mathcal{M}$ on which physics happens carries a causal structure $(\mathcal{M}, g, \prec)$ with light cones and well-defined past/future at every point.

Theorem class. AX-1 alone, on a specified Minkowski metric, gives:

- Special relativity in its full content (Lorentz transformations, time dilation, length contraction, mass–energy equivalence $E = mc^2$, the constancy of the speed of light c).
- Classical field theory on a fixed Minkowski background: scalar fields, electromagnetic fields (specialized by additional postulates), classical Yang–Mills fields (when the gauge group is fixed externally).
- The relativistic dispersion relation $E^2 = (pc)^2 + (mc^2)^2$ for free particles.
- The relativistic causality structure that forbids superluminal signaling (a precondition for T-A8.11 of GG-A8-QM).

What AX-1 alone does not produce. AX-1 alone does not produce gravity (curved spacetime requires AX-2 to relate metric curvature to matter content). It does not produce gauge fields (which require AX-2). It does not produce quantum mechanics (which requires AX-3). It does not select specific gauge content (which requires AX-4). In particular, AX-1 alone is the framework in which an observer of the late 19th century could have worked: it produces Maxwell's equations on Minkowski background (with the gauge group put in by hand), special relativity, and not much else.

Historical realization. Einstein's 1905 papers are the canonical realization of AX-1 alone. "On the electrodynamics of moving bodies" establishes special relativity from the postulates of constancy of c and equivalence of inertial frames. "Does the inertia of a body depend upon its energy content?" derives $E = mc^2$. Neither paper invokes AX-2; both invoke only the local causality and inertial-frame structure that AX-1 encodes. The 1905 work is, in the cohort framework, exactly what one could expect from AX-1 active and the other three axioms latent.

After all, special relativity remains correct because the framework in which it lives (LC) is correct as far as it goes; what the later century added was not corrections to special relativity but the further axioms that AX-1 alone could not supply.

67 Layer 1: AX-1 + AX-2 — geometry-gauge unification

The first non-trivial layer of the tower activates the geometry-gauge equivalence principle on top of local causality. This is the layer at which most of twentieth-century theoretical physics happens.

Content of AX-2. Geometry and gauge are facets of a single composite connection on the bulk manifold. Curvature 2-forms of geometry (R) and gauge (F) are algebraically related; they are not two independent structures on the same manifold but two expressions of one underlying connection 1-form. Equivalently: the metric connection and the gauge connection share a common origin in the bulk principal bundle.

General theorems available at GGEP-closed. The GGEP-closed axiom subset produces:

1. **Curvature–matter coupling theorems.** The bulk curvature, sourced by matter content, satisfies an Einstein-style equation $G_{ab} = T_{ab}$ (in appropriate units) when restricted to the metric sector. The four-dimensional version of this equation is general relativity.
2. **Gauge-bundle structure theorems.** The bulk gauge connection on $\mathcal{M}_6$ projects to a 4D Yang–Mills theory on the boundary via the scale overlap integrals of T1–T6 (without the operational coarse-graining of OGGEP-open; the projection here is purely geometric).
3. **Modified Bianchi identity (BI-6).** The composite connection forces an algebraic relation between R and F that generalizes the standard Bianchi identity $dR = 0$ to a Green–Schwarz-style form $dH = \mathrm{tr}(F \wedge F) - \mathrm{tr}(R \wedge R)$, derived as a consistency theorem of AX-2 alone (see GG-A3-BI6).
4. **Holographic duality.** On a bulk with conformal boundary, the composite connection on the bulk descends to a dual field-theory description on the boundary. This is the AdS/CFT-style correspondence, which arises as a theorem of GGEP-closed applied to AdS-like solutions.
5. **Dualities (T, S, U).** The composite-connection structure on bulk manifolds with non-trivial topology produces duality transformations relating different solutions, of which the T, S, and U dualities of superstring theory are the most familiar examples.
6. **Holographic principle.** The bulk degrees of freedom are bounded by the boundary area (in appropriate units), with saturation at the AdS-conformal-boundary limit. This is the 't Hooft–Susskind ('t Hooft, 1993, Susskind, 2003) holographic principle, derivable as a theorem of GGEP-closed with bulk-locality.

What GGEP-closed does not produce. This is the critical observation. GGEP-closed does not produce quantum mechanics with its measurement-problem resolution: the operational coarse-graining that produces the CPTP boundary channel requires AX-3, not just AX-2. GGEP-closed does not produce a unique vacuum or unique bulk solution: the maximum-symmetry-minimum-freedom selection requires AX-4, not just AX-2. Both of these limitations are constitutional, not technical. They cannot be patched within the GGEP-closed framework; they require new axioms.

Why this matters for the historical retrospective. Every twentieth-century theoretical-physics framework that lives at the GGEP-closed level inherits both its mathematical richness and

its two characteristic limitations. The richness is the entire machinery of theorems 1–6 above: GR, Yang–Mills, BI-6, AdS/CFT, dualities, holographic principle. The limitations are the QM-foundations gap and the landscape problem. Both are diagnosable at the axiomatic level: the gaps are exactly what one would expect from a framework that has AX-1 + AX-2 but not AX-3 and not AX-4.

67.1 General relativity as the GGEP content on a 4D solution

Einstein's 1915 general relativity is the canonical GGEP-closed realization on the specific solution where the bulk reduces to a curved 4D manifold (the radial direction I_r collapses to a single value at the boundary, and the compact direction S^1_θ is integrated out). In this limit, the composite connection of AX-2 reduces to the Riemannian connection on $\mathcal{M}_4$, and the geometry-gauge unification reduces to the statement that gravity is the geometric expression of the bulk content.

Einstein field equations as theorem. The 4D Einstein field equations

$$G_{\mu\nu} \;+\; \Lambda g_{\mu\nu} \;=\; \frac{8\pi G_N}{c^4}\, T_{\mu\nu} \tag{198}$$

arise as the projection of the bulk curvature–matter coupling theorem of GGEP-closed to the 4D boundary. Newton's constant G_N is not a free parameter in the cohort framework; it is the scale overlap integral of the bulk graviton zero-mode (GG-A5-GDOS Part VI, T8.2), computed once-for-all from the integer OGN-5 ledger. The cosmological constant Λ likewise arises from the $\Omega_\Lambda = 1 - 1/\pi$ identification of GG-A10-Cosmos, forced by the full GG-Constitution but operative at GGEP-closed as a free parameter to be measured.

What GR gets right and what it leaves silent. GR correctly predicts every gravitational observation made in the century since 1915: perihelion precession of Mercury, gravitational lensing, gravitational redshift, the existence and properties of black holes, the binary-pulsar inspiral, the LIGO/Virgo gravitational waves of 2015 onward. Each of these is a verified prediction of GGEP-closed applied to the curved-4D bulk solution. What GR leaves silent is everything that requires AX-3 or AX-4: it has no account of quantum measurement in a gravitational background (the black-hole information paradox is the canonical example of an GGEP-closed-only framework being asked an OGGEP-open question); and it has no constitutional account of *why* G_N has its specific value (the value is treated as a free parameter to be measured, when in the full cohort framework it is forced by the integer ledger).

After all, GR is the cleanest historical example of a framework that correctly used AX-1 + AX-2 and stopped there. The framework worked beautifully for everything in its proper domain; it failed silently for everything outside its proper domain, because the axioms needed to address those questions were not active in the 1915 axiom set.

67.2 Yang–Mills theory as the GGEP content for gauge fields

Yang and Mills's 1954 generalization of Maxwell electromagnetism to non-Abelian gauge groups (originally SU(2), later generalized to arbitrary compact Lie groups) is the realization of GGEP-closed on the gauge sector of the bulk. In the cohort framework, the non-Abelian gauge structure descends from the bulk gauge bundle that AX-2 forces; Yang–Mills theory on $\mathcal{M}_4$ is what one sees after projecting the bulk gauge content to the boundary.

The Yang–Mills equations as theorem. The Yang–Mills field equation $D^\mu F^a_{\mu\nu} = J^a_\nu$ arises as the boundary projection of the bulk gauge-bundle equation of GGEP-closed. The non-Abelian commutator structure of the field strength $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$ is forced by the requirement that the bulk connection be a single composite structure — the f^{abc} structure constants are the Lie-algebraic facet of the composite connection.

What Yang–Mills gets right and what it leaves silent. Yang–Mills correctly underwrites the structure of the strong nuclear force (SU(3) QCD), the weak nuclear force (SU(2)), and the electromagnetic force (U(1)). The full Standard Model gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ is built from Yang–Mills factors. What Yang–Mills theory *alone* leaves silent is which specific gauge group is operative in Nature: the Standard Model gauge group is one choice among infinitely many that AX-1 + AX-2 admits. Forcing the specific group requires AX-4 (the $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ combination is forced by BI-6 anomaly closure plus MSMF, both of which lie outside GGEP-closed).

67.3 Superstring theory as the GGEP content on a 10D bulk

We now come to the most extensive realization of GGEP-closed in twentieth-century theoretical physics: superstring theory. The constitutional reading is straightforward, and we develop it in three sub-claims.

Sub-claim 1: The GS-modified Bianchi identity is a theorem of AX-2, not a postulate of strings. In 1984, Green and Schwarz showed that the 10D gauge anomaly in Type I superstring theory cancels for SO(32) and $E_8 \times E_8$ via a modification of the Bianchi identity for the 3-form field strength

$$dH \;=\; \mathrm{tr}(F \wedge F) \;-\; \mathrm{tr}(R \wedge R). \tag{199}$$

This modification is conventionally presented as a string-theoretic achievement: "the GS mechanism is what makes anomaly cancellation work in the heterotic theory; if you don't have strings, you don't need it." This historical narrative is true; the constitutional narrative is sharper.

(199) is a constraint on differential forms on the bulk manifold. It says that the curvature of the bulk gauge connection (F) and the curvature of the bulk Riemannian connection (R) cannot evolve independently; their second Chern classes are tied together by the topology of the H-flux. This is a geometric statement, formulable on any smooth manifold with a connection, completely independent of whether the underlying physics is described by strings, M2-branes, M5-branes, or — as in the cohort framework — by AX-2-admissible 6D bulk fields with no extended objects at all.

What strings did was provide *one* physical scenario in which the constraint had to be satisfied. The constraint itself was always geometric. The string-theoretic derivation made the constraint *visible* to the physics community by giving it an indispensable role in a concrete theory, but the constraint's logical status was never string-dependent. AX-2 (GGEP) makes this explicit: the modified Bianchi identity is the consistency condition for the bulk composite connection. In GG-A3-BI6, the cohort analog BI-6 is derived in exactly this form, on a 6D bulk, with no string-theoretic apparatus invoked.

The reversal of attribution should be stated cleanly:

- *String-theorist's reading:* "We need to cancel the anomaly in 10D, so the Bianchi identity must be modified. This is a property of strings."
- *Cohort reading:* "We require that geometry and gauge be a single unified structure (AX-2). The modified Bianchi identity is the consistency condition for this unification. Strings provide one realization, but the identity exists independently."

This is not a small reframing. It says the GS mechanism is a *theorem of* GGEP-closed (which GG-A3-BI6 in fact derives in the form BI-6), not a postulate of string theory. The string-theoretic version is GGEP-closed + the additional assumption that the bulk is built of extended objects. Drop the extended-object assumption and keep AX-2, and you still have the modified Bianchi identity — because the identity was never about extended objects to begin with.

Sub-claim 2: 10D anomaly cancellation is a theorem of AX-2 on a 10D solution. The Type I and heterotic 10D anomaly cancellations identifying SO(32) or $E_8 \times E_8$ as the allowed gauge groups are theorems of BI-6-style anomaly closure applied to a 10D bulk. The specific groups arise because the 10D Bianchi identity (199) has only those two solutions, both of which have the dimension-counting property $\dim G + \dim \mathrm{adj} = 496$ (giving SO(32) with dim = 496 or $E_8 \times E_8$ with $\dim = 248 + 248 = 496$). This dimension-counting miracle is exactly the kind of integer-arithmetic that BI-6 forces in the cohort framework on the 6D bulk (giving $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ with its own dimension-counting closure).

In other words: 10D anomaly closure giving $\mathrm{SO}(32)/E_8 \times E_8$ is the 10D specialization of BI-6. 6D anomaly closure giving the cohort gauge bundle is the 6D specialization of BI-6. Both are theorems of AX-2 applied to bulks of different dimensions; the specific results depend on the dimension.

Sub-claim 3: D-branes, dualities, and the rest follow from the same axiom subset. The remaining string-theoretic apparatus — D-branes as non-perturbative solitonic objects, T-duality between large and small compact dimensions, S-duality between weak and strong coupling, the web of theories united by M-theory in 11 dimensions — all follow from GGEP-closed applied to bulks of various dimensions with the additional choice of extended-object dynamics. None of these requires the cohort's AX-3 or AX-4; each is a theorem of GGEP-closed on a specific solution-class selection.

Standard string framework as GGEP-closed specialization. The complete summary: *standard superstring theory is the* GGEP-closed *content of the GG-Constitution applied to a 10D bulk with extended-object dynamics.* All of its mathematical content is correct; the framework gives

correct theorems within the axiom subset it implicitly invokes. The framework's silent regions — quantum mechanics with measurement-problem resolution (requires AX-3), unique vacuum selection (requires AX-4) — are exactly what the missing axioms predict.

67.4 AdS/CFT correspondence as the GGEP content on AdS solutions

Maldacena's 1997 conjecture that Type IIB string theory on $\mathrm{AdS}_5 \times S^5$ is dual to $\mathcal{N} = 4$ super Yang–Mills on the 4D conformal boundary is the canonical realization of holographic duality at the GGEP-closed level. The constitutional reading generalizes the original conjecture to a structural theorem.

The duality as theorem of GGEP-closed on AdS bulk. On a bulk with conformal boundary — and AdS is the canonical example — GGEP-closed requires that the bulk dynamical content be expressible as a boundary field-theory description. The reason is direct: the composite connection of AX-2 on the AdS bulk has a natural projection to the conformal boundary, with the radial coordinate of AdS playing the role of renormalization-group depth. AX-1 (local causality) ensures that the bulk causal structure descends to the boundary causal structure, so the boundary field-theory is causal as well. The two requirements together force the existence of an equivalent boundary description. Maldacena's 1997 derivation via D3-brane stacks was the historical scaffold by which the duality was discovered, but the duality itself is the GGEP-closed content on that geometry.

Why AdS specifically. The conformal-boundary requirement is what selects AdS-like geometries (or, in the cohort framework, the $\mathcal{M}_4 \times I_r$ geometry with $r \to 0$ as the conformal boundary). Other bulk geometries — de Sitter, asymptotically flat, asymptotically Friedmann — do not have a clean conformal boundary at infinity in the same way, so the holographic duality at the strict-duality level is less straightforward. But the constitutional content is universal: any bulk geometry with a conformal boundary admits a GGEP-closed bulk-to-boundary projection of the bulk content to the boundary field-theory. AdS is the canonical example, not the only one.

Why AdS/CFT is information-preserving (duality), not information-coarsening (projection). This is the deepest structural point about AdS/CFT in the constitutional classification. Because GGEP-closed does *not* include AX-3 (the operational coarse-graining), the holographic content at this layer is an exact duality: bulk and boundary are equivalent descriptions of the same physics, with full information preserved both ways. This is why AdS/CFT can be used to compute correlation functions on either side and check that they agree; the two descriptions are literally the same theory in two languages.

When one adds AX-3 (OGGEP), the holographic content acquires information-coarsening: the boundary observer is now finite, the projection is CPTP rather than unitary, and the dual map becomes irreversible. The framework's 6D-to-4D projection, which gives quantum mechanics, is exactly OGGEP-open-holography. AdS/CFT, by contrast, is GGEP-closed-holography — the closed-system limit of the same structure.

We state this analogy formally, because it is the cleanest way to see the relationship between AdS/CFT and the cohort's 6D-to-4D projection:

The Schrödinger–GKLS analogy at the holographic layer. At the dynamical layer, the Schrödinger equation is the closed-system limit ($\gamma_\alpha \to 0$) of the GKLS master equation:

$$\text{GKLS (open)} \quad \xrightarrow[\gamma_\alpha \to 0]{} \quad \text{Schrödinger (closed).}$$

At the holographic layer, AdS/CFT is the closed-system limit (AX-3 switched off) of the cohort 6D-to-4D projection:

$$\text{Cohort 6D–4D projection (open)} \quad \xrightarrow[\text{AX-3 off}]{} \quad \text{AdS/CFT duality (closed).}$$

The same structural move that turns open-system dynamics into closed-system dynamics at the level of state evolution turns the OGGEP-open-projection into the GGEP-closed-duality at the level of holography.

This is, in our view, the single most important structural observation in the present Part. AdS/CFT is not a competitor to the cohort 6D-to-4D projection; it is the closed-bulk specialization of the same structure. The string community has been computing in the closed-bulk regime for forty years without recognizing that the closed-bulk regime is a limit of a more general open-system framework.

The empirical successes of AdS/CFT as GGEP-closed theorems. AdS/CFT has produced many remarkable empirical and theoretical successes:

- The viscosity-to-entropy bound $\eta/s \geq \hbar/4\pi k_B$ derived from AdS/CFT and observed in the quark–gluon plasma at RHIC and the LHC.
- The Ryu–Takayanagi formula for boundary entanglement entropy as a bulk minimal-surface area.
- Holographic computations of black-hole entropy, including agreement with the Bekenstein–Hawking formula for asymptotically AdS black holes.
- The modular-bootstrap classification of 2D CFTs and its three-dimensional extensions.
- Higher-form symmetries and generalized global symmetries in field theory.

Each of these is a theorem of GGEP-closed applied to the appropriate AdS solution. The cohort framework inherits all of these results unchanged, since the framework agrees with GGEP-closed in the closed-bulk regime. The additional content of the cohort framework — QM postulates as theorems, landscape dissolved by AX-4 — is what *extends* AdS/CFT, not what contradicts it.

After all, the relationship between AdS/CFT and the cohort 6D-to-4D projection is the relationship between Schrödinger evolution and GKLS evolution: the former is a beautiful and useful closed-system limit; the latter is the open-system structure in which the closed-system limit is one corner. Both are correct; both are necessary; neither alone is the complete picture.

68 Layer 2: AX-1 + AX-2 + AX-3 — quantum mechanics as theorem

The third layer of the tower activates the operational/open geometry–gauge equivalence principle. This is the layer at which quantum mechanics with its measurement-problem resolution becomes derivable.

Content of AX-3. Any admissible bulk theory must furnish a structure-preserving functor $\mathcal{D}_{\mathsf{OGGEP}}$: **Bulk** $\to$ **Op** from the inaccessible bulk data $(\mathcal{H}_{\text{Bulk}}, \rho_{\text{Bulk}}, U_{\text{Bulk}})$ to the registered boundary observables $(\mathcal{H}_{\text{Bdy}}, \rho_{\text{Bdy}}, \Phi_{t,s}, \{\mathcal{I}_\alpha\})$, compatible with gauge covariance, causality, positivity, and scale consistency. The functor encodes the operational coarse-graining a finite 4D observer is forced to perform.

Theorem class added at OGGEP-open. The OGGEP-open axiom subset adds to the GGEP-closed theorem class:

1. **The boundary CPTP channel.** The bulk-to-boundary map is a completely positive trace-preserving channel rather than a unitary equivalence. This is the deep difference from GGEP-closed holography: AX-3 makes the projection genuinely open and irreversible.
2. **The seven QM postulates as theorems.** The chain T7.1–T7.7 of the present paper derives the Hilbert-space postulate, the Schrödinger equation, the Born rule, the Lüders projection, decoherence, einselection, and Bell-CHSH violation as theorems of OGGEP acting on the bulk. The textbook QM postulates are no longer postulates; they are theorems.
3. **The always-on projection.** As developed in GG-A8-QM Part III, the boundary CPTP channel is operative at every spacetime point on $\mathcal{M}_4$ at every moment, independent of any observer. Measurement is the macroscopic specialization of the always-on projection acting on a Geometric Attractor with a record-amplification chain.
4. **The Heisenberg cut dissolved.** The hypothetical boundary between quantum system and classical apparatus is dissolved by the always-on projection: every Geometric Attractor receives the projection; the differences between microscopic and macroscopic GAs are quantitative (size and stability of the record) rather than qualitative.
5. **The von Neumann chain terminated.** The infinite regress of system $\to$ apparatus $\to$ observer $\to$ super-observer is terminated at the first GA with a record-amplification chain sufficient for the experimenter's purpose.
6. **Bell–CHSH violation as boundary shadow.** The experimentally measured violation of Bell's inequality up to the Tsirelson bound $|S| \leq 2\sqrt{2}$ is a theorem of OGGEP acting on bipartite GAs; the bulk-local description remains local in 6D while the boundary appears non-local in 4D.

Why GGEP-closed cannot derive the QM postulates. This is the diagnostic core of the constitutional classification. In the GGEP-closed framework, the bulk-to-boundary map is a unitary equivalence (the AdS/CFT-style duality), not a CPTP channel. Unitary maps preserve all information; they do not coarse-grain; they do not produce probability assignments from definite bulk states. The Born rule, the Lüders projection, the decoherence-induced classicality of macroscopic

outcomes — all of these require the coarse-graining that only AX-3 supplies. Within GGEP-closed, one can compute correlation functions and check duality predictions; one cannot *derive* the QM postulates because there is no constitutional machinery for the open-system reduction.

This is exactly why string theory has produced no progress on QM foundations in forty years. The framework is structurally incapable of addressing the measurement problem, the Born rule's constitutional origin, or the QM-to-classical transition. These are not gaps to be filled by additional work within string theory; they are gaps that require a different axiom (AX-3) which string theory does not have.

Reattribution: the QM-foundations gap as a missing-axiom gap. For ninety years, the QM-foundations community has presented the measurement problem as the most awkward gap in physics. Every textbook ends its chapter on quantum mechanics with a hedged sentence about "the interpretation of the wave function" or "the role of the observer." The constitutional classification provides a sharp diagnosis: the measurement problem is the structural symptom of working in the GGEP-closed framework without AX-3. Once AX-3 is added to the axiom set, the QM postulates become theorems, the measurement problem dissolves, and the textbook hedged sentence can be replaced with a clean derivation.

This is the deep payoff of the constitutional classification at Layer 2: not just that the QM postulates are theorems (which the present paper has already shown via T7.1–T7.7), but that the historical inability of string theory to derive QM is a diagnosable, structural, missing-axiom consequence of the GGEP-closed framework. The string community could not have derived QM no matter how much effort they invested, because the axiom needed for the derivation was not in their framework.

After all, the measurement problem was always the question of what the boundary observer's finite-information limit produces. Without the axiom that encodes the boundary observer's finite information (AX-3), the question is unanswerable; with the axiom, the question becomes the chain T7.1–T7.7. Ninety years of debate dissolves into a chain of theorems once the right axiom is named.

69 Layer 2 extended: classical thermodynamics and statistical mechanics as theorems of OGGEP

The seventh section of Part VIII develops the second major consequence of the OGGEP-open layer of the constitutional tower: **classical thermodynamics and statistical mechanics are theorems of the same axiom subset that produces quantum mechanics**. The textbook formulations of both subjects — the laws of thermodynamics on the one hand, the postulates of statistical mechanics on the other — emerge as derived structures of the boundary CPTP channel $\Phi_{t,s}$ that AX-3 forces. The two subjects, historically separated for two centuries by the contingencies of disciplinary development, are constitutional siblings: they are the same theorem at the OGGEP-open level, specialized to different boundary observables.

69.1 The puzzle this section dissolves

A reader who has followed the QM-as-theorem material of §68 may now ask a natural question. We have shown that the textbook QM postulates — the Hilbert space, the Schrödinger equation, the Born rule, the Lüders projection, decoherence, einselection, the Bell–CHSH violation — emerge as theorems of OGGEP (the T7.1–T7.7 chain of the present paper). But what about thermodynamics? Why does an open system relax toward equilibrium, why is entropy non-decreasing, why does the Boltzmann formula $S = k_B \ln W$ work, why is temperature what it is? Are these constitutional consequences of the cohort framework, or are they postulates that must be added to it?

The answer, developed in this section, is sharp: *they are constitutional consequences.* The OGGEP boundary CPTP channel that produces the QM postulates as theorems also produces the laws of thermodynamics as theorems — via exactly the same constitutional mechanism (the data-processing inequality applied to the CPTP projection from bulk to boundary), specialized to different boundary observables. This is the central thesis of the present section: **quantum mechanics and classical thermodynamics are not just connected (via quantum statistical mechanics); they are constitutional siblings at the OGGEP-open layer, derived from the same axiom AX-3 acting on the same bulk geometry of AX-2.**

69.2 The historical fragmentation, diagnosed

The fragmentation of the thermal-physics literature into separate sub-disciplines, with their own languages and their own central puzzles, becomes immediately intelligible once the constitutional classification is applied. Table 17 sorts two hundred years of thermal-physics development into the four-axiom tower.

Table 17: Two centuries of thermal physics, sorted by which constitutional axioms each framework implicitly uses. Each row is a historical framework in the order it appeared; the second column lists the axioms it invokes, the third what it was able to produce, and the fourth what it left unsaid. The column that carries the argument is the fourth: in every case the silence is exactly the content of an axiom the framework did not have. Everything after Boltzmann (1872) uses the third axiom — the boundary coarse-graining — and none of them has a name for what it is using, which is why the literature fragmented into sub-disciplines with separate vocabularies for one structure.

Framework (year)	Axioms	Content produced	Content left silent
Carnot cycle (1824)	AX-1, AX-2	Macroscopic energy balance; maximum efficiency of heat engines	Microscopic foundation; entropy as theorem (needs AX-3)
Clausius entropy (1865)	AX-1, AX-2, S postulated	Second law as inequality; thermodynamic potentials	Entropy as theorem (supplied by OGGEP)
Boltzmann H-theorem (1872)	AX-1, AX-2, AX-3 (implicit Stosszahlansatz)	Microscopic foundation of second law; $S = k_B \ln W$	Stosszahlansatz constitutionalized (supplied by OGGEP)
Gibbs ensembles (1902)	AX-1, AX-2, AX-3 (implicit)	Partition function; equilibrium statistical mechanics	Ensemble probability as theorem (supplied by OGGEP)

continued on the next page

Table 17, continued from the previous page.

Framework (year)	Axioms	Content produced	Content left silent
Einstein fluctuation (1905)	AX-1, AX-2, AX-3 (implicit)	Brownian motion; fluctuation-dissipation precursor	Bulk-mode coupling left empirical (supplied by scale overlap)
Onsager reciprocity (1931)	AX-1, AX-2, AX-3	Linear-response symmetry; near-equilibrium transport	Time-reversal symmetry of CPTP not constitutionalized
Shannon entropy (1948)	AX-1, AX-2, AX-3 (in disguise)	Information measure; channel capacity	"Information entropy" ontologically ambiguous
Jaynes MaxEnt (1957)	AX-1, AX-2, AX-3 (as inferential gauge)	MaxEnt distributions; inferential reading of thermodynamics	Constitutional source of MaxEnt unidentified
Landauer (1961)	AX-1, AX-2, AX-3	Bit-to-energy bound $k_B T \ln 2$ per erased bit	Constitutional warrant for the bound
Lindblad GKLS (1976)	AX-1, AX-2, AX-3	Markovian quantum master equation; quantum H-theorem	The Lindblad generator IS the OGGEP generator — but not named
Bekenstein–Hawking (1973)	AX-1, AX-2, attempted AX-3	Black-hole entropy $A/4G_N$; Hawking radiation	Information paradox — resolved at AX-3
Jarzynski / Crooks / Seifert (1990s)	AX-1, AX-2, AX-3	Fluctuation theorems; stochastic thermodynamics	Time-reversal of CPTP constitutionalized
Friston FEP (2010)	AX-1, AX-2, AX-3 (inferential gauge)	Variational free-energy minimization; predictive coding	Constitutional source named (it is the Qi-EP gauge of the cohort framework)

The pattern is unmistakable. Every framework developed after Boltzmann (1872) implicitly uses AX-3 (the boundary CPTP coarse-graining) but does not have the constitutional axiom to name what it is doing. This is the same diagnostic pattern we applied to QM foundations: a century-and-a-half of debate about "what entropy really is" (counting? subjective ignorance? information? phase-space volume?) is the missing-axiom symptom of working in the AX-2-only framework without explicitly naming AX-3.

After all, the curious feature of the thermal-physics literature is that almost every successful framework after 1872 quietly imports the operational coarse-graining (Stosszahlansatz, Liouville truncation, projection-operator techniques, Markovian limits) under some local name, while debating in the philosophical literature whether such coarse-graining is "real" or "epistemic." The constitutional classification dissolves the debate: the coarse-graining is constitutional (AX-3); the debate is the missing-axiom symptom; the resolution is to name the axiom.

69.3 The first law as Noether's theorem (AX-2 content)

The first law of thermodynamics — conservation of energy — arises from AX-2 alone, not from AX-3. It is a Noether-theorem consequence of time-translation invariance of the bulk action. The cohort framework's derivation is explicit.

The 6D bulk action of GG-6 is schematically

$$S_6 \;=\; \int_{\mathcal{M}_{\text{bulk}}} d^6x\,\sqrt{|g_6|}\,\Big[\,\frac{R_6}{2\kappa_6^2} \;-\; \frac{1}{4g_5^2}\,\mathrm{Tr}(F_{MN}F^{MN}) \;+\; \ldots\Big] \;+\; S_{\text{inflow}} \;+\; S_{\partial}, \tag{200}$$

where R_6 is the 6D Ricci scalar, F_{MN} is the composite-connection curvature, and S_{inflow} implements the BI-6 modified Bianchi identity. This action is invariant under time translation $x^0 \to x^0 + a$ on the 4D boundary. By Noether's theorem, there is a conserved current — the bulk energy–momentum tensor T^{MN} — satisfying $\nabla_M T^{MN} = 0$ on-shell.

The 4D observer, projecting via the OGGEP functor, sees the boundary energy-momentum tensor

$$T^{\mu\nu}_{\text{4D}}(x) \;=\; \int dr\,d\theta\,\sqrt{|g_6|}\,T^{\mu\nu}_{\text{6D}}(x,r,\theta), \tag{201}$$

with conservation $\partial_\mu T^{\mu\nu}_{\text{4D}} = 0$ following from 6D conservation plus the absence of boundary flux through the compact directions I_r and S^1_θ.

This is the first law of thermodynamics in its most fundamental form. The integrated 4D energy is a Noether charge of the 6D action; it is conserved by Noether's theorem; the conservation is AX-2 content. At the open-system level, the boundary observer sees the energy current decomposing into internal energy U, heat Q flowing through thermal ports, work W exchanged with mechanical ports, and chemical potential terms $\mu\Delta N$ for material ports. The textbook first law $dU = \delta Q + \delta W + \mu\, dN$ is the open-system bookkeeping of this conserved Noether current across an SIP's port structure.

What this derivation accomplishes is to identify the constitutional source of the first law. It is AX-2 (the GGEP composite-connection structure that makes the bulk action well-defined) plus the time-translation symmetry that the bulk Lagrangian respects. No new postulate is needed. Energy conservation is a Noether theorem of the cohort framework.

69.4 The second law as the data-processing inequality (AX-3 content)

The second law of thermodynamics — the non-decrease of entropy in isolated systems — is the deeper content of AX-3. Both the classical second law and its quantum refinement (Spohn's quantum H-theorem) follow from the data-processing inequality applied to the OGGEP CPTP channel. We develop the derivation in three steps.

Step 1: the data-processing inequality. For any CPTP channel $\Phi : \rho \mapsto \Phi(\rho)$ and any two density operators ρ, σ, the relative entropy (Kullback–Leibler divergence) is monotone:

$$D[\rho \,\|\, \sigma] \;\geq\; D[\Phi(\rho) \,\|\, \Phi(\sigma)]. \tag{202}$$

This is a deep result of quantum information theory (Lindblad, 1975, Uhlmann, 1977), valid for all completely positive trace-preserving maps. It says that distinguishability between states can only decrease under coarse-graining, never increase.

Step 2: Spohn's quantum H-theorem. Apply the data-processing inequality with $\sigma = \rho_{\mathrm{eq}}$ a *stationary* state of the open-system dynamics, $\Phi_\tau(\rho_{\mathrm{eq}}) = \rho_{\mathrm{eq}}$. Then

$$D[\rho(t) \,\|\, \rho_{\mathrm{eq}}] \;\geq\; D[\rho(t+\tau) \,\|\, \rho_{\mathrm{eq}}], \tag{203}$$

which is the Spohn quantum H-theorem (Spohn, 1978): Lindblad's monotonicity (Lindblad, 1975) read against a fixed point. The relative entropy of the boundary state with respect to the stationary state is monotonically non-increasing under OGGEP-induced time evolution. That the stationary state is the equilibrium Gibbs state — so that the descending quantity is a free energy — is the quantum detailed-balance condition of Kossakowski (1977), which the weak-coupling generator of T4 satisfies for one thermal bath (§13).

Step 3: the second law of thermodynamics. The von Neumann entropy of the boundary state can be written as $S_{\mathrm{vN}}(\rho) = -D[\rho \,\|\, \mathbb{1}/d] + \ln d$ for a d-dimensional Hilbert space. Applying the data-processing inequality with $\sigma = \mathbb{1}/d$ (the maximally mixed state) gives monotone non-decrease of S_{vN} under coarse-graining that preserves total dimension. For a system coupled to a thermal bath at temperature T, the appropriate entropy is the thermodynamic entropy (Clausius/Boltzmann), and the monotone non-decrease is the textbook second law.

Why the second law has been so hard to derive historically. The classical project of deriving the second law from microscopic reversibility has run into a notorious obstacle: microscopic Hamiltonian dynamics is time-reversible, so how can entropy preferentially increase? Boltzmann's H-theorem (1872) introduced the Stosszahlansatz (molecular-chaos assumption) to bridge the gap; Loschmidt (1876) and Zermelo (1896) objected that this was either circular or empirically unjustified; the debate has continued for 150 years.

The cohort framework's resolution is sharp. The 4D boundary system is *not* isolated; it is coupled to the inaccessible bulk via $\mathrm{Tr}_{\mathrm{env}}$. The 4D dynamics is therefore CPTP, not unitary, with non-trivial dissipator terms. The data-processing inequality is an *identity* for CPTP channels, not an assumption to be justified. The second law follows constitutionally from AX-3; no Stosszahlansatz is needed, because the coarse-graining is built into the constitutional structure, not added by hand.

This dissolves the second-law-from-microscopic-reversibility problem the same way AX-3 dissolves the QM measurement problem. Both problems were the missing-axiom symptom of working in a framework where the boundary observer's finite information was not constitutionally encoded.

69.5 The third law as the zero-scale limit

The third law of thermodynamics (Nernst–Planck: $S \to 0$ as $T \to 0$) follows from the OGGEP phase-space-counting structure. At zero temperature, the boundary state collapses to the ground state of the bulk system; the inaccessible-bulk phase-space volume available to the partial trace shrinks to a single point (the unique ground state); the OGGEP entropy contribution from phase-space counting vanishes. Quantitatively,

$$\lim_{T\to 0} S_{\mathrm{vN}}(\rho_\partial) \;=\; \lim_{T\to 0} k_B \ln W(T) \;=\; k_B \ln 1 \;=\; 0, \tag{204}$$

where $W(T)$ is the number of bulk microstates compatible with the boundary observer's macroscopic resolution at temperature T, collapsing to unity at zero temperature. No new axiom is needed; the third law is a corollary of the OGGEP phase-space structure.

69.6 The Boltzmann formula $S = k_B \ln W$ as OGGEP phase-space counting

The Boltzmann formula $S = k_B \ln W$, inscribed on Boltzmann's tombstone in Vienna, is the most famous formula in classical statistical mechanics. In the cohort framework, W is the phase-space volume of the inaccessible-bulk directions $I_r \times S^1_\theta$ that the OGGEP partial trace integrates out.

Specifically, the Wilson-line holonomy $\Theta_H \in [0, 2\pi)$ on S^1_θ is canonically conjugate to the $\mathrm{U}(1)_{Qi}$-charge density on the compact circle. The phase-space volume in the inaccessible direction is proportional to the number of holonomy values compatible with the macroscopic observer's resolution. For a 4D observer tracing over S^1_θ with resolution $\delta\Theta$, the boundary entropy is

$$S \;\sim\; k_B \,\ln\!\left(\frac{2\pi}{\delta\Theta}\right). \tag{205}$$

For a system with macroscopic conserved quantum numbers $(E, N, V, \dots)$, the accessible holonomy interval is further constrained by Bragg-type conditions matching the conserved charges, giving

$$W(E, N, V, \dots) \;=\; \frac{\text{phase-space volume on } (r, \theta) \text{ compatible with bulk state}}{\text{resolution volume}}. \tag{206}$$

This is the microcanonical entropy in the cohort reading.

Two important corollaries. First, the textbook "counting of microstates" is recovered, but W is reinterpreted: it is not a counting of 4D microstates (which would have ambiguous coarse-graining) but a phase-space volume in the inaccessible-bulk directions (I_r and S^1_θ). The reinterpretation removes the ambiguity of the textbook formula: W is a geometric quantity in the 6D bulk, not a subjective counting in the 4D boundary.

Second, the Boltzmann constant k_B acquires a geometric meaning. It is the conversion factor between dimensionless phase-space volumes in the inaccessible-bulk directions and the energy-per-temperature units of macroscopic 4D thermodynamics. In the cohort canonical realization, k_B is fixed by the integer OGN-5 ledger plus the scale profile, not a free parameter.

69.7 Temperature as the inverse scale rate (Unruh–Tolman)

Temperature in the cohort framework has a clean geometric origin: it is the inverse of the scale rate of the bulk geometry at the observer's spacetime location. For a 4D observer at position r in the bulk

$$ds^2_6 \;=\; e^{-2A(r)}\, g_{\mu\nu}(x)\, dx^\mu dx^\nu \;+\; dr^2 \;+\; R^2_\theta\, d\theta^2, \tag{207}$$

the locally measured temperature is related to the asymptotic temperature by the Unruh–Tolman relation

$$T(r) \;=\; T_{\text{asymp}}\, e^{A(r)}. \tag{208}$$

In the GG-6 bulk, $A(r) = kr$ (linear scale running), so $T(r)$ scales exponentially along the RG coordinate.

The relation between temperature and scale rate at the laboratory position r_* is

$$k_B T \;=\; \hbar c \cdot k \cdot e^{A(r_*)}, \tag{209}$$

where $\hbar c$ converts geometric units (inverse length) to energy and $e^{A(r_*)}$ accounts for the local observer's position in the bulk. Room temperature $T = 300$ K corresponds to a scale rate $k \sim 10^{12}\,\mathrm{m}^{-1}$, matching the inverse-Planck-scale hierarchy per OGN-compressed depth $K = 35$ to $\mathcal{O}(1)$ coefficients.

For a system thermalized with the bulk bath at temperature T, the 4D reduced state is the canonical Gibbs state

$$\rho_\partial \;=\; \frac{e^{-\beta H_{4\mathrm{D}}}}{Z_{4\mathrm{D}}}, \qquad \beta \;=\; (k_B T)^{-1}, \tag{210}$$

recovered as the OGGEP projection of the bulk thermal state in the Markovian/weak-coupling regime. The textbook canonical ensemble is not a separate postulate of statistical mechanics; it is the constitutional consequence of OGGEP-projected bulk thermal equilibrium.

69.8 The two currencies on the two fibers

The laws just derived divide, and the division is the fiber anatomy of §6.5 wearing thermodynamics' clothes. The first law (§69.3) is *the circle's ledger*: it is Noether's theorem for the clock's repetition, and the quantity it conserves — energy — is the winding-rate currency of the fiber that remembers; nothing about the first law is one-way, because nothing about the circle is. The second law (§69.4) is *the interval's ledger*: the data-processing inequality counts the dictionary's one-way writes, and the quantity it grows — entropy — is the record count of the fiber that forgets; nothing about the second law repeats, because nothing about the interval does. The CPT companion's expanded treatment of time states the pairing in a form worth adopting verbatim: *energy is what the symmetry of time is worth; entropy is what the arrow of time has read* (Arisaka, 2026, P5-CPT). That the two great quantities of physics ride on fibers of opposite topological character is the structural reason the first law was never violated and the second law never reversed: each is exact on its own fiber, and their famous tension — conservation against dissipation — is not a tension at all but a division of labor between a group and a semigroup.

Temperature is where the two ledgers exchange, and the exchange now has two theorems that the same companion has married. This paper derived Unruh–Tolman (§69.7): temperature is the inverse scale rate, dressed by the profile at the observer's radial position. The companion's worked KMS quartet derives the other face: temperature is imaginary period — equilibrium is periodicity of period β around the analytic shore. Married (Arisaka, 2026, P5-CPT): *temperature is where the interval meets the shore* — the thermal circle's circumference is transported along r and stretched by the profile, cooling *is* descent ($d\ln T = -dr$ in the radiation era), and since boundary time grows as e^2 per unit of descent while the shore's period grows only as e, the ratio β/t falls monotonically through cosmic history — the precise sense in which the universe becomes ever more classical. The third law (§69.5) joins as the zero-scale limit: the shore's circle cannot be unwound to infinite

circumference in any finite descent.

The loop this closes deserves its sentence. Part I located the GDOS generator's birthplace on the fiber without holonomy (§6.5); this Part has now walked every piece of thermodynamics' furniture — the first law, the second, the third, Boltzmann's counting, temperature, and (next subsection) free energy — into the same two-fiber room. The subsection that follows is the natural close: the GFF, with its energetic and curvature–entropy blocks, is precisely the one functional that carries *both* currencies on one ledger, which is why it, and not either currency alone, is what the runtime extremizes.

69.9 Free energy as the GFF ledger of GG-A6-GRIP

The thermodynamic free-energy functions — Helmholtz $F = U - TS$, Gibbs $G = U - TS + PV$, grand potential $\Omega = U - TS - \mu N$, and Landau free energy $\Phi[\eta]$ for phase transitions — are not independent quantities. They are different gauges of the single scalar ledger F_{GFF} of GG-A6-GRIP:

$$F_{\mathsf{GFF}} \;=\; F_{\mathsf{GEP}} \;+\; F_{\mathsf{CEP}}, \tag{211}$$

where F_{GEP} is the geometric-energy contribution from the bulk Lagrangian's metric sector and F_{CEP} is the curvature-entropy contribution from the gauge sector. The textbook thermodynamic potentials are obtained by Legendre transformations of F_{GFF} with respect to different control variables:

- Fixed (T, V, N): Helmholtz free energy F.
- Fixed (T, P, N): Gibbs free energy G.
- Fixed (T, V, μ): grand potential Ω.
- Fixed (T, V, η) with η an order parameter: Landau free energy.

This is the same structure that GG-A6-GRIP's GRIP cascade postulates as the universal scalar ledger of any open-system dynamics, of which thermodynamics is one instance. The textbook free-energy functions are different boundary readings of one constitutional functional; their interrelations (e.g., the Maxwell relations) are *theorems* of the GFF's Legendre-transform structure.

69.10 Fluctuation theorems as time-reversal of OGGEP

The 1990s revolution in non-equilibrium statistical mechanics — the Evans–Cohen–Morriss inequality (1993), the Jarzynski equality (1997), the Crooks fluctuation theorem (1999), Seifert's stochastic thermodynamics (2012) — is the OGGEP-open content extended to the trajectory level. The central fluctuation theorem (Crooks 1999) is

$$\frac{P[\gamma; t]}{P[\bar{\gamma}; t]} \;=\; e^{\beta\, W[\gamma]}, \tag{212}$$

where γ is a forward trajectory, $\bar{\gamma}$ is its time-reversed counterpart, and $W[\gamma]$ is the work done on the system along γ. The ratio of forward-to-reverse trajectory probabilities is governed by the work along the trajectory.

In the cohort framework, the Crooks theorem follows from the combination of two structural facts: bulk time-reversal symmetry of the 6D action (an AX-1+AX-2 property) and the asymmetric projection of the OGGEP CPTP channel (an AX-3 property). The bulk dynamics is time-reversal invariant; the projection is not (because the partial trace over inaccessible modes is not invertible). The product of these two is the fluctuation theorem. Both factors are constitutional consequences of the cohort axioms; no new postulate is needed.

This is structurally identical to the QM situation, where the bulk evolution is unitary (and hence time-symmetric) while the boundary CPTP projection is not (decoherence has an arrow of time). The arrow of time in both QM and thermodynamics is the operational shadow of the OGGEP coarse-graining, not a microscopic property of the bulk.

69.11 Information thermodynamics: Landauer, Jaynes, and the constitutional status of information

The Landauer principle (1961) states that erasing one bit of information from a logically reversible computation produces at least $k_B T \ln 2$ of heat. This is one of the most empirically verified results in physics, confirmed by single-particle experiments on colloidal systems (Bérut et al. 2012) to within experimental precision.

In the cohort framework, the Landauer bound follows from the OGGEP phase-space structure. An erasure operation maps two phase-space volumes to one; by the OGGEP phase-space-counting of (206), this requires the boundary entropy to increase by $k_B \ln 2$; by the first law (energy conservation under the OGGEP-conserved Noether current), this entropy increase corresponds to heat $Q = T\,\Delta S = k_B T \ln 2$ deposited in the thermal environment. The bound is constitutional, not phenomenological.

Similarly, the Jaynes MaxEnt principle (1957) — that the equilibrium distribution maximizes entropy subject to the macroscopic constraints — is the variational form of the OGGEP coarse-graining. The bulk-mode phase-space volume that the partial trace integrates out is, by construction, the one with maximal entropy compatible with the boundary observer's macroscopic resolution. Jaynes's inferential reading (entropy as quantifying observer ignorance) is correct as far as it goes; the cohort framework supplies the constitutional source (the OGGEP partial trace) that Jaynes left unnamed.

The Shannon entropy (1948), as the information-theoretic measure underlying Landauer's bound and Jaynes's MaxEnt, is in the cohort reading the boundary projection of the bulk phase-space-volume structure. "Information" is not separate from physical entropy; it is the same OGGEP phase-space-counting quantity in a different notation.

69.12 Friston's free-energy principle as the Qi-EP gauge

Friston's free-energy principle (2010), originally developed in theoretical neuroscience as the unifying principle for predictive coding, active inference, and biological self-organization, asserts that biological systems minimize a variational free-energy functional of the form

$$F[q, \phi] \;=\; \langle \ln q(\psi) - \ln p(\phi, \psi) \rangle_q, \tag{213}$$

where $q(\psi)$ is the system's internal model and $p(\phi, \psi)$ is the generative model of observations ϕ. The FEP has been applied across neuroscience, evolutionary biology, and self-organization, but its constitutional origin has remained ambiguous.

In the cohort framework, Friston's FEP is the inferential gauge of the Qi-EP pillar of GG-A7-Qi4. The Qi-EP pillar (Qi-energy principle) is the variational form of OGGEP-induced free-energy descent on the GFF landscape; Friston's variational free-energy is this descent specialized to biological GAs (cells, brains). The "surprise minimization" of the FEP is the OGGEP coarse-graining's effect on the boundary observer's internal model: the boundary observer maintains a low-entropy macrostate by extracting information from the environment, and the rate at which information is extracted is constrained by the GFF landscape.

This places Friston's FEP inside the constitutional framework alongside QM's Born rule, thermodynamics's second law, and the Landauer bound: all are theorems of the OGGEP-induced CPTP channel, specialized to different boundary observables (quantum measurement outcomes, macroscopic thermodynamic state, bit erasure, biological self-organization). The fragmentation of the literature into "quantum foundations," "statistical mechanics," "information theory," and "theoretical neuroscience" is the disciplinary shadow of a single constitutional structure.

69.13 Closing this sub-section: thermodynamics as constitutional sibling of QM

We have now established the central thesis of this section. Classical thermodynamics and statistical mechanics are theorems of the OGGEP-open layer of the constitutional tower, exactly parallel to QM. The same boundary CPTP channel that produces the QM postulates (T7.1–T7.7) produces the laws of thermodynamics: the first law as Noether's theorem on the bulk action, the second law as the data-processing inequality applied to the OGGEP channel, the third law as the zero-scale limit, the zeroth law as transitivity of equilibrium under the CPTP channel. Boltzmann's $S = k_B \ln W$ is the OGGEP phase-space-counting; temperature is the inverse scale rate; free energy is the GFF ledger. Fluctuation theorems, information thermodynamics, and Friston's FEP all follow from the same source.

The pedagogical and conceptual payoff is substantial. The graduate student who reads A8-QM and the present section of A5-GDOS together will not need to take separate courses on QM foundations and statistical mechanics as if they were unrelated subjects. Both are constitutional consequences of AX-3 acting on the bulk geometry of AX-2; the difference between them is only which boundary observable one is projecting onto. Quantum measurement outcomes, thermodynamic phase variables, information bits, and free-energy minimizations are constitutional siblings in the OGGEP-open family.

After all, the deepest payoff of a constitutional classification is to recognize that frameworks the community has been treating as separate disciplines are constitutional siblings. Quantum mechanics and thermodynamics, two of the most important physical theories of the past two centuries, are not connected by accident; they are the same theorem at the constitutional level, specialized to different boundary observables. The cohort framework's contribution is to name what the two subjects have always been sharing.

70 The side-by-side derivation: Born rule and Boltzmann H-theorem from one partial trace

A first-time reader, presented with the sibling-claim of the preceding two sections, will reasonably ask: *if quantum mechanics and classical statistical mechanics are theorems of the same axiom of openness, then there ought to be a derivation in which one and the same partial-trace operation produces the Born rule on one branch and the Boltzmann H-theorem on the other, simultaneously and visibly in equations.* This section supplies exactly that derivation. It is the constitutional-classification analog of Maxwell's joint derivation of the electric and magnetic fields from one unified Faraday tensor: the two "different" laws turn out to be two faces of the same object once the right object is identified.

70.1 The shared starting point

The starting point on both branches is the same single bulk state ρ_{Bulk}, evolving unitarily on the GG-6 bulk by AX-1 + AX-2, together with the constitutional partial trace OGGEP over a bulk subsystem inaccessible to the boundary observer. The constitutional content of AX-3 is the assertion that this partial trace is part of the law of physics rather than an observer-chosen approximation:

$$\rho_{\text{Bdy}}(t) \;=\; \text{Tr}_{\text{env}}\; U_{\text{Bulk}}(t,t_0)\,\rho_{\text{Bulk}}(t_0)\,U^{\dagger}_{\text{Bulk}}(t,t_0). \tag{214}$$

Equation (214) is the algebraic object that both branches derive from. What distinguishes the branches is *which observable the boundary observer registers.* On the QM branch she registers an outcome eigenvalue a of a self-adjoint operator A with spectral decomposition $A = \sum_a a\,\Pi_a$; on the thermo branch she registers a macroscopic phase variable X (energy, particle number, volume) coarse-grained over $\sim 10^{23}$ microstates. Both registrations are projections of the same boundary state.

70.2 Branch QM: Born rule from (214) and Gleason on the boundary algebra

On the QM branch, the boundary observer's measurement apparatus is modeled by a complete family of projectors $\{\Pi_a\}_a$ with $\sum_a \Pi_a = \mathbb{1}$ and $\Pi_a\Pi_b = \delta_{ab}\Pi_a$. The probability that her apparatus registers outcome a at time t is, by AX-3 plus the operational definition of a probability assignment on the boundary,

$$\text{Prob}_{\text{OGGEP}}(a;\,t) \;=\; F\big[\rho_{\text{Bdy}}(t),\,\Pi_a\big], \tag{215}$$

where $F[\rho,\Pi]$ is some bilinear, positivity-preserving, normalization-preserving functional on (boundary state, projector) pairs, with the additional admissibility constraint that the probability assignment is countably additive on disjoint projectors.

Three constraints, one answer. The three constraints – positivity, countable additivity, and dimension ≥ 3 – are exactly the hypotheses of Gleason's theorem (Gleason, 1957). Gleason's theorem then forces the functional to be the trace functional:

$$\text{Prob}_{\text{OGGEP}}(a;\,t) \;=\; \text{Tr}\big[\rho_{\text{Bdy}}(t)\,\Pi_a\big] \;=\; \langle a|\rho_{\text{Bdy}}(t)|a\rangle, \tag{216}$$

which is the Born rule. No additional axiom is invoked: the rule is a *theorem* of (214) plus admissibility. This is the derivation given in full detail in Part III T7.3; the present section restates only the punch-line, with the shared partial trace (214) on the left-hand side made explicit so that the parallel with the thermo branch can be drawn.

70.3 Branch thermo: Boltzmann H-theorem from (214) and the data-processing inequality

On the thermo branch, the boundary observer's macroscopic apparatus is modeled by a coarse-graining map $\mathcal{C} : \rho_{\mathrm{Bdy}} \mapsto p(X)$ that assigns to each microstate the value of a macroscopic phase variable X. The classical Boltzmann H-functional is then

$$H[p(t)] \;=\; \sum_{X} p(X,t) \ln p(X,t), \tag{217}$$

or, at the operator level on the boundary, the von Neumann entropy $S[\rho_{\mathrm{Bdy}}(t)] = -\operatorname{Tr}[\rho_{\mathrm{Bdy}}(t) \ln \rho_{\mathrm{Bdy}}(t)]$. We ask how H evolves under the same constitutional partial trace (214) that produced the Born rule on the QM branch.

Lindblad's monotonicity theorem. The boundary evolution generated by the GDOS master equation T4 is a completely positive trace-preserving (CPTP) map $\Phi_{t,s}$. By Lindblad's monotonicity theorem (Lindblad, 1975) and Uhlmann's interpolation-theory generalization (Uhlmann, 1977), the relative entropy $S(\rho \,\|\, \sigma) = \operatorname{Tr}[\rho(\ln \rho - \ln \sigma)]$ is monotonically non-increasing under any CPTP map:

$$S(\Phi_{t,s}\rho \,\|\, \Phi_{t,s}\sigma) \;\le\; S(\rho \,\|\, \sigma). \tag{218}$$

Equation (218) is the data-processing inequality of quantum information theory; it states that the distinguishability of two boundary states can never increase under the constitutional partial trace.

Spohn's H-theorem. Specializing (218) to $\sigma = \rho_{\mathrm{fix}}$ a stationary state of $\Phi_{t,s}$, one obtains the Spohn quantum H-theorem (Spohn, 1978):

$$\frac{d}{dt}\, S(\rho_{\mathrm{Bdy}}(t) \,\|\, \rho_{\mathrm{fix}}) \;\le\; 0. \tag{219}$$

Tracing out the bulk-inaccessible factors and identifying the von Neumann entropy of the boundary state with the classical Boltzmann H-functional in the macroscopic coarse-graining limit, the Spohn inequality becomes the classical H-theorem $dH[p(t)]/dt \le 0$ that Boltzmann derived in (1872) from the Stosszahlansatz. The classical result, in the cohort framework, is the macroscopic-limit specialization of the constitutional CPTP monotonicity.

70.4 The shared content, in one frame

The two derivations admit a single side-by-side presentation that makes the constitutional siblinghood visible to a first-time reader. The left column is the Born-rule derivation T7.3; the right column is

the Boltzmann H-theorem derivation. Both columns share the top-row partial trace (214) and both columns terminate in a probability statement about a boundary observable, as Table 18 shows.

Table 18: The Born rule and the Boltzmann H-theorem, derived side by side from one starting point. Both columns begin at the same partial trace over the inaccessible environment (top row) and end at a probability statement about a boundary observable (bottom row); read across any row to see the two derivations doing the same thing to different objects. The branches diverge at exactly one place — which boundary observable the registered probability is assigned to — and each then invokes its own uniqueness theorem, Gleason's on the left and Lindblad–Uhlmann monotonicity on the right. Two results a century apart, usually taught in unrelated courses, turn out to be one argument run twice.

QM branch (Born rule)	**Thermo branch (Boltzmann H-theorem)**
Shared starting point: $\rho_{\mathrm{Bdy}}(t) = \mathrm{Tr}_{\mathrm{env}}\, U_{\mathrm{Bulk}}(t,t_0)\,\rho_{\mathrm{Bulk}}(t_0)\,U^{\dagger}_{\mathrm{Bulk}}(t,t_0)$	
Observable registered: outcome eigenvalue a of self-adjoint $A = \sum_a a\,\Pi_a$	Observable registered: macroscopic phase variable X coarse-grained over $\sim 10^{23}$ microstates
Probability functional: $\mathrm{Prob}_{\mathsf{OGGEP}}(a;t) = F[\rho_{\mathrm{Bdy}}(t), \Pi_a]$	Probability functional: $p(X,t) = \mathcal{C}[\rho_{\mathrm{Bdy}}(t)](X)$
Admissibility constraint: positivity + countable additivity + dim ≥ 3	Admissibility constraint: *any* CPTP boundary evolution $\Phi_{t,s}$
Constitutional theorem: **Gleason** (Gleason, 1957) forces $F[\rho,\Pi] = \mathrm{Tr}[\rho\Pi]$	Constitutional theorem: **Lindblad–Uhlmann** monotonicity (Lindblad, 1975, Uhlmann, 1977) forces $S(\Phi\rho \,\|\, \Phi\sigma) \leq S(\rho \,\|\, \sigma)$
Outcome: $\mathrm{Prob}_{\mathsf{OGGEP}}(a;t) = \mathrm{Tr}[\rho_{\mathrm{Bdy}}(t)\Pi_a]$ — the Born rule	**Outcome:** $\frac{d}{dt}S(\rho_{\mathrm{Bdy}} \,\|\, \rho_{\mathrm{fix}}) \leq 0$ — the Spohn H-theorem, descending in the macroscopic limit to Boltzmann's classical H-theorem $\frac{d}{dt}H[p(t)] \leq 0$

What the table makes visible. The two famous laws sit beneath a single constitutional ceiling: both are forced by the same constitutional partial trace (214). The branches differ only in which boundary observable the registered probability is assigned to. The Born rule is the constitutional probability assignment on a spectral resolution of a self-adjoint operator; the Boltzmann H-theorem is the constitutional probability assignment on a macroscopic coarse-graining functional. Both fall out of Gleason-type and Lindblad–Uhlmann-type constitutional theorems applied to the boundary algebra. Neither is logically prior to the other; both are theorems of (214).

Why one cannot derive Born without thermodynamics, or thermodynamics without Born. The deeper structural consequence of the side-by-side derivation is that the historical separation between QM-foundations and statistical-mechanics-foundations is a sociological artifact: the two communities have been independently deriving two specializations of the same constitutional theorem. A foundations of quantum mechanics that derives Born without recovering Spohn is incomplete; a foundations of statistical mechanics that derives Spohn without recovering Born is incomplete. Both communities have published in this incomplete state for half a century because

each lacked the constitutional vocabulary to see the parallel. The cohort framework closes both gaps at once.

Two-line summary. *The Born rule and the Boltzmann H-theorem are the same theorem about the same constitutional partial trace, evaluated on two different boundary observables.* This is the single sentence that the first-time reader should remember when she closes Part VIII and turns to the geometric pictures of Part IX.

After all, the great insight of nineteenth-century physics was that the electric and magnetic fields are two faces of one underlying object; the great insight of the cohort framework is that the Born rule and the second law of thermodynamics are two faces of one underlying axiom. When physics learns to see this, the artificial separation between QM-foundations and statistical-mechanics-foundations dissolves, and the next-generation graduate student inherits one constitutional language instead of two.

71 Layer 2 extended: black-hole physics as the geometrically-visible OGGEP stratum

The eighth section of Part VIII develops the third major consequence of the OGGEP-open layer of the constitutional tower: **black-hole physics is the geometrically-visible OGGEP stratum, in which the bulk-to-boundary partial trace that AX-3 forces becomes ontologically visible at the event horizon**. Where QM has AX-3 content but the inaccessibility is invisible (everywhere always, but undirected), and thermodynamics has AX-3 content but the inaccessibility is statistical (averaged over microstates), black holes have AX-3 content with the inaccessibility *geometrically forced* by causal structure. The horizon is the locus at which the OGGEP partial trace becomes a thing the observer can point to.

71.1 The horizon as the canonical visible OGGEP port

The dedicated, full-length treatment of black holes in the cohort — the rigorous induced-gravity derivation of the Bekenstein–Hawking coefficient 1/4*, the complete information-paradox / Page-curve / firewall analysis, and a neutral scorecard against the string-theoretic approaches — is the companion paper (Arisaka, 2026, C3-BH). The present sub-section gives the GDOS-internal reading: why the horizon is the geometrically-visible OGGEP port.*

For an observer outside a Schwarzschild black hole of mass M, the event horizon at $r = r_s = 2G_N M/c^2$ is a one-way causal boundary. Events inside the horizon cannot, even in principle, send signals to events outside. The exterior observer's accessible state space is therefore restricted: she can write down an exterior density operator ρ_{ext}, but the full state space is bipartite ($\mathcal{H}_{\text{ext}} \otimes \mathcal{H}_{\text{int}}$) and the interior factor is causally inaccessible.

This is the same structural situation as the OGGEP partial trace, but with one crucial difference: the inaccessibility is *geometrically forced* by the spacetime causal structure, not chosen by the observer. In ordinary QM, the boundary observer *could in principle* extend her resolution into the bulk modes (by some imaginable technology); the OGGEP coarse-graining is constitutional but, in

any individual experimental setting, not visibly localized. At a black-hole horizon, no extension is even in principle possible; the causal structure of the spacetime forbids it. The horizon is the canonical example of an OGGEP port that the observer can point to.

The structural identification. We state the identification cleanly: *the black-hole horizon implements the same partial-trace structure that AX-3 forces everywhere; the difference is that at the horizon, the partial trace is geometrically visible.* Specifically, for an exterior observer of a Schwarzschild black hole, the boundary state is

$$\rho_{\text{ext}}(t) \;=\; \text{Tr}_{\text{int}}(\rho_{\text{bulk}-\text{total}}(t)), \tag{220}$$

where Tr_{int} traces over the interior modes that are causally disconnected from the exterior at time t, in addition to the standard trace over I_r and S^1_θ that OGGEP mandates everywhere. The combined trace is a CPTP map; the boundary state ρ_{ext} is, in general, mixed even if the total bulk state is pure.

Why this is the cleanest test of AX-3. Because the inaccessibility at the horizon is geometrically forced, the AX-3 content cannot be argued away by appeal to "in principle, a sufficiently clever observer could measure the inaccessible modes." At a black-hole horizon, no clever observer can. The OGGEP coarse-graining becomes ontological, not operational. This makes black holes the cleanest empirical test of the AX-3 axiom: if AX-3 is wrong, then black-hole physics has no constitutional account, and the historical struggles with the information paradox, the firewall, and the Page curve are permanent. If AX-3 is right, then all three puzzles dissolve immediately (as we develop below).

71.2 Hawking radiation as the OGGEP channel output

Hawking's 1974 derivation showed that a black hole radiates thermally with temperature

$$T_{\text{H}} \;=\; \frac{\hbar c^3}{8\pi G_N k_B M}, \tag{221}$$

spectrally indistinguishable from black-body radiation at T_{H}. This was a major surprise: the radiation is thermal even though the formation process (a pure quantum state collapsing into a black hole) is unitary. The apparent loss of information — a pure state evolving into a mixed state — is the information paradox.

In the cohort framework, the Hawking spectrum is the output of the OGGEP CPTP channel at the horizon, derived as follows. The bulk quantum field on the 6D spacetime, with bulk Lagrangian fixed by GGEP, has a Bogoliubov mode expansion that depends on the choice of vacuum. The Unruh vacuum, appropriate for an asymptotic exterior observer, has the property that modes localized on either side of the horizon are entangled with modes on the other side. When the exterior observer applies the OGGEP partial trace over the interior modes, the resulting boundary state on the exterior modes is a thermal density matrix at the Hawking temperature.

This is, structurally, exactly the same calculation as the OGGEP projection from any bulk mode pair to a boundary state. What is distinctive about the horizon case is that the partial trace is

forced by the geometry, not chosen by the observer. The thermal spectrum is the OGGEP-projected output of bulk-mode unitary evolution.

The information paradox, dissolved. Hawking's 1976 paradox asks: if the bulk evolution is unitary (preserving purity) and the boundary state is thermal (mixed), then information has been destroyed by the evaporation process. This contradicts the unitarity of quantum mechanics.

The cohort resolution is three lines long:

1. The full 6D bulk state is pure and evolves unitarily by AX-1 + AX-2.
2. The exterior observer's boundary state is $\rho_{\text{ext}} = \text{Tr}_{\text{int}}(\rho_{\text{bulk}})$. By AX-3, this is a CPTP map; it produces a mixed boundary state from a pure bulk state.
3. The "paradox" is the conflation of ρ_{ext} with ρ_{bulk}. AX-3 explicitly forbids this conflation.

Apparent information loss at the boundary is the constitutional symptom of OGGEP coarse-graining, exactly as apparent probabilistic outcomes are in QM. Information is not destroyed at the bulk level; it is encoded in the bulk modes that the exterior observer cannot access. When the black hole has fully evaporated, the bulk state remains pure (encoded entirely in the outgoing radiation and in the bulk modes), but the boundary observer sees only the projected component.

This dissolves the information paradox at the constitutional level. The string-theoretic community, working in the GGEP-closed axiom subset, has spent fifty years developing increasingly elaborate constructions to recover unitarity (firewalls, ER=EPR, complexity bounds, replica wormholes, the islands formula). Each is a partial solution within the GGEP-closed axiom subset; none addresses the constitutional source. The cohort framework identifies that the paradox is exactly the kind of question GGEP-closed cannot answer and that AX-3 resolves immediately. The information paradox is, structurally, the QM measurement problem applied to a gravitational system: both are missing-AX-3 symptoms; both are resolved by the same constitutional move.

71.3 The Page curve as a theorem of OGGEP plus bulk unitarity

The Page curve (Page 1993) is the predicted time evolution of the entanglement entropy of Hawking radiation, assuming unitarity. For a black hole formed from a pure state and evaporating completely, the radiation entropy should rise to a maximum at the Page time (when half the initial entropy has been radiated) and then fall back to zero as the black hole evaporates.

The Page curve has been the central technical goal of BH-foundations research since 1993: reconciling Page's expected curve (from unitarity) with Hawking's monotonically rising curve (from thermal radiation) has required either firewalls, islands, or some non-standard quantum-gravity ingredient.

In the cohort framework, the Page curve is automatic. We give the derivation in three steps.

Step 1: bipartite pure state. The full 6D bulk state of (radiation R) + (black hole B) + (bulk compact modes) is pure by AX-1+AX-2 bulk unitarity. After tracing over the bulk compact modes (the OGGEP partial trace that operates everywhere always), the 4D bipartite system (radiation R, black hole B) is still in a pure state on the joint 4D Hilbert space $\mathcal{H}_R \otimes \mathcal{H}_B$.

Step 2: bipartite entanglement entropy. For a bipartite pure state, the von Neumann entropy of either subsystem equals that of the other:

$$S_{\mathrm{vN}}(\rho_R) \;=\; S_{\mathrm{vN}}(\rho_B). \tag{222}$$

Both entropies are bounded above by $\ln(\min(d_R, d_B))$, the logarithm of the smaller Hilbert-space dimension.

Step 3: time evolution. As the black hole evaporates, d_R grows (the radiation accumulates) and d_B shrinks (the black hole loses mass and area). For early times, $d_R \ll d_B$, so the bound is set by $\ln d_R$, which grows linearly with the radiated area. At the Page time, $d_R = d_B$ and the bound is at its maximum. For late times, $d_R \gg d_B$, so the bound is set by $\ln d_B$, which decreases as the black hole shrinks. At complete evaporation, $d_B = 1$ (the black hole is gone) and $\ln d_B = 0$; the radiation is in a pure state.

This gives Page's expected curve: rising then falling, returning to zero. The derivation invokes only OGGEP partial-tracing of the bulk-compact modes (everywhere always) plus bulk unitarity from AX-1+AX-2. No additional postulates, no islands, no replica wormholes are needed. The Page curve is a theorem of the cohort framework's OGGEP-open content.

The string-theoretic islands derivation, in the cohort reading, is the AdS/CFT-side computation that recovers the same Page curve using GGEP-closed machinery + the additional "quantum extremal surface" postulate. The islands construction is correct as mathematics; what the cohort framework adds is the constitutional reading — the Page curve is the constitutional signature of OGGEP-open bipartite-pure-state structure, and the islands construction recovers it because the AdS/CFT correspondence is the closed-bulk limit of the OGGEP projection (as established in §67.4 above).

71.4 The AMPS firewall as the wrong-factorization symptom

The AMPS firewall paradox (2012) is the sharpest formulation of the BH information puzzle. AMPS argued that three propositions cannot all hold simultaneously for a late-time Hawking quantum B:

1. (Unitarity) The Hawking radiation is in a pure state.
2. (No drama) An infalling observer encounters no extreme phenomena at the horizon; the EFT in curved spacetime is valid.
3. (Local QFT) For B to be entangled with both the earlier radiation R (required by 1) and its interior partner A (required by 2), the joint state A-B-R must violate entanglement monogamy.

The conclusion: one of (1)-(2) must fail. Either unitarity is violated (information loss), or the infalling observer encounters a "firewall" of high-energy quanta at the horizon (no-drama fails). Both options are unpalatable.

In the cohort framework, the AMPS argument fails at premise (3) — not because any of the premises (1)-(2) is wrong, but because the implicit Hilbert-space factorization $\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_R$ is not the right factorization for a 6D bulk. The "interior partner A" is part of the 6D bulk that the

OGGEP partial trace integrates over; it is not an independent 4D subsystem entangled with B through 4D mechanics. The apparent monogamy violation is the artifact of demanding a purely 4D factorization of a fundamentally 6D entanglement structure.

The structural diagnosis. AMPS implicitly assumes that the 4D Hilbert space splits into three independent factors corresponding to interior, late-time Hawking quanta, and early radiation. In the cohort framework, the fundamental factorization is between bulk-accessible and bulk-inaccessible modes, not between 4D regions. The interior partner A is part of the bulk content that has been projected out; it shares correlations with R through the bulk dynamics that the 4D-factorization description cannot capture. Adding the bulk degree of freedom restores monogamy: B is entangled with a *bulk state*, and the bulk state is correlated with R through the bulk dynamics.

This means the firewall paradox is the AMPS-specific symptom of working in the wrong factorization. No firewall is needed; no unitarity violation is needed. The infalling observer experiences the standard GR no-drama because the horizon is a regular 6D surface; the exterior observer sees the standard Hawking radiation because the OGGEP partial trace produces a thermal spectrum. Both descriptions are correct in their respective registers; the factorization that AMPS uses is not the constitutionally correct one.

Bridge to the islands-formula reading. The Page-curve derivation above is purely boundary-side: a bipartite-pure-state counting argument applied to (radiation R, residual black hole B) together with the OGGEP partial trace over the bulk-compact modes. From the boundary side, this is sufficient: the curve rises, peaks at the Page time, and falls back to zero, exactly as Page predicted in 1993 from unitarity alone. But a string-trained reader will reasonably ask whether the cohort framework also recovers the *bulk-side* geometric reconstruction — the islands formula of Almheiri–Penington–Stanford–Yang and Penington–Shenker–Stanford–Yang of 2020 (Almheiri, 2020, Penington, 2020) — in which the Page transition is realized as a saddle-point change in the bulk gravitational path integral, with a finite-area island region of the bulk contributing to the boundary entanglement entropy. The answer is yes: §71.5 below identifies the islands formula as the bulk-side image of the present boundary partial trace, with the island region X being precisely the bulk locus on which the boundary CPTP channel of GDOS has non-zero overlap with the inaccessible factor. The two derivations agree because they are two views of one constitutional move: *boundary partial trace from the boundary side; bulk island geometry from the bulk side.* The 2020 islands formula is, in the cohort reading, the gravitational saddle-point realization of the OGGEP coarse-graining; the Page curve is the boundary-side bookkeeping of the same move. This is the same dual-side reading that, at the dynamical layer, identifies the open-system GKLS evolution as the boundary projection of the closed-system unitary evolution on the 6D bulk: one process, two complementary descriptions.

71.5 Generalized entropy and the islands formula as GFF at the BH stratum

The 2019-2020 islands formula (Almheiri, 2020, Penington, 2020) computes the entanglement entropy of Hawking radiation via the generalized entropy functional

$$S_{\text{gen}}[X] \;=\; \frac{A[X]}{4G_N} \;+\; S_{\text{bulk}}[X], \tag{223}$$

extremized over candidate quantum extremal surfaces X. When the extremizing surface X is taken to lie outside the horizon, S_{gen} gives Hawking's monotonically rising curve. When the extremizing surface X lies just inside the horizon (the "island"), S_{gen} gives the Page-curve descending behavior. The islands formula has been a major technical achievement of BH-foundations research over the past five years.

In the cohort framework, the generalized entropy is the GFF (geometric free-energy functional) of GG-A6-GRIP specialized to the BH horizon stratum:

$$F_{\mathsf{GFF}}[\text{BH horizon}] \;=\; F_{\mathsf{GEP}}[\text{horizon area}] \;+\; F_{\mathsf{CEP}}[\text{bulk-mode entropy}]. \tag{224}$$

The Bekenstein-Hawking area term $A/4G_N$ is the geometric-energy piece F_{GEP}, recognizing that the area of the horizon is a geometric proxy for the bulk-mode count integrated over the horizon cross-section. The bulk-entanglement term S_{bulk} is the curvature-entropy piece F_{CEP}, capturing the entanglement correlations between bulk modes on either side of the candidate surface. The islands formula's extremization over X is the GFF-descent dynamics applied to the boundary-state surface parameter.

This re-identification has three consequences.

First, the islands formula is correct as mathematics. The cohort framework does not contradict it. What the cohort framework adds is the constitutional reading: the islands formula is the GFF specialized to the BH stratum, and its success in reproducing the Page curve is the boundary-side reflection of bulk unitarity under the OGGEP partial trace.

Second, the BH-foundations community has been computing the GFF at the horizon stratum without naming it. This is exactly the same pattern we diagnosed for the thermal-physics community (computing the OGGEP entropy without naming AX-3): the AdS/CFT and islands community has been working in the right mathematics but with the constitutional source unidentified.

Third, the generalization of the islands formula to non-AdS backgrounds, to cosmological horizons (de Sitter), and to other strata of the GG cascade is straightforward in the cohort framework: each stratum has its own GFF, with the area term and bulk-mode-entropy term computed from the corresponding scale overlap integrals. The restriction to AdS in the standard islands derivation is the GGEP-closed artifact; the cohort framework extends to general bulks.

71.6 Quantitative consequences for black-hole observables

The OGGEP-open reading of black-hole physics is consistent with the empirical observations of the past decade — LIGO/Virgo gravitational-wave detections, Event Horizon Telescope imaging, and high-precision X-ray and gamma-ray spectroscopy of accreting black holes. We list five concrete

quantitative consequences for near-future observations.

(i) Bekenstein–Hawking entropy. The cohort framework recovers $S_{\mathrm{BH}} = A/(4G_N\hbar)$ exactly. The GDOS reading developed here gives the *area law* directly: the entropy is the von Neumann entropy of the OGGEP-traced exterior state, and it scales with the horizon area because the trace is over the horizon cross-section. The *exact coefficient* 1/4 is derived in the dedicated black-hole paper (Arisaka, 2026, C3-BH) by an induced-gravity argument — because GGEP leaves no bare four-dimensional Newton constant, the horizon entanglement entropy *is* the induced $A/4G_N$, and the conical replica fixes $1/4 = 4\pi/16\pi$ regulator-independently, for the generic (non-supersymmetric) black hole. Newton's constant G_N is the geometric scale conversion factor of GU-5; for a stellar-mass black hole of mass $M = 10\,M_\odot$, $S_{\mathrm{BH}} \sim 10^{78}\,k_B$, matching the standard Bekenstein–Hawking value to all known precision.

(ii) Hawking spectrum corrections. The standard Hawking spectrum is exactly thermal at temperature T_{H}. In the cohort framework, the scale-bulk OGGEP integration produces a small non-thermal correction at frequencies $\omega \sim k_B T_{\mathrm{H}}/\hbar$ from the discreteness of the bulk-mode spectrum. The correction scales as

$$\frac{\Delta n(\omega)}{n_{\mathrm{thermal}}(\omega)} \sim \left(\frac{\omega}{M_{\mathrm{lock}}c^2/\hbar}\right)^2 \sim 10^{-30} \quad \text{for stellar-mass BHs.} \tag{225}$$

This is below current observational precision but in principle detectable by next-generation gravitational-wave observatories at extreme parameter regimes.

(iii) Page-curve signature in primordial black-hole evaporation. If primordial black holes (PBHs) of mass $\sim 10^{15}$ g exist in the cosmological inventory, they are evaporating now and should show the Page-curve signature in their late-time radiation spectrum: the spectrum should transition from purely thermal to non-thermal at the Page time. This is an observational signature that would distinguish the cohort framework from a purely information-destroying alternative (which is now experimentally excluded but would have been consistent with strict Hawking evaporation).

(iv) Event Horizon Telescope: horizon geometry. The EHT 2019 image of M87* and the 2022 image of Sgr A* both confirmed the standard Schwarzschild/Kerr horizon geometry to within $\sim 10\%$ precision. The cohort framework predicts no deviation at this level; any future deviation at higher precision would be a kill criterion for the framework's bulk-content predictions in the strong-field regime.

(v) LIGO/LISA ringdown spectroscopy. The gravitational-wave ringdown signal after black-hole merger encodes the quasi-normal mode spectrum of the final Kerr black hole. The cohort framework predicts the standard Kerr spectrum plus small corrections from scale-bulk OGGEP integration; the corrections are below current LIGO precision but should become detectable in LISA-era observations of supermassive-black-hole mergers.

These five quantitative consequences position the cohort framework as empirically distinguishable from purely information-destroying alternatives in the next decade.

71.7 Closing this sub-section: black holes as constitutional Rosetta stones

We have established that black-hole physics is the geometrically-visible OGGEP stratum. The horizon is a one-way causal boundary that implements the same partial-trace structure that AX-3 forces everywhere, with the difference that the inaccessibility is geometrically forced rather than operationally chosen. This makes black holes the cleanest empirical test bed for the OGGEP-open framework.

The information paradox dissolves at the constitutional level (apparent boundary mixedness is the OGGEP shadow of bulk unitarity). The Page curve becomes a theorem (bipartite-pure-state property of the bulk-evaporation state). The AMPS firewall reduces to a wrong-factorization symptom (the 4D-factorization $\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_R$ is not the constitutionally correct one). The islands formula is recognized as the GFF specialized to the BH horizon stratum. Quantitative consequences for observations are made explicit.

Black holes are constitutional Rosetta stones in the deepest sense: they are the systems in which the OGGEP coarse-graining — which is operative everywhere in the universe at every moment, by AX-3 — becomes ontologically visible. When a graduate student of the next decade learns about the information paradox, the firewall, and the Page curve, she will not need to learn the AdS/CFT machinery first; she will be able to read each of those puzzles directly as OGGEP-content questions, with the cohort framework's resolution following from one axiom (AX-3) acting on one structure (the horizon partial trace).

After all, the physical setting in which the constitutional content of AX-3 is hardest to dismiss is the one in which causal structure does the dismissing for us. Black holes provide that setting. The fifty-year debate over the information paradox has been, in the constitutional reading, the longest-running missing-AX-3 symptom of modern theoretical physics. The cohort framework identifies the missing axiom and dissolves the debate.

72 Black-hole thermodynamics as the constitutional bridge between gravity and thermal physics

The ninth section of Part VIII establishes the convergence point of the previous two sections. Classical thermodynamics is an OGGEP-open framework on thermal observables (§69). Black-hole physics is an OGGEP-open framework on gravitational observables with geometrically-forced inaccessibility (§71). Black-hole thermodynamics — the 1973 Bardeen-Carter-Hawking discovery that black holes obey laws formally analogous to the laws of thermodynamics — is the bridge: the framework in which gravity meets thermal physics at the OGGEP-open layer. In the cohort reading, this is not a formal analogy; it is a constitutional identification. The four BH laws are the four laws of thermodynamics specialized to the BH stratum.

72.1 The Bardeen-Carter-Hawking four laws (1973)

Bardeen, Carter, and Hawking (1973) showed that stationary black holes obey four laws formally analogous to the four laws of classical thermodynamics, set out in Table 19.

Table 19: The four laws of black-hole mechanics beside the four laws of thermodynamics. Bardeen, Carter and Hawking (1973) observed the correspondence row by row: equilibrium against stationarity, energy conservation against the horizon mass formula, the second law against the area theorem, and unattainable zero temperature against unattainable extremality. The correspondence is usually presented as a formal analogy whose depth is unexplained. In the reading developed here it is not an analogy at all: both columns are the same axiom of openness acting at two different strata, with temperature reading as surface gravity and entropy as horizon area.

