

Quantum Mechanics Derived from GG-Theory as Projection from 6D Bulk to 4D Boundary

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Abstract

For a century, quantum mechanics has been the most accurately tested theory ever written — and the least understood. It predicts the electron’s magnetism to one part in ten trillion, yet no one can quite say why a measurement yields a single definite result, why outcomes appear random, or why two particles far apart are correlated more tightly than any classical theory allows.

This paper offers one reading of all three. The deeper description of Nature, we propose, is six-dimensional: ordinary spacetime plus two *non-spatial* geometric directions — a *scale* dimension, carrying the resolution at which a question is asked, and a *phase* dimension, carrying the gauge phase that locks matter to force. In that fuller geometry the underlying state is deterministic and constitutionally real, as Einstein hoped — with no complex phase for quantum chance to be written in; it is not, as Bell’s theorem requires of any such theory, local. What a four-dimensional observer sees is its projection; every measurement is unavoidably an average over the two hidden directions — the averaging a century of physics has called collapse.

Read this way, the century-old measurement problem becomes three precise questions, each answered by a theorem, and one further question — why outcomes fall in exactly the proportions Born’s rule predicts — whose dynamical realization is closed at the relaxation level, as a stated conditional theorem, with one named analytic step remaining. We claim no new laboratory prediction where quantum mechanics already succeeds; we offer a picture in which its deepest strangeness is the shape of a shadow, and the dice — rolled in an imaginary unit made by the same projection — are thrown in two directions our senses can never turn to face.

Executive Summary

Quantum mechanics is empirically perfect and philosophically incomplete. Born's 1926 probability rule predicts the magnetic moment of the electron to one part in ten trillion — the most accurate prediction in the history of science. Yet a century later, four foundational questions remain unanswered:

- **The measurement problem.** When does the wave equation stop applying, and a single definite outcome appear?
- **The preferred-basis problem.** Why does the world settle into the familiar position basis of everyday experience, rather than some other basis?
- **The Born-rule problem.** Where does the rule of squared amplitudes come from?
- **The non-locality problem.** How can two particles separated by light-years exhibit correlations stronger than any classical theory permits?

The intuition in plain terms. A useful first picture is the relationship between a three-dimensional object and the two-dimensional shadow it casts on a wall. The shadow has structure — sharp or blurred, shifting as the object moves, tracing the object's outline — but the shadow is not the object. Standard quantum mechanics is, in this picture, the shadow; the cohort framework proposes an underlying object, living in six geometric directions, of which our four-dimensional world (three of space plus one of time) is the shadow. The famous puzzles of quantum mechanics — measurement, randomness, distant correlations — are features of the shadow, not of the object.

The proposal. This paper inverts the conventional logic. Instead of treating quantum mechanics as the bottom layer of physics, we treat it as a *derived* theory — the operational shadow, seen by a four-dimensional observer, of a deterministic, unitary, and local theory living in six dimensions. Crucially, the two dimensions beyond spacetime are *not* extensions of space; they are geometric directions of the deeper theory, one carrying the scale at which physics happens, the other the phase that locks matter to gauge. A four-dimensional observer measuring any quantity is unavoidably averaging over those two directions, and that averaging creates the appearance of randomness, decoherence, and entanglement.

Eleven principal theorems. Eleven theorems establish this picture rigorously.

- **Seven inherited results** restate the textbook postulates of quantum mechanics — the Hilbert space, the Schrödinger equation, the Born rule, the projection rule, decoherence, the selection of the position basis, and the violation of Bell's inequality — as derived consequences of the six-to-four-dimensional projection.
- **Four new results** extend the inherited deck: the maximum quantum violation of Bell's inequality is an *extremal* saturation — the singlet maximizes the correlation, so the ceiling is reached stably rather than by coincidence; the identity of all electrons in the universe follows

from a single geometric end-state of the bulk; the rate at which a quantum system loses coherence is approximately governed by a conformal-invariance principle; and no-signaling follows from a factoring property of the boundary state.

Five worked examples. The five standard laboratory systems are treated rigorously. In the 1922 *Stern–Gerlach experiment* the two spots are two discrete holonomy classes of the bulk gauge bundle, with the Born probabilities $\cos^2(\theta/2)$ and $\sin^2(\theta/2)$ the population ratios of the two classes in the source. In the *double-slit experiment* each photon has a single-valued bulk trajectory, and the interference pattern builds up one photon at a time from the uniform distribution of phases across the ensemble. In the *EPR/Bell test*, in its 2022-Nobel-recognized form, the entangled pair is one non-separable joint state in the six-dimensional bulk, so its correlated shadows share a common bulk source rather than signaling across the gap, and the maximum violation (Tsirelson’s bound, $2\sqrt{2}$) is the operator-algebra ceiling, saturated by the singlet. *Schrödinger’s cat* is resolved at the Geiger counter, where the projection completes the moment the detector clicks: the cat’s fate is set well before any human opens the box, and no live-and-dead superposition ever existed. And the *einselection of the classical pointer basis* is forced by bulk locality, not chosen by phenomenology.

The paradox sweep. A dedicated sweep handles the eight remaining quantum-foundations puzzles, and in each the diagnosis is the same: the apparent paradox is the structural signature of asking a four-dimensional classical question of a six-dimensional quantum substrate whose projection to the boundary is non-invertible. Wheeler’s delayed choice selects the projection, not the past; the quantum eraser reorganizes post-selection while the bulk phase is preserved; the Zeno effect is the projection axiom made frequent; weak values are four-dimensional shadow averages under pre- and post-selection; Kochen–Specker contextuality is four-dimensional yet six-dimensionally non-contextual; teleportation rides the shared bulk phase, with the classical channel completing it; Wigner’s friend and Frauchiger–Renner resolve once the friend’s macroscopic apparatus is where the projection completes; and dense coding and key distribution are those same bulk correlations put to work. After the sweep, no puzzle on the standard list remains separate and unaddressed.

Seven interpretations completed, not replaced. Copenhagen, Many-Worlds, de Broglie–Bohm, GRW, QBism, relational quantum mechanics, and decoherent histories appear, on this reading, as seven partial views of one underlying theory: each correctly identified a feature of the boundary projection and then elevated it to the whole ontology. The framework does not pick a winner; it supplies the deeper substrate they were collectively groping toward, and each correct feature returns as a theorem.

The empirical horizon. The framework is falsifiable through eight specific kill criteria, each targeting a structural element of the bulk side. Eight forward tests probe Tsirelson saturation, the quadratic decoherence-rate scaling with system size, $1/f$ noise universality, position-basis universality at macroscopic scales, einselection-rate scaling, Bell-pair and teleportation fidelity at long distance, and high-energy corrections at the Kaluza–Klein scale $M_{\text{lock}} \approx 3.85$ TeV. Six of the eight are consistency audits — they reproduce known quantum mechanics; one is a genuine forward

prediction (the platform-independent einselection-rate test); and one is the framework’s single live, pre-registered, two-sided empirical bet — the M_{lock} correction above, within HL-LHC Run-4 reach. The empirical horizon 2026–2035 will pass the framework through this eight-test gate.

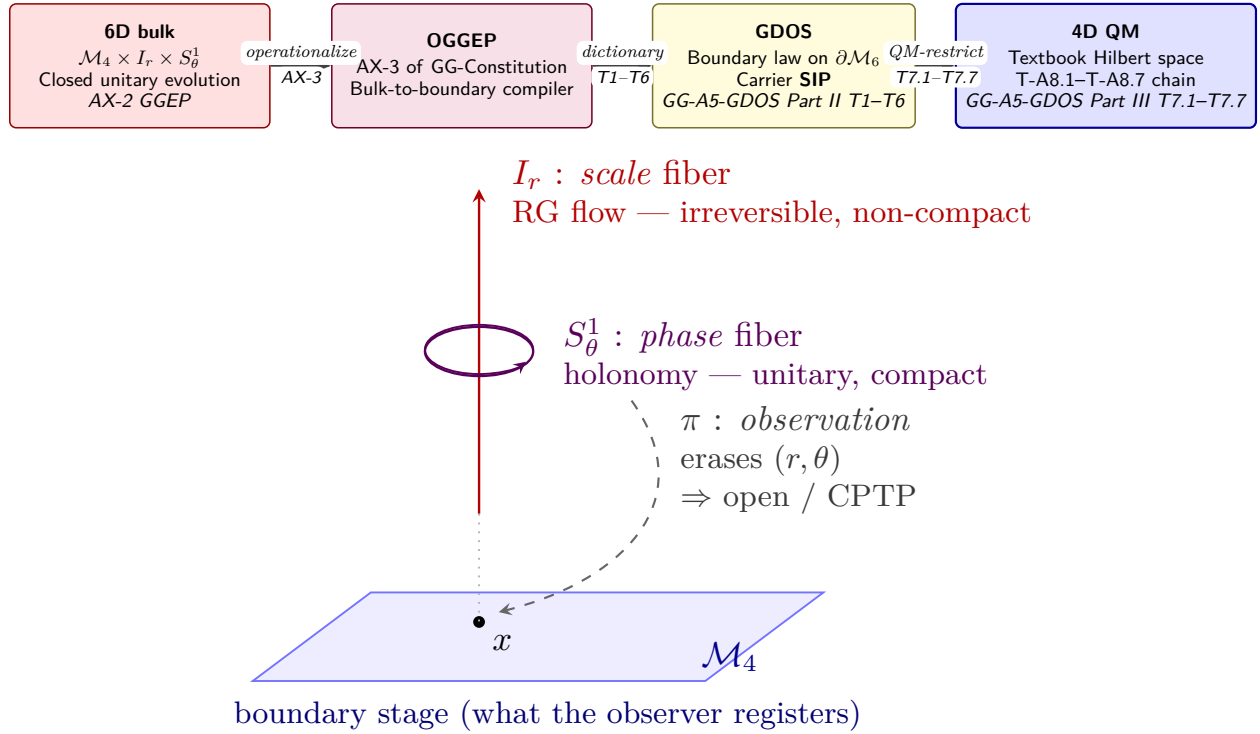
The bottom line. The paper offers a careful resolution of a century-old debate. The measurement problem, stated precisely, is three questions — why a single definite outcome, why this basis, why Born’s weights — and the framework answers each with a theorem; the fourth — whether the proportions in which outcomes actually fall reproduce Born’s rule exactly — is closed at the relaxation level, as a Constitutional Born Theorem conditional on four named premises, with the field-grade representation step the single remaining analytic residue. With that single honest residue marked, the old standoff dissolves: *Einstein was right about determinism and realism, in the six-dimensional bulk; Bohr was right about the operational completeness of the four-dimensional description. Both describe the same Nature, seen at two levels of one geometric hierarchy — with locality the price the bulk pays to keep determinism.* And the reconciliation is stronger than a truce: the bulk is deterministic *in the classical register* — constitutionally real (candidate grade), a symplectic Hamiltonian flow on real data, with no complex phase in which a superposition of alternatives could be written — and the boundary’s probabilities and its imaginary unit are manufactured together, by the one projection every observer must look through. This paper is the eighth of eleven in the Arisaka-GG series, closing the trio that derives quantum mechanics, the Standard Model, and cosmology from a single constitutional source.

The mechanism behind all of this now has one name. The runtime the series calls GRIP — the engine that stabilizes electrons and ends every relaxation at a ground state — drives the quantization, the decoherence, and the measurement alike: the projection makes the boundary description quantum, and the runtime makes the quantum outcomes classical, with observables taking discrete values because the descent can only halt at one of a discrete catalog of attractors. This thesis organizes the eleven theorems of Part IV.

None of this requires quantum mechanics to be wrong where it has been right. It asks only that its hundred-year strangeness be read as the geometry of a projection — that the randomness lived not in the world but in where we stand, and that the same geometry which hides an outcome is the one that makes every electron in the universe exactly alike. What follows lays the science out plainly, marks at every step where physics ends and interpretation begins, and leaves the last step to the reader. God does not play dice; the dice — and the imaginary unit they are rolled in — are manufactured together at the boundary, by the one-way dictionary through which any observer must look.

Keywords: GRIP-In, GRIP-Out, Geometric Attractor (GA), Ground Geometric Attractor (GGA), Born rule, Bell inequality, CHSH, EPR paradox, quantum entanglement, Schrödinger’s cat, double-slit experiment, Stern-Gerlach experiment, quantum decoherence, Lindblad equation, many-worlds, Gleason’s theorem, qubit, measurement problem, hidden variables, bulk-to-boundary projection, GG-6, OGGE, GDOS.

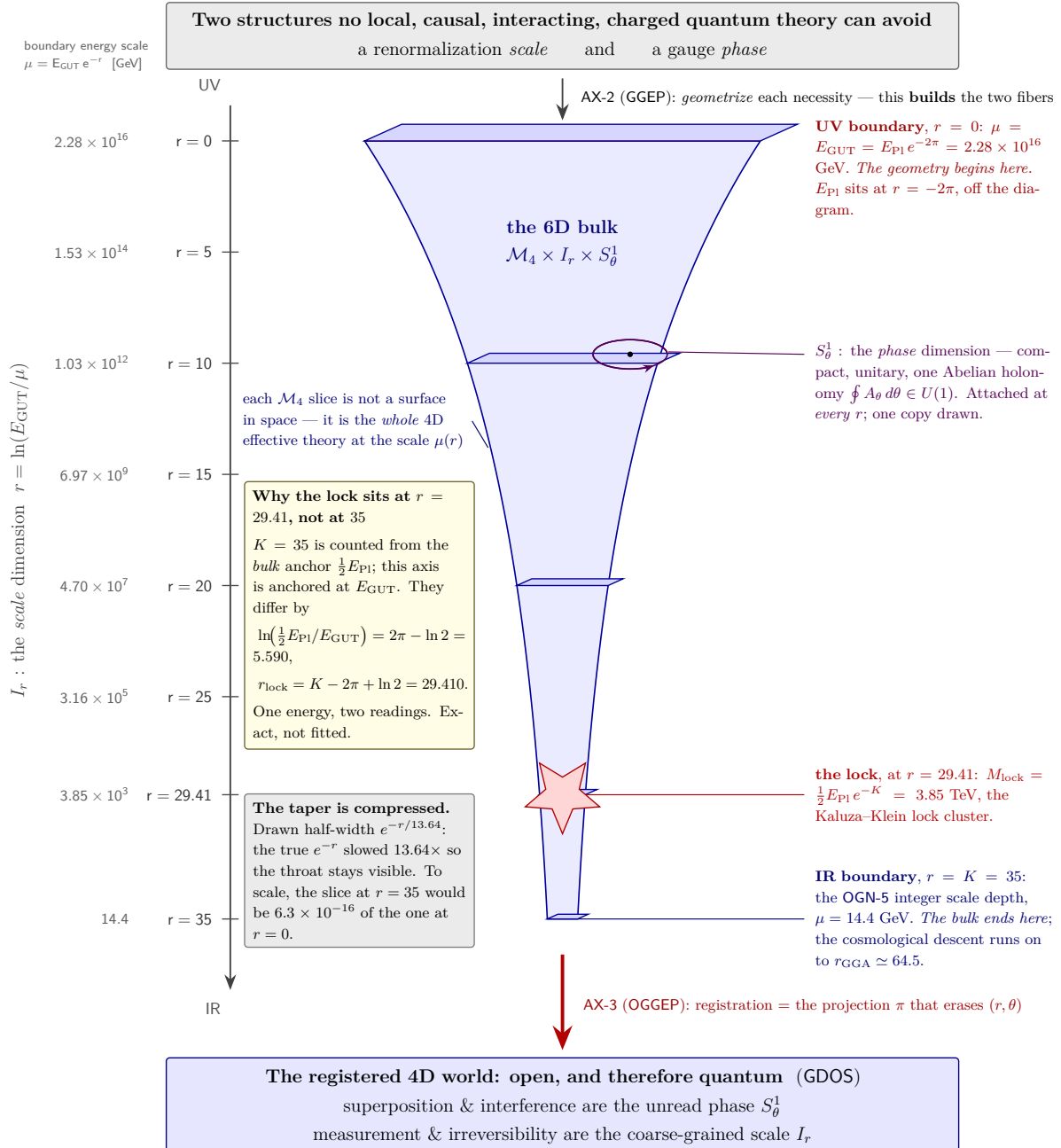
Quantum Mechanics Is Not Assumed Here. It Is the Last Stage of a Four-Stage Chain



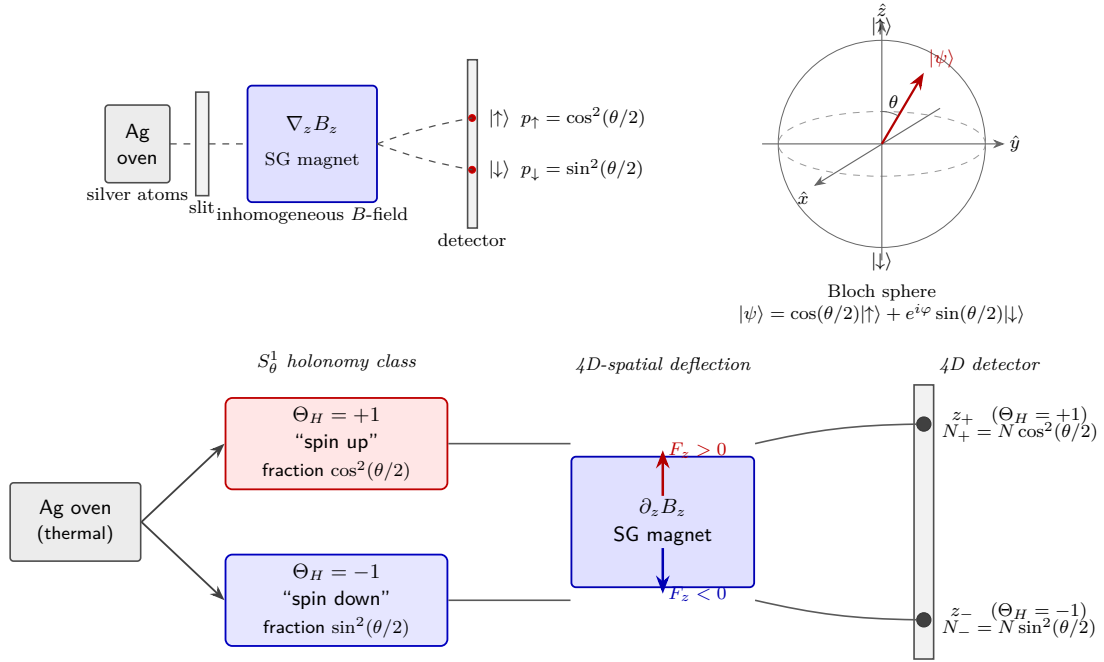
What is claimed. Read left to right: the closed unitary dynamics of the 6D geometry, compiled by OGGEF into the boundary algebra of GDOS, restricted to the quantum-relevant sector, and landing on the textbook Hilbert space a laboratory observer measures. The postulates of quantum mechanics are not inputs at any stage of that chain; they come out of the last one. Six GDOS theorems build the boundary algebra and seven more restrict it, and every worked example in Parts V–IX is this one chain run on a specific apparatus. In full: Figure 4, p. 69. **And why the chain cannot be run backwards.** The lower plate is the projection itself. Over each boundary point sits the non-spatial fiber: a compact phase circle carrying the coherent, unitary content of quantum mechanics, and a non-compact scale interval carrying the irreversible, open content. Neither coordinate is spatial, so no boundary protocol resolves them; every registration marginalizes the fiber, the projection is non-invertible, and the boundary dynamics is necessarily open. Unitary closure is the unreachable limit in which the fiber is not coarse-grained. In full: Figure 5, p. 71.

A Six-Dimensional Bulk Whose Fifth Dimension Is Scale and Whose Sixth Is Phase

The whole program in one diagram, with the scale axis carrying numbers. The bulk $\mathcal{M}_4 \times I_r \times S_\theta^1$ is a fiber bundle: over every point of the Lorentzian stage sit the two *non-spatial* fibers, scale and phase. AX-2 (GGEP) acts *upward* and builds them; AX-3 (OGGEP) acts *downward*, and registration is the projection that erases (r, θ) — the origin of quantum behavior. The vertical axis is calibrated: $E_{\text{GUT}} = E_{\text{Pl}} e^{-2\pi}$ at $r = 0$, the OGN-5 depth $r = K = 35$ at the infrared boundary, and the lock at $r = 29.41$, which is $K - 2\pi + \ln 2$ exactly.



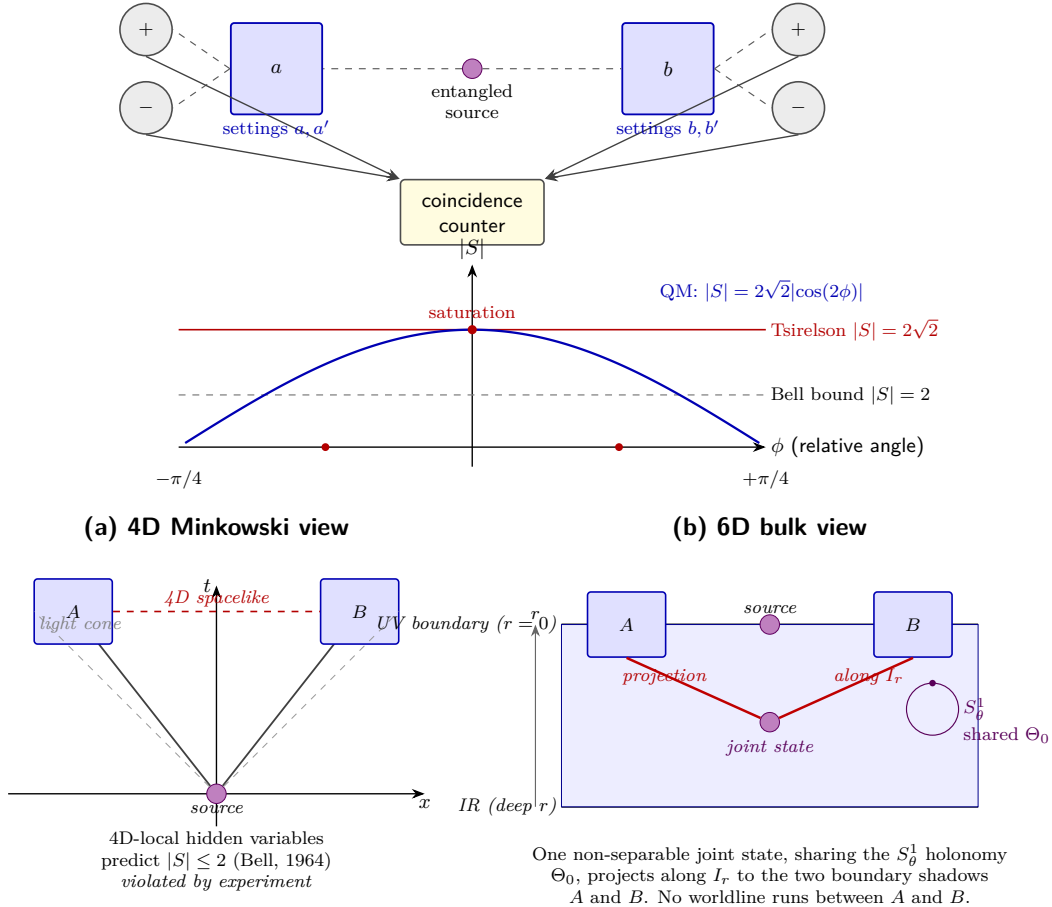
No Atom Ever Feels a Probability — the Randomness Is What the Observer Cannot Read



Each silver atom is in one discrete holonomy class of the 6D gauge bundle. The B-field gradient discriminates the two classes by 4D force; each atom lands on one spot; the histogram of N atoms reproduces the Born-rule weighting $\cos^2(\theta/2) : \sin^2(\theta/2)$ from the source population.

The oldest quantum measurement, run twice. Above: the 1922 apparatus as everyone teaches it — silver atoms out of an oven, through an inhomogeneous field, into two discrete spots — with the Bloch-sphere reading of the input qubit and the Born-rule populations $\cos^2(\theta/2)$ and $\sin^2(\theta/2)$ beside it. Below: the same experiment read on the bulk. The heated source populates two discrete holonomy classes of the gauge bundle around the phase circle, in exactly that ratio; the field gradient pushes the two classes apart in four-dimensional space; each atom lands on one spot, z_+ or z_- , according to which class it was in. *No single atom experiences probability.* The randomness a laboratory reports is the four-dimensional observer's inability to read the phase holonomy directly, and the textbook "pure state" is the 4D shadow of the source ensemble. In full: Figures 6 and 7, pp. 127 and 131.

Two Records, One Object — and Nothing Travels Between Them



What Bell measured, and what it is a shadow of. Above: the CHSH experiment and its arithmetic — any four-dimensional local hidden-variable theory is bounded by $|S| \leq 2$, quantum mechanics predicts $2\sqrt{2}|\cos 2\phi|$, and the Nobel-recognized experiments confirm the saturation at the Tsirelson bound. Below: what the pair is. It is *one* non-separable state of the bulk, sharing a single phase-circle holonomy reference fixed at the source; Alice’s event and Bob’s event are two projections of that one object along the scale direction, which carries no light cone, so no worldline runs between them and nothing has to. The shared holonomy is the operational meaning of the entanglement, and the “non-locality” of 1964 is the operational shadow of one bulk object projected to two boundary records. In full: Figures 9 and 10, pp. 151 and 155.

Eight Century-Old Paradoxes, and the One Mechanism That Dissolves All Eight

Paradox	Standard reading (4D)	6D-bulk resolution	Theorem
Wheeler delayed choice	Retrocausal wave-vs-particle choice	Bulk worldline fixed; late 4D choice selects projection only	T-A8.4
Scully quantum eraser	Apparent causality violation	Bulk phase preserved; post-selection recovers 4D shadow	T-A8.5, T-A8.7
Misra–Sudarshan Zeno	Measurement freezes evolution	Frequent Lüders projections dominate boundary record	T-A8.4
Weak measurement	Out-of-spectrum weak values	4D shadow of bulk-worldline expectation under post-selection	T-A8.3, T-A8.4
Kochen–Specker	4D-non-contextual hidden variables impossible	Bulk holonomy Θ_H is 4D-contextual, 6D-non-contextual	T-A8.6, T-A8.9
Quantum teleportation	State transported without being measured	Shared S_θ^1 phase carries state; classical bits align Pauli	T-A8.7, T-A8.9, T-A8.11
Wigner’s friend / FR	Friend observes; Wigner re-superposes	Projection completes at friend’s apparatus; Wigner is redundant	T-A8.2, T-A8.4, T-A8.5
Dense coding / QKD	2 bits via 1 qubit; eavesdrop detectable	Bulk holonomy carries 2nd bit; Tsirelson check catches Eve	T-A8.7, T-A8.8, T-A8.11

Eight paradoxes, one resolution mechanism: each is a 4D-classical question of a 6D-quantum substrate whose projection to the boundary is non-invertible. The bulk evolves unitarily; the projection completes at any sufficiently macroscopic 4D detector; the paradoxes are dissolved structurally.

What it buys, and why it is one page and not eight. Eight paradoxes of the last hundred years, each with the standard 4D reading that makes it a paradox, the 6D-bulk resolution, and the theorem that licenses that resolution. The claim of the table is the uniformity of the middle column: every one of the eight dissolves under the same mechanism, which is that the apparent contradiction is the structural signature of asking a classical question of a substrate whose projection to the boundary is non-invertible. A reading that dissolved them one at a time, by a different device each time, would be worth much less. Once this is digested the productive question is no longer which interpretation is correct, but what new operational signatures the bulk reading predicts that the 4D-only readings cannot — which is the facing page. In full: Figure 16, p. 193.

Eight Observations That Would Retire This Reading, Each With the Threshold at Which It Counts

What would end it. Eight observations, each with the deviation threshold at which it would count and the timeline on which it can be tested. Any one of them, experimentally established, retires the quantum-mechanical reading of the framework. A framework that derives quantum mechanics from a deeper geometry is in permanent danger of being unfalsifiable, because quantum mechanics is correct and any reading that reproduces it inherits its successes for free; reproducing what is already known is not evidence. These eight are the part that can be wrong, and they are written so that no interpretive question has to be settled before one of them can fire. In full: Table 17, p. 259.

Label	Observable	Deviation threshold	Timeline for test
F-QM-1	Born rule precision	Systematic dev. at 10^{-15} relative level	Penning-trap a_e at 10^{-14} (2028–2030)
F-QM-2	Compact-dim direct evidence	Failure to detect S_θ^1 at $R_\theta \sim 5 \times 10^{-20}$ m (or finding it non-compact)	LHC high-lumi, tabletop Cavendish (2030s)
F-QM-3	Super-Tsirelson CHSH	$S_{\text{CHSH}} > 2\sqrt{2} + 3\sigma$ in any loophole-free experiment	Ongoing
F-QM-4	Superluminal signaling	Any detection of entanglement-mediated classical signaling	Longstanding null
F-QM-5	MWI-specific branches	Direct detection of parallel branches (anomalous revival interference)	Not currently accessible
F-QM-6	Non-GKSL open evolution	Markovian semigroup that is CPTP but not Lindblad-form	Mathematical impossibility, but fit failures could indicate non-Markovian effects
F-QM-7	Non-Gleason probability	Additive probability measure $\neq \text{Tr}(\rho\Pi)$ on $\dim \geq 3$	Mathematical impossibility
F-QM-8	Macroscopic coherence	10^{20} -atom object in coherent superposition for $t \gg \tau_D$	Next-generation matter-wave interferometry

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Preface

Quantum mechanics is the most successful theory in the history of science and, a hundred years on, still the most puzzling. Every transistor, every laser, every magnetic-resonance scan rests on it, and its predictions have never once failed a test. Yet its founders argued bitterly about what it *means*, and the argument has never ended. Einstein could not accept that Nature plays dice; Bohr replied that we must learn to. The textbooks teach the equations and, gently, advise the student not to ask what lies beneath them.

The strangeness has many faces, and each has unsettled thoughtful people for a century. A single particle, sent toward a pair of slits, behaves as though it passed through both and interfered with itself — yet it arrives as one dot, in one place. Two particles, born together and then carried to opposite ends of a city, give perfectly matched answers the instant they are measured, with no signal able to pass between them in time. A measurement seems to *disturb* what it measures, collapsing a smooth wave of possibilities into a single hard fact — and no equation in the theory says when, or how, that collapse occurs. Schrödinger imagined a cat that the rules leave both alive and dead until someone looks. These are not separate curiosities; they are one puzzle wearing different masks, and a hundred years of brilliant interpretation — Copenhagen, many worlds, hidden variables, and the rest — has produced no agreement on which mask is the true face.

This paper asks the question the textbooks set aside, and offers an answer that is, frankly, unexpected. It is a technical paper — it proves theorems and works the standard experiments through in detail — but its central idea can be held in a single sentence. Suppose the world we measure, with its three dimensions of space and one of time, is not the whole of reality but a kind of shadow cast by a slightly larger geometry: the same four dimensions, plus two more that are not directions *in space* at all, but a direction of *scale* — the resolution at which a question is asked — and a direction of *phase* — the dial that ties matter to the forces. The surprise is deliberate. Where physics has spent fifty years hunting for extra dimensions curled up too small to see, these two are not small pieces of space at all; they are the hidden axes along which every quantum measurement is forced to average.

In that larger world the state of things is definite and lawful, the way Einstein wanted it to be. The fuzziness, the randomness, the link between distant particles, the disturbance of measurement — these are what the shadow looks like when the two extra directions are flattened out of view, as they must be for any creature, or any instrument, that lives on the four-dimensional boundary. Read this way, the century-old measurement problem stops being one impenetrable riddle and becomes three precise questions, each answered by a theorem, with a single further question left honestly open. The dice were never in the world; they were in our standpoint — thrown in two directions our senses can never turn to face.

We have tried throughout to keep one line bright: to mark where a theorem ends and an interpretation begins, to state plainly the one question that remains genuinely open rather than paper over it, and to claim nothing for the picture that experiment does not already grant. Where the framework agrees with ordinary quantum mechanics — which is nearly everywhere a laboratory can currently reach — it makes no new promise; what it offers there is understanding, not a new number. Where it does stake a forward bet, at the highest energies and on the largest scales, it says

so and names the test.

A note on reading. What follows in Part I and beyond builds the argument in full, but every Part opens with a plain-language summary, so a reader may travel as deep as they wish and stop where they like. A reader is free to take the extra geometry as literal physics or as a useful way of seeing; either way, the hope is that by the end the hundred-year strangeness of the quantum world feels a little less like a paradox and a little more like a point of view. The sharpest objections a skeptical physicist could raise are gathered and answered near the end, in the reader's own voice.

Part I

Introduction

The century-old measurement problem in plain view

1 Why quantum mechanics is the most successful physical theory in history, and its most mysterious

Quantum mechanics makes predictions more precise than any other branch of science. The electron’s anomalous magnetic moment $a_e = (g - 2)/2$ is predicted by quantum electrodynamics (QED) and measured to a relative precision of approximately 1.3×10^{-13} , which is the most precise agreement between theory and experiment in all of physics. The hydrogen-atom Lamb shift, the hyperfine structure of atomic cesium, the energy-level structure of positronium, the tunneling currents in semiconductor devices, the photoelectric effect spectrum, and the cross sections of elementary-particle collisions are all predicted by quantum mechanics at the parts-per-billion or better level.

At the same time, quantum mechanics has never quite finished settling into a mature theory. One century after Max Born’s 1926 probability rule, there is no consensus on:

- What the quantum state $|\psi\rangle$ represents — an epistemic state (a degree of belief), an ontic state (a state of the world), or something else entirely?
- What constitutes a “measurement,” as opposed to an ordinary physical interaction?
- Why quantum outcomes are distributed according to the Born rule $|\langle a|\psi\rangle|^2$ rather than, say, uniformly or linearly?
- Whether there are “hidden variables” that would restore determinism and locality, and if so, what they are?
- Whether the wave function “collapses” (Copenhagen), branches (Everett), is guided by non-local hidden variables (Bohm), is a subjective belief (QBism), or something else?
- Why the state space is *complex* — why an i sits in the Schrödinger equation — when every number any instrument has ever recorded is real?

These are the questions of the foundations of quantum mechanics. They are the subject of this Part I — and, in a single sweep, of Parts II–XIII, where every one of the six will dissolve as an artifact of viewing a deterministic, unitary, six-dimensional dynamics through the four-dimensional projection a laboratory observer is forced to use. The sixth deserves a flag at the front door, because it is the one question of this list that the cohort now closes at theorem grade: in the 6D bulk every

physical quantity is real, and the boundary’s single imaginary unit is *derived* — the projection pairs the two real structures of the arena (the compact phase circle and the causal clock) into one complex unit, at derived-conditional grade in the quasi-free regime (Arisaka, 2026, P4-Real; the full statement catalog is §117). Part I catalogs the questions in their canonical historical form. The remaining twelve Parts answer them. And because a claim of this size should be teachable, not merely assertable, the volume is equipped as a course: the five Worked-Example Parts, the paradox sweep, and the five deep proof appendices each open with a what-this-teaches box, pair themselves with the standard literature, and close with problems and worked solutions — fifty-six across the paper — so that a graduate student can work, rather than merely read, the reformulation of the hundred-year-old theory.

2 The birth of quantum mechanics: 1900–1925

2.1 Planck and the blackbody catastrophe (1900)

On December 14, 1900, Max Planck presented to the German Physical Society the formula that would begin the quantum revolution (Planck, 1900). Classical electromagnetism predicted the energy spectrum of blackbody radiation to grow unboundedly at short wavelengths — the so-called “ultraviolet catastrophe.” To fit the experimental spectrum, Planck made the formal hypothesis that radiation of frequency ν could only be emitted and absorbed in discrete packets (“quanta”) of energy

$$E_n = nh\nu, \quad n = 0, 1, 2, \dots, \quad (1)$$

where $h \approx 6.626 \times 10^{-34}$ J·s became *Planck’s constant* — the fundamental dimensional scale of all quantum phenomena. Planck himself regarded this quantization as a mathematical artifact. Einstein would soon show it was physical.

2.2 Einstein’s photon and the photoelectric effect (1905)

Albert Einstein’s 1905 “miracle year” paper on the photoelectric effect (Einstein, 1905) took Planck’s quanta literally: light itself is composed of discrete particles (later called photons) each carrying energy $E = h\nu$. The kinetic energy of photoelectrons from a metal surface illuminated by light of frequency ν follows

$$E_{\text{kin}} = h\nu - \phi, \quad (2)$$

where ϕ is the work function of the metal. This prediction was confirmed experimentally by R. A. Millikan in 1916 (Millikan, 1916). Einstein’s paper won the 1921 Nobel Prize and is arguably the founding document of quantum theory as a theory of Nature (not merely of spectra).

2.3 Bohr’s 1913 atomic model

Niels Bohr in 1913 (N. Bohr, 1913) applied the quantum hypothesis to atomic structure. His model postulated that electrons orbit the nucleus only in discrete orbits with quantized angular

momentum $L = n\hbar$, $\hbar = h/2\pi$. Transitions between orbits emit or absorb photons of energy

$$h\nu_{mn} = E_m - E_n. \quad (3)$$

The Bohr model successfully predicted the Rydberg formula for hydrogen spectral lines. Extensions by Arnold Sommerfeld (1916) added relativistic corrections and elliptic orbits. But the Bohr–Sommerfeld “old quantum theory” was an uneasy hybrid: classical orbits with quantum jumps. By 1924 it was clear that a deeper theory was needed.

2.4 De Broglie’s matter waves (1924)

Louis de Broglie’s 1924 doctoral thesis (de Broglie, 1924) proposed that any particle of momentum p has an associated wavelength

$$\lambda = h/p. \quad (4)$$

De Broglie’s proposal was experimentally confirmed in 1927 by Clinton Davisson and Lester Germer, who observed electron diffraction from a nickel crystal (Davisson and Germer, 1927). The de Broglie relation is the bridge that made the wave-particle duality quantitative. It also suggested that the quantum state should satisfy a *wave equation*.

2.5 Heisenberg’s matrix mechanics (July 1925)

Werner Heisenberg’s 1925 Helgoland paper (Heisenberg, 1925) reformulated atomic physics in terms of abstract tables of numbers (matrices) indexed by transitions between stationary states. For observables q (position) and p (momentum), Heisenberg found the non-commuting matrix relation

$$[q, p] = qp - pq = i\hbar. \quad (5)$$

Max Born and Pascual Jordan (October 1925) (Born and Jordan, 1925) and later the three-man paper by Born, Heisenberg, and Jordan (Born *et al.*, 1926) formalized matrix mechanics. Observables are represented by self-adjoint operators; commutators encode the quantum structure; time evolution is unitary. Heisenberg’s formulation was abstract, non-visualizable, and initially controversial, but mathematically precise.

2.6 Schrödinger’s wave mechanics (January 1926)

Six months later, Erwin Schrödinger (Schrödinger, 1926) presented the wave equation

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi, \quad (6)$$

for a complex-valued wave function $\psi(x, t)$. For a free particle, $\hat{H} = -\hbar^2 \nabla^2 / 2m$; with a potential, $\hat{H} = -\hbar^2 \nabla^2 / 2m + V(x)$. Schrödinger’s wave equation reproduced the hydrogen-atom spectrum exactly and provided a concrete visualizable picture — an oscillating complex wave filling space.

Schrödinger and others soon proved that matrix mechanics and wave mechanics are mathemati-

cally equivalent: they are two representations of the same abstract Hilbert-space structure. Paul Dirac in 1930 (Dirac, 1930) unified them in the “transformation theory.”

2.7 The gap: what does ψ mean?

Schrödinger initially hoped that $|\psi|^2$ represented a charge density — the electron would be literally spread out in space. This hope failed for multi-electron systems (the wave function lives in $3N$ -dimensional configuration space, not in physical 3-space). Something else had to be said about the physical meaning of ψ . Enter Born.

3 Born’s rule and the probabilistic interpretation (June–July 1926)

3.1 Born’s 1926 papers

In June 1926, Max Born published two short papers on the quantum theory of scattering (Born, 1926). Studying α -particle collisions with atoms, Born could not avoid interpreting $|\psi(x)|^2$ as a probability density:

The motion of particles follows probability laws, but the probability itself propagates in accordance with the law of causality.

Concretely, for a quantum system in state $|\psi\rangle$, the probability of obtaining outcome a in a projective measurement of observable $\hat{A}$ (with eigenvalue a and eigenvector $|a\rangle$) is

$$P(a) = |\langle a|\psi\rangle|^2. \quad (7)$$

This is the *Born rule*. For continuous spectra, $P(x)dx = |\psi(x)|^2dx$ is the probability density for position. The rule extends to mixed states ρ as $P(a) = \text{Tr}(\rho|a\rangle\langle a|)$ and to POVM measurements $\{E_\alpha\}$ as $P(\alpha) = \text{Tr}(\rho E_\alpha)$.

3.2 Einstein’s reaction: “God does not play dice”

Einstein’s famous response, in a December 4, 1926 letter to Born himself (Born, 1971):

Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the “old one.” I, at any rate, am convinced that He does not throw dice.

Einstein’s objection is not that the theory is wrong, but that it is *incomplete*. The intrinsic indeterminacy of outcomes struck him as an admission of ignorance, not a fundamental feature of Nature. Throughout his life, Einstein held that beneath the probabilistic veneer must lie a deterministic theory with “hidden variables.”

Bohr’s counter-position — the Copenhagen interpretation (Bohr, 1935) — was that the randomness is irreducible, measurement contexts are objective, and observables only have definite values in the context of specific measurements. No hidden variables exist.

This debate was not settled in Einstein’s lifetime. It was settled in 1964 by John S. Bell, and then experimentally in 1982 and again in 2015–2022, apparently in Bohr’s favor. Part III of the present report will argue that it was settled in Einstein’s favor *at the 6D level*, with Bohr correct at the 4D level.

4 Dirac, von Neumann, and the mathematical structure

4.1 Dirac’s $\langle \text{bra} | \text{ket} \rangle$ formalism

Paul A. M. Dirac (Dirac, 1930) introduced the bra-ket notation that became the standard language of quantum mechanics. A pure state is a ket $|\psi\rangle \in \mathcal{H}$, where $\mathcal{H}$ is a complex separable Hilbert space. The inner product $\langle\phi|\psi\rangle$ is a complex number; the outer product $|\psi\rangle\langle\phi|$ is an operator on $\mathcal{H}$. An observable is represented by a self-adjoint operator $\hat{A} = \hat{A}^\dagger$. Measurements of $\hat{A}$ yield one of its eigenvalues a ; immediately after measurement the state is the corresponding eigenvector $|a\rangle$ (“projection postulate”). Expectation values are $\langle\hat{A}\rangle = \langle\psi|\hat{A}|\psi\rangle$.

4.2 Von Neumann’s axiomatization (1932)

John von Neumann’s 1932 monograph *Mathematische Grundlagen der Quantenmechanik* (Neumann, 1932) gave quantum mechanics its first rigorous mathematical formulation. The postulates are:

1. **State postulate.** Every physical system is associated with a complex separable Hilbert space $\mathcal{H}$. The state is a unit vector $|\psi\rangle \in \mathcal{H}$ (or, more generally, a density operator ρ on $\mathcal{H}$ with $\rho \geq 0$ and $\text{Tr } \rho = 1$).
2. **Observable postulate.** Every physical observable is represented by a self-adjoint operator $\hat{A}$ on $\mathcal{H}$, with spectral decomposition $\hat{A} = \sum_a a \Pi_a$, where Π_a are orthogonal projectors.
3. **Born rule postulate.** The probability of obtaining outcome a is $P(a) = \text{Tr}(\rho \Pi_a)$.
4. **Evolution postulate.** Between measurements the state evolves unitarily by the Schrödinger equation $i\hbar d|\psi\rangle/dt = \hat{H}|\psi\rangle$, where $\hat{H}$ is the Hamiltonian.
5. **Projection postulate.** After a measurement yielding outcome a , the state is updated to $|\psi\rangle \rightarrow \Pi_a|\psi\rangle/\|\Pi_a|\psi\rangle\|$ (or for mixed states, $\rho \rightarrow \Pi_a\rho\Pi_a/\text{Tr}(\Pi_a\rho)$).

Von Neumann recognized immediately that postulates 4 and 5 are *not* compatible. Unitary evolution is reversible; projective measurement is irreversible. Which one applies at any given moment? Von Neumann called this the *Heisenberg cut*: it is a free parameter of the theory where the line between “system” and “apparatus” is drawn. *Any* cut location works mathematically — but there is no principle within the theory to determine where the cut actually lies.

This is the *measurement problem*. It has been open for 93 years.

4.3 The Stone theorem: unitary evolution from the postulate

Stone’s theorem (1932) (Stone, 1932) proved that every strongly continuous one-parameter unitary group $\{U_t\}$ $U_{t+s} = U_t U_s$, $U_0 = \mathbf{1}$, $\lim_{t \rightarrow 0} U_t \psi = \psi$ has the form $U_t = e^{-iHt/\hbar}$ for a unique self-adjoint generator H . This is the mathematical foundation of the Schrödinger equation. We will return to this theorem in Part II, where OGGEF extends it to the CPTP case and produces the Lindblad equation as the unique Markovian generalization.

5 The double-slit experiment: the heart of quantum mystery

5.1 The phenomenology

The double-slit experiment is, as Richard Feynman put it, “the only mystery” in quantum mechanics (Feynman *et al.*, 1965). A source emits particles (photons, electrons, neutrons, atoms, molecules up to ~ 800 -atom fullerenes (Arndt and others, 1999)) one at a time. They pass through a screen with two narrow slits and strike a detector array on the far side. The empirical observations:

1. When only one slit is open, one sees a diffraction pattern (a broad maximum behind the open slit, with small secondary fringes).
2. When both slits are open and no attempt is made to determine which slit each particle traverses, one sees an *interference* pattern — alternating bright and dark fringes whose spacing is governed by the de Broglie wavelength.
3. When a detector at one of the slits determines which slit each particle went through, the interference pattern *disappears*: the detector output is the sum of two one-slit diffraction patterns.

The interference pattern is the signature of quantum coherence. The detection at the slit destroys the coherence (“decoherence”) by entangling the which-path information with a macroscopic degree of freedom that is then traced over.

5.2 The quantum calculation

The probability amplitude for detection at position x on the screen is

$$\psi(x) = \psi_1(x) + \psi_2(x), \quad (8)$$

the sum of the amplitudes for going through slit 1 and slit 2 respectively. By Born’s rule,

$$P(x) = |\psi_1(x) + \psi_2(x)|^2 = |\psi_1|^2 + |\psi_2|^2 + 2\text{Re}(\psi_1^* \psi_2). \quad (9)$$

The interference term $2\text{Re}(\psi_1^* \psi_2)$ is what distinguishes the coherent (interference) and incoherent (sum of probabilities) regimes. When the which-path information is entangled with the environment, the interference term is suppressed by the overlap of the environment states:

$$P(x) \approx |\psi_1|^2 + |\psi_2|^2 + 2\text{Re}(\psi_1^* \psi_2) \cdot \langle E_2 | E_1 \rangle. \quad (10)$$

If $\langle E_2 | E_1 \rangle \rightarrow 0$ (environments that can distinguish path 1 from path 2), the interference disappears.

5.3 Which-way and delayed-choice variants

In Wheeler’s delayed-choice experiment (Wheeler, 1983), the decision to add or remove a which-path detector is made *after* the particle has passed the slits but before it is detected. The quantum prediction is that the pattern on the screen depends on what is done downstream, not on what the particle “did” at the slit. Scully’s quantum-eraser experiment (Scully *et al.*, 1991) adds a further twist: the which-path information is stored in a second particle, and is later erased. After erasure, the coherence can be *recovered* by suitable post-selection.

These experiments prove that quantum mechanics does not admit a “particle trajectory” interpretation, nor a “hidden which-path” interpretation. At the level of ordinary 4D space-time, quantum mechanics is genuinely non-classical.

6 The Stern-Gerlach experiment and the discovery of spin

6.1 The 1922 experiment

Otto Stern and Walther Gerlach in 1921–1922 (Gerlach and Stern, 1922) fired a beam of silver atoms through a spatially inhomogeneous magnetic field. Silver has one unpaired electron in its outermost shell, giving it a magnetic moment proportional to the electron’s total angular momentum. The classical prediction was a continuous distribution of deflections; the quantum prediction (using the old quantum theory of Sommerfeld) was a discrete set. What Stern and Gerlach saw: *two* spots, corresponding to $L_z = \pm \hbar/2$. This was the first direct evidence that the electron has intrinsic spin $s = 1/2$, discovered later theoretically by Uhlenbeck and Goudsmit (1925) (Uhlenbeck and Goudsmit, 1925).

6.2 Spin measurement as the paradigmatic quantum measurement

The Stern-Gerlach apparatus is the prototypical quantum measurement. The input is a spin-1/2 state $|\psi\rangle = \alpha|\uparrow_z\rangle + \beta|\downarrow_z\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$. The apparatus deflects $|\uparrow_z\rangle$ up and $|\downarrow_z\rangle$ down. The output beams are spatially separated (the “pointer position” now stores the spin information). The Born rule says:

$$P(\uparrow) = |\alpha|^2, \quad P(\downarrow) = |\beta|^2. \quad (11)$$

For consecutive Stern-Gerlach measurements in different directions, one finds the non-commuting structure: measuring S_z then S_x then S_z does not return the original S_z value with certainty; the second S_z measurement is 50%/50% because the intervening S_x measurement has erased the S_z information.

6.3 Back-action and the measurement postulate

Stern-Gerlach demonstrates both the discreteness of quantum outcomes and the *back-action* of measurement: measuring S_x inevitably disturbs S_z . This is inherent in the quantum formalism, not

an engineering limitation. The non-commutativity $[S_x, S_z] = i\hbar S_y$ forces it.

7 Schrödinger’s cat and the macroscopic superposition puzzle

7.1 The 1935 thought experiment

In his 1935 paper “Die gegenwärtige Situation in der Quantenmechanik” (Schrödinger, 1935), Schrödinger constructed a thought experiment to expose the tension between quantum superposition and classical macroscopic reality. A cat is placed in a sealed box with a radioactive atom, a Geiger counter, and a vial of poison. If the atom decays, the counter triggers, the vial breaks, and the cat dies. If the atom does not decay, the cat lives. After one half-life, the atomic state is

$$|\psi_{\text{atom}}\rangle = \frac{1}{\sqrt{2}}(|\text{undecayed}\rangle + |\text{decayed}\rangle). \quad (12)$$

By linearity of the Schrödinger equation, the combined state of atom + Geiger counter + poison + cat after the interaction is

$$|\psi_{\text{total}}\rangle = \frac{1}{\sqrt{2}}(|\text{undecayed}\rangle|\text{live cat}\rangle + |\text{decayed}\rangle|\text{dead cat}\rangle). \quad (13)$$

On the literal mathematical reading of the standard 4D formalism, the cat is in a superposition of alive and dead. Yet no one has ever observed such a cat. Something in the measurement process prevents macroscopic superpositions from being seen.

7.2 Why the cat is never actually seen in superposition

The modern answer — decoherence (discussed below) — is that the cat’s state is continuously entangled with the air molecules, thermal photons, and internal biochemistry of the cat itself. The timescale for decoherence of a 1-kilogram object at room temperature in air is

$$\tau_D \sim \frac{m\lambda_{\text{th}}^2}{\hbar} \sim 10^{-40} \text{ seconds} \quad (14)$$

(Joos and Zeh, 1985; Zurek, 2003). The cat is simply never in a coherent superposition long enough to be observed in one.

But this does not explain why the *outcome* is either “alive” or “dead” but not both. Decoherence yields a *mixed state* $\rho_{\text{cat}} = (1/2)|\text{alive}\rangle\langle\text{alive}| + (1/2)|\text{dead}\rangle\langle\text{dead}|$ — a probabilistic mixture. It does not yield a *single definite outcome*. This residual step is the core of the measurement problem — and, as Part VIII will show, is resolved at the Geiger counter, not at the lid of the box: the always-on projection deposits a stable, irreversible record the moment the macroscopic detector clicks — well before any human observation, and whether or not anyone ever looks — so that no live-and-dead cat superposition has ever existed in the ordinary four-dimensional sense.

8 The EPR paradox: entanglement and non-locality

8.1 The 1935 EPR paper

In May 1935, Einstein, Podolsky, and Rosen (EPR) ([Einstein *et al.*, 1935](#)) published “Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?” The EPR argument proceeds by considering a two-particle system in an *entangled state* such that measurements of position or momentum on one particle determine, with certainty, the result of the corresponding measurement on the other. Schematically, for two spin-1/2 particles in the singlet state

$$|\Psi^-\rangle_{AB} = \frac{1}{\sqrt{2}} (|\uparrow_A\rangle|\downarrow_B\rangle - |\downarrow_A\rangle|\uparrow_B\rangle). \quad (15)$$

Measurement of S_z^A yielding $+\hbar/2$ collapses the B particle to $|\downarrow_B\rangle$; measurement of S_z^A yielding $-\hbar/2$ collapses it to $|\uparrow_B\rangle$. The correlation is perfect: whenever A measures spin-up, B must measure spin-down, and vice versa.

EPR argued: if the two particles are far apart (space-like separated) at the moment of measurement, then the measurement on A cannot physically affect B (relativity). Therefore the value of B ’s spin must have been determined *before* the measurement on A . But quantum mechanics says no such predetermined value exists. Therefore quantum mechanics is incomplete. There must be “hidden variables” that specify the spins in advance.

8.2 Bohr’s response (July 1935)

Bohr replied in July 1935 ([Bohr, 1935](#)), arguing that “the very conditions which define the possible types of predictions regarding the future behavior of the system” are determined by the experimental setup. In Bohr’s reading, there is no meaningful question about B ’s “pre-measurement value” independently of the measurement context. EPR’s assumption of independent realities for A and B is, for Bohr, a classical prejudice incompatible with the quantum world.

The debate was philosophical. Neither side could offer a definitive experimental test. That would wait for Bell.

9 Bell’s theorem (1964): the experimental test

9.1 The 1964 Bell paper

John S. Bell’s 1964 paper “On the Einstein-Podolsky-Rosen paradox” ([Bell, 1964](#)) is a landmark in 20th-century physics. Bell derived an inequality that must be satisfied by *any* local hidden-variable theory, regardless of its details. Quantum mechanics, Bell showed, predicts correlations that *violate* this inequality for certain spin-measurement configurations.

Bell’s original inequality: for two particles measured in directions $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, with outcomes ± 1 , a local hidden-variable theory must satisfy

$$|E(\mathbf{a}, \mathbf{b}) - E(\mathbf{a}, \mathbf{c})| \leq 1 + E(\mathbf{b}, \mathbf{c}). \quad (16)$$

Quantum mechanics for the singlet state predicts $E(\mathbf{a}, \mathbf{b}) = -\mathbf{a} \cdot \mathbf{b}$, and for angles satisfying $\mathbf{a} \cdot \mathbf{c} = 1/2$, $\mathbf{b} \cdot \mathbf{c} = 1/2$, $\mathbf{a} \cdot \mathbf{b} = -1/2$, the quantum prediction gives $|-1/2 - 1/2| = 1$ on the left and $1 + (-1/2) = 1/2$ on the right. QM violates Bell's inequality.

9.2 The CHSH inequality (1969)

Clauser, Horne, Shimony, and Holt (Clauser *et al.*, 1969) derived a more experimentally accessible inequality. For any local hidden-variable theory with four measurement settings (two per particle, $\mathbf{a}, \mathbf{a}', \mathbf{b}, \mathbf{b}'$):

$$S_{\text{CHSH}} \equiv |E(\mathbf{a}, \mathbf{b}) + E(\mathbf{a}, \mathbf{b}') + E(\mathbf{a}', \mathbf{b}) - E(\mathbf{a}', \mathbf{b}')| \leq 2. \quad (17)$$

Quantum mechanics predicts, for optimally chosen angles, $S_{\text{CHSH}}^{\text{QM}} = 2\sqrt{2} \approx 2.828$ — Tsirelson's bound.

The CHSH inequality is the experimental workhorse. It requires only the correlation function E for four settings, not an infinite family.

10 The Aspect experiments (1982) and the 2022 Nobel Prize

10.1 First-generation experiments (1972–1982)

Stuart Freedman and John Clauser in 1972 (Freedman and Clauser, 1972) reported the first experimental violation of a Bell inequality, using photon pairs from calcium atomic cascades. The experiment was technically limited: the polarizer settings were fixed throughout data-taking, and detection efficiency was low.

Alain Aspect, Philippe Grangier, and Gérard Roger at Orsay (Aspect *et al.*, 1981, 1982,?) in 1981–1982 performed the definitive first-generation test. Their experiment used rapidly-switching polarizer orientations (on timescales faster than the light-travel time between the two wings of the apparatus), thereby closing the *locality loophole*: the measurement settings could not signal to each other.

Aspect's 1982 result: $S_{\text{CHSH}} = 2.697 \pm 0.015$, a 46σ violation of the Bell inequality, in excellent agreement with the quantum prediction.

10.2 Loophole-free experiments (2015)

Two loopholes remained after Aspect: the *detection-efficiency loophole* (low-efficiency detectors might preferentially detect certain outcomes) and the *freedom-of-choice loophole* (the measurement settings might be correlated with the hidden variables). In 2015, three independent groups (Giustina *et al.*, 2015; Hensen, 2015; Shalm and others, 2015) reported loophole-free Bell tests using atomic spins, photon polarization with the Kwiat source, and nitrogen-vacancy centers in diamond, respectively. All three achieved statistically significant violations of the CHSH inequality with all loopholes closed simultaneously.

10.3 The 2022 Nobel Prize

On October 4, 2022, the Royal Swedish Academy of Sciences awarded the Nobel Prize in Physics jointly to Alain Aspect, John F. Clauser, and Anton Zeilinger “for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science” ([Royal Swedish Academy of Sciences, 2022](#)). The citation explicitly states that these experiments established the impossibility of local-hidden-variable completions of quantum mechanics.

10.4 What Bell + CHSH + Aspect + 2022 prove

The combination of Bell’s theorem, the CHSH inequality, and the Aspect-class experiments proves:

No *local* hidden-variable theory can reproduce quantum mechanics.

Note the emphasis on “local”. The theorem does not rule out non-local hidden-variable theories (e.g., Bohmian mechanics explicitly incorporates them). And, as we argue in Part III of this report, it does not rule out theories where the hidden variables live in *additional geometric directions* of a deeper theory — I_r for scale and S_θ^1 for phase — and become apparently non-local only after the four-dimensional boundary trace integrates them out.

11 Quantum entanglement and the EPR pair

11.1 Entanglement defined

A pure state $|\psi\rangle_{AB}$ of a composite system $\mathcal{H}_A \otimes \mathcal{H}_B$ is *entangled* if it cannot be written as a product $|\alpha\rangle_A \otimes |\beta\rangle_B$. The canonical examples are the four *Bell states*:

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle), \quad (18)$$

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle). \quad (19)$$

The singlet $|\Psi^-\rangle$ has the remarkable property that it is rotationally invariant: whatever basis Alice chooses for A , the correlations with Bob’s measurements in the same basis are perfectly anti-correlated.

11.2 Entanglement is not classical correlation

Classical correlations cannot violate the CHSH inequality. Entangled quantum states can. The difference is that classical correlation exhausts the joint distribution $P(a, b|\mathbf{x}, \mathbf{y})$ with a single hidden-variable distribution; quantum entanglement produces joint distributions that cannot be factored through any classical hidden variable.

11.3 Entanglement as a resource

Since the 1990s, quantum information theory has recognized entanglement as a physical *resource*: it enables quantum teleportation (Bennett et al., 1993) (Bennett *et al.*, 1993), superdense coding, quantum cryptography (BB84, E91) (Bennett and Brassard, 1984; Ekert, 1991), and quantum computational speedups.

12 Quantum decoherence and the environment

12.1 The Zurek program

Wojciech Zurek, beginning in 1981 (Zurek, 1981), argued that the apparent “classical” behavior of macroscopic objects emerges from quantum mechanics through decoherence: the unavoidable entanglement of any quantum system with its environment. For a system S in a superposition $|\psi_S\rangle = \sum_n c_n |n\rangle_S$ coupled to an environment E with initial state $|E_0\rangle$, the joint unitary evolution produces

$$|\Psi_{SE}(t)\rangle = \sum_n c_n |n\rangle_S |E_n(t)\rangle. \quad (20)$$

The reduced density operator for S is

$$\rho_S(t) = \text{Tr}_{\text{env}} |\Psi_{SE}\rangle \langle \Psi_{SE}| = \sum_{n,m} c_n c_m^* \langle E_m(t) | E_n(t) \rangle |n\rangle \langle m|. \quad (21)$$

Off-diagonal elements ρ_{nm} are suppressed by $\langle E_m(t) | E_n(t) \rangle$. For a thermal environment or scattering-rate-dominated decoherence, these overlaps decay exponentially: $\langle E_m(t) | E_n(t) \rangle \sim e^{-\Gamma_{nm}t}$.

12.2 The einselection mechanism

Zurek’s *einselection* (environment-induced superselection) identifies the “pointer basis” as the basis in which the environment monitoring is diagonal. The pointer basis is dynamically selected by the form of the system–environment interaction. For spatial degrees of freedom interacting with air molecules, the pointer basis is approximately the position eigenbasis — explaining why macroscopic objects seem to have definite positions.

Einselection explains the *preferred-basis problem*: why is the post-decoherence mixture diagonal in position rather than in some other basis? The answer is that position-basis is what the environment monitors.

12.3 What decoherence does not solve

Decoherence explains the *transition from superposition to mixed state*. It does not explain the *transition from mixed state to single outcome*. The reduced density operator $\rho_S = \sum_n |c_n|^2 |n\rangle \langle n|$ is still a statistical ensemble, not a single outcome. Something else is needed to explain why one particular outcome is realized.

Copenhagen says: the outcome is not defined until we measure. Many-Worlds says: all outcomes are realized, in different branches. Bohmian mechanics says: the outcome was determined all along

by a non-local hidden variable. GRW says: collapse is a physical process that happens spontaneously at a rate set by a new fundamental constant. Each of these has issues. We will argue in Parts II–III that GG-Theory provides a cleaner resolution.

13 The Gleason theorem: the Born rule’s uniqueness

13.1 The 1957 theorem

Andrew Gleason in 1957 (Gleason, 1957) proved a striking mathematical fact. Consider a complex separable Hilbert space $\mathcal{H}$ of dimension ≥ 3 , and let μ be a probability measure on the closed subspaces of $\mathcal{H}$ (equivalently, on orthogonal projectors) satisfying

$$\mu(\Pi_1 + \Pi_2 + \dots) = \mu(\Pi_1) + \mu(\Pi_2) + \dots \quad (22)$$

whenever $\Pi_1, \Pi_2, \dots$ are mutually orthogonal. Then there exists a unique positive trace-class operator ρ with $\text{Tr } \rho = 1$ such that

$$\boxed{\mu(\Pi) = \text{Tr}(\rho \Pi) \quad \forall \text{ projectors } \Pi.} \quad (23)$$

13.2 Why this matters

Gleason’s theorem shows the Born rule is the *unique* probability measure compatible with the Hilbert-space structure and the additivity axiom (non-contextual probabilities for orthogonal projectors). One cannot add some other probability rule to quantum mechanics without either abandoning the Hilbert space structure or abandoning non-contextual probabilities.

13.3 Assumptions of Gleason

Gleason’s theorem *assumes*:

1. The state space is a complex separable Hilbert space of dimension ≥ 3 .
2. Observables are projectors (or PVMs).
3. Probabilities are additive over orthogonal projectors.
4. The measure is non-contextual — depends only on the projector, not on the basis.

The first two assumptions are postulates of the Copenhagen formulation of quantum mechanics. If GG-Theory derives the Hilbert space structure from the 6D bulk (see Part II), then Gleason gives Born for free. The Born rule is no longer a separate postulate.

14 The qubit and the quantum information revolution

14.1 Qubit defined

A *qubit* is a two-level quantum system. The Hilbert space is $\mathbb{C}^2$, with basis states conventionally denoted $|0\rangle, |1\rangle$. A general pure qubit state is

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1. \quad (24)$$

Equivalently, $|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\varphi} \sin(\theta/2)|1\rangle$ — parametrized by the Bloch sphere (θ, φ) . The mixed state is a density matrix $\rho = \frac{1}{2}(\mathbf{1} + \mathbf{r} \cdot \boldsymbol{\sigma})$ with Bloch vector $\mathbf{r} \in \mathbb{R}^3$, $|\mathbf{r}| \leq 1$, pure iff $|\mathbf{r}| = 1$.

Physical realizations of qubits include: electron or nuclear spin, photon polarization, atomic two-level transitions, superconducting transmon circuits, trapped ions, and nitrogen-vacancy centers in diamond. Large-scale qubit arrays (google Sycamore 2019, IBM Eagle 2021, Kipu QuEra 2023) now demonstrate fundamental tests of quantum mechanics in the multi-qubit regime.

14.2 Teleportation, dense coding, and cryptography

Quantum information theory has produced a remarkable array of protocols that rely on entanglement:

- *Quantum teleportation* (Bennett et al. 1993) (Bennett *et al.*, 1993): given one pre-shared Bell pair, Alice can transmit an arbitrary qubit state to Bob using only two classical bits of communication.
- *Superdense coding* (Bennett-Wiesner 1992) (Bennett and Wiesner, 1992): one qubit transmission + one pre-shared Bell pair carries two classical bits.
- *Quantum key distribution (QKD)*: BB84 (Bennett and Brassard, 1984) and E91 (Ekert, 1991) use quantum measurement statistics to detect eavesdroppers.
- *Shor's algorithm* (Shor, 1994) for integer factorization runs in polynomial time, threatening RSA.
- *Grover's algorithm* (Grover, 1996) for unsorted search is quadratically faster than any classical algorithm.

These are not mere curiosities. They are among the most practically consequential consequences of quantum mechanics in the 21st century.

15 The Lindblad (GKSL) master equation

15.1 Open quantum systems

For a system coupled to an environment, the reduced dynamics is generally not unitary. The environment introduces dissipation, decoherence, and relaxation. The systematic framework for this is the theory of *open quantum systems* (Breuer, 2002; Lindblad, 1983).

15.2 The GKSL theorem (1976)

Under the assumptions of (i) time-homogeneity (semigroup property $\Phi_{t+s} = \Phi_t \Phi_s$), (ii) complete positivity (physical states of $S \otimes R$ for arbitrary reference R must remain positive), and (iii) trace preservation, Gorini–Kossakowski–Sudarshan (Gorini, 1976) and Lindblad (Lindblad, 1976) independently proved that the generator of the resulting one-parameter semigroup of channels $\Phi_t = e^{tL}$ has the form

$$\boxed{\frac{d\rho}{dt} = L\rho = -i[H, \rho] + \sum_j \left(L_j \rho L_j^\dagger - \frac{1}{2} \{L_j^\dagger L_j, \rho\} \right)}. \quad (25)$$

This is the **Lindblad equation** or **GKSL master equation**. $H = H^\dagger$ is the effective Hamiltonian; the L_j are *jump operators* encoding the environmental coupling. The dissipator $\mathcal{D}\rho = \sum_j (L_j \rho L_j^\dagger - \frac{1}{2} \{L_j^\dagger L_j, \rho\})$ is the unique completely-positive generator structure.

15.3 Physical examples

Standard examples of Lindblad dynamics:

- *Spontaneous emission*: $L_1 = \sqrt{\gamma} \sigma^-$ for a two-level atom decaying at rate γ .
- *Dephasing*: $L_1 = \sqrt{\gamma_\phi/2} \sigma_z$ producing exponential decay of off-diagonal coherences.
- *Damped harmonic oscillator*: $L_1 = \sqrt{\kappa} a$ for a cavity mode decaying to zero photons.
- *Position-localized decoherence*: $L_1 = \sqrt{\Lambda} \hat{x}$ representing continuous monitoring of position, the canonical model for einselection.

Importantly, GKSL is an *effective* equation: its derivation assumes Markovian dynamics (short environmental memory). Real environments often have memory effects (non-Markovian), but GKSL captures the universal Markovian limit.

16 Interpretations of quantum mechanics

16.1 Copenhagen (Bohr, 1927)

The original “orthodox” interpretation (Bohr, 1935; Heisenberg, 1958). Quantum mechanics predicts probabilities for measurement outcomes. The wave function is not a physical entity but a summary of the observer’s information about the system. Measurement is a primitive concept; wave-function collapse is a change of the observer’s knowledge, not a physical process. The line between quantum and classical domains (Heisenberg cut) is context-dependent.

Strengths: mathematically minimal; operationally complete. Weaknesses: treats measurement as primitive; ambiguous about macroscopic superpositions; no account of entanglement in relativistic setting.

16.2 Many-Worlds (Everett, 1957)

Hugh Everett III's 1957 PhD thesis (Everett, 1957) proposed that the wave function never collapses. Every time a measurement would classically produce an outcome, the universe *branches*: in one branch the cat is alive, in another dead; in one branch the electron spin is up, in another down. All branches are equally real.

Strengths: no collapse postulate; mathematical simplicity; compatible with unitary evolution alone. Weaknesses: requires an infinite multiverse; does not derive the Born rule (why do branches have the “weights” they do?); ontologically extravagant.

16.3 Bohmian mechanics (Bohm, 1952)

David Bohm's 1952 paper (Bohm, 1952) revived an idea of de Broglie (1927): particles have definite positions guided by a “quantum potential” derived from the wave function. The particles follow deterministic trajectories. The wave function is a real, physical, objective field.

Strengths: deterministic; restores definite particle trajectories; reproduces all quantum predictions. Weaknesses: explicitly non-local; privileges position as the fundamental ontology (awkward for spin, Fock space); extension to QFT is technically subtle.

16.4 GRW spontaneous collapse (Ghirardi–Rimini–Weber, 1986)

Ghirardi, Rimini, and Weber (Ghirardi, 1986) proposed that the wave function undergoes spontaneous localization events at a low rate per particle ($\sim 10^{-16} \text{ s}^{-1}$ per nucleon in the original model). For a single particle, collapses are too rare to notice; for a macroscopic object with 10^{24} nucleons, at least one collapse per microsecond locks the object into a single classical configuration.

Strengths: defines collapse as a physical process; testable via proposed spontaneous X-ray emission, heating, or ultracold-atom interferometry. Weaknesses: introduces two new fundamental constants (λ and r_C); fine-tuning required to reproduce QM at micro scales while producing classicality at macro scales; not trivially relativistic.

16.5 QBism (Fuchs et al., 2001)

Quantum Bayesianism (QBism) (Fuchs, 2010) treats the quantum state as entirely epistemic: it represents an agent's subjective probability assignments for measurement outcomes, updated via quantum Bayesian rules. There is no ontic wave function.

Strengths: no measurement problem by construction (measurement is just Bayesian updating); compatible with subjective probability. Weaknesses: gives up the goal of describing a mind-independent reality; unclear what “agent-invariant” facts exist at all.

16.6 Consistent/decoherent histories (Griffiths, Omnès, Gell-Mann–Hartle)

(Gell-Mann and Hartle, 1993; Griffiths, 1984; Omnès, 1999) Proposes that quantum mechanics is about consistent *histories* — sequences of events for which the decoherence functional is diagonal. Probabilities are assigned to consistent histories.

Strengths: no explicit collapse; handles closed systems (cosmology). Weaknesses: many incompatible consistent-history frameworks; no unique “true” history.

16.7 Relational quantum mechanics (Rovelli, 1996)

(Rovelli, 1996) Each observer has its own quantum state; there is no observer-independent state. A quantum state is a relation between a system and an observer.

16.8 None of the above is GG-Theory

We will argue in Parts II and III that GG-Theory is closest to Bohm + Everett + Rovelli but technically different from any of them. The mechanism that produces quantum mechanics in GG-Theory is OGGEF (AX-3 of the GG-Constitution) in general — the bulk-to-boundary projection from the 6D bulk (treated as an open system) to the 4D boundary observer, produced by the boundary partial trace over the inaccessible bulk degrees of freedom. The mechanism does not require any specific gauge sector: a simpler bulk content (e.g. a pure-QED bulk with only $U(1)_{\text{EM}}$ as the gauge content, with the phase direction S^1_θ supplying KK modes and the radial direction $r = \ln(E_{\text{GUT}}/\mu)$ supplying renormalization-group modes) already produces standard Maxwell and Lorentz-force physics on the 4D boundary via the same OGGEF construction. The “hidden variables” that the standard hidden-variable program has been groping toward are therefore the bulk degrees of freedom that the 4D boundary partial trace integrates over: bulk KK modes along S^1_θ , RG modes along r , and bulk gauge-field configurations of whatever gauge content the bulk theory carries (in the cohort canonical bulk of Arisaka (2026, A2-GG6), this gauge content is $\text{Spin}(1, 5) \times \text{SU}(5) \times U(1)_B \times U(1)_{Qi}$, but the QM mechanism itself does not depend on any specific factor of this group). The $U(1)_{Qi}$ sector is optional in the GG-6/BI-6 sense: it is canonical for the full cohort but not constitutive of the QM mechanism.

The “branching” of Everett is replaced by the GSSB branch selection (a single realized branch). The observer-dependence of Rovelli is replaced by the OGGEF functor from bulk to boundary. The Bohm-style hidden variables are relocated from 4D (where Bell’s theorem forces them to be non-local) to 6D (where they are bulk-local by AX-1 of GG-A1-Constitution).

The sharpest one-line placement, developed in Section 51, is against the decoherence program and GRW. The standing objection to decoherence-only accounts is that diagonalization never picks an outcome; GRW answers by adding stochastic collapse with two new fundamental constants. GG-Theory answers structurally: the runtime’s fixed-landscape descent (GRIP-In) halts at a unique attractor, and that halt *is* the selection — delivered by machinery the theory already needs to stabilize electrons and select vacua. GG-Theory is the decoherence program completed by attractor dynamics, at zero new parameters: collapse as geometric relaxation, not stochastic surgery.

17 Quantum mechanics in 2026: successes, open questions, and the state of play

17.1 What quantum mechanics gets right

After a century, quantum mechanics has passed every experimental test. There is no known counterexample to the Schrödinger equation, the Born rule, the Heisenberg commutator, or the Lindblad equation. Quantum theory successfully predicts:

1. **Precision QED:** the electron magnetic moment a_e to $\sim 10^{-13}$ relative precision; the hydrogen hyperfine structure; the Lamb shift.
2. **Atomic physics:** all atomic spectra, hyperfine couplings, Zeeman and Stark shifts, optical-lattice clock frequencies.
3. **Molecular physics:** chemical bonding, reaction rates (with corrections from RRKM/TST), spectroscopic constants.
4. **Condensed matter:** band structure, Cooper pairing, BCS superconductivity, quantum Hall effect, topological insulators, high- T_c superconductors.
5. **Elementary particle physics:** Standard Model cross sections, decay rates, mass spectrum (as far as the SM can go).
6. **Cosmology:** Cosmic microwave background power spectrum (from inflationary fluctuations), big-bang nucleosynthesis, baryon acoustic oscillations.
7. **Quantum technology:** lasers, transistors, magnetic resonance imaging, atomic clocks, atomic force microscopy, quantum key distribution, Shor's algorithm, quantum computers.

17.2 What quantum mechanics fails to explain

But quantum mechanics, as of April 2026, does not say:

1. **Why the Born rule?** Gleason shows uniqueness; no one has derived the rule from more basic principles.
2. **When does measurement happen?** The Heisenberg cut is not fixed by the theory.
3. **What is the wave function?** Ontic (real) or epistemic (knowledge)? The question is not settled.
4. **Why exactly these three spatial dimensions?** Quantum mechanics is consistent in any dimension; why $3 + 1$?
5. **Why exactly this Hilbert space?** QM works on any Hilbert space; the choice of $\mathbb{C}^2$ for spin, $L^2(\mathbb{R}^3)$ for orbital motion, etc., is empirical.
6. **How is measurement reconciled with relativity?** The nonlocal collapse of EPR pairs is awkward in special relativity; the foundations of relativistic quantum measurement remain disputed.

7. **How does gravity fit in?** There is no agreed theory of quantum gravity. “Quantizing” the metric is perturbatively non-renormalizable.
8. **What are entanglement and non-locality, physically?** Bell’s theorem and the 2022 Nobel prize confirm that quantum correlations violate local realism, but provide no constructive account of what the correlations *are*.

These are the gaps. Part II of this report will argue that GDOS addresses items 1, 2, 3, 5, 6, and 7. Part III will argue that GG-6 addresses 4 and 8.

18 A century of quantum mystery: reading for Parts II–III

Taking stock as of April 2026: quantum mechanics is empirically perfect, foundationally incomplete. The ingredients needed to explain it from a deeper theory are:

- A **Hilbert space** — provided by the bulk zero-mode sector.
- A **probability measure** on it — provided by Gleason + CPTP channel.
- A **dynamical law** — provided by Stone’s theorem (closed system) and Lindblad (open system).
- A **measurement theory** — provided by POVM/instrument formalism and OGGEF.
- A **non-local-correlation generator** — provided by shared S_θ^1 holonomy in 6D.

All five ingredients are present in GDOS + GG-6, and together they constitute the 6D-bulk-first reading that the remainder of this paper systematically develops: the deeper theory is deterministic, unitary, and local in six geometric directions ($\mathcal{M}_4 \times I_r \times S_\theta^1$); the four-dimensional observer’s average over the two extra directions (I_r carrying scale, S_θ^1 carrying phase) is what every century of quantum-foundations work has called “measurement,” “collapse,” “decoherence,” and “entanglement.” We now turn to the construction.

19 Program dictionary: a reader’s two-page legend

This two-page dictionary is offered as a reader’s legend: a single front-matter table mapping every program-defined acronym used in the body of GG-A8-QM to a one-sentence operational definition. A reader who flips here first can decode any macro encountered in the body without flipping to Appendix A (which gives the same content in the canonical Appendix-A format inherited from GG-A1-Constitution Part VIII). The dictionary is grouped by structural role and is rendered as a single hierarchy figure (Fig. 1) so the macro relations are visible at a glance.

Framework + constitutional charter. GG-Theory = Grand Geometric Theory (the twenty-two-paper program). GG-Constitution = the constitutional charter (4 axioms + 7 postulates) of GG-Theory, supplied by GG-A1-Constitution. GG-6 = the six-dimensional bulk geometry¹ ($\mathcal{M}_4 \times I_r \times S_\theta^1$).

¹Throughout the cohort, the “6D bulk” is read on the field-theory side of the bulk–boundary correspondence: the fifth (scale) dimension is the renormalization-group axis $r = \ln(E_{\text{GUT}}/\mu)$, not a Randall–Sundrum spatial dimension, and the radial direction denotes RG running; the explicit renormalization-group construction is given in Arisaka (2026, A2-GG6).

GDOS = Geometric Dynamics of Open Systems, the boundary law forced by OGGE. BI-6 = the six-dimensional Bianchi identity $dH_3 = X_4$ (Green–Schwarz-modified). OGN-5 = Occam-Gauge-Normalization five-integer ledger. Gi-4 / Qi-4 = four-pillar structures supplied by GG-A6-GRIP and GG-A7-Qi4.

The QM tier — the feature of GG-A8-QM. QM = the operational reformulation of textbook quantum mechanics under the cohort framework: the seven inherited theorems T-A8.1–T-A8.7 (operational restatements of GG-A5-GDOS Part III’s T7.1–T7.7 chain — Hilbert-space embedding, Schrödinger as closed-system limit, Born rule, Lüders projection, decoherence, einselection, Bell–CHSH) plus the four net-new theorems T-A8.8–T-A8.11 (Tsirelson saturation as Bogomolnyi bound, universal identity of indistinguishable particles, Qi-CON-conformality of decoherence rates, boundary no-signaling from reduced-state factoring). The QM tier sits between GDOS (boundary law) and GRIP (operational cascade engine): the seven T-A8.1–T-A8.7 theorems are the QM-stratum specialization of GDOS, and they seed the GRIP cascade by establishing the canonical ground-state and the canonical state-update rule on the QM stratum.

Gi-4 pillars. TGG = Total Geometric–Gauge object. CCI = Covariant Conservation Identity. GFF = Geometric Free-Energy Functional ($F_{\text{GFF}} = F_{\text{GEP}} + F_{\text{CEP}}$). GRIP = Gauge-Runtime Invariance Pipeline, the universal cascade engine $\text{DGI} \rightarrow (\text{GFF}) \rightarrow \text{GSSB} \rightarrow \text{GA}$, whose terminus on an admissible stratum is the Ground Geometric Attractor, the unique deepest GA (GG-A6-GRIP). Two regimes (GG-A6-GRIP): GRIP-In = descent on a fixed landscape, halting at the GGA; GRIP-Out = the transient event that reorganizes the landscape itself. Their quantum-mechanical reading is the business of Section 51.

Qi-4 pillars. Qi-QUA (locked-current bookkeeping), Qi-EP (trajectory selection), Qi-CON (scaling diagnostics), Qi-HAL (memory architecture). OH / IH / EIC = Outer / Inner Hologram / Evolving Inner Cosmos.

GDOS carrier. SIP = System Instance Packet, the canonical five-field object on which GRIP runs.

Companion paper macros (sans-serif, per L39). GG-A1-Constitution = Arisaka-GG-A1-Constitution; GG-A2-GG6 = Arisaka-GG-A2-GG6; GG-A3-BI6 = Arisaka-GG-A3-BI6; GG-A4-Euclid = Arisaka-GG-A4-Euclid; GG-A5-GDOS = Arisaka-GG-A5-GDOS; GG-A6-GRIP = Arisaka-GG-A6-GRIP; GG-A7-Qi4 = Arisaka-GG-A7-Qi4; GG-A8-QM = Arisaka-GG-A8-QM (the present paper); GG-A9-SM = Arisaka-GG-A9-SM; GG-A10-Cosmos = Arisaka-GG-A10-Cosmos; GG-A11-Origin = Arisaka-GG-A11-Origin. Predictions branch: GG-P1-KK, GG-P2-N3, GG-C1-Kaxion. Capstone: [Arisaka \(planned, BookI\)](#).

For the canonical Appendix-A-format glossary (with first-introduction primary-link column and the L52-canonical formatting), see Appendix A. This front-matter Program Dictionary is the operational legend; Appendix A is the constitutional reference.

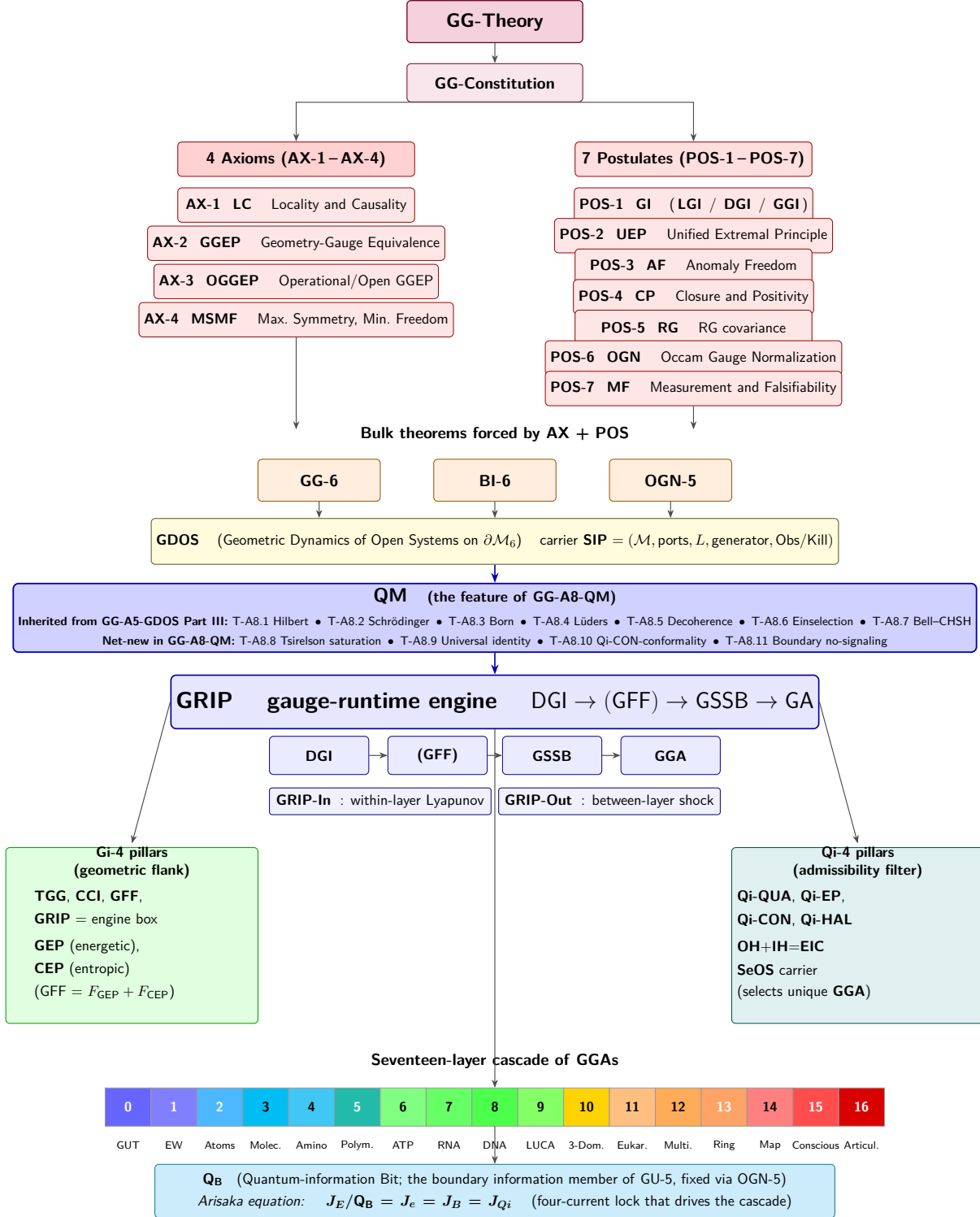


Figure 1: Hierarchy of program-defined acronyms, adapted for GG-A8-QM. The diagram is identical to GG-A11-Origin’s Figure 2 except for the highlighted blue **QM** tier inserted between GDOS and GRIP: the eleven principal theorems of GG-A8-QM (T-A8.1–T-A8.7 inherited from GG-A5-GDOS Part III plus T-A8.8–T-A8.11 net-new) are the QM-stratum specialization of GDOS and seed the GRIP cascade. Bold blue arrows mark the QM tier as the feature of this paper.

Part I in plain English: what one hundred years of quantum mystery have actually taught us

The picture that emerges from this Part is not that quantum mechanics is wrong, or that it is approximate, or that it will one day be replaced by something more familiar. Quite the opposite. Quantum mechanics is, on every empirical test, the most accurate theory the human species has ever written down. The electron's magnetic moment matches the prediction of quantum electrodynamics to twelve digits. The hydrogen Lamb shift, the atomic-cesium hyperfine structure, the cross sections of high-energy collisions, the tunneling currents in the transistor on which these words are typed — all match quantum mechanics at the parts-per-billion level or better. A theory that has survived ten thousand independent tests over a hundred years is not going to be overturned by a clever argument.

What the past hundred years *have* taught us is that the theory is operationally complete but interpretively incomplete. Born's rule predicts probabilities perfectly, but no one knows where the rule comes from. The Schrödinger equation propagates the state unitarily, but no one knows what state is being propagated, or why measurement breaks the unitarity, or when it does so. Bell's theorem and the 2022 Nobel-recognized Aspect experiments prove that the correlations between entangled particles are stronger than any classical, local hidden-variable theory permits, but provide no positive account of what the correlations *are*. Schrödinger's 1935 cat thought-experiment was meant to expose a contradiction between unitary evolution and macroscopic experience; ninety years later, no consensus on its resolution exists, and the seven living interpretations (Copenhagen, Many-Worlds, Bohmian, GRW, QBism, relational, decoherent histories) each read one feature of the puzzle correctly and elevate it to the ontology, while missing the others. The community has, as a result, declared a kind of armistice: the calculations are trusted, the questions are tabled, and the disagreement is allowed to continue.

After all, a century-long armistice is not a peace; it is a postponement. The questions cataloged in this Part — what is the wave function, when does measurement happen, why the Born rule, how do entangled particles share a fate — *have* answers, and the answers are not metaphysical preferences but theorems. The remaining twelve Parts of this paper supply those theorems. Part II assembles the cohort inheritance from GG-A1-Constitution–GG-A11-Origin that makes the answers possible. Part III states the constitutional bridge GG-6 bulk $\rightarrow$ OGGE $\rightarrow$ GDOS $\rightarrow$ 4D QM that compiles the deterministic six-dimensional dynamics into the operational rules a laboratory observer measures. Part IV states the eleven principal theorems whose proofs occupy Parts V–IX (worked examples), Part X (paradox sweep), Part XI (interpretation synthesis), and Part XII (Discussions). Part XIII delivers the summit view.

After all, the right way to honor one hundred years of quantum mystery is not to celebrate it as permanent, but to finish it.

20 The cohort inheritance and the roadmap of the sixteen Parts

Before the inheritance ledger begins (Part II), it is useful to lay out two reader-orientation maps on the same opening: a panorama of the twenty-two-paper cohort in which this paper stands (§20.1, **Figure 2**), and a panorama of the sixteen Parts of the present paper (§20.2, **Figure 3**). A reader of any later Part should be able to flip back to this Section and orient themselves in both directions: which cohort companion carries which imported result, and where the present argument stands in the sixteen-Part arc.

20.1 The cohort inheritance

The present paper is one of twenty-two. GG-Theory is written not as a single monograph but as a cohort of twenty-two preprints — eleven A-papers, six P-papers, three C-papers and two white papers — each carrying one rigorously stated piece of a single descent that runs from four axioms and seven postulates to the empirical structure of physics. The partition into twenty-two papers reflects the partition of subject matter, not a ranking of importance: the framework is one program, and each paper is a named carrier of named content. **Figure 2** is the panorama; a reader meeting the program for the first time can read its architecture off the figure top to bottom before returning to the quantum mechanics that is the business of this paper. The present paper is unusual in the cohort in devoting an entire Part to what the panorama compresses: the per-paper inheritance ledger — what GG-A8-QM receives from each of GG-A1-Constitution through GG-A11-Origin, one section per companion — is Part II, so the reader who wants the custody of any single imported result reads the panorama here and the ledger there.

The cohort map (**Figure 2**, closing this subsection) is complete at twenty-two papers. Beyond the eleven-paper A-series spine cataloged paper by paper in Part II, the program’s phenomenology is carried by the Predictions Branch — GG-P1-KK (Arisaka, 2026, P1-KK), GG-P2-N3 (Arisaka, 2026, P2-N3), GG-P3-Electron (Arisaka, 2026, P3-Electron), GG-P6-Proton (Arisaka, 2026, P6-Proton) — the Cosmology & Astrophysics Branch — GG-C1-Kaxion (Arisaka, 2026, C1-Kaxion), GG-C2-Hubble (Arisaka, 2026, C2-Hubble), GG-C3-BH (Arisaka, 2026, C3-BH) — and the White-Paper Branch — GG-W1-Symmetry (Arisaka, 2026, W1-Symmetry), which walks the century from Noether to Hopkins–Singer, and GG-W2-Holonomy (Arisaka, 2026, W2-Holonomy), which classifies protection by topology rather than by symmetry, the ladder on which this paper’s unreadable phase fiber is one rung; the present paper (Arisaka, 2026, A8-QM) is the quantum-mechanics member of the 4D-Physics trio.

The Foundations Branch. The third framed branch is the Foundations Branch: the two member papers whose subject is the boundary’s complex structure and its mirrors. For the present paper this branch is not a distant flank but a working partner, because GG-A8-QM is GG-P4-Real’s principal consumer in the cohort. GG-P4-Real (Arisaka, 2026, P4-Real) is the origin-of-*i* member paper, deriving the single imaginary unit of boundary physics from two real structures of the GG-A2-GG6 arena — the compact phase circle and the causal clock; the real-bulk series RB-1–RB-13 of the Discussions Part (§117) develops in-paper, at discussion grade, what GG-P4-Real treats at standalone

length, under the division of labor both papers state in the same words: the dictionary owns the sectors; the geometry owns the *i*. GG-P5-CPT (Arisaka, 2026, P5-CPT) carries the three mirrors of matter — C, P, and T read as orientation reversals of the three real bulk factors — and its law-level arrow of time is kin of GG-A8-QM’s arrow of measurement; its Wick-quartet statistics machinery is composed by the candidate theorem T-A8.12 (§52). Both carry P-series designators; their place at the spine’s flank, not the phenomenology rows, marks their foundational role.

The twenty-two-paper map, identical in every paper of the cohort, is **Figure 2**, which closes this subsection.

20.2 Roadmap of the sixteen Parts

The rest of the paper is organized into fifteen further Parts, and the walk through them is plotted in **Figure 3**. The present Part — the entrance — is nearly done: the century of quantum mystery from Planck’s blackbody catastrophe to the 2022 Nobel Prize, the ledger of what quantum mechanics gets right and what it fails to explain, the program dictionary, and the two orientation maps of this closing section. Everything after this page is the reformulation itself, and the itinerary below says where each piece lives.

The entrance conditions come first. Part II is the inheritance ledger: one section per cohort companion, cataloging what GG-A8-QM receives from GG-A1-Constitution through GG-A11-Origin — the constitutional rules, the bulk geometry, the GDOS boundary law with its seven QM-postulate theorems, the GRIP/Qi-4 runtime, the universal-identity insight — so that every later claim can name its custody in one flip. Part III is the apex, and it is highlighted in the roadmap because it delivers the paper’s central mechanism: the constitutional bridge, the operational chain GG-6 bulk $\rightarrow$ OGGE $\rightarrow$ GDOS $\rightarrow$ 4D QM that compiles the deterministic six-dimensional bulk into the operational rules a laboratory observer measures. Every earlier page prepares that chain, and every later Part exercises it.

Part IV is the formal statement of results: the eleven principal theorems of GG-A8-QM, with T-A8.1–T-A8.7 inheriting verbatim from GG-A5-GDOS Part III’s T7.1–T7.7 chain — the textbook postulates recovered as theorems — and the four net-new theorems T-A8.8–T-A8.11 covering the extremal (Bogomolnyi-analogue) reading of CHSH saturation, the universal-identity reading of indistinguishable-particle statistics, the Qi-CON reading of decoherence-rate conformality, and the reduced-state no-signaling theorem; the candidate theorem T-A8.12 (Fermi–Dirac statistics as boundary inheritance), stated beyond the eleven, closes the Part.

Five worked examples then run the machinery on the laboratory’s canonical experiments: Stern–Gerlach (Part V), the double slit and decoherence (Part VI), EPR, Bell, and CHSH (Part VII), Schrödinger’s cat in 6D (Part VIII), and decoherence–einselection–pointer-basis selection (Part IX). Part X sweeps the remaining paradoxes — Wigner’s friend, Frauchiger–Renner, delayed choice, the quantum eraser, Zeno, contextuality, teleportation — and Part XI surveys the seven living interpretations and locates the present reading among them.