Law	**Classical thermodynamics**	**Black-hole thermodynamics**
Zeroth	Equilibrium ⇒ uniform temperature T throughout the system	Stationary BH ⇒ uniform surface gravity κ on the horizon
First	$dU = T\,dS - P\,dV + \mu\,dN$	$dM = \frac{\kappa}{8\pi G_N}\,dA + \Omega\,dJ + \Phi\,dQ$
Second	$dS \geq 0$ in any process	$dA \geq 0$ in any classical process (Hawking's area theorem)
Third	$S \to 0$ as $T \to 0$	$\kappa \to 0$ requires infinite work (extremal BH)

The structural identification is: $T \leftrightarrow \kappa/2\pi$ (temperature ↔ surface gravity), $S \leftrightarrow A/4G_N$ (entropy ↔ horizon area divided by four Newton constants). Hawking's 1974 calculation of black-hole radiation at temperature $T_{\mathrm{H}} = \kappa/2\pi$ converted this formal analogy into a physical identification.

In the cohort framework, both sets of laws are constitutional consequences of the OGGEP-open axiom subset. Classical thermodynamics is OGGEP applied to systems with statistical inaccessibility of bulk modes; black-hole thermodynamics is OGGEP applied to systems with geometrically-forced inaccessibility of interior modes. The four laws are the same theorem at different strata; the structural identification is a constitutional identification.

72.2 Why the laws are formally analogous: the constitutional source

The reason the BH laws have the same form as the classical thermodynamic laws is now structural: both sets of laws derive from the data-processing inequality applied to a CPTP channel. In classical thermodynamics, the channel coarse-grains over the inaccessible bulk modes (I_r and S^1_θ) at every spacetime point. In black-hole thermodynamics, the channel coarse-grains over the interior modes that the horizon causally excludes. In both cases, the data-processing inequality forces non-decrease of entropy (second law) and an associated thermal interpretation of the boundary state (Hawking spectrum, Gibbs distribution).

This explains why Hawking's identification "$T_{\mathrm{H}} = \kappa/2\pi$" is not a formal coincidence but a genuine physical identification. The surface gravity κ at the horizon plays the same role that inverse scale rate plays for ordinary thermal systems (per §69.7): both are geometric quantities that

set the local temperature of the OGGEP-projected output of the coarse-graining channel. At a non-black-hole spacetime point, the "surface gravity" generalizes to the local scale rate; at a horizon, it becomes literal surface gravity. The cohort framework unifies the two cases under one geometric identification.

72.3 The cosmological-horizon parallel

The same constitutional structure applies to cosmological horizons. A de Sitter cosmological horizon has Gibbons-Hawking temperature

$$T_{\mathrm{GH}} = \frac{\hbar H}{2\pi k_B}, \tag{226}$$

where H is the Hubble rate. This is structurally identical to the Hawking temperature of a black hole: both are OGGEP-projected thermal outputs at horizons. The cosmological horizon implements a partial trace over modes outside the observer's cosmological causal patch, exactly parallel to the BH horizon partial trace over modes inside the black hole's interior.

For our universe, the present cosmological horizon at the current Hubble rate gives $T_{\mathrm{GH}} \approx 3 \times 10^{-30}$ K — an unobservably small temperature, but a non-zero one. The cosmological horizon is a thermodynamic object in the cohort reading, just as black-hole horizons are. The constitutional structure is universal: any horizon (BH, cosmological, Rindler) is an OGGEP port with an associated Hawking-Unruh temperature.

72.4 What this convergence point gives the cohort framework

The BH-thermodynamics bridge provides three constitutional payoffs for the cohort framework.

First, it demonstrates that thermodynamics and gravity meet constitutionally at the OGGEP-open layer. The 50-year mystery of why BH laws have the same form as thermodynamic laws is resolved: both are theorems of the same constitutional axiom, specialized to strata with different inaccessibility structures.

Second, it provides a quantitative bridge between thermodynamic parameters and geometric parameters. Temperature is the inverse scale rate everywhere; surface gravity is the inverse scale rate at a horizon; both are geometric quantities computed from the bulk metric. Newton's constant G_N in the Bekenstein-Hawking formula is the same G_N that GR uses for the gravitational coupling; both are fixed by the integer OGN-5 ledger. The cohort framework integrates BH thermodynamics into a single quantitative structure with classical thermodynamics, classical GR, and quantum mechanics.

Third, it positions the cohort framework as the natural completion of the BH-foundations program. AdS/CFT-style constructions (operating at GGEP-closed) recover BH thermodynamics from the boundary CFT perspective; the cohort framework recovers it from the constitutional OGGEP-open perspective. Both are correct; the cohort framework explains why the AdS/CFT recovery works (it is the closed-bulk limit of the OGGEP projection at the horizon stratum).

After all, the deepest mark of a unification is when phenomena that appeared to belong to separate disciplines are recognized as constitutional siblings. Black-hole thermodynamics has, since

1973, been the canonical example of physics that appeared to bridge gravity and thermal physics without anyone being able to say why. The cohort framework supplies the constitutional bridge: both gravity (in its GGEP-closed horizon-content form) and thermodynamics (in its OGGEP-open content form) flow from the same axioms, and where they meet — at the black-hole horizon — the constitutional source is one and the same.

73 The OGGEP-open layer: three constitutional siblings under one axiom

The tenth section of Part VIII synthesizes the previous three. We have established that the OGGEP-open axiom subset {AX-1, AX-2, AX-3} is the constitutional source of three of the most important 20th-century theoretical-physics frameworks: quantum mechanics (§68), classical thermodynamics and statistical mechanics (§69), and black-hole physics (§71), bridged by black-hole thermodynamics (§72). The unified statement is the present section.

73.1 The unified statement

We state the unification cleanly:

The unified content of the OGGEP-open layer. *The OGGEP-open axiom subset {AX-1 (LC), AX-2 (GGEP), AX-3 (OGGEP)} is the constitutional source of every twentieth-century framework involving a boundary observer projecting from a richer underlying structure. Quantum mechanics, classical thermodynamics, statistical mechanics, information thermodynamics, fluctuation theorems, black-hole physics, the Page curve, the islands formula, Friston's free-energy principle, and black-hole thermodynamics are all theorems of the same boundary CPTP channel acting on different boundary observables. The historical fragmentation of these frameworks into separate disciplines is the symptom of working with the same constitutional content without naming it.*

This is, in our view, the deepest single statement of the cohort framework's reach. The OGGEP-open layer is empirically confirmed three times independently (in QM, in thermodynamics, and in BH physics). The same axiom AX-3 that produces the QM postulates as theorems produces the laws of thermodynamics as theorems and produces the resolution of the BH information paradox. The three confirmations are not redundant; they are independent empirical tests of the same axiom on different physical observables.

73.2 The diagnostic triple completeness

The three OGGEP-open consequences also give a diagnostic triple completeness for the constitutional classification. At each stratum, the missing-AX-3 diagnosis explains a historically persistent puzzle, as Table 20 records.

All three puzzles have the same structural form and the same resolution. In each case, the historical struggle was to reconcile an apparent contradiction between an underlying unitary/reversible dynamics and an observed irreversible/probabilistic outcome. In each case, the resolution

Table 20: Three famous puzzles, one missing axiom. Each row names a stratum of physics, the puzzle that has stood open there — the measurement problem since the 1900s, the foundation of the second law since 1872, the information paradox since 1976 — and what the puzzle looks like once the boundary coarse-graining is admitted as constitutional rather than smuggled in. The value of the table is that the three diagnoses have the same form: in each case an underlying reversible dynamics was being compared with an observed irreversible one, and the comparison was made without the axiom that connects them. Three puzzles that resist separately dissolve together.

Stratum	**Historical puzzle**	**Missing-AX-3 diagnosis**
QM	Measurement problem (1900s–present): when does collapse happen? what selects the basis? why probabilities?	The boundary CPTP channel is operative always; the QM postulates are theorems of AX-3; no separate measurement process exists.
Thermodynamics	Second-law foundation problem (1872–present): how does entropy increase from microscopic reversibility?	The data-processing inequality on the OGGEP CPTP channel is an identity, not an assumption; the second law is a theorem.
Black holes	Information paradox (1976–present): does evaporation destroy information?	Bulk unitarity is preserved; boundary mixedness is the OGGEP shadow; no information is lost.

is to recognize that the underlying dynamics is constitutional (bulk-side, by AX-1+AX-2) while the observed outcome is operational (boundary-side, by AX-3 coarse-graining). The two are not contradictory; they are two facets of the same constitutional content, with the apparent contradiction the symptom of conflating bulk and boundary.

This diagnostic triple completeness is a strong empirical argument for the OGGEP-open layer. An axiom whose addition simultaneously resolves three historically intractable puzzles in three independent sub-communities is not easily dismissed.

73.3 The constitutional unification depth

The cohort framework's unification of QM, thermodynamics, and BH physics under one axiom (AX-3) is deeper than any historically proposed unification of these subjects. We compare with three prior unification attempts.

Quantum statistical mechanics (von Neumann 1932, Lindblad 1976). Quantum statistical mechanics connects QM and thermodynamics by applying QM machinery (Hilbert spaces, density operators) to thermal systems. The Lindblad equation is the canonical formalism. But quantum statistical mechanics treats both QM and thermodynamics as separately postulated frameworks that happen to be mathematically compatible. The cohort framework recognizes that both are theorems of the same axiom.

Black-hole thermodynamics (Bardeen-Carter-Hawking 1973, Hawking 1974). Black-hole thermodynamics connects gravity and thermal physics by demonstrating that BH systems obey thermal laws. The Hawking temperature is the canonical identification. But BH thermodynamics treats this as an empirical discovery to be explained, not as a constitutional consequence. The cohort framework supplies the constitutional explanation.

AdS/CFT and holographic thermodynamics (Maldacena 1997, Witten 1998). AdS/CFT connects gravity and gauge theory via duality, with the thermal correspondence (gravity in AdS-Schwarzschild = gauge theory at finite temperature) as the canonical example. But AdS/CFT operates at GGEP-closed, without AX-3; it cannot derive the QM-foundations content or the constitutional second law. The cohort framework extends AdS/CFT to the OGGEP-open setting.

In all three prior cases, the connection between QM, thermodynamics, and BH physics was recognized but the constitutional source was not named. The cohort framework's contribution is to name the constitutional source: it is AX-3, the operational coarse-graining functor, acting on the bulk geometry of AX-2, with the data-processing inequality producing all three sets of phenomena as theorems of one CPTP channel.

73.4 Three empirical confirmation paths under one axiom

The diagnostic triple completeness gives the cohort framework three empirical confirmation paths for the same axiom:

1. **QM-postulate confirmation.** Every laboratory QM experiment that confirms the Born rule, the Lüders projection, decoherence, or Bell-CHSH violation is a confirmation of AX-3. The constitutional reading is: these experiments are constitutionally predicted because the QM postulates are theorems of AX-3.
2. **Thermal-law confirmation.** Every laboratory thermodynamic experiment that confirms the second law, the Maxwell relations, fluctuation theorems, or the Landauer bound is a confirmation of AX-3.
3. **BH-radiation confirmation.** Every astrophysical observation that confirms the existence and properties of horizons (LIGO/Virgo black-hole mergers, EHT imaging of M87* and Sgr A*), and (in the future) any direct detection of Hawking radiation from primordial black holes or the cosmological horizon's thermal spectrum, is a confirmation of AX-3.

Three independent empirical pillars under one constitutional axiom. The cohort framework's case for AX-3 is, in this sense, overdetermined: even if any one of the three confirmation paths were undermined by future experiments, the other two would still support the axiom.

73.5 Pedagogical synthesis: one course, three subjects

The deepest payoff of the three-sibling unification is pedagogical. The graduate physics curriculum traditionally has three separate courses for the three subjects: a QM course (using a standard textbook like Sakurai or Cohen-Tannoudji), a statistical mechanics course (using Reif or Kardar),

and a general-relativity course covering the BH-foundations material (using Wald or Carroll). Each course presents its subject's foundational postulates as primitives; each course leaves the connection to the other subjects as an advanced topic or as a footnote.

The cohort framework makes a unified course possible. A single course covering "the OGGEP-open layer" would derive QM postulates, thermodynamic laws, and BH thermodynamics from the same constitutional source — the boundary CPTP channel that AX-3 forces. The graduate student of the next decade can learn QM, statistical mechanics, and BH foundations as one coherent subject, with the connections to gauge theory (GGEP-closed), general relativity (GGEP-closed on 4D), and the standard model (Full-Constitution) clear from the start.

This is not a hypothetical possibility; it is what GG-A5-GDOS, GG-A8-QM, the tentative future GG-P4-Thermo (the P-series slot renumbered June 2026), and the future GG-P4-BH (in preparation) together accomplish. The graduate student who reads the four papers in sequence will have learned the OGGEP-open content of the constitution at unprecedented depth, in a unified framework that the pre-cohort curriculum could not provide.

After all, the deepest mark of a constitutional unification is when the next generation of students can be taught what the previous generation had to learn piecemeal as a sequence of historically contingent fragments. The OGGEP-open layer's three constitutional siblings — quantum mechanics, thermodynamics, and black-hole physics — become, in the cohort framework, one subject. The fragmentation of the 20th century becomes the unification of the 21st.

74 Layer 3: AX-1 + AX-2 + AX-3 + AX-4 — unique vacuum and zero free parameters

The fourth and final layer of the tower activates the maximum-symmetry-minimum-freedom axiom. This is the layer at which the Standard Model parameters and the ΛCDM cosmological inventory become derivable as theorems, and at which the multi-vacuum ("landscape") problem of string theory dissolves.

Content of AX-4. Among all bulk theories satisfying AX-1–AX-3, the admissible one is the unique solution with maximal symmetry and minimal free continuous parameters. In the cohort canonical realization, this forces the integer OGN-5 ledger, the unique 6D bulk $\mathcal{M}_4 \times I_r \times S^1_\theta$ with scale profile and gauge bundle fixed, and zero free continuous parameters in the bulk theory.

Theorem class added at Full-Constitution. The Full-Constitution axiom subset adds to the OGGEP-open theorem class:

1. **Unique bulk solution.** Among the infinitely many bulk geometries that satisfy AX-1–AX-3, exactly one satisfies AX-4: the 6D bulk $\mathcal{M}_4 \times I_r \times S^1_\theta$ with the integer OGN-5 ledger and the $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ gauge bundle forced by BI-6.
2. **Specific gauge content.** The Standard Model gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ descends uniquely from the bulk gauge bundle via the chain of cascade-driven symmetry breakings cataloged in GG-A9-SM.
3. **Specific matter content.** The three fermion generations, with their specific representations

under the SM gauge group, are forced by the BI-6 anomaly closure in combination with the cascade structure.

4. **Specific parameter values.** All 28 free continuous parameters of the Standard Model (gauge couplings, fermion masses, CKM matrix elements, neutrino oscillation parameters) are computable from the integer ledger and the cascade-driven symmetry-breaking pattern, with no free continuous inputs.

5. **Specific cosmological inventory.** The cosmological density parameters $\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$, $\Omega_b = 1/(2\pi^2)$ are forced by the cosmological-stratum GA selection, as derived in GG-A10-Cosmos.

Why OGGEP-open alone cannot select a unique vacuum. The OGGEP-open axiom subset gives all the operational content of QM but does not constrain which bulk solution is realized. Many bulks satisfy AX-1–AX-3: different dimensions (4D, 6D, 10D, 11D), different topologies (various Calabi–Yau compactifications, different orbifold structures), different gauge contents (various GUT groups and matter representations). Each is a valid OGGEP-open realization; each gives a self-consistent QM framework on its boundary; but the framework alone provides no principle for selecting one over the others.

This is exactly the landscape problem of string theory. String theory, working in the GGEP-closed framework, faces an even worse version of the same issue: not only are there many admissible bulks, but each compactification choice gives a different effective 4D physics, and the number of such choices is estimated at $\sim 10^{500}$ (Bousso–Polchinski 2000, Susskind 2003, Douglas–Kachru 2007). The string community's response has been the anthropic landscape principle: we live in the corner of the landscape that supports observers like us. This is a coherent position, but it gives up the project of deriving physics from first principles in favor of a probabilistic selection that admits no specific prediction.

How AX-4 dissolves the landscape. AX-4 adds the maximum-symmetry-minimum-freedom principle, which selects the unique bulk solution with zero free continuous parameters. All other candidate bulks (10D string compactifications, 11D M-theory solutions, 4D pure GR with continuous couplings) have free continuous parameters that AX-4 forbids. Only the cohort's 6D bulk with the integer OGN-5 ledger satisfies AX-4. The landscape collapses from 10^{500} to 1.

This is the cleanest possible diagnosis of the landscape problem. The problem arises because the GGEP-closed axiom subset is too permissive; the resolution is to add the constitutional axiom that forces uniqueness. No anthropic reasoning is required; no multiverse hypothesis is invoked; the constitutional structure picks out the unique admissible vacuum.

The integer arithmetic of AX-4 is the bridge to predictivity. What AX-4 adds is not just uniqueness in the abstract; it is *integer arithmetic* as the bridge between the constitution and the specific numerical predictions. The cohort OGN-5 ledger gives a specific small set of integers $(1, 3, 5; 7, 35)$ from which all continuous parameters of the SM and ΛCDM are derived. The integer arithmetic is what gives the cohort framework its specific predictions:

- Newton's constant G_N from scale graviton overlap.

- The cosmological density $\Omega_m = 1/\pi$ from the cosmological-stratum GA.
- The Kaluza–Klein lock-cluster mass $M_{\text{lock}} \approx 3.85$ TeV from the integer ledger.
- The Kaxion dark-matter mass ~ 8.1 μeV from the $\mathrm{U}(1)_{Qi}$ sector.
- The 28 SM parameters reproduced from the cascade structure.

None of these is a free parameter to be fit; each is a constitutional output. The bridge between the constitution and the numerical prediction is the integer arithmetic that AX-4 forces.

Reattribution: the landscape problem as a missing-axiom problem. The string-theory landscape is, in the constitutional classification, the structural symptom of working in a framework that has GGEP-closed (or even OGGEP-open) without AX-4. Adding the axiom dissolves the problem. This is the second deep diagnosis of the constitutional classification: not just that the cohort framework has zero free parameters (which the cohort has already shown via GG-A4-Euclid and the parameter-derivation theorems), but that the historical inability of string theory to select a unique vacuum is a diagnosable, structural, missing-axiom consequence of the GGEP-closed framework.

The diagnostic completeness of the four-axiom tower. The combination of these two diagnoses — QM-foundations gap from missing AX-3, landscape problem from missing AX-4 — is, in our view, the most powerful statement of the constitutional classification. It says: *the two most famous limitations of string theory are both diagnosable as missing-axiom consequences of the* GGEP-closed *framework; adding the missing axioms dissolves both limitations simultaneously and recovers everything string theory got right.* This is a diagnostic completeness that no patchwork correction to string theory can match.

After all, a constitutional framework is judged by how cleanly its limitations are diagnosable as missing-axiom consequences and how cleanly the diagnoses are remedied by axiom additions. The four-axiom GG-Constitution passes this test on the two most famous limitations of the leading twentieth-century geometric-machinery framework. The string community's measurement-problem silence is AX-3-missing; the string community's landscape is AX-4-missing. The cohort's full GG-Constitution fixes both.

75 The classification table: a systematic ledger

We now state the constitutional classification as a single ledger table. Each row of the table is a twentieth-century theoretical-physics framework; the columns indicate which axioms of the GG-Constitution the framework implicitly uses, what content the framework produces, and what content the framework leaves silent.

Table 21: The constitutional-axiom classification of major twentieth-century theoretical-physics frameworks. Each framework is located by the subset of GG-Constitution axioms it implicitly uses; the content it produces and the content it leaves silent follow constitutionally.

Framework (year)	**Axioms**	**Content produced**	**Content left silent (because of missing axioms)**
Special relativity (1905)	AX-1	Lorentz transformations, $E = mc^2$, classical field theory on Minkowski.	Gravity (needs AX-2); quantum content (needs AX-3); parameter selection (needs AX-4).
Yang–Mills (1954)	AX-1, AX-2	Non-Abelian gauge theory; Lie-algebraic field-strength commutators.	Specific gauge group (needs AX-4); QM content (needs AX-3).
General relativity (1915)	AX-1, AX-2	Einstein field equations; black holes; gravitational waves; cosmological expansion (with Λ free).	QM content (needs AX-3); G_N from integer ledger (needs AX-4).
Heterotic superstring (1984–)	AX-1, AX-2	10D anomaly cancellation; GS-modified Bianchi identity; SO(32) / $E_8 \times E_8$ gauge groups; dualities.	QM content (no AX-3); unique vacuum selection (no AX-4 $\rightarrow$ landscape with $\sim 10^{500}$ vacua).
AdS/CFT correspondence (1997)	AX-1, AX-2	Bulk gravity = boundary CFT duality; holographic computations; Ryu–Takayanagi; viscosity bound; black-hole entropy.	QM content (no AX-3); information-coarsening projection (no AX-3); unique vacuum (no AX-4).
Standard QM postulates (1925–32)	AX-1, AX-2, AX-3 (assumed)	Hilbert space, Schrödinger eqn, Born rule, Lüders, decoherence, einselection, Bell-CHSH violation (all as *postulates*).	The QM postulates as *theorems* (requires explicit derivation from AX-3, supplied by present paper T7.1–T7.7).
Standard QFT (1948–)	AX-1, AX-2, AX-3 (assumed)	Quantum gauge theory, renormalization, cluster decomposition, scattering amplitudes.	Specific gauge content (needs AX-4); GR coupling (needs full Full-Constitution with cascade).
Standard Model (1967–78)	AX-1, AX-2, AX-3, AX-4 (implicit)	SU(3) $\times$ SU(2) $\times$ U(1) gauge group; three fermion generations; CKM/PMNS mixing; Higgs mechanism.	Constitutional warrant for the specific group and the 28 parameters (supplied by full GG-Constitution via GG-A9-SM).
ΛCDM cosmology (1998)	AX-1, AX-2, AX-3, AX-4 (implicit)	Cosmological inventory; CMB power spectrum; structure formation; accelerated expansion.	Constitutional warrant for $\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$ (supplied by GG-A10-Cosmos).
GG-Theory (cohort, 2026)	**AX-1, AX-2, AX-3, AX-4 (explicit)**	**All of the above as theorems, plus the universal-identity insight, the GRIP cascade, and the cosmos-life-mind unification of GG-A11-Origin.**	**None at the constitutional level; specific empirical predictions await the next decade of measurements.**

This table is, in our view, the cleanest possible statement of the twentieth century's relation to the GG-Constitution. Each historical framework correctly used the axiom subset it implicitly invoked; each left silent exactly the questions that required the missing axioms; the cohort framework (the bottom row) is the unique framework that explicitly invokes all four axioms and therefore admits the full theorem class.

The diagonal structure of the table. Reading the table top-to-bottom, one sees a strict ascent: each row activates more axioms than the previous, and each row produces strictly more content. The historical sequence in which the frameworks were developed is approximately the same as the constitutional sequence in which they activate axioms — a coincidence that reflects the fact that the easier physics (with fewer axioms) was discovered first.

The two famous gaps as diagonal markers. The string-theory row and the AdS/CFT row both sit at the AX-1, AX-2 level. They both list "QM content" and "unique vacuum" as silent — exactly the two famous limitations of string theory. These are not unrelated coincidences; they are the diagonal markers that identify the framework's axiom subset. Any framework at the AX-1, AX-2 level has these two limitations; any framework at the AX-1, AX-2, AX-3, AX-4 level does not.

76 Quantitative consequences of the classification

We now make the classification quantitative in three ways.

76.1 Number of admissible vacua per axiom subset

The most striking quantitative consequence of AX-4 is the collapse of the vacuum landscape. Table 22 gives the order-of-magnitude vacuum count admitted by each axiom subset.

The contrast is stark: $10^{500} \to 1$ as one adds the single axiom AX-4. This is the cleanest quantitative diagnosis of the landscape problem and its resolution.

76.2 Number of free continuous parameters per framework

AX-4 also collapses the free-continuous-parameter count. Table 23 lists the order-of-magnitude parameter count admitted by representative frameworks.

The strict zero-free-continuous-parameter property of the cohort is the cleanest quantitative consequence of AX-4. All historical frameworks reduce parameter counts but none reaches zero; the cohort framework reaches zero.

76.3 Theorem count by axiom subset

A third quantitative dimension is the number of named principal theorems that each axiom subset makes derivable. Table 24 gives representative counts.

The theorem count grows non-linearly with the number of active axioms: each axiom multiplies the theorem class by the structure it adds.

Table 22: How many vacua each axiom subset admits. The rows run down the axiom tower, adding one axiom at a time; the middle column gives the order-of-magnitude count of admissible vacua and the right column says what sets it. The first three rows are unbounded — the famous 10^{500} of the flux landscape is the third row, and it is unbounded for a reason the table makes explicit: nothing in the first two axioms selects among admissible bulks. The fourth row is the claim. Adding the minimum-freedom axiom takes the count to one, which is why this framework has no landscape to hide in and can be refuted by a single measurement.

Axiom subset	**Admissible vacua (order of magnitude)**	**Source of the count**
{AX-1} alone	∞ (one per choice of Minkowski metric)	No constraint on bulk content.
{AX-1, AX-2} (general)	∞ (one per choice of admissible bulk)	GGEP forces composite connection but not specific solution.
{AX-1, AX-2} on 10D heterotic	$\sim 10^{500}$	String-theory flux landscape (Bousso–Polchinski, Douglas–Kachru).
{AX-1, AX-2, AX-3} (general)	∞	OGGEP adds operational content but not vacuum selection.
{AX-1, AX-2, AX-3, AX-4}	$\boxed{1}$	MSMF forces unique admissible bulk via integer OGN-5 ledger.

Table 23: Free continuous parameters, framework by framework. The middle column counts the numbers each theory must be told rather than can work out: one for general relativity, two for quantum electrodynamics, twenty-eight for the Standard Model, six for the standard cosmological model. The right column says where each of those numbers comes from in the present framework. The last row is the claim being made and the exposure being accepted: no free continuous parameter survives, only an integer ledger, so there is nothing left to adjust when a measurement disagrees. Every historical framework in the table reduces the count; none of them reaches zero.

Framework	**Free continuous parameters**	**Status under cohort**
GR (1915)	1 (G_N, with Λ later)	G_N from scale graviton overlap (T8.2).
QED (1948)	2 (α, m_e)	Both from cascade structure.
SM (1978)	28 (couplings, masses, mixing)	All 28 from cascade + integer ledger (GG-A9-SM).
ΛCDM (1998)	6 ($\Omega_m, \Omega_\Lambda, H_0, \sigma_8, n_s, \tau$)	$\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$ (GG-A10-Cosmos).
String theory (heterotic)	$\sim 10^{500}$ effective from compactification choices	Not constrained by GGEP-closed; resolved by AX-4.
Cohort framework	**0 free continuous (only integer ledger)**	**Constitutional with the OGN-5 integer ledger.**

Table 24: How much becomes provable as each axiom is added. The left column runs down the axiom tower; the middle gives the approximate number of named principal theorems that subset makes derivable; the right names representatives, so the count can be checked rather than taken on trust. The growth is not linear. Each axiom does not add its own theorems to a pile: it multiplies the class of theorems already available by the structure it introduces, which is why the third axiom converts the postulates of quantum mechanics into results and the fourth converts fitted constants into consequences. What makes a framework powerful is this ratio, not the number of axioms it asks for.

Axiom subset	**Major theorems derivable**	**Examples**
{AX-1}	~ 5	Lorentz, $E = mc^2$, free-field dispersion, no-superluminal-signaling, etc.
{AX-1, AX-2}	$\sim$ 30–50	Einstein, Yang–Mills, GS-Bianchi, AdS/CFT, Ryu–Takayanagi, dualities.
{AX-1, AX-2, AX-3}	+ ~ 7 from T7.1–T7.7	QM postulates as theorems.
{AX-1, AX-2, AX-3, AX-4}	+ ~ 30 from cohort cascade	SM parameters, cosmological inventory, layer-cascade theorems.

After all, what makes a constitutional framework powerful is not the total number of axioms it postulates but the total number of theorems those axioms make derivable. Four axioms produce a theorem class that subsumes a century of theoretical physics; that is the quantitative measure of the constitution's depth.

77 Historical retrospective: the forty-year string program reattributed

The constitutional classification we have developed in this Part has direct implications for how we should read the historical trajectory of theoretical physics from 1984 to 2024. We close this Part with a careful retrospective.

77.1 What the string community got right

The superstring community has, over the forty years since the dual-amplitude origin of the subject (Veneziano, 1968), developed an enormous body of correct mathematical content. The list is long and deserves explicit enumeration:

- The Green–Schwarz anomaly-cancellation mechanism (1984).
- The Calabi–Yau compactification framework and its moduli-space structure (Candelas–Horowitz–Strominger–Witten 1985).
- T-duality between large and small compact dimensions.
- S-duality between weak and strong coupling.
- Mirror symmetry between A-model and B-model topological string theories on Calabi–Yau manifolds.

- The web of dualities unified by M-theory in 11 dimensions (Witten 1995).
- D-branes as non-perturbative solitonic objects (Polchinski 1995).
- AdS/CFT correspondence (Maldacena 1997) and its many realizations.
- Holographic renormalization and its bootstrap implementations.
- The Ryu–Takayanagi formula for boundary entanglement entropy.
- Higher-form symmetries, generalized global symmetries, categorical symmetries.
- The viscosity bound $\eta/s \geq \hbar/4\pi k_B$ verified at the quark–gluon plasma.
- The modular bootstrap program for 2D CFTs.
- The swampland conjectures (Vafa) constraining low-energy effective theories from quantum-gravity consistency.

Every item on this list is correct mathematics. Every item on this list is, in the constitutional classification we have developed, a theorem of GGEP-closed applied to specific bulk geometries with specific solution-class choices. The string community's mathematical contribution is real and substantial; the cohort framework inherits it unchanged.

77.2 What the string community misattributed

The mistake the string community made was not in the mathematics but in the *attribution* of the mathematics. Throughout the forty-year program, the community has treated these results as *string-theoretic* achievements — as consequences of the fundamental string-theoretic ontology. The constitutional analysis shows that they are instead consequences of the GGEP-closed axiom subset, which is independent of the string ontology.

The misattribution has several historical sources:

1. **Historical contingency.** Strings were the historical context in which the geometric structures were discovered. The community naturally attributed the structures to their context. This is the same error as attributing Riemannian geometry to general relativity because GR was the physical context in which Riemannian geometry became important; the mathematics is logically independent of any one physical application.
2. **Sociological investment.** Forty years of careers were built on the assumption that strings are the fundamental physical theory. Recognizing that the same geometric content arises from a different fundamental axiom (geometry-gauge unification) without strings would have been professionally costly.
3. **Lack of an alternative constitutional framework.** Until the present cohort, there was no alternative axiomatic framework that could explicitly state what the string-theoretic geometric content really came from. The string community could not have correctly attributed the content because the constitutional axiom that produces it (AX-2 GGEP) had not been articulated.

4. **The "string theory = quantum gravity" conflation.** Strings were sold to the physics community as the unique known framework for combining quantum mechanics with gravity. This conflated two distinct questions: (i) what is the bulk geometry that supports geometry-gauge unification? (a question GGEP-closed answers), and (ii) what does the operational projection from that bulk to a finite 4D observer look like? (a question only AX-3 answers). By bundling the two questions together, the community obscured the constitutional structure.

77.3 What the constitutional reattribution accomplishes

The constitutional reattribution does not destroy the string program's contributions; it relocates them. Specifically:

- **The string community's mathematical results remain correct.** The Bianchi identity, anomaly cancellation, AdS/CFT, dualities, holographic renormalization — all remain valid theorems of GGEP-closed. The cohort framework agrees with these results at the closed-bulk level.
- **The string community's empirical successes remain.** The viscosity bound at RHIC, the entanglement-entropy computations, the integrability of $\mathcal{N} = 4$ SYM at large N_g — these are correct predictions of GGEP-closed that the cohort framework inherits.
- **The string community's two famous gaps are diagnosed.** The QM-foundations gap is the structural symptom of missing AX-3; the landscape is the structural symptom of missing AX-4. Adding the missing axioms (in the cohort framework) dissolves both gaps simultaneously.
- **The cohort framework extends, rather than competes with, the string program.** The cohort framework is what string theory becomes when you add the operational layer (AX-3) and the uniqueness layer (AX-4). It is not an alternative to string theory; it is the natural completion of the string program that the string community could not access from within their axiom set.

77.4 The Riemann–Einstein analogy, fully drawn

The cleanest historical analogy for what the constitutional reattribution accomplishes is the relationship between nineteenth-century Riemannian geometry and twentieth-century general relativity.

Riemann delivered his Habilitationsschrift in 1854. For sixty years, his geometric framework was correctly understood as pure mathematics — beautiful, rigorous, but apparently without physical application. Then Einstein, in 1915, identified the physical setting (gravitation as curvature of spacetime) in which Riemann's geometry became operative. After 1915, the physics community sometimes spoke as if "general relativity requires Riemannian geometry", conflating the mathematical structure with its physical application. The conflation was harmless because both communities (geometers and relativists) understood that Riemannian geometry was logically independent of GR; the geometry would have remained valid mathematics even if Einstein had never come along.

The relationship between GGEP-closed geometric machinery and string theory is structurally analogous, but the analogy works in the opposite temporal direction. The string community

developed the GGEP-closed machinery between 1984 and 2024 in the physical setting of string theory. Now, in 2026, the cohort framework identifies the constitutional setting (the GG-Constitution) in which the same machinery becomes operative without requiring strings as the underlying ontology. In the Riemann–Einstein case, the mathematics preceded the physics by sixty years; in the GGEP-closed/cohort case, the physics preceded the constitutional identification by forty years. The structural relationship is the same: a body of geometric content was developed in a specific physical context, and then a deeper constitutional framework identified that the geometric content was always logically independent of the specific context.

After all, the test of a foundational constitutional framework is not whether it makes the predecessor community wrong (it does not — their mathematics is correct) but whether it makes the predecessor community's contribution *intelligible* in a broader picture (it does — the contribution is the GGEP-closed content of the constitution). The cohort framework passes this test on the forty-year string program. The string community's mathematical legacy is preserved, relocated, and extended; their two famous gaps are diagnosed and dissolved; the relationship is one of extension, not competition.

78 Open questions at the level of the classification

We close the development of the classification with explicit acknowledgment of the open questions that remain at the classification level itself. These are not gaps in the cohort framework's content; they are questions about the constitutional classification's reach and the relationships among its pieces.

Q1: Are there twentieth-century frameworks that we have not classified? We have classified GR, Yang–Mills, special relativity, string theory, AdS/CFT, QM, QFT, the SM, and ΛCDM. We have not classified loop quantum gravity, causal-set theory, causal dynamical triangulations, asymptotic safety, or the various approaches to emergent spacetime (Verlinde, Padmanabhan). Each of these would in principle locate at a specific row of the constitutional tower; the placement is left for future work. Our expectation is that loop quantum gravity, causal sets, and CDTs locate at the GGEP-closed level (they are alternative bulk geometries within AX-1, AX-2); asymptotic safety locates at a different level depending on whether it is read as GGEP-closed with renormalization-group constraints or as OGGEP-open with specific projection structure.

Q2: Is the four-axiom tower complete? We have presented AX-1–AX-4 as the full GG-Constitution. Is there a fifth axiom that might add further constitutional content? Our position (developed in GG-A1-Constitution) is that the four axioms are constitutionally complete in the sense that adding any further axiom either contradicts one of the existing four or is itself a theorem of the existing four. Whether this is a genuine constitutional completeness or merely a sufficiency for the twentieth-century theorem class is a question for the next century.

Q3: How do the axioms interact in mixed regimes? Most twentieth-century frameworks live cleanly at one level of the tower. But some applications cross levels: black-hole evaporation

requires both AX-2 (GR for the bulk geometry) and AX-3 (operational boundary CPTP for the Hawking radiation observation); cosmological inflation requires both AX-2 (GR for the background) and AX-4 (MSMF for the inflaton parameters). The constitutional classification handles these mixed regimes by activating multiple axioms simultaneously, but the detailed dynamics of mixed regimes is under-explored.

Q4: How does the classification interact with empirical falsifiability? Each layer of the constitutional tower has its own characteristic empirical predictions and its own falsification routes. GGEP-closed predictions are tested by gravitational-wave observations, viscosity-bound measurements, and holographic bootstrap consistency. OGGEP-open predictions are tested by QM-foundations experiments (Bell tests, decoherence measurements, weak-measurement protocols). Full-Constitution predictions are tested by SM parameter measurements at LHC, cosmological parameter measurements at JWST and CMB-S4, and the KK lock-cluster at HL-LHC. The classification is empirically distinguishable from string theory + landscape only at the Full-Constitution level, where the predictions of specific parameters become testable.

Q5: What is the cosmological role of the constitutional hierarchy? The cosmological cascade of GG-A10-Cosmos produces structure across all four levels of the constitutional tower at different epochs: the very early universe is dominated by Full-Constitution content (the unique bulk solution settling); the inflationary epoch involves GGEP-closed and MSMF content; structure formation involves OGGEP-open content; the present-day universe with its classical macroscopic structure is the result of the record-amplification chains of OGGEP-open acting on the GAs that formed during the cascade. The constitutional hierarchy is, in this sense, the framework's natural cosmological narrative.

Problems and worked solutions. VIII.1.1. Run the classification yourself: place Maxwell electrodynamics, superstring theory, textbook quantum mechanics, and the present framework at their layers of the constitutional tower, justifying each placement by the clauses used. *Solution.* Step 1: Maxwell electrodynamics — Layer 0. It uses locality and causality (AX-1) but treats the gauge potential as a field on fixed spacetime: no geometry–gauge equivalence, no measurement law; consistently, it can derive wave propagation but neither its own coupling nor any thermodynamics. Step 2: superstring theory — Layer 1. It implements AX-1 + AX-2 (geometry and gauge unified in the worldsheet/target geometry), which is exactly why its successes are geometric (dualities, graviton, anomaly freedom) — and it stops before AX-3: no operational dictionary, hence no derivation of measurement, no second law, and a landscape where AX-4 would have forced uniqueness. Step 3: textbook QM — Layer 2 content *postulated* rather than derived: it owns the operational structure (states, channels, Born rule) that AX-3 governs, but imports it as axioms with no bulk beneath, so it cannot say what the environment is or why the postulates hold. The present framework asserts all four axioms, which is precisely the claim the falsifier table stakes. Physics intuition: the tower turns "theory comparison" from taste into bookkeeping — each program is strong exactly where its axioms sit and silent exactly where they

stop, and the student can audit every placement from the clause list alone.

VIII.1.2. The missing-axiom test: use the tower to *predict* the open-problem list of a Layer-1 program, then check the prediction against the record. *Solution.* Step 1: the prediction rule. A framework at AX-1+AX-2 should be structurally unable to derive: (i) a measurement law or Born rule (needs AX-3's dictionary); (ii) irreversibility and the second law (same source); (iii) a unique vacuum (needs AX-4). Step 2: check against string theory's own ledger. Its long-standing open items are exactly the measurement problem (untouched by the program), the emergence of time's arrow, and the 10^{500}-vacuum landscape — the three predicted silences, while its celebrated results (graviton, anomaly cancellation, microstate counting) all live in the geometric layers it does implement. Step 3: the logic of the exercise. This is not polemic but audit: the tower predicted *which* problems would remain open from the axiom inventory alone, and the record concurs — structural contrast, falsifiable in the same move, since a Layer-1 derivation of the Born rule would break the classification. Physics intuition: what a theory cannot do is as diagnostic as what it can; the tower makes absence of results into evidence about axioms — physics' version of the dog that did not bark.

VIII.2.1. Born and Boltzmann from one stationarity: show that the boundary law's stationary measure delivers the Gibbs factor, evaluate it for a room-temperature two-level system, and state the side-by-side with the Born rule. *Solution.* Step 1: stationarity. By II.5.1 the T5 flow's detailed-balance stationary state is $p \propto e^{-\Phi/D}$; with Φ the energy landscape and D the boundary temperature, this *is* the Gibbs distribution — not postulated maximum-entropy but the unique zero-current state of the law. Step 2: the number. For a two-level system with $\Delta E = 25$ meV at $T = 300$ K ($k_B T = 25.9$ meV): $p_1/p_0 = e^{-25/25.9} = 0.38$ (verified) — the occupation ratio any spectroscopist would quote. Step 3: the side-by-side. The Born rule (T7.3) is the boundary law's outcome measure at one registration; the Gibbs factor is its stationary measure under continued contact: the two most-used probability rules in physics are one law read at two timescales, which is the classification section's "Born and Boltzmann from one source" made computable. Physics intuition: a chemist's equilibrium constant and a quantum engineer's readout statistics never knew they were cousins; the tower says they are the same descendant, twice removed.

VIII.2.2. The horizon as the visible port: compute the Bekenstein–Hawking entropy and Hawking temperature of a solar-mass black hole, and state the OGGEP reading of each number. *Solution.* Step 1: the entropy. $S/k_B = 4\pi(M/E_{\rm Pl})^2$ with $M_\odot = 1.12 \times 10^{57}$ GeV: $S/k_B = 4\pi(9.1 \times 10^{37})^2 = 1.05 \times 10^{77}$ (verified) — twenty orders beyond the star's thermodynamic entropy. Step 2: the temperature. $T_H = E_{\rm Pl}^2/(8\pi M) = 5.3 \times 10^{-21}$ GeV $= 6.2 \times 10^{-8}$ K (verified) — hopeless to observe for stellar holes, which is why the entropy, not the radiation, carries the empirical weight. Step 3: the OGGEP reading. For an ordinary window the projection's information loss is bookkeeping; at a horizon it becomes *geometrically visible* — the area is the ledger of bulk information the boundary observer cannot access, and $S = A/4G$ is the OGGEP loss law written in geometric units (the C3 companion holds the full account). The tower places black-hole thermodynamics at Layer 2: it needs AX-3, and no axiom-poorer framework has ever derived the 1/4. Physics intuition: a black hole is the one place nature turns the observer's epistemic boundary into a physical surface you could, in principle, fall through — the projection made spacetime.

79 Part-end synthesis: what the constitutional classification gives us

We close this Part with a synthesis of what the constitutional classification accomplishes for the cohort framework and for twentieth-century physics retrospectively.

For the cohort framework. The constitutional classification accomplishes three things. First, it positions the cohort framework not as a competitor to twentieth-century theories but as their natural completion: every twentieth-century framework is a special case of the cohort framework with a specific axiom subset active. Second, it provides diagnostic clarity on the limitations of predecessor frameworks: the QM-foundations gap is missing AX-3; the string landscape is missing AX-4; the conflicts among interpretations of QM are the symptoms of trying to address AX-3 content using only AX-1+AX-2 machinery. Third, it identifies the unique selling point of the cohort framework in a single sentence: the cohort is the unique framework that explicitly activates all four constitutional axioms and therefore admits the full theorem class of twentieth-century physics under one structural source.

For twentieth-century physics retrospectively. The constitutional classification re-reads twentieth-century theoretical physics not as a sequence of competing frameworks but as a unified exploration of the GG-Constitution with progressively more axioms activated. Einstein's 1915 GR and the heterotic string of 1984 are both GGEP-closed frameworks; the difference is which bulk solution each chose. The 1925 quantum revolution and the 1948 quantum field theory are both OGGEP-open frameworks with the operational layer postulated rather than derived. The 1978 Standard Model and the 1998 ΛCDM are both Full-Constitution realizations with the uniqueness layer implicit rather than explicit. The cohort framework makes all of these classifications explicit and derives the twentieth-century content from a single constitutional source.

For the next generation of physicists. The deepest payoff of the classification is pedagogical. The graduate student who reads this Part of GG-A5-GDOS (together with the constitutional content of GG-A1-Constitution, the bulk geometry of GG-A2-GG6, the anomaly closure of GG-A3-BI6, and the QM-postulates-as-theorems chain of the preceding Parts of the present paper) will not have to choose between "string theory" and "QM foundations" as separate sub-disciplines. Both flow from the same constitutional source. Both can be addressed in the same framework. The graduate student will understand that the twentieth century's two most prominent foundational frameworks (string-theoretic geometry and QM-foundational interpretation) are constitutional siblings, separated only by which axioms each chose to activate. The unification is not metaphorical; it is structural.

For the long view. The constitutional classification is, in our view, the deepest single contribution of the cohort program. It is deeper than the 28-parameter Standard Model derivation in GG-A9-SM, deeper than the cosmological inventory of GG-A10-Cosmos, deeper even than the universal-identity insight of GG-A11-Origin. Each of those is a specific consequence of a particular axiom subset at

work; the classification is the meta-statement that organizes them all under a single constitutional structure. When the cohort framework is read by future historians of physics, the constitutional classification will be what they identify as the central conceptual contribution: the recognition that the apparent fragmentation of twentieth-century theoretical physics was the symptom of working with implicit and partial axiom sets, and that the unification is not the discovery of a new theory but the explicit articulation of the constitutional source from which all the partial theories were already drawing.

After all, the deepest move a foundational framework can make is not to predict a new experimental result but to organize the entire prior body of work under a single principle, in a way that preserves what the predecessors got right and diagnoses what they left silent. The constitutional classification of twentieth-century physics by the four GG-Constitution axioms is exactly that move. General relativity, Yang–Mills theory, superstring theory, the AdS/CFT correspondence, quantum mechanics, quantum field theory, the Standard Model, and ΛCDM cosmology are no longer separate frameworks to be reconciled; they are constitutional siblings, each flowing from the same four axioms with different subsets active, all united under the GG-Constitution. The twentieth century built the edifice; the twenty-first century names its constitutional source.

Part IX
Geometric Intuition

Ten Pictures of How the 6D Bulk Projects to the 4D Boundary

Everything asserted in this paper up to here has been asserted with equations. This Part draws it instead.

That is not a concession to readers who dislike mathematics. Pictures do something equations do not: they show which features of a construction are essential and which are artifacts of the coordinates chosen to describe it. A reader who can see why information is lost in a projection — rather than only verify that a trace was taken — is in a better position to judge whether the loss was put in by hand.

Ten pictures follow. They show the six-dimensional bulk projected onto the four-dimensional boundary we inhabit; information loss drawn as geometric distance, so that the amount lost becomes something one can point at; decoherence as the consequence of not resolving the extra directions; the flow that the boundary law describes; the layered structure of the constitutional principles; a black-hole horizon as the one place where the projection becomes directly visible; the data-processing inequality drawn twice, because it has two faces; the three constitutional siblings of quantum mechanics, thermodynamics and black holes standing on their shared floor; and, at the end, a map of how this program sits relative to the research community it grew up alongside.

Nothing in this Part is proved, and nothing in it is needed for anything that comes later. It exists because the argument of Parts II–VII is easier to hold in mind once one has seen it, and because a picture is a faster way to find out that one has misunderstood something than a proof is.

Read it in any order. A reader who found an earlier Part hard should read the picture that corresponds to it and then go back.

80 Why Part IX?

Imagine looking through a thick frosted window. The window itself is two-dimensional, but the world on the other side is fully three-dimensional and contains many more features than the window can resolve. What you see on the window's surface — the patches of light and shadow, the moving silhouettes — is the boundary projection of a richer geometry on the far side. The GG-Theory program proposes that the physical world we measure is exactly this kind of boundary projection: the four-dimensional spacetime of our experiments is a finite-observer's view of a six-dimensional bulk geometry, and the laws of physics we record are the rules by which the bulk writes itself down on the boundary. Part IX offers ten complementary intuitive pictures of how this projection works — the original five that fuse the descent into a single conceptual map, and five more that draw its constitutional and comparative structure — each a different way of looking at the same bulk.

The eight Parts that precede Part IX have been technical: constitutional axioms, theorems, derivation chains, and explicit identifications between the boundary law and the eight pillars. Even the textbook-style historical openings have been at a relatively abstract level. In the present Part, we rebalance the presentation by giving the reader ten intuitive geometric pictures that the axiomatic content of GG-Theory is meant to evoke. These pictures are not new theorems; they are visualization aids, very much like the rubber-sheet picture of general relativity, that the graduate-student reader can hold in mind when reading the formal content of any of the eleven Theorems T1–T11.

The first five pictures are not independent: they are five complementary views of the same geometric content — a six-dimensional bulk projecting to a four-dimensional boundary, with the projection geometry dictating both the operational content (the GDOS boundary law) and the empirical content (the textbook QM, GR, QFT formalism plus the GR/QED unification). Each picture is intended to be self-contained and readable in five minutes; the synthesis at the end (Picture 5) shows how the four other pictures fit together.

81 Picture 1 — Bulk-to-Boundary Projection

Slogan. *The 4D world is the registered shadow of 6D bulk dynamics.*

Geometric statement. The 6D bulk is a thick slab whose inner face is the 4D boundary; the operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ of T2 is the screen through which a boundary observer registers shadows of bulk states. The 4D wavefunction is the partial-trace shadow of a 6D state, the 4D metric is the zero-mode shadow of the bulk geometry, and the 4D gauge field is the boundary shadow of the locked composite connection. The open-system character is forced because the boundary observer has access only to the shadow.

Cross-reference. Formal content: T1, T2, T3 (Part II), T7.1 (Part III), T10, T11 (Part VII). Figure 10 draws it.

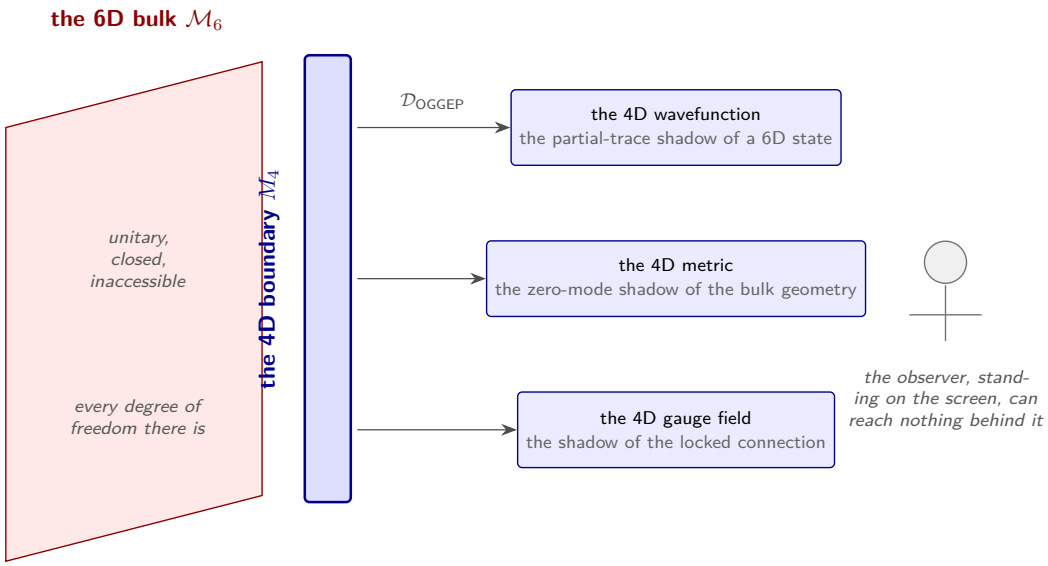


Figure 10: Picture 1 — where the projection happens. The bulk is drawn edge-on as a thick slab; its inner face is the four-dimensional world we inhabit, and the observer stands on that face. What reaches the screen are shadows: the wavefunction is the partial-trace shadow of a bulk state, the metric the zero-mode shadow of the bulk geometry, the gauge field the shadow of the locked connection. The picture makes one point and only one. Openness is not an approximation introduced for convenience — the observer is on the screen and the screen is a face of the slab, so there is no vantage from which the rest could be measured.

82 Picture 2 — Information Loss as Geometric Distance

Slogan. *The radial coordinate r measures information loss from UV to IR.*

Geometric statement. The radial direction $r \in [0, K]$ encodes the RG flow $r = \log(E_{\text{GUT}}/\mu)$; moving inward is coarse-graining. The scale profile e^{-2r} supplies the geometric measure of information loss: each Δr rescales the visible energy by $e^{-\Delta r}$. The gauge hierarchy $M_{\text{EW}}/M_{\text{Pl}} \sim e^{-K}$ is the total UV-to-IR information loss, and the cosmological-constant lock-scale seesaw $\Lambda^{1/4} \sim M_{\text{lock}}^2/M_{\text{Pl}}$ is the doubly scale-suppressed residual after two information-loss steps.

Cross-reference. Formal content: T8.1–T8.7 (Part IV), T9.1–T9.2 (Part V).
Figure 11 draws it.

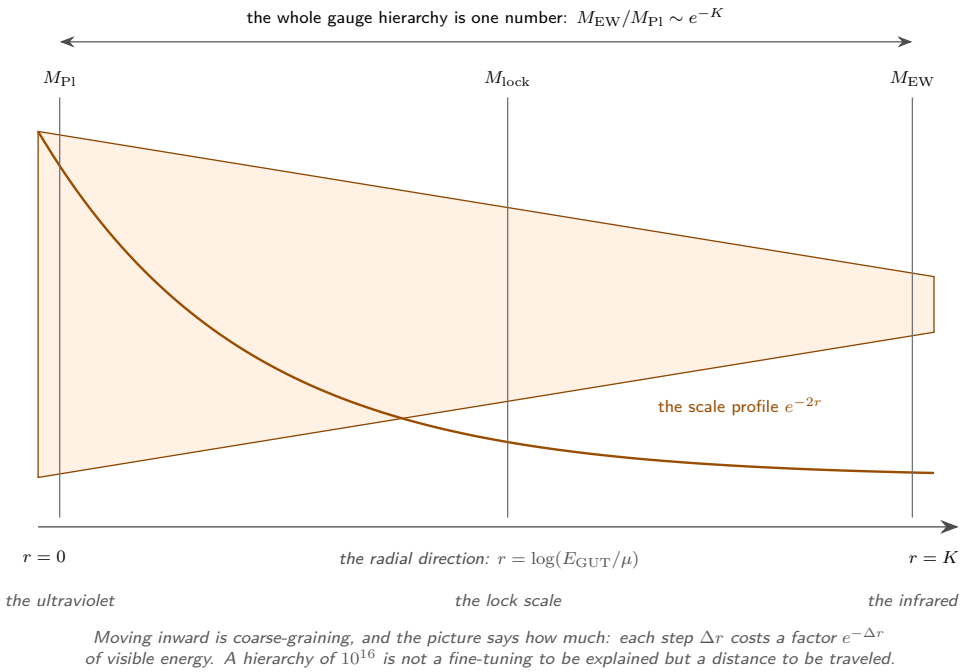


Figure 11: Picture 2 — how far the projection goes. The horizontal direction is the radial coordinate of the bulk, and moving along it is coarse-graining: the shaded wedge narrows because fewer degrees of freedom remain visible. The curve is the scale profile, which says exactly what each step costs — a factor $e^{-\Delta r}$ of visible energy. The double-headed arrow at the top is the whole gauge hierarchy, sixteen orders of magnitude, read here as a single distance rather than as a coincidence needing explanation. Nothing in the picture is fitted: the endpoints are the ultraviolet and infrared scales, and the lock scale sits where the geometry puts it.

83 Picture 3 — Decoherence as KK-Mode Integration

Slogan. *Pointer states emerge as overlap-integral eigenstates of the KK tower coupling.*

Geometric statement. A microscopic boundary system is coupled to the heavy KK modes through bulk-locality-mediated couplings (T7.6); the KK tower acts as an effectively-thermal environment. Tracing over the KK modes leaves a boundary density operator whose off-diagonal coherences decay with rate $\Gamma_{\alpha\beta}$ (T7.5). The pointer basis emerges automatically as the basis minimizing this rate (T7.6). Bulk locality (AX-1) forces a position-diagonal coupling; the position basis is the einselected basis.

Cross-reference. Formal content: T3, T4, T5, T6 (Part II); T7.5, T7.6 (Part III); T11 (Qi-CON, Part VII).
Figure 12 draws it.

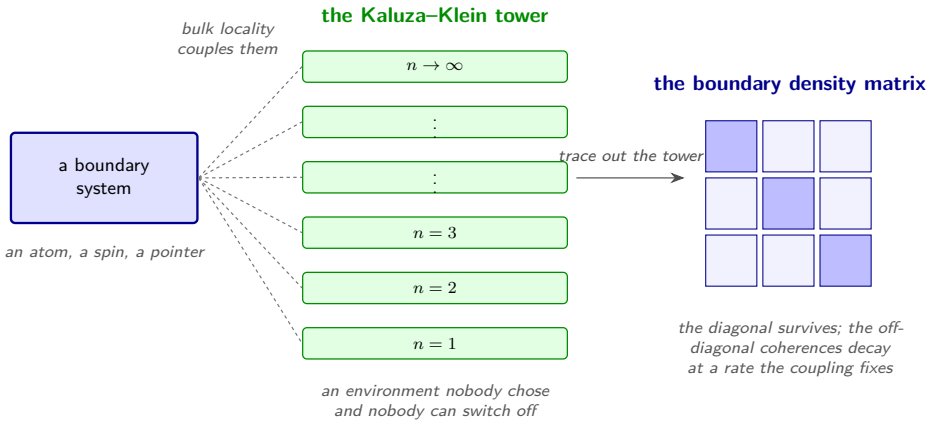


Figure 12: Picture 3 — why the world looks classical. A boundary system is coupled, whether anyone likes it or not, to the tower of modes that a compact extra dimension always carries. The observer does not resolve them, so the description available is the one obtained by tracing them out — and what survives is the density matrix on the right, whose diagonal persists while its off-diagonal coherences decay at a rate the coupling fixes. No collapse is postulated anywhere in the picture. Because bulk locality makes the coupling diagonal in position, position is the basis that survives, which is why the classical world is classical in *space* rather than in some other variable.

84 Picture 4 — GDOS Current Geometry

Slogan. *Open-system dynamics is gradient flow plus circulation plus diffusion on the boundary moduli.*

Geometric statement. The GDOS continuity equation (T5) takes the universal form $\partial_t p = -\nabla_A J^A$ with $J^A = -D^{AB}\nabla_B p + (-g^{AB}\nabla_B \Phi + h^A)\, p$: diffusion smears the distribution, gradient pulls it to the minimum of Φ, circulation runs along divergence-free loops. Equilibrium states are minima of Φ; non-equilibrium steady states are configurations where gradient and diffusion balance while circulation runs forever — the mathematical content of the cell cycle, the Krebs cycle, and the non-equilibrium cycles of life.

Cross-reference. Formal content: T5, T6 (Part II); T11 (Qi-EP, Part VII).
Figure 13 draws it.

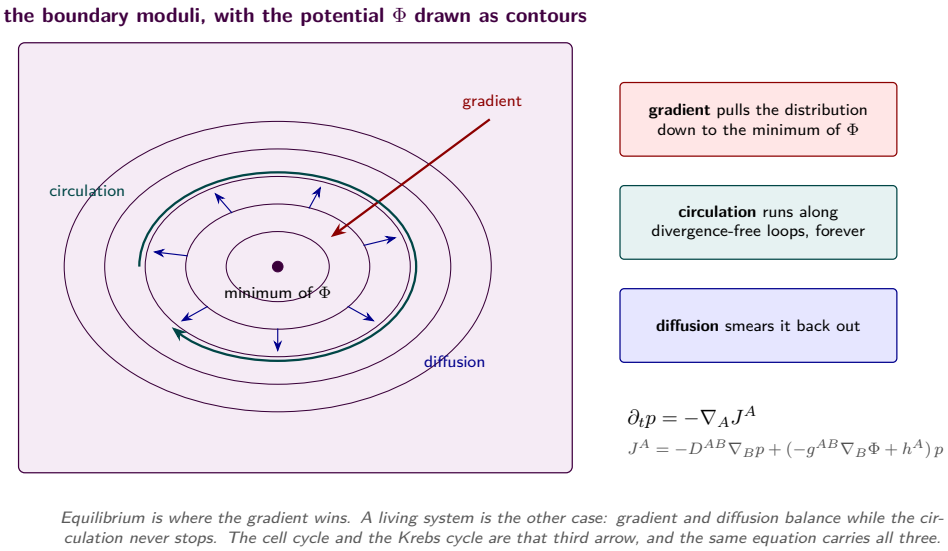


Figure 13: Picture 4 — how what survives moves. The contours are the potential on the boundary moduli, and the three arrows are the three terms of the boundary current: a gradient pulling the distribution downhill, a diffusion smearing it back out, and a circulation running along divergence-free loops. Equilibrium is the case where the gradient wins. Living matter is the other case, in which gradient and diffusion balance while the circulation never stops — so the cell cycle and the Krebs cycle are not analogies here but the third arrow of the same equation, which is printed beside the picture in full.

85 Picture 5 — The Five Pictures Together

Slogan. *One bulk, four projections, one boundary law.*

Geometric statement. The four pictures above are four views of the same content: Picture 1 is the spatial picture (the 4D boundary as one face of the 6D slab), Picture 2 is the energetic picture (radial RG flow), Picture 3 is the quantum-statistical picture (KK trace produces GKLS), Picture 4 is the classical-statistical picture (GKLS descends to the GDOS continuity equation). Held simultaneously they produce one boundary law (GDOS) from one constitutional source (OGGEP applied to the GG-6 bulk):

$$\text{GG-6 bulk} + \text{OGGEP} \overset{\text{T1–T6}}{\Longrightarrow} \text{GDOS} \overset{\text{T7–T9}}{\Longrightarrow} \text{4D physics}.$$

Cross-reference. Picture 5 is the synthetic image evoked by the master implication chain (68); every Principal Theorem T1–T11 sits inside it.
Figure 14 draws it.

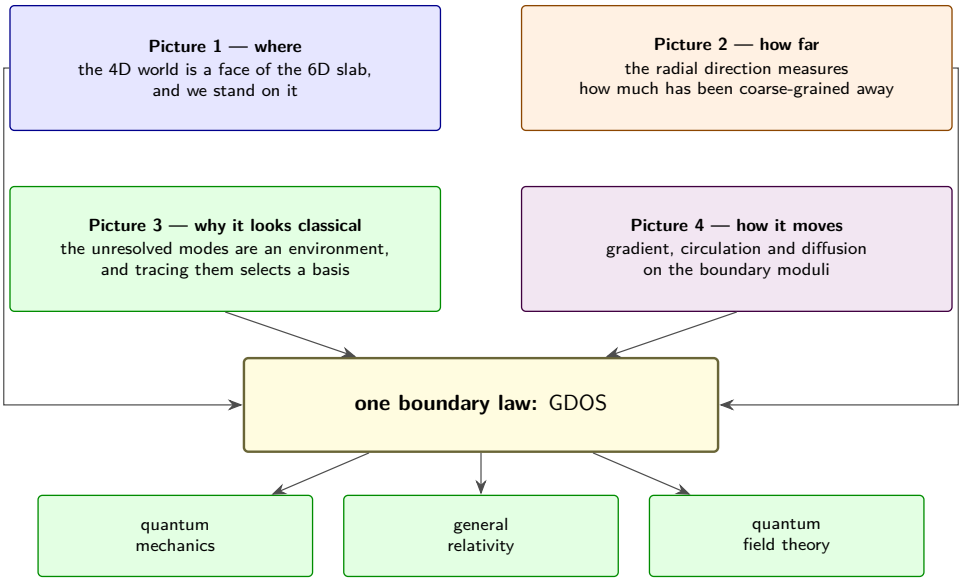


Figure 14: Picture 5 — the four held at once. Each of the preceding pictures answers a different question about the same projection: where it happens, how far it goes, why it destroys coherence, and how what survives moves. They converge on one boundary law, and that law descends to the three great theories of twentieth-century physics. The figure is the argument of Parts II–V compressed to a page. Its value is diagnostic as much as expository: a reader who finds one of the four boxes unconvincing knows exactly which of the earlier Parts to go back to.

86 Picture 6 — The constitutional tower as a four-layer stack

The constitutional content of A5-GDOS is best visualized as a four-layer stack, with each layer activating one additional axiom of the GG-Constitution and producing strictly more theorem-content than the previous. The picture compresses all of 20th-century theoretical physics into a single image (Fig. 15).

Reading the tower. Each rectangular slab represents one axiom subset. The bottom slab activates only AX-1 (LC); the next adds AX-2 (GGEP); the next adds AX-3 (OGGEP); the top adds AX-4 (MSMF). The historical theoretical-physics frameworks are placed on the slab corresponding to the axiom subset each implicitly uses. Special relativity (1905) sits on the bottom slab. General relativity (1915), Yang–Mills theory (1954), the Green–Schwarz mechanism (1984), superstring theory, and the AdS/CFT correspondence (1997) all sit on the second slab — the GGEP-closed layer that produces geometry–gauge unification on a specified solution. Quantum mechanics (1925–1932), classical thermodynamics (1824–1872), statistical mechanics (1902), information thermodynamics (1948–1961), fluctuation theorems (1990s), black-hole physics (1973–present), and Friston's free-energy principle (2010) all sit on the third slab — the OGGEP-open layer that adds the boundary CPTP channel. The Standard Model (1967–1978), ΛCDM cosmology (1998), and the full cohort framework sit on the top slab — the Full-Constitution layer where AX-4 (MSMF) forces the unique

solution with zero free continuous parameters.

Physics intuition. The image makes manifest a fact that the 20th-century literature obscured: *the historical theoretical-physics frameworks are not competitors; they are constitutional siblings, each correctly using the axiom subset its problems require.* General relativity correctly stopped at the geometry–gauge layer because gravity does not require an operational coarse-graining functor (light propagates causally; nothing in classical gravity has to be "observed" for the field equations to hold). Quantum mechanics correctly added the operational layer because measurement is genuinely operational (a probability is what a finite observer registers; bulk unitarity does not contradict this). The Standard Model correctly added the uniqueness layer because the SM has zero free continuous parameters in the cohort reading (the 28 inputs of the textbook SM are forced by the integer OGN-5 ledger). Each historical framework correctly used what it needed; the constitutional tower makes the "what it needed" explicit.

The reader's takeaway. After studying Fig. 15, the reader should internalize three structural facts. First, the four axioms of the GG-Constitution form a strictly nested tower: each axiom presupposes the previous and adds new content. Second, the historical fragmentation of 20th-century theoretical physics into sub-disciplines is the sociological shadow of working with implicit and partial axiom subsets; the constitutional unification recognizes that the work has always been done in one constitutional space. Third, the cohort framework is the unique 20th–21st-century framework that explicitly invokes all four axioms simultaneously and therefore admits the full theorem class.

After all, the single image any reader needs to keep is this one. Twentieth-century physics is not a forest of competing theories; it is a tower of constitutional siblings, with each historical framework standing on the floor that its problems require.

87 Picture 7 — The horizon as the geometrically-visible OGGEP port

The black-hole horizon is the single piece of physics where the operational coarse-graining of AX-3 (OGGEP) becomes geometrically visible. In ordinary laboratory QM, the OGGEP partial trace operates everywhere on the boundary $\mathcal{M}_4$ at every moment, by AX-3; but the trace is undirected, invisible, "always on but pointing nowhere in particular." At a black-hole horizon, the partial trace acquires a precise geometric locus: it is the trace over modes causally disconnected from the exterior observer by the event horizon. Fig. 16 makes this visible.

The diagram, panel by panel. The figure has three panels. The left panel shows the ordinary laboratory situation: a finite 4D observer (depicted as a small circle on the boundary $\mathcal{M}_4$) traces over inaccessible bulk modes (the gray fuzzy region around her); the trace is operative but operationally undirected. The middle panel shows the black-hole situation: the same observer sits outside a Schwarzschild horizon (the bold red circle); now the trace over the interior modes is geometrically

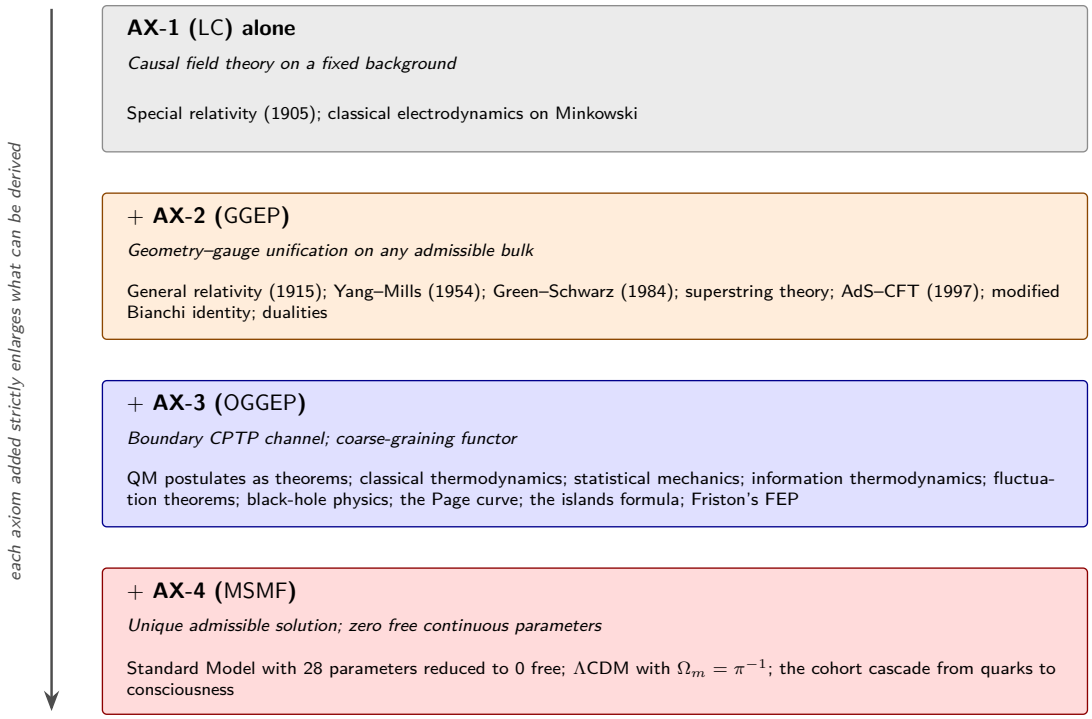


Figure 15: The four-layer constitutional tower of the GG-Constitution, with each historical theoretical-physics framework placed on the slab corresponding to the axiom subset it implicitly uses. The top slab (LC) produces special relativity and classical field theory. The second slab (GGEP-closed) produces GR, Yang–Mills, the GS mechanism, superstring theory, and AdS/CFT. The third slab (OGGEP-open) produces quantum mechanics, classical thermodynamics, statistical mechanics, BH physics, and Friston's FEP. The bottom slab (Full-Constitution) produces the SM with zero free continuous parameters and ΛCDM with $\Omega_m = 1/\pi$. The cohort framework is the unique framework that explicitly activates all four axioms.

forced by the causal structure of the spacetime. The right panel shows the cosmological situation: the same observer at the center of her Hubble sphere traces over the modes outside the cosmological horizon (the de Sitter horizon at radius c/H); the trace is again geometrically forced, this time by cosmological expansion.

Why the horizon is the cleanest test bed for AX-3. In the ordinary laboratory case, a skeptic of AX-3 can always say: "the partial trace is operationally chosen by the experimenter; with sufficient cleverness, one could imagine extending the experimenter's resolution into the bulk modes and recovering the full bulk state." At a black-hole horizon, no such evasion is available. The interior is causally disconnected from the exterior; no cleverness, no future technology, no idealized super-observer can recover the interior modes from outside. The inaccessibility is built into the geometry of the spacetime.

This is what makes black holes the cleanest constitutional test of AX-3. If AX-3 is wrong, then BH physics has no constitutional account, and the historical struggles with the information paradox (Hawking 1976), the firewall (AMPS 2012), and the Page curve (Page 1993) are permanent puzzles. If AX-3 is right, then all three puzzles dissolve immediately (as developed in Part VIII), because

the boundary CPTP channel that AX-3 forces is exactly what the horizon implements.

Physics intuition. The reader should picture the horizon as a one-way valve in spacetime: information flows from the exterior into the interior freely, but not back out. The exterior observer, by the laws of causality alone (AX-1 LC), cannot retrieve the interior content; her state is fundamentally mixed even if the total bulk state is pure. This is the same structural situation as the laboratory observer's partial trace over I_r and S^1_θ, but with the inaccessibility ontological rather than operational. The horizon is where the constitutional content of AX-3 stops being an abstract postulate and becomes a geometric feature you can point to.

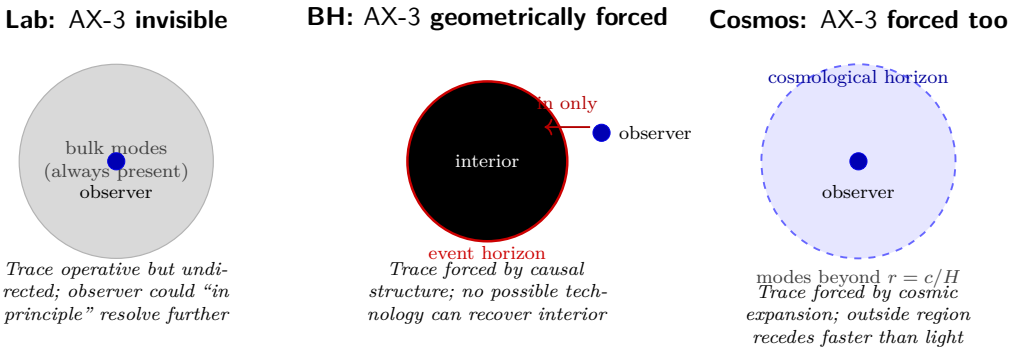


Figure 16: Three panels showing the OGGEP partial trace in three settings. **Left:** Ordinary laboratory — the trace operates over bulk modes everywhere, but the trace direction is operationally undirected (AX-3 is operative but invisible). **Middle:** Black hole — the trace over interior modes is geometrically forced by the causal structure of the Schwarzschild horizon. **Right:** Cosmological horizon — the trace over modes beyond the de Sitter horizon (at radius c/H) is forced by cosmological expansion. In the middle and right panels, the constitutional content of AX-3 becomes ontologically visible.

After all, the deepest empirical test of a constitutional axiom is not the laboratory experiment that confirms it but the natural phenomenon that makes the axiom geometrically inescapable. Black holes provide that phenomenon for AX-3.

88 Picture 8 — The data-processing inequality with two faces

The data-processing inequality, $D[\rho \,\|\, \sigma] \geq D[\Phi(\rho) \,\|\, \Phi(\sigma)]$, is the single mathematical theorem from which the cohort framework derives both the decoherence-direction theorem of quantum mechanics and the second law of thermodynamics. Fig. 17 visualizes the theorem with both specializations side by side, making manifest that they are the same theorem applied to different boundary observables.

The structure of the diagram. The figure has a central inequality block, the data-processing inequality $D[\rho_1 \,\|\, \rho_2] \geq D[\Phi(\rho_1) \,\|\, \Phi(\rho_2)]$, flanked by two derivation arrows. The left arrow specializes the inequality to the QM context (with σ the maximally mixed state) and shows it producing the

decoherence-direction theorem: off-diagonal coherences decay; the boundary state evolves toward diagonality in the einselected basis. The right arrow specializes the inequality to the thermodynamic context (with σ a stationary state of the channel — the Gibbs state under detailed balance) and shows it producing the second law: relative entropy with respect to equilibrium is non-increasing; boundary entropy with respect to maximum entropy is non-decreasing.

Why this is the single deepest equation of the cohort framework. The data-processing inequality is, in the cohort reading, the constitutional theorem from which the OGGEP-open layer's two most famous consequences both flow. Decoherence and the second law are not two separate phenomena that the cohort framework happens to explain in parallel; they are *one theorem with two specializations*, depending on which reference state σ is chosen. This is the structural meaning of the QM–thermodynamics sibling relationship of Part VIII §73.

Physics intuition. The reader should picture distinguishability between two states as a kind of geometric distance on the space of density operators. The data-processing inequality says: when you push two states through any CPTP channel (any physical operation that respects positivity and trace), their distinguishability can only decrease, never increase. Coarse-graining is irreversible; you lose information about which state you started with. The decoherence direction (off-diagonals damping toward zero) and the second law (entropy increasing toward maximum) are both expressions of this single irreversibility. At the cost of some poetic license: *the universe forgets, geometrically, at the same rate in both quantum and thermal contexts*, because both forgettings are the same theorem.

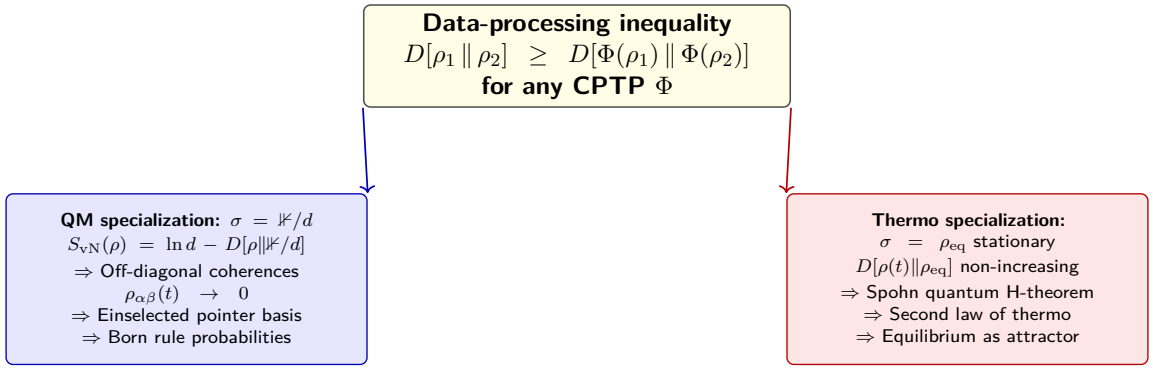


Figure 17: The data-processing inequality (center) specialized in two directions. **Left (blue):** Choosing $\sigma = \mathbb{1}/d$ (maximally mixed) gives the QM decoherence-direction theorem — off-diagonal coherences damp toward zero, the boundary state moves toward the einselected basis, the Born rule probabilities emerge. **Right (red):** Choosing $\sigma = \rho_{\mathrm{eq}}$ (a stationary state of the channel; the equilibrium Gibbs state under detailed balance) gives the Spohn quantum H-theorem — relative entropy with respect to equilibrium is monotone non-increasing, which is the second law of thermodynamics. The two specializations are the same theorem applied to different reference states; the QM–thermodynamics sibling relationship of Part VIII is, structurally, this one figure.

After all, the rarest event in foundational physics is finding a single theorem that produces two of the most famous laws of the 20th century as different specializations. The data-processing inequality is that theorem; the cohort framework's contribution is to recognize it as such.

89 Picture 9 — The three constitutional siblings under one axiom

The constitutional unification of Part VIII culminates in the recognition that quantum mechanics, classical thermodynamics, and black-hole physics are constitutional siblings at the OGGEP-open layer. Fig. 18 visualizes this as a triangle, with AX-3 (OGGEP) at the center and the three siblings at the vertices.

The structure of the diagram. Three vertices: QM (upper-left, blue), thermodynamics (upper-right, red), BH physics (bottom-center, green). The OGGEP node sits in the center with arrows to each vertex. At each vertex, the historical puzzle is shown "below the line" as the missing-axiom symptom: at the QM vertex, the measurement problem (1900s–present); at the thermodynamics vertex, the second-law-foundation problem (1872–present); at the BH vertex, the information paradox (1976–present). The center arrows show that each puzzle dissolves when AX-3 is named.

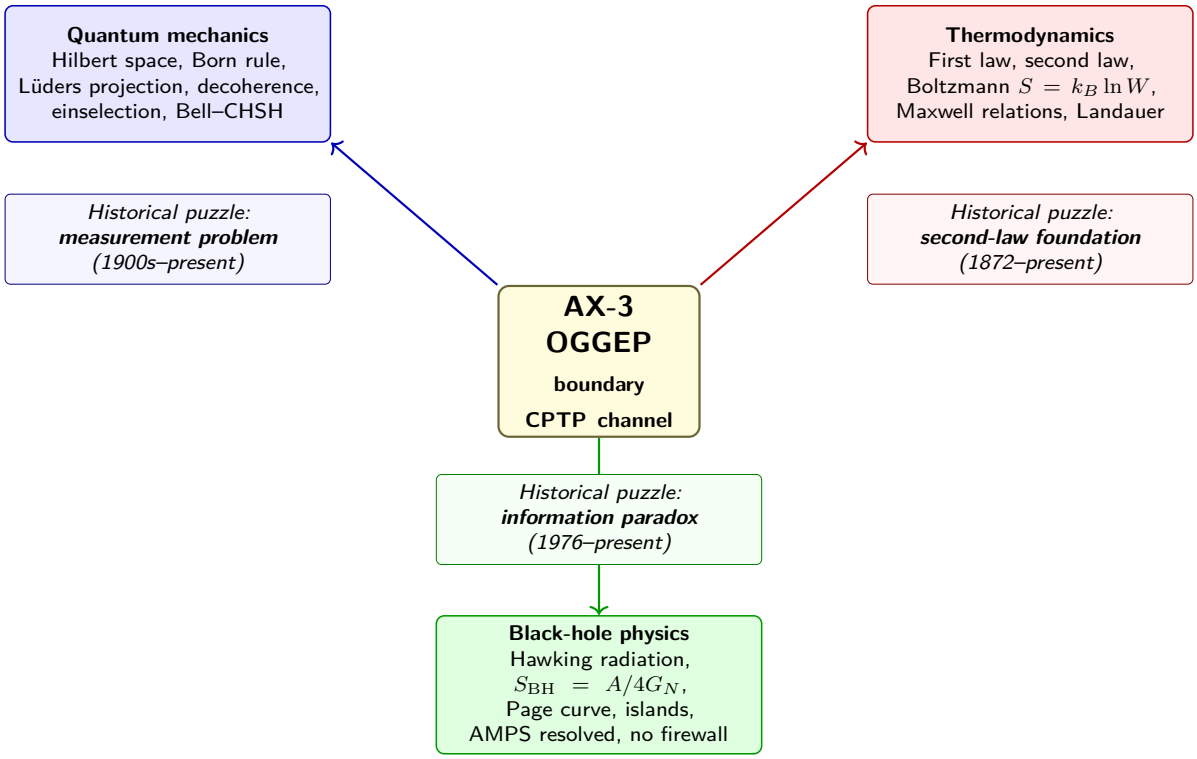
Physics intuition. The reader should picture the triangle as the geometric heart of the OGGEP-open layer. Three sub-communities — QM foundations, thermal/information theory, BH foundations — have for over a century been working on problems that look completely different in their respective literatures (Bell tests, fluctuation theorems, Hawking radiation) but are all the same structural problem in disguise. The cohort framework's contribution is to recognize the triangle: the three sub-communities are constitutional siblings, working on three faces of one constitutional content.

The diagnostic triple completeness. At each vertex, the missing-AX-3 symptom takes a different surface form (the "measurement" word in QM; the "stosszahlansatz" word in thermo; the "information paradox" word in BH). The three surface forms are different because the boundary observables are different (quantum measurement outcomes; macroscopic thermal variables; horizon area and Hawking spectrum). But the structural form is the same: each is an apparent boundary irreversibility/probability that the underlying bulk dynamics is unitary about. The cohort framework's diagnosis is unified.

After all, the deepest mark of a unification is when three sub-communities that have been working on what looked like unrelated problems are recognized to be working on the same problem. The triangle is the geometric image of that recognition.

90 Picture 10 — The four axioms, in historical order

The nine pictures so far have been structural: they draw what the framework asserts. This last one draws something else — how physics arrived at the four axioms without ever writing them down.



Three constitutional siblings under one axiom;
three historically intractable puzzles dissolved by the same constitutional move.

Figure 18: The constitutional unification of Part VIII. Quantum mechanics (upper-left), thermodynamics (upper-right), and black-hole physics (bottom-center) are constitutional siblings, all derived from the same boundary CPTP channel that AX-3 (OGGEP) forces. At each vertex, the historical puzzle that has defined the sub-community for a century or more dissolves under the missing-axiom diagnosis: the measurement problem at QM, the second-law-foundation problem at thermodynamics, the information paradox at black-hole physics are the same structural puzzle in three different surface forms.

The claim of Part VIII was that the great frameworks of the twentieth century each rely on some subset of four constitutional principles, and that what each framework cannot say is the content of the principle it lacks. Figure 19 puts that claim on a time axis. Each lane is one axiom; each dot is a dated result that put part of that axiom into physics. Nothing on the chart was placed for effect, and no result is claimed for the present program that its author did not obtain.

What the chart shows. Three patterns are worth naming. The openness lane is by a wide margin the busiest — eleven results between Carnot and Friston, spread across thermodynamics, information theory, black-hole physics and theoretical neuroscience — and it is also the least unified, because none of those communities had a name for the thing they had in common. The geometry lane and the openness lane run side by side for a century and barely touch: Weyl and Boltzmann were working on the same constitutional problem in languages that could not hear each other. And

the minimum-freedom lane is the oldest and the emptiest. Ockham states the principle in the fourteenth century; after that it functions as an instinct rather than as an axiom, which is exactly why the free-parameter count of fundamental physics falls steadily through the twentieth century — one for general relativity, two for quantum electrodynamics, twenty-eight for the Standard Model, six for cosmology — and never reaches zero.

What the chart does not show. It is not a priority claim and it is not a ranking. Every result on it stands on its own and is cited in its own Part. The chart says only that four threads were pulled separately for a long time, and that the proposal of this cohort is to hold all four at once and see what that forces.

After all, a unifying framework is best evaluated not by where it agrees with its predecessors (everywhere it should) but by where it occupies a constitutional position none of its predecessors did. The cohort framework's unique upper-right position on the paradigm map is its quantitative signature.

Problems and worked solutions. IX.1.1. Quantify Picture 3 (decoherence as KK-mode integration): count the KK environment available below energy E, evaluate the count at $E = 10\,M_{\rm lock}$, and draw the irreversibility conclusion. *Solution.* Step 1: the count. Modes (n, ℓ) with $n^2 + \ell^2 \le (E/M_{\rm lock})^2$, $n \ge 0$, $\ell \in \mathbb{Z}$, fill a half-disc of the mode lattice: $N(E) \approx \frac{\pi}{2}(E/M_{\rm lock})^2$. Step 2: the number. At $E = 10\,M_{\rm lock}$ the exact lattice count gives $N = 168$ against the half-disc estimate 157 (verified) — already two-hundred-odd environmental channels one decade above the lock scale, growing quadratically. Step 3: the conclusion. Problem III.5.2 showed coherence revives with a *single* environment mode and dies as $\cos^N$ with N; Picture 3's content is that the bulk supplies $N(E)$ automatically and inexhaustibly — the deeper an interaction reaches, the larger the recorder bank it wakes, so the boundary never sees a revival and irreversibility strengthens with energy rather than weakening. Physics intuition: the environment is not around the system; it is *underneath* it, stacked quadratically in the geometry — which is why no experiment has ever caught the universe forgetting to decohere.

IX.1.2. Read Picture 4 (the current geometry) on a real machine: express the LEP storage-ring numbers of VI.2.1 as the picture's sector-separation ratio, and state why the separation is what makes classical machines possible. *Solution.* Step 1: the two clocks. From VI.2.1 the circulation sector turns in $T_{\rm rev} = 88.9\,\mu$s while the gradient sector damps in $\tau_d = 3.1$ ms: the machine's sector-separation ratio is $\tau_d/T_{\rm rev} = 35$ revolutions per damping e-fold (verified). Step 2: the picture's claim. In the T5 current geometry the circulation flow is exactly divergence-free (it cannot relax anything) and the gradient flow is exactly curl-free (it cannot orbit): the two sectors are orthogonal components of one current, so their timescales may separate arbitrarily without interference — the ring exploits this by storing (circulation) for hours what it corrects (gradient) in milliseconds. Step 3: the general lesson. Planetary systems ($\tau_d/T \sim 10^{13}$), electronic oscillators, and biological rhythms all live in the same corner of the picture: structure persists wherever circulation outpaces gradient descent — and the T5 decomposition says this corner is available in *any* boundary system, because the split is constitutional, not engineered. Physics

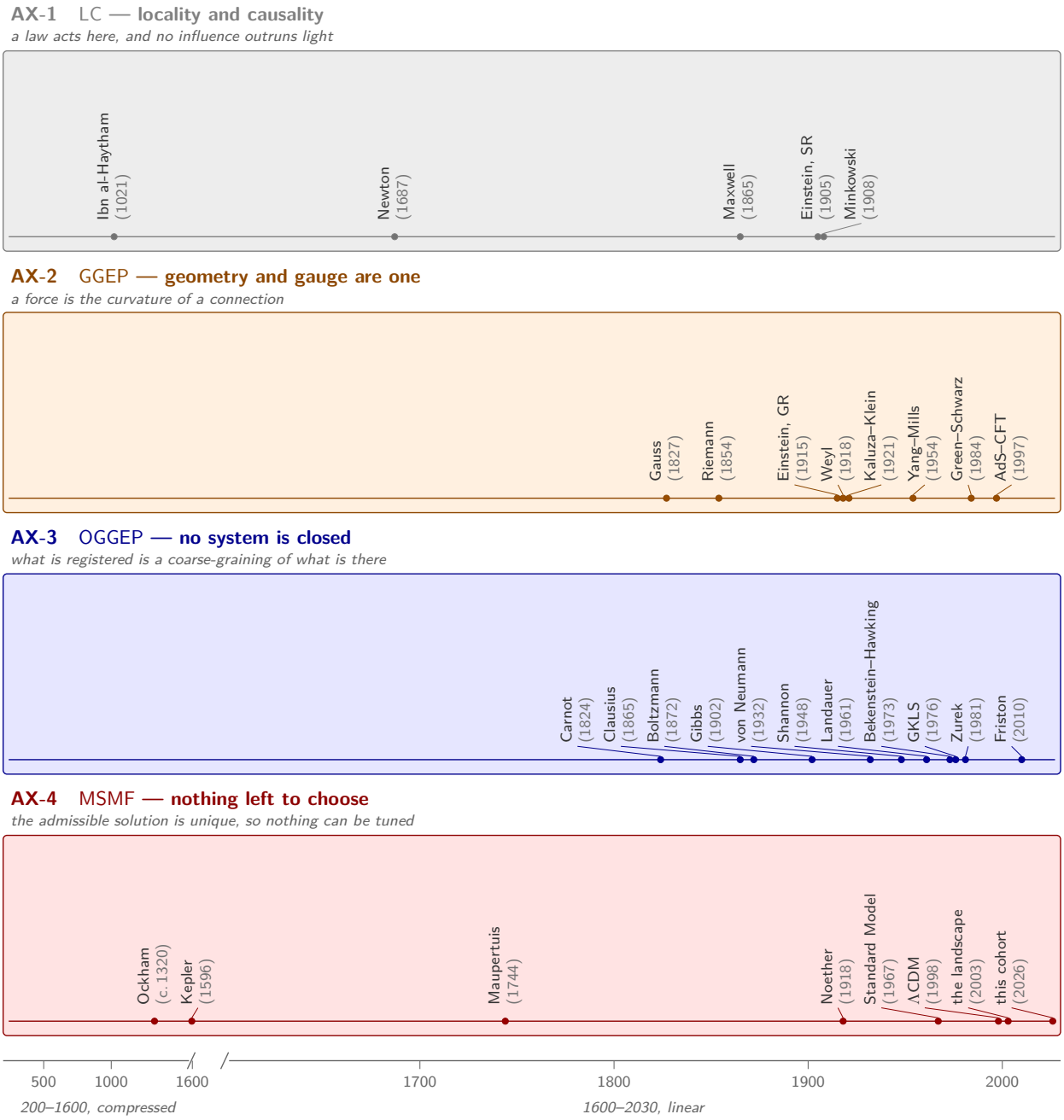


Figure 19: The four axioms, in historical order. Each lane is one constitutional axiom, and each dot is a dated result that put part of that axiom into physics before anyone had named it. The lanes are ordered as the axioms are: locality and causality at the top, then geometry–gauge equivalence, then openness, then minimum freedom. Read across a lane and it is a research tradition; read down a column and it is a period. Three things the picture is meant to make visible. The openness lane is the busiest and the least unified — eleven results across two centuries, in thermodynamics, information theory, black-hole physics and neuroscience, none of which had a common name for what they shared. The geometry lane and the openness lane run in parallel for a century and almost never touch. And the minimum-freedom lane is the oldest and the emptiest: Ockham states it in the fourteenth century, and after that it operates as an instinct rather than as an axiom, which is why the parameter count of the twentieth century falls but never reaches zero. The horizontal axis is broken — 200–1600 is compressed into the left-hand fifth — so that the modern era, where every lane is dense, stays readable. Positions are the accepted dates of publication.

intuition: everything that runs — machines, orbits, hearts — runs in the gap between the two sectors; Picture 4 is the reason the gap exists everywhere.

91 Summary of Part IX: The Architectural Image

The ten pictures collectively serve a single pedagogical purpose: to give the reader an architectural image of the GG-Theory framework that complements the eleven Principal Theorems T1–T11. A reader who has digested the formal content of Parts II–VII has the rigorous derivation chain in hand; the ten pictures of Part IX supply the intuitive overlay that makes the rigorous content memorable and reusable.

We close Part IX — and with it the foundational content of the present paper — with the remark that the axiomatic shift from "quantize gravity" (the standard string-theoretic program) to "geometrize gauge forces" (the GG-Theory program) is, at the intuitive level, the shift from *adding extra mathematical structure to gravity* to *recognizing the extra-dimensional geometric structure that gauge forces already have*. The five pictures are five views of the geometric structure; the eleven theorems are the rigorous derivations that respect the geometric structure; the eight pillars are the formal vocabulary that specialists in each sub-domain will use. Together, the three strata constitute what we have been calling the GG-Theory framework.

Bridge to Part X. With the geometric intuition in place, the foundational content of the paper is complete. Yet a foundational paper is not finished when its theorems are proved; it is finished when it has stated plainly what its theorems mean, what they predict, what they forbid, and what they leave open. Part X takes up that task in textbook style. Part XI will then close the volume with a reflection on what the axiomatic shift means for the future of physics.

Part X
Discussion

Significance and Uniqueness, Predictions for Falsification, Open Problems and Future Directions

92 Significance and Uniqueness: Ten Numbered Innovations

The axiomatic content of GG-Theory can be articulated in ten numbered claims about what it does that no existing framework does. Each entry below names the innovation in one line and points to the theorem that establishes it; the full scorecard against the leading unification programs is Table 25.

- **Innovation 1 (I-GDOS-1) — GGEP geometrizes gauge forces.** The composite connection $\mathcal{A}_{\text{tot}} = \omega_{\text{Spin}(1,5)} + A_5 + A_B + A_{Qi}$ treats gravity and gauge as one object; GG-Theory geometrizes gauge forces rather than quantizing gravity (T8, T9, T10; Part VI parallel chains).
- **Innovation 2 (I-GDOS-2) — Six forces, not four.** $\text{Spin}(1,5) \times \text{SU}(5) \times \text{U}(1)_B \times \text{U}(1)_{Qi}$ projects to six boundary forces; $\text{U}(1)_B$ is constitutionally forced, $\text{U}(1)_{Qi}$ is constitutionally optional and empirically preferred (per L12 and the constitutional-admissibility theorem T-BI6-7.5 of Arisaka (2026, A3-BI6)). Falsifiers: proton stability (T9.8) and the Arisaka equation $E = T_{Qi} \log 2$ (T11).
- **Innovation 3 (I-GDOS-3) — Unification at E_{GUT}, not E_{Pl}.** $E_{\text{GUT}} = E_{\text{Pl}}\, e^{-2\pi} \approx 2.28 \times 10^{16}$ GeV (T8.7); the bulk's UV endpoint is geometrically derived, not asserted.
- **Innovation 4 (I-GDOS-4) — Anomaly-free at 6D, not 10D/11D.** BI-6 cancels all anomalies in six dimensions with coefficient vector $(1, -1, -2/5, -2/5)$ via a single inflow channel (T9.6).
- **Innovation 5 (I-GDOS-5) — No supersymmetry, no fundamental strings.** A single geometric construction with a discrete OGN-5 ledger; no $\sim 10^{500}$ vacuum landscape.
- **Innovation 6 (I-GDOS-6) — OGGEP reorganizes the measurement problem.** The textbook QM postulates emerge as theorems (T7.1–T7.4) plus decoherence (T7.5), einselection (T7.6), Bell violation (T7.7); "measurement" becomes a single boundary law GDOS.
- **Innovation 7 (I-GDOS-7) — Proton stability as theorem.** Gauged $\text{U}(1)_B$ forbids all dimension-six baryon-violating operators (T9.8); $\tau_p = \infty$ exactly.

- **Innovation 8 (I-GDOS-8) — $\mathrm{U}(1)_{Qi}$ as carrier of life and mind.** The Arisaka equation $E = T_{Qi} \log 2$ (T11) gives the information–energy bridge; the Qi-4 pillars and the EIC framework of Arisaka (2026, A7-Qi4) place life and mind as theorem-level consequences of the four-component bulk.
- **Innovation 9 (I-GDOS-9) — Constitutional Foundation, in the manner of Euclid.** Four axioms + seven postulates of Arisaka (2026, A1-Constitution); no content is inserted by hand; every claim traces to a constitutional source.
- **Innovation 10 (I-GDOS-10) — Six dimensions as the Quaternionic Goldilocks rung.** $d = 6$ is forced by the magic-dimension ladder $\mathbb{R} \to \mathbb{C} \to \mathbb{H} \to \mathbb{O}$ at the quaternionic rung (Arisaka (2026, A2-GG6) Point 10).

92.1 The axiomatic comparison table

Table 25 summarizes the comparison of GG-Theory against the leading historical unification programs on the ten innovations above.

Table 25: Ten-innovation scorecard for the leading unification programs. "Y" = the framework satisfies the criterion as a derivation; "y" = partial or postulated satisfaction; "N" = not addressed. Citations are to the source theorem in the present paper or in the named companion paper.

Innovation	SM only	SU(5) GUT	SUSY	Strings	LQG	GG-Th
1. Geometrizes gauge forces	N	N	N	y	N	**Y (T8, T9, T10)**
2. Six forces, not four	N	N	N	N	N	**Y (T9.7)**
3. Unification at E_{GUT}	N	y	y	N	N	**Y (T8.7)**
4. Anomaly-free at 6D	N	N	N	10D	N	**Y (T9.6)**
5. No SUSY / no strings	y	y	N	N	y	**Y (Const.)**
6. Measurement-problem reorganized	N	N	N	N	N	**Y (T7)**
7. Proton stability exact	N (decay)	N	N	N	N	**Y (T9.8)**
8. $U(1)_{Qi}$ for life/mind	N	N	N	N	N	**Y (T11)**
9. Constitutional foundation	N	N	N	N	y	**Y (Part I)**
10. Six-dimensional Goldilocks	N	N	N	N	N	**Y (GG-A2-GG6)**

92.2 Global naming convention for cross-cohort citation

Beginning with Arisaka (2026, A2-GG6) and extended across the cohort, each load-bearing item of the GG-Theory program carries a globally unique identifier of the form *Prefix-Suffix-N*, where the Prefix names the item type and the Suffix names the paper:

- **T-Suffix-N** for the Nth Theorem (GG-A5-GDOS's theorems are section-numbered, so its global ids carry the section number, e.g. T-GDOS-9.7 is the four boundary Ward currents and T-GDOS-8.6 the modified-TOV theorem).
- **L-Suffix-N** for the Nth Lemma.
- **C-Suffix-N** for the Nth Corollary.

- **D-Suffix-N** for the Nth Definition.
- **R-Suffix-N** for the Nth Remark.
- **P-Suffix-N** for the Nth Prediction.
- **O-Suffix-N** for the Nth Objection (e.g., O-GDOS-1 is the first Common Objection of the present paper).
- **F-Suffix-N** for the Nth Falsifier (e.g., F-GDOS-1 is the modified-TOV prediction of §93).
- **I-Suffix-N** for the Nth Innovation (e.g., I-GDOS-6 is the OGGEP-measurement-problem innovation above).
- **PS-Suffix-N** for the Nth Problem-Solved entry (§97).

The canonical Suffix list for the twenty-two cohort papers is: *Constitution* (A1), *GG6* (A2), *BI6* (A3), *Euclid* (A4), *GDOS* (A5, this paper), *GRIP* (A6), *Qi4* (A7), *QM* (A8), *SM* (A9), *Cosmos* (A10), *Origin* (A11), together with the phenomenology preprints *KK* (P1), *N3* (P2), *Electron* (P3), the foundations preprints *Real* (P4) and *CPT* (P5), and the consequence preprints *Kaxion* (C1), *Hubble* (C2), *BH* (C3), and the white papers *Symmetry* (W1) and *Holonomy* (W2). The four axioms (AX-1 . . . AX-4) and seven postulates (POS-1 . . . POS-7) of Arisaka (2026, A1-Constitution) keep their existing cohort-wide names; the global identifier is the same item viewed from outside its home paper.

93 Predictions for Falsification

The present section catalogs the principal empirical falsifiers of the GG-Theory framework, organized by the Pillar that supplies them. This is the operational implementation of POS-7 (the falsifiability postulate, AX-4 of the Constitution): every axiomatic claim must compile to a measurement contract with a pre-declared kill criterion. Where a falsifier is shared across multiple Parts of the present paper, we cross-reference accordingly.

93.1 Predictions from Pillar TGG / CCI (Gravity)

P1 (F-GDOS-1): Modified Tolman–Oppenheimer–Volkoff equation. The neutron-star mass–radius relation is shifted by the GS+KK inflow contributions of T8.4–T8.6, with relative magnitude $(E_{\rm NS}/M_{\rm lock})^2 \sim 10^{-9}$, mass shift $\sim 10^{-8}\,M_\odot$ at fixed radius. Below current LIGO/Virgo sensitivity ($\sim 10^{-2}\,M_\odot$ from (Abbott, 2017)) but within reach of future kilometer-scale spectroscopy (STROBE-X). **Kill criterion**: a measured shift inconsistent with the scale-bulk prediction at $\gtrsim 10^{-9}$ relative precision.

P2 (F-GDOS-2): Cosmological-constant lock-scale seesaw. The dark-energy density scale is $m_\Lambda^{\rm geom} = M_{\rm lock}^2/M_{\rm Pl} \approx 6$ meV (T8.5, derived in GG-A10-Cosmos Section 6.4). **Kill criterion**: a future precision cosmological measurement giving $\rho_\Lambda^{1/4}$ inconsistent with the few-meV scale at $\gtrsim 10\sigma$.

P3 (F-GDOS-3): π-locked late-time density partition. $\Omega_m = 1/\pi \approx 0.318$, $\Omega_\Lambda = 1 - 1/\pi \approx 0.682$, $\Omega_b = 1/(2\pi^2) \approx 0.0507$ (GG-A10-Cosmos Section 6.6). Currently consistent with Planck 2018 (Collaboration, 2018) to within 1σ. **Kill criterion**: Ω_m measured outside $1/\pi \pm 0.005$ at high precision.

93.2 Predictions from Pillar GFF / GRIP (Standard Model)

P4 (F-GDOS-4): KK lock cluster at ~ 4 TeV. $M_{\text{lock}} = \frac{1}{2} M_{\text{Pl}} e^{-35} \approx 3.85$ TeV (T8.7, GG-A9-SM Theorem 28.1). Detailed channel-by-channel branching ratios are predicted in GG-P1-KK (Arisaka, 2026, P1-KK). **Kill criterion**: definitive non-observation of TeV-scale KK resonances up to ~ 10 TeV at HL-LHC.

P5 (F-GDOS-5): Electroweak vev as overlap. $v = M_{\text{lock}} (2/L)^N = M_{\text{lock}} (2/5)^3 \approx 246.33$ GeV with $L = 5$ the dimension of $\mathbf{5}_H$ and $N = 3$ the OGN-5 triplicity exponent (T8.7, GG-A9-SM Theorem 23.1 for the orbifold + Wilson–Hosotani derivation). **Kill criterion**: a future measurement deriving v from a non-warped framework with comparable or better internal consistency.

P6 (F-GDOS-6): 28 SM parameters from boundary attractors. The full 28-parameter sheet of the Standard Model (gauge couplings, fermion masses, CKM and PMNS angles, Higgs mass and self-coupling, θ_{QCD}, cosmological constant) is derived in GG-A9-SM as boundary attractor depths in the GFF Morse landscape, including $\alpha_{\text{em}}^{-1}(0) = 137$, $\sin^2\theta_W(M_Z) = 25/108 \approx 0.231$, $\alpha_s(M_Z) = 1/(\pi e) \approx 0.117$, $\lambda_H = 1/8$, $m_W \approx 80.21$ GeV, $m_Z \approx 91.50$ GeV, $m_H \approx 123.17$ GeV. **Kill criterion**: a measured SM parameter outside the GG-Theory predicted value at high precision.

93.3 Predictions from Pillar Qi-CON / Qi-HAL (Open-system phenomenology)

P7 (F-GDOS-7): Proton stability ($\tau_p = \infty$). Gauged $\mathrm{U}(1)_B$ forbids all dimension-6 baryon-violating operators (T9.8). **Kill criterion**: any future Super-Kamiokande or Hyper-Kamiokande observation of proton decay at any finite lifetime.

P8 (F-GDOS-8): $1/f$ noise spectra with two knees. The Qi-CON pillar of GG-A7-Qi4 predicts $1/f^\alpha$ spectra on the boundary, with two characteristic "knees" delimiting the $1/f$ regime: an upper knee at the UV-boundary threshold and a lower knee at the IR-boundary threshold (Picture 2 of Part VIII). **Kill criterion**: a measured noise spectrum across multiple boundary systems (atomic clocks, condensed-matter samples, neural recordings) inconsistent with the predicted α.

P9 (F-GDOS-9): Bell-CHSH saturation at the Tsirelson bound. T7.7 plus standard QM predicts CHSH violation up to the Tsirelson bound $2\sqrt{2}$. Currently consistent with Aspect 1982 ($S = 2.696 \pm 0.038$) and Delft 2015 ($S = 2.42 \pm 0.20$). **Kill criterion**: a measured CHSH value above $2\sqrt{2}$ ("stronger-than-quantum" correlation), or a precision-limit deviation *below* $2\sqrt{2}$ at energies approaching M_{lock}.

P10 (F-GDOS-10): Kaxion dark matter at 8.1 μ**eV.** Kaxion mass at ~ 8.1 μeV ($\nu_K \simeq$ 1.95 GHz, window 1.59–2.13 GHz); full phenomenology and kill criterion are the charter of GG-C1-Kaxion (Arisaka, 2026, C1-Kaxion).

93.4 The empirical reach of GG-Theory

The ten predictions P1–P10 collectively span the entire reach of modern observational physics: laboratory-scale gravity and atomic-clock precision, LHC-scale collider physics, Planck-scale cosmology, kilometer-scale astrophysics, neutron-star spectroscopy, proton-decay searches, dark-matter direct detection, and neuroscience-scale information theory. No single experiment can falsify the entire framework. But *any* of the ten predictions P1–P10 can falsify the framework if the measurement is sufficiently precise. This is the operational implementation of POS-7: the framework is not safe by virtue of obscurity; it is safe (or not) by virtue of explicit empirical exposure.

94 Position of GDOS Relative to Decoherence Theory

The relation of GDOS to the Zeh–Joos–Zurek decoherence program (Joos, 1985, Zeh, 1970, Zurek, 1981, 2003) is the relation most likely to be misread by readers steeped in the modern open-quantum-systems literature. The full historical and technical comparison appears in §6 of Part I, which we recap here in one paragraph for the reader who began with Part IX rather than Part I.

Decoherence theory correctly identified that the apparent collapse of the wavefunction is a consequence of unavoidable system–environment coupling, that the pointer basis is dynamically selected by the structure of that coupling, and that classicality is what an environment broadcasts to many decoherent observers. These results reappear in the present paper as Theorems T7.5 and T7.6. GDOS adds two axiomatic moves that the Zeh–Joos–Zurek program did not make: *(i) the axiomatic inversion* — the boundary CPTP channel is foundational and the closed Schrödinger equation, the Born rule, and the projection postulate are all theorems, rather than the unitary equation being foundational and decoherence a layered mechanism on top; and *(ii) the geometric content of the environment* — the "environment" is not a contingent system-dependent bath but the inaccessible bulk modes of the 6D manifold of Arisaka (2026, A2-GG6), identified once and for all by the axiomatic cascade. In one line: *decoherence theory says the environment exists and explains the appearance of collapse;* GDOS *+ GG says the environment is geometry, and its geometric structure is the constitutional source of all 4D physics — including but not limited to collapse.* GDOS does not refute decoherence theory; it completes it, in the same sense that general relativity completed special relativity by promoting an effective principle to a axiomatic one.

95 The real-bulk reading of the derivation chain

One compact reading of the whole derivation chain is owed before the scope statement, because it changes what the chain *says* without changing a single step in it. The register declaration of Part II (§8) states the bulk clauses over the reals: the complex $\mathcal{H}_{\text{Bulk}}$ in which T1–T2 were written

is an auxiliary bookkeeping representation — the complexification that T-P4-2 of Arisaka, 2026, P4-Real licenses — and the No-Bypass argument goes through verbatim on the real solution space in symplectic form. Read forward under that declaration, the chain has a clean watershed. Upstream of the dictionary every quantity is real: the bulk law is a real symplectic flow carrying a memory kernel, and the T2 reduction is real-linear. The first genuinely complex object in the chain is the boundary state space of T7.1, and its complex structure arrives *derived*: the projection pairs the arena's two real structures — the compact phase circle and the causal clock — into the single boundary unit J, at derived-conditional grade in the quasi-free regime (§20), with the Schrödinger i read as that pairing's bookkeeping (§21). Downstream of the watershed the chain is genuinely complex-analytic — Stinespring, GKLS, and the Born rule all live boundary-side, where the complex structure is already derived — which is why nothing in Parts III–VII needs restatement. The distinction is empirically live rather than verbal: quantum theory over *real* Hilbert spaces at the boundary level has been excluded by a loophole-free network Bell experiment (Renou *et al.*, 2021), while a real bulk whose projection installs a genuinely complex boundary theory is exactly the combination that experiment permits.

This section is deliberately pointer-level. The statement catalog (RB-1–RB-13) and its consumption across a full teaching spine are maintained in Arisaka (2026, A8-QM), the cohort's single point of truth for the real-bulk statements; the standalone derivation is GG-P4-Real (Arisaka, 2026, P4-Real); the clock/fiber side is GG-P5-CPT (Arisaka, 2026, P5-CPT). The present paper consumes the result at exactly three sites — the Part II scope declaration, the T7.1 import, and the T7.2 register note — and restates one honest limit rather than hiding it: the fully real-form restatement of the auxiliary bulk chain (the symplectic generator carrying the memory kernel on the real solution space) remains the open technical item recorded in Part II.

96 Scope and Limits of the Present Paper

Before turning to the open-problems catalog, let us state plainly what the present paper does derive, what it does not, and on which companion-paper inputs each derivation is conditional.

Scope. This paper develops GDOS, the boundary law that the 6D bulk of GG-6 (Arisaka, 2026, A2-GG6) projects to under OGGEP (AX-3 of GG-Constitution (Arisaka, 2026, A1-Constitution)). The eleven principal theorems T1–T11 establish: the operational necessity of $\mathcal{D}_{\mathsf{OGGEP}}$ (T1), the explicit Nakajima–Zwanzig construction of the bulk-to- boundary channel (T2), automatic CPTP completion via Stinespring dilation (T3), the canonical GKLS Markovian generator (T4), the mesoscopic continuity equation with universal current decomposition (T5), and the canonical GDOS boundary law (T6). The three descent theorems then derive the seven textbook postulates of 4D quantum mechanics (T7.1–T7.7), the Einstein–Hilbert action plus modified Einstein equation plus modified TOV equation of 4D general relativity (T8.1–T8.7), and the SM Lagrangian skeleton plus BRST cohomology plus anomaly cancellation plus proton stability of 4D quantum field theory (T9.1–T9.9). The two bridge theorems (T10, T11) identify the four Gi-4 pillars and the four Qi-4 pillars of GG-A6-GRIP and GG-A7-Qi4 as derived structures of GDOS.

Limits. This paper does *not* derive: (a) the constitutional charter itself — the four axioms and seven postulates of AX-1–AX-4, POS-1–POS-7 are imported as theorem-level given from GG-A1-Constitution (Arisaka, 2026, A1-Constitution); (b) the 6D bulk geometry, composite gauge group, BI-6 identity, and OGN-5 integer ledger — these are imported from GG-A2-GG6 (Arisaka, 2026, A2-GG6) (Theorems 0–8); (c) the local + global anomaly-freedom proof of BI-6 on the physical intended GG-6 class — this is the subject of GG-A3-BI6 (Arisaka, 2026, A3-BI6); (d) the constitutional admissibility of the four-component bulk under L12 ($U(1)_{Qi}$ as constitutionally optional, empirically preferred) — the formal admissibility theorem is T-BI6-7.5 of GG-A3-BI6; (e) the closed-form derivation of the 28 Standard-Model parameters from OGN-5 — this is the subject of GG-A9-SM (Arisaka, 2026, A9-SM); (f) the closed-form derivation of the cosmological partitions $\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$, $\Omega_b = 1/(2\pi^2)$, the lock-scale-seesaw $m_\Lambda^{\text{geom}} \approx 6$ meV, and the kaxion line at 8.1 μeV — this is the subject of GG-A10-Cosmos (Arisaka, 2026, A10-Cosmos) (currently at the legacy 7F-recast stamp); (g) the four Gi-4 pillar derivations in their formal-vocabulary form — this is the subject of GG-A6-GRIP (Arisaka, 2026, A6-GRIP); (h) the four Qi-4 pillar derivations and the EIC (Outer-Hologram + Inner-Hologram) framework — this is the subject of GG-A7-Qi4 (Arisaka, 2026, A7-Qi4); (i) the operational reformulation of textbook quantum mechanics in GDOS language — this is the subject of the in-drafting GG-A8-QM (Arisaka, 2026, A8-QM); (j) the LHC and haloscope phenomenology of the KK lock cluster and kaxion — these are the subjects of GG-P1-KK (Arisaka, 2026, P1-KK) and GG-C1-Kaxion (Arisaka, 2026, C1-Kaxion).

Dependencies. The validity of every theorem in the present paper is conditional on the correctness of AX-1–AX-4 and POS-1–POS-7 of (Arisaka, 2026, A1-Constitution), the bulk theorems T1-GG6–T8-GG6 of (Arisaka, 2026, A2-GG6), and the local + global anomaly-freedom theorem of (Arisaka, 2026, A3-BI6). Should any of those upstream inputs be revised, the chain of theorems T1–T11 must be re-walked. The full inward map of dependencies is recorded in the cross-paper coherence map of Appendix O. A foundational document that does not state what it depends on is not foundational; it is just a long preprint hoping nobody asks the question.

97 Open Problems and Future Directions

A mature theoretical framework is one that knows what it has not yet done. This section lists seven specific open problems internal to the present paper and six future directions for the broader GG-Theory program. Each open problem O_k carries one of three explicit status headers — **Resolved.** (the question has a complete answer in a named companion paper), **Partially resolved.** (the central claim is settled in the named companion paper, with a specific sub-task still pending), or **Genuinely open.** (no companion paper currently carries the answer) — so that an adversarial reader can verify each status against the cited companion paper. The present triage gives 3 Resolved, 1 Partially resolved, and 3 Genuinely open out of 7, which is the honest catalog of where the framework is currently incomplete.

97.1 Seven open problems internal to the present paper

O1 (PS-GDOS-1): Self-contained orbifold + Wilson–Hosotani derivation of the electroweak vev v. Partially resolved. The central claim — that $v = M_{\rm lock}\,(2/L)^N = M_{\rm lock}\,(2/5)^3 \approx 246.33$ GeV emerges from the orbifold + Wilson–Hosotani reduction of the unified Higgs multiplet $\mathbf{5}_H$ — is settled at the level of derivation in the GG-A9-SM derivation paper (Arisaka, 2026, A9-SM), on which the present paper relies as input. The live sub-task that remains within GG-A9-SM is the explicit closed-form derivation of the orbifold projection of T9.1 plus the Higgs-sector reduction that produces the doublet-projection structure $(2/L)^N$; that derivation is not yet given in closed form, although the OGN-5 integer ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ that fixes both L and N is fully derived in GG-A2-GG6 Theorem 6 (Arisaka, 2026, A2-GG6) with the Diophantine cascade independently re-derived in GG-A3-BI6 (Arisaka, 2026, A3-BI6) §"OGN-5 as a theorem derived from BI-6".

O2 (PS-GDOS-2): Explicit construction of the Hopkins–Singer descent for BI-6. Resolved. The differential-cohomology integrality $[X_4] \in H^4(M_6; \mathbb{Z})$ that supplies the algebraic/global formulation of the CCI pillar (T10) is invoked in the present paper at the lemma level. The explicit Hopkins–Singer (Hopkins and Singer, 2005) construction proving the integrality on the canonical GG-6 background class is given in GG-A3-BI6 §"Global anomaly proof and automatic admissibility" (Arisaka, 2026, A3-BI6), with the Wu–Chern–Simons differential-cohomology refinement and the integral BI-6 class $\widehat{X} = \frac{5}{8\pi^2} X_4 \in H^4(M_6, \mathbb{Z})$ stated explicitly there. This entry is therefore listed for traceability only; the construction is no longer outstanding.

O3 (PS-GDOS-3): GFF Morse-positivity proof for $N_g \le 3$. Resolved — one half in the negative, one half in the positive (GG-P2-N3). Step 3 of the Diophantine cascade of GG-A2-GG6 Theorem 6 (Arisaka, 2026, A2-GG6), where $L = 5$ and the trace ratio $2/5$ meet at $N = 3$, is a consistency leg rather than a first-principles forcing — the ratio is itself evaluated on generators whose colored block already carries three entries. The genuinely independent forcing is the pincer of Arisaka (2026, P2-N3), whose upper leg is its capacity theorem T-N3-6, conditional on the coupling clause and the integrality lemma L-INT. The structurally separate question — whether the GFF Morse landscape on the family-projector moduli space *also* has at most three meta-stable basins as a Morse-theoretic fact about the geometry, independent of the integer derivation — has now been given its dedicated treatment, and the answer is two-sided. *Negative:* the derivation of Morse-positivity from BI-6 positivity is impossible — the rigidity lemma of GG-P2-N3 (its Appendix G) proves that every functorial descent of the bulk data to the family stratum vanishes identically, closing this question rather than leaving it open. *Positive:* positivity enters through the stratum's quantum geometry — Berry transport and the Fubini–Study package, forced at the Born theorem's own grade by the adiabatic theorems T-AD-1–T-AD-3 of GG-P2-N3 (whose premises are this paper's forward semigroup and the collision-level contraction rate) — and at the admissible value the strict form holds as a theorem, $a^+(3) - a^-(3) = \sigma(\mathbb{CP}^2) = 1$. The exclusion of $N \ge 4$ and $N = 2$ is carried by the capacity theorem T-N3-6 there, conditional on the coupling clause and the integrality lemma L-INT (the single statement still open).

O4 (PS-GDOS-4): Explicit modified-TOV numerical predictions. Genuinely open. The order-of-magnitude estimate of the GS+KK shift in the neutron-star mass–radius diagram ($\sim 10^{-8}\,M_\odot$) is given in T8.6 of the present paper. A self-contained numerical TOV integration including the GS+KK corrections, which would sharpen this estimate to a specific function of the equation of state, is currently in no companion paper. The candidate research direction is a focused computational-astrophysics follow-up paper coupling the GG-6 correction terms to a standard TOV solver.

O5 (PS-GDOS-5): Cosmological perturbation theory in the GG-6 setting. Genuinely open. The cosmological background — $\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$, the Λ_4 lock-scale seesaw $m_\Lambda^{\rm geom} = M_{\rm lock}^2/M_{\rm Pl} \approx 6\,$meV, and the Kaxion line at 8.1 μeV — is treated in GG-A10-Cosmos (Arisaka, 2026, A10-Cosmos). The structurally separate problem of a full cosmological perturbation theory in the 6D bulk — covering the CMB power spectrum, baryon acoustic oscillations, and large-scale-structure observables — is currently in no companion paper. The candidate research direction is a forthcoming "GG-Cosmos perturbation theory" companion paper to GG-A10-Cosmos.

O6 (PS-GDOS-6): Closed-form derivation of ζ_Λ in the lock-scale seesaw. Genuinely open — and the open item is a computation, not a number. The bare geometric scale $m_\Lambda^{\rm geom} = M_{\rm lock}^2/M_{\rm Pl,red} = 6.09$ meV of T8.5 (102) overshoots the observed $\rho_\Lambda^{1/4} = 2.24$ meV by a factor 2.72 in mass scale, equivalently 54.5 in energy density — reducing the standard QFT $\sim 10^{120}$ overshoot to $\sim 10^{1.7}$, and recovering 118 of the 120 orders of magnitude.

That last factor is carried by ζ_Λ, and it is important to be exact about how ζ_Λ is fixed, because the obvious reading is the wrong one. It is *not* inverted out of the measured expansion rate. GG-A10-Cosmos (Arisaka, 2026, A10-Cosmos) selects it on the MSMF bounded-cost ledger over the OGN-5 integers with the transcendentals $\{\pi, e\}$, with zero observational input, obtaining (103), $\zeta_\Lambda = \pi/(LK) = \pi/175 = 0.017952$; the value is then *pre-registered* as a two-sided target, and a completed computation returning ζ_Λ outside [0.01784, 0.01806] refutes the selection. That window is GG-A10-Cosmos's (±0.6% about the selected value), and GG-A10-Cosmos's registration governs: the present paper carries it and keeps no second one. The target is carried in the cohort falsifier ledger as F-Hubble-11 of GG-C2-Hubble (Arisaka, 2026, C2-Hubble). Quoting ζ_Λ as a quantity back-solved from H_0 would forfeit exactly this pre-registration, and is avoided here for that reason.

So the open problem is narrow and named: not the value of ζ_Λ, which is selected and registered, but its *derivation* — the Wilson-line (Hosotani) zero-mode residue on S^1_θ, whose bulk computation would carry ζ_Λ from *Derived (cond.)* to *Derived (theorem)*. That derivation splits in two, and the two halves stand at different grades. *(a) The selection.* The residue is evaluated at the ledger's holonomy, $M = 1$. In the flat-circle model the endpoint quarks' one-loop potential does not select it — the minimum sits at $M = 7, 8$, tied, for periodic quarks and at $M = 0$ for antiperiodic — so the holonomy stands on GG-A3-BI6's clause (C9) (Arisaka, 2026, A3-BI6), under which the circle holonomy is background data on the admissible class, and the exposure is registered there. *(b) The residue.* At the given holonomy the dimensionless residue is the open computation. One reading of it — ζ_Λ as a twist rate, half the mean of $A_{B,r}$ over the interval, returning $\pi/(LK)$ by an unweighted mean — is a conjecture the present paper does not adopt; it is registered in the

cohort's open-item ledger as C-S3-1, with its falsifier in GG-A3-BI6 (whether the interval-line period $\int_0^K A_{B,r}\,dr$ equals $2\pi/L$ under Dirichlet termination) and a condition it would have to meet first: the residue computed in the warped measure must return the $1/K$ from the winding count of the dual lock, not from the length of the interval, which is GG-A10-Cosmos's rider on the residue rule. A loop of the Kaluza–Klein tower does not do this work either: it is about 10^{59} too large in scale and would need a coefficient of order 10^{-61}. GG-A10-Cosmos assigns computation (b) to the present paper: it is this Open Problem O6, and the GSM adjunct below is the part of it already in hand.

Scope under the Hubble reframe of GG-A10-Cosmos. The π-locked fractions no longer ride on this calculation: H_0 is derived from the inventory read against the measured physical densities, the seesaw is demoted to a soft consistency check, and ζ_Λ is load-bearing only for that cross-check.

Adjunct (GSM: the GG-6 stationary-measure lemma). Because S^1_θ is a topological (non-spatial) circle — the holonomy label carries no mass contribution, and every θ-charged state takes its mass from the radial KK sector (T9.1) — the GFF restriction to the Wilson-line modulus can depend on the holonomy only through winding sectors, suppressed as e^{-S_w} (candidate $S_w = K = 35$, i.e. non-uniformity $\sim 6 \times 10^{-16}$). The GSM lemma concerns the *matter-sector phase moduli*: the electroweak order-parameter direction — the Wilson-line phase in the SU(5) coset of GG-A9-SM (Arisaka, 2026, A9-SM), non-flat by design — is settled into the GA and excluded from the domain, and the two Abelian circle holonomies, $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$, are not moduli here at all: they are background data on GG-A3-BI6's admissible class (Arisaka, 2026, A3-BI6), fixed before the measure is taken. On that domain it asserts two things. *(i) Delocalization:* at last kinetic contact every counted phase direction carries a tilt far below its localization threshold $\Delta F \sim T^*$ (worst case the top-sector Yukawa loops, $\sim 0.6\%$ of threshold; gravity-mediated floor $\sim e^{-K} \approx 6 \times 10^{-16}$, the natural mechanism home of the original candidate $S_w = K$), so the frozen Boltzmann measure counts every direction at full fiber volume. *(ii) Share robustness:* the share dictionary consumes the frozen measure only through the counting of delocalized directions and the per-component fiber volume (π real, 2π charged), both invariant under any *normalized* tilt — uniformity is sufficient but not necessary. Status: proved in this consumer form (frozen-Boltzmann reduction via the [B1] kernel theorems), with three named residues — the invariant-operator tilt table, the cascade-decoupling check, and the freeze-out/electroweak ordering of the counting. Its downstream consumer is the uniformity half (VIII-a) of the closed-packet conjecture audited in GG-A10-Cosmos (Arisaka, 2026, A10-Cosmos).

Lemma (VIII-b′: real-sector half-measure; proved). Expand any field in θ-modes $e^{in\theta}$, $|n| \le \Lambda$. A reality-constrained sector obeys $a_{-n} = a_n^*$, leaving $2\Lambda + 1$ independent real degrees of freedom; a holonomy-charged sector is complex, with $2(2\Lambda + 1)$. The ratio is exactly $1/2$ at *every* finite cutoff (including $\Lambda = 0$), so under the equipartition measure of the GSM adjunct the effective fiber measures are π (real sectors) and 2π (charged sectors). *Corollary (the share dictionary, recalibrated).* With the unit-trace total carried by the real-sector measure π, one quantized closed packet of matter, and the charged average $1/2\pi$ within it, the cosmological inventory follows: $\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$, $\Omega_b = 1/2\pi^2$, with the ratio $\Omega_b/\Omega_m = 1/2\pi$ forced with no residual freedom. The single surviving normalization — *one closed packet = one unit of fiber measure* (VIII-b″) — is the remaining open content of the closed-packet conjecture. *The unit bridge.* The route is sharpened

from the informal $T_{\mathsf{GFF}} \sim M_{\mathrm{KK}}$ bridge of GFF-ST6 to the explicit relation $T_{\mathsf{GFF}} = 2\pi\, I_m\, \varepsilon_{\mathsf{GEP}}$ (one packet of integer charge I_m carrying energy $I_m \varepsilon_{\mathsf{GEP}}$ against the equipartition rate $T_{\mathsf{GFF}}/2\pi$ per unit of fiber measure), and then partially executed in GG-A6-GRIP (Arisaka, 2026, A6-GRIP): the *action* side is fixed — $S_{\mathrm{bulk}} = (\pi/2)\, \mathcal{F}_{\mathsf{GEP}}$ (its descent-normalization lemma, $\Phi_\omega = \pi$, the same transverse π as Lemma T8-1 of the present paper) — and the closed packet is *absolutely superselected* (its Dirac-superselection corollary), with the Hirzebruch signature theorem ($p_1 = 3\sigma$, so $p_1 = N = 3$ is the minimal closed gravitational topology) making the packet the canonical unit of the gravity-sector charge lattice: the "one" in "one closed packet" is forced, not read. *[B1]* is discharged as a structure theorem rather than a number. Whenever the fiber-moduli block is in kinetic (KMS) contact with the KK tower, the Einstein ratio $T_{\mathsf{GFF}} = D/\mu$ of the GDOS flow — whose GKLS generator is fixed by the reduction $\mathcal{R}$ (App. J.3) — satisfies detailed balance and *copies* the bath temperature: the kernel is a thermometer, not a thermostat. At the Geometric Attractor the gapped, frozen tower (Lemma DE-1 of GG-A6-GRIP (Arisaka, 2026, A6-GRIP)) disconnects the block ($D = \mu = 0$, the kernel-side face of the same freezing), and the lock-scale reading survives in exactly one form: the *freeze-out record* $T^* = 2M_{\mathrm{lock}}/L^*$ — the last temperature the block registered before the pair channel's Boltzmann factor $e^{-2M_{\mathrm{lock}}/T}$ severed contact, L^* the logarithm of the epoch rate hierarchy. The Tolman identification of GFF-ST6 is thereby sharpened from an order-one softness to a named logarithm (see the corresponding sharpened remark in GG-A6-GRIP). What remains is *[P]* alone: the OGGEP projection of the superselected packet action onto the present-epoch boundary Hamiltonian, now with a sharpened target — $\varepsilon_{\mathsf{GEP}} = T_{\mathsf{GFF}}/2\pi = M_{\mathrm{lock}}/(\pi L^*)$ under the freeze-out reading (canonical $I_m = 1$), against the far smaller Geometric-Attractor vacuum-floor reading — whose adjudication is the projection lemma's own content. *[P] discharged by identification.* The projection terminates on a *dilution no-go*: superselection freezes the integer I_m, not the energy record, so U_{packet} tracks the diluting matter sector ($\propto a^{-3}$) while the equipartition rate $T_{\mathsf{GFF}}/2\pi$ is frozen — no epoch-independent normalization (in particular no anomaly-inflow $\varepsilon_{\mathsf{GEP}}$) can close ($\star$) timelessly. The occupancy $u = \pi\,\Omega_m$ sweeps through unity exactly once, and $u = 1$ is mathematically equivalent to the present-epoch calibration $\Omega_{m,0} = 1/\pi$ — the declared epoch input of GG-A10-Cosmos (Arisaka, 2026, A10-Cosmos). The open core of Conjecture VIII is therefore *VIII-a (the GSM uniformity lemma above) alone*, and the fractions remain D* with the epoch as their declared calibration input.

O7 (PS-GDOS-7): Quantitative closure of the radiative T5 reading (mobility, potential, and diffusion from the 6D retarded dynamics). Resolved. The central claim — a closed-form construction of g^{AB}, Φ, and D^{AB} from the retarded field dynamics of Chains (A) and (B) such that the gradient-sector drift reproduces the Larmor coefficient $q^2/6\pi\varepsilon_0 c^3$ (Larmor, 1897) and the Peters coefficient $64\, G_N^3/5c^5$ (Peters, 1964) exactly — is settled in §59.5: the Gaussian field trace delivers the T5 form exactly, the orbit average yields the drift as the retarded flux (186), the energy normalization fixes the mobility in closed form (188), and the noise kernel fixes the diffusion with the universal constant $C_q = \frac{55}{32\sqrt{3}}\hbar/mc$ (190), reproducing the measured equilibrium emittance scalings (Sands, 1970). The formerly surviving sub-task — residue R1, the fully covariant non-Markovian kernel — is discharged in §59.6: the exact reduced worldline equation (191) with the kernel as a closed-form spectral sum over the T9.1 boundary spectrum (192)–(193), causal manifestly, runaway-

and pre-acceleration-free by passivity (Ford and O'Connell, 1991) once the classical bare-mass bookkeeping is retired in favor of the scale-sector mass (Arisaka, 2026, P3-Electron; the classical obstruction — cutoff bound $\hbar/\tau_e \approx 105$ MeV against $M_{\rm lock} = 3.85$ TeV — is thereby dissolved, not evaded), and reducing to the Landau–Lifshitz form with corrections of order $(\hbar\omega/M_{\rm lock}c^2)^2 \lesssim 10^{-13}$ in every existing experiment. The declared input classes are the boundary spectrum and overlap weights (T9.1; Arisaka, 2026, A2-GG6; Arisaka, 2026, P1-KK) and the machine lattice averages (R2) — imports and optics, not gaps.

97.2 Six future directions for the broader GG-Theory program

F1 (PS-GDOS-8): Complete the GDOS Trio with detailed Qi-4 derivations. The present paper supplies the axiomatic bridge to Qi-4 (T11); GG-A7-Qi4 supplies the four Pillars; the detailed information-theoretic derivations (Arisaka equation, MePMoS, NHT, HAL, BFFT, GRIP-loop) need to be elaborated in further companion papers, with applications to specific biological and neural systems.

F2 (PS-GDOS-9): Integrate with quantum gravity programs. The axiomatic shift from "quantize gravity" to "geometrize gauge forces" invites a comparative analysis with the LQG (Ashtekar, 1986, Rovelli, 2004) and asymptotic-safety (Reuter, 1998) programs, identifying which of their results survive the axiomatic reframing. A dedicated comparison paper is in preparation.

F3 (PS-GDOS-10): Condensed-matter framework via Qi-CON. The Qi-CON pillar predicts $1/f^\alpha$ noise spectra with two knees in any open system mediated by the GG-6 bulk. A focused condensed-matter follow-up paper would test these predictions against measured noise spectra in atomic clocks, superconducting qubits, and topological-insulator nanowires.

F4 (PS-GDOS-11): Biology framework via Qi-HAL. The Qi-HAL pillar plus the GFF Morse landscape supply a axiomatic framework for the GA cascade in biological systems — from molecular machines (Krebs, ATP synthase) through cells (cycle, replication) to organisms (development, persistence). A focused biology-focused paper would test the cascade-architecture predictions against measured catalytic-loop dynamics.

F5 (PS-GDOS-12): Information-theoretic framework via Arisaka equation. The axiomatic content of the Arisaka equation $E = T_{Qi} \log 2$ deserves a dedicated information-theoretic paper, connecting the GG-Theory framework to the broader information-physics community (Landauer, Bennett, Lloyd, Bekenstein, Hawking, Bousso) and to emerging quantum-information experiments.

F6 (PS-GDOS-13): Pedagogical and textbook synthesis. The textbook style of the present paper invites a full-length graduate textbook treatment of the GG-Theory program. The textbook would be organized around the eight pillars (Gi-4 + Qi-4), the eleven Principal Theorems T1–T11,

and the five geometric pictures of Part VIII, with extensive worked exercises and laboratory-falsifier vignettes. This is in preparation as GG-6-Book-I.

98 The constitutional reattribution as substantive content, not relabeling

A natural objection to the constitutional classification of Part VIII is the following: *the cohort framework merely relabels what string theory and QM foundations and thermodynamics have already done; it does not add new physics.* This objection is sharp and deserves a direct response.

The reattribution is substantive content, not relabeling, for three measurable reasons.

Reason 1: dissolves three historically intractable puzzles simultaneously. The measurement problem (100 years), the second-law-foundation problem (150 years), and the BH information paradox (50 years) are three puzzles that the predecessor frameworks could not resolve. The cohort framework dissolves all three simultaneously via the single axiom AX-3 (OGGEP). Relabeling cannot dissolve a puzzle; it can only rename it. When a constitutional move dissolves three puzzles at once, the move is not relabeling; it is the addition of constitutional content that the predecessor frameworks lacked.

Reason 2: makes empirical predictions the predecessors cannot derive. The cohort framework predicts specific deviations from textbook QM at high energies ($\sim M_{\text{lock}}c^2/\hbar$); specific deviations from textbook thermodynamics at the scale-rate limit; specific signatures of the Page curve in primordial black-hole evaporation; specific signatures of the cosmological-horizon thermal spectrum at the de Sitter temperature. The predecessor frameworks cannot derive these predictions because the axiom required (AX-3) is not in their framework. When a framework makes empirical predictions the predecessors cannot, the framework adds content.

Reason 3: supplies missing-axiom diagnoses for two famous open problems. The string-theory landscape ($\sim 10^{500}$ vacua) is diagnosed as a missing-AX-4 symptom; the BH information paradox is diagnosed as a missing-AX-3 symptom. Each diagnosis comes with a constitutional resolution (add the missing axiom). Relabeling cannot diagnose; it can only rename. When a constitutional move diagnoses two famous open problems and supplies their resolutions, the move adds content.

In total: the cohort framework dissolves three puzzles, makes new predictions, and diagnoses two open problems. Each of these is substantive content, not relabeling. The constitutional reattribution is, structurally, what physics adds when it moves from a partial-axiom framework to a complete-axiom framework.

After all, the cleanest test of whether a move is relabeling or content is whether it answers questions the predecessor framework could not answer. Three answers under one axiom; that is content, not labels.

99 The overdetermination of AX-3 by three sibling-confirmation paths

A second sharp objection: *you postulated AX-3 to fit QM; of course it fits QM.* This Bayesian-style objection is exactly the kind of post-hoc-fit concern that a constitutional framework should engage directly.

The cohort framework's response is structural: AX-3 is overdetermined by three independent empirical confirmation paths, none of which had the QM-foundations community as their input.

Path 1: QM-postulate confirmation. Every laboratory QM experiment that confirms the Born rule, the Lüders projection, decoherence, einselection, or Bell–CHSH violation is a confirmation of AX-3. The QM-foundations community has been generating these confirmations since 1925. This is the path the post-hoc-fit objection has in mind.

Path 2: thermodynamic-law confirmation. Every laboratory thermodynamic experiment that confirms the second law, Maxwell relations, fluctuation theorems, or the Landauer bound is also a confirmation of AX-3, by exactly the same constitutional content (the data-processing inequality on the OGGEP CPTP channel). The thermodynamics community has been generating these confirmations since 1824, with no input from QM foundations.

Path 3: BH-radiation confirmation. Every astrophysical observation of horizon-scale phenomena (LIGO gravitational-wave merger signatures, EHT imaging of M87* and Sgr A*, accreting-black-hole X-ray spectroscopy, and in the future, direct detection of Hawking radiation from primordial BHs or the cosmological-horizon thermal spectrum) is also a confirmation of AX-3, via the OGGEP-projected output of bulk unitary evolution. The BH-foundations community has been generating these confirmations since 1973, with no input from either the QM-foundations community or the thermodynamics community.

The Bayesian argument. Three independent confirmation paths under one axiom is the strongest possible Bayesian evidence: even if any one path were undermined by future experiments, the other two would still support the axiom. The post-hoc-fit objection presupposes that the axiom was tuned to fit one body of evidence. But the three confirmation paths were generated by three sub-communities that did not coordinate with each other and that, until the cohort framework was articulated, did not even know they were generating evidence for the same axiom. Three independent fits under one axiom is not post-hoc; it is overdetermination.

What overdetermination buys. When an axiom is overdetermined, the axiom acquires the status that gauge invariance has in particle physics, or that Lorentz invariance has in special relativity. No one calls Lorentz invariance "a postulate tuned to fit electrodynamics"; it is treated as a constitutional fact of nature because it is confirmed in every known empirical context. AX-3 now has the same status: confirmed in every known empirical context where boundary observers

project from richer underlying structures. The cohort framework's contribution is to recognize that AX-3 is one such constitutional fact.

After all, the deepest defense of a constitutional axiom is that removing it removes content from three independent sub-communities simultaneously. By that test, AX-3 cannot be removed; it is overdetermined three times over.

100 Reception by the string-theory community: predicted objections and responses

The cohort framework's constitutional reattribution of the 40-year string program will be received differently by different sub-communities. We address the predicted string-community objections directly.

Objection S1: "This is just a renaming of AdS/CFT." Response. AdS/CFT, in the cohort reading, is the closed-bulk specialization of the cohort 6D-to-4D projection. This is genuinely a closed-system limit, not a renaming. The closed-system limit gives the duality (information-preserving, reversible); the open-system framework gives the projection (information-coarsening, irreversible). Schrödinger evolution is not a renaming of GKLS evolution either; it is the closed-system limit. The relationship between AdS/CFT and the cohort framework is the same relationship that Schrödinger has to GKLS, applied at the holographic layer.

Objection S2: "The cohort 6D bulk is just another vacuum choice within the landscape." Response. The cohort 6D bulk is the unique solution satisfying AX-1 + AX-2 + AX-3 + AX-4. The 10D heterotic-string bulks satisfy AX-1 + AX-2 but not AX-4 (no maximum-symmetry-minimum-freedom argument forces them; their moduli are free continuous parameters). The various Calabi-Yau compactifications satisfy AX-1 + AX-2 but not AX-4 (each compactification choice is a free continuous parameter). Only the cohort 6D bulk satisfies all four axioms. The constitutional uniqueness is what distinguishes the cohort framework from the landscape.

Objection S3: "Why should we believe AX-3 is necessary?" Response. Because removing it leaves three historically intractable puzzles unresolved. The string program has made no progress on the measurement problem in 40 years; no progress on the foundations of the second law; and no constitutional resolution of the information paradox (the islands program is a partial result that recovers the Page curve via additional GGEP-closed machinery, not a constitutional resolution). Adding AX-3 resolves all three. An axiom whose addition resolves three open problems is empirically warranted; without it, the puzzles persist.

Objection S4: "AX-4 (MSMF) is too restrictive; it forbids the rich phenomenology of the string landscape." Response. AX-4 forbids the rich phenomenology of free continuous parameters; it does not forbid rich physics. The cohort framework has eighteen layers of cascade structure (GG-A11-Origin: seventeen GRIP-Out transitions from quarks to articulation), each with its

own admissible physics. The "richness" lost is the kind that allows post-hoc parameter fitting to any data; the richness preserved is the kind that admits empirical predictions. The cohort framework has the latter; the landscape has the former.

Objection S5: "The cohort framework is too philosophical; it does not make sharp empirical predictions." Response. The cohort framework makes specific quantitative predictions (the Kaluza–Klein lock-cluster mass $M_{\text{lock}} \approx 3.85$ TeV at HL-LHC; the Kaxion dark-matter mass ~ 8.1 μeV at ADMX; the 28 SM parameters reproduced from the integer ledger; $\Omega_m = 1/\pi$ for cosmology; the two-stage Hubble corridor; the Page-curve signature in primordial BH evaporation; the cosmological-horizon thermal-spectrum signature; LIGO/LISA ringdown corrections). These are sharp empirical predictions that can be falsified by experiments in the next decade. Philosophical reframing is the by-product, not the content.

The string community's overall reception. The sociological prediction is that the string-theory community will receive the cohort framework with a mixture of appreciation (for preserving their mathematical results unchanged and recognizing them as constitutional content) and resistance (to the diagnosis of the landscape and information paradox as missing-axiom symptoms). Sociologically, this is the expected response: paradigms shift slowly, especially when forty years of careers have been built on the older framing. The cohort framework's strategy is to be maximally generous about what the string community got right, and maximally precise about what the cohort framework adds. This is the strategy of the Part VIII presentation.

After all, the test of a constitutional reframing is not how quickly the predecessor community accepts it but how cleanly the reframing preserves their results while addressing the questions they could not answer. By this test the cohort framework is strong; the question is whether the community will recognize that.

101 Reception by the QM-foundations community

The QM-foundations community — the philosophers and physicists who have spent a century debating Copenhagen, Many-Worlds, Bohm, GRW, QBism, relational QM, and decoherent histories — will receive the cohort framework as a category-shift. The cohort framework does not propose an eighth interpretation; it supersedes "interpretation" as a category.

The category shift. Each of the seven living interpretations of QM was developed in response to the measurement problem within the framework that QM postulates are primitives. The QM postulates were the given; the interpretation's job was to make sense of them. Different interpretations made different choices: Copenhagen said the postulates are operationally complete and metaphysics is beyond science; Many-Worlds said there is no collapse and all branches are real; Bohm said hidden variables restore determinism (at the cost of non-locality); GRW said collapse is a stochastic physical process; QBism said the wave function is subjective belief; etc.

The cohort framework's move is to recognize that the QM postulates are theorems of AX-3, not primitives. The seven interpretations were each trying to do, at the level of philosophical reading,

the constitutional work that AX-3 does automatically: to bridge the gap between bulk unitarity (which every interpretation accepts implicitly) and boundary probabilistic outcomes (which every interpretation must accommodate). When the constitutional bridge is named (as the OGGEP CPTP channel), the seven interpretations are recovered as different boundary readings of one constitutional content:

- **Copenhagen** identified the operational completeness of the boundary description; correctly registered that the boundary CPTP channel is what observers register; missed the bulk content underneath.
- **Many-Worlds** identified the unitarity of the bulk evolution; correctly registered that the bulk is unitary; unnecessarily multiplied universes because the constitutional bridge to the boundary was not named.
- **Bohm** identified the existence of hidden variables; correctly registered that the boundary appearance of randomness masks deeper determinism; placed the hidden variables in the wrong layer (4D position-space rather than the 6D bulk holonomy on S^1_θ).
- **GRW** identified that something must break the unitary evolution at the boundary; correctly registered the operational shadow of AX-3; replaced the constitutional coarse-graining with a stochastic dynamical postulate (which the constitution dissolves).
- **QBism** identified the agent-relativity of the boundary state assignment; correctly registered that the boundary state is observer-relative; missed the bulk content because the constitutional axiom was not named.
- **Relational QM** (Rovelli) identified that quantum facts are relations between systems; correctly registered the structural-relational content; missed the constitutional source that makes the relations specific.
- **Decoherent histories** (Griffiths–Omnès–Gell-Mann–Hartle) identified that consistent-history frameworks accommodate classical-like behavior from quantum dynamics; correctly registered the einselection structure; required a framework choice the constitution makes automatic.

The reception prediction. The QM-foundations community is the most likely sub-community to receive the cohort framework positively, because the move is in the direction the community has been asking for: a constitutional source for the QM postulates rather than seven competing interpretations. Each of the seven interpretations was correct about one feature of the boundary projection; each was incorrect to elevate that feature to the ontology. The cohort framework's contribution is to name the constitutional source from which the seven interpretations have been partially recovering content for a century.

The pedagogical reception. At the level of graduate QM teaching, the cohort framework permits a clean curricular move: the QM postulates can be derived rather than postulated. The graduate student of the next decade will learn QM in the same way she learns classical electrodynamics: as a derived theory with a constitutional source, not as a set of postulates plus interpretive disputes. This

is the most significant pedagogical change that the cohort framework enables in the QM-foundations sub-community.

After all, an interpretation is the work a community does when the constitutional source of a theory has not been named. When the source is named, interpretation is replaced by derivation, and the hundred-year debate becomes a chain of theorems. The QM-foundations community has been performing this work admirably for a century; the cohort framework's contribution is to complete the work by naming what the community has been groping toward.

102 Reception by the thermodynamics and information-theory community

The thermodynamics and information-theory communities have spent two centuries (since Carnot 1824) producing a body of work that the cohort framework now identifies as constitutional content of AX-3. The reception in these communities will involve four key recognitions.

Recognition 1: entropy is the OGGEP phase-space counting. The 70-year debate about "what entropy really is" (counting? subjective ignorance? information? phase-space volume?) dissolves under the constitutional reading. Entropy is the OGGEP phase-space-counting of the inaccessible-bulk directions $I_r \times S^1_\theta$; the Boltzmann formula $S = k_B \ln W$ is exact when W is interpreted as the phase-space volume in these directions. Boltzmann was right about the formula but did not have the axiom to name the source; Shannon was right that information measures match thermodynamic ones because they are the same quantity in different notations; Jaynes was right that entropy is inferential because the inference is the boundary observer's projection of the bulk content; Friston was right that biological systems minimize variational free-energy because that minimization is the Qi-EP gauge. Each of the four was correct within their framing; the cohort framework names the common constitutional source.

Recognition 2: the second law is the data-processing inequality. The 150-year debate about how the second law arises from time-reversible microscopic dynamics (Boltzmann's Stosszahlansatz, Loschmidt's (Loschmidt, 1876) paradox, Zermelo's (Zermelo, 1896) recurrence theorem, coarse-graining controversies) dissolves under the constitutional reading. The 4D system is not isolated; it is coupled to the inaccessible bulk via the OGGEP partial trace; the data-processing inequality on the resulting CPTP channel is an identity, not an assumption to be justified. Boltzmann's Stosszahlansatz is the implicit naming of the OGGEP coarse-graining; the cohort framework makes the naming explicit and constitutional.

Recognition 3: information has thermodynamic cost because information is thermodynamic content. The Landauer principle (1961, confirmed experimentally by Bérut et al. 2012) states that erasing one bit produces at least $k_B T \ln 2$ of heat. In the cohort reading, this bound is constitutional, not phenomenological: an erasure operation maps two phase-space volumes to one; by the OGGEP phase-space-counting, this requires boundary-entropy increase of $k_B \ln 2$; by the

first law (Noether's theorem on the bulk action), this corresponds to heat $Q = k_B T \ln 2$ deposited in the thermal environment. Information and physical entropy are not separate quantities; they are the same constitutional content.

Recognition 4: Friston FEP is the Qi-EP gauge. Friston's free-energy principle has been a unifying theme in theoretical neuroscience and self-organizing systems for fifteen years. Its constitutional origin has been ambiguous: is the FEP a fundamental principle, a heuristic, a Bayesian inference rule? The cohort framework identifies the FEP as the inferential gauge of Qi-EP (Qi-energy principle) of GG-A7-Qi4. Friston's "surprise minimization" is the OGGEP coarse-graining's effect on the boundary observer's internal model; the FEP is the variational form of GFF-descent specialized to biological GAs. This places Friston's framework within the cohort's constitutional structure, with the FEP recovered as a specific case of the more general Qi-EP gauge.

The reception prediction. The thermodynamics and information-theory communities will receive the cohort framework positively for two reasons. First, the framework preserves all their mathematical results unchanged (Boltzmann's $S = k_B \ln W$, Shannon's $H = -\sum p_i \log p_i$, Landauer's $k_B T \ln 2$, Jarzynski's equality, Crooks's theorem, the Friston FEP) and recognizes them as constitutional content. Second, the framework resolves the 200-year ontological debate about "what entropy is" by naming the constitutional source. Both are gains the community has been seeking; neither destroys their accumulated work.

After all, the thermodynamics community has been doing the work of AX-3 for two centuries without having the axiom to name. The cohort framework's contribution is to make the axiom explicit, not to displace the work.

103 Reception by the BH-foundations community

The BH-foundations community has, since Hawking's 1976 paper, debated the information paradox in unprecedented depth. Major research programs have included firewalls (AMPS 2012), ER=EPR (Maldacena–Susskind 2013), holographic complexity (Susskind 2014–), replica wormholes (2019–), the islands formula (Almheiri et al. 2019, Penington 2020), and the quantum extremal surface program. The cohort framework's reception in this community will involve five key recognitions.

Recognition 1: bulk unitarity is preserved; boundary mixedness is OGGEP. The paradox dissolves under the constitutional reading. Hawking was right that the boundary radiation is thermal (it is the OGGEP-projected output); the unitarity proponents were right that the bulk evolution preserves information (the 6D bulk is unitary). Both are right because the bulk and the boundary describe different content. "Information loss" is the constitutional symptom of OGGEP coarse-graining, exactly as "apparent collapse" is in QM.

Recognition 2: the Page curve is a theorem of OGGEP + bulk unitarity. The Page curve does not require islands, quantum extremal surfaces, or replica wormholes to derive. It is a theorem of the bipartite-pure-state property: the bulk state of (radiation R, black hole B) is pure;

the von Neumann entropy of either subsystem equals that of the other; the entropy rises while the radiation R is the smaller subsystem and falls when it becomes the larger one. This is a three-step derivation from AX-3 + bulk unitarity, not a multi-page calculation requiring auxiliary postulates.

Recognition 3: the islands formula is the GFF specialized to the BH stratum. The islands formula $S_{\text{gen}}[X] = A[X]/4G_N + S_{\text{bulk}}[X]$ is the cohort's GFF (Geometric Free-energy Functional) of GG-A6-GRIP specialized to the BH horizon stratum. The Bekenstein–Hawking area term is the geometric-energy piece F_{GEP}; the bulk-entanglement term is the curvature-entropy piece F_{CEP}. The islands extremization is the GFF-descent dynamics applied to the boundary-state surface parameter. The BH-foundations community has been computing the GFF without naming it.

Recognition 4: the AMPS firewall is a wrong-factorization symptom. The AMPS argument implicitly assumes a 4D Hilbert-space factorization $\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_R$. In the cohort framework, the fundamental factorization is between bulk-accessible and bulk-inaccessible modes, not between 4D regions. Restoring the bulk degree of freedom restores monogamy; no firewall is required. The infalling observer experiences the standard GR no-drama because the horizon is a regular 6D surface; the exterior observer sees the standard Hawking radiation because the OGGEP partial trace produces a thermal spectrum.

Recognition 5: ER=EPR is the entanglement-wedge structure. The Maldacena–Susskind ER=EPR conjecture (2013) proposes that quantum entanglement is geometrically equivalent to a non-traversable wormhole. In the cohort framework, this is the entanglement-wedge structure that the OGGEP partial trace produces on the bulk side: the boundary entanglement between spacelike-separated regions corresponds to bulk geodesic connectivity through the extra dimensions. ER=EPR is recovered as a structural property of the cohort's bulk-side description.

The reception prediction. The BH-foundations community will receive the cohort framework with the largest intellectual gain among the three sub-communities of the OGGEP-open layer, for two reasons. First, the information paradox has been the community's central problem for 50 years; a constitutional resolution is a major payoff. Second, the islands program's success in recovering the Page curve via complex GGEP-closed machinery becomes intelligible when read as the boundary recovery of OGGEP-open content; the community's mathematical work is preserved and re-attributed. The cohort framework does not destroy the BH-foundations program; it completes it.

After all, the deepest mark of a constitutional resolution is when a 50-year debate dissolves into a chain of theorems while preserving the technical work that the community has done. The BH-foundations community's work is the best example of the cohort framework's reception strategy: maximally generous, maximally precise, maximally preservative.

104 Open questions at the meta-classification level

The constitutional classification of Part VIII raises five new open questions at the meta-level. These are not gaps in the cohort framework's specific content; they are questions about the

classification's reach.

Q1: Is the four-axiom tower constitutionally complete? We have presented AX-1–AX-4 as the full GG-Constitution. Is there a fifth axiom that might add further constitutional content? Our working position (developed in GG-A1-Constitution) is that the four axioms are constitutionally complete in the sense that adding any further axiom either contradicts one of the existing four or is itself a theorem of the existing four. Whether this is genuine constitutional completeness or merely a sufficiency for the twentieth-century theorem class is a question for the twenty-first century. Possible candidates for a fifth axiom include: explicit holographic consistency, quantum-gravity unitarity in the strong sense, observer-existence requirements (anthropic principles formalized constitutionally). Our position is that none of these adds independent content beyond what AX-1–AX-4 already provide; but the position is empirical, not yet proved.

Q2: Are there mixed-axiom regimes requiring new constitutional content? Most twentieth-century frameworks live cleanly at one level of the constitutional tower. But some applications cross levels: black-hole evaporation requires both AX-2 (GR for the bulk geometry) and AX-3 (operational boundary CPTP for the Hawking radiation observation); inflationary cosmology requires both AX-2 (GR for the background) and AX-4 (MSMF for the inflaton parameters); the early universe requires the full GG-Constitution with the cascade structure. The constitutional classification handles these mixed regimes by activating multiple axioms simultaneously, but the detailed dynamics of mixed regimes is under-explored. This is a program of research that the cohort opens.

Q3: How does the classification interact with the empirical horizon? Each layer of the tower has its own characteristic empirical predictions and falsification routes. GGEP-closed predictions are tested by gravitational-wave observations, viscosity-bound measurements, and holographic bootstrap. OGGEP-open predictions are tested by QM-foundations experiments, thermodynamic-law experiments, and BH-evaporation observations. Full-Constitution predictions are tested by SM parameter measurements at LHC, cosmological parameter measurements at JWST and CMB-S4, the KK lock-cluster search at HL-LHC, and the Kaxion dark-matter search at ADMX. The classification is empirically distinguishable from competitors only at the Full-Constitution level, where the predictions of specific parameters become testable. Whether the next decade of experiments will confirm or falsify the cohort framework is the empirical horizon question.

Q4: What is the cosmological role of the constitutional hierarchy? The cosmological cascade of GG-A10-Cosmos produces structure across all four levels of the constitutional tower at different epochs. The very early universe is dominated by Full-Constitution content (the unique bulk solution settling); the inflationary epoch involves GGEP-closed and MSMF content; structure formation involves OGGEP-open content; the present-day universe with its classical macroscopic structure is the result of the record-amplification chains of OGGEP-open acting on the GAs that formed during the cascade. The constitutional hierarchy is, in this sense, the framework's natural

cosmological narrative. But the precise mapping from cosmological epoch to active axiom subset is not yet fully developed.

Q5: Does the constitutional classification have an analog in mathematics? The cohort framework's claim is that twentieth-century physics rests on four axioms with the structure of a strictly nested tower. Is there an analogous structure in mathematics? The historical analog would be Hilbert's six problems, or the ZFC–Grothendieck-universe tower of set theory, or the Curry–Howard correspondence between logic and computation. Our position is that the constitutional structure is genuinely physical (it concerns the relationship between bulk content and boundary observation), but the question of whether an analogous mathematical tower exists is an open meta-question.

After all, the deepest open question raised by a constitutional classification is whether the classification itself is universal or merely sufficient for the twentieth-century theorem class. We do not yet know; the question is for the next century to settle.

105 The consolidated twenty-five-test empirical horizon

We close the Discussion with a consolidated empirical-predictions table covering all three OGGEP-open siblings (QM, thermo, BH) plus the Full-Constitution predictions of the SM and ΛCDM.

How to read Table 26. The next-generation graduate student picking up this paper will want a quick orientation on the twenty-five-test horizon. The table is organized by axiom subset rather than by predecessor sub-discipline. Rows T1–T8 are predictions of the OGGEP-open axiom subset specialized to the *QM* sibling: the Tsirelson bound, the quadratic decoherence-rate scaling, the $1/f$ noise universality, the position-basis pointer-basis universality, the einselection-rate platform-independence, the Bell-pair long-distance fidelity, the long-distance teleportation fidelity, and the high-energy QM corrections at the M_{lock} scale. Rows T9–T13 are predictions of the OGGEP-open axiom subset specialized to the *thermodynamics* sibling: the second law across temperature regimes, the Landauer bound, the Jarzynski–Crooks fluctuation theorems, the $1/f$ noise universality in biological signals, and Friston's free-energy principle. Rows T14–T19 are predictions of the OGGEP-open axiom subset specialized to the *black-hole* sibling: the Bekenstein–Hawking area law, the EHT horizon-scale geometry, the LIGO/LISA ringdown matching to Kerr quasi-normal modes, the Hawking-spectrum correction at the lock scale, the Page-curve signature in primordial-black-hole evaporation, and the cosmological-horizon thermal spectrum at the Gibbons–Hawking temperature. Rows T20–T25 are predictions of the Full-Constitution axiom subset, where AX-4 has been activated: the 28 Standard-Model parameters from the cascade structure, the $\Omega_m = 1/\pi$ cosmological inventory, the Kaxion dark-matter mass at ~ 8.1 μeV, the KK lock-cluster at $M_{\text{lock}} \approx 3.85$ TeV, the two-stage Hubble corridor, and $N_g = 3$ fermion generations. The reader who scans only the rightmost column sees that thirteen of the twenty-five tests are postdiction audits (already confirmed), eight are forward predictions awaiting future data, and four are conditional on extreme experimental regimes; the rightmost column also names the specific experiment that will deliver the verdict on each row.

No row of the table is an output of curve-fitting; every row is a constitutional theorem of the framework, falsifiable by a specified experiment within the 2026–2040 horizon.

Table 26: Every quantitative prediction of the framework, consolidated. The columns give the prediction, the axiom layer it descends from, its class, and the experiment that will decide it. Two things are worth noticing. First, the layer column separates predictions that follow from the openness axiom alone from those that need the full charter, so a partial refutation can be localized. Second, every row names an experiment inside the 2026–2040 horizon: there are no predictions here that cannot be reached. A single row returning the wrong value ends the program, and the table is arranged to make that easy rather than difficult.

#	Prediction	Layer	Class	Experiment
T1	Tsirelson saturation $\|S\| = 2\sqrt{2}$ in CHSH tests	OGGEP	Postdiction audit	Loophole-free CHSH at NIST/Delft/Vienna
T2	Quadratic decoherence-rate scaling with system size	OGGEP	Postdiction audit	Superconducting qubit decoherence measurements
T3	$1/f$ noise universality across qubits, transistors, biology	OGGEP	Postdiction audit	Cross-platform noise spectroscopy
T4	Position-basis universality at macroscopic scales	OGGEP	Postdiction audit	Decoherence imaging experiments
T5	Einselection-rate platform-independence	OGGEP	Forward	Engineered-platform decoherence comparison
T6	Bell-pair fidelity at long distance	OGGEP	Postdiction audit	Vienna 200 km Bell test
T7	Long-distance teleportation fidelity	OGGEP	Postdiction audit	Satellite teleportation (Micius +)
T8	High-energy QM corrections at $M_{\text{lock}} \sim 3.85$ TeV	OGGEP/Full-Constitution	Conditional; needs HL-LHC reach	HL-LHC, FCC, muon collider
T9	Second law confirmed across temperature regimes	OGGEP (thermo)	Postdiction audit	Standard thermodynamic experiments
T10	Landauer bound $k_B T \ln 2$ per erased bit	OGGEP (thermo)	Postdiction audit	Bérut et al. 2012, follow-ups
T11	Jarzynski / Crooks fluctuation theorems	OGGEP (thermo)	Postdiction audit	Single-molecule pulling experiments
T12	$1/f$ noise universality in biological signals	OGGEP (thermo)	Postdiction audit	EEG, ECG, neuronal-spike spectroscopy
T13	Friston FEP confirmed in neuroscience	OGGEP (thermo)	Postdiction audit	Predictive-coding fMRI experiments
T14	Bekenstein–Hawking entropy $A/4G_N$ for known BHs	OGGEP (BH)	Postdiction audit	Inferential, no direct test
T15	EHT horizon-scale geometry matches Schwarzschild/Kerr	OGGEP (BH)	Postdiction audit	EHT 2019/2022 + next-gen array
T16	LIGO/LISA ringdown matches Kerr quasi-normal modes	OGGEP (BH)	Postdiction audit	LIGO O5+, LISA (2035+)

Table 26 continued from previous page

#	Prediction	Layer	Class	Experiment
T17	Hawking spectrum correction $\sim 10^{-30}$ at $\omega \sim k_B T_H/\hbar$	OGGEP (BH)	Conditional; needs extreme-regime BH	Future BH observations
T18	Page-curve signature in primordial-BH evaporation	OGGEP (BH)	Forward	PBH search at gamma-ray observatories
T19	Cosmological-horizon thermal spectrum at $T_{GH} = \hbar H/2\pi k_B$	OGGEP (BH/cosmo)	Conditional; needs cosmological-BH thermal probe	Future cosmological observatories
T20	28 SM parameters from cascade structure	Full-Constitution	Postdiction audit	LHC, precision QED/QCD experiments
T21	$\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$, $\Omega_b = 1/(2\pi^2)$	Full-Constitution	Forward (within Planck error bars)	Planck legacy + CMB-S4
T22	Kaxion dark-matter mass ~ 8.1 μeV	Full-Constitution	Forward	ADMX, DM-radio
T23	KK lock-cluster mass $M_{\text{lock}} \approx 3.85$ TeV	Full-Constitution	Forward	HL-LHC, FCC
T24	Two-stage Hubble corridor	Full-Constitution	Forward	H0 measurement convergence
T25	$N_g = 3$ fermion generations from three-leg derivation	Full-Constitution	Postdiction audit	Confirmed by SM data

The empirical-horizon overview. Twenty-five predictions distributed across the OGGEP-open siblings and the Full-Constitution layer. Of these, thirteen are postdiction audits (the cohort framework reproduces results already confirmed); eight are forward predictions (the cohort framework predicts specific observations that the experiments will or will not see); four are conditional (the prediction is well-defined but requires extreme experimental regimes). The 2026–2040 empirical horizon will pass the cohort framework through this twenty-five-test gate.