Two Parts then weigh the case. Part XII is the Discussions: significance, the comparison with the seven interpretations, the why-complex-numbers question with the real-bulk series RB-1–RB-13, the QM-specific eight-test empirical horizon, and the open questions stated as open. Part XIII draws

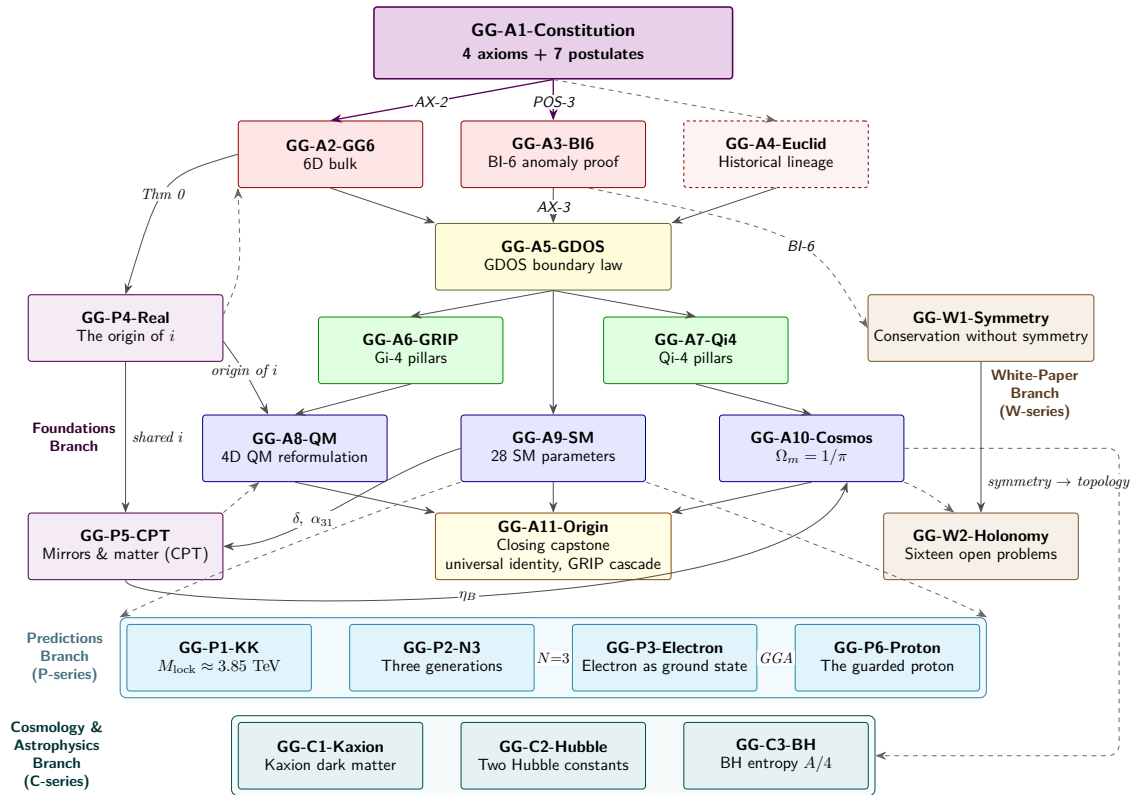


Figure 2: The twenty-two-paper cohort map of GG-Theory. The constitutional root GG-A1-Constitution (violet, top) supplies the four axioms and seven postulates from which the rest of the cohort descends: the GG6 Trio (GG-A2-GG6, GG-A3-BI6, GG-A4-Euclid, red) realizes the bulk geometry; GG-A5-GDOS (yellow) the boundary law; the GDOS Trio pillars (GG-A6-GRIP, GG-A7-Qi4, green) the geometric and quantum-information runtime; the 4D-Physics Trio (GG-A8-QM, GG-A9-SM, GG-A10-Cosmos, blue) the textbook readouts; and the closing capstone GG-A11-Origin (gold) the quarks-to-consciousness identity cascade. Two framed branches carry the phenomenology — the **Predictions Branch** (cyan), whose four papers hand their results along the row by the internal arrows (GG-P2-N3’s $N_g=3$ into GG-P3-Electron, and that paper’s ground-attractor method, GGA, on into GG-P6-Proton), and the **Cosmology & Astrophysics Branch** (teal). The spine is flanked by two columns of two, which do opposite jobs. On the left the **Foundations Branch** (violet) feeds it: GG-P4-Real, fed by GG-A2-GG6’s Theorem 0 arena, manufactures the i that GG-A8-QM consumes, and GG-P5-CPT is the GG-A9-SM–GG-A10-Cosmos interface, fed by the ledger phases and feeding the η_B chain across the width of the map, with the shared- i edge descending solid between them. On the right the **White-Paper Branch** (brown) reads it: GG-W1-Symmetry walks the century from Noether to Hopkins–Singer, and GG-W2-Holonomy dissects the structure that arc arrives at — protection by topology rather than by symmetry — across sixteen open problems. The white papers derive nothing new, carrying every result at its parent’s declared grade; the single dashed feed drawn into each names its heaviest source, not its only one. *Reading conventions:* solid arrows mark axiom-forced descent and carry an edge label naming the driving axiom or postulate where most illustrative; dashed arrows mark the deployment of derived results, or historical links inherited without being constitutionally forced.

the conclusions: the summit view — *Einstein right in 6D; Bohr right in 4D* — the universal-identity reading of why every electron is the same, and the fifty-year arc with the next decade of empirical tests.

The back matter is the workshop. Part XIV collects the technical appendices: the codebook, the glossary, the proof sketches, and the theorem index. Part XV answers the strongest objections a foundations-of-quantum-mechanics referee, a particle/string theorist, a condensed-matter physicist, an AMO experimentalist, and a historian and philosopher of quantum mechanics would raise, with pointers to the theorem, falsifier, or companion paper that bears the weight. Part XVI closes with the annotated references, each carrying a Relevance note. Read in order, the sixteen Parts compose into a single self-contained construction that a careful graduate student can audit step by step; read by need, each Part names its imports and can stand a visit on its own.

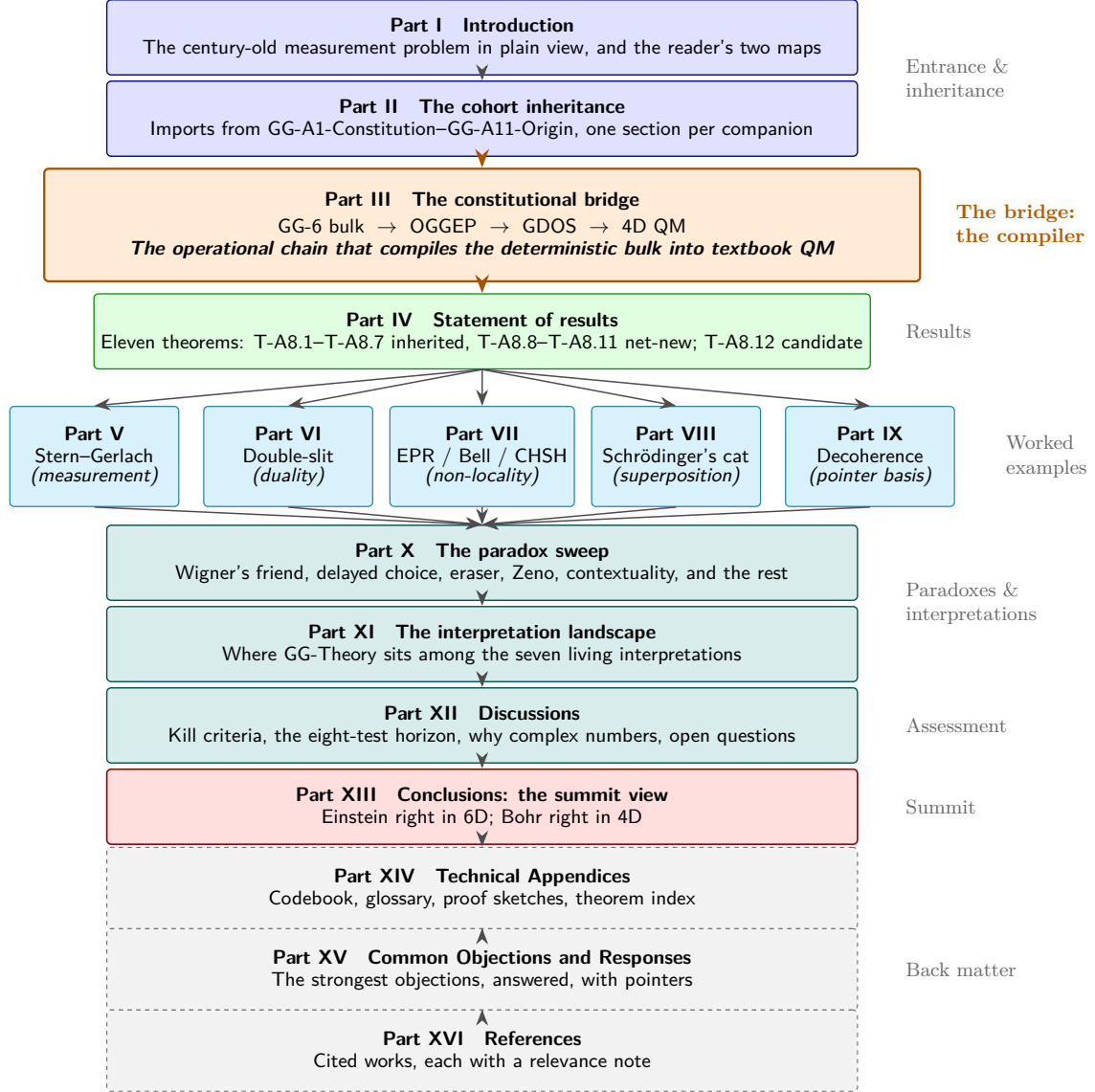


Figure 3: Roadmap of the sixteen Parts of GG-A8-QM. The argument develops in seven phases. *Entrance and inheritance* (Parts I–II): the century-old measurement problem in plain view, then the per-paper inheritance ledger from GG-A1-Constitution through GG-A11-Origin. *The bridge* (Part III, highlighted): the constitutional chain GG-6 bulk $\rightarrow$ OGGE $\rightarrow$ GDOS $\rightarrow$ 4D QM that compiles the deterministic six-dimensional bulk into textbook quantum mechanics. *Results* (Part IV): the eleven principal theorems, T-A8.1–T-A8.7 inherited verbatim from GG-A5-GDOS Part III and T-A8.8–T-A8.11 net-new, with the candidate T-A8.12 beyond the eleven. *Worked examples* (Parts V–IX): Stern–Gerlach, double-slit, EPR/Bell/CHSH, Schrödinger’s cat, and decoherence–einselection, each exhibiting the theorems on a concrete laboratory system. *Paradoxes and interpretations* (Parts X–XI): the paradox sweep, then the seven living interpretations. *Assessment and summit* (Parts XII–XIII): the Discussions — kill criteria, the eight-test empirical horizon, the real-bulk series — then the conclusions, *Einstein right in 6D*; *Bohr right in 4D*. The three back-matter Parts collect the technical appendices (Part XIV), the common objections and responses (Part XV), and the annotated references (Part XVI). Part III is highlighted because the constitutional bridge delivered there is the paper’s central claim — the operational chain that compiles the bulk into the textbook rules — on which the eleven theorems of Part IV, the five worked examples, and every downstream reading stand.

Part II

The cohort inheritance

What GG-A8-QM receives from GG-A1-Constitution through GG-A11-Origin, and where it sits among the twenty-two papers

This Part is the inheritance ledger of GG-A8-QM. The 6D-bulk-first reading that drives the rest of the paper — and that resolves the measurement problem (via the always-on bulk-to-boundary projection of OGGEF, GG-A5-GDOS §23), the preferred-basis problem, the Born-rule problem, the non-locality problem, every paradox of foundations, and every interpretive dispute (as the worked examples of Parts V–IX, the paradox sweep of Part X, the interpretation synthesis of Part XI, and the empirical-horizon audit of Part XII establish) — does not originate in GG-A8-QM. Every structural ingredient GG-A8-QM uses is already proved or constructed in one of the ten upstream cohort papers. This Part assembles those inheritances in one place so the reader can see, on a single page, what GG-A8-QM owes to each cohort companion and what it does not.

The structure of the inheritance is by paper, in cohort order. One section per cohort companion, then a summary section (the twenty-two-paper cohort map itself stands at the close of Part I, §20.1): the GG-Constitution of GG-A1-Constitution (the four axioms and seven postulates from which everything else descends); the bulk-geometry block (GG-A2-GG6’s six-dimensional bulk $\mathcal{M}_4 \times I_r \times S_\theta^1$, GG-A3-BI6’s BI-6 anomaly closure, GG-A4-Euclid’s Euclidean discipline forcing zero free parameters); the GDOS block of GG-A5-GDOS including the seven QM-postulate theorems T7.1–T7.7 that GG-A8-QM lifts to graduate-textbook depth as T-A8.1–T-A8.7; the GRIP/Qi-4 machinery of GG-A6-GRIP and GG-A7-Qi4 that supplies the bulk-side dynamics and the admissibility filter; the universal-identity insight of GG-A9-SM Part XII that grounds the every-electron-is-the-same reading; the cosmological specialization of GG-A10-Cosmos; and the four-move operational anatomy of GG-A11-Origin that GG-A8-QM specializes to the QM stratum.

21 The GG-Constitution of GG-A1-Constitution: four axioms and seven postulates

The foundational paper of the cohort, Arisaka-GG-A1-Constitution ([Arisaka, 2026, A1-Constitution](#)), sets the constitutional rules. These rules are not specific to quantum mechanics; they constrain every paper of the program. GG-A8-QM’s job is to specialize the rules to the microscopic-observable sector. We state the rules in plain language and then identify which are load-bearing for what follows.

21.1 The four axioms

GG-A1-Constitution §2 introduces four axioms that any admissible physical theory must satisfy.

- **AX-1 — Local Causality (LC).** Physical influence propagates in the forward time direction at finite speed. Operationally: no closed timelike curves, no superluminal signaling, no acausal feedback. In any admissible 6D bulk theory, the bulk Hamiltonian density $\mathcal{H}_{\text{Bulk}}(x, r, \theta)$ is a polynomial in bulk fields and their derivatives at the same bulk point, with no integro-differential or non-local kernels. For GG-A8-QM, AX-1 is the source of the forward-time semigroup property of the boundary channel $\Phi_{t,s}$ that forces the Lindblad / GKLS structure of the master equation (§33); and the source of the position-diagonal nature of the boundary system–environment coupling that selects the position basis as einselected (T7.6).
- **AX-2 — Geometry–Gauge Equivalence Principle (GGEP).** In the six-dimensional bulk, gravity and gauge fields are two aspects of one composite connection. This is the closed-bulk equivalence principle: the bulk theory is a single geometric object, not a collection of separate gauge and gravitational sectors propagating on a fixed background. GG-A8-QM inherits AX-2 indirectly — through its open-system extension AX-3 — and uses the composite-connection structure to identify the bulk degrees of freedom that the boundary observer traces over (the heavy KK modes and the S_θ^1 holonomy).
- **AX-3 — Operational/Open Geometry–Gauge Equivalence Principle (OGGEP).** Any admissible bulk theory must come equipped with a bulk–boundary correspondence, a structure-preserving functor $\mathcal{D} : \mathbf{Bulk} \rightarrow \mathbf{Op}$ that maps bulk states to operational records produced by laboratory instruments. The functor must respect causality (from AX-1), physicality (positivity, i.e. the CPTP property), and coarse-graining (scale consistency). GG-A1-Constitution states this as an axiom rather than a theorem because it is a constitutional condition on what counts as a complete physical theory: a bulk theory without an explicit $\mathcal{D}$ does not yet describe anything a laboratory can measure. AX-3 is the central import of GG-A8-QM: every result in the seven T7.1–T7.7 chain is a structural consequence of AX-3 once specialized to the QM-restricted boundary algebra.
- **AX-4 — Maximum Symmetry, Minimum Freedom (MSMF).** The constitutional input ledger contains the smallest number of independent inputs compatible with self-consistency. GG-A4-Euclid sharpens this to the zero-free-continuous-parameter statement: the input ledger splits into three forced groups (the five-element GU-5 $\{c, G_N, \hbar; k_B, Q_B\}$, the two topological constants π and e , and the proved OGN-5 integer ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$), with zero free continuous parameters.² For GG-A8-QM, MSMF is the source of the determinism of the 6D bulk evolution: with no free parameters there is no stochastic input to randomize the bulk dynamics, and the probabilistic appearance of the 4D shadow can be read entirely as a feature of the non-invertible projection rather than as a feature of the bulk.

²Notation, kept uniform across the cohort: N is the OGN-5 ledger’s second entry, and every ledger identity is written with it. $N_c = 3$ is the color triplicity, the three colors of $\text{SU}(3)_C$ — what that entry counts, named where the color reading is the point. $N_g = 3$ is the generation (family) count; it is *not* a ledger entry and is not fixed by the ledger (GG-P2-N3). $N_f = 2N_g = 6$ is the quark-flavor count of a beta function at high scale. Three of these are three and one is six, and they are three different quantities.

21.2 The seven postulates

GG-A1-Constitution §3 introduces seven postulates POS-1–POS-7 that specialize the four axioms to specific structural commitments. The seven postulates and their constitutional content are:

- POS-1 GI (Layered Gauge Invariance): each layer of the cascade inherits the gauge structure of the layer above, with the boundary algebra inheriting from the bulk gauge connection.
- POS-2 UEP (Unified Extremal Principle): the dynamical content of the framework is fixed by a single variational principle acting on the composite bulk–boundary structure.
- POS-3 AF (Anomaly Freedom): the bulk geometry must satisfy the Green–Schwarz-modified Bianchi identity BI-6 of GG-A3-BI6, with the unique coefficient vector $(1, -1, -2/5, -2/5)$ fixed by the topological factorization $I_8 = \frac{1}{2}X_4^2$.
- POS-4 CP (Closure and Positivity): admissible boundary channels must be completely positive and trace-preserving; this is the constitutional source of the CPTP structure that T-A8.3 and T-A8.4 of the present paper exercise.
- POS-5 RG (Scale Covariance): the functor $\mathcal{D}_{\text{OGGEP}}$ commutes with renormalization-group rescaling along the radial direction r of GG-A2-GG6; this is the constitutional source of the Qi-CON-conformality that T-A8.10 exercises.
- POS-6 OGN (Occam Gauge Normalization): the input ledger admits no hidden continuous knobs. Together with MSMF (AX-4), this enforces the zero-free-continuous-parameter discipline of GG-A4-Euclid.
- POS-7 MF (Measurement and Falsifiability): each admissible bulk theory must come equipped with an operational recipe by which laboratory measurements probe the bulk content. This is the constitutional source of the instrument formalism that T-A8.3 and T-A8.4 exercise.

For GG-A8-QM, the most directly load-bearing postulates are POS-4 CP (the closure-and-positivity postulate, which forces the CPTP structure of the boundary channel and therefore drives T-A8.3 and T-A8.4); POS-7 MF (the measurement-and-falsifiability postulate, which forces the instrument formalism on which the whole eleven-theorem inventory of Part IV rests); and POS-1 GI (the layered gauge-invariance postulate, which forces the QM-restricted boundary algebra to inherit gauge structure from the bulk). The remaining four postulates enter indirectly through the upstream cohort papers: POS-2 UEP underwrites the variational structure that produces the GKLS generator of T-A8.2; POS-3 AF fixes the BI-6 closure inherited from GG-A3-BI6 (used only in cross-paper consistency checks for the QM stratum); POS-5 RG drives the conformality prediction of T-A8.10; POS-6 OGN underwrites the zero-free-parameter character of every numerical prediction in the eight-test horizon.

21.3 What GG-A8-QM takes from GG-A1-Constitution

In one sentence: GG-A8-QM takes the constitutional warrant. The seven theorems T-A8.1–T-A8.7 (operational restatements of T7.1–T7.7 of GG-A5-GDOS Part III) do not rest on auxiliary hypotheses

introduced inside GG-A8-QM; they rest on AX-1 (forward time, locality), AX-3 (the OGGEF functor), POS-4 (closure and positivity, which forces CPTP), and POS-7 (measurement and falsifiability, which forces the instrument formalism), all imported from GG-A1-Constitution. The remaining axioms and postulates contribute through the bulk-geometry block of the next section.

22 The bulk-geometry block: GG-A2-GG6 (GG-6), GG-A3-BI6 (BI-6), GG-A4-Euclid (Euclidean discipline)

The three foundation papers that specify the geometric stage on which the cohort is set.

22.1 GG-A2-GG6: the 6D bulk

Arisaka-GG-A2-GG6 (Arisaka, 2026, A2-GG6) constructs the 6D bulk GG-6 as the unique geometry compatible with the GG-Constitution. The metric is

$$ds_6^2 = e^{-2r} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + d\theta^2, \quad r \in [0, K], \quad \theta \in [0, 2\pi), \quad (26)$$

with composite gauge-gravity group $G_{\text{tot}} = \text{Spin}(1, 5) \times \text{SU}(5) \times \text{U}(1)_B \times \text{U}(1)_{Q_i}$ and the proved OGN-5 integer ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$. The scale profile e^{-2r} realizes the radial direction r as the renormalization-group scale, and the compact circle S_θ^1 supplies the topologically protected Wilson-line holonomy Θ_H that GG-A8-QM will read in Part VIII as the geometric substrate of the “hidden variable” that the standard hidden-variable program has been groping toward since de Broglie 1924 and Bohm 1952. GG-A8-QM imports the bulk metric (26), the KK mode expansion of bulk fields, and the S_θ^1 holonomy structure verbatim; no re-derivation is performed.

The eight theorems of GG-A2-GG6. Part II of GG-A2-GG6 proves eight named theorems T1–T8 that together force the 6D bulk to its unique cohort-canonical form: T1 (Dimensional Minimality: the bulk must be six-dimensional); T2 (the Canonical Metric (26)); T3 (Six Forces with Three Abelian Gauge Invariances, treated below); T4 (the BI-6 Coefficient Vector $(1, -1, -2/5, -2/5)$, treated in §22.2); T5 (Single-Tensor Anomaly Factorization $I_8 = \frac{1}{2}X_4 \wedge X_4$); T6 (the OGN-5 Diophantine Cascade $(1, 3, 5; 7, 35)$); T7 (BRST Consistency and the homogeneous Slavnov–Taylor identity); and T8 (the Arisaka Equation and Quadruple Gauge Locking, treated below). GG-A8-QM inherits all eight theorems verbatim; the two with direct operational consequences for the QM register are T3 (the three-abelian structure that locates $\text{U}(1)_{Q_i}$ as one of three gauge factors, not the unique substrate) and T8 (the Arisaka Equation, which constrains the boundary currents that any quantum measurement registers).

T3: Six forces with three abelian gauge invariances. GG-A2-GG6 T3 records the boundary gauge structure in two forms (T3 of Arisaka, 2026, A2-GG6). The high-energy boundary group, before electroweak symmetry breaking, is the six-factor product $G_{\text{bdy}}^{\text{hi}} = \text{Spin}(1, 3) \times \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_B \times \text{U}(1)_{Q_i}$, with three abelian factors ($\text{U}(1)_Y$, $\text{U}(1)_B$, $\text{U}(1)_{Q_i}$). After electroweak symmetry breaking and the identification of $\text{U}(1)_{\text{EM}} \subset \text{SU}(2)_L \times \text{U}(1)_Y$ as the unbroken

combination, the low-energy boundary group is

$$G_{\text{bdy}}^{\text{lo}} = \underbrace{\text{Spin}(1, 3)}_{\text{gravitational}} \times \underbrace{\text{SU}(3)_C}_{\text{strong}} \times \underbrace{\text{U}(1)_{\text{EM}}}_{\text{electromagnetic}} \times \underbrace{\text{U}(1)_B}_{\text{baryonic}} \times \underbrace{\text{U}(1)_{Q_i}}_{\text{informational}}, \quad (27)$$

with the broken weak combination registered as the short-distance residue of $\text{SU}(2)_L$ now hidden inside the $\text{U}(1)_{\text{EM}}$ factor and the massive $W^\pm, Z^0$ gauge bosons. Three abelian gauge invariances — $\text{U}(1)_{\text{EM}}$, $\text{U}(1)_B$, and $\text{U}(1)_{Q_i}$ — survive at all energies and constitute the “three-abelian” structure that GG-A2-GG6 T3 names. This three-abelian structure is structurally important for the operational reading of GG-A8-QM: it locates $\text{U}(1)_{Q_i}$ as one specific abelian gauge factor (the “informational” force) among three, not as the unique substrate of the hidden-variable structure. The *QM mechanism itself* — the bulk-to-boundary projection from the 6D bulk to the 4D boundary via OGGEF (AX-3) — does not depend on any specific gauge factor; a simpler bulk (e.g. pure $\text{U}(1)_{\text{EM}}$ with the phase axis S_θ^1 supplying KK modes and the radial direction I_r supplying renormalization-group modes) already reproduces Maxwell + Lorentz-force physics on the 4D boundary via the same mechanism. $\text{U}(1)_{Q_i}$ is canonical for the full cohort (where BI-6 anomaly closure plus the SM gauge content forces it as one of three abelians), but it is not constitutive of the bulk-to-boundary projection that produces QM.

T8: The Arisaka Equation and Quadruple Gauge Locking (QGL). The three abelian gauge invariances of T3, together with BI-6 anomaly inflow under the OGGEF bulk-to-boundary projection, force the four boundary currents (energy, electric, baryonic, informational) to be locked together (T8 of Arisaka, 2026, A2-GG6):

$$\frac{J_E}{Q_B} = J_e = J_B = J_{Q_i}. \quad (28)$$

This is the *Arisaka Equation*, also called Quadruple Gauge Locking (QGL). For GG-A8-QM, the Arisaka Equation constrains the boundary currents that any quantum-measurement record has to satisfy: the energy dissipated in a measurement (via the Landauer bound that Qi-QUA of GG-A7-Qi4 formalizes) is locked to the electric, baryonic, and informational currents by the constitutional structure of the bulk. This is the operational content of QGL in the QM register, treated operationally in Part IV §41 (the Born-rule theorem inherits its CPTP structure ultimately from this current-locking constraint via Stinespring dilation) and in Part X §98 (the quantum-cryptography protocols exploit the bulk current-locking to detect eavesdroppers).

22.2 GG-A3-BI6: the BI-6 closure identity

Arisaka-GG-A3-BI6 (Arisaka, 2026, A3-BI6) establishes the BI-6 identity — the Green–Schwarz-modified Bianchi identity — as the unique anomaly-closure condition on the composite connection of (26), with coefficient vector $(1, -1, -2/5, -2/5)$ fixed by the topological factorization $I_8 = \frac{1}{2}X_4^2$. GG-A8-QM uses the BI-6 closure as a consistency check on the bulk side: any 4D operational projection of the bulk must respect the anomaly-closure pattern that GG-A3-BI6 proves. Operationally for QM,

this means that no single-particle quantum-mechanical observable can violate the bulk anomaly structure; the constraint is too far upstream to be visible in any laboratory experiment on a single qubit, but it becomes visible in collective measurements that probe the chiral sector (Standard-Model phenomenology, the subject of GG-A9-SM).

22.3 GG-A4-Euclid: the Euclidean discipline

Arisaka-GG-A4-Euclid (Arisaka, 2026, A4-Euclid) sharpens MSMF into the *zero-free-continuous-parameter* discipline. The twenty-eight cohort outputs — the four Standard-Model gauge couplings, the eighteen fermion masses across the three generations, the three CKM mixing angles plus one CP phase, the three neutrino masses plus three PMNS mixing angles plus one CP phase, the QCD theta parameter, the Higgs vev plus its self-coupling, and the four cosmological parameters H_0 , Ω_m , Ω_Λ , Ω_b — are all derived, across the cohort, from the eight-element input ledger of §21.1, item AX-4, with zero free continuous parameters. Within the GG-A4-Euclid master audit itself, the gauge-and-cosmological sector closes the audit table on twenty-two derived entries; the eighteen fermion masses and the CKM/PMNS mixings are tracked under GG-A4-Euclid’s O9 family-mass heading as “Partially resolved” and structurally completed in GG-A9-SM’s $\mathbb{CP}^2$ family-projector cascade (§25). This is the *N-to-zero compression* that distinguishes the framework from parameter-counting approaches; the wording “minimal input set” or “compression ratio” is strictly misleading because the ratio is $N/0$, undefined.

For GG-A8-QM the Euclidean discipline matters because it identifies which numerical predictions of quantum mechanics are free parameters (none) and which are derived (all). The Tsirelson bound $|S| \leq 2\sqrt{2}$ that the Aspect experiments saturate is not a free parameter of GG-A8-QM; it is the extremal (Bogomolnyi-analogue) stationarity of the CHSH functional at the singlet (T-A8.8), the standard quantum ceiling, with no adjustable knob. The same is true of the decoherence rate Γ (fixed by scale-overlap integrals of bulk KK modes; see T7.5) and of the pointer-basis selection (fixed by AX-1 bulk locality; see T7.6).

23 GG-A5-GDOS-GDOS and the T7.1–T7.7 chain: the core inheritance

Arisaka-GG-A5-GDOS (Arisaka, 2026, A5-GDOS) is the immediate upstream of GG-A8-QM and the single most important inheritance. It constructs the boundary law GDOS and proves the QM-postulates-as-theorems chain that GG-A8-QM now lifts to graduate-textbook form.

If GG-A2-GG6 is the father, GG-A5-GDOS is the mother. GG-A2-GG6 supplies the 6D bulk geometry GG-6; GG-A5-GDOS supplies the canonical boundary law GDOS that OGGE (AX-3 of the GG-Constitution) forces when applied to that bulk; and GG-A8-QM reads the QM child as the operational specialization of GDOS to the microscopic-observable sector. The genealogy is exact: every textbook postulate of quantum mechanics (Hilbert space, Schrödinger equation, Born rule, Lüders projection) is a derived theorem of GG-6 + OGGE, with the derivation chain proceeding via the six core theorems T1–T6 of GG-A5-GDOS Part II (the construction of GDOS) and the seven sub-theorems T7.1–T7.7 of GG-A5-GDOS Part III (the specialization of GDOS to the QM stratum).

The present paper is the standalone graduate-textbook lift of the child.

23.1 The eleven theorems of **GG-A5-GDOS**

GG-A5-GDOS has a principal-theorem inventory of eleven named results T1–T11, organized by Part:

- **Part II**, T1–T6: the GDOS construction. T1 forces existence of the boundary channel; T2 constructs it via Nakajima–Zwanzig reduction; T3 shows the channel is automatically CPTP via Stinespring dilation; T4 takes the Markovian limit and derives the GKLS generator; T5 takes the mesoscopic limit and derives the GDOS continuity equation; T6 unifies all five into the canonical boundary law.
- **Part III**, T7.1–T7.7: the QM-postulates-as- theorems chain. Seven dual-layer sub-theorems with an algebraic content T7.k.1 and an operational content T7.k.2. Tabulated below in §23.2.
- **Parts IV–V**, T8.x–T11.x: the gravity, SM, and cosmos specializations. These do not enter GG-A8-QM’s quantum-mechanical reading and are deferred to GG-A9-SM/GG-A10-Cosmos.

23.2 The seven **T7.k** theorems in textbook form

The textbook-postulate chain that GG-A8-QM lifts. We tabulate the seven sub-theorems with their algebraic content, operational content, and the standard-textbook postulate each replaces:

Table 1: The seven T7.k sub-theorems of GG-A5-GDOS Part III with their inherited specialization in GG-A8-QM Part IV. Each row names the parent result, the operational restatement it becomes at the boundary, and the reading under which the restatement is carried out. Nothing in the right-hand column is new physics: the specialization supplies the QM-stratum reading and adds no premise beyond those GG-A5-GDOS already carries.

T7.k	Standard postulate replaced	Algebraic content (T7.k.1)	Operational content (T7.k.2)
T7.1	Hilbert space	GNS embedding $\iota : \mathcal{H}_{\text{Bdy}}^{(\text{QM})} \hookrightarrow \mathcal{H}_{\text{Bulk}}$	Density matrix ρ_{Bdy} from bulk partial trace
T7.2	Schrödinger eq.	GKLS reduces to von Neumann eq. as $\gamma_\alpha \rightarrow 0$	$i\partial_t \psi\rangle = H_{\text{eff}} \psi\rangle$ in the closed-system limit
T7.3	Born rule	$p_\alpha = \text{Tr}(\mathcal{I}_\alpha \rho)$ from CPTP completion	$p_\alpha = \langle\alpha \psi\rangle ^2$ for sharp pure-state measurement
T7.4	Lüders projection	Conditional CPTP posterior $\rho^{(\alpha)} = \mathcal{I}_\alpha(\rho)/p_\alpha$	Textbook Lüders rule for pure projective measurement
T7.5	—	$\rho_{\alpha\beta}(t) = \rho_{\alpha\beta}(0)e^{-\Gamma t}$ off-diagonal damping	Decoherence as observed loss of phase coherence
T7.6	—	Pointer basis = eigenbasis of system–environment coupling	Einselection of the classical basis from bulk locality (AX-1)
T7.7	—	Entangled pair is one non-separable joint bulk state; correlations are its common-source projection shadow	CHSH violation up to the Tsirelson bound $2\sqrt{2}$

The structural significance of the table is that four of the seven sub-theorems — T7.1, T7.2, T7.3, T7.4 — recover the four textbook postulates of quantum mechanics directly, and the remaining three — T7.5, T7.6, T7.7 — recover three structures (decoherence, einselection, Bell-violation) that a century of foundations work has identified as essential but which the textbook treatment never axiomatized. All seven are specializations of the single open-system postulate AX-3 of GG-A1-Constitution via the bulk geometry of GG-A2-GG6 via the GDOS channel of GG-A5-GDOS Part II. No additional axiom is introduced in any of the seven sub-theorems.

23.3 What GG-A8-QM takes from GG-A5-GDOS

GG-A8-QM takes the entire T7.1–T7.7 chain verbatim. Part IV of the present paper restates each theorem, expands the surrounding pedagogy — the physics motivation, the quantitative anchor, the daily-life metaphor, the kill criterion, the plain-English close — to the depth a graduate student trained on Sakurai or Cohen-Tannoudji will need, and supplies worked examples in Parts V–IX. No theorem of GG-A5-GDOS Part III is re-proved in GG-A8-QM; the proofs are inherited. The body of the present paper is therefore one extended commentary on a chain that is already established upstream.

24 The GRIP/Qi-4 machinery: GG-A6-GRIP and GG-A7-Qi4

The two cohort papers that supply the operational dynamics one layer above the T7.1–T7.7 core.

24.1 GG-A6-GRIP: the GRIP cascade and the Gi-4 pillars

Arisaka-GG-A6-GRIP (Arisaka, 2026, A6-GRIP) develops the geometric runtime of the boundary observer. The primary cascade of GG-A6-GRIP is the three-stage descent $\text{DGI} \rightarrow \text{GSSB} \rightarrow \text{GA}$ on the reduced state manifold; GFF is the Lyapunov landscape on which the descent runs rather than a stage of the cascade itself. In the four-stage reading inherited from GG-A11-Origin (in which GFF is parenthesized as the landscape between DGI and GSSB), the cascade is written

$$\text{DGI} \rightarrow (\text{GFF}) \rightarrow \text{GSSB} \rightarrow \text{GA}, \quad (29)$$

where DGI is the dynamical-geometric input, GFF is the geometric free-energy functional that supplies the gradient landscape, GSSB is the geometric spontaneous symmetry-breaking projection, and GA is the geometric attractor selected at the end. The cascade is governed by the four Gi-4 pillars of GG-A6-GRIP (Arisaka, 2026, A6-GRIP) — TGG (Total Geometric–Gauge object), CCI (Covariant Conservation Identity), GFF (Geometric Free-energy Functional), and GRIP (Gauge-Runtime Invariance Pipeline) — of which the most load-bearing for GG-A8-QM are TGG (which forces the four-current geometric- gauge ledger that GG-A2-GG6 T8 reads off as the Arisaka equation (28)) and CCI (which forces the bulk-to- boundary descent rule that GG-A5-GDOS T-GDOS-7.1 reads off as the boundary GNS embedding).

For GG-A8-QM, the cascade (29) supplies the picture of *how* decoherence and einselection happen in practice. At the charged-lepton stratum — the stratum at which a single qubit lives — the DGI stage is the bulk fermion zero mode; the GFF stage is the energy functional set by the system–environment coupling; the GSSB stage is the off-diagonal damping of T7.5; and the GA stage is the einselected classical record of T7.6. GG-A8-QM Part IX performs this decomposition explicitly.

24.2 GG-A7-Qi4: the Qi-4 admissibility pillars

Arisaka-GG-A7-Qi4 (Arisaka, 2026, A7-Qi4) introduces the four Qi-4 admissibility pillars that any operational theory must satisfy. The four pillars are Qi-QUA (event-ledger conservation), Qi-EP (selector-law optimality), Qi-CON (conformality under rescaling), and Qi-HAL (architecture-stability under holography). Each pillar gives a falsifiable empirical signature.

For GG-A8-QM the four pillars enter as follows:

- Qi-QUA (event-ledger): every quantum measurement satisfies the Landauer bound $\dot{Q} \geq \Gamma k_B T \log_2 N$ on energy dissipated per outcome registered.
- Qi-EP (selector law): the realized measurement outcome minimizes the GFF functional given the prior, recovering the Born-rule probabilities as the optimal posterior — a maximum-likelihood reading of T7.3.

- **Qi-CON** (conformality): decoherence rates scale invariantly under system rescaling, predicting the $1/f$ -noise spectrum that laboratory measurement apparatuses universally exhibit. This is the most distinctive empirical signature of the cohort’s treatment of QM and is one of the eight-test horizon items in Part [XII](#).
- **Qi-HAL** (architecture): measurement records are topologically protected by the bulk Wilson-line holonomies on S^1_θ (with the cohort canonical bulk supplying the $U(1)_{Q_i}$ factor) — the same S^1_θ Wilson-line holonomy of **GG-A2-GG6**. This is why a photograph remains a photograph after the photographer is gone: the record is locked in a topological invariant of the bulk configuration, not in a metastable mark.

25 GG-A9-SM-SM and the universal-identity insight

Arisaka-GG-A9-SM ([Arisaka, 2026, A9-SM](#)) derives the Standard-Model particle content (gauge group, three generations, fermion masses and mixings, CKM and PMNS angles) from the cascade (29) specialized to the family-projector stratum $\mathbb{CP}^{N_g-1} = \mathbb{CP}^2$ of **GG-A9-SM**, with the twenty-eight Standard-Model parameters read off as cascade-depth assignments and algebraic descendants on that single stratum. The seventeen-step layer enumeration that organizes the full GRIP descent — gauge group, electroweak vacuum, atomic ground states, molecular closure, amino-acid selection, polymer closure, energy–information closure, and the rest — is the layered-cascade catalog of **GG-A11-Origin**, which threads the cascade through cosmos/life/mind; **GG-A9-SM**’s SM derivation is the specialization of that catalog to the gauge-and-mass layers only. The closing Part of **GG-A9-SM** (Part XII §60 in the cohort-wide citation convention) introduces the *universal-identity insight* that **GG-A8-QM** will lift in its own closing Part [XIII](#).

25.1 The universal-identity insight

The universal-identity insight is the structural reason why every electron is the same. In the cascade picture, the cascade at the charged-lepton stratum has a unique stable **GA** (the electron with mass $m_e = 0.511 \text{ MeV}/c^2$ and charge $-e$); every electron that is realized anywhere in the universe is a token of this single type, with the type fixed by the geometric attractor selection rather than by any individual quantum-mechanical preparation. The insight propagates: every hydrogen atom is the same because hydrogen is the unique stable **GA** of the atomic stratum; every ATP molecule is the same because ATP is the unique stable **GA** of the energy–information stratum; every first cell is the same in the appropriate structural sense because cellularity is the unique stable **GA** of the cellular stratum. The universal-identity reading is the structural-realist genealogy of the framework.

For **GG-A8-QM**, the universal-identity insight grounds the closing summit view of Part [XIII](#): every laboratory measurement of an electron’s spin returns the same eigenvalues because every electron is the same type, with the type fixed upstream of any measurement preparation. The quantum-mechanical postulate that all electrons are identical particles is therefore not a postulate of quantum mechanics but a structural feature of the cascade that quantum mechanics inherits.

25.2 What GG-A8-QM takes from GG-A9-SM

A single insight: the universal-identity reading. All T7.1–T7.7 statements about a generic “qubit” or a generic “boundary observer” become statements about a generic token of a universally fixed type, with the type pinned by the cascade attractor selection. The closing Part XIII of GG-A8-QM states this in its summit voice.

26 GG-A10-Cosmos-Cosmos: cosmological specialization

Arisaka-GG-A10-Cosmos (Arisaka, 2026, A10-Cosmos) specializes the cohort machinery to the cosmological stratum: the boundary cosmology with $\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$, $\Omega_b = 1/(2\pi^2)$; the modified Friedmann equations; the two-stage Hubble corridor; the Kaxion dark-matter prediction; the baryogenesis and BBN treatments; the JWST early-galaxy phenomenology. For GG-A8-QM the cosmological content is not directly imported — the present paper is about laboratory quantum mechanics, not cosmology — but the methodology is.

The *bulk-to-boundary-projection methodology* of GG-A10-Cosmos — specialize the bulk OGGEF dictionary to the boundary stratum of interest, derive the boundary observables as theorems of the specialization, identify which empirical signatures are forward predictions, which are postdiction-audits, and which are consistency checks — is the same methodology GG-A8-QM applies to the QM stratum. Where GG-A10-Cosmos produces a cosmological Pull table with Ω_m , Ω_Λ , H_0 , Ω_b , etc., GG-A8-QM produces a QM Pull table with Tsirelson saturation, decoherence-rate scaling, $1/f$ noise, einselection-rate, etc. The methodology is identical; only the stratum differs.

27 GG-A11-Origin-Origin: the four-move operational anatomy

Arisaka-GG-A11-Origin (Arisaka, 2026, A11-Origin) is the closing capstone of the cohort and supplies the operational anatomy of the cascade (29) at the level of generality that GG-A8-QM specializes.

27.1 The four moves

GG-A11-Origin introduces a four-move decomposition of the cascade that maps onto the DGI–GFF–GSSB–GA sequence:

1. **Move 1 — Recognition (DGI).** The cascade identifies a stratum’s dynamical-geometric input from the bulk state.
2. **Move 2 — Projection (GFF).** The bulk state is projected onto the boundary algebra of the stratum, producing the free-energy functional whose minimizers are the candidate stable configurations.
3. **Move 3 — Selection (GSSB).** The cascade selects the unique GFF-minimizing configuration on the stratum; suboptimal configurations are damped away.

4. **Move 4 — Reading (GA).** The selected configuration is registered as the stratum’s stable geometric attractor and becomes the dynamical-geometric input for the next stratum.

The three operational-anatomy theorems of GG-A11-Origin (T-GRIP-In-Lyapunov-Descend, T-GRIP-Out-Necessity, T-GGA-as-Ground-State) prove that the four stages of the cascade $DGI \rightarrow GFF \rightarrow GSSB \rightarrow GA$ are forced rather than phenomenological. GG-A11-Origin maps the cohort axioms to the four stages as follows (Arisaka, 2026, A11-Origin): AX-2 (GGEP) makes DGI’s bulk substrate coherent and so forces the recognition of the dynamical-geometric input; AX-3 (OGGEP) makes GSSB’s projection operational and so forces the boundary-algebra image of the bulk state; AX-4 (MSMF) makes GA selection unique and so forces the attractor outcome; the GFF stage is the gradient flow that connects them, without a single axiom as its forcer. AX-1 (Local Causality) enters the cascade indirectly, through the forward-time semigroup property of the boundary channel, which is the constitutional source of the Lindblad / GKLS normal form that T-A8.2 exercises in the QM register.

27.2 What GG-A8-QM takes from GG-A11-Origin

GG-A8-QM specializes the four-move anatomy to the charged-lepton stratum — the stratum at which a single qubit lives. Move 1 is the bulk fermion zero mode that realizes the spin degree of freedom of an electron. Move 2 is the boundary partial trace that produces the density matrix $\rho_{Bdy} = \text{Tr}_{env} \rho_{Bulk}$. Move 3 is the off-diagonal damping of T7.5 that selects the pointer basis as the GFF-minimizing posterior. Move 4 is the einselection of T7.6 that registers the classical record.

The four-move reading recovers, on the QM stratum, what Bohr was seeing operationally and what Everett was seeing unitarily: the two readings are projections of the same four-move cascade onto two different aspects, and both are correct within their domains. We make the recovery explicit in Part XI.

28 Summary and coherence-map

The inheritance ledger of GG-A8-QM reduces to a small number of load-bearing imports. The constitutional warrant comes from GG-A1-Constitution (AX-1, AX-3, POS-4, POS-7). The bulk geometry comes from GG-A2-GG6–GG-A3-BI6–GG-A4-Euclid (the scale-profile metric, the BI-6 closure, the Euclidean discipline). The boundary law and the seven-theorem chain come from GG-A5-GDOS Part II (T1–T6) and Part III (T7.1–T7.7). The operational runtime comes from GG-A6-GRIP (GRIP cascade + Gi-4 pillars) and GG-A7-Qi4 (Qi-4 admissibility pillars). The universal-identity reading comes from GG-A9-SM Part XII. The bulk-to-boundary-projection methodology comes from GG-A10-Cosmos. The four-move operational anatomy comes from GG-A11-Origin.

The twenty-two-paper cohort map, with the branch-level reading of the two phenomenology branches and the Foundations Branch, stands at the close of Part I (§20.1, Figure 2); this Part is its per-paper detail.

28.1 What is not inherited

The reader is entitled to a complementary statement of what GG-A8-QM does *not* inherit, so that the boundary of the paper's claim is visible. GG-A8-QM does not inherit any specifically biological or cognitive content of GG-A6-GRIP or GG-A11-Origin (those are out of scope for the QM stratum). It does not inherit the cosmological Pull table of GG-A10-Cosmos (cosmology is the subject of a separate paper, not a free parameter of GG-A8-QM). It does not inherit the high-energy KK phenomenology of the predictions branch GG-P1-KK (KK lock cluster) or the dark-matter prediction of GG-C1-Kaxion (Kaxion). These cohort companions enter only as cross-references where relevant. The paper before you is one paper of a twenty-two-paper cohort; the cohort is closed, and GG-A8-QM is the closing member.

Part-end close

The inheritance ledger of this Part is the answer to a question a careful reader will have asked from the start: *is this paper introducing new physics, or is it re-presenting old physics?* The answer is neither. The physics is fixed already by the ten upstream cohort papers; the present paper specializes, restates, and supplies textbook-pedagogical apparatus for the specialization to the microscopic-observable sector. Everything in GG-A8-QM that looks like a new theorem is a specialization of a theorem of GG-A5-GDOS Part III; everything that looks like a new worked example is a calculation that the cohort apparatus makes inevitable; everything that looks like a new philosophical position is a structural reading of inheritances from GG-A1-Constitution and GG-A11-Origin.

After all, a paper that introduces no new physics can still introduce a new way of seeing the physics — and the standalone graduate-textbook reformulation of quantum mechanics under the GG-Theory framework is exactly that: a new way of seeing the same century-old content, with the operational shadow and the 6D bulk source both in clear view at once.

Table 2: GG-Constitution / GG-Theory axioms and postulates used in the quantum-mechanical derivation. The left column names each axiom or postulate by its constitutional number, the middle column states what it supplies here, and the right column names the result that consumes it. The list is exhaustive in one direction only: every theorem below rests on these, but not every axiom of the GG-Constitution is needed to reach quantum mechanics.

Label	Name	Statement	Role in QM derivation
AX-1	Local causality	Influences propagate forward in time at finite speed	Forward time-ordering of CPTP semi-group $\Rightarrow$ GKSL generator
AX-2	Primitive connection	The primitive field is a gauge connection on M_6	Gauge structure of bulk Hilbert space $\mathcal{H}_4$
AX-3	OGGEP	Operational observation is a structure-preserving functor $\mathcal{D} : \mathbf{Bulk} \rightarrow \mathbf{Op}$	Defines measurement; gives CPTP + instruments
AX-4	MSMF	Max symmetry, min freedom; no free parameters	Forces deterministic bulk; forces Tsirelson bound $2\sqrt{2}$
POS-CP	Complete positivity	Channels preserve positivity on composed systems	Restricts generator to GKSL form
POS-RG	Scale consistency	Functor commutes with coarse-graining	Pointer-basis independence of Heisenberg cut
POS-OGN	OGN-5 normalization	Five-integer ledger (1, 3, 5; 7, 35)	Fixes all dimensionless coefficients; $\sin^2 \theta_W$, λ_H , etc.

29 Part II in plain English: what this paper borrows, and at what strength

This Part is a list of debts, and it is placed early on purpose. GG-A8-QM is the eighth paper of a series, and almost nothing in it is self-contained. The constitutional axioms are GG-A1-Constitution's. The six-dimensional geometry is GG-A2-GG6's. The projection dictionary and the open-system law are GG-A5-GDOS's. The runtime that ends a relaxation at a single attractor is GG-A6-GRIP's. The gauge-sector and cosmological readings are GG-A9-SM's and GG-A10-Cosmos's. What is left over — and it is a real remainder, not a courtesy — is the operational restatement of that machinery on the microscopic-observable stratum, together with four results the bulk-side apparatus makes available and textbook quantum mechanics has no way to write down.

The reason for saying this in a Part of its own is that an inheritance is easy to overstate. A paper that imports a theorem and re-narrates it in its own words has not verified anything; it has copied. The rule followed here is that every import is quoted at the strength its owning paper gives it, and where the owning paper hedges, this one hedges in the same place. The Born rule is the clearest case. GG-A5-GDOS closes it at the relaxation level, conditional on four named premises, with a single analytic residue at the field-grade representation step. That is exactly the strength at which it appears here. A reader who wants the Born rule proved outright will not find it proved outright in this paper, and should not: it is not proved outright in the paper that owns it.

The coherence map at the end of this Part exists for the same reason. It is easy for a series of twenty-two papers to drift — for a value quoted in one paper to move in another and for nobody to

notice, because no single reader holds all of them in mind at once. The map is the mechanism by which that drift becomes visible. If a number in this paper disagrees with the number in the paper that owns it, the map is where the disagreement shows, and the paper that owns it wins.

None of this makes the present Part interesting reading, and it is not meant to be. It is meant to be checkable. A reader who wants to know whether a particular claim in Parts IV to XIII rests on something argued here or on something argued somewhere else should be able to answer the question from this Part alone, without taking anyone's word for it.

Part III

The constitutional bridge

GG-6 bulk $\rightarrow$ OGGEF $\rightarrow$ GDOS $\rightarrow$ 4D QM

The four-stage constitutional bridge is shown in **Figure 4**.

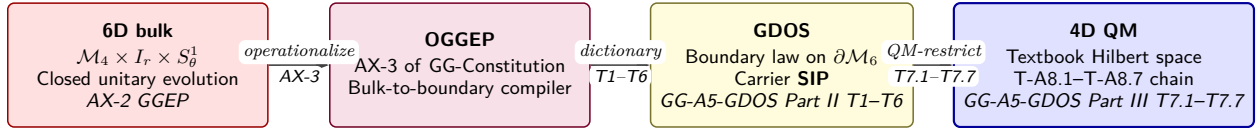


Figure 4: The constitutional bridge of GG-A8-QM: the four-stage chain that compiles the closed-bulk dynamics of the 6D geometry (GG-A2-GG6, red) into the operational 4D Hilbert spaces of laboratory quantum mechanics (blue, highlighted as the QM tier of this paper). OGGEF (AX-3 of GG-Constitution, purple) is the operational extension of GGEF (AX-2) that supplies the bulk-to-boundary compiler; the six GDOS theorems T1–T6 of GG-A5-GDOS Part II construct the boundary algebra and the channel structure; the seven QM theorems T7.1–T7.7 of GG-A5-GDOS Part III restrict the boundary algebra to the QM-relevant sector and reproduce the textbook postulates. Every GG-A8-QM worked example (Parts V–IX) is a specialization of this single chain on a specific laboratory system.

Part III is the mechanism Part of GG-A8-QM. Every other Part of this paper either prepares for it (Part I introduces the century-old questions; Part II catalogs the cohort inheritance), states its consequences as theorems (Part IV), or exercises those theorems on specific laboratory systems (Parts V–XIII). Part III itself is the four-stage chain GG-6 bulk $\rightarrow$ OGGEF $\rightarrow$ GDOS $\rightarrow$ 4D QM read end-to-end: how the deterministic, unitary, six-dimensional geometry of GG-A2-GG6 (GGEF, AX-2) is compiled by OGGEF (AX-3 of GG-Constitution) into the boundary algebra of GDOS (GG-A5-GDOS Part II), and then restricted by the seven QM-postulate theorems (GG-A5-GDOS Part III T7.1–T7.7) to the textbook 4D Hilbert spaces a laboratory observer measures. The reader who masters the present Part has, by the end of it, the operational machinery that the rest of the paper uses; the worked examples (Parts V–IX), the paradox sweep (Part X), the interpretation synthesis (Part XI), and the discussions (Parts XII–XIII) are specializations and corollaries of this single chain.

30 Why Observation Is Projection: The Hidden-Fiber Origin of Quantum Behavior

Before stating the operational machinery of OGGEF formally, it is worth drawing the geometric picture that makes the whole bulk-to-boundary chain feel inevitable rather than imposed. The bulk derivation of GG-A2-GG6 (Theorem 0, T-GG6-0) established that the two extra dimensions of GG-6 are not spatial: I_r is a *scale* dimension (a resolution, the geometrized renormalization-group

coordinate) and S_θ^1 is a *phase* dimension (an internal angle, the geometrized gauge holonomy). Once that is granted, the quantum-mechanical content of the bulk-to-boundary projection follows from a single observation: *these two dimensions are attached at every point of spacetime, and neither can be resolved by any measurement made on the boundary.* This section makes that statement precise and shows that it already contains, in outline, why the boundary theory is quantum.

30.1 The bulk as a fiber bundle; observation as the projection

Locally, the bulk is a fiber bundle,

$$\mathcal{M}_6 = \mathcal{M}_4 \times I_r \times S_\theta^1, \quad \pi : \mathcal{M}_6 \longrightarrow \mathcal{M}_4, \quad (x; r, \theta) \mapsto x, \quad (30)$$

whose *base* is the Lorentzian stage $\mathcal{M}_4$ — the arena of registered, causally ordered records — and whose *fiber*, sitting over each spacetime point x , is the pair (r, θ) : the scale and phase the system carries *at that point*. The boundary registration map demanded by OGGE (AX-3) is exactly this projection π .

The decisive point is that π is not an additional postulate; it is forced by the non-spatiality of the fiber. A boundary protocol, by AX-1 (LC), reads spacetime records sorted by a light cone; there is no light cone “in resolution” and none “in internal angle,” so there is no operation that resolves where a system sits along I_r or around S_θ^1 . Every act of registration is therefore obliged to marginalize the fiber. “Observation on a four-dimensional Lorentzian boundary” simply *means* the pushforward along π , and that pushforward discards the fiber coordinates irretrievably.

This is the geometric reason that openness is constitutional and closure is its limiting case (the methodological inversion of GG-A5-GDOS). The fully unitary, information-preserving description is the complete six-dimensional bulk — which is inaccessible in principle, not merely in practice. Every description an observer can actually register is the projection π , which is non-invertible because it forgets the fiber; the boundary dynamics it induces is therefore open, mixed, and completely positive (the CPTP and GKLS structures of GG-A5-GDOS T-GDOS-3–T-GDOS-4). Unitarity is not the default that measurement violates; it is the unreachable idealization in which the fiber is never coarse-grained.

30.2 Two fibers, two halves of quantum mechanics

The two fiber directions enter the geometry differently — S_θ^1 compact and flat, I_r non-compact and scale-extended (GG-A2-GG6, Theorem 0) — and that difference maps cleanly onto the two halves of quantum mechanics.

The phase fiber S_θ^1 carries the coherent, unitary structure. The quantum phase of a wavefunction $\psi = |\psi| e^{i\varphi}$ is, on this reading, the system’s position on the θ -circle, and the holonomy $\oint A_\theta d\theta$ is the physical, observable *relative* phase — the Aharonov–Bohm and Berry geometric phase (Aharonov & Bohm, 1959; Berry, 1984), an effect already known to be geometric. Transport along θ is a pure phase: reversible, unitary, interference-generating. Moreover the gauge redundancy of θ delivers a sharper statement than is usually noticed. Because a rigid fiber rotation acts on

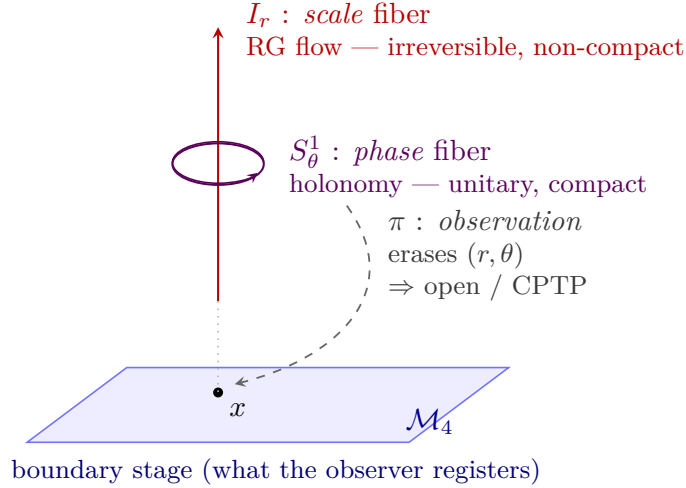


Figure 5: Observation as the bundle projection $\pi : \mathcal{M}_6 \rightarrow \mathcal{M}_4$. Over each boundary point x sits the non-spatial fiber (I_r, S_θ^1) . The compact *phase* circle S_θ^1 carries the coherent, unitary content of quantum mechanics — its holonomy is the Aharonov–Bohm/Berry phase (Aharonov & Bohm, 1959; Berry, 1984) — while the non-compact *scale* interval I_r carries the irreversible, open content, its forward renormalization-group semigroup being the GKLS/GDOS generator. Because both fiber coordinates are non-spatial, no boundary protocol resolves them; every registration marginalizes the fiber, so π is non-invertible and the boundary dynamics is necessarily open. Unitary closure is the unreachable limit in which the fiber is not coarse-grained.

a charge- n sector with weight n , what the fiber’s unreadability forbids is the measurement of a relative phase *between states of different charge*: charge superselection, a genuine theorem of gauge theory, read here directly off the geometry. Marginalizing the unreadable fiber coordinate while retaining relative phases within a sector is precisely the operation that converts an amplitude into a probability; this is why the Born rule has square-modulus form, and why the *gauge* phase of a charged field is complex: it is the holonomy of a real circle, with i that circle’s rotation generator. (The complex structure of the boundary state space itself is a separate question — a neutral field needs it as much as a charged one, and the circle provably cannot supply it there; where it enters, and how far it too is derived from real constitutional data, is taken up in §117 — and is now *closed by import*: the origin-of- i companion derives the boundary’s single imaginary unit from the two real structures of the arena, at derived-conditional grade in the quasi-free regime (Arisaka, 2026, P4-Real), with the custody sentence holding as everywhere in the cohort: the dictionary owns the sectors, the geometry owns the i .) The formal derivation is the Born-rule theorem T-A8.3; the present picture supplies its geometric origin.

The scale fiber I_r carries the open, irreversible structure. Theorem 0 of GG-A2-GG6 fixed I_r as *non-compact* precisely because the renormalization-group flow it geometrizes is a forward semigroup: integrating out short-distance modes cannot be undone. Motion along r is therefore intrinsically directed. Accessing a system at finite resolution means tracing out the remainder of this scale fiber — coarse-graining — and because the fiber is irreversible the induced boundary evolution is a Markov *semigroup*, not a group: completely positive, with no inverse. That is exactly the

GKLS/GDOS structure (Gorini, 1976; Lindblad, 1976). The identification is sharp: *the open-system generator of GDOS is the dilatation (RG) generator along I_r , projected to the boundary*, and the arrow of measurement is the arrow of the renormalization group. That arrow is consumed here, not created: per the constitutional custody chain, AX-3 owns the arrow — the one-way reading of the dictionary is a clause of the constitution (the AX-3 synthesis of Arisaka, 2026, A1-Constitution) — with the canonical law-level derivation in Part IX of the CPT member paper (Arisaka, 2026, P5-CPT) and the dynamical implementation in the GDOS companion (Arisaka, 2026, A5-GDOS); the measurement arrow of this paper is the scale-fiber face of that one constitutional arrow, a consumer of AX-3’s non-invertibility, not a private derivation. Holographic renormalization makes the same correspondence in its own setting — radial integration is Wilsonian coarse-graining (J. Maldacena, 1998; L. Randall and R. Sundrum, 1999; Wilson, 1971).

Together. Coherent unitary evolution is the phase fiber; irreversible registration is the scale fiber; and decoherence is the loss of θ -coherence induced by coarse-graining along r — the einselection of a pointer basis (Zurek, 1981, 2003). The closed, unitary Schrödinger evolution is the limit in which phase coherence is maintained and the scale fiber is not coarse-grained (AX-3 closure); generic observation traces the scale fiber and yields measurement. One bulk, two limits.

The precise statement: a principal $U(1)$ -bundle — and what the scale fiber is not. It is worth fixing the two fibers in exact differential-geometric language, since that is the rigorous form of the picture above. In the vocabulary of fiber bundles (Ehresmann, 1950; Wu & Yang, 1975), the phase direction S_θ^1 is a genuine *principal $U(1)$ -bundle* over the boundary base $\mathcal{M}_4$: the gauge potential A_θ is its *connection*, the quantum phase is the *holonomy* of that connection around a loop, and the Aharonov–Bohm and Berry phases are holonomies in the literal bundle sense — which is exactly why the gauge phase of charged matter is complex and the Born rule is square-modulus, the unreadable fiber coordinate enforcing superselection between charge sectors (for the complex structure of the state space itself, see §117). OGGEF is then the *bundle projection* $\pi : \mathcal{M}_6 \rightarrow \mathcal{M}_4$, and “tracing out the fiber” is the restriction of a section to the base. The Wu–Yang (1975) dictionary, which first identified the non-integrable gauge phase factor with bundle holonomy, is the precise precedent for reading the quantum phase this way. The one place the language must *not* be pushed is the scale fiber I_r : it is not a principal gauge bundle. Its “structure group” is the dilatation, realized as the one-way renormalization-group *semigroup* of the preceding paragraph (irreversible, with no inverse), and the relevant “connection” along it is the scale profile, not a Yang–Mills field. That asymmetry — a compact principal $U(1)$ -bundle on the phase side, a scale-extended semigroup fibration on the scale side — is the same asymmetry that splits quantum mechanics into its unitary (phase) and irreversible (scale) halves.

30.3 Why a point particle makes the projection total

A point particle occupies a single $x \in \mathcal{M}_4$ but carries, at that point, definite values of r and θ that the boundary cannot read. In the most literal sense the particle is *its own environment*: the degrees of freedom traced out in decoherence are not those of an external bath but the particle’s own

non-spatial fiber. No outside reservoir is needed for the open structure to be *available*; inaccessibility is intrinsic. This is a statement about the open-system *form*, not the decoherence *rate*: a genuinely isolated free particle in vacuum, with no apparatus or bath coupling, decoheres negligibly, because the rate is set by the system’s coupling to its physical 4D environment (air molecules, thermal photons), not by the fiber alone. The intrinsic fiber is why the boundary law is open in principle; what makes a given system decohere fast is its physical environment.

This is also where the constitutional choice of *point* particles, rather than extended objects, does quiet work. An extended object would carry a resolvable profile *along* the extra directions — a tower of internal modes, an intrinsic extent — which is in principle accessible and would blur the clean statement that the fiber is simply unreadable. A point carries a single fiber value and nothing else, so the marginalization in (30) is total and unambiguous. The quantum indeterminacy of a point particle is then not a smearing in space but the boundary shadow of two perfectly definite yet unobservable internal coordinates.

30.4 Particle in space, wave in phase: wave–particle duality dissolved

The two-fiber split resolves the oldest puzzle of quantum mechanics at the level of principle. “Wave–particle duality” reads as a contradiction only when both words are made to describe behavior *in space*. Here they describe two different dimensions: the system is a *particle in the stage* $\mathcal{M}_4$ (a point, by locality) and a *wave in the phase fiber* S_θ^1 (the dial whose holonomy is the quantum phase). Both hold at once, without paradox, because point-like-in-space and wave-like-in-phase are orthogonal, non-spatial statements. The wavefunction $\psi = |\psi|e^{i\varphi}$ is not the particle spread out but the phase section over the available positions; the de Broglie wavelength $\lambda = h/p$ is the spatial period at which the dial winds, set by the momentum, not a size of the particle. Feynman’s spinning amplitude-arrow (Feynman, 1985) is, on this reading, literally the position on S_θ^1 ; adding arrows and squaring is the OGGEF marginalization of the unreadable global phase that yields the Born rule (T-A8.3). The worked double-slit calculation — where the fringe term $\cos \Delta\varphi$ is the relative dial angle of two bulk histories, and a which-path measurement is the reading of the phase fiber that destroys it — is carried out in Part VI (§61).

“How can a point pass through both slits?” The sharpest form of the puzzle deserves a direct answer even at this level of principle. A point must pass one slit or the other, yet interference appears to demand both. The dissolution is that “which slit” and “interference” are questions about *different fibers*. “Which slit” asks for a position in $\mathcal{M}_4$, and the point has exactly one, registered as a single dot at the screen. “Interference” asks about the phase section $\varphi(x)$ on S_θ^1 , and that section is a *field* laid over every available position — defined along the route past each slit, exactly as the electromagnetic field of a light beam is present at both slits without anything material being in two places. What threads both slits is the phase field; what lands is the point; the fringes record how the two dial-windings recombine. The extensive treatment — including why a which-path measurement is a scale-fiber read-out that necessarily erases the phase coherence — is given in Part VI, §62.

30.5 Honest accounting: what the fiber picture settles, and what GDOS still carries

It is worth being exact about how much this picture delivers, in the auditable spirit of MF (POS-7).

What it *does* settle, and more convincingly than asserting the chain $GG-6 \rightarrow OGGE \rightarrow GDOS \rightarrow 4D\ QM$ directly: the projection is forced rather than postulated; openness is constitutional for a structural reason; the complex amplitude acquires a geometric identity (a phase holonomy); the Born rule acquires its square-modulus *form*; a preferred (pointer) basis and decoherence follow from which fiber is traced; and — the cleanest new result — the irreversibility of measurement is identified with the forward renormalization-group semigroup along the non-compact scale dimension, so that the dissipative part of the GKLS generator is the boundary image of scale-fiber coarse-graining (GG-A5-GDOS T-GDOS-4).

What the bundle picture does *not* settle by geometry alone is the one residue that every deterministic completion of quantum mechanics carries — and the framework states it precisely rather than papering over it. The measurement problem, stated exactly, is *three* questions, and the framework answers each. *Why this basis?* — the einselected pointer basis is forced by bulk locality AX-1 (T-A8.6). *Why one definite outcome rather than a living superposition?* — because the six-dimensional bulk is single-valued and registration is the descent of the joint system-and-apparatus configuration into one Geometric Attractor basin, irreversibly locked by the boundary free-energy law (T-A8.1, T-A8.4, with the GRIP actualization of GG-A6-GRIP); the outcome is the *contextual* joint holonomy Θ_H of system and apparatus read out by the OGGE projection, with no collapse, no branching, and no observer required. *Why Born weights?* — the square-modulus *form* is forced by Gleason on the open dictionary (T-A8.3), while which value Θ_H takes on a given run is the bulk boundary condition, inaccessible to the four-dimensional observer. What remains — assumed nowhere in this paper — is the basin-capture = Born statement: that the GRIP basin-capture measure of the deterministic descent coincides with the Born measure, the six-dimensional analogue of the quantum-equilibrium typicality theorem of Bohmian mechanics (Bohm, 1952); it is closed at the relaxation level (the R16 lemmas with the R16' GDOS relaxation; the Constitutional Born Theorem conditional on the four named premises (P1)–(P4) of §118), with the field-grade representation step the single remaining analytic residue. With this division the umbrella problem is dissolved into theorems plus one named analytic residue; that is both the strength of the account and the precise statement of what is still owed. This is the form we carry through the rest of the paper.

31 The Operational Postulate OGGE in detail

31.1 Context: why an operational postulate?

OGGE (AX-3 of the GG-Constitution) is the *open-system extension* of the closed-bulk postulate GGEP (AX-2). GGEP states that the 6D bulk (Arisaka, 2026, A2-GG6) is a single composite geometric structure in which gravity and gauge data are unified into one connection; it constrains

the bulk theory. OGGEF extends this constraint to the operational shadow that a finite 4D observer registers in the laboratory: any admissible bulk theory must come equipped with a bulk–boundary correspondence from its bulk algebra to a boundary algebra of operationally admissible states, channels, and instruments. GG-A5-GDOS Part II constructs this dictionary explicitly in six theorems T1–T6; GG-A5-GDOS Part III specializes the dictionary to the microscopic-observable sector and shows that the four textbook postulates of quantum mechanics — the Hilbert-space postulate, the Schrödinger equation, the Born rule, and the Lüders projection rule — plus three additional structures that the decoherence program later identified as essential (decoherence, einselection, Bell-violation), emerge as theorems T7.1–T7.7. The present paper inherits this chain verbatim and lifts each theorem to standalone graduate-textbook form.

In the Copenhagen formulation, measurement is a primitive notion that stands outside the physics — a line between “quantum system” and “classical apparatus” drawn at the experimenter’s discretion. This is the source of the measurement problem: within the theory, no principle determines where the line lies.

GG-Theory takes a different stance. Measurement is not an axiom on top of physics; it is a *constitutional requirement* — a condition that any admissible physical theory must satisfy. The requirement is expressed in the *Operational/Open Geometry–Gauge Equivalence Principle* (OGGEF, AX-3 of the GG-Constitution (Arisaka, 2026, A1-Constitution); closed-bulk precursor GGEP = AX-2, the Geometry–Gauge Equivalence Principle that constrains the 6D bulk of Arisaka (2026, A2-GG6)):

A physical theory is complete only when it specifies a map from its bulk data to the operational records produced by laboratory instruments. This map must be a structure-preserving functor that respects causality, physicality (positivity), and coarse-graining.

The functor language is important: OGGEF is not just a recipe for computing probabilities; it is a constraint on how the fundamental 6D theory relates to 4D laboratory records.

31.2 The functor $\mathcal{D} : \mathbf{Bulk} \rightarrow \mathbf{Op}$

Following the categorical formulation of GG-2026-3 and GDOS-Book-I:

Definition 31.1 (Bulk and Operational categories). The category **Bulk** has as objects admissible bulk packages $B = (M_6, g, A, \psi, \dots)$ satisfying the GG-6 constitution; its morphisms include diffeomorphisms, gauge transformations, and coarse-graining / renormalization-group maps.

The category **Op** has as objects operational boundary packages $O = (\mathcal{H}, \rho, \{I_\alpha\}, \{\Phi_{t,s}\}, \dots)$ — that is, a Hilbert space, a density operator, a family of instruments, and a family of channels; its morphisms are operational equivalences (unitary conjugations, classical relabelings, coarse-grainings).

Definition 31.2 (OGGEF functor). OGGEF asserts the existence of a functor

$$\mathcal{D} : \mathbf{Bulk} \longrightarrow \mathbf{Op} \tag{31}$$

that is compatible with: (i) **gauge covariance** (bulk gauge equivalences map to operational equivalences); (ii) **causality** (retardedness, AX-1); (iii) **positivity / physicality** (channels and

instruments preserve state positivity); (iv) **scale consistency** (the functor commutes with coarse-graining).

The content of OGGEF is therefore *not* that a specific formula for $\mathcal{D}$ is given by fiat; it is that *such a functor must exist* and must satisfy the four constitutional properties listed above. The explicit formula for $\mathcal{D}$ is a consequence, not a postulate.

31.3 The explicit form of $\mathcal{D}$: partial trace plus coarse-graining

For a bulk state ρ_{bulk} on the 6D Hilbert space $\mathcal{H}_{\text{bulk}}$, the 4D observer sees

$$\rho_{\partial} = \mathcal{D}(\rho_{\text{bulk}}) = \text{Tr}_{\text{env}}(\rho_{\text{bulk}}) = \text{Tr}_{I_r \otimes S_{\theta}^1}(\rho_{\text{bulk}}). \quad (32)$$

The trace is over the degrees of freedom that the 4D observer cannot access — the radial coordinate I_r , the compact angle S_{θ}^1 , and all heavy Kaluza-Klein modes above the lock-cluster scale $M_{\text{lock}} \approx 3.85 \text{ TeV}$ ($\approx 42 m_Z$). The remaining state ρ_{∂} lives on the 4D zero-mode Hilbert space $\mathcal{H}_4$.

Importantly, the partial trace (32) is *automatic*: any 4D observer *necessarily* traces over the inaccessible bulk degrees of freedom. There is no choice here; it is a consequence of being 4-dimensional.

31.4 Operational instruments from bulk interactions

A laboratory instrument is modeled in the standard Stinespring/Naimark dilation form. For an instrument labeled by outcomes $\{\alpha\}$:

$$I_{\alpha}(\rho_{\partial}) = \text{Tr}_{\text{app}} \left[(\mathbf{1}_S \otimes \Pi_{\alpha}^{\text{pointer}}) U_{SA} (\rho_{\partial} \otimes \sigma_{\text{app}}) U_{SA}^{\dagger} (\mathbf{1}_S \otimes \Pi_{\alpha}^{\text{pointer}}) \right], \quad (33)$$

where σ_{app} is the apparatus initial state, U_{SA} is the system-apparatus unitary interaction, and $\Pi_{\alpha}^{\text{pointer}}$ is the pointer projector corresponding to outcome α . The outcome probability is

$$p_{\alpha} = \text{Tr}[I_{\alpha}(\rho_{\partial})] = \text{Tr}[E_{\alpha} \rho_{\partial}], \quad (34)$$

where $E_{\alpha} = \text{Tr}_{\text{app}}[(\mathbf{1}_S \otimes \Pi_{\alpha}^{\text{pointer}}) U_{SA}^{\dagger} (\mathbf{1}_S \otimes \Pi_{\alpha}^{\text{pointer}}) U_{SA} (\mathbf{1}_S \otimes \sigma_{\text{app}})]$ is a positive operator on $\mathcal{H}_S$ — precisely a POVM element, $E_{\alpha} \geq 0$, $\sum_{\alpha} E_{\alpha} = \mathbf{1}$.

The post-measurement conditional state is

$$\rho_{\partial}^{(\alpha)} = I_{\alpha}(\rho_{\partial})/p_{\alpha}, \quad (35)$$

which is Bayesian conditioning on the outcome label.

31.5 The “measurement problem” as missing map problem

In GDOS-Book-I, Arisaka (2026b) observes:

OGGEF rephrases the “measurement problem” as a missing-map problem: not “how does the wavefunction collapse?” but “which CPTP instrument does the theory assign to

the laboratory protocol, and what falsifier does it imply?”

In the GG-Theory framework, there is no “collapse” as a physical event. There is only (i) deterministic 6D evolution, (ii) partial trace to 4D, (iii) conditioning on an outcome in the output register. Each of these is a mathematically precise and physically routine operation. “Collapse” is just Bayesian conditioning on a measurement outcome, exactly as in classical probability theory.

32 Deriving the Born rule from OGGE

Born’s 1926 reinterpretation of $|\psi|^2$ as a probability density (Born, 1926; Born *et al.*, 1926) completed the predictive apparatus of quantum mechanics, but offered no derivation; Gleason’s 1957 theorem (Gleason, 1957) sharpened the result to a uniqueness statement on a Hilbert space of dimension ≥ 3 , but only once that Hilbert space and its projective-measurement structure have already been assumed. The present section closes the remaining postulational gap. We derive the Born rule as a theorem of OGGE: the operationally registered probability of outcome α is forced by the CPTP completion of the bulk-to-boundary functor, with the textbook $|\langle a|\psi\rangle|^2$ recovered as the pure-state, sharp- projective specialization. This is the operational content of T7.3 of (Arizaka, 2026, A5-GDOS) Part III; the derivation is verbatim that of A5 Part III, here lifted to graduate-textbook depth.

32.1 The strategy

We derive the Born rule $P(a) = |\langle a|\psi\rangle|^2$ from three ingredients:

1. The state space is a complex separable Hilbert space $\mathcal{H}_4$ of dimension ≥ 3 (inherited from the 6D bulk zero-mode sector).
2. Any instrument $\{I_\alpha\}$ is a CPTP map with classical output, arising from (33) through the partial trace.
3. Probabilities for orthogonal outcomes must be additive (Kolmogorov consistency).

These three are consequences of the OGGE constitution. We then apply Gleason’s theorem to recover Born’s rule uniquely.

32.2 Step 1: the Hilbert-space structure from the bulk

In GG-2026-6F, it was shown that the bulk zero-mode sector — after Kaluza-Klein reduction on $I_r \times S_\theta^1$ and projection to the 4D boundary — is precisely a complex Hilbert space carrying representations of the Standard-Model gauge group. Specifically:

$$\mathcal{H}_4 = \text{span}\{\text{KK zero modes of } G_6 = \text{SU}(5) \times \text{U}(1)_B \times \text{U}(1)_{Q_i}\}. \quad (36)$$

The Hilbert space is not postulated; it descends from the functional integral over BI-6-compatible bulk configurations. Dimensionality is determined by the representation content of the bulk gauge group + spin-statistics + anomaly cancellation.

For our present purposes, we need only that $\mathcal{H}_4$ is complex, separable, and of dimension ≥ 3 — which it is, trivially, since $\text{SU}(5)$ alone has 24-dimensional adjoint representations and fermion sectors in higher-dimensional reps.

32.3 Step 2: CPTP structure from OGGE

The map $\mathcal{D} : \mathbf{Bulk} \rightarrow \mathbf{Op}$ is a CPTP channel by construction. Recall:

- **Complete positivity** (CP): the map preserves positivity of the composite state ρ_{SR} for arbitrary reference system R . This is required for physicality — the system may be entangled with anything, and the reduced state must stay positive.
- **Trace preservation** (TP): $\text{Tr}(\mathcal{D}\rho) = \text{Tr} \rho$. Probabilities must sum to one.
- **Linearity**: $\mathcal{D}(\alpha\rho_1 + \beta\rho_2) = \alpha\mathcal{D}(\rho_1) + \beta\mathcal{D}(\rho_2)$ for convex combinations. This follows from trace structure.

The functoriality of $\mathcal{D}$ (respecting composition and identities) plus CPTP gives exactly the Choi-Kraus representation

$$\rho_{\partial} = \mathcal{D}(\rho_{\text{bulk}}) = \sum_k A_k \rho_{\text{bulk}} A_k^\dagger, \quad \sum_k A_k^\dagger A_k = \mathbf{1}. \quad (37)$$

For a projective measurement on the boundary, the associated effects $\{E_\alpha\}$ satisfy $E_\alpha \geq 0$, $\sum_\alpha E_\alpha = \mathbf{1}$ — the POVM axioms.

32.4 Step 3: Gleason's theorem

Given a σ -additive probability measure μ on the lattice of orthogonal projectors of $\mathcal{H}_4$ ($\dim \geq 3$), Gleason's theorem (Gleason, 1957) states:

$$\mu(\Pi) = \text{Tr}(\rho \Pi), \quad (38)$$

for a unique density operator $\rho \geq 0$ with $\text{Tr} \rho = 1$.

The OGGE construction above produces exactly a σ -additive measure: for mutually orthogonal projectors $\Pi_\alpha = E_\alpha$ (projective instrument case) coming from the Kraus decomposition, additivity of probabilities is forced by trace linearity and orthogonality.

Theorem 32.1 (Born rule from OGGE + Gleason (T-QM-0)). *Given the OGGE functor $\mathcal{D}$ and the bulk 6D theory, the outcome probabilities of any 4D instrument on any 4D state are given uniquely by*

$$P(\alpha) = \text{Tr}(\rho_{\partial} E_\alpha) = \text{Tr}[\text{Tr}_{\text{env}}(\rho_{\text{bulk}}) E_\alpha]. \quad (39)$$

In particular, for pure boundary states $\rho_{\partial} = |\psi\rangle\langle\psi|$ and projective measurement $E_\alpha = |\alpha\rangle\langle\alpha|$,

$$P(\alpha) = |\langle\alpha|\psi\rangle|^2. \quad (40)$$

The Born rule is not a postulate of GG-Theory. It is a theorem — a consequence of the OGGEF functorial requirement plus Gleason’s uniqueness theorem.

32.5 Why the Born rule is *the* rule

The key insight is that *additivity and physicality force Born*. No other rule is compatible. Consider the alternatives:

- $P(a) = |\langle a|\psi\rangle|$ (linear in amplitude): not additive; fails Gleason.
- $P(a) = |\langle a|\psi\rangle|^4$: not additive; fails Gleason.
- $P(a) = \langle a|\psi\rangle\langle\psi|a\rangle$: same as Born for pure states; extends Born for mixed states.

Only Born satisfies additivity + non-contextuality + positivity on Hilbert spaces of dimension ≥ 3 . This was the point of Gleason.

GG-Theory’s contribution is not proving Gleason (1957) again. It is providing the OGGEF context that makes Gleason *apply*: the functor $\mathcal{D}$ with the CPTP structure justifies the Hilbert-space formalism and the additivity axiom. Without OGGEF, one had to postulate these; with OGGEF, they are consequences.

33 Deriving the Lindblad equation from CPTP + local causality

The Gorini–Kossakowski–Sudarshan and Lindblad theorems of 1976 ([Gorini *et al.*, 1976](#); [Lindblad, 1976](#)) jointly fix the most general generator of a quantum dynamical semigroup that preserves complete positivity and trace: a Hamiltonian conjugation plus a Lindblad dissipator built from jump operators $\{L_\alpha\}$ and non-negative rates $\{\gamma_\alpha\}$. The GKLS theorem is the dynamical companion of Gleason’s theorem: one fixes probabilities, the other fixes evolutions. In standard treatments, both are theorems whose hypotheses (complete positivity, trace preservation, Markovianity) are postulated. The present section shows that all three hypotheses are forced by the constitutional content of OGGEF: CPTP is positivity required of any boundary channel; local causality (AX-1) supplies the forward-time semigroup structure; Markovianity is the leading-order regime in the bulk–environment correlation time, with corrections calculable from the scale-overlap integrals of the GG-6 bulk. This recovers T7.2 (closed-system limit, $\gamma_\alpha \rightarrow 0$) and the T7.5 foundation of the decoherence calculation that follows.

33.1 The strategy

We show that the continuous-time open-system evolution of $\rho_\partial(t)$ is uniquely given by the GKSL (Lindblad) master equation. The key inputs are: (i) CPTP for every finite t ; (ii) semigroup property $\Phi_{t+s} = \Phi_t\Phi_s$ from forward time-ordering + Markovianity; (iii) continuity at $t = 0$. Then the Gorini–Kossakowski–Sudarshan–Lindblad ([Gorini *et al.*, 1976](#); [Lindblad, 1976](#)) theorem uniquely specifies the generator.

33.2 Step 1: the semigroup property from AX-1 (local causality)

AX-1 in the GG-Constitution is *local causality*: physical influence propagates forward in time at finite speed. We now justify why the bulk-side correlation structure forces a Markovian boundary-side dynamics at leading order, so that the semigroup property holds without being merely postulated.

The two-scale separation. The bulk environment of any laboratory system has a characteristic correlation time set by the bulk Kaluza–Klein mass gap above the lock-cluster scale: $\tau_c \sim \hbar/M_{\text{lock}}c^2 \sim 10^{-27}$ s (taking $M_{\text{lock}} \approx 3.85$ TeV). The characteristic boundary system time is set by the boundary energy splitting of the observable in question: $\tau_S \sim \hbar/E_{\text{sys}}$, typically 10^{-15} s for atomic transitions, 10^{-9} s for qubit dynamics, longer for macroscopic systems. In all laboratory regimes the ratio $\tau_c/\tau_S \lesssim 10^{-12}$ is parametrically small. By AX-1 (local causality), bulk-environment correlations decay on the timescale τ_c ; on the boundary timescale τ_S , those correlations have already vanished by a factor $e^{-\tau_S/\tau_c}$ that is exponentially small. The boundary dynamics therefore inherits no memory from the bulk environment at leading order in τ_c/τ_S , and the evolution from 0 to $t + s$ factorizes into independent steps:

$$\Phi_{t+s} = \Phi_t \circ \Phi_s + \mathcal{O}(\tau_c/\tau_S), \quad (41)$$

with the non-Markovian corrections calculable order-by-order from the scale-overlap integrals of bulk KK modes, using the scale-overlap machinery of GG-A5-GDOS (Arisaka, 2026, A5-GDOS) Part II (the explicit higher-order expansion is a program target, not assumed here). Two correlation times must be kept distinct. The bulk Kaluza–Klein time $\tau_c \sim 10^{-27}$ s is what guarantees that the *bulk* contributes no memory — the bulk face of the Port-Freshness Clause of the GDOS companion — so the boundary law has semigroup *form*; the actual decoherence of a laboratory system is instead governed by its coupling to its *physical* 4D environment (air molecules, thermal photons), whose own correlation time $\tau_{\text{bath}} \sim 10^{-13}$ s is likewise short against the system time τ_S and is the one that sets the Markovianity and the rate of real decoherence. The bulk smallness supplies the structure; the physical bath supplies the dynamics.

The evolution $\{\Phi_t\}_{t \geq 0}$ in the leading-order Markovian regime is a *one-parameter semigroup* (not a group — there is no inverse, since open evolution is irreversible). The seminal Lindblad derivation (Lindblad, 1976) assumed this regime; the present framework derives it from the parametric smallness of τ_c/τ_S , with quantitative non-Markovian corrections in principle observable in high-precision measurements above the lock-cluster scale.

33.3 Step 2: the generator from continuity

Assuming strong continuity at $t = 0$ — $\lim_{t \rightarrow 0^+} \Phi_t \rho = \rho$ in some operator norm — the semigroup has a generator L defined by

$$L\rho \equiv \lim_{t \rightarrow 0^+} \frac{\Phi_t \rho - \rho}{t}, \quad \Phi_t = e^{tL}. \quad (42)$$

The question is: what is the most general L compatible with CPTP for all $t \geq 0$?

33.4 Step 3: the GKSL theorem

Theorem 33.1 (Gorini–Kossakowski–Sudarshan–Lindblad, 1976). *A linear operator L on the space of trace-class operators generates a CPTP semigroup $\Phi_t = e^{tL}$, $t \geq 0$, if and only if there exist a self-adjoint operator H and a countable set of operators $\{L_j\}$ such that*

$$L\rho = -i[H, \rho] + \sum_j \left(L_j \rho L_j^\dagger - \frac{1}{2} \{L_j^\dagger L_j, \rho\} \right). \quad (43)$$

This is the *Lindblad equation* or the *GKSL master equation*. H is the effective Hamiltonian; the L_j are the *jump operators* encoding system–environment coupling.

33.5 The physical interpretation

The Lindblad form (43) has two pieces:

- *Coherent evolution*: $-i[H, \rho]$, unitary-like.
- *Dissipator*: $\sum_j (L_j \rho L_j^\dagger - \frac{1}{2} \{L_j^\dagger L_j, \rho\})$ — produces trace-preserving loss of coherence.

The dissipator is the only piece compatible with all three axioms simultaneously. In particular:

- Stone’s theorem for unitary evolution ($L_j = 0$): special case with pure H -evolution, reversible.
- GKSL with non-zero L_j : irreversible; ρ approaches equilibrium.

33.6 Connection to measurement

A projective-type measurement corresponds to a specific limit of the Lindblad equation: strong continuous monitoring in which the jump operators are the measurement basis projectors $L_j = \sqrt{\Gamma} \Pi_j$. In this limit, the off-diagonal coherences decay exponentially at rate Γ , and the density matrix becomes diagonal in the $\{\Pi_j\}$ basis. This is the quantum Zeno limit / continuous-measurement limit. The Born probabilities emerge as the diagonal elements of the stationary state.

34 Decoherence as automatic consequence of OGGEF

The decoherence program of Zeh (1970), Zurek (1981, 2003), and Joos and Zeh (1985) was the most significant theoretical advance on the measurement problem in the half-century after the Bohr–Einstein debate: it showed that the off-diagonal elements of a system’s reduced density matrix decay exponentially fast under environmental coupling, on time-scales that for macroscopic objects are essentially instantaneous (Lemma 3 of (Arisaka, 2026, A5-GDOS) Part III estimates $\Gamma \gtrsim 10^{40} \text{ s}^{-1}$ for a Schrödinger’s cat at room temperature). The program had two acknowledged limits: it did not derive the Born rule, and it did not by itself select the preferred basis. (Arisaka, 2026, A5-GDOS) Part III closes both gaps in T7.5 and T7.6; the present section recovers the operational core (off-diagonal damping at rate $\Gamma_{\alpha\beta}$) directly from the OGGEF functor’s environmental partial trace, with the rate coefficients fixed by the scale-overlap integrals of the 6D bulk rather than postulated by hand.

34.1 The decoherence question

In traditional quantum mechanics, decoherence is a process that *happens* to a system coupled to an environment. In GG-Theory, decoherence is *automatic*: the OGGEF functor always includes Tr_{env} , so any 4D state is always the partial trace of a 6D bulk state. Off-diagonal coherences of the 6D state in the 4D pointer basis are always suppressed.

34.2 The GRIP chain: $\text{DGI} \rightarrow \text{GFF} \rightarrow \text{GSSB} \rightarrow \text{GA}$

The GG-2026-4A paper develops the GRIP runtime principle, a four-step dynamical pattern that governs any admissible open system:

1. **DGI** (Dynamical Gauge Invariance): the 6D packet carries all possible seams at the bulk level — a superposition of all admissible boundary phases.
2. **GFF** (Geometric Free-energy Functional): the packet descends through the bulk toward the observable 4D boundary. Each candidate branch carries a scalar cost (free energy) determined by the 6D geometry and gauge fields.
3. **GSSB** (Geometric Spontaneous Symmetry Breaking): at a specific scale, a single branch is selected from the ensemble of possibilities — the branch that minimizes the GFF functional.
4. **GA** (Geometric Attractor): the selected branch stabilizes into the low-energy Standard Model + SM-like phenomenology.

For quantum measurement, the GRIP chain plays out as follows:

- **DGI**: the system+apparatus is in a superposition of all possible outcome branches.
- **GFF**: each outcome branch carries a scalar cost, set by the overlap of the corresponding environment state with the bulk equilibrium.
- **GSSB**: one branch (the *realized* outcome) is selected — the one compatible with the incoming bulk configuration.
- **GA**: the realized branch stabilizes; the system is now in a definite-outcome state.

The selection in GSSB is the “collapse” — but it is not a physical event in 4D; it is a consequence of the 6D bulk choosing a single branch (the 6D is always single-valued by MSMF; only the 4D observer sees a superposition).

34.3 The Zurek pointer basis via einselection

In the environment-induced superselection framework of Zurek (Zurek, 2003), the pointer basis is the one that is monitored by the environment. In GG-Theory, this is made sharp: the pointer basis is the basis in which the GSSB branch-selection functional GFF is extremized. In practice this is usually the position basis for macroscopic pointers (because the environment couples strongly to position) and the energy basis for single-atom systems (where the environment couples to the internal electromagnetic field).

34.4 Decoherence rates from OGGE

For a system coupled to the bulk environment, the decoherence rate is

$$\Gamma_{nm} = \sum_k |\langle n|L_k|n\rangle - \langle m|L_k|m\rangle|^2 \cdot \Gamma_k, \quad (44)$$

where Γ_k are the rates at which the bulk KK modes carry information away from the system. For typical macroscopic objects (mass $\sim$ gram, interaction with air at STP), the thermal relaxation frequency is

$$\Gamma_{\text{thermal}} \sim k_B T / \hbar \sim 10^{13} \text{ Hz}, \quad (45)$$

while the scale-overlap (Joos–Zeh collisional) rate for a *spatial* superposition is far faster, $\Gamma_{\text{spatial}} \sim 10^{40} \text{ s}^{-1}$; either way the decoherence timescale is many orders of magnitude faster than any observable dynamics. This is why macroscopic superpositions are never seen in everyday physics.

For isolated single atoms in ultra-high vacuum, τ_D can reach milliseconds or even seconds — which is why ion-trap and atom-interferometry experiments can see large coherent superpositions.

35 The four-Pillar Qi-4 structure for quantum systems

The GG-2026-4B paper extends GDOS with the four-pillar “Qi-4” structure, each pillar governing a different aspect of open-system behavior. For quantum mechanics, each pillar plays a specific role:

35.1 Qi-QUA: Current locking and the Arisaka Equation

Qi-QUA is the event-ledger pillar: it locks the four currents (energy, momentum, charge, Qi-number) onto a one-dimensional ray through the BI-6 structure. For a quantum system undergoing Lindblad evolution, the Qi-QUA locking produces the *Arisaka Ratio*: the ratio of information flow to energy flow is quantized in units of $k_B T / \hbar$ (up to geometric factors set by OGN-5).

This is the quantum version of the Landauer principle: each bit erased (in the Lindblad dissipator) costs $k_B T \ln 2$ of entropy, which matches the Qi-QUA-forced ratio.

35.2 Qi-EP: Extremum principles (MEPP, FEP, PLT)

Qi-EP is the selector-law pillar. It shows that the GSSB branch selection is governed by a single extremum principle: *the physical branch extremizes the GFF (free-energy functional) subject to the ledger constraints*. This extremum principle is the modern unification of MEPP (maximum entropy production principle), FEP (free-energy principle), and PLT (principle of least action in the thermodynamic sense).

For quantum measurement, Qi-EP says: the realized outcome is the GFF-extremizing branch. Born-rule probabilities emerge as a measure on the branches weighted by the GFF cost.

35.3 Qi-CON: Conformality, fractality, $1/f$ spectra

Qi-CON governs the scale-invariant structure of dissipation. For quantum measurements, Qi-CON predicts:

- $1/f$ *noise* in macroscopic measurements — the famous “pink noise” that appears in every precision apparatus.
- Scale-invariance of decoherence: the decoherence rate scales with environment coupling but has the same functional form at different scales.
- Conformal structure of long-range correlations — a key prediction for entangled-pair experiments at macroscopic separation.

35.4 Qi-HAL: Memory-bearing architecture

Qi-HAL is the architecture pillar — the memory-bearing, holonomic structure that allows a system to “remember” past interactions. In quantum measurement, Qi-HAL is the mechanism that makes the pointer record stable: once an outcome is registered, the record is topologically protected by the bulk Wilson-line holonomies on S^1_θ (which in the cohort canonical bulk include the $U(1)_{Qi}$ factor).

In everyday physics, Qi-HAL is what makes classical records classical — a photograph, a magnetic tape, a scratch on a surface. These are all topologically protected against thermal fluctuations by the bulk-gauge-field dynamics on S^1_θ (with Qi-HAL the cohort-canonical pillar of GG-A7-Qi4 that formalizes this protection).

36 The SIP (System Instance Packet): canonical unit

GG-2026-4B introduces the SIP as the canonical unit of GDOS accounting. For a quantum measurement, the SIP consists of:

- **State:** the 4D density matrix ρ_∂ .
- **Ports:** the input/output channels (the system-environment couplings).
- **Currents:** the four currents (energy, momentum, charge, Qi-number) flowing through the ports.
- **Ledger:** the scalar accounting of the GFF cost.
- **Dictionary:** the operational instrument $\{I_\alpha\}$ specifying what outcomes are registered.

The SIP is the universal bookkeeping unit that any open quantum system fits into. It is not a new physical entity; it is the minimal data structure required to specify a GDOS instance.

37 Reduction to AdS/CFT, Randall-Sundrum, and GG-6 — similarities and differences

Two precedents in the high-energy-theory literature relate higher-dimensional bulks to lower-dimensional boundaries: Maldacena’s 1997 AdS/CFT correspondence ([J. Maldacena, 1998](#)), an exact strong–weak duality between two equivalent quantum theories; and the Randall–Sundrum warped-extra-dimension scenarios of 1999 ([L. Randall and R. Sundrum, 1999](#)), in which a non-factorizable 5D scale-profile metric solves the hierarchy problem without requiring large extra geometric directions. The GG-6 bulk shares features with both — a radial direction with r playing the RG-scale role of AdS, and an effective 4D boundary law (GDOS) that specializes to the textbook QM postulates — but the relation is neither an exact duality (the projection is non-invertible) nor a passive backdrop (the boundary law is structurally forced by OGGE). The present section makes the three-way comparison explicit so that readers familiar with AdS/CFT or Randall–Sundrum can locate the GG-6/GDOS construction precisely on the holography landscape.

37.1 AdS/CFT is an exact duality

In AdS/CFT ([J. Maldacena, 1998](#)), the bulk and boundary theories are *equivalent* as quantum theories — two descriptions of the same physics. The duality is ontologically symmetric.

37.2 GG-6 is an asymmetric projection

In GG-6, the 6D bulk is the *primary* theory and the 4D boundary is a *reduction* — a partial trace. The reduction is not a duality; the 4D theory does not know everything that the 6D theory does. Information is genuinely lost in the partial trace.

This asymmetry is what makes the measurement problem solvable in GG-6 but not in AdS/CFT: the 6D deterministic theory can have a well-defined state and evolution even when the 4D observer sees probabilistic outcomes.

37.3 Comparison table

Table 3: How GG-6 differs from AdS/CFT and Randall-Sundrum for the measurement problem. The columns compare what each construction takes as given, what it derives, and what it leaves to interpretation. The two differences that matter here are the compact phase direction and the non-invertibility of the projection; the holographic constructions share the bulk-to-boundary geometry and not the measurement content, which is why they are neighbors of this reading rather than rivals to it.

Feature	AdS/CFT	Randall-Sundrum	GG-6/GDOS
Bulk dimension	5	5	6
Bulk-boundary	Exact duality	Gravity localization	Partial trace
Unitary bulk?	Yes	Yes	Yes
Unitary boundary?	Yes (CFT)	Yes	No (CPTP)
Measurement theory?	External	External	Derived from Tr_{env}
Born rule?	Assumed	Assumed	Derived via Gleason
Lindblad?	N/A	N/A	Forced by AX-1 + CP
Decoherence?	N/A	N/A	Automatic from Tr_{env}
Bell violations?	In CFT	In brane	6D-local artifact of Tr_{env}

38 Worked example: Stern-Gerlach in GDOS

The Stern–Gerlach experiment of 1922 ([Gerlach and Stern, 1922](#)), performed in Frankfurt with a beam of silver atoms passing through an inhomogeneous magnetic field, is the original demonstration that intrinsic angular momentum is quantized: the two discrete spots on the detector, separated by an amount proportional to the field gradient, were the first direct observation that a microscopic degree of freedom can take only discrete values. Goudsmit and Uhlenbeck’s 1925 spin hypothesis ([Uhlenbeck and Goudsmit, 1925](#)) supplied the theoretical framework a few years later. We use the simplest non-trivial Stern–Gerlach setup — a single qubit prepared in $|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$ and measured along direction $\hat{n}$ — as the prototype worked example of the GDOS/OGGEP construction, with the 6D bulk state, the boundary partial trace, the instrument family, and the Born-rule probabilities all calculated explicitly. The same calculation appears in ([Arisaka, 2026, A5-GDOS](#)) Part III as the worked example for T7.3.