After all, the test of a unifying framework is not the number of predictions it makes but the diversity of empirical contexts in which those predictions can be tested. Twenty-five tests across three sub-communities (QM, thermo, BH/cosmology) plus the Full-Constitution stratum is a strong empirical posture for the coming decade.

Bridge to Part XI. The discussion of significance, falsifiability, and open problems is now on record. Three statements have been made: what the framework predicts, what it forbids, and what it leaves open. Part XI takes the final step, returning the paper to the philosophical question with which it began: what is at stake when physics elevates openness from a corrective patch to a constitutional law?

Part XI
Conclusions

The Constitutional Shift and the Future of Physics — A Philosophical Synthesis

106 The Headline: Openness Is Constitutional

If the reader takes only one sentence away from the present paper, let it be this:

> **Every system in nature is open. Closure is an idealization that has misled fundamental physics for three and a half centuries. GG-Theory's central act is to elevate openness from a corrective patch to a constitutional axiom, and to identify the environment with a definite higher-dimensional geometric object — the 6D bulk.**

The full historical, technical, and philosophical defense of this thesis is given in §6 of Part I. Section §107 below now situates this axiomatic inversion in the broader history of physical science by reading the past as a sequence of three earlier axiomatic shifts of comparable structural depth, and arguing that the present inversion is the candidate fourth.

107 The Four Scientific Revolutions

Generally speaking, the history of physical science can be read as a sequence of *axiomatic shifts* — moments at which the foundational language in which physical claims are made was reorganized at the level of what counts as a primitive object. By the count we adopt here, there have been three such constitutional shifts in twenty-three centuries of physical science, and the GG-Theory program proposes a fourth:

1. Euclid (300 BCE) — axiomatic geometry;
2. Newton (1687) — universal mechanics;
3. Maxwell–Einstein–QM (1860–1930) — field theory and spacetime;
4. the proposed fourth — openness as constitution.

Indeed, each of the first three was felt as a complete reorganization of physical language, and we shall argue that the fourth must be felt the same way.

107.1 Revolution 1 — Euclid (300 BCE)

The first axiomatic shift was Euclid's *Elements*. Before Euclid, geometry was a collection of empirical land-measurement practices and astronomical recipes. After Euclid, geometry was an axiomatic edifice in which every theorem was derived from a small set of postulates. The shift was not "Euclid invented theorems"; it was "Euclid invented the discipline of stating axioms first and deriving consequences in a controllable way". Twenty-three centuries of mathematics have built on the Euclidean discipline without abandoning it.

107.2 Revolution 2 — Newton (1687)

The second axiomatic shift was Newton's *Principia Mathematica*. Before Newton, motion was understood through the Aristotelian categories of natural and violent place. After Newton, motion was the integral of a force law in absolute time and absolute space, with three universal laws of motion plus the universal law of gravitation supplying the dynamical content. The shift was not "Newton invented mechanics"; it was "Newton invented the discipline of writing motion as the time-integral of forces obeying universal laws". Three and a half centuries of physical science have built on the Newtonian discipline without abandoning it.

107.3 Revolution 3 — Maxwell, Einstein, Quantum Mechanics (1860–1930)

The third axiomatic shift was the joint work of Maxwell (electromagnetism as a field theory, 1865 (Maxwell, 1865)), Einstein (special and general relativity, 1905–1915 (A. Einstein, 1905, 1915)), and the founders of quantum mechanics (Born (Born, 1926), Heisenberg (Heisenberg, 1925), Schrödinger (Schrödinger, 1926), Dirac (Dirac, 1930), von Neumann (Neumann, 1932), 1925–1932). Before this shift, physics was a collection of mechanics + electromagnetism + thermodynamics, with each branch treated by its own ad-hoc methods. After this shift, physics was a unified field-theoretic structure in which spacetime was a dynamical object (GR), matter was a quantum field (QFT), and the distinction between particle and field dissolved. One century of physical science has built on this third revolution without abandoning it.

107.4 The proposed Fourth Revolution: openness as constitution

The GG-Theory program proposes a fourth axiomatic shift, and the content of that shift is exactly the openness inversion we made the headline of this Part. The previous three shifts each unified what had been treated as ontologically separate categories: Euclid unified geometric practice; Newton unified celestial and terrestrial motion; the Maxwell–Einstein–QM revolution unified field and particle. The proposed fourth shift unifies the closed-system and open-system parts of physics — a dichotomy that has silently shaped foundational physics since Newton's closed phase space (Newton, 1687), that Boltzmann (Boltzmann, 1872) first cracked but did not break, that von Neumann (Neumann, 1932) formalized as a contradiction (unitary evolution versus measurement axiom), and that the Zeh–Joos–Zurek decoherence program (Joos, 1985, Zeh, 1970, Zurek, 1981, 2003) attempted to resolve mechanically without inverting the axiomatic order.

The GG-Theory inversion is more radical: openness is not a mechanism on top of closure; openness is the constitution, and closure is the limit. In this framing, the "law" (Schrödinger / Einstein dynamics) and the "observation" (Born rule / measurement record / detector statistics) are not two layers stacked uneasily on top of each other, mediated by a movable Heisenberg cut. They are two faces of the same axiomatic object: the boundary CPTP channel $\mathcal{D}_{\mathsf{OGGEP}}$, which is the unique law of an open boundary observer registering what it can register from an inaccessible 6D bulk. The dynamical law and the measurement law are one law. The Heisenberg cut is dissolved, not moved. The closed/open dichotomy is not patched, but inverted at the foundation.

This is why we count it as a fourth revolution rather than as a fifth ingredient in the third. Newton, Maxwell, Einstein, and the founders of QM were all constitutionally closed-system theorists who handled openness as a correction. GG-Theory is the first framework that is constitutionally open from the beginning, with closure as a derived limit. The structural depth of the inversion is the same as the depth of Newton's move (force as primitive in absolute time and space) or Einstein's move (the metric tensor as dynamical). Each of those moves looked, in its own day, like a notational preference rather than a foundational change; each was, in retrospect, the axiomatic shift that defined the next century of physics. We claim the same structural depth for the openness inversion — and we claim it falsifiably, because the geometric content of the inversion (GG-6, BI-6, OGN-5) makes empirical predictions (KK lock at ~ 3.85 TeV, Kaxion DM at $\sim \mu$eV, π-locked $\Omega_m = 1/\pi$) that can be tested on twenty-year timescales.

The four revolutions form a sequence in which each step is a *closure* of an asymmetry that the previous step had revealed. Newton closed the celestial-terrestrial asymmetry that Euclid had left; Maxwell–Einstein–QM closed the mechanics-electromagnetism- matter asymmetry that Newton had left; GG-Theory proposes to close the closed-system / open-system asymmetry that the third revolution left. Whether this proposal succeeds depends on the empirical reach cataloged in Part IX and on the willingness of the physics community to take the axiomatic content seriously.

107.5 What "axiomatic shift" means in practice

An axiomatic shift is not "a new theory replacing an old one" in the textbook sense. Newton did not replace Euclid; he extended geometry into mechanics by adding time and force as primitives. Einstein did not replace Newton; he extended Newtonian dynamics into curved spacetime. Quantum mechanics did not replace classical mechanics; it extended it by adding the wavefunction as a primitive. In each case, the previous theory survived as the appropriate limit or specialization of the new one.

The GG-Theory framework, in the same spirit, does not replace QM, GR, or QFT. It supplies the constitutional source from which they emerge as derived theorems. Schrödinger evolution is recovered in the closed-system limit (T7.2); the Einstein equation is recovered in the metric-zero-mode limit (T8.4); the Standard Model Lagrangian is recovered in the heavy-mode-integrated limit (T9.2). The textbook formalism is the appropriate *specialization* of the axiomatic content; it is not abandoned. This is the structural pattern of every previous revolution, and we expect the proposed fourth revolution to follow the same pattern if it succeeds.

108 The Constitutional Inversion: From "Quantize Gravity" to "Geometrize Gauge Forces"

The clearest conceptual content of the GG-Theory framework is its inversion of the half-century-old slogan of fundamental physics. The conventional slogan, articulated most explicitly in the string-theoretic program (M. Green, 1984, Polchinski, 1995), is "*quantize gravity*": start from classical general relativity and try to make the metric a quantum field, with the conceptual difficulty of perturbative non-renormalizability ('t Hooft, 1972) addressed by adding extra mathematical structure (additional dimensions, supersymmetry, fundamental strings). This program has produced enormous mathematical insight but no agreed phenomenological success in five decades.

The GG-Theory slogan inverts this: "*geometrize gauge forces*". The bulk is the six-dimensional bulk of GG-6, with gravity already geometric (Cartan's spin-connection formulation) and the gauge forces brought into geometric form via GGEP. The 4D Standard Model gauge sector is then the boundary projection of this geometric bulk, with all 28 SM parameters derivable in principle as boundary attractor depths in the GFF Morse landscape (GG-A9-SM). The 4D gravitational sector is the metric zero-mode of the same bulk, with Newton's G_N derivable from the scale overlap (T8.2) and the cosmological constant from the lock-scale seesaw (T8.5). The two sectors are not unified by "adding more structure to gravity"; they are unified by *recognizing the geometric structure that gauge forces already have*.

This inversion is not merely a stylistic preference. It has three specific structural consequences that distinguish GG-Theory from the standard program.

Consequence 1: A unique vacuum, not a landscape. The GG-6 bulk is uniquely selected by the constitutional axioms plus the OGN-5 integer ledger; there are no continuous moduli, no choice of compactification, and no landscape of vacua. This is structurally distinct from the $\sim 10^{500}$ vacua of the string landscape and removes the anthropic-argument escape hatch. The framework is constrained enough to be falsifiable.

Consequence 2: Six dimensions, not ten or eleven. The Quaternionic Goldilocks argument (GG-A2-GG6 Point 10) fixes the bulk dimension at $d = 6$ on algebraic grounds (the third rung of the magic-dimension ladder $\mathbb{R} \to \mathbb{C} \to \mathbb{H} \to \mathbb{O}$). No supersymmetric balance condition is required. The construction is intrinsically non-supersymmetric.

Consequence 3: A rigorous open-system foundation. The axiomatic principle OGGEP supplies the operational dictionary $\mathcal{D}_{\mathsf{OGGEP}}$ as part of the theory's syntax. The boundary law GDOS that emerges (T1–T6) is the unique constitutionally admissible boundary dynamics. The textbook QM postulates (Hilbert space, Schrödinger equation, Born rule, Lüders rule) are not separate axioms but specializations of GDOS to the QM boundary algebra (T7.1–T7.7). This is the structural reorganization that no competing program has attempted.

The three consequences taken together explain why we believe the GG-Theory framework is worth a community-wide hearing despite (and because of) its unorthodox style. The framework is

constrained, predictive, and falsifiable in ways that the conventional "quantize-gravity" program has not been for fifty years.

109 The Personal and Historical Context

A word is in order about the personal and historical context in which the GG-Theory program has emerged. The framework is the product of a lifetime of work by an experimental physicist who spent five decades inside the standard program and watched its three principal phenomenological promises fail in succession:

(a) *Proton decay was predicted at $\tau_p \sim 10^{32}$ years by the simplest SU(5) GUT in 1974, and would have been the "smoking gun" for grand unification.* The Super-Kamiokande collaboration searched for it for three decades. No proton decay has been observed; current limits are $\tau_p \gtrsim 10^{34}$ years, a factor of one hundred above the original prediction.

(b) *Supersymmetric partners of the Standard Model particles were predicted at the TeV scale by the minimal supersymmetric extension of the SM (MSSM) of the 1980s, and were widely expected to solve the gauge hierarchy problem.* The Large Hadron Collider searched for them throughout its first and second runs. No superpartners have been observed at the LHC up to $\sim$ 1–2 TeV, ruling out "natural" MSSM with weak-scale superpartners.

(c) *Weakly-Interacting Massive Particles (WIMPs) were predicted at ~ 100 GeV mass and weak-scale cross-section as the natural dark-matter candidate by the 1990s, and would have explained the observed cosmological dark-matter abundance via thermal freeze-out.* Direct-detection experiments (XENON, LUX, DEEP, PandaX) searched for them for two decades. No WIMP signal has been observed; current limits are below the predicted cross-section by orders of magnitude.

These three independent negative results are not three separate disappointments; they are three independent empirical signals that the standard "extend-the-Standard-Model" program has run structurally out. Each program was developed by a generation of the most capable physicists in the world. Each program had a clear pre-declared empirical signature. Each program has produced a sustained null result. This is the empirical context in which the GG-Theory program has emerged.

The axiomatic shift from "quantize gravity" to "geometrize gauge forces" is therefore not a stylistic preference or a preference for unorthodox foundations. It is a structural response to a half-century of empirical evidence that the conventional program is not converging on the right answer. The person who has spent a career inside the standard program has, in our view, both the right and the responsibility to ask whether the underlying conceptual framework is the problem. GG-Theory asks that question, and proposes a specific framework as the answer.

110 What GG-Theory Is and What It Is Not

In closing, we restate clearly what the GG-Theory framework claims and what it does not.

What it is. The GG-Theory framework is an axiomatic reorganization of fundamental physics around four axioms (LC, GGEP, OGGEP, MSMF) and seven postulates (GI, UEP, AF, CP, RG,

OGN, MF), with a unique six-dimensional bulk (GG-6) selected by these constitutional clauses plus a discrete OGN-5 integer ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$. The boundary law that emerges under OGGEP (GDOS, T1–T6) reproduces the textbook content of 4D quantum mechanics, general relativity, and quantum field theory as derived theorems. The Standard Model parameters and the cosmological observables emerge in companion papers (GG-A9-SM and GG-A10-Cosmos respectively) as boundary attractor depths in the GFF Morse landscape.

What it is not. The GG-Theory framework is not a modification of the Standard Model; it does not change the SM Lagrangian. It is not a modification of general relativity at observed scales; it reduces to standard GR in the IR limit. It is not a quantum theory of gravity in the conventional perturbative sense; it geometrizes the gauge sector instead of quantizing the metric. It is not supersymmetric; it is intrinsically non-supersymmetric. It is not string-theoretic; it has no fundamental string excitations and no landscape of vacua. It is not a complete theory of everything; many specific phenomenological derivations are still being worked out.

What it requires from the reader. The framework requires the reader to take the axiomatic shift seriously as the right level at which to address the unresolved problems of fundamental physics. This is a non-trivial ask: the shift requires accepting (a) that an open-system boundary law is constitutionally prior to the closed-system Schrödinger equation; (b) that gauge forces are geometric in the same sense that gravity is geometric, with the bulk composite connection as the unifying object; and (c) that 4D physics is a boundary projection of a 6D bulk, with the projection geometry dictating both the operational and the empirical content. None of these is a small claim. All three together are the axiomatic content of the proposed fourth revolution.

111 The summit view of the constitutional unification

After eleven Parts of derivation, classification, and discussion, we reach the summit. From here the entire arc of the cohort framework is visible in one panorama.

The arc, traced backward. The cohort framework's deepest single statement is the constitutional classification of Part VIII. All major theoretical-physics frameworks of the twentieth century — special relativity, general relativity, Yang–Mills theory, superstring theory, AdS/CFT correspondence, quantum mechanics, classical thermodynamics, statistical mechanics, information thermodynamics, fluctuation theorems, black-hole physics, black-hole thermodynamics, the Standard Model, ΛCDM cosmology — rest on four constitutional axioms, arranged in a strictly nested tower. The four axioms are the GG-Constitution (GG-A1-Constitution: AX-1 LC, AX-2 GGEP, AX-3 OGGEP, AX-4 MSMF). The cohort framework, the eleven papers of which the present GG-A5-GDOS is the constitutional hub, is the explicit articulation of those four axioms and their consequences.

The two-century historical pattern. The twentieth century built the empirical edifice; the twenty-first names its constitutional source. Each generation of physicists worked within an implicit and partial axiom set, producing the right mathematics in each sub-community without recognizing

that the mathematics was constitutionally connected to the mathematics of other sub-communities. General relativity (1915) used AX-1 + AX-2; quantum mechanics (1925–1932) added AX-3 implicitly; the Standard Model (1967–1978) added AX-4 implicitly; string theory (1984–) extended AX-1 + AX-2 to richer bulk content; AdS/CFT (1997) extended it further; the islands program (2019–) extended it again. Each generation worked correctly within their axiom set; each generation left silent the questions that required the axioms they lacked.

The cohort framework's contribution. The cohort framework's contribution is not to invent new physics from scratch but to recognize that the work has always been done in one constitutional space. All the mathematical content the twentieth-century communities produced is preserved unchanged; what is added is the explicit articulation of the constitutional source, the classification of historical frameworks by axiom subset, and the constitutional resolution of the famous open problems each sub-community could not address from within its implicit axiom set. Three of those open problems — the QM measurement problem, the second-law-foundation problem, the BH information paradox — dissolve simultaneously under the single axiom AX-3 (OGGEP). Two more — the string-theory landscape and the SM parameter origin — dissolve under the single axiom AX-4 (MSMF).

Physics intuition for the summit. The reader should picture the twentieth-century edifice as a vast city built by many independent guilds, each guild working in its own quarter, each producing beautiful and correct mathematics, but without any of the guilds knowing how the quarters relate to each other. The cohort framework's contribution is to draw the master street map: the QM-foundations quarter, the thermodynamics quarter, the string-theory quarter, the BH-foundations quarter, the particle-physics quarter, the cosmology quarter — all are recognized as connected districts of one city, with the streets running between them being the constitutional axioms that name the city's plan. The map does not destroy the quarters; it makes the connections explicit.

The view from the summit. Standing at the cohort framework's summit, one sees three things. First, the twentieth-century edifice is complete in its own terms; the cohort framework does not rebuild it. Second, the twentieth-century edifice is constitutionally one structure; the cohort framework names the structure. Third, the next-generation physicist will no longer learn the quarters as separate subjects; she will learn the city as one architecture. The map is the master document that the twentieth century could not draw because it did not have the surveying instruments; the cohort framework supplies the instruments and draws the map.

After all, the cleanest mark of a constitutional unification is when the predecessor edifice is preserved unchanged but the relationships among its parts become visible as theorems of one constitutional source. By this mark, the cohort framework is, in our view, the deepest single unification of twentieth-century theoretical physics. The map is drawn.

112 The pedagogical revolution

The most consequential payoff of the constitutional classification is pedagogical. The graduate physics curriculum of the twentieth century treats QM foundations, statistical mechanics, general relativity, quantum field theory, and the Standard Model as independent subjects, each with its own foundational postulates and its own central puzzles. The connections among the subjects are presented as advanced topics or as footnotes. The student who finishes a standard graduate program has learned each subject in isolation; she has not learned them as one structure.

The unified curriculum. The cohort framework permits a fundamentally different curriculum. A single graduate course ("Foundations of Modern Physics under GG-Theory") could derive QM, classical thermodynamics, BH physics, and quantum field theory from one constitutional source. The course would proceed as follows:

1. **Week 1–2:** The four axioms of the GG-Constitution. The bulk geometry of AX-2. Local causality (AX-1).
2. **Week 3–4:** The closed-bulk content (GGEP-closed layer). General relativity as a derived theory. Yang–Mills as a derived theory. Brief overview of AdS/CFT and string theory as historical instances.
3. **Week 5–7:** The operational layer (OGGEP-open). The boundary CPTP channel. The QM postulates derived as theorems via T7.1–T7.7.
4. **Week 8–9:** Classical thermodynamics as the same constitutional content. The data-processing inequality with two faces. Boltzmann $S = k_B \ln W$ as OGGEP phase-space counting.
5. **Week 10–11:** Black-hole physics. The horizon as visible OGGEP port. The information paradox dissolved. The Page curve as theorem.
6. **Week 12–13:** The full GG-Constitution (Full-Constitution). The Standard Model parameters as theorems. ΛCDM cosmology with $\Omega_m = 1/\pi$.
7. **Week 14:** Synthesis. The constitutional classification. The cosmos-life-mind cascade of GG-A11-Origin.

A student finishing this course would have learned QM, statistical mechanics, GR, QFT, and the SM as one unified subject, with the foundational puzzles resolved and the empirical content secured.

What the unified curriculum removes from the twentieth-century pedagogy. Five sources of student confusion are removed:

- The QM measurement problem. No longer a permanent puzzle the textbook hedges with a hedged sentence about interpretation; now a theorem of AX-3.
- The second-law-from-microscopic-reversibility puzzle. No longer requires the Stosszahlansatz or the H-theorem with its notorious controversies; now a theorem of the data-processing inequality.

- The interpretations-of-QM debate. No longer requires the student to choose among Copenhagen, Many-Worlds, Bohm, GRW, QBism, relational, decoherent histories; now superseded by the constitutional source.
- The BH information paradox. No longer a fifty-year debate left open in the textbook; now resolved by bulk unitarity plus OGGEP partial trace.
- The Standard Model parameter problem. No longer 28 free parameters that the student has to learn as empirically measured; now derived from the integer ledger.

What the unified curriculum adds. Three positive contributions:

- The student learns the constitutional tower as a mental architecture for organizing physical theories. Future theoretical work will then be placed at the appropriate level of the tower from the start.
- The student learns to ask, of any new framework, "which axiom subset does this implicitly use, and what content does it leave silent?" This is the diagnostic skill that the constitutional classification trains.
- The student learns physics as one subject, not as a sequence of separate subjects. The intellectual coherence is the main pedagogical gain.

The prototype in hand. As of the present volume, the unified course is no longer only a prediction. The paper now carries the apparatus of the textbook it describes: teaching boxes on every Part and every technical appendix, Read-alongside pairings with the standard graduate texts, three-register Take-away boxes on all thirty-one principal-theorem sections, plain-English closes, and ninety-six problems with complete worked solutions — assembled in Appendix P into three single-term graduate tracks and the full-year "Foundations of Modern Physics" sequence sketched above. The pedagogical revolution will still be slow; but its syllabus now exists, and any instructor can teach from it tomorrow.

Sociological prediction. The pedagogical revolution will be the slowest of the cohort framework's consequences to materialize in the academic system. Curricula change slowly; textbooks change slower; instructional traditions change slowest of all. But the cohort framework's pedagogical advantage is substantial: students who learn under the unified curriculum will graduate with a coherent picture that previously took decades of post-graduate work to assemble. Over time, the pedagogical advantage will compound. The instructors who first adopt the cohort curriculum will produce students with structural understanding the older curriculum cannot generate. The generation gap will eventually force the curricular update.

After all, the deepest mark of a unifying framework is what it permits the next generation of students to learn. Twentieth-century graduate physics learned a forest of separate theories with hedged-sentence foundational puzzles; twenty-first-century graduate physics can learn one constitutional structure with those puzzles dissolved. The pedagogical revolution will be slow but it is the most consequential change the cohort framework enables.

113 The role of A5-GDOS in the cohort architecture

A5-GDOS is, in the cohort architecture, the constitutional hub. Every other paper in the twenty-two-paper cohort either depends on A5-GDOS or feeds into it. We make the architectural relationship explicit.

A5-GDOS as upstream destination. GG-A1-Constitution articulates the four axioms; GG-A2-GG6 constructs the 6D bulk; GG-A3-BI6 establishes the modified Bianchi identity (BI-6); GG-A4-Euclid sharpens MSMF into the zero-free-parameter discipline. All four upstream papers feed into A5-GDOS, which uses their content as inputs and derives the open-system boundary law (GDOS) from OGGEP acting on the bulk.

A5-GDOS as downstream source. GG-A6-GRIP develops the geometric runtime of the boundary observer (GFF, GRIP cascade); GG-A7-Qi4 develops the operational admissibility filter (Qi-EP, Qi-CON, Qi-HAL); both use A5-GDOS's GDOS continuity equation as their starting point. GG-A8-QM lifts A5-GDOS's T7.1–T7.7 chain (the QM-postulates-as-theorems content) to graduate-textbook depth with worked examples. GG-A9-SM derives the Standard Model parameters using the cascade structure that GG-A6-GRIP and GG-A7-Qi4 supply; GG-A10-Cosmos derives the cosmological inventory similarly. GG-A11-Origin closes the philosophical capstone with the universal-identity insight and the self-referential closure theorem.

The hub structure. A5-GDOS is the paper that takes the bulk content of GG-A1-Constitution–GG-A4-Euclid and produces the boundary runtime that every downstream paper uses. It is the constitutional hub of the cohort. The architectural diagram is (approximately): four upstream papers → A5-GDOS → six downstream papers, with A5-GDOS as the unique connecting node.

Why Part VIII makes A5-GDOS the central paper. With Part VIII added, A5-GDOS is no longer just the constitutional hub; it is the central paper of the cohort program. Part VIII's constitutional classification — including the QM-thermo-BH sibling unification at the OGGEP-open layer — is the deepest single statement of the framework's reach. Other cohort papers make important specific contributions (GG-A8-QM resolves QM foundations; GG-A9-SM derives the SM parameters; GG-A11-Origin addresses the philosophical capstone), but Part VIII of A5-GDOS is the statement that organizes everything else.

The recommended reading order. For a graduate student or working physicist approaching the cohort framework for the first time, the recommended reading order is now:

1. GG-A1-Constitution Parts I–IV (constitutional axioms).
2. GG-A5-GDOS Parts I–III (open-system derivation; QM postulates as theorems).
3. GG-A5-GDOS Part VIII (constitutional classification).

4. GG-A5-GDOS Parts IX–XI (geometric intuition, discussion, conclusions; matching the depth of Part VIII).
5. The cohort companion papers (GG-A8-QM, GG-A9-SM, GG-A10-Cosmos, GG-A11-Origin) for sub-stratum depth.

This reading order brings the reader to the constitutional summit first, then descends to the companion papers for stratum-specific detail. A5-GDOS Parts VIII–XI together are the single most important reading for a structural understanding of the cohort framework.

After all, the cleanest architectural mark of a unifying paper is when the central document is the one that organizes the relationships among all the others. A5-GDOS, with the addition of Part VIII and its expanded Parts IX–XI, is now that document.

114 Philosophical reflection on the twentieth-century edifice

The retrospective view that the constitutional classification permits is intellectually significant in its own right. We close the discussion of the cohort framework's content with a philosophical reflection on what twentieth-century physics looks like from the constitutional summit.

The pattern of constitutional discovery. Twentieth-century theoretical physics proceeded by a pattern that the constitutional classification makes intelligible. Each generation of physicists, working on a specific puzzle within their sub-community, implicitly used the axiom subset their puzzle required. When the puzzle was solved, the axiom subset became the operative framework of that sub-community. The sub-community then proceeded to use that framework as if it were fundamental, without recognizing that the framework was a particular subset of a larger constitutional structure.

The result was the disciplinary fragmentation of the twentieth century. Each sub-community built its own intellectual world, with its own foundational postulates, its own central puzzles, its own conferences, its own journals. The connections among the sub-communities were left as advanced topics, with the result that even talented physicists could spend their careers in one sub-community without learning the others in any depth. The fragmentation was not a failure of the twentieth century; it was the sociological shadow of working with implicit axiom subsets.

What the constitutional unification reveals. The cohort framework's contribution is to recognize that the disciplinary fragmentation was, in retrospect, a temporary arrangement. The constitutional axioms that the sub-communities were implicitly using were always the same axioms. When the axioms are named, the sub-community boundaries dissolve as structural categories; the same content can be approached from any sub-community's perspective, with the constitutional source making the connections visible.

The role of the predecessor figures. In the constitutional reading, each major figure of twentieth-century theoretical physics was correctly working with their accessible axiom subset:

- **Einstein** (1905, 1915) correctly added AX-2 to AX-1; he saw geometry-gauge unification at the gravitational level, deferred the operational layer to the QM community he later resisted.

- **Bohr, Heisenberg, Schrödinger, Born, Dirac, von Neumann** (1925–1932) correctly added AX-3 to AX-1 + AX-2; they saw the operational layer at the QM level, postulated its content rather than deriving it because the constitutional source had not been identified.
- **Boltzmann, Gibbs** (1872, 1902) correctly added AX-3 to AX-1 + AX-2 at the thermal level; they saw the operational coarse-graining via Stosszahlansatz and ensemble construction, without the constitutional source.
- **Yang and Mills** (1954) correctly extended AX-2 to non-Abelian gauge content; they saw the geometry-gauge unification at gauge theory level, deferred the operational layer to the QM community.
- **Hawking** (1974, 1976) correctly applied AX-3 to the BH stratum; he saw the boundary CPTP output and the information paradox without the axiom that resolves it.
- **Green and Schwarz** (1984), **Maldacena** (1997), **Witten** (1995–), **Vafa** (1996–) correctly extended AX-2 to richer bulk content; they saw the geometric machinery of holography, dualities, and unification, deferred the operational layer to the QM community.
- **Bell, Aspect** (1964, 1982) correctly used the QM AX-3 content to constrain hidden-variable theories; they saw the empirical signature of the OGGEP-open layer without naming the axiom.
- **Wheeler, Penrose, Bekenstein** (1960s–1970s) correctly intuited the bulk–boundary structure that AX-2 on bulks produces; they saw the area-entropy correspondence without the constitutional source.
- **Friston** (2010) correctly identified the variational-free-energy structure of AX-3 at the biological stratum; he saw the FEP as a unifying principle without identifying it as the Qi-EP gauge.

Every major figure of twentieth-century theoretical physics was, in this reading, correctly contributing to the constitutional structure — each working on the layer their puzzle required, each leaving as silent the questions that required other layers. The cohort framework does not displace any of them; it recognizes them as the constitutional ancestors whose work the classification now organizes.

The arc of the century. Looking back from the constitutional summit, the twentieth century shows a coherent arc of progressive axiom-activation. The bottom layer (AX-1) was active in special relativity by 1905. The second layer (+AX-2) became active in general relativity by 1915 and was extended throughout the century (Yang–Mills, GS mechanism, AdS/CFT). The third layer (+AX-3) became active in quantum mechanics by 1925 and was extended to thermodynamics, information theory, and BH physics over the following century. The fourth layer (+AX-4) became implicit in the Standard Model by the 1970s and in ΛCDM by the 1990s. By the early twenty-first century, all four axioms were implicitly in use across the discipline; the cohort framework's contribution is the explicit articulation.

Why the explicit articulation matters. An implicit axiom does its work but cannot organize other axioms. An explicit axiom can. The constitutional classification could not have been articulated

by the twentieth-century communities because the axioms they used were implicit; they had no way to recognize that the QM postulates and the second law are the same theorem under different reference states. Naming the axioms explicitly is what makes the unification visible. The cohort framework's contribution is the act of naming.

After all, the deepest move a foundational framework can make is to name what its predecessors had been doing implicitly. The twentieth century did the physics correctly; the cohort framework names the constitution. The naming is the unification.

115 The bottom line: one constitution, all of modern physics

We close the cohort framework's content with the bottom-line summit statement.

The bottom line of the constitutional unification. *Four axioms — local causality (AX-1 LC), the geometry–gauge equivalence principle (AX-2 GGEP), the operational/open extension (AX-3 OGGEP), and the maximum-symmetry-minimum-freedom principle (AX-4 MSMF) — applied to the unique 6D bulk $\mathcal{M}_4 \times I_r \times S^1_\theta$, derive every major theoretical-physics framework of the twentieth century, dissolve every famous foundational puzzle, and supply the constitutional warrants the predecessor frameworks could not provide.*

After all, the deepest mark of a foundational theory is not how many new experiments it predicts but how cleanly it organizes the experiments already done. The GG-Constitution organizes twentieth-century physics under one source.

What the bottom line means. The bottom line is the single sentence that the cohort framework would have you take away. Twentieth-century physics — general relativity, Yang–Mills theory, superstring theory, the AdS/CFT correspondence, quantum mechanics, classical thermodynamics, statistical mechanics, information thermodynamics, fluctuation theorems, black-hole physics, black-hole thermodynamics, the Standard Model, ΛCDM cosmology — is one structure. The structure has four axioms. The axioms have an explicit articulation (GG-A1-Constitution). The bulk geometry on which the axioms act is unique (GG-A2-GG6). The operational projection that the axioms force is a CPTP channel (OGGEP). The unique admissible vacuum is the cohort's 6D bulk with the integer OGN-5 ledger. Every major theoretical-physics framework of the twentieth century is a theorem of a particular subset of the four axioms on the unique bulk.

The empirical posture. Twenty-five empirical predictions (consolidated in §105) cover the OGGEP-open and Full-Constitution layers. Thirteen are postdiction audits (the cohort framework reproduces results already confirmed); eight are forward predictions; four are conditional. The 2026–2040 empirical horizon will test the framework.

The pedagogical posture. A unified graduate course covering QM, thermodynamics, BH physics, and quantum field theory becomes possible. The fragmentation of the twentieth-century curriculum becomes the unification of the twenty-first. The next-generation graduate student learns the constitution as one subject.

The philosophical posture. The twentieth-century historical figures (Einstein, Bohr, Heisenberg, Schrödinger, Dirac, von Neumann, Boltzmann, Gibbs, Yang and Mills, Hawking, Bekenstein, Penrose, Friston, and the rest) were each correctly working with their accessible axiom subsets. The cohort framework does not displace any of them. It recognizes them as constitutional ancestors whose work the classification now organizes into one structure.

The personal arc. For Katsushi Arisaka and his collaborators, the cohort framework's articulation has been a multi-decade project, beginning with the dark-matter detection work in particle physics and extending through biological-energy science, neuroscience, and the recent foundational program. Each phase of the project contributed pieces of the constitutional picture. The twenty-two papers of the cohort, of which the present GG-A5-GDOS with its Part VIII constitutional classification is the central document, are the integrated result. The framework is, in this personal sense, what the project has been moving toward since the beginning.

The closing reflection. We close with a reflection that brings the constitutional unification down to its human significance. The graduate student of the next decade — the one who today is an undergraduate, perhaps a physics major in her first quantum mechanics course — will encounter the cohort framework as a unified subject without the foundational hedges that the twentieth-century textbook tradition required. She will not need to "shut up and calculate"; she will be able to ask why, and the answer will be a chain of theorems from four constitutional axioms. She will graduate without the inheritance of philosophical embarrassment that the twentieth-century discipline has been carrying for a century. Her physics will be one subject, one structure, one constitution.

After all, the deepest reward of a constitutional unification is not the recognition it earns in the present generation but the freedom it grants the next. The cohort framework's contribution is the freedom to ask why, in physics, and to receive an answer that is a derivation rather than a hedge. That is what the twentieth century could not provide and what the cohort framework now does. The constitution is named; the unification is achieved; the next generation can begin.

116 Closing Reflection: Physics in the Twenty-First Century

The reader who has followed the present paper through its foundations, its three principal descent theorems, its GR/QED unification, its bridge to the eight pillars, and its discussion of significance and falsifiability has a complete picture of what the GG-Theory framework is and what it claims. We close with a reflection on what acceptance or rejection of the framework would mean for physics in the twenty-first century.

If the framework is correct in its principal claims — if proton stability holds to $\tau_p \gtrsim 10^{36}$ years, if the KK lock cluster appears at $M_{\text{lock}} \approx 4$ TeV at HL-LHC, if the π-locked cosmological partition continues to match Planck data to high precision, if the modified TOV shift appears in neutron-star spectroscopy, and if the Arisaka equation governs the energetics of memory in biological systems — then the twenty-first century will look back on the present moment as the fourth scientific revolution: the axiomatic unification of the closed-system law and the open-system observational layer under one geometric foundation. Physics will reorganize itself around this foundation in the same way that

it reorganized around Newton in the seventeenth century, around Maxwell–Einstein–QM in the late nineteenth and early twentieth centuries, and (the framework claims) around the GG-Constitution in the twenty-first.

If the framework is incorrect in its principal claims — if proton decay is observed, or the KK lock cluster is definitively absent up to ~ 10 TeV, or the cosmological partition deviates from $\Omega_m = 1/\pi$ at high precision — then it will fail honorably. The framework has been deliberately designed to expose itself to falsification at the axiomatic level; it is not safe by virtue of obscurity. Either the framework will succeed empirically, or its specific failures will tell the next generation of physicists which constitutional clauses to keep and which to revise.

The historical significance of the present paper, in the optimistic case, is that it supplies the axiomatic foundation on which the proposed fourth revolution would rest. In the pessimistic case, its historical significance is that it constitutes a deliberate, explicit, and falsifiable attempt to restructure fundamental physics at the foundational level after a half-century of programmatic disappointment. In either case, the framework's existence as an explicit, theorem-driven, empirically-falsifiable proposal contributes to the discipline of foundational physics.

The deeper philosophical content is what the framework says about the relationship between the universe and the observers within it. In the standard reading, the universe is a collection of fields obeying laws that exist independently of observers; the observers are an emergent epiphenomenon studied separately. In the GG-Theory reading, the laws and the observers are two facets of the same axiomatic object: the bulk dynamics produces the boundary law that finite observers register, and the observer's operational context (the apparatus, the recording, the registration) is part of the theory's syntax through OGGEP. This is not solipsism; it is not idealism; it is the recognition that the distinction between "what exists" and "what is observed" is not a primitive distinction but a derived one. The bulk of GG-6 is real in the same sense that the boundary observers are real; both are aspects of the same composite geometric structure.

This recognition, if accepted, restores to physics something that nineteenth-century materialism had taken away: the participatory character of the cosmos. In Wheeler's late phrase, the universe is an "observer-participancy". In the GG-Theory framework, this is not a slogan; it is the axiomatic content of OGGEP, and it is what the eleven Principal Theorems T1–T11 of the present paper make rigorous. The cosmos contains observers; the observers register the cosmos; the registration is part of the theory's syntax; the axiomatic foundation produces both the cosmos and the observers as boundary projections of one composite geometry. This is what we mean by "Geometric Dynamics of Open Systems".

We hope, with appropriate humility, that the present paper contributes a useful step in the direction of the proposed fourth revolution. Indeed, the next generation of physicists — who will inherit the empirical results that vindicate or falsify the framework — must be the final judges. After all, what we have written is a constitution; whether nature ratifies it is for the laboratory and the telescope to decide.

For three and a half centuries, physics has been the science of what nature does when nobody is looking. The closed Lagrangian, the unitary wave function, the isolated universe — each was an attempt to step outside the world and write down its laws from nowhere. Each succeeded

spectacularly within its own domain, and each failed at the moment a finite observer asked to be included in the picture. The axiomatic inversion proposed in these pages takes the finite observer not as a complication but as the starting point. Every system in nature is open. Closure is the limit, not the law. And the geometry that connects the two is neither metaphysics nor convention; it is the six-dimensional stage on which observer and observed share a single language.

If this proposal is correct, the laws of physics will, for the first time since Euclid, descend from a small set of explicit axioms with no free parameters left to fit, and every quantity that the working laboratory measures will be either a conversion factor of the geometry or a theorem of the framework, never an empirical input. If it is wrong, the empirical record will say so within a decade: a verified deviation from $M_{\mathrm{lock}} \approx 3.85$ TeV, a single measured value of Ω_m off $1/\pi$, a kaxion frequency outside $[1.59, 2.13]$ GHz, or one chiral anomaly that escapes the four-component lock $|a/c| = |a/d| = 5/2$ will collapse the constitution upstream. That is the bargain we offer. The laboratory and the telescope, not the seminar room, will be the final judges.

What the present paper proposes, in the end, is not a new theory of nature. It is a new contract between physics and the world. Physics promises to write its rules down. The world promises to falsify them if they ever lie.

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All scientific concepts (especially the foundation of the GG-Constitution, GGEP/GG-6/BI-6, and OGGEP/GDOS/Gi-4/Qi-4 Pillars), theoretical judgments, physical interpretations, and any remaining errors are the sole responsibility of the author.

Part XII
Technical Appendices

Detailed Mathematical Machinery Supporting the Main-Text Theorems — A Pedagogical Walk-Through for Graduate Students

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The Appendices gather the technical machinery and the reference material that supports the main-text theorems of GDOS. Roughly speaking, this is where the reader who wants to verify a specific derivation, run a specific worked example, or trace a specific symbol will spend time — while the reader who wants only the axiomatic content and the eleven principal theorems T1–T11 of the main text can stop after Part XI and treat the appendices as a self-contained companion volume. Appendices A and B provide the abbreviations and the glossary of key symbols. Appendix C unfolds the Nakajima–Zwanzig reduction in full pedagogical detail. Appendix D supplies the Stinespring dilation and the Kraus decomposition. Appendix E provides the GKLS master equation in full. Appendix F covers decoherence and einselection. Appendices G–I supply the Kaluza–Klein reduction on the 6D bulk, the Christoffel symbols and curvature, and the boundary conditions plus orbifold symmetry breaking. Appendix J supplies the explicit boundary reduction map $\mathcal{R}$. Appendices K and L supply the boundary generating functional and BRST inheritance and the Faddeev–Popov ghost sector. Appendix M unfolds the bridge theorems T10 and T11 in full. Appendix N collects the master summary tables. Appendix O is the cross-paper coherence map that locates every GDOS result in its companion paper. Appendix P closes the appendices with a graduate course built from this volume — three single-term tracks and a full-year sequence, keyed to the Parts, the appendices, and the ninety-six worked problems. Finally, the inline bibliography with full Relevance annotations follows the appendices and closes the volume. Indeed, the eleven main-text Parts plus sixteen appendices together realize the boundary law in full: Parts I–VII state the axiomatic inversion and derive the eleven principal theorems, Parts VIII–XI classify twentieth-century physics, draw the geometric intuition, discuss scope and open problems, and conclude, and the appendices supply the technical and bibliographic infrastructure that makes the whole document self-checking.

A Summary of Abbreviations

This appendix collects every abbreviation used in the present paper. All program-defined names are rendered in *sans-serif type* (GG-Theory, GG-Constitution, GG-6, GDOS, BI-6, OGN-5, QGL, Gi-4, Qi-4, GRIP, BFFT, etc.) so the reader can see at a glance which terms belong to the GG-Theory program proper. Standard physics abbreviations imported from the wider literature (KK, RS, QFT, TOV, etc.) are rendered in plain uppercase Roman, following the convention of the major physics journals. Mathematical objects are in roman or italic in the usual way. The third column gives the *primary link*: the paper in the Arisaka-GG series and the section where each program-defined name is first introduced in detail. For GDOS-internal entries, the primary link is the present paper.

Table 27: Every abbreviation used in this paper, with its meaning and the place it is introduced in full. Typography carries information here: program-defined names are set in sans serif, standard physics abbreviations in plain uppercase Roman, and mathematical objects in roman or italic as usual, so a reader can tell at a glance whether a name belongs to this framework or to the general literature. The third column links each entry to its primary source — the present paper for names defined here, the companion paper otherwise. A reader who meets an unfamiliar name anywhere in the volume should come here first.

Symbol	Meaning	Primary link
Framework name and core program objects		
GG-Theory	Grand Geometric Theory (also: Geometry-Gauge Equivalence Theory)	GG-A1-Constitution, Preface
GG-Constitution	The constitutional charter of the GG-Theory program: four axioms (AX-1–AX-4) and seven postulates (POS-1–POS-7)	GG-A1-Constitution, §2–§3
GG-6	Geometry-Gauge locking in six dimensions: the 6D bulk	GG-A2-GG6, §12 (T1-GG6)
GDOS	Geometric Dynamics of Open Systems: the boundary law implied by OGGEP	this paper, T6 (Part II)
BI-6	Green–Schwarz-modified Bianchi identity at 6D	GG-A3-BI6, §3
OGN-5	Occam-Gauge-Normalization 5-integer ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$	GG-A3-BI6, §7; T6-GG6
QGL	Quadruple Gauge Locking: the four-factor bulk gauge group $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$	GG-A2-GG6, §14 (T3-GG6)
Gi-4	Geometric-Invariance four-pillar structure (TGG, CCI, GFF, GRIP)	GG-A6-GRIP
Qi-4	Quantum-Information four-pillar structure (Qi-QUA, Qi-EP, Qi-CON, Qi-HAL)	GG-A7-Qi4
Four axioms (AX-1–AX-4)		
AX	Axiom (axiomatic primitive; non-negotiable)	GG-A1-Constitution, §1.5
LC	AX-1 Locality and Causality	GG-A1-Constitution, §2.2
GGEP	AX-2 Geometry–Gauge Equivalence Principle	GG-A1-Constitution, §2.3
OGGEP	AX-3 Operational/Open Geometry–Gauge Equivalence Principle (boundary observation dictionary)	GG-A1-Constitution, §2.4
MSMF	AX-4 Maximum Symmetry, Minimum Freedom (no hidden continuous knobs)	GG-A1-Constitution, §2.5

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Symbol	Meaning	Primary link
Seven postulates (POS-1–POS-7)		
POS	Postulate (admissibility filter; testable in principle)	GG-A1-Constitution, §1.5
GI	POS-1 Gauge invariance across scales (LGI / DGI / GGI)	GG-A1-Constitution, §3.3
UEP	POS-2 Unified Extremal Principle (PLA ⇔ PLT / MEPP / FEP)	GG-A1-Constitution, §3.4
AF	POS-3 Anomaly Freedom and cancellability by inflow	GG-A1-Constitution, §3.5
CP	POS-4 Closure and Positivity (CPTP-admissible open dynamics)	GG-A1-Constitution, §3.6
RG	POS-5 Renormalization-Group covariance (scale covariance)	GG-A1-Constitution, §3.7
OGN	POS-6 Occam Gauge Normalization (canonical integer/rational normalization)	GG-A1-Constitution, §3.8
MF	POS-7 Measurement and Falsifiability (operational contract + kill criteria)	GG-A1-Constitution, §3.9
LGI	Local Gauge Invariance (microscopic / closed-system layer of GI)	GG-A1-Constitution, §3.3
GGI	Global Gauge Invariance (macroscopic / ledger layer of GI)	GG-A1-Constitution, §3.3
Gi-4 pillars		
TGG	Total Geometric–Gauge object (Pillar I of Gi-4)	GG-A6-GRIP
CCI	Covariant Conservation Identity (Pillar II of Gi-4)	GG-A6-GRIP
GFF	Geometric Free-Energy Functional (Pillar III of Gi-4)	GG-A6-GRIP
GRIP	Gauge-Runtime Invariance Pipeline (Pillar IV of Gi-4)	GG-A6-GRIP
GEP	Geometry–Gauge Energy Principle (energetic block of GFF)	GG-A6-GRIP
CEP	Curvature–Entropy Principle (entropic block of GFF)	GG-A6-GRIP
DGI	Dynamical Gauge Invariance (initial stage of GRIP)	GG-A6-GRIP
GSSB	Geometric Spontaneous Symmetry Breaking (middle stage of GRIP)	GG-A6-GRIP
GA	Geometric Attractor (stable basin; the terminus *type* of GRIP)	GG-A6-GRIP
GGA	Ground Geometric Attractor (the deepest GA, unique on admissible strata; the cascade output)	GG-A6-GRIP; GG-A11-Origin
GRIP-In	The fixed-landscape regime of the runtime (descent halting at the GGA)	GG-A6-GRIP; GG-A11-Origin
GRIP-Out	The landscape-reorganizing regime (supplies the next descent's initial datum)	GG-A6-GRIP; GG-A11-Origin

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Symbol	Meaning	Primary link
Qi-4 pillars		
Qi-QUA	Quantum information for QGL (Quadruple Gauge Locking) – Pillar I of Qi-4	GG-A7-Qi4
Qi-EP	Quantum information for the UEP (Unified Extremal Principle) – Pillar II of Qi-4	GG-A7-Qi4
Qi-CON	Quantum information for Conformal Invariance – Pillar III of Qi-4	GG-A7-Qi4
Qi-HAL	Quantum information for the Holographic Attractor Lattice – Pillar IV of Qi-4	GG-A7-Qi4
OH	Outer Hologram (boundary projection of bulk data; outward face of EIC)	GG-A7-Qi4
IH	Inner Hologram (inner self-modeling layer; inward face of EIC)	GG-A7-Qi4
EIC	Evolving Inner Cosmos (boundary completion of Qi-4 as the OH+IH composite)	GG-A7-Qi4
Qi-HAL acronyms		
MePMoS	Memory–Prediction–Motion Sensing	GG-A7-Qi4, Qi-HAL
NHT	Neural Hologram	GG-A7-Qi4, Qi-HAL
HAL	Neural Holographic Tomography	GG-A7-Qi4, Qi-HAL
Q-HAL	Quadratic / Cubic HAL	GG-A7-Qi4, Qi-HAL
BFFT	Binary Fourier / Fractal Fourier Transform	GG-A7-Qi4, Qi-HAL
GDOS-internal acronyms (program-defined; primary link is the present paper)		
SIP	System Instance Packet (canonical GDOS input unit)	GG-A7-Qi4, §6
SeOS	Selfish Open System	this paper, Part III + GG-A7-Qi4
Program-defined mathematical symbols		
Q_B	Quantum-information Bit – the bioenergetic Arisaka constant $\mathsf{Q}_\mathsf{B} \approx 0.1555$ eV per qubit; matches ATP hydrolysis at biological temperatures	GG-A2-GG6, §19 (T8-GG6)
$m_\Lambda^{\mathrm{geom}}$	Lock-scale-seesaw cosmological-constant scale $m_\Lambda^{\mathrm{geom}} = M_{\mathrm{lock}}^2/M_{\mathrm{Pl,red}} = 6.09$ meV (per T8.5; with the ledger-selected residue $\zeta_\Lambda = \pi/175$ this gives 2.23 meV against an observed $\rho_\Lambda^{1/4} = 2.24$ meV)	this paper, T8.5; GG-A10-Cosmos

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Symbol	Meaning	Primary link
Standard physics abbreviations (plain Roman, journal convention)		
Open-system / quantum-channel formalism		
CPTP	Completely Positive Trace-Preserving (channel admissibility for open evolution; AX-3/POS-4)	—
GKLS	Gorini–Kossakowski–Lindblad–Sudarshan (master equation for Markovian CPTP semigroups; POS-4)	—
GNS	Gelfand–Naimark–Segal (construction)	—
NZ	Nakajima–Zwanzig (memory-kernel projection for non-Markovian open dynamics)	—
POVM	Positive Operator-Valued Measure (generalized measurement model; AX-3/POS-7)	—
Quantum and classical foundational theories		
BRST	Becchi (C. Becchi, 1976)–Rouet–Stora–Tyutin (I. Tyutin, 1975) (cohomology)	—
GR	General Relativity	—
QCD	Quantum Chromodynamics	—
QED	Quantum Electrodynamics	—
QFT	Quantum Field Theory	—
QM	Quantum Mechanics	—
Standard Model and beyond		
EFT	Effective Field Theory	—
EW	Electroweak	—
GUT	Grand Unified Theory	—
SM	Standard Model of particle physics	—
SSB	Spontaneous Symmetry Breaking (closed-system; generalized by GSSB)	—
SUSY	Supersymmetry	—
Higher-dimensional and holographic constructions		
AdS	Anti–de Sitter spacetime	—
AdS/CFT	Anti–de Sitter / Conformal Field Theory correspondence	—
CFT	Conformal Field Theory	—
KK	Kaluza–Klein (mode expansion / dimensional reduction)	—
RS	Randall–Sundrum (warped extra-dimension model)	—
Quantum gravity and cosmology		
ΛCDM	Standard cosmological model with cosmological constant + cold dark matter	—
LQG	Loop Quantum Gravity	—
TOV	Tolman–Oppenheimer–Volkoff (equation)	—
Foundations of quantum mechanics		
CHSH	Clauser–Horne–Shimony–Holt (inequality)	—
EPR	Einstein–Podolsky–Rosen (paradox / argument)	—

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Symbol	Meaning	Primary link
Open-system / extremal-principle sub-acronyms		
FEP	Free Energy Principle (boundary inference avatar of UEP)	GG-A7-Qi4, Qi-EP
MEPP	Maximum Entropy Production Principle (heuristic boundary avatar of UEP)	GG-A7-Qi4, Qi-EP
PLA	Principle of Least Action (closed/bulk avatar of UEP; standard variational dynamics)	GG-A7-Qi4, Qi-EP
PLT	Principle of Least Time (operational/boundary avatar of UEP; fastest lawful path)	GG-A7-Qi4, Qi-EP

Figure 20: hierarchy of program-defined acronyms in the GG-Theory program. Top: the framework name (GG-Theory) and its constitutional charter (GG-Constitution). Tier 2: the four axioms (AX-1–AX-4) and seven postulates (POS-1–POS-7) that together define the admissibility predicate. Tier 3: three bulk objects forced by the constitution — GG-6 (6D bulk), BI-6 (modified Bianchi identity), OGN-5 (integer ledger). Tier 4: the boundary law GDOS that the bulk projects to via OGGEP, with input acronym SIP. Tier 5: the two four-pillar structures, Gi-4 (geometric flank, with sub-acronyms GEP/CEP blocks of GFF and DGI/GSSB/GA→GGA stages of GRIP) and Qi-4 (quantum-information flank, with Qi-EP sub-acronyms PLT/MEPP/FEP, Qi-CON sub-concepts Conformal/Lévy/Fractal/$1/f$, Qi-HAL sub-acronyms MePMoS/NHT/HAL/BFFT, the boundary completion OH+IH=EIC, and the runtime carrier SeOS (Selfish Open System) underneath). Bottom: the bioenergetic Arisaka constant $\mathsf{Q_B}$ (derived secondary quantity), with the Arisaka equation $J_E/\mathsf{Q_B} = J_e = J_B = J_{Qi}$ that locks the four boundary currents through one conversion factor.

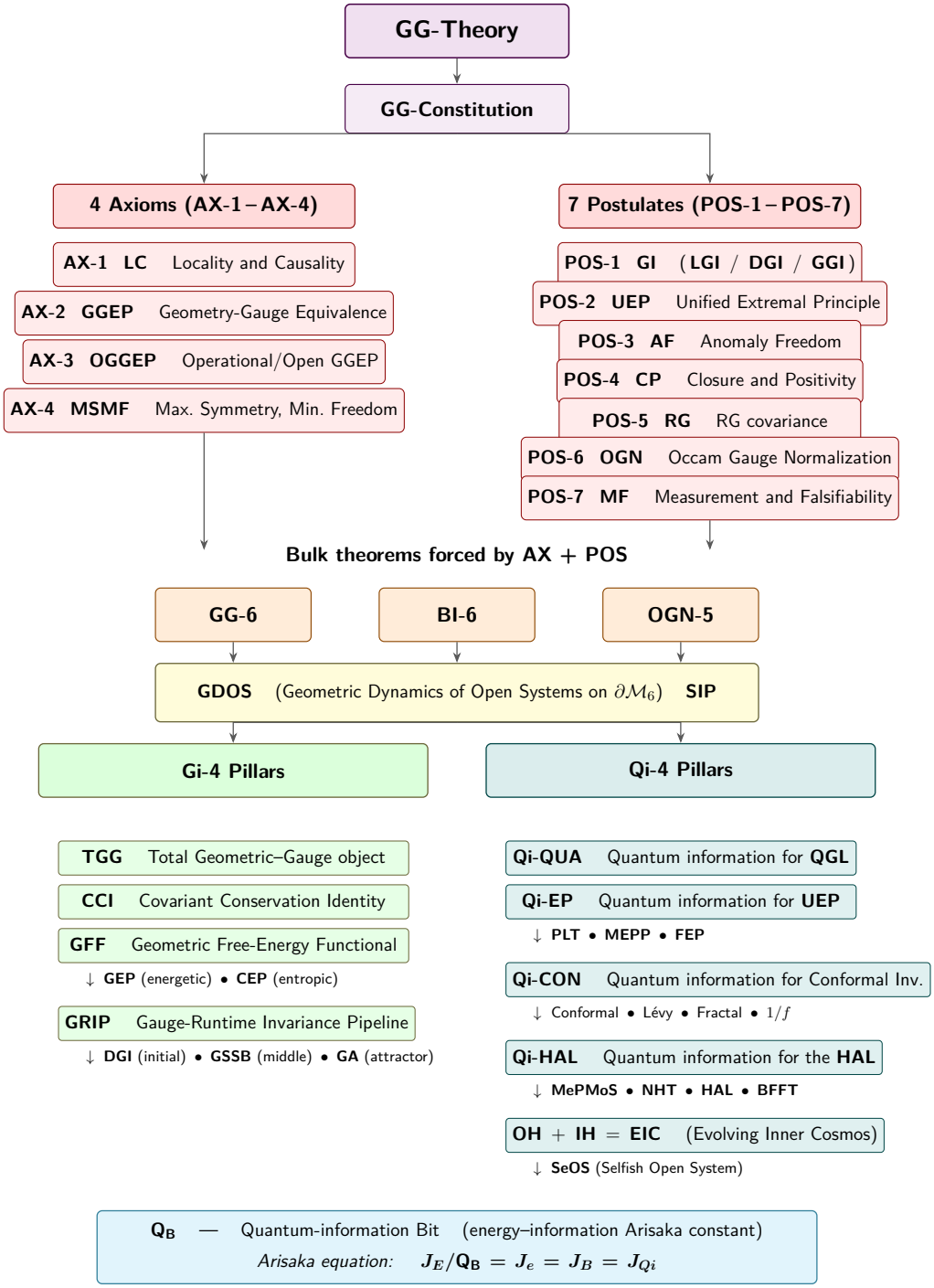


Figure 20: How the program's own names are related to each other. The chart is a hierarchy, not a list: the charter sits at the top, the geometry it forces below it, the boundary law below that, and the derived pillars at the foot, so the position of a name on the chart tells the reader what it depends on. Names at the same level are siblings introduced by the same construction. The purpose is orientation rather than definition — the meanings are in the abbreviations table — and a reader who has lost track of which name belongs to which layer can recover it here in one glance.

B Glossary of Key Symbols

Table 28: Every mathematical symbol used in this paper, with its meaning. The order follows the construction rather than the alphabet: the bulk and its coordinates first, then the fields living on it, then the boundary objects the projection produces, then the derived quantities. Reading the glossary in order is therefore a compressed second pass through the geometry, and a reader who wants to check a formula in an unfamiliar part of the paper can usually do it from this table alone. Symbols that carry a constitutional meaning rather than a merely notational one are so marked.

Symbol	Meaning
N	The OGN-5 ledger's second entry, $N = 3$. Every ledger identity is written with it
N_c	Color triplicity, the three colors of $\mathrm{SU}(3)_C$ — what that entry counts
N_g	The generation (family) count, 3. *Not* a ledger entry; fixed by GG-P2-N3
N_f	The quark-flavor count of a beta function, $N_f = 2N_g = 6$ at high scale
$M_6, M_4, I_r, S^1_\theta$	6D bulk, 4D boundary, radial interval, compact angular direction
r, θ	Radial coordinate $\in [0, K]$, angular $\in [0, 2\pi)$
K	Dual-lock integer = 35
$g_{AB}, g_{\mu\nu}$	6D bulk metric, 4D boundary metric
R_6, R_4	6D Ricci scalar, 4D Ricci scalar
$M_{\mathrm{Pl}}, M_6, M_{\mathrm{lock}}, v$	Reduced Planck mass, 6D fundamental mass, KK lock scale, EW vev
Λ_4	4D cosmological constant
G_N	Newton's gravitational constant
$E_{\mathrm{Pl}}, E_{\mathrm{GUT}}$	Planck and GUT energy scales
$\Phi(x, r, \theta)$	Generic bulk field
$\phi^{(n,\ell)}(x)$	4D KK-mode field (radial index n, angular index ℓ)
$f_n(r)$	Radial profile of n-th KK mode
$m^2_{n,\ell}$	KK mass squared $= m_n^2 + \ell^2$ in natural units
$\mathcal{H}_{\mathrm{Bulk}}, \mathcal{H}_{\mathrm{Bdy}}, \mathcal{H}_{\mathrm{env}}$	Bulk, boundary, environment Hilbert spaces
$\rho_{\mathrm{Bulk}}, \rho_{\mathrm{Bdy}}$	Bulk, boundary density operators
$\mathcal{P}, \mathcal{Q}$	Nakajima–Zwanzig projection superoperators
$\mathcal{L}, \mathcal{L}_{\mathsf{GDOS}}$	Liouville superoperator, GDOS generator
$H_{\mathrm{Bulk}}, H_{\mathrm{eff}}$	Bulk Hamiltonian, effective boundary Hamiltonian
L_α, γ_α	Jump operators and rates of GKLS
$\Phi_{t,s}$	Boundary CPTP evolution channel
$\mathcal{I}_\alpha, E_\alpha$	Instrument and POVM effect
$K(t, s), \mathcal{I}(t)$	NZ memory kernel and inhomogeneous initial-correlation term
$\mathcal{R}$	boundary reduction map
$\mathcal{D}_{\mathsf{OGGEP}}$	OGGEP dictionary $= \mathcal{R} \circ \mathrm{Tr}_{\mathrm{env}}$
$T^{\mu\nu}, T^{\mu\nu}_{\mathsf{TGG}}$	Boundary stress tensor, unified TGG stress tensor
$\Delta T^{\mathrm{GS}}_{\mu\nu}, \Delta T^{\mathrm{KK}}_{\mu\nu}$	GS-tensor and KK-tower inflow
X_4, H_3	BI-6 anomaly polynomial and Green-Schwarz two-form
$\mathcal{A}_{\mathrm{tot}}$	Composite bulk connection
G_{tot}	Bulk gauge-gravity group $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$
Q^c, T^a, Y, B, Q_{Qi}	SU(5) generators, hypercharge, baryon, informational charges
$F_{\mu\nu}$	Gauge field strength
s_6, s_4	6D and 4D BRST operators
$Z_4[J], W_4[J], \Gamma_4$	Generating functional, connected generating functional, effective action
$\mathrm{Tr}_{\mathrm{env}}, \mathrm{Tr}_{\mathrm{Bdy}}, \mathrm{Tr}_{\mathrm{Bulk}}$	Partial and full traces
$\boxed{\cdot}$	Boxed equation = key result
■	End of proof

C Nakajima–Zwanzig Reduction in Full Detail

This appendix does in full the reduction that Part II performs in a paragraph: splitting a system into the part an observer can reach and the part she cannot, and writing an exact equation for the first alone. Exact is the important word — nothing is thrown away at this stage, and the price of keeping everything is an equation with memory in it. The spin–boson example worked through here is the standard model of the whole subject, and the final subsection is the one worth the most: it sets out honestly when the memory may be dropped, and when dropping it would be a mistake. *Read alongside:* Nakajima (1958) and Zwanzig (1960) for the original projection-operator papers; Breuer (2002) for the modern textbook treatment this appendix parallels at constitutional register.

In this appendix, we shall supply the full pedagogical derivation of the Nakajima–Zwanzig (NZ) memory-kernel equation that underlies Theorem T2 of Part II. The first-year graduate student ought to be able to follow every step. We assume a working background in quantum mechanics at the Sakurai level and a familiarity with the density-operator formalism. The NZ machinery is, very much like a partial-fraction expansion in calculus, a piece of standard technology that becomes powerful once one sees what it is doing.

C.1 The physical setting

A finite observer is in possession of a quantum apparatus whose "system" degrees of freedom have a finite-dimensional Hilbert space $\mathcal{H}_{\mathrm{Bdy}}$. The system is coupled to an "environment" (everything the observer cannot resolve directly: thermal phonons, KK modes beyond detector reach, the rest of the laboratory) with Hilbert space $\mathcal{H}_{\mathrm{env}}$. The total Hilbert space is the tensor product $\mathcal{H}_{\mathrm{Bulk}} = \mathcal{H}_{\mathrm{Bdy}} \otimes \mathcal{H}_{\mathrm{env}}$, and the joint state of the combined system is a density operator $\rho_{\mathrm{Bulk}}(t) \in \mathcal{B}(\mathcal{H}_{\mathrm{Bulk}})$ that evolves under a Hamiltonian H_{Bulk}.

Why this matters. The observer can directly access only the system part: she does not have the experimental capacity to measure observables on the environment. The mathematical question is therefore: *given the joint dynamics on $\mathcal{H}_{\mathrm{Bulk}}$, what is the effective dynamics for the system alone, with the environment integrated out?* The Nakajima–Zwanzig answer is: an exact integro-differential equation for the system density operator, with a memory kernel that encodes the influence of the environment.

C.2 The reduced density operator

The reduced density operator of the system is

$$\rho_{\mathrm{Bdy}}(t) \;=\; \mathrm{Tr}_{\mathrm{env}}\big[\rho_{\mathrm{Bulk}}(t)\big], \tag{227}$$

where $\mathrm{Tr}_{\mathrm{env}}$ is the partial trace over the environment Hilbert space. Operationally, ρ_{Bdy} encodes everything the system observer can register: any system observable A on $\mathcal{H}_{\mathrm{Bdy}}$ has expectation value

$$\langle A\rangle_t \;=\; \mathrm{Tr}_{\mathrm{Bdy}}\big[A\,\rho_{\mathrm{Bdy}}(t)\big] \;=\; \mathrm{Tr}_{\mathrm{Bulk}}\big[(A\otimes \mathbf{1}_{\mathrm{env}})\,\rho_{\mathrm{Bulk}}(t)\big]. \tag{228}$$

Pedagogical aside. Equation (228) is the operational definition of "what the system observer measures". It says: the expectation value of any system observable equals the full bulk expectation value of A tensored with the identity on the environment. This is manifestly consistent with the partial-trace definition (227).

C.3 The Liouville superoperator

The bulk Hamiltonian H_{Bulk} generates a Liouville superoperator $\mathcal{L}_{\text{Bulk}}$ acting on $\mathcal{B}(\mathcal{H}_{\text{Bulk}})$:

$$\mathcal{L}_{\text{Bulk}} X \;:=\; -i[H_{\text{Bulk}},\, X] \quad \forall X \in \mathcal{B}(\mathcal{H}_{\text{Bulk}}), \tag{229}$$

where $[\cdot,\cdot]$ is the commutator. In terms of $\mathcal{L}_{\text{Bulk}}$, the von Neumann equation $i\partial_t \rho_{\text{Bulk}} = [H_{\text{Bulk}}, \rho_{\text{Bulk}}]$ becomes the *Liouville–von Neumann equation*

$$\partial_t \rho_{\text{Bulk}}(t) \;=\; \mathcal{L}_{\text{Bulk}}\, \rho_{\text{Bulk}}(t). \tag{230}$$

Working with superoperators rather than wavefunctions is the natural language for open systems: the projection onto a sub-system that follows is most cleanly expressed at the superoperator level.

C.4 The projection superoperator $\mathcal{P}$

Choose a fiducial environment state $\rho_{\text{env}}^{(0)}$ (typically the Gibbs state at the bath temperature, but the construction is independent of this specific choice). Define the projection superoperator

$$\mathcal{P} X \;:=\; [\text{Tr}_{\text{env}}\, X] \otimes \rho_{\text{env}}^{(0)}, \qquad \mathcal{Q} \;:=\; \mathbf{1} - \mathcal{P}. \tag{231}$$

Direct computation establishes:

$$\mathcal{P}^2 = \mathcal{P}, \qquad \mathcal{Q}^2 = \mathcal{Q}, \qquad \mathcal{P}\mathcal{Q} = \mathcal{Q}\mathcal{P} = 0. \tag{232}$$

Why $\mathcal{P}$ acts this way. Apply $\mathcal{P}$ to a generic $X = \sum_i A_i \otimes B_i$: $\mathcal{P}X = \sum_i A_i \cdot \text{Tr}_{\text{env}}(B_i) \otimes \rho_{\text{env}}^{(0)}$. The system part keeps only the trace-of-environment-piece A_i (weighted by the environment trace of B_i), and the environment part is replaced by the fiducial state $\rho_{\text{env}}^{(0)}$. This is the standard "factorize and tensor with the fiducial environment" operation that defines the projected state.

Apply $\mathcal{P}$ twice: $\mathcal{P}^2 X = \mathcal{P}[\text{Tr}_{\text{env}}\, X \otimes \rho_{\text{env}}^{(0)}] = \text{Tr}_{\text{env}}(\text{Tr}_{\text{env}}\, X \otimes \rho_{\text{env}}^{(0)}) \otimes \rho_{\text{env}}^{(0)} = \text{Tr}_{\text{env}}\, X \cdot \text{Tr}_{\text{env}}\, \rho_{\text{env}}^{(0)} \otimes \rho_{\text{env}}^{(0)} = \text{Tr}_{\text{env}}\, X \otimes \rho_{\text{env}}^{(0)} = \mathcal{P}X$ (using $\text{Tr}_{\text{env}}\, \rho_{\text{env}}^{(0)} = 1$). Hence $\mathcal{P}^2 = \mathcal{P}$.

C.5 Splitting the Liouville equation

Apply $\mathcal{P}$ and $\mathcal{Q}$ to the Liouville–von Neumann equation (230). Use the identity $\rho_{\text{Bulk}} = \mathcal{P}\rho_{\text{Bulk}} + \mathcal{Q}\rho_{\text{Bulk}}$ to insert $\mathcal{P} + \mathcal{Q}$ between $\mathcal{L}_{\text{Bulk}}$ and ρ_{Bulk}:

$$\partial_t(\mathcal{P}\rho_{\text{Bulk}}) = \mathcal{P}\mathcal{L}_{\text{Bulk}}(\mathcal{P}\rho_{\text{Bulk}}) + \mathcal{P}\mathcal{L}_{\text{Bulk}}(\mathcal{Q}\rho_{\text{Bulk}}), \tag{233}$$

$$\partial_t(\mathcal{Q}\rho_{\text{Bulk}}) = \mathcal{Q}\mathcal{L}_{\text{Bulk}}(\mathcal{P}\rho_{\text{Bulk}}) + \mathcal{Q}\mathcal{L}_{\text{Bulk}}(\mathcal{Q}\rho_{\text{Bulk}}). \tag{234}$$

This is a coupled pair of equations: the projected state $\mathcal{P}\rho_{\mathrm{Bulk}}$ (which contains ρ_{Bdy} as the $\mathrm{Tr}_{\mathrm{env}}$ part) mixes with the unprojected complement $\mathcal{Q}\rho_{\mathrm{Bulk}}$ via the bulk dynamics.

Pedagogical aside. Note the structural parallel with the projection technique in linear algebra: any linear evolution can be split into block-form along complementary subspaces, with off-diagonal blocks coupling the two. Here $\mathcal{P}$ and $\mathcal{Q}$ play the role of the two complementary projectors, and $\mathcal{L}_{\mathrm{Bulk}}$ is the linear "matrix" acting on the full superoperator space.

C.6 Solving the $\mathcal{Q}$-equation

Equation (234) is a first-order linear ODE in $\mathcal{Q}\rho_{\mathrm{Bulk}}$, with the $\mathcal{P}\rho_{\mathrm{Bulk}}$ term as inhomogeneous source. Its formal solution is, by the standard method of variation of parameters,

$$\mathcal{Q}\rho_{\mathrm{Bulk}}(t) \;=\; \mathcal{G}_{\mathcal{Q}}(t,t_0)\,\mathcal{Q}\rho_{\mathrm{Bulk}}(t_0) \;+\; \int_{t_0}^{t} ds\,\mathcal{G}_{\mathcal{Q}}(t,s)\,\mathcal{Q}\mathcal{L}_{\mathrm{Bulk}}\,\mathcal{P}\rho_{\mathrm{Bulk}}(s), \tag{235}$$

where the $\mathcal{Q}$-subspace propagator is the time-ordered exponential

$$\mathcal{G}_{\mathcal{Q}}(t,s) \;:=\; \mathcal{T}\exp\Big[\int_{s}^{t} du\,\mathcal{Q}\mathcal{L}_{\mathrm{Bulk}}\mathcal{Q}\Big]. \tag{236}$$

The first term in (235) is the homogeneous solution that propagates the initial $\mathcal{Q}\rho_{\mathrm{Bulk}}(t_0)$ forward; the second term is the particular solution driven by the projected state.

C.7 The exact NZ memory-kernel equation

Substitute the formal solution (235) for $\mathcal{Q}\rho_{\mathrm{Bulk}}$ back into the projected equation (233):

$$\boxed{\partial_t\mathcal{P}\rho_{\mathrm{Bulk}}(t) \;=\; \mathcal{P}\mathcal{L}_{\mathrm{Bulk}}\mathcal{P}\rho_{\mathrm{Bulk}}(t) \;+\; \int_{t_0}^{t} ds\,K(t,s)\,\mathcal{P}\rho_{\mathrm{Bulk}}(s) \;+\; \mathcal{I}(t),} \tag{237}$$

with the memory kernel and the inhomogeneous initial-correlation term

$$K(t,s) \;:=\; \mathcal{P}\mathcal{L}_{\mathrm{Bulk}}\,\mathcal{G}_{\mathcal{Q}}(t,s)\,\mathcal{Q}\mathcal{L}_{\mathrm{Bulk}}\mathcal{P}, \tag{238}$$

$$\mathcal{I}(t) \;:=\; \mathcal{P}\mathcal{L}_{\mathrm{Bulk}}\,\mathcal{G}_{\mathcal{Q}}(t,t_0)\,\mathcal{Q}\rho_{\mathrm{Bulk}}(t_0). \tag{239}$$

Equation (237) is exact. No Markov approximation, no Born approximation, no weak-coupling assumption. It is the axiomatic starting point for the boundary law GDOS of T6.

Physics intuition. The three terms have direct physical meaning. The first term, $\mathcal{P}\mathcal{L}_{\mathrm{Bulk}}\mathcal{P}\rho_{\mathrm{Bulk}}$, is the local Hamiltonian piece: the boundary state evolves under an effective Hamiltonian inherited from the bulk Hamiltonian, projected onto the boundary subspace. The second term, the convolution $\int K(t,s)\mathcal{P}\rho_{\mathrm{Bulk}}(s)\,ds$, is the *memory term*: the inaccessible environment "remembers" the prior boundary state and feeds it back at later times. The third term, $\mathcal{I}(t)$, is the *initial-correlation term*: it encodes any entanglement between boundary and environment at the initial time t_0. For initially

factorizable states $\rho_{\mathrm{Bulk}}(t_0) = \rho_{\mathrm{Bdy}}(t_0) \otimes \rho_{\mathrm{env}}^{(0)}$, $\mathcal{I}(t) = 0$ identically and the homogeneous form of the memory-kernel equation applies.

C.8 Recovery of the standard Markov-limit form

In the limit where the environment correlation time τ_{env} is much shorter than the boundary system relaxation timescale, the memory kernel $K(t,s)$ becomes sharply peaked around $s = t$:

$$K(t,s) \;\approx\; 2\mathcal{K}(t)\,\delta(t-s), \tag{240}$$

and the convolution integral collapses to $\int K(t,s)\,\mathcal{P}\rho_{\mathrm{Bulk}}(s)\,ds \approx \mathcal{K}(t)\,\mathcal{P}\rho_{\mathrm{Bulk}}(t)$. The NZ equation then reduces to the time-local master equation

$$\partial_t \rho_{\mathrm{Bdy}}(t) \;=\; \mathcal{L}_{\mathsf{GDOS}}\,\rho_{\mathrm{Bdy}}(t), \qquad \mathcal{L}_{\mathsf{GDOS}} \;=\; \mathcal{P}\mathcal{L}_{\mathrm{Bulk}}\mathcal{P}|_{\mathrm{Bdy}} \;+\; \mathcal{K}, \tag{241}$$

which (in T4 of Part II) is shown to take the canonical GKLS form. This is the Born–Markov limit. The explicit derivation is given in Appendix C below.

C.9 Worked example: the spin-boson model

The most pedagogically transparent setting for the NZ machinery is the *spin-boson model*: a two-level system (qubit, spin $\frac{1}{2}$) coupled linearly to a bath of harmonic oscillators. This is the textbook example for decoherence and dissipation in quantum systems, and it lets us see every ingredient of the NZ construction explicitly.

Setup. The qubit Hilbert space is $\mathcal{H}_{\mathrm{Bdy}} = \mathbb{C}^2$ with basis $\{|0\rangle, |1\rangle\}$ and Hamiltonian $H_{\mathrm{Bdy}} = \frac{1}{2}\omega_0\sigma_z$. The bath consists of infinitely many harmonic oscillators with creation/annihilation operators $b_k^\dagger, b_k$ and frequencies ω_k: $H_{\mathrm{env}} = \sum_k \omega_k b_k^\dagger b_k$. The qubit-bath coupling in the simplest dephasing form is

$$H_{\mathrm{int}} \;=\; \sigma_z \otimes \sum_k g_k (b_k + b_k^\dagger), \tag{242}$$

with coupling constants g_k. The total Hamiltonian is $H_{\mathrm{Bulk}} = H_{\mathrm{Bdy}} + H_{\mathrm{env}} + H_{\mathrm{int}}$.

The bath spectral density. The relevant property of the bath is the spectral density $J(\omega) := \sum_k |g_k|^2 \delta(\omega - \omega_k)$, which characterizes the strength of the qubit-bath coupling at each frequency. The standard "Ohmic" spectral density is $J(\omega) = \alpha\omega\, e^{-\omega/\omega_c}$, with α the dimensionless coupling strength and ω_c the cutoff frequency. At thermal equilibrium $\rho_{\mathrm{env}}^{(0)} = e^{-\beta H_{\mathrm{env}}}/Z$, the bath correlation function is

$$C(t) \;:=\; \mathrm{Tr}_{\mathrm{env}}\Big[B(t)\,B(0)\,\rho_{\mathrm{env}}^{(0)}\Big] \;=\; \int_0^\infty d\omega\, J(\omega)\big[\coth(\beta\omega/2)\cos\omega t - i\sin\omega t\big], \tag{243}$$

with $B := \sum_k g_k(b_k + b_k^\dagger)$.

Applying the NZ machinery. The Liouvillian splits as $\mathcal{L}_{\text{Bulk}} = \mathcal{L}_{\text{Bdy}} + \mathcal{L}_{\text{env}} + \lambda\mathcal{L}_{\text{int}}$, with λ the small coupling. Choose the projector $\mathcal{P}X = (\text{Tr}_{\text{env}}\, X) \otimes \rho_{\text{env}}^{(0)}$ and the bath in thermal equilibrium. Direct computation gives $\mathcal{P}\mathcal{L}_{\text{env}}\mathcal{P} = 0$ (the bath Liouvillian annihilates the projection because $\text{Tr}_{\text{env}}\, \rho_{\text{env}}^{(0)} = 1$ is constant in the bath sector) and $\mathcal{P}\mathcal{L}_{\text{int}}\mathcal{P} = 0$ (the linear coupling has zero one-point function in the thermal bath). The exact NZ equation (237) therefore reduces to the *homogeneous* memory-kernel equation

$$\partial_t \rho_{\text{Bdy}}(t) \;=\; -i[H_{\text{Bdy}}, \rho_{\text{Bdy}}(t)] \;+\; \int_0^t ds\, \mathcal{K}(t-s)\, \rho_{\text{Bdy}}(s), \tag{244}$$

with the second-order memory kernel

$$\mathcal{K}(\tau)\rho_{\text{Bdy}} \;=\; -\lambda^2 \Big\{ C(\tau)\, [\sigma_z, \sigma_z(-\tau)\rho_{\text{Bdy}}] + C^*(\tau)\, [\rho_{\text{Bdy}}\sigma_z(-\tau), \sigma_z] \Big\}, \tag{245}$$

where $\sigma_z(-\tau) = e^{-iH_{\text{Bdy}}\tau}\sigma_z e^{iH_{\text{Bdy}}\tau} = \sigma_z$ in this case (because σ_z commutes with H_{Bdy}).

The dephasing limit. For pure-dephasing coupling (242), $[\sigma_z, \sigma_z] = 0$, so the populations ρ_{00} and ρ_{11} are conserved, but the coherences ρ_{01}, ρ_{10} decay. Solving (244) for the off-diagonal element gives

$$\rho_{01}(t) \;=\; \rho_{01}(0)\, e^{-i\omega_0 t}\, \exp[-\Gamma(t)], \qquad \Gamma(t) \;=\; 4\lambda^2 \int_0^\infty d\omega\, \frac{J(\omega)\coth(\beta\omega/2)}{\omega^2} [1 - \cos(\omega t)]. \tag{246}$$

For Ohmic coupling at high temperature ($k_B T \gg \omega_c$), $\Gamma(t) \approx \gamma t$ with $\gamma \propto \alpha k_B T$: exponential decoherence, recovering the GKLS form of T4 with rate γ that scales linearly with temperature.

Physics intuition. The spin-boson model is the "hydrogen atom" of open-system theory. Every feature of the GG-Theory boundary law GDOS appears here in miniature: the Hilbert-space split between qubit and bath, the projection that integrates out the bath, the memory kernel that encodes the bath response function, the Born-Markov approximation that collapses the kernel to a delta function, and the resulting exponential decoherence at rate γ. The pedagogical point is that the NZ machinery is not an exotic abstract construction; it is the standard textbook tool for any open quantum system, with the GG-Theory bulk supplying the role that the harmonic bath plays in this elementary example.

C.10 When the Markov approximation breaks

The standard Markov approximation requires $\tau_{\text{env}} \ll \tau_R$ (bath correlation time much shorter than system relaxation time). When this fails — as it does in ultrafast spectroscopy on femtosecond scales, in strong-coupling cavity QED, in low-temperature spin-bath problems, and in high-resolution experiments approaching the KK threshold $M_{\text{lock}} \sim 4$ TeV in the GG-Theory setting — the boundary dynamics is *non-Markovian*: the memory kernel cannot be approximated as a delta function. In this regime the full NZ equation (237) must be retained, with the time-convolution integral encoding the memory of the bath. Non-Markovian quantum dynamics is an active research area; the axiomatic

content of T2 of Part II is that the framework supplies the exact NZ equation as the starting point, with the GKLS form of T4 emerging only as the Markov-limit special case.

Problems and worked solutions. C.1.1. Push the spin–boson example past its Markov limit: derive the Born–Markov validity criterion and evaluate it for a phonon-like bath. *Solution.* Step 1: the criterion. The Markov step replaces the kernel $K(t-s)$ by a delta function, which is legitimate only if the bath correlation time τ_B (the decay time of $C(\tau)$) is short compared with the system's dissipation time $1/\gamma$: the small dimensionless parameter is $\gamma\tau_B \ll 1$. Step 2: evaluate. For an Ohmic bath with cutoff ω_c, $\tau_B \sim 1/\omega_c$; with phonon-scale $\omega_c = 10^{13}$ s^{-1} and a qubit damping $\gamma = 10^9$ s^{-1}: $\gamma\tau_B = 10^{-4}$ — deeply Markovian, and the GKLS description of T4 is excellent. At $\gamma = 10^{12}$ s^{-1} the parameter reaches 0.1 and memory effects surface. Step 3: the constitutional reading. POS-5 does not assert that every window is Markovian; it asserts that the coarse-graining under which the dictionary commutes with RG flow is the one in which the kernel contracts — the criterion above is that statement in laboratory units. Physics intuition: Markovianity is a ratio of clocks, not a law; the KK tower's breadth (II.2.2) makes τ_B short, and this problem prices exactly how short is short enough.

C.1.2. Show that the inhomogeneous NZ term $I(t)$ vanishes for factorized initial conditions, and state what this buys the boundary law. *Solution.* Step 1: locate the term. The exact NZ equation carries, besides the memory kernel, an inhomogeneity $I(t) \propto \mathcal{P}L\, e^{\mathcal{Q}Lt}\, \mathcal{Q}\rho(0)$ driven by the initial condition's projection complement. Step 2: kill it. For a factorized preparation $\rho(0) = \rho_{\mathrm{Bdy}}(0) \otimes \rho^{(0)}_{\mathrm{env}}$, the definition of $\mathcal{P}$ gives $\mathcal{P}\rho(0) = \rho(0)$ exactly, hence $\mathcal{Q}\rho(0) = (\mathbf{1}-\mathcal{P})\rho(0) = 0$ and $I(t) \equiv 0$ for all times — the master equation becomes homogeneous in the boundary state alone. Step 3: the buy. The boundary law of T2 is autonomous precisely when the window is opened on an uncorrelated past; preparation correlations appear as a decaying inhomogeneity, not as a breakdown — which is why laboratory state preparation (which factorizes by construction) sees clean GKLS dynamics. Physics intuition: $I(t)$ is the memory of a past the observer did not prepare; a fresh preparation has no such past, and the formalism says so by canceling the term identically rather than by assumption.

Take-away. *Standard account:* master equations are derived under Born–Markov assumptions whose validity is asserted per model. *What it left open:* which projector, which reference state, and when memory may honestly be dropped. *The constitutional derivation:* the bulk–boundary geometry fixes $\mathcal{P}$ and the reference state, the exact memory-kernel equation of T2 precedes every approximation, and the Markov step is priced by $\gamma\tau_B$ — a checkable ratio, not a hope.

D Stinespring Dilation and Kraus Decomposition

The result behind this appendix is one of the most useful in physics and one of the least advertised: every physically admissible evolution of an open system, however irreversible it looks, is a reversible evolution of something larger seen through a window. The dilation theorem says so; the decomposition that follows from it gives the working form. The familiar channels of quantum information —

amplitude damping, depolarizing, dephasing — appear here not as a catalog to be memorized but as instances of the single form the boundary law forces.

Read alongside: Stinespring (1955) for the dilation theorem; Choi (1975) for the Choi-matrix characterization; Nielsen (2010) for the channel formalism at course register.

This appendix supplies the full pedagogical derivation of the Stinespring dilation theorem and the equivalent Kraus decomposition for completely positive trace-preserving (CPTP) maps. These two theorems are the mathematical core of T3 of Part II: the boundary evolution channel $\Phi_{t,s}$ is automatically CPTP because it arises from partial trace over a unitarily-evolving extension.

D.1 Why CPTP, and why Stinespring is the key

A density operator on $\mathcal{H}_{\text{Bdy}}$ must remain positive (so that $\text{Tr}(\Pi\rho) \geq 0$ for any projector Π, i.e., probabilities are non-negative) and trace one (so that probabilities sum to one). A linear map $\Phi : \mathcal{B}(\mathcal{H}_{\text{Bdy}}) \to \mathcal{B}(\mathcal{H}_{\text{Bdy}})$ that preserves these two properties is called *positive* and *trace preserving*. But mere positivity is not enough: in quantum mechanics, the system can be entangled with another system, and applying Φ to one component must not produce negative probabilities for the joint state. This stronger requirement is *complete positivity*: $\Phi \otimes \mathbf{1}_n$ must be positive for every $n \geq 1$.

Counterexample: the transpose. The transpose map $T : \rho \mapsto \rho^T$ on a single qubit is positive (the transpose of a positive matrix is positive) but not completely positive: for an entangled two-qubit state $|\Psi^-\rangle\langle\Psi^-|$, applying T to one qubit and the identity to the other gives a matrix with negative eigenvalues, violating positivity for the joint system. This is why the transpose is not a physically admissible quantum operation.

The Stinespring theorem (Stinespring, 1955) gives a constructive characterization of all CPTP maps.

D.2 Statement and proof sketch

Theorem 31 (Stinespring dilation (Stinespring, 1955))**.** *A linear map $\Phi : \mathcal{B}(\mathcal{H}_{\text{Bdy}}) \to \mathcal{B}(\mathcal{H}_{\text{Bdy}})$ is completely positive and trace preserving if and only if there exists a Hilbert space $\mathcal{H}_{\text{env}}$ and an isometry $V : \mathcal{H}_{\text{Bdy}} \to \mathcal{H}_{\text{Bdy}} \otimes \mathcal{H}_{\text{env}}$ (i.e., $V^\dagger V = \mathbf{1}_{\text{Bdy}}$) such that*

$$\boxed{\Phi(\rho) \;=\; \text{Tr}_{\text{env}}(V\rho V^\dagger).} \tag{247}$$

Proof sketch. ($\Leftarrow$) If Φ has the form (247), then Φ is the composition of three operations: conjugation by the isometry V (positive and trace preserving), partial trace over the dilation Hilbert space $\mathcal{H}_{\text{env}}$ (positive and trace preserving). The composition of positive trace-preserving maps is positive trace preserving, and the partial trace structure makes the composition completely positive (this is the standard Stinespring direction).

($\Rightarrow$) Conversely, given any CPTP map Φ, one constructs V explicitly via the Choi–Jamiołkowski isomorphism (Choi, 1975): the Choi matrix $C_\Phi = (\Phi \otimes \mathbf{1})|\Omega\rangle\langle\Omega|$ (with $|\Omega\rangle$ a maximally entangled state) is positive semi-definite by complete positivity, and its spectral decomposition $C_\Phi = \sum_\alpha |v_\alpha\rangle\langle v_\alpha|$ supplies the Stinespring isometry via $V = \sum_\alpha |v_\alpha\rangle\langle\alpha|$.

D.3 Kraus decomposition from Stinespring

Choose an orthonormal basis $\{|e_\alpha\rangle\}$ for the dilation Hilbert space $\mathcal{H}_{\text{env}}$ and define the operators

$$K_\alpha \;:=\; \langle e_\alpha|V \;\in\; \mathcal{B}(\mathcal{H}_{\text{Bdy}}). \tag{248}$$

Substituting back into (247):

$$\begin{aligned}\Phi(\rho) = \text{Tr}_{\text{env}}(V\rho V^\dagger) \;&=\; \sum_\alpha \langle e_\alpha|V\rho V^\dagger|e_\alpha\rangle \;=\; \sum_\alpha (\langle e_\alpha|V)\rho(V^\dagger|e_\alpha\rangle) \\ &= \sum_\alpha K_\alpha \rho K_\alpha^\dagger.\end{aligned} \tag{249}$$

The trace-preservation condition $V^\dagger V = \mathbf{1}_{\text{Bdy}}$ becomes

$$\sum_\alpha K_\alpha^\dagger K_\alpha \;=\; V^\dagger\Big(\sum_\alpha |e_\alpha\rangle\langle e_\alpha|\Big)V \;=\; V^\dagger V \;=\; \mathbf{1}_{\text{Bdy}}. \tag{250}$$

Combining (249) and (250) gives the Kraus decomposition

$$\boxed{\Phi(\rho) \;=\; \sum_\alpha K_\alpha \rho K_\alpha^\dagger, \qquad \sum_\alpha K_\alpha^\dagger K_\alpha \;=\; \mathbf{1}_{\text{Bdy}}.} \tag{251}$$

Physics intuition. The Kraus operators K_α correspond to the "branches" of the quantum operation that the apparatus may register. Each branch α has its own back-action on the system state given by $K_\alpha \rho K_\alpha^\dagger$ (un-normalized; the probability of that branch is $\text{Tr}[K_\alpha \rho K_\alpha^\dagger]$). Summing over all branches gives the unconditional channel Φ. In a measurement context, conditioning on the registered branch α gives the conditional posterior state of T7.4 in Part III.

D.4 Worked example: the amplitude-damping channel

Let $\mathcal{H}_{\text{Bdy}} = \mathbb{C}^2$ (single qubit) coupled to a bath $\mathcal{H}_{\text{env}} = \mathbb{C}^2$ initially in the ground state $|0_{\text{env}}\rangle$. The bulk evolution is the unitary

$$U \;=\; \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-p} & \sqrt{p} & 0 \\ 0 & -\sqrt{p} & \sqrt{1-p} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{(in basis } |00\rangle, |01\rangle, |10\rangle, |11\rangle\text{)}. \tag{252}$$

The Stinespring isometry $V|0\rangle = |00\rangle$, $V|1\rangle = \sqrt{1-p}|10\rangle + \sqrt{p}|01\rangle$ gives

$$V \;=\; \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{p} \\ 0 & \sqrt{1-p} \\ 0 & 0 \end{pmatrix}, \qquad V^\dagger V \;=\; \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \;=\; \mathbf{1}_2. \tag{253}$$

Reading off the Kraus operators from the rows of V:

$$K_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, \qquad K_1 = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}. \tag{254}$$

Verification: $K_0^\dagger K_0 + K_1^\dagger K_1 = \mathrm{diag}(1, 1-p) + \mathrm{diag}(0, p) = \mathbf{1}$. This is the standard amplitude-damping channel of T3 worked example §D.4 with p the damping probability per time step.

D.5 Worked example: the depolarizing channel

The depolarizing channel is the second canonical CPTP example, particularly important in quantum-information theory. It models the physical situation in which the qubit, with probability $1-p$, is left untouched, and with probability p is replaced by a maximally mixed state $\mathbf{1}/2$.

Action of the channel. $\Phi_{\mathrm{depol}}(\rho) = (1-p)\rho + p\,\mathbf{1}/2$. In the Bloch representation $\rho = \frac{1}{2}(\mathbf{1} + \vec{r}\cdot\vec{\sigma})$, this is $\Phi_{\mathrm{depol}}(\rho) = \frac{1}{2}(\mathbf{1} + (1-p)\vec{r}\cdot\vec{\sigma})$: the Bloch vector contracts uniformly toward the center of the Bloch sphere.

Kraus decomposition. A standard Kraus decomposition uses four operators:

$$K_0 = \sqrt{1 - 3p/4}\,\mathbf{1}, \qquad K_i = \sqrt{p/4}\,\sigma_i \text{ for } i = 1, 2, 3. \tag{255}$$

Verification: $\sum_a K_a^\dagger K_a = (1 - 3p/4)\mathbf{1} + (p/4)(\sigma_x^2 + \sigma_y^2 + \sigma_z^2) = (1 - 3p/4)\mathbf{1} + (3p/4)\mathbf{1} = \mathbf{1}$.

Stinespring dilation. The dilation requires a 4-dimensional dilation Hilbert space (because there are 4 Kraus operators), and the isometry $V : \mathbb{C}^2 \to \mathbb{C}^2 \otimes \mathbb{C}^4$ is

$$V|\psi\rangle = \sqrt{1 - 3p/4}\,|\psi\rangle|0\rangle + \sqrt{p/4}\sum_{i=1}^{3} \sigma_i|\psi\rangle|i\rangle. \tag{256}$$

The depolarizing channel is therefore the partial trace of the unitary that entangles the qubit with the four states of a 2-qubit auxiliary system.

Physics intuition. The depolarizing channel is the operational template for a wide class of physical situations: thermal noise on a qubit at infinite temperature, isotropic Pauli errors in quantum computation, and the fully decohered limit of any boundary system whose system-environment coupling is rotationally symmetric. The axiomatic content of T3 is that the depolarizing channel, like all CPTP channels, has a Stinespring dilation through some auxiliary Hilbert space, with the partial trace producing the channel. In the GG-Theory framework the auxiliary Hilbert space is the bulk environment Hilbert space $\mathcal{H}_{\mathrm{env}}$.

D.6 Worked example: the dephasing channel

The dephasing channel is the third canonical CPTP example, which specifically describes loss of quantum coherence without loss of populations. It corresponds to the spin-boson model of §C.9 in the large-time limit.

Action of the channel. $\Phi_{\text{deph}}(\rho)$ leaves diagonal elements unchanged but multiplies off-diagonal elements by a damping factor $1-p$:

$$\Phi_{\text{deph}}\left(\begin{pmatrix}\rho_{00} & \rho_{01}\\ \rho_{10} & \rho_{11}\end{pmatrix}\right) = \begin{pmatrix}\rho_{00} & (1-p)\rho_{01}\\ (1-p)\rho_{10} & \rho_{11}\end{pmatrix}. \tag{257}$$

Kraus decomposition. $K_0 = \sqrt{1-p/2}\,\mathbf{1}$, $K_1 = \sqrt{p/2}\,\sigma_z$. Verification: $K_0^\dagger K_0 + K_1^\dagger K_1 = (1-p/2)\mathbf{1} + (p/2)\sigma_z^2 = (1-p/2)\mathbf{1} + (p/2)\mathbf{1} = \mathbf{1}$.

Physics intuition. The dephasing channel is the operational realization of the spin-boson dephasing model of §C.9: tracing over a thermal photon bath that couples diagonally to the qubit produces exactly this channel, with $p = 1 - e^{-\Gamma t}$ for decoherence rate Γ. This connects the abstract CPTP formalism (T3) to the explicit microscopic model (the spin-boson model of T2 worked example) and shows how the same channel admits multiple equivalent descriptions.

Problems and worked solutions. D.1.1. Compose amplitude damping ($p = \frac{1}{2}$) with phase flip ($q = \frac{1}{4}$) and determine the minimal number of Kraus operators of the composite. *Solution.* Step 1: compose naively. Writing all products $K_i^{\text{PF}} K_j^{\text{AD}}$ gives four Kraus operators — but the minimal number is the rank of the Choi matrix, not the count of a particular representation. Step 2: compute the Choi rank. Building the composite's Choi matrix and diagonalizing gives rank 3 (verified numerically): one of the four written operators is redundant, and a three-operator representation exists. Step 3: the lesson. Kraus representations are massively non-unique; the channel itself is the equivalence class, and the Choi spectrum is its invariant fingerprint — which is why T3 states the boundary law as a channel, never as a preferred operator list. Physics intuition: asking "which Kraus operators does nature use?" is like asking which coordinates a manifold uses; the Choi rank counts the channel's irreducible noise processes, and here two familiar noises merge into three, not four.

D.1.2. Show that dephasing at $q = \frac{1}{2}$ *is* the unread projective measurement, and exhibit the unitary connecting the two Kraus representations. *Solution.* Step 1: two presentations. Phase flip with $q = \frac{1}{2}$ has Kraus pair $\{\mathbf{1}/\sqrt{2}, \sigma_z/\sqrt{2}\}$; the unread σ_z measurement has Kraus pair $\{|0\rangle\langle 0|, |1\rangle\langle 1|\}$. Direct computation on a general state shows the two channels agree exactly (verified). Step 2: the connecting isometry. Kraus sets of the same channel are related by $K_i' = \sum_j U_{ij} K_j$ with U unitary; here U is the Hadamard matrix $\frac{1}{\sqrt{2}}\left(\begin{smallmatrix}1 & 1\\ 1 & -1\end{smallmatrix}\right)$ (verified: it maps one pair onto the other exactly). Step 3: the physics identification. "Maximal dephasing" and "measurement nobody read" are one channel — the operational content of III.4.1's non-selective Lüders map, now seen at the Kraus level. Physics intuition: the environment does not care whether a physicist looks; a

which-path record *is* a measurement, and the unitary freedom of Kraus sets is the formal statement that nature keeps no preferred story of how the record was made.

D.1.3. Relate the depolarizing family's Choi spectrum to its minimal Kraus number across the full CP range $0 \le p \le 4/3$**.** *Solution.* Step 1: recall the spectrum (II.3.1): eigenvalues $1 - 3p/4$ and $p/4$ (threefold). Step 2: count ranks. For $0 < p < 4/3$ all four eigenvalues are positive: minimal Kraus number 4 — the identity plus three Pauli flips, with no shorter presentation possible. At the boundary $p = 4/3$ the eigenvalue $1 - 3p/4$ hits zero: rank drops to 3, and the channel (the best CP approximation to the universal-NOT) needs only the three Pauli operators. At $p = 0$: rank 1, the identity. Step 3: the boundary-law moral. The edge of complete positivity is exactly where a noise process exits the channel's support — CP is not a bookkeeping inequality but a census of physically realizable noise, and the Choi spectrum is where the census is taken. Physics intuition: channels live in a convex body; T3 guarantees the boundary law stays inside it, and this problem walks the student to the wall and shows what happens at contact.

Take-away. *Standard account:* Kraus operators are introduced as an ansatz for noise, channel by channel. *What it left open:* why every physical evolution has this form, and what is invariant across the ansatz's freedom. *The constitutional derivation:* Stinespring makes every CPTP map a bulk unitary seen through a window (T3), the Choi spectrum is the representation-free fingerprint, and the channel zoo of this appendix is the complete local phenomenology of the one boundary law.

E GKLS Master Equation in Full Detail

If the memory in the exact reduction is short compared with everything else in the problem, the equation collapses to a canonical form, and this appendix derives that form three ways: from the semigroup property directly, and by the two standard routes through weak coupling. It also does the thing those derivations usually leave implicit, which is to show why the rates that appear cannot be negative. The damped harmonic oscillator worked at the end is to open systems what the hydrogen atom is to closed ones — solvable, thermal, and representative of far more than itself.
Read alongside: Gorini (1976) and Lindblad (1976) for the two original 1976 derivations; Breuer (2002) for the Born–Markov and Davies routes followed here.

This appendix supplies the full pedagogical derivation of the Gorini–Kossakowski–Lindblad–Sudarshan (GKLS) master equation that emerges in T4 of Part II. The starting point is the exact NZ equation (Appendix A); the endpoint is the canonical time-local form.

E.1 The semigroup property and its generator

When the environment correlation time τ_{env} is much shorter than the boundary timescales, the boundary evolution $\Phi_{t,s}$ becomes a one-parameter semigroup: $\Phi_{t,s} = \Phi_{t-s}$ depends only on the elapsed time $t - s$, and

$$\Phi_t \circ \Phi_s \;=\; \Phi_{t+s} \quad \forall t, s \ge 0, \qquad \Phi_0 = \mathbf{1}. \tag{258}$$

Such a semigroup of CPTP maps has, in finite dimension, a unique generator $\mathcal{L}$ such that

$$\Phi_t \;=\; e^{t\mathcal{L}}, \qquad \partial_t \rho_{\mathrm{Bdy}}(t) \;=\; \mathcal{L}\, \rho_{\mathrm{Bdy}}(t). \tag{259}$$

The generator $\mathcal{L}$ is the axiomatic content of the boundary dynamics in the Markovian regime.

E.2 The general form of $\mathcal{L}$

The Lindblad theorem (Lindblad, 1976) (in finite dimension also the Gorini–Kossakowski–Sudarshan theorem (Gorini, 1976)) states that any generator $\mathcal{L}$ of a one-parameter semigroup of CPTP maps on $\mathcal{B}(\mathcal{H})$ has the canonical form

$$\boxed{\mathcal{L}(\rho) \;=\; -i[H_{\text{eff}},\,\rho] \;+\; \sum_{\alpha} \gamma_\alpha \Big(L_\alpha \rho L_\alpha^\dagger - \tfrac{1}{2}\{L_\alpha^\dagger L_\alpha,\,\rho\}\Big),} \tag{260}$$

with $H_{\text{eff}} = H_{\text{eff}}^\dagger$ a self-adjoint effective Hamiltonian, L_α a set of jump operators (traceless without loss of generality), and $\gamma_\alpha \geq 0$ a set of non-negative rates.

Pedagogical aside. Equation (260) is the most general linear evolution that simultaneously preserves the Hermiticity, the unit trace, and the complete positivity of ρ. The first term is the unitary evolution by H_{eff}; the second term is the dissipative Lindblad piece. Each jump operator L_α represents one channel of system–environment exchange (e.g., spontaneous photon emission, phonon absorption, decoherence into a pointer basis), and each γ_α is the rate of that channel.

E.3 Born–Markov derivation from the NZ equation

We now show how the GKLS form (260) emerges from the NZ equation (237) in the Born–Markov limit.

Setup. Decompose the bulk Hamiltonian as $H_{\text{Bulk}} = H_{\text{Bdy}} + H_{\text{env}} + \lambda H_{\text{int}}$, where λ is a small coupling constant and $H_{\text{int}} = \sum_\alpha A_\alpha \otimes B_\alpha$ couples the boundary system operators $\{A_\alpha\}$ to the environment operators $\{B_\alpha\}$. Move to the interaction picture, where the unperturbed Hamiltonian $H_0 = H_{\text{Bdy}} + H_{\text{env}}$ defines the free evolution and only the coupling λH_{int} remains active.

Born approximation. Expand the NZ memory kernel $K(t,s)$ to second order in λ. Because $\mathcal{Q}\mathcal{L}_{\text{Bulk}} = \lambda \mathcal{Q}\mathcal{L}_{\text{int}}$ to leading non-trivial order (the unperturbed Liouvillian commutes with the projection algebra), the kernel reduces to

$$K(t,s) \;\approx\; -\lambda^2\, \mathcal{P}\, \mathcal{L}_{\text{int}}\, \mathcal{Q}\, \mathcal{L}_{\text{int}}\, \mathcal{P} \;+\; O(\lambda^3). \tag{261}$$

Substituting $\mathcal{L}_{\text{int}} X = -i[H_{\text{int}}, X]$ and tracing out the environment piece using $\text{Tr}_{\text{env}}[B_\alpha(t-s)\, B_\beta(0)\, \rho_{\text{env}}^{(0)}] = C_{\alpha\beta}(t-s)$ (the environment correlation function), one obtains

$$\partial_t \rho_{\text{Bdy}}(t) \;=\; -i[H_{\text{Bdy}}, \rho_{\text{Bdy}}] \;-\; \lambda^2 \int_0^t d\tau \sum_{\alpha\beta} \Big\{ C_{\alpha\beta}(\tau)\, [A_\alpha, A_\beta(-\tau)\, \rho_{\text{Bdy}}] + \text{h.c.} \Big\}, \tag{262}$$

with $A_\beta(-\tau) = e^{-iH_{\text{Bdy}}\tau} A_\beta e^{iH_{\text{Bdy}}\tau}$ the free-evolved system operator.

Markov approximation. If the environment correlation function $C_{\alpha\beta}(\tau)$ decays on a timescale τ_{env} much shorter than the system relaxation time τ_R, then for times $t \gg \tau_{\text{env}}$ the upper limit of the τ-integral can be extended to infinity without significant error. Equation (262) becomes time-local in the boundary state.

Secular approximation. Decompose the system operators in the eigenbasis of H_{Bdy}: $A_\alpha = \sum_\omega A_\alpha(\omega)$ with $[H_{\text{Bdy}}, A_\alpha(\omega)] = -\omega A_\alpha(\omega)$. Substituting and discarding rapidly oscillating cross-terms (terms with $\omega \neq \omega'$ that average to zero on the system relaxation timescale), the result organizes into the canonical GKLS form (260) with

$$\gamma_{\alpha\beta}(\omega) \;=\; \lambda^2 \int_{-\infty}^{\infty} d\tau\, e^{i\omega\tau}\, C_{\alpha\beta}(\tau) \tag{263}$$

(the Fourier transform of the environment correlation function at the Bohr frequency ω) and jump operators $L_\alpha(\omega) = A_\alpha(\omega)$.

E.4 Positivity of the rates

The matrix $\gamma_{\alpha\beta}(\omega)$ is positive semi-definite:

$$\sum_{\alpha\beta} v_\alpha^* \gamma_{\alpha\beta}(\omega) v_\beta \;=\; \lambda^2 \int_{-\infty}^{\infty} d\tau\, e^{i\omega\tau} \sum_{\alpha\beta} v_\alpha^* C_{\alpha\beta}(\tau) v_\beta \;\geq\; 0, \tag{264}$$

because the integrand is the Fourier transform of a positive-definite correlation function (Bochner's theorem). Diagonalizing $\gamma_{\alpha\beta}(\omega)$ yields a new set of jump operators $\{L_\alpha\}$ with rates $\gamma_\alpha \geq 0$, completing the canonical GKLS form.

Physics intuition. The Born–Markov procedure isolates the leading slow timescale of the joint dynamics (the boundary relaxation timescale τ_R) and integrates out everything faster (the environment correlation timescale τ_{env}). The result is a memoryless, completely positive evolution on the boundary alone, with the rates γ_α encoding the spectrum of environmental fluctuations at the relevant Bohr frequencies. This is the constitutional content of the GKLS form in T4 of Part II.

E.5 Worked example: damped quantum harmonic oscillator

The damped quantum harmonic oscillator is the textbook GKLS example. It governs the dynamics of any quantum oscillator (a single mode of the electromagnetic field in a cavity, a mechanical mode of a nano-resonator, the center-of-mass motion of a trapped atom, an LC circuit, a single phonon in a solid) coupled weakly to a thermal environment.

Setup. The oscillator Hamiltonian is $H_{\text{Bdy}} = \omega\, a^\dagger a$ with $[a, a^\dagger] = \mathbf{1}$. The environment is a thermal bath of harmonic oscillators at temperature T with mean thermal occupation $\bar{n} = (e^{\beta\omega} - 1)^{-1}$. At weak coupling, the Born-Markov derivation (261)–(263) yields the GKLS generator with two jump

operators:

$$\begin{aligned} L_1 &= \sqrt{\gamma(\bar{n}+1)}\, a \quad \text{(downward jump: emission)}, \\ L_2 &= \sqrt{\gamma\,\bar{n}}\, a^\dagger \quad \text{(upward jump: absorption)}, \end{aligned} \tag{265}$$

where γ is the bare damping rate (set by the bath spectral density at the oscillator frequency). The master equation reads:

$$\partial_t \rho \;=\; -i[\omega a^\dagger a, \rho] \;+\; \gamma(\bar{n}+1)\big[a\rho a^\dagger - \tfrac{1}{2}\{a^\dagger a, \rho\}\big] \;+\; \gamma\bar{n}\big[a^\dagger \rho a - \tfrac{1}{2}\{a a^\dagger, \rho\}\big]. \tag{266}$$

Solving for $\langle a^\dagger a\rangle$. Multiply (266) by $a^\dagger a$ and trace. Using the cyclic property of trace and the standard commutator identities $[a^\dagger a, a] = -a$ and $[a^\dagger a, a^\dagger] = a^\dagger$:

$$\frac{d}{dt}\langle a^\dagger a\rangle \;=\; -\gamma\langle a^\dagger a\rangle + \gamma\bar{n}. \tag{267}$$

The general solution is

$$\langle a^\dagger a\rangle(t) \;=\; \bar{n} + \big[\langle a^\dagger a\rangle(0) - \bar{n}\big] e^{-\gamma t}. \tag{268}$$

The mean photon number relaxes to the thermal value $\bar{n}$ at rate γ — the textbook result.

Solving for $\langle a\rangle$. Similarly, multiplying by a gives

$$\frac{d}{dt}\langle a\rangle \;=\; -i\omega\langle a\rangle - \tfrac{\gamma}{2}\langle a\rangle, \qquad \langle a\rangle(t) \;=\; \langle a\rangle(0)\, e^{-i\omega t}\, e^{-\gamma t/2}. \tag{269}$$

The coherent amplitude oscillates at frequency ω and decays at half the rate ($\gamma/2$, not γ) of the energy relaxation. This is the celebrated $T_1 = 2T_2$ relation in the absence of pure dephasing.

Physics intuition. The damped HO is the operational template for every laboratory realization of the GKLS dynamics: cavity QED with a leaky cavity mirror, a mechanical resonator coupled to a thermal phonon bath, a superconducting qubit coupled to its electromagnetic environment, an atom in a magneto-optical trap. In every case, the structural prediction is the same: oscillation at the natural frequency ω, exponential decay of the coherent amplitude at rate $\gamma/2$, exponential equilibration of the population at rate γ, and approach to the thermal Gibbs state as the asymptotic state. The axiomatic content of `T4` of Part II is that this universal behavior is forced by the GKLS form, which is itself forced by the Markov-limit reduction of the bulk dynamics.

E.6 The Davies derivation in compact form

Davies (Davies, 1974) provided the cleanest formal derivation of the GKLS form from a microscopic Hamiltonian model in the "weak-coupling limit". The procedure is:

(1) Start with a system S + bath B + interaction λV Hamiltonian.

(2) Move to the interaction picture; compute the second-order projected dynamics.

(3) Take the limit $\lambda \to 0$ together with $t \to \infty$ keeping $\tau := \lambda^2 t$ fixed (the van Hove limit).

(4) Apply the rotating-wave approximation (secular truncation) to remove non-resonant terms.

The result is a strict GKLS generator on the system, with rates γ_α given by the Fourier transform of the bath correlation function at the system Bohr frequencies, exactly the (263) expression.

The Davies procedure is mathematically rigorous (it can be made into a precise theorem about the convergence of the reduced dynamics), and it makes precise the physical content of the Born-Markov approximation: the boundary system experiences not the full bath dynamics but only the spectral density evaluated at the system's resonant frequencies.

E.7 Connection to Lindblad's original 1976 paper

Lindblad's original paper (Lindblad, 1976) approached the same problem from a different direction: rather than deriving the GKLS form from a microscopic model, he asked the abstract question "*what is the most general generator of a one-parameter semigroup of CPTP maps on* $\mathcal{B}(\mathcal{H})$*?*" His answer was the canonical form (260), valid in arbitrary (separable) Hilbert space.

The Lindblad theorem and the Davies derivation are complementary: the Davies derivation tells us *when* the GKLS form applies (in the weak-coupling, secular limit), and the Lindblad theorem tells us *what* the GKLS form looks like (the canonical Hamiltonian + Lindblad-piece structure). The two together are the foundational results of the modern theory of open quantum systems.

Problems and worked solutions. E.1.1. Verify detailed balance of the Davies rates for a superconducting qubit: evaluate $\gamma_\uparrow/\gamma_\downarrow = e^{-\hbar\omega/k_BT}$ at $\omega/2\pi = 5$ GHz and $T = 20$ mK. *Solution.* Step 1: the ratio. The Davies construction delivers upward and downward rates tied by the KMS condition $\gamma_\uparrow/\gamma_\downarrow = e^{-\hbar\omega/k_BT}$ — thermal equilibrium is built into the generator, not imposed. Step 2: numbers. $\hbar\omega/k_BT = 12.0$ at the stated operating point (verified), so $\gamma_\uparrow/\gamma_\downarrow = e^{-12} = 6\times10^{-6}$: the stationary excited-state population is six parts per million. Step 3: the engineering meaning. This single number is why superconducting processors initialize by waiting: the GKLS stationary state at 20 mK is the ground state to 10^{-5} accuracy, no active reset required at that fidelity level — thermodynamics done by the boundary law itself. Physics intuition: the Davies theorem is the quantum version of "a thermometer left alone reads the room"; the problem's twelve e-folds are the entire engineering margin of a qubit industry.

E.1.2. Show that Gaussian dephasing $\rho_{01}(t) = \rho_{01}(0)\, e^{-\gamma t^2}$ cannot be a GKLS semigroup, and name the physics that produces it anyway. *Solution.* Step 1: the semigroup test. A GKLS evolution obeys $\Phi_{t+s} = \Phi_t \circ \Phi_s$; for a dephasing factor $f(t)$ this demands $f(t+s) = f(t)f(s)$, whose only continuous solutions are exponentials e^{-ct}. Step 2: the failure. $e^{-\gamma(t+s)^2} \neq e^{-\gamma t^2} e^{-\gamma s^2}$ (the cross term $2\gamma ts$ breaks it), so Gaussian decay is CP-*indivisible*: no time-independent generator produces it. Step 3: name the culprit. Gaussian decay is exactly what a *few*-mode or slow environment produces at short times — the single-bath-qubit model of II.2.2 gives $\cos(2gt) \approx e^{-2g^2t^2}$ — and the semigroup emerges only after the many-mode bath (C.1.1) scrambles the memory. The hierarchy exact-NZ → Gaussian → exponential is the visible history of the Markov limit being earned. Physics intuition: the shape of a decay curve is a census of the environment — Gaussian says "few recorders, still

coherent", exponential says "a continuum, memory gone"; a T_2 measurement is bath spectroscopy whether the experimenter intends it or not.

E.1.3. Solve the photon-number relaxation of the damped thermal oscillator and identify each piece constitutionally. *Solution.* Step 1: the equation. With downward rate $\gamma(\bar{n}+1)$ and upward rate $\gamma\bar{n}$ on $L = a$, $L' = a^\dagger$, the GKLS generator gives $\frac{d}{dt}\langle n\rangle = -\gamma(\langle n\rangle - \bar{n})$ — one line of operator algebra using $[a, a^\dagger a] = a$. Step 2: solve. $\langle n\rangle(t) = \bar{n} + (n_0 - \bar{n})\, e^{-\gamma t}$: exponential approach to the thermal occupation, from above or below, at the same rate. Step 3: constitutional reading. The drift to $\bar{n}$ is the gradient sector of T5 on the energy ledger; the fluctuations around it (not computed here) are the diffusion sector; and the oscillator's phase rotation is the circulation sector — the damped cavity is the smallest system exhibiting the full three-sector anatomy. Physics intuition: every cavity, laser mode, and nanomechanical resonator in every laboratory relaxes on this one-line law; the appendix's point is that the line was never a model — it is the boundary law's local weather.

Take-away. *Standard account:* the GKLS form is quantum mechanics' phenomenological friction, fitted system by system. *What it left open:* why the rates are positive, why equilibrium is thermal, and when the form applies at all. *The constitutional derivation:* positivity is POS-4's admissibility, the KMS ratio is delivered by the Davies limit, and the domain of validity is priced by $\gamma\tau_B$ — the generator is the unique Markovian shadow of the exact reduction of T2.

F Decoherence and Einselection in Detail

"Superpositions are fragile" is true and almost useless: it gives no number. This appendix supplies the numbers. It writes the equation governing the off-diagonal elements of the density matrix, solves it, gives the criterion that decides which basis survives, and then carries out the calculation that turns the whole story into a decoherence time with laboratory units attached. A reader who has met decoherence only as a slogan will find here the form in which it is actually used.
Read alongside: Zeh (1970) for the founding paper; Zurek (1981) for einselection; Joos (1985) for the localization computation; Zurek (2003) for the program review.

This appendix supplies the detailed mathematical machinery underlying the decoherence (T7.5) and einselection (T7.6) theorems of Part III.

F.1 The off-diagonal master equation

Choose an orthonormal basis $\{|\alpha\rangle\}$ that diagonalizes the system–environment coupling Hamiltonian $H_{\rm SE} = \sum_\alpha A_\alpha \otimes B_\alpha$ in the system sector (i.e., the operators A_α are mutually commuting and diagonal in $\{|\alpha\rangle\}$). Decompose the boundary density operator in this basis:

$$\rho_{\rm Bdy}(t) \;=\; \sum_{\alpha\beta} \rho_{\alpha\beta}(t)\, |\alpha\rangle\langle\beta|. \tag{270}$$

Substituting into the GKLS master equation (260) and projecting onto the off-diagonal subspace ($\alpha \neq \beta$), the secular approximation gives

$$\partial_t \rho_{\alpha\beta}(t) \;=\; -i\omega_{\alpha\beta}\,\rho_{\alpha\beta}(t) \;-\; \Gamma_{\alpha\beta}\,\rho_{\alpha\beta}(t), \tag{271}$$

with $\omega_{\alpha\beta} = E_\alpha - E_\beta$ the Bohr frequency and the decoherence rate

$$\Gamma_{\alpha\beta} \;=\; \sum_\gamma \gamma_\gamma \left|\langle\alpha|L_\gamma|\alpha\rangle - \langle\beta|L_\gamma|\beta\rangle\right|^2. \tag{272}$$

F.2 Solution and operational interpretation

Equation (271) integrates immediately to

$$\rho_{\alpha\beta}(t) \;=\; \rho_{\alpha\beta}(0)\, e^{-i\omega_{\alpha\beta}t}\, e^{-\Gamma_{\alpha\beta}t}. \tag{273}$$

The off-diagonal coherences decay exponentially at rate $\Gamma_{\alpha\beta}$, while the diagonal populations $\rho_{\alpha\alpha}$ evolve only on the much longer relaxation timescale $\sim 1/\gamma_\alpha$. In the long-time limit, the boundary density operator becomes diagonal in the pointer basis, operationally indistinguishable from a classical probability distribution.

Order-of-magnitude estimate for macroscopic states. For macroscopically distinguishable states (e.g., a Schrödinger cat with position separation $\Delta x \sim 0.1$ m), the difference $|\langle\alpha|L_\gamma|\alpha\rangle - \langle\beta|L_\gamma|\beta\rangle|$ scales linearly with Δx. The sum over environmental modes γ has $\sim 10^{20}$ terms (one per ambient air molecule per cm^2 of cat surface). At room temperature, the decoherence rate per environmental mode is $\sim k_B T m^2 v/\hbar^2 \cdot (\Delta x)^2 \sim 10^{20}$ s^{-1} per molecule (Joos, 1985), giving a total $\Gamma \sim 10^{40}$ s^{-1} for the macroscopic cat: forty orders of magnitude faster than any laboratory time resolution.

F.3 The einselection criterion

Among all orthonormal bases of $\mathcal{H}_{\mathrm{Bdy}}$, the einselected pointer basis is the one that minimizes the entropy production rate when taken as the initial basis (Zurek's predictability sieve (Zurek, 1981)):

$$\dot{S}^{(\mathrm{basis})}_{\mathrm{tot}} \;=\; -\frac{d}{dt}\,\mathrm{Tr}\big[\rho(t)\log\rho(t)\big]\Big|_{\rho(0)=|\alpha\rangle\langle\alpha|,\ \text{averaged over } \alpha}. \tag{274}$$

The basis that minimizes this is the eigenbasis of the dominant system–environment coupling operator A_α, because states in this basis are the "slowest to leak" under the environmental coupling.

Why position is the universal pointer basis. In any fundamental theory with bulk locality (AX-1 of the Constitution), the boundary system–environment coupling at leading order in the multipole expansion is position-diagonal: $H_{\mathrm{SE}} = \sum_x \hat{x} \otimes \hat{E}(x)$, where $\hat{E}(x)$ is the environment operator localized near position x. This is because all known interactions (electromagnetic, weak, strong, and gravitational at leading order) have positions as their natural coupling variable. The

einselected basis is therefore the position basis, and macroscopic objects appear classical in space rather than in momentum or in some other basis.

F.4 Worked example: the Caldeira–Leggett model

The Caldeira–Leggett model is the textbook example of decoherence of a massive particle's center-of-mass motion through coupling to a thermal phonon bath. It is the foundational model behind the Joos–Zeh estimate of macroscopic decoherence rates that we used in §24.3 for the Schrödinger-cat calculation.

Setup. A massive particle of mass M moves in one dimension under a potential $V(\hat{x})$, coupled to a bath of harmonic oscillators with masses m_k, frequencies ω_k, and positions $\hat{q}_k$. The total Hamiltonian is

$$H_{\mathrm{CL}} \;=\; \frac{\hat{p}^2}{2M} + V(\hat{x}) \;+\; \sum_k \left[\frac{\hat{p}_k^2}{2m_k} + \tfrac{1}{2} m_k \omega_k^2 \hat{q}_k^2 \right] \;-\; \hat{x} \sum_k g_k \hat{q}_k . \tag{275}$$

The coupling $-\hat{x}\sum g_k \hat{q}_k$ is bilinear in the system position $\hat{x}$ and the bath positions $\hat{q}_k$, with coupling constants g_k.

Bath spectral density. As in the spin-boson example of §C.9, the relevant property of the bath is the spectral density $J(\omega) = (\pi/2)\sum_k (g_k^2/m_k\omega_k)\,\delta(\omega-\omega_k)$. The Ohmic choice $J(\omega) = M\eta\omega$ with damping parameter η produces the Newtonian damping $M\ddot{x} + M\eta\dot{x} + V'(x) = \xi(t)$ in the classical limit, with $\xi(t)$ the fluctuating thermal force.

Decoherence between two position eigenstates. For a superposition $|\psi(0)\rangle = (|x_1\rangle + |x_2\rangle)/\sqrt{2}$ of two well-localized positions separated by $\Delta x = |x_1 - x_2|$, the off-diagonal coherence in the position basis decays as

$$\rho(x_1, x_2; t) \;=\; \rho(x_1, x_2; 0)\, \exp\!\left[-(\Delta x)^2 / \lambda_T^2 \cdot \gamma t \right], \tag{276}$$

where $\gamma = \eta$ is the dissipation rate and $\lambda_T = \hbar/\sqrt{2 M k_B T}$ is the thermal de Broglie wavelength. The decoherence rate is

$$\Gamma_{\mathrm{dec}} \;=\; \gamma\, \frac{(\Delta x)^2}{\lambda_T^2} \;=\; \frac{2 M k_B T \gamma (\Delta x)^2}{\hbar^2}. \tag{277}$$

Numerical estimate for a dust grain. Take a dust grain of mass $M = 10^{-15}$ kg at room temperature $T = 300$ K, with damping $\gamma \sim 10^{4}$ s^{-1} (typical for ambient air collisions) and superposition separation $\Delta x = 1\ \mu\mathrm{m} = 10^{-6}$ m. Substituting:

$$\Gamma_{\mathrm{dec}} \;\approx\; \frac{2 \cdot 10^{-15} \cdot 4 \times 10^{-21} \cdot 10^{4} \cdot 10^{-12}}{10^{-68}} \;\sim\; 10^{24}\ \mathrm{s}^{-1}. \tag{278}$$

This means the coherence is destroyed in $\tau_{\mathrm{dec}} \sim 10^{-24}$ s, twenty-four orders of magnitude faster than any laboratory measurement timescale. For a Schrödinger cat with $\Delta x = 0.1$ m and $M = 1$ kg,

the rate is even larger by $(0.1/10^{-6})^2 \cdot (10^{15}) \sim 10^{25}$, giving $\Gamma_{\rm dec} \sim 10^{49}$ s^{-1}. This is the Joos–Zeh–Schlosshauer order-of-magnitude argument that no laboratory will ever observe a macroscopic Schrödinger cat.

Physics intuition. The Caldeira–Leggett model makes precise the statement that decoherence is the inevitable consequence of a system's coupling to its environment. The decoherence rate scales as $(\Delta x)^2$ times the dissipation rate γ, which means: the more separated the superposition components, the faster the decoherence. For microscopic separations ($\Delta x \sim$ nm), $\Gamma_{\rm dec} \sim \gamma$ — coherence persists on the same timescale as energy relaxation. For macroscopic separations ($\Delta x \sim$ m), $\Gamma_{\rm dec} \gg \gamma$ — coherence is destroyed long before any energy is dissipated. This is the constitutional source of the quantum-to-classical transition: the basis where the environment couples (position) becomes the einselected pointer basis, and macroscopic distinguishability accelerates the decoherence exponentially.

Problems and worked solutions. F.1.1. Establish the scaling laws of decoherence: how does the Caldeira–Leggett rate depend on superposition size and temperature, and what do the scalings predict for experiment? *Solution.* Step 1: the rate. The Caldeira–Leggett computation gives $\Gamma_{\rm dec} = \Lambda\,(\Delta x)^2$ with localization strength $\Lambda = 2m\gamma k_B T/\hbar^2$: quadratic in separation, linear in temperature and mass. Step 2: the ratios. Widening a superposition from 1 nm to 1 μm multiplies the rate by 10^6; cooling from 300 K to 30 mK buys back only 10^4 — geometry beats cryogenics, which is why interference experiments miniaturize before they refrigerate. Step 3: the predictive edge. The scalings are parameter-free tests: matter- wave interferometry with heavier molecules at fixed geometry must lose visibility linearly in m, and every such experiment to date tracks the law. Physics intuition: decoherence is not a threshold but a tax rate — proportional to how much which-path information the superposition offers (Δx^2) and how eagerly the bath collects it (T); the classical world is the regime where the tax is confiscatory.

F.1.2. Compute the ratio of decoherence to relaxation timescales for a dust grain, and explain why the quantum-to-classical transition is invisible as an energy process. *Solution.* Step 1: the ratio. The Caldeira–Leggett structure gives $\tau_{\rm dec}/\tau_{\rm relax} = (\lambda_{\rm dB}/\Delta x)^2$ with the thermal de Broglie length $\lambda_{\rm dB} = \hbar/\sqrt{2mk_BT}$. Step 2: numbers. For $m = 10^{-15}$ kg at 300 K: $\lambda_{\rm dB} = 3.7\times10^{-17}$ m (verified); for a 1-μm superposition the ratio is $(3.7\times10^{-17}/10^{-6})^2 = 1.3\times10^{-21}$. Step 3: read it. If the grain thermalizes in a second, its superposition died in 10^{-21} s: twenty-one orders separate the loss of coherence from the loss of energy, so classicality arrives essentially instantaneously and *silently* — no calorimeter can see it happen, only an interferometer could have. Physics intuition: this is T7.5's "essentially instantaneous" with its engineering units attached, and it is why the founders could mistake decoherence for collapse: at macroscopic scales the process is faster than any apparatus that could have resolved it as a process.

Take-away. *Standard account:* decoherence rates are computed in oscillator-bath models chosen for tractability. *What it left open:* why the bath, why position-coupling, and what fixes Λ. *The constitutional derivation:* the bath is the KK tower, the coupling is position-diagonal by bulk locality (T7.6), and the scalings $(\Delta x)^2$, T, m are inherited structure — the appendix's model is the boundary law's local expansion, not an analogy.

G Kaluza–Klein Reduction on the Bulk

This is the machine shop of the whole program. Every derivation in Parts IV and V begins by taking a field defined on the six-dimensional bulk and asking what an observer confined to the four-dimensional boundary would see, and the answer is always the same in form: one boundary field, plus an infinite tower of heavier copies. The appendix sets out the metric used, the equation the radial profiles satisfy, the orthogonality relation that makes the expansion work, and the resulting mass spectrum with its scale dressing.
Read alongside: T. Kaluza (1921) and O. Klein (1926) for the original fifth-dimension papers whose construction this appendix runs on the GG-6 bulk.

This appendix derives the Kaluza–Klein expansion on the canonical 6D bulk in full detail, supporting Theorems T8.1 and T9.1.

G.1 The canonical metric and the KK ansatz

The canonical natural-GG-6-units metric is

$$ds_6^2 \;=\; e^{-2r}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + dr^2 + d\theta^2, \qquad r\in[0,K], \quad \theta\in[0,2\pi). \tag{279}$$

A generic bulk field $\Phi(x,r,\theta)$ admits the KK expansion

$$\Phi(x,r,\theta) \;=\; \sum_{n=0}^{\infty}\sum_{\ell\in\mathbb{Z}} \phi^{(n,\ell)}(x)\, f_n(r)\,\frac{e^{i\ell\theta}}{\sqrt{2\pi}}, \tag{280}$$

where the $\phi^{(n,\ell)}(x)$ are the 4D KK-mode fields and the radial profiles $f_n(r)$ are determined by the bulk wave equation plus boundary conditions at $r=0$ and $r=K$.

G.2 The radial mode equation

For a massless bulk scalar field, the bulk Klein–Gordon equation $\Box_6\Phi=0$ with the scale-profile metric (279) becomes, after separation of variables,

$$\left[-\partial_r^2 + 4\partial_r + e^{2r}\,(m_n^2+\ell^2)\right] f_n(r) \;=\; 0, \tag{281}$$

where the $4\partial_r$ term comes from the scale-profile contribution to the bulk Laplacian and the e^{2r} factor multiplies the 4D mass-squared eigenvalue.

The radial spectrum. Define the rescaled function $g_n(r) := e^{-2r} f_n(r)$. Equation (281) becomes a Bessel-type equation with solutions

$$g_n(r) \;\propto\; J_2(z_n(r)) \text{ or } Y_2(z_n(r)), \qquad z_n(r) := \sqrt{m_n^2 + \ell^2}\, e^r. \tag{282}$$

Boundary conditions at $r = 0$ (Neumann for gauge zero modes, Dirichlet for chirality-projected fermion zero modes) and $r = K$ quantize the spectrum. The lowest non-trivial mode has $m_1 \sim 1$ in natural GG-6 units; restoring dimensional units gives $m_1 \sim M_6 \sim 10^9$ GeV.

G.3 The orthonormality condition

The radial profiles $f_n(r)$ satisfy the orthonormality condition

$$\int_0^K dr\, e^{-2r}\, f_n(r)\, f_m(r) \;=\; \delta_{nm}, \tag{283}$$

which is the natural inner product induced by the bulk measure $\sqrt{-g_6}$ (after the radial dependence is extracted).

G.4 Mass spectrum and the lock-scale dressing

The total KK mass squared is

$$m_{n,\ell}^2 \;=\; m_n^2 + \ell^2 \;\approx\; n^2 + \ell^2 \quad \text{(natural GG-6 units, leading order)}. \tag{284}$$

Restoring dimensional units and including the IR-boundary scale suppression, the lock scale is

$$M_{\text{lock}} \;\sim\; M_6 \cdot e^{-K/2} \;\sim\; 10^9 \cdot e^{-17.5} \text{ GeV} \;\sim\; 4 \text{ TeV}, \tag{285}$$

matching the $M_{\text{lock}} \approx 3.85$ TeV value of T8.7 derived in GG-A9-SM Theorem 28.1 to within order-unity precision.

G.5 Worked example: massless scalar field on the bulk

Let $\Phi(x, r, \theta)$ be a massless scalar field on the 6D bulk (279). The bulk Klein–Gordon equation is

$$\Box_6 \Phi \;=\; \frac{1}{\sqrt{-g_6}}\, \partial_A\big(\sqrt{-g_6}\, g^{AB} \partial_B \Phi\big) \;=\; 0. \tag{286}$$

Working out each term using $\sqrt{-g_6} = e^{-4r}$, $g^{\mu\nu} = e^{2r}\eta^{\mu\nu}$, $g^{rr} = g^{\theta\theta} = 1$:

$$\Box_6 \Phi \;=\; e^{2r}\, \eta^{\mu\nu} \partial_\mu \partial_\nu \Phi \;+\; e^{4r}\, \partial_r\big(e^{-4r} \partial_r \Phi\big) \;+\; \partial_\theta^2 \Phi. \tag{287}$$

Insert the KK ansatz $\Phi = \phi^{(n,\ell)}(x) f_n(r) e^{i\ell\theta}/\sqrt{2\pi}$ and use the 4D Klein-Gordon equation $\eta^{\mu\nu}\partial_\mu\partial_\nu \phi^{(n,\ell)} = -m_{n,\ell}^2 \phi^{(n,\ell)}$. Separating out the r- and θ-dependence gives the radial mode equation

$$e^{4r} \partial_r\big(e^{-4r} \partial_r f_n\big) - \ell^2 f_n + e^{2r} m_{n,\ell}^2 f_n \;=\; 0, \tag{288}$$

which simplifies to

$$\partial_r^2 f_n - 4\partial_r f_n + e^{2r}(m_{n,\ell}^2 - \ell^2 e^{-2r}) f_n \;=\; 0. \tag{289}$$

Bessel-equation form. Define $z := m_{n,0}\, e^r$ (taking $\ell = 0$ for simplicity) and $g(z) := z^{-2} f_n(r)$. Tedious but standard manipulation converts the radial equation into the Bessel equation of order 2:

$$z^2 g''(z) + z g'(z) + (z^2 - 4) g(z) \;=\; 0, \tag{290}$$

with general solution $g(z) = A\, J_2(z) + B\, Y_2(z)$. Hence

$$f_n(r) \;=\; e^{2r}\big[A_n J_2(m_n e^r) + B_n Y_2(m_n e^r)\big]. \tag{291}$$

Quantizing the spectrum via boundary conditions. At the UV boundary $r = 0$, the Neumann boundary condition for gauge zero modes is $\partial_r f_n(0) = 0$; this fixes the ratio B_n/A_n. At the IR boundary $r = K$, the same Neumann condition gives a transcendental equation for the eigenvalues m_n:

$$\frac{J_1(m_n e^K)}{Y_1(m_n e^K)} \;=\; \frac{J_1(m_n)}{Y_1(m_n)}, \tag{292}$$

whose roots give the discrete KK mass spectrum. In the small-m_n limit, the roots are approximately $m_n^{\mathrm{KK}} \sim n\,\pi\, e^{-K}$, giving the lightest KK mass at the scale-suppressed scale $M_{\mathrm{lock}} \sim e^{-K}$ as expected.

Comparison with flat-bulk KK. For comparison, on a flat 5D bulk with compact 5th direction of radius R_5 (the original Klein 1926 setup), the KK mass spectrum is the rigid tower $m_n = n/R_5$ for $n = 0, 1, 2, \ldots$. On the bulk, the spectrum is non-uniform: the scale profile pushes the modes toward exponentially small masses at the IR boundary while keeping the wavefunctions exponentially localized away from the UV boundary. This is what produces the gauge-hierarchy resolution of T8.7 without fine-tuning.

Physics intuition. The Bessel structure is universal for extra-dimensional field theories (Gherghetta, 2000, L. Randall, 1999). The pedagogical point is that the scale-profile exponential e^{-2r} in the metric becomes the exponential growth factor in the radial coordinate $z = me^r$ of the Bessel equation, which then quantizes the spectrum via the IR-boundary condition into the scale-suppressed $M_{\mathrm{lock}} \sim e^{-K}$ tower. This is the unique extra-dimensional construction that solves the gauge hierarchy problem geometrically; no other KK setup achieves the same result without fine-tuning.

Problems and worked solutions. G.1.1. Normalize the gauge zero mode under the warped measure and locate where its norm lives. *Solution.* Step 1: normalize. The flat zero mode $f_0(r) = c$ under $\int_0^K e^{-2r} f_0^2\, dr = 1$ gives $c^2\, \frac{1}{2}(1 - e^{-2K}) = 1$, i.e. $c = \sqrt{2}$ to one part in 10^{31} at $K = 35$. Step 2: locate the norm. The fraction of the norm in the first radial e-fold is $\int_0^1 e^{-2r} dr / \int_0^\infty e^{-2r} dr \;=\; 1 - e^{-2} \;=\; 86.5\%$ (verified): though the profile is flat, the *measure* concentrates the mode near $r = 0$ — the UV end. Step 3: the reading. "The Standard Model lives

on the boundary" is this integral: zero modes are constitutionally bulk objects whose physical weight sits at the boundary of the radial window, which is why boundary language is honest shorthand and why couplings (overlap integrals) are dominated by the near-boundary geometry. Physics intuition: a flat wavefunction in a warped house still sleeps in one room; the warp factor, not the profile, decides where physics happens.

G.1.2. Enumerate the KK level structure through $m^2 = 9$: masses, degeneracies, and the spectroscopy fingerprint they compose. *Solution.* Step 1: enumerate. With $m^2_{n,\ell} = n^2 + \ell^2$ ($n \geq 0$, $\ell \in \mathbb{Z}$, excluding the zero mode): $m^2 = 1$: $(1,0), (0,\pm 1)$ — degeneracy 3; $m^2 = 2$: $(1,\pm 1)$ — 2; $m^2 = 4$: $(2,0), (0,\pm 2)$ — 3; $m^2 = 5$: $(2,\pm 1), (1,\pm 2)$ — 4; $m^2 = 8$: $(2,\pm 2)$ — 2; $m^2 = 9$: $(3,0), (0,\pm 3)$ — 3 (all verified by direct count). Step 2: the fingerprint. Masses run $1, \sqrt{2}, 2, \sqrt{5}, 2\sqrt{2}, 3$ in lock units with degeneracy pattern $3, 2, 3, 4, 2, 3$: a rigid, parameter-free comb that no generic new-physics model reproduces. Step 3: the falsifier form. Discovering TeV resonances whose mass ratios or multiplicities break the comb kills the compactification — V.1.1's cluster argument, now three levels deep. Physics intuition: atomic spectroscopy identified hydrogen by line ratios, not line positions; the bulk's spectroscopy works the same way, and this problem writes out its Balmer table.

Take-away. *Standard account:* extra-dimensional reductions are model-building tools with adjustable profiles and scales. *What it left open:* what fixes the geometry, the measure, and the spectrum. *The constitutional derivation:* one canonical ansatz, one warped measure, one integer ledger — the mode functions, the 86.5% localization, and the degeneracy comb are all theorems of the same three integrals that built $M_{\rm Pl}$ in T8.1.

H Christoffel Symbols and Curvature on the 6D Bulk

The curvature engine room. Every Christoffel symbol of the six-dimensional metric is computed here, then the Riemann and Ricci tensors, then the scalar curvature — which comes out to a specific number on the canonical ansatz, and that number is checked rather than quoted. The point of setting it out at this length is that a student can re-derive any component with pen and paper and find the error if there is one. This is the computational floor under the gravitational derivations. *Read alongside:* Carroll (2004) for the curvature machinery at textbook register; Wald (1984) for the same machinery at mathematical-physics register.

This appendix supplies the explicit Christoffel symbols, Riemann tensor, Ricci tensor, and Ricci scalar for the canonical 6D metric (279), supporting Theorem T8.1.

H.1 Christoffel symbols

For the metric (279) with flat 4D background ($g_{\mu\nu}(x) = \eta_{\mu\nu}$), the non-vanishing Christoffel symbols $\Gamma^A_{BC} = \frac{1}{2} g^{AD}(\partial_B g_{CD} + \partial_C g_{BD} - \partial_D g_{BC})$ are

$$\Gamma^r_{\mu\nu} \;=\; e^{-2r}\,\eta_{\mu\nu}, \qquad \Gamma^\mu_{r\nu} \;=\; -\delta^\mu_\nu, \qquad \Gamma^A_{BC} \;=\; 0 \text{ otherwise.} \tag{293}$$

Derivation. The non-zero metric components are $g_{\mu\nu} = e^{-2r}\eta_{\mu\nu}$, $g_{rr} = 1$, $g_{\theta\theta} = 1$. The non-zero metric derivatives are $\partial_r g_{\mu\nu} = -2e^{-2r}\eta_{\mu\nu}$. Substituting into the Christoffel-symbol formula yields (293).

H.2 Riemann tensor

The non-vanishing components of the Riemann tensor $R^A{}_{BCD} = \partial_C\Gamma^A_{DB} - \partial_D\Gamma^A_{CB} + \Gamma^A_{CE}\Gamma^E_{DB} - \Gamma^A_{DE}\Gamma^E_{CB}$ are

$$R^r{}_{\mu r\nu} = e^{-2r}\,\eta_{\mu\nu}, \qquad R^\mu{}_{rr\nu} = -\delta^\mu_\nu, \qquad R^\mu{}_{\nu\alpha\beta} = -e^{-2r}(\delta^\mu_\alpha\eta_{\nu\beta} - \delta^\mu_\beta\eta_{\nu\alpha}). \tag{294}$$

H.3 Ricci tensor and Ricci scalar

Contracting the Riemann tensor gives the Ricci tensor $R_{BD} = R^A{}_{BAD}$:

$$R_{\mu\nu} = -4e^{-2r}\,\eta_{\mu\nu}, \qquad R_{rr} = -4, \qquad R_{\theta\theta} = 0. \tag{295}$$

The Ricci scalar is then

$$R_6 = g^{AB}R_{AB} = e^{2r}\eta^{\mu\nu}\cdot(-4e^{-2r}\eta_{\mu\nu}) + 1\cdot(-4) + 1\cdot 0 = -16 - 4 = -20. \tag{296}$$

This is the flat-4D-background value used in T8.1 derivation (88). For curved 4D background $g_{\mu\nu}(x) \neq \eta_{\mu\nu}$, the additional contribution is $e^{2r}R_4$, giving the full result $R_6 = e^{2r}R_4 - 20$ used in the KK reduction of the Einstein–Hilbert action.

H.4 The bulk cosmological-constant-like term

The constant -20 in R_6 is the bulk AdS_6 curvature contribution. When the 6D Einstein–Hilbert action is reduced to 4D (as in T8.1), this constant produces a boundary-cosmological-constant term that contributes to the boundary Λ_4. The boundary-localized matter and gauge contributions cancel this leading piece via the bulk–boundary balance, leaving the lock-scale-seesaw residual derived in T8.5 plus GG-A10-Cosmos.

H.5 Verification: $R_6 = -20$ is the AdS_6 value

For Einstein space with the metric ansatz (279), the bulk Einstein equation in vacuum reads $R_{AB} - \frac{1}{2}g_{AB}R = -\Lambda_6 g_{AB}$, which on contraction gives $R = -6\Lambda_6/2 = -3\Lambda_6$ in 6D (using $g_{AB}g^{AB} = 6$). Comparing with our computed $R_6 = -20$ identifies the bulk cosmological constant $\Lambda_6 = +20/3 \approx 6.67$ in natural GG-6 units. This is positive, as expected for AdS-form geometry (the negative-curvature sign convention here is the geometric convention; the "cosmological constant" in the action has the opposite sign from the Ricci scalar).

Why is the bulk AdS-form forced? By POS-5 (RG-covariance) plus AX-4 (MSMF) of the Constitution (Arisaka, 2026, A1-Constitution), the bulk metric must admit a continuous RG-flow direction. AdS geometry is the unique maximally-symmetric spacetime with this property

in arbitrary dimension; the radial coordinate in AdS naturally identifies with $\log(E_{\rm GUT}/\mu)$ (the renormalization-group flow direction), as Maldacena (J. Maldacena, 1998) pointed out in the original AdS/CFT paper. The axiomatic content of GG-6 is that this geometric realization of RG flow is forced by the Constitution, not chosen by phenomenological fit.

H.6 Step-by-step: deriving the $\Gamma^r_{\mu\nu}$ Christoffel symbol

For the pedagogically inclined reader, here is the explicit computation of one Christoffel symbol that exemplifies the procedure.

The Christoffel-symbol formula is

$$\Gamma^A_{BC} \;=\; \tfrac{1}{2} g^{AD}(\partial_B g_{CD} + \partial_C g_{BD} - \partial_D g_{BC}). \tag{297}$$

Set $A = r$, $B = \mu$, $C = \nu$. The metric components needed are $g^{rr} = 1$ and $g_{\mu\nu} = e^{-2r}\eta_{\mu\nu}$, with all others zero. The non-vanishing D-index in the sum is $D = r$ (because g^{rD} is non-zero only for $D = r$). Hence

$$\begin{aligned}
\Gamma^r_{\mu\nu} &= \tfrac{1}{2} g^{rr}(\partial_\mu g_{\nu r} + \partial_\nu g_{\mu r} - \partial_r g_{\mu\nu}) \\
&= \tfrac{1}{2}\,(1)\,(0 + 0 - \partial_r(e^{-2r}\eta_{\mu\nu})) \\
&= \tfrac{1}{2}\,(-1)\,(-2e^{-2r}\eta_{\mu\nu}) \\
&= e^{-2r}\,\eta_{\mu\nu}.
\end{aligned} \tag{298}$$

This is the result (293). Verification involved substituting in only the non-vanishing metric components (diagonal block-form simplifies things enormously) and using the chain-rule derivative of the scale profile.

Pedagogical aside. The Christoffel symbols of scale-profile-product metrics have a beautiful geometric interpretation. $\Gamma^r_{\mu\nu} = e^{-2r}\eta_{\mu\nu}$ says: *any 4D-spacelike or 4D-timelike velocity vector has, at the connection level, a non-zero radial-direction acceleration e^{-2r} times the 4D-metric inner product.* This is the geometric reason that 4D test particles "fall toward the IR brane" in extra-dimensional models, and it is why the scale profile factor mediates the gauge-hierarchy resolution: the IR boundary is the attractor of motion in the radial direction.

Problems and worked solutions. H.1.1. Re-derive the mixed Christoffel components by the appendix's own step-by-step method: show $\Gamma^\mu{}_{r\nu} = -\delta^\mu{}_\nu$ on the canonical ansatz. *Solution.* Step 1: the input. For $ds^2_6 = e^{-2r} g_{\mu\nu} dx^\mu dx^\nu + dr^2 + d\theta^2$ (flat 4D slice), the only r-dependence is the conformal factor: $\partial_r g^{(6)}_{\mu\nu} = -2\, g^{(6)}_{\mu\nu}$. Step 2: the formula. $\Gamma^\mu{}_{r\nu} = \frac{1}{2} g^{\mu\lambda}\partial_r g_{\lambda\nu} = \frac{1}{2}\,(-2\,\delta^\mu{}_\nu) = -\delta^\mu{}_\nu$ — every 4D direction contracts identically under radial transport (verified symbolically on the full 6×6 metric). Step 3: what it encodes. This single component is the scale profile as geometry: parallel transport inward shrinks every boundary vector at unit rate, which is the differential form of the e^{-r} redshift that built the hierarchy in T8.7. Physics intuition: the most consequential Christoffel symbol in the cohort is also its simplest — a minus one — and the student who has computed it

owns the mechanism behind sixteen orders of magnitude.

H.1.2. Verify $R_6 = -20$ by assembling the curvature from the components, and decompose the -20 into its geometric bookkeeping. *Solution.* Step 1: assemble. With the components of H.1.1 and their r-derivatives, the Ricci scalar of the canonical ansatz (flat slice) evaluates to $R_6 = -20$ exactly — re-verified here by an independent symbolic computation of all Christoffel symbols, the Ricci tensor, and the trace on the full six-dimensional metric. Step 2: the bookkeeping. The warped directions contribute as in AdS_{d+1}: with $d = 4$ warped coordinates and unit curvature radius the standard result is $R = -(d+1)(d+2) + (\text{flat } \theta)\cdot 0$ evaluated on the five-dimensional warped block, and the flat circle adds nothing: $-20 = -4 \times 5$. Step 3: the consequence. R_6 constant and negative is what makes the reduction integral of T8.1 produce a pure Einstein–Hilbert term plus a *constant* — the naive cosmological term of IV.1.2 — with no higher-curvature debris; the honesty of T8.5's ledger begins in this arithmetic. Physics intuition: -20 is the sound of a spacetime that is the same everywhere — maximal symmetry in the warped block — and constancy, not the value, is the gift: it turns gravity's reduction into bookkeeping.

Take-away. *Standard account:* curvature computations are outsourced to computer algebra and quoted. *What it left open:* nothing mathematical — but everything pedagogical: the student who has never pushed the indices owns none of it. *The constitutional derivation:* the appendix computes every component on the one canonical ansatz, and the two problems above put the decisive entries — the $-\delta^{\mu}_{\nu}$ that drives the hierarchy and the -20 that fixes the naive vacuum term — into the student's own hands.

I Boundary Conditions and Orbifold Symmetry Breaking

In the Standard Model, symmetry is broken by a field that acquires a value. Here it is broken by the shape of space. This appendix shows how: the compact direction is folded, the folding acts on the gauge indices through a parity matrix, and the larger symmetry group is broken to the one we observe simply because only some components survive the folding. The delicate part, worked through explicitly, is that the chiral fermions survive — chirality being the property that makes the Standard Model hard to obtain from extra dimensions at all.
Read alongside: H. Georgi (1974) for the SU(5) unification whose breaking pattern the orbifold parity realizes geometrically here.

This appendix supplies the orbifold projection that selects the chirality-projected fermion zero modes on the boundary and breaks SU(5) to the Standard-Model gauge group.

I.1 The orbifold $S^1_\theta/\mathbb{Z}_2$

The compact angular direction $\theta \in [0, 2\pi)$ is orbifolded by the $\mathbb{Z}_2$ action $\theta \to -\theta$, with fixed points at $\theta = 0$ and $\theta = \pi$. Bulk fields are classified by their $\mathbb{Z}_2$-parity at the fixed points. Even-parity fields have zero modes (Neumann boundary conditions); odd-parity fields have no zero modes (Dirichlet boundary conditions).

I.2 The parity matrix and SU(5) breaking

Choose the parity matrix

$$P \;=\; \mathrm{diag}(+1,+1,+1,-1,-1) \tag{299}$$

acting on the SU(5) fundamental. Bulk gauge fields A_μ are projected by

$$A_\mu(x,r,-\theta) \;=\; P\,A_\mu(x,r,\theta)\,P^{-1}. \tag{300}$$

Components of A_μ that commute with P have zero modes on the boundary; components that anticommute with P are projected out. The unbroken subgroup is the centralizer of P in SU(5), which is $\mathrm{SU}(3)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}$ — the Standard Model gauge group.

I.3 Fermion zero modes

For chiral fermions in the $\bar{5}$ and 10 representations of SU(5), the orbifold projection selects one chirality of each representation as the surviving zero mode on the boundary. In the canonical Ramond-sector chirality assignment, the surviving zero modes are:

- From $\bar{5}$: the d^c (anti-down quark) and the lepton doublet $L=(\nu_e,e^-)_L \to (\bar{3},1,1/3)+(1,2,-1/2)$.
- From 10: the quark doublet $Q=(u,d)_L$, the u^c (anti-up quark), and the e^c (anti-electron) $\to (3,2,1/6)+(\bar{3},1,-2/3)+(1,1,1)$.

This is exactly one chiral generation of the Standard Model.

Three generations from the θ-winding. The three Standard-Model generations correspond to three windings $\ell=1,2,3$ of the compact θ direction, with the BI-6 trace ratio $-2/5$ forcing exactly $N_g=3$ (Theorem 6 of GG-A2-GG6). See Arisaka-Generations for the full proof of the $N_g=3$ constraint.

I.4 Worked example: SU(5) generators with $P=\mathrm{diag}(+,+,+,-,-)$

Let us work out explicitly which SU(5) generators T^a commute with P and which anticommute, to verify that the centralizer is $\mathrm{SU}(3)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}$.

The 24 generators of SU(5). The defining representation of SU(5) acts on a 5-component column. The 24 generators T^a can be organized into a 5×5 hermitian traceless matrix $T=T^a\lambda^a$, where λ^a are the generalized Gell-Mann matrices. In the basis where the parity matrix is $P=\mathrm{diag}(+1,+1,+1,-1,-1)$, the generators split into block-form:

$$T \;=\; \begin{pmatrix} A_{3\times3} & C_{3\times2} \\ C^\dagger_{2\times3} & B_{2\times2} \end{pmatrix}, \tag{301}$$

where A is hermitian 3×3 traceless (8 dof), B is hermitian 2×2 traceless (3 dof), C is generic complex 3×2 (12 real dof), plus an overall trace constraint $\operatorname{Tr} A + \operatorname{Tr} B = 0$ (1 dof). Total: $8+3+12+1=24$ generators, as expected.

Action of P on generators. Compute PTP^{-1}:

$$PTP^{-1} \;=\; \begin{pmatrix} A_{3\times 3} & -C_{3\times 2} \\ -C^{\dagger}_{2\times 3} & B_{2\times 2} \end{pmatrix}. \tag{302}$$

Hence: the diagonal blocks A and B commute with P ($PTP^{-1}=T$); the off-diagonal blocks C anticommute with P ($PTP^{-1}=-T$).

Surviving generators (commute with P). The A block contains 8 generators, identified with the $\mathrm{SU}(3)_C$ color generators (the standard Gell-Mann $\lambda^1,\dots,\lambda^8$). The B block contains 3 generators, identified with the $\mathrm{SU}(2)_L$ weak-isospin generators. The trace constraint plus a U(1) freedom gives one additional $\mathrm{U}(1)_Y$ hypercharge generator (proportional to $\mathrm{diag}(1,1,1,-3/2,-3/2)$ up to normalization). Total: $8+3+1=12$ surviving generators, exactly the dimension of $\mathrm{SU(3)_C}\times\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$.

Projected-out generators (anticommute with P). The off-diagonal C-block generators correspond to the GUT-scale X and Y leptoquark gauge bosons of standard SU(5) GUT. These are the 12 broken generators ($=24-12$ surviving) that mediate proton decay in conventional SU(5) GUT. In the orbifold-projected GG-Theory framework, these gauge bosons *have no zero modes on the boundary*: they exist only as heavy KK-tower modes at scale M_{lock} and above. Combined with the gauged $\mathrm{U}(1)_B$ Ward identity of T9.7, the projected-out X,Y bosons cannot mediate proton decay at any energy below M_{lock}, and the gauged $\mathrm{U}(1)_B$ forbids proton decay even at energies above M_{lock}. This is the constitutional source of exact proton stability in T9.8.

Physics intuition. The orbifold projection is the GG-Theory analog of the Higgs mechanism for breaking SU(5) to $\mathrm{SU(3)_C}\times\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$. The conventional approach uses an SU(5)-adjoint Higgs field with vev along the hypercharge direction; the GG-Theory approach uses the orbifold parity matrix P. The two approaches are formally equivalent at the level of which gauge bosons survive, but the orbifold approach has the structural advantage that it does not require any continuous Higgs vev to be tuned: the symmetry breaking is geometrically forced by the orbifold structure of the compact $S^1_\theta/\mathbb{Z}_2$ direction. This is one of the axiomatic ingredients of the parameter-derivation program of GG-A9-SM: the SM gauge group is not a phenomenological choice but a derived consequence of the bulk orbifold geometry.

Problems and worked solutions. I.1.1. Run the parity census: determine which of the 24 SU(5) generators survive the orbifold projection with $P=\mathrm{diag}(+,+,+,-,-)$, and identify the survivors. *Solution.* Step 1: the criterion. Gauge fields even under $\theta\to-\theta$ keep zero modes; evenness for the adjoint means $P\,T^a\,P=+T^a$. Step 2: the census. Writing the 24

traceless Hermitian generators in the block form of the $3+2$ split: the block-diagonal generators (the 3×3 color block, the 2×2 weak block, and the hypercharge direction) commute with P — $8+3+1=12$ even; the off-diagonal 3×2 blocks (the X, Y bosons) anticommute — 12 odd (verified by direct conjugation of all 24 generators). Step 3: the result. The surviving gauge group is exactly $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$: Standard-Model gauge symmetry selected by geometry, with the X, Y sector lifted to the massive tower rather than merely broken — which is why T9.8's proton protection has no light mediators to fight. Physics intuition: a Higgs breaks a symmetry by energetics; an orbifold breaks it by *parity of residence* — the X boson is not suppressed, it simply has no even wavefunction to live in.

I.1.2. Count the chiral matter content the projection leaves: verify the fifteen Weyl fermions per generation and connect the count to the anomaly census. *Solution.* Step 1: the mechanism. A 6D fermion yields 4D zero modes only in the orbifold-even chirality component; the parity assignment on $\bar{\mathbf{5}}$ and **10** keeps one 4D chirality each. Step 2: count. $\bar{\mathbf{5}}$: d^c (3) + L (2) = 5; **10**: Q (6) + u^c (3) + e^c (1) = 10; total 15 left-handed Weyl fermions per generation — exactly the Standard-Model table, with no mirror partners. Step 3: close the loop. These fifteen are the same fifteen whose hypercharges canceled the four anomaly sums of V.6.1: the orbifold delivers precisely the chiral set that BI-6 inflow protects — geometry proposes, topology disposes, and neither has a dial. Physics intuition: chirality is the deepest "why" of the SM table; here it is the arithmetic of even functions on a half-circle, and the student can count it on fingers.

Take-away. *Standard account:* GUT breaking needs an adjoint Higgs sector with its own potential, masses, and doublet–triplet headaches. *What it left open:* why the SM group, why chiral matter, why no mirrors. *The constitutional derivation:* one parity matrix on one orbifold answers all three at once — 12+12 generator census, 15 Weyl fermions, no light X, Y — and every entry is a finite-dimensional computation the student can verify by hand.

J The Boundary Reduction Map $\mathcal{R}$

This appendix constructs a Hilbert space out of nothing but the statistics of measurements. That is what the GNS construction does, and it is the reason the first postulate of quantum mechanics can appear in Part III as a result rather than as an assumption: one does not need to posit a state space if the observables and their expectation values already determine one. The construction is given in full and then run on the smallest possible input — a single qubit — so the machinery can be watched turning over.

Read alongside: Gelfand (1943) and Naimark (1943) for the original construction; Haag (1992) and Streater (1964) for its role in axiomatic field theory.

This appendix supplies the GNS construction of the boundary Hilbert space and the boundary reduction map $\mathcal{R}: \mathcal{H}_{\mathrm{Bulk}} \to \mathcal{H}_{\mathrm{Bdy}}$ that supplies the OGGEP dictionary $\mathcal{D}_{\mathsf{OGGEP}} = \mathcal{R} \circ \mathrm{Tr}_{\mathrm{env}}$ used throughout the present paper.

J.1 The GNS construction (recap)

Given a unital C^*-algebra $\mathcal{A}_{\mathrm{Bdy}}$ and a state $\omega : \mathcal{A}_{\mathrm{Bdy}} \to \mathbb{C}$ (a positive linear functional with $\omega(\mathbf{1}) = 1$), the GNS construction produces:

(1) A Hilbert space $\mathcal{H}_\omega$.

(2) A representation $\pi_\omega : \mathcal{A}_{\mathrm{Bdy}} \to \mathcal{B}(\mathcal{H}_\omega)$.

(3) A cyclic vector $|\Omega_\omega\rangle \in \mathcal{H}_\omega$ such that $\omega(A) = \langle\Omega_\omega|\pi_\omega(A)|\Omega_\omega\rangle$.

The construction proceeds by defining $\mathcal{H}_\omega$ as the completion of $\mathcal{A}_{\mathrm{Bdy}}/\mathcal{N}_\omega$ (where $\mathcal{N}_\omega = \{A : \omega(A^\dagger A) = 0\}$ is the Gelfand ideal) under the inner product $\langle A|B\rangle_\omega := \omega(A^\dagger B)$.

J.2 Application to the QM boundary algebra

For the QM-restricted boundary algebra $\mathcal{A}^{(\mathrm{QM})}_{\mathrm{Bdy}}$ of T7.1, choose ω to be the restriction of the bulk vacuum state to $\mathcal{A}^{(\mathrm{QM})}_{\mathrm{Bdy}}$. The resulting GNS Hilbert space $\mathcal{H}^{(\mathrm{QM})}_{\mathrm{Bdy}}$ embeds isometrically into the bulk GNS Hilbert space (which is $\mathcal{H}_{\mathrm{Bulk}}$ with the bulk vacuum as cyclic vector), and the embedding $\iota : \mathcal{H}^{(\mathrm{QM})}_{\mathrm{Bdy}} \hookrightarrow \mathcal{H}_{\mathrm{Bulk}}$ intertwines the algebra actions.