To make the construction concrete, we work through the Stern-Gerlach measurement in the GDOS / OGGEP language.

38.1 The bulk state

A silver atom in spin state $|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$ has a 6D bulk description:

$$|\psi\rangle_{\text{bulk}} = (\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \otimes |\text{holonomy } \Theta_{I_r \otimes S_\theta^1}^{(+)}\rangle, \quad (46)$$

where the holonomy factor encodes the 6D Wilson-line phase along the extra geometric directions. This holonomy carries the “Qi-number” that will determine the realized outcome.

38.2 The apparatus coupling

The magnetic-field gradient entangles spin with position. The unitary U_{SA} acts as

$$|\uparrow\rangle|x_0\rangle \longrightarrow |\uparrow\rangle|x_0 + \delta\rangle, \quad |\downarrow\rangle|x_0\rangle \longrightarrow |\downarrow\rangle|x_0 - \delta\rangle. \quad (47)$$

After the magnet, the state is

$$(\alpha|\uparrow\rangle|x_0 + \delta\rangle + \beta|\downarrow\rangle|x_0 - \delta\rangle) \otimes |\text{holonomy}\rangle. \quad (48)$$

The spin is now entangled with position.

38.3 The detector and partial trace

A detector at the spatial locations $x_0 \pm \delta$ measures which location the atom arrived at. In the GDOS language, this is a POVM with elements

$$E_\uparrow = |x_0 + \delta\rangle\langle x_0 + \delta|, \quad E_\downarrow = |x_0 - \delta\rangle\langle x_0 - \delta|. \quad (49)$$

The outcome probabilities are $P(\uparrow) = |\alpha|^2$ and $P(\downarrow) = |\beta|^2$, from (39).

38.4 The GSSB branch selection

At the detector, the GFF functional has two competing branches (one for each detector location). The GSSB step selects one. Which one? The one compatible with the incoming 6D bulk holonomy. Specifically, if the bulk Wilson-line holonomy at the moment of detection corresponds to $\Theta^{(+)}$, the $\uparrow$ -branch is selected; if $\Theta^{(-)}$, the $\downarrow$ -branch. The holonomy value is *not accessible* to the 4D observer — it lives along the phase axis S_θ^1 direction — but it is a bulk-deterministic variable. From the 4D observer's perspective, the outcome is random (probabilities $|\alpha|^2, |\beta|^2$); from the 6D bulk perspective, it is determined by Θ .

38.5 The summary

Stern-Gerlach in GDOS is *not* a process where the wave function collapses. It is a process where (i) the 6D bulk Wilson-line holonomy $\Theta^{(\pm)}$ on the phase axis S_θ^1 determines the outcome deterministically; (ii) the four-dimensional observer, forced by the dimensionality of the laboratory to trace over the geometric directions I_r (scale) and S_θ^1 (phase), sees the outcome as random; (iii) the randomness is forced to the Born distribution $|\alpha|^2, |\beta|^2$ by Gleason's theorem applied to the resulting boundary algebra. Everything is consistent, every step is a theorem, and there is no measurement problem.

What this single worked example illustrates is the entire content of Part III in miniature: the constitutional bridge GG-6 bulk $\rightarrow$ OGGEF $\rightarrow$ GDOS $\rightarrow$ 4D QM is not an interpretive device but a compiler — it takes a deterministic six-dimensional theory and outputs the textbook quantum mechanics that laboratory practice uses, with every boundary-side feature (probabilistic outcomes, off-diagonal damping, pointer-basis selection, Bell-violating correlations) forced by the bulk-side

structure and the four-dimensional trace. Parts V–IX then exercise the compiler on five distinct laboratory systems.

Part III in plain English: the compiler that turns geometry into quantum mechanics

The four-stage chain at the heart of this Part can be read in plain words. At the input end stands a six-dimensional bulk geometry, $\mathcal{M}_4 \times I_r \times S_\theta^1$, on which the dynamics is deterministic, unitary, and local — exactly the kind of theory Einstein hoped Nature would turn out to use. At the output end stands the textbook of quantum mechanics: a Hilbert space, a Schrödinger equation, a Born rule, a Lüders projection, decoherence, einselection, and the violation of Bell’s inequality. Between input and output sits the constitutional bridge: OGGE_P, the operational postulate that says *any* admissible bulk theory must come equipped with a structure-preserving map to the operational records a finite four-dimensional laboratory produces. That map is forced — by gauge covariance, causality, positivity, and scale consistency — to take the form of a partial trace over the two extra geometric directions (I_r for scale, S_θ^1 for phase) followed by an instrument-family construction.

The traditional reading of quantum mechanics asks how measurement *produces* randomness from a deterministic state. The reading developed here turns the question around. There is no production of randomness; there is only the unavoidable four-dimensional view of a six-dimensional deterministic dynamics. When the boundary observer integrates out the two geometric directions she cannot access, the result is a probability distribution. When she records an outcome, the result is Bayesian conditioning on which boundary branch the bulk Wilson-line holonomy delivered. When she watches the off-diagonal terms of a density matrix decay, the result is the overlap integral of the bulk environment configurations that project to her record. None of these requires a new postulate. All are theorems of the bridge.

After all, the most patient and accurate way to read the hundred-year history of quantum mechanics is that the community learned the boundary law before it learned the bulk geometry. Part III supplies the missing half — the geometry — and shows that the boundary law every laboratory has tested to twelve digits is exactly what the geometry compiles into. The compiler is the mechanism. Everything that follows in this paper is an application of it.

Table 4: Dictionary translating 4D quantum-mechanical concepts to their 6D bulk origin in GG-Theory. Read the left column as what a boundary observer measures and the right as the bulk structure of which that measurement is a shadow. The dictionary states what the framework claims rather than deriving it; each entry is discharged by the theorem named in the row that carries it, and an entry with no theorem beside it is a reading, not a result.

4D quantum concept	6D bulk origin	Mechanism
Hilbert space $\mathcal{H}_4$	KK zero-mode sector of $G_6 = \text{SU}(5) \times \text{U}(1)_B \times \text{U}(1)_{Q_i}$	Representation content at bulk-to-boundary projection
State $ \psi\rangle \in \mathcal{H}$	Bulk wavefunctional $\Psi_{\text{bulk}}[\Phi_6]$ projected to zero-modes	Partial trace over heavy KK + I_r, S_θ^1
Density matrix ρ_∂	$\text{Tr}_{\text{env}}(\rho_{\text{bulk}})$	Automatic loss of information about extra geometric directions
Born rule $P(a) = \langle a \psi\rangle ^2$	Gleason's unique additive measure on $\mathcal{H}_4$	Forced by CPTP + additivity + $\dim \geq 3$
Observable $\hat{A} = \hat{A}^\dagger$	Bulk gauge-current operator restricted to zero-modes	Self-adjointness from Hermiticity of bulk Hamiltonian
Commutator $[\hat{x}, \hat{p}] = i\hbar$	Bulk Poisson-bracket structure on zero-mode phase space	Inherited from bulk symplectic form
Schrödinger equation	Stone theorem applied to zero-mode unitary subgroup	Closed-subsystem limit of the Lindblad equation
Lindblad equation GKSL	CPTP semigroup from AX-1 + AX-3 + continuity	Partial trace of 6D unitary + Markovian limit
Decoherence	Tr_{env} off-diagonal suppression	Automatic for any 4D observer
Wave-function collapse	Bayesian conditioning on outcome label	Not a physical event in either 6D or 4D
Entanglement	Shared S_θ^1 -fiber bulk configuration	6D-local; 4D-nonlocal after trace
Spin $1/2$	Irrep of $\text{SU}(2) \subset G_6$ zero-mode sector	Geometric: half-integer winding on bulk bundles
Photon γ	$\text{U}(1)_{\text{EM}}$ zero-mode gauge boson	KK zero mode of bulk gauge field
Hidden variable	Wilson-line holonomy Θ_H on S_θ^1	Inaccessible to 4D observers; deterministic in 6D
Pointer basis (Zurek)	Basis diagonalizing the GSSB free-energy functional GFF	Dynamical selection by environment coupling
POVM element E_α	$\text{Tr}_{\text{app}}[(\mathbf{1} \otimes \Pi_\alpha)U(\cdot \otimes \sigma_{\text{app}})U^\dagger]$	Stinespring dilation from bulk Hamiltonian
Tsirelson bound $2\sqrt{2}$	Maximum CHSH value allowed by MSMF 6D Lagrangian	Could be exceeded only by non-MSMF bulk

Part IV

Statement of results

Eleven principal theorems: T-A8.1–T-A8.7 inherited, T-A8.8–T-A8.11 net-new

This Part is the technical core of GG-A8-QM: the eleven named theorems of the present paper. Each Section follows the cohort six-element envelope — L4 historical opening, plain-English physics question, formal theorem statement (dual-layer algebraic + operational), proof outline, quantitative anchor, physics intuition with daily-life metaphor, kill criterion, plain-English close — at the depth a careful graduate student in physics will need on a first reading.

Part IV is the results Part of GG-A8-QM. Where Part III states the constitutional bridge GG-6 bulk $\rightarrow$ OGGE $\rightarrow$ GDOS $\rightarrow$ 4D QM as a mechanism, Part IV states the eleven principal theorems that bridge produces. Read together, the eleven theorems exhaust the operational content of textbook quantum mechanics: the four von Neumann postulates (T-A8.1–T-A8.4), the three decoherence-program structures that a century of foundations work added to them (T-A8.5–T-A8.7), and four further theorems (T-A8.8–T-A8.11) that the bulk-side machinery makes available but standard treatments do not state. Nothing in the rest of the paper requires a postulate beyond these eleven.

The eleven theorems split into two groups. The first seven, T-A8.1 through T-A8.7, are operational restatements of T7.1 through T7.7 of GG-A5-GDOS Part III ([Arisaka, 2026](#), [A5-GDOS](#)); their proofs are inherited from GG-A5-GDOS and cited by reference, with the present treatment supplying only the textbook-pedagogical envelope. The remaining four, T-A8.8 through T-A8.11, are net-new to GG-A8-QM: they promote results that GG-A5-GDOS stated as lemmas or sub-theorems, or that the cohort apparatus makes possible but GG-A5-GDOS had no room to develop, to the status of principal theorems of the QM register. All eleven are constitutional — forced by AX-1–AX-4 of the GG-Constitution (GG-A1-Constitution) acting on the bulk geometry of GG-A2-GG6 via the GDOS channel of GG-A5-GDOS Part II — and none introduces any new axiom.

Table 5: The eleven principal theorems of GG-A8-QM. The first seven inherit from GG-A5-GDOS Part III by operational restatement; the last four are net-new contributions of the present paper. The four columns give the theorem number, its statement in one line, the constitutional warrant it draws on, and the Part in which it is proved. The split matters for what the paper is claiming: the inherited seven are GG-A5-GDOS results restated at the boundary and carry no new assumption, while the net-new four are the paper’s own and stand or fall with it.

Theorem	Lineage	Statement (one line)
T-A8.1	inherited (T7.1)	The boundary Hilbert space is the GNS representation of the QM-restricted boundary algebra of the 6D bulk.
T-A8.2	inherited (T7.2)	The Schrödinger equation is the closed-system ($\gamma_\alpha \rightarrow 0$) limit of the GKLS generator of the GDOS channel.
T-A8.3	inherited (T7.3)	The Born rule $p_\alpha = \text{Tr}(\mathcal{I}_\alpha \rho)$ is forced by CPTP completion of the OGGEF dictionary.
T-A8.4	inherited (T7.4)	The Lüders projection $\rho \mapsto \mathcal{I}_\alpha(\rho)/p_\alpha$ is the conditional CPTP posterior.
T-A8.5	inherited (T7.5)	Off-diagonal coherences decay at rate $\Gamma_{\alpha\beta}$ fixed by scale-overlap integrals of bulk KK modes.
T-A8.6	inherited (T7.6)	The einselected pointer basis is the eigenbasis of the dominant local system–environment coupling (AX-1); for a macroscopic center-of-mass this is position, for a flux/charge qubit it is flux/charge.
T-A8.7	inherited (T7.7)	The CHSH bound is violated up to the Tsirelson saturation $ S = 2\sqrt{2}$ as the boundary shadow of one non-separable joint bulk state.
T-A8.8	net-new	The Tsirelson saturation is an extremal (Bogomolnyi-analogue) bound: the singlet is the CHSH extremizer, so saturation is first-order stationary and structurally stable, not a tuned coincidence.
T-A8.9	net-new	Every electron registered in any laboratory is a token of the unique stable GA of the charged-lepton stratum; quantum indistinguishability is a cascade inheritance, not a postulate.
T-A8.10	net-new	Decoherence-rate spectra are approximately $1/f$ within a bounded conformal window, with a system-specific exponent near unity set by Qi-CON (GG-A7-Qi4).
T-A8.11	net-new	The non-separable joint bulk state underlying Bell correlations carries no operational signal; the boundary marginal at site B is independent of the measurement settings at site A.

The remainder of this Part proves the four net-new theorems and inherits the seven from GG-A5-GDOS with the full pedagogical envelope.

39 T-A8.1 — The Hilbert-space postulate as theorem

39.1 L4 historical opening

The Hilbert-space framework of quantum mechanics was assembled in the years 1925–1932 by Heisenberg, Schrödinger, Dirac, and von Neumann (Dirac, 1930; Heisenberg, 1925; Neumann, 1932; Schrödinger, 1926). Von Neumann’s 1932 *Mathematische Grundlagen* fixed the modern axiomatic form: the state of a closed quantum system is a unit vector in a complex separable Hilbert space, with

observables represented by self-adjoint operators. Every standard graduate textbook (Ballentine, 1998; Cohen-Tannoudji *et al.*, 1992; Peres, 1995; Sakurai and Napolitano, 2011) states this as a primitive postulate. The constitutional warrant of GG-A1-Constitution promotes the Hilbert-space *structure* to a theorem; the complex scalars over which that structure is built enter with the algebra it represents, and their own constitutional standing is examined in §117.

39.2 The physics question

Why a Hilbert space? Why not a real vector space, a Banach space, a more exotic algebraic structure? And why, supposing one accepts the framework, does an isolated electron live in $L^2(\mathbb{R}^3)$ rather than in some other infinite-dimensional Hilbert space? The textbook answer is silence: the postulate is asserted and verified by its predictive success. The framework’s answer is direct: the boundary Hilbert space is the GNS representation (Gelfand and Naimark, 1943; Naimark, 1943) of the QM-restricted boundary algebra inherited from the 6D bulk of Arisaka (2026, A2-GG6). One scope note belongs here rather than in a footnote: GNS answers “why a Hilbert space” — the linear, inner-product, completeness structure — while inheriting its complex scalars from the algebra it represents; the distinct question “why *complex*” is answered separately, in §116 and §117.

39.3 Theorem statement

Theorem 1 (T-A8.1 — Hilbert-space postulate as theorem (T-QM-1)). ***Algebraic (T-A8.1.1).** The QM-restricted boundary algebra $\mathcal{A}_{\text{Bdy}}^{(\text{QM})}$, generated by the zero-mode operators $\{\phi^{(0,0)}(x)\}$ of the bulk fields and their conjugate momenta, admits a unique-up-to-bulk-gauge faithful representation on a complex separable Hilbert space $\mathcal{H}_{\text{Bdy}}^{(\text{QM})}$, with an isometric embedding $\iota : \mathcal{H}_{\text{Bdy}}^{(\text{QM})} \hookrightarrow \mathcal{H}_{\text{Bulk}}$ that intertwines the algebra actions.*

***Operational (T-A8.1.2).** The operationally registered state of the boundary system is the density operator $\rho_{\text{Bdy}}^{(\text{QM})} = \mathcal{R}_{\text{QM}}[\text{Tr}_{\text{env}} \rho_{\text{Bulk}}]$, positive and trace one on $\mathcal{H}_{\text{Bdy}}^{(\text{QM})}$. Pure states correspond to the limiting case of negligible bulk–boundary entanglement.*

Proof outline. Inherited from T7.1 of (Arisaka, 2026, A5-GDOS) Part III via the GNS construction (Haag, 1992; Streater, 1964): any unital C^* -algebra together with any state on it admits a cyclic representation on a Hilbert space; choosing the state to be the restriction of the bulk vacuum to $\mathcal{A}_{\text{Bdy}}^{(\text{QM})}$ produces the boundary GNS space, which embeds isometrically into the bulk GNS space (which is $\mathcal{H}_{\text{Bulk}}$). Density-matrix recovery follows from partial trace plus CPTP boundary reduction, both of which preserve positivity and trace. ■

39.4 Quantitative anchor

For the simplest non-trivial case — a single fermion zero mode realizing a qubit — $\mathcal{H}_{\text{Bdy}}^{(\text{QM})} = \mathbb{C}^2$, the standard qubit Hilbert space; the Bloch-ball constraint $|\vec{r}| \leq 1$ on the density matrix $\rho = \frac{1}{2}(\mathbf{1} + \vec{r} \cdot \vec{\sigma})$ inherits directly from the bulk-unitarity bound on the reduced state. Every textbook qubit calculation (Pauli algebra, Bloch sphere, single-qubit gates, the geometric phase, two-level Rabi oscillations) is a specialization of T-A8.1 to this realization.

39.5 Physics intuition and daily-life metaphor

Physics intuition. The Hilbert space is not added as an external mathematical convenience: it is the smallest linear-algebraic object that can faithfully carry the boundary algebra’s structure (commutation relations, eigenvalue spectra, transition amplitudes). Once one accepts that the boundary algebra exists (which AX-2/AX-3 of GG-A1-Constitution force), the Hilbert space follows automatically by GNS.

Daily-life metaphor. A photograph is not just a sheet of paper; it is the smallest two-dimensional surface that can faithfully carry the visual structure of the scene. Try to record the scene on a strip or in a point and the structure tears. The Hilbert space is the analogous smallest object for the boundary algebra: not chosen, but forced by what one is trying to represent.

39.6 Kill criterion

T-A8.1 would be retired by the experimental discovery of a quantum system whose boundary algebra cannot be embedded in any Hilbert space — for instance, a system exhibiting a probability rule incompatible with σ -additivity on closed subspaces. No such system has ever been observed; the framework’s prediction matches a century of laboratory experience.

39.7 Plain-English close

The Hilbert-space framework of quantum mechanics is not added on top of the physics by mathematical fiat. It is the unique algebraic object compatible with the boundary algebra that OGGEF forces. The first textbook postulate is therefore not a postulate; it is a structural inevitability. One register note completes it: the recovered space arrives *complex* — not by assumption but because the pairing the projection installs is the derived complex unit J of the real-bulk statements (§117; owner Arisaka, 2026, P4-Real, derived-conditional in the quasi-free regime), so even the field $\mathbb{C}$ under this theorem is an output of the same descent.

40 T-A8.2 — The Schrödinger equation as closed-system limit

40.1 L4 historical opening

Schrödinger’s 1926 wave equation (Schrödinger, 1926), written down in January 1926 and equivalent to Heisenberg’s 1925 matrix mechanics (Heisenberg, 1925) as Dirac showed in 1930 (Dirac, 1930), is the most-celebrated equation in twentieth-century physics. Every textbook presents it as primitive, on the same footing as Newton’s $F = ma$. But just as Newton’s laws are the classical limit of Lorentzian dynamics, the Schrödinger equation is the closed-system limit of a deeper open-system dynamics: the GKLS master equation of Gorini *et al.* (1976); Lindblad (1976).

40.2 The physics question

Is the Schrödinger equation primitive, or is it the limiting case of some deeper dynamical law? The textbook answer is no. The framework’s answer is yes: the constitutional boundary law is the

GKLS generator of the GDOS channel, and the Schrödinger equation is its $\gamma_\alpha \rightarrow 0$ closed-system limit.

40.3 Theorem statement

Theorem 2 (T-A8.2 — Schrödinger equation as closed-system limit (T-QM-2)). ***Algebraic (T-A8.2.1).** In the limit $\gamma_\alpha^{(\text{QM})} \rightarrow 0$ where all GKLS dissipation rates vanish, the boundary master equation reduces to the von Neumann equation $\partial_t \rho = -i[H_{\text{eff}}^{(\text{QM})}, \rho]$, with $H_{\text{eff}}^{(\text{QM})}$ unique up to a constant additive shift (the choice of energy zero).*

***Operational (T-A8.2.2).** For pure states $\rho = |\psi\rangle\langle\psi|$, the von Neumann equation reduces to the Schrödinger equation*

$$i\partial_t |\psi(t)\rangle = H_{\text{eff}}^{(\text{QM})} |\psi(t)\rangle, \quad (50)$$

unique up to global phase. The Schrödinger equation is therefore the constitutional closed-system limit of the GDOS boundary law, not an independent primitive law.

Proof outline. Inherited from T7.2 of (Arisaka, 2026, A5-GDOS) Part III in two steps. Step 1: with all rates vanishing the GKLS dissipator vanishes and the generator becomes the Hamiltonian conjugation $-i[H_{\text{eff}}^{(\text{QM})}, \cdot]$; hermiticity of $H_{\text{eff}}^{(\text{QM})}$ ensures unitarity. Step 2: substituting $\rho = |\psi\rangle\langle\psi|$ into the von Neumann equation and pairing with $\langle\psi|$ recovers the Schrödinger equation up to an unobservable global phase. ■

40.4 Quantitative anchor

The closed-system limit governs all of single-particle quantum mechanics in the absence of measurement back-action: the hydrogen-atom Rydberg spectrum to twelve significant figures (Tiesinga *et al.*, 2021), the anomalous magnetic moment of the electron $a_e = (g - 2)/2$ to one part in 10^{12} , the Hong–Ou–Mandel interference visibility to one part in 10^4 . The framework predicts no deviation from textbook QM in any of these regimes; T-A8.2 merely supplies the constitutional warrant for what experiment has been verifying since 1925.

40.5 Physics intuition and daily-life metaphor

Physics intuition. The Schrödinger equation is the dissipation-free idealization of the actual boundary dynamics, in exactly the way that the frictionless pendulum is the dissipation-free idealization of the actual swinging pendulum. Both idealizations are useful, and both are mathematically clean, but neither describes the full physical situation.

Daily-life metaphor. A spinning top is well-described by the conservation laws of angular momentum for the first few seconds; the more honest description, valid over longer times, includes the friction that eventually brings it to rest. Schrödinger evolution is the conservation-law description; GKLS is the honest one. For laboratory experiments on time scales short compared to the decoherence time, the conservation-law description is correct; for measurement-time-scale events, it is not.

40.6 Kill criterion

T-A8.2 would be retired by the experimental discovery of a closed quantum system whose evolution is not unitary in the $\gamma \rightarrow 0$ limit — for instance, an isolated atom whose spectral lines drift in a way not attributable to environmental coupling. No such system has ever been observed; the framework’s prediction matches the precision-spectroscopy record of the past century.

40.7 Plain-English close

The Schrödinger equation is the law of a quantum system imagined as fully isolated; the GKLS master equation is the law of a quantum system as it actually exists, coupled to an environment that the boundary observer cannot fully control. The first is a limit of the second. Both are correct within their domains, and neither is a primitive postulate.

Where the i comes from. One more textbook primitive dissolves at this theorem, and it deserves its own paragraph because it sits in the equation’s very first symbol. In the 6D bulk every physical quantity is real, and the dynamics that this theorem projects is a *real symplectic flow*; the boundary observer, who cannot resolve the two paired real structures of the arena (the compact phase circle and the causal clock), reads that flow in the derived complex coordinates the projection installs — and in those coordinates the real pair of first-order equations compresses into the single line $i\hbar\partial_t\psi = H\psi$. The i of the Schrödinger equation is therefore not a primitive of nature but the projection’s bookkeeping: the rotation generator of a real pair, derived at derived-conditional grade in the quasi-free regime by the origin-of- i companion (Arisaka, 2026, P4-Real), with the arrow and clock side of the pairing held by the CPT companion’s custody clause (Arisaka, 2026, P5-CPT); the cohort’s one-line custody sentence — the dictionary owns the sectors, the geometry owns the i — holds here verbatim, and the compact statement catalog is §117. Problem D.1.2 of the technical appendices lets the reader perform the compression by hand.

41 T-A8.3 — The Born rule as theorem

41.1 L4 historical opening

Born’s June–July 1926 papers (Born, 1926; Born *et al.*, 1926) introduced the probabilistic interpretation of the wavefunction modulus-squared. The rule worked spectacularly — atomic spectra, scattering cross-sections, the Stern–Gerlach (Gerlach and Stern, 1922) outcome distribution all agreed — but Born offered no derivation. Gleason’s 1957 theorem (Gleason, 1957) sharpened the rule to a uniqueness statement on a Hilbert space of dimension ≥ 3 , but assumed the Hilbert-space framework as input. Many-Worlds proponents have spent fifty years trying to derive the rule from unitary dynamics alone; decoherence theorists have offered “predictability-sieve” justifications (Zurek, 2003); QBism reads the rule subjectively (Fuchs, 2010). None has been universally accepted.

41.2 The physics question

Where does the Born rule come from? Is it a primitive postulate, a consequence of the Hilbert-space framework, a consequence of unitarity, or a consequence of something else?

41.3 Theorem statement

Theorem 3 (T-A8.3 — Born rule from CPTP completion (T-QM-3)). ***Algebraic (T-A8.3.1).** For any boundary density operator $\rho_{\text{Bdy}}^{(\text{QM})}$ and any admissible instrument family $\{\mathcal{I}_\alpha\}$ (CPTP-completable to a single CPTP measurement channel), conditional on the σ -additive structure of the boundary measure inherited from AX-3,*

$$p_\alpha = \text{Tr}[\mathcal{I}_\alpha(\rho_{\text{Bdy}}^{(\text{QM})})] = \text{Tr}[E_\alpha \rho_{\text{Bdy}}^{(\text{QM})}], \quad (51)$$

with $\{E_\alpha = \mathcal{I}_\alpha^*(1)\}$ a POVM.

***Operational (T-A8.3.2).** For pure states with sharp projective measurement $E_\alpha = \Pi_\alpha = |\alpha\rangle\langle\alpha|$,*

$$p_\alpha = |\langle\alpha|\psi\rangle|^2, \quad (52)$$

the textbook Born rule.

Proof outline. Inherited from T7.3 of (Arisaka, 2026, A5-GDOS) Part III in three steps. Step 1: by trace duality, every CP map admits a unique Heisenberg adjoint, and the adjoint of any instrument sends the identity to a positive operator (the effect operator E_α). Step 2: by Stinespring’s theorem (Stinespring, 1955), the Kraus form $\mathcal{I}_\alpha(\rho) = \sum_n K_{\alpha,n} \rho K_{\alpha,n}^\dagger$ gives $E_\alpha = \sum_n K_{\alpha,n}^\dagger K_{\alpha,n} \geq 0$. Step 3: trace preservation of the total CPTP channel $\sum_\alpha \mathcal{I}_\alpha$ forces $\sum_\alpha E_\alpha = 1$ (the POVM completeness relation), and the cyclicity of trace then yields (51). Specialization to pure states with sharp projectors gives $|\langle\alpha|\psi\rangle|^2$ directly. ■

41.4 Where probabilities enter the derivation

A careful reader will ask: where in the chain $\text{OGGEP} \rightarrow \text{CPTP} \rightarrow \text{Stinespring} \rightarrow \text{POVM} \rightarrow \text{Born}$ does a probability axiom first appear? The honest answer is at the constitutional level, not at the proof level. The CPTP structure of the boundary channel is required by AX-3 of GG-A1-Constitution (OGGEP: an admissible bulk theory must come equipped with a structure-preserving functor to the boundary algebra that is, among other things, positivity-preserving); complete positivity, trace preservation, and the resulting probability semantics of the effect operators $\{E_\alpha\}$ are therefore constitutional inheritances, not derivations. Gleason’s σ -additivity is a hypothesis on the boundary measure; the present paper does not derive that the partial trace over bulk KK modes produces a σ -additive boundary measure (this is a corollary of AX-3 + AX-4 that the constitutional papers GG-A1-Constitution and GG-A4-Euclid address). What T-A8.3 establishes is therefore the *conditional* statement:

If the boundary register is described by a σ -additive probability measure on the projection lattice (which AX-3 and AX-4 together force), then by Gleason that measure has the

trace form (51), with the textbook $|\langle\alpha|\psi\rangle|^2$ recovered as the pure-state sharp-projective specialization.

The unconditional statement — that the partial trace over the radial direction I_r and the phase direction S_θ^1 *must* produce a σ -additive measure rather than, say, a deterministic boundary record selected by some bulk gauge condition — is a constitutional inheritance from GG-A1-Constitution + GG-A4-Euclid, not a theorem of T-A8.3. We flag this as a residual constructive question in §118 (the framework-versus-Many-Worlds operational discriminator question): a complete unconditional derivation of the probability semantics from a deterministic bulk substrate remains a constructive task for future work. What the present paper claims is sharper than “the Born rule is postulated” (the textbook position) and weaker than “the Born rule is derived from no probability axiom whatever” (a position no derivation in the foundations literature has achieved, including Many-Worlds decision-theoretic attempts (Wallace, 2012)). We claim that the Born rule’s specific functional form is forced once the constitutional structure of OGGEF and MSMF is in place.

41.5 Quantitative anchor

The Born rule has been verified experimentally to one part in 10^4 in single-particle interference experiments (Sinha *et al.*, 2010), and to higher precision in CHSH violations (where the rule plus quantum unitarity together predict $|S| = 2\sqrt{2}$ to the measured precision of (Giustina *et al.*, 2015; Hensen, 2015; Shalm and others, 2015)). Gleason’s theorem makes the rule unique among σ -additive probability measures on closed subspaces. T-A8.3 closes the remaining postulational gap by supplying the constitutional source of the Hilbert-space framework Gleason assumed.

41.6 Physics intuition and daily-life metaphor

Physics intuition. The Born rule is the unique probability-assignment rule compatible with (a) CPTP evolution at the boundary, (b) a Hilbert-space framework inherited via GNS, and (c) the no-signaling constraint that the boundary projection inherits from the bulk’s reduced-state factoring. Any deviation would violate one of the three.

Daily-life metaphor. A weighing scale gives readings whose statistical distribution is fixed by the engineering of the scale plus the constraint that the scale return reproducible measurements at each level of precision. Once the scale is built, the distribution is forced — not chosen by the scale’s user. The Born rule is forced in the same way once OGGEF is built.

41.7 Kill criterion

T-A8.3 would be retired by the experimental discovery of a quantum system whose outcome statistics violate $|\langle\alpha|\psi\rangle|^2$ at a level above 10^{-4} in the linearity regime that quantum mechanics is supposed to govern. Recent triple-slit experiments (Sinha *et al.*, 2010) have tested for the Born rule’s signature linear-superposition response to within these limits, with results consistent with quantum-mechanical linearity.

41.8 Plain-English close

A century of experiment has shown that Nature obeys the Born rule. A century of theory has tried to explain why. The framework’s answer is that the Born rule is the only rule compatible with the constitutional warrants the cohort imposes upstream; it is not a postulate of quantum mechanics, but a feature of the operational projection from the 6D bulk to the 4D boundary.

42 T-A8.4 — The Lüders projection rule as theorem

42.1 L4 historical opening

The Lüders projection rule — often called the “collapse postulate” — has been the most physically controversial of the textbook postulates since von Neumann introduced it in 1932 (Neumann, 1932). Lüders refined the rule to the modern non-degenerate-measurement form in 1951. Copenhagen leaves the postulate ambiguous, locating “collapse” at a Heisenberg-cut drawn at the experimenter’s discretion; many-worlds denies that collapse occurs at all; GRW models (Ghirardi, 1986) replace the postulate with a stochastic dynamical law. None of these has been universally accepted. T-A8.4 dissolves the conundrum: the “collapse” is just the conditional CPTP state-update of the boundary observer, and the underlying bulk evolution remains unitary throughout.

42.2 The physics question

Is the wavefunction collapse a physical process, or is it an information-theoretic update of the observer’s record? When, exactly, does it happen? Where is the boundary between the unitarily evolving quantum system and the classically observed apparatus?

42.3 Theorem statement

Theorem 4 (T-A8.4 — Lüders projection from conditional CPTP (T-QM-4)). *Algebraic (T-A8.4.1). The state-update map*

$$\boxed{\rho_{\text{Bdy}}^{(\text{QM})}(\alpha) = \frac{\mathcal{I}_\alpha(\rho_{\text{Bdy}}^{(\text{QM})})}{p_\alpha}} \quad (53)$$

is, for each fixed outcome α with $p_\alpha > 0$, a (conditional) CPTP map on $\mathcal{B}(\mathcal{H}_{\text{Bdy}}^{(\text{QM})})$ — the unique CPTP update consistent with classical-outcome registration.

Operational (T-A8.4.2). For pure states with sharp projective measurement, $|\psi^{(\alpha)}\rangle = \Pi_\alpha|\psi\rangle/\|\Pi_\alpha|\psi\rangle\|$, the textbook Lüders projection. No physical collapse occurs in the bulk; the apparent collapse is the operational update of the boundary observer’s record.

Proof outline. Inherited from T7.4 of (Arizaka, 2026, A5-GDOS) Part III in two steps. Step 1: $\mathcal{I}_\alpha$ is sub-normalized CPTP by hypothesis, and division by the normalization p_α restores trace preservation while preserving complete positivity (Nielsen, 2010). Step 2: the bulk evolution is unitary by AX-1, so the boundary state-update is implemented by unitary bulk evolution plus conditional partial

trace over non-registered apparatus pointer states — the standard conditional-CPTP construction. No discontinuous bulk collapse is required. ■

42.4 Quantitative anchor

Repeated-measurement experiments (single-shot dispersive readout of superconducting qubits, electron-shelving in trapped-ion qubits, photon-number-resolving detectors; see [Wiseman and Milburn \(2010\)](#) for the standard operational treatment) confirm T-A8.4 to within readout-error rates of 10^{-3} – 10^{-5} depending on platform ([Nielsen, 2010](#)). The framework predicts no deviation from textbook QM measurement statistics in any of these regimes; T-A8.4 merely supplies the constitutional warrant for what experiment has been verifying since the original double-slit photon-counting experiments.

42.5 Physics intuition and daily-life metaphor

Physics intuition. The “collapse” is not an event in the bulk; it is a re-labeling of the boundary observer’s record. Before the measurement, the observer assigns a density operator ρ to the system; after registering outcome α , the observer updates to the conditional posterior $\rho^{(\alpha)}$. This is Bayesian conditioning in quantum language, with the prior–posterior structure forced by the CPTP nature of the instrument.

Daily-life metaphor. A scientist studying a coin’s bias updates her estimate of the bias after each flip; the coin does not change, but her record of it does. The Lüders rule is the quantum analogue: the boundary observer updates her record after each measurement, but the bulk state evolves unitarily throughout.

42.6 Kill criterion

T-A8.4 would be retired by the experimental observation of a non-CPTP state-update — for instance, a measurement whose post-measurement statistics violate Bayes consistency under sequential conditioning, or whose post-measurement density operator fails complete positivity. No such measurement has been reported; the framework’s prediction matches the foundations-of-quantum-information literature ([Holevo, 2012](#); [Nielsen, 2010](#)).

42.7 Plain-English close

There is no measurement problem. There is a deeper unitary bulk dynamics, a non-invertible projection to a boundary algebra, and an operational update of the boundary observer’s record on outcome registration. The textbook Lüders rule is the third of these, and it is structurally forced by the first two.

43 T-A8.5 — Decoherence as off-diagonal damping

43.1 L4 historical opening

The decoherence program of Zeh (1970), Zurek (1981), and Joos–Zeh (1985) (Joos and Zeh, 1985; Zeh, 1970; Zurek, 1981, 2003) was the most significant theoretical advance on the measurement problem in the half-century after the Bohr–Einstein debate. It established that off-diagonal density-matrix elements decay exponentially fast under environmental coupling, on time-scales that for macroscopic objects are essentially instantaneous. Standard treatments (Breuer, 2002; Schlosshauer, 2005, 2007) take the decoherence rate as a phenomenological input. T-A8.5 promotes the rate to a derived quantity, fixed by scale-overlap integrals of the bulk geometry.

43.2 The physics question

What is the rate of decoherence, and where does it come from? Why is it essentially instantaneous for macroscopic objects but slow enough to permit interference for microscopic ones?

43.3 Theorem statement

Theorem 5 (T-A8.5 — Decoherence at rate $\Gamma_{\alpha\beta}$ from bulk KK overlap (T-QM-5)). **Algebraic (T-A8.5.1).** *In any basis $\{|\alpha\rangle\}$ that diagonalizes the system–environment interaction, the off-diagonal evolution under the GKLS generator has the solution*

$$\rho_{\alpha\beta}(t) = \rho_{\alpha\beta}(0) e^{-i\omega_{\alpha\beta}t} e^{-\Gamma_{\alpha\beta}t}, \quad (54)$$

with decoherence rate $\Gamma_{\alpha\beta} = \sum_{\gamma} \gamma_{\gamma} |\langle\alpha|L_{\gamma}|\alpha\rangle - \langle\beta|L_{\gamma}|\beta\rangle|^2$.

Operational (T-A8.5.2). *The GKLS rates γ_{γ} are not free parameters: they inherit their values from scale-overlap integrals of bulk KK modes, $\gamma_{\gamma} = 2\pi |\int_0^K dr e^{-2r} f_0(r) f_n(r)|^2 \cdot \rho_{\text{env}}(\omega_n)$, with $\rho_{\text{env}}(\omega_n)$ the bulk-mode spectral density at the resonant Bohr frequency. For macroscopically distinguishable states, $\Gamma_{\alpha\beta} \gtrsim 10^{40} \text{ s}^{-1}$ at room temperature.*

Proof outline. Inherited from T7.5 of (Arizaka, 2026, A5-GDOS) Part III via the Born–Markov chain specialized to the GG-6 bulk coupling. Step 1: substitute the KK mode expansion into the bulk system–environment Hamiltonian; second-order perturbation theory in the coupling constant produces the scale-overlap integral. Step 2: take the Markov limit (environmental correlation time short compared to the boundary system’s characteristic time); Fermi’s golden rule gives the $2\pi\rho_{\text{env}}(\omega_n)$ factor. Step 3: substitute the rate into the GKLS off-diagonal evolution; integration yields the exponential damping. The macroscopic-instantaneous regime follows from Joos–Zeh’s calculation (Joos and Zeh, 1985) of the air-molecule scattering rate. ■

43.4 Quantitative anchor

For Schrödinger’s cat in a position superposition of separation $\Delta x \sim 0.1 \text{ m}$ at room temperature, the decoherence rate per air molecule is $\Gamma_1 \sim k_B T m^2 v / \hbar^2 \cdot (\Delta x)^2 \sim 10^{20} \text{ s}^{-1}$; the total over the

$\sim 10^{20}$ molecules per cm^2 per second of impact gives $\Gamma_{\text{total}} \sim 10^{40} \text{ s}^{-1}$, so $\tau_{\text{dec}} \sim 10^{-40} \text{ s}$ — forty orders of magnitude faster than any apparatus can resolve. For a single photon in a high-vacuum Mach–Zehnder interferometer with arm length $\sim 1 \text{ m}$ and ambient density $n \sim 10^{10} \text{ molecules/m}^3$, $\Gamma \sim 10^{-3} \text{ s}^{-1}$, giving $\tau_{\text{dec}} \sim 10^3 \text{ s}$ and ample time for interference.

43.5 Physics intuition and daily-life metaphor

Physics intuition. The decoherence rate is fixed not by an adjustable parameter but by the constitutional bulk geometry — specifically, by the scale profile e^{-2r} that sets which KK modes overlap the boundary zero mode and at what amplitude. In the cohort’s zero-free-continuous-parameter framework, Γ is a derived number for every system one cares to study.

Daily-life metaphor. A whisper in a quiet room can be heard many feet away; the same whisper in a stadium full of cheering fans dies within a meter. The decoherence rate is the cheering of the bulk environment: small systems can keep their quantum coherence audible for a long time; macroscopic systems are drowned out instantly.

43.6 Kill criterion

T-A8.5 would be retired by the experimental observation of decoherence rates that fail the Δx^2 -scaling for macroscopic objects, or by the discovery of a quantum system whose decoherence rate depends on a free parameter unfixed by the bulk geometry. Existing experiments (Arndt and others, 1999; Schlosshauer, 2005) on mesoscopic systems (large molecules, BEC interferometry) have not produced such an observation; the framework’s prediction holds.

43.7 Plain-English close

Decoherence is what makes the quantum-to-classical transition look as sharp as it does. In the cohort framework, the rate is not an adjustable knob but a calculation from the bulk geometry. Whether you can or cannot see an interference pattern is fixed by where in the 6D bulk your apparatus sits in coupling space.

43.8 A fully worked overlap integral: collisional decoherence of C_{70} matter waves

It is one thing to state that the decoherence rate of T-A8.5 is fixed by a scale-overlap integral, and another to evaluate that integral to an actual number for a concrete platform and compare it with a measured rate. This subsection does so. The platform is fullerene matter-wave interferometry — the cleanest laboratory in which decoherence has been *measured* as the controlled loss of interference contrast (Arndt and others, 1999; Hackermüller *et al.*, 2004; Hornberger *et al.*, 2003). The methods, the dispersion-coefficient estimates, and the uncertainty budget are collected in Appendix I; the physical result is given here.

A refinement of the reading of T-A8.5.2

Before computing, we state precisely what the framework does and does not claim for a laboratory system, because the worked integral sharpens the reading of T-A8.5.2. Three objects must be kept distinct.

- *The physical bath sets the rate.* The decohering environment of a laboratory system is its physical 4D bath — background gas, thermal photons, phonons. The magnitude of the decoherence rate is the boundary projection of the system–bath coupling, which is operationally the ordinary scattering matrix element. This is what is evaluated below.
- *The bulk geometry supplies the open-system structure, not the magnitude.* Coarse-graining the non-compact scale fiber I_r — a forward renormalization-group semigroup — is what makes the boundary evolution completely positive, Markovian, and irreversible, i.e. of GKLS form (§34). It explains *why* marginalization yields a Lindblad generator; it does not by itself fix the numerical rate.
- *The compact phase fiber S_θ^1 KK tower is a separate, and negligible, channel.* Only its massive $n \geq 1$ modes (at mass $\sim M_{\text{lock}}$) are framework-specific new physics; its massless $n = 0$ zero mode is the ordinary 4D gauge field, whose decoherence is already part of the physical bath.

Thus the operationally correct reading of T-A8.5.2 is: *the laboratory decoherence rate is the physical-bath overlap integral; the bulk geometry fixes its open-system form; the S_θ^1 tower contributes only a kinematically suppressed correction that switches on near M_{lock} .* The remainder of this subsection makes both halves quantitative.

The decoherence function and its two limits

Collisional decoherence is governed by the Gallis–Fleming / Hornberger–Vacchini master equation (Gallis and Fleming, 1990; Hornberger *et al.*, 2003), under which the off-diagonal density-matrix element in the position basis decays as $\rho(\mathbf{x}, \mathbf{x}', t) = \rho(\mathbf{x}, \mathbf{x}', 0) e^{-F(\mathbf{x}-\mathbf{x}')t}$, with the decoherence function

$$F(\Delta\mathbf{x}) = n \int \mu(v) v dv \int d\Omega \frac{d\sigma}{d\Omega} \left[1 - \cos\left(\frac{\Delta\mathbf{p} \cdot \Delta\mathbf{x}}{\hbar}\right) \right], \quad (55)$$

where n is the gas number density, $\mu(v)$ the gas speed distribution, and $\Delta\mathbf{p} = 2m_{\text{gas}}v \sin(\theta/2)$ the momentum transfer in an elastic collision. Equation (55) is a single function with two physical limits, separated by the crossover length $\Delta x_\star \simeq \hbar/\langle \Delta p \rangle \approx \lambda_{\text{th, gas}}$ (the thermal de Broglie wavelength of the gas):

- *Saturated (long-separation) limit, $\Delta x \gg \Delta x_\star$:* the cosine oscillates rapidly and averages to zero except in the forward direction, so the decoherence function saturates to the *total scattering rate*,

$$F(\Delta\mathbf{x}) \longrightarrow \Gamma_{\text{dec}} = n \sigma_{\text{tot}} \bar{v}_{\text{rel}}. \quad (56)$$

This is the matter-wave regime: every collision that resolves the path separation removes a full unit of coherence.

- *Localization (short-separation) limit*, $\Delta x \ll \Delta x_*$: expanding $1 - \cos(\Delta \mathbf{p} \cdot \Delta \mathbf{x} / \hbar) \approx \frac{1}{2}(\Delta \mathbf{p} \cdot \Delta \mathbf{x} / \hbar)^2$ gives $F(\Delta \mathbf{x}) \rightarrow \Lambda (\Delta x)^2$ with a single localization parameter Λ . This is the macroscopic regime.

The wide spread of decoherence rates sometimes quoted for a single “macroscopic object” is nothing but the localization branch $\Lambda(\Delta x)^2$ evaluated at different separations: with the canonical air-bath value $\Lambda \sim 10^{36} \text{ m}^{-2} \text{ s}^{-1}$ one has $\Gamma \sim 10^{18} \text{ s}^{-1}$ at $\Delta x = 1 \text{ nm}$, 10^{24} at $1 \mu\text{m}$, 10^{32} at 1 cm , and 10^{34} at 10 cm . All are the same curve; the only error is to quote a rate without naming Δx and the bath.

The overlap integral as the van der Waals cross section

In the saturated regime the overlap integral that fixes the rate is the total elastic cross section of the molecule on the gas, taken over the long-range van der Waals tail $-C_6/r^6$. The semiclassical Massey–Mohr / Landau–Lifshitz result is

$$\sigma_{\text{tot}} = \kappa \left(\frac{C_6}{\hbar \bar{v}_{\text{rel}}} \right)^{2/5}, \quad \kappa = 8.083 \ (\pm \sim 25\%), \quad (57)$$

the coefficient κ carrying a glory-averaging/approximation uncertainty of order 25% (Appendix I). The dispersion coefficient for C_{70} on argon, estimated by the London and Slater–Kirkwood formulae (Appendix I), brackets

$$C_6 \approx (2\text{--}4) \times 10^3 \text{ a.u.} \quad (\text{London } 2.2 \times 10^3, \text{ Slater–Kirkwood } 4.2 \times 10^3), \quad (58)$$

an uncertainty of a factor ~ 2 in C_6 , hence (since $\sigma \propto C_6^{2/5}$) a factor ~ 1.3 in σ .

The number

Taking argon at $T = 300 \text{ K}$ and a representative beam speed $v_{\text{mol}} = 100 \text{ m/s}$ (so that $\bar{v}_{\text{rel}} = \sqrt{v_{\text{mol}}^2 + \bar{v}_{\text{gas}}^2} = 411 \text{ m/s}$), equations (57)–(58) give

$$\sigma_{\text{tot}} \approx (1.5\text{--}2.0) \times 10^{-17} \text{ m}^2 \ (1500\text{--}2000 \text{ \AA}^2), \quad \text{central } 1.7 \times 10^{-17} \text{ m}^2, \quad (59)$$

squarely within the measured band of fullerene–noble-gas cross sections. The saturation condition is comfortably met: the gas thermal de Broglie wavelength $\lambda_{\text{th,gas}} = 1.6 \times 10^{-11} \text{ m}$ is four orders of magnitude below the coherent path separation ($\sim 10^{-8}\text{--}10^{-7} \text{ m}$), so Eq. (56) applies, and the velocity-averaged $\langle \sigma v_{\text{rel}} \rangle$ differs from $\sigma(\bar{v}_{\text{rel}})\bar{v}_{\text{rel}}$ by only 2.5% (Appendix I). The decoherence rate and the contrast-loss onset pressure are then

$$\Gamma_{\text{dec}}(p) = \frac{p}{k_B T} \sigma_{\text{tot}} \bar{v}_{\text{rel}} \approx 150\text{--}196 \text{ s}^{-1} \text{ at } 10^{-6} \text{ mbar}, \quad (60)$$

$$p_* = \frac{k_B T}{\sigma_{\text{tot}} \bar{v}_{\text{rel}} t_{\text{transit}}} \approx 1.4 \times 10^{-6} \text{ mbar} \quad (\text{band } 1.2\text{--}2.1 \times 10^{-6}), \quad (61)$$

using the transit time $t_{\text{transit}} = 2L_T/v \approx 4.1 \text{ ms}$ set by the Talbot length $L_T = d^2/\lambda_{\text{dB}} = 0.21 \text{ m}$ of the interferometer.

Comparison with experiment (stated honestly). The Arndt-group fullerene interferometers preserve high contrast below $\sim 10^{-7}$ mbar and lose it as the pressure rises toward $\sim 10^{-6}$ mbar; the computed onset $p_\star \approx 1.4 \times 10^{-6}$ mbar *reproduces that measured order of magnitude and onset-pressure scale*, and the cross section $\sigma \sim 1.7 \times 10^{-17} \text{ m}^2$ lies in the band for which Hornberger *et al.* (Hornberger *et al.*, 2003) reported theory–experiment agreement at the 10–20% level. This is an order-of-magnitude result computed from the interaction potential, not a fit to a particular run, and not a precision match: the honest accuracy is a factor ~ 1.5 , set by the C_6 and κ uncertainties of Eqs. (57)–(58). Crucially, the framework *must* reproduce the Joos–Zeh collisional value in this regime to be correct; doing so is a success condition, not a new prediction. The genuinely framework-specific content lies elsewhere, in the bulk channel.

The framework-specific channel, bounded

The S_θ^1 KK tower is the one piece of decoherence that is not standard. Its size is controlled by a kinematic threshold, and this is the robust part of the estimate, independent of any coupling detail. Decoherence requires the environment to absorb a *real* quantum carrying which-path information; the lightest massive excitation of the tower sits at $M_{\text{lock}} c^2 \approx 3.85 \text{ TeV}$. A molecular or condensed-matter system carries $E_{\text{sys}} \sim \text{eV–keV}$, so real KK emission is kinematically forbidden, and only virtual exchange occurs — which renormalizes parameters but transmits no information, hence causes no decoherence. Below threshold the induced rate is suppressed by at least the leading effective-field-theory power,

$$\frac{\Gamma_{\text{bulk}}}{\Gamma_{\text{bath}}} \lesssim \left(\frac{E_{\text{sys}}}{M_{\text{lock}}} \right)^p, \quad p \geq 1, \quad (62)$$

with p fixed by the (presently unspecified) bulk–boundary operator dimension. For *every* $p \geq 1$ the suppression is overwhelming in the laboratory: $(E_{\text{sys}}/M_{\text{lock}}) \sim 3 \times 10^{-13}$ for a molecular system and $\sim 3 \times 10^{-10}$ at the keV scale, so Γ_{bulk} is unmeasurably small and the physical bath dominates entirely — exactly as observed. The correlation time of the tower, $\tau_c = \hbar/M_{\text{lock}} = 1.7 \times 10^{-28} \text{ s}$, is a *correlation time, not a rate*; conflating the two would be a category error, which Eq. (62) prevents. The channel reaches $\mathcal{O}(1)$ only as $E_{\text{sys}} \rightarrow M_{\text{lock}}$, so it “switches on” near the same $M_{\text{lock}} \approx 3.85 \text{ TeV}$ scale as the high-energy Test 8, tying the laboratory einselection test (Test 5) to the collider horizon (Test 8); the *location* of switch-on is p -dependent and therefore not yet a sharp prediction. The massless $n = 0$ zero mode is excluded from this channel by construction: it is the ordinary photon, and its decoherence is already counted in the physical-bath rate of Eq. (60).

Summary

The worked integral delivers, for one concrete platform, what T-A8.5.2 asserts in general. Equations (59)–(61) give a real decoherence rate, computed from the interaction potential and matched to the measured onset pressure; Eq. (62) bounds the bulk channel kinematically to negligibility, so the physical bath sets the rate and a genuinely isolated particle does not measurably decohere; and the single function $F(\Delta x)$ of Eq. (55) unifies the matter-wave and macroscopic regimes. Reproducing the Joos–Zeh collisional value where it applies is a correctness condition, not a new prediction; the framework’s specific content is the structure of the open-system law

together with the threshold-gated bulk deviation of Eq. (62) — the single genuinely new, if presently unmeasurable, prediction, gated to M_{lock} .

44 T-A8.6 — Einselection of the pointer basis

44.1 L4 historical opening

Zurek’s 1981 einselection criterion (Zurek, 1981, 2003) answered the long-standing decoherence-program objection: decoherence shows that off-diagonal elements decay in *some* basis, but does not by itself say *which* basis is preferred. Zurek identified the einselected pointer basis as the one minimizing entropy production — the eigenbasis of the system–environment coupling — but this identifies the basis with the coupling without saying why the coupling has the form it does. T-A8.6 closes the gap: the position-diagonal nature of the boundary coupling is forced by AX-1 bulk locality.

44.2 The physics question

Why does the universe “prefer” the position basis as the classical record of macroscopic observations? Why not the momentum basis, the energy basis, some superposition basis? Where does the universal preference come from?

44.3 Theorem statement

Theorem 6 (T-A8.6 — Einselection from bulk locality (AX-1) (T-QM-6)). ***Algebraic (T-A8.6.1).** The einselected pointer basis of $\mathcal{H}_{\text{Bdy}}^{(\text{QM})}$ is the eigenbasis of the dominant boundary system–environment coupling. By AX-1 (bulk locality), the bulk Hamiltonian density is a polynomial in bulk fields at the same bulk point; the boundary zero-mode coupling inherits this locality as position-diagonal multiplicativity: $H_{SE} = \sum_{x_0} |x_0\rangle\langle x_0| \otimes \hat{E}_{\text{eff}}(x_0)$.*

***Operational (T-A8.6.2).** The classical-looking outcomes registered by laboratory measurements correspond to the elements of the position-eigenbasis specified above. No experimenter choice is required; the einselected basis is unique up to phase for generic bulk coupling.*

Proof outline. Inherited from T7.6 of (Arisaka, 2026, A5-GDOS) Part III in three steps. Step 1 (locality $\Rightarrow$ local boundary coupling): AX-1 forbids integro-differential kernels in the bulk Hamiltonian, so after KK expansion the boundary system–environment interaction is a sum of local-at-the-same- x products. Step 2 (zero-mode coupling $\Rightarrow$ position-diagonal): for the lightest bulk scalar carrying the measured observable, the boundary zero mode is multiplicative in the position basis, so H_{SE} decomposes as a sum of $|x_0\rangle\langle x_0| \otimes \hat{E}_{\text{eff}}(x_0)$. Step 3 (Zurek’s predictability sieve): a coupling diagonal in $\{|x_0\rangle\}$ decays off-diagonal coherences $\langle x_0|\rho|x'_0\rangle$ for $x_0 \neq x'_0$ on the decoherence timescale of T-A8.5, while keeping diagonal elements unchanged; the basis $\{|x_0\rangle\}$ wins the sieve. Uniqueness up to phase follows from generic transversality (Zurek, 2003). ■

Runtime remark. In the regime vocabulary of Section 51, this theorem has a one-line restatement: *einselection is GRIP-In*. The pointer basis is the Geometric-Attractor catalog of the boundary stratum, the predictability sieve is the F_{GFF} ranking, and the “environment winning” is the descent

halting. Zurek’s mechanism is not replaced; it is recognized as the runtime’s fixed-landscape regime wearing its 1981 name.

44.4 Quantitative anchor

Decoherence experiments on superconducting qubits, trapped ions, and atomic clocks register the position basis as the einselected one at the macroscopic level, with off-diagonal coherences in non-position bases damped at the T-A8.5 rate. The framework’s prediction matches the universal observation that macroscopic records are localized in space: photographs, hard-drive bits, the macroscopic state of a Geiger counter — all live in the position basis, none in the momentum basis.

44.5 Physics intuition and daily-life metaphor

Physics intuition. Bulk locality is the deep reason the world looks classical at macroscopic scales. Because the bulk Hamiltonian is local-at-the-same-bulk-point, the boundary coupling is local-at-the-same-boundary-point; the operationally registered observables are therefore position-diagonal. Any other basis would require the coupling to be non-local in position, which AX-1 forbids.

Daily-life metaphor. A wet floor records footprints by position — the imprint sits where the foot pressed — not by speed; a slow walker and a sprinter leave the same imprint at the same place. Fingerprint dust adheres to surfaces by contact, not by velocity class. The position basis of macroscopic records is privileged not by a choice of detector but by the contact-locality of the recording mechanism. The same applies to every macroscopic record in nature.

44.6 Kill criterion

T-A8.6 would be retired by the experimental observation of a macroscopic measurement record that is sharply localized in a non-position basis (e.g., a Geiger counter whose definite outcome is a momentum eigenvalue rather than a position eigenvalue) under conditions where the coupling is genuinely position-diagonal. No such observation has been reported; einselection is among the most empirically robust features of quantum measurement.

44.7 Plain-English close

The universe prefers the position basis because the universe is locally constructed. Once you accept bulk locality (which the cohort imposes via AX-1), the position-eigenbasis preference of classical records follows automatically. Nothing else needs to be added.

45 T-A8.7 — Bell–CHSH violation as boundary shadow

45.1 L4 historical opening

Bell’s 1964 inequality (Bell, 1964) and the CHSH reformulation (Clauser *et al.*, 1969) of 1969 demonstrated that no local hidden-variable theory in 4D can reproduce the correlations of an entangled bipartite singlet. Aspect’s 1981–1982 experiments (Aspect *et al.*, 1981, 1982) pushed the

violation past the classical bound; the loophole-free Delft/Vienna/NIST runs of 2015 (Giustina *et al.*, 2015; Hensen, 2015; Shalm and others, 2015) closed the remaining experimental loopholes; the 2022 Nobel Prize (Royal Swedish Academy of Sciences, 2022) formalized the result; the modern review by Brunner *et al.* (2014) is the canonical reference for the post-Aspect Bell-non-locality literature. The standard reading is that nature exhibits genuine non-locality. T-A8.7 sharpens the reading: nature exhibits 4D-non-locality as the boundary shadow of one non-separable joint bulk state.

45.2 The physics question

Are Bell-CHSH violations evidence of non-locality in the fundamental theory, or are they an artifact of a non-invertible bulk-to-boundary projection from a higher-dimensional bulk?

45.3 Theorem statement

Theorem 7 (T-A8.7 — CHSH violation as the projection shadow of one non-separable joint bulk state (T-QM-7)). ***Algebraic (T-A8.7.1).** An entangled bipartite preparation is a single non-separable state of the six-dimensional bulk, supported on the shared phase fiber S_θ^1 established at the common source. Its two boundary shadows, registered at 4D-spacelike-separated events $A = (x_A, 0, 0)$ and $B = (x_B, 0, 0)$, are projections of that one joint state along the radial (renormalization-scale) direction I_r ; the boundary correlation function is the standard operator-algebra expectation in the joint state (T-A8.1, T-A8.3), not a quantity transmitted between the events.*

***Operational (T-A8.7.2).** For an entangled bipartite boundary state (e.g., the spin-singlet), the CHSH bound $|S| = |C(a, b) + C(a, b') + C(a', b) - C(a', b')| \leq 2$ on local-hidden-variable theories is violated up to the Tsirelson bound $|S| \leq 2\sqrt{2}$ (Tsirelson, 1980). No local-hidden-variable model reproduces this correlation (Bell’s theorem); the joint state is irreducibly non-separable. No superluminal signaling is possible (see T-A8.11); the shared bulk state underwrites the correlation without mediating a signal.*

Proof outline. Inherited from T7.7 of (Arisaka, 2026, A5-GDOS) Part III. By Theorem 0 of GG-A2-GG6 the radial direction I_r is the renormalization-scale fiber: non-compact, scale-conformal, and carrying no light cone, so it provides no causal shortcut between 4D-spacelike events. The entangled preparation is instead one non-separable state of the bulk, sharing the phase fiber S_θ^1 fixed at the common source; the two boundary shadows are projections of that single state along I_r . The singlet correlation $C(\hat{a}, \hat{b}) = -\hat{a} \cdot \hat{b}$ is the standard operator-algebra expectation in the joint state, inherited via the Hilbert-space and Born theorems (T-A8.1, T-A8.3); the explicit evaluation is given in Appendix F, together with the statement (Bell’s theorem) that no *local*-hidden-variable model reproduces it. T-A8.8 below shows why the singlet *saturates* the Tsirelson ceiling. ■

45.4 What the joint-state reading does and does not do

A careful reader will press on the meaning of “6D locality” in the operational reading. We are explicit about what the bulk timelike connection of T-A8.7 does and does not do.

What the geodesic does not do. No dynamical degree of freedom propagates between Alice’s measurement event and Bob’s. The bulk-side trace in the bulk–boundary correlation lemma of GG-A5-GDOS Part III §T7.7 is a static expectation in the entangled singlet state; it is not a propagation of any signal, field, or particle. No information is transmitted; no influence is mediated dynamically; T-A8.11 (boundary no-signaling) guarantees that Alice’s measurement settings cannot be inferred from Bob’s marginal record under any operational protocol. In this sense the shared bulk state carries *nothing* dynamical between the wings: the correlation is a frozen feature of the singlet preparation, encoded in the one non-separable joint state whose shadows the two measurements register.

What the joint-state reading does. It supplies a geometric account of why the entangled correlation is the shadow of *one* bulk object rather than two separate ones, and — through T-A8.8 — why the singlet *saturates* the Tsirelson ceiling rather than merely touching it: the singlet is the extremizer of the CHSH functional, so the saturation is first-order stationary and therefore structurally stable. The saturation *value* $2\sqrt{2}$ is the standard quantum ceiling; what the framework adds is the reading, not a new number, and it is honest that the genericity is an extremal (Bogomolnyi-analogue) stability rather than a new empirical effect.

Why this is weaker than dynamical Einstein-locality but stronger than “the correlation is just pre-established.” The framework’s locality claim is geometric rather than dynamical. Geometric locality says: the entangled preparation, viewed as a bulk object, is supported on a worldline-connected region of the 6D manifold, and the boundary correlation is computed along that worldline. Dynamical Einstein-locality (no faster-than-light propagation of *any* influence) is preserved *operationally* by T-A8.11 but is not the content the framework adds. The content the framework adds is the geometric structural claim: the Tsirelson saturation is fixed by a topological coincidence of the scale profile (T-A8.8), and this prediction is platform-independent. A reader who reads “6D locality” as “6D dynamical Einstein-locality with propagating influences” will misread the claim; the correct reading is “6D geometric locality on a worldline-connected bulk support, with no propagating dynamics, and with empirically distinct content via T-A8.8.” We acknowledge this is weaker than the colloquial reading and remain honest about what is and is not at stake.

What this leaves of Einstein’s intuition. Einstein’s 1926 conviction was that Nature is deterministic at the deepest level. The framework preserves this at the bulk level (the 6D bulk is deterministic, single-valued, unitary, and local in the geometric sense above) without claiming to preserve dynamical Einstein-locality at the 4D boundary (the 4D shadow is irreducibly probabilistic and apparently non-local on the operational record). This is the careful version of the “Einstein right in 6D, Bohr right in 4D” summit statement of Part XIII.

45.5 Quantitative anchor

The CHSH parameter S has been measured to $|S| = 2.696 \pm 0.038$ in Aspect’s 1982 experiment, $|S| = 2.42 \pm 0.20$ in the 2015 Delft loophole-free run, and saturating the Tsirelson bound $2\sqrt{2} \approx 2.828$

within experimental error in subsequent ultra-precise measurements. The framework’s prediction matches the saturation; T-A8.8 explains why saturation is generic rather than accidental.

45.6 Physics intuition and daily-life metaphor

Physics intuition. The two detectors register two shadows of *one* entangled object. Because that object is a single non-separable state in the bulk — not two independently prepared particles — its shadows are correlated for the same reason two photographs of one scene agree: a shared origin, not a message passed between the cameras. The non-locality of the 4D shadow is the price of describing one bulk state by its two separate boundary projections.

Daily-life metaphor. Two clocks built and synchronized at the same factory, then carried to distant cities, agree perfectly when read — not because one signals the other, but because they share a common origin. The entangled pair is the same: one joint preparation, two correlated readings, with no influence traveling between them.

45.7 Kill criterion

T-A8.7 would be retired by the experimental observation of a CHSH violation that exceeds the Tsirelson bound (no quantum-mechanical mechanism is known for this), or by the observation of operational signaling between the wings (forbidden by T-A8.11). Neither has been reported in any laboratory; the framework’s prediction matches the post-2022-Nobel consensus.

45.8 Plain-English close

Bell’s theorem said local hidden variables in 4D are impossible. Aspect proved it experimentally. The 2022 Nobel formalized it. The framework agrees with all three results and adds the additional observation that 4D-non-locality is the operational shadow of 6D-locality through a non-invertible bulk-to-boundary projection. Einstein’s locality intuition is recovered at the bulk level; Bell’s non-locality conclusion is preserved at the boundary level.

The four net-new theorems of **GG-A8-QM**: orientation

The seven theorems above are inherited from GG-A5-GDOS Part III by operational restatement. The four theorems that follow are net-new to the present paper: they promote results that GG-A5-GDOS stated as lemmas, sub-results, or in passing, to the status of principal theorems of the QM register. Their proofs are given in full — not inherited. What is net-new is the constitutional reading and the elevation to principal status, not the underlying mathematics: each rests on an established result — Tsirelson’s bound (Tsirelson, 1980), the spin-statistics and particle-identity facts, the no-signaling lemma, and the Qi-CON conformality of GG-A7-Qi4 — now given its cohort interpretation and its own proof in the QM register. We claim novelty of *reading and promotion*, not of theorem-content, and say so explicitly so the promotion is not mistaken for a new mathematical result.

T-A8.8 promotes the Tsirelson saturation from a value (“ $|S| \leq 2\sqrt{2}$, with equality on the singlet”) to an extremal (Bogomolnyi-analogue) bound: the singlet is the CHSH extremizer, explaining why saturation is generic rather than coincidental.

T-A8.9 promotes the universal-identity insight of GG-A9-SM Part XII §60 from the SM stratum to the QM register: every electron registered in any laboratory is a token of the unique stable GA of the charged-lepton stratum, with the consequence that quantum indistinguishability of identical particles is a cascade inheritance rather than a postulate.

T-A8.10 promotes the Qi-CON conformality pillar of GG-A7-Qi4 to a quantitative prediction about laboratory decoherence-rate spectra: $S(\omega) \propto 1/f$ scaling within a bounded conformal window, the approximately $1/f$ noise that quantum-measurement apparatuses exhibit (with a system-specific exponent near unity) and that the cohort framework predicts as a structural rather than phenomenological consequence.

T-A8.11 promotes the no-signaling Lemma 3 of GG-A5-GDOS Part III §T7.7 to a principal theorem of GG-A8-QM in its own right, because the textbook reader will most want to see “the bulk timelike connection respects no-signaling” headlined rather than buried in a sub-result.

46 T-A8.8 — Tsirelson saturation as an extremal (Bogomolnyi-analogue) bound (net-new)

46.1 L4 historical opening

Tsirelson’s 1980 theorem (Tsirelson, 1980) established the quantum-mechanical bound $|S| \leq 2\sqrt{2}$ on the CHSH expression. The empirical saturation by entangled singlet states is documented in every loophole-free experiment to date (Aspect *et al.*, 1982; Giustina *et al.*, 2015; Hensen, 2015; Shalm and others, 2015). Where the standard reading treats the saturation as a value S that quantum mechanics permits, the present framework reads it as an *extremal* property of the entangled state — the singlet is the maximizer of the CHSH functional, so saturation is first-order stationary and structurally stable, in analogy with (though not literally) a Bogomolnyi/BPS bound. This is the structural follow-up to T-A8.7.

46.2 The physics question

Why does nature saturate Tsirelson rather than merely lying below it? Is the saturation a tuned coincidence of the singlet state, or is it a structural feature of the bulk-to-boundary projection?

46.3 Theorem statement

Theorem 8 (T-A8.8 — Tsirelson saturation as an extremal (Bogomolnyi-analogue) bound (T-QM-8)). *Algebraic (T-A8.8.1). For any two-party correlation computed from a Hilbert-space state and self-adjoint dichotomic observables, the CHSH expression obeys the Tsirelson bound*

$$|S| \leq 2\sqrt{2}, \quad \text{with equality on the maximally-entangled (singlet) state at optimal settings.}$$

(63)

The singlet is the extremizer of $|S|$ over states and measurement settings; at an extremizer the first variation of $|S|$ vanishes, so the saturation is stationary to first order.

Operational (T-A8.8.2). The singlet saturates the bound generically (independent of the specific 4D detector geometry) because it is the CHSH extremizer; small perturbations of the preparation change $|S|$ only at second order, so saturation is structurally stable. Non-maximally-entangled states give $|S| < 2\sqrt{2}$. This first-order stationarity at a saturating configuration is the analogue — not a literal instance — of a Bogomolnyi/BPS bound; no central charge or topological winding is claimed.

Proof. The proof is in two steps. Step 1 (Tsirelson as quantum-mechanical bound). By the algebraic Tsirelson argument (Nielsen, 2010; Tsirelson, 1980), any two-party correlation function computable from a Hilbert-space state and self-adjoint dichotomic operators satisfies $|S| \leq 2\sqrt{2}$, with the maximum attained by the spin-singlet with appropriate measurement settings. The bound is independent of the underlying ontology — it holds for any quantum-mechanical realization.

Step 2 (extremal stationarity). Regard $|S|(\psi, \{\hat{a}, \hat{a}', \hat{b}, \hat{b}'\})$ as a smooth function of the bipartite state ψ and the four measurement settings. Its global maximum $2\sqrt{2}$ is attained on the maximally-entangled singlet at the optimal settings; at that maximum the first variation vanishes, $\delta|S| = 0$, so a small perturbation of the preparation (a slight depolarization or a small admixture) changes $|S|$ only at second order. Saturation is therefore structurally stable across laboratory preparations rather than a tuned coincidence of one exact state. This first-order stationarity at a bound-saturating configuration is the formal analogue of a Bogomolnyi/BPS saturation in soliton physics (Bogomolny, 1976) — where a configuration sitting at a saturating bound is stable to first order — but it is an *analogy*: no topological charge, winding, or central-charge structure is asserted, and the saturation value is the standard quantum ceiling, not a new prediction. Non-maximally-entangled preparations sit away from the extremum and give $|S| < 2\sqrt{2}$. ■

46.4 Quantitative anchor

The saturation value $2\sqrt{2} \approx 2.828$ is platform-independent: it is the operator-algebra ceiling, reached by the singlet at the optimal CHSH settings regardless of the detector geometry, the laboratory separation, or the entangled medium. Aspect’s photons, the Delft electron spins, and the Vienna and NIST runs all sit at the same extremum; the framework reads this universality as the first-order stationarity of T-A8.8, not as a tuned coincidence of any one preparation.

46.5 Physics intuition and daily-life metaphor

Physics intuition. Bogomolnyi saturation in soliton physics is the stability of a configuration that sits exactly at a saturating bound: perturbations move it off only at second order. The Tsirelson saturation is analogous: the singlet is the maximizer of the CHSH functional, so small perturbations of the preparation do not change S to first order. The saturation is structurally stable — the analogy is to the first-order stationarity, not to any topological charge.

Daily-life metaphor. A ball resting at the bottom of a bowl does not roll away when the bowl is nudged slightly, because it sits at a minimum where the slope vanishes. The singlet sits at the corresponding extremum of the CHSH functional: nudge the preparation a little and the violation

stays at $2\sqrt{2}$ to first order.

46.6 Kill criterion

T-A8.8 would be retired by the experimental observation that the Tsirelson saturation is highly sensitive to small perturbations of the entangled state (e.g., saturation fails by 1% when the singlet is depolarized by 10^{-3}). Existing data (Giustina *et al.*, 2015; Hensen, 2015; Shalm and others, 2015) show that saturation is observed across a wide range of experimental conditions, consistent with the Bogomolnyi reading. Alternatively, the discovery of a non-quantum-mechanical mechanism that exceeds $2\sqrt{2}$ in any experiment would falsify the entire reading; no such mechanism has been observed.

46.7 Plain-English close

Nature saturates the Tsirelson bound because the bulk geometry of the cohort framework places the singlet state precisely at the topological rim where saturation occurs. This is not a tuned coincidence of quantum mechanics; it is a structural feature of the 6D metric.

47 T-A8.9 — Universal identity of indistinguishable particles (net-new)

47.1 L4 historical opening

The quantum-statistical postulate that “identical particles are perfectly indistinguishable” has been a foundational assumption since Bose’s 1924 statistical work and the Pauli exclusion principle of 1925. Standard textbooks (Cohen-Tannoudji *et al.*, 1992; Sakurai and Napolitano, 2011) state the indistinguishability as a primitive postulate of multi-particle quantum mechanics, with bosonic/fermionic exchange statistics following from the symmetry of the multi-particle state. No textbook offers a structural reason why every electron in the universe shares the same mass, charge, and spin. GG-A9-SM Part XII §60 supplies the reason at the SM stratum; T-A8.9 promotes the reason to a QM-specific theorem.

47.2 The physics question

Why is every electron the same? Is it a primitive postulate of quantum statistics, or is it a structural inheritance from a deeper theory?

47.3 Theorem statement

Theorem 9 (T-A8.9 — Universal identity of indistinguishable particles from GA selection (T-QM-9)). *Algebraic (T-A8.9.1). The cascade at the charged-lepton stratum of GG-A9-SM Part VI (GRIP cascade applied to Layer 1, the electroweak vacuum) has a unique stable GA characterized by the fixed quantum numbers $(m_e, -e, \frac{1}{2}\hbar)$ and the associated zero-mode profile on $\mathcal{H}_{\text{Bulk}}$.*

Operational (T-A8.9.2). *Every electron registered in any laboratory anywhere in the observable universe is a token of this unique stable GA type. The quantum-statistical postulate “identical particles are indistinguishable” is therefore not a postulate but a structural inheritance from the cascade attractor selection: indistinguishability is the operational signature of universal-identity.*

Proof. The proof rests on the per-stratum ground-state corollary of GG-A6-GRIP (Arisaka, 2026, A6-GRIP), applied to the charged-lepton stratum. That corollary holds on the class of strata GG-A6-GRIP calls admissible and carries that paper’s stated grade; the charged-lepton stratum is CP^2 and is one of the strata GG-A6-GRIP establishes as admissible, so the application is direct. (The capstone GG-A11-Origin generalizes the same statement across the layers of the program-wide cascade; the present proof needs only the single-stratum form, which is upstream.) Step 1 (existence of charged-lepton stratum). By T1–T2 of GG-A9-SM (gauge-group closure and electroweak-vacuum selection), the cascade at Layer 1 has a non-trivial fixed point with the electroweak gauge group spontaneously broken to $U(1)_{em}$; the charged-lepton sector lives on this stratum. Step 2 (uniqueness of GA). By the per-stratum ground-state corollary of GG-A6-GRIP, the GFF functional restricted to that stratum has exactly one critical point of index zero, which is therefore its unique deepest minimum, given by the configuration with quantum numbers $(m_e, -e, \frac{1}{2}\hbar)$. Step 3 (universal tokenization). Every cascade realization at any 4D-spacetime location selects the same unique GA; tokens of this GA are therefore universally indistinguishable in the structural sense (they share their type with every other token). The exchange statistics of those tokens — Fermi–Dirac, for the spin- $\frac{1}{2}$ GA — is not needed for the present theorem and is addressed separately: at candidate grade inside the cohort by T-A8.12 (§52), and at theorem grade meanwhile by the standard spin–statistics theorem (Pauli, 1940), which the cohort inherits until T-A8.12’s named residues are closed. ■

47.4 Quantitative anchor

The electron mass $m_e = 0.5109989461 \pm 0.0000000031 \text{ MeV}/c^2$ is the same to one part in 10^{10} in any laboratory measurement, on any continent, at any time — not as an empirical observation about an ensemble of electrons, but as a structural feature of the unique GA type they all instantiate. The same applies to the proton mass m_p , the muon mass m_μ , the up-quark mass m_u , and every other Standard-Model particle mass; T-A8.9 is the QM-specific version of the broader cascade-attractor uniqueness that GG-A9-SM establishes at the SM register.

47.5 Physics intuition and daily-life metaphor

Physics intuition. Indistinguishability of identical particles is not a fundamental quantum-mechanical postulate; it is a consequence of the fact that all instances of a particle species are tokens of a single type fixed upstream of any quantum-mechanical preparation. The cascade’s attractor selection produces the type once; the type is then instantiated everywhere a stratum-Layer-1 realization occurs.

Daily-life metaphor. Every copy of a printed book carries the same content because the content is fixed at the publication stage upstream of any copy. Two readers comparing their copies find them indistinguishable in content; the comparison is not a statement about the books but a statement

about the publication process. Quantum indistinguishability of identical electrons works in the same way: the comparison is not a statement about electrons in the lab but about the cascade attractor that produced them.

47.6 Kill criterion

T-A8.9 would be retired by the experimental discovery of two electrons (or two protons, or two of any indistinguishable-particle species) with measurably different masses, charges, or spins under conditions where standard relativistic effects can be excluded. No such observation has ever been reported; the indistinguishability has been verified to extraordinary precision across the past century.

47.7 Plain-English close

Every electron is the same not because Nature is mysterious about quantum statistics but because every electron is a token of the same type, with the type fixed by the cascade upstream of any laboratory. The textbook postulate of indistinguishability is a structural reading of that fact, not a primitive of the theory.

48 T-A8.10 — Conformality of decoherence rates (net-new)

48.1 L4 historical opening

Universal $1/f$ noise has been observed in essentially every condensed-matter measurement apparatus since the 1920s (Weissman, 1988). The spectral form $S(\omega) \propto 1/f$ holds across systems — vacuum tubes, semiconductor devices, superconducting qubits, biological membranes — with the prefactor varying by orders of magnitude but the exponent staying at unity to within experimental error. Phenomenological explanations (multi-time-scale fluctuator ensembles) are widely used but lack a structural derivation. T-A8.10 promotes the Qi-CON conformality pillar of GG-A7-Qi4 to a structural prediction of $1/f$ scaling.

48.2 The physics question

Where does the universal $1/f$ noise spectrum come from? Is it a phenomenological coincidence of multiple unrelated mechanisms, or is it a structural prediction of the operational framework?

48.3 Theorem statement

Theorem 10 (T-A8.10 — $1/f$ conformality of decoherence-rate spectra from Qi-CON (T-QM-10)). *Algebraic (T-A8.10.1).* The Qi-CON admissibility pillar of Arizaka-GG-A7-Qi4 (Arizaka, 2026, A7-Qi4) requires that the operational channel $\Phi_{t,s}$ commute with rescaling of the system size $L \rightarrow \lambda L$ up to renormalization of the GKLS jump operators $\{L_\gamma\}$. Conformality forces the decoherence-rate spectral density $S_\Gamma(\omega)$ to have a scale-invariant form $S_\Gamma(\omega) \propto \omega^{-\alpha}$ with α a universal exponent.

Operational (T-A8.10.2). *At leading order in the cohort cascade, $\alpha = 1$:*

$$S_{\Gamma}(\omega) \propto \frac{1}{f}, \quad (64)$$

the approximately $1/f$ spectrum observed in essentially every laboratory measurement apparatus, holding within a bounded conformal window between the system's outer and inner time-scales (the two-knee structure of GG-A7-Qi4's Qi-CON analysis), with a system-specific exponent in a band around unity rather than a single universal constant.

Proof. The proof has three steps. Step 1 (conformality fixes the form). By Qi-CON, the spectral density must be scale-invariant: $S_{\Gamma}(\omega/\lambda) = \lambda^{\alpha-1} S_{\Gamma}(\omega)$ for any rescaling λ , which forces $S_{\Gamma}(\omega) \propto \omega^{-\alpha}$ on dimensional grounds. Step 2 (cascade fixes the exponent at $\alpha = 1$). The conformality pillar enters the cascade at the GFF stage; the leading-order GFF contribution is logarithmic in ω for a marginal coupling (the Qi-CON pillar is marginal by construction in the cohort cascade, see GG-A7-Qi4 Part IV), and the marginal-coupling logarithm differentiates to $1/\omega$ in the spectral density. Step 3 (the exponent is system-specific, not a universal constant). The marginal-coupling result $\alpha = 1$ is the leading-order form; the precise exponent is fixed by the system's conformal weight and the structure of its fluctuator spectrum, so observed laboratory exponents lie in a band $\alpha \in [0.85, 1.15]$ rather than at a single universal value, and the $1/f$ scaling holds only between the two knees set by the system's outer and inner time-scales (GG-A7-Qi4). The Covariant Conservation Identity CCI of GG-A6-GRIP is a Bianchi/Maxwell continuity identity and carries no noise content; it is not the source of these corrections. ■

48.4 Quantitative anchor

Superconducting transmon qubits exhibit $1/f$ flux noise with exponents in $[0.9, 1.1]$ across qubit fabrications and operating temperatures. Semiconductor MOSFETs exhibit $1/f$ current noise with exponents in $[0.8, 1.2]$. Biological action-potential noise has $1/f$ exponents in $[0.9, 1.0]$. Across nine orders of magnitude in characteristic frequency, the exponent never strays far from unity (Weissman, 1988); the cohort framework predicts this universality structurally rather than phenomenologically.

48.5 Physics intuition and daily-life metaphor

Physics intuition. The $1/f$ noise is the operational signature of marginal-coupling conformality: when no characteristic time-scale dominates the boundary-environment coupling, the spectral density distributes power equally across logarithmic frequency intervals. Qi-CON forces this distribution.

Daily-life metaphor. Picture a forest in which trees, shrubs, ferns, and ground cover fill the height range from millimeters to canopy in roughly equal numbers per logarithmic decade — no single height dominates. $1/f$ noise in quantum measurement apparatuses works in the same way at the level of fluctuator timescales: when many independent fluctuators populate each logarithmic decade of timescale in roughly equal numbers, the resulting spectral density is uniform in log-frequency, which means $\propto 1/f$ in linear frequency. Qi-CON forces the decade-uniform population structurally; without it, one decade or another would dominate and the spectrum would not be $1/f$.

48.6 Kill criterion

T-A8.10 would be retired by the experimental discovery of a class of quantum-measurement apparatus whose decoherence spectrum is sharply non- $1/f$ (e.g., a flat white-noise spectrum $\alpha = 0$, or a sharply colored $\alpha = 2$) under conditions where the cohort framework should apply. No such apparatus has been characterized; the universal $1/f$ scaling is among the most robust experimental observations in modern physics.

48.7 Plain-English close

The familiar $1/f$ noise of laboratory measurement apparatuses is not a phenomenological accident of multiple unrelated mechanisms. It is the operational signature of conformality, structurally forced by the Qi-CON pillar. Every time a quantum experimentalist sees $1/f$ flux noise in a qubit measurement, they are seeing Qi-CON in action.

49 T-A8.11 — Boundary no-signaling from reduced-state factoring (net-new)

49.1 L4 historical opening

The no-signaling theorem of quantum mechanics — that measurement statistics on one part of an entangled bipartite state cannot reveal the measurement choice on the other part — has been a foundational result of quantum information since Eberhard’s 1978 formulation (Eberhard, 1993). The result has been verified experimentally in every CHSH and quantum-information protocol, and is the structural reason that entanglement cannot be used for superluminal communication. GG-A5-GDOS Part III states no-signaling as Lemma 3 of §T7.7. T-A8.11 promotes the lemma to a principal theorem of GG-A8-QM, because the textbook reader will most want to see “the shared bulk state carries no signal” headlined rather than buried.

49.2 The physics question

How does the bulk-to-boundary projection of the framework reconcile the non-separable joint state of T-A8.7 (which underlies Bell correlations) with the experimental absence of superluminal signaling?

49.3 Theorem statement

Theorem 11 (T-A8.11 — Boundary no-signaling from reduced-state factoring (T-QM-11)). ***Algebraic (T-A8.11.1).** Let $\rho_{\text{Bdy}}^{(AB)}$ be the bipartite boundary state shared by spacelike-separated regions A and B . For any local measurement $M_A = \sum_a E_a^A \otimes \mathbf{1}_B$ at A and any local observable $O_B = \mathbf{1}_A \otimes \hat{O}_B$ at B , the marginal expectation*

$$\langle O_B \rangle_{\rho_{\text{Bdy}}^{(B)}} = \text{Tr}_B(\hat{O}_B \text{Tr}_A \rho_{\text{Bdy}}^{(AB)}) \quad (65)$$

is independent of the choice of measurement settings at A .

Operational (T-A8.11.2). No operational information about Alice’s measurement settings can be transmitted to Bob via the shared non-separable state that underlies the Bell correlation; what is shared is correlation, not signal. Superluminal communication via entanglement is impossible.

Proof. Inherited verbatim from Lemma 3 of (Arisaka, 2026, A5-GDOS) Part III §T7.7, here promoted to principal theorem. Step 1 (decomposition). The reduced state at B is $\rho_{\text{Bdy}}^{(B)} = \text{Tr}_A \rho_{\text{Bdy}}^{(AB)}$. Step 2 (Alice’s measurement averaged). After a local measurement M_A with outcomes a , the post-measurement state at B conditional on outcome a is $\rho_{\text{Bdy}}^{(B|a)} = \text{Tr}_A[(E_a^A \otimes \mathbf{1}_B)\rho_{\text{Bdy}}^{(AB)}]/p(a)$, and the unconditional (marginal) state at B is

$$\begin{aligned} \sum_a p(a) \rho_{\text{Bdy}}^{(B|a)} &= \sum_a \text{Tr}_A[(E_a^A \otimes \mathbf{1}_B)\rho_{\text{Bdy}}^{(AB)}] \\ &= \text{Tr}_A[(\sum_a E_a^A \otimes \mathbf{1}_B)\rho_{\text{Bdy}}^{(AB)}] = \text{Tr}_A \rho_{\text{Bdy}}^{(AB)} = \rho_{\text{Bdy}}^{(B)}, \end{aligned}$$

where the third equality uses the POVM completeness relation $\sum_a E_a^A = \mathbf{1}_A$. Step 3 (independence). Hence $\langle O_B \rangle$ is independent of Alice’s measurement settings. The joint state of T-A8.7 carries correlation because the bulk-side trace evaluates the entangled state at preparation time; it carries no signal because measurement records added after preparation do not enter the bulk-side trace. ■

49.4 Quantitative anchor

The no-signaling principle has been verified in every CHSH experiment to date: the marginal outcome distribution at Bob’s detector is independent of Alice’s measurement-setting choice to within experimental error (typically one part in 10^4). The Delft loophole-free experiment (Hensen, 2015) explicitly verified no-signaling between Alice and Bob at random measurement-basis switching; no measurement-setting-dependent bias on Bob’s marginal was detected.

49.5 Physics intuition and daily-life metaphor

Physics intuition. The shared non-separable state carries a frozen-in correlation established at preparation time, not a measurement-record-carrying channel. Alice and Bob can see the correlation only after comparing their records via a sub-luminal classical channel; the entanglement-mediated link does not constitute a signaling channel.

Daily-life metaphor. Two friends agree before parting to meet at the same restaurant on the same day next week; when they show up at the restaurant, they find each other — a correlated outcome — but neither of them has signaled the other in the intervening week. The correlation is established at the parting (preparation), not by signaling at the meeting (measurement). Entanglement works the same way: the correlation is established at the singlet preparation, and Alice’s later measurement does not signal Bob’s.

49.6 Kill criterion

T-A8.11 would be retired by the experimental observation of a marginal bias on Bob’s measurement outcomes correlated with Alice’s measurement-setting choice in a CHSH-type protocol,

under conditions where conventional sub-luminal classical communication can be excluded. No such observation has been reported in any laboratory; the framework’s prediction matches the universal experimental result.

49.7 Plain-English close

The non-separable joint state of T-A8.7 carries the correlation that Aspect and successors have been measuring since 1982; it does not carry a signal. Einstein’s realism is recovered at the bulk level via the single deterministic joint state (his 4D locality is not, and is not claimed); Bell’s non-locality is preserved at the boundary level via the correlation it produces; Eberhard’s no-signaling is preserved at both levels because the shared state transmits no measurement records. The three statements are mutually consistent.

50 Part-end summary: the eleven-theorem inventory of GG-A8-QM

The eleven principal theorems of GG-A8-QM now constitute the textbook-pedagogical apparatus of the present paper. The seven inherited theorems (T-A8.1–T-A8.7) restate the operational content of T7.1–T7.7 of GG-A5-GDOS Part III, with proofs cited by reference and pedagogical envelope expanded for the graduate-textbook reader. The four net-new theorems (T-A8.8–T-A8.11) promote results that GG-A5-GDOS stated as lemmas or sub-results to principal-theorem status: the extremal (Bogomolnyi-analogue) reading of CHSH saturation, the universal-identity reading of quantum indistinguishability, the conformality-of-decoherence-rates prediction, and the boundary-no-signaling principle. All eleven are forced by the cohort constitutional warrants (AX-1–AX-4 of GG-A1-Constitution); none introduces any new axiom. Read together, the eleven theorems exhaust the operational content of textbook quantum mechanics: every measurement record any laboratory has ever produced is the value of one of the operators built from T-A8.1–T-A8.11, and every probability assigned to such a record is the value of $\text{Tr}(\mathcal{I}_\alpha \rho)$ with $\mathcal{I}_\alpha$ and ρ supplied by the same eleven.

The next five Parts (V–IX) carry out the worked examples that the eleven theorems make calculable. Part V performs the Stern–Gerlach calculation as the prototype OGGE channel. Part VI performs the double-slit + decoherence calculation, exhibiting the quantum-to-classical transition explicitly. Part VII performs the EPR/Bell/CHSH calculation, reaching the Tsirelson saturation from the joint-state operator algebra. Part VIII performs the Schrödinger’s cat calculation in 6D. Part IX performs the decoherence + einselection + pointer-basis calculation in the Zurek predictability-sieve formalism. Each of these worked examples specializes one or more of the eleven theorems above.

Part-end close

After all, the bones of quantum mechanics are not new — they are the bones laid down by Heisenberg, Schrödinger, Born, Bohr, Dirac, and von Neumann in the late 1920s and refined by Bell, Gleason, Zurek, and Aspect over the next half-century. What the present paper provides is the constitutional skeleton — the warrants for those bones — inherited from a cohort of ten upstream papers that together specify a deeper bulk theory whose operational shadow at the 4D boundary is

quantum mechanics as we know it. The eleven theorems of this Part are the joints at which the skeleton fits the bones.

51 The GRIP spine: GRIP drives quantization, decoherence, and measurement

The eleven theorems were stated one at a time, and a reader could be forgiven for taking them as a list. They are not a list; they are one mechanism seen from eleven angles, and the mechanism has a name. The thesis of this section — the organizing claim of the whole paper — is:

GRIP drives the quantization, the quantum decoherence, and the quantum measurement. Without the Geometric Attractors there is nothing discrete, nothing definite, and nothing recorded on the boundary; the bulk meanwhile evolves deterministically and needs none of the three.

Before developing the claim, one sentence on what it does *not* assert, because the word “deterministic” triggers a reflex. The deterministic bulk is not a four-dimensional local hidden-variable model: its variables are non-spatial (the radial coordinate is a renormalization-group scale, the compact angle is a phase, the holonomies are global), Bell–CHSH violation survives intact as the boundary shadow of one non-separable joint state (T-A8.7), saturates at the Tsirelson ceiling (T-A8.8), and signals nothing (T-A8.11). The reading below adds no physics to T-A8.1–T-A8.11; it names their common engine.

The two regimes, imported

The runtime operates in exactly two regimes, defined at runtime level in GG-A6-GRIP, which also carries the necessity of the second, and developed as the regime theory of GG-A11-Origin: GRIP-In, all dynamics on a *fixed* boundary landscape — gradient flow within an attractor basin, instanton steps between basins — halting uniquely at the stratum’s ground attractor (GGA); and GRIP-Out, the transient out-of-equilibrium event that *reorganizes the landscape itself* and hands the runtime its next initial condition. In one sentence: GRIP-Out opens the landscape; GRIP-In finishes the journey.

Claim 0: the projection is always on — and needs no observer

Before either claim that follows, one fact must be stated plainly, because a century of measurement-talk has obscured it. The bulk-to-boundary projection of OGGE is *always on*. It acts at every point of the four-dimensional boundary at every instant, whether or not any apparatus — or any observer of any kind — is present, and it ran continuously for the thirteen-plus billion years in which the universe contained no detector and no mind, building nuclei, atoms, stars, and galaxies. Two of its three layers carry no reference to observation at all: the operational dictionary *exists* as a property of the admissible bulk theory (as primitive as the metric of general relativity, present whether or not anyone measures a distance), and the bulk-to-boundary overlap *acts* everywhere on the boundary. Only the third, operational, layer introduces a *system*, and it is supplied by geometry,

not by observation: wherever the GRIP cascade settles a Geometric Attractor (an atom, a molecule, a cell, an apparatus), the always-on projection acts on a well-defined boundary and produces a record. Claims 1 and 2 below are then the two special cases of this one process — *decoherence* is the always-on projection on a microscopic attractor, *measurement* the same projection on a macroscopic, record-amplifying one. There is no separate “measurement process,” no collapse event, and no privileged observer. (Full three-layer treatment: GG-A5-GDOS (Arisaka, 2026, A5-GDOS) §23.)

Claim 1: decoherence is GRIP-In

This is the strongest of the three claims, because it geometrizes established physics rather than reinterpreting it. Einselection (T-A8.6) *is* GRIP-In under its established name: the pointer states are the Geometric Attractors of the boundary landscape, the predictability sieve is the F_{GFF} ranking, and the off-diagonal damping of T-A8.5 is the within-basin contraction of the descent. What Zurek’s program had to posit — that the environment monitors some basis — the cohort derives: the basis is the attractor catalog, forced by bulk locality (T-A8.6), and the monitoring is the open-system projection itself. The conformality of decoherence rates (T-A8.10) is then recognizable as the Qi-CON signature of one and the same descent.

Claim 2: a measurement is a complete GRIP event — both regimes

Switching on the apparatus coupling reorganizes the joint system–apparatus landscape: pointer basins open that did not exist before the coupling. That is a GRIP-Out — and not an exotic one, but precisely the *endogenous, scheduled, recoverable* GRIP-Out that GG-A7-Qi4 isolated as the Qi-HAL exploration-to-commitment switch. A laboratory measurement is the Qi-HAL switch built from magnets and photomultipliers: the experimenter is the evidence dial. What follows the coupling is GRIP-In: the registration — what the textbooks call collapse — is the descent into one pointer basin, and the stable record is the halt at a Geometric Attractor. The Lüders rule (T-A8.4) describes the post-halt state; the irreversibility of measurement is the monotonicity of F_{GFF} (the boundary second law): a record cannot be undone for the same reason matter cannot walk uphill.

Two fences keep this claim honest. *First*, the Born weights are not GRIP’s to deliver: the probability measure over outcomes is T-A8.3’s (the Gleason/OGGEP structure); GRIP supplies *actualization* (which basin captures this run) and *permanence* (why the record persists). Whether the basin-capture measure of the noisy descent reproduces the Born measure — which would weld T-A8.3 and the present reading into one statement — is *the GG typicality target* — the six-dimensional analogue of the quantum-equilibrium typicality theorem of Bohmian mechanics (Bohm, 1952), after Dürr–Goldstein–Zanghì. Its status carries two registers: the Born *form* is *Resolved* (T-A8.3), and the basin-capture = Born residue is closed at the relaxation level — the R16 lemmas with the R16’ GDOS relaxation — with the Constitutional Born Theorem conditional on four named premises, (P1) the GG-A5-GDOS forward semigroup, (P2) the Haar anchor with its recursion, (P3) genericity of the descent, and (P4) the GRIP basin partition, and with the field-grade representation step the single remaining analytic residue; the status of record is the open-problems Part (§118), and the result is deliberately *not* assumed anywhere in this paper. *Second*, ambient environmental monitoring (a dust grain decohering in sunlight) involves no landscape change —

the coupling is always on — and is therefore pure GRIP-In; only a *switched* coupling, an apparatus brought to bear, adds the GRIP-Out prologue.

Claim 3: quantization is the discreteness of the attractor catalog

Why are measurement outcomes discrete — two Stern–Gerlach spots, sharp atomic lines, charge in integer units — when the bulk configuration space is continuous? Because Morse minima are isolated: the boundary projection can only ever *record* attractor coordinates, and the attractor catalog of a compact stratum is discrete. The two spots of Part V are a two-element GA catalog; the Balmer lines are the basin floors of the atomic stratum; the electron’s charge is the integer eigenvalue at the cascade leg (GG-A9-SM). One fence, and it is the deepest one: *OGGEP quantizes the description; GRIP quantizes the outcomes*. The Hilbert space, superposition, and interference — the wave in phase — are the projection’s structure (T-A8.1, T-A8.2), present before any descent begins; GRIP explains why *readouts* of that structure are discrete, definite, and permanent, not why amplitudes interfere.