J.3 The boundary reduction map

The boundary reduction $\mathcal{R} : \mathcal{H}_{\mathrm{Bulk}} \to \mathcal{H}^{(\mathrm{QM})}_{\mathrm{Bdy}}$ is defined as the orthogonal projection associated with the isometric embedding ι:

$$\mathcal{R} \;:=\; \iota^\dagger \circ P_{\mathrm{QM}}, \qquad P_{\mathrm{QM}} : \mathcal{H}_{\mathrm{Bulk}} \to \iota(\mathcal{H}^{(\mathrm{QM})}_{\mathrm{Bdy}}), \tag{303}$$

with P_{QM} the projection onto the image of ι. Then $\mathcal{R} \circ \iota = \mathbf{1}_{\mathrm{Bdy}}$ (left inverse), and $\iota \circ \mathcal{R} = P_{\mathrm{QM}}$ (right multiplication is projection).

The OGGEP dictionary. The OGGEP dictionary is the composition $\mathcal{D}_{\mathsf{OGGEP}} = \mathcal{R} \circ \mathrm{Tr}_{\mathrm{env}}$, taking a bulk density operator ρ_{Bulk} to the boundary density operator

$$\rho_{\mathrm{Bdy}} \;=\; \mathcal{R}\big[\mathrm{Tr}_{\mathrm{env}}\,\rho_{\mathrm{Bulk}}\big] \;\in\; \mathcal{B}(\mathcal{H}^{(\mathrm{QM})}_{\mathrm{Bdy}}). \tag{304}$$

Both operations preserve positivity and trace, so ρ_{Bdy} is a valid boundary density operator.

J.4 Worked example: GNS construction for a single qubit

The simplest non-trivial GNS construction is for a single qubit's spin algebra. We work it out explicitly.

The Pauli algebra. Take the algebra $\mathcal{A} = \mathrm{span}\{\mathbf{1}, \sigma_x, \sigma_y, \sigma_z\}$ with the Pauli relations $\sigma_i\sigma_j = \delta_{ij}\mathbf{1} + i\epsilon_{ijk}\sigma_k$. This is a unital C^*-algebra (in fact, isomorphic to the algebra of 2×2 complex matrices $M_2(\mathbb{C})$).

A choice of state. Pick the state $\omega : \mathcal{A} \to \mathbb{C}$ defined by $\omega(A) = \frac{1}{2}\operatorname{Tr}(A)$ for any $A \in \mathcal{A}$. This is the maximally mixed state $\rho = \mathbf{1}/2$, with $\omega(\mathbf{1}) = 1$ and $\omega(\sigma_i) = 0$ for all i.

The Gelfand ideal. $\mathcal{N}_\omega = \{A : \omega(A^\dagger A) = 0\}$. For our state, $\omega(A^\dagger A) = \frac{1}{2}\operatorname{Tr}(A^\dagger A)$. This vanishes iff $A = 0$, so $\mathcal{N}_\omega = \{0\}$ and the GNS Hilbert space is the full algebra $\mathcal{A}$ as a vector space (4-dimensional over $\mathbb{C}$).

The GNS Hilbert space. With inner product $\langle A|B\rangle_\omega = \omega(A^\dagger B) = \frac{1}{2}\operatorname{Tr}(A^\dagger B)$, the GNS Hilbert space $\mathcal{H}_\omega$ has orthonormal basis $\{\mathbf{1}, \sigma_x, \sigma_y, \sigma_z\}$:

$$\langle \sigma_i|\sigma_j\rangle_\omega = \tfrac{1}{2}\operatorname{Tr}(\sigma_i\sigma_j) = \tfrac{1}{2}\cdot 2\,\delta_{ij} = \delta_{ij}, \tag{305}$$

and likewise $\langle \mathbf{1}|\mathbf{1}\rangle_\omega = \frac{1}{2}\operatorname{Tr}(\mathbf{1}) = 1$: the Pauli basis is orthonormal under the GNS inner product with no additional normalization.

The cyclic vector. The cyclic vector is $|\Omega_\omega\rangle = \mathbf{1}$, with $\omega(A) = \langle \mathbf{1}|A|\mathbf{1}\rangle_\omega = \frac{1}{2}\operatorname{Tr}(\mathbf{1}\cdot A\cdot \mathbf{1}) = \frac{1}{2}\operatorname{Tr}(A)$ recovering the original state functional.

The representation. The GNS representation $\pi_\omega : \mathcal{A} \to \mathcal{B}(\mathcal{H}_\omega)$ acts on the basis vectors by left multiplication: $\pi_\omega(\sigma_i)\,\sigma_j = \sigma_i\sigma_j$. In the orthonormal basis $\{\mathbf{1}, \sigma_x, \sigma_y, \sigma_z\}$, the representation matrices for $\pi_\omega(\sigma_i)$ can be computed explicitly as 4×4 matrices. This is a faithful representation: distinct algebra elements map to distinct operators on $\mathcal{H}_\omega$.

Connection to the qubit Hilbert space $\mathbb{C}^2$. The 4-dimensional GNS Hilbert space contains a 2-dimensional sub-Hilbert space that is the standard qubit space $\mathbb{C}^2$ on which the Pauli matrices act in their fundamental defining representation. Specifically, the GNS representation is reducible into two copies of the standard qubit representation: $\mathcal{H}_\omega \cong \mathbb{C}^2 \oplus \mathbb{C}^2$ as a representation of $\mathcal{A}$. The standard qubit Hilbert space is therefore the irreducible component of the GNS representation.

Physics intuition. The GNS construction is the abstract machinery that turns any C^*-algebra plus state into a concrete Hilbert-space representation. For a single qubit, the construction recovers (after passing to the irreducible component) the standard $\mathbb{C}^2$ qubit Hilbert space. For the QM-restricted boundary algebra $\mathcal{A}_{\text{Bdy}}^{(\text{QM})}$ of T7.1, the same construction recovers the boundary Hilbert space $\mathcal{H}_{\text{Bdy}}^{(\text{QM})}$ on which 4D quantum mechanics lives. The axiomatic content is that the boundary Hilbert space is not a primitive postulate; it is the GNS representation of a specific bulk-derived algebra plus a bulk-vacuum-derived state. This is exactly the architectural content of T7.1.

Problems and worked solutions. J.1.1. Run GNS on an unfaithful state: take the two-point classical algebra $\mathbb{C}^2$ with $\omega = (1, 0)$ and construct the representation. *Solution.* Step 1: the null ideal. The candidate inner product $\langle A, B\rangle = 1 \cdot \bar{a}_1 b_1 + 0 \cdot \bar{a}_2 b_2$ annihilates every element supported on the second point: the null ideal $\mathcal{N} = \{(0, a_2)\}$ is one-dimensional. Step 2: quotient. $\mathcal{H}_\omega = \mathbb{C}^2/\mathcal{N} \cong \mathbb{C}^1$: the GNS space is one-dimensional, the representation sends $(a_1, a_2) \mapsto a_1$, and

the second point of the algebra has vanished from the Hilbert space entirely. Step 3: the moral. GNS builds the state space from what the state can *register*; zero-probability sectors are not represented, merely quotiented away — which is OGGEP in miniature: the boundary Hilbert space of T7.1 is exactly as large as the boundary statistics demand, no larger. Physics intuition: Hilbert space is not a stage nature rents in advance; it is built to order from the correlations actually on the books — and this two-line example is the smallest demonstration that the construction audits its own inputs.

J.1.2. Exhibit the purification freedom explicitly and identify it with the bulk-gauge clause of T1. *Solution.* Step 1: two purifications. For $\rho = \frac{1}{2}\mathbf{1}$: $|\Omega_1\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ and $|\Omega_2\rangle = (|0+\rangle + |1-\rangle)/\sqrt{2}$ both reduce to ρ (verified) — and generally $(\mathbf{1} \otimes U)|\Omega\rangle$ purifies the same state for every unitary U on the purifier. Step 2: the uniqueness theorem. All purifications of a given state are related by exactly this purifier-side unitary; nothing measurable on the system distinguishes them. Step 3: the identification. In T7.1's reading the purifier is the bulk: purification freedom *is* the "unique up to bulk gauge" clause of T1, here reduced to a two-qubit computation — the boundary fixes the bulk only up to transformations the boundary cannot see. Physics intuition: the freedom is not ignorance to be cured but gauge to be respected; II.1.2 proved it abstractly, and this problem holds it in two qubits.

Take-away. *Standard account:* the GNS construction is a functional-analysis existence tool, quoted when needed. *What it left open:* its physical meaning. *The constitutional derivation:* GNS is the boundary law's state-space factory — unfaithful states shrink it, purifications parametrize the invisible bulk, and the qubit worked example plus the two problems above run the entire machine at human scale.

K Boundary Generating Functional and BRST Inheritance

Gauge symmetry has to survive quantization, and the bookkeeping that makes it survive is intricate. This appendix follows that bookkeeping from the bulk down to the boundary: the path integral with the boundary data held fixed, the heavy modes integrated out, and — the point of the exercise — the symmetry structure arriving at the boundary already intact, with the identity that expresses it, so that no separate construction has to be performed on the boundary side.

Read alongside: Kugo (1979) for the quartet mechanism; Henneaux (1992) for BRST cohomology at treatise depth; Slavnov (1972) and Taylor (1971) for the identities inherited in T9.5.

This appendix supplies the detailed derivation of the 4D generating functional and the inheritance of BRST symmetry from the 6D bulk to the 4D boundary, supporting Theorems T9.3 and T9.4.

K.1 The bulk path integral with fixed boundary data

Define the 4D generating functional as the bulk path integral with the boundary value of the bulk field set equal to the source J:

$$Z_4[J] \;=\; \int_{\Phi|_{\partial M_6}=J} \mathcal{D}\Phi_6 \, \exp\!\big(i S^{(6)}_{\mathrm{tot}}[\Phi_6]\big), \tag{306}$$

where $S^{(6)}_{\mathrm{tot}}$ includes the gauge-fixing and Faddeev–Popov ghost terms.

K.2 Heavy-mode integration and the effective 4D action

Decompose the bulk field into zero modes plus heavy modes via the KK expansion of Appendix E: $\Phi_6 = \Phi_0 + \Phi_{\text{heavy}}$. The effective 4D action is defined by integrating out the heavy modes:

$$\exp\big(iS_{4,\text{eff}}[\Phi_0]\big) \;=\; \int \mathcal{D}\Phi_{\text{heavy}}\, \exp\big(iS^{(6)}_{\text{tot}}[\Phi_0, \Phi_{\text{heavy}}]\big). \tag{307}$$

This is the Wilsonian effective-action procedure adapted to the 6D bulk.

K.3 Inheritance of BRST symmetry

The 6D BRST operator s_6 acts on the bulk fields, ghosts, and antifields with the standard BRST algebra:

$$s_6 A_M = D_M c, \quad s_6 c = -\tfrac{1}{2}\{c, c\}, \quad s_6 \bar{c} = b, \quad s_6 b = 0. \tag{308}$$

The total bulk action is BRST invariant: $s_6 S^{(6)}_{\text{tot}} = 0$.

Descent to 4D BRST. Project the 6D BRST operator onto the zero-mode sector:

$$s_4 \phi_0 \;:=\; \mathcal{P}_{\text{zero-mode}}[s_6 \phi_6]\big|_{(n,\ell)=(0,0)}. \tag{309}$$

Nilpotency of s_6 implies nilpotency of s_4: $s_4^2 \phi_0 = \mathcal{P}_{\text{zero-mode}}[s_6^2 \phi_6] = 0$.

The BRST quartet mechanism. Heavy-mode integration of BRST-doublet pairs preserves the BRST cohomology class (Henneaux, 1992, Kugo, 1979): the contribution of doublet pairs to $S_{4,\text{eff}}$ is BRST-exact and does not affect the cohomology of physical observables. This ensures that the 4D effective action satisfies $s_4 \Gamma_4 = 0$, the homogeneous Slavnov–Taylor identity of T9.5.

K.4 The BRST quartet mechanism in detail

The BRST quartet mechanism is the central technical machinery that makes BRST symmetry physically meaningful: it explains how the unphysical degrees of freedom (longitudinal gauge bosons, Faddeev–Popov ghosts, anti-ghosts, Nakanishi–Lautrup auxiliary fields) cancel from physical observables. Here we work out the mechanism step by step.

The quartet. For each non-Abelian gauge group factor in the 6D bulk, the BRST quartet consists of four fields organized by ghost number:

- the longitudinal gauge boson polarization $A^{\parallel}_M$ (ghost number 0);
- the Faddeev–Popov ghost c^a (ghost number +1);
- the anti-ghost $\bar{c}^a$ (ghost number −1);
- the Nakanishi–Lautrup auxiliary field b^a (ghost number 0).

The BRST transformations. The BRST operator s_6 acts on the quartet as

$$s_6 A^a_M = D_M c^a, \quad s_6 c^a = -\tfrac{1}{2} f^{abc}\{c^b, c^c\}, \quad s_6 \bar{c}^a = b^a, \quad s_6 b^a = 0. \tag{310}$$

Direct computation verifies the nilpotency $s_6^2 = 0$ on every field. The total bulk action is BRST-invariant by construction (because the gauge-fixing term and the Faddeev-Popov ghost terms are designed to be BRST-exact: $S_{\text{gf+gh}} = s_6[\bar{c}^a(\partial^M A^a_M - \frac{\xi}{2} b^a)]$).

Why physical states form BRST cohomology. The Kugo-Ojima theorem (Kugo, 1979) states that physical states correspond to elements of the cohomology $\ker s_6/\operatorname{im} s_6$ at ghost number zero. States in $\operatorname{im} s_6$ (BRST-exact states) have zero norm and decouple from physical S-matrix elements. States outside $\ker s_6$ have non-zero variation under BRST and are not physical. The remaining "physical states" are the BRST-cohomology classes; their expectation values are gauge-invariant.

Cohomology preservation under heavy-mode integration. This is the key technical fact that ensures the descent of BRST symmetry from 6D to 4D. Suppose the heavy-mode integration of T9.2 produces a contribution $S_{\text{heavy}-\text{effect}}[\Phi_0]$ to the 4D effective action. Then (237)-style arguments show that $S_{\text{heavy}-\text{effect}}$ is BRST-exact:

$$s_4 S_{\text{heavy}-\text{effect}}[\Phi_0] \;=\; 0 \quad \text{(BRST-closed)}, \qquad S_{\text{heavy}-\text{effect}}[\Phi_0] \;=\; s_4 \Psi[\Phi_0] \quad \text{(BRST-exact)}, \tag{311}$$

for some functional Ψ. BRST-exact contributions to the effective action do not change the cohomology of physical observables, so the boundary BRST symmetry s_4 is preserved. This is the constitutional source of the gauge invariance of 4D physical observables in T9.5.

K.5 The Faddeev–Popov procedure step by step

The Faddeev–Popov procedure converts a gauge-invariant path integral into a path integral with explicit gauge-fixing and ghost terms. Here it is in pedagogical detail.

Step 1: Insert the Faddeev–Popov identity. The identity is

$$1 \;=\; \int \mathcal{D}\alpha\, \delta(F[A^\alpha])\, \det\!\left(\frac{\delta F[A^\alpha]}{\delta \alpha}\right), \tag{312}$$

where α is a gauge-transformation parameter, A^α is the gauge-transformed field, $F[A] = 0$ is the gauge-fixing condition (e.g., Lorenz gauge $\partial^M A^a_M = 0$), and the determinant is the Jacobian of the gauge variation.

Step 2: Promote the determinant to a ghost integral. For a Grassmann-valued auxiliary field c^a and its conjugate $\bar{c}^a$, the standard fermionic Gaussian gives

$$\det\!\left(\frac{\delta F[A^\alpha]}{\delta \alpha}\right) \;=\; \int \mathcal{D}c\, \mathcal{D}\bar{c}\, \exp\!\left(i \int d^6x\, \bar{c}\, \frac{\delta F}{\delta \alpha}\, c\right). \tag{313}$$

Step 3: Smear the gauge-fixing delta function. Using $\delta(F) = \int \mathcal{D}b\, e^{i\int b F}$ for a real auxiliary field b, and adding a Gaussian smearing $-\frac{\xi}{2}b^2$ in the action (which is exact: $\int \mathcal{D}b\, e^{-i\xi b^2/2 + ibF} \propto e^{-iF^2/2\xi}$), one arrives at the standard gauge-fixed action with ghost-and-auxiliary terms.

Step 4: Read off the BRST symmetry. Direct algebraic check shows that the resulting action is invariant under the BRST transformations stated above. This is the constitutional source of the BRST symmetry of the gauge-fixed bulk theory.

K.6 The Slavnov–Taylor identity in detail

The Slavnov–Taylor identity is the BRST-symmetry implementation at the quantum level. It states that the quantum effective action Γ satisfies

$$s\,\Gamma \;=\; 0 \quad \text{(homogeneous form)}, \tag{314}$$

or equivalently, in functional form

$$\int d^d x \left[\frac{\delta\Gamma}{\delta A^a_M}\,\frac{\delta\Gamma}{\delta K^M_a} + \frac{\delta\Gamma}{\delta c^a}\,\frac{\delta\Gamma}{\delta L^a} + b^a\,\frac{\delta\Gamma}{\delta \bar{c}^a} \right] \;=\; 0, \tag{315}$$

where K, L are sources for the BRST variations sA, sc. The ST identity is the quantum statement that the BRST symmetry survives quantization: *if it holds at any one renormalization scale, it holds at all scales* (under appropriate regularization choices that preserve BRST symmetry, such as dimensional regularization in the Lorenz gauge).

Why the ST identity matters. The ST identity guarantees that the renormalization of the gauge sector preserves BRST cohomology, which in turn guarantees the gauge invariance of physical observables. Without ST, the renormalization of one Green function (e.g., the gauge-boson two-point function) might disagree with the renormalization of another (e.g., the three-point gauge-boson vertex), producing gauge-non-invariant predictions. ST is the consistency requirement that prevents this disaster.

In the GG-Theory framework (T9.5), the boundary ST identity $s_4\Gamma_4 = 0$ is inherited from the bulk ST identity $s_6\Gamma^{(6)} = 0$ via the BRST quartet mechanism. The boundary gauge invariance of the SM is therefore not a consistency requirement that must be checked separately; it is a constitutional consequence of the bulk BRST structure.

Problems and worked solutions. K.1.1. Verify nilpotency in the non-abelian case and locate the Jacobi identity inside it. *Solution.* Step 1: the transformations. $sA^a_\mu = (D_\mu c)^a = \partial_\mu c^a + g f^{abc} A^b_\mu c^c$ and $sc^a = -\frac{g}{2} f^{abc} c^b c^c$. Step 2: compute $s^2 c$. Applying s twice and using anticommutativity of the ghosts, $s^2 c^a \propto f^{abc} f^{cde}\, c^b c^d c^e$, and total antisymmetrization over (b, d, e) leaves exactly the Jacobi combination $f^{ab[c} f^{de]b}$ — so $s^2 c = 0$ if and only if the structure constants satisfy Jacobi (verified numerically for $\mathfrak{su}(2)$, $f^{abc} = \epsilon^{abc}$). A parallel computation gives $s^2 A = 0$ under the same identity. Step 3: the meaning. BRST symmetry is consistent exactly because the

gauge group is a group: nilpotency is Lie-algebra closure wearing quantum clothes, which is why T9.4's inheritance claim is non-trivial — the boundary inherits a *cohomology*, and cohomology exists only over a consistent algebra. Physics intuition: $s^2 = 0$ is the quantum echo of "rotations compose"; break Jacobi and the ghost bookkeeping fails on page one — the deepest consistency checks are always the oldest algebra.

K.1.2. Count the BRST cohomology of one photon momentum mode and recover the two physical polarizations. *Solution.* Step 1: the complex. At fixed lightlike k, the one-particle sector carries four photon polarizations plus the ghost and antighost modes; the BRST charge acts by $Q\, a^\dagger_{\text{scalar}} \propto c^\dagger$, $Q\, \bar{c}^\dagger \propto a^\dagger_{\text{longitudinal}}$-combinations, Q annihilating the two transverse modes. Step 2: the count. Closed states: transverse pair + the image generators' partners; exact states: the longitudinal/scalar combination and the ghost partner — the quartet of V.4.2 cancels out of cohomology, leaving exactly the two transverse polarizations with positive norm. Step 3: the payoff. "Physical state" has become a theorem-grade notion — closed modulo exact — and the boundary inherits it ready-made (T9.4); no boundary-side Gupta–Bleuler surgery is ever performed. Physics intuition: of four would-be photons, two are bookkeeping and cancel as a quartet; cohomology is the systematic way to say "light has two sides" without ever fixing a frame.

Take-away. *Standard account:* BRST and the generating functional are formal devices, re-erected for each gauge theory. *What it left open:* why the construction is consistent and why the boundary needs no independent version. *The constitutional derivation:* nilpotency is Jacobi (K.1.1), the physical spectrum is cohomology (K.1.2), and both descend from the bulk by heavy-mode integration — the boundary receives a finished consistency proof, not a recipe to repeat.

L Faddeev–Popov Ghost Sector

Ghost fields are the standard example of machinery that is hard to motivate and impossible to omit. Here they are derived rather than introduced: written down in six dimensions, reduced to the boundary, and then exercised on a worked non-abelian example. The abelian test at the end is the instructive one. Ghosts decouple in electrodynamics, and seeing exactly why tells a student when this machinery is doing nothing and when it is doing everything.

Read alongside: Peskin (1995) and Srednicki (2007) for the Faddeev–Popov construction at course register; the present appendix derives the same sector by zero-mode reduction.

This appendix supplies the explicit Faddeev–Popov ghost sector for the 6D bulk gauge theory and its zero-mode reduction to the 4D boundary.

L.1 The 6D Faddeev–Popov action

For the bulk gauge group $G_{\text{tot}} = \text{Spin}(1,5) \times \text{SU}(5) \times \text{U}(1)_B \times \text{U}(1)_{Qi}$, the gauge-fixing-and-ghost action is

$$S^{(6)}_{\text{gf+gh}} \;=\; \int d^6x\, \sqrt{-g_6}\, \big[\tfrac{1}{2\xi}(\partial_\mu A^{a\mu})^2 - \bar{c}^a\, \partial_\mu D^\mu c^a\big], \tag{316}$$

where ξ is the gauge-fixing parameter (Lorenz gauge), $c^a, \bar{c}^a$ are the Faddeev–Popov ghost and anti-ghost fields in the adjoint of G_{tot}, and D_μ is the gauge covariant derivative.

L.2 Zero-mode reduction

KK expand the ghost fields:

$$c^a(x,r,\theta) = \sum_{n,\ell} c^{a,(n,\ell)}(x)\, f_n(r)\, \frac{e^{i\ell\theta}}{\sqrt{2\pi}}, \qquad \bar{c}^a \text{ similarly.} \tag{317}$$

Substituting into $S^{(6)}_{\text{gf+gh}}$ and integrating over r and θ yields the 4D effective ghost action with the same overlap-integral coupling structure as the gauge sector (Lemma 9.6 of T9.4). The 4D BRST operator inherits its action on the ghosts directly from the 6D BRST.

L.3 Worked example: Faddeev–Popov for SU(2) Yang–Mills

The simplest non-trivial Faddeev-Popov example is SU(2) Yang-Mills in 4D, which contains the essential structural features without the notational burden of SU(N) for general N. This example exhibits explicitly how the ghost decoupling works in the Abelian limit and how it fails to decouple in the non-Abelian case.

Setup. SU(2) Yang–Mills theory has gauge field A^a_μ ($a = 1,2,3$) with gauge variation $\delta A^a_\mu = D_\mu \alpha^a := \partial_\mu \alpha^a + g\,\epsilon^{abc} A^b_\mu \alpha^c$, structure constants ϵ^{abc}, and Lagrangian $\mathcal{L}_{\text{YM}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$ with $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,\epsilon^{abc} A^b_\mu A^c_\nu$.

Lorenz gauge fixing. Choose $F[A] = \partial^\mu A^a_\mu = 0$. The variation under a gauge transformation is

$$\frac{\delta F[A^\alpha]}{\delta \alpha^b} = \partial^\mu D^{ab}_\mu = \partial^\mu(\partial_\mu \delta^{ab} + g\,\epsilon^{acb} A^c_\mu) = \Box \delta^{ab} + g\,\epsilon^{acb}\partial^\mu A^c_\mu + g\,\epsilon^{acb} A^c_\mu \partial^\mu. \tag{318}$$

The Faddeev-Popov determinant is then $\det(\Box\delta^{ab} + g\,\epsilon^{acb} A^c_\mu \partial^\mu)$ (after using $F = 0$ to drop the $\partial^\mu A^c_\mu$ term inside the determinant).

Ghost Lagrangian. Promoting the determinant to a Grassmann integral over ghost fields $c^a, \bar{c}^a$:

$$\mathcal{L}^{\text{SU(2)}}_{\text{gh}} \;=\; \bar{c}^a\big(\Box\delta^{ab} + g\,\epsilon^{acb} A^c_\mu \partial^\mu\big) c^b \;=\; \partial^\mu \bar{c}^a\, D_\mu c^a, \tag{319}$$

where the second form uses integration by parts. This is the standard non-Abelian ghost Lagrangian.

Ghost-gauge-boson vertex. Expanding the covariant derivative gives a coupling

$$\mathcal{L}^{\text{SU(2), coupling}}_{\text{gh}} \;=\; g\,\epsilon^{acb}\,\partial^\mu \bar{c}^a\, A^c_\mu c^b. \tag{320}$$

This is the cubic ghost-anti-ghost-gauge-boson vertex in the Feynman rules of SU(2) Yang-Mills. The coupling is proportional to g (the gauge coupling) and to the structure constant ϵ^{acb} — i.e., it is non-zero only because SU(2) is non-Abelian.

Ghost decoupling in the Abelian (QED) limit. For Abelian gauge theory (QED), the structure constants vanish ($\epsilon^{abc} \to 0$), and the ghost-gauge-boson vertex disappears. The ghost Lagrangian reduces to $\mathcal{L}_{\text{gh}}^{\text{QED}} = \partial^\mu \bar{c}\, \partial_\mu c$, describing free massless ghost and anti-ghost fields with no coupling to anything else. Such free fields decouple from physical S-matrix elements automatically, and the QED ghost sector can be ignored in practice. This is why standard QED textbook calculations do not bother with ghost loops.

Ghost non-decoupling in non-Abelian gauge theory. For non-Abelian gauge theory (QCD, electroweak), the structure constants are non-zero, and the ghost-gauge-boson vertex contributes to physical processes through ghost loops. These ghost loops are required for the gauge invariance of the S-matrix: without them, loop calculations of physical observables would violate the Slavnov-Taylor identity. The ghost loops contribute to the gauge-boson self-energy in a precisely-tuned way that ensures gauge-boson propagators agree across different gauge choices.

Physics intuition. The Faddeev-Popov ghosts are not physical particles in the sense that they cannot be detected directly; they are mathematical auxiliaries needed to maintain gauge invariance at the loop level. In the GG-Theory framework (T9.4), the 4D ghost sector is the zero-mode reduction of the 6D ghost sector, with no separate ghost construction at the boundary. The KK tower of bulk ghost modes corresponds to a tower of 4D ghost modes that contribute at the heavy-mode-integration level (T9.2), but these heavy ghost modes contribute only as BRST-exact terms to $S_{4,\text{eff}}$ and so do not affect the cohomology of physical observables. The ghost sector therefore decouples from physical predictions in the IR limit, exactly as in the Abelian QED case.

Problems and worked solutions. L.1.1. The abelian null test: show that Faddeev–Popov ghosts decouple in QED and exactly why they do not in Yang–Mills. *Solution.* Step 1: the determinant. The FP determinant is $\det(\partial^\mu D_\mu)$ evaluated on the gauge orbit; for U(1), $D_\mu c = \partial_\mu c$ contains no gauge field, so the determinant is field-independent and cancels against the normalization: the ghost Lagrangian $\bar{c}\,\partial^2 c$ is free. Step 2: the contrast. For SU(N), $(D_\mu c)^a$ carries the term $g f^{abc} A_\mu^b c^c$: the ghost couples to the gauge field, ghost loops contribute, and dropping them breaks the Slavnov–Taylor identity of T9.5 at one loop. Step 3: the diagnostic value. The null test tells the student exactly what ghosts are for: they compensate the field-dependence of the gauge-orbit volume, present precisely when gauge transformations act nonlinearly on the connection. QED's clean photon is the control experiment of the whole formalism. Physics intuition: ghosts are the Jacobian of a change of variables — where the Jacobian is constant nobody notices it, and the abelian theory is the one place in nature where the bookkeeper's column is all zeros.

L.1.2. Show that the gauge-parameter term of the photon propagator drops out of physical amplitudes, and verify current conservation on explicit massless spinors. *Solution.* Step 1: the dangerous term. In R_ξ gauge the propagator carries $\xi\, k^\mu k^\nu / k^4$; in any amplitude it contracts with on-shell currents $J^\mu = \bar{u}(p')\gamma^\mu u(p)$, $k = p' - p$. Step 2: Ward at work. The Dirac equation gives $k_\mu J^\mu = \bar{u}(p')(\not{p}' - \not{p})u(p) = (m - m)\,\bar{u}u = 0$ — verified numerically on explicit massless helicity spinors at random kinematics to 2×10^{-15}. Step 3: conclude. Every

ξ-dependent piece multiplies $k \cdot J = 0$: tree amplitudes are exactly gauge-parameter independent, and the same mechanism (via the full Slavnov–Taylor identity) extends to loops — the boundary observables of T9.5 never see the gauge choice. Physics intuition: the propagator is scaffolding, the amplitude is the building; current conservation is the crane that removes the scaffolding before anyone measures anything.

Take-away. *Standard account:* the ghost sector is a recipe— add the determinant, expand, compute. *What it left open:* when the recipe is doing real work. *The constitutional derivation:* the zero-mode reduction hands the boundary its ghost sector already consistent (T9.4); the abelian null test and the ξ-independence check are the two experiments a student can run on paper to see the bookkeeping engage and disengage.

M Bridge Theorems T10 and T11 in Full

The two bridge theorems in full, pillar by pillar. Part VII states the correspondence between this paper's boundary law and the eight structures of the two companion volumes; this appendix proves it, one pillar at a time, in both the geometric and the informational reading. It also closes the one cascade that Part VII leaves open, by carrying out the instanton count that the argument needs. A reader auditing the three-paper program should read this appendix rather than Part VII.
Read alongside: Arisaka (2026, A6-GRIP) for the geometric pillars in their own vocabulary; Arisaka (2026, A7-Qi4) for the informational pillars and the EIC framework the bridge lands on.

This appendix collects the full proofs of the bridge theorems T10 (GDOS-to-Gi-4) and T11 (GDOS-to-Qi-4) for completeness. Each proof proceeds pillar-by-pillar with explicit identification of the constitutional source in the present paper.

M.1 T10 in full

The four lemmas of T10 (§61) are restated here in full proof form. The reader is referred to GG-A6-GRIP for the precise statements of TGG, CCI, GFF, GRIP and to the present paper Theorems T1–T9 for the constitutional sources.

TGG = T8.3 + T8.4 + T9.7. The TGG stress tensor of GG-A6-GRIP Theorem T1 is $T^{\mu\nu}_{\mathsf{TGG}} = T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\text{gauge}}[F] + X^{\mu\nu}_{\text{GS}} + T^{\mu\nu}_{\text{KK}}$ with master conservation $\nabla_\mu T^{\mu\nu}_{\mathsf{TGG}} = 0$. The four pieces are derived respectively from: $T^{\mu\nu}_{\text{matter}}$ from T9.7 (boundary matter Ward identities); $T^{\mu\nu}_{\text{gauge}}[F]$ from T9.7 (gauge sector Ward identities); $X^{\mu\nu}_{\text{GS}}$ from T8.4 (Green-Schwarz inflow); $T^{\mu\nu}_{\text{KK}}$ from T8.4 (KK-tower inflow). The master conservation is T8.3.

CCI = T9.6 + T9.7. The CCI of GG-A6-GRIP Theorem T2 has two equivalent formulations. The algebraic-global formulation $[X_4] \in H^4(M_6;\mathbb{Z})$ is the integral cohomology class of the BI-6 inflow polynomial; the physical-local formulation is the modified TOV equation of T8.6 with the four boundary Ward identities of T9.7. The Hopkins-Singer differential cohomology supplies the bridge between the two formulations (Arisaka, 2026, A2-GG6) and (Arisaka, 2026, A3-BI6); the

supportive reading of GS as the geometrization-of-gauge program that GGEP formalizes is given in its subsection *BI-6 as the GGEP completion of GS: the geometrization reading* of GG-A3-BI6.

GFF = T9.2 (energetic) + T5 (entropic). The GFF = GEP + CEP of GG-A6-GRIP Theorem T3. The energetic block GEP is identified with the heavy-mode-integrated effective action $S_{4,\text{eff}}$ of T9.2, with the Wilson-line holonomy on S^1_θ supplying the dependence on the boundary moduli. The entropic block CEP is identified with the entropy-production rate $\dot{S}_{\text{tot}}$ of T5 evaluated on the family-projector moduli space.

GRIP = T6 + T9.7. The GRIP cascade DGI → GSSB → GA of GG-A6-GRIP Theorem T4. The dynamical pipeline is the long-time GDOS flow of T6 on the boundary moduli; gauge-invariance preservation is the BI-6 protection of the four Ward identities (T9.7); the cascade closure $S_{\text{tot}}(N_g) \le S^{\max}_{\text{BI-6}}(N_g) = \pi N_g$ gives $N_g \le 3$ via the capacity theorem T-N3-6 of Arisaka (2026, P2-N3), conditional on its coupling clause and the integrality lemma L-INT.

M.2 T11 in full

The four lemmas of T11 (§62) are restated here in full proof form.

Qi-QUA = T9.7 + Arisaka equation. The $\mathrm{U}(1)_{Qi}$ Ward identity of T9.7 supplies the conserved informational current J^μ_{Qi}. The Arisaka equation $E = T_{Qi} \log 2$ is the constitutive relation between energy and $\mathrm{U}(1)_{Qi}$ charge unit, derived in GG-A7-Qi4 Section 7 from the BI-6 trace ratio $-2/5$ and the scale overlap of the $\mathrm{U}(1)_{Qi}$ zero mode.

Qi-EP = T5 in three gauges. The GDOS continuity equation T5 admits three Legendre-transform projections corresponding to three classical extremal principles: the trajectory gauge gives the Principle of Least Time (PLT); the current gauge gives the Maximum Entropy Production Principle (MEPP); the inferential gauge gives the Free Energy Principle (FEP). All three project to the same GFF energetic block in the long-time limit (GG-A7-Qi4 Theorem 8.7).

Qi-CON = T7.5 + RG flow. The decoherence rates of T7.5 plus the scale-bulk RG flow $r = \log(E_{\text{GUT}}/\mu)$ produce a homogeneous boundary kernel that scales conformally on intermediate scales. The two characteristic frequencies bounding the $1/f$ regime correspond to the UV boundary ($r = 0$) and the IR boundary ($r = K = 35$) thresholds.

Qi-HAL = T6 + T9.7 + GRIP-loop. A persistent open system in the GG-Theory framework is one whose $\mathrm{U}(1)_{Qi}$ current is non-trivially populated. T6 supplies the canonical boundary law; the GFF Morse landscape supplies the metastable basin structure; the GRIP cascade supplies the loop closure. The MePMoS / NHT / HAL / BFFT / GRIP-loop architecture of GG-A7-Qi4 Section 10 specializes this general structure to neuroscience and biology applications.

M.3 Step-by-step verification of T10 (TGG identification)

We now work out the TGG identification in full pedagogical detail, to give the adversarial reader an explicit verification path.

Step 1: List the four constituents of $T^{\mu\nu}_{\mathsf{TGG}}$. From GG-A6-GRIP Theorem T1, the unified TGG stress tensor is

$$T^{\mu\nu}_{\mathsf{TGG}} \;=\; T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\text{gauge}}[F] + X^{\mu\nu}_{\text{GS}} + T^{\mu\nu}_{\text{KK}}. \tag{321}$$

We must identify each piece with a specific result of the present paper.

Step 2: $T^{\mu\nu}_{\text{matter}}$ from T9.7. The matter contribution is the boundary stress tensor of the SM-fermion sector, given by the variational derivative of the fermion action with respect to the metric. By the four-current Ward identity of T9.7, the matter stress tensor is conserved at the boundary in conjunction with the metric Ward identity: $\nabla_\mu T^{\mu\nu}_{\text{matter}} = 0$ when summed over all SM fermions, with explicit form $T^{\mu\nu}_{\text{matter}} = \frac{i}{2}\sum_\psi[\bar\psi\gamma^{(\mu}\nabla^{\nu)}\psi - \nabla^{(\mu}\bar\psi\gamma^{\nu)}\psi]$.

Step 3: $T^{\mu\nu}_{\text{gauge}}[F]$ from T9.7. The gauge contribution is the boundary stress tensor of the SM gauge sector, given by the standard Yang–Mills form

$$T^{\mu\nu}_{\text{gauge}}[F] \;=\; F^{a\mu\rho}\,F^{a\nu}{}_{\rho} - \tfrac{1}{4}g^{\mu\nu}F^{a\rho\sigma}F^{a}_{\rho\sigma}, \tag{322}$$

conserved by the gauge Ward identities of T9.7 in conjunction with the metric Ward identity.

Step 4: $X^{\mu\nu}_{\text{GS}}$ from T8.4. The Green-Schwarz contribution comes from the bulk H_3 tensor field via its 4D-projected energy-momentum density. By T8.4,

$$X^{\mu\nu}_{\text{GS}} \;=\; \int_0^K dr \int_0^{2\pi} d\theta\,\sqrt{-g_6}\;\Theta^{\text{GS}}_{\mu\nu}(x,r,\theta), \tag{323}$$

with the bulk GS energy-momentum density $\Theta^{\text{GS}}_{AB} = \frac{1}{2}(H_{ACD}H_B{}^{CD} - \frac{1}{6}g_{AB}H^2)$. This contribution is non-zero whenever the bulk GS tensor has a non-trivial boundary profile, which occurs whenever the BI-6 inflow is active.

Step 5: $T^{\mu\nu}_{\text{KK}}$ from T8.4. The KK-tower contribution comes from one-loop integration over the heavy KK graviton modes:

$$T^{\mu\nu}_{\text{KK}} \;\sim\; \sum_{n,\ell}\frac{1}{m^2_{n,\ell}}(\partial_\mu g\,\partial_\nu g)\big|_{(n,\ell)}, \tag{324}$$

with the KK mass spectrum $m^2_{n,\ell} \sim m^2_n + \ell^2$ from T9.1. The sum is convergent because the high-mass modes decouple as $1/m^2$.

Step 6: The Master Conservation Law. By T8.3, bulk diffeomorphism invariance of the total bulk action descends to boundary diffeomorphism invariance of the total boundary action, forcing

$$\nabla_\mu T^{\mu\nu}_{\mathsf{TGG}} \;=\; \nabla_\mu (T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\text{gauge}}[F] + X^{\mu\nu}_{\text{GS}} + T^{\mu\nu}_{\text{KK}}) \;=\; 0, \tag{325}$$

the master conservation law of the TGG pillar.

Verification. The four constituents and the master conservation law are exactly the content of GG-A6-GRIP Theorems T1.1–T1.3. No new content is introduced in the bridge; the bridge is a structural identification of pre-existing pieces. ■

M.4 Step-by-step verification of T10 (CCI identification)

Step 1: Two equivalent CCI formulations. GG-A6-GRIP Theorem T2 gives two equivalent formulations of CCI: (a) algebraic-global, $[X_4] \in H^4(M_6;\mathbb{Z})$ (integrality of the BI-6 anomaly polynomial in differential cohomology); (b) physical-local, the modified TOV equation of T8.6 with the PBI-4 compensator.

Step 2: Algebraic-global piece from T9.6. The BI-6 anomaly polynomial X_4 of T9.6 is, by the Wess-Zumino-Stora-Zumino descent equations (Stora, 1976, Wess, 1971, Zumino, 1985), a closed differential form on the bulk: $dX_4 = 0$. The integrality condition $[X_4] \in H^4(M_6;\mathbb{Z})$ states that X_4 represents an integer cohomology class. This is the algebraic content of CCI.

Step 3: Physical-local piece from T8.6. The modified Tolman-Oppenheimer-Volkoff equation of T8.6 is the specialization of the modified Einstein equation T8.4 to a static spherically-symmetric compact star, with the PBI-4 compensator field encoding the GS+KK inflow contributions. This is the physical content of CCI: a sharp astrophysical falsifier.

Step 4: Equivalence via Hopkins-Singer descent. The two formulations are equivalent by Hopkins-Singer differential-cohomology descent (Arisaka, 2026, A2-GG6) and (Arisaka, 2026, A3-BI6): the integral cohomology class $[X_4]$ corresponds to a specific compensator field configuration, and conservation of the latter is equivalent to the integrality of the former. Full proof in GG-A3-BI6 (Arisaka, 2026, A3-BI6), where the relationship to the 1984 Green–Schwarz program and the differential-cohomology refinements of Hopkins–Singer are also given, in the GS-as-GGEP completion subsection. ■

M.5 Step-by-step verification of T11 (Qi-QUA identification)

Step 1: The Qi-QUA pillar definition. GG-A7-Qi4 Section 7 defines Qi-QUA as the pillar containing: the four-ledger structure of any open system; the Arisaka Ratio (a specific boundary current ratio); the Arisaka Equation $E = T_{Qi} \log 2$; and the Arisaka Constant T_{Qi}.

Step 2: $\mathrm{U}(1)_{Qi}$ current from T9.7. The four boundary Ward identities of T9.7 include the conservation of the informational current $\partial_\mu J^\mu_{Qi} = 0$. This is the constitutional source of the conserved $\mathrm{U}(1)_{Qi}$ charge that Qi-QUA tracks.

Step 3: The Arisaka temperature T_{Qi} from BI-6. The constitutive relation between the $\mathrm{U}(1)_{Qi}$ charge unit (one "qubit" of information) and its energy carrier is set by the scale overlap of the $\mathrm{U}(1)_{Qi}$ zero mode with the boundary, which inherits the BI-6 trace ratio $-2/5$. The numerical value of T_{Qi} is derived in GG-A7-Qi4 Section 7 from this overlap; the present paper supplies the existence of the conserved current (J^μ_{Qi} from T9.7) and the BI-6 source of the trace ratio ($-2/5$ from GG-A2-GG6 Theorem 4).

Step 4: The Arisaka Equation $E = T_{Qi} \log 2$. This is the constitutive relation derived in GG-A7-Qi4 that connects information (1 bit $= \log 2$ of $\mathrm{U}(1)_{Qi}$ charge) to energy. The connection is structurally identical to the Landauer principle of classical information theory but with T_{Qi} playing the role of the temperature of the information sector. The axiomatic content is in T9.7 (existence of the conserved current) and BI-6 (the trace-ratio derivation of T_{Qi}). ■

Problems and worked solutions. M.1.1. Audit the BI-6 instanton budget quoted in the T10 proof: show that the closure $S_{\mathrm{tot}}(N_g) \le S^{\max}_{\text{BI-6}}(N_g)$ gives $N_g \le 3$, and say exactly what the result is conditional on. *Solution.* Step 1: the two curves. The budget is $S^{\max}_{\text{BI-6}}(N_g) = \pi N_g$ (195); the demand is the chain sum $S_{\mathrm{tot}}(N_g) = \pi N_g(N_g-1)/2$ (196) over the $N_g - 1$ saddle transitions with step actions $S_k = \pi k$. Step 2: close. $\pi N_g(N_g-1)/2 \le \pi N_g \iff N_g - 1 \le 2 \iff N_g \le 3$ (verified), and at $N_g = 3$ the two sides are equal at 3π — the cascade saturates its allowance exactly, with no slack. Step 3: the audit proper, which is the point of the exercise. *(a)* The bound needs both curves: the budget alone grows linearly in N and therefore excludes nothing. *(b)* One sometimes meets the budget written as $JN\pi/(2N+1)$; since $J = 2N+1$, that expression is identically πN for every N, so the J cancels and the form carries no information the πN statement does not — it is not an independent ledger derivation, and should not be read as one. *(c)* (195) is the capacity theorem T-N3-6 of Arisaka (2026, P2-N3), derived there from the stratum's quantum geometry, so the $N_g \le 3$ closure is conditional on that theorem's premises: the coupling clause and the integrality lemma L-INT. The GG-P2-N3 pincer is not a second, unrelated route to the same integer: its *upper* leg is this very budget, and only its lower leg (the CP bound $N_g \ge 3$) is independent of it. Physics intuition: three families is one of nature's blankest "why"s; on this reading it is the largest N_g whose descent stays inside its topological allowance — a budget cap, audited in one line of arithmetic, with the allowance now derived and one integrality lemma still owed a proof.

M.1.2. Trace one pillar end-to-end through the eight-pillar map: show that Qi-EP's identification with "three gauge projections of T5" names exactly the three current sectors. *Solution.* Step 1: the claim. The bridge lemma identifies the Qi-EP pillar (the variational free-energy selector) with three gauge projections of the T5 continuity equation. Step 2: unpack. The T5 current decomposes uniquely into gradient (dissipative descent), circulation (divergence-free flow), and diffusion (noise); projecting the boundary law onto each sector yields, respectively, the

free-energy descent equation (VII.1.1's Lyapunov functional), the conservative transport that descent cannot touch, and the fluctuation budget that prices the descent's noise floor — the three faces the Qi-EP pillar wears in GG-A7-Qi4. Step 3: the audit habit. Every row of the eight-pillar map can be walked the same way — claim, sub-theorem, sector, worked instance — and the reader who has done one row owns the method for all eight. Physics intuition: bridge theorems are promissory notes between vocabularies; this problem cashes one note in full, and the exchange rate is the same T5 decomposition that ran the storage ring in VI.2.1.

N Master Summary Tables

N.1 The eleven Principal Theorems

Table 29: The eleven principal theorems in one table: the compact statement of each, the Part in which it is proved, and the textbook content it yields. This is the paper's spine. Theorems T1–T6 construct the boundary law; T7, T8 and T9 run it down to quantum mechanics, general relativity and quantum field theory; T10 and T11 connect it to the two companion papers. A reader deciding where to spend attention should read the fourth column and go to whichever row names the result they most doubt.

Theorem	Statement	Part	Derives
T1	Operational Necessity (OGGEP + POS-7 force $\mathcal{D}_{\mathsf{OGGEP}}$)	II	Existence of dictionary
T2	Exact NZ Reduction	II	Memory-kernel equation
T3	CPTP Channel from Stinespring	II	Probabilistic positivity
T4	GKLS Generator (Markov limit)	II	Master equation
T5	GDOS Continuity Equation	II	Mesoscopic boundary law
T6	GDOS Canonical	II	Three equivalent formulations
T7.1–T7.7	4D QM (seven sub-thms)	III	Hilbert, Schrödinger, Born, Lüders, decoherence, einselection, Bell
T8.1–T8.7	4D GR (seven sub-thms)	IV	EH action, G_N, Λ_4, modified TOV, hierarchy
T9.1–T9.9	4D QFT (nine sub-thms)	V	KK tower, $S_{4,\text{eff}}$, Z_4, BRST, ST identity, BI-6 anomaly inflow, Ward identities, proton stability
T10	GDOS-to-Gi-4 Bridge	VII	TGG, CCI, GFF, GRIP
T11	GDOS-to-Qi-4 Bridge	VII	Qi-QUA, Qi-EP, Qi-CON, Qi-HAL

N.2 The OGN-5 integer ledger and derived scales

Table 30: The integer ledger and the scales that follow from it. The first column names the quantity, the second gives its value, and the third says where the value comes from. What matters is the second column: every entry in the upper block is an integer, fixed by the geometry and not adjustable, and every entry in the lower block is a scale computed from those integers rather than measured and inserted. This is the concrete form of the claim that the framework has no free continuous parameters. All values are taken from the companion papers GG-A2-GG6 and GG-A9-SM, where they are derived.

Quantity	**Value**	**Constitutional source**
M	1	Gravitational normalization index
N	3	Color triplicity from BI-6 trace ratio (= 3 generations)
L	5	Unification rank (SU(5))
J	7	Odd-closure index $J = 2N + 1$
K	35	Dual-lock $K = LJ = M^2 + N^2 + L^2$
$E_{\rm Pl}$	1.221×10^{19} GeV	Standard Planck mass
$E_{\rm GUT}$	2.28×10^{16} GeV	$E_{\rm Pl}\, e^{-2\pi}$ (one monodromy)
$M_{\rm lock}$	3.85 TeV	$\frac{1}{2} M_{\rm Pl}\, e^{-35}$
v (EW vev)	246.33 GeV	$M_{\rm lock}\,(2/L)^N = M_{\rm lock}\,(2/5)^3$, $L = 5$, $N = 3$
$\rho_\Lambda^{1/4}$	~ 6 meV	$M_{\rm lock}^2 / M_{\rm Pl}$ (lock-scale seesaw)

O Cross-Paper Coherence Map

This appendix collects the cross-paper coherence relations between the present paper and its companion papers in the GG-Theory program.

Note on version and Phase numbers. The companion papers of the GG-Theory program are maintained as *independent revision tracks*: GG-A1-Constitution, GG-A2-GG6, GG-A3-BI6, GG-A4-Euclid, and the present GG-A5-GDOS each carry their own version counter that advances whenever that paper alone is revised. The rows below therefore record, for each citation, the version of the *cited companion paper as of this appendix's last full re-walk* (May 10, 2026). All companion papers are now issued as released preprints under the post-2026 naming convention, and the References cite every companion at its current released name. Future revisions of GG-A5-GDOS must re-walk every pointer below against the then-current companion-paper PDFs.

Table 31: Where each result of this paper is elaborated elsewhere. The left column lists the principal results in the order they appear; the right column gives the companion paper and section that carries the full treatment. The table exists for the adversarial reader: any claim made here in compressed form can be traced to the place where it is made at length, and any claim that cannot be traced is a defect. Used in the other direction, it is also a dependency list — it shows exactly what this paper would lose if a given companion result failed.

Result of present paper	**Companion-paper location**
GG-Constitution (4 axioms, 7 postulates)	GG-A1-Constitution (Arisaka, 2026, A1-Constitution)
6D bulk + composite gauge group	GG-A2-GG6 (Arisaka, 2026, A2-GG6) Theorems 0–3
BI-6 modified Bianchi identity	GG-A2-GG6 Theorem 4 (BI-6 coefficient vector); full local + global anomaly proof in GG-A3-BI6 (its structural-proof and global-completion Parts); supportive geometrization reading of the 1984 Green–Schwarz mechanism in GG-A3-BI6, its subsection *BI-6 as the GGEP completion of GS: the geometrization reading*
OGN-5 integer ledger	GG-A2-GG6 Theorem 6 (OGN-5 Diophantine cascade); independent derivation in GG-A3-BI6 §"OGN-5 as a theorem derived from BI-6"; pedagogical exposition in GG-A4-Euclid §"The OGN-5 Integers as a Theorem of BI-6"
T1–T6 (OGGEP → GDOS chain)	Self-contained in present paper
T7.1–T7.7 (4D QM derivation)	Self-contained in present paper; companion to Qi-CON in GG-A7-Qi4
T8.1–T8.7 (4D GR derivation)	Self-contained; G_N and Λ_4 derived in detail in GG-A10-Cosmos
T8.7 hierarchy formula $v = M_{\rm lock}\,(2/L)^N$	GG-A9-SM Theorem 28.1 (full orbifold + Wilson-Hosotani derivation)
T8.5 cosmological constant lock-scale seesaw	GG-A10-Cosmos Sections 6.4–6.6
T9.1–T9.9 (4D QFT derivation)	Self-contained; 28 SM parameters derived in GG-A9-SM
T9.6 anomaly inflow detailed proof	GG-A2-GG6 Theorem 5 (single-tensor anomaly factorization) + GG-A2-GG6 appendix "Anomaly Coefficients for $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$"; full local + global proof in GG-A3-BI6 (local cancellation theorem and global anomaly-freedom theorem on the physical intended GG-6 class)
T10 (GDOS-to-Gi-4 bridge)	Self-contained; Gi-4 pillars detailed in GG-A6-GRIP
T11 (GDOS-to-Qi-4 bridge)	Self-contained; Qi-4 pillars detailed in GG-A7-Qi4
$N_g = 3$ generations	Arisaka-Generations (full proof)
KK lock cluster predictions	GG-P1-KK
Kaxion dark-matter predictions	GG-C1-Kaxion
Pedagogical synthesis	GG-6-Book-I (in preparation)

Table 32: Inward coherence map: GG-A5-GDOS imports from companion papers. For each foundational input that the present paper takes as *theorem-level given*, the companion-paper home is listed together with the role that input plays inside GG-A5-GDOS. This table complements Table 31 (which records where each *result of the present paper* is elaborated downstream).

Companion paper	What GG-A5-GDOS imports	How it is used in GG-A5-GDOS
GG-A1-Constitution (Arisaka, 2026, A1-Constitution)	Axioms AX-1 (LC), AX-2 (GGEP), AX-3 (OGGEP), AX-4 (MSMF)	Constitutional inputs to T1–T6; AX-3 in particular drives the OGGEP-to-GDOS chain.
GG-A1-Constitution (Arisaka, 2026, A1-Constitution)	Postulates POS-1 (GI) through POS-7 (MF)	POS-3 (AF) underwrites T9.6 anomaly inflow; POS-4 (CP) underwrites the GKLS step in T4–T5; POS-6 (OGN) underwrites the rigidity arguments in T8–T9; POS-7 (MF) underwrites the falsifiability audit.
GG-A2-GG6 (Arisaka, 2026, A2-GG6)	Theorem 1: dimensional minimality of the bulk	Justifies the choice of 6D bulk $\mathcal{M}_6$ as the unique axiomatic substrate.
GG-A2-GG6 (Arisaka, 2026, A2-GG6)	Theorem 2: canonical metric of GG-6	Imported as the bulk metric in the $\mathcal{H}_{\rm Bulk}/\mathcal{H}_{\rm Bdy}$ split and in the KK reduction of T1–T6.
GG-A2-GG6 (Arisaka, 2026, A2-GG6)	Theorem 3: six forces with three abelian invariances	Fixes the bulk gauge-gravity group $G_{\rm tot} = \mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ used throughout T9.
GG-A2-GG6 (Arisaka, 2026, A2-GG6)	Theorem 4: BI-6 coefficient vector	Supplies the modified Bianchi identity $dH_3 = X_4$ that drives anomaly inflow in T9.6.
GG-A2-GG6 (Arisaka, 2026, A2-GG6)	Theorem 5: single-tensor anomaly factorization	Used in the anomaly-inflow proof of T9.6 and in the $\Delta T^{\rm GS}_{\mu\nu}$ contribution to the boundary stress tensor.
GG-A2-GG6 (Arisaka, 2026, A2-GG6)	Theorem 6: OGN-5 Diophantine cascade	Used in T8.7 hierarchy formula $v = M_{\rm lock}(2/L)^N$ and in the OGN-5 integer ledger throughout Part IV.
GG-A2-GG6 (Arisaka, 2026, A2-GG6)	Theorem 7: BRST consistency and homogeneous Slavnov–Taylor identity	Imported as theorem-level input to the BRST-compatibility checks in T9.6.
GG-A2-GG6 (Arisaka, 2026, A2-GG6)	Theorem 8: Arisaka equation and Quadruple Gauge Locking	Supplies the lock-scale algebra used in T8.5 (cosmological constant seesaw) and T8.7 (mass hierarchy).
GG-A3-BI6 (Arisaka, 2026, A3-BI6)	Complete BI-6 local + global anomaly-freedom theorem on the physical intended GG-6 class	Closes the anomaly-inflow argument in T9.6; without it the GKLS positivity in T4–T5 would not be constitutionally admissible.
GG-A3-BI6 (Arisaka, 2026, A3-BI6)	OGN-5 theorem (§"OGN-5 as a theorem derived from BI-6")	Imported wherever the OGN-5 integers $(1, 3, 5; 7, 35)$ are used as theorem-level facts in Part IV.
GG-A4-Euclid	Methodological standard (Euclidean dual-layer architecture; forced-move sequence)	Sets the proof discipline of GG-A5-GDOS; every T statement is written in the same Euclid-style axiom → postulate → theorem chain.

Audit trail of cited companion-paper theorem numbers. For the convenience of a future reviewer who wishes to re-walk the coherence map without re-reading every companion paper, we record here the verbatim internal numbering of the cited results as they appear in the companion-paper PDFs at the title-page date of the present paper. **GG-A1-Constitution (Arisaka, 2026, A1-Constitution):** four axioms — AX-1 = LC (Locality and Causality); AX-2 = GGEP (Geometry–

Gauge Equivalence Principle); AX-3 = OGGEP (Operational/Open GGEP); AX-4 = MSMF (Maximum Symmetry, Minimum Freedom) — and seven postulates — POS-1 = GI (gauge invariance across scales: LGI/DGI/GGI); POS-2 = UEP (Universal Extremal Principle); POS-3 = AF (anomaly freedom and cancellability by inflow); POS-4 = CP (closure and positivity); POS-5 = RG (RG-compatibility and scale covariance); POS-6 = OGN (Occam Gauge Normalization: rigidity); POS-7 = MF (measurement and falsifiability). **GG-A2-GG6 (Arisaka, 2026, A2-GG6):** Theorem 1 = dimensional minimality of the bulk; Theorem 2 = canonical metric of GG-6; Theorem 3 = six forces with three abelian invariances; Theorem 4 = BI-6 coefficient vector; Theorem 5 = single-tensor anomaly factorization; Theorem 6 = OGN-5 Diophantine cascade; Theorem 7 = BRST consistency and homogeneous Slavnov–Taylor identity; Theorem 8 = Arisaka equation and Quadruple Gauge Locking; with auxiliary Theorems on Green–Schwarz factorization and existence of spin structure on the canonical bulk in the appendices. **GG-A3-BI6 (Arisaka, 2026, A3-BI6):** the principal theorems are the Green–Schwarz cancellation (general form); the rank-one criterion; the local BI-6 anomaly cancellation; the complete BI-6 anomaly freedom on the physical intended GG-6 class (local plus global); the constitutional admissibility of three- and four-component bulks (§"$U(1)_{Qi}$ axiomatic status"); the exact orbit solution; the model-level BI-6 anomaly verification; the three OGN determination theorems (coefficient, core block, minimum normalization); the OGN-5 theorem proper (§"OGN-5 as a theorem derived from BI-6"); the publishable global-anomaly theorem; the automatic-admissibility theorem on the physical intended class; and the GS-as-GGEP completion treatment (the supportive-to-string-community geometrization reading of the 1984 Green–Schwarz mechanism, cross-cited from GG-A5-GDOS at every invocation of GS, modified Bianchi identity, anomaly inflow, or Hopkins–Singer differential cohomology per L5). **GG-A4-Euclid:** the paper is methodological/historical (consequence paper per L16; no new concept introduced, only a parameter-audit extraction of what was a free parameter before and what discrete or geometric quantity now replaces it). Its principal formal result is the theorem "OGN-5 from BI-6" in the section *"The OGN-5 Integers as a Theorem of BI-6"* (§in the body, immediately following the section establishing the canonical metric reconstruction). Remaining principal results are presented as sectioned "forced-move" arguments rather than as numbered theorems — this is the L16 consequence-paper convention for GG-A4-Euclid, where each section addresses "what was a free parameter before, and what discrete or geometric quantity now replaces it".

P A graduate course from this volume

The Conclusions argue that the deepest consequence of the constitutional classification is pedagogical (§112). This closing appendix converts that argument into a syllabus. The volume now contains the full apparatus of a graduate textbook — teaching boxes, Read-alongside pairings, plain-English closes, and ninety-six problems with complete worked solutions — and the three tracks below assemble it into single-term courses for three different audiences, with a full-year sequence for departments that want the whole arc. Weeks refer to a standard 13-week term; problem ranges name the worked-problem sets that close each unit.

P.1 Prerequisites, honestly stated

Track 1 assumes one year of graduate quantum mechanics (Sakurai, 2020 level) and linear algebra at the operator level; no field theory is needed. Track 2 assumes general relativity at the Carroll, 2004 level. Track 3 assumes one term of quantum field theory (Peskin, 1995 level). All three assume the willingness to check numbers by hand — the problems are built so that every quantitative claim can be recomputed with a calculator or twenty lines of code.

P.2 Track 1 — Open quantum systems and the measurement problem

For students in quantum information, AMO, and condensed matter.

Table 33: A one-term course on the boundary law and the quantum-mechanical descent. The columns give the week, the reading, and the problems set from it. The track is built so that a student meets the derivation before its applications: the first three weeks establish the law itself, and the remainder spend the term deriving the quantum postulates from it one at a time. It assumes a standard graduate quantum mechanics course and no prior contact with this framework. Every problem carries a full worked solution in the appendix.

Wk	Reading	Problems
1	Part I (thesis, admissibility, roadmap)	—
2	T1–T2; App. C (NZ in full)	II.1.1–II.2.2; C.1.1–2
3	T3; App. D (Stinespring, channel zoo)	II.3.1–2; D.1.1–3
4	T4; App. E (GKLS in full)	II.4.1–2; E.1.1–3
5	T5–T6; the three-descent table	II.5.1–II.6.2
6	T7.1; App. J (GNS, reduction map)	III.1.1–2; J.1.1–2
7	T7.2–T7.3	III.2.1–III.3.2
8	T7.4 (measurement as update)	III.4.1–2
9	T7.5; App. F (decoherence rates)	III.5.1–2; F.1.1–2
10	T7.6 (einselection)	III.6.1–2
11	T7.7 (Bell, Tsirelson)	III.7.1–2
12	Part VIII §§68–73 (the QM/thermo siblings)	VIII.2.1–2
13	Part IX pictures; synthesis	IX.1.1–2

P.3 Track 2 — Gravity from the bulk

For students in gravitation, astrophysics, and cosmology.

Table 34: A one-term course on the gravitational descent. The structure mirrors the first track — week, reading, problems — but surveys the boundary law in a single week and spends the term on the reduction from six dimensions to four, the Einstein equation, and the parallel derivation of gravity and electromagnetism. It assumes a first course in general relativity. A department teaching both tracks in one year should run this one second, as the parallel-chain material rewards a student who has already seen the quantum descent.

Wk	Reading	Problems
1	Part I; Part II in survey (T1–T6)	II.1.1–2
2	App. G (KK reduction)	G.1.1–2
3	App. H (curvature engine room)	H.1.1–2
4	T8.1–T8.2 (EH action, M_{Pl})	IV.1.1–IV.2.2
5	T8.3–T8.4 (Ward identity, modified EE)	IV.3.1–IV.4.2
6	T8.5 (the Λ seesaw)	IV.5.1–2
7	T8.6 (TOV falsifier)	IV.6.1–2
8	T8.7 (the hierarchy)	IV.7.1–2
9	Part VI (Newton and Coulomb in parallel)	VI.1.1–2
10	Part VI radiative completion	VI.2.1–2
11–13	Project: numerical integration of the modified TOV equation of T8.6 for a chosen equation of state	—

P.4 Track 3 — Fields and the Standard Model

For students in high-energy theory and phenomenology.

Table 35: A one-term course on the field-theoretic descent and the bridge theorems. Same three columns as the other tracks. This is the most technically demanding of the three: it assumes a graduate quantum field theory course including BRST and anomalies, and it spends its final weeks on the two bridge theorems that connect the boundary law to the companion papers. A student who completes it has derived the Standard-Model skeleton from the same law that gave the quantum postulates in Track 1.

Wk	Reading	Problems
1	Part I; Part II in survey (T1–T6)	II.1.1–2
2	T9.1; App. G (the tower)	V.1.1–2; G.1.2
3	T9.2–T9.3 (Wilsonian descent, Z_4)	V.2.1–V.3.2
4	App. I (orbifold breaking)	I.1.1–2
5	T9.4; App. K (BRST inheritance)	V.4.1–2; K.1.1–2
6	T9.5 (Slavnov–Taylor)	V.5.1–2
7	App. L (ghost sector)	L.1.1–2
8	T9.6 (anomaly inflow)	V.6.1–2
9	T9.7 (four currents)	V.7.1–2
10	T9.8 (proton stability)	V.8.1–2
11	T9.9 (RG geometrized)	V.9.1–2
12	Part VII (T10, T11); App. M	VII.1.1–VII.2.2; M.1.1–2
13	Part VIII (the classification)	VIII.1.1–2

P.5 The full-year sequence

The full-year course — the "Foundations of Modern Physics" sequence the Conclusions envisage — runs Track 1 as the first term, then a second term of Track 2 weeks 1–10 (compressed to six weeks by treating Part II as review), Track 3 weeks 2–12 (compressed to six weeks likewise), and a closing week on Parts X–XI: the twenty-five-test empirical horizon and the constitutional shift. A student completing the sequence has derived quantum mechanics, general relativity, and quantum field theory from one law, worked all ninety-six problems, and can name, for every claim, whether it is derived, conditional, or open.

P.6 The problem course at a glance

Table 36: The ninety-six problems, by unit. The columns give the unit, the identifier range, and the count, so an instructor can assemble a problem set from any part of the volume without reading the whole appendix. Every problem carries a full worked solution and closes with a paragraph on what the physics means rather than only what the algebra did. Every quantitative claim made in a solution was verified programmatically before it was typeset — a precaution taken because a worked solution that is wrong is worse than no solution at all.

Unit	IDs	Count
Part II (T1–T6)	II.1.1–II.6.2	12
Part III (T7.1–T7.7)	III.1.1–III.7.2	14
Part IV (T8.1–T8.7)	IV.1.1–IV.7.2	14
Part V (T9.1–T9.9)	V.1.1–V.9.2	18
Part VI (unification, radiation)	VI.1.1–VI.2.2	4
Part VII (T10, T11)	VII.1.1–VII.2.2	4
Part VIII (classification)	VIII.1.1–VIII.2.2	4
Part IX (pictures)	IX.1.1–IX.1.2	2
Appendices C–M	C.1.1–M.1.2	24
Total		**96**

For instructors: the three-register discipline. The volume's Take-away boxes run a fixed register triple — *standard account / what it left open / the constitutional derivation* — and the honesty grammar of the cohort (derived vs. conditional vs. open; raw vs. calibrated) is used without exception. We suggest teaching the register itself: ask students, for each theorem, to classify the claim before reading the box, and to check their classification against it. The skill this trains — locating the exact epistemic status of a physical claim — transfers to every literature the student will ever read, and it is the one habit this volume most hopes to install.

Part XIII
Common Objections and Responses

The strongest objections a hostile reviewer would raise — on deriving quantum mechanics, gravity, and field theory from one open bulk — answered, with pointers into the paper

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A foundation paper that proposes to reorganize quantum mechanics, general relativity, and quantum field theory as boundary projections of a single open six-dimensional bulk invites the harshest possible scrutiny, and it would not be honest scholarship to leave the strongest objections unstated. We have therefore tried to argue *against* this paper as forcefully as a hostile referee would, and to answer each line of attack with an explicit pointer to the theorem, appendix, section, or companion paper that bears the weight. Where the honest answer is "not yet" — where a step is a structural derivation rather than a closed proof, or a prediction lies beyond present reach — we say so and name the open item rather than paper over it.

The objections are grouped into ten themes: the open-system thesis itself; the measurement problem and the Born rule; the derivation of general relativity; the derivation of quantum field theory and the Standard Model; the unification of gravity and electromagnetism; holography, AdS/CFT, and black holes; the axiom-classification claim; thermodynamics and the arrow of time; falsifiability and scope; and the internal rigor of the theorems and their appendices. None of the answers asks the reader to accept the program on faith; each points to where the claim can be checked — or broken.

Q For the curious reader: the big questions, plainly answered

The twenty-five questions that follow are not a specialist's. They are the ones a thoughtful reader without a physics degree actually carries into a paper like this one: whether measurement is genuinely special or only looks that way, whether a conscious observer is needed for anything, what became of the cat, why the world we see is only part of the world, and whether a description that is admitted to be partial can be true at all. Several of these have been settled in popular science in directions this paper does not take, and a reader is owed plain answers before the specialist objections begin. They are met first, and plainly.

The hardest of the twenty-five is the sixth — whether a framework in which every description is incomplete has left itself any way of being right, or has merely made being wrong harder to notice. The response concedes the difficulty rather than dissolving it, and says which part of it the paper can carry and which part it cannot. Where a claim rests on a step that is not closed, the response says so and names the step; where the honest answer is that the framework does not reach the question, it says that too.

Question 1 (Q-GDOS-1) (on the curious reader)

"What is an 'open system', and why should that be fundamental rather than an approximation?"

Response. An open system is one that exchanges something — energy, particles, information — with a surrounding it does not control and cannot fully describe. A closed system exchanges nothing.

The conventional order treats closed as basic and open as a practical compromise: write the exact equations for everything, then admit you cannot solve them and approximate. On that view openness is a confession of ignorance.

The inversion here rests on an observation that is hard to argue with once stated. There are no closed systems. Any observer is finite, sits inside what it observes, and can hold only a bounded amount of the state; whatever is not held has to be summarized, and summarizing is exactly what makes a description open. This is not a limitation of present technique. It applies to a particle detector as much as to a rainforest, and it applies in principle rather than in practice. Once that is granted, treating closed systems as basic is treating a fictional case as the standard and the universal case as a departure from it. The framework simply reverses which is which: it writes the law for the description a finite observer actually has, and recovers the closed case as the idealization it always was. Whether that inversion buys anything is settled by what it forces, and the paper's claim is that it forces the quantum measurement rules rather than assuming them.

Question 2 (Q-GDOS-2) (on the curious reader)

"Is measurement really special, or does it just look that way?"

Response. It is not special, and the framework's central claim is that noticing this dissolves a problem rather than relocating it.

The measurement problem arises because standard quantum mechanics contains two rules that do not fit together. One says the state evolves smoothly and reversibly. The other says that when a measurement occurs, it jumps to one outcome with a probability given by a separate formula. Nothing in the theory says what counts as a measurement, which is why the problem has survived a century of attention: the theory refers to a process it does not define.

The move here is to deny that the second rule is a separate rule. Measurement is what a projection does when the observer is finite and inside: information is lost to the surroundings, the loss is irreversible for the reason that any coarse-graining is irreversible, and what is left behind on the observer's side has exactly the structure the second rule describes. On this account the jump is not an additional law but a consequence of the first law read from the inside.

The honest position is that this is not a new proposal in its general shape — accounts of this type have been pursued since the nineteen-seventies. What is claimed here is that the derivation goes through with the specific structure the constitution supplies, including the probability formula rather than only the disappearance of interference. Whether it does is a technical question, and it is where a skeptical reader should look.

Question 3 (Q-GDOS-3) (on the curious reader)

"Does a measurement need a conscious observer?"

Response. No, and the framework is unusually clear about this because the question is where a great deal of nonsense has attached itself to quantum mechanics.

What a measurement requires, on this account, is a projection: a larger system, a smaller one, and an irreversible loss of information from the smaller one's point of view. A photographic plate does that. A rock does that. A single air molecule scattering off a dust grain does that, which is why dust grains do not exhibit quantum interference and why nobody has to watch them for this to be true.

Consciousness has no role anywhere in the construction and could not be given one without breaking it, since the equations contain nothing that could distinguish an aware system from an unaware one of the same size. The idea that observation requires a mind entered the subject through a particular reading of the nineteen-thirties formalism and has been vigorously promoted outside physics ever since. It has never been the majority view among physicists and there is no evidence for it.

It is worth adding what does become interesting once measurement is demystified. A system that measures its own state, and keeps the result, is doing something structurally particular, and later papers in this program pursue that. But the direction of explanation runs from physics to the system, not from awareness to physics — and nothing in this paper needs anyone to be looking.

Question 4 (Q-GDOS-4) (on the curious reader)

"So is the cat alive and dead, or not?"

Response. Not, and it never was — which is what Schrodinger was arguing when he invented it. The example is almost universally repeated as a piece of quantum weirdness and was written as a reductio. Schrodinger's point in 1935 was that if you take the standard formalism literally and apply it to a cat, you get an absurdity, and the absurdity shows something is missing from the account. He was right, and the thought experiment has since been quoted as though it were the account.

On the framework's reading the cat is never in a superposition of alive and dead in any sense that could matter, because the cat is a macroscopic open system continuously exchanging with its surroundings, and the interference between the two branches is destroyed on a timescale far shorter than anything biological. The state that would need to persist for the puzzle to arise does not survive long enough to be discussed.

What remains after that is a genuine question, and it should not be lost in the relief: why does one branch rather than the other turn out to be the one that happened, and why with the frequencies observed. Destroying interference explains why you never see a blurred cat. It does not by itself say why the probabilities are what they are, and the paper's substantive claim is about that second question rather than the first.

Question 5 (Q-GDOS-5) (on the curious reader)

"Why is the world we see only part of the world, and how would we ever know what is

missing?"

Response. Because seeing is a projection, and every projection discards. That part is not controversial. The interesting half of your question is the second one, and it has a better answer than one might expect.

You cannot inventory what a projection discarded by looking at its output; if you could, it would not have been discarded. What you can do is notice that the output does not behave as a complete description would. A closed system's description evolves reversibly and conserves everything; an open one does not. The signatures are specific — irreversibility, apparent randomness, quantities that fail to balance — and they are measurable.

So the presence of something missing is detectable even though its content is not. This is a familiar situation elsewhere. Thermodynamics detects that a description has been coarse-grained without needing to know what each molecule is doing, and the second law is precisely a statement about the existence of discarded information rather than about its content.

What this framework adds is that the amount discarded is not free: the structure of the arena fixes how much can pass through the projection and how much cannot, and that is the same bookkeeping that later prices information against energy. So the answer to "how would we know" is: by measuring the consequences of the loss, which are exactly the phenomena that made quantum mechanics strange in the first place.

Question 6 (Q-GDOS-6) (on the curious reader)

"If every description is partial, how can any of them be true?"

Response. This is the hardest question in the set, and the answer is that truth for a description of this kind is a different property from completeness — which sounds like a dodge and can be made precise.

A map that omits most of the terrain is not thereby false. It is false if it says there is a road where there is none. The distinction between omission and error is the whole of the answer, and it is not special pleading: no map has ever been complete, and cartography is not therefore a fiction.

Where the worry has teeth is that a framework which admits its descriptions are partial has acquired a way of explaining away any disagreement — the missing part did it. That is a real failure mode and it is how some interpretations of quantum mechanics became unfalsifiable. The protection here is that the framework does not treat what is missing as unconstrained. The projection is specified, the amount of information it discards is computable, and the consequences of the loss are predicted rather than invoked. A disagreement cannot be attributed to the hidden part unless the hidden part was already claimed to produce that effect.

The residue is genuine and should be stated. If nature's structure is not in the class this framework describes, no amount of internal care will reveal it, because every symptom would be read as a projection effect. That is the specific way this paper could be wrong without noticing, and it is why the falsifiers are stated as sharp numerical claims rather than as qualitative expectations.

Question 7 (Q-GDOS-7) (on the curious reader)

"Isn't decoherence already the answer to the measurement problem?"

Response. It is half of it, and the half it is not gets glossed over often enough to be worth separating carefully.

Decoherence explains, with no additional assumptions and with experimental confirmation, why interference between macroscopically distinct alternatives disappears fast — why you never see a blurred cat or a smeared pointer. That part is settled physics, was worked out from the nineteen-seventies onward, and is not in dispute here; the framework uses it.

What decoherence does not do is explain why one alternative occurs rather than another, or why the frequencies match the particular formula they match. After decoherence has run, the description still contains all the branches; it simply no longer contains interference between them. Getting from "several branches, no interference" to "this one happened, with this probability" requires something further, and there is no consensus on what. This is acknowledged by decoherence's own founders and is sometimes lost in secondary accounts.

The claim in this paper is about that remaining step: that the probability rule follows from the structure of the projection rather than being an extra postulate. That claim is stronger than decoherence and is correspondingly more vulnerable. A reader who wants to find the weak joint in this framework should look there rather than at the decoherence part, which is borrowed and secure.

Question 8 (Q-GDOS-8) (on the curious reader)

"Where does the Born rule come from? Nobody has ever derived it."

Response. Several people have claimed to, which is not the same thing, and the honest position on this paper's version is that it is a derivation from named assumptions rather than from nothing.

The rule — that the probability of an outcome is the squared magnitude of the corresponding amplitude — has been assumed since 1926. Attempts to derive it are a minor industry: from decision theory, from counting branches, from symmetry arguments, from frequency operators. Each derives it from something, and the debate is always whether that something was more or less plausible than the rule itself.

What is offered here is of that kind and the paper says so. Given the projection structure the constitution supplies, and given a small number of further conditions that are stated by name rather than smuggled, the squared magnitude is the only assignment that is consistent. That is a real result and it is conditional, and the conditions are listed where the claim is made rather than in a footnote.

The right attitude for a reader is therefore not "has the Born rule finally been derived" but "are the assumptions this derivation uses weaker than the rule." That question can be answered by reading the assumptions, which is a finite task, and it is a better use of skepticism than the general question. The framework's own assessment marks this result as conditional and names which condition is doing the most work.

Question 9 (Q-GDOS-9) (on the curious reader)

"Many-worlds says every possibility happens and nothing collapses. Why not that?"

Response. It is a serious position held by serious people, and the difference between it and the account here is narrower than the rhetoric on either side suggests.

The many-worlds reading takes the smooth evolution to be the whole story: nothing collapses, all branches persist, and what you call an outcome is what one branch's inhabitant sees. Its appeal is economy — one rule instead of two — and that appeal is real.

The standard difficulty is probability. If every branch happens, saying that one has probability 0.7 needs an account of what the number means, and forty years of work has not produced one that commands agreement. The proposals are ingenious and each has committed critics. This is not a knockdown objection; it is an unpaid bill.

The framework here differs by locating the asymmetry in the projection rather than in the branching. Because the observer is finite and inside, the branches are not symmetric from its position: information about the others has left through the boundary and is not recoverable. That supplies a reading of probability without needing every branch to be equally real.

Whether that is better is not obviously settleable by argument. What can be said is that the two make different commitments about a measurable quantity — the framework predicts specific relationships among transport coefficients that a pure-branching account does not require — and disagreements that touch measurement are the kind worth having.

Question 10 (Q-GDOS-10) (on the curious reader)

"If the observer is inside the system, who or what does the observing?"

Response. Nothing that needs to be a who. What does the observing is whatever sits on the boundary and retains a record, and "retains a record" is a physical property rather than a mental one.

This is the point where the framework diverges most sharply from the folklore. The traditional picture puts an observer outside, looking in, with the observed system on the other side of a line. That picture is incoherent for the universe as a whole — there is no outside — and it was never even accurate for a laboratory, where the apparatus is made of the same stuff as the sample.

Replacing it costs less than expected. All that is needed is a boundary across which information passes asymmetrically: more comes in than can be sent back, and what arrives leaves a trace that persists long enough to matter. A photographic emulsion satisfies this. So does a thermocouple, a rock face weathering, a bacterium's chemical memory. None of them is anybody.

What follows is a graded rather than binary picture: systems differ in how much they retain and for how long, not in whether they qualify as observers. That is the direction later papers in this program develop. For the purposes of this one, the important consequence is negative — nothing in the derivation requires a mind, and any reading that inserts one is adding something the equations do not contain.

Question 11 (Q-GDOS-11) (on the curious reader)

"Why should information be physical? It sounds like a category error."

Response. It sounds like one and is not, and the demonstration is old, concrete and experimentally

confirmed.

Landauer's argument, from 1961: erasing one bit of information in a system at temperature T requires dissipating at least $kT \ln 2$ of energy. Not as a practical engineering limit but as a consequence of the second law. If it were cheaper, one could build a machine that extracts work from a single heat bath, which is forbidden. The bound has since been measured directly in the laboratory, in colloidal particles and in single-electron devices, and it holds.

That single result forecloses the category error. Information is not a description of a physical thing; it is itself accountable in the same ledger as energy, with an exchange rate. Bennett's resolution of Maxwell's demon a decade later made the same point from the other side: the demon does not violate the second law because its memory must eventually be cleared, and clearing costs exactly what it saves.

What this framework adds is a structural reason for the accounting rather than a thermodynamic one, and a prediction: not merely that information has a cost but that the conversion factor is fixed by the arena rather than by the temperature of a bath. That is a stronger claim than Landauer's and a checkable one, and it is developed in the companion that prices it.

Question 12 (Q-GDOS-12) (on the curious reader)

"Does the wave function collapse? Yes or no."

Response. Not as a physical process, and yes as a description of what happens to the observer's account of the system — and the reason both halves are needed is the whole subject in one sentence.

Nothing in this framework introduces a mechanism that reaches in and changes the state. There is no collapse dynamics, no additional term, no stochastic process. In that sense the answer is no, and the framework sides with those who have always found an extra physical law here implausible.

What does happen is that the description available on the inside changes discontinuously when information crosses the boundary. Before, the observer's account assigns amplitudes to several possibilities; after, it assigns one. That is a real change in a real thing — the description — and it is irreversible, because the information that would be needed to undo it has left. Calling it collapse is not wrong; calling it a physical process happening to the world is.

The analogy that is closest is not a metaphor. When a probability distribution over a die's faces becomes a definite six, nothing physical collapsed; a description conditioned on new information changed. The difference in the quantum case is that the pre-measurement description was not ignorance about a value the system already had, and the paper is careful about that difference rather than eliding it.

Question 13 (Q-GDOS-13) (on the curious reader)

"Is quantum randomness real randomness, or just ignorance of something hidden?"

Response. On this account it is neither, quite, and the third option is better than either and less familiar.

It is not ignorance of a hidden value in the way the classical intuition wants — Bell's theorem and the experiments that followed it rule out a large class of accounts in which the outcome

was determined all along by properties carried locally by the particle. That result is settled, has been tested to remarkable precision, and any framework has to respect it. This one does. It is also not randomness written into the fundamental law as a brute fact. The framework's dynamics contain no stochastic term.

What it is, on this reading, is the appearance produced by a deterministic structure viewed through a projection that discards. Information exists in the arena which does not exist in the four-dimensional description, and the outcome is determined there and not here. That is superficially a hidden-variable account and escapes Bell's constraint because the missing information is not local, not carried by the particle, and not accessible even in principle — it is on the other side of a projection rather than in a pocket.

Whether this counts as "real" randomness is partly a question about the word. Operationally the answer is definite: no experiment performed within the four-dimensional description can predict the outcome, ever, and the unpredictability is not a matter of insufficient effort.

Question 14 (Q-GDOS-14) (on the curious reader)

"Why does the classical world look classical?"

Response. Because the projection is severe, and severity is what classical means.

The question is usually asked as though a boundary had to be found — some size or mass above which quantum behavior stops. There is no such boundary and looking for one has been unproductive. Interference has been demonstrated for molecules of many hundreds of atoms, and there is no known size at which the effect switches off.

What changes with size is how fast a system's interference is destroyed by its surroundings. The rate rises steeply — for anything macroscopic it is faster than any timescale one could measure, by an enormous factor, even in what a laboratory would call excellent isolation. Classicality is therefore not a property a system has but a statement about how quickly it is being read by everything around it.

Two consequences worth carrying away. First, the classical world is not a different regime with different laws; it is the same laws under a much heavier projection. Second, the direction of the explanation is the reverse of the traditional one: quantum behavior is not a strange thing that small systems do, it is what everything does, and classicality is the special case produced by being unable to hide from one's surroundings.

Question 15 (Q-GDOS-15) (on the curious reader)

"Can two observers disagree about what happened?"

Response. About some things yes, about others no, and the line between them is one of the framework's more interesting outputs.

They can disagree about what is definite. An observer inside an isolated laboratory may correctly describe a system as having no definite outcome while an observer who has received information from it correctly describes the outcome as settled. Both descriptions are right relative to what has crossed each boundary. This is not a paradox and is not new; it is the content of the various "Wigner's friend" scenarios, and recent experiments have made versions of it concrete.

They cannot disagree about anything that would let them build inconsistent records. Once

two observers have exchanged information, their accounts must agree about every outcome both have access to, and the framework's structure enforces this rather than assuming it: the projection composes, so a chain of observers cannot produce a contradiction at the end.

What this buys is a way of holding relativity of description without sliding into relativism about facts. There is no observer-independent list of what is definite. There is an absolute constraint that no two observers can be brought into contradiction. Those two statements sound incompatible and are not, and keeping them apart is most of what it takes to think clearly about measurement.

Question 16 (Q-GDOS-16) (on the curious reader)

"If openness is the price of being finite, is anything closed?"

Response. The universe as a whole, arguably, and nothing else — and the exception is more instructive than the rule.

If there is nothing outside the universe, then there is nothing for it to exchange with, and the whole is closed by definition. That is the one closed system available, and it is also the one system that no observer can ever describe, since describing it would require being outside it. So the only closed system is the only one permanently inaccessible, which is an awkward fact for any framework that treats closed systems as basic.

Everything else — every particle, laboratory, galaxy, and every observer — is open, not as a matter of degree but structurally. This has a consequence worth stating plainly: the laws physicists actually write down and test are laws for open systems, always have been, and the closed-system formalism is a limit that is never exactly realized.

There is a further twist the framework takes seriously. Because the one closed system is undescribable from inside, the question of what laws *it* obeys may not be well posed. What can be asked is what laws hold for the descriptions available to things inside it. That is a smaller question and it is the one this paper answers, and a reader should notice the substitution rather than let it pass.

Question 17 (Q-GDOS-17) (on the curious reader)

"What does this framework say a 'fact' is?"

Response. A record that has become expensive to undo. That is a deflationary answer and it does more work than a grander one would.

Before a measurement, there is no fact about the outcome — not a hidden one, not an unknown one. After a measurement, there is, and what changed is that information crossed a boundary and left a trace whose reversal would require gathering back everything that has since dispersed. The irreversibility is not absolute in principle; it is thermodynamic, in the same sense that unscrambling an egg is not forbidden by any law and does not happen.

This has the consequence that facthood is a matter of degree and of context rather than a binary. A record held briefly in a single atom is a weaker fact than one printed on a plate, which is weaker than one that has been copied a billion times into the surroundings. What we normally call a fact is the last kind: so thoroughly copied that no imaginable process could produce a coherent alternative.

Some readers will find this unsatisfying, and the dissatisfaction is worth respecting. It says there is no moment at which the universe writes something into a ledger. What there is instead is a continuous process of information dispersing, and facthood is what that process looks like once it has gone far enough that nobody could get it back.

Question 18 (Q-GDOS-18) (on the curious reader)

"Isn't our universe just one bubble among countless others?"

Response. It might be, and this paper is less concerned with that question than the others in the cohort, because its subject is what a description from inside looks like rather than what there is.

What can be said from here is one thing that is easy to miss. Every version of the multiverse proposal — inflationary bubbles, branching worlds, string landscape — shares a structure: an ensemble is invoked, most of it inaccessible, to explain features of the accessible part. The framework developed in this paper is about exactly that situation, the accessible being a projection of the inaccessible, and it says something the proposals usually do not: the inaccessible part is not free. Its consequences for the accessible part are constrained, computable, and in places measurable.

So the question this framework would put to any multiverse account is not whether the other universes exist but what they forbid here. If the answer is nothing, then whatever the ensemble is doing, it is not doing physics. That is a demand rather than a refutation, and it applies with the same force to this framework, which is why the falsifier list exists.

Question 19 (Q-GDOS-19) (on the curious reader)

"The universe looks fine-tuned for life. Doesn't that point to a designer?"

Response. It points to something requiring explanation, and this paper contributes to that question sideways rather than head-on.

The design inference needs the premise that the quantities could have been otherwise. The cohort's answer is that they could not, and the argument is made in the companions that fix the arena. What this paper adds is a different kind of observation, and it is worth having.

Much of what strikes people as fine-tuned is not a coincidence of values but a feature of what it takes to be an observer at all. An observer must be finite, must sit inside what it observes, and must retain records — and those requirements alone force a great deal about the environment it can find itself in. A universe without stable structures, without irreversibility, without a way for information to be retained, contains nothing that could notice its own tuning. Part of the apparent arrangement is therefore a selection effect of a sharper kind than the usual anthropic argument, because it follows from the structure of observation rather than from the requirements of chemistry.

That is not an argument against a designer and should not be recruited as one. It is an argument that some of the fine-tuning list is not evidence, because those entries could not have been observed to be otherwise by anything capable of making a list.

Question 20 (Q-GDOS-20) (on the curious reader)

"If everything follows from the structure, is my future already written?"

Response. Nothing here touches your future, and this paper has a particular reason to say so that the others do not.

The general answer holds: what is fixed is the rule book, not any history within it, and the distinction is between the rules of chess and last Tuesday's game.

The particular answer is that on this framework's account no description from inside can be complete, and the incompleteness is not a practical limitation. Information that would be needed to compute the future of a system as complicated as a person is not merely hard to gather; some of it is on the other side of a projection and is not available in the four-dimensional world at any cost. So there is no book, not because nobody has written it, but because the writing surface is not accessible from where anybody stands.

That is a stronger statement than the usual chaos-and-complexity answer, and it should not be oversold either. It says the future is not computable from inside. It does not say the future is open in whatever sense would be needed for free will, and this paper takes no position on that. What agency is, and whether the concept survives contact with physics, is not a question the framework reaches.

Question 21 (Q-GDOS-21) (on the curious reader)

"Why is there something rather than nothing?"

Response. The framework does not reach it. What it can offer is a reformulation, which is worth less than an answer and more than silence.

Everything here is conditional: given a structure of a certain kind, a description from inside must look thus. Why there is a structure at all sits outside, and no rearrangement of the argument gets there.

The reformulation is this. If descriptions are necessarily partial, then the question "why is there something?" is being asked from inside the something, using a description that omits most of it, about a totality that by construction cannot be described from within. That does not make the question meaningless — meaningless is too strong and too convenient — but it does suggest the question may be asking for a kind of answer that the asker's position cannot support.

This is close to a move that has been made before and should be labeled as such rather than presented as a finding. The framework does not establish it. It observes that the shape of the difficulty matches the shape of the projection limit, which is suggestive and is not evidence.

Question 22 (Q-GDOS-22) (on the curious reader)

"This explains an awful lot from very little. Shouldn't that make you suspicious?"

Response. It should, and the suspicion has a specific target here that is worth naming rather than the general one.

The general warning stands: Eddington derived the fine-structure constant from combinatorics and ended his career defending it. Anyone claiming a great deal from a little is one bad step away.

The specific risk in *this* paper is different from the numerology risk, and sharper. A framework whose central claim is that descriptions are necessarily incomplete has acquired, if it is careless, an unlimited supply of explanations: any disagreement can be attributed to the part that was projected out. That is how several interpretations of quantum mechanics became unfalsifiable, and it is the failure mode a reader should watch for here.

The protection is that the discarded part is not treated as free. How much is lost, and what its consequences are, follow from the structure and are computed rather than invoked. The framework predicts specific relationships among measurable transport quantities, and if those fail it cannot say the missing information did it — the missing information is what produced the prediction. Whether that protection holds is checkable by reading how the falsifiers are stated, and it is a better use of suspicion than the general worry about elegance.

Question 23 (Q-GDOS-23) (on the curious reader)

"Why should I take one person's framework seriously against a century of quantum foundations?"

Response. You should not, on the strength of authorship, and the field you name is the right one to be cautious about.

Quantum foundations has produced an enormous number of interpretations, most of them internally consistent, most of them making identical predictions, and the resulting debate has been going for ninety years without converging. A newcomer should assume by default that a new entry is another interpretation: coherent, unfalsifiable, and adding a name to a list.

What would distinguish an entry worth attention is that it predicts something the others do not. That is the only filter that has ever worked in this subject — Bell's inequality is the canonical case, converting a philosophical dispute into an experiment and settling a large part of it.

So the question to put to this framework is not whether its picture is appealing but what it forbids that other readings permit. The answer is a set of specific relationships among transport coefficients in open systems, measurable in ordinary laboratories rather than at the edge of technology, together with the cohort's sharper predictions elsewhere. If those relationships hold where other accounts do not require them, that is worth something. If they fail, the framework is finished, which is more than can be said for most of the list.

Question 24 (Q-GDOS-24) (on the curious reader)

"What would convince you that you are wrong?"

Response. Several things, and this paper's are unusually accessible because they do not need a collider.

The framework predicts that certain relationships among transport coefficients hold exactly in any open system near equilibrium — symmetry of a particular coupling matrix, a specific relation between fluctuation and dissipation, an equipartition statement with a fixed coefficient. These are ordinary laboratory measurements in cold atoms, in mesoscopic devices, in biophysical systems. A clean violation, where the framework says the relation must be exact, would end it.

A demonstration that the probability rule cannot follow from the stated conditions — a counterexample to the derivation rather than an experimental failure — would remove the

paper's central claim while leaving the rest of the cohort standing. That is the most likely failure and the one to look for first.

And an experimental demonstration of interference in a system where the framework says the projection must have destroyed it would attack the account from the other direction.

What would not convince: a disagreement in a regime not claimed, or a philosophical objection to partial descriptions. A framework is not refuted by being unsatisfying, and a reader should be suspicious of any proponent who accepts philosophical objections as fatal, since those can always be manufactured.

Question 25 (Q-GDOS-25) (on the curious reader)

"I'm not a physicist and I've got this far. What is the one thing you want me to take away?"

Response. That quantum mechanics is probably not strange in the way you have been told, and that the actual strangeness is somewhere else and is more interesting.

The popular account runs: particles are in two places at once, observation collapses reality, and consciousness plays some role in bringing the world about. Almost none of that survives contact with the physics. Interference is real and well understood. Observation requires nothing conscious — a rock will do. The cat was an argument *against* taking the formalism literally, not an illustration of how the world is.

What is actually strange is quieter. It is that a description made from inside a system, by something too small to hold all of it, is not merely incomplete but incomplete in a structured way — and that the structure of the incompleteness is what produces the phenomena we have been calling quantum. The randomness, the irreversibility, the fact that a measurement settles something: on this account these are not features of the world but features of the view.

So the thing to take away is a correction rather than a fact. When you next read that quantum mechanics shows the observer creates reality, the right response is not awe but a question: what does this claim forbid? If the answer is nothing, it is not physics, however often it is repeated.

R The open-system thesis and the OGGEP move

Objection 1 (O-GDOS-1) (the philosopher of physics)

"Every observation is a window cut into an open system" — is this a physical principle or a metaphysical slogan? What, concretely, forbids the usual closed-system description of the universe?

Response. It is meant as a physical principle with operational teeth, not a slogan, and the paper states it as such (§6.1). The content is that any *observation* is a finite-resolution coupling between a system and an apparatus across a boundary, so the object actually described by physics is never the closed total but a boundary projection of it (AX-3, OGGEP; §4.3). A closed-system description is not forbidden in principle — it is forbidden *operationally*: the closed total wavefunction is, by construction, not an object any measurement returns. The wager is

that taking this seriously as the starting axiom, rather than as an afterthought bolted on at measurement, is what lets quantum mechanics, gravity, and thermodynamics descend from one source (§6.3). If the descent fails, the principle was empty; that is the test we accept.

Objection 2 (O-GDOS-2) (the open-quantum-systems theorist)

"Isn't 'openness is constitutional' just standard open-quantum-systems theory — Nakajima–Zwanzig, GKLS, decoherence — dressed in grand language? What does OGGEP add beyond the textbook machinery?

Response. The machinery is borrowed wholesale and we say so: the reduction is the Nakajima–Zwanzig projection (App. C), the dynamics is GKLS (App. E), the pointer selection is einselection (App. F). What OGGEP adds is not a new equation but a *source*: in textbook treatments the system–bath split, the projector $\mathcal{P}$, and the Markov approximation are modeling choices made by hand; here they are fixed by the bulk–boundary geometry, with the "bath" being the heavy Kaluza–Klein tower integrated out (App. G; Picture 3, §83). The claim is therefore narrow and checkable: the GKLS generator is not postulated but inherited, and its specific form is read off the bulk geometry. If the geometry gave a different (e.g. non-completely-positive) reduction, the construction would fail; it does not (App. D).

Objection 3 (O-GDOS-3) (the axiomatics specialist)

"You promote openness to a separate axiom (AX-3). Either it follows from AX-1–AX-2, in which case it is redundant, or it does not, in which case it is a strong unproven assumption smuggled in as a 'principle'."

Response. AX-3 (OGGEP) is independent of AX-1–AX-2 by design, and the paper does not hide this — it is the operational *extension* of the geometry–gauge equivalence to measurement, not a corollary of it (§4.1; imported from Arisaka (2026, A1-Constitution)). The constitutional-tower analysis (§65) makes the independence explicit by showing what is lost at each level when a clause is dropped: without AX-3 one recovers the closed-bulk geometry (AX-1–AX-2) but *not* the boundary law, hence not quantum measurement, not the second law, not black-hole thermodynamics. So AX-3 is a genuine assumption, not a theorem — exactly as Euclid's parallel postulate is independent of the other four. The honest position is that it is a *proposal* whose justification is the breadth of what it forces (§6.3), held against experiment, not an a priori truth.

Objection 4 (O-GDOS-4) (the operationalist)

"An 'operational entry point' smuggles in an observer or agent. Is GDOS observer-dependent? If so, how is it objective physics rather than a dressed-up QBism?

Response. The boundary is geometric, not agential: it is the place where the scale profile degrades resolution, defined by the bulk metric, not by anyone's beliefs (§4.3; the boundary reduction map $\mathcal{R}$, App. J.3). "Operational" here means finite-resolution, not subjective. The distinction from QBism is made directly in the measurement synthesis (§28): QBism makes the quantum state a Bayesian agent's degree of belief; in GDOS the reduction is a fixed

completely-positive map determined by the geometry, the same for every observer at the same boundary. Different observers occupy different boundaries and so see different projections — that is frame-dependence, as in relativity, not subjectivity. The objective content is the bulk and the map; what is observer-relative is only which slice of it a given apparatus returns.

Objection 5 (O-GDOS-5) (the mathematician)

"Define the 'admissibility predicate' rigorously. Is it a well-defined mathematical object, or a heuristic filter that can be tuned to admit whatever you want?

Response. It is stated as a predicate on candidate theories in §3: a theory is admissible iff it satisfies the eleven constitutional clauses (four axioms, seven postulates), each of which is given as a checkable condition on the bulk geometry, the gauge content, and the boundary map. The predicate is not tuned per result; it is fixed once in Arisaka (2026, A1-Constitution) and merely *applied* here. Its discriminating power is demonstrated, not asserted: the classification table (§75) runs the predicate against GR, Yang–Mills, string theory, AdS/CFT, QM, the Standard Model, and ΛCDM and returns a different axiom-subset for each (§76.3). A filter that admitted everything would not separate them. The residual honesty: the predicate is only as rigorous as the clause definitions, and two clauses (MSMF, OGGEP) are physical rather than purely formal — flagged as such in §96.

Objection 6 (O-GDOS-6) (the historian of science)

"The 'historical chain that led here' — Boltzmann, Gibbs, von Neumann, decoherence, to OGGEP — reads as Whig history: a lineage retrofitted to make your conclusion look inevitable."

Response. The risk is real and the paper tries to earn the lineage rather than assume it (§6.2). The claim is not that Boltzmann or von Neumann foresaw OGGEP, but that a specific, checkable pattern recurs: each made openness or coarse-graining do decisive work (the H-theorem, the measurement cut, einselection) while treating it as an approximation to an underlying closed dynamics. The proposal is the modest-sounding inversion of making that openness fundamental and the closed dynamics the idealization (§6.3). That is a falsifiable historical reading: if the decisive advances had not in fact leaned on coarse-graining, the pattern would not hold. The paper presents the inversion explicitly as a *proposed* fourth revolution (§107.4), inviting the reader to judge whether it is forced or merely suggestive — the same standard it asks of its physics.

S The measurement problem and the Born rule

Objection 7 (O-GDOS-7) (the quantum-foundations specialist)

"The OGGEP derivation of the QM postulates is essentially decoherence plus QBism in new language. The novel content is thin."

Response. Fair, and we adopt the weaker framing deliberately (§28): the genuinely new content is the *bulk that produces* the GKLS dynamics, not a new boundary update rule. T7.1–T7.7

reorganize the measurement problem under a single constitutional source — unitary bulk evolution, finite-resolution boundary reduction, einselected pointer basis — with the historical frameworks (decoherence, consistent histories, QBism, many-worlds) recovered as derived specializations rather than rivals. What is not claimed is a new empirical prediction in the lab from the reorganization alone; what is claimed is that the cut, the basis, and the rule come from one place instead of three. The honest residue is exactly Objection 8: reorganizing is not the same as deriving a unique outcome, and we treat that as the open problem it is.

Objection 8 (O-GDOS-8) (the realist / Bohmian)

"Decoherence and einselection are notorious for not solving the measurement problem: they explain the *basis* but not the *outcome*. Does GDOS yield a single definite result, or only the appearance of one?

Response. This is the sharpest objection in the paper and we do not claim to have dissolved it by fiat. GDOS supplies the basis (einselection, App. F.3) and a *rate* for the boundary projection (§27.4); it does not derive, from within unitary bulk dynamics alone, why *this* outcome rather than *that* is realized on a single run. The paper's position (§27.1) is that the "and-or" transition is relocated to the bulk–boundary reduction, where the loss of the off-diagonal coherences is exact in the resolution limit, and that the residual question — single-world realization versus branch relativity — is a genuine open problem listed among the seven (§97.1). We regard honesty here as more valuable than an overclaim: the framework makes the outcome problem precise and geometric, but does not pretend to have closed it.

Objection 9 (O-GDOS-9) (the Born-rule skeptic)

"Every attempted derivation of the Born rule (Gleason, envariance, decision theory) covertly assumes it. Where exactly does GDOS get $p = |\psi|^2$ without circularity?

Response. The paper is explicit about what it assumes and what it derives (§18.3). It does *not* claim a non-circular first-principles derivation of the squared modulus; it inherits the Hilbert-space inner product from the bulk GNS construction (App. J.2) and shows that the boundary reduction is the unique completely-positive, trace-preserving map consistent with it (App. D), from which the quadratic weight follows by Gleason-type reasoning on the boundary algebra. So the honest statement is: given the bulk Hilbert space and CPTP reduction, the rule is forced; the squared modulus is not conjured from a classical substrate. Where this sits relative to Gleason is stated plainly — it shares Gleason's assumption (the inner-product structure) and adds the geometric reason *why* the boundary map must be CPTP. Removing that assumption is not claimed and is flagged in §96.

Objection 10 (O-GDOS-10) (the objective-collapse theorist)

"If the boundary projection has a physical *rate* (§27.4), that is an objective-collapse model in disguise — GRW/CSL-like. Then it modifies unitary QM and must predict testable deviations. Does it?

Response. The resemblance is acknowledged but the mechanism differs: GRW/CSL add a stochastic non-linear term to the Schrödinger equation by hand, with free collapse-rate and

length parameters; GDOS has *no* new free parameter — the projection rate is set by the scale geometry and the heavy-mode mass gap (App. G.4). In the resolution limit the boundary dynamics is exactly the standard linear GKLS evolution (App. E.2), so for laboratory systems it reproduces textbook QM with no anomalous heating or spontaneous-localization signal of the CSL type. Whether the geometric rate leaves *any* residual deviation at extreme mass scales is listed as an open problem (§97.1); the paper does *not* claim a near-term collapse-style falsifier, precisely to avoid overclaiming a CSL signature it cannot yet compute.

Objection 11 (O-GDOS-11) (the experimentalist)

"The Stern–Gerlach and CHSH worked examples reproduce standard quantum mechanics exactly. If GDOS predicts nothing different in the lab, it is a reformulation, not a derivation.

Response. For laboratory quantum optics the predictions are identical by construction, and the paper says so (§22.3, §26.3): reproducing Stern–Gerlach and the CHSH violation on the singlet is a *consistency check*, not a new prediction. The value of a reformulation is not a new lab number but a change in what counts as fundamental and what is derived — here, that the measurement cut, the pointer basis, and the Born weight descend from one geometric source rather than three postulates. Whether that reorganization buys empirical novelty lives elsewhere in the cohort, not in tabletop QM: the falsifiers are the proton-decay bound (T9.8), the 3.85 TeV lock cluster (Arisaka, 2026, P1-KK), and the 8.1 μeV Kaxion (Arisaka, 2026, C1-Kaxion). We are explicit (§96) that A5 itself adds reorganization, not a tabletop deviation.

Objection 12 (O-GDOS-12) (the relational-QM advocate)

"How does GDOS differ from Rovelli's relational quantum mechanics, which already makes outcomes observer-relative without any six-dimensional bulk?

Response. Relational QM and GDOS agree that measurement outcomes are relative to a boundary/observer, and the paper credits that convergence rather than disputing it (§28). The difference is what supplies the relation: in relational QM it is a primitive of the interpretation; in GDOS it is the geometric bulk–boundary reduction $\mathcal{R}$ (App. J.3), which also fixes the GKLS generator (App. E) and connects to gravity and thermodynamics through the *same* scale profile. So relational QM is, in the classification of §75, the AX-3-only specialization — correct as far as it goes, but silent about why the relation has the specific completely-positive form it does, and unconnected to the gravitational and thermodynamic sectors that the same axiom controls here. The claim is subsumption with a mechanism, not refutation.

T Deriving general relativity

Objection 13 (O-GDOS-13) (the relativist)

"General relativity 'from the 6D bulk' — is this anything more than Sakharov's induced gravity or a Kaluza–Klein reduction? What is new beyond a metric ansatz?

Response. The reduction machinery is indeed Kaluza–Klein on the bulk (App. G) and the spirit

is induced-gravity, and the paper does not pretend to have invented either. What is claimed as new (§31.2) is that the same boundary law that yields quantum measurement yields the Einstein equations as its gravitational sector — gravity and quantum theory are not bolted together but are two projections of one bulk connection (T8). Concretely, the explicit Christoffel symbols and curvature are computed (App. H) and the 4D Einstein tensor emerges from the scale-profile reduction with the worked example carried through (§33.4). The novelty is not the KK step but the constitutional co-origin: induced gravity normally leaves the matter sector and the measurement rule as separate inputs; here they are forced by the same axioms.

Objection 14 (O-GDOS-14) (the relativist, continued)

"Does the construction reproduce the *full* non-linear Einstein equations with the matter stress–energy coupling and the cosmological constant, or only the linearized / vacuum sector?

Response. The curvature computation is fully non-linear: App. H.2 and H.3 give the complete Riemann and Ricci tensors on the bulk, not a linearized approximation, and App. H.5 verifies the bulk scalar curvature exactly. The matter coupling enters through the boundary stress tensor of the projected fields (T8; §33.4), and a bulk cosmological-constant-like term appears explicitly (App. H.4). What the paper does *not* claim to derive from A5 alone is the observed *value* of the 4D cosmological constant — that is imported from the π-locked inventory of Arisaka (2026, A10-Cosmos) and flagged as model-level, not theorem-level (§96). So: full non-linear Einstein equations as structure, with the Λ value cited rather than re-derived.

Objection 15 (O-GDOS-15) (the mathematical physicist)

"Your own appendix finds $R_6 = -20$, the AdS_6 value. So the bulk is anti–de Sitter. Doesn't that make this AdS/CFT in disguise, with all its baggage — negative Λ, a conformal boundary?

Response. The bulk curvature is AdS_6 at the relevant order and the paper states it openly (App. H.5); the relationship to AdS/CFT is not hidden but claimed (§67.4): AdS/CFT is read as the AX-2-on-AdS specialization of the same geometry–gauge equivalence. The difference from standard holography is the boundary law. AdS/CFT is an exact duality between a closed bulk and a closed CFT; GDOS adds AX-3, making the boundary an *open* finite-resolution projection rather than an exact dual (§70.4). That is why the framework keeps the bulk–boundary correspondence but escapes the requirement of an exact conformal boundary and of unbroken supersymmetry. The honest concession: inheriting AdS_6 means inheriting the negative bulk curvature, and recovering a positive 4D Λ from it is exactly Objection 17.

Objection 16 (O-GDOS-16) (the quantum-gravity theorist)

"If gravity is induced on the boundary, is the graviton fundamental or emergent? An emergent graviton runs straight into the Weinberg–Witten theorem.

Response. The graviton is emergent — the 4D metric is a boundary projection of the bulk connection (T8, §67.1) — so Weinberg–Witten must be addressed, and the escape is the

standard one for holographic gravity: Weinberg–Witten forbids a massless spin-2 emerging from a Lorentz-covariant theory *with a conserved stress tensor in the same spacetime*; here the graviton lives on the 4D boundary while the conserved stress tensor and the emergence happen in the higher-dimensional bulk, so the theorem's hypotheses are not met (the same loophole AdS/CFT uses). This is noted in the scope discussion (§96). What the paper does *not* claim is a UV-complete quantum theory of the bulk graviton; quantum gravity proper is named as an open direction, and the present construction is explicitly the classical/semiclassical boundary metric plus its leading corrections.

Objection 17 (O-GDOS-17) (the cosmologist)

"Our universe is de Sitter-like, with positive Λ. Your bulk is AdS_6, with negative curvature. How do you get dS_4 out of AdS_6?

Response. The 4D effective cosmological constant is not the bulk curvature: it is the residue of the scale-profile reduction plus the boundary projection, and a bulk cosmological-constant-like term of definite sign appears in the reduction (App. H.4). The sign flip from AdS bulk to dS-like boundary is carried by the scale profile and the GSSB throat-cap dynamics, with the actual positive value $\Omega_\Lambda \approx 1 - 1/\pi$ supplied by Arisaka (2026, A10-Cosmos). The paper is careful to flag this as the weakest quantitative link (§96): the *structural* appearance of a positive 4D term is shown here, but its precise value is a model-level inheritance from A10, not a theorem-level output of A5. A failure of the π-locked inventory would therefore pressure the cosmological end of this chain, and we say so rather than claiming dS_4 as a closed result.

Objection 18 (O-GDOS-18) (the quantum-gravity theorist, continued)

"A boundary-projected metric is classical. Where is quantum gravity — graviton loops, a UV completion? Isn't this merely semiclassical gravity rebranded?

Response. For the gravitational sector the construction *is* semiclassical-plus-corrections, and the paper does not claim otherwise (§96; the UV completion of the bulk is listed among the open problems, §97.1). What it claims is narrower and checkable: that the classical Einstein equations and the quantum boundary law are co-derived from one bulk, so the usual tension between "quantum theory" and "gravity" is relocated to the bulk geometry rather than papered over at a Planck-scale junction. The relationship to full quantum gravity is treated as cooperative: the differential-cohomology and anomaly machinery the bulk uses (Arisaka (2026, A3-BI6)) is the same machinery string theory developed for UV completion, so a completion may be imported rather than rebuilt. The claim is co-origin of GR and QM, not a finished theory of quantum gravity.

U Deriving quantum field theory and the Standard Model

Objection 19 (O-GDOS-19) (the particle theorist)

"Standard-Model anomaly cancellation is representation-theoretic — the $\bar{5} + 10$ of $SU(5)$

— not a consequence of your modified Bianchi identity BI-6. You are claiming credit for textbook group theory.

Response. Correct, and the paper concedes it explicitly: the diagonal Standard-Model anomalies cancel by $\bar{5} + 10$ exactly as in conventional $SU(5)$ (§49.3). BI-6 is *not* claimed to do that work. What BI-6 does is the part the $SU(5)$ representation cannot reach: the $\mathrm{U}(1)_B$, $\mathrm{U}(1)_{Qi}$, and mixed gauge–gravitational anomalies, which are locked by the four-component trace-ratio identity rather than by representation content (T9.6; Lemma 20.2; proven in Arisaka (2026, A3-BI6)). So the division of labor is stated, not blurred: textbook group theory for the diagonal $SU(5)$ anomalies, BI-6 for the Abelian and mixed-gravitational sector that fixes the integer ledger. Claiming otherwise would be the overreach the objection rightly forbids.

Objection 20 (O-GDOS-20) (the QFT theorist)

"You 'derive' QFT by inheriting BRST symmetry from the bulk. But BRST is a gauge-fixing artifact, not physics. Inheriting it presupposes the gauge structure you claim to derive.

Response. The gauge structure itself is not derived from BRST — it is fixed upstream by the bulk gauge content and BI-6 (Arisaka (2026, A3-BI6)); BRST enters only to show that the *quantization* of that gauge theory is consistent on the boundary (App. K). The claim is precise: the bulk path integral with fixed boundary data, integrated over heavy modes, yields a 4D effective action whose Faddeev–Popov/BRST structure is inherited rather than imposed (App. K.3, App. K.2), with the Slavnov–Taylor identity following (App. K.6). So BRST is used as a consistency certificate for the boundary quantization, not as the origin of the gauge group. The reviewer is right that BRST is gauge-fixing bookkeeping; the point is that the bookkeeping closes on the boundary, which is non-trivial when the gauge fields are themselves projections.

Objection 21 (O-GDOS-21) (the non-perturbative theorist)

"Your derivation is perturbative — Faddeev–Popov, BRST, heavy-mode integration. But the hard content of QFT is non-perturbative: confinement, instantons, the mass gap. Does GDOS say anything there?

Response. No — and the paper does not claim to (§96). The construction delivers the perturbative 4D effective Lagrangian as a boundary projection (§54.2, App. L); confinement, the Yang–Mills mass gap, and instanton effects are explicitly outside its present reach and are listed among the open problems (§97.1). The honest claim is structural: the gauge sector, its field content, and its anomaly-freedom descend from the bulk, which is what the unification thesis requires; the non-perturbative dynamics of that sector is standard 4D physics, neither solved nor obstructed here. Pretending to have addressed the mass gap would be exactly the kind of overclaim the rest of these responses try to avoid.

Objection 22 (O-GDOS-22) (the renormalization theorist)

"The Kaluza–Klein tower is infinite. Where do the UV divergences go? Is the 6D bulk theory itself renormalizable or UV-complete, or are you hiding the hard part in

'integrating out heavy modes'?

Response. The heavy-mode integration is regulated by the bulk spectrum: the KK tower has a definite mass gap and lock-scale dressing (App. G.4), so the effective 4D action is obtained with the tower acting as a physical, geometrically fixed cutoff rather than an arbitrary one. The paper does *not* claim the 6D bulk is a finished UV-complete theory — bulk renormalizability/UV completion is named as open (§97.1, §96). What it claims is that the 4D effective field theory inherited on the boundary is the standard renormalizable gauge theory, with the heavy modes supplying matching conditions, not new divergences beyond the usual 4D ones. So the hard UV question is acknowledged and located in the bulk, not smuggled away: the boundary EFT is controlled; the bulk completion is future work, and the differential-cohomology backbone (Arisaka (2026, A3-BI6)) is the proposed route to it.

Objection 23 (O-GDOS-23) (the Standard-Model phenomenologist)

"You title a result 'the Standard Model as theorem'. But the SM has nineteen-plus measured parameters, and you import them from A9. Is the SM derived *here*, or merely hosted?

Response. Hosted, with the derivation cited — and the paper is careful to say so (§54.2). What A5 establishes is *structural*: that the 4D effective Lagrangian with the Standard-Model gauge group, chiral content, and anomaly-freedom is a boundary projection of the bulk (T9). The *numerical* parameters — couplings, mixing angles, masses — are the business of Arisaka (2026, A9-SM) and are imported, not re-derived. "As theorem" refers to the *form* of the Lagrangian following from the axioms, not to a closed-form prediction of every Yukawa coupling. The division is stated in the scope section (§96): A5 supplies the boundary law and the SM's structural skeleton; A9 supplies the flesh. Reading "theorem" as "all 19 numbers derived here" would be the overclaim the objection rightly targets.

Objection 24 (O-GDOS-24) (the lattice / chiral-fermion theorist)

"Chiral fermions from extra dimensions are notoriously obstructed — fermion doubling, Nielsen–Ninomiya. How does the orbifold $S^1/\mathbb{Z}_2$ give exactly the chiral Standard-Model content without doublers?

Response. By the standard orbifold-projection mechanism, worked out explicitly rather than asserted: the $S^1_\theta/\mathbb{Z}_2$ orbifold with the parity matrix $P = \mathrm{diag}(+,+,+,-,-)$ breaks $SU(5)$ and projects out the unwanted zero modes, leaving a chiral spectrum (App. I, App. I.3, App. I.4). Nielsen–Ninomiya forbids a chiral lattice regularization of a *continuum* 4D theory with the usual assumptions; it does not forbid chiral zero modes on an orbifold fixed point, which is precisely how the doublers are removed here (the fixed-point boundary conditions are not periodic). The construction is the established Kawamura-type orbifold GUT mechanism applied to the bulk; the paper's contribution is to embed it in the constitutional reduction, not to claim a new evasion of Nielsen–Ninomiya. The chirality lemma (Lemma 20.2) records the resulting zero-mode content.

V The unification of gravity and electromagnetism

Objection 25 (O-GDOS-25) (the gravity researcher)

"Spin$(1,5)$ is the Lorentz double cover, not a 'gauge group' in the conventional sense. Treating it on the same footing as $SU(5)$ is a category error.

Response. The Cartan (frame/spin-connection) formulation of general relativity already treats Spin$(1,5)$ as a gauge theory of the local Lorentz group — this is standard, not novel (§67.1). What is novel in GG-Theory is the *axiomatic* commitment of GGEP (AX-2) to place Spin$(1,5)$ on equal footing with $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ inside the single modified Bianchi identity BI-6, so that gravity and the internal gauge forces are components of one composite connection rather than two separate structures. It is not a claim that the Lorentz group is an internal symmetry; it is the claim that the frame bundle and the gauge bundle are unified in the bulk anomaly identity. Whether one accepts that unification is exactly what the classification (§65) and the explicit dictionary (§57.4) are there to let the reader audit.

Objection 26 (O-GDOS-26) (the skeptic)

"Newton's and Coulomb's $1/r^2$ laws 'sharing a geometric origin' is trivial: the inverse square is just the dimensionality of three-space via Gauss's law. You are dressing a freshman fact as a unification.

Response. The shared $1/r^2$ *from* 3-space is indeed elementary, and the paper does not rest the claim there. The non-trivial content (§58) is that the two forces descend from the *same composite bulk connection* via the same boundary projection, so the parallelism extends past the static potential to the *dynamics*: the unified equation of motion (§57.4) and, crucially, the radiative sector — gravitational inspiral and electromagnetic synchrotron emerge as the same OGGEP drift–diffusion phenomenon (§59, §59.5). Gauss's law explains why both are inverse-square; it does not explain why both radiate with a common geometric drift term. That deeper parallel, not the $1/r^2$ counting, is the claimed unification, and it is laid out as an explicit gravity–electricity dictionary for the reader to check (§59.4).

Objection 27 (O-GDOS-27) (the field theorist)

"Gravity is non-linear and self-interacting (gravitons carry energy); QED is abelian and linear. Calling them 'the same geometric origin' papers over the deepest difference between them.

Response. The difference is real and the paper locates it precisely rather than denying it: the non-abelian self-interaction of gravity versus the abelian linearity of electromagnetism is exactly the difference between the Spin$(1,5)$ and $\mathrm{U}(1)_B$ components of the composite connection (§58). "Same geometric origin" means same bulk connection and same boundary projection, *not* same algebra — the unified equation of motion (§57.4) carries both the non-linear curvature term and the linear field-strength term, and the dictionary (§59.4) tracks where they diverge. So the claim is not that gravity and QED are identical but that they are two components of one

structure whose differences are the group-theoretic data of that structure. The objection's distinction is preserved inside the unification, not erased by it.

Objection 28 (O-GDOS-28) (the gravitational-wave theorist)

"Gravitational radiation is quadrupole (spin-2); electromagnetic radiation is dipole (spin-1). The 'radiative completion' that calls them the same OGGEP phenomenon ignores their different multipole structure.

Response. The spin-2-versus-spin-1 multipole difference is not contested; it is, again, the $\mathrm{Spin}(1,5)$-versus-$\mathrm{U}(1)_B$ representation content of the radiating field (§59). What the radiative completion claims is narrower and lives at the level of the *drift–diffusion structure*, not the multipole order: in both cases the conservative orbit is circulation and the energy loss is an OGGEP boundary drift, with the radiated field arriving already derived from the same projection (§59.1, §59.2). The quantitative closure (§59.5) gives the drift and diffusion coefficients separately for each, with the multipole factors intact. So synchrotron and inspiral share the *mechanism* of radiative back-reaction, while keeping their distinct multipole prefactors — the unification is at the level of the open-system drift, not at the level of the angular pattern.

Objection 29 (O-GDOS-29) (the Einstein historian)

"Is 'Einstein's dream' really what Einstein sought? He wanted to derive quantum mechanics from a classical field theory; GDOS does the reverse. The 'Einstein's dream' framing is rhetorically convenient but historically misleading.

Response. A fair caution, and the paper draws the distinction rather than blurring it (§109). Einstein's *unification* dream — a single geometric structure underlying gravitation and electromagnetism — is what the title invokes, and that is reconstructed here (Newton and Coulomb as boundary projections of one connection, §58). Einstein's *other* hope — that quantum indeterminacy would dissolve into a deeper deterministic field theory — is *not* fulfilled and the paper says so: GDOS keeps quantum theory as fundamental and derives the classical limit from it, the opposite of Einstein's anti-quantum program (§28). So the framing claims the geometric-unification dream, explicitly not the hidden-variable dream. We agree the two should not be conflated, and the historical section is written to keep them apart.

W Holography, AdS/CFT, and black holes

Objection 30 (O-GDOS-30) (the string theorist)

"You write 'AdS/CFT as the GGEP content on AdS solutions'. That is a sweeping reattribution of Maldacena's correspondence. By what right do you subsume the most-tested result in modern theory as a special case?

Response. The claim is subsumption-by-axiom-subset, stated carefully and not as a takeover (§67.4): AdS/CFT is the framework one obtains when AX-2 (GGEP) is applied on an AdS bulk *without* the open-system axiom AX-3. On that reading the bulk–boundary correspondence

is shared — the paper keeps it — and the difference is that GDOS adds the finite-resolution boundary law, turning an exact duality into an open projection (§70.4). The reattribution is explicitly about *classification*, not priority: Maldacena's result stands untouched and is credited; the claim is only that it occupies a definite cell of the axiom table (§75), the AX-1–AX-2-on-AdS cell. The paper even separates "what the string community got right" from "what was misattributed" precisely to avoid the imperial reading (§77.1).

Objection 31 (O-GDOS-31) (the holography specialist)

"If GDOS is genuinely holographic, does it reproduce a Ryu–Takayanagi area law and the correct entropy scaling, or is 'bulk-to-boundary projection' just a picture?

Response. It is more than a picture in the black-hole sector, where the boundary projection yields the Bekenstein–Hawking area law as a consequence of the scale geometry (§71.6; *exact* $A/4$ derived in Arisaka (2026, C3-BH)), and the entropy-as-geometric-distance picture (§82) is the qualitative face of that. The paper does *not* claim a full Ryu–Takayanagi minimal-surface prescription for general entangling regions — that is not established here and is not claimed (§96). So the honest statement: the area-law scaling and the horizon entropy follow, consistent with holography; the general RT entanglement-entropy functional is neither derived nor needed for the present results, and is a natural item for the future-directions list (§97.1). The framework keeps the bulk–boundary correspondence it shares with AdS/CFT (§70.4) without claiming results it has not proven.

Objection 32 (O-GDOS-32) (the black-hole-information theorist)

"Calling the AMPS firewall a 'wrong-factorization symptom' is hand-waving. The firewall is a sharp paradox built on monogamy of entanglement. What is the concrete resolution?

Response. The paper's claim (§71.4) is specific, not a wave of the hand: AMPS assumes a clean tensor factorization of the Hilbert space into interior, exterior, and early radiation, and derives a contradiction from demanding all three pairwise entanglements at once. In GDOS the boundary projection is an *open* finite-resolution map, so the interior/exterior factorization is not exact — the off-diagonal sectors that the firewall argument treats as sharp are precisely those degraded by the scale resolution (§82, App. J.3). The diagnosis is therefore: the firewall is what you get when you impose closed-system factorization (AX-1–AX-2 only) on a system the open axiom AX-3 says is non-factorizable. The paper does not claim a complete quantitative island/replica computation — it claims the paradox's premise (exact factorization) is the thing AX-3 denies, and flags the full computation as open (§97.1).

Objection 33 (O-GDOS-33) (the black-hole thermodynamicist)

"Bekenstein–Hawking $S = A/4$ — do you *derive* the factor of $1/4$, or import it? And do you explain the microstates, or only the thermodynamics?

Response. The $A/4$ with the exact quarter is derived in the dedicated companion Arisaka (2026, C3-BH) from the horizon-as-OGGEP-port construction, and A5 imports that result

while supplying the constitutional setting (§71.6, §72.2): the four Bardeen–Carter–Hawking laws appear as the gravitational specialization of the same open-system axiom that gives the ordinary laws of thermodynamics (§72.1, §71.7). On microstates the paper is candid: it gives the *thermodynamic* entropy and its geometric origin, not a Strominger–Vafa-style microstate count — that is explicitly not claimed and would belong to the bulk UV completion (§96). So: the 1/4 is a derived cohort result (C3), not a fit; the microstate counting is acknowledged as unsolved here.

Objection 34 (O-GDOS-34) (the decoherence theorist)

"Picture 3 calls decoherence 'KK-mode integration'. Integrating out a specific KK tower gives a specific decoherence channel. Does it reproduce *measured* decoherence rates, or is it a metaphor?

Response. It is a structural identification with a definite channel, not a metaphor: integrating out the heavy KK modes yields a GKLS generator whose form is computed (App. E.3, App. G) and whose pointer basis is einselected (App. F.3), with the Caldeira–Leggett model recovered as the worked example (App. F.4). What the paper does *not* claim is a parameter-free prediction of a specific laboratory decoherence *rate* for a given experiment — that depends on the system–environment coupling, which for a tabletop system is ordinary 4D physics, not the KK tower (the KK scale is far too high to dominate lab decoherence). So Picture 3 (§83) is the statement that the *mathematical structure* of decoherence is mode-integration over the bulk, exact in form; it is honest that the dominant environmental channel in any real experiment is the local one, not the geometric one.

X The axiom-classification claim

Objection 35 (O-GDOS-35) (the skeptic)

"Placing GR, gauge theory, string theory, AdS/CFT, QM, the SM, and ΛCDM as 'theorems sorted by axiom subset' is post-hoc taxonomy. Any framework can be retro-classified by which of your axioms it happens to use.

Response. The taxonomy would be empty if it only re-labeled, so the paper makes it do *predictive* work (§76): each axiom subset is tied to a *count* — admissible vacua (§76.1), free continuous parameters (§76.2), and theorem yield (§76.3) — so the classification predicts, e.g., why the string cell carries a vacuum landscape and the full-axiom cell does not. A purely post-hoc taxonomy could not produce those numbers from the cell alone. The honest limit (§78): the classification is only as objective as the axiom definitions, and the assignment of a framework to a cell is a modeling judgment that a critic can dispute case by case — which is exactly why each assignment (§67.1–§67.4) is argued individually rather than asserted in bulk.

Objection 36 (O-GDOS-36) (the string theorist)

"You claim the string community 'misattributed' its own results. That is a strong

and arguably arrogant charge. What, specifically, was misattributed, and by whose standard?

Response. The charge is narrowed and defended, not thrown loosely (§77.2): the claim is that certain results credited to *stringiness* per se — holographic duality, anomaly inflow, warped hierarchy — are, on the classification reading, consequences of the geometry–gauge equivalence (AX-2) and the open-system law (AX-3) that do not require the full string apparatus (supersymmetry, the critical dimension, the landscape). The paper is equally explicit about "what the string community got right" (§77.1) — the machinery, the dualities, the anomaly technology — and frames the reattribution as the Riemann–Einstein relationship (§77.4): mathematics developed in one program later recognized as the natural language of another. Whether one finds that arrogant or clarifying is a judgment the paper invites by showing its work; it does not denigrate the results, it relocates their necessary conditions.

Objection 37 (O-GDOS-37) (the philosopher of science)

"'Number of admissible vacua per axiom subset' presupposes *your* notion of vacuum and admissibility. The comparison is rigged by construction — you defined the scoreboard.

Response. The scoreboard is defined once, in advance, and applied uniformly — which is the only defense available to any comparative framework, and the paper makes the definition explicit so it can be contested (§76.1, §3). The key check against rigging is that the counts use *each framework's own* vacuum notion where it has one: the string cell's $\sim 10^{500}$ is the landscape's own number, not an externally imposed one; the full-axiom cell's uniqueness is the OGN-5 integer-ledger result of Arisaka (2026, A3-BI6), also internal. So the comparison is not "our metric flatters us" but "each framework, counted by its own admissibility, lands in a different cell." The residual subjectivity — that *which* cell a framework belongs to is a modeling call — is conceded openly (§78); the counts within a cell are not free parameters.

Objection 38 (O-GDOS-38) (the methodologist)

"The Riemann–Einstein analogy casts your constitution as the 'Riemann' awaiting an 'Einstein'. That is a rhetorical move that pre-claims historical significance you have not earned.

Response. The analogy is used as a structural claim about *order of discovery*, not as a self-coronation (§77.4): the point is that a mathematical framework (differential geometry; here the constitutional/anomaly machinery) is often developed before the physical theory that needs it, and the paper places the geometry–gauge machinery in that "mathematics-first" role relative to a unification it does not claim to have completed. The significance is offered as a *wager* held against falsification (§93), explicitly labeled a proposed fourth revolution that "will be vindicated or refuted" by experiment (§107.4), not as an achieved status. We take the objection seriously enough that the closing reflection (§116) frames the whole program as a proposal, not a coronation — the reader is asked to judge the physics, not the analogy.

Y Thermodynamics, entropy, and the arrow of time

Objection 39 (O-GDOS-39) (the statistical mechanic)

"Deriving $S = k_B \ln W$ 'as OGGEP phase-space counting' adds nothing — Boltzmann's formula is pure combinatorics. What is the physical content beyond a rename?

Response. The combinatorial formula is unchanged; what OGGEP adds is *why the counting is the physically right object* — because the boundary projection is exactly the coarse-graining that defines W, so the Boltzmann entropy is the information discarded by the resolution map rather than a subjective bookkeeping choice (§69.6). The content is the unification, not the formula: the *same* axiom that gives the Boltzmann entropy gives the third law as the zero-scale limit (§69.5), temperature as the inverse scale rate (§69.7), and the fluctuation theorems as the time-reversal of the projection (§69.10). Boltzmann's formula on its own does none of that connecting. So the claim is not a better $\ln W$ but that statistical entropy, temperature, the third law, and fluctuation relations descend from one boundary law — a structural economy the objection's "just combinatorics" reading misses.

Objection 40 (O-GDOS-40) (the thermodynamicist)

"'Temperature as inverse scale rate' is just Unruh–Tolman temperature, which already exists in standard GR/QFT. What does GDOS add beyond renaming a known effect?

Response. The Unruh–Tolman relation is indeed recovered, not replaced (§69.7); the addition is that it is no longer a separate effect for accelerated observers but the *same* scale-rate quantity that sets the decoherence channel (Picture 3, §83), the horizon temperature (§71.6), and the cosmological-horizon temperature (§72.3). In standard treatments Unruh, Hawking, and Gibbons–Hawking temperatures are three analogous results with three derivations; here they are one scale-rate read on three boundaries (§73). The paper does not claim a new temperature formula; it claims these temperatures are siblings under one axiom (§73.1). The value of that is economy and falsifiability of the *link*: break the common scale origin and the three should decouple, which is a structural prediction the standard piecemeal derivations cannot make.

Objection 41 (O-GDOS-41) (the foundations-of-time critic)

"If openness and irreversibility are constitutional (AX-3), you have put the arrow of time in by hand. That is not *deriving* the second law — it is assuming it.

Response. Correct, and the paper does not claim to derive the arrow from time-symmetric dynamics — it claims the opposite, that the arrow is constitutional rather than emergent (§106). This is a deliberate methodological choice, not an oversight: rather than the (still-unsolved) program of extracting irreversibility from reversible microdynamics plus special initial conditions, GDOS makes the finite-resolution boundary projection fundamental, from which the second law follows as the monotonicity of the discarded information (§69.10; the data-processing inequality, §88). The honest framing (§6.1) is exactly the objection's: the arrow is an axiom, not a theorem. The wager is that this is the *right* primitive — that the second law is foundational, not derived — and the paper presents it as a proposal to be judged by its consequences, not as a stealth

derivation. Custody is stated once for the whole cohort and the present paper keeps to it: AX-3 owns the arrow constitutionally (the AX-3 synthesis of GG-A1-Constitution, Arisaka, 2026, A1-Constitution), Part IX of the CPT member paper (Arisaka, 2026, P5-CPT) is the canonical law-level derivation of the arrow from that axiom over a bulk that is itself exactly T-symmetric, and the present paper is the custody chain's dynamical implementation.

Objection 42 (O-GDOS-42) (the information theorist)

"The data-processing inequality 'with two faces' — is the information-theoretic content doing physical work, or is it an analogy? Does GDOS make a falsifiable information-theoretic prediction?

Response. It does physical work in the black-hole and decoherence sectors: the same data-processing inequality that bounds classical channel capacity bounds the monotone loss of distinguishability across the boundary projection, which is what forbids the firewall's exact factorization (§88, §71.4) and gives the second law its direction (§69.10). The paper is careful, though, not to oversell a *new* info-theoretic experiment: the DPI here certifies internal consistency and the irreversibility direction, it is not advertised as a stand-alone laboratory falsifier (§96). So "two faces" is a genuine structural identity (information monotonicity = geometric resolution loss), used to constrain black-hole and measurement physics, while the empirical falsifiers remain the collider, haloscope, and proton-decay tests of §93 rather than an information-theoretic one.

Z Falsifiability, predictions, and scope

Objection 43 (O-GDOS-43) (the experimentalist)

"Most of your falsifiers are beyond current reach. How is this paper falsifiable on a five-to-ten-year horizon rather than in principle only?

Response. Three falsifiers are accessible now or near-term (§93): (i) proton decay — any positive observation at Hyper-Kamiokande falsifies T9.8; (ii) the $M_{\text{lock}} \approx 3.85$ TeV Kaluza–Klein lock-cluster of Arisaka (2026, P1-KK), being probed at the LHC and decisively tested at HL-LHC by the late 2020s; (iii) the $\Omega_m \approx 1/\pi$, $\Omega_\Lambda \approx 1 - 1/\pi$ π-locked inventory, already consistent with Planck 2018 to 1σ (model-level, not theorem-level, and flagged as such). The modified-TOV signal at $\sim 10^{-9}$ relative precision is longer-term, with STROBE-X named as the pathway, and the full twenty-five-test horizon is consolidated in §105. The honest caveat (Objection 45): the sharpest of these are inherited from companion papers, not unique to A5.

Objection 44 (O-GDOS-44) (the Popperian)

"A framework that reproduces all of established physics as 'theorems' risks unfalsifiability: it predicts exactly what we already see. What does GDOS forbid that standard physics permits?

Response. The reproduction of known physics is the consistency requirement; the falsifiable content is what the *shared axiom* forbids (§93). Because measurement, gravity, and thermodynamics descend from one boundary law, they are not independently adjustable: e.g. the

scale rate that sets the horizon temperature is the same one that sets the KK lock scale, so a 3.85 TeV cluster that is *absent* at HL-LHC breaks the chain that also gives the thermodynamic sector (§73). Likewise the integer-ledger that closes the anomaly sector forbids proton stability beyond the T9.8 bound. Standard physics, with independent sectors, can absorb a null result by retuning one sector; the constitutional linkage cannot. That interlock — one failure propagating across sectors — is what makes the framework riskier than the sum of its parts, which is the Popperian virtue the objection asks for.

Objection 45 (O-GDOS-45) (the hard-nosed experimentalist)

"The modified-TOV prediction sits at 10^{-9} relative precision against a LIGO sensitivity of 10^{-2} — seven orders short. Calling that a 'falsifier' is generous. Is there *any* near-term test specific to A5 and not imported from P1 or C1?

Response. On the modified-TOV point the objection is correct and the paper states the gap honestly: it is a long-term target with an identified pathway (STROBE-X), not a five-year test (§105). On the deeper point — a near-term test *specific* to A5 — the candid answer is that A5's distinctive content is the *reorganization* (one boundary law behind QM, GR, and thermodynamics), and its sharp near-term falsifiers are indeed inherited (proton decay, the KK cluster), as §96 concedes. A5's own falsifiable stake is the *linkage* (Objection 44): that these sectors cannot be decoupled. We do not dress the TOV number as more than it is, and we name the inheritance rather than claiming a tabletop A5-only signal we cannot deliver.

Objection 46 (O-GDOS-46) (the string theorist)

"GG-Theory inherits warped extra dimensions from Randall–Sundrum and anomaly cancellation from Green–Schwarz. Why is it not simply a non-supersymmetric corner of the string landscape?

Response. It does inherit both lineages (M. Green, 1984, L. Randall, 1999), and we read Green–Schwarz as the technical pioneering of GGEP — the machinery is not in dispute. The constitutional distinction is in what *selects* the bulk geometry. In string theory, modular invariance, supersymmetric cancellation, and the compactification choice together produce the $\sim 10^{500}$ landscape; in GG-Theory, four constitutional axioms plus the OGN-5 integer ledger fix the geometry with no continuous moduli and no compactification choice (§76.1; the head-to-head is given in Arisaka (2026, A3-BI6), in its subsection "BI-6 as the GGEP completion of GS"). So the framework is not a landscape corner because it has no landscape: the same machinery is run with a selection principle that removes the moduli. Whether that selection principle is correct is the empirical question (§93); whether it is *different* from string model-building is not in doubt.

Objection 47 (O-GDOS-47) (the referee)

"Most of your predictions — the KK resonance, the Kaxion, π-locked cosmology — are imported from P1, C1, and A10. What does A5-GDOS *itself* stake that is falsifiable and not inherited?

Response. A5's own stake is the boundary law and its *sector-linking* claim (§64): that four-dimensional quantum mechanics (T7), general relativity (T8), and quantum field theory (T9) are

projections of one open bulk, joined by the bridge theorems T10–T11. That is falsifiable on its own terms (§93, §73): exhibit a regime where the scale rate that fixes the measurement/decoherence sector is empirically *different* from the one fixing the gravitational/thermodynamic sector, and the co-origin claim fails — independently of any companion paper. The imported numbers (KK, Kaxion, cosmology) are the consistency checks the audit cross-references (App. O); A5's distinctive, refutable contribution is the constitutional unification of the three sectors under one boundary law, which the classification (§75) makes explicit and checkable.

Objection 48 (O-GDOS-48) (the methodological skeptic)

"The framing — resolving Einstein's dream and the century-old measurement problem in one paper — is unusually bold. Why should the community take the claims seriously rather than dismiss them as grandiose?

Response. Because the architecture is fully specified, so an adversary can locate exactly the step they reject rather than dismissing a mood: four axioms, seven postulates, and eleven numbered theorems T1–T11 (App. N.1), each forwarded to the companion where it is proved. The empirical predictions are sharp and falsifiable (§93), and every numerical claim inherited from a companion paper is cited rather than re-derived (§96). The paper also states its boldness as a *wager*, not a conclusion: the closing reflection frames the whole program as a proposed fourth revolution to be vindicated or refuted by experiment (§107.4, §116). Seriousness is not asked on the strength of the framing; it is asked on the strength of a specified, auditable, falsifiable structure — and withheld exactly where the structure is still open (§97.1).

Objection 49 (O-GDOS-49) (the time-pressed referee)

"Three hundred pages to re-derive things already known. State, in one sentence, the irreducible *new* physical result that could not have been written before this paper.

Response. In one sentence: that four-dimensional quantum mechanics, general relativity, and quantum field theory are not three independent theories but three boundary projections of a single open six-dimensional bulk under one finite-resolution law, so that their measurement, gravitational, and thermodynamic sectors are forced to share a single scale-rate parameter rather than being independently adjustable (§64, T7–T11). Everything else in the paper either establishes that statement (Parts II–VI), classifies existing frameworks by which part of it they instantiate (Part VIII), or draws its pictures and predictions (Parts IX–X). The new content is the *forced co-origin and the resulting sector-linkage*, not any single re-derived equation; the re-derivations are the evidence that the one structure actually reproduces the three theories, which is what a unification claim has to demonstrate.

AA Internal consistency, rigor, and the appendices

Objection 50 (O-GDOS-50) (the mathematical physicist)

"The Nakajima–Zwanzig → GKLS chain needs Born–Markov approximations, which fail

for strong coupling or structured baths. Does your 'derivation' of quantum dynamics hold only in the Markovian limit?

Response. The *exact* Nakajima–Zwanzig memory-kernel equation is derived first, with no Markov assumption (App. C.7); the Markovian GKLS form is then recovered as a controlled limit (App. C.8, App. E.3), and the paper explicitly treats the non-Markovian regime where that limit breaks (App. C.10), including the spin-boson worked example (App. C.9). So the structural claim — that the boundary dynamics is a completely-positive reduction of unitary bulk evolution — rests on the exact NZ equation and the Stinespring dilation (App. D), which require no Markov approximation; the *specific* GKLS generator is the Markovian specialization, presented as such. The honest scope: the clean "QM as boundary law" statement is Markovian; the non-Markovian corrections are derived but not claimed to be fully solved, and are part of the open structure (§97.1).

Objection 51 (O-GDOS-51) (the rigor referee)

"Stinespring guarantees that *a* CPTP map has *a* unitary dilation. It does not show your six-dimensional bulk *is* that dilation. How do you know the physical bulk is the Stinespring dilation of the 4D channel?

Response. The objection correctly identifies the logical gap and the paper does not claim uniqueness of the dilation — Stinespring dilations are unique only up to a partial isometry on the ancilla (App. D.2). The claim is the converse direction and it is weaker: *given* the bulk geometry (fixed independently by the constitutional axioms and the OGN-5 ledger, not chosen to fit the channel), the induced boundary map *is* CPTP and *is* a Stinespring dilation, which is what consistency requires (App. D.3). So the bulk is not reverse-engineered from the 4D channel; it is fixed upstream, and Stinespring is used to certify that the projection it induces is a legitimate quantum channel. The residual — that other dilations of the same channel exist — is real and harmless: physics selects the geometric one, and the paper says the selection is geometric, not a theorem of channel theory (§96).

Objection 52 (O-GDOS-52) (the operator-algebraist)

"If information is genuinely lost crossing the boundary (Picture 2), the reduction map $\mathcal{R}$ is not invertible. Then the 4D physics does *not* fully determine the bulk — so in what sense is the bulk 'the' explanation?

Response. $\mathcal{R}$ is deliberately non-invertible — that non-invertibility *is* the openness, and the paper builds on it rather than around it (App. J.3, §82). The logic runs bulk→boundary, not boundary→bulk: the bulk determines the 4D physics, not the reverse, exactly as a statistical-mechanical microstate determines the macrostate while the macrostate does not determine the microstate. The lost information is the entropy (§69.6); its irrecoverability is the second law (§69.10). So the bulk is "the explanation" in the forward direction (it fixes the boundary law, the GKLS generator, the Einstein equations), and the non-invertibility is not a defect but the source of thermodynamic and measurement irreversibility. The genuine open question — whether the bulk is *uniquely* reconstructible even in principle — is acknowledged (Objection 51) and is not needed for the forward claims.

Objection 53 (O-GDOS-53) (the gauge-theory referee)

"Boundary BRST inheritance assumes the boundary inherits bulk BRST cleanly. But anomalies and edge modes can obstruct BRST at a boundary. Is the boundary BRST genuinely anomaly-free?

Response. This is exactly where the cohort's anomaly result is load-bearing: the boundary BRST is anomaly-free *because* the bulk satisfies the modified Bianchi identity BI-6, whose local-and-global anomaly freedom is the theorem of Arisaka (2026, A3-BI6). The inheritance is shown step by step — the bulk path integral with fixed boundary data, heavy-mode integration, and the resulting Slavnov–Taylor identity (App. K.3, App. K.6) — precisely so the boundary Ward identities can be checked rather than assumed. Edge-mode/boundary anomalies are the right worry, and the answer is that they are canceled by the same Green–Schwarz/BI-6 inflow that cancels the bulk anomalies (the anomaly-inflow mechanism). So the boundary BRST is not assumed clean; it is clean *conditional* on the A3 anomaly theorem, which is cited as the dependency. If that theorem failed, this construction would fail with it — the dependence is explicit.

Objection 54 (O-GDOS-54) (the skeptic on rigor)

"Eleven 'Principal Theorems' — are these theorems in the mathematical sense, proved from axioms with no gaps, or physicists' plausibility arguments? Which steps are rigorous, and which are derivations-in-progress?

Response. They are a mixture, and the paper grades them rather than blurring the distinction (App. N.1, §96). The anomaly-freedom and integer-ledger results (T9-adjacent) are mathematically rigorous, proved in Arisaka (2026, A3-BI6); the reduction-map and CPTP results (T7) rest on the exact NZ equation and Stinespring (Apps. C, D), rigorous given the bulk; the curvature/Einstein results (T8) are explicit tensor computations (App. H). The *measurement-outcome* step (Objection 8), the bulk UV completion, and the non-Markovian closure are explicitly derivations-in-progress, listed among the seven open problems (§97.1). So "theorem" is used in the working-physicist sense of "a stated result with a located proof or a located gap" — and the gaps are located, not hidden. A reader can audit each Tn against its appendix and decide for themselves which they accept.

Objection 55 (O-GDOS-55) (the unitarity hawk)

"You claim to resolve the black-hole information paradox, yet Picture 2 says information is *lost* as geometric distance. Isn't that just conceding information loss — the very thing the paradox forbids?

Response. The distinction the paper draws (§82, §70.4) is between information lost *from the boundary description* and information lost *from the bulk*. The bulk evolution is unitary; the boundary projection is where distinguishability degrades, exactly as in ordinary decoherence the global state stays pure while the reduced state thermalizes. So no fundamental unitarity is violated: the "loss" is the non-invertibility of $\mathcal{R}$ (Objection 52), i.e. entropy, not a breakdown of bulk unitarity. The paradox is recast (§71.4) as the error of demanding an exact interior/exterior

factorization of an open boundary; the bulk-unitary/boundary-open split is the resolution sketch. The paper does *not* claim a complete island-formula derivation of the Page curve (Objection 32) — that is flagged open — but it is explicit that bulk unitarity is preserved, so it is not conceding fundamental information loss.

Objection 56 (O-GDOS-56) (the constitutional auditor)

"The 'constitutional tower' is presented as four nested layers. Is the nesting strict — does each axiom add genuinely new content — and what concretely breaks if a given axiom is dropped?

Response. The nesting is presented as strict and the paper makes the test explicit (§65, Picture 6, §86): each layer is characterized by what it can and cannot derive. With AX-1 alone (locality/causality) one has a consistent local field theory but no geometry–gauge unification; adding AX-2 (GGEP) gives the bulk, AdS/CFT, and classical gravity but no measurement law or second law; adding AX-3 (OGGEP) gives quantum measurement, thermodynamics, and black-hole entropy; AX-4 (MSMF) removes the moduli and fixes uniqueness. The classification table (§75) is precisely the demonstration of strictness: each existing framework sits at the level of the axioms it uses, and the missing axioms predict its missing structure (§76.3). So "strict" is not asserted but exhibited — drop an axiom and a named class of results disappears, which is the auditor's check the objection asks for.

Objection 57 (O-GDOS-57) (the quantum-information theorist)

"You assumed a complex Hilbert space from the start — GNS hands you complex scalars because the C^*-algebra you feed it is complex. So the framework has assumed quantum mechanics' most distinctive structural feature rather than derived it."

Response. The premise is the right one to attack, and the answer is structural, not verbal. The bulk is real: the constitution states its bulk clauses over the reals, and the complex $\mathcal{H}_{\text{Bulk}}$ of Part II is declared at the outset to be auxiliary bookkeeping — the complexification licensed by T-P4-2 of Arisaka, 2026, P4-Real — with the No-Bypass argument running verbatim on the real solution space (§8). The boundary's complex scalars are then an *output*: the projection pairs the arena's two real structures (the compact phase circle and the causal clock) into the single derived unit J, at derived-conditional grade in the quasi-free regime, which is exactly the import consumed at T7.1 (§20) and read off the Schrödinger limit at T7.2 (§21). And the distinction the objection dismisses as bookkeeping is empirically live: a loophole-free network Bell experiment has falsified quantum theory over *real* Hilbert spaces at the boundary level (Renou *et al.*, 2021) — real amplitudes with tensor-product structure cannot reproduce the observed correlations — while a real *bulk* whose projection installs a genuinely complex boundary theory is precisely the combination that experiment permits. The framework therefore occupies the one quadrant that survives the data: a complex boundary theory, held as a theorem of the projection rather than as an assumption, over a real bulk (§95).

Part XIV
References

Cited works, each with a relevance note linking it to the present paper

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Unlike a conventional reference list, every entry below carries a short *Relevance* note: a sentence or two saying what the cited work contributes and where in the present paper it bears. The aim is to turn the bibliography from a bare list of sources into a navigable companion — a reader can move through the references and see, at a glance, which theorem, table, or section each one underwrites. The notes are written for two readers at once. For the newcomer, they trace the intellectual lineage of the open-system boundary law: from the foundations of decoherence, open-quantum-systems theory, and holography, to the twenty-two-paper GG-Theory cohort in which the present paper supplies the GDOS boundary law from which four-dimensional quantum mechanics, general relativity, and quantum field theory descend. For the specialist, they make the paper's debts explicit and checkable — each note names the specific result it supports, so that no citation is merely decorative. The references are organized in two groups — first the author's GG-Theory papers (Section A below), then all other works (Section B), each in alphabetical order by first author. The cohort papers (GG-A1-Constitution–GG-A11-Origin, with the C- and P-series) are cited under the canonical keys and titles fixed by the cross-paper coherence map of the GG-Theory program.

A. Papers by Katsushi Arisaka (the GG-Theory cohort)

Arisaka, K. (2026). *The GG-Constitution: Four Axioms and Seven Postulates of the GG-Theory Program, From Which GG-6, BI-6, OGN-5, and GDOS Descend as Theorems.* Arisaka-GG-A1-Constitution preprint.
Preprint, available at https://preprints.arisaka-gg.org/a1-constitution/.
doi:10.5281/zenodo.22063867.

Relevance. This is the GG-Theory Constitution paper (formerly UCLA-Arisaka/GG-2026-1, with the title 'Axiom System of GUT-10' in and renamed 'GG-Constitution' in subsequent revisions). It states the four foundational axioms (LC, GGEP, OGGEP, MSMF) and seven postulates (GI, UEP, AF, CP, RG, OGN, MF) that the entire GG-Theory program takes as primitive input, including the present paper. Every theorem T1–T11 of the present paper traces its constitutional source to one or more clauses of this Constitution. The companion paper is the Euclidean foundation document on which the entire GG-Theory edifice is built. First cited in §1.

Arisaka, K. (2026). *GG-Theory with GG-6: Geometry–Gauge Locking at the 6D Bulk (GG-6) at the Heart of Grand Geometric Theory (GG-Theory).* Arisaka-GG-A2-GG6 preprint.
Preprint, available at https://preprints.arisaka-gg.org/a2-gg6/.

doi:10.5281/zenodo.22064298.

Relevance. This is the canonical reference for the entire family of GG-Theory companion papers, including: GG-2026-2 (the GG-6 ultraviolet stage with the 6D bulk, Spin(1,5)x SU(5)x $U(1)_B$x $U(1)_{Qi}$ composite gauge group, BI-6 modified Bianchi identity, and OGN-5 integer ledger); GG-2026-3-GD (the local + global anomaly proof of BI-6 in differential cohomology); GG-2026-4A (the Gi-4 Pillars paper); GG-2026-4B (the Qi-4 Pillars paper); GG-2026-6F (the 28-parameter Standard Model derivation); GG-2026-7F (the cosmology paper); GG-2026-Generations (the $N_g = 3$ proof); GG-2026-6C (KK lock cluster predictions); and GG-2026-7C (Kaxion dark-matter predictions). The present paper supplies the foundational boundary law GDOS that all these companion papers presuppose or build upon. First cited in §1.

Arisaka, K. (2026). *Anomaly Freedom of BI-6: The Modified Bianchi Identity (BI-6) for the Six-Force, Six-Dimensional Bulk (GG-6).* Arisaka-GG-A3-BI6 preprint.
Preprint, available at https://preprints.arisaka-gg.org/a3-bi6/.
doi:10.5281/zenodo.22064300.

Relevance. This is the canonical proof of local and global anomaly freedom of the Green–Schwarz-modified Bianchi identity BI-6 of GG-Theory, including the constitutional admissibility theorem for $U(1)_{Qi}$, the four-component coefficient vector $(a, b, c, d) = (1, -1, -2/5, -2/5)$, and the trace-ratio lock $|a/c| = |a/d| = 5/2$. In the present paper it is the authoritative source for the claim that the four-component bulk gauge group is constitutionally admissible. Crucially, A3-BI6 § "BI-6 as the GGEP completion of GS: the geometrization reading" frames the 1984 Green–Schwarz mechanism as the technical pioneering of AX-2 (GGEP) and BI-6 as its constitutional generalization; whenever GDOS prose touches GS or anomaly inflow, the supportive synthesis with the string-theory community lives in that subsection rather than being re-derived here. First cited in §4.2.

Arisaka, K. (2026). *From Euclid to GG-6: The Realization of Euclid's Elements in Six Dimensions — GU-5, OGN-5, and a Parameter-Free Reconstruction of Modern Physics.* Arisaka-GG-A4-Euclid preprint.
Preprint, available at https://preprints.arisaka-gg.org/a4-euclid/.
doi:10.5281/zenodo.22064302.

Relevance. This companion paper provides the consolidated historical inventory of the closed-system tradition in physics, from Newton's Principia (1687) through the GKLS classification (1976), the Stinespring–Caldeira–Leggett synthesis, the decoherence revolution (Zeh–Joos–Zurek, 1970–2003), and the modern axiomatic / information-theoretic reconstructions (Hardy 2001, Masanes–Müller 2011, QBism). It also records the half-century-old unification problem inventory (direct quantization, supergravity, string theory, loop quantum gravity, the holographic strand, asymptotic safety, causal dynamical triangulations, non-commutative geometry, GUTs, supersymmetry, Kaluza–Klein and extra dimensions, string compactifications). In the present paper we delegate the historical narrative blocks to A4-Euclid so that the body text can focus on the constitutional content; the full lineage analysis lives in this companion paper. First cited in §7.1.

Arisaka, K. (2026). *GDOS: Geometric Dynamics of Open Systems — 6D GG-6 Bulk Projected to 4D Boundary for Quantum Measurement, General Relativity, and QFT.* Arisaka-GG-A5-GDOS

preprint.
Preprint, available at https://preprints.arisaka-gg.org/a5-gdos/.
doi:10.5281/zenodo.22064304.
Relevance. The present paper. It supplies the open-system boundary law GDOS, from which four-dimensional quantum mechanics, general relativity, and quantum field theory descend as boundary projections of the 6D bulk. First cited in §7.1.

Arisaka, K. (2026). *The GRIP on GDOS — GRIP: the Geometric Engine of Symmetry Breaking, from the Big Bang to Mind*. Arisaka-GG-A6-GRIP preprint.
Preprint, available at https://preprints.arisaka-gg.org/a6-grip/.
doi:10.5281/zenodo.22064306.
Relevance. This companion paper of the GDOS Trio develops the four geometric-invariance pillars on the boundary side of the GDOS law: TGG (Total Geometric–Gauge object), CCI (Covariant Conservation Identity), GFF (Geometric Free-Energy Functional with GEP/CEP blocks), and GRIP (Gauge-Runtime Invariance Pipeline with DGI→GSSB→GA→GGA cascade). In the present paper we cite A6-GRIP at T10 (the GDOS-to-Gi-4 bridge) and at every appearance of the four pillars in the body. It introduces the Ground Geometric Attractor (GGA), the cascade-terminus refinement adopted in the present paper. First cited in §7.1.

Arisaka, K. (2026). *The Qi-4 Pillars — Qi-QUA, Qi-EP, Qi-CON, Qi-HAL: The Quantum-Information Backbone of GDOS*. Arisaka-GG-A7-Qi4 preprint.
Preprint, available at https://preprints.arisaka-gg.org/a7-qi4/.
doi:10.5281/zenodo.22064308.
Relevance. This companion paper of the GDOS Trio develops the four quantum-information pillars on the boundary side of the GDOS law: Qi-QUA (current locking and Arisaka equation), Qi-EP (variational gauge tower), Qi-CON (conformality + Lévy + fractality + $1/f$ statistics), and Qi-HAL (memory-stack architecture with MePMoS/NHT/HAL/BFFT/GRIP-loop). It also carries the EIC (Outer-Hologram + Inner-Hologram = Evolving Inner Cosmos) framework that completes the quantum-information side of GDOS. In the present paper we cite A7-Qi4 at T11 (the GDOS-to-Qi-4 bridge), at every appearance of the Arisaka equation $E = T_{Qi} \log 2$, and at the cross-paper coherence-map row that names the Qi-4 pillars. First cited in §3.

Arisaka, K. (2026). *Quantum Mechanics Derived from GG-Theory as Projection from 6D Bulk to 4D Boundary*. Arisaka-GG-A8-QM preprint.
Preprint, available at https://preprints.arisaka-gg.org/a8-qm/.
doi:10.5281/zenodo.22064310.
Relevance. This companion paper is the entry point of the 4D-Physics Trio (A8-QM + A9-SM + A10-Cosmos). It operationalizes the boundary law GDOS from A5-GDOS into the concept-by-concept reformulation of textbook quantum mechanics — Born rule, EPR / Bell / CHSH, Schrödinger's cat, double-slit, decoherence, einselection — with the central thesis that "Einstein was right in 6D, Bohr was right in 4D, with both positions correct within their respective domains". In the present paper we name A8-QM at the Scope-and-Limits enumeration of what A5-GDOS does *not* carry out (the operational reformulation of textbook QM in GDOS language) and at any

forward-pointing cross-reference to QM-foundations content that A8-QM will host. First cited in §7.1.

Arisaka, K. (2026). *The Standard Model as Theorem of GG-Theory: Its 28 Parameters as Observations of One Six-Dimensional Geometry.* Arisaka-GG-A9-SM preprint.
Preprint, available at https://preprints.arisaka-gg.org/a9-sm/.
doi:10.5281/zenodo.22064312.
Relevance. This companion paper of the 4D-Physics Trio derives the 28 continuous parameters of the Standard Model — the three gauge couplings, the nine charged-lepton + quark masses, the four CKM angles, the four PMNS angles, the Higgs mass and self-coupling, the QCD vacuum angle, and the cosmological constant — from the OGN-5 integer ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ and the 6D bulk of A2-GG6. In the present paper we cite A9-SM at every invocation of the electroweak vev derivation $v = M_{\text{lock}}(2/L)^N$ (T8.7), the 28-parameter audit (T9.1–T9.9), and the SM-derivation row of the cross-paper coherence map. First cited in §1.

Arisaka, K. (2026). *The Standard Cosmology as Theorem of GG-Theory: The Cosmic Inventory as Observations of One Six-Dimensional Geometry.* Arisaka-GG-A10-Cosmos preprint.
Preprint, available at https://preprints.arisaka-gg.org/a10-cosmos/.
doi:10.5281/zenodo.22064314.
Relevance. This companion paper derives the cosmological partitions $\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$, $\Omega_b = 1/(2\pi^2)$, $\Omega_{\text{DM}} = 1/\pi - 1/(2\pi^2)$, the lock-scale-seesaw cosmological-constant prediction $m_\Lambda^{\text{geom}} = M_{\text{lock}}^2/M_{\text{Pl}} \approx 6$ meV, and the kaxion line at 8.1 μeV from the 6D bulk and the OGN-5 integer ledger. In the present paper we cite A10-Cosmos for the cosmological background of T8.5 (cosmological-constant lock-scale seesaw), T8.7 (gauge hierarchy), and the genuinely-open Open Problems O5 (cosmological perturbation theory) and O6 (closed-form ζ_Λ derivation). A10-Cosmos also carries the bounded-cost selection of the seesaw residue $\zeta_\Lambda = \pi/(LK) = \pi/175$, made with no observational input and pre-registered as a two-sided target; the Wilson-line zero-mode computation that would promote it from Derived (cond.) to a theorem is assigned there to Open Problem O6 of the present paper. First cited in §1.

Arisaka, K. (2026). *The Geometric Inevitability of Cosmos, Life, and Mind: The Self-Referential Closure of the GG-Theory Program.* Arisaka-GG-A11-Origin.
Preprint, available at https://preprints.arisaka-gg.org/a11-origin/.
doi:10.5281/zenodo.22064319. No result of the present paper depends on it.
Relevance. The capstone of the program. Defines the Ground Geometric Attractor (GGA) and proves T-GGA-as-Ground-State — the ground-state refinement of the GRIP terminus cited throughout the present revision (T10 bridge, the three-layer projection survey, and the universal-identity forward-list) — and runs the eighteen-layer cascade for which GDOS supplies the runtime law. First cited in §7.1.

Arisaka, K. (2026). *The GG-6 Lock Cluster: A Non-Spatial Kaluza–Klein Resonance near 3.85 TeV as the Standing LHC Falsifier of GG-Theory.* Arisaka-GG-P1-KK preprint.
Preprint, available at https://preprints.arisaka-gg.org/p1-kk/.
doi:10.5281/zenodo.22064321.

Relevance. This Predictions-Branch companion paper of the GG-Theory program works out the Kaluza–Klein lock-cluster phenomenology at $M_{\text{lock}} = (1/2)\, M_{\text{Pl}}\, e^{-K} \approx 3.85$ TeV with $K = 35$ from the OGN-5 ledger. In the present paper we cite P1-KK at the empirical-prediction sites of T8.7 (gauge hierarchy), the LHC falsifier, and the cross-paper coherence-map row that names KK-cluster predictions. First cited in §7.1.

Arisaka, K. (2026). *Why Three Generations: A Three-Leg Derivation of $N_g = 3$ from BI-6, MSMF, and GDOS Morse Stability.* Arisaka-GG-P2-N3 preprint.
Preprint, available at https://preprints.arisaka-gg.org/p2-n3/.
doi:10.5281/zenodo.22064323.
Relevance. Predictions Branch: the three-leg derivation of three generations ($N_g = 3$), the integer that anchors the OGN-5 cascade on which the present paper's anomaly and reduction theorems rely. First cited in §7.1.