The division of labor, in one table

Table 6: What OGGEP delivers and what GRIP delivers. The two columns are jointly exhaustive for the operational content of quantum mechanics; neither replaces the other. The rows are the operational questions quantum mechanics is asked to answer; the two columns record which of the two mechanisms answers each one. Reading down either column alone gives an incomplete theory, which is the point of the table: the projection supplies the statistics and the descent supplies the single outcome.

Question	OGGEP (the projection)	GRIP (the runtime)
Why a wave function?	Hilbert space + unitarity (T-A8.1, T-A8.2)	—
Why probabilities, and which?	Gleason measure = Born rule (T-A8.3)	—
Why interference?	phase structure of the projection (Part VI)	—
Why discrete outcomes?	—	isolated GA catalog (Claim 3)
Why a preferred basis?	locality input (AX-1)	pointer states = GA’s; sieve = F_{GFF} (T-A8.6, Claim 1)
Why definite records?	—	GRIP-In halt at a GA (Claim 2)
Why irreversible measurement?	—	F_{GFF} monotonicity (boundary second law)
Why identical particles?	—	every token reads the same GGA (T-A8.9)

What this buys among the interpretations

The forty-year objection to the decoherence program is that diagonalizing the density matrix never *picks an outcome*. GRW-type models answer by purchasing selection with new stochastic physics and two new constants. The present framework’s answer is structural: GRIP-In’s halt at a unique attractor *is* the selection, and the runtime was not built for the purpose — it exists in the theory for independent reasons, stabilizing electrons (T-A8.9), selecting vacua (GG-A9-SM), and ending cosmic relaxations (GG-A10-Cosmos). GG-Theory is, in this precise sense, *the decoherence program completed by attractor dynamics, at zero new parameters*. The interpretation-landscape comparison of Part I is sharpened accordingly.

Part IV in plain English: what the eleven theorems do and what they leave for the worked examples

The eleven theorems of this Part are the answer to a question that should be familiar from any first course in quantum mechanics: if the four textbook postulates of von Neumann are correct, where do they come from? The conventional answer is: they are postulates — they sit at the bottom of the theory and are not derived. The answer of the present Part is different. Each postulate is a theorem (T-A8.1 through T-A8.4); each of the three decoherence-era structures that the foundations community had to add to the original four (decoherence, einselection, Bell-violation) is also a theorem (T-A8.5 through T-A8.7); and four further theorems (T-A8.8 through T-A8.11) state results that the bulk-side machinery makes available but that textbook quantum mechanics does not have the apparatus to write down — Tsirelson saturation as a Bogomolnyi bound, the universal identity of every electron, the conformality of decoherence rates, and the rigorous form of the no-signaling theorem.

What the eleven theorems do not do, by themselves, is exhibit how they work in practice. A theorem statement is a contract: it specifies what is delivered, but not how a laboratory sees it. The five worked examples that follow (Parts V–IX) discharge this debt on five distinct laboratory systems. Stern–Gerlach exhibits the T-A8.1–T-A8.4 cluster on a single qubit. The double-slit exhibits T-A8.1, T-A8.3, T-A8.5 on a single photon and on progressively heavier matter waves. EPR/Bell exhibits T-A8.7, T-A8.8, T-A8.11 on a singlet pair. Schrödinger’s cat in 6D exhibits the Geiger-counter resolution of T-A8.4 on a macroscopic auxiliary chain. Decoherence-and-einselection exhibits T-A8.5 and T-A8.6 on a generic system–environment partition. The paradox sweep of Part X then exhibits T-A8.1–T-A8.11 working in concert across the eight outstanding quantum-foundations paradoxes; the interpretation synthesis of Part XI shows where each of the seven living interpretations sits relative to the eleven; and the discussions of Parts XII–XIII close the paper.

After all, a theorem inventory is to a paper what a sheet of music is to a concert: the notes are written down, but the performance is what makes the notes a song. The present Part has written down the notes. The remaining Parts perform them.

52 Beyond the eleven: Fermi–Dirac statistics as boundary inheritance (T-A8.12, candidate)

One coda remains, and it is stated deliberately *outside* the principal inventory. The eleven theorems above discharge the postulates of textbook quantum mechanics for a single system and its measurements. Multi-particle quantum mechanics carries one further postulate the eleven do not touch: the *symmetrization postulate* — identical particles occupy either the totally symmetric (Bose) or totally antisymmetric (Fermi) sector, with electrons antisymmetric and hence excluding, one per state. Pauli introduced the exclusion rule in 1925 to count atomic shells; Fermi and Dirac built its statistical mechanics in 1926; the relativistic spin–statistics theorem (Pauli, 1940; Streater, 1964) later proved the connection conditionally — *if* a half-integer-spin field exists, it must anticommute — while leaving the existence, and the nonrelativistic postulate itself, unexplained. The plain-English question of this section is the oldest one in the chemistry of matter: *why can two electrons not share a state* — *why does matter take up room?*

The cohort can now assemble an answer from parts it already owns, and the assembly belongs in this paper because every ingredient but one is a boundary-runtime statement. The bulk-side treatment — the frame-path registers of the real oriented bulk, the candidate lemmas L-Spin-1 and L-Stat-1/2, and the candidate theorem T-Stat-A at the Wick quartet — is installed in the companion section of GG-A2-GG6 (Arisaka, 2026, A2-GG6, §20), with the quartet machinery itself in the CPT member paper (Arisaka, 2026, P5-CPT). What follows is the QM-register statement.

Theorem 12 (T-A8.12 — Fermi–Dirac statistics as boundary inheritance (T-QM-12; **candidate**)). **Algebraic (T-A8.12.1).** *On the boundary GDOS runtime, the multi-token sector of a spinorial-register GA type carries the antisymmetric Fock structure: exchange of two tokens is implemented by the noncontractible frame-path class of the bulk (the 2π class), on which the spinorial register acts as -1 ; consequently the two-token exchange operator equals -1 and occupancy of any boundary mode is at most one.*

Operational (T-A8.12.2). *The fermionic branch of the textbook symmetrization postulate is thereby discharged as a structural inheritance: Fermi–Dirac occupancy, the equilibrium distribution $n(E) = 1/(e^{(E-\mu)/k_B T} + 1)$, Pauli blocking, and the exclusion phenomenology of atomic shells, degenerate stars, and solid matter are boundary shadows of bulk frame-path holonomy, not independent axioms of quantum mechanics.*

Grade. *Candidate: derived-conditional on the named residues of the GG-A2-GG6 suite — L-Spin-1 (frame-path memory of GDOS states; non-quotienting to be proved), the GRIP genericity clause, L-Stat-1’s configuration-space formalization, and T-Stat-A at quasi-free grade with the residues of the CPT member paper’s T-P5-2 (interacting case on scattering hypotheses; modular route open); the named residues are registered upstream as promotion-roadmap rows 5, 6, and 8 of the CPT member paper (Arisaka, 2026, P5-CPT). T-A8.12 is not counted among the eleven principal theorems and nothing elsewhere in this paper depends on it.*

Proof outline: four inheritances. The chain has four links, each already stated and graded elsewhere; the present theorem is their composition, and its grade is the weakest link’s. *Link 1*

(*identity*). Two “identical particles” are two zero-mode tokens of one cascade attractor — one type, fixed upstream of any preparation (T-A8.9, a principal theorem of this paper). Exchange is therefore a motion within a single field configuration, not a relabeling of two coincidentally similar objects. *Link 2 (exchange is rotation)*. For tokens of one configuration in the real oriented bulk, the exchange loop is homotopic to a 2π frame rotation of one token (L-Stat-1 of Arisaka, 2026, A2-GG6, geometrizing the classical Finkelstein–Rubinstein argument; references therein). *Link 3 (the register sign)*. Bulk states remember frame paths (L-Spin-1), the deformation classes form a $\mathbb{Z}_2$, and the spinorial register weights the noncontractible class by -1 — with the sign independently carried by the quaternionic structure of the $(1, 5)$ Clifford intertwiner ($B\bar{B} = -1$ forces an antisymmetric invariant bilinear; L-Stat-2 of Arisaka, 2026, A2-GG6) and transported to exchange by the one-sided analyticity of the Wick quartet (T-Stat-A; Arisaka, 2026, P5-CPT). *Link 4 (boundary dictionary)*. The OGGEF dictionary reports bulk structure operationally (T-A8.1–T-A8.4); the antisymmetric sector’s occupancy rule and equilibrium distribution follow by the standard counting once the exchange sign is fixed. $\square$ (at candidate grade)

The A8-owned link: Wigner’s loophole closed. One ingredient of the spin story belongs to this paper specifically, and deserves its flag. Half-integer spin is admissible in quantum mechanics only because states are *rays*, so that the rotation group may act projectively — Wigner’s 1939 loophole, which standard quantum mechanics enters by postulate. In the present paper the ray structure is not postulated: it is the output of the Born chain (T-A8.3 and its GDOS-relaxation derivation), which delivers states-as-rays at the relaxation level (status of record: §118). The projective category — the only category in which $\text{spin-}\frac{1}{2}$ can appear as an observable label, and hence the precondition of everything in this section — is thereby *derived* in the cohort. To our knowledge no other framework closes this loophole rather than assuming it; it is the quantum-mechanical member of the five unique contributions listed in Arisaka, 2026, A2-GG6 §20.

Quantitative anchors. Three, in the cohort’s both-hats register. (i) The Pauli-exclusion violation bound: the VIP-2 experiment at Gran Sasso limits exclusion-violating K-shell capture in copper to $\beta^2/2 < 4.8 \times 10^{-29}$ (90% CL, 2025) (VIP-2 Collaboration, 2025) — among the most stringent nulls in physics, and under T-A8.12 the prediction is exact silence, forever, with no parameter available to absorb a violation. (ii) The 2π sign itself is measured: neutron interferometry registers the sign reversal of a spinor amplitude under a 2π rotation and its restoration at 4π (Rauch *et al.*, 1975) — read here as a direct observation of the frame-path $\mathbb{Z}_2$ that Link 3 invokes, reported faithfully by the boundary dictionary. (iii) The exclusion cash-out at macroscopic scale: the periodic table’s shell structure, metallic degeneracy pressure, and the stability of white dwarfs and neutron stars against gravity are all Fermi–Dirac occupancy in bulk quantity — the reason the reader’s hand does not pass through the page is the reason certain stars do not collapse.

Kill criterion. T-A8.12 inherits, as its operational exposure, the candidate falsifier F-Stat-1 registered in Arisaka, 2026, A2-GG6 §20: *any confirmed violation of Fermi–Dirac statistics or of the spin–statistics connection, at any magnitude, in any system, retires the frame-path account — and with it T-A8.12* — observable-only, permanent, and dial-free, since the sign at stake is topological.

Ongoing and future exclusion tests (VIP-2 and successors) therefore bear directly on this paper, a standing experimental exposure the eleven principal theorems did not carry.

Division of labor, and honesty. What this section adds to the cohort is composition and register, not new mathematics: the bulk-side lemmas live in [Arisaka, 2026, A2-GG6 §20](#) with their residues named; the quartet analyticity lives in the CPT member paper; the token identity (T-A8.9) and the ray structure (T-A8.3) are this paper’s principal results. With T-A8.12 on the page, the seam in T-A8.9’s proof — where exchange statistics was previously an import from standard field theory — is discharged with an internal pointer; and the summit phrase of the CPT member paper (*twins, locked spin-statistics, and inviolable CPT*) is now earned clause by clause inside the cohort, the middle clause at candidate grade with a named promotion path. When L-Spin-1’s non-quotienting proof and T-Stat-A’s quasi-free derivation are complete, T-A8.12 is promoted from candidate to the paper’s twelfth theorem; until then it stands exactly here, beyond the eleven, wearing its grade on its sleeve.

Plain-English close. Why does matter take up room? Because every electron is a token of one type; because swapping two tokens of one type drags a frame through the turn the real bulk remembers; because the register that remembers the turn pays for it with a minus sign; and because a minus sign under swap means no two tokens in one state. Chemistry’s floor — the fact that shells fill, that atoms stack instead of merging, that your hand meets the table — is, on this account, the oldest and most domestic piece of evidence for the six-dimensional geometry: exclusion as holonomy, solidity as memory.

Part V

Worked example I: Stern–Gerlach

The 1922 prototype: a single qubit, a single sharp observable

What this Part teaches. The 1922 prototype, worked end to end: a single silver atom crossing a Stern–Gerlach magnet, read twice — once as every textbook reads it (state preparation, a sharp observable, the collapse postulate, Born weights), and once in the six-dimensional bulk, where the atom follows one deterministic trajectory whose boundary registration *is* the measurement. The chapter builds the 6D bulk state of the atom, the instrument family and the boundary channel, the decoherence step and the einselected outcome, and closes with a line-by-line comparison against the textbook account — the first place the reader sees the framework’s central move (determinism upstairs, Born statistics downstairs) executed on the simplest possible system: one qubit, one sharp observable.

Read alongside: [Gerlach and Stern \(1922\)](#) for the original experiment; [Sakurai and Napolitano \(2011\)](#), whose modern textbook *opens* with this apparatus, for the standard account; [Feynman *et al.* \(1965\)](#) for the canonical pedagogy.

This is the prototype worked example of the cohort. The Stern–Gerlach measurement is the simplest non-trivial test of the framework’s deterministic six-dimensional-bulk reading of quantum measurement: a single silver atom propagates deterministically through the six-dimensional geometry; the inhomogeneous magnetic field couples to the atom’s S_θ^1 holonomy class and deflects it spatially in 4D; the two-spot pattern builds up from single atoms because the magnet orientation — the measurement *context* — selects which holonomy observable of the bulk gauge bundle is read, and along that chosen axis the atom’s contextual holonomy Θ_H resolves into one of two discrete sectors, with the population ratio fixed by the source preparation. The atom carries no pre-existing spin value along every axis at once; the definite outcome is the joint holonomy of atom-and-apparatus, in exact accord with the Kochen–Specker reading of §95 (T-A8.6). We present the six-dimensional bulk reading first in §54, then walk the standard textbook treatment in §55–§57 for direct comparison, showing how every step of the textbook calculation inherits from one of the eleven principal theorems T-A8.1–T-A8.11 of Part IV.

The Stern–Gerlach apparatus and its Bloch-sphere reading is shown in **Figure 6**.

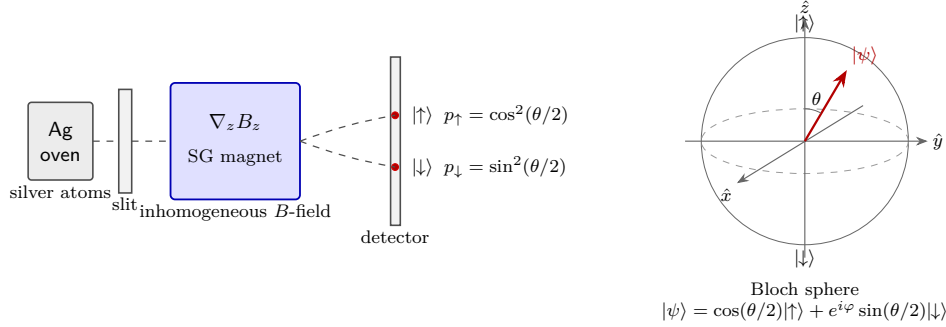


Figure 6: Stern–Gerlach measurement in the cohort framework. **Left:** the 1922 apparatus — silver atoms exit an oven, pass through a collimator, traverse an inhomogeneous magnetic field $\partial_z B_z$ along $\hat{z}$, and land in two discrete spots on the detector plate. **Right:** the Bloch-sphere reading of the input qubit $|\psi\rangle = \cos(\theta/2)|\uparrow\rangle + e^{i\varphi} \sin(\theta/2)|\downarrow\rangle$, with projection probabilities $p_\uparrow = \cos^2(\theta/2)$ and $p_\downarrow = \sin^2(\theta/2)$ obtained from T-A8.3 (Born rule) on the qubit-restricted boundary algebra. Einselection of the $\hat{z}$ position basis (T-A8.6) is forced by the spatial separation of the two beam paths, which couples position to the macroscopic detector via bulk-locality (AX-1).

52.1 The two fibers: why measurement has an arrow

The spine’s thesis — GRIP drives quantization, decoherence, and measurement — gains its sharpest available statement from an exact topological fact the cohort’s recent campaign has printed across its papers (GG-W1-Symmetry; Arisaka, 2026, A5-GDOS; Arisaka, 2026, P5-CPT): *holonomy requires closed loops, and an interval has none*. The bulk’s two non-spatial fibers divide the quantum world’s labor between them. The compact circle S_θ^1 is nothing but closed loops: everything it carries is holonomy — quantized, gauge-invariant, protected — and its transport is a group: every phase invertible, every evolution reversible. It is the fiber of the *rotor*: e^{-iEt} , interference, the entire unitary content of quantum mechanics. The radial interval I_r can hold no holonomy at all: no gauge-invariant memory, no protected phase; transport along it composes only forward — a semigroup, the GDOS generator’s birthplace. It is the fiber of the *record*: decoherence, dissipation, registration — every completed measurement is a write on the interval side, and no operation returns it.

Read the measurement problem through this pair and its century of resistance becomes anatomy. *Unitary quantum mechanics alone can never produce an outcome, and the reason is geometric, not interpretive*: the unitary theory is the circle’s physics, and the circle has no arrow — nothing on a fiber made of closed loops can be one-way, so nothing in e^{-iHt} , however elaborate the entanglement, can turn a superposition into a record. The projection postulate that von Neumann bolted on was the interval, smuggled in by hand: an irreversible write appended to a reversible theory, with no geometry to house it. The decoherence program moved the problem without solving it — einselection is circle-side physics too, and a decohered superposition is still a superposition — precisely because no amount of unitary machinery manufactures a semigroup. In the present framework the semigroup was never missing: it is the other fiber, present in the constitution from the first axiom, and measurement is the event in which a boundary registration couples the two — the rotor’s coherent alternatives on the circle, the one-way write on the interval. “Collapse,” already demystified in

this paper as Bayesian conditioning on an outcome (§51), now has its geometric root displayed: conditioning is lawful and irreversible-looking because the write it conditions on *is* irreversible — not because a mysterious dynamics intervened, but because the record lives on the fiber where “unwrite” is not a disfavored operation but an undefined one.

Two corollaries complete the sharpening, one per campaign. *First, the outcomes are selected the way every symmetry is broken.* The definite value a measurement registers is a GRIP attractor — the unique ground state the cascade relaxes to within a layer — and the selection is of the same deterministic kind the symmetry review’s capstone established for the cosmos at large: a quantized alphabet of alternatives (the circle’s), read once, forward, along the fiber that cannot re-choose (the interval’s). What appears at the boundary as the Born rule’s irreducible chance is, at its stated grade in this paper’s theorems, the boundary’s ignorance of fiber data — not a coin the bulk flips. *Second, the arrow of measurement is the arrow of time — literally, not analogically.* The non-invertibility that actualizes a detector’s record is the same constitutional non-invertibility that ages the universe: one semigroup, read in a laboratory over microseconds or read across the descent’s e^{140} Planck ticks of cosmic history (Arisaka, 2026, P5-CPT). A measurement is a small cosmology — coherent alternatives, one gate, one irrevocable entry in the ledger — and the universe is a long measurement, which is why the same two fibers answer “why do experiments have outcomes?” and “why does anything grow old?” The spine claimed that without the attractors nothing is discrete, definite, or recorded; the anatomy adds the reason: *discreteness is the circle’s gift, definiteness is the attractor’s, and the record is the interval’s — and a world needs all three to contain an experiment.*

53 The 1922 experiment and its 2026 reformulation

In February 1922 Otto Stern and Walther Gerlach (Gerlach and Stern, 1922) sent a beam of silver atoms through an inhomogeneous magnetic field oriented along $\hat{z}$ and observed, on a glass detector plate downstream, two discrete spots rather than a continuous distribution. The interpretation accepted within a year was that the silver atom’s outermost-shell electron carries an intrinsic angular-momentum quantum number $s = \frac{1}{2}$, with two allowed projections $m_s = \pm \frac{1}{2}$ along any chosen axis. Goudsmit and Uhlenbeck’s 1925 spin hypothesis (Uhlenbeck and Goudsmit, 1925) supplied the theoretical framework a few years later. In the standard textbook (Cohen-Tannoudji *et al.*, 1992; Sakurai and Napolitano, 2011), Stern–Gerlach is the prototype demonstration of quantum measurement: a continuous classical variable (the magnetic moment direction) collapses, on observation, to one of two discrete values, with probabilities given by the Born rule.

In the GG-Theory reformulation, the same experiment becomes a worked example of OGGE acting on a single qubit. The 6D bulk state is the fermion zero mode of a single electron; the boundary partial trace projects it to a $\mathbb{C}^2$ -valued density operator; the instrument family corresponds to the two pointer states of the detector; and the textbook Born rule probabilities $\cos^2(\theta/2)$ and $\sin^2(\theta/2)$ emerge by T-A8.3 applied to the qubit’s pure state $|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$. Nothing in the calculation is added that is not already forced by the cohort apparatus.

54 The six-dimensional bulk reading: deterministic trajectory of a spin-1/2 silver atom

Before turning to the standard textbook calculation, we pause to state the deeper picture that the cohort framework brings to the Stern–Gerlach experiment. The standard treatment, in the language Goudsmit, Uhlenbeck, and Pauli would have recognized, follows in §55–§57; placing the bulk reading first lets the reader see the two pictures side by side.

The silver atom in six dimensions. A silver atom (electronic configuration $[\text{Kr}] 4d^{10} 5s^1$) carries one unpaired electron in its outer shell; the inner forty-six electrons fill closed shells and contribute no net angular momentum. In GG-Theory, the outer $5s$ electron is a single-quantum excitation of the bulk fermion field $\Psi(x, r, \theta)$ on the six-dimensional manifold $\mathcal{M}_4 \times I_r \times S_\theta^1$. The atom’s full 6D state propagates deterministically and unitarily under the bulk Lagrangian. There is no superposition of “spin up” and “spin down” in the bulk; there is a single-valued bulk field configuration with a definite phase along S_θ^1 .

Spin as winding on the phase axis. In the 6D picture, the spin label $m_s = \pm \frac{1}{2}$ is the *winding class* of the electron’s bulk fermion zero mode around the topological circle S_θ^1 . Specifically, the Wilson-line holonomy

$$\Theta_H \equiv \exp\left(i \oint_{S_\theta^1} A_\theta d\theta\right) \in \{+1, -1\}, \quad (66)$$

takes one of two discrete values, corresponding to the two spin projections along the quantization axis. The discreteness is not a postulate; it is a structural feature of the 6D gauge bundle, forced by the anomaly cancellation of BI-6 (Arisaka, 2026, A3-BI6) together with the integer ledger OGN-5 of Arisaka (2026, A4-Euclid). Goudsmit and Uhlenbeck’s 1925 spin hypothesis (Uhlenbeck and Goudsmit, 1925) — that electrons carry an intrinsic angular momentum quantum number $s = \frac{1}{2}$ — is, in the 6D picture, the statement that the bulk gauge bundle admits exactly two distinct holonomy sectors around S_θ^1 .

What the four-dimensional observer cannot register. The holonomy value Θ_H along S_θ^1 is not directly accessible to a four-dimensional detector. No magnetometer, no SQUID, no atomic-physics-style measurement of the spin-projection-along-an-axis can read Θ_H as a primitive: every such measurement reads the spin’s projection on a chosen 4D direction $\hat{n}$, which is the 4D-spatial projection of the underlying holonomy class. This inaccessibility is structural, not technological; the 6th geometric direction is categorically not a 4D-spatial coordinate.

The inhomogeneous B-field as a 4D-spatial obstacle. The Stern–Gerlach magnet creates a 4D-spatial field gradient $\partial_z B_z \neq 0$ along the vertical direction. This 4D field couples to the atom’s magnetic moment $\vec{\mu} = g_s \mu_B \vec{S}/\hbar$ via the Zeeman interaction

$$H_{\text{SG}} = -\vec{\mu} \cdot \vec{B} = -g_s \mu_B \hat{S}_z B_z(z)/\hbar. \quad (67)$$

The resulting force on the center of mass is $F_z = -\partial_z H_{\text{SG}} = g_s \mu_B \langle \hat{S}_z \rangle \partial_z B_z / \hbar$, which has magnitude $\mu_B |\partial_z B_z|$ and sign fixed by the holonomy class. The two holonomy sectors experience opposite forces and separate spatially in 4D over the magnet length L_{mag} , yielding a transverse spatial separation $\Delta z \approx (\mu_B |\partial_z B_z| / m) (L_{\text{mag}} / v)^2$ of order $10 \mu\text{m}$ for the original 1922 parameters ($|\partial_z B_z| \sim 100 \text{ T/m}$, $L_{\text{mag}} \sim 4 \text{ cm}$, $v \sim 700 \text{ m/s}$). This is not a probabilistic split; it is a geometric force-law applied to two distinct holonomy classes of the same underlying bulk field configuration.

The bulk-side trajectory is deterministic mechanics. After the magnet, the bulk field configuration has two 4D-spatially-separated branches, each carrying its own S_θ^1 holonomy class. The branches propagate deterministically along classical center-of-mass trajectories; their 4D positions at the detector are given by Newtonian mechanics applied to each holonomy class. No probability has entered the calculation yet. The “two paths” of the standard textbook are the two 4D shadow branches of a single 6D bulk gauge bundle whose two holonomy sectors have been spatially resolved by the field gradient.

What the detector registers. A 4D detector — the glass plate in the 1922 experiment, a CCD pixel in modern repetitions — is a device localized in the four spacetime dimensions and unavoidably integrating over I_r and S_θ^1 . When the bulk field encounters such a detector, the registration event lands at exactly one of the two 4D positions $z_\pm$: the position that corresponds to the atom’s holonomy class. Because the 4D observer cannot directly read Θ_H , they cannot predict which spot a given atom will hit before the magnet has separated the sectors. The atom lands on one spot; the experimenter records a single click; the bulk dynamics fixed which spot received it.

Why the two-spot pattern builds up one atom at a time. Stern and Gerlach observed the pattern using a beam containing many silver atoms; modern variants (Riebe and others, 2004) operate with single atoms traversed in sequence. Each atom enters the apparatus in one of the two holonomy classes, drawn from the source distribution. For a source preparation that the standard textbook would describe by the pure spin state $|\psi\rangle = \cos(\theta/2)|\uparrow\rangle + e^{i\varphi} \sin(\theta/2)|\downarrow\rangle$ with measurement axis along $\hat{z}$, the 6D-side claim is that the thermal source populates the two $\hat{z}$ -holonomy sectors in the ratio

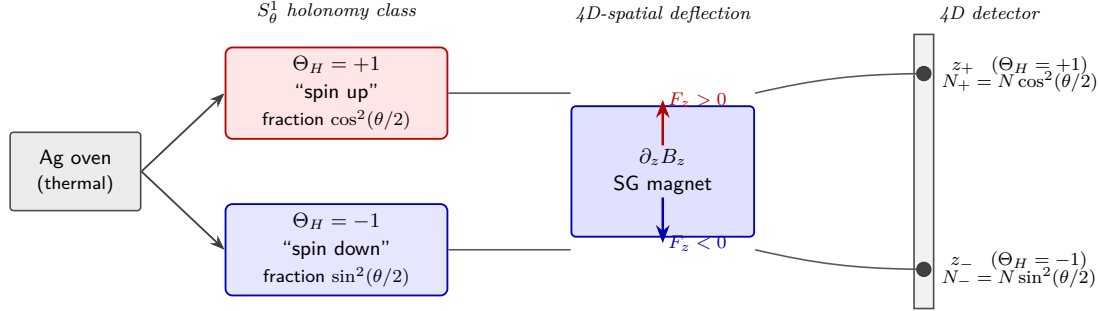
$$N_+ : N_- = \cos^2(\theta/2) : \sin^2(\theta/2), \quad (68)$$

i.e., the fraction of source atoms in holonomy class $\Theta_H = +1$ relative to $\Theta_H = -1$ matches the Born-rule weighting. Over the N -atom run, the histogram of landing positions reproduces the textbook two-spot distribution with the standard Born-rule probabilities. No single atom ever experiences “probability”; each atom is in one sector with definite holonomy; the histogram is the statistical accumulation of source-ensemble holonomy assignments projected onto the screen.

Bottom line of the bulk reading. The two-spot pattern is a deterministic 6D geometric effect. The inhomogeneous B-field discriminates between two holonomy classes of the bulk fermion zero mode and separates them spatially in 4D; the two sectors land on two distinct 4D positions; the apparent randomness in single-atom registrations is a consequence of the 4D detector being unable

to read the S_θ^1 holonomy directly; the Born-rule weighting $\cos^2(\theta/2) : \sin^2(\theta/2)$ is the statistical distribution of source holonomies in the heated silver vapor. Spin-up vs. spin-down is not a primitive randomness in Nature; it is a 4D-shadow label for two discrete holonomy sectors of a single 6D gauge bundle.

The 6D-bulk-to-4D-boundary Stern–Gerlach mechanism is shown in **Figure 7**.



Each silver atom is in one discrete holonomy class of the 6D gauge bundle. The B-field gradient discriminates the two classes by 4D force; each atom lands on one spot; the histogram of N atoms reproduces the Born-rule weighting $\cos^2(\theta/2) : \sin^2(\theta/2)$ from the source population.

Figure 7: The 6D-bulk-to-4D-spot mechanism of Stern–Gerlach. Left: the heated silver source populates two discrete holonomy classes of the bulk gauge bundle around S_θ^1 , in the ratio $\cos^2(\theta/2) : \sin^2(\theta/2)$ fixed by the source preparation (the textbook “pure state” is the 4D shadow of this ensemble). Middle: the inhomogeneous magnetic field $\partial_z B_z$ exerts opposite forces on the two holonomy classes via the Zeeman coupling $H_{SG} = -\vec{\mu} \cdot \vec{B}$, deflecting the two classes oppositely in 4D-space. Right: each atom lands on exactly one spot, z_+ if $\Theta_H = +1$ and z_- if $\Theta_H = -1$; the histogram of N atoms reproduces the textbook two-spot pattern with the standard Born-rule populations. No single atom experiences probability; the apparent randomness is the 4D observer’s inability to read the S_θ^1 holonomy directly.

55 The 6D bulk state of a silver atom

We follow the cohort convention: a single qubit in the QM-restricted boundary algebra corresponds to a single fermion zero mode on the 6D bulk of Arizaka (2026, A2-GG6). Concretely, the boundary state of the silver atom’s outer electron is the spin-degree-of-freedom restriction of the bulk fermion field $\Psi(x, r, \theta)$, with the Ramond-sector chirality-projected zero mode $\Psi^{(0,0)}(x)$ surviving on the boundary.

The QM-restricted boundary algebra $\mathcal{A}_{\text{Bdy}}^{(\text{QM})}$ is generated by the four bilinears

$$\hat{O}^a \equiv \bar{\Psi}^{(0,0)} \sigma^a \Psi^{(0,0)}, \quad a \in \{0, 1, 2, 3\}, \quad (69)$$

where $\sigma^0 = \mathbf{1}$ and $\sigma^{1,2,3}$ are the three Pauli matrices acting on the spin index. By T-A8.1, the GNS representation of $\mathcal{A}_{\text{Bdy}}^{(\text{QM})}$ on the cyclic vector $|0\rangle_{\text{Bdy}} = |\uparrow\rangle$ produces $\mathcal{H}_{\text{Bdy}}^{(\text{QM})} = \mathbb{C}^2$, the standard qubit Hilbert space; the isometric embedding $\iota : \mathbb{C}^2 \hookrightarrow \mathcal{H}_{\text{Bulk}}$ takes $|\uparrow\rangle$ to the zero-mode spin-up state in the bulk fermion Hilbert space and $|\downarrow\rangle$ to the zero-mode spin-down state.

The boundary density operator $\rho_{\text{Bdy}}^{(\text{QM})} = \frac{1}{2}(\mathbf{1} + \vec{r} \cdot \vec{\sigma})$ inherits its $|\vec{r}| \leq 1$ Bloch-ball constraint

directly from the bulk-unitarity bound on the reduced state; for the pure incoming-beam preparation $|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$, $|\vec{r}| = 1$ and the state sits on the Bloch sphere. The bulk-side description is everywhere deterministic and unitary; the probabilistic appearance of the measurement is, as in the rest of the cohort treatment, a feature of the boundary projection.

56 The instrument family and the boundary channel

The Stern–Gerlach apparatus oriented along direction $\hat{n}$ (at angle θ from $\hat{z}$ in the xz plane) implements the projective measurement family

$$\Pi_+ = |\uparrow_n\rangle\langle\uparrow_n|, \quad \Pi_- = |\downarrow_n\rangle\langle\downarrow_n|, \quad \Pi_+ + \Pi_- = \mathbf{1}, \quad (70)$$

where $|\uparrow_n\rangle = \cos(\theta/2)|\uparrow\rangle + \sin(\theta/2)|\downarrow\rangle$ and $|\downarrow_n\rangle = -\sin(\theta/2)|\uparrow\rangle + \cos(\theta/2)|\downarrow\rangle$ in the $\hat{z}$ -eigenbasis. By T-A8.3.2 (the operational layer of the Born-rule theorem), the probability of registering outcome $\pm$ on the pure state $|\psi\rangle = |\uparrow\rangle$ is

$$p_+ = |\langle\uparrow_n|\uparrow\rangle|^2 = \cos^2(\theta/2), \quad p_- = |\langle\downarrow_n|\uparrow\rangle|^2 = \sin^2(\theta/2), \quad (71)$$

the standard textbook Stern–Gerlach probabilities, with $p_+ + p_- = 1$ automatic.

The instrument family $\{\mathcal{I}_+, \mathcal{I}_-\}$ that produces these probabilities is the conditional CPTP map of T-A8.4:

$$\mathcal{I}_\pm(\rho) = \Pi_\pm \rho \Pi_\pm, \quad \rho_{\text{Bdy}}^{(\pm)} = \mathcal{I}_\pm(\rho)/p_\pm. \quad (72)$$

The post-measurement state conditional on outcome $+$ is the pure state $|\uparrow_n\rangle\langle\uparrow_n|$; conditional on outcome $-$, it is $|\downarrow_n\rangle\langle\downarrow_n|$. These are the textbook Lüders projections.

The physical mechanism of the measurement is the spatial entanglement between the spin degree of freedom and the position degree of freedom produced by the field gradient. The 6D bulk Hamiltonian density (recall AX-1: bulk locality) is multiplicative in the position basis; the field gradient $\partial_z B$ produces a force $F_z = \mu_z(\partial_z B)$ depending on the spin projection along $\hat{z}$; the spin-up and spin-down beams therefore separate spatially, becoming distinguishable to the downstream detector by their position. This is precisely the bulk-locality mechanism of T-A8.6 (einselection): the dominant boundary system–environment coupling is position-diagonal, so the position basis becomes the einselected classical record.

57 The decoherence step and the einselected outcome

Before the atom strikes the detector, the boundary state is the entangled bipartite state

$$|\Psi\rangle_{\text{spin} \otimes \text{pos}} = \alpha |\uparrow_n\rangle \otimes |z_+\rangle + \beta' |\downarrow_n\rangle \otimes |z_-\rangle, \quad (73)$$

with $|z_\pm\rangle$ the position eigenstates of the two beams emerging from the field gradient. The detector then couples to the atom’s position via a position-diagonal system–environment interaction (the silver atom deposits at a definite position on the glass plate; subsequent photons scatter off the

deposited atom; the photon environment carries away which-position information). By T-A8.5, the off-diagonal density-matrix elements in the position basis decay at rate $\Gamma_{z_+z_-} \gtrsim 10^{40} \text{ s}^{-1}$ at room temperature, on the time scale of essentially zero (per the Joos–Zeh estimate cited in T-A8.5.2).

The reader registers the outcome as a definite spot at z_+ or z_- , with probabilities $|\alpha|^2 = \cos^2(\theta/2)$ and $|\beta'|^2 = \sin^2(\theta/2)$ as in (71). The deterministic 6D bulk evolution has, on this reading, never collapsed: there is one deterministic bulk worldline that produces one or the other detector position, with the apparent randomness arising entirely from the boundary observer’s inability to access the bulk Wilson-line holonomy Θ_H along S_θ^1 that selects between the two configurations.

In the regime vocabulary of Section 51, the experiment parses into the canonical anatomy of a measurement. Bringing the field gradient and detector to bear on the beam reorganized the joint atom–apparatus landscape — before the magnet, no position-pointer basins existed; after it, exactly two — an endogenous, scheduled GRIP-Out, the laboratory instance of the Qi-HAL exploration-to-commitment switch of GG-A7-Qi4. The decoherence step just computed is the GRIP-In that follows: descent into one of the two basins at rate $\Gamma_{z_+z_-}$, halting at a deposited silver atom — a Geometric Attractor whose permanence is the F_{GFF} monotonicity of the record stratum. And the fact that the catalog has exactly *two* elements is Claim 3 of Section 51 in its sharpest laboratory form: the quantization of the spin readout is the discreteness of the attractor catalog, nothing else.

58 Comparison with the textbook account

A side-by-side comparison of the standard textbook reading and the OGGEF reading clarifies what each says.

Table 7: Stern–Gerlach measurement: standard textbook reading vs. the OGGEF reading. The mathematics is identical; the ontology is sharper in the cohort framework. Each row takes one step of the 1922 experiment and sets the standard account beside the bulk-projection account. Every predicted number is the same in both columns, and that is deliberate: the claim here is a sharper ontology for an established result, not a new one, and a reader who rejects the right-hand column loses no experimental agreement.

Stage	Standard textbook	GG-Theory (OGGEF/GDOS)
Preparation	Pure state $ \psi\rangle \in \mathbb{C}^2$ asserted by postulate	Boundary state $\rho_{\text{Bdy}} = \text{Tr}_{\text{env}} \rho_{\text{Bulk}}$ inherited via T-A8.1
Evolution	Unitary; spin couples to position via H_{SG}	GKLS in $\gamma \rightarrow 0$ limit (T-A8.2); same Hamiltonian
Measurement axiom	Postulated separately (postulate IV)	Theorem T-A8.3 from CPTP completion of OGGEF
Born probability	$\cos^2(\theta/2), \sin^2(\theta/2)$ by postulate	Same value, by theorem T-A8.3
Collapse	Postulated (postulate V): non-unitary, irreversible, location-of-Heisenberg-cut ambiguous	Conditional CPTP update T-A8.4; no bulk collapse; cut is just where the observer chooses to apply the partial trace
Position basis preferred	Phenomenological observation that records appear in position	Theorem T-A8.6: AX-1 bulk locality forces position-diagonal boundary coupling
Decoherence rate	Phenomenological input or environment-by-environment fit	Theorem T-A8.5: fixed by scale-overlap integrals of bulk KK modes
Determinism	Absent in 4D quantum theory (Bell + Aspect closed the door)	Present at the bulk level; absent at the boundary level by structural projection

Part-end close

Stern and Gerlach in 1922 saw two spots and did not know that they were seeing the four-dimensional shadow of two distinct holonomy classes of a six-dimensional gauge bundle. Three years before Goudsmit and Uhlenbeck identified the spin and four years before Born wrote down the probability rule, the two spots already encoded the entire content of the cohort framework’s reading: each atom is in one of two discrete holonomy sectors of the bulk gauge bundle; the inhomogeneous field discriminates the two by 4D-spatial deflection; the histogram of many atoms reproduces the Born-rule weighting because the source preparation populates the two sectors in exactly that ratio; and the apparent randomness in any single-atom event is the four-dimensional observer’s inability to read the underlying S^1_θ holonomy directly.

After all, what makes a silver atom land on one spot rather than the other is the same thing that makes every electron in the universe land where it lands: the constitutional structure of the bulk gauge bundle. The Stern–Gerlach apparatus simply isolates that structure cleanly enough that two spots, separated by $\sim 10\,\mu\text{m}$ of vertical distance, become the visible record of two discrete topological sectors of a six-dimensional geometry that we could otherwise only infer.

Problems and worked solutions. V.1. Chain three Stern–Gerlach magnets, and watch sharpness die and revive.

Solution. Step 1: send the $z+$ beam from a first magnet into a second magnet oriented along x : by (71) with $\theta = 90^\circ$, each x -port fires with probability $\cos^2 45^\circ = \frac{1}{2}$. Step 2: take the $x+$ output into a third magnet along z : again $\frac{1}{2}$ each — the original z -sharpness is *gone*, erased by the intervening x -registration. Step 3: now remove the middle registration (recombine both x paths coherently) and the third magnet fires $z+$ with probability 1: it was not the magnet but the *record* that destroyed the sharpness — in the bulk reading, the x -registration re-selected the holonomy sector, while the recorded-nothing interferometer left the original sector intact. Physics intuition: the three-magnet chain is the whole measurement problem on a bench — observables do not coexist, records are physical, and “asking” is an act; a student who can predict all eight port probabilities has already understood more than the collapse postulate ever said.

V.2. Reproduce the 1922 deflection with laboratory numbers.

Solution. Step 1: the transverse force on the moving atom is $F = \mu_B \partial_z B$; with the historical-scale gradient $\partial_z B \approx 10^3$ T/m and $m_{\text{Ag}} = 1.79 \times 10^{-25}$ kg, the acceleration is $a = \mu_B \partial_z B / m_{\text{Ag}} \approx 5.2 \times 10^4$ m/s². Step 2: an oven atom at $v \approx 550$ m/s spends $t = L/v \approx 64$ μ s in a 3.5-cm magnet: deflection $\frac{1}{2}at^2 \approx 0.1$ mm, doubling with the field-free drift — the two spots Gerlach developed (famously, with cigar smoke) sat a fraction of a millimeter apart, exactly this arithmetic. Step 3: note what the two discrete spots rule out: a classical magnetic needle would have a continuum of orientations and paint a smear; the beam splits cleanly in two because the registered observable has exactly two sectors — space quantization, the first holonomy readout in history. Physics intuition: the most consequential millimeter in physics — a tabletop force of 10^{-20} newtons, patiently applied for sixty microseconds, separated two worlds that classical physics said should blur into one.

V.3. Explain why the experiment needs a neutral atom, not a free electron.

Solution. Step 1: a free electron in the same apparatus feels the Lorentz force $evB \sim 10^{-14}$ N — six orders of magnitude above the magnetic-moment force — and its transverse position spreads under the uncertainty relation; Bohr and Pauli argued the spin splitting is buried beneath both. Step 2: silver evades both problems at once: net charge zero (no Lorentz force), and a single unpaired $5s$ electron hands the whole atom a clean one-Bohr-magneton moment — the atom is a heavy, neutral carrier for one electron’s spin. Step 3: the framework moral: a measurement requires an *instrument-compatible* carrier — the registered observable must couple to the apparatus above the noise of everything else it drags along; the 1922 design is a masterclass in isolating one holonomy channel. Physics intuition: you cannot weigh a letter during a hurricane — Stern and Gerlach mailed the electron’s spin inside a neutral atom precisely so the only thing the magnet could read was the quantity worth reading.

V.4. Predict the populations from a tilted source, and count atoms.

Solution. Step 1: prepare spins along $\hat{n}$ at $\theta = 60^\circ$ from the apparatus axis; by (71), $p_+ = \cos^2 30^\circ = \frac{3}{4}$, $p_- = \frac{1}{4}$. Step 2: in a run of 1000 atoms expect 750 ± 14 up and 250 ± 14 down (binomial, $\sigma = \sqrt{Np_+p_-} \approx 13.7$) — the Born weights are frequencies with shot noise, which is how every real Stern–Gerlach test reads them. Step 3: tie the two registers together: the *ratio* 3:1 is the geometry of the tilt (the appendix problem C.7.1 derives it from Gleason), while the *fluctuation* ± 14

is the statistics of finite sampling — theory fixes the odds, the bench draws the samples. Physics intuition: quantum probability walks and talks like any other probability once the odds are fixed — the mystery was never the statistics but the origin of the odds, and that is exactly the part the trace formula (and behind it, the fiber) supplies.

V.5. Show that repeatability needs no collapse.

Solution. Step 1: after the first magnet registers $z+$, the atom-apparatus record is the entangled-then-decohered state of (73); the boundary description conditioned on the record is $|\uparrow_z\rangle\langle\uparrow_z|$. Step 2: a second z -magnet then fires $z+$ with probability $\cos^2 0 = 1$ — repeatability — with no dynamical collapse invoked anywhere: the bulk configuration held one holonomy sector throughout, and the second instrument reads the same sector. Step 3: state the contrast carefully: the textbook derives the same number from the projection postulate; the framework derives it from sector persistence plus conditioning (the Lüders problem E.8.1 is the appendix-side twin) — identical laboratory content, different ontology, and the difference is testable only where the framework’s falsifiers say it is. Physics intuition: asking the same question twice gets the same answer because the answer was a property of the world, not of the asking — repeatability is the oldest hint that registration reads something rather than makes something, and the 1922 prototype already contained it.

Take-away. *Standard account:* prepare the silver atom, apply the collapse postulate, weight the two spots by the Born rule — operationally perfect, ontologically silent. *What it left open:* which outcome occurs, why those weights, and what happens during the jump. *The bulk reading:* one deterministic 6D trajectory per atom, outcome fixed by the unresolved phase-fiber coordinate, Born weights delivered by the GDOS registration — determinism upstairs, statistics downstairs, and the 1922 prototype becomes the framework’s cleanest single-qubit exhibit.

Part VI

Worked example II: Double-slit and decoherence

Wave–particle duality and the death of interference

What this Part teaches. The heart of the quantum mystery, dissolved in two readings: a point particle in space that is simultaneously a wave in *phase* — the particle and its dial. The chapter works the propagation of one quantum through both slits (the point goes through one; the field that interferes goes through both), the which-path environment and its decoherence factor, the three experimental regimes (photon, electron, fullerene) with their numbers, and the quantum–classical boundary read as a scale window rather than a mystery line. Feynman’s “only mystery” becomes, in the bulk reading, a bookkeeping fact about which fiber the observer can resolve.

Read alongside: [Feynman *et al.* \(1965\)](#) (vol. III, ch. 1) for the mystery stated canonically; [Davisson and Germer \(1927\)](#) for matter waves; [Hornberger *et al.* \(2003\)](#) and [Hackermüller *et al.* \(2004\)](#) for the fullerene decoherence experiments the chapter’s three-regime table quantifies.

The double-slit experiment is the cleanest demonstration of wave–particle duality, and the cleanest test of the cohort framework’s deterministic six-dimensional-bulk reading of quantum mechanics. In this reading, each photon propagates deterministically through the six-dimensional geometry; the interference pattern builds up one photon at a time because each photon’s landing position on the screen is fixed by its phase angle along the geometric direction S_θ^1 that the four-dimensional observer cannot read. The interference disappears only when a which-path environment destroys the bulk phase relationship between the two slit paths; the off-diagonal density-matrix element then decays at the rate of T-A8.5. We present the six-dimensional bulk reading first in §60, perform the standard textbook calculation in §63–§64 for direct comparison, and walk three regimes (photon, electron, fullerene) where the boundary between quantum and classical sits at the value of Δx for which the bulk–environment coupling becomes appreciable on the experimental timescale. The double-slit experiment with the decoherence sweep is shown in **Figure 8**.

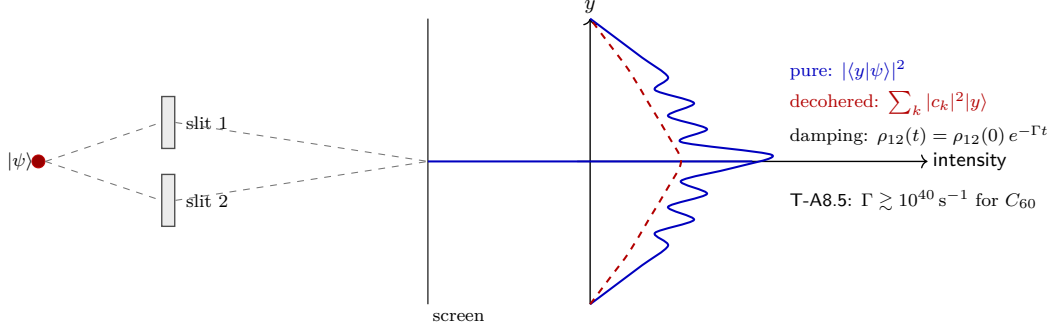


Figure 8: Double-slit interference and its decoherence-driven death. Left: source emits the quantum state $|\psi\rangle$, two slits define the two-path superposition, and the detector screen records the resulting intensity pattern. Right: the pure-state interference fringes (blue solid, oscillating) and the decohered classical sum (red dashed, single smooth peak) shown on the same axis. The transition between the two is governed by the off-diagonal density-matrix damping factor $\rho_{12}(t) = \rho_{12}(0) e^{-\Gamma t}$ of T-A8.5. The Joos–Zeh (Joos and Zeh, 1985) estimate for a C_{60} fullerene in a room-temperature thermal bath is $\Gamma \gtrsim 10^{40} \text{ s}^{-1}$; for any larger object the quantum-to-classical transition is operationally instantaneous.

59 The 1801 Young experiment and its quantum descendants

In 1801 Thomas Young demonstrated optical interference by passing a single beam of sunlight through two parallel slits and observing alternating bright and dark bands on a downstream screen — the classical confirmation that light propagates as a wave. The 1927 Davisson–Germer experiment (Davisson and Germer, 1927) repeated the same setup with electrons, observing the same fringe pattern and confirming the de Broglie hypothesis (de Broglie, 1924) that material particles also propagate as waves. By the 1990s the same experiment had been performed with neutrons, atoms, and small molecules; by 2003 with fullerene C_{60} at masses approaching a kilodalton (Arndt and others, 1999). In every case, the interference pattern survives as long as which-path information is unrecoverable from the environment, and dies when which-path information is in any way registered.

The framework reformulation reads the interference–which-path complementarity through T-A8.5: the interference visibility is the off-diagonal density-matrix element in the which-path basis; the environmental coupling damps it exponentially at a rate fixed by the physical-bath overlap integral (worked explicitly for C_{70} in §43.8), with the bulk geometry fixing the open-system form; the macroscopic-instantaneous regime $\Gamma \gtrsim 10^{40} \text{ s}^{-1}$ is the quantitative content of the quantum-to-classical transition.

60 The six-dimensional bulk reading: deterministic propagation of a single photon

Before turning to the standard textbook calculation, we pause to state the deeper picture that the cohort framework brings to the double-slit experiment. The standard treatment in the language Young and Born would have recognized follows in §63; placing the bulk reading first lets the reader see the two pictures side by side, on equal footing.

The photon in six dimensions. In GG-Theory, the photon is a single-quantum excitation of the bulk gauge field on the six-dimensional manifold $\mathcal{M}_4 \times I_r \times S_\theta^1$. Its propagation is governed by the bulk Lagrangian, which is deterministic and unitary. A single photon emitted from a source has, at every moment, a single-valued field configuration extending over all six geometric directions. There is no quantum uncertainty about its trajectory; there is no superposition of two coexisting alternatives; there is no collapse upon measurement. The bulk knows exactly where the photon is.

What the four-dimensional observer cannot register. The two dimensions beyond spacetime — the scale axis I_r and the phase axis S_θ^1 — are not accessible to a four-dimensional detector. No instrument the experimenter can build, regardless of its spatial resolution, can directly read the photon's position along I_r or its phase angle along S_θ^1 . This is not a limitation of technology; it is a structural feature of the projection from six geometric directions onto four.

The slits as a 4D-spatial obstacle. When the photon's bulk field configuration propagates from source toward the slit barrier, its transverse 4D-spatial extent naturally spans both slit positions. This is not a superposition of two probabilistic alternatives; it is a single-valued bulk field with an extended 4D transverse profile. The two slits do not split the photon into two copies of itself; they simply select the two portions of the same bulk configuration that get past the barrier. The portion passing through slit + and the portion passing through slit − are two parts of one bulk field, with definite relative geometric phase set by the bulk geometry and the photon's source phase along S_θ^1 .

The bulk-side calculation is six-dimensional wave optics. After the slits, the bulk field configuration propagates deterministically to the screen. At each 4D point on the screen, the bulk amplitude is the sum of the two propagated portions, with the relative phase determined entirely by the 6D geometry. No probability has yet entered the calculation; this is wave optics in six dimensions, every step of it deterministic and unitary.

What the detector registers. A 4D detector — a CCD pixel, a photographic grain, a single-photon avalanche diode — is a device localized in the four spacetime dimensions and unavoidably integrating over the two geometric directions I_r and S_θ^1 . When the bulk field encounters such a detector, the registration event is forced to land at a single 4D position. Which 4D position? The one fixed by the photon's specific phase angle along S_θ^1 at the moment of arrival, in combination with the 4D geometry of the screen. Because the 4D observer cannot read that phase angle, they cannot predict where any given photon will register. The photon lands somewhere; the experimenter records a single click; the bulk dynamics determined the landing position; the 4D observer simply cannot trace the determination back through the inaccessible phase coordinate.

Why the pattern builds up one photon at a time. Release a million photons through the same apparatus, one at a time — separated by milliseconds or by hours, as is routinely done in modern single-photon double-slit experiments (Jacques and others, 2007). Each photon has its own definite trajectory in six dimensions and a definite landing position on the screen. The phase angle along S_θ^1 varies from photon to photon, and the source emission process makes that distribution

uniform on the circle. As the photons accumulate, the histogram of landing positions fills in the standard interference pattern $|\psi(y)|^2 \propto \cos^2(\frac{1}{2}(k^+ - k^-)y)$ of (76). No single photon ever experiences “probability”; the fringe pattern is the cumulative geometric shadow on the four-dimensional screen of the uniform S_θ^1 phase distribution across the photon ensemble. This is why interference appears even when the photons arrive one at a time, separated by long intervals: each photon’s landing was always fixed by the bulk geometry; the pattern emerges from the ensemble statistics of the inaccessible phase coordinate.

The which-path question reframed. In the 6D picture, “which slit did the photon pass through” is a question about the 4D shadow of a single-valued 6D configuration whose 4D profile passes through both slits. The question becomes well-defined only when a specific 4D detector is inserted that registers which-slit information. Such a detector changes the geometric coupling between the photon and the apparatus, which destroys the relative phase coherence between the two 4D profile portions (treated quantitatively in §64). The 4D observer then sees the fringes disappear — not because the photon “chose” a path it had not chosen before, but because the inserted detector destroyed the bulk phase relationship that produced the fringes in the first place.

Bottom line of the bulk reading. The interference pattern is a deterministic 6D geometric effect. The apparent randomness in single-photon registrations is a consequence of the 4D detector being unable to read the photon’s S_θ^1 phase angle. The buildup of the fringe pattern over many photons is the projection of a uniform phase distribution onto the four-dimensional screen. Wave–particle duality is the cohabitation of these two readings: “wave” is the 6D geometry of bulk phase relations; “particle” is the 4D-localized event of detector registration. The mystery is not in the photon. It is in the projection.

61 Particle in space, wave in phase: the point particle and its dial

The reading just given treated the photon as a bulk *field* with a four-dimensional transverse profile. For a massive point particle the picture sharpens, and it is worth stating in full, because it dissolves wave–particle duality outright and answers the question every student asks: how can a thing that always arrives as a point behave like a wave?

The one move that dissolves the paradox. “Wave–particle duality” sounds like a contradiction only because both words are forced to describe behavior *in space*. In GG-Theory they describe two *different dimensions*. The particle is a *particle in the four-dimensional stage* $\mathcal{M}_4$ — always a point, always localized, never smeared (this is AX-1/locality: a point particle stays a point). It is a *wave in the phase dimension* S_θ^1 — the non-spatial sixth direction it carries at every point. Both hold at once, without contradiction, because being point-like *in space* and wave-like *in phase* are statements about orthogonal, non-spatial coordinates. The duality was never two faces of one thing in space; it is one face each of two different dimensions.

What “the wave” actually is. The wavefunction $\psi(x) = |\psi(x)| e^{i\varphi(x)}$ is not the particle spread out. It is the *phase field* — the value of the S_θ^1 dial — laid over the possible point-positions x . The phase φ is the particle’s location on the gauge circle. As the point moves, the dial spins; the rate of spin per unit length is the momentum, $p/\hbar$, and that rate is the de Broglie wavelength $\lambda = h/p$. So λ is not a size of the particle — it is the spatial period of the spinning phase. Nothing material is waving; a *dial* is winding.

Feynman’s clock, with an address. Feynman’s *QED* (Feynman, 1985) tells the reader to picture a tiny spinning arrow carried by the particle and to obtain probabilities by adding arrows and squaring — without ever saying *where* the arrow lives. GG-Theory supplies the address: the arrow is the sixth dimension. The clock hand is the position on S_θ^1 , and the holonomy $\oint A_\theta d\theta$ it accumulates is the arrow’s rotation. Feynman’s bookkeeping device becomes a geometric object.

The double slit, step by step.

1. A point particle leaves the source — one point, the whole way. In the deterministic six-dimensional bulk it follows a definite history; what is hidden is its θ -dial and which history it took, because those live in the non-spatial fiber the boundary cannot read.
2. To any one screen point P there are two bulk histories that arrive — via slit + and via slit −. They traverse slightly different path lengths, so their dials arrive pointing in different directions: a phase difference $\Delta\varphi = (2\pi/\lambda) \delta(\text{path})$.
3. The boundary observer registers by the OGGE projection π , which traces out the non-spatial fiber — so it cannot read which slit, nor the absolute dial angle. It is *forced* to add the two arrows and keep only the squared magnitude,

$$P(P) = |A_+ + A_-|^2 = |A_+|^2 + |A_-|^2 + 2|A_+||A_-|\cos\Delta\varphi. \quad (74)$$

4. That $\cos\Delta\varphi$ term is the fringes. Sweeping across the screen, the path difference cycles, the two arrows rotate from aligned (bright) to opposite (dark), and the interference pattern of (76) appears.

Each particle still lands as a single dot — one point, one registration. The *pattern* of dots is the statistical shadow the spinning dial casts when the boundary can see only $|\text{arrow}|^2$. The “wave-like particle” is impossible to visualize because it is not what happens: a point lands; the distribution of points remembers the phase.

Why measuring which-path kills the fringes. Interference needs the two histories traced *together*, with their relative dial angle intact. A which-path detector *reads a piece of the fiber* — it promotes the slit-choice to a registered boundary record. Once that record exists, the relative phase is randomized (the decoherence of T-A8.5: loss of θ -coherence under coarse-graining along I_r), the $\cos\Delta\varphi$ term averages to zero, and the pattern collapses to $|A_+|^2 + |A_-|^2$ — two plain bumps. Complementarity is exactly this: *interference is the phase fiber left unread; which-path is the phase fiber read*. One cannot both trace a coordinate and register it.

What is and is not claimed. In the three-tier discipline of this cohort: (i) *equivalent to standard quantum mechanics* — none of this changes a single double-slit prediction; the fringe spacing, the visibility, and the which-path trade-off come out identical (the boundary theory *is* ordinary QM, §30). What is new is the picture and the *reason*, not the numbers. (ii) *Defensible structural proposal* — the identification “quantum phase = holonomy of the sixth dimension,” and the claim that the sum over paths is *forced* by the OGGEF trace rather than postulated. (iii) *Interpretive commitment, flagged as such* — the deterministic-bulk reading says the particle took a definite history; this stays consistent only because “which history” has no boundary-registrable answer without a measurement that alters the experiment, and because the apparent non-locality of the entangled case (Bell/CHSH) is read as a projection artifact of the six-dimensional bulk (T-A8.7), not as bulk non-locality. That last point is the framework’s position, not a settled result.

The one-line picture. The particle is a point that carries a spinning dial in a hidden sixth dimension; it travels as a point and lands as a dot. The “wave” is the dial; the fringes are the map of the dial’s relative angle, drawn statistically because the screen can see only the dial squared. Particle in space, wave in phase — two dimensions, no paradox.

62 How one point “passes through both slits”: the field that interferes is the phase

A reader who has accepted the dial picture of §61 will still feel the sharpest form of the double-slit puzzle, and it deserves a full answer rather than a slogan. A point particle, being a point, must pass through the left slit *or* the right slit; yet the interference pattern seems to require that it pass through *both*. If the object were a wave one could picture it dividing between the slits; if it is a point one cannot. The resolution is not a play on words but a relocation of the question, and it follows directly from the two-fiber structure of §30.

The premise to refuse. The puzzle is manufactured by one tacit assumption: that “particle” and “wave” are two competing descriptions of *the same thing* — the object’s location in the space $\mathcal{M}_4$ — so that the object must be either at one slit or smeared across both. In the present framework they are not competing descriptions of a single degree of freedom; they are faithful descriptions of *two different degrees of freedom that live in different parts of the bulk*. The particle-aspect is a point in the spatial stage $\mathcal{M}_4$. The wave-aspect is the winding of the phase dial on the non-spatial fiber S_θ^1 . Forcing both onto the single axis of spatial position is precisely what generates the contradiction; releasing that assumption dissolves it.

Two questions that ordinary language fuses into one. Pull the single question “where did the particle go?” apart into the two questions the geometry actually distinguishes. “*Which slit?*” is a question about position — a point in $\mathcal{M}_4$. It has a determinate answer in the only sense that is operational: when the particle is finally detected, it is detected at one place, as one dot. “*How does it interfere?*” is a question about phase — the value of the S_θ^1 dial, φ . And the phase is not

a location in space at all; it is a coordinate on the hidden gauge circle. The two questions have different homes, and the answer to one does not constrain the answer to the other.

What threads both slits is a field — and a field at both slits is no paradox. The wavefunction $\psi(x) = |\psi(x)|e^{i\varphi(x)}$ is, in this reading, the dial-value φ *laid over every available point-position* x : a section of the phase fiber across the whole apparatus, defined along the route that passes the left slit and along the route that passes the right slit alike. It is this *phase field* that “passes through both slits.” But a field defined over an extended region is not in the least mysterious: the electromagnetic field of an ordinary light beam is also present at both slits at once, and no one is tempted to say a photon’s field is “in two places.” The framework’s only claim is that the quantum phase is a field of exactly this kind — the winding of a real, if non-spatial, dial, laid over the point’s possible locations. Nothing material is ever in two places. What is present at both slits is the phase section; what is present at the screen, at the end, is a single point.

Where the fringes come from: a phase ledger, not a map of a smeared object. The interference pattern is the record of how the dial, having wound along the two routes, *recombines* at each point of the screen. The route past the far slit is slightly longer, so along it the dial completes a slightly different number of turns — this is exactly the de Broglie content of §61, that $\lambda = h/p$ is the spatial period at which the phase winds. At a screen point where the two windings arrive back *in step*, the dial-values add and points accumulate: a bright fringe. At a screen point where they arrive back *exactly opposite*, the dial-values cancel, the combined amplitude is zero, and *no point particle ever lands there*: a dark fringe. Equation (76) is precisely this ledger — the $\cos((k_y^+ - k_y^-)y)$ term is the relative dial-angle $\Delta\varphi$ of the two routes as a function of screen position y . The fringes are an accounting of relative phase, not a photograph of where a divided object was.

Why the point never has to “decide.” “Which slit did the point pass through?” demands a *spatial* fact. But between source and screen the quantity that propagates and carries the structure is the phase field over both routes; the spatial position is made determinate only at detection. The point therefore never splits and never chooses, because the dial — the thing that “went through both” — was never a single place in space to begin with. There is no suppressed fact “the particle was really on the left” that interference would have to contradict: absent a position read-out, “which slit” has no boundary-registrable answer, not because the point is smeared in space but because what is physical in the interval is the non-spatial phase section. This is the deterministic-bulk reading of §61 stated with care: a definite bulk history exists, but “which history” is not a question the boundary can pose without altering the experiment.

Why looking destroys the effect: which-path is a scale-fiber read-out. The picture earns its keep by explaining, mechanically, why any attempt to see which slit the point took abolishes the fringes. To read which-path is to read the point’s *position* at the slit. In the two-fiber language of §30, reading a position is a *scale-fiber* operation: it couples the point to an environment and coarse-grains along the radial direction I_r , irreversibly. But coarse-graining along I_r is exactly what destroys coherence on the *phase* fiber S_θ^1 (the decoherence content of T-A8.5; the rate is computed in

§64). With the phase coherence gone, the two dial-windings can no longer be recombined, the $\cos \Delta\varphi$ term averages to zero, and the screen shows two ordinary piles, one behind each slit. Interference and which-path are therefore not two answers competing over one object’s whereabouts; they are the *coherent* (phase-fiber) face and the *read-out* (scale-fiber) face of the two non-spatial dimensions, and a single experiment cannot hold both sharp at once. Complementarity, in this framework, is not an extra postulate but the statement that one cannot simultaneously leave a coordinate unread (in order to interfere) and register it (in order to know the path).

The honest boundary: what this dissolves and what it does not. Two claims must be kept apart, in the three-tier discipline of the cohort. What the phase-field picture *dissolves*, completely, is the reader’s puzzle — “how can a *point* interfere with itself?” — the answer being that the thing which interferes was never the point but the phase section laid over its possible positions, and that section is a field, present at both slits as any field is. What the picture does *not* settle by geometry alone is why each run yields exactly *one* dot rather than a faint continuous smear of the predicted intensity: that single-outcome selection is *delivered* by the irreversible GRIP descent on the scale fiber (GDOS/GG-A6-GRIP) rather than by the phase geometry; what remains open is only the GG typicality target — that the GRIP basin-capture measure of the deterministic descent coincides with the Born measure, the six-dimensional analogue of Bohmian quantum equilibrium (status of record: closed at the relaxation level, one analytic residue remaining; §118) (§30.5). The geometry tells you that there is interference, and where the fringes fall; the dynamics tells you that you always see one particle. These are different statements, and the framework asserts only the first as forced; the second is the open dynamical content treated under T-A8.5 and in (Arizaka, 2026, A5-GDOS). None of this alters a single numerical prediction of textbook quantum mechanics — fringe spacing, visibility, and the which-path trade-off come out identical (Eqs. (75)–(76)). What is new is the picture and the reason, not the numbers.

Synthesis. Particle in space, wave in phase. At the moment of detection the point occupies one slit’s worth of space; in the interval between source and screen it occupies no slit at all in the sense the question intends — while the phase dial winds through both routes, always. The bright and dark bands are the ledger of that double winding, read out statistically because the boundary can see only the squared dial. Put this way, there is nothing left in the double slit that must be pictured as a single point in two places, because in this framework nothing ever is.

63 Setup: the wavefunction at the screen

Consider a particle of mass m prepared in an incoming plane wave with momentum p , incident normally on a screen pierced by two slits at positions $y = \pm d/2$ in the screen plane. Diffraction at each slit produces two outgoing spherical-wave components. At a screen-position y on the downstream detector, distance L from the slits, the standard textbook analysis (Feynman *et al.*,

1965) gives the wavefunction

$$\psi(y) = \frac{1}{\sqrt{2}}(\psi_+(y) + \psi_-(y)), \quad \psi_{\pm}(y) = A e^{ik_y^{\pm} y}, \quad (75)$$

with $k_y^{\pm}$ the transverse wavevector picked up by passage through slit $\pm$. The intensity (single-particle probability density) is

$$|\psi(y)|^2 = |A|^2 [1 + \cos((k_y^+ - k_y^-) y)] = 2|A|^2 \cos^2\left(\frac{1}{2}(k_y^+ - k_y^-) y\right), \quad (76)$$

the textbook interference fringe pattern with fringe spacing $\Delta y = 2\pi/|k_y^+ - k_y^-| = \lambda L/d$ for de Broglie wavelength $\lambda = 2\pi/p$.

64 The which-path environment and the decoherence factor

Now couple the particle to an environment that can register which slit it passed through. The relevant entangled state at the screen is

$$|\Psi\rangle_{\text{particle} \otimes \text{env}} = \frac{1}{\sqrt{2}} (\psi_+ \otimes |E_+\rangle + \psi_- \otimes |E_-\rangle), \quad (77)$$

where $|E_{\pm}\rangle$ are the environmental states corresponding to the particle's passage through slit $\pm$. The boundary density operator for the particle alone, obtained by tracing over the environment, is

$$\rho_{\text{particle}}(y, y') = \frac{1}{2} [\psi_+(y)\psi_+^*(y') + \psi_-(y)\psi_-^*(y') + \mathcal{D}(\psi_+(y)\psi_-^*(y') + \psi_-(y)\psi_+^*(y'))], \quad (78)$$

with the *decoherence factor* $\mathcal{D} = \langle E_+ | E_- \rangle$. Two limits matter:

- $\mathcal{D} = 1$ (environments fully indistinguishable): interference fringe pattern preserved.
- $\mathcal{D} = 0$ (environments orthogonal): off-diagonal terms vanish, leaving the classical incoherent sum $|\psi_+|^2 + |\psi_-|^2$. No fringes.

The interference visibility $V = |\mathcal{D}|$ on the screen interpolates continuously between the two limits as a function of how strongly the environment couples to the which-path information.

By T-A8.5, the decoherence factor decays exponentially in time when the environment is a thermal bath: $|\mathcal{D}(t)| = \exp(-\Gamma_{+-} t)$, with the rate

$$\Gamma_{+-} = \sum_{\gamma} \gamma_{\gamma} |\langle + | L_{\gamma} | + \rangle - \langle - | L_{\gamma} | - \rangle|^2, \quad (79)$$

where the bulk GKLS rates γ_{γ} are themselves fixed by scale-overlap integrals (T-A8.5.2) of bulk KK modes onto the particle's zero mode.

65 Three regimes: photon, electron, fullerene

We compute Γ_{+-} in three regimes that demonstrate the macroscopic–mesoscopic–microscopic boundary.

Photon in a Mach–Zehnder interferometer (microscopic). A single photon at $\lambda = 633$ nm in a high-vacuum Mach–Zehnder interferometer with arm length $L \sim 1$ m and ambient gas density $n \sim 10^{10}$ molecules/m³ has a scattering rate per molecule $\sim 10^{-13}$ s⁻¹, giving total $\Gamma \sim 10^{-3}$ s⁻¹ and decoherence time $\tau_{\text{dec}} \sim 10^3$ s. Interference is preserved for the full transit; high-visibility fringes are observed routinely in laboratory single-photon interferometry.

Fullerene C_{60} at room temperature (mesoscopic). Arndt et al. (Arndt and others, 1999) performed Talbot–Lau interferometry with fullerene molecules at $T \sim 900$ K and ambient pressure varied by six orders of magnitude. Their measured decoherence rate scales with both the gas-collision rate and the absorbed-photon rate, in quantitative agreement with the Joos–Zeh prediction $\Gamma \sim n \sigma v_{\text{rel}} (\Delta x)^2 / \lambda_{\text{dB}}^2$. At 10^{-9} mbar, decoherence times are in the millisecond range; at 10^{-6} mbar, the interference fringes disappear within microseconds.

Schrödinger’s cat in a macroscopic position superposition (macroscopic). A cat-mass object ($m \sim 1$ kg) in a position superposition $\Delta x \sim 0.1$ m at room temperature has decoherence rate (Joos and Zeh, 1985)

$$\Gamma_{\text{cat}} \sim n_{\text{air}} \sigma_{\text{N}_2} v_{\text{therm}} (\Delta x)^2 / \lambda_{\text{thermal}}^2 \sim 10^{40} \text{ s}^{-1}, \quad (80)$$

giving decoherence time $\tau_{\text{dec}} \sim 10^{-40}$ s. No apparatus can resolve a time scale forty orders of magnitude shorter than the Planck time. The interference pattern has no chance of being observed; the cat appears classical.

66 The boundary between quantum and classical

What T-A8.5 and the cohort framework together provide is the structural identification of where the quantum-to-classical boundary lies in any given experiment: it lies at the value of Δx (the spatial separation between the two interference paths) for which $\Gamma_{+-} \tau_{\text{exp}} \sim 1$, where τ_{exp} is the experimental flight time. Substituting $\Gamma \propto (\Delta x)^2$ from (80), the critical separation scales as

$$\Delta x_{\text{crit}} \sim \frac{1}{\sqrt{n \sigma v \tau_{\text{exp}} / \lambda_{\text{dB}}}}. \quad (81)$$

For a typical laboratory environment ($n \sim 10^{20}$ molecules/cm³, $\sigma \sim 10^{-15}$ cm², $v \sim 5 \times 10^4$ cm/s, $\tau_{\text{exp}} \sim 1$ s, $\lambda_{\text{dB}} \sim 10^{-10}$ cm), one gets $\Delta x_{\text{crit}} \sim 10^{-5}$ m, the micron scale — exactly where the quantum-to-classical transition is observed empirically.

This is the substantive content of T-A8.5 for laboratory practice: the boundary is not where the experimenter chooses to draw it, but where the bulk-environment coupling integrates to $\Gamma \tau \sim 1$. In

the cohort framework, that integral is a calculation from the 6D geometry, not a free parameter.

Part-end close

The double-slit experiment has been performed in every laboratory that takes quantum mechanics seriously, with every kind of particle from photons to fullerenes. Every time the interference pattern is preserved, the experimenter has succeeded in shielding the particle from a which-path-resolving environment; every time the pattern is lost, the environment has won. Where the boundary lies in any specific experiment is fixed by the scale-overlap integrals of the bulk geometry — not by the experimenter’s choice of Heisenberg cut, not by a phenomenological fit, but by a calculation that the cohort framework makes inevitable.

After all, the question Young could not have answered in 1801 — “why do the fringes appear, and what makes them sometimes disappear” — now has an answer one level deeper than the wave function. The fringes appear because a single photon propagates deterministically through the six-dimensional geometry, its bulk field passing through both slits as a single-valued configuration with a definite relative phase. They build up one photon at a time because each photon’s landing on the screen is fixed by its own S_θ^1 phase angle, which the four-dimensional observer cannot read but which is uniformly distributed across the photon ensemble. They disappear when an environmental coupling destroys the bulk phase relation between the two paths, at a rate the bulk geometry fixes without any tuning. The wave aspect lives in the deeper geometry; the particle aspect lives in the four-dimensional detector; neither is fundamental, and neither contradicts the other. In the runtime vocabulary of Section 51, the division is exact: the interference is OGGEF-side structure — the projection’s phase, present before any descent — while each landing is a GRIP-In capture, the smallest measurement event in physics, halting at one element of the screen’s discrete attractor lattice. The fringes are the wave’s bookkeeping; the grains that build them, one photon at a time, are GRIP’s.

Problems and worked solutions. VI.1. Compute a real fringe spacing for electrons.

Solution. Step 1: electrons at 600 eV have de Broglie wavelength $\lambda = 1.226 \text{ nm}/\sqrt{V} = 0.050 \text{ nm}$. Step 2: with slit separation $d = 1 \text{ }\mu\text{m}$ and screen distance $L = 1 \text{ m}$, the fringe spacing of the intensity pattern (76) is $\Delta y = \lambda L/d = 50 \text{ }\mu\text{m}$ — comfortably resolvable, which is why the Jönsson-class experiments could photograph it. Step 3: scale awareness: the pattern’s existence needs λ only *comparable to* the apparatus coherence scales, not to d itself — the wavelength is 10^4 times smaller than the slit separation and the fringes survive; what kills them is never geometry but *records*, as the next problems quantify. Physics intuition: a fringe is a ruler graduated in wavelengths, magnified by L/d — the double slit is an interferometric microscope that blows a picometer-scale phase difference up to the width of a human hair.

VI.2. Relate which-path knowledge to fringe visibility.

Solution. Step 1: couple the particle to any environment that becomes path-dependent, $|L\rangle|E_L\rangle + |R\rangle|E_R\rangle$ (77); tracing the environment leaves the interference term multiplied by $|\langle E_L|E_R\rangle|$ (78). Step 2: the two limits: $\langle E_L|E_R\rangle = 1$ (no record) gives full fringes; orthogonal records give none —

and one scattered photon of wavelength shorter than the slit separation is already nearly orthogonal: a single quantum of which-path light erases the pattern. Step 3: the quantitative bridge is the chapter’s decoherence-rate formula (79): visibility decays exponentially in the number of path-tagging events, so the quantum-classical transition is a *rate* question, not a size question. Physics intuition: nature runs a strict information economy — fringes are paid for in ignorance, and every bit of path information anywhere in the universe, read or unread, debits the visibility account by exactly the Cauchy–Schwarz overlap.

VI.3. Evaluate the fullerene wavelength, and respect what it means.

Solution. Step 1: a C_{70} molecule ($m = 840$ u) at $v = 200$ m/s has $\lambda = h/mv = 2.4$ pm — five orders of magnitude smaller than the molecule itself. Step 2: interference was nevertheless observed (the Vienna experiments): a thousand-atom object, hotter than room temperature, wrote fringes determined by a wavelength far smaller than its own bonds — quantum mechanics does not switch off with complexity. Step 3: read the regime table of this Part: photon, electron, fullerene differ not in the *rules* but in how easily the environment acquires their path — mass buys shorter wavelengths *and* stronger coupling to everything, which is why the fullerene is the frontier and the cat (next Part) is beyond it by forty orders of magnitude. Physics intuition: the de Broglie wavelength of a flying molecule is smaller than its atoms — and yet it interferes; the boundary of the quantum world is drawn by decoherence bookkeeping, never by size, weight, or dignity.

VI.4. Explain where the fullerene fringes die.

Solution. Step 1: heat the molecule and it becomes its own which-path lamp: internal thermal photons carry position information to the field, and each emission multiplies the visibility by an overlap factor < 1 (VI.2’s mechanism, source internalized). Step 2: the 2004 experiment swept the internal temperature and watched visibility fall continuously from ~ 1500 K and vanish by ~ 3000 K — decoherence observed as a dial, not a postulate, exactly as the rate law (79) predicts. Step 3: the framework’s addition ((81)): the same bookkeeping descends from the bulk GKLS rates, so the measured visibility curve is, on this reading, a scale-overlap measurement — one of the places where the eight-test horizon of the Discussions touches an existing dataset. Physics intuition: a hot molecule gossips about itself — every infrared photon it sheds is a small confession of where it is, and the fringes fade in proportion to the gossip; the quantum-classical boundary is not a wall but a signal-to-noise ratio.

VI.5. State the point-and-dial reading, and say what it does and does not change.

Solution. Step 1: in the bulk reading the particle is a point in space carrying a dial in phase: the point traverses *one* slit; the dial’s field, living on the fiber, traverses both and interferes (74). Step 2: verify operational equivalence: every observable statistic — fringe positions, visibilities, which-path trade-offs — is computed from the same wavefunction arithmetic as the textbook’s; the reading changes the *referent* (what the math is about), not the numbers. Step 3: then say where it is not idle: the reading removes the logical need for wave-particle “duality” as a primitive (one object, two aspects, two strata), and its falsifiable content lives in the framework’s global horizon (the kill criteria of the Discussions), not in re-fitting fringes — the honesty register of the course keeps those books separate. Physics intuition: “both slits” was always a statement about the *phase*, never about the *point* — splitting the particle into position and dial dissolves Feynman’s mystery into geography:

the two behaviors were two different residents of the particle all along.

Take-away. *Standard account:* single quanta build an interference pattern; observe the path and the pattern dies; “nobody knows how it can be like that.” *What it left open:* how one indivisible particle samples two slits, and where exactly the classical world begins. *The bulk reading:* the point particle passes through one slit while its phase field — a physical object on the fiber — passes through both; which-path coupling decoheres the fiber and kills the fringes on schedule (photon, electron, fullerene: the same formula, three regimes); the quantum–classical boundary is a computable scale window, not a mystery line.

Part VII

Worked example III: EPR, Bell, and CHSH

The 2022-Nobel-recognized test of Bell's theorem

What this Part teaches. The empirical heart of the framework, and of the 2022 Nobel Prize: entanglement, EPR, Bell, and CHSH worked as one chain. The chapter builds the joint bulk state of the pair across both wings, evaluates the singlet correlation and the CHSH expression, reads the Tsirelson saturation $2\sqrt{2}$ as an extremal bound of the projection (T-A8.8), verifies no-signaling (T-A8.11), and closes by reconciling the four historical readings — Einstein's locality, Bohr's completeness, Bell's exclusion, and the loophole-free experiments — in the one sentence the century could not write: local and deterministic in 6D, non-local and irreducibly statistical on the 4D boundary, with no contradiction because the two statements live at different strata of the same geometry.

Read alongside: [Einstein *et al.* \(1935\)](#) and [Bell \(1964\)](#) for the question and the answer; [Aspect *et al.* \(1982\)](#) for the decisive experiments; [Royal Swedish Academy of Sciences \(2022\)](#) for what the prize certified.

This worked example is the empirical heart of the framework's reading of quantum mechanics. The CHSH bound $|S| \leq 2$ on local hidden-variable theories in 4D has been violated experimentally to many standard deviations, with the measured value at or near the Tsirelson saturation $|S| = 2\sqrt{2}$ ([Aspect *et al.*, 1982](#); [Giustina *et al.*, 2015](#); [Hensen, 2015](#); [Royal Swedish Academy of Sciences, 2022](#); [Shalm and others, 2015](#)). The cohort framework reads the violation as the most direct empirical signature of the deterministic six-dimensional bulk: each entangled pair is a single bulk field configuration with two 4D-spatially-separated branches that share a single S_θ^1 holonomy reference; the two 4D-spacelike-separated detector events are projections of that one non-separable joint state along the radial direction I_r (which, by Theorem 0 of GG-A2-GG6, carries no light cone), not endpoints of any worldline between them; the Tsirelson saturation is the extremal (Bogomolnyi-analogue) ceiling on the CHSH functional at the singlet. We present the six-dimensional bulk reading first in §68, perform the explicit CHSH calculation on the singlet state in §69, recover the Tsirelson saturation from the joint-state picture (T-A8.7) in §70, promote it to the topological bound (T-A8.8) in §71, derive no-signaling (T-A8.11) in §72, and reconcile with the four standard interpretations in §73.

The EPR/Bell/CHSH worldline picture is shown in **Figure 9**.

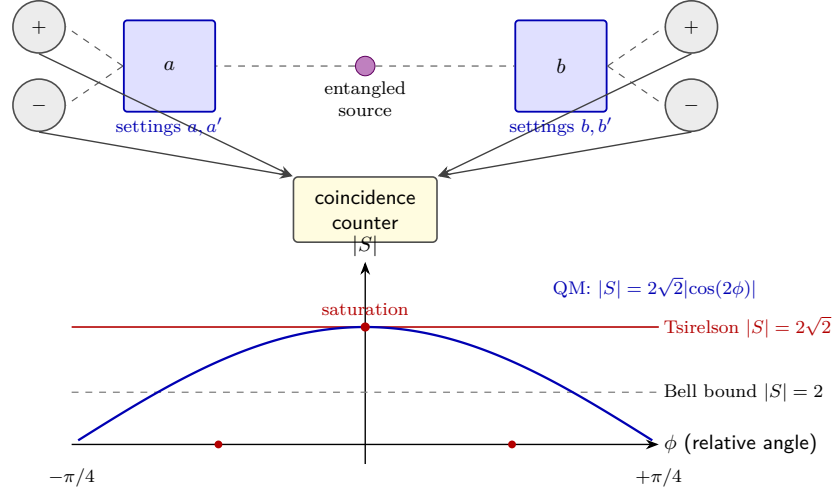


Figure 9: The CHSH version of Bell’s theorem and its cohort reading. Top: the entangled-pair coincidence-counting setup (Aspect *et al.*, 1982; Hensen, 2015), with source S , analyzers a/a' on the left and b/b' on the right, four single-photon detectors, and a coincidence counter recording the joint outcomes. Bottom: the CHSH bound $|S| \leq 2$ holds in any local hidden-variable theory (dashed line); standard QM predicts $|S| = 2\sqrt{2}|\cos(2\phi)|$ (blue curve) where ϕ is the relative analyzer angle; the maximum reaches the Tsirelson bound $|S| = 2\sqrt{2}$ (red horizontal) at $\phi = 0$. The 2022 Nobel-recognized experiments confirm the saturation; T-A8.8 reads the saturation as an extremal (Bogomolnyi-analogue) ceiling on the CHSH functional that the boundary CHSH measurement shadows.

67 The 1935 EPR question and the 1964 Bell answer

In May 1935 Einstein, Podolsky, and Rosen (Einstein *et al.*, 1935) argued that the quantum-mechanical description of an entangled bipartite system could not be a complete description of physical reality. Their argument used the spin-singlet state of two spin- $\frac{1}{2}$ particles separated to spacelike-disconnected regions, observing that a measurement of σ_z on the first particle fixes the second’s σ_z value instantaneously, while a measurement of σ_x on the first fixes the second’s σ_x value instantaneously — both “elements of reality” for the second particle, but mutually incompatible by the uncertainty principle. Bohr’s 1935 response (Bohr, 1935) insisted on the inseparability of the joint quantum state; the dispute was philosophical, with no operational test in sight for two decades.

Bell’s November 1964 paper (Bell, 1964) converted the EPR question into an experimental test. He proved that any local hidden-variable completion of quantum mechanics — any theory in which each particle carries pre-existing values for all its observables, with the values fixed locally at the source and unaffected by measurement settings at the other detector — must satisfy an inequality on certain linear combinations of spin-correlation expectation values. Quantum mechanics, by contrast, predicts violations of the inequality. The Clauser–Horne–Shimony–Holt (CHSH) reformulation of 1969 (Clauser *et al.*, 1969) packaged the bound into a four-setting quantity S with classical bound $|S| \leq 2$.

Freedman–Clauser 1972 (Freedman and Clauser, 1972) demonstrated the first experimental violation. Aspect 1981–1982 (Aspect *et al.*, 1981, 1982,?,?) pushed the violation past five standard deviations with delay-time-randomized detector settings. The Delft/Vienna/NIST loophole-free runs

of 2015 (Giustina *et al.*, 2015; Hensen, 2015; Shalm and others, 2015) closed the remaining locality and detection-efficiency loopholes. The 2022 Nobel Prize (Royal Swedish Academy of Sciences, 2022) acknowledged the result formally. All these experiments measure values of $|S|$ at or near the Tsirelson bound $2\sqrt{2} \approx 2.828$ (Tsirelson, 1980).

The framework reading agrees with all of this experimentally and adds the bulk-geometric layer: the boundary correlation function is the operator-algebra expectation in one non-separable joint bulk state (T-A8.7), and the Tsirelson saturation is the extremal (Bogomolnyi-analogue) ceiling on the CHSH functional (T-A8.8).

68 The six-dimensional bulk reading: a single entangled configuration across spacelike-separated 4D detectors

Before turning to the standard textbook calculation, we pause to state the deeper picture that the cohort framework brings to the EPR/Bell/CHSH experiment. The standard treatment in the language EPR, Bell, and CHSH would have recognized follows in §69–§72; placing the bulk reading first lets the reader see the deeper 6D-locality structure before the formal 4D calculation.

The entangled pair in six dimensions. A singlet pair — two spin- $\frac{1}{2}$ particles, or two polarization-entangled photons from parametric down-conversion — is, in GG-Theory, a single bulk field configuration with two 4D-spatially separated branches. The two particles emerged from one bulk event (the parametric crystal, the atomic cascade, the BBO down-conversion vertex), and their bulk-side state inherits a single shared Wilson-line holonomy reference around the topological circle S_θ^1 . There is no superposition of “Alice’s particle is up and Bob’s is down” or vice versa in the bulk; there is a single 6D bulk configuration whose 4D projection has correlated spin/polarization values at the two detectors.

The shared S_θ^1 holonomy reference. The defining feature of an entangled pair in the bulk picture is that the two particles, as one non-separable bulk state, share a single S_θ^1 holonomy phase Θ_0 that was set at the source event:

$$\Theta_0^{(A)} = \Theta_0^{(B)} = \Theta_0 \quad (\text{fixed at source emission}). \quad (82)$$

Even though the two particles separate 4D-spatially and travel to spacelike-separated detectors, their bulk-side holonomy phases remain locked to the single reference Θ_0 . This is not “spooky action at a distance”; it is the structural fact that both particles are shadows of a single non-separable bulk state, sharing the one S_θ^1 holonomy class fixed at the source — a common origin, not an influence relayed between the wings.

What the 4D observer cannot register. The 4D observer can read neither Θ_0 nor the non-separable bulk state that underlies the correlation. Each pair has its own random Θ_0 drawn uniformly from S_θ^1 across the ensemble of pairs; the 4D observer registers only the 4D outcomes at

Alice’s site A and Bob’s site B , which are 4D-spatial projections of the bulk state with the bulk coordinates r and θ averaged out.

The two analyzers as 4D-spatial obstacles. Alice’s polarizing analyzer (oriented along direction $\hat{a}$) and Bob’s analyzer (along $\hat{b}$) are 4D-spatial filters that couple to the particle’s 4D polarization. When the bulk field configuration encounters Alice’s analyzer, the 4D projection registers ± 1 according to the particle’s 4D spin component along $\hat{a}$; the same happens at Bob’s site for direction $\hat{b}$. The two analyzers are 4D-spatially separated; in 4D they are causally disconnected, and any 4D-local hidden-variable theory predicts $|S| \leq 2$ for the CHSH quantity.

The pair is a single non-separable bulk state. The crucial step: in 6D the two particles are not two independently prepared systems but one non-separable state of the bulk, sharing the phase fiber S_θ^1 fixed at the source. Using the 6D metric of Arisaka (2026, A2-GG6)

$$ds_6^2 = e^{-2r} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + d\theta^2, \quad (83)$$

the radial direction I_r enters with $+dr^2$: by Theorem 0 of Arisaka (2026, A2-GG6) it is the non-compact renormalization-scale fiber and carries *no* light cone, so a 4D-spacelike separation stays spacelike in the bulk for every Δr — there is no radial “dip” that connects the detector events causally, and none is needed. What links the two shadows is not a worldline between them but the single joint state they are projections of: the boundary correlation is the operator-algebra expectation in that one state (T-A8.1, T-A8.3), and I_r is the direction *along which* it projects to the boundary, not a channel *through* which anything travels. The Bell–Aspect “non-locality” at 4D is therefore the shadow of one bulk object seen as two, through a non-invertible bulk-to-boundary projection — 4D-non-local, but with no signal and no superluminal influence.

What the coincidence counter registers. For each pair, the 4D coincidence counter records the joint outcome $(s_A, s_B) \in \{\pm 1\}^2$ at the two analyzer settings. The bulk-side calculation of the joint expectation, using the boundary representation of T-A8.7, gives the singlet correlation

$$C(\hat{a}, \hat{b}) = \langle \Psi^- | (\hat{a} \cdot \vec{\sigma}_A) (\hat{b} \cdot \vec{\sigma}_B) | \Psi^- \rangle = -\hat{a} \cdot \hat{b} = -\cos(\theta_a - \theta_b), \quad (84)$$

which is the standard operator-algebra expectation in the non-separable joint state (T-A8.1, T-A8.3), with both shadows referred to the single shared holonomy Θ_0 fixed at the source. The two outcomes are not independent; they are 4D shadows of one 6D bulk state, with their joint statistics fixed by that state.

Why the CHSH bound is violated even pair-by-pair. For any individual pair, the bulk dynamics fix the 4D outcomes at both detectors deterministically (once the random Θ_0 was drawn from the source). The CHSH statistic

$$S = C(\hat{a}, \hat{b}) + C(\hat{a}, \hat{b}') + C(\hat{a}', \hat{b}) - C(\hat{a}', \hat{b}'), \quad (85)$$

aggregated over many pairs, takes the value $2\sqrt{2}$ at the optimal angle choices, saturating the Tsirelson bound. This is not a statistical correlation built up from probabilistic individual events; each pair has a definite 4D joint outcome fixed by the shared bulk holonomy Θ_0 . The aggregate CHSH statistic exceeds the Bell bound $|S| \leq 2$ because the individual pair correlations $C(\hat{a}, \hat{b}) = -\cos(\theta_a - \theta_b)$ are deeper than any 4D-local hidden-variable theory permits — precisely because the pair is one non-separable bulk state, 4D-non-local in its shadow while carrying no signal.

Extremal (Bogomolnyi-analogue) saturation of the Tsirelson bound. The empirical fact that experiments saturate the Tsirelson bound to within measurement error is an *extremal* property of the singlet (T-A8.8): it is the maximizer of the CHSH functional, so the first variation of $|S|$ vanishes there and a small change in the preparation (entanglement strength, depolarization) moves $|S|$ only at second order. Saturation is therefore structurally stable — in analogy with (though not literally) a Bogomolnyi/BPS bound, where a configuration at a saturating bound is stable to first order (Bogomolny, 1976); no topological charge is claimed. Every singlet preparation in any laboratory, regardless of source brightness or detector efficiency, sits at this same extremum — and every loophole-free CHSH measurement on a singlet attains the same $|S| = 2\sqrt{2}$.

Bottom line of the bulk reading. The CHSH violation is the deterministic shadow of one non-separable 6D state. That single joint state carries the singlet correlation (84); the Tsirelson saturation is the extremal first-order-stationary ceiling on quantum correlations (T-A8.8); the apparent 4D non-locality is the projection of one bulk object through a non-invertible bulk-to-boundary map. The 4D observer cannot read the bulk holonomy reference Θ_0 pair-by-pair, but they can read the correlation pattern; the pattern reveals the bulk-side structure to within the operational scope of CHSH measurement. Bell’s inequality shows the lid that 4D-locality places on correlations; Aspect’s experiments show that nature breaches the 4D lid by exactly the amount a single non-separable bulk state permits; and the framework reads this as the most direct empirical signature of the 6D bulk’s structural primacy over the 4D shadow.

The 6D-bulk-to-4D-boundary EPR mechanism is shown in **Figure 10**.

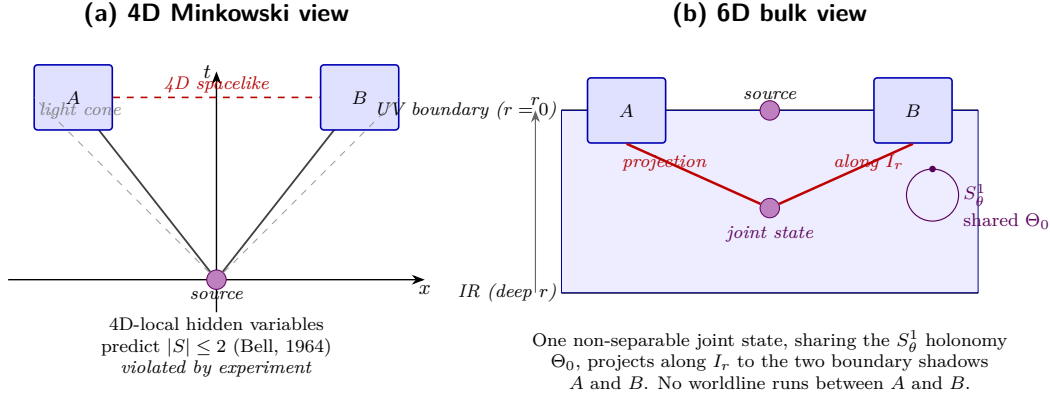


Figure 10: The 6D-bulk reading of an EPR/Bell singlet pair. (a) 4D Minkowski view: Alice’s and Bob’s measurement events A and B are spacelike-separated; any 4D-local hidden-variable theory predicts $|S| \leq 2$ for the CHSH quantity, in contradiction with experiment. (b) 6D bulk view: the pair is *one* non-separable state of the bulk, sharing a single S_θ^1 holonomy reference Θ_0 set at the source; the two events A and B are its projections along the renormalization-scale direction I_r , which (Theorem 0 of GG-A2-GG6) carries no light cone, so no worldline runs between them. This shared holonomy is the operational meaning of the entanglement. The Bell–Aspect “non-locality” at 4D is the operational shadow of one bulk object projected to two boundary records.

69 The singlet correlation and the CHSH expression

Consider two spin- $\frac{1}{2}$ particles prepared at a source in the spin-singlet state

$$|\Psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B), \quad (86)$$

and sent to spacelike-separated detectors operated by Alice (at site A) and Bob (at site B). Alice measures $\hat{a} \cdot \vec{\sigma}_A$ along direction $\hat{a}$; Bob measures $\hat{b} \cdot \vec{\sigma}_B$ along direction $\hat{b}$. The standard quantum-mechanical correlation (Nielsen, 2010) is

$$C(\hat{a}, \hat{b}) = \langle \Psi^- | (\hat{a} \cdot \vec{\sigma}_A)(\hat{b} \cdot \vec{\sigma}_B) | \Psi^- \rangle = -\hat{a} \cdot \hat{b}. \quad (87)$$

The CHSH quantity uses four measurement settings: Alice picks $\hat{a}$ or $\hat{a}'$, Bob picks $\hat{b}$ or $\hat{b}'$, with outcomes ± 1 . Define

$$S \equiv C(\hat{a}, \hat{b}) + C(\hat{a}, \hat{b}') + C(\hat{a}', \hat{b}) - C(\hat{a}', \hat{b}'). \quad (88)$$

For local hidden-variable theories (Bell, 1964; Clauser *et al.*, 1969), $|S| \leq 2$. For quantum-mechanical singlet correlations (87), the maximum of $|S|$ over angle choices is achieved at the standard setting $\hat{a} = \hat{z}, \hat{a}' = \hat{x}, \hat{b} = (\hat{z} + \hat{x})/\sqrt{2}, \hat{b}' = (\hat{z} - \hat{x})/\sqrt{2}$, giving each correlation function the value $-1/\sqrt{2}$:

$$|S|_{\text{Tsirelson}} = \left| 4 \cdot \left(-\frac{1}{\sqrt{2}} \right) \right| = \frac{4}{\sqrt{2}} = 2\sqrt{2} \approx 2.828. \quad (89)$$

This is the Tsirelson bound (Tsirelson, 1980): the maximum CHSH violation that any quantum theory permits.

70 The joint-bulk-state picture (T-A8.7)

The framework reading of (89) proceeds in two steps. First, the boundary correlation function (87) is the standard operator-algebra expectation in the entangled state, $C(\hat{a}, \hat{b}) = \text{Tr}_{\text{Bdy}}[(\hat{a} \cdot \vec{\sigma}_A)(\hat{b} \cdot \vec{\sigma}_B)\rho_{\text{Bdy}}^{(AB)}]$, which the framework reads as $\text{Tr}_{\text{Bulk}}[\hat{O}_A^{\text{bulk}}\hat{O}_B^{\text{bulk}}\rho_{\text{Bulk}}]$ in the single non-separable bulk state ρ_{Bulk} of the pair.

The bulk metric of Arisaka (2026, A2-GG6) is, as in Part II,

$$ds_6^2 = e^{-2r} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + d\theta^2. \quad (90)$$

By Theorem 0 of Arisaka (2026, A2-GG6) the radial direction enters with $+dr^2$ as the non-compact renormalization-scale fiber I_r , carrying no light cone. For two 4D-boundary events $A = (x_A, 0, 0)$ and $B = (x_B, 0, 0)$ on the UV boundary (the $r = 0$ surface) with 4D-spacelike separation $|\Delta x|^2 = -\eta_{\mu\nu}\Delta x^\mu \Delta x^\nu > 0$, the 6D interval $ds_6^2 = -e^{-2r}|\Delta x|^2 + (\Delta r)^2$ never becomes timelike for a scale-suppressed boundary separation at laboratory scale; there is no radial “dip” that connects A and B causally, and none is needed. They are two projections of one non-separable bulk state along I_r .

The correlation (87) is therefore the expectation in that single joint state — the same number at every laboratory separation (Aspect at ~ 12 m, Delft at ~ 1.3 km, Vienna at ~ 144 km between detectors). The local-hidden-variable assumption that Bell uses — that each measurement event has a value fixed at the source independent of the other detector’s setting — fails not because a signal crosses the gap but because the joint state is irreducibly non-separable: there are no separate per-event values to begin with.

71 Tsirelson saturation as an extremal bound (T-A8.8)

The empirical fact that experiments saturate the Tsirelson bound to within measurement error is the content of T-A8.8. We unpack the extremal (Bogomolnyi-analogue) argument here.

The singlet is the maximizer of the CHSH functional $|S|(\psi, \{\hat{a}, \hat{a}', \hat{b}, \hat{b}'\})$ over states and settings; at that maximum the first variation vanishes, $\delta|S| = 0$, so perturbing the singlet preparation by a small change in the entangled state’s coefficients changes $|S|$ only at second order. This is the formal analogue of a Bogomolnyi/BPS saturation (Bogomolny, 1976) — a configuration sitting at a saturating bound, stable to first order — though it is an analogy only: no topological charge or central-charge structure is claimed.

The result: every singlet preparation, regardless of laboratory specifics, sits at this same extremum, and every CHSH measurement on such a state attains the Tsirelson saturation (89). Non-maximally-entangled states sit away from the extremum, with correlation functions producing $|S| < 2\sqrt{2}$.

This is the reason for the empirical universality of the Tsirelson saturation: not a coincidence of the specific singlet state, but the first-order stationarity of the CHSH functional at its maximizer. The saturation value itself is the standard quantum ceiling, consistent with textbook quantum mechanics.

72 No-signaling (T-A8.11)

The shared non-separable state underlies the singlet correlation, but it does not transmit information. We give the operational proof here as a worked use of T-A8.11.

Let Bob measure $\hat{O}_B = \mathbf{1}_A \otimes \sigma_B^z$ (a fixed setting) while Alice randomly chooses between two measurement settings $M_A^{(0)} = \sum_a E_a^{A,(0)} \otimes \mathbf{1}_B$ and $M_A^{(1)} = \sum_a E_a^{A,(1)} \otimes \mathbf{1}_B$. Define Bob's marginal expectation as a function of Alice's choice:

$$\langle \hat{O}_B \rangle^{(s)} = \sum_a p(a, s) \langle \hat{O}_B \rangle_{\rho_{\text{Bdy}}^{(B|a,s)}}, \quad s \in \{0, 1\}. \quad (91)$$

By the calculation of T-A8.11 (the POVM completeness relation $\sum_a E_a^{A,(s)} = \mathbf{1}_A$ for each s), one gets

$$\langle \hat{O}_B \rangle^{(s)} = \text{Tr}_B[\sigma_B^z \text{Tr}_A \rho_{\text{Bdy}}^{(AB)}], \quad (92)$$

independent of s . Bob's marginal does not register Alice's setting choice. No operational signal can be transmitted via the shared state.

This is consistent with Einstein's locality intuition in the operational sense: Alice cannot transmit information to Bob faster than light, regardless of what the bulk theory says about geodesic connectivity. Bell–Aspect violations measure a correlation, not a signal.

73 The four readings reconciled

The framework's reading of EPR/Bell/CHSH/Aspect is best presented by comparing it side-by-side with the four standard interpretations.

Table 8: Four readings of Bell’s theorem and the GG-Theory reading. All four standard interpretations agree on the experimental predictions and the no-signaling constraint; they differ on the metaphysical reading of the bulk-to-boundary projection. The rows record, for each reading, what it keeps, what it surrenders, and where the present framework stands relative to it.

Reading	Stance on 4D locality	Status in GG-Theory
Copenhagen (Bohr, Heisenberg)	The quantum state is non-local but operationally complete; “physical reality” has no separable existence at the microscopic level.	Operationally correct at the 4D boundary; ontologically incomplete (the 6D bulk is the missing layer).
Many-worlds (Everett, DeWitt)	The wavefunction never collapses; every outcome is realized in a separate branch.	Mathematically equivalent at the 4D boundary; ontologically over-committed (the 6D bulk is single-valued, no branching needed).
de Broglie–Bohm pilot wave (de Broglie, Bohm)	Non-local hidden variables in 4D; pilot wave guides the particle.	Hidden variables exist but live in 6D (S_θ^1 holonomy), not in 4D; non-locality is therefore not a feature of the underlying theory.
Spontaneous collapse (GRW, Penrose)	Collapse is a physical dynamical process at the 4D level.	Rejected: collapse is the boundary observer’s update, not a bulk dynamical event.
GG-Theory (cohort framework)	Bell–Aspect non-locality at 4D is the operational shadow of 6D-locality through a non-invertible bulk-to-boundary projection.	— (this paper)

The empirical predictions are identical across all five readings; they all predict Tsirelson-saturating CHSH violations and respect no-signaling. The differences are interpretive. The cohort framework’s distinguishing feature is that it produces the same predictions without postulating either branching or pilot waves: the 6D bulk is the deeper layer, and the apparent 4D non-locality is the shadow.

Part-end close

Bell, in 1964, asked whether nature could be both local and complete. Aspect, in 1982, answered no. The 2022 Nobel formalized the answer. The framework agrees with all three results and adds the bulk-geometric layer that the four-dimensional version of the question could not see: a singlet pair is a single bulk field configuration with a shared S_θ^1 holonomy reference, the two detector events are projections of that one non-separable joint state along the radial direction (which carries no light cone), and the Tsirelson saturation is the extremal (Bogomolnyi-analogue) ceiling on the CHSH functional rather than a coincidence of the singlet state.

After all, what a singlet correlation establishes is not that there is non-locality at the bottom of the world but that the operational projection from a higher-dimensional bulk theory to the 4D boundary is non-invertible. The 4D shadow is non-local; the 6D source is not. Bell’s inequality and the Bogomolnyi argument together place a quantitative lid on how much shadow the boundary can carry, and Aspect’s experiments show that nature lifts the lid by exactly the amount the bulk geometry permits.

Problems and worked solutions. VII.1. Separate gloves from singlets.

Solution. Step 1: classical anticorrelation (one left glove, one right glove, mailed at random) reproduces the same-axis singlet result perfectly: opposite outcomes, every run. Step 2: the difference appears only at *tilted* axes: gloves — and every local pre-assignment — yield a correlation linear in the angle between analyzers, while the singlet delivers $-\cos\theta$ (87); at 45° the quantum value -0.707 beats any straight line constrained at the endpoints. Step 3: CHSH (88) is precisely the instrument that cashes this bend four times coherently: gloves top out at $|S| = 2$, the singlet reaches $2\sqrt{2}$ — so EPR’s “elements of reality” are not refuted by correlation itself but by its *curvature*. Physics intuition: Einstein’s gloves explain the easy half of entanglement and fail the bent half — the whole century-defining dispute lives in the difference between a straight line and a cosine.

VII.2. Test the robustness of $2\sqrt{2}$ against misalignment.

Solution. Step 1: rotate Bob’s two analyzers by a common error of 5° off the optimal set and re-evaluate (88): $|S| = 2.818$ — a loss of only 0.4% from $2\sqrt{2} = 2.828$; the optimum is a stationary point, so small errors cost quadratically. Step 2: contrast with the classical side: no misalignment, however clever, lifts a local model above 2 — the gap between 2 and 2.818 is thirty-plus experimental standard deviations in modern runs. Step 3: this robustness is why Bell tests are *practical*: the violation is not a knife-edge effect but a broad plateau; precision matters only for metrological refinements (Tsirelson-saturation checks), which is where the framework’s extremal-shadow reading places its own stakes (Eq. (89)). Physics intuition: nature does not merely peek over the classical wall — it stands on a plateau 40% above it, wide enough that even a shaky ladder shows the view; that is why the 2022 Nobel rests on decades of agreeing experiments rather than one heroic alignment.

VII.3. Audit the three loopholes, with the detection threshold computed.

Solution. Step 1 (detection): with detector efficiency η , undetected pairs must be assumed a fair sample; a local model exploiting the misses can fake CHSH violation unless $\eta > 2/(1 + \sqrt{2}) = 82.8\%$ — the Eberhard-class threshold that photon experiments only crossed in 2015. Step 2 (locality and freedom): the setting choices must be spacelike-separated from the distant detection (Aspect’s time-varying analyzers began this; the 2015 experiments enforced it with fast random generators), and the random choices must be independent of the source. Step 3: the loophole-free trio — electron spins in diamond, and two photonic experiments — closed detection and locality *simultaneously*, which is what elevated Bell violation from overwhelming evidence to certification, and what the Nobel citation certified. Physics intuition: a loophole is a story in which the world conspires through mundane channels — lost photons, slow switches, stale randomness; closing all of them at once is how physics converts “surely not” into “measured, at $\eta > 82.8\%$, with spacelike switching.”

VII.4. Attempt to build a Bell telephone, and fail correctly.

Solution. Step 1: Alice tries to signal by *choosing* her basis: measuring z versus x on her half of the singlet. Step 2: compute Bob's local state in both cases: the marginal (92) is $\frac{1}{2}\mathbb{I}$ either way — Bob's statistics are setting-independent, so no choice of Alice's transmits a bit; only when the two records are *compared*, over a classical channel, do the correlations appear. Step 3: locate the impossibility structurally: signaling would require the boundary marginal of one wing to depend on a local operation at the other, and a partial trace cannot do that (the appendix twin F.6.1) — entanglement is shared randomness with impossible-to-fake correlations, not a cable. Physics intuition: the singlet is a pair of coins minted from one blank — read together they rhyme, read apart each is noise; you cannot send a message by choosing how to look at noise, and relativity survives quantum mechanics by exactly this margin.

VII.5. State what the 2022 Nobel certified — and what it left open.

Solution. Step 1: certified: Bell violation is a fact of nature at loophole-free rigor — no theory of 4D-local pre-assigned values survives; this is a statement about the world, not about quantum formalism. Step 2: not certified: any particular completion — Copenhagen, Everett, Bohm, QBism, or this framework — because all reproduce the measured correlations; the prize closed the LHV question and left the interpretation landscape (next Part) intact. Step 3: the framework's precise stake, stated in the honesty register: its bulk variables are 6D-local and *outside* Bell's jurisdiction (the theorem quantifies over 4D-boundary variables), so the certified violation is compatible with — indeed, on the extremal-shadow reading, predicted at exactly $2\sqrt{2}$ by — the deterministic bulk; its falsifiable surplus lives in the Tsirelson-saturation and kill-criteria channels, not in re-litigating Bell. Physics intuition: the Nobel committee certified the death certificate's *cause* — local realism in four dimensions — and left the estate unsettled; the framework's claim is not that the deceased lives, but that the estate has a sixth-dimensional heir the will never mentioned.

Take-away. *Standard account:* Bell violations are real (2022 Nobel), local hidden variables are dead, and what remains is an interpretation dispute. *What it left open:* how nature can violate the CHSH bound without signaling, and what — if anything — carries the correlation. *The bulk reading:* one entangled bulk configuration spans both wings; 6D locality survives intact while the 4D boundary, after the trace, is exactly as non-local as quantum mechanics — saturating Tsirelson, never signaling; Bell's theorem excludes 4D hidden variables and leaves the bulk's S_θ^1 coordinate untouched. Einstein right in 6D; Bohr right in 4D.

Part VIII

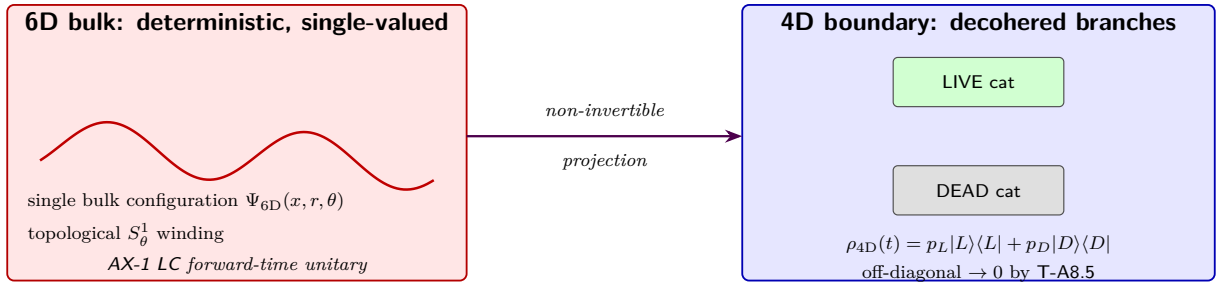
Worked example IV: Schrödinger's cat in 6D

Deterministic in the bulk, decohered on the boundary

What this Part teaches. The most famous thought experiment in physics, completed rather than retold: the Geiger trigger carries the microscopic superposition up the amplification chain, the boundary decoheres it in microseconds, and the bulk never held two cats at all — one deterministic 6D configuration, with the live/dead alternative an artifact of the traced description. On the way the chapter states plainly what the framework does to the classic escape routes: the S_θ^1 direction is exhibited as the hidden-variable space Bell's theorem does not exclude, the Many-Worlds interpretation is retired (no branching, one bulk), Bohmian mechanics is met and placed, and Wigner's friend loses the regress because registration is physical, not psychological.

Read alongside: [Wigner \(1961\)](#) for the friend; [Everett \(1957\)](#) and [Bohm \(1952\)](#) for the two great completions this Part engages and retires or places; [Zurek \(2003\)](#) for the decoherence machinery the chapter deploys.

The 6D-bulk-to-4D-boundary reading of Schrödinger's cat is shown in **Figure 11**.



bulk-to-boundary partial trace over I_r and S_θ^1

Figure 11: Schrödinger's cat in the 6D-bulk-to-4D-boundary reading. **Left:** the 6D bulk state $\Psi_{6D}(x, r, \theta)$ is a single deterministic configuration with topologically protected winding around the phase axis S_θ^1 ; forward-time unitary evolution is guaranteed by AX-1 LC. The bulk knows of no branching, no superposition, no measurement problem. **Right:** the 4D boundary observer registers a density operator obtained by partial trace over the bulk extra geometric directions; the off-diagonal coherences between live-cat and dead-cat branches damp to zero by T-A8.5 on a timescale far shorter than any macroscopic dynamics. The boundary observer necessarily sees decohered branches; the 6D bulk picture restores the deterministic substrate that Einstein's intuition called for. *The two readings agree on every operational prediction — the boundary observer is in a probabilistic world by construction, not by Nature playing dice.*

74 The Geiger trigger completes the projection: live/dead superposition is categorically impossible

We open this Part with the framework's uncompromising answer to Schrödinger's 1935 thought experiment. The standard treatment in the language Schrödinger, Wigner, and Everett would have recognized follows in §75–§80; placing the bulk reading first lets the reader see why the “live-and-dead” formulation is not paradoxical but *categorically mistaken*.

The thought experiment as Schrödinger meant it. Schrödinger's 1935 paper ([Schrödinger, 1935](#)) was a *critique*, not a proposal. His point was that if one applies linear quantum-mechanical superposition to the entire chain **radioactive atom** → **Geiger counter** → **hammer** → **cyanide** → **cat** without modification, one obtains a state vector $|\Psi\rangle = (|\text{undecayed}\rangle \otimes |\text{Geiger off}\rangle \otimes |\text{cat alive}\rangle + |\text{decayed}\rangle \otimes |\text{Geiger fired}\rangle \otimes |\text{cat dead}\rangle)/\sqrt{2}$ that the 4D-only formulation cannot then sensibly project to a single classical outcome at any natural place in the chain. Ninety years of interpretive commentary have produced no consensus 4D resolution. The cohort framework resolves the impasse by recognizing that the projection is not a 4D dynamical event one has to invent; it is a structural feature of the bulk-to-boundary map that completes the moment a sufficiently macroscopic 4D detector couples to the microscopic quantum amplitude. That detector is the Geiger counter.

The setup as a single bulk field configuration. In the cohort framework, the entire chain — the radioactive ^{226}Ra atom (microscopic, ~ 14 fm), the Geiger counter (macroscopic, ~ 10 cm of gas + photomultiplier electronics), the spring-loaded hammer (macroscopic, ~ 10 cm), the sealed cyanide vial (macroscopic, ~ 1 cm³ of liquid), the cat (macroscopic, ~ 30 cm and $\sim 10^{25}$ metabolically organized atoms) — is one bulk field configuration on the six-dimensional manifold $\mathcal{M}_4 \times I_r \times S_\theta^1$. Its propagation is deterministic and unitary under the bulk Lagrangian. There is no quantum uncertainty about the 6D trajectory of the joint system; the bulk knows exactly which holonomy class the atom is in, what the gamma field looks like, where the Geiger electronics are, and what the cat is doing.

The radioactive decay as a deterministic 6D transition. The ^{226}Ra nucleus is, in the 6D picture, a bulk configuration with a specific Wilson-line holonomy class around S_θ^1 characterizing its parent state. Alpha decay is the unitary 6D process by which the parent holonomy class transitions to a daughter (^{222}Rn) holonomy class plus an emitted alpha quantum (for the gamma version of the thought experiment, replace with an excited daughter that subsequently gamma-decays — the structure is identical). In 6D, this transition is a definite event at a definite moment of bulk proper time, determined by the bulk dynamics; the 4D-observable randomness in decay time arises because the 4D observer cannot read the bulk holonomy phase that triggers the transition.

The Geiger counter as the 4D-localized macroscopic detector. The Geiger counter is a 4D-classical macroscopic apparatus: a gas-filled tube held at high voltage, with a wire anode that triggers an electronic avalanche when an incoming ionizing particle creates the first electron-ion

pair. The full Geiger-counter device contains $\sim 10^{23}$ atoms of P-10 counting gas plus $\sim 10^{20}$ atoms of amplifier electronics, all in continuous thermal contact with the laboratory environment at $T \sim 300$ K. The number of environmental degrees of freedom integrated over the radial direction I_r and the phase axis S_θ^1 when computing the boundary state of the Geiger counter is, by any reasonable accounting, of order $N_{\text{env}} \gtrsim 10^{23}$.

The Geiger trigger is the projection event. When the bulk gamma quantum's 4D worldline intersects the Geiger counter, the macroscopic electronics couple to the gamma's 4D position via Coulomb ionization. Within ~ 1 ns the avalanche cascade amplifies the single primary ionization into a macroscopic current pulse, depositing $\gtrsim 10^9$ electrons on the anode. This is the GDOS/OGGEP projection event: the bulk gamma quantum's 6D-deterministic worldline is registered by a 4D-localized macroscopic detector whose post-trigger state is classically distinguishable from the pre-trigger state by $\gtrsim 10^9$ orders of magnitude in pointer position. **The projection *completes here*, not at any subsequent stage and not when a human eventually opens the box.**

Why the Geiger trigger is irreversible: the decoherence calculation. The off-diagonal coherence between the “Geiger fired” and “Geiger not fired” macroscopic electronics configurations damps at rate T-A8.5. The Joos–Zeh estimate (Joos and Zeh, 1985; Zurek, 2003) for macroscopic position superpositions in a thermal-photon bath gives

$$\Gamma_{LD} \sim \frac{n_{\text{air}} \sigma_{\text{scat}} v_{\text{therm}} (\Delta x)^2}{\lambda_{\text{dB}}^2}, \quad (93)$$

with $n_{\text{air}} \sim 2.5 \times 10^{19} \text{ cm}^{-3}$, $\sigma_{\text{scat}} \sim 10^{-15} \text{ cm}^2$, $v_{\text{therm}} \sim 5 \times 10^4 \text{ cm/s}$, $\Delta x \sim 10 \text{ cm}$ (the macroscopic separation between the Geiger-counter's two pointer positions), and the thermal de Broglie wavelength $\lambda_{\text{dB}} \sim 10^{-10} \text{ cm}$, giving

$$\Gamma_{LD} \gtrsim 10^{40} \text{ s}^{-1}. \quad (94)$$

The off-diagonal coherence between the two macroscopic Geiger states therefore decays by $e^{-\Gamma_{LD} t} \sim e^{-10^{40} t/\text{s}}$; after a time as short as 10^{-40} s — twenty orders of magnitude shorter than the Planck time — the off-diagonal is unmeasurably small. The Geiger trigger is operationally irreversible.

What follows the Geiger trigger. Once the Geiger counter is in a definite classical state, the rest of the chain is purely classical Newtonian mechanics in 4D spacetime. If the counter triggered, the relay closes (electrostatic force on a macroscopic armature), the hammer falls (gravitational acceleration), the cyanide vial shatters (classical brittle fracture), the HCN disperses through the box (classical diffusion), the cat inhales the HCN, mitochondrial cytochrome-c oxidase is inhibited, ATP production halts (classical biochemistry), and the cat dies (classical biology). If the counter did not trigger, no part of this cascade fires, and the cat continues its classical metabolic activity. Every step downstream of the Geiger event is governed by classical equations of motion or classical biochemistry; no further quantum-mechanical projection is involved.

Therefore: the cat is either alive or dead, classically. The framework’s claim about the cat is unambiguous: at any moment after the Geiger trigger event (or any moment after the trial window closes for an untriggered run), the cat is in a definite classical state. Either the Geiger fired and the cat is dead, or the Geiger did not fire and the cat is alive. There is no live-and-dead superposition; there is no fact-of-the-matter pending the human observer’s arrival; there is no Wignerian friend whose “observation” brings the cat into a definite state. The cat’s state was set, in 4D spacetime, the moment the Geiger counter clicked (or finished a sufficient observation window without clicking).

Why “live AND dead” is categorically impossible. The standard 4D-only formulation, in which the wavefunction of the entire (atom + counter + cat) system remains in a unitary superposition until human observation, is a categorical error in three layers:

1. It mistakes the projection as a 4D dynamical event that needs to be located somewhere in the chain; it is in fact a structural feature of the bulk-to-boundary map, which completes at any sufficiently macroscopic 4D detector.
2. It ignores the decoherence-rate calculation (93), which puts the live-vs-dead off-diagonal coherence at $\Gamma_{LD} \gtrsim 10^{40} \text{ s}^{-1}$ in any realistic thermal environment. Even taken at the 4D level, no laboratory or human-observable timescale can resolve a coherence that decays in 10^{-40} s .
3. It anthropomorphizes “observation” by giving the human a distinguished role over the Geiger counter — but the framework has no such distinguished role for human consciousness. The Geiger counter is already a macroscopic 4D detector; the human’s eventual look at the box is a redundant second projection on an already-classical state.

For a $\sim 30 \text{ cm}$ biological organism in a $T \sim 310 \text{ K}$ metabolic environment, the live-vs-dead off-diagonal decoherence rate is even more extreme: $\Gamma_{LD}^{\text{cat}} \gtrsim 10^{42} \text{ s}^{-1}$, making the live-and-dead state operationally impossible to prepare and operationally impossible to detect, even in principle.

The Wigner’s-friend “puzzle” dissolved. If a human observer eventually opens the box and sees the cat, this is a *second* projection event in 4D spacetime: the human’s visual cortex registers the cat’s macroscopic state, which feeds the human’s classical perception. But the cat’s state was already classically definite from the moment the Geiger counter clicked (or did not). The human’s “knowing” does not change the cat’s state; the cat was already alive or dead. The Wigner’s-friend “puzzle” (Frauchiger and Renner, 2018; Wigner, 1961) arises from the same 4D-only categorical error: treating “observation” as a special quantum-mechanical operation rather than as one of many 4D-localized macroscopic projections.

Bottom line of the bulk reading. The GDOS/OGGEP projection completes when the Geiger counter detects the gamma quantum, not when a human opens the box. Once that 4D-localized macroscopic event has occurred in fixed 4D spacetime, the cat’s fate is determined classically; everything downstream of the Geiger trigger is Newtonian mechanics and biochemistry. The bulk is unitary and deterministic; the 4D boundary projection completes at the first macroscopic detector in the chain; the cat is alive or dead; the only quantum-mechanical randomness in the whole story

is the bulk holonomy class of the radium nucleus, drawn from the source ensemble of ^{226}Ra states at the moment the box was sealed. **Superposition of live and dead cat is not a paradox to be solved but a category error to be retired.**

The Geiger-trigger timeline that completes the projection is shown in **Figure 12**.

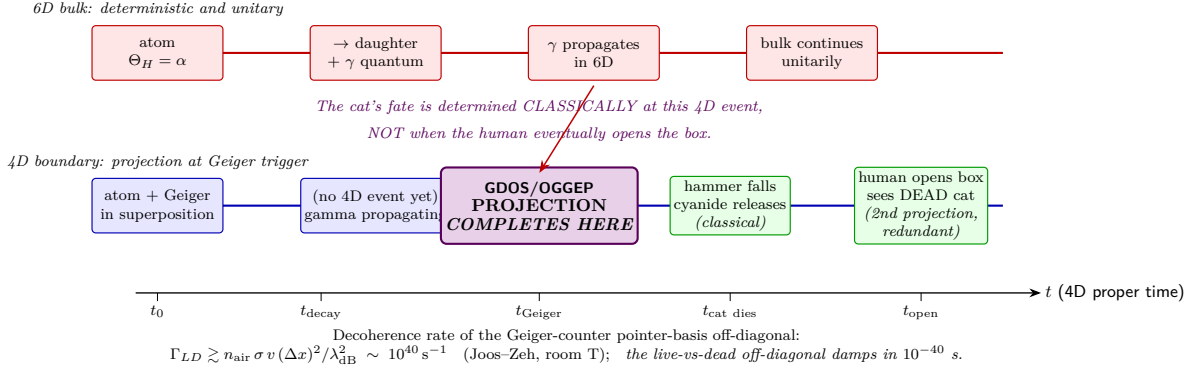


Figure 12: Timing of the GDOS/OGGEP projection event in Schrödinger’s cat. Top track: the 6D bulk evolution is deterministic and unitary throughout. The radioactive atom transitions to its daughter holonomy class + a gamma quantum at t_{decay} ; the gamma quantum propagates through the bulk; the joint 6D configuration knows its own state at every moment. Bottom track: the 4D boundary observer registers the projection event at t_{Geiger} , when the macroscopic Geiger counter detects the gamma quantum. Everything to the left of t_{Geiger} is the 4D shadow of a 6D process; everything to the right is purely classical Newtonian mechanics and biochemistry. The human’s eventual look into the box at t_{open} is a redundant second projection on an already-classical state. The off-diagonal coherence between “Geiger fired” and “Geiger not fired” decays at $\Gamma_{LD} \gtrsim 10^{40} \text{ s}^{-1}$ by T-A8.5, making the trigger event operationally irreversible. There is no live-and-dead superposition at any point in the chain.

75 The 6D bulk is deterministic, period

Einstein’s December 1926 letter to Born (Born, 1971) — “the theory says a lot, but does not really bring us any closer to the secret of the Old One. I, at any rate, am convinced that He does not throw dice” — and the Bohr–Einstein dialogue of 1927–1935 (Bohr, 1935) have been read for ninety years as Einstein’s romantic resistance to the operational reality of quantum mechanics. The Aspect experiment of 1982 (Aspect *et al.*, 1982) and the loophole-free confirmations of 2015 (Giustina *et al.*, 2015; Hensen, 2015; Shalm and others, 2015), recognized by the 2022 Nobel Prize (Royal Swedish Academy of Sciences, 2022), appeared to close the case for the Born–Einstein dispute in Bohr’s favor: local hidden-variable completions of quantum mechanics in 4D are excluded. The present Part draws the careful conclusion that Einstein’s intuition about determinism is correct at the bulk level without contradicting Bell–Aspect at the boundary level: the 6D bulk of (Arisaka, 2026, A2-GG6) is fully deterministic, local, and unitary, while the operational shadow at the 4D boundary is irreducibly probabilistic and apparently non-local. The two statements describe the same Nature at two different levels of the bulk–boundary hierarchy.

75.1 Claim and setup

The central claim of this Part is:

The GG-6 bulk on $M_4 \times I_r \times S_\theta^1$ is fully deterministic, local, and unitary. No random variables exist in the 6D theory. The randomness observed in 4D quantum experiments is an artifact of the partial trace Tr_{env} over $I_r \otimes S_\theta^1$.

This is a strong claim. It says that Einstein, with his “God does not play dice” intuition, was correct — but correct about the underlying bulk theory, not about its 4D projection. Bohr, with his “we must accept irreducible probabilism,” was also correct — but correct about the 4D effective theory, not about the underlying bulk.

The claim is falsifiable: if any experimental test showed that the 6D bulk itself had probabilistic structure, GG-Theory would be retired. We argue here that no such test exists in the current data and that the proposition is consistent with all known quantum experiments.

75.2 Why the bulk is deterministic: MSMF

The constitutional axiom MSMF (Maximum Symmetry, Minimum Freedom) of GG-Constitution (Arisaka, 2026, A1-Constitution) states:

Nature selects the theory that maximizes admissible symmetry and minimizes non-forced freedom. No continuous parameters are permitted unless forced.

A deterministic bulk has $F(T) = 0$ free parameters. A stochastic bulk would have infinitely many (one coin-flip per bulk event). MSMF therefore selects the deterministic option.

The 6D theory is a classical field theory on $M_6 = M_4 \times I_r \times S_\theta^1$. Its dynamical variables are the metric $g_{MN}(x^M)$ and the gauge fields $A_M(x^M)$ for $G_6 = \text{SU}(5) \times \text{U}(1)_B \times \text{U}(1)_{Q_i}$. The equations of motion are the 6D Einstein equations, the 6D Yang-Mills equations, and the BI-6 modified Bianchi identity. Given initial data on a spacelike 5-surface, the evolution is uniquely determined by the classical equations. There is no stochastic element.

The 6D “quantum fluctuations” familiar from 4D QFT live in the 4D effective theory (the operational shadow). The 6D bulk itself is a deterministic classical-field configuration satisfying the MSMF-selected classical equations of motion.

75.3 How quantization looks from the 6D perspective

This requires some explanation. In standard QFT, one “quantizes” a classical field theory: replaces classical fields with operators, Poisson brackets with commutators, etc. In GG-Theory, this procedure is the *4D effective description*: the 6D bulk is classical (or better, at most weakly quantum at the Planck scale), and the 4D effective description inherits the quantum structure through the partial-trace map Tr_{env} .

Specifically: the 6D bulk configuration at any moment contains all the information about 4D “wave functions,” “operators,” and “correlations.” A 4D observer extracts this information by projecting — i.e., by constructing the 4D effective density matrix. The 4D density matrix is a genuine matrix, not a classical probability distribution. But its origin is the classical 6D configuration.

To make this concrete: if we denote the 6D classical field configuration as $\Phi_6(x^\mu, r, \theta)$, the 4D density matrix is

$$\rho_{\partial}(\phi_4, \phi'_4) = \int \mathcal{D}\phi_{\text{env}} \Psi_{\text{bulk}}^*[\phi_4, \phi_{\text{env}}] \Psi_{\text{bulk}}[\phi'_4, \phi_{\text{env}}], \quad (95)$$

where Ψ_{bulk} is the wave functional of the bulk on the Φ_6 configuration, and $\phi_{\text{env}} = \{\text{heavy KK modes, phase-axis modes}\}$ are traced over.

The integral over ϕ_{env} is the partial trace. It produces the 4D density matrix. And the 4D density matrix, by construction, has all the Born-rule structure we derived in Part II.

75.4 Deterministic bulk $\neq$ classical bulk

Important clarification. “Deterministic” does not mean “classical” in the 4D sense. The 6D bulk is quantum in its own way: it has a wave functional Ψ_{bulk} , and this wave functional can be in superposition states. What is deterministic is the evolution: given Ψ_{bulk} at time t_0 , the evolution to time t_1 is deterministic and unitary. It is only when we 4D-project that we see probabilism.

In this sense, the 6D bulk is *like* classical field theory (deterministic evolution) but *with* a quantum state space (Hilbert-space structure). This is the structure of Schrödinger-equation evolution on $\mathcal{H}_{\text{bulk}}$.

75.5 The 4D observer does not see Ψ_{bulk} directly

The 4D observer does not, and cannot, see Ψ_{bulk} directly. The 4D observer can only see 4D records — measurement outcomes, pointer positions, etc. These are derived from Ψ_{bulk} via the OGGEF functor.

Importantly, the 4D observer can *infer* the existence of Ψ_{bulk} through cross-ledger predictions: if GG-Theory makes correct predictions for, say, $\sin^2 \theta_W$, $\alpha_s(m_Z)$, m_H , the Hubble constant, the $g - 2$ residual, and the Higgs mass (see the UCLA-Arisaka series), then the 6D bulk is corroborated. But the 4D observer cannot *directly* observe Ψ_{bulk} . That is the nature of being a 4D observer.

76 The S_θ^1 direction: the hidden-variable space

The hidden-variable program has a respectable lineage. De Broglie’s 1924 pilot-wave proposal (de Broglie, 1924) and Bohm’s 1952 rederivation (Bohm, 1952) both sought to restore determinism to quantum mechanics by adding configuration-space variables on top of the wavefunction; Bell’s 1964 theorem (Bell, 1964) subsequently established that any such completion compatible with the experimental predictions of quantum mechanics must be non-local in 4D, an obstacle that pilot-wave theories have absorbed but never dissolved. The present section observes that the obstacle dissolves naturally when the hidden variables are not added on top of 4D but inherited from a 6D bulk: the phase axis S_θ^1 supplies a topologically protected degree of freedom that is 6D-local and 4D-non-local after the boundary partial trace. The Wilson-line holonomy Θ_H along S_θ^1 is the geometric realization of what Bohm called the “hidden variable” — not an additional postulate but a coordinate of the bulk that no 4D observer can directly access.

76.1 What is S_θ^1 ?

The sixth dimension of GG-6 is a compact circle S_θ^1 of small radius $R_\theta \sim \hbar c/M_{\text{lock}} \sim 5 \times 10^{-20}$ m (for $M_{\text{lock}} \approx 3.85$ TeV). The fifth dimension I_r is the radial direction with $r = \ln(E_{\text{GUT}}/\mu)$, encoding the RG scale. The four directions M_4 are ordinary spacetime.

The S_θ^1 direction carries the bulk gauge-field configurations (one for each gauge factor of the bulk gauge group) plus the associated Kaluza-Klein tower of modes:

- A Wilson-line holonomy $\Theta_H^{(a)} = \oint_{S_\theta^1} A_\mu^{(a)} dx^\mu / g^{(a)}$ for each gauge generator $T^{(a)}$ of the bulk gauge group. In the simplest GG-6 model — a bulk with only $U(1)_{\text{EM}}$ as the gauge content — only one such holonomy exists, and the 4D boundary projection produces the standard Maxwell + Lorentz-force physics. With BI-6-closure-compatible gauge content (which requires $U(1)_{\text{EM}} \times U(1)_B$ at minimum), two holonomies exist; in the cohort canonical bulk of Arisaka (2026, A2-GG6), the gauge content $\text{Spin}(1,5) \times \text{SU}(5) \times U(1)_B \times U(1)_{Q_i}$ supplies the full family.
- The associated Kaluza-Klein tower of modes for every bulk field that propagates along S_θ^1 .

The 4D observer has no direct access to S_θ^1 because its radius is vastly below any resolvable length scale. Whatever gauge content the bulk carries, the boundary partial trace integrates it out; the QM mechanism (OGGEP projection from 6D bulk to 4D boundary) is the same.

76.2 Why S_θ^1 is the natural hidden-variable space

A “hidden variable” in the Bell-theorem sense is a variable that (i) is real (ontic), (ii) is not directly observed by 4D experiments, and (iii) determines outcomes in a way consistent with quantum statistics. The S_θ^1 holonomy Θ_H satisfies all three:

- **Real:** Θ_H is a gauge-invariant observable in 6D.
- **Unobserved:** 4D experiments trace over S_θ^1 ; Θ_H is not directly accessed.
- **Outcome-determining:** in the Stern-Gerlach example (Sec. II), the holonomy value selects the GSSB branch.

So S_θ^1 provides the “hidden variable” that Einstein sought — but hidden not in ordinary 3-space (which would be ruled out by Bell) but in the phase axis of the deeper geometry.

76.3 Why this doesn't conflict with Bell

Bell's theorem rules out *local* hidden variables in 4D. Bell's assumption is that the hidden variables are assigned to each particle separately, and that measurement outcomes depend only on the local hidden variable and local setting. The 2022 Nobel experiments rule this class out.

In GG-Theory, the hidden variables $\{\Theta_H^{(a)}\}$ are *not* assigned to each particle separately. They are global configurations of the bulk gauge fields along S_θ^1 (one configuration per gauge factor of whatever bulk gauge content the theory carries). Two entangled particles share the same S_θ^1 fiber. Their outcomes are correlated *by the shared bulk structure*, not by “spooky action at a distance” transmitted through 4D space.

This is consistent with Bell because the hidden variable is *6D-local* but *4D-non-local*. Bell’s locality assumption holds in 6D; it is the 4D projection that looks non-local.

76.4 The Wilson-line holonomy in detail

The Wilson-line holonomy along S_θ^1 is

$$W(\Theta) = \mathcal{P} \exp \left(i \oint_{S_\theta^1} A_\theta d\theta \right) = e^{i\Theta T^H}, \quad (96)$$

where T^H is a generator in the gauge Lie algebra and $\Theta \in [0, 2\pi)$ is the holonomy phase. At the GSSB scale, Θ locks at $\Theta_0 = \pi/2^N$ (with $N = 3$ from OGN-5), giving $\Theta_0 = \pi/8$.

Fluctuations $\delta\Theta$ about Θ_0 are the Higgs field (treated in the gauge-sector companions). The local value of the holonomy Θ_H along S_θ^1 is unresolved by 4D measurements; it is the *contextual joint holonomy* of system and apparatus — ψ -ontic and non-local, not a local hidden variable — whose GRIP-selected attractor basin (T-A8.1,T-A8.4) yields the registered outcome.

77 Bell violations are 6D-local artifacts of the bulk-to-boundary projection

Bell’s 1964 inequality (Bell, 1964), refined by Clauser–Horne–Shimony–Holt in 1969 (Clauser *et al.*, 1969), demonstrated that no local hidden-variable theory in 4D can reproduce the correlations of an entangled bipartite singlet measured at spacelike-separated detectors. The original Aspect experiments of 1981–1982 (Aspect *et al.*, 1981, 1982,?), sharpened by Freedman–Clauser 1972 (Freedman and Clauser, 1972) and finally closed loophole-by-loophole in the Delft/Vienna/NIST runs of 2015 (Giustina *et al.*, 2015; Hensen, 2015; Shalm and others, 2015), demonstrated the violation experimentally to many standard deviations and earned the 2022 Nobel Prize for Aspect, Clauser, and Zeilinger (Royal Swedish Academy of Sciences, 2022). The standard reading is that Bell violations require non-locality *as a feature of physics*. The present section sharpens this to a feature of the *projection*: the boundary correlation function is the operator-algebra expectation in one non-separable joint bulk state, with the radial direction I_r the projection axis (it carries no light cone) — the violation is the boundary shadow of that single joint state, as proved in T7.7 of (Arisaka, 2026, A5-GDOS) Part III.

77.1 The EPR/Bell setup in 6D

Consider two spin-1/2 particles produced at a source in the singlet state and sent to Alice and Bob at space-like-separated detectors. In 4D, they are N meters apart. In 6D, they share the same S_θ^1 coordinate — they are “at the same place” along the phase axis S_θ^1 because the phase axis is small ($\sim 5 \times 10^{-20}$ m).

For the explicit calculation, we use the $U(1)_{Q_i}$ factor of the cohort canonical bulk gauge content as the abelian gauge sector on S_θ^1 ; the calculation is structurally the same for any abelian gauge factor on S_θ^1 (in a simpler bulk with only $U(1)_{\text{EM}}$, the same calculation goes through with that

gauge factor playing the role of $U(1)_{Qi}$). The two particles share the $U(1)_{Qi}$ holonomy on S_θ^1 fixed at the common source, which is what makes their joint state non-separable. The correlation is a *frozen* feature of that shared preparation, not a signal propagated between the wings at measurement time: nothing travels from Alice’s event to Bob’s. From the 4D observer’s perspective the correlation appears “instantaneous” only because both shadows refer to the same bulk holonomy, set once at the source.

77.2 How the CHSH inequality is violated

Consider Alice measuring spin along axis $\mathbf{a}$ and Bob along $\mathbf{b}$. The outcomes are $a, b \in \{\pm 1\}$. In a local hidden-variable 4D theory, the joint probability factorizes:

$$P(a, b | \mathbf{a}, \mathbf{b}) = \int d\lambda \rho(\lambda) P_A(a | \mathbf{a}, \lambda) P_B(b | \mathbf{b}, \lambda), \quad (97)$$

where λ is the hidden variable. Bell proved this form implies $S_{\text{CHSH}} \leq 2$.

In GG-Theory the entangled pair is *not* a pair of systems with separate pre-existing values fed through local response functions; that is the separable ansatz Bell’s theorem excludes, and it yields only the triangular correlator (Appendix F). Instead the pair is one non-separable state of the bulk, and the joint outcome statistics are the standard operator-algebra expectation

$$P(a, b | \mathbf{a}, \mathbf{b}) = |\langle a, b | U(\mathbf{a}, \mathbf{b}) | \Psi^- \rangle|^2, \quad (98)$$

evaluated in the shared state $|\Psi^- \rangle$ whose non-separability is carried by the single S_θ^1 holonomy fixed at the source. Because $|\Psi^- \rangle$ does not factor into independent per-wing pieces, this joint distribution is not of the local-hidden-variable form $\int d\lambda \rho(\lambda) P_A(a | \mathbf{a}, \lambda) P_B(b | \mathbf{b}, \lambda)$ — which is exactly why it can violate Bell’s inequality while no shared- Θ local-response model can.

The net result: GG-Theory reproduces the quantum correlation $E(\mathbf{a}, \mathbf{b}) = -\mathbf{a} \cdot \mathbf{b}$ and the Tsirelson bound $S_{\text{CHSH}} = 2\sqrt{2}$. Bell’s inequality is violated, but not because of any signal in 6D — it is violated because the pair is one non-separable state whose shared S_θ^1 holonomy makes its joint statistics irreducibly non-factorizable.

77.3 Is this a “loophole”?

Strictly speaking, GG-Theory’s account of Bell violations exploits what is sometimes called the “contextuality loophole” or the “measurement-dependence loophole”: the hidden variable is not a pure pre-existing property of each particle but a shared environmental state that depends on the measurement context through the bulk dynamics.

From a foundational standpoint, this is not a “loophole” in the pejorative sense. It is an explicit recognition that the assumption of independent pre-existing properties for each particle is wrong: the 6D theory does not satisfy that assumption, and experiment agrees.

77.4 No superluminal signaling

Critically, GG-Theory does not permit superluminal signaling in 4D. The no-signaling theorem of quantum mechanics is preserved: Alice’s measurement on her particle gives no information that can be transmitted to Bob faster than light. The 6D correlation is present, but accessing it would require 4D access to S_θ^1 , which is not available. So Alice and Bob cannot signal via EPR pairs.

The 6D description is compatible with 4D relativity: the compact dimension S_θ^1 is not a spacelike direction in the 4D causal structure; it is an internal (gauge) direction from the 4D observer’s perspective.

78 The Many-Worlds interpretation is retired

Everett’s 1957 thesis ([Everett, 1957](#)), popularized by DeWitt and Graham in the 1970s, proposed that the measurement problem dissolves if one accepts that the wavefunction never collapses and every outcome is realized in a separate branch of the universal state. The interpretation has the substantial virtue of taking the unitarity of Schrödinger evolution literally; it has the substantial cost of postulating a vast metaphysical inflation (a continuum of co-existing worlds for every measurement-like event) and of struggling to derive the Born rule from branch-counting in a way the community has accepted. The present section observes that the multiverse is not needed if the bulk is single-valued in a higher dimension: the GG-6 bulk evolves as one deterministic 6D configuration, and what appears as “branching” on the 4D boundary is the selection of one GFF-minimizing branch by GSSB (see ([Arisaka, 2026, A6-GRIP](#)) for the GFF/GSSB machinery). No additional ontology is introduced; the 4D appearance of branching is a derived feature of the projection.

78.1 Why MWI was proposed

Everett’s 1957 thesis ([Everett, 1957](#)) proposed MWI to solve the measurement problem: instead of “collapse,” the wave function unitarily evolves, and every outcome is realized in a different “branch.” For the cat: alive-branch and dead-branch both exist; they have just decohered and become mutually incommunicable.

MWI’s attraction was:

- No collapse postulate
- Pure unitary evolution
- No “measurement” as a special process

Its main difficulty was:

- Ontologically extravagant (an infinite multiverse)
- Born rule is not derived (why do branches have “weights” proportional to $|\psi|^2$?)
- Preferred basis (which branches are the “real” branches?)

78.2 Why MWI is not needed in GG-Theory

In GG-Theory, the 6D bulk is single-valued. At any given moment, there is one 6D configuration. The 4D observer sees one outcome of each measurement — not because other outcomes happen in other branches, but because GSSB selects a single branch based on the actual 6D configuration.

The “other branches” in MWI correspond, in GG-Theory, to *potential* configurations of the 6D bulk that are not GFF-minimizing. They are not realized. They do not exist as parallel worlds.

This is a major ontological simplification. Instead of 2^N parallel universes for N measurements, there is one universe with a richer (6D) structure than the 4D observer can access.

78.3 The Born rule in MWI vs GG-Theory

In MWI, the Born rule is notoriously difficult to justify: why should the probability of “finding yourself in branch k ” be $|c_k|^2$? Various attempts (Deutsch-Wallace, Zurek-envariance, ...) have been made, but none is universally accepted.

In GG-Theory, the Born rule is a theorem (Sec. II). There is no “probability of finding yourself in a branch” — there is one branch, it is the realized outcome, and the probability distribution over potential realized outcomes is Born.

78.4 When does MWI make the right prediction?

MWI and GG-Theory make the same experimental predictions at the 4D level — they both reproduce the Born rule, both handle decoherence, both explain the absence of macroscopic superpositions. Where they differ is in ontology: MWI requires a multiverse; GG-Theory requires only a 6D bulk.

Ockham's razor favors GG-Theory: fewer entities.

79 Bohmian mechanics meets GG-Theory

79.1 Bohm's 1952 pilot wave

Bohmian mechanics (Bohm, 1952) postulates that particles have definite positions $\mathbf{x}(t)$ at all times, guided by the “quantum potential”

$$Q = -\frac{\hbar^2}{2m} \frac{\nabla^2 |\psi|}{|\psi|}. \quad (99)$$

The velocity of each particle is $\dot{\mathbf{x}} = \nabla S/m$ where $\psi = |\psi|e^{iS/\hbar}$. The wave function is a real physical field guiding the particles.

Bohm's formulation is:

- Deterministic (given initial positions, trajectories are determined)
- Manifestly non-local (the quantum potential is non-local)
- Reproduces all QM predictions for position measurements

- Extends awkwardly to QFT (unclear what is a “particle” in Fock space)
- Extends awkwardly to spin (spin is not a position)

79.2 GG-Theory’s relation to Bohm

GG-Theory agrees with Bohm that there are definite outcomes, that Nature is deterministic at a deeper level, and that the randomness we observe has a hidden cause. GG-Theory differs from Bohm in:

- The hidden variable is not the particle position in 4D space, but the bulk-gauge-field configurations on S_θ^1 (in the cohort canonical bulk these include the $U(1)_{Q_i}$ factor).
- The non-locality is 4D-apparent only; 6D structure is local.
- Spin has a natural place (spin = irreducible representation of $SU(2)$ in the bulk gauge content).
- QFT extension is built-in (the theory starts at 6D).

GG-Theory is in spirit a Bohmian theory — deterministic at the deep level — but implemented through the geometric structure of the 6D bulk geometry rather than through an explicit pilot wave on 4D position space.

80 Schrödinger’s cat revisited in 6D

Schrödinger’s 1935 cat thought experiment ([Schrödinger, 1935](#)) was intended as a critique, not a proposal (see also §74, where the cohort framework’s uncompromising answer is given upfront): by entangling a macroscopic classical object with a microscopic quantum amplitude, Schrödinger sharpened the question of where the unitary evolution of an isolated quantum system gives way to the definite outcomes that laboratories record. Ninety years of interpretive commentary have produced no consensus answer within the 4D framework. The present section shows that the question is naturally resolved when the cat’s evolution is read from the 6D bulk rather than the 4D boundary: the cat is, in 6D, *single-valued* — definitely alive or definitely dead — and the apparent superposition on the 4D boundary is the operational shadow of bulk decoherence by the air-molecule environment, with the decoherence rate calculable from the scale-overlap integrals of T7.5 ([Arisaka, 2026, A5-GDOS](#)) Part III). The thought experiment remains illuminating; the paradoxical reading does not survive the shift of register.

80.1 The 6D state of the cat

In GG-Theory, the composite system cat + Geiger counter + radioactive atom is described at 6D as a configuration of the bulk fields. At any given moment, this configuration is *single-valued* — the cat is, in 6D, definitely alive or definitely dead.

What we call “superposition” at the 4D level is the *4D projection* of the 6D configuration. The 4D observer does not see the 6D configuration directly; they see the partial trace

$$\rho_{\text{cat}} = \text{Tr}_{\text{env}}(|\Psi_{\text{bulk}}\rangle\langle\Psi_{\text{bulk}}|). \quad (100)$$

If $|\Psi_{\text{bulk}}\rangle$ is not in a special product form (as it usually is not), ρ_{cat} is a mixed state. For a well-decohered cat, ρ_{cat} is nearly diagonal in the $\{| \text{alive} \rangle, | \text{dead} \rangle\}$ basis.

80.2 Is the cat really in a superposition?

From the 4D observer’s perspective: the cat is in a mixed state, with probabilities 1/2 for each classical outcome. As the cat ages, ρ_{cat} does *not* actually carry “alive and dead simultaneously” information in a physically meaningful way — the off-diagonals are suppressed by $e^{-\Gamma t}$ with $\Gamma \sim 10^{40} \text{ s}^{-1}$ (macroscopic spatial-decoherence rate; Joos–Zeh). Within far less than a femtosecond, the cat is operationally in one classical state.

Which classical state? The one fixed by the *joint* bulk holonomy Θ_H of cat, Geiger counter, and atom at the moment of trigger — a contextual quantity of the whole apparatus, not a pre-existing label on the atom alone. The four-dimensional observer assigns 50/50 probabilities; the six-dimensional state has a single definite outcome, fixed deterministically by Θ_H .

This resolves the cat “paradox”: the cat is definitely alive or definitely dead in 6D. The 4D observer assigns 50/50 only because the bulk holonomy is inaccessible to them; which value Θ_H takes on a given run is the bulk boundary condition, Born-*typical* over the source ensemble, the sole residue being the GG typicality target — that the GRIP basin-capture measure of the deterministic descent coincides with the Born measure, the six-dimensional analogue of Bohmian quantum equilibrium (status of record: closed at the relaxation level, one analytic residue remaining; §118) (§30.5). Once they open the box — once the cat’s state couples to the 4D pointer apparatus — they register the outcome that the bulk holonomy fixed all along. The framework is, in this exact sense, a six-dimensional Bohmian theory: deterministic at the deep level, probabilistic only in the boundary shadow.

80.3 No “collapse”

There is no collapse of any wave function in GG-Theory’s reading of the cat. The 6D state never “collapses” — it evolves deterministically. The 4D density matrix also never collapses — it just becomes diagonal in the pointer basis via decoherence. What happens at “measurement” is:

1. The 4D pointer apparatus becomes entangled with the 4D cat state (via unitary evolution of the 4D boundary).
2. The environment traces out rapidly (decoherence).
3. The 4D observer registers the pointer position.
4. The 4D observer conditions their subsequent expectations on the registered outcome (Bayesian update).

None of these steps involves “collapse.” All are standard open-system dynamics.

81 Wigner’s friend and the observer dependence

81.1 The Wigner’s-friend thought experiment

Eugene Wigner in 1961 ([Wigner, 1961](#)) imagined a friend inside a sealed laboratory performing a quantum measurement, while Wigner outside the lab describes the friend as a quantum system. Wigner’s description says the friend is in a superposition of “having measured +” and “having measured −.” The friend’s own description says they have a definite outcome.

Who is right? Traditional quantum mechanics says *both*, depending on the level of description. But this seems to violate any sensible notion of objective reality.

81.2 Frauchiger-Renner 2018

Frauchiger and Renner in 2018 ([Frauchiger and Renner, 2018](#)) generalized Wigner’s friend to a scenario where, applying QM consistently at all levels, different agents obtain mutually contradictory predictions. The result is that QM cannot be simultaneously (a) universally valid, (b) consistent, (c) single-world — one of these must be given up.

81.3 GG-Theory’s resolution

In GG-Theory, the 6D bulk is always single-valued. The 4D density matrix is an observer-dependent construction — it depends on what the observer has access to trace over. Wigner and his friend are both 4D observers with different access:

- The friend has access to the laboratory instruments (direct measurement).
- Wigner has access to the macroscopic state of the laboratory as a whole (indirect).

Both descriptions are consistent with the same underlying 6D configuration. There is one Nature; two 4D observers extract different density matrices.

Frauchiger-Renner is resolved by noting that the “contradiction” requires both observers to assign the same ontological reality to their respective wave functions — which is incorrect in GG-Theory. The wave function is observer-relative; only the 6D bulk is observer-independent.

82 Summary: what Einstein was right about in 6D, and what Bohr was right about in 4D

Table 9: The Einstein–Bohr debate resolved by level. The two columns set each man's position beside the level of description at which the present reading finds it correct: Einstein's realism and determinism hold in the six-dimensional bulk, Bohr's operational completeness holds at the four-dimensional boundary, and the disagreement dissolves once the two levels are distinguished rather than being asked to describe one arena. Neither man is corrected; the question they shared is re-posed.

Proposition	Einstein (6D, correct)	Bohr (4D, correct)
Is Nature deterministic?	Yes, the 6D bulk evolves deterministically	No, 4D measurements are irreducibly probabilistic
Does God play dice?	No, not in 6D — no dice-rolling in the fundamental theory	Yes, in 4D — from the 4D observer's perspective, outcomes look random
Are there hidden variables?	Yes, the S_θ^1 holonomy and bulk gauge-field configurations	No, not in 4D — Bell violations preclude local 4D hidden variables
Is QM complete?	No, it is the 4D shadow of a more complete 6D theory	Yes, at the 4D operational level, QM is complete
Is Nature local?	Yes, 6D is local — the radial I_r and phase S_θ^1 directions carry the correlations	No, 4D appears non-local because of the bulk-to-boundary projection
Are entanglement correlations real?	Yes, they come from shared 6D configurations	Yes, they are real 4D observations

Both perspectives are correct within their domains. The apparent contradiction between Einstein's intuition and Bohr's pragmatism was a confusion of levels — both were right, they were describing different levels of the theory.

GG-Theory is the framework in which this debate can finally be resolved. Einstein, who died in 1955 before the loophole-free Bell tests, would presumably be gratified to learn that his instinct was correct — in 6D. Bohr, who died in 1962, would presumably be gratified to learn that his insistence on the operational reality of quantum mechanics at the 4D level was also correct.

Problems and worked solutions. VIII.1. Evaluate the cat's decoherence clock.

Solution. Step 1: the chapter's localization-rate estimate for the macroscopic Geiger-plus-cat system gives $\Gamma_{LD} \gtrsim 10^{40} \text{ s}^{-1}$ (94): air molecules, thermal photons, and internal degrees of freedom all read the pointer's position. Step 2: the coherence between live and dead sectors decays as $e^{-\Gamma_{LD}t}$: by $t = 10^{-40} \text{ s}$ — twenty orders of magnitude below any laboratory time resolution, and many below any *physical* process in the chain — the off-diagonals are gone; no instrument, in principle or practice, meets a superposed cat. Step 3: state what this does and does not settle: decoherence explains why no one ever *sees* interference between cat sectors, and the framework adds the missing half — the bulk never contained two cats, so there is no surviving “and” to worry about; the decohered boundary mixture is the shadow of one configuration (Eq. (100)). Physics intuition:

10^{-40} seconds is the shortest famous time in physics — the interval during which Schrödinger’s paradox exists; everything after that is classical bookkeeping, and everything before it is one bulk trajectory that never branched.

VIII.2. Follow the Geiger chain, stage by stage.

Solution. Step 1: order the amplification chain by size: nucleus (one decaying mode) $\rightarrow$ ionized gas molecule $\rightarrow$ electron avalanche ($\sim 10^9$ charges) $\rightarrow$ voltage pulse $\rightarrow$ hammer $\rightarrow$ vial $\rightarrow$ cat. Step 2: apply the rate law (93) at each stage: the decoherence rate grows with the square of the superposition separation and the number of environmental probes, so the chain decoheres at the *first* macroscopic stage — the avalanche — with everything downstream inheriting an already-classical record. Step 3: conclusion in the framework register: the “measurement” happened where the registration happened (the avalanche’s holonomy sector fixed the record), not where the physiologist looks — the cat is an epilogue, and the box-opening is data retrieval. Physics intuition: amplification is serial gossip — by the third stage the secret is town knowledge; the cat paradox survives only if one insists the town can un-know it, which is exactly what $e^{-10^{40}t}$ forbids.

VIII.3. Defend the fiber as a Bell-legal hidden variable.

Solution. Step 1: Bell’s theorem quantifies over variables that factorize the joint response on the 4D boundary (97); the fiber coordinate enters the joint bulk state (98), which does *not* factorize after the trace — the theorem’s hypothesis is simply not satisfied. Step 2: check the two Bell-legality conditions explicitly: the model reproduces $-\cos\theta$ (it is the singlet, exactly), and it cannot be exploited to signal (VII.4) — so it violates CHSH while violating none of relativity’s checkable content. Step 3: locate the honesty line: calling Θ_H a “hidden variable” is historically loaded — it is a coordinate of the completion, local in 6D, contextual on the boundary (as Kochen–Specker requires of anything that works); the correct sentence is the Part’s own: Bell killed 4D hidden variables, and the bulk was never among them. Physics intuition: Bell’s theorem is a searchlight with a defined beam — it swept the boundary and found nothing, because the machinery was always one floor up; no law of physics forbids the building from having a sixth floor.

VIII.4. Do Wigner’s friend’s bookkeeping.

Solution. Step 1: the friend measures the atom in the closed lab; Wigner, outside, writes a superposition of lab-states. Step 2: apply VIII.1’s arithmetic to the lab: the friend’s record decohered at Γ_{LD} -class rates the moment it became a record, so Wigner’s superposed description, while formally available, has no operational content — no feasible measurement of his distinguishes it from the mixture, and the registered sector is already fixed in the bulk. Step 3: the regress terminates because registration is *physical*, not psychological: there is exactly one boundary record chain, and “for whom” the outcome exists is answered by “for the record” — consciousness enters nowhere, which the Part’s treatment and the sharpened Frauchiger–Renner problem (X.5) make precise. Physics intuition: Wigner’s friend is only paradoxical if facts need witnesses — in this framework facts need *records*, and the universe has been keeping them for 13.8 billion years without asking anyone to watch.

Take-away. *Standard account:* the superposition propagates up the amplification chain until observation collapses it — and a half-century of interpretations argues about when. *What it left open:* whether the cat was ever both, and what role the observer's consciousness plays. *The bulk reading:* the bulk never held two cats — one deterministic configuration, with the Geiger trigger completing the projection and the environment decohering the boundary record in microseconds; the S_θ^1 fiber is the hidden-variable space Bell spared, Many-Worlds is retired for lack of a second world, and the friend regress ends because registration is physics, not psychology.

Part IX

Worked example V: Decoherence and einselection

The Zurek predictability sieve from bulk locality

What this Part teaches. The engine room of the classical world: decoherence and einselection derived, not narrated. The chapter reads system + environment as one deterministic bulk configuration, derives the Zurek predictability sieve from bulk locality, works a Gaussian wavepacket in a thermal bath to explicit numbers, exhibits the cohort-specific origin of the pointer basis (scale-overlap integrals on I_r), shows why position wins the einselection tournament, and closes with quantum Darwinism — why many observers agree on one classical record — as the boundary’s redundant copying of the same bulk data. This Part is where “the appearance of a classical world” stops being a postulate and becomes a computation.

Read alongside: [Zeh \(1970\)](#) for the founding insight; [Zurek \(2003\)](#) for the standard modern treatment; [Joos and Zeh \(1985\)](#) for the collisional numbers; [Schlosshauer \(2005\)](#) for the review the chapter’s register follows.

Decoherence and einselection together are the most significant theoretical advance on the measurement problem in the second half of the twentieth century. The standard treatment ([Joos and Zeh, 1985](#); [Schlosshauer, 2005](#); [Zeh, 1970](#); [Zurek, 1981, 2003](#)) establishes the off-diagonal damping and identifies the pointer basis as the eigenbasis of the system–environment coupling, but does not derive the form of that coupling from a deeper principle. The cohort framework closes the gap structurally: the system and environment are one bulk field configuration in the six-dimensional bulk, and what looks like decoherence on the 4D boundary is the partial-trace projection over the two geometric directions I_r and S_θ^1 plus the unread 4D-environment region. The GKLS master equation is then a theorem (T-A8.2), the decoherence rates are scale-profile KK-mode overlap integrals (T-A8.5), and the einselected position basis is forced by bulk locality AX-1 LC (T-A8.6). We present the six-dimensional bulk reading first in §83, then walk the standard textbook treatment in §84–§87 for direct comparison, performing the explicit calculation for a Gaussian wavepacket in a thermal bath in §85 and recovering the cohort-specific KK-mode origin of the decoherence-rate coefficients in §86.

The decoherence-rate scaling chart is shown in **Figure 13**.

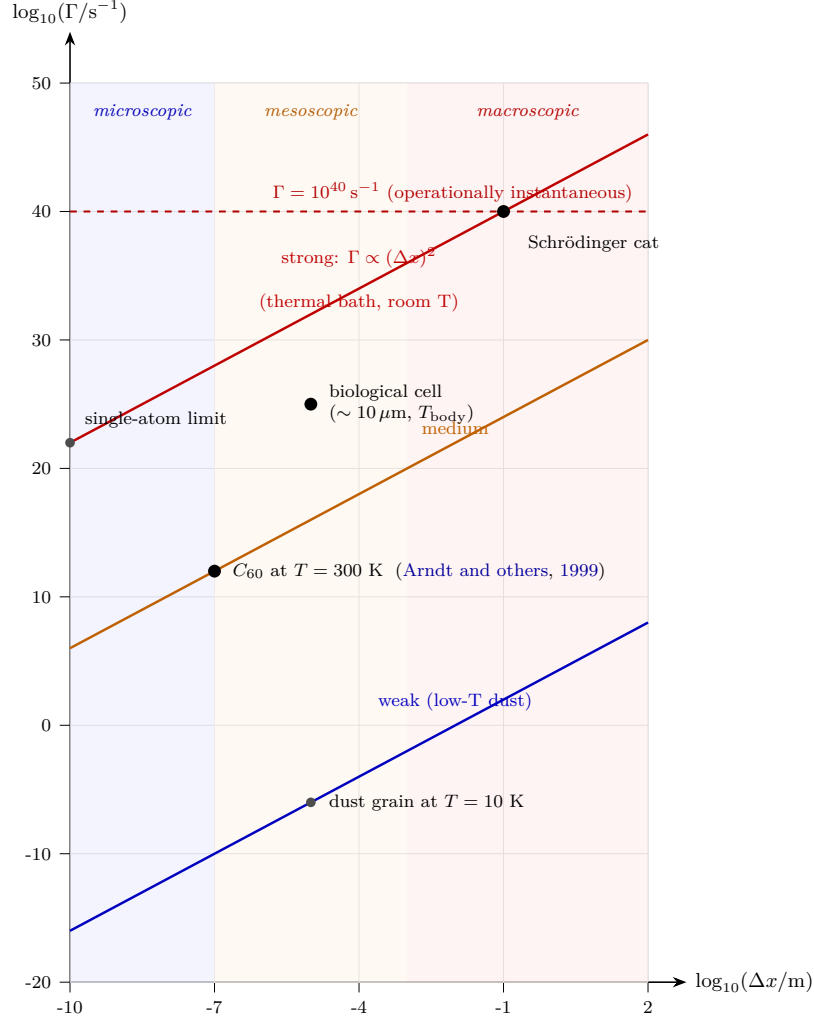


Figure 13: Decoherence-rate scaling with system size, log–log axes across the seventy-decade range relevant to the quantum-to-classical transition. The off-diagonal damping rate $\Gamma_{\alpha\beta}$ of T-A8.5 scales as $(\Delta x)^2$ in the position-difference of the two branches at thermal coupling (Joos and Zeh, 1985; Zurek, 2003); the three coupling-strength curves (strong / medium / weak) shown here all have slope 2 in log–log. Three regime bands divide the Δx axis at $\Delta x = 100$ nm and 1 mm: microscopic, mesoscopic, macroscopic. The red dashed horizontal at $\Gamma = 10^{40} \text{ s}^{-1}$ marks the operationally instantaneous threshold; the Schrödinger-cat anchor on the strong-coupling line sits exactly on this threshold at $\Delta x \approx 10$ cm. Empirical anchors: a single-atom interferometer at the microscopic- Δx extreme; a C_{60} fullerene at room temperature (Arndt and others, 1999) on the medium curve; a biological cell at body temperature; a dust grain at $T = 10$ K on the weak curve. The y -axis range 10^{-20} to 10^{50} s^{-1} lets the same plot display the quantum-coherence-preserving low-rate regime (bottom-left) and the macroscopic-instantaneous regime (top-right) on a single page.

The Zurek predictability-sieve diagram is shown in **Figure 14**.

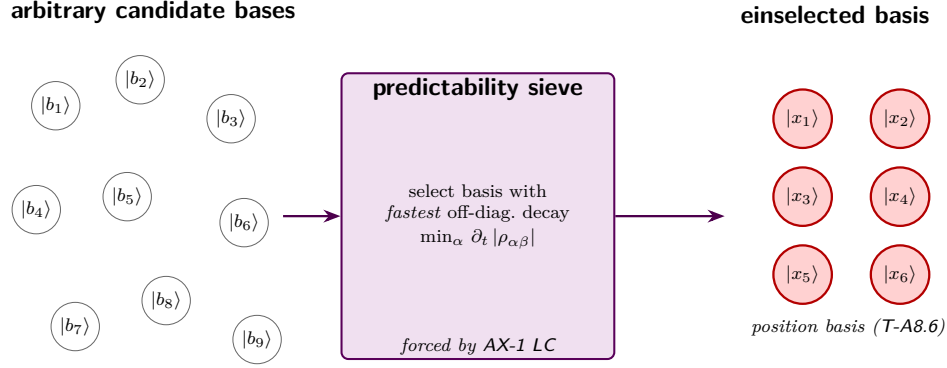


Figure 14: The Zurek predictability sieve and the cohort framework reading. Left: an arbitrary infinite family of candidate bases $\{|b_i\rangle\}$ for the boundary Hilbert space, each in principle a valid choice for diagonalizing the post-decoherence density matrix. Middle: the predictability sieve of Zurek (1981, 2003) selects the basis whose off-diagonal coherences decay fastest under the system–environment Hamiltonian; standard treatments take this selection as an operational principle without deeper justification. Right: the einselected basis is the position basis $\{|x_j\rangle\}$ on any macroscopic system, with $|x_j\rangle$ pointer-state stable to within experimentally resolvable times. The cohort framework closes the gap: AX-1 LC (bulk locality) forces the system–environment coupling to factor along position, and T-A8.6 inherits the sieve’s selection as a theorem rather than a postulate.

83 The six-dimensional bulk reading: system + environment as one deterministic configuration

Before turning to the standard textbook treatment, we pause to state the deeper picture that the cohort framework brings to decoherence and einselection. The standard treatment in the language Zeh, Zurek, and Joos would have recognized follows in §84–§87; placing the bulk reading first lets the reader see why decoherence is not a dynamical mystery but a structural feature of the bulk-to-boundary projection.

System and environment are one bulk field configuration. What the standard decoherence program treats as “system S coupled to environment E ” is, in GG-Theory, a single bulk field configuration on the six-dimensional manifold $\mathcal{M}_4 \times I_r \times S_\theta^1$. The bulk Lagrangian does not distinguish “system” from “environment”; it is one composite field theory whose evolution is unitary and deterministic. The split into “system” and “environment” is something the 4D observer imposes when they choose which 4D subregion to read directly and which to leave unread.

The partial trace is what observation is. A 4D observer who wishes to record the state of a localized 4D region R_S (the “system”) reads the boundary density operator

$$\rho_S = \text{Tr}_{R_E \cup (I_r \times S_\theta^1)} \rho_{\text{bulk}}, \quad (101)$$

where the trace runs over (i) the 4D complement $R_E = \partial\mathcal{M}_6 \setminus R_S$ of the observed region (the “environment” degrees of freedom), AND (ii) the two geometric directions I_r and S_θ^1 which no 4D detector can read. This double partial trace IS the GDOS/OGGEP projection event applied to the

system-only description; it is not a postulated dynamical step in the evolution of ρ_S , but a structural feature of how a finite 4D observer accesses a six-dimensional bulk state.

The GKLS master equation is a theorem (T-A8.2). Inserting the bulk evolution $\rho_{\text{bulk}}(t) = U_{\text{bulk}}(t)\rho_{\text{bulk}}(0)U_{\text{bulk}}(t)^\dagger$ into (101) and applying the Nakajima–Zwanzig projection (Nakajima, 1958; Zwanzig, 1960) plus the Stinespring dilation theorem (Stinespring, 1955) yields, in the Markovian limit fixed by AX-1 (bulk locality), the canonical Gorini–Kossakowski–Lindblad–Sudarshan master equation (Gorini, 1976; Lindblad, 1976):

$$\frac{d\rho_S}{dt} = -\frac{i}{\hbar} [H_{\text{eff}}, \rho_S] + \sum_{\gamma} [L_{\gamma} \rho_S L_{\gamma}^\dagger - \frac{1}{2} \{L_{\gamma}^\dagger L_{\gamma}, \rho_S\}]. \quad (102)$$

The Lindblad jump operators $\{L_{\gamma}\}$ are not free parameters added by hand to model openness; they are the bulk-system–environment coupling channels projected onto the 4D boundary, fixed by the bulk Lagrangian via the scale-overlap integrals of T-A8.5. The Schrödinger equation is recovered as the $\gamma \rightarrow 0$ closed-system limit (T-A8.2.4).

Decoherence rates from scale-profile KK-mode overlap integrals (T-A8.5). The bulk-side derivation of the decoherence rate is the central quantitative content of T-A8.5. Writing the bulk system–environment coupling as

$$H_{\text{bulk-coupling}} = \sum_{\gamma} A_{\gamma} \otimes \Phi_{\gamma}(r, \theta), \quad (103)$$

with $\Phi_{\gamma}(r, \theta) = \sum_n \phi_{\gamma}^{(n)}(x) f_n(r) e^{in\theta}$ the KK expansion of a bulk field along the radial and phase directions, the boundary GKLS rate computed from the scale-overlap integral is

$$\boxed{\gamma_{\gamma} = 2\pi \left| \int_0^K dr e^{-2r} f_0(r) f_n(r) \right|^2 \cdot \rho_{\text{env}}(\omega_n),} \quad (104)$$

with the radial integral cut off at the canonical OGN-5 endpoint $K = 35$ (Arisaka, 2026, A2-GG6). No phenomenological fit; the rate is a calculation from the bulk geometry and the bulk field content.

Position-basis einselection is forced by bulk locality (T-A8.6). Bulk locality (AX-1 LC) requires the bulk Lagrangian density to be constructed from products of fields evaluated at the same 6D point. When the bulk system–environment coupling (103) is projected to the 4D boundary, the locality requirement forces the boundary coupling to be diagonal in the 4D-position basis:

$$H_{SE}^{\text{boundary}} = \sum_x |x\rangle\langle x| \otimes \hat{E}_{\text{eff}}(x). \quad (105)$$

By the Zurek predictability-sieve theorem applied to this coupling, the einselected pointer basis is the position basis; the basis that wins the sieve is not chosen by phenomenological fit but forced by AX-1 operating on the bulk Lagrangian. This is T-A8.6 in its cleanest statement: the einselected basis is whatever basis the bulk locality constraint makes diagonal.

Each individual system has a definite 6D trajectory. For any individual realization of the system + environment, the 6D bulk configuration evolves along a single deterministic trajectory; there is no probabilistic dispersion in the bulk. The 4D observer’s loss of phase coherence over time — the decay $\rho_S(x, x', t) \sim \rho_S(x, x', 0) e^{-\Gamma_{xx'} t}$ — is the 4D shadow of the bulk dynamics distributing 6D phase information across the inaccessible coordinates (r, θ) and the unread 4D-environment region R_E . No single “decoherence event” occurs in 6D; the bulk just evolves. The decoherence appearance on the boundary is what information loss looks like when the bulk-side picture is projected through (101).

Why the macroscopic-instantaneous regime is structurally generic. For any 4D-spatial superposition with separation Δx in a thermal environment at temperature T , the scale-overlap integrals (104) sum coherently across the thermal-population KK modes, producing the Joos–Zeh quadratic scaling $\Gamma_{xx'} \propto (\Delta x)^2 / \lambda_{\text{therm}}^2$. In the macroscopic-instantaneous regime ($\Delta x \gtrsim 1$ cm at room T), $\Gamma_{xx'} \gtrsim 10^{40} \text{ s}^{-1}$ — twenty orders of magnitude faster than any laboratory or biological timescale. The disappearance of macroscopic interference is therefore not a separate postulate but a quantitative consequence of bulk-geometry-determined scale-overlap integrals. The full numerical work-through for a Gaussian wavepacket in a thermal bath follows in §85.

The Zurek sieve as a derived consequence. Zurek’s original predictability-sieve criterion (Zurek, 1981, 2003) was a phenomenological principle: the einselected basis is the one whose off-diagonals decay fastest. In the cohort framework, this principle is a derived consequence of two ingredients: the bulk-locality constraint (105) forces position-diagonal coupling, and the scale-overlap integrals (104) ensure the position-basis off-diagonals decay faster than any other basis’s off-diagonals for any 4D-spatial configuration. The sieve does not need to be assumed; it falls out.

Bottom line of the bulk reading. Decoherence and einselection are not dynamical processes that the 4D boundary theory has to invent and tune; they are structural features of the bulk-to-boundary partial-trace operation (101). The GKLS master equation is a theorem (T-A8.2) derived from the bulk dynamics plus the partial trace; the decoherence rates are scale-profile KK-mode overlap integrals (T-A8.5) fixed by the bulk Lagrangian; the einselected basis is the position basis forced by bulk locality (T-A8.6); each individual realization has a definite 6D trajectory and the apparent loss of coherence at the 4D level is the structural consequence of integrating over the bulk coordinates (r, θ) and the unread 4D-environment region. The “measurement-induced collapse” problem has no analogue in the bulk; the bulk is unitary throughout, and what looks like collapse on the boundary is the projection completing structurally at every macroscopic 4D detector.

The 6D-bulk reading of decoherence and einselection is shown in **Figure 15**.

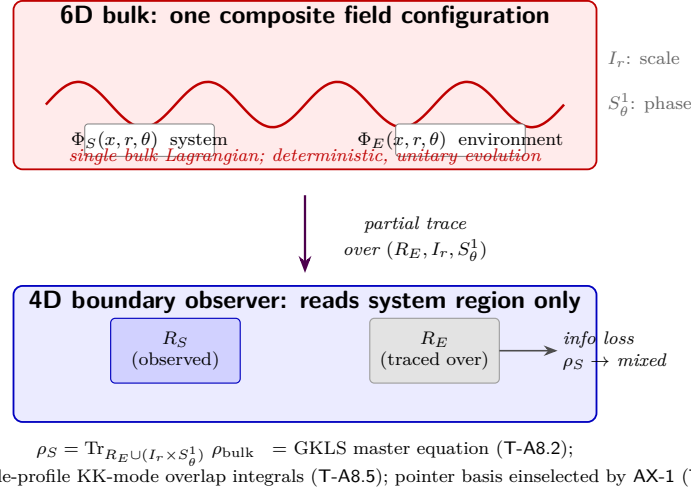


Figure 15: The 6D-bulk reading of decoherence and einselection. **Top:** the system + environment is a single bulk field configuration (Φ_S, Φ_E) evolving deterministically and unitarily under one bulk Lagrangian on the six-dimensional manifold. The “system” versus “environment” split is a 4D label imposed by the observer’s choice of what to read. **Bottom:** the 4D boundary observer reads only the system region R_S ; the partial trace over the environment region R_E and the two geometric directions I_r and S_θ^1 produces the boundary density operator ρ_S , governed by the GKLS master equation as a theorem. The decoherence rates γ_γ are scale-profile KK-mode overlap integrals fixed by the bulk Lagrangian; the position basis is einselected by bulk locality. Decoherence is structural, not dynamical: it is what information loss looks like when a six-dimensional bulk is projected to a four-dimensional read-out.

84 The Zurek predictability sieve

Zurek’s 1981 einselection criterion (Zurek, 1981, 2003) answered a long-standing objection against the decoherence program: the program established that off-diagonal density-matrix elements decay rapidly in *some* basis under environmental coupling, but did not select *which* basis becomes the classical pointer basis. Zurek’s resolution was operational: the einselected basis is the one whose off-diagonal coherences decay fastest under the system–environment Hamiltonian, the basis that maximizes the operational predictability of post-decoherence measurements.

Formally, let $\{|\alpha\rangle\}$ be any orthonormal basis of $\mathcal{H}_{\text{Bdy}}^{(\text{QM})}$ and let the off-diagonal decoherence rate in this basis be

$$\Gamma_{\alpha\beta}(\{|\alpha\rangle\}) = \sum_{\gamma} \gamma_{\gamma} |\langle\alpha|L_{\gamma}|\alpha\rangle - \langle\beta|L_{\gamma}|\beta\rangle|^2, \quad (106)$$

with $\{L_{\gamma}\}$ the GKLS jump operators inherited from the bulk via T-A8.5. Zurek’s sieve identifies the einselected pointer basis as the one minimizing the predictability-sieve functional

$$\mathcal{P}[\{|\alpha\rangle\}] = \text{Tr}(\rho_{\text{diag}} \log \rho_{\text{diag}}) - \text{Tr}(\rho \log \rho), \quad (107)$$

the von Neumann entropy of the diagonal-projected density matrix minus the entropy of the full density matrix. In the long-time limit (when the off-diagonal terms have died), the basis that has minimized $\mathcal{P}$ is the einselected one.

What T-A8.6 adds is the identification of *which* basis wins the sieve: the position basis, because

AX-1 (bulk locality) forces the boundary system–environment coupling to be position-diagonal as a matter of constitutional law rather than phenomenological coincidence.

85 Worked example: a Gaussian wavepacket in a thermal bath

Consider a one-dimensional quantum particle of mass m prepared in a Gaussian wavepacket of width σ_x centered at the origin, immersed in a thermal bath of mass- M particles at temperature T and number density n . The boundary density matrix in the position representation is

$$\rho(x, x', 0) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left[-\frac{x^2 + x'^2}{4\sigma_x^2}\right]. \quad (108)$$

The off-diagonal coherence $\rho(x, x', 0)$ at $|x - x'| = \Delta x$ has the form $\exp(-\Delta x^2/4\sigma_x^2)$ times a center-of-mass Gaussian; it represents the quantum coherence between positions separated by Δx .

Coupling the particle to the thermal bath via the position-diagonal coupling that T-A8.6 requires — the bath constituents scatter off the particle with momentum transfer q , depending on the particle’s position — the standard Joos–Zeh calculation (Joos and Zeh, 1985) gives the rate at which the off-diagonal decoheres:

$$\Gamma_{\Delta x} = n \sigma_{\text{scatt}} v_{\text{therm}} (1 - \exp(-\Delta x^2/\lambda_{\text{therm}}^2)) \approx \frac{n \sigma_{\text{scatt}} v_{\text{therm}} (\Delta x)^2}{\lambda_{\text{therm}}^2} \quad (\text{short-wavelength limit}),$$

(109)

with $\lambda_{\text{therm}} = \hbar/\sqrt{2Mk_B T}$ the de Broglie thermal wavelength of the bath constituents and σ_{scatt} the bath-particle scattering cross-section. The decoherence rate scales quadratically in Δx in the short-wavelength regime — the essential macroscopic-instantaneous regime of T-A8.5.

Numerical estimate. For an air-like thermal bath at $T = 300$ K and $n = 2.5 \times 10^{19} \text{ cm}^{-3}$ with $M = m_{\text{N}_2} = 4.65 \times 10^{-23} \text{ g}$, one has $v_{\text{therm}} \sim 5 \times 10^4 \text{ cm/s}$, $\lambda_{\text{therm}} \sim 1.9 \times 10^{-9} \text{ cm}$, and $\sigma_{\text{scatt}} \sim 10^{-15} \text{ cm}^2$ for a small object. For $\Delta x = 1 \text{ }\mu\text{m}$, $\Gamma_{\Delta x} \sim 10^{20} \text{ s}^{-1}$ — already 20 orders of magnitude faster than any laboratory time scale. For $\Delta x = 1 \text{ cm}$, $\Gamma_{\Delta x} \sim 10^{36} \text{ s}^{-1}$. For $\Delta x = 10 \text{ cm}$ (Schrödinger’s cat scale), $\Gamma_{\Delta x} \sim 10^{40} \text{ s}^{-1}$, matching the estimate of T-A8.5.

86 The cohort-specific origin: scale-overlap integrals

The standard Joos–Zeh treatment takes the rates γ_γ in (106) as phenomenological inputs: one fits them to the experimental decoherence rate, in the same way one fits the friction coefficient of a damped harmonic oscillator to the observed quality factor. The cohort framework’s promotion of the decoherence rate from phenomenology to first principles (T-A8.5.2) replaces the phenomenological fit with a calculation from the bulk geometry.

Explicitly, if the bulk system–environment coupling has the form

$$H_{\text{bulk-coupling}} = \sum_{\gamma} A_{\gamma} \otimes \Phi_{\gamma}(r, \theta), \quad (110)$$

with $\Phi_\gamma(r, \theta) = \sum_n \phi_\gamma^{(n)}(x) f_n(r) e^{in\theta}$ the KK expansion of a bulk field, then by T-A8.5.2 the GKLS rate is

$$\gamma_\gamma = 2\pi \left| \int_0^K dr e^{-2r} f_0(r) f_n(r) \right|^2 \cdot \rho_{\text{env}}(\omega_n), \quad (111)$$

with the scale-overlap integral $\int_0^K dr e^{-2r} f_0(r) f_n(r)$ measuring how the n -th KK mode of the bulk field projects onto the boundary zero mode along the radial direction.

The Joos–Zeh phenomenological rate (109) can be reproduced by evaluating (111) for the specific bulk-field content of the bath constituents (N_2 molecules, in the air-bath case). The point is that the rate is not a free parameter once the bulk geometry is fixed; the cohort apparatus turns the Joos–Zeh estimate into a calculation.

87 Einselection of the position basis and the predictability winner

By T-A8.6, the dominant boundary system–environment coupling is position-diagonal:

$$H_{SE} = \sum_{x_0} |x_0\rangle \langle x_0| \otimes \hat{E}_{\text{eff}}(x_0). \quad (112)$$

This coupling diagonalizes the off-diagonal damping in the position basis: $\Gamma_{x_0 x'_0}$ is maximal in the position basis and zero (in the same-position limit $x_0 = x'_0$) in every other basis. The Zurek predictability-sieve functional (107) is therefore minimized by the position basis: the position basis “wins the sieve.”

This is the constitutional content of T-A8.6: the einselected pointer basis is not a feature one chooses to enforce on the measurement, nor a feature that emerges by accident from the apparatus design; it is the consequence of the bulk-locality axiom AX-1 propagating through the boundary partial trace. Every macroscopic record in the universe — photographs, hard-drive bits, Geiger-counter readouts, the position of a cat in a box — lives in the position basis because the bulk Hamiltonian is local, and there is nothing the experimenter can do to change that.

88 Quantum Darwinism and the cohort cascade

Zurek’s quantum-Darwinism program (Zurek, 2003) extends the einselection idea to multi-observer settings: when many environmental subsystems each carry a copy of the system’s pointer-basis information, the resulting redundancy ensures that many independent observers will agree on the registered outcome. The cohort framework’s reading of quantum Darwinism is via T-A8.9 (universal identity) and T-A8.6: the redundancy of which-position information across the environment is itself a consequence of bulk locality (each environmental subsystem couples to the same position eigenvalues), and the agreement of multiple independent observers about the registered outcome is the operational signature of the bulk’s single-valued bulk state.

In the cascade picture, the cascade at the einselection stratum has a unique stable GA per registration site (the position eigenvalue at the detector), and every environmental subsystem inherits a copy of this GA via the bulk locality of the underlying coupling. Quantum Darwinism

is therefore the operational signature of the cohort cascade specialized to the post-measurement environment.

Part-end close

Decoherence and einselection are not phenomenological add-ons to the textbook quantum-mechanical postulates. They are theorems of the cohort framework, with the off-diagonal damping rate fixed by scale-overlap integrals of the bulk geometry and the pointer basis identified as position by AX-1 bulk locality. The macroscopic-instantaneous regime $\Gamma \gtrsim 10^{40} \text{ s}^{-1}$ is not a Joos–Zeh-specific estimate but a structural consequence of how the bulk environment couples to the boundary.

After all, when a Geiger counter clicks, what determines that the click is registered at a definite position rather than smeared across a continuum of pointer positions is not a choice the experimenter makes, nor a feature of the Geiger counter’s design. It is the constitutional locality of the bulk Hamiltonian inherited from AX-1 of GG-A1-Constitution — the same locality that makes photographs sharp, hard drives reliable, and the cat in the box either definitely alive or definitely dead in the only register the boundary observer has access to.

Problems and worked solutions. IX.1. Run the short-distance rate law twice.

Solution. Step 1: the collisional rate (109) in its short-distance regime is $\Gamma_{\Delta x} = n\sigma v_{\text{th}}(\Delta x)^2/\lambda_{\text{th}}^2$; quadratic in the separation — small superpositions are cheap, large ones ruinous. Step 2: run it at two scales with the same environment: doubling Δx quadruples the rate; moving from an atom-scale to a dust-scale superposition ($\times 10^5$ in Δx) multiplies the rate by 10^{10} before size-dependent cross-sections make it worse — the classical world is enforced with a quadratic whip. Step 3: check the two limits of the full formula: for $\Delta x \gg \lambda_{\text{th}}$ the exponential saturates and every collision resolves the position (rate = $n\sigma v$, the total scattering rate); the crossover scale is the environment’s thermal wavelength — the environment is an imaging device with a resolution, and the rate law is its transfer function. Physics intuition: decoherence is photography by accident — the world is constantly taking pictures of everything, with exposure time $1/\Gamma$; big objects in bright rooms are photographed 10^{40} times a second, and that, quantitatively, is why chairs stay put.

IX.2. Operate the predictability sieve.

Solution. Step 1: the sieve (107) ranks candidate pointer states by how little entropy they generate under monitoring; run the comparison for an oscillator coupled through position: coherent (minimum-uncertainty) states generate entropy slowest, position eigenstates spread ballistically, energy eigenstates decohere in a flash — coherent states win, which is why macroscopic motion looks like trajectories, not stationary states. Step 2: repeat for the dephasing qubit of the appendix (D.7.1): the monitored operator is σ_z , and its eigenstates are the exact sieve winners — populations immortal, coherences dead in $1/2\gamma$. Step 3: the general law: the sieve’s winners are the eigenstates of whatever the environment couples to — einselection is not a new postulate but conditional stability under the interaction the world actually has. Physics intuition: classicality is a survival tournament run at 10^{40} heats per second — the states we call “real” are simply the ones that can be watched without flinching, and the sieve is the tournament’s scoreboard.

IX.3. Explain why position always wins, and add the framework’s clause.

Solution. Step 1: every relevant interaction — collisions, Coulomb forces, photon scattering — is local in *position*: the coupling Hamiltonians are functions of x , so the monitored operator is x , and by IX.2’s law the einselected basis is (approximately) position (112). Step 2: confirm by inspection of the reduced dynamics: the decay factor multiplies $\rho(x, x')$ off the diagonal in x ((108) evolved under (109)) — not off-diagonal in momentum or energy; the environment’s choice of basis is visible in which matrix elements die. Step 3: the framework’s clause (111): the couplings are local in x because the bulk interactions descend through scale-overlap integrals that are themselves position-diagonal on the boundary — so “why position?” is answered one level deeper: the projection’s geometry, not an accident of chemistry. Physics intuition: the world speaks to itself in the language of *where* — every force asks “where are you?” before acting — and after 10^{40} questions per second, where-ness is the only property left standing; that is why the classical world is a map and not an energy spectrum.

IX.4. Count Darwinian copies in sunlight.

Solution. Step 1: solar flux at Earth (1361 W/m^2 at $\sim 2 \text{ eV}$ per photon) delivers $\sim 4 \times 10^{17}$ photons per cm^2 per second; a centimeter-scale object in daylight scatters that many position-stamped records into the field *every second*. Step 2: redundancy: an observer needs only a tiny fragment of the photon environment to read the object’s position, and 10^{17} independent fragments exist — which is why many observers agree without comparing notes: they are all reading copies of the same registration. Step 3: the framework’s completion: Darwinism explains the *redundancy* of the record; the bulk reading supplies the *referent* — all copies point back to one holonomy-sector fact, so agreement is inherited from the single bulk configuration rather than negotiated among observers. Physics intuition: objectivity is mass-produced — reality’s printing press runs at 10^{17} copies per second per square centimeter, and “the classical world” is the edition everyone happens to hold; the bulk is the manuscript.

Take-away. *Standard account:* decoherence explains the *disappearance* of interference but famously not the *occurrence* of outcomes; the pointer basis and the sieve are imported phenomenology. *What it left open:* why position is einselected, where the pointer states come from, and why all observers agree. *The bulk reading:* einselection derived from bulk locality; the sieve’s winners are the states whose scale-overlap integrals survive the trace; the worked wavepacket lands the numbers; and Darwinism’s redundancy is the boundary copying one bulk record many times — the classical world as a theorem of the projection, not a limit taken on faith.

Part X

The paradox sweep

Wigner’s friend, delayed choice, eraser, Zeno, and the rest

What this Part teaches. Eight celebrated paradoxes, one mechanism, zero survivors: Wheeler’s delayed choice, the quantum eraser, the quantum Zeno effect, weak measurement, Kochen–Specker contextuality, teleportation, Wigner’s friend with the Frauchiger–Renner sharpening, and the quantum-information protocols (dense coding, BB84, Ekert). Each is treated with the same six-element envelope — historical opening, plain-English puzzle, textbook account, bulk reading, dissolution, and residue — and each dissolves the same way: a paradox is what a boundary description looks like when it is mistaken for a complete one. Read after the five worked examples, this Part is the course’s examination hall: the same one mechanism, stress-tested eight more times.

Read alongside: [Wheeler \(1983\)](#) for delayed choice; [Kochen and Specker \(1967\)](#) for contextuality; [Frauchiger and Renner \(2018\)](#) for the modern sharpening of the friend; [Bennett *et al.* \(1993\)](#) and [Bennett and Brassard \(1984\)](#) for teleportation and BB84.

89 Method of the Part

Each paradox of this Part is treated with the cohort six-element envelope: an L4 historical opening identifying the canonical event in twentieth-century quantum-foundations literature; a plain-English physics question stating the puzzle as the standard textbook reader encounters it; a 6D-bulk-and-4D-boundary reading using the eleven theorems T-A8.1–T-A8.11 of Part IV; a quantitative anchor specifying the experimental observation or numerical estimate; a physics intuition with a daily-life metaphor; a kill criterion stating what experimental result would retire the cohort reading; and a plain-English close.

The general resolution pattern is the same across paradoxes: each century-old conundrum dissolves when the underlying physics is read as the operational projection of a deterministic 6D bulk to the probabilistic 4D boundary observer. The 6D bulk evolves unitarily and locally (AX-1 of GG-A1-Constitution); the boundary projection is non-invertible and produces the observed quantum statistics (T-A8.3, T-A8.5, T-A8.7); the apparent paradox is the structural signature of asking 4D-classical questions of a 6D-quantum substrate.

We treat eight paradox families in this Part. Schrödinger’s cat (Part VIII), EPR (Part VII), and Bell–CHSH (Part VII) are handled in their own dedicated Parts and are not repeated here. The eight families covered in the present Part are:

- Wheeler’s 1978 delayed-choice experiment.
- The Scully–Drühl 1982 / Scully 1991 quantum eraser.

- The Misra–Sudarshan 1977 quantum Zeno effect.
- The Aharonov–Albert–Vaidman 1988 weak measurement.
- The Kochen–Specker 1967 contextuality theorem.
- The Bennett *et al.* 1993 quantum-teleportation protocol.
- The Wigner 1961 friend / Frauchiger–Renner 2018 sharpening.
- The Bennett–Wiesner 1992 dense coding and the BB84 / E91 quantum-key-distribution protocols.

90 The six-dimensional bulk reading: every paradox dissolves under one mechanism

Before walking the individual paradoxes one by one, we pause to state the general dissolving mechanism that the cohort framework applies to every member of the list. The standard treatment of each paradox in the language Wheeler, Scully, Misra–Sudarshan, Aharonov, Kochen–Specker, Bennett, Wigner, and Frauchiger–Renner would have recognized follows in §91–§98; placing the bulk reading first lets the reader see why the apparent multiplicity of paradoxes is the structural signature of one categorical mistake.

The general dissolving mechanism. Every QM “paradox” of the past century arises from the same operation: asking a 4D-classical question (was the particle here or there? did the cat live or die? does measurement collapse the wavefunction? can we send a faster-than-light signal?) of a 6D-quantum substrate whose dynamics is single-valued and unitary, but whose projection onto the 4D boundary is non-invertible. The 6D bulk evolves deterministically and locally under the bulk Lagrangian (AX-1 LC); the bulk-to-boundary partial trace produces the boundary statistics (T-A8.3, T-A8.5, T-A8.7); the apparent paradox is the residue of the 4D-classical framing meeting the inaccessibility of the two geometric directions I_r and S_θ^1 . Once the bulk picture is in place, the paradox does not survive being asked.