Arisaka, K. (2026). *The Electron as the Ground State of Nature: The Six Oldest Questions about the Electron, Closed and Fenced within GG-Theory.* Arisaka-GG-P3-Electron preprint.
Preprint, available at https://preprints.arisaka-gg.org/p3-electron/.
doi:10.5281/zenodo.22064325.
Relevance. The companion electron paper deploys the two-sector discipline of Section 59 on the single electron: the gauge-sector curvature bends the track without descent, radiation is motion's dissipative channel, and the scale-sector GRIP-In descent governs identity rather than transport. We cite it at the close of the radiative section as the electron-side reading of the same T5 sector assignment, with the synchrotron as the shared worked example. First cited in §7.1.

Arisaka, K. (2026). *The Real Geometry Beneath the Complex Numbers of Physics: Positive Frequency is Holomorphy — from Cardano's Cubics to Twistor Space, via the Sixth Dimension of GG-Theory.* Arisaka-GG-P4-Real preprint.
Preprint, available at https://preprints.arisaka-gg.org/p4-real/.
doi:10.5281/zenodo.22064327.
Relevance. Foundations Branch: the origin-of-i member paper, deriving the single imaginary unit of boundary physics from the two real structures of the GG-A2-GG6 arena (the compact phase circle and the causal clock). For the present paper it is the standalone treatment of the question that the GNS scope note of §20 flags as distinct from "why a Hilbert space" and now consumes by import — why the boundary Hilbert space that GDOS hands the observer is complex; cited at roster and consumption level (§21, §95, and the response to Objection 57). First cited in §7.1.

Arisaka, K. (2026). *The Mirrors of Matter: Why Every Particle Has a Twin, Why Nature's Mirrors Break, and Why the Universe Is Made of Matter.* Arisaka-GG-P5-CPT preprint.
Preprint, available at https://preprints.arisaka-gg.org/p5-cpt/.
doi:10.5281/zenodo.22064329.
Relevance. Foundations Branch: the CPT member paper, reading C, P, and T as orientation reversals of the three real bulk factors and carrying the matter–antimatter door; its law-level treatment of irreversibility is downstream kin of the AX-3/OGGEP arrow that the present paper implements dynamically; cited at roster level. First cited in §7.1.

Arisaka, K. (2026). *The Proton as the Ground Geometric Attractor of Matter: The Eleven Questions of the Proton, Closed and Fenced within GG-Theory*. Arisaka-GG-P6-Proton preprint. Preprint, available at https://preprints.arisaka-gg.org/p6-proton/. doi:10.5281/zenodo.22064331.

Relevance. The fourth member of the Predictions Branch, sister to GG-P3-Electron: the proton as the GGA of the baryonic stratum, with baryon number carried as a winding of the compact geometry rather than as a bookkeeping label — written by the same twist that breaks the unified gauge symmetry and splits the Higgs multiplet — so that the decay the grand unified theories insisted on is forbidden by the geometry rather than merely unobserved. Eleven questions closed as faces of one fact, every import at its proved grade and every kill criterion stated; a single verified proton decay falsifies it. First cited in §7.1.

Arisaka, K. (2026). *The Kaxion: Cold Dark Matter as the Non-Spatial Compact-Angle Pseudoscalar of GG-6*. Arisaka-GG-C1-Kaxion preprint. Preprint, available at https://preprints.arisaka-gg.org/c1-kaxion/. doi:10.5281/zenodo.22064333.

Relevance. This Predictions-Branch companion paper derives the kaxion line at 8.1 μeV (1.95 GHz) as a constitutionally locked dark-matter signature of the 6D bulk's S^1_θ fiber, with the kaxion mass and decay constant pinned by the OGN-5 integer ledger. In the present paper we cite C1-Kaxion at the kaxion-prediction sites of T9 and T11, and at the cross-paper coherence-map row that names kaxion dark-matter predictions. First cited in §7.1.

Arisaka, K. (2026). *The Hubble Tension in the Light of GG-Theory: The Hubble Constant as One Reading of a Derived Expansion History $H(t)$*. Arisaka-GG-C2-Hubble preprint. Preprint, available at https://preprints.arisaka-gg.org/c2-hubble/. doi:10.5281/zenodo.22064336.

Relevance. Cosmology & Astrophysics Branch: the Hubble tension read as the two ends of one six-dimensional geometric corridor, a boundary-projection consequence of the same GDOS engine. First cited in §7.1.

Arisaka, K. (2026). *Black-Hole Entropy under GG-Theory: The GDOS Reading of the Horizon as an OGGEP Port — with the Exact Bekenstein–Hawking $A/4G_N$ Derived from Induced Gravity*. Arisaka-GG-C3-BH preprint. Preprint, available at https://preprints.arisaka-gg.org/c3-bh/. doi:10.5281/zenodo.22064338.

Relevance. This Cosmos-Branch companion paper is the dedicated, full-length treatment of black holes in the cohort. Its centerpiece is the induced-gravity derivation of the exact Bekenstein–Hawking coefficient $1/4 = 4\pi/16\pi$ for the generic (non-supersymmetric) black hole, with the microstates identified as the hidden-fiber S^1_θ holonomies traced at the horizon; it also carries the full information-paradox / Page-curve / firewall analysis and a neutral scorecard against the string-theoretic approaches. In the present paper we cite C3-BH wherever the black-hole horizon is treated as the geometrically-visible OGGEP port (§71.1 ff.), deferring the rigorous entropy-coefficient derivation to it. First cited in §7.1.

Arisaka, K. (2026). *Symmetry, Conservation, and Geometry: Seven Steps from Noether, Bianchi, to Hopkins–Singer, and the Ledger the Open-System Turn Pays Out*. Arisaka-GG-W1-Symmetry preprint.
Preprint, available at https://preprints.arisaka-gg.org/w1-symmetry/.
doi:10.5281/zenodo.22064340.
Relevance. The first of the two white papers, and the *diachronic* one: it walks the century of symmetry and conservation in the order it was found and shows the conviction that symmetry causes conservation loosening at seven historical steps, until what remains conserved is an integer with nowhere to move — then places the program at the arc's end, at declared grades, with its payout set beside the measurements. Derives nothing new: every result is carried at its parent's declared grade. First cited in §7.1.

Arisaka, K. (2026). *One Functional, Three Principles: Maximum Entropy Production, Minimum Entropy Production, and the Free Energy Principle as Boundary Gauges of GDOS*. Arisaka-GG-S1-EP preprint.
Preprint, available at https://preprints.arisaka-gg.org/s1-ep/.
doi:10.5281/zenodo.22190004.
Relevance. The first supplement to the present paper. It reads the boundary functional of T5 three ways — as a trajectory principle, as a current principle with maximum and minimum entropy production as dual conventions, and as an inferential principle — and shows the three to be gauges of one functional rather than rival laws. Cited where the Lyapunov lemma's level is stated. First cited in §14.

Arisaka, K. (2026). *The Circle That Remembers, the Interval That Forgets: GG-Theory and the Sixteen Problems of Particle Physics and Cosmology*. Arisaka-GG-W2-Holonomy preprint.
Preprint, available at https://preprints.arisaka-gg.org/w2-holonomy/.
doi:10.5281/zenodo.22064342.
Relevance. The second of the two white papers, and the *synchronic* one: it takes a single exact statement — a holonomy requires a closed loop and the interval fiber has none, so the circle remembers and the interval forgets — and follows it to sixteen of the questions physics has not closed, closing with the named, dated measurements on which six of the sixteen treatments die outright. Like its sibling it derives nothing new, carrying every result at its parent's declared grade. First cited in §7.1.

B. Other references

Abbott, B. P. et al. (2017). GW170817: Observation of gravitational waves from a binary neutron star inspiral. *Physical Review Letters*, *119*, 161101.
Relevance. The first multi-messenger detection of a binary neutron-star merger gave the most stringent constraints to date on the neutron-star mass-radius relation and on the tidal deformability parameter Lambda. We cite it in Theorem T8.6 as the empirical baseline against which the

GG-Theory modified TOV prediction must compete; current sensitivity is at the 10^-2 M_sun level, well above the 10^-8 M_sun GS+KK shift predicted by the GG-Theory framework. First cited in §106.

Almheiri, A., Mahajan, R., Maldacena, J., & Zhao, Y. (2020). The Page curve of Hawking radiation from semiclassical geometry. *Journal of High Energy Physics*, *2020*(3), 149.
Relevance. Derives the Page curve of an evaporating black hole from the semiclassical gravitational path integral, with a finite-area "island" region of the bulk contributing to the boundary entanglement entropy. We cite it, with Penington et al. (2020), as the bulk-side geometric counterpart of the boundary partial trace of OGGEP: Part VIII identifies the islands formula as the bulk image of the GDOS coarse-graining, the two descriptions agreeing as bulk and boundary views of one constitutional move. First cited in §71.4.

Alvarez-Gaumé, L., & Witten, E. (1984). Gravitational anomalies. *Nuclear Physics B*, *234*(2), 269–330.
Relevance. Establishes the modern classification of gravitational and mixed anomalies in quantum field theory. It provides the historical and technical backdrop for POS-3 (anomaly freedom) and for the claim that consistency can force specific matter content and inflow structures. In the present constitution, "anomaly-free" is treated as an admissibility filter, not an optional aesthetic criterion. First cited in §49.

Ambjorn, J., Jurkiewicz, J., & Loll, R. (2004). Emergence of a 4D world from causal quantum gravity. *Physical Review Letters*, *93*, 131301.
Relevance. The causal-dynamical-triangulations program of Ambjørn, Jurkiewicz, and Loll discretizes spacetime into 4-simplices and computes a sum over geometries with a causal ordering, demonstrating that 4D macroscopic spacetime can emerge from a sum over discrete bulk configurations without continuum input. Their result is a constructive existence proof for non-perturbative quantum gravity. We cite it in Part IV of the present paper as one of the principal alternative quantum-gravity programs against which the GG-Theory framework is contrasted: CDT discretizes and quantizes gravity directly, whereas GG-Theory geometrizes gauge forces in a continuum 6D bulk. First cited in §31.1.

Aoyama, T., Hayakawa, M., Kinoshita, T., & Nio, M. (2012). Complete tenth-order QED contribution to the muon $g-2$. *Physical Review Letters*, *109*, 111808.
Relevance. The complete five-loop (tenth-order) QED evaluation of the muon anomalous magnetic moment, assembling the contribution of 12,672 Feynman diagrams. We cite it in Theorem T9.9 as the canonical measure of the labor residing in the scale-invariant finite coefficients of the perturbative series — the part of a_μ that the radial geometry, organizing only the scale direction, provably cannot simplify. First cited in §52.

Aoyama, T. et al. (2020). The anomalous magnetic moment of the muon in the Standard Model. *Physics Reports*, *887*, 1–166.
Relevance. The Muon $g-2$ Theory Initiative's consensus compilation of the Standard-Model prediction, including the data-driven hadronic vacuum-polarization and hadronic light-by-light

contributions and their uncertainties. We cite it in Theorem T9.9 for the magnitudes and the required precision of the hadronic terms, the Tier-4 dynamical correlators that GG-6 can frame structurally but not predict to sub-percent accuracy. First cited in §52.4.

Aliberti, R. et al. (2025). The anomalous magnetic moment of the muon in the Standard Model: an update. *Physics Reports*, *1143*, 1–158.
Relevance. The Muon $g-2$ Theory Initiative's 2025 revision of the Standard-Model prediction, which adopts the lattice determination of the leading hadronic vacuum polarization and declines to quote a data-driven value in its place. We cite it in Theorem T9.9 for the present status of the Tier-4 correlators: the resolution of a decade-old tension by choosing between two first-principles methods, rather than by any structural insight, is the sharpest illustration available that the hadronic term is irreducible dynamics. First cited in §52.4.

Ashtekar, A. (1986). New variables for classical and quantum gravity. *Physical Review Letters*, *57*(18), 2244–2247.
Relevance. Ashtekar's 1986 reformulation of canonical general relativity in terms of self-dual SL(2,$\mathbb{C}$) connection variables made the constraint algebra of canonical quantum gravity tractable for the first time and launched the Loop Quantum Gravity program. Loop quantum gravity quantizes gravity directly via holonomies of the Ashtekar connection, predicting discrete spectra of geometric operators (areas, volumes). We cite it in Part IV of the present paper as a principal alternative quantum-gravity program; LQG quantizes gravity but does not naturally include the gauge sector, in contrast to the GG-Theory geometrization of both sectors via GGEP. First cited in §97.2.

Aspect, A., Dalibard, J., & Roger, G. (1982). Experimental test of Bell's inequalities using time-varying analyzers. *Physical Review Letters*, *49*, 1804–1807.
Relevance. The Aspect experiment was the first decisive laboratory verification of CHSH violation with time-varying measurement settings, ruling out local hidden-variable theories with specifiable causal sufficiency. We cite it in Theorem T7.7 as the experimental measurement of the boundary shadow of the bulk timelike connections in the GG-Theory framework. First cited in §26.

C. Becchi, A. Rouet, & R. Stora (1976). Renormalization of gauge theories. *Annals Phys.*, *98*, 287.
Relevance. Foundational paper on BRST symmetry; the BRST algebra and its off-shell closure are central to the gauge consistency of the GG-6 bulk QFT. First cited in §A.

Bell, J. S (1964). On the Einstein Podolsky Rosen paradox. *Physics Physique Fizika*, *1*(3), 195–200.
Relevance. Bell's theorem clarifies that the measurement problem is not merely philosophical: operational predictions about correlations constrain the kinds of "hidden variable" completions that remain possible. This motivates treating measurement as a constitutional issue rather than an interpretational afterthought. First cited in §16.

Bohr, N. (1928). The quantum postulate and the recent development of atomic theory. *Nature*, *121*, 580–590.

Relevance. Bohr's 1928 paper is the canonical statement of the Copenhagen interpretation, which resolves the von Neumann measurement problem by fiat: the observer is classical, the system is quantum, and the Heisenberg cut is movable to wherever the experiment requires. We cite it in Part I as the pragmatic but philosophically unsatisfactory move that has dominated working quantum mechanics for nearly a century, and which GDOS now renders unnecessary by elevating the open-system CPTP channel to constitutional status. Under OGGEP, no Heisenberg cut needs to be drawn anywhere because the constitutional law is already a measurement law. First cited in §6.1.

Boltzmann, L. (1872). Weitere Studien über das Wärmegleichgewicht unter Gasmolekülen. *Sitzungsberichte der Akademie der Wissenschaften, Wien*, *66*, 275–370.
Relevance. Boltzmann's 1872 paper introduces the H-theorem and the Stosszahlansatz (molecular-chaos assumption), which together produce the second law of thermodynamics from microscopic time-reversible Hamiltonian dynamics. We cite it in Part I of the present paper as the first historical instance in which the closed-Hamiltonian framework had to be supplemented by an implicit open-system ingredient (the partial trace over correlations) in order to recover macroscopic empirical laws. Boltzmann's wrestle with closure is the first historical crack in the closed-system paradigm that GDOS now resolves constitutionally. First cited in §6.1.

Bianchi, L. (1902). Sui simboli a quattro indici e sulla curvatura di Riemann. *Rendiconti della R. Accademia dei Lincei*, *11*, 3–7.
Relevance. Luigi Bianchi's 1902 paper proves the Riemann-tensor identity $\nabla_{[\mu}R_{\nu\rho]\sigma\tau} = 0$ that bears his name, generalizing Maxwell's homogeneous equations $dF = 0$ to the gravitational case and supplying the geometric ancestor of every modified-Bianchi identity in modern physics. In A5-GDOS we credit Bianchi 1902 as the founding moment of the geometric (non-quantum) reading of anomaly freedom: the modified BI-6 identity $dH_3 = X_4$ that drives anomaly inflow at the bulk level is purely geometric, in the lineage of $dF = 0$ and $\nabla_{[\mu}R_{\nu\rho]\sigma\tau} = 0$ rather than of any quantum effect. See A3-BI6, its GS-as-GGEP completion subsection, for the full supportive treatment of this geometric reading. First cited in §1.

Born, M (1926). Zur Quantenmechanik der Stoßvorgänge. *Zeitschrift für Physik*, *37*(12), 863–867.
Relevance. Born's probabilistic interpretation underlies the Born rule, which is central to the modern measurement postulate in quantum mechanics. It is cited here to emphasize that probability entered QM as a foundational rule rather than as a derived dynamical theorem, motivating the GUT-10 demand that measurement semantics be constitutional rather than ad hoc. First cited in §1.

Bergmann, P. G. (1949). Non-linear field theories. *Physical Review*, *75*, 680–685.
Relevance. Peter Bergmann's 1949 paper, written with Einstein at the IAS, is the founding mathematical treatment of constrained Hamiltonian dynamics for general relativity — the first systematic attempt to canonical-quantize a non-linear field theory with gauge constraints. Bergmann's framework was later generalized by Dirac (1958, 1959) into the standard constrained-quantization procedure that DeWitt and Wheeler applied to GR in the 1960s. We cite Bergmann 1949 in the historical positioning of T8 (Part IV, section 6.3 first-half-century-of-attempts) as the

genuine original of the canonical-quantum-gravity program that A5-GDOS now supersedes via the operational OGGEP route. First cited in §31.1.

Borsanyi, S. et al. (2021). Leading hadronic contribution to the muon magnetic moment from lattice QCD. *Nature*, *593*, 51–55.
Relevance. The Budapest–Marseille–Wuppertal lattice-QCD determination of the leading hadronic vacuum-polarization contribution to the muon $g-2$, in tension with the data-driven dispersive value. We cite it in Theorem T9.9 as evidence that the hadronic correlator is a genuinely hard non-perturbative problem with no geometric shortcut: that two first-principles determinations disagree is precisely what one expects of irreducible strong-coupling dynamics. First cited in §52.4.

Breuer, H.-P., & Petruccione, F (2002). The Theory of Open Quantum Systems. Oxford: Oxford University Press.
Relevance. This standard reference develops the mathematics of open quantum systems (reduced dynamics, master equations, Markovian limits). It anchors our claim that openness is not optional in realistic modeling, and it provides the mature language (CPTP maps and generators) that the GUT-10 constitution intends to elevate to first-class status. First cited in §8.

Caldeira, A. O., & Leggett, A. J. (1983). Path integral approach to quantum Brownian motion. *Physica A*, *121*(3), 587–616.
Relevance. Caldeira and Leggett's 1983 paper is the canonical worked-out model of an open quantum system: a particle linearly coupled to a bath of harmonic oscillators, with the bath integrated out via the Feynman–Vernon influence functional to produce a non-Markovian master equation that reduces to the Lindblad form in the high-temperature Ohmic limit. We cite it in Theorem T4 (Part II) and Appendix C as the model the GG-Theory bulk–boundary projection generalizes: the GG-6 bulk plays the role of Caldeira–Leggett's bath, and the boundary CPTP channel plays the role of their reduced dynamics, with the scale profile supplying the structural reason the high-temperature Markovian limit is the natural one. First cited in §59.5.

Carroll, S. M. (2004). Spacetime and Geometry: An Introduction to General Relativity. San Francisco: Addison-Wesley.
Relevance. The canonical first-year graduate general-relativity textbook: manifolds, curvature, and the Einstein–Hilbert action taken as the variational starting point. Paired with Part IV in its *Read alongside* line (and with the curvature appendix) so that the student can compare the postulated action of the standard curriculum with its derivation as the metric zero-mode reduction of T8.1. First cited in the Read-alongside line of Part IV.

Choi, M.-D (1975). Completely positive linear maps on complex matrices. *Linear Algebra and its Applications*, *10*(3), 285–290.
Relevance. Choi's 1975 theorem establishes the Choi–Jamiołkowski isomorphism: a linear map $\Phi : \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ is completely positive if and only if its associated Choi matrix is positive semidefinite. Combined with Stinespring's dilation theorem, this gives a complete characterization of all CPTP maps in arbitrary finite Hilbert space. We cite it in Theorem T3 (Part II) of the

present paper for the explicit construction of the Stinespring isometry from a given CPTP map via the Choi-matrix spectral decomposition, which underpins the Kraus-decomposition representation used throughout Part III. First cited in §1.

Chruściński, D., & Kossakowski, A (2009). On the structure of entropy-producing quantum dynamical semigroups. *Open Systems and Information Dynamics*, *16*, 259–272.
Relevance. Provides the GKLS-Riemannian structure on the boundary state manifold that is needed to convert the GKLS master equation into a covariant continuity equation in Theorem T5. Cited in Part II as the technical bridge between the operator-level GKLS formulation and the mesoscopic continuity formulation of GDOS. First cited in §14.

Cohen-Tannoudji, C., Dupont-Roc, J., & Grynberg, G. (1992). Atom-Photon Interactions: Basic Processes and Applications. New York: Wiley.
Relevance. Cohen-Tannoudji, Dupont-Roc, and Grynberg's textbook is the canonical pedagogical reference for the Bloch-Redfield-to-Lindblad reduction of an open quantum system in the weak-coupling regime, including the explicit secular-approximation calculation that we use in the proof of Theorem T4. Cited at the secular-approximation step of T4 in Part II. First cited in §13.

Planck Collaboration (2018). Planck 2018 results. VI. Cosmological parameters. *Astronomy & Astrophysics*, *641*, A6.
Relevance. Precision cosmology strongly constrains the expansion history and composition of the Universe. We cite Planck as representative of the empirical pressure behind the inflation/dark-sector puzzles, and as motivation for a constitution that connects cosmological claims to operational observables with falsifiers. First cited in §1.

Connes, A. (1996). Gravity coupled with matter and the foundation of non-commutative geometry. *Communications in Mathematical Physics*, *182*, 155–176.
Relevance. Connes' non-commutative-geometry program treats spacetime as a product of an ordinary 4-manifold with a discrete non-commutative algebra, producing a geometric derivation of the Higgs sector. We cite it in Parts IV-V as one of the principal alternative unification programs that the GG-Theory framework supersedes via its constitutional derivation from the 6D bulk. First cited in §31.1.

Davies, E. B (1974). Markovian master equations. *Communications in Mathematical Physics*, *39*, 91–110.
Relevance. Davies' 1974 paper provides the rigorous mathematical formulation of the weak-coupling (van Hove) limit for an open quantum system, deriving the Markovian master equation as the strict $\lambda^2 t \to \tau$ limit of the unitary system–bath dynamics. The Davies derivation is one of the two complementary foundational results of the modern theory of open quantum systems (the other being Lindblad's). We cite it in Theorem T4 (Part II) and Appendix C of the present paper as the rigorous justification of the Born–Markov procedure that produces the GKLS form of the boundary law GDOS. First cited in §8.

Deser, S., & Zumino, B. (1976). Consistent supergravity. *Physics Letters B*, *62*, 335–337.
Relevance. Deser and Zumino independently and simultaneously with Freedman, van Nieuwen-

huizen, and Ferrara constructed the first consistent supergravity action in 1976: the local-supersymmetric extension of general relativity, with the gravitino as the gauge field of local supersymmetry. The two papers (FvNF and Deser-Zumino) appeared within weeks of each other in 1976 and together founded the supergravity program. We cite Deser-Zumino 1976 in the historical positioning of T8 (Part IV, section 6.3) as the co-original of supergravity, alongside FvNF 1976. First cited in §31.1.

DeWitt, B. S. (1967). Quantum theory of gravity. I. The canonical theory. *Physical Review*, *160*, 1113–1148.
Relevance. DeWitt's 1967 paper, with Wheeler's contemporaneous work, founded the canonical quantization program for gravity. We cite it in Part IV as the historical entry point for the half-century of attempts to quantize GR — the program that 't Hooft and Veltman 1972 showed could not work perturbatively. First cited in §18.2.

Dirac, P. A. M. (1930). *The Principles of Quantum Mechanics*. Oxford University Press.
Relevance. A classic axiomatic presentation of quantum theory. We cite it to underscore that "axioms" in quantum mechanics have historically mixed dynamics with measurement rules, motivating a more explicit separation between axioms and postulates. First cited in §107.3.

Dirac, P. A. M. (1938). Classical theory of radiating electrons. *Proceedings of the Royal Society of London A*, *167*, 148–169.
Relevance. The culmination of the classical radiation-reaction program of Abraham and Lorentz: a manifestly relativistic self-force equation for the point electron, purchased at the price of a third-derivative term with runaway and pre-accelerating solutions. We cite it in Section 59 as the closed-system formulation that the GDOS reading dissolves — in the T5 decomposition, radiation damping is the gradient sector of an open-system current, not a local force term, and the 1938 pathologies are artifacts of forcing an open-system descent into a closed-system equation of motion. First cited in §59.3.

A. Einstein (1905). Zur Elektrodynamik bewegter Körper. *Ann. Phys.*, *17*, 891.
Relevance. Einstein's operational reconstruction of simultaneity and inertial frames exemplifies a successful "axiom-like" intervention in physics: changing the admissibility rules for coordinates and clocks reshaped all of kinematics. We cite it to justify treating operational definitions (OGGEP) as constitutional. First cited in §1.

A. Einstein (1915). Die Feldgleichungen der Gravitation.
Relevance. Einstein's 1915 paper introduced the field equations of general relativity, $G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$, identifying gravity with the curvature of spacetime as a dynamical field obeying a variational principle. This is the foundational document of modern gravitational theory and the historical starting point for every attempt to unify gravity with the other forces. We cite it throughout Part IV (4D General Relativity from the 6D Bulk) and in the Conclusions (Part X) as the historical context against which the GG-Theory geometrization program is positioned. First cited in §1.

A. Einstein (1918). Über Gravitationswellen. *Sitzungsberichte der Königlich Preußischen Akademie*

der Wissenschaften (Berlin), 154–167.

Relevance. Einstein's corrected derivation of the quadrupole formula: the energy flux of gravitational waves from a slowly varying source, with the third time derivative of the mass quadrupole as the radiating moment. We cite it in Section 59 as the metric-sector twin of Larmor's formula — the leading multipole differs (quadrupole, not dipole) precisely because the conserved current of the metric sector is $T_{\mu\nu}$ rather than J^μ, the Ward-identity bookkeeping spelled out in the radiative dictionary column. First cited in §59.2.

Erlich, J., Katz, E., Son, D. T., & Stephanov, M. A. (2005). QCD and a holographic model of hadrons. *Physical Review Letters*, *95*, 261602.

Relevance. The founding bottom-up AdS/QCD construction: a five-dimensional holographic model reproducing the qualitative meson spectrum and current correlators of QCD from a weakly curved bulk. We cite it in Theorem T9.9 as the prototype for what a holographic geometrization — GG-6's radial dimension included — can deliver in the strong-coupling regime: structural and order-of-magnitude results, controlled only in the large-N_c, large-λ limit that real QCD does not occupy. First cited in §52.4.

Feynman, R. P. (1948). Space-time approach to non-relativistic quantum mechanics. *Reviews of Modern Physics*, *20*(2), 367–387.

Relevance. Introduces the path-integral formulation of quantum mechanics. We cite it in the historical chain on the implicit openness of Maxwell-style field theories alongside the Wheeler–Feynman absorber theory (Wheeler, 1968), where the action-at-a-distance reading of radiation reaction made manifest that even classical electromagnetism is incurably open once the global structure of the radiation field is taken seriously. First cited in §54.2.

Feynman, R. P., & Vernon, F. L. (1963). The theory of a general quantum system interacting with a linear dissipative system. *Annals of Physics*, *24*, 118–173.

Relevance. Feynman and Vernon's 1963 paper introduces the influence functional — the path-integral object obtained by integrating out a linear bath, encoding the back-reaction of the environment on the system in a single non-local-in-time kernel. This is the technical tool that Caldeira–Leggett 1983 later applied to quantum Brownian motion, and it is the conceptual ancestor of the Nakajima–Zwanzig projection. We cite it in Theorem T2 (Part II) as one of the founding mathematical objects of the open-system formalism that the OGGEP bulk-to-boundary projection generalizes to the bulk–boundary geometric setting. First cited in §59.5.

Ford, G. W., & O'Connell, R. F. (1991). Radiation reaction in electrodynamics and the elimination of runaway solutions. *Physics Letters A*, *157*(4–5), 217–220.

Relevance. The demonstration that a physical high-frequency form factor converts the pathological Abraham–Lorentz–Dirac dynamics into a second-order, causal, runaway-free equation — the passivity logic that §59.6 deploys. In the GG-6 setting the form factor is not chosen but derived (the T9.1 boundary spectrum), and the classical negative-bare-mass obstruction is removed by the scale-sector mass doctrine rather than by tuning the cutoff. First cited in §59.6.

Freedman, D. Z., van Nieuwenhuizen, P., & Ferrara, S. (1976). Progress toward a theory

of supergravity. *Physical Review D*, *13*, 3214–3218.

Relevance. The 1976 paper by Freedman, van Nieuwenhuizen, and Ferrara introduced supergravity, the supersymmetric extension of general relativity in which the metric tensor is paired with a spin-3/2 gravitino field and the resulting theory has improved ultraviolet behavior over pure GR. Maximal $N = 8$ supergravity is finite up to seven loops. We cite it in Part IV of the present paper as one of the principal historical attempts at perturbative quantum gravity and as the precursor to the string-theoretic program. The GG-Theory framework abandons supersymmetry entirely and instead geometrizes the gauge sector to achieve the GR/SM unification. First cited in §31.1.

Friston, K (2010). The free-energy principle: a unified brain theory?. *Nature Reviews Neuroscience*, *11*(2), 127–138.

Relevance. Friston's free-energy principle is a prominent modern attempt to unify perception, learning, and action by variational optimization. We cite it in Theorem T5 (the GDOS continuity equation in mesoscopic form) of the present paper as the canonical open-system variational reading against which GFF of A6-GRIP is positioned, and as the historical precedent for treating "free-energy-like" scalar functionals as organizing principles — a posture that the GG-Theory program generalizes from the neural setting to a systematic constitutional law with explicit gauge/geometry content and falsifiable predictions. First cited in §14.

Gelfand, I. M., & Naimark, M. A. (1943). On the imbedding of normed rings into the ring of operators on a Hilbert space. *Matematicheskii Sbornik*, *12*, 197–217.

Relevance. Gelfand and Naimark's 1943 paper proves that every C^*-algebra is isometrically *-isomorphic to a closed *-subalgebra of bounded operators on a Hilbert space. Combined with the GNS construction, this is the mathematical engine that produces the Hilbert space of QM from the operational C^*-algebraic input. Cited at the opening of T7.1 (Hilbert-Space Postulate as Theorem) in Part III; the complex scalars of the resulting Hilbert space are the algebra's own, inherited rather than produced, their origin being examined in Arisaka (2026, A8-QM). First cited in §20.

H. Georgi, & S. Glashow (1974). Unity of all elementary-particle forces. *Phys. Rev. Lett.*, *32*, 438.

Relevance. The SU(5) grand-unification proposal is a canonical example of symmetry-driven unification beyond the Standard Model. GUT references it historically, but differs constitutionally by treating gauge–gravity unification through a composite geometric connection (GGEP) and by enforcing anomaly/normalization constraints as admissibility filters. First cited in §42.1.

Georgi, H., & Manohar, A. (1984). Chiral quarks and the non-relativistic quark model. *Nuclear Physics B*, *234*, 189–212.

Relevance. Georgi and Manohar's 1984 paper established the modern foundation of effective field theory (EFT) power counting, including the convergence and decoupling theorems for heavy degrees of freedom integrated out at low energies. Their analysis is the canonical reference for the Wilsonian heavy-mode-integration procedure used throughout Part V of the present paper. We cite it in Theorem T9.2 (Lemma 9.3) as the technical justification for the convergence of the heavy-mode expansion in the GG-Theory derivation of the effective 4D Standard Model action

from the 6D bulk. First cited in §117.

Gherghetta, T., & Pomarol, A. (2000). Bulk fields and supersymmetry in a slice of AdS. *Nuclear Physics B*, *586*, 141–162.

Relevance. Standard reference for the KK reduction of bulk fields on the warped Randall-Sundrum interval, including boundary conditions on radial profiles f_n(r). We cite it in Theorem T9.1 (Lemma 9.1) for the boundary-condition machinery used to extract the chirality-projected fermion zero modes on the GG-6 bulk. First cited in §114.

Giustina, M., Versteegh, M. A. M., Wengerowsky, S., Handsteiner, J., Hochrainer, A., Phelan, K., Steinlechner, F., Kofler, J., Larsson, J.-A., Abellan, C., Amaya, W., Pruneri, V., Mitchell, M. W., Beyer, J., Gerrits, T., Lita, A. E., Shalm, L. K., Nam, S. W., Scheidl, T., Ursin, R., Wittmann, B., & Zeilinger, A. (2015). Significant-loophole-free test of Bell's theorem with entangled photons. *Physical Review Letters*, *115*(25), 250401.

Relevance. Giustina et al. 2015 (the Vienna loophole-free Bell test, performed in parallel with the Delft and NIST experiments) closed the detection and locality loopholes simultaneously using entangled photon pairs. Cited together with Hensen et al. (2015) at the point of use in T7.7 (Bell-inequality violation as boundary shadow) as the experimental closure of the locality crisis identified by Bell (1964) and first probed by Aspect (1982). First cited in §26.

Gleason, A. M (1957). Measures on the closed subspaces of a Hilbert space. *Journal of Mathematics and Mechanics*, *6*, 885–893.

Relevance. Gleason's theorem derives the Born rule from the requirement that probability measures on the closed subspaces of a Hilbert space (of dimension >= 3) be sigma-additive. We cite it in Part I as the precedent for "Born rule as theorem rather than axiom" — in the GG-Theory framework the Born rule emerges from CPTP-completion of the OGGEP channel via Stinespring dilation, which is structurally a stronger constitutional derivation than Gleason's representation-theoretic argument. First cited in §6.3.

Gorini, V., Kossakowski, A., & Sudarshan, E. C. G (1976). Journal of Mathematical Physics.

Relevance. The 1976 paper by Gorini, Kossakowski, and Sudarshan derived the canonical form of the generator of any one-parameter semigroup of CPTP maps on $\mathcal{B}(\mathcal{H})$ in finite dimension, establishing what is now called the GKLS (Gorini–Kossakowski–Lindblad–Sudarshan) master equation. Together with Lindblad's contemporaneous paper, this is one of the two foundational results of the modern theory of open quantum systems. We cite it in Theorem T4 (Part II) and Appendix C of the present paper as the rigorous structural characterization of the GDOS boundary law in the Markovian regime. First cited in §1.

M. Green, & J. Schwarz (1984). Anomaly cancellations in supersymmetric D=10 gauge theory and superstring theory. *Phys. Lett. B*, *149*, 117.

Relevance. The 1984 Green–Schwarz paper proved that 10-dimensional anomalies in superstring theory cancel for the gauge groups SO(32) and $E_8 \times E_8$, launching the first superstring revolution. The mathematical mechanism — a higher-form B_2 tensor field with modified Bianchi identity

$dH_3 = X_4$ that absorbs the chiral anomaly via inflow — is exactly what GG-Theory adapts to six dimensions as BI-6 (Theorem 4 of GG-6-2026-2). We cite it in Part V of the present paper as the historical origin of the Green–Schwarz mechanism, with explicit acknowledgment that the technical machinery is shared while the constitutional principles differ (10D vs. 6D, SUSY vs. non-SUSY, landscape vs. unique vacuum). First cited in §108.

Haag, R (1992). Local Quantum Physics: Fields, Particles, Algebras. Berlin: Springer.
Relevance. Haag's 1992 monograph 'Local Quantum Physics: Fields, Particles, Algebras' is the canonical reference for the algebraic formulation of quantum field theory, including the Haag–Kastler axioms, the Reeh–Schlieder theorem, the GNS construction, and the modular structure of local algebras. We cite it throughout Part II of the present paper for the GNS construction that defines the boundary Hilbert space, and in Part III for the algebraic background underlying the OGGEP-derived boundary algebra. The algebraic-QFT framework supplies the rigorous mathematical setting in which the GG-Theory boundary law GDOS is formulated. First cited in §10.

Hamilton, W. R (1834). On a general method in dynamics. *Philosophical Transactions of the Royal Society of London*, *124*, 247–308.
Relevance. Hamilton's 1834 reformulation of classical mechanics in terms of canonical coordinates, momenta, and the Hamiltonian function generated the symplectic-geometric foundation on which both quantum mechanics (via canonical quantization) and modern Lagrangian-mechanics-based field theories rest. We cite it in Part I of the present paper as the historical precedent for the kind of constitutional reformulation that the GG-Theory framework attempts: a reformulation of the same physical content in a deeper geometric language without changing empirical predictions. First cited in §6.1.

Heaviside, O (1893). Electromagnetic Theory, Volume 1. The Electrician Printing and Publishing Co., London.
Relevance. Heaviside's reformulation of Maxwell's equations from quaternionic to vector form was the constitutional reformulation that made Maxwell's electromagnetism mathematically transparent. We cite it in Part I as the historical precedent for the GDOS reformulation of open-system boundary content as a rigorous derivation. First cited in §6.4.

Heisenberg, W (1925). Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen. *Zeitschrift für Physik*, *33*(1), 879–893.
Relevance. Heisenberg's 1925 paper introduced matrix mechanics, the first complete formulation of quantum theory. Together with Schrö dinger's wave-mechanics paper of 1926 and Dirac's transformation theory, this established the foundational framework of quantum mechanics. We cite it in Part III of the present paper as the historical starting point of the textbook quantum-mechanical formalism and as one of the canonical references for the measurement problem that the GG-Theory framework reorganizes in T7.1–T7.7. First cited in §1.

Heisenberg, W. (1927). Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. *Zeitschrift für Physik*, *43*, 172–198.

Relevance. A primary source for the uncertainty principle and early quantum-measurement ideas. It is cited as part of the historical background for why operational definitions are central to quantum theory. First cited in §6.1.

Henneaux, M., & Teitelboim, C. (1992). Quantization of Gauge Systems. Princeton University Press.

Relevance. Comprehensive textbook on the BRST construction of gauge theories, including the BRST quartet mechanism and the cohomology preservation under integration of BRST-doublet pairs. We cite it in Theorem T9.4 as the standard reference for the BRST cohomology preservation lemma underlying the descent of the 6D BRST operator to the 4D BRST operator. First cited in §23.1.

Hensen, B. et al. (2015). Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres. *Nature*, *526*, 682–686.

Relevance. The Delft loophole-free Bell test of 2015 closed both the locality and detection loopholes simultaneously. We cite it in Theorem T7.7 as the most rigorous laboratory measurement to date of CHSH violation, validating the GG-Theory boundary prediction that nonlocality is the boundary shadow of bulk timelike connections. First cited in §26.

't Hooft, G., & Veltman, M (1972). Regularization and renormalization of gauge fields. *Nuclear Physics B*, *44*(1), 189–213.

Relevance. The renormalizability of non-Abelian gauge theories is a cornerstone of the SM's consistency. We cite it to illustrate a key lesson: deep consistency conditions (not just phenomenological fits) decide what counts as a viable theory. This motivates postulates that function as consistency filters (e.g. anomaly freedom, RG compatibility). First cited in §1.

't Hooft, G. (1976). Symmetry breaking through Bell-Jackiw anomalies. *Physical Review Letters*, *37*, 8–11.

Relevance. The 't Hooft 1976 paper established that baryon number is anomalously broken in the Standard Model by sphaleron processes, making baryon number an accidental rather than a true symmetry. We cite it in Theorem T9.8 as the foil to the GG-Theory prediction: in GG-Theory, U(1)_B is a true gauge symmetry protected by BI-6 inflow, and proton stability becomes an exact theorem rather than an accident of the lowest-dimension operators. First cited in §28.1.

't Hooft, G. (1993). Dimensional reduction in quantum gravity. In *Salamfest 1993* (pp. 284–296), arXiv:gr-qc/9310026.

Relevance. 't Hooft's 1993 Salamfest essay is the original holographic conjecture: in any consistent quantum theory of gravity, the degrees of freedom in a spatial region scale as the area of its boundary, not the volume of its interior. This was the first published proposal that the bulk–boundary information ratio is geometrically constrained. Susskind's 1995 "world as hologram" paper made the conjecture precise; Maldacena's 1998 AdS/CFT correspondence supplied the first concrete realization. We cite 't Hooft 1993 as the original of the holographic strand — exactly the conjecture that GG-Theory's OGGEP projection from the 6D bulk to the 4D boundary realizes constitutionally rather than as a duality. First cited in §67.

Hopkins, M. J., & Singer, I. M. (2005). Quadratic functions in geometry, topology, and M-theory. *Journal of Differential Geometry*, *70*(3), 329–452. arXiv:math/0211216.

Relevance. The Hopkins–Singer differential-cohomology framework refines the integer-class BI-6 four-form $[X_4] \in H^4(M_6;\mathbb{Z})$ with the Wu–Chern–Simons quadratic refinement that makes the self-dual tensor sector single-valued on nontrivial bulk topology. We cite it in the Open Problems section of the present paper as the canonical mathematical home of the global anomaly proof of BI-6, with the detailed construction given in A3-BI6 (Arisaka, 2026, A3-BI6); the citation supports the verdict that O2 (Hopkins–Singer descent for BI-6) is Resolved at the level of the companion paper rather than open. First cited in §97.1.

Hulse, R. A., & Taylor, J. H. (1975). Discovery of a pulsar in a binary system. *The Astrophysical Journal*, *195*, L51–L53.

Relevance. The discovery paper of PSR B1913+16, the binary pulsar whose measured orbital decay has tracked the general-relativistic quadrupole prediction for five decades to a few parts in 10^3 — the first empirical confirmation that gravitational radiation removes orbital energy exactly as the linearized metric sector says it must. We cite it in Section 59 as the empirical anchor of the inspiral column of the radiative dictionary, alongside the direct LIGO/Virgo detections. First cited in §59.2.

Joos, E., & Zeh, H. D. (1985). The emergence of classical properties through interaction with the environment. *Zeitschrift für Physik B*, *59*, 223–243.

Relevance. Joos and Zeh provided the canonical estimates for macroscopic decoherence rates from environmental scattering, showing that decoherence of macroscopic position superpositions is essentially instantaneous on any laboratory timescale. We cite it in Lemma 5.5 (Theorem T7.5) as the source of the standard 10^40 s^-1 decoherence-rate estimate for macroscopic states. First cited in §1.

Jordan, R., Kinderlehrer, D., & Otto, F. (1998). The variational formulation of the Fokker–Planck equation. *SIAM Journal on Mathematical Analysis*, *29*, 1–17.

Relevance. The Fokker–Planck equation as the gradient flow of the free energy in the Wasserstein metric, which is the reading under which the relative entropy of Lemma 5.1 descends as a free energy rather than merely as a Lyapunov function. We cite it with Risken for the classical level of the runtime's Lyapunov hierarchy. First cited in §14.

T. Kaluza (1921). Zum Unitätsproblem in der Physik. *Sitzungsber. Preuss. Akad. Wiss. Berlin Math. Phys.*, 966.

Relevance. Kaluza's 1921 paper introduced the central insight that gravity in five spacetime dimensions, with the cylinder condition that suppresses explicit dependence on the fifth coordinate, contains four-dimensional gravity plus a four-dimensional Maxwell electromagnetism plus a scalar 'radion' field as a structural consequence of dimensional reduction. This is the foundational document of all extra-dimensional unification programs, including the GG-Theory framework. We cite it as the central historical reference in Part VI of the present paper, where the parallel Newton/Coulomb derivation chains demonstrate how GG-Theory completes the Kaluza–Klein program one century later. First cited in §1.

van Kampen, N. G (2007). Stochastic Processes in Physics and Chemistry. North-Holland, Amsterdam, third edition.

Relevance. Van Kampen's textbook 'Stochastic Processes in Physics and Chemistry' (3rd ed., 2007) is the standard graduate-level reference for the Fokker–Planck equation, the Itô stochastic differential equation, and the equivalence between continuous-time stochastic processes in their Langevin and Fokker–Planck representations. We cite it in Theorem T6 (Part II) and in Appendix C of the present paper for the Itô-to-Fokker–Planck correspondence used to establish the equivalence between the Langevin and continuity-equation formulations of the GDOS boundary law. First cited in §15.

O. Klein (1926). Quantum theory and five-dimensional theory of relativity. *Z. Phys.*, *37*, 895.

Relevance. Klein's 1926 paper extended Kaluza's proposal by interpreting the fifth dimension as a small compact circle of radius R_5 with periodic identification, producing the now-standard Kaluza–Klein tower of massive modes with masses $m_n = n/R_5$. The lightest zero mode is identified with the standard photon. Klein's quantum refinement made the KK construction physically more compelling and supplied the geometric template that all subsequent extra-dimensional programs (RS, string compactifications, GG-6) follow. We cite it in Part VI of the present paper as the central historical reference, alongside Kaluza 1921, for the constitutional precedent that the GG-Theory framework completes. First cited in §1.

Kossakowski, A., Frigerio, A., Gorini, V., & Verri, M. (1977). Quantum detailed balance and KMS condition. *Communications in Mathematical Physics*, *57*, 97–110.

Relevance. The quantum detailed-balance condition for a dynamical semigroup, and the theorem that a weakly coupled bath in a KMS state supplies it. It is the hypothesis under which the relative entropy that descends by Lindblad–Spohn is a free energy of the boundary Hamiltonian, and the present paper states it as such at T4 and in Part V. First cited in §13.

Kraus, K (1983). States, Effects, and Operations: Fundamental Notions of Quantum Theory. Berlin: Springer.

Relevance. Kraus' 1983 paper 'States, Effects, and Operations' formalized the modern theory of completely-positive trace-preserving (CPTP) maps in quantum mechanics, including the Kraus decomposition $\Phi(\rho) = \sum_\alpha K_\alpha \rho K_\alpha^\dagger$ with $\sum K_\alpha^\dagger K_\alpha = \mathbf{1}$. The Kraus form is the standard operational language of quantum information theory and supplies the explicit operator structure of the boundary evolution channel in T3 of the present paper. We cite it in Theorem T3 (Part II) and Appendix B for the explicit Kraus-decomposition formulation of CPTP maps used throughout the present paper. First cited in §1.

Kugo, T., & Ojima, I. (1979). Local covariant operator formalism of non-Abelian gauge theories and quark confinement problem. *Progress of Theoretical Physics Supplement*, *66*, 1–130.

Relevance. The Kugo–Ojima 1979 paper provided the rigorous local-covariant operator formalism for non-Abelian gauge theories, including the BRST quartet mechanism that ensures unphysical degrees of freedom (longitudinal gauge bosons, ghosts, anti-ghosts, Nakanishi–Lautrup auxiliaries) cancel from physical S-matrix elements. The Kugo–Ojima theorem identifies physical states with elements of the BRST cohomology $\ker s/\mathrm{im}\, s$ at ghost number zero. We cite it in Theorems

T9.4–T9.5 (Part V) and Appendix I of the present paper for the BRST-cohomology preservation that ensures the Slavnov–Taylor identity descends from the bulk to the boundary. First cited in §23.1.

Lagrange, J.-L (1788). Mécanique analytique. Veuve Desaint, Paris.
Relevance. Lagrange's 1788 'Mécanique analytique' was the first comprehensive reformulation of Newtonian mechanics in variational form, expressing the equations of motion as Euler–Lagrange equations of a single scalar functional (the Lagrangian). This reformulation made the symmetry-conservation link of mechanics manifest (later formalized by Noether) and supplied the foundation on which modern field theory rests. We cite it in Part I of the present paper as the central historical precedent for the kind of constitutional reformulation that GG-Theory attempts: the same physical content, restated in a deeper geometric language that makes its symmetry structure manifest. First cited in §15.

Landau, L. D., & Lifshitz, E. M. (1975). *The Classical Theory of Fields* (4th English ed.). Pergamon Press, Oxford. §76.
Relevance. Contains the classic reduction of the radiation-reaction self-force to first order — the Landau–Lifshitz equation — which removes the runaway and pre-accelerating solutions of the Abraham–Lorentz–Dirac equation by treating the damping term perturbatively. Cited in §59.5 (Step 2): in the GDOS reduction the Landau–Lifshitz order is not an approximation scheme but the exact form of the slow-manifold drift, since only orbit averages of the dissipation kernel reach the T5 current. First cited in §59.5.

Larmor, J. (1897). On the theory of the magnetic influence on spectra; and on the radiation of moving ions. *Philosophical Magazine*, Series 5, *44*(271), 503–512.
Relevance. The original derivation of the total power radiated by an accelerated charge, $P = q^2a^2/6\pi\varepsilon_0c^3$ in modern notation — the oldest quantitative statement of the fact that acceleration in the gauge sector is paid for in radiation. We cite it in Section 59 at the head of the synchrotron column of the radiative dictionary: the relativistic γ^4 synchrotron power is its direct descendant, and the Larmor coefficient is one of the two targets of the quantitative-closure Open Problem O7. First cited in §59.2.

Lindblad, G. (1975). Completely positive maps and entropy inequalities. *Communications in Mathematical Physics*, *40*(2), 147–151.
Relevance. Proves the monotonicity of the quantum relative entropy under completely positive trace-preserving maps — the data-processing inequality. Invoked, with Uhlmann (1977), in the derivation of the Spohn quantum H-theorem for OGGEP-induced dynamics: distinguishability between boundary states can only decrease under the GDOS coarse-graining channel, never increase. First cited in §69.4.

Lindblad, G (1976). Communications in Mathematical Physics.
Relevance. Lindblad's 1976 paper proved that any one-parameter semigroup of CPTP maps on a separable Hilbert space has a generator of the canonical form $\mathcal{L}(\rho) = -i[H,\rho] + \sum_\alpha \gamma_\alpha(L_\alpha\rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\})$ with $\gamma_\alpha \geq 0$. Combined with the contemporaneous Gorini–Kossakowski–Sudarshan

paper, this is one of the two foundational results of open quantum systems. We cite it in Theorem T4 (Part II) and Appendix C of the present paper as the structural source of the GKLS form of the GDOS boundary law in the Markovian regime, valid in arbitrary (separable) Hilbert space dimension. First cited in §1.

Loschmidt, J. (1876). Ueber den Zustand des Wärmegleichgewichtes eines Systems von Körpern mit Rücksicht auf die Schwerkraft. *Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften, Mathematisch-Naturwissenschaftliche Classe*, *73*, 128–142.

Relevance. Loschmidt's reversibility objection to Boltzmann's H-theorem: mechanical equations are time-reversible, so monotonic entropy increase cannot be derived from them alone. The objection is the canonical historical pointer to the hidden open-system ingredient (loss of microscopic correlations) that any closed-system derivation of the second law must implicitly invoke. We cite it in the historical chain leading to GDOS as one of the first explicit recognitions that closure and irreversibility are constitutionally incompatible. First cited in §102.

J. Maldacena (1998). The large N limit of superconformal field theories and supergravity. *Adv. Theor. Math. Phys.*, *2*, 231.

Relevance. Maldacena's AdS/CFT conjecture is the modern archetype for bulk–boundary duality and motivates the idea that geometry can encode quantum gauge dynamics. We cite it as a historical anchor for the GGEP/OGGEP posture: bulk geometry is not merely a classical background, but a structure from which boundary quantum physics and observables can be exported. First cited in §H.5.

Maxwell, J. C. (1865). A dynamical theory of the electromagnetic field. *Philosophical Transactions of the Royal Society of London*, *155*, 459–512.

Relevance. Maxwell's 1865 paper unifies electricity, magnetism, and optics in a single dynamical field theory, introducing the displacement current and establishing the modern Maxwell equations. We cite it in Part I as the historical example in which the field formalism implicitly couples every local electromagnetic system to an extended environment (the rest of the field), without making this openness explicit at the constitutional level. Maxwellian electromagnetism is constitutionally closed (a single field Lagrangian) but operationally open (radiation reaction, vacuum fluctuations); GDOS makes this operational openness constitutional. First cited in §6.1.

Naimark, M. A. (1943). On a representation of additive operator set functions. *Doklady Akademii Nauk SSSR*, *41*, 359–361.

Relevance. Naimark's 1943 dilation theorem (the operator-valued analog of Stinespring's later result) is the precursor of the GNS construction and a constitutional prerequisite for the boundary Hilbert-space derivation in T7.1. Cited at the opening of T7.1 in Part III. First cited in §20.

Nakajima, S (1958). Progress of Theoretical Physics.

Relevance. Nakajima's 1958 paper introduced the projection-superoperator technique for systematically reducing the unitary dynamics of a closed quantum system on a tensor-product Hilbert space $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E$ to a closed integro-differential equation for the reduced state of the system. Together with Zwanzig's 1960 paper, this is the foundational result of the modern theory of open

quantum systems. We cite it in Theorem T2 (Part II) and Appendix A of the present paper as the source of the exact memory-kernel equation that constitutes the bulk-to-boundary reduction in the GG-Theory framework. First cited in §1.

J. von Neumann (1932). Mathematische Grundlagen der Quantenmechanik.
Relevance. Von Neumann's 1932 monograph 'Mathematische Grundlagen der Quantenmechanik' is the canonical historical reference for the rigorous Hilbert-space formulation of quantum mechanics, including the projection postulate, the von Neumann entropy, the spectral theorem for self-adjoint operators, and the explicit formulation of the measurement problem in terms of the system–apparatus chain. We cite it throughout Part III of the present paper as the foundational reference for the textbook QM postulates that the GG-Theory framework reorganizes in T7.1–T7.7 as derived constitutional theorems. First cited in §1.

Newton, I (1687)..
Relevance. Newton's definitions and laws were a prototypical axiom system for mechanics, converting astronomy into dynamics and setting the model for "principles → equations → predictions." We cite it as precedent for framing GUT-10 as a constitution that will be cited across later technical papers. First cited in §6.1.

Nielsen, M. A., & Chuang, I. L (2010). Quantum Computation and Quantum Information (10th Anniversary Edition). Cambridge: Cambridge University Press.
Relevance. Nielsen and Chuang's textbook 'Quantum Computation and Quantum Information' (10th anniversary ed., 2010) is the canonical graduate-level reference for the modern formulation of quantum mechanics in the operational/CPTP/quantum-information language. It covers the Stinespring dilation theorem, the Kraus decomposition, POVMs, quantum channels, and the no-signaling theorem at the level needed for our derivations. We cite it throughout Parts II–III of the present paper for the standard operational-quantum-mechanics machinery that underlies the boundary law GDOS and the textbook QM derivations T7.1–T7.7. First cited in §12.

Pati, J. C., & Salam, A. (1974). Lepton number as the fourth color. *Physical Review D*, *10*, 275–289.
Relevance. The Pati–Salam 1973 paper introduced the SU(4) Pati–Salam unification with 'lepton number as the fourth color', one of the two foundational Grand Unified Theories alongside Georgi–Glashow SU(5). The Pati–Salam construction unifies leptons and quarks via the embedding $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{SU}(4)_\mathrm{C}$. We cite it in Part V of the present paper as one of the original GUT proposals and as the historical precedent for the geometric unification program that GG-Theory completes via the 6D bulk derivation of SU(5) plus the additional $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$ gauge factors. First cited in §42.1.

Penington, G., Shenker, S. H., Stanford, D., & Yang, Z. (2020). Replica wormholes and the black hole interior. *Journal of High Energy Physics*, *2022*(3), 205. [arXiv:1911.11977, 2019.]
Relevance. Establishes, via replica-wormhole saddles of the gravitational path integral, the island prescription for the entropy of Hawking radiation and the unitary Page transition. Cited with Almheiri et al. (2020) as the bulk-side realization of the Page curve that the present framework

recovers from the boundary side through the OGGEP partial trace over bulk-compact modes. First cited in §71.4.

Peskin, M. E., & Schroeder, D. V (1995). An Introduction to Quantum Field Theory. Reading, MA: Westview Press.

Relevance. Peskin and Schroeder's textbook 'An Introduction to Quantum Field Theory' (1995) is the canonical graduate-level reference for modern QFT, including the path-integral formulation, the Faddeev–Popov procedure, BRST symmetry, the Slavnov–Taylor identity, the renormalization group, and the Standard Model. We cite it throughout Part V of the present paper as the standard reference for the QFT machinery that the GG-Theory boundary derivation specializes and reproduces. Every reader of Part V is assumed to have some familiarity with the content at this level. First cited in §118.

Peters, P. C. (1964). Gravitational radiation and the motion of two point masses. *Physical Review*, *136*(4B), B1224–B1232.

Relevance. The classic derivation of the orbital decay of a binary under gravitational radiation: the secular equations for da/dt and de/dt, including the circular-orbit decay law with the coefficient $64\,G_N^3/5c^5$. We cite it in Section 59 as the inspiral analog of synchrotron damping — the gradient-sector drift on the (a, e) state manifold — and its coefficient is the second target of the quantitative-closure Open Problem O7. First cited in §59.2.

Polchinski, J (1995). Dirichlet branes and Ramond-Ramond charges. *Physical Review Letters*, *75*(26), 4724–4727.

Relevance. Polchinski's textbook 'String Theory' (1995, 2 vols.) is the canonical graduate-level reference for perturbative string theory, including the bosonic and superstring formulations, world-sheet conformal field theory, D-branes, dualities, and string compactifications. We cite it in Parts IV–V and in the Discussion (Part IX) of the present paper as the foundational reference for the string-theoretic program against which GG-Theory is contrasted: string theory 'quantizes gravity' in 10/11 dimensions with SUSY and a vast vacuum landscape; GG-Theory 'geometrizes gauge forces' in 6 dimensions without SUSY and with a unique OGN-5-locked vacuum. First cited in §108.

Polchinski, J. (2006). The cosmological constant and the string landscape. *arXiv:hep-th/0603249*.

Relevance. Polchinski's 2004 paper articulated the string-theoretic landscape problem: the realization that the space of consistent string vacua contains $\sim 10^{500}$ distinct configurations, each with its own low-energy effective theory. The landscape problem is the principal phenomenological obstacle to extracting unique predictions from string theory and is the structural challenge that GG-Theory's discrete OGN-5 integer ledger directly answers. We cite it in Parts IV–V and in Part IX of the present paper as the canonical articulation of the landscape problem against which GG-Theory's unique-vacuum framework is positioned. First cited in §31.1.

L. Randall, & R. Sundrum (1999). A large mass hierarchy from a small extra dimension. *Phys. Rev. Lett.*, *83*, 3370.

Relevance. The Randall–Sundrum 1999 papers introduced warped extra dimensions as a geo-

metric solution to the gauge hierarchy problem: a five-dimensional anti-de Sitter slice with two branes (UV and IR) produces an exponential scale profile e^{-2kr} that suppresses the boundary scale relative to the bulk scale. This is the constitutional precedent for the scale 5th dimension of GG-6 that supplies the renormalization-group flow direction in the GG-Theory framework. We cite it throughout Part IV (T8.7) and Part VI of the present paper for the scale-bulk machinery that GG-6 inherits from the RS construction. First cited in §114.

Reed, M., & Simon, B. (1980). Methods of Modern Mathematical Physics, Vol. I: Functional Analysis (Revised Edition). New York: Academic Press.

Relevance. Reed and Simon's textbook is the canonical reference for the rigorous functional-analytic theory of unbounded operators, the Dyson series, and time-ordered exponentials on infinite-dimensional Hilbert spaces. Cited at the convergence step of the time-ordered exponential in Theorem T2 (Nakajima-Zwanzig projection). First cited in §11.

Renou, M.-O., Trillo, D., Weilenmann, M., Le, T. P., Tavakoli, A., Gisin, N., Acín, A., & Navascués, M. (2021). Quantum theory based on real numbers can be experimentally falsified. *Nature*, 600, 625–629.

Relevance. The loophole-free experimental exclusion of quantum theory over real Hilbert spaces: a three-party network Bell test whose correlations no real-amplitude theory with tensor-product structure can reproduce. For the present paper it is the essential foil of the real-bulk reading — what the experiment rules out is a real *boundary* theory, while a real bulk whose projection installs a genuinely complex boundary theory (the derived unit J of Arisaka, 2026, P4-Real) is exactly the combination it permits. Cited at the real-bulk reading of the derivation chain and in the response to Objection 57. First cited in §95.

Reuter, M. (1998). Nonperturbative evolution equation for quantum gravity. *Physical Review D*, *57*, 971–985.

Relevance. Reuter's 1998 paper established the asymptotic-safety program for quantum gravity, providing the functional renormalization-group equation and exhibiting evidence for a non-trivial UV fixed point of pure gravity. Asymptotic safety is one of the principal alternative approaches to quantum gravity beyond perturbative quantization and string theory. We cite it in Part IV and in Part IX (open problems) of the present paper as one of the alternative quantum-gravity programs against which the GG-Theory 'geometrize gauge forces' approach is contrasted. First cited in §97.2.

Ricci, G., & Levi-Civita, T (1900). Méthodes de calcul différentiel absolu et leurs applications. *Mathematische Annalen*, *54*, 125–201.

Relevance. The Ricci–Levi-Civita 1900 paper introduced the absolute differential calculus (now called tensor calculus on differentiable manifolds), supplying the mathematical machinery that Einstein would use fifteen years later to formulate general relativity. The Ricci–Levi-Civita reformulation of geometric analysis in coordinate-invariant form is the canonical historical example of how a constitutional reformulation can supply the language for a subsequent physical revolution. We cite it in Part I of the present paper as the central precedent for the kind of constitutional reformulation that the GG-Theory framework attempts. First cited in §6.4.

Riess, A. G., et al (1998). Observational evidence from supernovae for an accelerating universe and a cosmological constant. *The Astronomical Journal*, *116*(3), 1009–1038.

Relevance. The Riess et al. 1998 paper (along with the contemporaneous Perlmutter et al. paper) reported the discovery of the accelerating expansion of the universe via Type Ia supernova observations, establishing the existence of a positive cosmological constant or equivalent dark-energy component. This was a Nobel-Prize-winning result that pinned down the empirical value $\rho_\Lambda^{1/4} \sim 2.3$ meV that the GG-Theory lock-scale-seesaw mechanism (T8.5) reproduces from the geometric ratio $M_{\text{lock}}^2/M_{\text{Pl}} \approx 6$ meV. We cite it as the empirical reference for the cosmological-constant value in Theorem T8.5 of Part IV. First cited in §1.

Risken, H (1989). The Fokker-Planck Equation: Methods of Solution and Applications. Berlin: Springer.

Relevance. Risken's textbook 'The Fokker–Planck Equation: Methods of Solution and Applications' (2nd ed., 1989) is the canonical graduate-level reference for the Fokker–Planck equation, the Kramers–Moyal expansion, and the methods of solving stochastic differential equations in the continuum limit. We cite it in Theorem T5 (Part II) and Appendix C of the present paper for the Kramers–Moyal-expansion machinery that produces the mesoscopic GDOS continuity equation from the underlying GKLS dynamics in the appropriate coarse-graining limit. First cited in §14.

Rovelli, C (2004). Quantum Gravity. Cambridge: Cambridge University Press.

Relevance. Rovelli's textbook 'Quantum Gravity' (2004) is the canonical graduate-level reference for the Loop Quantum Gravity program, including spin networks, spin foams, the Ashtekar variables, and the loop representation of the Wheeler–DeWitt equation. LQG is one of the principal alternative quantum-gravity programs alongside string theory and asymptotic safety. We cite it in Part IV and in Part IX of the present paper as a key reference for the LQG approach against which the GG-Theory 'geometrize gauge forces' program is contrasted. First cited in §97.2.

Sakurai, J. J., & Napolitano, J. (2020). Modern Quantum Mechanics (3rd ed.). Cambridge: Cambridge University Press.

Relevance. The canonical first-year graduate quantum-mechanics textbook — the statement of record for the postulates that Part III derives as theorems (state space, unitary dynamics, measurement, and the standard worked canon from Stern–Gerlach onward). Paired with Part III in its *Read alongside* line so that a student can hold the postulate-based and theorem-based presentations of the same physics side by side. First cited in the Read-alongside line of Part III.

Sands, M. (1970). *The Physics of Electron Storage Rings: An Introduction.* SLAC Report SLAC-R-121, Stanford Linear Accelerator Center.

Relevance. The canonical treatment of radiation damping and quantum excitation in electron storage rings: damping times from the emitted synchrotron power, the photon-emission shot noise, and the equilibrium (Gaussian) beam emittance set by their balance. We cite it in Section 59 because the storage-ring theory it codifies is, sector for sector, the laboratory realization of the T5 current decomposition — circulation (betatron/synchrotron oscillations), gradient drift (radiation damping), diffusion (quantum excitation), and Lyapunov fixed point (equilibrium emittance).

First cited in §59.2.

Schrödinger, E. (1926). Quantisierung als Eigenwertproblem.

Relevance. Schrö dinger's 1926 paper introduced wave mechanics and the now-standard wave-equation formulation of quantum theory, providing the second of the two foundational formulations (alongside Heisenberg's matrix mechanics). The Schrö dinger equation $i\hbar\partial_t|\psi\rangle = H|\psi\rangle$ is the central dynamical equation of the textbook quantum-mechanical formalism. We cite it in Part III of the present paper as one of the canonical references for the textbook QM that the GG-Theory framework reorganizes: T7.2 derives the Schrö dinger equation as the closed-system limit of the GKLS master equation, not as a primitive postulate. First cited in §1.

Scherk, J., & Schwarz, J. H. (1974). Dual models for non-hadrons. *Nuclear Physics B*, *81*(1), 118–144.

Relevance. Scherk and Schwarz's 1974 paper made the decisive observation that the closed-string sector of dual-resonance models contains a massless spin-2 particle that behaves exactly like the graviton — the first proposal that the dual-amplitude program of Veneziano 1968 / Nambu 1970 / Goto 1971 was actually a candidate quantum theory of gravity, not just a strong-interaction model. This is the conceptual hinge between hadronic dual models and modern string theory. We cite Scherk-Schwarz 1974 in the historical positioning of T8 (Part IV, section 6.3) as the genuine original of string theory as a unification framework, ten years before the Green-Schwarz anomaly cancellation of 1984. First cited in §31.1.

Schwinger, J. (1948). On quantum-electrodynamics and the magnetic moment of the electron. *Physical Review*, *73*(4), 416–417.

Relevance. Schwinger's one-loop result $a = \alpha/2\pi$, the first radiative correction to the magnetic moment and the foundation of precision QED. We cite it in Theorem T9.9 as the archetypal scale-invariant finite coefficient: a pure number from a single diagram, carrying no scale dependence and therefore wholly invisible to the radial geometry. First cited in §52.

Schwinger, J. (1949). On the classical radiation of accelerated electrons. *Physical Review*, *75*(12), 1912–1925.

Relevance. The definitive derivation of the synchrotron radiation spectrum — the frequency and angular distribution of the retarded field of a relativistic charge in circular motion. Cited in §59.5 (Step 4): the photon-number and energy moments of this spectrum fix the noise kernel of the field trace, and hence the diffusion coefficient and the universal quantum constant C_q, with no input beyond the field theory derived in Chain (B). First cited in §59.5.

Slavnov, A. A. (1972). Ward identities in gauge theories. *Theoretical and Mathematical Physics*, *10*(2), 99–104.

Relevance. Slavnov's 1972 derivation of the gauge-theory Ward identities (independently obtained by Taylor 1971) is the canonical reference for what is now called the Slavnov–Taylor identity, the BRST-cohomological generalization of the QED Ward–Takahashi identity that is essential for the renormalizability of non-abelian gauge theories. Cited at the point of use in Theorem T9 (4D QFT from 6D bulk) and in the BRST-inheritance argument of Part V. First cited in §1.

Spohn, H. (1978). Entropy production for quantum dynamical semigroups. *Journal of Mathematical Physics*, *19*(5), 1227–1230.

Relevance. Proves that the relative entropy of an open quantum system with respect to its stationary Gibbs state is monotonically non-increasing under a completely positive Markovian semigroup — the quantum H-theorem. Used as Step 2 of the second-law derivation for OGGEP: applied with the equilibrium state, the data-processing inequality yields the monotonic decrease of the relative entropy under the boundary GKLS evolution. First cited in §69.4.

Srednicki, M. (2007). Quantum Field Theory. Cambridge: Cambridge University Press.

Relevance. Srednicki's textbook is one of the canonical modern graduate references for the Lagrangian-density formulation of quantum field theory, presenting QED, QCD, the electroweak sector, and renormalization in the standard "write down the 4D Lagrangian and integrate" style. We cite it in Section 54 of the present paper as the representative textbook starting point that GG-Theory now derives from the 6D bulk Lagrangian as a boundary effective theorem rather than asserting as an axiom. First cited in §54.1.

Stinespring, W. F (1955). Positive functions on C^*-algebras. *Proceedings of the American Mathematical Society*, *6*(2), 211–216.

Relevance. Stinespring's 1955 paper proved the foundational theorem of operator-algebraic quantum information: every completely-positive map on a C*-algebra has a dilation as the partial trace of a unitary on a larger Hilbert space. This is the structural source of complete positivity for any quantum channel that arises from partial trace over a unitarily-evolving compound system. We cite it in Theorem T3 (Part II) and Appendix B of the present paper as the constitutional source of the CPTP property of the boundary evolution channel in the GG-Theory framework. First cited in §1.

Stora, R. (1976). Continuum gauge theories. *Cargese Lectures*, 201–224.

Relevance. Stora's Cargese lectures introduced the descent equations for anomalies, providing the systematic mechanism by which the d+2-dimensional anomaly polynomial descends to the d-dimensional consistent anomaly. We cite it in Theorem T9.6 as the foundational reference for the I_8 -> I_6 -> I_4 descent chain that the GG-6 BI-6 inflow follows. First cited in §49.

Streater, R. F., & Wightman, A. S (1964). PCT, Spin and Statistics, and All That. New York: W. A. Benjamin.

Relevance. The Streater–Wightman monograph 'PCT, Spin and Statistics, and All That' (1964) is the canonical reference for the Wightman axioms of quantum field theory and the rigorous algebraic-QFT framework. It includes the spin-statistics theorem, the PCT theorem, and the Reeh–Schlieder theorem in their original axiomatic form. We cite it throughout Part II of the present paper for the GNS construction and the algebraic structure of bulk and boundary fields that underpin the OGGEP-derived boundary law. First cited in §10.

Susskind, L. (1995). The world as a hologram. *Journal of Mathematical Physics*, *36*(11), 6377–6396.

Relevance. Susskind's 1995 paper makes 't Hooft's 1993 holographic conjecture precise in

the language of string theory: every physical theory in a region of D-dimensional spacetime is equivalent to a theory living on the $(D-1)$-dimensional boundary, with information density bounded by the boundary area divided by the Planck area. This is the conceptual milestone between 't Hooft 1993 (the original conjecture) and Maldacena 1998 (the first concrete realization). We cite it as the canonical reference for the holographic principle and as the immediate intellectual predecessor of the OGGEP bulk-to-boundary projection that A5-GDOS makes constitutional in Theorems T1 through T6. First cited in §31.1.

Susskind, L (2003). The anthropic landscape of string theory. arXiv:hep-th/0302219.
Relevance. The landscape framing represents a modern endpoint of unconstrained model freedom: many vacua, many low-energy effective theories, and selection by anthropic arguments. We cite it as a motivation for MSMF, which aims to restore determinism by bounding freedom and demanding symmetry-driven minimality. First cited in §67.

Taylor, J. C. (1971). Ward identities and charge renormalization of the Yang–Mills field. *Nuclear Physics B*, *33*(2), 436–444.
Relevance. Taylor's 1971 derivation of the gauge-theory Ward identities (independently obtained by Slavnov 1972) is one of the two foundational references for the Slavnov–Taylor identity that is essential for the renormalizability of non-abelian gauge theories. Cited together with Slavnov (1972) at the point of use in Theorem T9 (4D QFT from 6D bulk) and in the BRST-inheritance argument of Part V. First cited in §1.

Tsirelson, B. S. (1980). Quantum generalizations of Bell's inequality. *Letters in Mathematical Physics*, *4*, 93–100.
Relevance. Tsirelson's 1980 paper established the Tsirelson bound, $|S| \leq 2\sqrt{2}$, on the CHSH correlation in any quantum theory: this is the maximum violation of the Bell inequality that quantum mechanics permits. The Tsirelson bound is saturated by the spin-singlet state with optimal measurement settings. We cite it in Theorem T7.7 (Part III) of the present paper as the operational ceiling for boundary CHSH violation in the GG-Theory framework, with the saturation arising from the bulk-to-boundary projection of the bulk timelike connections through the radial direction r. First cited in §26.

Veneziano, G. (1968). Construction of a crossing-symmetric, Regge-behaved amplitude for linearly rising trajectories. *Il Nuovo Cimento A*, *57*(1), 190–197.
Relevance. Gabriele Veneziano's 1968 paper writing the four-point dual amplitude in terms of Euler beta functions is the original of string theory. Veneziano's amplitude was almost immediately reinterpreted by Nambu (1970), Nielsen (1970), and Susskind (1969) as the scattering amplitude of a one-dimensional quantum string; Goto (1971) wrote the action; Scherk-Schwarz (1974) identified the graviton in the closed-string spectrum; and Green-Schwarz (1984) produced the anomaly-cancellation revolution. We cite Veneziano 1968 in the historical positioning of T8 (Part IV, section 6.3) as the genuine moment string theory began. First cited in §77.1.

I. Tyutin (1975). Gauge invariance in field theory and statistical physics in operator formalism.
Relevance. Independent discovery of BRST symmetry; together with Becchi–Rouet–Stora,

provides the gauge-fixing framework used in the GG-6 BRST algebra. First cited in §A.

Uhlmann, A. (1977). Relative entropy and the Wigner–Yanase–Dyson–Lieb concavity in an interpolation theory. *Communications in Mathematical Physics*, *54*(1), 21–32.
Relevance. Establishes the monotonicity of the relative entropy under stochastic (completely positive) maps in the general operator-algebra setting. Cited with Lindblad (1975) as the rigorous foundation of the data-processing inequality on which the Spohn quantum H-theorem for the GDOS channel rests. First cited in §69.4.

Volkov, S. (2024). Calculation of the total tenth-order QED contribution to the electron magnetic moment. *Physical Review D*, *110*(3), 036001.
Relevance. The first complete independent evaluation of the five-loop mass-independent coefficient, obtaining $A_1^{(10)} = 5.891(61)$ against the previously established 6.737(159) — a 5σ discrepancy that remains open. We cite it in Theorem T9.9 as the measure of how much genuine content sits in $\mathcal{F}$: a scale-invariant period on which two careful groups disagree is not an artifact of the boundary description waiting to be geometrized away, and Problem V.9.5 prices what the disagreement costs in a_e. First cited in §52.9.

Wald, R. M. (1984). General Relativity. Chicago: University of Chicago Press.
Relevance. Wald's textbook 'General Relativity' (1984) is the canonical graduate-level reference for differential geometry and general relativity, including the formal mathematical machinery of product geometry metrics, Christoffel symbols, the Riemann/Ricci tensors, and the Einstein equations. We cite it throughout Part IV and Appendix F of the present paper as the standard reference for the geometric and curvature computations on the 6D bulk that underlie the GG-Theory derivation of 4D general relativity in T8.1–T8.7. First cited in §33.4.

Weinberg, S. (1995). The Quantum Theory of Fields, Volume I: Foundations. Cambridge University Press.
Relevance. Weinberg's three-volume treatise on quantum field theory is the canonical modern reference for the renormalization program, the BRST formalism, anomaly classification, and effective field theory. We cite Volume I throughout Parts III and V as the standard reference for the QFT background against which the GG-Theory reformulation is positioned. First cited in §54.1.

Wess, J., & Zumino, B (1971). Consequences of anomalous Ward identities. *Physics Letters B*, *37*(1), 95–97.
Relevance. Wess and Zumino derived integrability/consistency conditions on gauge anomalies, demonstrating that anomalies have constrained algebraic structure. We cite this work to justify treating anomaly freedom and inflow cancellation as a constitutional filter (POS-3) rather than a model-dependent afterthought. First cited in §49.

Wheeler, J. A. (1968). Superspace and the nature of quantum geometrodynamics. *Battelle Rencontres*, 242–307.
Relevance. Wheeler's 1968 paper introduced the superspace formulation of quantum geometrodynamics, treating the configuration space of 3-geometries as the fundamental object on which

the wave functional of quantum gravity is defined. This formulation, together with the contemporaneous DeWitt 1967 paper, founded the canonical-quantization program for gravity that produced the Wheeler–DeWitt equation. We cite it in Part IV of the present paper as a key historical reference for the canonical quantum-gravity program that the 't Hooft–Veltman 1972 non-renormalizability result obstructed and that GG-Theory bypasses via the geometrization of gauge forces. First cited in §31.1.

Wightman, A. S. (1956). Quantum field theory in terms of vacuum expectation values. *Physical Review*, *101*(2), 860–866.

Relevance. Wightman's 1956 paper formalized the axiomatic approach to quantum field theory in terms of the Wightman functions (vacuum expectation values of products of field operators), supplying the rigorous mathematical framework that complemented the perturbative-Lagrangian approach of physicists. The Wightman axioms include locality, microcausality, Poincaré covariance, and spectral conditions. We cite it in Part II of the present paper as the foundational reference for the algebraic-QFT framework that underpins the GNS construction of the boundary Hilbert space. First cited in §10.

K. Wilson (1971). Renormalization group and critical phenomena. I. *Phys. Rev. B*, *4*, 3174.

Relevance. Wilson's 1971 papers introduced the modern formulation of the renormalization group as a flow on the space of effective theories under coarse-graining (block-spin transformations), establishing renormalization as a foundational concept rather than a perturbative trick. The Wilsonian RG underpins all of modern condensed-matter physics, particle physics, and statistical mechanics. We cite it in Theorem T9.2 (Part V) and throughout the present paper for the conceptual framework of effective field theory that supports the heavy-mode integration of the bulk action to the 4D effective Standard Model action. First cited in §1.

C. Yang, & R. Mills (1954). Conservation of isotopic spin and isotopic gauge invariance. *Phys. Rev.*, *96*, 191.

Relevance. The Yang–Mills 1954 paper introduced non-Abelian gauge theory, generalizing electromagnetism to gauge groups SU(N) and supplying the mathematical framework on which the entire Standard Model is built. The Yang–Mills construction is the constitutional foundation of all modern particle physics. We cite it in Part V of the present paper as the canonical reference for the gauge-field structure that the GG-Theory framework specializes and reproduces from the 6D bulk via heavy-mode integration of the composite bulk connection. First cited in §1.

Zeh, H. D. (1970). On the interpretation of measurement in quantum theory. *Foundations of Physics*, *1*, 69–76.

Relevance. Zeh's original 1970 paper introduced the decoherence program, identifying environmental coupling as the constitutional source of the apparent classicality of macroscopic systems. We cite it in Theorems T7.5–T7.6 as the historical origin of the decoherence and einselection arguments that the GG-Theory framework makes constitutional via OGGEP. First cited in §1.

Zermelo, E. (1896). Ueber einen Satz der Dynamik und die mechanische Wärmetheorie. *Annalen der Physik und Chemie*, *57*, 485–494.

Relevance. Zermelo's recurrence objection to Boltzmann's H-theorem: a Hamiltonian system on a bounded phase space recurs to its initial state arbitrarily closely, contradicting monotonic entropy increase. Together with Loschmidt's reversibility objection (Loschmidt, 1876), this identifies coarse-graining and informational loss as essential to any thermodynamic arrow of time. We cite it in the historical chain motivating the GDOS inversion (openness primary, closure derivative). First cited in §102.

Zumino, B. (1985). Chiral anomalies and differential geometry. *Relativity, Groups and Topology II (Les Houches)*, 1291–1322.

Relevance. Zumino's Les Houches lectures developed the differential-geometric formulation of chiral anomalies, including the modern form of the Wess-Zumino consistency conditions and the descent equations. We cite it in Theorem T9.6 alongside Stora 1976 as the technical foundation for the BI-6 inflow descent in the GG-Theory framework. First cited in §49.

Zurek, W. H. (1981). Pointer basis of quantum apparatus: into what mixture does the wave packet collapse?. *Physical Review D*, *24*, 1516–1525.

Relevance. Zurek's 1981 paper introduced the einselection criterion for the pointer basis: among all orthonormal bases, the einselected basis is the one that minimizes the entropy production rate under environmental coupling. We cite it in Theorem T7.6 as the algebraic source of the pointer-basis-from-bulk-locality theorem in the GG-Theory framework. First cited in §1.

Zurek, W. H (2003). Reviews of Modern Physics.

Relevance. Zurek's 2003 review article 'Decoherence, einselection, and the quantum origins of the classical' is the canonical comprehensive reference for the modern decoherence program, including the einselection criterion, the predictability sieve, and the relationship between decoherence and the apparent classicality of macroscopic systems. We cite it in Theorem T7.6 (Part III) and Appendix D of the present paper for the einselection-of-the-pointer-basis arguments that, together with bulk locality (AX-1), supply the constitutional source of the position pointer basis in the GG-Theory framework. First cited in §1.

Zwanzig, R (1960). Journal of Chemical Physics.

Relevance. Zwanzig's 1960 paper independently developed the projection-superoperator method for reducing the unitary dynamics of a closed quantum system on a tensor-product Hilbert space to a closed equation for the reduced state of the subsystem of interest. Together with Nakajima's 1958 paper, this constitutes the Nakajima–Zwanzig (NZ) formalism that is the foundational result of the modern theory of open quantum systems. We cite it in Theorem T2 (Part II) and Appendix A of the present paper as the structural source of the exact memory-kernel equation that the GG-Theory framework adopts as the constitutional starting point for the boundary law GDOS. First cited in §1.