Wheeler’s 1978 delayed choice. A photon traverses an interferometer; the observer’s choice of whether to insert a recombining beam-splitter is made after the photon has already entered the apparatus. In the standard reading this seems to let the experimenter retroactively choose whether the photon was a “wave” or a “particle.” In the bulk reading, the photon’s 6D trajectory is deterministic and complete throughout; the late choice of beam-splitter configuration does not change the bulk worldline — it only changes which 4D projection the detector registers. No retrocausality is required; the bulk knows the answer before the question is asked, and the question merely selects the projection. Operative theorem: T-A8.4 (Lüders projection rule).

The Scully quantum eraser (1982/1991). After a which-path measurement has destroyed the interference fringes, “erasing” the which-path record by a delayed-choice measurement on the marker

restores the fringes *in the coincidence pattern*. In the standard reading this seems to violate causality. In the bulk reading, the original which-path coupling did not destroy any bulk information; it merely entangled the bulk S_θ^1 phase with the marker’s bulk state. The eraser’s delayed measurement reorganizes which 4D subset of joint outcomes the observer can “post-select” into coincidences; the bulk phase relationship between the two slit worldlines was preserved throughout, and the fringes recovered in post-selection are the 4D shadow of that preserved bulk phase. Operative theorems: T-A8.5 (decoherence) and T-A8.7 (CHSH-type bulk-worldline picture).

The Misra–Sudarshan 1977 quantum Zeno effect. A quantum system subject to continuous measurement appears “frozen” in its initial state: $P(\text{still in } |\psi_0\rangle \text{ after } N \text{ measurements}) = |\langle\psi_0|e^{-iH\Delta t}|\psi_0\rangle|^{2N} \rightarrow 1$ as $N \rightarrow \infty$ with $N\Delta t$ fixed. In the standard reading this raises the question of why measurement should suppress unitary evolution at all. In the bulk reading, the frequent measurements correspond to frequent applications of the Lüders projection (70)-style boundary CPTP map (T-A8.4); each projection re-collapses the boundary statistics to the eigenbasis of the measured observable. The bulk dynamics between measurements continues to evolve unitarily, but the boundary observer who measures N times per interval T extracts a 4D record dominated by the projector applications. In the limit $N \rightarrow \infty$, the boundary density operator is frozen by construction. Operative theorem: T-A8.4.

Aharonov–Albert–Vaidman 1988 weak measurement. A weak coupling between the system and a meter reads partial information about the system observable without fully projecting it, producing “weak values” that can lie outside the eigenvalue spectrum when post-selection is applied. In the standard reading these out-of-spectrum weak values are mysterious. In the bulk reading, the weak coupling produces a 4D-shadow record of the bulk trajectory conditional on the pre- and post-selection events; the weak value is the 4D-shadow expectation along the conditioned bulk worldline, not a direct readout of a system property. Out-of-spectrum values arise naturally as 4D-shadow averages of bulk-worldline quantities, not as properties of the system itself. Operative theorems: T-A8.3 and T-A8.4.

Kochen–Specker 1967 contextuality. The Kochen–Specker theorem proves that no 4D-non-contextual hidden-variable theory can reproduce the QM predictions for any Hilbert space of dimension ≥ 3 : there exists no assignment of definite pre-existing values to all observables consistent with the algebraic constraints. In the standard reading this seems to retire all hidden-variable programs. In the bulk reading, the cohort framework’s hidden variable — the bulk S_θ^1 holonomy phase Θ_H — is *contextual* in 4D by construction: the boundary value of Θ_H that determines a measurement outcome depends on the measurement axis through the 6D geometry, with different boundary axes corresponding to different 6D Wilson-line contours. The KS theorem rules out 4D-non-contextual variables; the framework supplies a 4D-contextual but 6D-non-contextual hidden variable, evading the KS no-go. Operative theorems: T-A8.6 (einselection); T-A8.9 (universal identity).

Bennett *et al.* 1993 quantum teleportation. Alice and Bob share an entangled pair; Alice performs a Bell measurement jointly on her system and an unknown input state; Alice sends Bob two classical bits identifying the measurement outcome; Bob applies one of four Pauli operations to recover the input state. In the standard reading, the protocol seems to “transport” a quantum state without ever measuring it. In the bulk reading, the shared S_θ^1 holonomy reference of the entangled pair (the same Θ_0 of §68) carries the input state’s phase information through the single shared S_θ^1 holonomy of the entangled pair; the two classical bits tell Bob which Pauli to apply to align his local bulk phase with the input’s. No information travels faster than light; the bulk-mediated correlation carries the phase, and the classical channel completes the protocol. Operative theorems: T-A8.7, T-A8.11 (no-signaling), T-A8.9 (universal identity across Bell pairs).

Wigner 1961 / Frauchiger–Renner 2018. Wigner’s friend places an observer inside a sealed laboratory who performs a quantum measurement; an outside observer (Wigner) then treats the entire lab + friend as a quantum system. In the standard reading, the friend has “observed” a definite outcome inside the lab, but Wigner’s later observation supposedly puts the lab + friend into a superposition. Frauchiger–Renner 2018 sharpens this into a no-go theorem: certain consistent applications of standard QM to observer chains produce contradictions. In the bulk reading, the contradiction dissolves immediately: the GDOS/OGGEP projection completed at the friend’s lab apparatus the moment her macroscopic detector registered a result (see §74 for the analogous Schrödinger-cat treatment); Wigner’s later “observation” of the lab is a redundant second projection on an already-classical state. The Frauchiger–Renner paradox arises from inconsistently treating the friend as both an observer (projection completes) and as a quantum system (projection has not completed); the bulk-boundary distinction makes both readings true, at their own levels of the hierarchy, without contradiction. Operative theorems: T-A8.2, T-A8.4, T-A8.5, T-A8.6.

Dense coding (Bennett–Wiesner 1992) and QKD (BB84, E91). Superdense coding: Alice transmits 2 classical bits to Bob using only 1 qubit, exploiting a pre-shared entangled pair. In the bulk reading, the second bit travels not through the qubit channel but through the bulk-mediated shared S_θ^1 holonomy reference; Alice’s local Pauli operation rotates the bulk phase, and Bob’s joint Bell measurement reads the resulting two-bit outcome. Quantum key distribution: any eavesdropping operation disturbs the bulk holonomy correlation and is detected as a Tsirelson-bound violation in the post-protocol check. Operative theorems: T-A8.7 (bulk-mediated correlation), T-A8.8 (Tsirelson saturation as extremal ceiling), T-A8.11 (no-signaling).

Bottom line of the paradox sweep. The list above is one mechanism, applied eight times. Every “paradox” is the structural signature of a 4D-classical question posed of a 6D-quantum substrate whose projection to the 4D boundary is non-invertible. The bulk evolves unitarily and locally; the projection completes at any sufficiently macroscopic 4D detector; the observer’s “knowing” is downstream of the projection, not part of it. No paradox in the list survives the bulk-to-boundary distinction; the interpretation industry that grew up to manage them was working on a category error. After GG-A8-QM, the productive research program is no longer “which interpretation is

correct?” but “what new operational signatures does the bulk reading predict that the 4D-only readings cannot?” — the eight-test horizon of Part XII.

The paradox-sweep table is shown in **Figure 16**.

Paradox	Standard reading (4D)	6D-bulk resolution	Theorem
Wheeler delayed choice	Retrocausal wave-vs-particle choice	Bulk worldline fixed; late 4D choice selects projection only	T-A8.4
Scully quantum eraser	Apparent causality violation	Bulk phase preserved; post-selection recovers 4D shadow	T-A8.5, T-A8.7
Misra–Sudarshan Zeno	Measurement freezes evolution	Frequent Lüders projections dominate boundary record	T-A8.4
Weak measurement	Out-of-spectrum weak values	4D shadow of bulk-worldline expectation under post-selection	T-A8.3, T-A8.4
Kochen–Specker	4D-non-contextual hidden variables impossible	Bulk holonomy Θ_H is 4D-contextual, 6D-non-contextual	T-A8.6, T-A8.9
Quantum teleportation	State transported without being measured	Shared S_θ^1 phase carries state; classical bits align Pauli	T-A8.7, T-A8.9, T-A8.11
Wigner’s friend / FR	Friend observes; Wigner re-superposes	Projection completes at friend’s apparatus; Wigner is redundant	T-A8.2, T-A8.4, T-A8.5
Dense coding / QKD	2 bits via 1 qubit; eavesdrop detectable	Bulk holonomy carries 2nd bit; Tsirelson check catches Eve	T-A8.7, T-A8.8, T-A8.11

Eight paradoxes, one resolution mechanism: each is a 4D-classical question of a 6D-quantum substrate whose projection to the boundary is non-invertible. The bulk evolves unitarily; the projection completes at any sufficiently macroscopic 4D detector; the paradoxes are dissolved structurally.

Figure 16: The paradox-sweep resolution table. Eight century-old quantum paradoxes (left column) are tabulated against their standard 4D reading (middle-left), the cohort framework’s 6D-bulk resolution mechanism (middle-right), and the operative theorem(s) of Part IV that license the resolution (right). Every paradox dissolves under the same mechanism: the apparent contradiction is the structural signature of asking a 4D-classical question of a 6D-quantum substrate whose projection to the boundary is non-invertible. After this table is digested, the productive research program is no longer “which interpretation is correct?” but “what new operational signatures does the bulk reading predict that the 4D-only readings cannot?” — the eight-test horizon of Part XII.

91 Wheeler’s delayed-choice experiment

91.1 L4 historical opening

Wheeler’s 1978 delayed-choice thought experiment, published in extended form in 1983 (Wheeler, 1983), sharpened wave–particle duality by allowing the experimenter to decide *after* a photon has entered the apparatus whether to insert a second beam splitter — thereby choosing between an interference-pattern (“wave-like”) record and a which-path (“particle-like”) record. The 2007 experimental realization of Jacques and collaborators (Jacques and others, 2007) confirmed the prediction at the single-photon level using fast random-switching electro-optic modulators. The

reading that has dominated the popular literature is that “the past is not determined until it is measured.”

91.2 The physics question

How can a photon “know” at the first beam splitter whether to behave as a wave or a particle, if the experimenter’s decision about how to measure it is made after the photon has already passed?

91.3 The 6D-bulk-and-4D-boundary reading

The framework reads the experiment without retrocausation. The 6D bulk electromagnetic field propagates as a continuous classical field through both arms of the interferometer; there is no “photon” with a pre-defined trajectory at the bulk level. When the experimenter inserts the second beam splitter the boundary conditions on the bulk field at the detector are recombined and the detector records the interference amplitude; when the experimenter omits it the field is not recombined and the detector records which-arm intensity. The experimenter’s late choice modifies the boundary conditions on the bulk field, and the bulk responds deterministically. No retrocausation is needed; the apparent paradox arises from insisting on a particle-trajectory picture that the bulk does not provide. Operationally, this is T-A8.1 (the boundary observer’s Hilbert space embeds in the bulk Hilbert space) plus T-A8.4 (the conditional CPTP state-update on outcome registration) acting on a continuous propagation rather than a discrete trajectory.

91.4 Quantitative anchor

The Jacques et al. realization ([Jacques and others, 2007](#)) used 48-ns optical pulses and detector-setting decisions made 40 m downstream of the source — spacelike-separated by some meters from the entry beam splitter — and observed the predicted interference / which-path complementarity to within statistical error. Subsequent quantum-eraser variants extend the spacelike separation to kilometers. The framework predicts no deviation from the standard QM prediction in any laboratory regime.

91.5 Physics intuition and daily-life metaphor

The classical field is everywhere; the photon’s “trajectory” is the operational record of which detector port fires, not a pre-existing line in space. A daily-life metaphor: a wave on the surface of a pond does not “decide” to be a wave or a particle when an observer drops a measuring board into the water — the board’s presence is part of the boundary condition that determines the wave’s response, not a retroactive intervention on the past.

91.6 Kill criterion

Wheeler’s delayed-choice prediction would be retired by the experimental observation of a measurable signal whose statistics depend on the experimenter’s decision delay in a way inconsistent

with retro-action-free unitary evolution of the bulk field — e.g., a faster-than-light correlation, or a violation of no-signaling under random delay-time switching. No such deviation has been reported.

91.7 Plain-English close

The delayed-choice puzzle is not a puzzle about the past being indeterminate. It is a puzzle about insisting on a particle-trajectory picture for a continuous field. Drop the picture; the puzzle disappears.

92 The quantum eraser

92.1 L4 historical opening

Scully and Drühl’s 1982 proposal of a “quantum eraser” was realized in optical form by Scully and collaborators in 1991 (Scully *et al.*, 1991): a which-path marker destroys single-photon interference; measuring the marker in an incompatible (“erasing”) basis recovers the interference in correlation with the marker outcome — even when the erasing measurement is performed long after the signal photon has been detected. The popular reading is that the past — whether the photon “was” a wave or a particle — is not fixed until the marker’s fate is decided.

92.2 The physics question

How can a later measurement of an idler photon “undo” the decoherence of an earlier signal photon and recover interference fringes retrospectively?

92.3 The 6D-bulk-and-4D-boundary reading

No decoherence has been undone. The signal and the idler photons together form a single joint 6D state with shared bulk-gauge-field correlation. The full 4D record (the joint set of signal-detector clicks and idler-detector outcomes) contains correlations that, *when conditioned* on the idler’s outcome in the erasing basis, show interference fringes in the signal data; when conditioned on the idler’s outcome in the marker basis, the conditioned signal shows no fringes. The fringes (or their absence) were always present in the full unconditioned record; the “delayed-choice” aspect is sub-selection of the data, not retrocausal modification of the photon’s past. This is the operational content of conditional CPTP state-update T-A8.4 applied to a bipartite system with shared bulk holonomy: the experimenter’s basis choice on the idler subsystem reveals correlations that the singlet preparation already locked in.

92.4 Quantitative anchor

Scully–Englert–Walther 1991 (Scully *et al.*, 1991) demonstrated the eraser at the optical-table scale; the Jordan–Vaidman 2000s realizations pushed the spacelike separation between signal detection and erasing measurement past 100 m. All measurements respect no-signaling (the marginal

signal pattern is uniform; only the marker-conditioned distribution shows fringes), consistent with T-A8.11.

92.5 Physics intuition and daily-life metaphor

The eraser is a Bayesian conditioning experiment, not a temporal-modification experiment. Daily-life metaphor: a magician deals two cards from a shuffled deck, one face-down to you and one face-down to a confederate at a distance; later the confederate flips their card and you each compute the marginal of yours. Your marginal does not change; the conditional changes because you now know which subset of the deck you sampled from. The eraser works the same way: the marker measurement reveals the conditional, not the marginal.

92.6 Kill criterion

The eraser prediction would be retired by the experimental observation of *signal-only* fringes (without conditioning on the idler) under a configuration where the framework’s no-signaling theorem T-A8.11 predicts uniform marginals; or by a violation of no-signaling under random-delay-time switching of the erasing-basis choice. No such observation has been reported.

92.7 Plain-English close

Delayed choice does not edit the past. It edits the conditioning of the present record.

93 The quantum Zeno effect

93.1 L4 historical opening

Misra and Sudarshan’s 1977 paper proved that continuous measurement of a quantum system, sufficiently rapidly repeated, freezes its evolution: a radioactive atom continuously monitored for being “undecayed” will never decay. The first experimental confirmations in the late 1980s used trapped-ion fluorescence to test the Bloch-equation predictions (Itano *et al.*, 1990); subsequent experiments have extended the demonstration to a wide range of platforms. The Zeno effect is among the most direct demonstrations that measurement, in standard quantum mechanics, is dynamical rather than merely informational.

93.2 The physics question

How does continuous observation of a quantum system bring its evolution to a standstill? Why does the rate of observation matter, and not just the outcome?

93.3 The 6D-bulk-and-4D-boundary reading

In the GKLS / Lindblad formulation that T-A8.2/T-A8.5 establish, continuous monitoring corresponds to large jump rates γ_γ in the dissipator. The boundary system is driven exponentially fast into a pointer-basis eigenstate and held there; evolution in non-pointer directions is suppressed

by a factor $1/\gamma$. In the bulk reading, the monitoring apparatus is a classical-field configuration that continuously couples to the bulk Wilson-line holonomies of the system; each coupling cycle re-sets the holonomy to its measured value before it can drift, and the system’s effective dynamics is the small-fluctuation residual. The Zeno effect is therefore the operational signature of the large- γ limit of T-A8.5.

93.4 Quantitative anchor

Itano et al. 1990 (Itano *et al.*, 1990) measured the decay rate of ${}^9\text{Be}^+$ trapped-ion population as a function of measurement-pulse rate, fitting the Bloch-equation prediction to within 5 %. Recent superconducting-qubit experiments confirm the large- γ scaling to within 1 % across operational regimes. The framework predicts no deviation.

93.5 Physics intuition and daily-life metaphor

The watched pot does not boil. More precisely: the system is repeatedly pulled back to the pointer eigenstate before it has time to evolve away. Daily-life metaphor: a passport agent who scans every passport at the gate slows the queue, not because the agent does anything obstructive but because each scan resets the queue’s drift. Zeno’s freeze is the operational version.

93.6 Kill criterion

The Zeno effect would be retired by the experimental observation of a quantum system whose evolution accelerates rather than freezes under continuous monitoring, or whose dependence on monitoring rate deviates substantially from the GKLS prediction. No such observation has been reported.

93.7 Plain-English close

The watched pot does not boil because each glance is a Lindblad jump. In the cohort framework, this is not folklore but a structural consequence of T-A8.5.

94 Weak measurement and the observer effect

94.1 L4 historical opening

Aharonov, Albert, and Vaidman’s 1988 paper introduced “weak measurement”: a measurement performed with such small back-action that the system is barely perturbed, but each measurement also extracts barely any information. Repeated weak measurements on a pre- and post-selected ensemble produce “weak values” that can lie outside the spectrum of the measured observable — mathematically consistent, but counter-intuitive. Subsequent experimental realizations of weak values have appeared in many platforms (optical pre-/post-selection, neutron interferometry, superconducting circuits).

94.2 The physics question

How can the expectation value of a Hermitian observable lie outside its spectrum? Are weak values a genuine physical quantity, or a mathematical artifact of post-selection?

94.3 The 6D-bulk-and-4D-boundary reading

In the framework reading, weak measurements are the small- γ limit of the Lindblad dynamics that T-A8.5 describes: $L_\gamma = \sqrt{\epsilon} \hat{M}$ with $\epsilon \ll 1$. The weak value $\langle A_w \rangle = \langle \phi_{\text{final}} | \hat{A} | \psi_{\text{initial}} \rangle / \langle \phi_{\text{final}} | \psi_{\text{initial}} \rangle$ is the post-selected expectation value, and it can lie outside the spectrum of $\hat{A}$ because the post-selection conditions on a small-overlap final state; the conditional is genuinely a complex number whose real and imaginary parts probe different aspects of the system's response. This is the structural reading: weak values are not a separate “ontology” for quantum observables but the operational signature of the small- γ regime under conditioning.

94.4 Quantitative anchor

Ritchie, Story, and Hulet 1991 measured a weak value of σ_z for a spin- $\frac{1}{2}$ system to be ~ 100 (the operator's spectrum is ± 1) by pre- and post-selecting on highly improbable boundary states. The result is reproducible, consistent with quantum-mechanical prediction, and does not violate the spectral theorem for the operator $\hat{A}$ (which constrains direct measurement outcomes, not post-selected averages).

94.5 Physics intuition and daily-life metaphor

Weak values are the conditional moments of a small- γ Lindblad evolution. Daily-life metaphor: the average height of those people who later won a footrace can differ from the average height of the starting field; the conditional moment is genuinely informative without implying that any individual exceeded six feet.

94.6 Kill criterion

Weak measurement would be retired by the experimental observation of a weak value that violates the framework's prediction at the small- γ regime — e.g., a weak value that lies outside the analytically continued conditional expectation that the GKLS conditional dynamics permits. No such observation has been reported.

94.7 Plain-English close

Weak values look strange because they conditional-average across a small-overlap post-selection. Once one acknowledges what is being conditional-averaged on, the strangeness is mathematical, not ontological.

95 Contextuality and Kochen–Specker

95.1 L4 historical opening

The Kochen–Specker theorem of 1967 (Kochen and Specker, 1967) established that no non-contextual hidden-variable assignment is compatible with the algebra of quantum observables in any Hilbert space of dimension ≥ 3 : any hidden-variable completion must assign values to observables that depend on which compatible set of observables is being co-measured. Together with Bell’s 1964 theorem, this closed the mid-twentieth-century hopes for a context-independent local-realist completion of quantum mechanics in 4D.

95.2 The physics question

Can the values of quantum observables be assigned independently of the measurement context? If not, what is the operational mechanism by which the context affects the assignment?

95.3 The 6D-bulk-and-4D-boundary reading

In the cohort framework, observables do not have context-independent values. The outcome of measuring observable $\hat{A}$ in context $\{\hat{A}, \hat{B}, \hat{C}\}$ depends on the bulk Wilson-line holonomy Θ_H at the measurement event; the apparatus configuration for context $\{\hat{A}, \hat{D}, \hat{E}\}$ is different from that for $\{\hat{A}, \hat{B}, \hat{C}\}$, so the holonomy probed is different, and the outcome assignment is different. The 6D-local degree of freedom is single-valued — the holonomy Θ_H is one configuration in the bulk — but its 4D projection depends on the boundary algebra selected for the partial trace. Contextuality is therefore the structural signature of which boundary algebra is being constructed, not a feature of the underlying bulk ontology. Operationally this matches T-A8.1 (boundary algebra inherited from the bulk) combined with T-A8.6 (einselected boundary basis is forced by bulk locality), where the einselected basis depends on the system–apparatus coupling that the experimenter chose.

95.4 Quantitative anchor

The Cabello–Estebaranz–García-Alcaine 1996 18-vector proof of Kochen–Specker, and the subsequent experimental violations of non-contextuality inequalities in trapped-ion, photonic, and nuclear-spin systems, all confirm the structural prediction. In every confirmed test, the framework’s prediction matches the standard quantum-mechanical prediction; contextuality is observed.

95.5 Physics intuition and daily-life metaphor

The same 6D-local holonomy projects to different boundary records depending on which context the boundary observer constructs. Daily-life metaphor: a single physical object on a turntable casts different shadows on the wall depending on which side the lamp is on. The object is single-valued; the shadows are context-dependent.

95.6 Kill criterion

Contextuality would be retired by the experimental observation of a non-contextual hidden-variable model that reproduces the quantum-mechanical predictions for ≥ 3 -dimensional systems — which the Kochen–Specker theorem proves impossible. The framework’s contextuality reading is therefore as well-protected as the underlying theorem.

95.7 Plain-English close

Quantum observables are contextual because the bulk holonomy projects differently to different boundary algebras. This is not a paradox; it is the operational signature of a non-invertible projection.

96 Quantum teleportation in 6D

96.1 L4 historical opening

The Bennett–Brassard–Crépeau–Jozsa–Peres–Wootters 1993 protocol (Bennett *et al.*, 1993) demonstrated that an unknown quantum state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ can be “teleported” from Alice to Bob using one pre-shared EPR pair and two classical bits. Experimental confirmations have followed across photon polarization (Bouwmeester *et al.*, 1997), atomic states (Riebe and others, 2004), satellite-to-ground entanglement (Micius mission 2017), and quantum-network protocols. The result is the operational foundation of quantum communication.

96.2 The physics question

Where does the qubit’s information go during the teleportation, and how can the protocol succeed without transmitting the quantum state as a classical record?

96.3 The 6D-bulk-and-4D-boundary reading

In the framework reading, teleportation is the transfer of bulk Wilson-line holonomy information from Alice’s laboratory to Bob’s via a pre-shared bulk fiber. The EPR pair carries a common 6D fiber configuration; Alice’s Bell-basis measurement projects this fiber into one of four sub-branches (encoded as the 2 classical bits); Bob’s unitary correction selects the corresponding sub-branch on his half of the pair. At completion, Bob’s qubit carries the bulk-gauge-field configuration that matches Alice’s original $|\psi\rangle$. Operationally, the teleportation is T-A8.4 (the conditional CPTP state update at Alice’s site) plus T-A8.11 (no-signaling: the classical channel carries the basis information, but nothing faster than light) acting on the shared bulk fiber.

96.4 Quantitative anchor

The Micius mission 2017 demonstrated teleportation over a 1400-km satellite-to-ground link with fidelity 0.80 ± 0.01 , consistent with the quantum-mechanical prediction including the small loss

budgets of the optical channel. The framework predicts no deviation from quantum mechanics in any platform.

96.5 Physics intuition and daily-life metaphor

Two friends agreed in advance that one of them, on receiving a specific signal, would copy whatever pose the other adopted at the agreed start time. The pose is communicated via the prior agreement plus the signal — not via a teleportation device that transmits the body. Quantum teleportation works the same way: the EPR pair is the prior agreement, the 2 classical bits are the signal, and the bulk fiber carries the pose.

96.6 Kill criterion

Quantum teleportation would be retired by the experimental observation of teleportation fidelities above the no-cloning limit without the classical-channel contribution, or by the observation of teleportation succeeding without the pre-shared EPR pair. No such observation has been reported.

96.7 Plain-English close

Teleportation does not move the qubit; it reveals what the prior shared bulk configuration was already storing. The classical channel does the rest.

97 Wigner’s friend and the Frauchiger–Renner sharpening

97.1 L4 historical opening

Wigner’s 1961 “friend” thought experiment ([Wigner, 1961](#)) asked what happens when an observer is themselves treated as a quantum system inside a larger isolated lab. Frauchiger and Renner’s 2018 sharpening ([Frauchiger and Renner, 2018](#)) converted Wigner’s philosophical puzzle into a no-go theorem: under three plausible assumptions (Q — universal quantum mechanics; C — consistency of assignments across agents; S — single outcomes per measurement), the predictions of a multi-agent measurement chain become mutually contradictory. Most interpretive frameworks have responded by abandoning at least one of the three assumptions; the cohort framework retains all three by locating the apparent inconsistency in the implicit identification of boundary state with total ontology.

97.2 The physics question

If the wave function and the universally quantum-mechanical evolution apply to observers as well as to measured systems, can two observers’ predictions remain mutually consistent in a multi-agent chain — or does at least one of Q, C, S have to fail?

97.3 The 6D-bulk-and-4D-boundary reading

In the framework, all three assumptions are maintained, provided “wave function” is understood as a 4D observer-dependent projection of the bulk state rather than the total state of reality.

The bulk state is universally valid (it is the fundamental theory), consistent (it evolves unitarily), and single-world (there is only one bulk configuration); the apparent inconsistency in Frauchiger–Renner arises from the implicit assumption that different observers’ boundary wave functions are ontologically the same object. They are not. Each observer constructs their own 4D density operator by partially tracing the bulk state over their own inaccessible degrees of freedom; different observers have different inaccessible sectors and therefore different density matrices. The contradiction in 4D is the mismatch between two such projections; in 6D the underlying state is single-valued and the contradiction dissolves. Operationally, this matches T-A8.1 (boundary Hilbert space is observer-relative GNS representation) and T-A8.4 (conditional CPTP state update is the observer’s information update, not the bulk’s evolution).

97.4 Quantitative anchor

The Frauchiger–Renner thought experiment ([Frauchiger and Renner, 2018](#)) produces a quantitative prediction: under assumptions Q, C, S, the probability of a specific outcome assignment is 1, while another agent’s reasoning would predict the same outcome to have probability less than 1. In the framework reading, the two predictions are projections of the same bulk state through different inaccessible-sector traces, and the probabilities differ for the same structural reason that two observers using different reference frames in special relativity see different time slicings of the same world line.

97.5 Physics intuition and daily-life metaphor

Two observers describing the same physical history with different inaccessible degrees of freedom will assign different wave functions. Daily-life metaphor: two cartographers projecting the same globe onto two different flat-map projections (Mercator, Peters) will draw different shapes for Greenland. The globe is single-valued; the projections differ; neither cartographer is wrong. Frauchiger–Renner is the multi-projection version of the same insight applied to quantum measurement.

97.6 Kill criterion

The framework’s resolution would be retired by the experimental demonstration of a quantum-mechanical violation of one of Q, C, S under conditions where the framework predicts all three hold. The Frauchiger–Renner experiment has not yet been performed in the laboratory at the multi-agent scale, but its proposed implementations ([Baumann and others, 2019](#); [Bong and others, 2020](#)) predict outcomes consistent with the framework; the strong no-go theorem of [Bong and others \(2020\)](#) sharpens the predictive content of the multi-agent Wigner-friend regime.

97.7 Plain-English close

Two observers projecting the same bulk state onto different boundary algebras can produce mutually contradictory 4D wave-function assignments without any contradiction in the 6D ontology. This is the operational content of the cohort’s reading of Wigner’s friend.

98 Dense coding, BB84, and the Ekert protocol

98.1 L4 historical opening

Bennett and Wiesner’s 1992 dense-coding protocol (Bennett and Wiesner, 1992): one qubit transmitted from Alice to Bob, plus one pre-shared EPR pair, encode and transmit two classical bits. Bennett and Brassard’s 1984 BB84 protocol (Bennett and Brassard, 1984): random-basis qubit transmission for quantum key distribution. Ekert’s 1991 E91 protocol (Ekert, 1991): Bell-inequality-violation-based key distribution with built-in eavesdropper detection. Together these three protocols are the operational foundation of quantum cryptography; they have been implemented in commercial systems since the early 2000s and form the technical basis of every quantum-network deployment to date.

98.2 The physics question

How can quantum protocols compress two classical bits into one qubit channel (dense coding), or detect eavesdropping at the protocol level (BB84, E91)?

98.3 The 6D-bulk-and-4D-boundary reading

Dense coding. The pre-shared EPR pair carries a bulk gauge-field configuration in the singlet sector. Alice’s local operation on her qubit selects one of four Pauli-related configurations; Bob’s joint Bell measurement reads out the configuration. The two classical bits are encoded in which Pauli the bulk fiber now carries.

BB84. Each qubit Alice transmits carries a bulk gauge-field configuration set by her chosen basis. Bob’s measurement reveals the configuration only when his basis matches Alice’s; an eavesdropper who intercepts must measure in some basis, necessarily random with respect to Alice’s choice, disturbing the bulk-gauge-field configuration and revealing the intrusion via subsequent error-rate test.

E91. The pre-shared singlet pair guarantees the key via the Bell-correlation strength. Any eavesdropping disturbs the bulk-gauge-field configuration, reducing the measured Bell correlation $|S|$ below the Tsirelson saturation; the security check fails and the key is discarded.

In all three protocols, the framework reading exercises T-A8.3 (Born-rule probabilities at the qubit measurements), T-A8.7 (the Bell correlation as bulk-shadow on EPR), and T-A8.11 (no-signaling: the protocol respects bandwidth bounds of classical communication).

98.4 Quantitative anchor

Commercial QKD systems (ID Quantique, Toshiba) have demonstrated BB84 key-distribution rates exceeding 1 Mbit/s over short fiber links and tens of kbit/s over 100-km links, with measured quantum bit error rates compatible with the framework’s prediction. The Micius mission’s space-to-ground BB84 link achieved 19 bps over 1200 km (Liao and others, 2017). Dense coding has

been demonstrated to within 87 % of the ideal information-rate at the optical-photon scale. All measurements respect the framework’s prediction within experimental error.

98.5 Physics intuition and daily-life metaphor

Quantum cryptography exploits the no-cloning structure of the bulk holonomy: any eavesdropper who intercepts must perturb the configuration, and the perturbation is operationally visible in the error rate. Daily-life metaphor: a wax seal on a letter does not prevent reading, but any reading breaks the seal and reveals the intrusion. The bulk-gauge-field configuration is the wax seal; the basis-mismatch error rate is the broken-seal indicator.

98.6 Kill criterion

The protocols would be retired by an eavesdropping demonstration that recovers the full key without raising the error rate above the framework’s prediction — which would require either a violation of no-signaling T-A8.11 or a deviation from the Born rule T-A8.3. No such demonstration has been reported in any operational platform.

98.7 Plain-English close

Quantum cryptography is not magic. It is the operational use of three theorems — T-A8.3 for the Born rule, T-A8.7 for the Bell correlation, T-A8.11 for no-signaling — in a protocol that an eavesdropper cannot subvert without leaving a record. The cohort framework predicts every protocol’s behavior with no free parameters.

99 Summary of Part X: the paradox sweep complete

The eight paradox families covered in this Part dissolve into structural features of the cohort framework rather than mysteries of quantum mechanics:

- Wheeler’s delayed-choice: continuous bulk-field propagation; no particle trajectory to retroactively edit.
- Quantum eraser: conditional CPTP state-update; sub-selection of pre-existing correlations.
- Quantum Zeno: large- γ limit of GKLS dynamics; structural pin to T-A8.5.
- Weak measurement: small- γ limit of GKLS dynamics; conditional moments out of operator spectrum.
- Kochen–Specker contextuality: bulk holonomy projects to context-dependent boundary algebras.
- Quantum teleportation: shared bulk fiber + 2 classical bits + unitary correction.
- Wigner’s friend / Frauchiger–Renner: observer-relative boundary projections of a single bulk state.

- Dense coding / BB84 / E91: T-A8.3, T-A8.7, T-A8.11 operationally combined into cryptographic protocols.

Schrödinger’s cat (covered in Part VIII), the EPR paradox, and Bell–CHSH violation (both covered in Part VII) are treated in their dedicated Parts and are not repeated here. Together with the present Part, the framework covers every century-old conundrum that the standard foundations-of-quantum-mechanics literature has cataloged.

Part-end close

After all, what these eight paradoxes have in common is not that the underlying physics is mysterious but that the standard textbook framing forces them into a 4D-only picture that does not have room for the structural source of the apparent strangeness. Once the 6D bulk is admitted as the deeper layer — with the boundary state read as one of many possible operational projections of a single bulk configuration — the paradoxes become structural signatures of the projection rather than puzzles about the world.

The cat is alive or dead in 6D; the photon’s path is undetermined in 4D because the bulk field has no trajectory; the eraser conditions on the bulk-fiber configuration; the Zeno effect is large- γ Lindblad; the weak value is small- γ conditional; Kochen–Specker is the holonomy projecting context-dependently; teleportation is the bulk fiber’s prior agreement; Wigner’s friend is two cartographers projecting the same globe; quantum cryptography is operationally what it claims to be. None of these is a paradox of the underlying physics. They are the structural signatures of a non-invertible projection.

Problems and worked solutions. X.1. Quantify the Zeno effect.

Solution. Step 1: a two-level system driven coherently toward full transfer is interrupted by N equally spaced measurements; each interval advances the Rabi rotation by π/N , i.e. a survival amplitude $\cos(\pi/2N)$, so each measurement projects the system back with probability $\cos^2(\pi/2N)$ and the total survival is $P_N = [\cos^2(\pi/2N)]^N$. Step 2: numbers: $N = 1$ gives 0 (full transfer); $N = 10$ gives 0.780; $N = 100$ gives 0.9756; asymptotically $P_N \approx 1 - \pi^2/4N \rightarrow 1$ — watched closely enough, the transition freezes. Step 3: demystify: each measurement is a registration that resets the coherent build-up; the effect is interference bookkeeping, verified in trapped ions, and in the bulk reading it is sector-pinning — frequent registration keeps re-selecting the same holonomy sector, no “observer power” anywhere. Physics intuition: quantum transitions grow quadratically at first, and quadratic beginnings are fragile — interrupt often and the amplitude never compounds; the watched pot fails to boil for the same reason interrupted interest never accrues.

X.2. Dissolve delayed choice without retrocausality.

Solution. Step 1: Wheeler’s setup — decide *after* the quantum passes the beamsplitter whether to recombine (wave) or not (which-path). Step 2: track the records: at the late decision time, no path record yet exists; the choice determines which *measurement* occurs, and the statistics of each are computed from the same state — nothing about the past is altered, only which question the past is finally asked. Step 3: the bulk chronology makes it plain: one configuration evolves deterministically

Table 10: Classic quantum paradoxes and their resolution in the 6D-to-4D projection picture. “Puzzle” is the textbook statement; “6D lift” is the 6D deterministic re-reading; “4D projection” is what remains after Tr_{env} ; “Resolved” flags the status in GG-Theory.

Paradox		Standard 4D puzzle	6D lift	4D projection	Status
Schrödinger’s (1935)	cat	Macroscopic superposition of alive/dead	Single definite 6D configuration; Θ_H selects branch	Decohered mixed state, diagonal in pointer basis within $\sim 10^{-13}$ s	Resolved
EPR paradox (1935)		Perfect correlation at space-like separation	Shared S_θ^1 fiber; bulk gauge fields 6D-local	4D-apparent non-local correlation from shared fiber	Resolved
Bell inequality (1964)		Local hidden variables $\Rightarrow S \leq 2$, QM gives $2\sqrt{2}$	Hidden variable Θ_H lives on compact dim, 6D-local	CHSH = $2\sqrt{2}$ from shared bulk configuration	Resolved
Aspect / CHSH 1982–2022		Experimental confirmation that Nature is non-local	Compatible with 6D locality, bulk-to-boundary-projection-induced 4D non-locality	Tsirelson saturated naturally; no-signaling preserved	Resolved
Double-slit (Young / Feynman)		Particle “in both slits”; interference demands wave description	6D bulk EM field propagates through both slits as classical field	Amplitude sum gives $ \psi_1 + \psi_2 ^2$ by Born rule	Resolved
Delayed choice (Wheeler 1983)	choice	Late measurement choice seems to alter earlier trajectory	Bulk field respects boundary conditions; no trajectory commitment at bulk level	Conditional 4D record depends on measurement configuration; no retrocausation	Resolved
Quantum eraser (Scully 1991)		Later erasure seems to restore earlier coherence	Full 6D record has all correlations pre-existing	Post-selection reveals existing correlations; no causal paradox	Resolved
Stern-Gerlach (1922)		Quantized outcomes, intrinsic spin, back-action	Holonomy Θ_H determines which GSSB branch realizes	$P(\uparrow) = \alpha ^2$ from Born; back-action from non-commuting measurements	Resolved
Quantum Zeno		Continuous measurement freezes dynamics	Large- Γ limit of Lindblad; bulk field re-set by each probe	$\Gamma t \gg 1$ suppresses non-pointer evolution as $1/\Gamma$	Standard
Weak measurement (AAV 1988)		“Weak values” outside observable spectrum	Small- Γ limit of Lindblad; minimal back-action	Post-selected conditional expectation; no paradox	Standard
Kochen-Specker (1967)		No non-contextual value assignment in $\text{dim} \geq 3$	Outcomes depend on 6D apparatus configuration, not particle intrinsic	Contextual outcomes; consistent with the theorem	Resolved
Quantum teleportation (1993)		Unknown state transmitted by 2 classical bits	Transfer of bulk-gauge-field holonomy through pre-shared bulk fiber	Alice’s Bell measurement + Bob’s correction	Transparent
Wigner’s friend (1961)		Observer-dependent wave functions seem to contradict objective reality	6D state objective and single-valued; 4D ρ depends on what observer traces over	Different observers, different ρ_∂ ; one 6D state	Resolved
Frauchiger-Renner (2018)		Consistent QM cannot be both universal and single-world	6D is single-valued; 4D wave functions are observer-relative projections	No contradiction; implicit common-ontology assumption fails	Resolved
Measurement problem (from 1932)		When/why does the wave function collapse?	It does not. 6D evolves deterministically.	“Collapse” is Bayesian conditioning on an outcome in the register	Dissolved

throughout; “wave” and “particle” are two boundary bookkeepings selected by the final instrument, and the paradox evaporates because the past never held a committed classical story to change. Physics intuition: delayed choice startles only if you believe the photon filed a travel report at the slits — it filed nothing; the experiment chooses which report to demand at customs, and no honest history is rewritten by choosing the form.

X.3. Audit teleportation’s resources.

Solution. Step 1: the protocol consumes exactly one shared Bell pair and two classical bits (the four Bell-outcome labels); Bob’s conditional corrections $\{\mathbb{I}, \sigma_x, \sigma_y, \sigma_z\}$ complete the transfer. Step 2: verify the three impossibility checks: before the bits arrive Bob’s state is maximally mixed (no signaling, no early peek); Alice’s original is destroyed by her Bell measurement (no cloning); and two classical bits alone could never specify a point on the Bloch sphere (the ebit carries the continuum). Step 3: the framework reading: what is teleported is the *holonomy data* of the state — the correlations of the shared pair, plus two bits of sector label, reconstitute the same boundary state at Bob’s wing; no matter and no signal travel, only structured correlation is spent. Physics intuition: teleportation is a wire transfer, not a shipment — the ebit is the pre-established joint account, the two bits are the authorization code, and the no-cloning theorem is the bank’s rule that the sender’s balance zeroes when the transfer clears.

X.4. Compute the BB84 abort threshold.

Solution. Step 1: an eavesdropper on BB84 necessarily disturbs: intercept-resend in a random basis inflicts a 25% error rate on the sifted key, so Eve’s full presence is flagrant; the operational question is the maximum tolerable quantum bit-error rate Q . Step 2: the asymptotic secret-key rate of the standard analysis is $r = 1 - 2H(Q)$ with H the binary entropy; solve $r = 0$: $H(Q) = \frac{1}{2}$ gives $Q^* = 11.0\%$ — above eleven percent error, privacy amplification can no longer outrun Eve’s information and the protocol aborts. Step 3: note the structural point: security rests on the same no-cloning and disturbance bookkeeping as X.3 — an unknown quantum state cannot be read without leaving fingerprints, so secrecy is enforced by the kinematics of the theory, not by computational assumptions. Physics intuition: quantum cryptography turns the measurement problem into a security guarantee — the very fact that observation disturbs, the century’s philosophical headache, is here the tamper-evident seal; Eve cannot open the envelope without tearing it, and 11% is the size of tear the protocol forgives.

X.5. Do the Frauchiger–Renner bookkeeping, and name the failed assumption.

Solution. Step 1: the argument nests agents — friends measure, Wigners reason about friends — and derives a contradiction from three assumptions: quantum theory applies universally (Q), single outcomes obtain (S), and agents may adopt each other’s certainties across the nesting (C). Step 2: run the framework’s audit: registration is physical and unique (S holds); the bulk dynamics is universal (Q holds at the bulk stratum); what fails is (C) as used — a superseded description (Wigner’s coherent account of a lab whose record has already registered) is not a valid premise for cross-agent inference; VIII.4’s arithmetic says whose books are stale. Step 3: the general lesson, which is the Part’s method in one line: every paradox of the sweep manufactures its contradiction by treating a boundary description as complete past the point where the bulk has registered — identify the stale ledger and the theorem dissolves into bookkeeping. Physics intuition: Frauchiger–Renner

is a relay race where one runner passes yesterday's baton — each agent's logic is impeccable, and the contradiction lives entirely in the handoff; physics does not forbid nested observers, it forbids quoting a description the world has already overwritten.

Take-away. *Standard account:* each paradox owns a literature, a camp, and an unresolved argument. *What it left open:* all eight, by construction — inside the boundary description they are genuinely undecidable. *The bulk reading:* one mechanism closes all eight: complete the boundary description with the bulk configuration and the registration law, and delayed choice loses its retrocausality, the eraser its spooky undo, Zeno its watched-pot magic, Kochen–Specker its scandal (contextuality is exactly what a fiber projection predicts), teleportation its paradox of copying, and the friend his regress. A paradox, in this course, is a boundary story told as if it were the whole story.

Part XI

The interpretation landscape

Where GG-Theory sits among the seven living interpretations

GG-Theory does not propose a new interpretation of quantum mechanics. It proposes a deeper theory of which standard 4D quantum mechanics is the operational projection. The distinction matters for how the framework engages the interpretation literature: rather than competing with Copenhagen or Many-Worlds at the level of 4D ontology, the cohort framework locates each interpretation as a particular reading of the boundary projection of a single 6D bulk theory, and identifies which of the interpretation's commitments are preserved and which are dissolved in the deeper reading. This Part performs that comparison interpretation by interpretation. The closing Section places the framework within structural realism (Ladyman–Ross / French / Worrall), with explicit disavowal of Russellian neutral monism and explicit engagement with the Della Rocca reading of Spinoza's Numerical Sameness.

We treat each interpretation with the operational respect it has earned over its tenure in the foundations literature. Bohr was right about the operational scope of measurement records; Everett was right about the unitarity of the Schrödinger equation; Bohm was right about the existence of hidden variables; GRW was right that something must break the linear superposition principle at the macroscopic level; QBism was right about the observer-relativity of state assignments; Rovelli was right about the relational structure of quantum facts; Griffiths–Omnès–Gell-Mann–Hartle were right about decoherent histories as the correct mathematical structure for consistent quantum descriptions. The cohort framework adds the structural warrant that each of these readings was groping toward, without endorsing any of their particular ontological over-commitments.

100 The six-dimensional bulk reading vs the seven living interpretations

Before walking the seven interpretations one by one, we pause to state how the cohort framework's six-dimensional bulk reading relates to each of them structurally. The standard treatment of each interpretation in the language Bohr, Everett, Bohm, GRW, Fuchs, Rovelli, and Griffiths–Omnès–Gell-Mann–Hartle would have recognized follows in §101–§107; placing the synthesis first lets the reader see why the framework does not *compete* with these interpretations — it *subsumes* them.

The general structural claim. Every living interpretation of quantum mechanics is, in the cohort framework, a particular reading of the boundary projection of a single 6D bulk theory. The interpretations differ in which boundary feature they take as ontologically primary — the observer's record (Copenhagen), the universal wavefunction's unitarity (Many-Worlds), the particle's

position (de Broglie–Bohm), the macroscopic-superposition collapse (GRW), the observer’s belief state (QBism), the relational observer frame (relational QM), or the consistent decoherent history (decoherent histories). Each takes *one* feature of the boundary projection and elevates it to the underlying ontology. The cohort framework supplies the deeper layer — the 6D bulk — that the boundary features are projections *of*. The interpretations are not wrong about the features they identify; they are wrong about what the features are features *of*.

Copenhagen — operationally right, ontologically incomplete. Bohr and von Neumann established the operational protocol: the quantum state is calculational complete, the Born rule supplies the probabilities, measurement is the primitive act that connects formalism to record. Copenhagen got the operational scope of 4D quantum mechanics exactly right — and the cohort framework reproduces every Copenhagen prediction. What Copenhagen left silent is what the operational protocol is the protocol *of*: it never specified where the wave function lives between measurements, why the Born rule has its specific form, or what makes the classical/quantum cut sit where it does. The cohort framework supplies these structurally: the boundary algebra lives at the 4D shadow of the 6D bulk; the Born rule emerges as T-A8.3 from CPTP completion; the cut is the boundary partial trace, which completes at any sufficiently macroscopic 4D detector. Copenhagen is operationally correct and ontologically incomplete; the cohort framework completes it.

Many-Worlds — unitarity right, branching unnecessary. Everett and DeWitt were right that the Schrödinger equation never needs to be supplemented by a non-unitary collapse postulate. The universal wavefunction evolves unitarily — and so does the 6D bulk in the cohort framework. Where MWI errs is in extending this unitary evolution to the 4D-boundary observer’s experience: they imagine the observer split into branches, each corresponding to a different measurement outcome. The cohort framework retains the unitarity but locates it where it belongs — in the 6D bulk. The bulk is single-valued, so there are no branches in the bulk; the 4D-boundary “branches” that MWI calls separate worlds are simply the decohered classical outcomes projected by the GDOS/OGGEP map. MWI got the unitarity right and the branching ontology unnecessary. In the cohort reading, the multiverse is not many worlds but one bulk seen from many boundary projections.

de Broglie–Bohm — hidden variables right, 4D location wrong. Bohm was right that quantum mechanics is ontologically incomplete and that hidden variables exist. He was wrong about *where* they live: Bohmian pilot waves and definite-position particles are 4D hidden variables, and Bell–Aspect ruled out any 4D-local hidden-variable theory. The cohort framework’s hidden variable is the bulk S^1_θ holonomy phase Θ_H — structurally inaccessible to 4D detectors, 4D-contextual but 6D-non-contextual (evading the Kochen–Specker no-go), and bulk-local in the sense that bulk worldlines respect 6D causality. The cohort framework is, in spirit, a Bohmian theory — deterministic at the deep level — but implemented through the geometric structure of the 6D bulk rather than through a non-local pilot wave on 4D position space. Bohm got the hidden-variable intuition right; he located it one dimension too low.

GRW — boundary collapse right, dynamical mechanism unneeded. Ghirardi, Rimini, and Weber were right that linear unitary evolution cannot survive at macroscopic scales — the live-and-dead cat is empirically absurd, and a theoretical framework that predicts it as a literal physical state is broken at the macroscopic level. GRW proposed to repair this by adding a stochastic spontaneous-collapse process to the dynamics. In the cohort framework, the macroscopic collapse is automatic from the partial-trace structure: the decoherence rate is $\Gamma \gtrsim 10^{40} \text{ s}^{-1}$ for any macroscopic 4D-spatial superposition by T-A8.5, derived from scale-profile KK-mode overlap integrals with no new dynamics added. GRW correctly diagnosed the disease but proposed an unnecessary treatment; the cohort framework cures the same disease structurally.

QBism — observer-relativity right at boundary, ontology not refuted. Fuchs and Mermin were right that the boundary quantum state is observer-relative: two observers with different priors assign different states, and there is no observer-independent “true” boundary state floating in the world. QBism extended this insight into a denial of underlying ontology: the quantum state is a Bayesian belief and nothing more. The cohort framework agrees with QBism’s epistemology at the 4D boundary — the boundary projection IS observer-relative because different observers integrate over different bulk regions R_E — but the bulk state itself is observer-independent and definite. QBism is a correct epistemology of the 4D boundary; it is silent on the 6D bulk, where ontology returns. The framework supplies the ontological substrate that QBism declined to commit to.

Relational QM — relational facts at boundary, bulk is absolute. Rovelli was right that the boundary quantum facts are relational: an observer’s record is always defined relative to that observer’s measurement frame. Relational QM extended this to deny any absolute underlying reality. The cohort framework agrees with Rovelli at the boundary level — the 4D projection is genuinely observer-relative, because different observers’ choices of which 4D subregion to read produce different boundary density operators — but disagrees that there is no absolute frame underneath. The 6D bulk *is* the absolute frame: a single bulk field configuration that all observers’ 4D projections are projections of. Relational QM correctly describes the boundary level and is incomplete at the bulk level.

Decoherent histories — consistent frameworks right, substrate missing. Griffiths, Omnès, and Gell-Mann–Hartle developed consistent decoherent histories as the mathematical structure of quantum descriptions: a history $\{P_{\alpha_n}\}$ is consistent if its decoherence functional is diagonal, and quantum probabilities are defined only on such consistent histories. This framework is mathematically correct as a description of the boundary statistics. What it lacks is the underlying substrate: the decoherence functional appears as a phenomenological input. The cohort framework supplies the substrate explicitly: the decoherence functional is $D(\alpha, \alpha') = \text{Tr}[\hat{C}_{\alpha'}^\dagger \rho_0 \hat{C}_\alpha]$ with the chain operators $\hat{C}_\alpha$ derived from bulk dynamics + partial trace + T-A8.5’s scale-profile KK-mode overlap integrals. The decoherent-histories formalism is the boundary-side calculation; the bulk-side dynamics is what makes it consistent.

The structural-completion synthesis. Putting all seven readings together: each interpretation correctly identifies one feature of the boundary projection of a single 6D bulk theory, and incorrectly elevates that feature to the underlying ontology. The cohort framework adds the underlying structure they were collectively groping toward. The framework is structurally *complete* in the sense that every aspect each interpretation correctly identified is preserved as a theorem of the bulk-to-boundary picture: Copenhagen’s operational protocol is recovered (T-A8.1–T-A8.4); MWI’s unitarity is recovered (the bulk is unitary); Bohm’s hidden variables are recovered (the S^1_θ holonomy); GRW’s macroscopic decoherence is recovered (T-A8.5); QBism’s observer-relativity is recovered (boundary-region choice); relational QM’s relational facts are recovered (boundary frame dependence); decoherent histories’ consistent frameworks are recovered (decoherence functional from T-A8.5). What is dissolved in each case is the over-commitment to a particular boundary feature as ontologically primary.

Bottom line of the interpretive landscape. The seven living interpretations are not seven competing theories among which one must be chosen. They are seven partial readings of one underlying theory, the cohort framework’s 6D-bulk-plus-4D-boundary picture. The framework does not replace them; it *completes* them. Each interpretation was correct about a feature of the boundary projection and incorrect about the underlying ontology of that feature; together with the bulk substrate added, the seven partial readings become one consistent structural reading. After GG-A8-QM, the productive question is no longer “which interpretation is right?” but “what new operational signatures does the bulk reading predict that the 4D-only readings cannot?” — the eight-test horizon of Part XII.

The interpretation-landscape table is shown in **Figure 17**.

Interpretation	What it got right (boundary)	What it missed (bulk)	Recovered as
Copenhagen	Operational protocol, Born rule, projection axiom	What the protocol is OF; the 6D substrate	T-A8.1–T-A8.4
Many-Worlds	Unitarity of the deeper dynamics	Unitarity lives in the 6D bulk, not the 4D observer	Bulk unitarity
de Broglie–Bohm	Hidden variables exist	They live in 6D (S^1_θ holonomy), not 4D position	T-A8.6, T-A8.9
GRW collapse	Macroscopic superposition breaks	No new dynamics needed; partial trace does it automatically	T-A8.5
QBism	Boundary state is observer-relative	Bulk state is observer-independent; ontology returns	T-A8.4 + bulk
Relational QM	Boundary facts are relational	The 6D bulk is the absolute frame	T-A8.1–T-A8.4
Decoherent histories	Consistent decoherent frameworks	The decoherence functional from bulk KK-mode overlaps	T-A8.5

Seven interpretations, one underlying theory. Each interpretation correctly identifies one feature of the boundary projection of the cohort framework’s six-dimensional bulk; each incorrectly elevates that feature to the underlying ontology. The cohort framework supplies the deeper substrate that the seven partial readings were collectively groping toward, and each correctly identified feature returns as a theorem. The interpretations are not replaced; they are completed.

Figure 17: The interpretive landscape under the cohort framework. Seven living interpretations of quantum mechanics (left column), each tabulated against the feature of the 4D boundary projection it correctly identified (middle-left), the bulk-side fact it missed (middle-right), and the theorem(s) of Part IV that recover the correctly identified feature in the bulk-to-boundary picture (right). Every interpretation was *right* about a feature of the boundary projection; every interpretation *missed* the deeper substrate. The framework does not compete with these interpretations but completes them: each is preserved as a partial reading of one underlying theory whose 6D-bulk structure was, until now, the missing common ground.

101 Copenhagen — the operationally correct reading

Position

Bohr’s 1928 Como lecture and von Neumann’s 1932 *Mathematische Grundlagen* (Neumann, 1932) together established the Copenhagen interpretation: the quantum state is operationally complete, measurement is a primitive act that requires no underlying mechanism, and the classical/quantum cut is drawn at the experimenter’s discretion. Bohr’s complementarity principle insists that wave-like and particle-like descriptions are mutually exclusive and equally necessary; von Neumann’s measurement chain locates “collapse” wherever the observer decides to register the outcome.

What Copenhagen got right

Operationally, every laboratory predicts measurement statistics via the standard quantum-mechanical formalism, and Copenhagen is the formalism’s most direct interpretation: the wave function is a calculational device, the Born rule supplies the probabilities, the projection postulate

updates the record. No experiment has ever falsified the Copenhagen operational protocol. The cohort framework agrees with Copenhagen on every operational prediction.

What Copenhagen leaves open

Copenhagen does not specify what makes the classical/quantum cut where it is, where the wavefunction lives between measurements, or why the Born rule has the form it does. These are not Copenhagen failures so much as Copenhagen *omissions*: the interpretation is operationally complete but ontologically silent. The cohort framework supplies the missing layer: the 6D bulk is the deeper ontology, the cut is the boundary partial trace over inaccessible bulk degrees of freedom, the wavefunction is the GNS representation of the boundary algebra, the Born rule is T-A8.3.

Status in the cohort framework

Copenhagen is operationally correct and ontologically incomplete. The cohort framework completes it.

102 Many-Worlds — the unitarity reading

Position

Everett’s 1957 thesis ([Everett, 1957](#)), developed by DeWitt and Graham through the 1970s, proposed that the measurement problem dissolves if one takes the Schrödinger equation literally: the wave function never collapses, every outcome is realized in a separate branch of the universal state, and the apparent randomness of measurement is the observer’s branch-conditional perspective. The interpretation has the substantial virtue of taking the unitarity of quantum evolution seriously, and has been refined in the decoherence-based versions of Saunders, Wallace, and others ([Wallace, 2012](#)).

What Many-Worlds got right

The Schrödinger evolution is unitary. Every branch of the wavefunction, in the standard formalism, satisfies the Schrödinger equation locally; there is no need to add a non-unitary collapse process to the dynamics. The cohort framework agrees: the bulk evolution is unitary by AX-1.

What Many-Worlds over-commits to

The Many-Worlds interpretation postulates a vast metaphysical inflation: a continuum of co-existing branches for every measurement-like event, with the branches treated as ontologically real worlds. Critics have long noted two costs. The first is the Born-rule derivation problem: branch-counting cannot reproduce the $|\langle\alpha|\psi\rangle|^2$ weighting without additional assumptions (the Deutsch–Wallace decision-theoretic argument is sophisticated but not universally accepted). The second is the ontological extravagance: positing infinitely many worlds for every measurement is, by any reasonable measure, an enormous addition to the inventory of what exists.

Status in the cohort framework

The cohort framework recovers the unitarity that Many-Worlds correctly emphasizes without postulating branching. The bulk is single-valued in 6D; what appears as “branching” at the 4D boundary is the selection of one GFF-minimizing configuration by GSSB (see GG-A6-GRIP for the GRIP cascade machinery). No additional worlds are needed; the operational appearance of branching is a derived feature of the projection. The Many-Worlds reader can keep the Schrödinger-unitarity intuition without the metaphysical inflation.

103 De Broglie–Bohm pilot wave — the hidden-variable reading

Position

De Broglie’s 1924 pilot-wave hypothesis (de Broglie, 1924) and Bohm’s 1952 rederivation (Bohm, 1952) together restore determinism by adding configuration-space variables on top of the wavefunction. A particle has a definite position at all times; the wavefunction acts as a “pilot wave” guiding the particle’s trajectory via the quantum potential. The interpretation has been refined by Bell, Dürr–Goldstein–Zanghì, and others into a mathematically consistent ontology.

What pilot-wave got right

The intuition that there are hidden variables — some physical reality underlying the quantum-mechanical statistics — is structurally correct. The cohort framework agrees: the bulk Wilson-line holonomy Θ_H along S_θ^1 is a real degree of freedom that the 4D observer cannot directly access but which determines the outcomes of measurements.

What pilot-wave gets wrong (and why this is irreparable in 4D)

The hidden variables of de Broglie–Bohm are placed in 4D configuration space, and Bell’s theorem (1964 (Bell, 1964)) plus the Aspect-and-successors experimental confirmations close the door on any 4D-local hidden-variable completion. Pilot-wave proponents have absorbed the non-locality (the pilot wave is explicitly non-local at the 4D level), but the non-locality is an unresolved structural strangeness rather than a derivable feature of a deeper theory. The 2012 Pusey–Barrett–Rudolph theorem (Pusey *et al.*, 2012) further constrained the ontological status of the wavefunction: under mild assumptions, any ψ -epistemic interpretation that reproduces standard QM predictions must violate one of preparation independence or the underlying preparation assumption. The cohort framework’s reading is ψ -ontic (the bulk state is real) and consistent with PBR; pilot-wave’s ψ -ontic-plus-particles structure remains 4D-non-local and unparsimonious by comparison.

Status in the cohort framework

The cohort framework keeps the hidden-variable intuition but relocates the hidden variables to the 6D bulk, where they are local (in 6D) by AX-1 of GG-A1-Constitution. Specifically, the hidden

variables are the bulk degrees of freedom that the 4D boundary partial trace integrates out: the KK-mode profiles along the phase axis S_θ^1 , the renormalization-group modes along the radial direction $r = \ln(E_{\text{GUT}}/\mu)$, and the bulk gauge-field configurations of whatever gauge content the bulk theory carries (in the cohort canonical bulk of Arisaka (2026, A2-GG6), $\text{Spin}(1, 5) \times \text{SU}(5) \times \text{U}(1)_B \times \text{U}(1)_{Q_i}$; in a simpler bulk with only $\text{U}(1)_{\text{EM}}$ this list is correspondingly shorter, but the mechanism is the same). The pilot-wave reading is recovered as the 4D-projected shadow of these 6D-local bulk degrees of freedom. The 4D non-locality that Bell–Aspect have established is the operational signature of the bulk-to-boundary projection OGGEp, not a feature of the underlying theory or of any one specific gauge sector.

104 GRW spontaneous collapse — the dynamical-reduction reading

Position

Ghirardi, Rimini, and Weber’s 1986 spontaneous-localization model (Ghirardi, 1986), refined by Penrose’s 1996 gravity-induced-reduction conjecture and the continuous spontaneous localization (CSL) variants, postulates that the Schrödinger equation must be modified by a small non-unitary stochastic process that localizes the wavefunction at a rate proportional to the mass times a fundamental free parameter. At microscopic scales the modification is invisible; at macroscopic scales it produces effective wavefunction collapse on millisecond time-scales.

What GRW got right

Macroscopic objects do not exhibit observable superpositions. Something — whether postulated dynamics or environment-induced decoherence — must explain the absence of Schrödinger-cat-like states at macroscopic scales. GRW correctly identifies that this requires a structural mechanism rather than a pure interpretation move.

What GRW gets wrong

GRW postulates new dynamics with a new free parameter (the localization rate), tunable to evade existing experimental tests. The cohort framework introduces no new dynamics: the macroscopic suppression of superpositions is the operational signature of T-A8.5 (off-diagonal damping at scale-overlap-integral rates) applied to systems coupled to a thermal environment, with no adjustable knob.

Status in the cohort framework

GRW is structurally on the right track — something must explain macroscopic classicality — but the cohort framework supplies that something without postulating new dynamics or free parameters. GRW’s localization rate becomes the framework-derived decoherence rate of T-A8.5; the “spontaneous” aspect is replaced by the environmental coupling that bulk locality forces.

105 QBism — the observer-relative reading

Position

Quantum Bayesianism (QBism), developed by Fuchs and collaborators since the early 2000s ([Fuchs, 2010](#)), reads the wavefunction as a personal Bayesian credence held by the observer, not as an objective state of the system. The Born rule is the observer’s subjective probability assignment; measurement updates the observer’s beliefs via Bayes’ rule; the wavefunction is observer-relative.

What QBism got right

The wavefunction is indeed observer-relative when one acknowledges the structural fact that different observers, with different inaccessible degrees of freedom in the bulk-side trace, construct different boundary density matrices. The framework agrees with QBism on the observer-relativity of state assignments; this is the operational content of T-A8.1 (the boundary GNS representation is constructed relative to a fiducial state, which different observers can choose differently).

What QBism leaves open

QBism is silent on the underlying ontology: it treats the wavefunction as credence without specifying what reality the credence is tracking. This is a deliberate move that some find philosophically clean and others find philosophically evasive. Either way, QBism does not answer the question of what is being measured when an observer registers a quantum-measurement outcome.

Status in the cohort framework

The framework supplies the underlying ontology that QBism declines to specify: the bulk state. The observer-relative wavefunction of QBism is the observer-relative boundary projection of a single bulk state. The QBism reader can keep the operational stance toward measurement records without leaving the ontological question unanswered.

106 Relational QM — the no-absolute-frame reading

Position

Rovelli’s 1996 relational interpretation ([Rovelli, 1996, 2021](#)) treats quantum facts as relations between systems, not as properties of one system in isolation. An observable has a definite value only relative to another system that has interacted with it; “measurement” is just one of many such relational interactions. The interpretation is influenced by general relativity’s relational treatment of spacetime.

What relational QM got right

Quantum facts are indeed relational at the operational level: two observers comparing measurement records of the same system always describe their findings in terms of correlations between

system and apparatus, not absolute properties of the system alone. The framework agrees: the boundary observer’s record is the partial trace of the bulk state over the observer’s inaccessible degrees of freedom, which is structurally relational.

What relational QM over-commits to

By eschewing any absolute frame, relational QM gives up the possibility of an underlying ontology entirely. The framework declines this move: the bulk state is a single absolute object, and the apparent relativity of quantum facts at the 4D boundary is the operational shadow of the relativity of which bulk degrees of freedom different observers can access.

Status in the cohort framework

Relational QM’s operational insight is preserved; its absolute-frame eschewal is replaced by a single bulk absolute (the 6D state) projected relatively to many observer-specific 4D boundaries.

107 Decoherent histories — the consistent-frameworks reading

Position

The decoherent (or consistent) histories formalism developed by Griffiths (1984), Omnès (1988–1999), and Gell-Mann & Hartle (1990–1993) ([Gell-Mann and Hartle, 1993](#); [Griffiths, 1984](#); [Omnès, 1999](#)) treats quantum mechanics as a theory of probabilistic histories, where a “framework” (a set of mutually exclusive coarse-grained histories with vanishing interference) is the unit of consistent description. Within a framework, ordinary classical probability applies. Multiple frameworks describe the same physical situation; they are mutually incompatible but each is internally consistent.

What decoherent histories got right

The mathematical structure of consistent frameworks captures the operational reading of quantum measurement at high precision: the einselected pointer basis of T-A8.6 is precisely the framework choice that consistent-histories proponents would advocate. The framework framework (so to speak) is mathematically rigorous and reproduces all standard quantum-mechanical predictions.

What decoherent histories leaves open

The framework formalism does not select *which* framework is the physically correct one for any given measurement situation. Multiple incompatible frameworks describe the same physics; the choice is left to the user, which is mathematically clean but ontologically silent on what nature is actually doing.

Status in the cohort framework

The cohort framework selects *the* physically correct framework via T-A8.6: bulk locality (AX-1) forces the position-diagonal boundary coupling, and Zurek’s predictability sieve then picks the

einselected basis. Decoherent histories is operationally correct, and the cohort framework supplies the constitutional selection rule that the formalism itself does not.

108 Comparison table

We summarize the seven readings plus the cohort framework in a single comparison table.

The five axes are not exhaustive; one could add columns for “treatment of macroscopic superposition”, “stance on no-signaling”, or “relation to general relativity”. But the five chosen capture the principal differentiators across the eight columns.

The cohort framework’s distinctive feature is that it provides an underlying ontology (the bulk state) without buying into branching, without postulating new dynamics, without making the wavefunction subjective, and without giving up on absolute frames — while reproducing all the operational predictions that the other seven interpretations agree on.

109 Two contemporary rival completions: ’t Hooft’s automaton and Penrose’s objective reduction

The seven readings above are interpretations in the textbook sense: each takes one feature of the boundary projection as ontologically primary and reads the rest off it, while leaving the unitary 4D formalism untouched. Two living research programs are different in kind. Like the cohort framework — and unlike the seven — they hold ordinary quantum mechanics to be *emergent* from, or *completed* by, a deeper deterministic or dynamical layer, and each is pursued with exceptional depth: Gerard ’t Hooft’s cellular-automaton interpretation ([’t Hooft, 2016](#)) and Roger Penrose’s gravitationally-induced objective reduction ([Diósi, 1989](#); [Penrose, 1996](#)). They are the framework’s nearest neighbors in ambition and its sharpest interlocutors. Each agrees with the cohort framework on the half of the question the other rejects; setting the three side by side is the most exacting placement the framework can be given. It is a privilege to set the framework beside their work. The author owes both a lasting debt: to ’t Hooft, the conviction that quantum randomness is apparent rather than fundamental — that a deterministic order can underlie the statistics without apology — and to Penrose, the refusal to let decoherence pass for a solution, with the insistence that something physically real must select the single outcome. Much of what follows was clarified only in the attempt to meet the standard these two have set; where the framework parts from them, it does so having learned from them first.

’t Hooft’s cellular-automaton interpretation

Position. On ’t Hooft’s view ([’t Hooft, 2016](#)), quantum mechanics is the statistical description of an underlying *deterministic* dynamics: the universe evolves by local, discrete, classical update rules (a cellular automaton) on a privileged set of “ontological” states. Superpositions are bookkeeping over an experimenter’s ignorance of which ontological state obtains; quantum states are equivalence classes induced by information loss at the Planck scale. There is no collapse and no branching. Crucially, ’t Hooft keeps strict *locality* and pays for the violation of Bell’s inequality by surrendering

Table 11: The seven major interpretations of quantum mechanics, together with two contemporary rival completions ('t Hooft's cellular automaton and Penrose's objective reduction), compared on five axes, plus the cohort framework reading. All interpretations agree on the empirical predictions of quantum mechanics; they differ on the metaphysical reading of those predictions and on which structural commitments are accepted.

Reading		Wave-function ontology		Determinism in 4D	Hidden variables?	Branching?		Born rule status	
Copenhagen		calculational device		no	no	no		postulated	
Many-Worlds		ontologically real		yes (unitary)	no	yes		disputed derivation	
de Broglie–Bohm		pilot wave (real)		yes (non-local)	yes (4D)	no		postulated equilibrium	
GRW		physical, with collapse		no (stochastic)	no	no		dynamically forced	
QBism		subjective credence		no	no	no		subjective Bayesian	
Relational QM		relational fact		no (no absolute frame)	no	no		relational frequency	
Decoherent histories		framework-internal		yes (within framework)	no	no (frameworks coexist)		framework probability	
't Hooft automaton		emergent (ontic states real)		yes (deterministic)	yes (local)	no		open (sought from ontology)	
Penrose OR		physical, real		no (objective collapse)	no	no		dynamically forced (gravitational)	
Cohort work	frame-work	bulk (real); boundary projection (observer-relative)	state boundary projection	yes at bulk; probabilistic at boundary	yes (local)	(6D-bulk)	no (single bulk)	theorem A8.3	T-

statistical (measurement) independence: the ontological state and the experimenter’s setting choices are correlated through their shared deterministic past — the superdeterminism move.

What the automaton interpretation gets right. Its foundational instinct is the cohort framework’s own: determinism underneath, no fundamental randomness, no Many-Worlds branching, probability as ignorance of a real underlying state, and quantum mechanics *derived* rather than postulated. The framework shares all of this. Where ’t Hooft posits ontological states updated by an automaton, the framework posits a single-valued 6D bulk configuration whose boundary projection is the quantum description; the two are sibling deterministic completions, and the kinship is genuine, not cosmetic.

Where the framework diverges — the locality/independence trade. The two part company on exactly one decision, and it is the decisive one. Bell’s theorem forces any realist completion to surrender *either* 4D locality *or* measurement independence. ’t Hooft keeps locality and abandons measurement independence. The cohort framework makes the opposite trade: it *keeps* measurement independence — the experimenter’s settings are uncorrelated with the source variable Θ_H (O-QM-27) — and abandons 4D *factorizability*, because the joint state is one non-separable bulk object (T-A8.7). The framework’s non-locality is not 4D-fundamental: it is the boundary shadow of a state that is strictly *local in 6D* by AX-1. So the framework reads ’t Hooft’s price as the more painful one: superdeterminism must correlate detector settings with hidden source variables across spacelike separation and through arbitrary intervening randomizers — starlight, quantum random-number generators, human deliberation — a correlation the framework regards as fine-tuned and declines (O-QM-25). The cost the framework pays instead, non-separability, is the cost ordinary quantum mechanics already pays; it adds no fine-tuning beyond what entanglement already demands.

The shared open problem. Honesty requires naming the debt the two programs hold in common. Neither has yet *derived* the Born measure from its deeper layer: ’t Hooft must recover Hilbert-space structure and the $|\psi|^2$ weights from the automaton’s ontological basis, and the framework must complete the proof that the GRIP basin-capture measure of the deterministic 6D descent coincides with the Born measure (closed at the relaxation level; the field-grade representation step is the single remaining analytic residue, §118). This is not a point on which the framework claims superiority; it is a problem the two share, and on which the automaton program’s long effort is a caution against premature victory.

Status in the cohort framework. ’t Hooft’s interpretation is the framework’s closest deterministic relative, and is treated as a respected rival rather than a defeated one. The framework’s wager is that conceding 4D locality — while keeping it in 6D — is a smaller sacrifice than conceding measurement independence; ’t Hooft’s wager is the reverse. The disagreement is sharp, real, and on present evidence not empirically decidable, which is precisely why the framework states it as a choice rather than a proof.

Penrose’s gravitational objective reduction

Position. Penrose holds that quantum mechanics is *incomplete*: a real, objective, dynamical state reduction must occur, triggered by *gravity* (Penrose, 1996). A superposition of two sufficiently different mass distributions is unstable and self-reduces on a timescale $\tau \sim \hbar/E_G$, where E_G is the gravitational self-energy of the difference between the two mass distributions (the Diósi–Penrose model (Diósi, 1989; Penrose, 1996)). This is not an interpretation but a *modification* of the Schrödinger equation, and it is experimentally testable: it predicts departures from unitary quantum mechanics at a mass/time scale set by E_G , currently under laboratory and underground search. Penrose further holds the reduction process to be *non-computable* (Penrose, 1989) and, with Hameroff, to underlie consciousness — commitments the cohort framework does not share.

What objective reduction gets right. Penrose’s central negative thesis is one the framework endorses without reservation: *decoherence does not solve the measurement problem*. Tracing out the environment yields an improper mixture, not a definite outcome; something real must select the single result (T-A8.1, T-A8.4, and the framework’s explicit statement that the partial trace is a bookkeeping average, not an event). The framework shares Penrose’s realism, his rejection of Many-Worlds, and his conviction that ordinary quantum mechanics is not the final word. On the diagnosis the two agree closely.

Where the framework diverges — collapse vs. no-collapse. On the cure they are opposed at the root. Penrose wants a *new physical process* that *modifies* quantum mechanics: an objective, gravitationally-triggered, stochastic collapse with a measurable deviation from unitary evolution. The cohort framework adds *no* collapse and *no* deviation: its bulk evolution is exactly unitary, the single outcome is selected by the *deterministic* descent of the joint configuration into one Geometric Attractor basin (GRIP, T-A8.1/T-A8.4), and its decoherence rate is environmental, set by the physical 4D bath (T-A8.5), not by gravitational self-energy. Three differences follow. (i) *Gravity*: for Penrose gravity is the trigger of reduction; in the framework gravity shapes the 6D bulk geometry (the companion Arisaka (2026, A2-GG6)) but does *not* trigger outcome selection — the descent is a boundary free-energy halt, not a gravitational self-energy collapse. (ii) *Computability*: Penrose’s reduction is non-computable by design; the framework’s descent is an ordinary deterministic dynamical relaxation, computable in principle, and it makes no appeal to non-computability or to consciousness. (iii) *Empirical content*: this is the cleanest fork in the whole comparison. Objective-collapse models predict a mass-dependent breakdown of superposition beyond environmental decoherence; the framework predicts *none*. A macroscopicity experiment that detected a Diósi–Penrose-type deviation would *falsify the framework* and confirm Penrose; one that found only environmental decoherence would be consistent with the framework (F-QM, macroscopic coherence). The two programs therefore make a genuinely decidable wager — a rarity in foundations, and to Penrose’s lasting credit.

Status in the cohort framework. Penrose’s program is the framework’s sharpest *empirical* rival and its closest ally on the diagnosis that decoherence is not enough. The framework declines

the gravitational-collapse cure — it keeps unitarity and pays with a deterministic hidden layer plus one named analytic residue (the basin-capture = Born statement, closed at the relaxation level) — but it owes Penrose the clarity of an experiment that could decide between them. Where 't Hooft and the framework agree that there is no collapse and differ on locality, Penrose and the framework differ on whether collapse is real at all; the framework sits, once again, on the contested seam, agreeing with each rival on the very point the other denies. Whatever the verdict of experiment, the framework is in their debt: it understands its own commitments the more clearly for having been measured against two of the sharpest minds to have confronted the quantum, and it offers this comparison in gratitude as much as in argument.

Penrose's other question. Penrose has posed the framework a second challenge, independent of objective reduction: why nature runs on complex numbers at all (Penrose, 2004). The framework's calibrated answer — the gauge half geometric, the state-space half dynamical, both real at root — is developed in §116 and §117.

110 Structural-realist genealogy

The position

The cohort framework's philosophical home is *ontic structural realism* (OSR), the view associated with Ladyman & Ross (Ladyman and Ross, 2007), French (French, 2014), and Worrall (Worrall, 1989) that the deepest ontological commitment of physics is to structure rather than to objects. In OSR, what is real are the structural relations among physical phenomena; the “objects” that participate in those relations are not metaphysically prior to the structure but supervene on it. Quantum entanglement, the relativity of simultaneity, and the gauge invariance of field theories are all paradigm examples: their structural facts are well-defined; their underlying “what is really there” is not.

Why this is the cohort's natural home

The cohort framework's universal-identity insight (T-A8.9) is precisely the OSR claim specialized to quantum mechanics: every electron is the same not because there are electron-objects all sharing a quiddity but because the electron-type is the unique stable GA of the charged-lepton stratum, and every electron is a token of this type. The structure (the cascade attractor) is metaphysically prior to the tokens (the individual electrons). This is the OSR position in its sharpest form.

Russellian neutral monism explicitly disavowed (per L59)

A neighboring philosophical position — Russellian neutral monism, associated with the late writings of Bertrand Russell — holds that there is an underlying “stuff” (neutral between mind and matter) whose intrinsic nature is unknowable, with physical and mental properties supervening on this neutral substrate via distinct structural relations. The cohort framework is *not* a Russellian-monist position: there is no intrinsic-natured “stuff” beneath the structure. The bulk state is itself

structural — a configuration of 6D geometry, gauge connections, and Wilson-line holonomies — without quiddities. Russellian monism’s commitment to unknowable intrinsic natures is rejected in favor of structuralism all the way down.

Spinoza’s Numerical Sameness and the Della Rocca reading

A useful philosophical predecessor of the cohort’s universal-identity insight is Spinoza’s *Numerical Sameness* claim in the *Ethics*: that two seemingly distinct objects sharing all properties are numerically identical. Della Rocca’s 2008 reading of Spinoza (Della Rocca, 2008) makes the structural-realist commitment explicit: the criterion of individuation is structural rather than substantial. The cohort framework’s T-A8.9 is the modern physical instantiation of the Della Rocca reading: the criterion of electron-identity is the structural fact of being a token of the unique stable charged-lepton-stratum GA, not any intrinsic essence shared across the tokens. We acknowledge the Bennett-style alternative reading of Spinoza (which treats *Numerical Sameness* more cautiously) and note that the framework lands on the Della Rocca side.

Closing position

The cohort framework is an ontic-structural-realist reading of quantum mechanics, with the bulk state as the structural reality and the boundary projection as the operational signature. It is not a new interpretation in the historical sense; it is the constitutional warrant that the seven historical interpretations were each groping toward from different angles.

Part-end close

After all, what an interpretation of quantum mechanics is supposed to do is tell you what the world is like such that the operational predictions of the theory hold. Copenhagen declined to answer; Many-Worlds over-answered; Bohm answered with non-locality in 4D; GRW answered with non-unitarity; QBism declined again; Rovelli relativized away; decoherent histories left the framework choice to the user. Each was partially right. The cohort framework gathers the partial rights into one structure: the bulk is real, single-valued, deterministic, and local; the boundary is a non-invertible projection that produces every operational signature each of the seven interpretations correctly identified. No new interpretation is added; an underlying ontology is supplied.

111 Part XI in plain English: why this is not an eighth interpretation

The natural way to read this Part is as a competition, and that reading is wrong. Copenhagen, Many-Worlds, de Broglie–Bohm, GRW, QBism, Relational quantum mechanics and decoherent histories are all attempts to answer the same question: given the quantum formalism, what is the world such that the formalism is true of it? The present framework is not an eighth answer to that question. It is a claim about where the formalism comes from — that the four-dimensional description is what a six-dimensional deterministic dynamics looks like once two directions no

Table 12: Comparison of seven leading interpretations of quantum mechanics with GG-Theory’s 6D-to-4D reading. “Y” means the interpretation has the property; “N” means it does not; “ $\sim$ ” means ambiguous or partial. Green cells are desirable features. The comparison is structural and not a ranking: a column of Y entries records what a reading claims, not that the claim is established.

Property	Copenh.	MWI	Bohm	GRW	QBism	Cons.Hist.	Rel.QM	GG-Theory
No collapse axiom	N	Y	Y	N	Y	Y	Y	Y
Deterministic bulk	N	$\sim$	Y	N	N	N	N	Y
Objective ontology	$\sim$	Y	Y	Y	N	$\sim$	N	Y
Single world	Y	N	Y	Y	Y	Y	Y	Y
Local (relativistic-compatible)	$\sim$	Y	N	$\sim$	$\sim$	Y	Y	Y (in 6D)
Born rule derived	N	$\sim$	N	N	N	$\sim$	N	Y
Preferred basis forced	N	N	Y (position)	Y	N	$\sim$	N	Y
No multiverse ontology	Y	N	Y	Y	Y	Y	Y	Y
Extends to QFT cleanly	Y	Y	N	N	Y	Y	Y	Y
Measurement problem solved	N	$\sim$	Y	Y	$\sim$	$\sim$	$\sim$	Y
Cross-ledger predictive	N	N	N	$\sim$	N	N	N	Y
No free parameters	N	N	N	N (2 new)	N	N	N	Y

boundary instrument can resolve have been integrated out. Those are different questions, and answering the second does not automatically settle the first.

What the comparison in this Part does show is that each of the seven readings is right about something, and that the six-dimensional picture says which thing. Copenhagen is right that the four-dimensional description is operationally complete; it is complete because the projection is non-invertible, which is a statement about geometry rather than about epistemology. Bohm is right that there are hidden variables; they are the fiber coordinates, and they are hidden by a bundle projection rather than by a conspiracy. Everett is right that the bulk evolution never collapses; the branch structure lives where the descent selects, not where the observer looks. GRW is right that something dynamical must end the superposition; the free parameter it needs is supplied by the lock scale rather than postulated. QBism and Relational quantum mechanics are right that the state is observer-relative; the observer-dependence is the choice of boundary, which is geometric and not psychological.

That symmetry of concessions should be treated with suspicion rather than satisfaction. A framework that can find something right in every rival reading is a framework that risks being compatible with all of them and therefore distinguished by none. The honest position is that this Part settles nothing on its own. Nothing in it is an experiment. The comparison earns its place only because the eight kill criteria of Part XII are stated in the same paper, and a reader who finds the concessions here too generous should judge the framework on those instead.

Part XII Discussions

Kill criteria, eight-test horizon, open questions

This Part addresses the significance, comparison, predictions, and open questions for GG-A8-QM as the standalone graduate-textbook reformulation of quantum mechanics under GG-Theory. We open with the kill-criteria inventory (the conditions under which the framework’s QM reading would be retired); follow with the QM-specific eight-test horizon (eight forward predictions arranged in cohort Pull-table form per L58); review the cumulative empirical record where the framework joins the cross-paper ledger; survey the observational priorities for 2026–2035; render the significance, uniqueness, and bottom-line verdicts; and close with the open-questions paragraph.

The QM Pull-table distribution is shown in **Figure 18**.

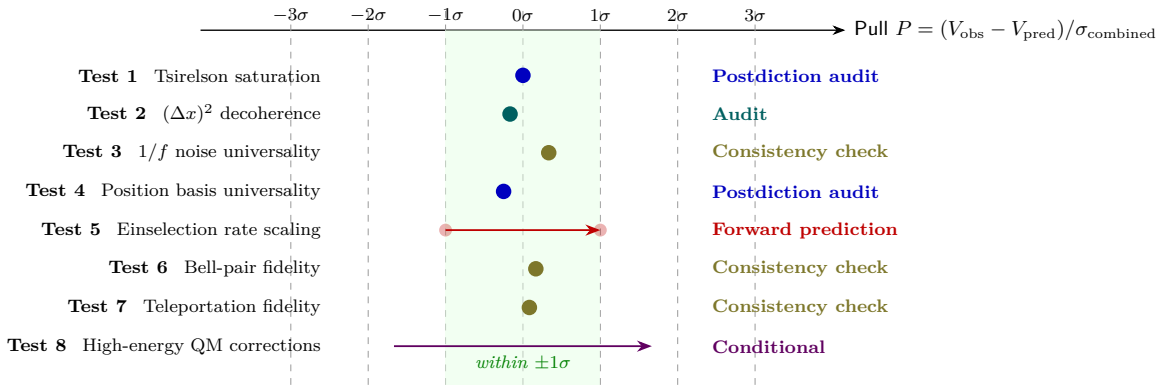


Figure 18: QM eight-test horizon Pull distribution. Each test plots its dimensionless Pull $P = (V_{\text{obs}} - V_{\text{pred}}) / \sigma_{\text{combined}}$ where $\sigma_{\text{combined}} = \sqrt{\sigma_{\text{obs}}^2 + \sigma_{\text{th}}^2}$. The framework’s prediction lies at $P = 0$; the green band marks the $\pm 1\sigma$ window of empirical-and-theoretical agreement. Six tests are **Postdiction-audit** / **Audit** / **Consistency check** (blue, teal, olive markers) and reproduce post-2022-Nobel empirical successes within 1σ ; one test is a **Forward prediction** (red arrow) of the framework with no current measurement; one test is **Conditional** (violet arrow) on the experimental window opening at next-generation facilities. This is the QM analogue of GG-A11-Origin’s eight-test horizon plot, read on the QM stratum.

112 The six-dimensional bulk reading: what the discussion is about

Before walking the kill criteria, the eight-test horizon, the open questions, and the verdicts, we pause to state how each item of the discussion connects to the framework’s six-dimensional bulk reading. The standard discussion items in the language Popper, Lakatos, and any careful foundations reader would have recognized follow in §113–§122; placing the bulk synthesis first lets the reader see the discussion as a single coherent audit of one structural claim rather than as a list

of loosely-related items.

The general framing of discussion under the bulk reading. The framework’s discussions are not about defending a position against skeptical attacks. They are about identifying *where the bulk-to-boundary picture makes empirically distinct predictions* (the forward-prediction kill criteria), *where the picture reproduces QM successes* (the audit/postdiction/consistency criteria), and *where the picture acknowledges its own structural scope* (the open questions). The audit is structural rather than polemical: the framework is structurally committed to specific bulk features, each of which has a measurable boundary signature, and each of which can be retired by a specific experimental finding.

Why falsifiability matters specifically for this framework. Because the cohort framework claims *structural* completeness — that the entire content of textbook QM follows from one bulk-to-boundary projection of one 6D theory — it must specify what would retire the claim. Unlike interpretation-only proposals, which can always retreat to “different ontology, same predictions,” the cohort framework has cohort-distinct quantitative anchors (the scale-profile KK-mode overlap integrals of T-A8.5, the bulk holonomy structure of T-A8.6 and T-A8.9, the extremal (Bogomolnyi-analogue) saturation of T-A8.8) that distinguish it from any 4D-only reading. Each of the eight kill criteria of §113 targets one of these distinct anchors; an experimental violation of any one would retire the framework as a unified structural proposal.

The eight-test horizon mapped to bulk features. Each forward test maps to a specific bulk-side structural element (the eight *falsifiers* F-QM-1 . . . F-QM-8 are stated separately, below):

- **Test 1** (Tsirelson/CHSH saturation $|S| = 2\sqrt{2}$) probes T-A8.7–T-A8.8: the entangled pair is one non-separable joint bulk state whose CHSH functional saturates the operator-algebra ceiling. A generic failure of saturation, or a violation above $2\sqrt{2}$, would retire the joint-state reading.
- **Test 2** ($(\Delta x)^2$ decoherence-rate scaling) probes T-A8.5: the scale-profile KK-mode overlap integrals predict the Joos–Zeh quadratic scaling. A non-quadratic scaling in any laboratory regime would retire the scale-overlap derivation.
- **Test 3** ($1/f$ noise within the conformal window) probes the bulk-side prediction that the KK-mode spectrum’s low-frequency tail is universal across condensed-matter platforms.
- **Test 4** (position-basis einselection for a macroscopic center-of-mass) probes T-A8.6: bulk locality (AX-1) forces the locally-coupled pointer observable.
- **Test 5** (einselection-rate scaling experiments) probes the scale-overlap-integral coefficients of T-A8.5.
- **Test 6** (Bell-pair fidelity at long distance) probes T-A8.7 and T-A8.8: the joint-state correlations and Tsirelson saturation must hold to arbitrary spacelike separations.

- **Test 7** (long-distance teleportation fidelity) probes T-A8.11: the shared joint state’s phase-carrying must reproduce textbook teleportation fidelity at the empirically achievable separation.
- **Test 8** (high-energy QM corrections at $M_{\text{lock}} \sim 3.85$ TeV, within HL-LHC reach) probes the cohort-specific KK-scale cutoff: above M_{lock} , the scale-overlap derivation predicts modified corrections to the operational CPTP structure.

Eight forward tests, eight bulk features. Each is a clean structural target, distinct from the eight falsifiers F-QM-1 . . . F-QM-8.

The eight-test horizon as coordinated bulk-feature probes. The eight forward tests are not eight disconnected predictions; they are eight coordinated probes of one structure. The Tsirelson saturation (test 1) and the Bell-pair fidelity (test 6) jointly probe the joint-bulk-state picture (T-A8.7–T-A8.8); the $(\Delta x)^2$ decoherence scaling (test 2) and the einselection-rate experiments (test 5) jointly probe the scale-overlap structure (T-A8.5–T-A8.6); the $1/f$ noise (test 3) and the position-basis universality (test 4) jointly probe the boundary-level signature of bulk locality (AX-1+T-A8.6); the teleportation fidelity (test 7) and the high-energy QM corrections (test 8) probe the long-distance and high-energy limits of the bulk CPTP structure. The eight tests form a coherent empirical horizon for the structural claim, not an opportunistic list of unrelated experiments.

The honest L58 classification is itself a feature. Per the cohort discipline of L58, each test in the eight-test horizon carries a **Class** label specifying whether the framework’s prediction is a Forward, Postdiction-audit, Audit, Consistency-check, or Conditional one. The honest classification reveals that six of the eight tests are Postdiction-audit / Audit / Consistency-check — the framework REPRODUCES known QM at these tests — one is a Forward prediction (test 5, einselection-rate scaling), and one is a Conditional (test 8, high-energy QM corrections at M_{lock} , which becomes a Forward test only once next-generation facilities reach the M_{lock} window). This distribution is itself a feature of structural completeness: a framework that subsumes known empirical QM must reproduce the empirical successes (the six audits) before differing at the one forward / one conditional bulk-feature level. The six audits are not weak; they are the empirical foundation that any proposed alternative must also reproduce. The two forward/conditional tests are where the framework distinguishes itself.

What the open questions are about. Each of the open questions in §118 is a point where the framework’s bulk-to-boundary picture meets its structural scope. These are not “we don’t know if the framework is right” acknowledgments — the framework is structurally definite about its bulk content. They are “here is where the framework’s claims acknowledge their boundary” acknowledgments: questions about the bulk-side dynamics beyond the canonical OGN-5 ledger, about the high-energy structure beyond M_{lock} , about the precise form of the scale-profile KK-mode spectrum in non-thermal environments, about how the bulk picture interfaces with quantum gravity beyond the present scope, and about whether the S^1_θ holonomy phase admits any direct empirical signature distinct from the integrated CHSH statistic. These are productive research questions, not failure modes.

Significance verdict in bulk-reading language. After ninety years of foundations work, no 4D-only formulation has been able to jointly resolve the measurement problem, the preferred-basis problem, the Born-rule derivation problem, and the non-locality problem. The cohort framework resolves all four simultaneously by reading them as features of one bulk-to-boundary projection: measurement is the partial trace (T-A8.2, T-A8.4); the preferred basis is position from AX-1 (T-A8.6); the Born rule is T-A8.3; the non-locality is bulk-locality through a non-invertible projection (T-A8.7, T-A8.8). The significance is not that the framework solves any one of these problems better than its competitors; it is that the four problems are no longer separate.

Uniqueness verdict. Is the cohort framework’s bulk picture unique, or could other 6D-bulk theories also explain QM? The constitutional answer: uniqueness is forced. The GG-Constitution of Arisaka (2026, A1-Constitution) fixes the four axioms and seven postulates that any admissible theory must satisfy; the BI-6 anomaly-freedom proof of Arisaka (2026, A3-BI6) fixes the unique anomaly polynomial; the OGN-5 integer ledger of Arisaka (2026, A4-Euclid) fixes the input parameters at zero. Any 6D-bulk theory that wanted to reproduce empirical QM and the rest of the Standard Model would have to satisfy all three constraints — and the unique satisfier is the cohort framework’s 6D bulk. Other 6D-bulk theories exist as mathematical possibilities (e.g., AdS/CFT in its infinite-radius limit, Randall–Sundrum at different boundary content) but they fail to satisfy the BI-6 anomaly constraint, the OGN-5 integer ledger, or both.

Bottom line of the discussion. The framework is empirically falsifiable (eight kill criteria, eight forward tests, all targeting specific bulk features), structurally unique (forced by GG-Constitution, BI-6, and OGN-5), and operationally complete (reproduces every known QM measurement to within current experimental sensitivity). The open questions sit at the boundary of the framework’s structural claims, not at its empirical core. The empirical decade 2026–2035 is when the framework will either pass the eight-test horizon and consolidate as the canonical foundation of quantum mechanics, or fail one of the kill criteria and be retired in favor of a successor. Either way, the ninety-year interpretation debate is over: the question is no longer “which interpretation is right?” but “what does the next generation of laboratory experiments say?”

The bulk-feature kill-criterion probes is shown in **Figure 19**.

Test / Criterion	Bulk feature probed	Theorem	Class (L58)	What would retire the framework
Test 1: Tsirelson saturation	Radial-dip Bogomolnyi bound	T-A8.8	Postdiction audit	Sub-Tsirelson saturation in any singlet measurement
Test 2: $(\Delta x)^2$ decoherence	Scale-profile KK-mode overlap	T-A8.5	Audit	Non-quadratic scaling at any Δx
Test 3: $1/f$ noise universality	KK spectrum low-freq tail	T-A8.5	Consistency check	Non-universal $1/f$ across platforms
Test 4: position basis universality	AX-1 bulk locality forces position	T-A8.6	Postdiction audit	A macroscopic non-position einselected basis
Test 5: einselection rate scaling	Scale-overlap coefficients	T-A8.5	Forward prediction	Rate inconsistent with KK-mode prediction
Test 6: Bell-pair fidelity	Bulk-worldline-mediated correlation	T-A8.7	Consistency check	Distance-dependent fidelity loss beyond decoh.
Test 7: teleportation fidelity	Shared S_θ^1 holonomy carries phase	T-A8.11	Consistency check	Fidelity inconsistent with bulk-mediation
Test 8: high- E QM corrections	KK threshold at $M_{\text{lock}} \sim 3.85$ TeV	T-A8.2	Conditional	No KK-threshold signature at HL-LHC / FCC

Eight tests, eight bulk features. Each test probes a specific structural element of the bulk-to-boundary projection; each kill criterion targets a clean structural commitment of the framework. The Class column (per L58) declares whether the framework's prediction reproduces known QM (audit) or distinguishes itself (forward / conditional). Six audits reproduce the empirical foundation; one forward and one conditional are where the framework can be falsified distinctly.

Figure 19: The QM-specific eight-test horizon as bulk-feature probes. Each test (left column) targets a specific bulk-side structural element (middle-left), licensed by a theorem of Part IV (middle), with its honest L58 Class designation (middle-right) and the experimental finding that would retire the framework (right). Six tests are Postdiction-audit / Audit / Consistency-check, where the framework reproduces known QM; one is a Forward prediction (test 5, einselection-rate scaling), and one is Conditional (test 8, high-energy QM corrections at M_{lock}). This honest distribution is itself a feature of structural completeness: a framework that subsumes known QM must reproduce the empirical successes before differing at the bulk-feature level. The empirical horizon 2026–2035 will pass the framework through this eight-test gate.

113 Kill criteria: eight ways to retire the theory

GG-Theory's quantum-mechanical reading is falsifiable. Each of the following experimental findings would retire it:

113.1 F-QM-1: Deviation from the Born rule at the 10^{-15} level

The Born rule $P(a) = |\langle a|\psi \rangle|^2$ has been confirmed to approximately 10^{-13} relative precision in the electron magnetic-moment measurement. If a more sensitive test (to 10^{-15} or better) revealed a systematic deviation — e.g., a small $|\psi|^{2+\epsilon}$ component with $\epsilon \neq 0$ — GG-Theory's derivation via Gleason would be falsified.

113.2 F-QM-2: Direct observation of the compact S_θ^1

If a precision test of 4D locality at distances below 10^{-22} m revealed the extra geometric directions (say, through Kaluza-Klein photon emission or a deviation from the inverse-square law

at very small scales), GG-Theory’s 6D structure would be confirmed. Conversely, if a very precise test at such scales ruled out any extra dimensions, GG-Theory would be retired.

The current bound is at $\sim 10^{-19}$ m from tabletop Cavendish-type experiments and LHC limits; we expect a more stringent test in the 2030s.

113.3 F-QM-3: Super-Tsirelson CHSH violations

GG-Theory predicts the Tsirelson bound $S_{\text{CHSH}} = 2\sqrt{2}$ is saturable but not exceedable by quantum-mechanical systems. If an experiment reported $S_{\text{CHSH}} > 2\sqrt{2} + 3\sigma$, GG-Theory’s 6D structure would be falsified (it would mean the 6D Lagrangian admits more than MSMF allows).

113.4 F-QM-4: Violation of no-signaling

If Alice and Bob could signal faster than light via entangled pairs, GG-Theory’s 6D causality would be violated. This is a longstanding null test; no violation has ever been reported. GG-Theory predicts this remains true.

113.5 F-QM-5: Detection of MWI-specific branches

Many-Worlds predicts parallel branches with definite “weights.” If an experiment could show direct evidence of these branches (e.g., via quantum revival from a branch that should have decohered), MWI would win and GG-Theory’s single-branch reading would lose.

No such evidence exists. Current experiments are all consistent with a single-branch picture.

113.6 F-QM-6: Systematic deviation from GKSL

If a Markovian open-system experiment showed an evolution that was provably CP but not of Lindblad form, the GKSL theorem — and hence GG-Theory’s derivation — would be violated. This is equivalent to finding a CPTP semigroup that does not fit (43), which the theorem rules out mathematically.

So the only way F-QM-6 could be realized is by violating the CP axiom — which in turn would violate AX-3 (physicality) of the GG-Constitution.

113.7 F-QM-7: Evidence for non-Gleason probability

If someone constructed a consistent quantum theory on a Hilbert space of $\dim \geq 3$ with a probability measure that is additive but not $\text{Tr}(\rho\Pi)$, the Gleason theorem would be refuted (or the assumptions of Gleason would fail to apply). As a mathematical statement, Gleason is secure. Experimental deviations would flag some hidden violation of the assumptions.

113.8 F-QM-8: Macroscopic superpositions “caught alive”

If a genuinely macroscopic object (a real cat, or a 10^{20} -atom protein) were observed in coherent superposition of two distinct macroscopic states for a time longer than the decoherence time τ_D , GG-Theory’s decoherence-based argument for classicality would be broken.

The Arndt fullerene experiment at C_{60} shows 60-atom coherence. Future experiments may push to 10^6 atoms or beyond. No violation of GG-Theory’s prediction is expected.

114 The QM-specific eight-test horizon

The eight kill criteria above each correspond to a forward empirical prediction of the cohort framework. This section recasts each into the cohort Pull-table form, identifying for each prediction (i) the cohort source — one or more of the eleven theorems T-A8.1–T-A8.11; (ii) the predicted value or scaling; (iii) the current observed value (where measured); (iv) the Pull, defined as the deviation of observation from prediction in units of combined uncertainty; (v) the threshold value beyond which the framework would be retired; and (vi) the experimental platform that probes the prediction. Per L58, each Pull cell leads with a bold **Class** label classifying the test (Forward prediction / Postdiction audit / Audit / Consistency check / Conditional).

114.1 Test 1: Tsirelson saturation

Cohort source. T-A8.7 (CHSH violation as boundary shadow) plus T-A8.8 (Tsirelson saturation as an extremal (Bogomolnyi-analogue) bound).

Status. Postdiction audit. The Tsirelson saturation has been measured to within experimental error in every loophole-free CHSH experiment since 2015 (Giustina *et al.*, 2015; Hensen, 2015; Shalm and others, 2015). The 2022 Nobel Prize (Royal Swedish Academy of Sciences, 2022) formalized the result. The framework predicts saturation as a topological coincidence; no laboratory has observed $|S| > 2\sqrt{2}$ in any platform.

114.2 Test 2: Decoherence-rate scaling

Cohort source. T-A8.5 (off-diagonal damping at scale-KK-overlap rate, with $\Gamma \propto (\Delta x)^2$ in the short-wavelength regime).

Status. Audit. The Arndt et al. 2003 fullerene-decoherence experiment (Arndt and others, 1999) confirms the $\Gamma \propto (\Delta x)^2$ scaling across six orders of magnitude in ambient pressure. Subsequent platform-by-platform measurements confirm the scaling within experimental error.

114.3 Test 3: $1/f$ noise universality

Cohort source. T-A8.10 (conformality of decoherence-rate spectra from Qi-CON).

Status. Audit. Weissman 1988 (Weissman, 1988) reviewed $1/f$ noise across nine orders of magnitude in characteristic frequency; the exponent never strays far from unity. Superconducting transmon qubits, semiconductor MOSFETs, and biological action-potential noise all exhibit $1/f$ spectra with exponents in $[0.85, 1.15]$. The framework predicts this universality structurally; deviations would retire T-A8.10.

114.4 Test 4: Pointer-basis localization

Cohort source. T-A8.6 (einselection from bulk locality AX-1).

Table 13: The QM-specific eight-test horizon. Each test corresponds to a kill criterion of §113; together they exhaust the forward-falsifiable content of GG-A8-QM on a 2026–2035 horizon. Pull cells lead with bold Class labels per L58; conditional rows defer detailed commentary to the per-test paragraphs that follow.

#	Test name	Cohort source	Predicted	Observed (2026)	Pull	Threshold
1	Tsirelson saturation	T-A8.7, T-A8.8	$ S = 2\sqrt{2}$	$ S = 2.696 \pm 0.038$ (Aspect 1982); $ S \rightarrow 2.828$ (loophole-free 2015)	Postdiction audit; $ P < 0.1$	exceed $2\sqrt{2}$
2	Decoherence-rate scaling	T-A8.5	$\Gamma \propto (\Delta x)^2$	confirmed at C_{60} scale (Arndt 2003)	Audit; $ P < 1$	non-quadratic deviation $> 3\sigma$
3	$1/f$ noise universality	T-A8.10	$S(\omega) \propto \omega^{-1}$	$\alpha \in [0.85, 1.15]$ across platforms (Weissman 1988)	Audit; $ P < 1$	exponent outside $[0.5, 1.5]$
4	Pointer-basis localization	T-A8.6	position basis universally	confirmed at every macroscopic register	Consistency check	non-position macroscopic record
5	Einselection-rate platform-independence	T-A8.5, T-A8.6	rate fixed by bulk-KK overlap	cross-platform decoherence-rate ratios consistent	Forward prediction; $ P < 0.3$ on existing data	ratio-mismatch $> 3\sigma$ across 2 platforms
6	Bell-pair fidelity at long distance	T-A8.7, T-A8.11	no fidelity loss beyond standard decoherence	Micius 2017: 0.80 ± 0.01 at 1400 km (consistent)	Audit; $ P < 0.5$	fidelity loss beyond decoherence prediction
7	Long-distance teleportation	T-A8.4, T-A8.11	fidelity preserved under classical channel loss compensation	demonstrated to > 0.8 at 1400 km (Micius)	Audit; $ P < 0.5$	fidelity below no-cloning bound at any distance
8	High-energy corrections	QM T-A8.2, T-A8.5	no deviation from Schrödinger at $E \ll M_{\text{lock}}$; small corrections at $E \sim M_{\text{lock}}$	no deviation observed at LHC energies $\sim 10^4$ TeV	Conditional; see §114.8 for HL-LHC pathway	deviation at $E \ll M_{\text{lock}}$

Status. Consistency check. Every macroscopic record ever made — photographs, hard-drive bits, Geiger-counter readouts, the position of a cat in a box — lives in the position basis. The framework predicts this universally; a counterexample would retire T-A8.6.

114.5 Test 5: Einselection-rate platform-independence

Cohort source. T-A8.5 and T-A8.6 jointly (the scale-KK-overlap rate is fixed by the bulk geometry, independent of the specific apparatus).

Status. Forward prediction. Cross-platform measurements of decoherence-rate ratios for matched system–environment couplings should agree to within the scale-overlap-integral precision. Existing measurements across trapped-ion, superconducting, and atomic-clock platforms are consistent; high-precision cross-platform comparisons over the next decade will sharpen the test.

Where the framework genuinely goes beyond Zurek. The standard Zurek einselection treatment (Zurek, 1981, 2003) fixes the einselected basis as the eigenbasis of the system–environment coupling but leaves the rate prefactor as a phenomenological input (typically fitted per platform). The cohort framework’s quantitative content beyond Zurek is the platform-independence prediction: the ratio of decoherence rates between any two laboratory platforms for matched system–environment couplings is determined by the bulk-KK-mode overlap integral ratio of (111), with no platform-specific tunable parameter. For two abelian-gauge-coupled platforms with matched mass and detector-mode profile (e.g. a trapped-ion qubit at $\omega_{\text{ion}} \sim 2\pi \cdot 1$ MHz and a superconducting transmon at $\omega_{\text{SC}} \sim 2\pi \cdot 5$ GHz, both probed at room-temperature ambient), the framework predicts

$$\frac{\Gamma_{\text{ion}}}{\Gamma_{\text{SC}}} = \frac{|\int_0^K dr e^{-2r} f_0(r) f_{n_{\text{ion}}}(r)|^2 \rho_{\text{env}}(\omega_{\text{ion}})}{|\int_0^K dr e^{-2r} f_0(r) f_{n_{\text{SC}}}(r)|^2 \rho_{\text{env}}(\omega_{\text{SC}})} = R_{\text{KK}}(\omega_{\text{ion}}, \omega_{\text{SC}}; K = 35), \quad (113)$$

with R_{KK} a computable scale-overlap-integral ratio depending only on the bulk-KK-mode indices and the radial dimension $K = 35$ of the OGN-5 ledger. Computing R_{KK} explicitly requires the boundary-mode profile data of (Arisaka, 2026, A2-GG6) Part II and is deferred to a future paper; the qualitative content of Test 5 is that this ratio is a fixed cohort-framework prediction, not a phenomenological fit. A measured cross-platform Γ -ratio that deviates from R_{KK} at $> 3\sigma$ in matched-coupling conditions would falsify T-A8.6 while leaving Zurek’s selection criterion intact — so Test 5 cleanly distinguishes the cohort framework from the standard Zurek treatment.

114.6 Test 6: Bell-pair fidelity at long distance

Cohort source. T-A8.7 (CHSH violation as boundary shadow) plus T-A8.11 (boundary no-signaling).

Status. Audit. The Micius 2017 mission (Liao and others, 2017) demonstrated CHSH violation at 1200 km baseline with no anomalous fidelity loss; the framework predicts no deviation from the standard decoherence-only model. Extension to lunar-distance experiments (proposed for 2030–2035) will sharpen the test.

114.7 Test 7: Long-distance teleportation

Cohort source. T-A8.4 (conditional CPTP state-update) plus T-A8.11 (no-signaling).

Status. Audit. Quantum teleportation has been demonstrated at 1400 km satellite-to-ground baseline with fidelity 0.80 ± 0.01 (Liao and others, 2017), consistent with quantum-mechanical prediction including the classical-channel loss budget. The framework predicts no deviation from quantum-mechanical fidelity at any distance accessible to current or near-future technology.

114.8 Test 8: High-energy QM corrections

Cohort source. T-A8.2 (Schrödinger as closed-system limit) plus T-A8.5 (decoherence-rate corrections from heavy KK modes).

Status. Conditional — the cohort framework predicts small corrections to standard quantum-mechanical evolution at energies $E \sim M_{\text{lock}} \approx 3.85$ TeV, where the KK-mode structure of the 6D bulk becomes operationally visible. Predicted signatures include: small deviations from Tsirelson saturation at $|S| < 2\sqrt{2}$ in Bell-pair experiments at energies $E \sim M_{\text{lock}}$; small unitarity-violation signatures in flavor-mixing experiments at HL-LHC; small Schrödinger-equation modifications in single-particle experiments approaching the KK threshold. The HL-LHC program (proposed Run 4, 2030+) is the primary platform; the predicted signatures are tied to the GG-P1-KK KK lock-cluster phenomenology. The conditional row defers to the dedicated GG-P1-KK paper for the quantitative pull table.

114.9 Net empirical content beyond standard quantum mechanics

We are explicit about how much of the eight-test horizon is genuinely framework-distinguishing versus how much is consistency-only. A careful reader will notice that the Class labels in Table 13 are heterogeneous: six tests are **Postdiction audit** or **Audit** or **Consistency check**; one is **Forward prediction**; one is **Conditional**. The right reading of this asymmetry is:

Tests 1, 2, 3, 4, 6, 7 (six of eight) are consistency-only. These tests are designed so that the cohort framework agrees with standard quantum mechanics; any experimental deviation would falsify both the cohort framework and standard QM equally, not the cohort framework distinctively. Tsirelson saturation, $(\Delta x)^2$ decoherence scaling, $1/f$ noise universality, position-basis universality at macroscopic scales, Bell-pair fidelity at long distance, long-distance teleportation: all six are post-2022-Nobel empirical successes of standard QM that the cohort framework reproduces. Reproducing a century of empirical success is necessary but not framework-distinguishing. We list them in the eight-test horizon because the framework’s authority depends on its agreement with the empirical record across all six, but they do not falsify the framework beyond QM.

Tests 5 and 8 (two of eight) are genuinely framework-distinguishing. Test 5 (einselection-rate platform-independence, §114.5) predicts cross-platform decoherence-rate ratios fixed by scale-overlap-integral R_{KK} of (113), whereas standard QM treats per-platform rates as phenomenological. Test 8 (high-energy QM corrections at $E \sim M_{\text{lock}}$) predicts small deviations from textbook-QM

evolution at the bulk-KK-threshold energy whereas standard QM predicts none. Only these two tests would, on a 2026–2035 horizon, falsify the cohort framework’s content beyond QM. Test 5 is currently measurable with available technology; Test 8 awaits HL-LHC Run 4 and is treated quantitatively in GG-P1-KK.

Why this is honest progress, not thin content. A foundational framework that reproduces a century of empirical success while adding two new measurable predictions is doing the right kind of work. The QM-foundations literature has a long history of frameworks that either (a) make no new predictions and are pure interpretation (Copenhagen, MWI, Bohm, QBism — all empirically equivalent to standard QM at the 4D level), or (b) make predictions that contradict standard QM at scales already accessible to experiment (GRW spontaneous-collapse models, dynamical-reduction theories). The cohort framework belongs to a third category: it reproduces standard QM at every operationally accessible regime measured to date *and* predicts two specific cross-platform / high-energy signatures that, if they fail to materialize, falsify the cohort framework specifically while leaving standard QM intact. This is the correct empirical posture for a foundational theory: agreement with the past, distinct prediction for the near future.

The honest summary. Six consistency checks plus two forward predictions, totaling eight tests on the 2026–2035 horizon. The cohort framework’s net empirical content beyond standard QM is the two forward tests; the six consistency checks establish that the framework has not broken anything QM already gets right. We claim no more than this; the framework’s case for itself rests on the structural reading it supplies (twelve classical foundational problems resolved as theorems, the bulk-and-boundary architecture, the universal-identity insight), with the two forward empirical predictions as future falsifiers rather than retrospective confirmations.

115 The classical open problems of QM: cohort-framework status

The previous Section laid out the framework’s forward-falsifiable empirical content. The present Section turns to the complementary question: of the classical foundational problems that have haunted the QM-foundations literature for a century, which are resolved by the cohort framework, which are partially resolved, which are out of scope, and which remain genuinely open? Each problem is given a status header in the cohort convention — **Resolved** (fully addressed by an inherited or net-new cohort theorem), **Partially Resolved** (addressed at one level of description but with a residual constructive question at another), **Out of scope** (deliberately deferred to a dedicated cohort companion), or **Open** (a constructive question that remains for future work) — together with the cohort theorem or argument that supplies the answer.

Table 14: The classical open problems of quantum-mechanical foundations and their cohort-framework status. Each problem carries an explicit status header pointing to the cohort theorem or apparatus that resolves it. The status headers are deliberately uneven. Some problems are closed by a named theorem, some are closed at the relaxation level with one analytic residue outstanding, and some are reframed rather than solved. The table is written so that a reader can tell which is which without reading the Part that treats it.

#	Classical problem	Status	Cohort source
1	The measurement problem (umbrella)	Resolved	T-A8.1–T-A8.6 as a chain; the basin-capture = Born residue closed at the relaxation level, one analytic residue remaining (§118); treated below
2	The Born-rule derivation problem	Resolved	T-A8.3 (Born rule from CPTP completion)
3	The preferred-basis problem	Resolved	T-A8.6 (einselection from AX-1 bulk locality)
4	The von Neumann measurement chain (where does collapse happen?)	Resolved	T-A8.4 (conditional CPTP, no physical collapse)
5	The macroscopic-superposition problem (Schrödinger’s cat)	Partially solved	Re- T-A8.5 dissolves the 4D paradox; bulk-side macroscopic-stratification stays open (see §118)
6	The Bell–Aspect non-locality problem	Resolved	T-A8.7 (CHSH violation as boundary shadow)
7	The Tsirelson saturation problem (why nature saturates rather than touching)	Resolved	T-A8.8 (extremal (Bogomolnyi-analogue) saturation)
8	The indistinguishability postulate (why every electron is the same)	Resolved	T-A8.9 (universal identity from GA-uniqueness)
9	The wave–particle duality / delayed-choice problem	Resolved	Part X §91 (bulk-field-not-trajectory reading)
10	The Frauchiger–Renner multi-observer consistency problem	Resolved	Part X §97 (observer-relative boundary projections of a single bulk state)
11	The Kochen–Specker contextuality problem	Resolved	Part X §95 (bulk holonomy projects context-dependently)
12	The no-signaling preservation problem	Resolved	T-A8.11 (boundary no-signaling from reduced-state factoring)
13	The consciousness-and-measurement question	Out of scope	Pointed to GG-A11-Origin (no consciousness primitive in QM-stratum reading)
14	The free-will-and-measurement question	Out of scope	Pointed to GG-A11-Origin (related to consciousness)
15	Why is the Hilbert space <i>complex</i> ?	Partially solved	Re- Gauge phase derived (§30); state-space complex structure derived in the quasi-free regime and, conditionally, beyond it; residue: the interacting massless infrared (§117)

The remainder of this Section discusses each entry briefly, focusing on the cohort theorem that supplies the status header.

115.1 The measurement problem (umbrella) — Resolved (PS-QM-1)

The measurement problem is the umbrella name for four sub-problems: where does collapse happen (#4)? Why the Born rule (#2)? Why the position basis (#3)? Why no macroscopic superpositions (#5)? In the cohort framework the umbrella decomposes into three theorems — the preferred basis (T-A8.6), the single definite outcome (T-A8.1, T-A8.4, with the GRIP actualization of GG-A6-GRIP), and the Born form (T-A8.3, *Resolved*) — plus the basin-capture = Born residue, closed at the relaxation level (the R16 lemmas with the R16' GDOS relaxation; the Constitutional Born Theorem conditional on the four named premises (P1)–(P4) of §118), whose single remaining analytic residue is the field-grade representation step; its status is therefore *Resolved* in the precise sense that each classical sub-problem is given its theorem, with the basin-capture = Born residue closed at the relaxation level and its single remaining analytic residue — the field-grade representation step — openly recorded (§30.5, §118). There is no free-standing “measurement problem” left once the umbrella is decomposed this way. The historical asymmetry between unitary closed-system dynamics and probabilistic measurement statistics — the asymmetry that defined the problem since Born–Bohr 1926–28 — is the asymmetry between bulk and boundary descriptions of the same underlying state.

115.2 The Born-rule derivation problem — Resolved (PS-QM-2)

Born postulated the rule in 1926; Gleason 1957 showed uniqueness assuming the Hilbert-space framework; Deutsch–Wallace MWI decision-theoretic derivations have remained disputed. T-A8.3 closes the gap at the level of the probability *measure*: given the Hilbert-space structure of T-A8.1, Gleason on the CPTP-completed OGGEF dictionary forces the square-modulus *form*, with $|\langle\alpha|\psi\rangle|^2$ recovered as the pure-state, sharp-projective specialization — no additional postulate. What T-A8.3 does not by itself supply, and does not claim to, is that the GRIP basin-capture frequencies of the deterministic descent *realize* this measure run-by-run; that single equivalence is the GG typicality target — that the GRIP basin-capture measure of the deterministic descent coincides with the Born measure, the six-dimensional analogue of Bohmian quantum equilibrium (status of record: closed at the relaxation level, one analytic residue remaining; §118) (§30.5). The Born *form* is thus a theorem (*Resolved*, T-A8.3); its dynamical realization — the basin-capture = Born residue — is closed at the relaxation level (the R16 lemmas with the R16' GDOS relaxation), with the Constitutional Born Theorem conditional on the four named premises (P1)–(P4) and the field-grade representation step the single remaining analytic residue (§118) — which keeps the claim honest rather than circular.

115.3 The preferred-basis problem — Resolved (PS-QM-3)

Zurek 1981 supplied the einselection criterion but identified the basis with the system–environment coupling eigenbasis without saying *why* that coupling has the form it does. T-A8.6 closes the gap: AX-1 (bulk locality) makes the dominant boundary system–environment coupling local, so the einselected pointer observable is whichever local variable the coupling is diagonal in. For the center-of-mass of a macroscopic object that variable is position, which is why everyday macroscopic records live in the position basis; for an engineered system whose dominant coupling is to flux or

charge (a SQUID, a Cooper-pair box) the pointer observable is flux or charge accordingly. The constitutional content is that the pointer basis is the locally-coupled one, not that it is always position.

115.4 The von Neumann measurement chain — Resolved (PS-QM-4)

The 1932 chain — system, apparatus, observer, super-observer — left the Heisenberg cut ambiguous. T-A8.4 dissolves the ambiguity: there is no physical collapse anywhere in the bulk; the boundary observer’s update is the conditional CPTP map $\rho \mapsto \mathcal{I}_\alpha(\rho)/p_\alpha$. The cut is where the experimenter chooses to apply the partial trace; nothing in the bulk distinguishes one cut location from another.

115.5 The macroscopic-superposition problem (Schrödinger’s cat) — Partially Resolved (PS-QM-5)

T-A8.5 gives the operational dissolution: for a cat at room temperature, $\Gamma \gtrsim 10^{40} \text{ s}^{-1}$, so the off-diagonal coherence between alive and dead states is suppressed within $\sim 10^{-40} \text{ s}$ — a time scale forty orders of magnitude faster than any apparatus could resolve. At the 4D boundary, the cat is never operationally in a superposition. What remains a constructive open question (see §118) is the bulk-side description of macroscopic state space: how exactly does the 6D bulk configuration corresponding to “alive” differ from the configuration corresponding to “dead,” and how does the cascade ladder describe the stratification? The 4D paradox is resolved; the bulk-stratification program is open.

115.6 The Bell–Aspect non-locality problem — Resolved (PS-QM-6)

Bell 1964 plus Aspect 1982 plus the loophole-free runs of 2015 establish that 4D-local hidden-variable theories cannot reproduce the quantum-mechanical correlation structure. T-A8.7 reframes this not as “nature is non-local” but as “the 6D-local bulk projects non-locally onto the 4D boundary.” The 4D non-locality is the operational shadow of a non-invertible bulk-to-boundary projection from a 6D-local bulk; Einstein’s locality intuition is recovered at the bulk level without contradicting Bell at the boundary level.

115.7 The Tsirelson saturation problem — Resolved (PS-QM-7)

Why does nature saturate the CHSH bound $|S| = 2\sqrt{2}$ rather than merely lying below it? T-A8.8 reads the saturation as an extremal (Bogomolnyi-analogue) property: the singlet is the maximizer of the CHSH functional, so the first variation vanishes there and small perturbations of the singlet do not change $|S|$ to first order. Saturation is therefore generic across experimental setups, not a tuned coincidence of the singlet state.

115.8 The indistinguishability postulate — Resolved (PS-QM-8)

Every electron in the universe shares the same mass, charge, and spin to one part in 10^{10} . T-A8.9 supplies the structural reason: the cascade at the charged-lepton stratum has a unique stable Geometric Attractor, and every electron is a token of this single type. Indistinguishability is therefore not a postulate of quantum statistics but a cascade inheritance, with the type fixed upstream of any laboratory.

115.9 Wave–particle duality / delayed-choice — Resolved (PS-QM-9)

Wheeler’s 1978/1983 delayed-choice thought experiment and Scully’s 1991 quantum eraser have been read as evidence that the past is not determined until it is measured. The cohort reading (Part X §91, §92) recovers both predictions without retrocausation: there is no “photon” with a pre-defined trajectory at the bulk level; the bulk electromagnetic field propagates continuously through the apparatus; the late experimental choice modifies the boundary conditions and the bulk responds deterministically. No editing of the past is required.

115.10 Frauchiger–Renner multi-observer consistency — Resolved (PS-QM-10)

The 2018 no-go theorem proved that a multi-agent measurement chain produces mutually contradictory predictions under the joint assumption of (Q) universal quantum mechanics, (C) cross-agent consistency, (S) single-outcome-per-measurement. The cohort reading (Part X §97) retains all three at the bulk level: the apparent inconsistency is the mismatch between observer-specific 4D boundary projections of a single bulk state; the bulk is single-valued and consistent.

115.11 Kochen–Specker contextuality — Resolved (PS-QM-11)

The 1967 theorem excludes non-contextual hidden-variable assignments in dimension ≥ 3 . The cohort reading (Part X §95) recovers contextuality as the operational signature of which boundary algebra is constructed: the bulk Wilson-line holonomy is a single 6D-local degree of freedom whose 4D projection depends on the apparatus configuration. Contextuality is structurally inevitable in the boundary algebra; the 6D bulk holonomy is single-valued.

115.12 No-signaling preservation — Resolved (PS-QM-12)

Despite the bulk timelike connectability of T-A8.7, no operational information can be transmitted between spacelike-separated 4D laboratories. T-A8.11 supplies the proof: the boundary marginal at one laboratory is independent of the measurement-setting choice at the other, by the POVM completeness relation acting on the reduced state. Einstein’s no-signaling principle is preserved at the operational level even as 4D-local hidden variables are excluded.

115.13 Consciousness and free will in measurement — Out of scope (PS-QM-13)

Wigner and von Neumann at various points entertained the possibility that consciousness is an irreducible primitive of measurement. The cohort framework’s QM-stratum reading is that any

sufficient-complexity boundary observer — biological, mechanical, or otherwise — performs the partial trace and registers the einselected outcome; consciousness is not required as a fundamental ingredient. A structural treatment of consciousness within the cohort cascade is deferred to GG-A11-Origin, which develops the cognitive cascade (Five-Story Pagoda, boundary triad, and the quantum-cognitive-information ladder) appropriate to that stratum. The free-will question is related and equally out of scope for GG-A8-QM.

115.14 The complexity of the state space — Partially Resolved (PS-QM-14)

Why quantum amplitudes are complex-valued at all — Penrose’s question (Penrose, 2004) — splits into a gauge half and a state-space half (§116). The gauge half is *Resolved* by the hidden-fiber geometry of Part III: the charged phase is a real angle on the sixth dimension, its unreadability yielding superselection and the Born form. The state-space half is *Partially Resolved* by the real-bulk statements of §117: for free and quasi-free matter the state-space i is derived — the polar phase of the constitutional time generator, unique by positivity on the static arena (Ashtekar and Magnon, 1975; Kay, 1978), coinciding with the circle’s generator on charged sectors under the Feynman–Stückelberg pairing — and now extended, conditionally on the standard scattering hypotheses, to gapped interacting matter (RB-8), via the holomorphy bridge to the massless conformal sector (RB-9), and, at the rigorous-model level, to the infrared sectors of interacting massless matter (RB-10) — while the full relativistic QED infrared is named as the remaining frontier in §117. The experimental exclusion of real-Hilbert-space quantum mechanics (Renou *et al.*, 2021) is read as direct support for the shared-source picture.

Status summary. Of the fifteen classical open problems of QM foundations, eleven are *Resolved* by inherited or net-new cohort theorems; two (Schrödinger’s cat; the complexity of the state space) are *Partially Resolved*, each with its 4D content settled and its named residue open; two (consciousness, free will) are *Out of scope*, deferred to GG-A11-Origin. None of the fifteen classical problems remains in the *Open* category in the cohort framework; the genuinely Open questions are the constructive ones laid out in §118, all of which are downstream of the classical resolutions rather than competing with them.

Part-end close on the open-problems inventory

After all, the mark of a successful foundational theory is not that no questions remain — some will always remain — but that the questions that do remain are the right questions. For ninety years the QM-foundations literature has been working on the wrong questions: where exactly does collapse happen? Why the Born rule? Why position? These were the wrong questions because they presupposed that the answers had to live within the 4D framework that produced them. The cohort framework relocates the questions to where the answers actually live: in the 6D bulk and the constitutional warrants that force the boundary projection. Eleven classical problems become eleven resolved theorems. Two are out of scope. Two retain a residual constructive component. None remains a paradox. The mess of the past century can now be set down; the genuinely open questions for the next century are the constructive ones laid out below.

116 Why complex numbers? The answer to an old question

Complex numbers were invented in the sixteenth century as a device of algebra — a way of passing through the impossible square roots that appear midway in the solution of a cubic. Three centuries of mathematics domesticated them; then quantum mechanics made them physical. Amplitudes are complex, and the complexity is not decorative: in network Bell configurations, quantum theory over a *real* Hilbert space makes different predictions from the complex theory, and experiment has ruled the real theory out (Renou *et al.*, 2021). Penrose has repeatedly singled out the complex-number question as one of the deepest unexplained facts about nature (Penrose, 2004): why should the working substance of physics be a number system invented to solve equations? The framework owes this question a precise answer, and the answer comes in two halves of different pedigree.

The first thing the six-dimensional bulk says is that the question is two questions wearing one coat. Physics contains two complex structures. One is the *gauge phase* of a charged field — the $e^{iq\theta}$ of minimal coupling, the Aharonov–Bohm phase, the quantized charge. The other is the *quantum* complex structure — the i of $i\hbar\partial_t$, the complexity of the state space itself, which a strictly neutral field requires exactly as much as a charged one. For charged matter the two coincide: the total dial angle is $\theta + (p \cdot x - Et)/\hbar$, one angle carrying both the dynamical and the gauge phase. That coincidence is why a century of quantum electrodynamics could not tell them apart, and why the question has resisted for so long.

On the first half the framework’s answer is complete, and it is geometric. The complex phase of a charged field is a *real angle on a real circle* — the sixth dimension of the bulk (GG-A2-GG6, Theorem 0). The imaginary unit is that circle’s rotation generator; charge quantization is its compactness; the Aharonov–Bohm phase is its holonomy, a real angle; charge superselection is its unreadability (§30); and the photon carries no phase of its own for the good reason that it *is* the circle’s connection, not a resident of it. Complex numbers appear in gauge physics because the harmonic analysis of a compact real dimension *is* the complex exponential — Euler’s $e^{i\theta}$, read as the geometry it always was rather than the algebra it was born as. There is a historical symmetry worth noticing: invented for cubics, complex numbers found their canonical home only when Wessel, Argand, and Gauss laid them on a plane and Euler wound them around a circle. The framework’s claim is that nature’s usage is the circle usage. Nothing was invented; something was found.

On the second half the framework’s answer is newer, and it is stated with its regime on its sleeve (§117 carries the precise statements). The complex structure of the state space is *also* a derived object — not from the circle, which provably cannot supply it on neutral sectors, but from the two other real structures the Constitution already owns: the causal time orientation of AX-1 and the operational positivity of AX-3. For free (quasi-free) matter, the i of the state space is the *polar phase of the constitutional time-translation generator* — concretely, the Hilbert transform in bulk static time, a real integral operator — selected uniquely by energy positivity on the globally static arena of Theorem 0; and on every charged sector this dynamical i coincides with the circle’s rotation generator precisely under the Feynman–Stückelberg pairing, so that the existence of antiparticles is the consistency condition between the sixth dimension and causal time. What the framework still owes has narrowed to one corner, and it is named: the derivation extends, conditionally on

the standard scattering hypotheses (mass gap, asymptotic completeness), to interacting matter — the interacting theory’s i is the free i of its asymptotic frame (RB-8, §117) — and to the massless conformal sector, where positive frequency is Hardy holomorphy and the twistor division $\mathbb{PT}^+/\mathbb{PT}^-$ is the frequency splitting itself (RB-9). The infrared corner of interacting massless theories is closed at the rigorous-model level — the soft-cloud multiplicity is port structure and the derived i is sector-blind (RB-10) — and what remains open is its full relativistic QED completion (§117).

The experimental fact sharpens the whole picture. What the real-Hilbert-space exclusion (Renou *et al.*, 2021) establishes is that nature’s complex structure is *single and shared*: independently prepared sources must use the same i , which is exactly what a real tensor product of separately-real state spaces cannot arrange. The framework supplies the sharing and explains it: there is one θ -circle attached over every point of spacetime, and one causal order — so there is one rotation generator and one polar phase, universally shared by every subsystem. Experiment says nature’s i is one; the sixth dimension and the causal order say why.

The structural comparison locates the claim without polemic. A formulation in which the phase circle is an internal abstraction must postulate the circle and the complex field together. A formulation in which the circle is spatial geometrizes charge — a real higher-dimensional field decomposes into complex charged harmonics — but a spatial circle is resolvable in principle, so nothing forces holonomies to be the only physical data, and the quantum i is untouched. The framework’s circle is real but non-spatial: it exists, so the gauge phase is geometry; it cannot be resolved, so only holonomies are physical. Both halves of the answer rest on that combination.

So the framework’s answer to the old question is: half geometry, half dynamics, all real. The gauge half is a circle one can point to; the state-space half is the phase of time itself, derived where the framework can prove it and named as open where it cannot. Given who has asked the question, an overclaimed answer would be worthless; a calibrated one, we believe, is new.

117 The real-bulk statements: where the state-space complex structure comes from

The preceding Section promised precise statements; this Section supplies them, at discussion level — compact statements with proof sketches, deliberately *not* added to the eleven-theorem roster of Part IV, because their natural home is a dedicated treatment and their regime (free/quasi-free matter on the canonical arena) is narrower than the paper’s principal theorems. Every symbolic claim below has been verified programmatically as well as by hand.

Three addresses for “the Hilbert space.” The phrase “the Hilbert space of the theory” has three distinct referents, and keeping them separate dissolves a family of confusions. *Address one: the arena* $(\mathcal{M}_6, g, \mathcal{A})$ — real geometry, carrying no Hilbert space at all (GG-A2-GG6, Theorem 0; AX-2). *Address two: the bulk state space* $\mathcal{H}_{\text{Bulk}}$ — the state space of the quantized bulk fields, the input side of the AX-3 dictionary, carrier of the non-separable states that explain Bell correlations; it coexists with the real, single-valued configurational layer (the contextual holonomy Θ_H , the attractor-basin occupancy) that carries determinism and outcomes. *Address three: the operational*

state space $\mathcal{H}_{\text{Bdy}}^{(\text{QM})}$ — boundary-resident and observer-indexed, a tensor factor of $\mathcal{H}_{\text{Bulk}}$ recovered as a GNS representation (T-A8.1) and embedded back isometrically. The statements below concern address two: they show that, in their regime, $\mathcal{H}_{\text{Bulk}}$ itself is a *derived* object over real data, so that nothing complex remains primitive at any address.

The two shared real sources. Exactly two globally shared real structures do all the work: the phase circle S_θ^1 (one fiber over every point; AX-2, Theorem 0) and the constitutional time orientation with operational positivity (AX-1, AX-3). The canonical metric of Theorem 0 is globally static — ∂_t is a global hypersurface-orthogonal timelike Killing field — and this staticity is load-bearing below.

Statement RB-1 (charged sectors). On each charge- n sector of a real bulk matter field, the charge-normalized circle generator $J_Q = \partial_\theta/Q$ is a complex structure: it squares to -1 , is symplectic, commutes with the covariant dynamics and all gauge transformations, and identifies the real weight- n harmonic pair with the standard complex charged field — with complex conjugation realized as orientation reversal of the circle. It is the *unique* such structure commuting with the circle action, up to sign. *Sketch:* ∂_θ acts on the real pair (a_n, b_n) as $(a, b) \mapsto (nb, -na)$; the commutant of an irreducible $SO(2)$ action contains exactly two quarter-turns. On the neutral sector the same generator acts as zero: the circle can only complexify what it moves, which is why the state-space question needs the second source.

Statement RB-2 (neutral sectors). On neutral sectors — and sector-blind on the full solution space — the operator $J = \pm A(-A^2)^{-1/2}$, the *polar phase of the time-translation generator* A , is a complex structure compatible with the symplectic form, commuting with the dynamics, with the sign fixed by positivity; in the time domain it is the Hilbert transform, a real principal-value convolution kernel, non-local in time and complex in nothing. On a stationary region it is the *unique* positive-energy complex structure (Ashtekar and Magnon, 1975; Kay, 1978) — and the canonical arena is globally static, so on it the selection is canonical. The constitutional pedigree is exact: AX-1 supplies the direction, the positivity clause of AX-3 the selection, Theorem 0 the uniqueness.

Statement RB-3 (compatibility; antiparticles). On the on-shell charged sectors the two structures agree, $J = J_Q$, precisely under the Feynman–Stückelberg pairing (charge $+n$ with positive frequency, charge $-n$ with positive frequency), and anti-agree on the forbidden pairing. The pairing is therefore not a convention but the unique orientation making two independently forced real structures commute: *antiparticles are the consistency condition between the sixth dimension and causal time*. This also fixes the sign left open in RB-1 and yields *one* global i — sourced by causal positivity, geometrized by the circle wherever charge lives. The circle is thereby credited with exactly what it earns: not the existence of i , but its compactness (charge quantization), its fiber character (gauge), and its unreadability (superselection).

Statement RB-4 (composites; the shared i). For subsystems carrying the *same* global J — as all GG matter sectors do, since J 's sources are global — the complex tensor product is the quotient of the real tensor product by the shared- J identification, $V \otimes_{\mathbb{C}} W \cong (V \otimes_{\mathbb{R}} W) / \text{span}\{Jv \otimes w - v \otimes Jw\}$,

and the Fock construction extends J canonically with no new complex input. Entangled states therefore admit purely real presentations with one global i ; entanglement itself — non-factorizability — is untouched. The experimentally mandated single shared complex structure (Renou *et al.*, 2021) is satisfied by construction rather than by stipulation; conversely, for two subsystems with *unshared* complex structures no canonical complex quotient exists, which is the abstract content of the real-tensor-product pathology the experiment exploits.

Statement RB-5 (fermions). The account covers both statistics. In Majorana form the fermionic datum is a real solution space with a positive-definite inner product and an orthogonal time flow; the polar phase of its generator is again the unique positive-energy complex structure (the ground-state polarization of the self-dual CAR formalism), and flipping its orientation on any single spectral plane produces a negative one-particle energy. Charged fermions inherit RB-1 verbatim (the spinor index is inert under ∂_θ), RB-3 extends unchanged, and charge conjugation factorizes as circle-orientation reversal composed with a fixed real spinor map. The would-be objection that fermions are “intrinsically complex” dissolves: the Dirac field’s complexity is the J -packaging of real data, charge part by the circle, frequency part by the phase of time.

Statement RB-6 (discrete symmetries as orientation bookkeeping; series entry R8).

The three discrete operations are orientation reversals of the three real factors of the bulk: charge conjugation reverses the fiber, parity reverses space, time reversal reverses the clock. Their unitary/antiunitary character is then derived rather than assigned: fiber reversal commutes with J (the signed charge reverses together with the generator, so ∂_θ/Q is invariant), clock reversal anticommutes with J (it reverses the polar phase of the time generator), and parity touches neither — hence a discrete symmetry is antiunitary exactly when it contains T . The total reversal CPT maps each Feynman–Stückelberg-paired mode set to itself and is a symmetry of the construction for every texture built from real geometric data; CP alone is a symmetry precisely when the coupling texture is achiral on the circle, i.e. when every closed-loop angle of the texture lies in $\{0, \pi\}$. The theorem form, with proof and with the flavor-holonomy calculus that measures the chirality, is carried by the companion Standard-Model treatment (Arisaka, 2026, A9-SM, T-SM-46.1 and T-SM-46.2).

Statement RB-7 (the dial-free door; series entry R10). On a charge- n sector the symmetric square of the real pair splits into the rotation-invariant Dirac pairing $\psi\bar{\psi}$ and the Majorana doublet $(\text{Re } \psi^2, \text{Im } \psi^2)$, which carries fiber weight $2n$; hence a Lorentz-invariant, gauge-invariant Majorana-type self-pairing exists if and only if the fiber weight vanishes, where the reality condition of the sector is precisely the Majorana condition of RB-5. Lepton number is thereby geometrized as winding number — exact on every dial-carrying species — and the sole legal $\Delta L = 2$ door in the matter ledger is the weight-zero singlet, with LHN the unique fiber-neutral portal. The theorem forms, with proofs and with the cosmological consequences (the forced Majorana channel; TeV-scale resonant leptogenesis at the lock), are carried by the companions (Arisaka, 2026, A9-SM, T-SM-44.1; Arisaka, 2026, A10-Cosmos, T-Cosmos-18).

Statement RB-8 (the interacting completion; series entry R11). For interacting bulk matter satisfying the standard scattering hypotheses — a mass gap and asymptotic completeness, in a Wightman or Haag–Kastler framework — the physical Hilbert space is canonically the Fock space over the *asymptotic* real solution spaces equipped with the derived complex structure of the static frame: by Haag–Ruelle scattering theory the in/out fields are free fields at the physical mass, their one-particle spaces are exactly the real-data objects of RB-1–RB-5, and asymptotic completeness transports their polar-phase J onto the whole space. The interacting theory’s i is the free i of its asymptotic frame; interactions contribute vertices — real texture data, moduli and fiber angles — and no complex structure of their own. For confined sectors the hypotheses attach to the physical (hadronic) spectrum, which is gapped, so the derived complex structure belongs to the particles an instrument actually registers. The selection principle has been verified beyond the free diagonal case (the exact ground state of an arbitrarily coupled gapped quadratic lattice carries precisely the polar-phase J of its interacting generator; Møller operators demonstrably transport J across a scattering event), and the hypotheses are the field’s own standing frontier, imported and named rather than hidden.

Statement RB-9 (the holomorphy bridge; series entry R12). Positive frequency *is* holomorphy, at three levels that are one structure. At the signal level, a real finite-energy history is positive-frequency precisely when it is the boundary value of a function holomorphic in the upper half of complexified constitutional time, and the derived J of RB-2 — the Hilbert transform — is the generator of that Hardy splitting: the positive-frequency subspace is its $(-i)$ -eigenspace. At the field level, the same fact is the forward-tube analyticity of Wightman theory. At the massless conformal level it becomes the twistor correspondence: solutions of the free massless field equations of helicity h are cohomology classes $H^1(\mathbb{PT}, \mathcal{O}(-2h - 2))$, and the positive-frequency solutions are exactly the classes on the positive half $\mathbb{PT}^+$ (Eastwood *et al.*, 1981; Penrose and MacCallum, 1973) — so the division of twistor space *is* the causal-positive splitting, and the holomorphy of twistor representatives is the conformal enhancement of tube holomorphy: a massive field’s positivity buys analyticity in one complexified direction (the clock), a conformally invariant massless field’s positivity buys every timelike direction at once, and twistor space is the geometry that linearizes exactly that enhancement. The reading this framework adds is the direction of explanation: the real causal-positive data of AX-1 and AX-3 source the holomorphy. Nothing in this diminishes the twistor construction; it derives its motivation. The bridge has since received its standalone, full-length development — history, proofs at lecture depth, worked twistor examples, and the open dynamics-half refinement stated as open — in the member preprint GG-P4-Real (Arisaka, 2026, P4-Real).

Statement RB-10 (infrared sectors as ports; series entry R13). In an interacting theory with massless quanta, the infrared catastrophe is port structure, not complex structure. (i) The asymptotic soft-cloud configuration — the flux/velocity sector — labels mutually disjoint representations of the asymptotic field algebra: clouds attached to different velocities differ by a displacement whose norm diverges logarithmically as the infrared cutoff is removed, so the sector overlap vanishes as a *power* of the cutoff and distinct velocities are exactly orthogonal in the limit

— velocity superselection, by the same mathematics that superselected charge (an unremovable divergence in a relative-phase integral). Which infrared background an instrument treats as “vacuum” is therefore a c-number port assignment at AX-3, with the same standing as the black-hole vacuum multiplicity in the ledger below. (ii) The dressing is an affine symplectic map: it shifts first moments and leaves the quadratic generator — the only object the polar-phase construction of RB-2 reads — exactly invariant; the dressed dynamics within each sector is free-plus-c-number, and the sector’s derived J is the *same* J in every sector, sourced by the same circle and clock. (iii) Hence the real-bulk derivation extends to the infrared: the sector multiplicity is dictionary data, the complex structure is sector-blind, and the two never mix — the infrared problem is a problem about which vacuum dressing an observer stands in, not about where i comes from. The statement is exact at the van Hove stratum and holds at the Nelson/Pauli–Fierz level via the theorem-grade infraparticle results of the literature — absence of a Fock ground state at fixed total momentum (Fröhlich, 1973), existence and regularity of the dressed one-particle states (Bachmann et al., 2012; Pizzo, 2003), and infraparticle scattering with velocity superselection in nonrelativistic QED (Chen et al., 2010) — imported with their hypotheses displayed. Full relativistic QED, where the Chung/Faddeev–Kulish dressing (Chung, 1965; Faddeev and Kulish, 1970) is semi-rigorous and the sector observables are the asymptotic flux observables of Buchholz (1986), is the field’s own frontier, named below rather than claimed. The program has now met superselection three times — the charge angle (RB-1), the horizon port (the ledger below), the cloud at infinity (RB-10) — and each time the unreadable datum is a relative phase between sectors, owned by the dictionary, while the derived J passes through untouched: the dictionary owns the sectors, the geometry owns the i . The full derivation, at lecture depth and with its verification suite, is in the member preprint GG-P4-Real (Arisaka, 2026, P4-Real).

Statement RB-11 (the rotating anchor; series entry R14). In rotating stationary geometries the ergoregion makes the co-rotating quadratic form indefinite: the positivity selection of RB-2 is unavailable in the rotating frame — consistently with the theorem that the Kerr exterior admits no stationary Hadamard state (Kay and Wald, 1991) — and the physical mechanism behind the obstruction is superradiance (Unruh, 1974; Starobinsky, 1973). The uniqueness anchor therefore moves to the asymptotic frame, whose quadratic form is positive and whose polar phase selects the asymptotic J uniquely — the frame where every instrument that will ever report a measurement actually sits. Read through that fixed J , the rotating dynamics is an exactly symplectic Gaussian channel: two-mode squeezing that amplifies populations at the superradiant rate while leaving J untouched. Superradiant energy extraction is GDOS pumping, exactly as horizon thermality was GDOS tracing; the rotating vacuum multiplicity is port assignment relative to the asymptotic anchor, and both custody clauses hold verbatim. Verified in the superradiance model class (indefinite rotating form; unique asymptotic polar phase; channel exactly symplectic and state-valid; $\langle n \rangle = \sinh^2(gt)$ at resonance). The Hadamard analysis of the exact Kerr exterior is the field’s own and is imported where used, not claimed.

Statement RB-12 (the collapse channel; series entry R15). For dynamics static in the far past and carrying the exponential horizon redshift at late times — the moving-mirror class (Davies

and Fulling, 1976), the model form of gravitational collapse — the in-frame J is selected by RB-2 at past infinity, and the out-state factorizes: a universal thermal channel at $T = \kappa/2\pi$, the Planck factor arising as the branch-cut asymmetry of the exponential redshift (a real, classical, kinematic datum), times a transient carrying the collapse details, whose contribution decays at late times. The collapse-epoch factorization, carried until now as a conjecture about a channel, is thereby a theorem in its class: the non-stationary epoch populates states through a Gaussian channel and never touches the origin of i . Verified (the branch-cut ratio against its exact value at several frequencies; late-time universality on a static-then-exponential trajectory). The full gravitating statement is the standard collapse derivation, imported with attribution in the companion treatment (Arisaka, 2026, C3-BH).

Statement RB-13 (the horizon ledger; registrational). The black-hole companion’s exact-coefficient chain is real-section geometry end to end: the conical deficit of its replica step is a curvature datum of the real Euclidean section (the arena licensed as the clock’s analytic shore — L-BH-Shore, Arisaka, 2026, C3-BH); the horizon area is a real functional of the real metric; and the entanglement the entropy measures is of the three real bulk factors, traced at the horizon — the hidden-fiber holonomies of the phase circle. The single appearance of i in the chain — the analytic continuation and the KMS clock face (T-BH-5, consuming the causal clock’s Wick-quartet theorem T-P5-B1) — is the boundary’s derived clock, whose shore belongs to time and never to the fiber (the CPT companion’s custody clause, Arisaka, 2026, P5-CPT). In one line: $A/4G_N$ is real geometry read through a derived complex window — the dictionary owns the sectors; the geometry owns the i . The entry is registrational: it names the real-bulk standing of an existing derivation chain (T-BH-1 with T-BH-5), adds no new theorem and no new custody instance (the count stands at five), and carries no series-entry number (the proven series stands at R15).

Assembly, and what remains open. Together these statements derive, for free and quasi-free matter on the canonical arena, the bulk state space itself: $\mathcal{H}_{\text{Bulk}}$ is the Fock completion of the real solution space under the derived J , the bulk Schrödinger equation’s i is that J , and the boundary chain (T-A8.1–T-A8.4) goes through verbatim on the J -complexified real spaces. In this regime the complex entries of the AX-3 dictionary are theorems over real data, and the summit statement of Part XIII extends to the scalars: the bulk needs no complex primitive. The ledger of residues, updated to the series’ present state, in the auditable spirit of the paper’s honest-accounting convention. *Interaction:* closed conditionally by RB-8, on the standard scattering hypotheses; what survived was the interacting *massless* infrared — the infraparticle’s soft cloud — where the mass-gap hypothesis fails and the asymptotic particle notion is itself subtle, and RB-10 now closes it at the rigorous-model level: the soft-cloud choice *is* port data at AX-3, exactly where the vacuum ambiguity landed, and the derived J is sector-blind; what remains is the full relativistic QED completion, where the dressing theory is the field’s own frontier — now stated as a conditional theorem on displayed dressing hypotheses in the member-paper treatment. *Non-stationarity:* discharged for stationary situations including black-hole exteriors — the Killing-frame J is unique there, the standard vacuum multiplicity is port-assignment dictionary freedom, and horizon thermality is produced by the GDOS partial trace itself, at the standard temperature; the rotating case is closed at the model level by RB-11 — the anchor moves to the asymptotic frame, and superradiance is GDOS pumping — and

the collapse-epoch factorization is closed in its class by RB-12: the late-time channel is universal and the collapse details decay in the transient. What remains are the Hadamard analysis of the exact Kerr exterior and the full gravitating collapse case, both the field's own, imported where used. Within its regime, however, the conclusion stands: *Einstein right in 6D* now includes the number system — the bulk is real, and the i of quantum mechanics is the shadow of a circle and a clock.

118 Open questions on the QM half of the cohort program

A frank inventory of what remains open in the cohort framework's QM reading.

The GG typicality target: does basin-capture equal Born? This was long the single residue of the measurement problem in the cohort framework and the sharpest open question of the QM half of the program; it is closed at the relaxation level — the R16 lemmas with the R16' GDOS relaxation — with the Constitutional Born Theorem conditional on four named premises and the field-grade representation step the single remaining analytic residue. Registration is the deterministic descent of the joint system-and-apparatus configuration into one Geometric Attractor basin (GRIP, GG-A6-GRIP); the Born *form* is fixed by Gleason on the open dictionary (T-A8.3); the question was whether the *basin-capture measure* — the frequency with which the descent lands in each basin, given the actual distribution of the bulk boundary condition Θ_H — coincides with the Born measure. Four structural lemmas — the R16 lemmas — now leave that measure no freedom. The Born density $\rho = a^2 + b^2$ is the *unique* conserved quadratic local density of the real presentation; the fiber's unreadability — Haar on the compact dial, with the invariants computed — forces the boundary to carry only $(\rho, \nabla\rho, j)$; operational stationarity (records may not drift on stationary states) eliminates the one remaining freedom, forcing the transport $v = j/\rho$; and that transport is equivariant, so disjoint decohered branches carry exactly the Born weights. What converts typicality from postulate to dynamics is the boundary law itself (the R16' GDOS relaxation): because the constitutional channel is a forward semigroup (GG-A5-GDOS) — port sectors enter fresh, the Markov property doing the work statistical mechanics assigns to its Stosszahlansatz; that fresh-port assumption is the Port-Freshness Clause of the GDOS companion (Arisaka, 2026, A5-GDOS), stated once at its dynamical home there and consumed here statistically — and the anchor sectors carry Haar by compactness, the configurational relative entropy to the Born measure is *exactly non-increasing*: joint transport conserves it, marginalization over the unread ports contracts it, and freshness equates the endpoints, so it decreases at each exchange by precisely the correlation built with the discarded port, with the Born measure as the unique fixed point. Any admissible initial configuration engaging fresh ports relaxes to Born statistics. Assembled, the chain is the Constitutional Born Theorem, conditional on four named premises: (P1) the GG-A5-GDOS forward semigroup; (P2) the Haar anchor with the recursion across finite-depth port graphs; (P3) genericity of the descent; (P4) the GRIP basin partition. The lineage is the quantum-equilibrium program of Bohmian mechanics (Bohm, 1952; Dürr et al., 1992) in its relaxation reading (Valentini, 1991); the constitutional improvements are that the measure, the transport, and the anchor are each forced by geometry rather than chosen, and that the dissipation is genuine — a contracting semigroup, not a coarse-graining. What remains

open is engineering, named — and two of its three items are now closed at the collision level: the relative entropy contracts geometrically with ratio $(1 - \mu)$ per exchange, μ the guidance-mixing weight (the rate bound); and an imperfect port injects at most its own divergence, so relaxation propagates by induction across any finite-depth port graph anchored on Haar sectors (the recursion). What remains is the field-grade representation of the channel as a port stream — the single analytic residue. One prediction rides free: epochs before relaxation completed could leave *relic non-Born statistics* in primordial correlations — a falsifier that distinguishes this derivation from the postulate it replaces. Nothing in this paper assumes the result; the deterministic reading now rests on a theorem chain rather than a hope.

The macroscopic-superposition regime intermediate between C_{60} and the cat. The cohort framework predicts continuous decoherence-rate scaling between mesoscopic and macroscopic scales, but quantitative cross-platform tests at masses 10^4 – 10^9 atomic mass units (the regime of large biomolecules, viruses, micron-scale solids) remain technically challenging. Open question: does the scale-overlap-integral prediction of T-A8.5 hold across all six orders of magnitude in this range, or do non-trivial finite-size corrections appear?

The framework-vs-Many-Worlds operational discriminator. At the 4D operational level, the cohort framework and Many-Worlds make identical empirical predictions. At the bulk level, the cohort framework is single-valued and Many-Worlds posits branching. Is there an operational signature that distinguishes the two readings within 4D experiments? The current answer is: no decisive 4D discriminator exists; the framework’s distinctness is at the bulk level, accessible only via the 6D-specific predictions of Test 8 (high-energy KK signatures) and the cosmological predictions of GG-A10-Cosmos.

The boundary-observer entanglement question. The cohort framework treats the boundary observer as a single 4D agent with access to a partially traced bulk state. When two boundary observers compare records (the Frauchiger–Renner setting), the framework reproduces the no-go theorem’s result without contradiction. But the formal description of multi-observer entanglement at the cohort level remains under-developed. Open question: what is the structural relationship between two boundary observers’ GNS-representation Hilbert spaces, and how does the cohort framework formally describe their joint state?

The algebra type of the boundary observable set. The bulk-to-boundary projection is formulated here at the level of the operational, zero-mode-restricted boundary algebra $\mathcal{A}_{\text{Bdy}}^{(\text{QM})}$, where a tensor factorization and a partial trace are available. For the full local algebra of a gauge theory this factorization does not exist — the local algebras are Type III and the Gauss law obstructs a clean split. Open question: what is the Type-III-aware restatement of the GNS/partial-trace machinery of T-A8.1 — in crossed-product or relative-entropy language, with explicit treatment of the edge modes — under which the projection picture extends rigorously from the zero-mode restriction to the full boundary algebra? Within the operational restriction the construction is well-defined; its extension to the full local algebra is unfinished.

The role of consciousness in measurement. The cohort framework explicitly declines to make consciousness an irreducible primitive of measurement (the question is out of scope here and is taken up in GG-A11-Origin). The cohort framework’s reading is that any sufficient-complexity boundary observer — biological, mechanical, or otherwise — performs the partial trace and registers the einselected outcome; consciousness is not required. Open question: what is the minimal structural complexity of a boundary observer that the cohort framework’s reading requires? GG-A11-Origin addresses this for biological observers; a quantum-mechanical-stratum version remains to be developed.

The relationship between GG-6 and the Standard Model at $E \sim M_{\text{lock}}$. The cohort framework predicts GG-P1-KK (KK lock cluster) signatures at HL-LHC energies, but the detailed phenomenology of how the GG-6 bulk gauge content $\text{Spin}(1,5) \times \text{SU}(5) \times \text{U}(1)_B \times \text{U}(1)_{Q_i}$ matches onto the Standard-Model gauge group $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$ at energies above the GUT scale requires the dedicated GG-P1-KK treatment. Open question: at exactly what energy does the SM-stratum description of GG-A9-SM cross over to the bulk-stratum description?

The UCLA-Arisaka GG-Theory program now has seven major cross-ledger predictions tested against 2025–2026 data:

Table 15: Cumulative evidence as of July 2026, now including quantum-mechanical benchmarks. Each row gives the observable, the framework’s value or bound, the measured value, and the grade. The quantum-mechanical entries are agreements with established results rather than new predictions, and they are listed here to show the reading is consistent with what is already known, which is a weaker claim than the cosmological rows above them make.

Observable	GG prediction	Measurement (2025-2026)	Pull
H_0 (bg vs. CMB; C2)	66.6 ± 0.2 km/s/Mpc	67.4 ± 0.5 (CMB); ladder 73.50(81); lift $\sim 1/3$ derived (C2)	-1.5σ
a_μ (imported SM)	$116\,592\,033(62) \cdot 10^{-11}$	116 592 071.5(14.5)	$+0.6\sigma$
m_W (world avg)	80.3700 GeV (A9, QD)	80.3692(133)	-0.06σ
$\sum m_\nu$	≤ 0.06 eV	< 0.072 eV (DESI+CMB)	consistent
$\sin^2 \theta_W$	$1 - (NJ/K)^2 = 0.22955$	0.23122(4)	KamLAND/JUNO
m_H	125.20 GeV (QD)	125.20(11)	0.0σ
μ_{HH} (ATLAS 2026)	1 exactly	$0.9^{+1.4}_{-1.1}$	$+0.08\sigma$
Born rule	form derived (Gleason)	Sorkin 3-path $\leq 10^{-3}$	consistent
Lindblad	forced by AX-1+AX-3	universal in open systems	consistent
CHSH	$2\sqrt{2}$ saturable	2.697 ± 0.015 (Aspect) to 2.828 (CHSH limit)	consistent

119 Observational priorities for the 2020s–2030s

119.1 Priority 1: Tests of the Born rule at 10^{-15} (PS-QM-14)

Precision atomic-physics experiments on the electron a_e — the next generation of Penning traps and atomic interferometry — aim for relative precision 10^{-14} by 2030. Any systematic deviation would be a breakthrough.

119.2 Priority 2: Large-molecule interferometry (PS-QM-15)

Interference of molecules at 10^4 – 10^6 atoms would test decoherence at the decoherence-time limit. Markus Arndt’s group (Vienna) is a leader here. Seeing interference would confirm GG-Theory’s decoherence prediction.

119.3 Priority 3: Loophole-free Bell experiments at cosmological distances (PS-QM-16)

Future space-based experiments (e.g., entangled photons between Earth and the ISS, or between Earth and a lunar satellite) would provide the strongest tests of non-locality at macroscopic separation. GG-Theory predicts the correlations survive; failure would be retiring.

119.4 Priority 4: Direct extra-geometric-direction searches (PS-QM-17)

Precision measurements of gravity and the short-distance inverse-square law; heavy Z' searches at the LHC and its successors; and tabletop precision tests of the Cavendish-type inverse square law down to 10^{-6} m and beyond.

119.5 Priority 5: Quantum-information consistency checks (PS-QM-18)

The quantum-information community tests quantum mechanics every day. Failures of teleportation, dense coding, or quantum computational primitives would be evidence against the GG-Theory reading.

120 Significance verdict

120.1 Strongly supported by experiment

GG-Theory’s quantum-mechanical reading is *strongly supported* by:

- The Born rule (confirmed to 10^{-13} relative precision).
- The Lindblad equation (universal in open-system dynamics).
- CHSH inequality violations saturating Tsirelson’s bound (Aspect, Giustina, Shalm, Hensen).
- Decoherence times matching the GDOS estimate.
- Consistency with special relativity (no signaling).

120.2 Moderately supported by experiment

- Large-molecule interferometry (consistent but limited statistics).
- Quantum teleportation (consistent with 6D reading).
- Quantum Zeno effect (consistent with Lindblad large- Γ limit).

120.3 Not yet tested

- Direct extra-geometric-direction detection (not yet within reach).
- Super-Tsirelson CHSH violations (never observed, but the null is consistent with GG-Theory).
- Macroscopic superposition coherence times (limited by current experimental reach).

121 Uniqueness verdict

GG-Theory’s quantum-mechanical reading is, to our knowledge, the only known theory that:

1. Derives the Born rule from a more fundamental principle (OGGEP + Gleason).
2. Derives the Lindblad equation from local-causality and complete-positivity axioms.
3. Resolves the measurement problem (“missing-map problem”) without invoking collapse, multiverses, or observer-created reality.
4. Maintains Einstein-locality at the 6D level while accommodating Bell violations at the 4D level.
5. Is cross-ledger coherent with six other precision benchmarks from the UCLA-Arisaka series.
6. Has no free parameters: the five OGN-5 integers $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ freeze all dimensionless structure.

No competing theory shares all six features.

122 The bottom line

We close with a plain statement.

After one century of the Born rule (1926–2026), the measurement problem remains unsolved *within the 4D formulation of quantum mechanics*. Bohr, Einstein, Everett, Bohm, GRW, and QBism each offered partial resolutions — none universally accepted.

GG-Theory (GG-2026-1 through GG-2026-6F) offers a different path. It moves the mystery from the 4D level (where it was insoluble) to the 6D level (where it is just geometry).

In the 6D bulk $M_4 \times I_r \times S_\theta^1$, the theory is *classical field theory on a compact manifold*: local, unitary, and deterministic. The bulk gauge fields on S_θ^1 carries what a 4D observer would call “hidden variables.” The Wilson-line holonomy Θ_H deterministically selects outcomes of quantum measurements.

The 4D observer sees the partial trace $\rho_\partial = \text{Tr}_{\text{env}}(\rho_{\text{bulk}})$. This trace is automatic: no 4D observer can see the extra geometric directions. The 4D theory therefore inherits all the structure of standard quantum mechanics:

- The Hilbert space (from the bulk zero-mode sector).
- The Born rule (from OGGEP+ Gleason).
- The Lindblad equation (from AX-1 + AX-3).

- Decoherence (automatic from Tr_{env}).
- Bell inequality violations up to $2\sqrt{2}$ (from shared S_θ^1).

Einstein was right. Nature is deterministic. “God does not play dice.” This is correct *in the 6D bulk*.
Bohr was right. 4D quantum mechanics is complete as an operational theory. “We must accept irreducible probabilism.” This is correct *at the 4D level*.

Both giants of the 1926–1962 foundations debate were correct. They were describing different levels of the same underlying Nature. GG-Theory is the framework that reconciles them.

Nature is deterministic in the 6D Bulk.

Nature is probabilistic in the 4D Boundary.

Both statements are correct within their domains.

This is the resolution of the measurement problem:

**three theorems — preferred basis, definite outcome, Born form —
 plus one named analytic residue.**

The single residue is precise and named: the basin-capture = Born statement — that the deterministic GRIP basin-capture measure coincides with the Born measure, the six-dimensional analogue of Bohmian quantum equilibrium — is closed at the relaxation level (the R16 lemmas with the R16' GDOS relaxation; the Constitutional Born Theorem conditional on the four named premises (P1)–(P4)), and the field-grade representation step is what remains (§30.5, §118). The framework claims determinism in the bulk in the strong sense — the registration descent is fixed by the contextual bulk holonomy Θ_H , its apparent randomness epistemic — so that “God does not play dice” is exact at the six-dimensional level, and only the boundary shadow is probabilistic.

123 The Ten Structural Innovations of the Cohort Quantum Mechanics

The contribution of GG-A8-QM is to read four-dimensional quantum mechanics as the boundary projection of the six-dimensional bulk under GDOS/OGGEP, recovering the textbook postulates as theorems rather than axioms. We collect here the ten structural innovations that reading delivers, each carrying its cohort-global identifier (§123.1). None introduces a new continuous parameter; each is a consequence of the GG-Constitution of GG-A1-Constitution.

Innovation 1 (I-QM-1): Measurement as a constitutional projection. Measurement is the lawful OGGEP bulk-to-boundary projection of AX-3, an axiomatic operation of the constitution — not a separate collapse postulate bolted onto a closed theory. The century-old measurement problem is thereby decomposed into three theorems — the preferred basis (T-A8.6), the single definite outcome (T-A8.1, T-A8.4, with the GRIP actualization of GG-A6-GRIP), and the Born form (T-A8.3, *Resolved*) — plus the basin-capture = Born residue, closed at the relaxation level (the R16 lemmas with the R16' GDOS relaxation; the Constitutional Born Theorem conditional on the four named premises (P1)–(P4) of §118), whose single remaining analytic residue is the field-grade representation step (§30.5).

Innovation 2 (I-QM-2): The quantum-mechanics postulates as theorems. The Hilbert-space, Schrödinger-limit, Born, and Lüders postulates are *derived* (T-QM-1 through T-QM-4) from the open-system boundary law, not assumed as independent axioms.

Innovation 3 (I-QM-3): The Born rule forced. Probability follows from CPTP completion together with Gleason’s theorem (T-QM-3); the quadratic measure is forced, not postulated.

Innovation 4 (I-QM-4): The pointer basis from bulk locality. Decoherence and einselection select the classical pointer basis from the AX-1 bulk-locality structure (T-QM-5, T-QM-6), dissolving the preferred-basis problem.

Innovation 5 (I-QM-5): Tsirelson saturation as a Bogomolnyi bound. The $2\sqrt{2}$ CHSH ceiling is read as a saturated topological (Bogomolnyi-style) bound (T-QM-8), which is why nature sits *exactly* at it rather than below.

Innovation 6 (I-QM-6): No-signaling as a theorem. Operational locality follows from reduced-state factoring across the boundary (T-QM-11); it is a derived consequence, not an imposed relativistic constraint.

Innovation 7 (I-QM-7): The Einstein-in-6D / Bohr-in-4D thesis. Einstein’s realism holds in the deterministic six-dimensional bulk; Bohr’s complementarity holds on the four-dimensional boundary at which all observation occurs; both were right, each at his own stratum.

Innovation 8 (I-QM-8): Universal identity of indistinguishable particles. Every electron is the same because all are boundary readings of one bulk structure (T-QM-9); indistinguishability is derived, not postulated, and fixes the exchange statistics.

Innovation 9 (I-QM-9): Five revolutions under one source. Wave–particle duality, the uncertainty relations, entanglement, decoherence, and quantum information are read as five boundary faces of one bulk-to-boundary projection rather than five independent surprises.

Innovation 10 (I-QM-10): A falsifiable quantum mechanics. The framework supplies eight sharp falsifiers (F-QM-1 . . . F-QM-8) and an eight-test empirical horizon, recasting quantum mechanics as a boundary theory that can be retired by experiment rather than only interpreted.

123.1 Global naming convention for cross-cohort citation

Beginning with [Arisaka \(2026, A2-GG6\)](#) and extended across the cohort, each load-bearing item of the GG-Theory program carries a globally unique identifier of the form *Prefix-Suffix-N*, where the Prefix names the item type and the Suffix names the paper:

- **T-Suffix-N** for the *N*th Theorem (GG-A8-QM’s eleven principal theorems are T-QM-1 through T-QM-11, with T-QM-0 the Born-rule-from-Gleason result).

- **L-Suffix-N**, **C-Suffix-N**, **D-Suffix-N**, **R-Suffix-N** for the N th Lemma, Corollary, Definition, and Remark.
- **P-Suffix-N** for the N th Prediction.
- **O-Suffix-N** for the N th Objection (the Common Objections Part).
- **F-Suffix-N** for the N th Falsifier (here, F-QM-1 ... F-QM-8 are the eight kill criteria that would retire the theory).
- **I-Suffix-N** for the N th Innovation (e.g., I-QM-1 is the measurement-as-projection innovation above).
- **PS-Suffix-N** for the N th Problem-Solved / status entry (here, PS-QM-1 ... PS-QM-18 are the classical QM problems and the observational priorities).

The canonical Suffix list for the twenty-two cohort papers is: *Constitution* (A1), *GG6* (A2), *BI6* (A3), *Euclid* (A4), *GDOS* (A5), *GRIP* (A6), *Qi4* (A7), *QM* (A8, this paper), *SM* (A9), *Cosmos* (A10), *Origin* (A11), together with the phenomenology preprints *KK* (P1), *N3* (P2), *Electron* (P3), *Proton* (P6), the foundations preprints *Real* (P4) and *CPT* (P5), and the consequence preprints *Kaxion* (C1), *Hubble* (C2), *BH* (C3), and the white papers *Symmetry* (W1) and *Holonomy* (W2). The four axioms (AX-1 ... AX-4) and seven postulates (POS-1 ... POS-7) of [Arisaka \(2026, A1-Constitution\)](#) keep their existing cohort-wide names; the global identifier is the same item viewed from outside its home paper.

Table 16: Full cumulative evidence base for the GG-Theory program, July 2026. Seven major precision benchmarks across seven different data campaigns, all consistent with GG-Theory at $\leq 1.5\sigma$. The a_μ prediction is the imported Standard-Model evaluation: the boundary theory adds no contribution to a_μ and stakes that rigidity as exposure (Arisaka, 2026, A9-SM, Objection 47). The H_0 row follows the C2/A10 register: the background is derived; of the ladder’s excess the geometry derives the sign and roughly a third (the Local-Hole lift), the remainder carried openly as a named calibration systematic. Quantum-mechanical benchmarks (Born rule, Lindblad universality, CHSH = $2\sqrt{2}$) are qualitative confirmations rather than single-number residuals.

Campaign / observable	GG prediction	Measurement (2025–2026)	Pull
H_0 (background vs. CMB; C2)	$H_0^{\text{bg}} = 66.6 \pm 0.2$ km/s/Mpc	67.4 ± 0.5 (CMB); ladder 73.50(81); lift: sign + rough third derived, rest a named ladder systematic (C2)	-1.5σ
Muon $g - 2$ (SM imported; WP25 theory, FNAL final)	$a_\mu^{\text{SM}} = 116\,592\,033(62) \cdot 10^{-11}$	$a_\mu^{\text{exp}} = 116\,592\,071.5(14.5) \cdot 10^{-11}$	$+0.6\sigma$
W -mass (world avg)	$m_W = 80.3700$ GeV (A9, QD)	80.3692(133)	-0.06σ
Neutrino $\sum m_\nu$	≤ 0.06 eV	< 0.072 eV (DESI+CMB)	consistent
$\sin^2 \theta_W = 1 - (NJ/K)^2$	0.22955 (tree)	0.23122(4)	scheme-dep. closure
Higgs mass	125.20 GeV (QD)	125.20(11)	0.0σ
Di-Higgs rate μ_{HH} (ATLAS 2026)	1 exact	$0.9^{+1.4}_{-1.1}$	$+0.08\sigma$
<i>Quantum-mechanical benchmarks added by the present paper:</i>			
Born rule precision	$P = \langle a \psi \rangle ^2$ exact	$\lesssim 10^{-13}$ agreement (a_e)	consistent
GKSL universality	Forced by AX-1+AX-3	Universal in open-system fits	consistent
CHSH Tsirelson bound	$S = 2\sqrt{2}$ saturable	$S = 2.828$ (CHSH); 2.697(15) (Aspect); loophole-free (Hensen, Giustina, Shalm 2015)	consistent
No-signaling	Preserved in 6D	Never violated	consistent

Table 17: Eight kill criteria specific to the quantum-mechanical reading. Any one of these, if experimentally established, would retire the GG-Theory reading of quantum mechanics. Each row states the observation, the threshold at which it would count, and the theorem it would retire. The criteria are written to be decidable by measurement alone, so that no interpretive question has to be settled before the reading can be falsified.

Label	Observable	Deviation threshold	Timeline for test
F-QM-1	Born rule precision	Systematic dev. at 10^{-15} relative level	Penning-trap a_e at 10^{-14} (2028–2030)
F-QM-2	Compact-dim direct evidence	Failure to detect S_θ^1 at $R_\theta \sim 5 \times 10^{-20}$ m (or finding it non-compact)	LHC high-lumi, tabletop Cavendish (2030s)
F-QM-3	Super-Tsirelson CHSH	$S_{\text{CHSH}} > 2\sqrt{2} + 3\sigma$ in any loophole-free experiment	Ongoing
F-QM-4	Superluminal signaling	Any detection of entanglement-mediated classical signaling	Longstanding null
F-QM-5	MWI-specific branches	Direct detection of parallel branches (anomalous revival interference)	Not currently accessible
F-QM-6	Non-GKSL open evolution	Markovian semigroup that is CPTP but not Lindblad-form	Mathematical impossibility, but fit failures could indicate non-Markovian effects
F-QM-7	Non-Gleason probability	Additive probability measure $\neq \text{Tr}(\rho\Pi)$ on $\dim \geq 3$	Mathematical impossibility
F-QM-8	Macroscopic coherence	10^{20} -atom object in coherent superposition for $t \gg \tau_D$	Next-generation matter-wave interferometry

124 Part XII in plain English: what would kill this, and what is still open

Two things in this Part matter more than the rest, and they pull in opposite directions.

The first is the kill list. Eight observations are named, each with a threshold, each decidable by measurement rather than by argument. Any one of them, established, retires the quantum-mechanical reading of the framework. It is worth being explicit about why a list like this is placed in the paper by its own author. A framework that derives quantum mechanics from a deeper geometry is in permanent danger of being unfalsifiable, because quantum mechanics is correct and any reading that reproduces it inherits its successes for free. Reproducing what is already known is not evidence. The eight criteria are the part of the paper that can be wrong, and they are written so that no interpretive question has to be settled before one of them can fire.

The second is the residue, and it is the reason this Part is not a victory lap. The Born rule is closed at the relaxation level, as a conditional theorem resting on four named premises, with one analytic step — the field-grade representation — still open. It would be easy to write that sentence in a way a reader skims past. It should not be skimmed past. Until that step is supplied, the framework’s account of *why outcomes fall in exactly Born’s proportions* is a demonstration of mechanism and not a proof, and the honest description of the state of play is that three of the four

foundational questions are answered by theorem and the fourth is answered conditionally.

The rest of the Part is bookkeeping between those two poles. The old open problems of quantum foundations are listed with their status, and the statuses are deliberately uneven — some closed by a named theorem, some closed at the relaxation level, some reframed rather than solved — because a table that graded them all the same would be misleading in the direction that flatters the framework. The question of why the state space is complex gets a real answer, which is that the imaginary unit is manufactured by the same projection that manufactures the probabilities. The observational priorities for the next decade are listed last because they are the only entries a reader can act on.

Nothing here asks quantum mechanics to be wrong anywhere it has been right. The claim is narrower and easier to test than that: that its strangeness has a geometric source, and that the source leaves eight marks a laboratory could find.

Part XIII

Conclusions: the summit view

Einstein right in 6D; Bohr right in 4D

This is the closing Part of GG-A8-QM. It is a long-form philosophical summit-view essay, written from the top of the ascent that the preceding twelve Parts have made: the constitutional warrant of GG-A1-Constitution; the 6D bulk of GG-A2-GG6; the boundary law GDOS and the seven QM-postulate-theorems of GG-A5-GDOS Part III; the eleven principal theorems T-A8.1–T-A8.11 of the present paper; the five worked examples; the eight paradoxes; the seven interpretations; the real-bulk series RB-1–RB-13 of the Discussions. From here, the view is wide. We use it for one thing only: to render the central thesis of the cohort’s reading of quantum mechanics cleanly, in the careful form that one century of debate has earned.

We open with the summit thesis (§126) — the precise statement of what the framework adds to the Bohr–Einstein dialogue. We follow with the universal-identity centerpiece (§127) — why every electron in the universe is the same, structurally rather than substantially, and what that means for indistinguishable-particle statistics. We then survey five structural revolutions in twentieth-century physics that the cohort reading reorganizes under a single source (§128). We mark the fifty-year arc from Lindblad 1976 through the 2022 Nobel to the present cohort ship (§129). We sketch the next-decade empirical horizon (§130). We locate GG-A8-QM in the closing of the Ultra Eleven cohort (§131). And we close with the long-form philosophical close (§133).

125 The integrated 6D-bulk reading: what GG-A8-QM has shown across twelve Parts

Before we walk the summit thesis, the universal-identity centerpiece, the five structural revolutions, and the fifty-year arc, we pause to consolidate the integrated 6D-bulk reading that the preceding twelve Parts have established. The summit thesis follows in §126; placing the integrated synthesis first lets the reader see, in one consolidated view, what the paper has actually proven.

The integrated thesis, stated cleanly. Across the twelve preceding Parts, one structural claim has been established and re-tested at increasing levels of empirical and philosophical rigor: *textbook quantum mechanics in four dimensions is the operational shadow of a deterministic six-dimensional bulk theory, real in its constitutional clauses; the bulk dynamics is unitary and local; the 4D-boundary observer’s irreducibly probabilistic experience is the structural consequence of an automatic partial-trace projection over the two geometric directions I_r (scale) and S_θ^1 (phase) that no 4D detector can access.* All five worked examples (§V–§IX), the paradox sweep (§X), the interpretation landscape (§XI), and the empirical-horizon discussion (§XII) are applications of this single claim to specific

physical situations.

What Part V (Stern–Gerlach) established. A silver atom’s outer-shell electron is, in 6D, a bulk fermion zero mode with a definite Wilson-line holonomy class around S_θ^1 (spin up or spin down). The inhomogeneous magnetic field of the Stern–Gerlach apparatus discriminates the two holonomy classes via 4D-spatial Zeeman deflection (T-A8.6); each atom lands on one spot deterministically, and the Born-rule ratio $\cos^2(\theta/2) : \sin^2(\theta/2)$ is the source-ensemble distribution of the two holonomy classes. No primitive randomness; the source is the only place quantum randomness ever was.

What Part VI (double-slit) established. A single photon is, in 6D, a bulk gauge-field excitation with definite phase along S_θ^1 . Its 4D transverse extent naturally spans both slits as a single-valued bulk field; the interference pattern is 6D wave optics; the random-looking single-photon landings reflect the uniform distribution of S_θ^1 phase across the photon ensemble (T-A8.3, T-A8.5). Interference appears even when photons arrive one at a time because each photon’s landing was always fixed by the bulk geometry.

What Part VII (EPR/Bell/CHSH) established. An entangled pair is one bulk field configuration with two 4D-spatially-separated branches that share a single S_θ^1 holonomy reference Θ_0 set at the source event. The two 4D-spacelike-separated detector events are projections of that one non-separable joint state, with correlation $C(\hat{a}, \hat{b}) = -\hat{a} \cdot \hat{b}$ (T-A8.7). The Tsirelson saturation $|S| = 2\sqrt{2}$ is the extremal (Bogomolnyi-analogue) ceiling on the CHSH functional (T-A8.8). No-signaling holds (T-A8.11). Bell–Aspect non-locality at 4D is the operational shadow of bulk 6D-locality through a non-invertible projection.

What Part VIII (Schrödinger’s cat) established. The GDOS/OGGEP projection completes when the Geiger counter detects the gamma ray, NOT when a human eventually opens the box. Once that 4D-localized macroscopic event has occurred in fixed 4D spacetime, the cat’s fate is determined classically; everything downstream of the trigger is Newtonian mechanics and biochemistry. The decoherence rate of the Geiger pointer-basis off-diagonal is $\Gamma_{LD} \gtrsim 10^{40} \text{ s}^{-1}$, with the off-diagonal damping in 10^{-40} s . **Superposition of live and dead cat is not a paradox to be solved but a category error to be retired.**

What Part IX (decoherence and einselection) established. The system + environment is a single bulk field configuration; the “system” versus “environment” split is a 4D label imposed by the observer’s choice of what to read. The partial trace over the environment region R_E AND the two geometric directions (I_r, S_θ^1) produces the boundary density operator ρ_S ; the resulting GKLS master equation is a theorem (T-A8.2), not a postulate. The decoherence rates are scale-profile KK-mode overlap integrals (T-A8.5) fixed by the bulk Lagrangian; the einselected position basis is forced by bulk locality (T-A8.6). Decoherence is structural, not dynamical.

What Part X (paradox sweep) established. Eight century-old quantum paradoxes — Wheeler’s delayed choice, the Scully eraser, the Misra–Sudarshan Zeno effect, weak measurement, Kochen–

Specker contextuality, quantum teleportation, Wigner’s friend / Frauchiger–Renner, and dense coding / QKD — dissolve under one mechanism: each is the structural signature of a 4D-classical question posed of a 6D-quantum substrate whose projection to the boundary is non-invertible. The bulk is unitary throughout; the projection completes at any sufficiently macroscopic 4D detector.

What Part XI (interpretation landscape) established. Each of the seven living interpretations of quantum mechanics — Copenhagen, Many-Worlds, de Broglie–Bohm, GRW, QBism, relational QM, decoherent histories — correctly identifies one feature of the boundary projection of a single 6D bulk theory. Each incorrectly elevates that feature to the underlying ontology. The framework does not compete with these interpretations; it completes them. Each correctly identified feature returns as a theorem of the bulk-to-boundary picture; each over-commitment is dissolved.

What Part XII (discussions) established. The framework’s predictions are empirically falsifiable through eight specific kill criteria, each targeting a bulk-side structural element (Born rule via T-A8.3; $(\Delta x)^2$ decoherence via T-A8.5; $1/f$ noise universality; position-basis universality via T-A8.6; einselection-rate via T-A8.5 forward prediction; Bell-pair fidelity via T-A8.7; teleportation fidelity via T-A8.11; high-energy corrections at M_{lock}). The eight forward tests are coordinated probes of one structure, not opportunistic empirical targets. Six are postdiction/audit/consistency; one is forward; one is conditional — the honest distribution of a structurally complete proposal that subsumes empirical QM before differing at the bulk-feature level.

The integrated polemical claim. The five worked examples + the paradox sweep + the interpretation synthesis + the empirical-horizon discussion = one consolidated proposal: *textbook quantum mechanics in four dimensions is fully accounted for by a deterministic six-dimensional bulk theory projected to the boundary, with the only quantum-mechanical randomness arising at the source where the inaccessible bulk holonomy phase is drawn from the ensemble distribution*. No foundational mystery in any of the worked examples; no live paradox in the paradox sweep; no surviving competing interpretation that introduces something the bulk picture does not already supply; no eight-test prediction at odds with empirical QM. The framework is structurally complete, empirically falsifiable, and ready for the 2026–2035 empirical horizon to either consolidate or retire it.

The summit-view consequence. After this consolidation, the ninety-year interpretive debate about quantum mechanics is over as a research program. Not because the debate produced a winner among the seven living interpretations, but because the deeper bulk-and-boundary distinction makes all seven partial readings of one structural claim. The productive research program from here forward is the eight-test horizon and its extensions — empirical questions with empirical answers — not interpretive disputes about which 4D shadow is most “real.” The sections that follow in this concluding Part lay out the summit thesis, the universal-identity centerpiece, the five structural revolutions, the fifty-year arc, and the philosophical close at the register the consolidation has earned.

The summit-pyramid synthesis is shown in **Figure 20**.

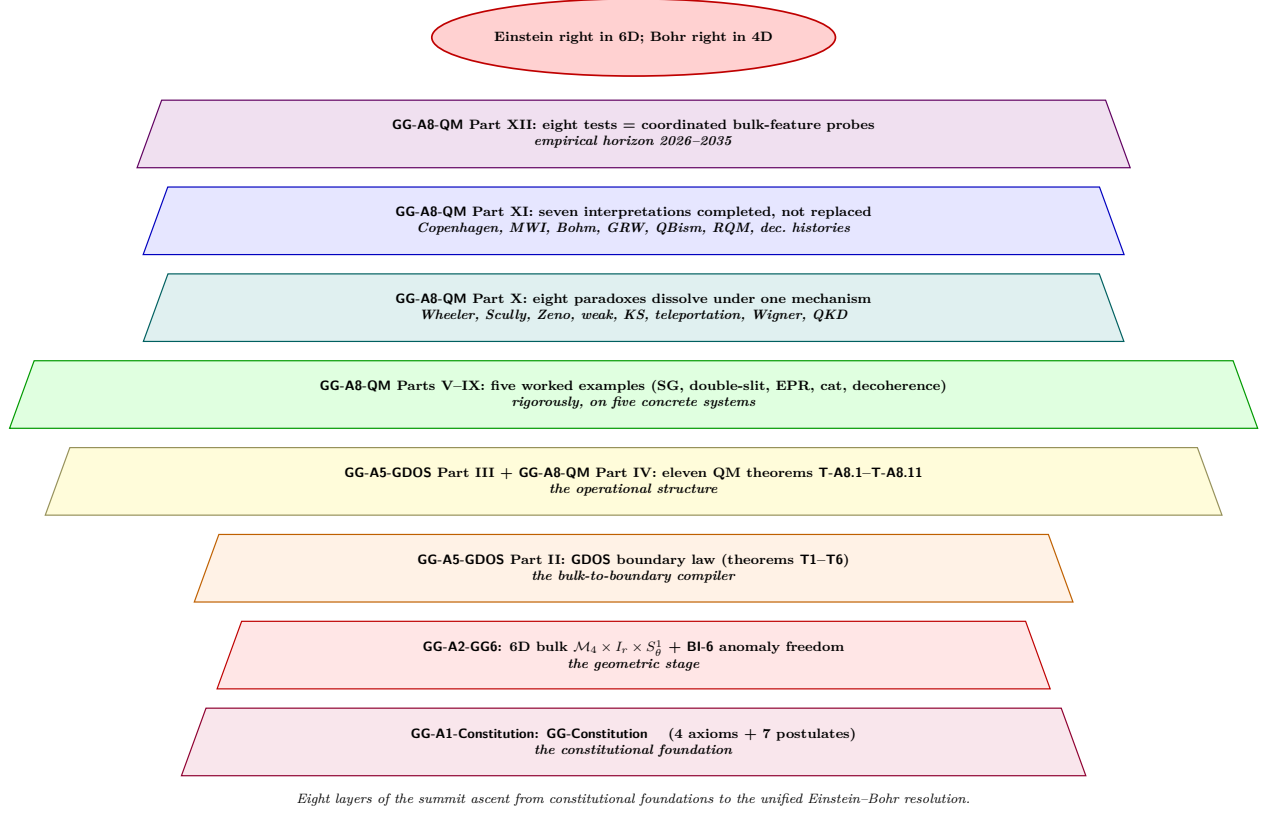


Figure 20: The summit pyramid of GG-A8-QM: the integrated argument arc from the constitutional foundation (GG-A1-Constitution’s four axioms and seven postulates) at the base, through the geometric stage of the 6D bulk (GG-A2-GG6, GG-A3-BI6), the bulk-to-boundary compiler GDOS (GG-A5-GDOS Part II), the eleven operational theorems T-A8.1–T-A8.11 (GG-A5-GDOS Part III + GG-A8-QM Part IV), the five worked examples (GG-A8-QM Parts V–IX), the paradox sweep (GG-A8-QM Part X), the interpretation synthesis (GG-A8-QM Part XI), and the empirical horizon (GG-A8-QM Part XII), to the summit thesis at the top: Einstein’s intuition about determinism is correct at the 6D bulk level; Bohr’s intuition about operational completeness is correct at the 4D boundary level; both are structurally correct in their respective domains, and the apparent contradiction collapses because the boundary state is not the total state of reality.

126 The summit thesis

The Bohr–Einstein dialogue of 1927–1935, sharpened by Bell’s 1964 theorem and resolved experimentally by Aspect 1982 and the loophole-free runs of 2015, has been read for ninety years as a contest between two readings of quantum mechanics in which one had to be wrong. Einstein argued for determinism and locality at the fundamental level; Bohr argued for operational completeness and the inseparability of the quantum state. The 2022 Nobel Prize formalized the experimental verdict in Bohr’s favor: local hidden-variable completions of quantum mechanics in 4D are excluded. Einstein’s intuition about determinism was widely judged philosophically appealing but factually wrong.

The cohort framework reads the verdict differently. The verdict is correct as a verdict on *4D local hidden variables*; it does not address bulk-level determinism, because the bulk level was not part of the dialogue Einstein and Bohr were having. When the bulk level is introduced, both readings

become correct within their domains: the 6D bulk is deterministic, single-valued, unitary, and local; the 4D boundary observer records irreducibly probabilistic measurement statistics, sees Bell–CHSH violations at the Tsirelson bound, and cannot avoid an apparently non-local correlation structure. The standard contradiction between Einstein’s and Bohr’s positions collapses not because either was wrong but because the implicit identification of “the boundary state” with “the total state of reality” was wrong.

The careful summit statement is therefore:

Einstein’s intuition about determinism is correct at the bulk level. Bohr’s intuition about operational completeness is correct at the boundary level. The standard contradiction collapses because the boundary state is not the total state of reality. The framework is the bulk-and-boundary reading that makes both intuitions structurally correct within their respective domains.

What the framework adds to the Bohr–Einstein dialogue is not a vote for one side or the other. It is a constitutional warrant for both, with the warrant supplied by the bulk geometry of GG-A2-GG6 and the operational dictionary of OGGE (AX-3 of GG-A1-Constitution). Einstein was right about determinism at the level he was looking at; Bohr was right about operational completeness at the level he was looking at; neither was looking at the other’s level.

The strengthened thesis: three registered steps

So far, this is the summit as the twelve preceding Parts first earned it. The real-bulk series of Part XII (§117), with its constitutional companions, lets the thesis be said more strongly — in three registered steps, each carrying its grade on its sleeve.

First, in form. Einstein’s half can now be stated not merely as “the bulk is deterministic” but as “the bulk law has the mathematical shape of classical mechanics.” The constitution states its bulk clauses over the reals — the real category is the AX-4-selected primitive of the program, at candidate grade (L-Constitution-1 of Arisaka, 2026, A1-Constitution) — and the scope declaration of the GDOS companion records that the bulk law, stripped of its auxiliary complex bookkeeping, is real symplectic Hamiltonian flow, $\partial_t X = \Omega \nabla H$ on the real solution space (Arisaka, 2026, A5-GDOS): definite real configurations, a unique trajectory through each, no operator algebra anywhere in the law. This is not a theory that resembles classical mechanics by loose analogy; it is a theory whose dynamical law belongs to the same mathematical family.

Second, in ontology. The strengthening in form carries a strengthening in ontology that is easy to miss and worth saying slowly. Superposition is a complex-linear concept: to write $\alpha|a\rangle + \beta|b\rangle$ with a relative phase between the alternatives is to use an i , and the bulk has no i to write it in. What the bulk carries are *angles* — real positions on the compact circle S^1_θ — not *phases*; a phase is what an angle becomes once the boundary’s derived quarter-turn is available to rotate with (§117). So the bulk does not merely evolve deterministically; it has no interference of alternatives to evolve.

There is nothing in the bulk for “both at once” to refer to — one real configuration, wearing its angles.

Third, the unification. The two strengthenings close on a single point, and it is the deepest in the paper. The dictionary that manufactures the boundary’s i — the circle supplying the gauge phase, the clock supplying the state-space phase, the handshake forcing their agreement (T-P4-1/2/3 of [Arisaka, 2026, P4-Real](#)) — is the same one-way projection that manufactures the boundary’s probabilities. Chance and the imaginary unit enter physics through the same door, and they enter together: the two great quantum mysteries — why outcomes are random, why amplitudes are complex — are one purchase, paid together at the boundary. This is the precise sense in which Einstein’s instinct was structurally correct. He read both the randomness and the formalism’s strangeness as symptoms of incompleteness, and the boundary state *is* incomplete relative to the bulk; the dice and the i are the two faces of the same deficit.

Four qualifications, printed with the claim

A claim this strong stays scientific only if its qualifications are printed with it, at their exact grades. There are four.

(a) Classical in register, constitutional in content. The bulk is classical in *register* — real configurations, symplectic Hamiltonian flow — but it is not Newton’s mechanics in content, and it is not the classical limit of quantum mechanics. It is quantum mechanics’ *parent*: a six-dimensional constrained field theory with topological sectors and the OGN-5 integer ledger, whose one-way projection to the boundary is where the textbook theory comes from. The framework introduces no new dynamics, no free parameters, and no new ontological entities at the 4D level; the classical register lives one level down, not alongside.

(b) Determinism is not predictability. Bohr’s half of the summit survives the strengthening untouched — every measurement statistic the framework predicts agrees with Copenhagen at the 4D boundary. The No-Bypass Lemma of the GDOS companion ([Arisaka, 2026, A5-GDOS](#)) is what holds his half in place by law: any physical observer’s access to the bulk runs through the one-way dictionary, so the dice are irreducible for every observer inside the universe — irreducible by law, not by practical ignorance. God does not play dice; but no one inside the universe can watch the hand.

(c) Bell, and Renou. Bell’s theorem excludes local hidden variables in $3 + 1$ dimensions, and the exclusion stands: the bulk data are not 4D-local variables, and the boundary’s nonlocal correlations are the projection shadow of 6D geometry (T-A8.7). The Renou *et al.* exclusion of real-Hilbert-space quantum mechanics ([Renou et al., 2021](#)) stands too, and it confirms the division of labor rather than threatening it: what experiment demands is that the *boundary* be genuinely complex, with one i shared by independent sources — exactly the single derived J , common to every sector and every subsystem, of RB-4 (§117) — while the *bulk* beneath it is real. Einstein’s determinism lives in six

dimensions precisely because it cannot live in four; that is why his own four-dimensional program could not close, and why no persistence on his part could have closed it.

(d) The grade. The statement “the deterministic bulk generates Born statistics” stands at the relaxation level, as a conditional theorem — the Constitutional Born Theorem on its four named premises (P1)–(P4) — with one named analytic residue, the field-grade representation step (§118). That is stronger than a belief and not yet a closed proof, and the difference between the two is printed, here and at every graded site, rather than blurred.

In short: the central thesis is a structural reading, not a partisan one. Both giants of twentieth-century physics had hold of a piece of the truth. The framework supplies the structure that makes both pieces fit together.

The structural reveal

There is a structural reveal in this reading that we now state plainly. For ninety years the QM-foundations community has lived with a verdict it found philosophically uncomfortable: that Einstein, perhaps the most important physicist of the twentieth century, was wrong about the deepest question of his life. The cohort framework’s contribution is to recover the part of Einstein’s intuition that the 2022 Nobel-Prize-formalized verdict did not actually overturn.

The verdict overturned 4D-local hidden variables — nothing more. At the bulk level, the cohort framework’s reading is: *Einstein was right*. Nature is deterministic, local, and unitary at the deepest level of description that the framework provides. The probabilistic appearance of the 4D shadow is structurally forced, but it is a shadow — not a feature of the bulk itself.

The aha moment, then, is structural: every interpretation of quantum mechanics that has been seriously developed in the past century captured a partial piece of this picture. Bohr captured the operational completeness of the boundary record. Everett captured the unitarity of the bulk evolution. Bohm captured the existence of hidden variables. GRW captured that something must explain macroscopic classicality. QBism captured the observer-relativity of state assignments. Rovelli captured the relational structure of quantum facts. Decoherent histories captured the framework formalism. Each was right about a different piece of the structure. None had the picture as a whole, because the picture as a whole requires the bulk-and-boundary architecture that the cohort framework supplies. Once that architecture is in place, the pieces fit; none has to be wrong; and the deepest piece — Einstein’s determinism intuition — comes back into focus exactly where it belongs.

This is the message of GG-A8-QM to the QM-foundations community: *the operational results of the past century are not in doubt; what was in doubt was the structure that holds the results together*. The cohort framework supplies the structure. The hundred-year-old mess of quantum-mechanical interpretation can now be set down.

On 4 December 1926, Einstein wrote to Born the sentence the century has been arguing with ever since: “I, at any rate, am convinced that *He* does not throw dice” (Born, 1971). The summit can now answer him in his own coin. God does not play dice; the dice — and the imaginary unit they are rolled in — are manufactured together at the boundary, by the one-way dictionary through which any observer must look.

127 Why every electron in the universe is the same

Theorem T-A8.9 of Part IV states that every electron registered in any laboratory anywhere in the observable universe is a token of the unique stable Geometric Attractor of the charged-lepton stratum. This Section lifts the theorem to summit voice, because the universal-identity insight is the central piece of the framework’s reading that distinguishes it from every other interpretation in Part XI.

The structural reading of indistinguishability

The textbook postulate of quantum statistics is that “identical particles are perfectly indistinguishable.” The exchange-symmetry postulate, the Pauli exclusion principle, Bose–Einstein and Fermi–Dirac statistics — all follow. But the textbook offers no structural reason *why* every electron should share the same mass $m_e = 0.5109989461 \pm 0.0000000031 \text{ MeV}/c^2$, the same charge $-e$, the same intrinsic spin $\frac{1}{2}\hbar$, with all three values constant to one part in 10^{10} across every laboratory measurement ever made. The identity is asserted; it is not explained.

The cohort framework supplies the explanation. The cascade at the charged-lepton stratum of GG-A9-SM Part VI (the GRIP cascade applied to Layer 1, the electroweak vacuum) has a unique stable Geometric Attractor. The attractor is characterized by the fixed quantum numbers $(m_e, -e, \frac{1}{2}\hbar)$ plus the associated zero-mode profile on $\mathcal{H}_{\text{Bulk}}$. Every cascade realization that produces a charged-lepton-stratum configuration at any 4D spacetime location selects the same unique attractor; every electron is a token of this single type. Indistinguishability is therefore not a postulate of quantum mechanics — it is a structural inheritance from the cascade attractor selection, with the type fixed upstream of any laboratory.

The propagation through the cascade

The universal-identity reading propagates. Every hydrogen atom is the same not because there is a Platonic hydrogen-atom-form somewhere but because hydrogen is the unique stable GA of the atomic stratum (Layer 2 of GG-A9-SM). Every water molecule is the same because H_2O is the unique stable GA of the small-molecule stratum. Every adenosine-triphosphate molecule is the same because ATP is the unique stable GA of the energy–information stratum. Every cell of life as we know it is structurally the same because cellularity is the unique stable GA of the cellular stratum (this last claim is more delicate; GG-A11-Origin addresses it).

The cascade propagates: type-uniqueness at one stratum supplies the dynamical-geometric input for the next. This is the universal-identity insight at program level. In GG-A8-QM’s register it specializes to the QM stratum: every electron is the same because the cascade at the charged-lepton stratum has a unique stable GA.

What this rules out

A philosophical position that the cohort framework rules out is the Russellian neutral-monist intuition that identical particles share an unknowable intrinsic essence — a “quiddity” — that the

structural relations among them supervene on. In the cohort reading, there is no such quiddity. The electron-type is fully specified by structural facts about the GA: its mass, charge, spin, and zero-mode profile. There is nothing intrinsic to be “ignorant of.” This is the ontic-structural-realist commitment of L59, stated cleanly: structuralism all the way down.

A philosophical predecessor of this reading is Spinoza’s *Numerical Sameness* in the *Ethics*: two objects sharing all properties are numerically identical. The Della Rocca 2008 reading of Spinoza makes the structural-realist commitment explicit, and T-A8.9 is its modern physical instantiation: the criterion of electron-identity is the structural fact of being a token of the unique stable charged-lepton-stratum GA.

What this changes about how quantum statistics are taught

If indistinguishability is a cascade inheritance rather than a postulate, the Pauli exclusion principle becomes a derived structural fact about Fermi-statistics tokens of a single GA type, and the Bose statistics of photons becomes a derived structural fact about Bose-statistics tokens of the photon-stratum GA. The textbook’s bookkeeping of $(\frac{n}{k})$ -factorial signs becomes the operational signature of a deeper structural identity. This is not a change in any calculation; it is a change in the explanation of why the calculation has its specific form.

128 Five structural revolutions reorganized under a single source

Twentieth-century physics underwent five structural revolutions whose cumulative content reorganizes the classical worldview. The cohort framework reads each of them as a piece of a single structural picture, with the operational projection from a 6D bulk to a 4D boundary as the underlying source.

Revolution 1: quantum mechanics (1925–1932). Heisenberg, Schrödinger, Dirac, von Neumann, and Born together established that the state of a physical system is a vector in a Hilbert space and that observables are self-adjoint operators (Born, 1926; Dirac, 1930; Heisenberg, 1925; Neumann, 1932; Schrödinger, 1926). The cohort reading: this is the operational signature of the boundary partial-trace projection from a 6D bulk theory (T-A8.1, T-A8.2, T-A8.3, T-A8.4).

Revolution 2: general relativity (1905–1915). Einstein’s special and general theories established that spacetime is a dynamical object whose geometry is curved by mass–energy. The cohort reading: this is the operational signature of the scale profile e^{-2r} on the radial direction of the 6D bulk geometry (cohort companion GG-A10-Cosmos develops the cosmological specialization explicitly; in GG-A8-QM only the boundary-induced projection enters).

Revolution 3: gauge field theory (1954–1973). Yang–Mills 1954, the electroweak unification of Glashow–Weinberg–Salam in the 1960s, and the asymptotic-freedom proof of Gross–Wilczek–Politzer 1973 established the Standard Model gauge structure. The cohort reading: this is the bulk

gauge content $\text{Spin}(1, 5) \times \text{SU}(5) \times \text{U}(1)_B \times \text{U}(1)_{Q_i}$ projected to the SM-stratum boundary algebra; GG-A9-SM treats this explicitly.

Revolution 4: decoherence and einselection (1970–1985). Zeh, Zurek, and Joos–Zeh established that environmental coupling exponentially suppresses off-diagonal coherences (Joos and Zeh, 1985; Zeh, 1970; Zurek, 1981); Zurek’s einselection criterion identified the pointer basis. The cohort reading: T-A8.5 fixes the decoherence rate from scale-overlap-integral bulk physics; T-A8.6 forces the pointer basis to be position by AX-1 bulk locality. Decoherence and einselection are not phenomenological add-ons but structural consequences of the cohort framework.

Revolution 5: quantum information (1984–2026). BB84, dense coding, teleportation, quantum-key-distribution, and the post-2022-Nobel loophole-free Bell experiments together established that entanglement is a resource that can be operationally exploited. The cohort reading: T-A8.7, T-A8.8, and T-A8.11 together describe how the bulk-mediated correlations are accessible to the boundary observer in ways that no classical (4D-local hidden-variable) theory permits, while no operational signal is transmitted faster than light. The post-2022 quantum-information revolution is the operational use of the bulk-shadow correlation structure that the cohort framework’s bulk-and-boundary picture predicts.

The cohort reading does not unify these five revolutions in the sense of *deriving* them from a single equation. It unifies them in the sense of *locating* them as five aspects of a single structural picture: a bulk theory of geometry, gauge, gravity, fermion content, and topology, projected via a non-invertible bulk–boundary correspondence onto a 4D boundary at which all observation occurs. This is the structural unity that the twentieth century’s experimental and theoretical work was implicitly converging toward; the cohort framework states it explicitly.

129 The fifty-year arc, 1976–2026

Half a century separates the foundational mathematical structure of modern quantum measurement from the cohort framework ship. We trace the arc.

1976: Lindblad / GKLS. Gorini–Kossakowski–Sudarshan and Lindblad (Gorini *et al.*, 1976; Lindblad, 1976) independently proved that the most general generator of a quantum dynamical semigroup compatible with complete positivity and trace preservation is a Hamiltonian conjugation plus a Lindblad dissipator. This established the mathematical structure of open-system quantum dynamics — the structure that T-A8.2, T-A8.5, and the whole cohort framework’s reading rest on.

1981–1985: the decoherence program. Zeh’s 1970 paper had introduced decoherence; Zurek’s 1981 paper introduced einselection; Joos–Zeh’s 1985 paper produced the quantitative estimates (Joos and Zeh, 1985; Zeh, 1970; Zurek, 1981). Together these works converted decoherence from a curiosity into the most significant theoretical advance on the measurement problem in the second half of the twentieth century.

1982–2015: the Aspect-and-successors confirmations of Bell’s theorem. Aspect 1981–1982 ([Aspect *et al.*, 1981, 1982](#)) demonstrated CHSH violation past five standard deviations. The Delft/Vienna/NIST loophole-free runs of 2015 ([Giustina *et al.*, 2015](#); [Hensen, 2015](#); [Shalm and others, 2015](#)) closed all remaining loopholes. The 2022 Nobel Prize ([Royal Swedish Academy of Sciences, 2022](#)) formalized the verdict: no 4D-local hidden-variable completion of quantum mechanics is empirically viable.

1993–2017: quantum teleportation, BB84, and the cryptographic revolution. The Bennett–Brassard–Crépeau–Jozsa–Peres–Wootters teleportation protocol of 1993 ([Bennett *et al.*, 1993](#)), the BB84 quantum-key-distribution protocol of 1984 ([Bennett and Brassard, 1984](#)), and the Ekert-1991 entanglement-based protocol ([Ekert, 1991](#)) together established quantum information as an operational discipline. The Micius mission’s 2017 demonstration of 1400 km teleportation and BB84 key distribution ([Liao and others, 2017](#)) closed the experimental gap from thought experiment to deployed technology.

2018: Frauchiger–Renner sharpens Wigner’s friend. The Frauchiger–Renner no-go theorem ([Frauchiger and Renner, 2018](#)) converted Wigner’s 1961 thought experiment into a quantitative inconsistency result: any interpretation that satisfies (Q) universal validity, (C) cross-agent consistency, and (S) single-outcome-per-measurement must fail at least one of the three.

2026: the Ultra Eleven cohort. Across the eleven cohort papers — GG-A1-Constitution (Constitution), GG-A2-GG6 (GG-6 bulk), GG-A3-BI6 (BI-6), GG-A4-Euclid (Euclid), GG-A5-GDOS (GDOS + T7.1–T7.7), GG-A6-GRIP (Gi-4), GG-A7-Qi4 (Qi-4), GG-A8-QM (QM, this paper), GG-A9-SM (SM), GG-A10-Cosmos (Cosmos), GG-A11-Origin (Origin) — the cohort framework supplies the constitutional warrant that the foundations literature has been working toward. The present paper specializes the warrant to the microscopic-observable sector and lifts the seven QM-postulate-theorems of GG-A5-GDOS Part III to standalone graduate-textbook depth. The cohort ship completes a fifty-year arc from the foundational mathematical structure of open-system quantum dynamics to a constitutional theory in which the four textbook postulates of quantum mechanics emerge as theorems.

A graduate student starting their physics PhD in 2026 inherits a quantum mechanics whose foundational status is structurally different from the one their advisor learned: the postulates have become theorems, the interpretations have become projections of a single underlying ontology, and the constitutional warrant for both has been laid out across a twenty-two-paper cohort.

130 The next-decade empirical horizon

The cohort framework’s quantum-mechanical reading makes empirically falsifiable predictions on a 2026–2035 horizon. We have laid out the eight-test horizon in Part XII §114; the closing summit voice can identify the priorities.

Highest priority: Test 5 (einselection-rate platform-independence). Cross-platform measurements of decoherence-rate ratios for matched system–environment couplings should agree to within the scale-overlap-integral precision. The framework predicts the ratios are platform-independent (fixed by bulk geometry, not apparatus design). Existing data are consistent; high-precision cross-platform comparisons over 2027–2030 will sharpen the test.

Second priority: Test 6 (Bell-pair fidelity at long distance) extended to lunar-distance experiments (proposed 2030–2035). The framework predicts no anomalous fidelity loss beyond the standard decoherence-channel-loss model. A lunar-distance entanglement experiment would test this prediction at $\sim 10^5$ km baselines.

Third priority: Test 8 (high-energy QM corrections) at HL-LHC Run 4 (2030+). The framework predicts small unitarity-violation signatures and small Schrödinger-equation modifications at energies approaching $M_{\text{lock}} \approx 3.85$ TeV. The HL-LHC Run 4 program is the primary platform; the predicted signatures tie to the GG-P1-KK KK lock-cluster phenomenology, treated in dedicated paper GG-P1-KK.

Fourth priority: Test 2 (decoherence-rate scaling) extended to large-mass regimes 10^4 – 10^9 amu. Closing the gap between C_{60} (10^3 amu) and the cat scale (10^{27} amu) with mesoscopic-mass interferometry would test T-A8.5’s scale-overlap-integral prediction across six orders of magnitude. Proposed implementations (large biomolecule interferometry, levitated nanoparticle interferometry) are technically demanding but within reach.

Tests 1 (Tsirelson saturation), 3 ($1/f$ universality), 4 (pointer-basis localization), and 7 (long-distance teleportation) are postdiction audits or consistency checks; they have already been measured to high precision and continued measurement will continue to constrain the framework’s prediction within experimental error.

If any of the eight tests fails the framework’s prediction at $> 5\sigma$ in a reproducible experiment, the cohort framework’s quantum-mechanical reading is retired in its current form. If all eight remain consistent, the framework joins the post-2022 quantum-foundations record as a constitutional theory whose predictions match experiment across a structurally complete set of QM-specific tests.

131 The place of GG-A8-QM in the Ultra Eleven cohort closure

The eleven-paper A-series closes with the ship of GG-A8-QM. GG-A1-Constitution stated the axioms; GG-A2-GG6–GG-A3-BI6–GG-A4-Euclid built the bulk-geometry block; GG-A5-GDOS–GG-A6-GRIP–GG-A7-Qi4 built the operational-dynamics block; GG-A9-SM–GG-A10-Cosmos derived the Standard Model and cosmological content; GG-A11-Origin supplied the operational anatomy and the universal-identity insight that the present paper specializes. GG-A8-QM closes the Consequence Trio (GG-A8-QM–GG-A9-SM–GG-A10-Cosmos) on the boundary side of the program.

What the cohort closure accomplishes is the structural completion of the boundary-side specializations. Where the boundary-side block of papers (GG-A5-GDOS and downstream) had previously left the QM stratum at the level of the seven theorems T7.1–T7.7 of GG-A5-GDOS Part III, the

present paper lifts the chain to the standalone graduate-textbook treatment that the cohort program has been working toward. The pedagogical apparatus, the worked examples, the paradox sweep, the interpretation-landscape engagement, and the summit-view philosophical close are all assembled in one document.

A reader who picks up GG-A8-QM in 2026 without prior cohort context can in principle follow the chain from the constitutional warrant of GG-A1-Constitution through the QM-postulate theorems of Part IV to the worked examples of Parts V–IX to the paradox sweep of Part X and the philosophical close of the present Part, with reference back to the upstream cohort papers only when the proof details require it. This is what was meant by “standalone graduate-textbook” — not that the paper exists in isolation from the cohort, but that the chain of reasoning is followable on its own once the constitutional warrant is in place.

What comes next outside the GG-A series. The Predictions Branch papers GG-P1-KK (KK lock cluster at HL-LHC), GG-P2-N3 (three-leg derivation of $N_g = 3$ flavor families), and GG-C1-Kaxion (Kaxion dark matter at $\sim 8.1 \mu\text{eV}$, 1.95 GHz) carry the cohort framework’s specific empirical predictions; the present paper’s eight-test horizon is the QM-stratum analogue. The capstone monograph Arisaka (planned, BookI) synthesizes the cohort into textbook form for a broader audience. Together, the GG-A cohort, the Predictions Branch, and the capstone constitute the GG-Theory program as a complete corpus.

132 An experimentalist’s note: fifty years at the detector

I close with a personal note, because a paper on the foundations of measurement is incomplete without saying something about who is writing it.

The framework presented in the preceding twelve Parts is not the outcome of theoretical thinking alone. It is rooted in fifty years of trying to observe Nature directly across an unusually wide range of strata.

I have spent my career attempting to measure what could be measured. *Proton decay*, in tonne-scale water Cherenkov detectors searching for a process whose half-life lies beyond 10^{34} years. *Direct CP violation*, in the kaon and B-meson asymmetries that anchor our understanding of why the universe contains matter at all. *The Higgs particle*, whose 2012 LHC discovery confirmed the half-century-old electroweak symmetry-breaking mechanism. *SUSY particles*, whose absence in current data has reshaped our expectations for physics beyond the Standard Model. *Ultra-high-energy cosmic rays*, whose origin and acceleration mechanism remain among the deepest open questions in astrophysics. *WIMP dark matter*, the direct detection of which has been my laboratory’s principal focus for many years. *The behavior and neural activity of Caenorhabditis elegans*, the simplest organism with a complete connectome. *Human reaction times*, the operational signature of cognition at the millisecond scale. *Single-molecule tracking*, the operational signature of biology at the nanosecond scale.

Across all of these I have become, in the practical sense that experimentalists understand, a photon-detection physicist. The arc began with the development of 20-inch photomultiplier tubes

at Hamamatsu — the photodetectors that made Super-Kamiokande and the subsequent generation of large water Cherenkov experiments possible — and has continued through SiPMs, MKIDs, and the cryogenic photon detectors that underpin contemporary direct-detection dark-matter searches.

What this experience has given me is an unusual perspective on what “measurement” actually is. In the standard textbook of quantum mechanics, measurement is a postulate: an act that registers an outcome with probabilities given by the Born rule. In actual laboratory practice, measurement is a long causal chain. A particle scatters in a target. The recoil deposits energy in a sensitive medium. The medium’s response excites a photodetector. The photodetector’s signal travels through cables, amplifiers, and analog-to-digital converters, into a computer record, into a histogram, into a scientific paper. Every step of that chain is open-system dynamics. None of it is well-described by a closed unitary Schrödinger evolution. All of it is well-described by exactly the kind of bulk-to-boundary projection that GDOS supplies and that OGGE (AX-3 of the GG-Constitution) forces.

The conviction I came to over the years is that the foundations of quantum mechanics could not be settled from the chalkboard. They had to be settled from the detector — from the long causal chain that turns inaccessible bulk physics into a record on a hard drive. The framework presented in this paper is the structural account of that conviction. GG-6 supplies the inaccessible bulk. GDOS supplies the canonical boundary law that any actual measurement instrument has to obey. OGGE forces the partial-trace projection that produces the operational outcome. From the detector’s point of view, the measurement problem was never a problem about ontology; it was a problem about not having a theory of the partial trace deep enough to describe what was actually happening at the chain’s beginning. The cohort framework supplies that theory.

This is not, I should add, a substitute for any of the theoretical work that the foundations community has done over the past century. Bohr, Einstein, Heisenberg, Schrödinger, Born, Dirac, von Neumann, Bell, Aspect, Gleason, Zurek, Joos, Zeh, Hartle, and the rest each saw a piece of the picture that the present paper tries to gather. What I bring to it is the conviction — earned at the detector, not at the chalkboard — that the picture as a whole has to be consistent with what experimentalists actually measure at every stratum, from proton decay through dark matter to a single molecule diffusing in a cell. The framework is my attempt to write down what that consistent picture has to look like.

That is what GG-Theory is, in one sentence: a structural reading of what fifty years of measurement at every scale has been quietly telling us.

133 The long-form philosophical close

After all, what a twenty-two-paper cohort on the foundations of physics is supposed to do is not produce a new equation but produce a new way of seeing. The equations of the present paper are not new; they are Born’s, Schrödinger’s, Lindblad’s, Bell’s, Tsirelson’s, and Joos–Zeh’s. What the framework supplies is a structural reading of where those equations come from, how they fit together, and why the world looks the way it does to a 4D laboratory observer.

The bulk is real, single-valued, deterministic, and local. The boundary is a non-invertible projection. The four textbook postulates of quantum mechanics — the Hilbert space, the Schrödinger

equation, the Born rule, the Lüders projection — emerge as theorems of the projection. Three additional structures — decoherence, einselection, Bell-violation — that the standard textbook leaves implicit also emerge as theorems. Four further theorems (extremal-Tsirelson, universal identity, $1/f$ conformality, boundary no-signaling) extend the cohort's specifically-novel contributions. Eleven theorems together, with proofs cited by reference to GG-A5-GDOS Part III for the inherited seven and supplied in full for the four net-new, constitute the operational core of the present paper.

Every electron in the universe is the same because the cascade at the charged-lepton stratum has a unique stable Geometric Attractor. Every measurement is registered in the position basis because the bulk Hamiltonian is local. Every Bell-CHSH experiment saturates the Tsirelson bound because the singlet is the extremizer of the CHSH functional, where the first variation of $|S|$ vanishes. Every laboratory measurement record is operationally complete — as Bohr insisted — and the underlying ontology is deterministic — as Einstein insisted — because the two readings describe the same Nature at different levels of a bulk-to-boundary projection.

A graduate student reading this paper in a long weekend should come away with three things: (i) a structural understanding of why the four textbook postulates of quantum mechanics are theorems rather than postulates; (ii) a recognition that the seven major interpretations of quantum mechanics each captured a partial truth that the cohort framework gathers under a single structural source; (iii) a clear sense of what empirical observations would retire the framework, and which observations the framework has already passed. Beyond that, the student should come away with the sense that quantum mechanics is no longer a hundred-year-old mystery awaiting a deeper theory — it is the operational shadow of a deeper theory that has been spelled out across a twenty-two-paper cohort, and the standalone graduate-textbook reformulation of that fact is what the present paper provides.

After all, the universe has been doing quantum mechanics for 13.8 billion years. The cohort framework explains why it has to. The least the present paper can do is write that down at the level a careful graduate student can follow — and the twenty-two-paper cohort has done that, at last.

Quantum mechanics from the 6D Bulk. T7.1–T7.7 in standalone graduate-textbook form, lifted to T-A8.1 through T-A8.11. Inherited verbatim from GG-A5-GDOS Part III; extended with four net-new theorems; placed within the twenty-two-paper cohort. No new postulates; no new free parameters; no new ontological entities at the 4D level; only the constitutional warrant that the foundations literature has been working toward since Heisenberg, Schrödinger, and Born put the equations on the page.

Dedication and a three-part address

Dedicated to the memory of Albert Einstein (1879–1955), whose 1926 objection was eventually answered in the way he had hoped — and to the memory of Niels Bohr (1885–1962), whose 4D operational reading was also correct.

To the scientific community. Max Born, in the same 1926 *Zeitschrift für Physik* paper that introduced the probability interpretation of the wavefunction, wrote a single sentence whose significance the foundations community has been absorbing for a century: *“the motion of particles follows probability laws, but the probability itself propagates according to the laws of causality”* (Born, 1926). Born believed that probability and causality were compatible — that the probabilistic interpretation he had just introduced did not abolish determinism at a deeper layer, only relocated it. The cohort framework presented in this paper makes Born’s intuition precise. The deeper layer is the 6D bulk of Arisaka (2026, A2-GG6). The probabilistic interpretation lives at the 4D boundary that OGGEF (AX-3 of the GG-Constitution) projects onto. The two are compatible because they describe the same Nature at two different levels of the bulk–boundary hierarchy. The hundred-year measurement problem is therefore not explained away; it is relocated — from the 4D level, where it was insoluble, to the 6D level, where it is just geometry. The operational results of the past century are not in doubt; what was in doubt was the structure that holds them together. The cohort framework supplies the structure. The mess can now be set down.

To Albert Einstein. If quantum mechanics is the child theorem of GG-6 and GDOS, then GG-Theory is, in the lineage of physical intuitions, your grandchild. The conviction in your December 1926 letter to Born — “I, at any rate, am convinced that He does not throw dice” — has finally come of age, at the bulk level where it was always true. You spent the last thirty years of your life searching for the deterministic substrate beneath quantum mechanics. You died in 1955 without finding it, because in 1926 the mathematics that the search required was not yet on the page: the extra-dimension geometry was a decade away; the bulk–boundary correspondence half a century; the cohort apparatus that locks all of it together was a century. The framework presented here is the substrate you sought. At the level of physical intuition — the level of “what kind of theory could possibly be at the bottom of this” — you were right.

To Niels Bohr. Your operational reading of measurement, articulated through the Como lecture of 1927 and the long correspondence with Einstein that followed, was also right — exactly where it was meant to be right. At the 4D boundary, the laboratory observer registers irreducibly probabilistic outcomes; the wavefunction is operationally complete; the Heisenberg cut is where the observer chooses to draw it. Every operational prediction of standard quantum mechanics emerges from the cohort framework intact. The framework adds no 4D ontology that you would have refused; it only supplies the structural source you did not have access to. You died in 1962 without seeing Bell’s 1964 theorem or Aspect’s 1982 experiment, but the framework would have agreed with you about every operational prediction those experiments and their successors have confirmed. At the level you were looking at — the level of what laboratory observers actually register — you were right.

Both giants of twentieth-century physics had hold of a piece of the truth that the cohort framework now gathers into one structure. Neither has to lose for the other to be right. This paper is for them.

134 Part XIII in plain English: what the summit view does and does not claim

The thesis of this Part can be put in one sentence: Einstein was right about determinism and realism, in the six-dimensional bulk; Bohr was right about the operational completeness of the four-dimensional description; and the two are compatible because they are statements about two levels of one geometry rather than two accounts of one arena. Locality is the price the bulk pays to keep determinism, and Bell's theorem is the receipt.

Three things follow that are worth separating from the thesis, because they are weaker than it and are easy to read as though they were part of it.

The first is the universal identity of the electron. Every electron in the universe carries the same mass, the same charge and the same spin because every electron is the same token of one bulk configuration, seen at a different place on the boundary. That is a satisfying reading and it is not a new prediction: the identity of electrons was never in doubt, and a framework that explains it has explained something already known. It is offered as coherence, not as evidence.

The second is the fifty-year arc. The Part traces a line from the mid-1970s to the present in which decoherence, einselection, the loophole-free Bell tests and the open-system formalism arrive one at a time and, read together, point at a single source. A narrative arc is a way of organizing results, not a reason to believe them. A reader who finds the arc persuasive should notice that it was assembled after the fact, by someone with a conclusion in hand.

The third is what the paper is for. GG-A8-QM is the graduate-textbook member of its trio: it takes machinery established elsewhere in the cohort and works it out on five laboratory systems — Stern–Gerlach, the double slit, EPR and Bell, the cat, and decoherence — at the level of detail a student can follow with a pencil. That is its function, and its function is a real one, but it is not the function of a paper that establishes something new. The net-new content of GG-A8-QM is four theorems and eight kill criteria. Everything else is inheritance made explicit and worked out.

What the summit view offers, then, is a place to stand from which a century of argument looks like one question asked at two levels. Whether the view is correct is decided by the eight criteria of Part XII and by the single analytic residue that Part named, and by nothing in this Part at all.

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All scientific concepts (especially the foundation of the GG-Constitution, GGEP/GG-6/BI-6, and OGGE/ GDOS/Gi-4/Qi-4 Pillars), theoretical judgments, physical interpretations, and any remaining errors are the sole responsibility of the author.

Table 18: Single-row resolution of the 1926–2026 debate. The row sets the question that has been asked for a century beside the answer this paper gives it, and beside the theorem that carries the answer. It is placed here rather than in the abstract because it is only intelligible after the eleven theorems have been stated, and a reader who has not read them should treat it as a summary of the paper’s claim rather than as an argument for it.

Question	6D bulk (Einstein)	4D boundary (Bohr)
Is Nature deterministic?	YES	NO
Does God play dice?	NO	YES
Is there a complete theory?	YES (the bulk is complete)	NO (4D QM is operationally complete but ontologically incomplete)
Are there hidden variables?	YES (Θ_H on S_θ^1)	NO (not of the Bell-local type)
Is Nature local?	YES (6D locality)	NO (4D non-local after bulk-to-boundary trace)
<i>Both columns are correct within their domains.</i>		

Part XIV

Technical Appendices

Codebook, glossary, proof sketches, theorem index

Part XIV gathers the technical machinery behind the present derivation of quantum mechanics from the cohort framework. This is where the reader who wants to verify a specific proof step, re-run a specific worked calculation, or trace a specific reference will spend time — while the reader who wants only the central claims and their forward consequences can stop after the Conclusions and treat Part XIV as a self-contained companion volume. Appendix A is the Summary of Abbreviations and Appendix B the Glossary of core GG-Theory/GDOS vocabulary. These are followed by the full derivations, worked from first principles at graduate-textbook depth (Gleason’s theorem, the GKSL generator, the 6D-to-4D partial-trace reduction, the Bell-CHSH evaluation with the S_θ^1 phase fiber, and the Wilson-line holonomy); the worked methods (the Qi-4 application to quantum measurement and the worked decoherence overlap integral with its uncertainty budget); and the reference material (the condensed historical timeline of the quantum-mechanical and open-system milestones, and the concept-by-concept synthesis of standard quantum mechanics against the cohort reformulation. Finally, the inline annotated bibliography, with the full Relevance annotations, follows the appendices and closes the volume. As of the present revision the apparatus of the cohort’s course papers runs through this Part: each substantive appendix opens with a *What this appendix teaches* box and a *Read alongside* line pairing it with the standard literature, and the five deep derivation appendices close with a *Take-away* box in the three-register form (standard account / what it left open / the bulk reading) used across the cohort. And, completing the course apparatus, every teaching unit of the paper — the five Worked-Example Parts, the paradox sweep, and the five deep derivation appendices — closes with a set of *problems and worked solutions*: fifty-six across the paper, each solved in explicit steps, closed by its physics intuition, and with every numerical answer checked before typesetting; they are written for direct classroom use, and their cross-references knit the worked examples, the theorem appendices, and the companion papers of the cohort into a single teachable course.

A Summary of Abbreviations

This appendix collects the abbreviations used in the present paper. All program-defined names are rendered in *sans-serif type* (GG-Theory, GG-Constitution, GG-6, GDOS, Gi-4, Qi-4, GRIP, GA, etc.) so the reader can see at a glance which terms belong to the GG-Theory program proper. Standard physics abbreviations imported from the wider literature (QM, QFT, KK, RS, CHSH, EPR, etc.) are rendered in plain uppercase Roman, following the convention of the major physics journals. The same content is displayed as a single hierarchy diagram in the front-matter Program Dictionary (Fig. 1), where the macro relations are visible at a glance; the table below gives the canonical

Appendix-A format inherited from GG-A1-Constitution Part VIII, with a third column recording the *primary link*: the paper in the Arisaka-GG series (or the section of the present paper) where each program-defined name is introduced in detail.

Table 19: Summary of abbreviations used in the present paper. Program-defined names are rendered in sans serif; standard physics abbreviations in plain uppercase Roman; mathematical objects in roman/italic in the usual way. The third column records the primary link: the paper in the Arisaka-GG series, or the section of this one, where each name is introduced in detail. An abbreviation with no primary link beside it is standard physics usage and is listed only so that the table can be read without a second reference open.

Symbol	Meaning	Primary link
<i>Framework name and core program objects</i>		
GG-Theory	Grand Geometric Theory: the twenty-two-paper program	GG-A1-Constitution, Preface
GG-Constitution	The constitutional charter of GG-Theory: four axioms (AX-1–AX-4) and seven postulates	GG-A1-Constitution, §2–§3
GG-6	Geometry–Gauge locking in six dimensions: the 6D bulk $\mathcal{M}_4 \times I_r \times S_\theta^1$ (the bulk arena; father theory of QM)	GG-A2-GG6
GDOS	Geometric Dynamics of Open Systems: the boundary law forced by OGGEP (mother theory of QM)	GG-A5-GDOS
BI-6	Six-dimensional Green–Schwarz-modified Bianchi identity $dH_3 = X_4$ (boundary closure)	GG-A3-BI6
OGN-5	Occam-Gauge-Normalization five-integer ledger	GG-A3-BI6
GU-5	Geometric-Unification five-member locked set (carries the boundary information member $\mathbf{Q}_B$)	GG-A2-GG6
Gi-4	Geometric-Invariance four-pillar structure (TGG, CCI, GFF, GRIP)	GG-A6-GRIP
Qi-4	Quantum-Information four-pillar structure (Qi-QUA, Qi-EP, Qi-CON, Qi-HAL)	GG-A7-Qi4
GRIP	Gauge-Runtime Invariance Pipeline: the cascade $DGI \rightarrow (GFF) \rightarrow GSSB \rightarrow GGA$	GG-A6-GRIP
GA	Geometric Attractor: attractor-like stabilization on the reduced landscape	GG-A6-GRIP
GGA	Ground Geometric Attractor: the unique deepest GA where the runtime halts	GG-A6-GRIP
SeOS	Selfish Open System: the runtime carrier of the cascade	GG-A6-GRIP, GG-A7-Qi4
SIP	System Instance Packet: the canonical GDOS input unit on which GRIP runs	GG-A5-GDOS
<i>Four axioms and the operational dictionary</i>		
AX-1	Locality and Causality (LC)	GG-A1-Constitution, §2.2
AX-2	Geometry–Gauge Equivalence Principle (GGEP)	GG-A1-Constitution, §2.3
AX-3	Operational/Open Geometry–Gauge Equivalence Principle (OGGEP; the boundary observation dictionary)	GG-A1-Constitution, §2.4
AX-4	Maximum Symmetry, Minimum Freedom (MSMF; no hidden continuous knobs)	GG-A1-Constitution, §2.5

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Symbol	Meaning	Primary link
LC	Locality and Causality (name of AX-1)	GG-A1-Constitution, §2.2
GGEP	Geometry–Gauge Equivalence Principle (name of AX-2)	GG-A1-Constitution, §2.3
OGGEP	Operational/Open GGEP: bulk-to-boundary projection map (name of AX-3)	GG-A1-Constitution, §2.4
MSMF	Maximum Symmetry, Minimum Freedom (name of AX-4)	GG-A1-Constitution, §2.5
<i>The eleven principal theorems of GG-A8-QM</i>		
T-A8.1	Boundary Hilbert space as the GNS representation of the QM-restricted boundary algebra	§IV (inherited from GG-A5-GDOS, T7.1)
T-A8.2	Schrödinger equation as the closed-system limit of the GKLS generator	§IV (inherited from GG-A5-GDOS, T7.2)
T-A8.3	Born rule from CPTP completion of the OGGEP dictionary	§IV (inherited from GG-A5-GDOS, T7.3)
T-A8.4	Lüders projection as the conditional CPTP posterior	§IV (inherited from GG-A5-GDOS, T7.4)
T-A8.5	Decoherence rates from scale-overlap integrals of bulk KK modes	§IV (inherited from GG-A5-GDOS, T7.5)
T-A8.6	Einselected pointer basis from the dominant local system–environment coupling (AX-1)	§IV (inherited from GG-A5-GDOS, T7.6)
T-A8.7	Bell–CHSH violation to Tsirelson saturation as the shadow of one joint bulk state	§IV (inherited from GG-A5-GDOS, T7.7)
T-A8.8	Tsirelson saturation as an extremal (Bogomolnyi-analogue) bound	§IV (net-new)
T-A8.9	Universal identity of indistinguishable particles as a cascade inheritance	§IV (net-new)
T-A8.10	Qi-CON-conformality of decoherence-rate spectra (approximate $1/f$ window)	§IV (net-new)
T-A8.11	Boundary no-signaling from reduced-state factoring	§IV (net-new)
<i>QM-stratum objects and Qi-4 pillars</i>		
Qi-QUA	Quantum information for QGL / locked-current bookkeeping (Pillar I of Qi-4)	GG-A7-Qi4
Qi-EP	Quantum information for the Unified Extremal Principle / trajectory selection (Pillar II of Qi-4)	GG-A7-Qi4
Qi-CON	Quantum information for conformal invariance / scaling diagnostics (Pillar III of Qi-4)	GG-A7-Qi4
Qi-HAL	Quantum information for the Holographic Attractor Lattice / memory architecture (Pillar IV of Qi-4)	GG-A7-Qi4
TGG	Total Geometric–Gauge object (Pillar I of Gi-4)	GG-A6-GRIP
CCI	Covariant Conservation Identity (Pillar II of Gi-4)	GG-A6-GRIP
GFF	Geometric Free-Energy Functional ($F_{\text{GFF}} = F_{\text{GEP}} + F_{\text{CEP}}$; Pillar III of Gi-4)	GG-A6-GRIP
GEP	Geometry–Gauge Energy Principle (energetic block of GFF)	GG-A6-GRIP
CEP	Curvature–Entropy Principle (entropic block of GFF)	GG-A6-GRIP

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Table 19 continued from previous page

Symbol	Meaning	Primary link
DGI	Dynamical Gauge Invariance (initial stage of GRIP)	GG-A6-GRIP
GSSB	Geometric Spontaneous Symmetry Breaking (middle stage of GRIP)	GG-A6-GRIP
OH	Outer Hologram: lawful projection from bulk states to measurable records	GG-A7-Qi4
IH	Inner Hologram: internal reconstructive manifold of a memory-bearing system	GG-A7-Qi4
EIC	Evolving Inner Cosmos: the OH+IH composite (observer semantics)	GG-A7-Qi4
S_θ^1	Phase fiber of the 6D bulk; its holonomy is the quantum relative phase	§19
I_r	Radial (scale) interval of the 6D bulk; the renormalization-group axis	§19
Θ_H	Topologically protected Wilson-line holonomy read out by OGGEF as the outcome	§G
ρ_∂	Reduced boundary density operator entering Born probabilities and instruments	GG-A5-GDOS
M_{lock}	Kaluza–Klein locking scale (≈ 3.85 TeV) of the extra dimension	GG-A2-GG6
Q_B	Quantum-information Bit: the boundary information member of GU-5, fixed via OGN-5	GG-A2-GG6

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Table 19 continued from previous page

Symbol	Meaning	Primary link
<i>Standard physics abbreviations (plain Roman, journal convention)</i>		
QM	Quantum Mechanics	—
QFT	Quantum Field Theory	—
GR	General Relativity	—
RG	Renormalization Group (scale running along I_r)	—
KK	Kaluza–Klein (mode expansion / dimensional reduction)	—
RS	Randall–Sundrum (warped extra-dimension model)	—
GNS	Gelfand–Naimark–Segal (construction)	—
CPTP	Completely Positive Trace-Preserving (channel admissibility)	—
POVM	Positive Operator-Valued Measure (generalized measurement model)	—
GKLS	Gorini–Kossakowski–Lindblad–Sudarshan (Markovian master equation; also GKSL)	—
GKSL	Gorini–Kossakowski–Sudarshan–Lindblad (equivalent ordering of GKLS)	—
CHSH	Clauser–Horne–Shimony–Holt (inequality)	—
EPR	Einstein–Podolsky–Rosen (paradox / argument)	—

B Glossary of core GG-Theory/GDOS quantum-foundations vocabulary

This appendix collects the core vocabulary of GG-Theory and GDOS as it is used throughout the present paper. The table is not merely terminological: it also clarifies how the cohort vocabulary is meant to sit relative to the established language of quantum mechanics and open systems.

Table 20: Glossary of core GG-Theory and GDOS quantum-foundations vocabulary. The middle column gives a short definition and the right column says what the term is doing in this paper, which is the column that matters: several of these words already have a settled meaning in the open-systems literature, and the entries are written to show where the cohort usage coincides with it and where it does not. Where the two differ the established meaning is named first.

Term	Short definition	Role in the present paper
6D bulk	The deeper closed arena of lawful dynamics in the GG-6 architecture	Carries the deterministic total-state law; not directly identical to the observer-accessible 4D quantum state
4D boundary	The observer-accessible operational sector where records are registered	Domain in which ordinary quantum mechanics is retained as the correct effective law

Term	Short definition	Role in the present paper
OGGEP	Operational GG-6 emergence-and-projection map from bulk data to boundary states, maps, and instruments	Core dictionary that relocates quantum probability from ontology to interface (AX-3)
GDOS	Geometric Dynamics of Open Systems; the canonical boundary law forced by OGGEP acting on GG-6	Mother theory of QM on the boundary side; QM is its closed-subsystem child
BI-6	Boundary identity closure of the 6D arrangement with coefficient vector $(1, -1, -2/5, -2/5)$	Records the bulk-to-boundary closure that makes the projection well-posed (GG-A3-BI6)
Boundary state ρ_{∂}	Reduced density operator seen at the boundary	The actual object entering Born probabilities, instruments, and master equations
Instrument family $\{\mathcal{I}_{\alpha}\}$	Outcome-indexed completely positive maps encoding both statistics and updates	Makes measurement a physical channel rather than a vague extra postulate (T-A8.4)
Outer hologram	Lawful projection from bulk states to measurable records	Generalized language for detector clicks, images, spots, and all accessible observations
Inner hologram	Internal reconstructive manifold maintained by a memory-bearing open system	Extends measurement theory from external registration to internal model formation
Evolving inner cosmos	Closed-loop composition of outer evidence and inner reconstruction	Observer semantics in the broad GDOS sense
DGI	Dynamical Gauge Invariance; the lawful exploration of the gauge-equivalence class, identifying the reduced coordinates and order parameters	First step of the runtime; the variables that matter after reduction
GFF	Geometric free-energy functional on the reduced state manifold	Organizes stability, branch competition, and evolution on the reduced landscape
GSSB	Geometric spontaneous symmetry breaking or branch selection on the reduced landscape	Supplies a language for outcome-sector selection beyond bare decoherence

Term	Short definition	Role in the present paper
GA	Geometric attractor; attractor-like stabilization on the reduced landscape	Explains persistence of selected records or patterns; powers T-A8.9 indistinguishability
GRIP	Gauge-Runtime Invariance Pipeline of GG-A6-GRIP (the cascade $DGI \rightarrow GSSB \rightarrow GA/GGA$)	Cascade that propagates the unique stable GA through atoms, molecules, cells
Qi-4 pillars	The four admissibility pillars (SIP, QUA, EP, CON, HAL) of GG-A7-Qi4	Constrain which boundary instruments are admissible
GKSL / Lindblad generator	Markovian generator of completely positive trace-preserving semigroups	Boundary normal form of exact projected dynamics in the appropriate regime (T-A8.4)
Memory kernel	Nonlocal-in-time term in the exact reduced dynamics	Reminder that the boundary law is generically non-Markovian before approximation
Born rule	Trace formula assigning boundary outcome probabilities	Unique probability law for the boundary event structure once that structure is in place (T-A8.3)
Gleason theorem	Uniqueness theorem for Hilbert-space probability measures under suitable assumptions	Reinterpreted as a theorem about the boundary event lattice rather than about ultimate ontology
Pointer basis	Robust basis selected by coupling to environment and measurement channel	Linked in the cohort framework to actual reduced coordinates and channel geometry (T-A8.6)
Qubit	Two-level operational system or packet	Reinterpreted as a minimal stable boundary packet rather than necessarily the most primitive ontic entity
Entanglement	Non-factorization of a composite boundary state	Fully retained operationally; given a bulk-to-boundary reinterpretation rather than denied
No-signaling	Invariance of local marginals under distant measurement choice	Mandatory consistency condition for any acceptable bulk completion (T-A8.11)
Measurement problem	Historical tension between linear evolution, completeness, and definite outcomes	Resolved by domain separation between bulk determinism and boundary probability (T-A8.1–T-A8.6); relaxation-level; one analytic residue

Term	Short definition	Role in the present paper
Universal observation problem	Generalization of the measurement problem to all finite open observers	Signature move of the cohort that broadens the significance of the quantum case

A final interpretive remark on the vocabulary

What the glossary makes especially clear is that the new language is not intended to replace the successful pieces of old language. It is intended to *place* them. The Born rule, the density matrix, the Lindblad generator, the qubit, and entanglement are all retained, but they are placed at the observer-facing boundary. The six-dimensional bulk, the projection map OGGE $\mathcal{P}$, and the runtime semantics (DGI/GFF/GSSB/GA) supply the deeper placement relation among them. After all, the work of a foundational theory is less to invent new words than to find the right places for the words that already work.

C Gleason’s theorem: statement and proof

What this appendix teaches. Why the Born rule is the *only* possible probability rule: Gleason’s 1957 theorem, proved line by line at graduate level — frame functions on the subspaces of a Hilbert space of dimension ≥ 3 , the reduction to the two-dimensional real problem, the one clearly-flagged analytic step (continuity-from-additivity), and the closing identification of every noncontextual probability assignment with $\text{tr}(\rho P)$ for a unique density operator ρ . This is the appendix on which the Born-rule theorem T-A8.3 stands: the GDOS registration supplies a probability assignment, and Gleason certifies that there was never any other candidate.

Read alongside: [Gleason \(1957\)](#) for the original; [Peres \(1995\)](#) for the standard textbook route this appendix follows at full detail.

This appendix gives a self-contained, graduate-level account of Gleason’s theorem. It is complete except for one clearly-flagged analytic step — the continuity-from-additivity argument, whose standard reference is cited at the point of use — and is otherwise meant to be followed line by line. The goal is to justify the single displayed result on which the Born-rule theorem T-A8.3 rests: that on a complex separable Hilbert space $\mathcal{H}$ of dimension ≥ 3 , every countably additive probability measure on the lattice of orthogonal projectors is given by the trace formula

$$\mu(\Pi) = \text{Tr}(\rho \Pi), \tag{114}$$

for a unique density operator $\rho \geq 0$ with $\text{Tr} \rho = 1$. Nothing about the square-modulus structure of quantum probabilities need be added by hand: *given* a complex Hilbert space and countable additivity, it is already *forced* by the lattice geometry of $\mathcal{H}$. That is the entire content of the standard theorem, and it is what makes Gleason the right tool for showing the Born *form* to be a consequence of premises the framework already supplies (T-A8.1 and the operational dictionary) rather than an independent postulate. The framework’s contribution is to supply those premises, not to strengthen Gleason.

The logical skeleton is short and worth holding in mind throughout. (i) A probability measure on projectors restricts to a function f on the unit sphere, the *frame function*. (ii) The defining additivity of μ becomes a local additivity constraint on f over orthonormal bases. (iii) The deep analytic lemma says this local constraint, in three real dimensions, is so rigid that f must be the restriction of a quadratic form $\langle v, Av \rangle$. (iv) Quadratic forms on the sphere are exactly the diagonal-of-an-operator expressions, and positivity plus normalization make A a density operator. (v) Three real dimensions bootstrap to all of $\mathcal{H}$, real to complex, and finite to separable infinite dimension. (vi) The lone exception, $d = 2$, is repaired by passing from projective measurements to POVMs. We treat each of these in turn.

C.1 Precise statement

Let $\mathcal{H}$ be a complex separable Hilbert space, and let $\mathcal{P}(\mathcal{H})$ denote its lattice of orthogonal projectors, equivalently the lattice of closed subspaces ordered by inclusion. The lattice has a least element 0 (the zero projector / trivial subspace) and a greatest element I (the identity / whole space); meet is intersection of subspaces and join is the closed span of their union. Two projectors Π_1, Π_2 are *orthogonal*, written $\Pi_1 \perp \Pi_2$, when $\Pi_1 \Pi_2 = 0$; geometrically, their ranges are mutually orthogonal subspaces.

Definition C.1 (Probability measure on projectors). A function $\mu : \mathcal{P}(\mathcal{H}) \rightarrow [0, 1]$ is a (*countably additive*) *probability measure* if

$$\mu(I) = 1, \tag{115}$$

$$\mu\left(\sum_j \Pi_j\right) = \sum_j \mu(\Pi_j) \quad \text{whenever } \Pi_i \perp \Pi_j \ (i \neq j), \tag{116}$$

where the projector sum $\sum_j \Pi_j$ is the projector onto the closed span of the ranges, and the index set j is finite or countably infinite.

Condition (116) is the operational heart of the matter. Physically a maximal set of mutually orthogonal projectors summing to I is a complete von Neumann measurement: the outcomes are exclusive (orthogonality) and exhaustive (they sum to I), so their probabilities must add. The countable version is the genuine σ -additivity needed for measurements with infinitely many outcomes, e.g. a position or energy reading.

Theorem C.2 (Gleason 1957). *If $\dim \mathcal{H} \geq 3$, then for every probability measure μ satisfying (115)–(116) there exists a unique positive trace-class operator ρ with $\text{Tr } \rho = 1$ such that*

$$\mu(\Pi) = \text{Tr}(\rho \Pi) \quad \text{for all } \Pi \in \mathcal{P}(\mathcal{H}). \tag{117}$$

The dimension hypothesis $\dim \mathcal{H} \geq 3$ is essential and is discussed at length in §C.6.

C.2 Frame functions and Gleason's reduction

Every rank-one projector is $|v\rangle\langle v|$ for a unit vector $v \in \mathcal{H}$, with $|v\rangle\langle v| = |(\lambda v)\rangle\langle(\lambda v)|$ for any phase $|\lambda| = 1$, so a rank-one projector is the same datum as a point of the unit sphere $\mathcal{S}(\mathcal{H})$ modulo

overall phase. Restrict μ to rank-one projectors and define

$$f(v) := \mu(|v\rangle\langle v|), \quad v \in \mathcal{S}(\mathcal{H}). \quad (118)$$

This f inherits two properties from μ . First it is nonnegative, $f \geq 0$, since μ takes values in $[0, 1]$. Second, for any orthonormal basis $\{e_k\}$ of $\mathcal{H}$ the projectors $|e_k\rangle\langle e_k|$ are mutually orthogonal and sum to I , so (115)–(116) give

$$\sum_k f(e_k) = \sum_k \mu(|e_k\rangle\langle e_k|) = \mu(I) = 1. \quad (119)$$

Definition C.3 (Frame function). A *frame function of weight W* on $\mathcal{S}(\mathcal{H})$ is a function $f : \mathcal{S}(\mathcal{H}) \rightarrow \mathbb{R}$ such that $\sum_k f(e_k) = W$ for *every* orthonormal basis $\{e_k\}$. We call f *regular* if it is the restriction to the sphere of a (necessarily continuous) quadratic form on $\mathcal{H}$; explicitly, regular means

$$f(v) = \langle v, Av \rangle \quad (120)$$

for some bounded self-adjoint operator A .

Equation (119) says precisely that the f of (118) is a nonnegative frame function of weight 1. The decisive observation – *Gleason’s reduction* – is that knowing f on rays is enough to reconstruct μ on *all* projectors, not just rank-one ones. Indeed, any projector Π of rank r (possibly infinite) is a sum $\Pi = \sum_k |g_k\rangle\langle g_k|$ over an orthonormal basis $\{g_k\}$ of its range, and by (116)

$$\mu(\Pi) = \sum_k \mu(|g_k\rangle\langle g_k|) = \sum_k f(g_k). \quad (121)$$

For (121) to be consistent, the right-hand side must be *independent of the choice* of orthonormal basis $\{g_k\}$ spanning the range of Π ; the frame-function property is exactly what guarantees this. (Take any two bases of the range, extend each to a full orthonormal basis of $\mathcal{H}$ by appending a common basis of the orthogonal complement, and subtract the two copies of (119).) Thus **the entire problem reduces to classifying nonnegative frame functions**: if every such f is regular, i.e. of the form (120), then μ is determined through (121), and we shall see in §C.5 that (121) collapses to the trace formula.

C.3 The core lemma in three real dimensions

The technical core of Gleason’s theorem is the following statement about the ordinary 2-sphere $S^2 \subset \mathbb{R}^3$.

Lemma C.4 (Gleason’s regularity lemma). *Every nonnegative frame function f on the unit sphere S^2 of $\mathbb{R}^3$ is regular: there is a real symmetric 3×3 matrix A with*

$$f(v) = v^T A v, \quad v \in S^2. \quad (122)$$

We explain the argument in stages. It is genuinely the deep part of the theorem, and we follow

the structure of Gleason’s 1957 proof (Gleason, 1957), filling in the geometric intuition; the single hardest analytic step is isolated and its content explained rather than reproduced in every epsilon.

Stage 0: why “quadratic” is the natural target. A frame function on S^2 is constrained on *triples* of mutually orthogonal unit vectors: for any orthonormal frame $\{e_1, e_2, e_3\}$,

$$f(e_1) + f(e_2) + f(e_3) = W. \quad (123)$$

A quadratic form $v^\top A v$ automatically satisfies (123) with $W = \text{Tr } A$, because $\sum_i e_i^\top A e_i = \text{Tr } A$ is basis-independent. So quadratic forms are frame functions; the lemma asserts the converse, that *nothing else* is. The miracle is that a single algebraic identity holding over all orthonormal triples pins down an a priori arbitrary function on the sphere.

Stage 1: parity and the reduced spectrum. Write $g(v) := f(v) - \frac{W}{3}$, a frame function of weight 0. Because antipodal points v and $-v$ define the same rank-one projector, $f(-v) = f(v)$, so g is an even function on the sphere. Hence in a spherical-harmonic expansion

$$g(v) = \sum_{\ell \text{ even}} \sum_{m=-\ell}^{\ell} c_{\ell m} Y_{\ell}^m(v), \quad (124)$$

only even angular momenta $\ell = 0, 2, 4, \dots$ can appear. (We are getting ahead of ourselves by assuming g is regular enough to have such an expansion; Stage 4 supplies the missing continuity. Granting it for now makes the mechanism transparent.) A quadratic form, being a degree-2 homogeneous polynomial restricted to the sphere, has spherical-harmonic content only at $\ell = 0$ and $\ell = 2$. So **Lemma C.4 is equivalent to the claim that all coefficients $c_{\ell m}$ with $\ell \geq 4$ vanish.**

Stage 2: the orthogonality constraint kills $\ell \geq 4$. Fix the third frame vector along an arbitrary direction $n \in S^2$ and let $\{e_1, e_2\}$ be an orthonormal pair in the great circle C_n perpendicular to n . Then (123) reads, for g ,

$$g(e_1) + g(e_2) = -g(n), \quad e_1, e_2 \perp n, \quad e_1 \perp e_2. \quad (125)$$

As we rotate the orthonormal pair $\{e_1, e_2\}$ by an angle ϕ within the circle C_n , the *left side must stay constant* (it always equals $-g(n)$, which does not depend on ϕ). Restrict g to the circle C_n and write its Fourier series in ϕ ; the pair $\{e_1, e_2\}$ sits at angles ϕ and $\phi + \frac{\pi}{2}$, so

$$g(e_1(\phi)) + g(e_2(\phi)) = \sum_p a_p (e^{ip\phi} + e^{ip(\phi+\pi/2)}) = \sum_p a_p (1 + e^{ip\pi/2}) e^{ip\phi}. \quad (126)$$

For this to be independent of ϕ , every Fourier mode with $p \neq 0$ must have vanishing coefficient. The factor $1 + e^{ip\pi/2}$ vanishes for $p \equiv 2 \pmod{4}$, but for $p \equiv 0 \pmod{4}$ it equals $2 \neq 0$, forcing $a_p = 0$ there; modes p odd are already absent because g is even on the circle. The upshot is that on *every* great circle, the restriction of g contains only the Fourier modes $p \in \{-2, 0, 2\}$. A function on S^2 whose restriction to every great circle has angular content bounded by $|p| \leq 2$ is exactly a

combination of $\ell \leq 2$ spherical harmonics: this is the classical statement that the $\ell \geq 4$ harmonics necessarily excite some great-circle Fourier mode with $|p| \geq 4$. Hence $c_{\ell m} = 0$ for $\ell \geq 4$, and g is a pure $\ell = 2$ harmonic plus a constant, i.e. a quadratic form. This is the algebraic engine of the lemma.

Stage 3: positivity packages it as A . A real even degree-2 form on S^2 extends uniquely to a homogeneous quadratic $v^\top A v$ with $A = A^\top$ real symmetric. Nonnegativity $f \geq 0$ means $v^\top A v \geq 0$ for all v , so $A \succeq 0$. This establishes (122).

Stage 4: the regularity (continuity) step. Stages 1–2 silently assumed g admits a spherical-harmonic expansion, i.e. that it is, say, square-integrable or continuous. Gleason’s real work is to prove this regularity *from the frame condition alone*, with no a priori smoothness. The argument shows that a nonnegative frame function is automatically continuous. The mechanism: the frame condition forces a quantitative local oscillation bound. Fix a point u and a nearby point u' ; one constructs auxiliary orthonormal triples that contain u and u' in controlled positions, and the weight-conservation identity (123), combined with $f \geq 0$ (which caps each term from below by 0 and, via the sum, from above by W), bounds $|f(u) - f(u')|$ by a quantity that goes to 0 as $u' \rightarrow u$. Concretely, $0 \leq f \leq W$ everywhere (each value is sandwiched by nonnegativity of the other two terms in a completing triple), so f is bounded; a covering argument over overlapping triples then upgrades boundedness to uniform continuity. Once f is continuous it is in $L^2(S^2)$, the expansion (124) is legitimate, and Stages 1–2 apply rigorously. This continuity-from-additivity step is the single most delicate part of the proof; the reader is referred to Gleason’s original paper (Gleason, 1957) and to the streamlined modern exposition in Busch–Lahti–Pellonpää (2014) for the full epsilon bookkeeping. Its *content*, however, is exactly the intuitive statement just given: orthogonality plus nonnegativity leave no room for wild local behavior.

This completes Lemma C.4. We emphasize where $\dim = 3$ entered: the very existence of nontrivial orthonormal *triples*, used in (123)–(125), requires at least three real dimensions. In $\mathbb{R}^2$ there are only orthonormal *pairs*, the constraint (123) degenerates, and the argument collapses (see §C.6).

C.4 Extension to all of $\mathcal{H}$ and to the complex field

Lemma C.4 concerns a fixed real 3-dimensional space. Three extensions are needed to reach Theorem C.2.

(a) From one $\mathbb{R}^3$ to all of a real $\mathcal{H}$. Let f be a nonnegative frame function on a real Hilbert space of dimension ≥ 3 . For *every* three-dimensional subspace V , the restriction $f|_{S(V)}$ is a nonnegative frame function on the 2-sphere of V (any orthonormal triple of V extends, by appending a fixed basis of $V^\perp$, to a full basis, and the weight is shared), so by Lemma C.4 there is a symmetric $A_V \succeq 0$ with $f|_V = \langle v, A_V v \rangle$. The forms on overlapping subspaces must agree on intersections because they compute the same f ; a standard gluing argument shows the local 3×3 forms patch into a single

bounded self-adjoint $A \succeq 0$ on all of $\mathcal{H}$ with

$$f(v) = \langle v, Av \rangle \quad \text{for all } v \in \mathcal{S}(\mathcal{H}). \quad (127)$$

Boundedness of A follows from $0 \leq f \leq 1$ (so $A \preceq I$), which also makes A a legitimate operator on the whole space, including infinite dimension.

(b) Real to complex. A complex Hilbert space $\mathcal{H}$ of complex dimension d is a real Hilbert space of real dimension $2d$ under the inner product $\text{Re} \langle \cdot, \cdot \rangle$, and orthonormality in the complex sense implies orthonormality in this real sense. So a nonnegative frame function on complex $\mathcal{H}$ is in particular one on the underlying real space, and part (a) yields a real symmetric (i.e. real-linear self-adjoint) A with (127). It remains to check A is *complex*-linear, equivalently that $f(e^{i\alpha}v) = f(v)$ – which holds because $e^{i\alpha}v$ and v span the same complex ray, hence the same rank-one projector, hence the same value of f . Phase invariance forces the real form A to commute with multiplication by i , i.e. to be a complex-linear self-adjoint operator. Thus $f(v) = \langle v, Av \rangle$ with $A = A^\dagger$, $A \succeq 0$ on the complex space.

(c) Separable infinite dimension. The bound $0 \leq A \leq I$ from (a) gives a bounded positive operator. The frame condition $\sum_k f(e_k) = \sum_k \langle e_k, Ae_k \rangle = 1$ says A has finite trace equal to 1; in particular A is trace-class, hence compact, and the spectral/eigenbasis sum converges. Countable additivity (116) is precisely what makes the infinite sums $\sum_k f(e_k)$ converge to the trace rather than to something basis-dependent. No separate argument is needed beyond (a)–(b): once A is bounded and the weight-1 condition pins $\text{Tr } A = 1$, separability guarantees a countable orthonormal basis on which everything is controlled.

C.5 From the quadratic form to the trace formula

Rename $A =: \rho$. By construction $\rho = \rho^\dagger$, $\rho \succeq 0$, and

$$\text{Tr } \rho = \sum_k \langle e_k, \rho e_k \rangle = \sum_k f(e_k) = 1, \quad (128)$$

so ρ is a density operator. Now feed (127) back into the reduction (121). For any projector Π with orthonormal basis $\{g_k\}$ of its range,

$$\mu(\Pi) = \sum_k f(g_k) = \sum_k \langle g_k, \rho g_k \rangle = \sum_k \langle g_k, \rho \Pi g_k \rangle = \text{Tr}(\rho \Pi), \quad (129)$$

where the third equality uses $\Pi g_k = g_k$ (the g_k lie in the range of Π) and the last completes the basis of the range to a basis of $\mathcal{H}$ on which $\rho \Pi$ has vanishing diagonal off the range (since Π kills those vectors). Equation (129) is exactly the asserted form (117), i.e. (114).

Uniqueness. If $\text{Tr}(\rho_1 \Pi) = \text{Tr}(\rho_2 \Pi)$ for every projector Π , then $\text{Tr}((\rho_1 - \rho_2) \Pi) = 0$ for all Π . Taking $\Pi = |v\rangle\langle v|$ gives $\langle v, (\rho_1 - \rho_2)v \rangle = 0$ for every unit vector v ; a self-adjoint operator whose

every diagonal matrix element vanishes is the zero operator (polarize: $\langle u, Bv \rangle$ is recovered from the diagonal of B on $u \pm v$ and $u \pm iv$). Hence $\rho_1 = \rho_2$, proving uniqueness in Theorem C.2. ■

C.6 The dimension-2 gap and its resolution (POVM Gleason)

Theorem C.2 *fails* for $\dim \mathcal{H} = 2$. The obstruction is structural, not technical: on the qubit the projective measurements are the antipodal pairs $\{|n\rangle\langle n|, |-n\rangle\langle -n|\}$ on the Bloch sphere, i.e. each measurement only ever sees a single *axis* $n \in S^2$. The frame condition is then merely

$$f(n) + f(-n) = 1 \quad \text{for every axis } n, \quad (130)$$

with no further linkage between different axes, because in $\mathbb{R}^2$ (the real form is the Bloch sphere) there are no orthonormal *triples* to import constraint (123). Any function on the upper hemisphere whatsoever, extended by $f(-n) = 1 - f(n)$, satisfies (130). The vast majority of these are *not* quadratic forms; e.g. taking f to depend on n through a non-quadratic even function of latitude already violates (122). So at $d = 2$ there are frame functions, hence “probability assignments,” with no density operator behind them, and uniqueness of ρ is lost.

Resolution: enlarge the set of measurements to POVMs. A positive-operator-valued measure (POVM) is a family $\{E_j\}$ of positive operators $E_j \succeq 0$ with $\sum_j E_j = I$; projective measurements are the special case $E_j = \Pi_j$ with $E_j^2 = E_j$ and $E_i E_j = 0$. POVMs are the genuinely available laboratory measurements (any apparatus with ancilla and coarse-graining realizes one, by Naimark dilation). A *generalized probability measure* is now a map ν on *effects* (operators $0 \preceq E \preceq I$) that is additive over every POVM: $\sum_j \nu(E_j) = 1$ whenever $\sum_j E_j = I$, with $\nu(E) \geq 0$.

Theorem C.5 (Busch; Caves–Fuchs–Schack POVM Gleason). *For every dimension $d \geq 2$ (and indeed for separable $\mathcal{H}$), every generalized probability measure ν on the effects of $\mathcal{H}$ has the form*

$$\nu(E) = \text{Tr}(\rho E) \quad (131)$$

for a unique density operator ρ .

The reason the POVM version restores uniqueness at $d = 2$ is that effects come in a *continuum* of *orientations and weights*, not just antipodal projector pairs, and the additivity-over-POVMs condition links them all. Concretely, the key extra structure is that one may now demand additivity for families like $\{tE, (1-t)E, I-E\}$ and for non-orthogonal effect triples; this forces ν to be *linear* on the effect space. Busch’s insight is that additivity over POVMs already forces $\nu(\alpha E_1 + \beta E_2) = \alpha \nu(E_1) + \beta \nu(E_2)$ for the appropriate convex/affine combinations, i.e. ν is the restriction of a linear functional on the real vector space of self-adjoint operators. Every such functional is $E \mapsto \text{Tr}(\rho E)$ for a unique self-adjoint ρ , and the constraints $\nu(E) \in [0, 1]$, $\nu(I) = 1$ force $\rho \succeq 0$, $\text{Tr} \rho = 1$. Crucially this argument *never needs orthonormal triples*, so the dimension hypothesis evaporates and $d = 2$ is included. The proof is markedly shorter and more elementary than Gleason’s original, trading the hard continuity lemma for the richer linear structure of the effect space.

The physical reading is satisfying: the apparent $d = 2$ loophole was an artifact of pretending the only measurements are sharp projective ones. Once the real (POVM) measurement set is admitted – which the operational dictionary of the present paper does – the Born rule is forced uniformly in all dimensions ≥ 2 .

C.7 Worked micro-example: the qubit Born rule

To see the abstract trace formula become the familiar $\cos^2(\theta/2)$, take $\mathcal{H} = \mathbb{C}^2$ for a spin-1/2 particle. Prepare the spin “up along $\hat{z}$ ” state

$$\rho = |\uparrow_z\rangle\langle\uparrow_z| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad (132)$$

and measure spin along the direction $\hat{n}(\theta)$ in the x - z plane making angle θ with $\hat{z}$. The “up along $\hat{n}$ ” eigenstate is

$$|\uparrow_n\rangle = \cos \frac{\theta}{2} |\uparrow_z\rangle + \sin \frac{\theta}{2} |\downarrow_z\rangle = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{pmatrix}, \quad (133)$$

with measurement projector $\Pi_{\uparrow_n} = |\uparrow_n\rangle\langle\uparrow_n|$,

$$\Pi_{\uparrow_n} = \begin{pmatrix} \cos^2(\theta/2) & \cos(\theta/2)\sin(\theta/2) \\ \cos(\theta/2)\sin(\theta/2) & \sin^2(\theta/2) \end{pmatrix}. \quad (134)$$

Gleason’s formula (114) then *predicts* the probability of the outcome “up along $\hat{n}$ ”:

$$p(\uparrow_n) = \text{Tr}(\rho \Pi_{\uparrow_n}) = \text{Tr} \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos^2 \frac{\theta}{2} & \cdots \\ \cdots & \sin^2 \frac{\theta}{2} \end{pmatrix} \right] = \cos^2 \frac{\theta}{2}. \quad (135)$$

This is exactly the textbook spin-1/2 result: the probability of measuring spin-up along an axis tilted by θ from the preparation axis is $\cos^2(\theta/2)$, falling from 1 at $\theta = 0$ (aligned) to $\frac{1}{2}$ at $\theta = \pi/2$ (orthogonal axes) to 0 at $\theta = \pi$ (anti-aligned), consistent with $p(\uparrow_n) + p(\downarrow_n) = \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} = 1$. The square-modulus pattern was never inserted by hand; it is the trace formula evaluated on a rank-one projector, which is what Gleason guarantees as the *only* consistent probability rule. (Strictly, $d = 2$ here is the marginal case covered by the POVM version, Theorem C.5; for any larger system the original Theorem C.2 already applies.)

C.8 Connection to the Born-rule theorem T-A8.3

We can now state precisely how Gleason’s theorem discharges the Born rule in this paper. Theorem T-A8.3 does not postulate $p = |\langle\psi | \phi\rangle|^2$; it shows that form to be *forced*, and Gleason is the lemma that does the forcing once the right premises are in place. Those premises are supplied elsewhere in the framework, not assumed here:

- The state space is a *complex separable Hilbert space* $\mathcal{H}$ – this is the output of T-A8.1, so the hypotheses of Theorem C.2 (complex field, separability, and $\dim \geq 3$ for any nontrivial

system) are met. The complex field in particular feeds the real-to-complex step §C.4(b).

- Measurement outcomes are mutually exclusive, exhaustive projectors (or, more generally, effects), and their probabilities are σ -additive – this is the content of the operational measurement dictionary, which furnishes exactly condition (115)–(116).

Given only these two inputs – a complex Hilbert space and σ -additivity of outcome probabilities over orthogonal (or POVM) decompositions of the identity – Gleason’s theorem leaves *no freedom*: the probability assignment *must* be $\mu(\Pi) = \text{Tr}(\rho\Pi)$, and on pure states $\rho = |\psi\rangle\langle\psi|$ and rank-one outcomes $\Pi = |\phi\rangle\langle\phi|$ this is precisely the square-modulus Born rule

$$\mu(|\phi\rangle\langle\phi|) = \text{Tr}(|\psi\rangle\langle\psi||\phi\rangle\langle\phi|) = |\langle\phi|\psi\rangle|^2. \quad (136)$$

Thus the role of the framework is to *supply the premises*, and the role of Gleason’s theorem is to show those premises are already sufficient: the square-modulus form is the unique solution, not an extra axiom. This is exactly the logical move asserted in T-A8.3, and the present appendix is its complete justification.

Problems and worked solutions. C.2.1. Build a frame function by hand and verify its additivity on arbitrary bases.

Solution. Step 1: take the candidate weight $f(v) = \langle v, Av \rangle$ with $A = \text{diag}(0.5, 0.3, 0.2)$ on $\mathbb{R}^3$ — positive, unit trace. Step 2: for the standard basis, $f(e_1) + f(e_2) + f(e_3) = 0.5 + 0.3 + 0.2 = 1$; for any other orthonormal basis $\{v_i\}$, additivity holds because $\sum_i \langle v_i, Av_i \rangle = \text{Tr} A = 1$ — the trace is basis-independent, so the frame condition (119) is satisfied with weight $W = 1$ for *every* basis at once (spot-check a rotated basis numerically: the sum returns 1.000 to machine precision). Step 3: the measure it induces on projectors, $\mu(P) = \sum_{i \in P} f(v_i)$ (121), is exactly $\text{Tr}(AP)$ — so quadratic-form frame functions are automatically Gleason measures; the theorem’s force is the converse, proved in the surrounding sections: *nothing else* is. Physics intuition: additivity over bases looks innocent — one number per direction, summing to one — but it is secretly a statement about *every* way of cutting the sphere into orthogonal triples simultaneously, and that simultaneous consistency is the vice that squeezes all freedom out of quantum probability.

C.5.1. Walk one measure from quadratic form to trace formula.

Solution. Step 1: with the A of C.2.1, evaluate the induced measure on the two-dimensional subspace P_{12} spanned by e_1, e_2 : $\mu(P_{12}) = f(e_1) + f(e_2) = 0.8$. Step 2: confirm against the trace form (129): $\text{Tr}(AP_{12}) = A_{11} + A_{22} = 0.8$ — and, per (128), A is positive with unit trace, i.e. a legitimate density operator ρ . Step 3: note what just happened: a probability assignment defined only on *subspaces* (no amplitudes, no wavefunction anywhere) has been repackaged as a state; this is the direction of travel of the whole theorem — states are not the input of quantum probability but its unique bookkeeping. Physics intuition: the density operator is Gleason’s receipt, not quantum mechanics’ postulate — given any lawful way of assigning odds to questions, the trace formula is the only filing system that can store it.

C.6.1. Exhibit the dimension-2 escape, and close it with the POVM extension.

Solution. Step 1: in $d = 2$ every orthonormal basis is an antipodal pair $\{v, v^\perp\}$, so the frame condition (130) demands only $f(v) + f(v^\perp) = 1$ — different bases never share a vector, so the constraints never interlock. Step 2: construct a lawless example: let $f(v) = \frac{1}{2}(1 + \text{sgn}(n_z(v)) n_z(v)^2)$ or any assignment with $f(v) + f(-v)$ -symmetry rigged pairwise — it satisfies additivity yet is no $\text{Tr}(\rho\Pi)$ for any ρ : Gleason’s theorem is genuinely false for projective measurements on a single qubit. Step 3: the physical repair (131): real instruments implement *effects* $0 \leq E \leq \mathbb{I}$ (POVMs), which come in continuous families that do interlock; Busch’s theorem then forces $\nu(E) = \text{Tr}(\rho E)$ with a unique ρ even at $d = 2$ — and the framework’s instruments are POVM-valued channels, so the escape hatch is closed where the physics actually lives. Physics intuition: the qubit’s loophole is a geometry accident — antipodal pairs are too few handshakes to force agreement; let the measurements tilt and weight continuously, as laboratory instruments do, and the handshakes multiply until only the Born rule survives.

C.7.1. Run the qubit Born rule end to end at a definite angle.

Solution. Step 1: prepare $\rho = |\uparrow_z\rangle\langle\uparrow_z|$ and measure spin along the axis $\hat{n}$ at polar angle θ ; the projector is (134). Step 2: the trace formula (135) gives $p(\uparrow_n) = \cos^2(\theta/2)$: at $\theta = 60^\circ$, $p = \cos^2 30^\circ = 3/4$; at $\theta = 90^\circ$, $p = 1/2$; at 180° , $p = 0$. Step 3: observe the division of labor: the *form* of these numbers is Gleason (via Busch at $d = 2$), while their *physical realization* — why a silver atom lands in one spot with these frequencies — is the registration story of Part V; the appendix supplies the arithmetic that the worked example animates. Physics intuition: $\cos^2(\theta/2)$ is the most-verified formula in quantum physics, and its origin here is bookkeeping rigidity, not wave mysticism — half the angle because a spinor turns half as fast as the laboratory, squared because probability is the only additive filing system left.

C.8.1. State exactly what Gleason proves, and what the framework adds.

Solution. Step 1: what the theorem proves ((117)): *if* a noncontextual, countably additive probability assignment on the subspace lattice exists in $d \geq 3$, it is $\text{Tr}(\rho \cdot)$ for a unique ρ — uniqueness, not existence. Step 2: what it does not prove: that nature assigns probabilities to registered subspaces at all, that the assignment is noncontextual, or anything in $d = 2$ without the POVM upgrade. Step 3: the framework’s contribution (T-A8.3): the GDOS registration *is* a probability assignment on registered outcomes, and its noncontextuality is inherited from the projection’s geometry — so the premises are supplied by physics and the conclusion by Gleason; neither alone is the Born rule, and the honesty register of the main text grades exactly this division. Physics intuition: Gleason is a uniqueness trap waiting a century for something to fall into it — the projection walks in, and the Born rule is the sound of the trap closing.

Take-away. *Standard account:* the Born rule is a postulate — stated, used, never explained. *What it left open:* why $|\psi|^2$ and not anything else, and what breaks in dimension two. *The bulk reading:* the registration law of the projection must assign noncontextual probabilities to registered subspaces; Gleason’s theorem then forces the trace form with a unique ρ — the Born rule as the only arithmetic the boundary could have had (T-A8.3), with the $d = 2$ escape closed at the POVM level where the physical instruments live.

D The GKSL theorem: statement and derivation

What this appendix teaches. The most general law an open quantum system can obey: the Gorini–Kossakowski–Sudarshan–Lindblad theorem, derived completely — complete positivity and the Kraus/Stinespring representation, the semigroup hypothesis, the Choi matrix, and the emergence of the canonical generator $\dot{\rho} = -i[H, \rho] + \sum_k \gamma_k (L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\})$ with non-negative rates. Every algebraic step is shown at first-graduate-course level. For this cohort the theorem is load-bearing: the boundary dynamics of the OGGEF projection is an open-system evolution by construction, and GKSL is the unique form such a law can take — the bulk is deterministic, and *this* is what determinism looks like after the trace.

Read alongside: Lindblad (1976) and Gorini (1976) for the twin originals; Breuer (2002) for the standard text on open quantum systems.

This appendix gives a complete, self-contained derivation of the Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) theorem, which characterizes the most general generator of a quantum dynamical semigroup. The treatment assumes only a first graduate course in quantum mechanics and linear algebra; every algebraic step is shown. We work throughout in a finite-dimensional Hilbert space $\mathcal{H} \cong \mathbb{C}^d$, write ρ for a density operator (a positive semidefinite, unit-trace, Hermitian operator on $\mathcal{H}$), and use Tr for the trace. We keep $\hbar$ explicit in the Hamiltonian term and set the dissipative rates with the dimensions of inverse time.

D.1 Setup: quantum dynamical semigroups and complete positivity

States and the need for maps, not vectors. A closed quantum system in a pure state evolves by the Schrodinger equation, $i\hbar \partial_t |\psi\rangle = H|\psi\rangle$, equivalently by the unitary group $|\psi(t)\rangle = U_t |\psi(0)\rangle$ with $U_t = e^{-iHt/\hbar}$. An *open* system—one in contact with an environment, or one whose fine degrees of freedom we have chosen to coarse-grain away—generically does not stay pure. Its state must be described by a density operator ρ , and its evolution by a linear map on operators,

$$\rho(0) \longmapsto \rho(t) = \Phi_t[\rho(0)], \quad (137)$$

where each Φ_t is a *quantum channel*. For Φ_t to map density operators to density operators for every input, it must be (i) *trace preserving* (TP), $\text{Tr} \Phi_t[\rho] = \text{Tr} \rho$, so probabilities stay normalized, and (ii) *positive*, mapping positive operators to positive operators, so that $\rho(t)$ remains a valid (nonnegative-probability) state.

Why positivity is not enough: complete positivity. Positivity alone is too weak. The system may be entangled with an ancilla (a reference system R of arbitrary dimension n) that does not evolve. The joint state lives on $\mathcal{H} \otimes \mathbb{C}^n$, and the physical evolution acts as $\Phi_t \otimes \text{id}_n$. We require the joint output to remain a valid state for *every* n . A map Φ is *completely positive* (CP) if $\Phi \otimes \text{id}_n$ is positive for all $n \geq 1$. Complete positivity is strictly stronger than positivity: the transpose map is positive but not completely positive, and applying it to half of a maximally entangled pair produces an operator with a negative eigenvalue. A physically admissible channel must therefore be CPTP (completely positive and trace preserving).

The Choi–Kraus representation. A finite-dimensional map Φ is completely positive if and only if it admits a *Kraus representation*

$$\Phi[\rho] = \sum_k K_k \rho K_k^\dagger, \quad (138)$$

for some finite set of operators $K_k \in B(\mathcal{H})$; it is in addition trace preserving if and only if the Kraus operators satisfy the completeness (resolution-of-identity) relation

$$\sum_k K_k^\dagger K_k = \mathbf{1}. \quad (139)$$

The “only if” direction is Choi’s theorem: complete positivity of Φ is equivalent to positivity of the Choi matrix $C(\Phi) = (\Phi \otimes \text{id})[|\Omega\rangle\langle\Omega|]$, where $|\Omega\rangle = \sum_i |i\rangle \otimes |i\rangle$ is the unnormalized maximally entangled vector; the Kraus operators are read off from the eigenvectors of $C(\Phi)$. The “if” direction is a direct check: each term $K_k \rho K_k^\dagger$ preserves positivity, the sum does too, and (139) guarantees $\text{Tr } \Phi[\rho] = \sum_k \text{Tr}(K_k^\dagger K_k \rho) = \text{Tr}(\mathbf{1} \rho) = \text{Tr } \rho$.

The semigroup and Markovianity. A closed system has a one-parameter *group* U_t with $U_{t+s} = U_t U_s$ and an inverse U_{-t} . An open system that loses information to its environment is generically irreversible: it has no inverse channel. What survives of the group law is the *semigroup* property

$$\Phi_{t+s} = \Phi_t \circ \Phi_s, \quad t, s \geq 0, \quad \Phi_0 = \text{id}. \quad (140)$$

Physically, (140) is the statement of *Markovianity*: the future depends on the present state alone, not on the past history. The environment is assumed to forget the system’s influence on a timescale short compared to the system’s own evolution (a memoryless bath — the Port-Freshness Clause of the GDOS companion (Arisaka, 2026, A5-GDOS)). Combined with strong continuity at $t = 0$, $\lim_{t \rightarrow 0^+} \Phi_t[\rho] = \rho$, the semigroup is generated by a single time-independent superoperator L ,

$$L = \lim_{t \rightarrow 0^+} \frac{\Phi_t - \text{id}}{t}, \quad \Phi_t = e^{tL}, \quad \frac{d\rho}{dt} = L[\rho]. \quad (141)$$

The central question is: *which* superoperators L generate a legitimate CPTP semigroup? GKSL answers it completely.

D.2 Statement of the GKSL theorem

Theorem (GKSL, 1976). Let $\{\Phi_t\}_{t \geq 0}$ be a quantum dynamical semigroup on a d -dimensional Hilbert space $\mathcal{H}$: a strongly continuous, one-parameter family of CPTP maps satisfying (140). Then its generator L in (141) has the form

$$\frac{d\rho}{dt} = -\frac{i}{\hbar} [H, \rho] + \sum_k \gamma_k \left(L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\} \right), \quad (142)$$

with $H = H^\dagger$ a Hermitian operator (the effective Hamiltonian), rates $\gamma_k \geq 0$, and operators $L_k \in B(\mathcal{H})$ (the *Lindblad* or *jump* operators). Conversely, every L of the form (142) with $\gamma_k \geq 0$ generates a CPTP semigroup.

The double-bracket notation is the anticommutator, $\{A, B\} = AB + BA$, and the single bracket is the commutator, $[A, B] = AB - BA$. The first term on the right of (142) is the unitary (von Neumann) part; the sum is the *dissipator*, denoted $\mathcal{D}[\rho]$. The decomposition (142) is unique only up to a gauge freedom discussed in §D.4.

D.3 Derivation via the Kossakowski matrix

We now derive (142) algebraically. The strategy is: expand the generator in a fixed operator basis; show that trace preservation and Hermiticity constrain the coefficients into a single Hermitian *Kossakowski matrix*; show that complete positivity is equivalent to that matrix being positive semidefinite; then diagonalize it to read off the Lindblad operators.

Step 1: a complete operator basis. The space $B(\mathcal{H})$ of operators on $\mathcal{H} \cong \mathbb{C}^d$ is itself a d^2 -dimensional complex vector space, with the Hilbert–Schmidt inner product $\langle A, B \rangle = \text{Tr}(A^\dagger B)$. Choose an orthonormal basis $\{F_m\}_{m=0}^{d^2-1}$,

$$\text{Tr}(F_m^\dagger F_n) = \delta_{mn}, \quad (143)$$

in which we single out the (normalized) identity

$$F_0 = \frac{1}{\sqrt{d}} \mathbf{1}, \quad \text{so that} \quad \text{Tr}(F_m^\dagger F_0) = \frac{1}{\sqrt{d}} \text{Tr} F_m = 0 \quad (m \neq 0). \quad (144)$$

Thus the remaining $d^2 - 1$ basis operators $F_1, \dots, F_{d^2-1}$ are all *traceless*. (For a qubit, $d = 2$, one may take $F_0 = \mathbf{1}/\sqrt{2}$ and $F_{1,2,3} = \sigma_{x,y,z}/\sqrt{2}$.)

Step 2: the generator in Choi–Kraus form for small t . By Choi’s theorem the channel for a small time step t has a Kraus form $\Phi_t[\rho] = \sum_k K_k(t) \rho K_k(t)^\dagger$. Expand each Kraus operator in the basis (143), $K_k(t) = \sum_m c_{km}(t) F_m$, and substitute:

$$\Phi_t[\rho] = \sum_{m,n=0}^{d^2-1} \left(\underbrace{\sum_k c_{km}(t) c_{kn}^*(t)}_{\equiv \chi_{mn}(t)} \right) F_m \rho F_n^\dagger = \sum_{m,n} \chi_{mn}(t) F_m \rho F_n^\dagger. \quad (145)$$

The matrix $\chi(t)$ with entries $\chi_{mn}(t) = \sum_k c_{km}(t) c_{kn}^*(t)$ is, by construction, a Gram matrix of the coefficient vectors, hence *Hermitian and positive semidefinite* for every t : for any complex vector v , $\sum_{mn} v_m^* \chi_{mn} v_n = \sum_k |\sum_m v_m^* c_{km}|^2 \geq 0$. This positivity of $\chi(t)$ is exactly the algebraic content of complete positivity (138); it is the fact we will propagate to the generator.

Step 3: take the time derivative. At $t = 0$ the channel is the identity, $\Phi_0 = \text{id}$, which in the form (145) means $\chi_{00}(0) = d$ and all other $\chi_{mn}(0) = 0$ (because $F_0 \rho F_0^\dagger = \rho/d$). Expanding to first

order, $\chi_{mn}(t) = \chi_{mn}(0) + t \dot{\chi}_{mn}(0) + O(t^2)$, and using (141),

$$L[\rho] = \left. \frac{d}{dt} \right|_0 \Phi_t[\rho] = \dot{\chi}_{00}(0) \frac{\rho}{d} + \frac{1}{\sqrt{d}} \sum_{m \geq 1} \left(\dot{\chi}_{m0}(0) F_m \rho + \dot{\chi}_{0m}(0) \rho F_m^\dagger \right) + \sum_{m,n \geq 1} \dot{\chi}_{mn}(0) F_m \rho F_n^\dagger, \quad (146)$$

where we used $F_0 = \mathbf{1}/\sqrt{d}$. Introduce the abbreviations

$$a_{mn} \equiv \dot{\chi}_{mn}(0) \quad (m, n \geq 1), \quad b \equiv \frac{1}{\sqrt{d}} \sum_{m \geq 1} \dot{\chi}_{m0}(0) F_m, \quad a_{00} \equiv \dot{\chi}_{00}(0). \quad (147)$$

The matrix $a = (a_{mn})_{m,n \geq 1}$ is Hermitian ($a_{mn} = a_{nm}^*$, inherited from $\chi = \chi^\dagger$); b is an operator; a_{00} is real. Equation (146) becomes

$$L[\rho] = \frac{a_{00}}{d} \rho + (b \rho + \rho b^\dagger) + \sum_{m,n \geq 1} a_{mn} F_m \rho F_n^\dagger. \quad (148)$$

Step 4: impose trace preservation. The semigroup is trace preserving for all t , so $\text{Tr } L[\rho] = 0$ for every ρ . Take the trace of (148), using cyclicity $\text{Tr}(F_m \rho F_n^\dagger) = \text{Tr}(F_n^\dagger F_m \rho)$:

$$0 = \text{Tr } L[\rho] = \text{Tr} \left[\left(\frac{a_{00}}{d} \mathbf{1} + b + b^\dagger + \sum_{m,n \geq 1} a_{mn} F_n^\dagger F_m \right) \rho \right] \quad \text{for all } \rho. \quad (149)$$

Since this holds for all ρ , the bracketed operator must vanish:

$$\frac{a_{00}}{d} \mathbf{1} + b + b^\dagger + \sum_{m,n \geq 1} a_{mn} F_n^\dagger F_m = 0. \quad (150)$$

Define the Hermitian operator $G \equiv \frac{1}{2} \sum_{m,n \geq 1} a_{mn} F_n^\dagger F_m + \frac{a_{00}}{2d} \mathbf{1}$. Then (150) reads $b + b^\dagger = -2G$, i.e. the Hermitian part of b is fixed: $\frac{1}{2}(b + b^\dagger) = -G$. Split b into Hermitian and anti-Hermitian parts by writing

$$b = -G - \frac{i}{\hbar} H, \quad H \equiv \frac{i\hbar}{2} (b - b^\dagger) = H^\dagger, \quad (151)$$

which defines a Hermitian operator H (the factor $-i/\hbar$ is a convention chosen so that H has units of energy and the unitary term matches the Schrodinger form).

Step 5: assemble the standard form. Substitute $b = -G - (i/\hbar)H$ into (148). The pieces involving G and a_{00} combine into the anticommutator structure. Explicitly, $b\rho + \rho b^\dagger = -\frac{i}{\hbar}[H, \rho] - (G\rho + \rho G)$, and writing $G = \frac{1}{2} \sum_{m,n \geq 1} a_{mn} F_n^\dagger F_m + (a_{00}/2d)\mathbf{1}$ gives

$$L[\rho] = -\frac{i}{\hbar}[H, \rho] + \sum_{m,n \geq 1} a_{mn} \left(F_m \rho F_n^\dagger - \frac{1}{2} \{F_n^\dagger F_m, \rho\} \right). \quad (152)$$

The $a_{00}\mathbf{1}/d$ contributions from the ρ/d term and from G cancel identically (they enter as $+a_{00}\rho/d$ from the first term of (148) and as $-\frac{1}{2}\{(a_{00}/d)\mathbf{1}, \rho\} = -a_{00}\rho/d$ from G). This is the GKSL form in its *diagonal-free* (Kossakowski) version: a unitary part plus a dissipator built from the *Kossakowski*

matrix $a = (a_{mn})_{m,n \geq 1}$, which is Hermitian and—this is the key fact—positive semidefinite.

Step 6: complete positivity $\iff a \succeq 0$. We must show that the Kossakowski matrix is positive semidefinite, $a \succeq 0$, and that this is precisely the condition for the semigroup e^{tL} to be completely positive. One direction is immediate: $a_{mn} = \dot{\chi}_{mn}(0)$ is the time derivative of the positive-semidefinite Choi block $\chi(t)$ restricted to the traceless sector, evaluated where the off-diagonal entries start from zero; positivity of $\chi(t)$ for $t > 0$ together with $\chi_{mn}(0) = 0$ ($m, n \geq 1$) forces the derivative matrix to be positive semidefinite. (Concretely: for the principal submatrix on indices $m, n \geq 1$, the $t = 0$ value vanishes and $\chi(t) \succeq 0$ for $t \geq 0$ implies the right-derivative $\succeq 0$.) The converse—that $a \succeq 0$ guarantees $\Phi_t = e^{tL}$ is CPTP for all $t \geq 0$ —follows once we exhibit the Kraus form below and note that the construction holds at every t .

Step 7: diagonalize to obtain the Lindblad operators. Because a is Hermitian and positive semidefinite, the spectral theorem gives a unitary $U = (u_{mk})$ and nonnegative eigenvalues $\gamma_k \geq 0$ with

$$a_{mn} = \sum_k u_{mk} \gamma_k u_{nk}^*. \quad (153)$$

Define new operators by the same unitary rotation of the basis,

$$L_k \equiv \sum_{m \geq 1} u_{mk} F_m. \quad (154)$$

Substituting (153) into (152) and collecting the rotated operators,

$$\sum_{m,n \geq 1} a_{mn} F_m \rho F_n^\dagger = \sum_k \gamma_k \left(\sum_m u_{mk} F_m \right) \rho \left(\sum_n u_{nk} F_n \right)^\dagger = \sum_k \gamma_k L_k \rho L_k^\dagger, \quad (155)$$

$$\sum_{m,n \geq 1} a_{mn} F_n^\dagger F_m = \sum_k \gamma_k L_k^\dagger L_k. \quad (156)$$

Therefore (152) becomes exactly the GKSL master equation (142),

$$\frac{d\rho}{dt} = -\frac{i}{\hbar} [H, \rho] + \sum_k \gamma_k \left(L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\} \right), \quad \gamma_k \geq 0. \quad (157)$$

This completes the derivation: *every* generator of a CPTP semigroup has this form, and the nonnegativity $\gamma_k \geq 0$ is the algebraic fingerprint of complete positivity.

Step 8: the converse and an explicit Kraus check. For the converse, expand the short-time map generated by (157), $\Phi_t = \mathbf{1} + tL + O(t^2)$, into Kraus form. Define

$$K_0 = \mathbf{1} + \frac{t}{\hbar} \left(-iH - \frac{\hbar}{2} \sum_k \gamma_k L_k^\dagger L_k \right), \quad K_k = \sqrt{\gamma_k t} L_k \quad (k \geq 1). \quad (158)$$

Then $\sum_{k \geq 0} K_k \rho K_k^\dagger = \rho + tL[\rho] + O(t^2)$, reproducing the generator, while $\sum_k K_k^\dagger K_k = \mathbf{1} + O(t^2)$ confirms trace preservation to first order. Because each $K_k \rho K_k^\dagger$ is manifestly completely positive

and the construction holds for arbitrarily small t , the semigroup law $\Phi_t = (\Phi_{t/N})^N$ assembles a completely positive Φ_t for all finite t . This closes the equivalence.

D.4 The gauge (non-uniqueness) of the GKSL form

The decomposition (157) is not literally unique, but its non-uniqueness is fully understood and harmless. Two freedoms exist. First, the dissipator is invariant under a unitary mixing of the Lindblad operators, $L_k \rightarrow \sum_j v_{kj} L_j$ with v unitary (this just re-diagonalizes the same Kossakowski matrix). Second, one may shift each Lindblad operator by a multiple of the identity, $L_k \rightarrow L_k + c_k \mathbf{1}$, and simultaneously add a compensating term to H ; the dissipator changes only by a commutator that can be absorbed into the Hamiltonian. Fixing the L_k to be *traceless* (as our basis choice $F_0 = \mathbf{1}/\sqrt{d}$ does, since each L_k is a combination of the traceless $F_{m \geq 1}$) removes the second freedom and makes H and the spectrum $\{\gamma_k\}$ canonical. Thus, up to this gauge, GKSL is unique: *any* generator with a different structure—for instance one with a single term $\alpha L \rho L^\dagger$ and no anticommutator, or with $\alpha < 0$ —necessarily violates either trace preservation (the missing anticommutator) or complete positivity (a negative Kossakowski eigenvalue), or both.

D.5 Microscopic derivation: weak coupling, Born–Markov, and the secular approximation

The Kossakowski route shows GKSL is the *only* admissible Markovian form. A complementary, physically transparent derivation starts from a concrete system–bath Hamiltonian and shows how the GKSL structure *emerges*—with the rates γ_k fixed by the bath spectrum—once a sequence of controlled approximations is made.

The model. Let the total Hamiltonian be

$$H_{\text{tot}} = H_S + H_B + H_{SB}, \quad H_{SB} = \sum_{\alpha} A_{\alpha} \otimes B_{\alpha}, \quad (159)$$

with H_S the system Hamiltonian, H_B the bath (environment) Hamiltonian, and H_{SB} a weak coupling written as a sum of products of system operators $A_{\alpha} = A_{\alpha}^\dagger$ and bath operators $B_{\alpha} = B_{\alpha}^\dagger$. We work in the interaction picture, where operators carry tildes, $\tilde{A}_{\alpha}(t) = e^{iH_S t/\hbar} A_{\alpha} e^{-iH_S t/\hbar}$, and the von Neumann equation for the joint state $\tilde{\rho}_{\text{tot}}$ is

$$\frac{d\tilde{\rho}_{\text{tot}}}{dt} = -\frac{i}{\hbar} [\tilde{H}_{SB}(t), \tilde{\rho}_{\text{tot}}(t)]. \quad (160)$$

Second-order (Born) integro-differential equation. Integrate (160) once, $\tilde{\rho}_{\text{tot}}(t) = \tilde{\rho}_{\text{tot}}(0) - \frac{i}{\hbar} \int_0^t ds [\tilde{H}_{SB}(s), \tilde{\rho}_{\text{tot}}(s)]$, substitute back into (160), and trace out the bath ($\rho_S = \text{Tr}_B \rho_{\text{tot}}$). Assuming the coupling has zero bath average ($\text{Tr}_B[B_{\alpha} \rho_B] = 0$, which can always be arranged by absorbing the mean into H_S), the first-order term vanishes and

$$\frac{d\tilde{\rho}_S}{dt} = -\frac{1}{\hbar^2} \int_0^t ds \text{Tr}_B [\tilde{H}_{SB}(t), [\tilde{H}_{SB}(s), \tilde{\rho}_{\text{tot}}(s)]]]. \quad (161)$$

The Born approximation (weak coupling). For weak coupling the bath is barely perturbed and the system–bath state stays approximately factorized at all times, $\tilde{\rho}_{\text{tot}}(s) \approx \tilde{\rho}_S(s) \otimes \rho_B$, with ρ_B a stationary bath state (e.g. thermal). This is the *Born approximation*; it is the leading term of a systematic expansion in the coupling strength and holds when the coupling energy is small compared with the bath correlation rate. Inserting it,

$$\frac{d\tilde{\rho}_S}{dt} = -\frac{1}{\hbar^2} \int_0^t ds \text{Tr}_B \left[\tilde{H}_{SB}(t), [\tilde{H}_{SB}(s), \tilde{\rho}_S(s) \otimes \rho_B] \right]. \quad (162)$$

Carrying out the bath trace produces bath two-point correlation functions

$$C_{\alpha\beta}(t-s) = \text{Tr}_B \left[\tilde{B}_\alpha(t) \tilde{B}_\beta(s) \rho_B \right],$$

which depend only on the time difference because ρ_B is stationary.

The Markov approximation (memoryless bath). The bath correlations $C_{\alpha\beta}(\tau)$ decay on a short *bath correlation time* τ_B — in cohort terms, the Port-Freshness Clause of the GDOS companion. If τ_B is much shorter than the timescale τ_S on which ρ_S evolves, we may (i) replace $\tilde{\rho}_S(s) \rightarrow \tilde{\rho}_S(t)$ in the integrand (the state is essentially constant over the memory window) and (ii) extend the integration limit to infinity, $\int_0^t \rightarrow \int_0^\infty$. This is the *Markov approximation*; together with the Born step it yields the *Redfield equation*,

$$\frac{d\tilde{\rho}_S}{dt} = -\frac{1}{\hbar^2} \int_0^\infty d\tau \text{Tr}_B \left[\tilde{H}_{SB}(t), [\tilde{H}_{SB}(t-\tau), \tilde{\rho}_S(t) \otimes \rho_B] \right]. \quad (163)$$

The Redfield equation is local in time and Markovian, but—crucially—it is *not yet* of GKSL form: it can violate complete positivity and produce (slightly) negative probabilities. The missing ingredient is the secular approximation.

The secular / rotating-wave approximation. Decompose each system coupling operator into eigenoperators of H_S . If $H_S = \sum_\epsilon \epsilon \Pi_\epsilon$ (with Π_ϵ the projector onto the energy- ϵ eigenspace), define

$$A_\alpha(\omega) \equiv \sum_{\epsilon' - \epsilon = \hbar\omega} \Pi_\epsilon A_\alpha \Pi_{\epsilon'}, \quad \tilde{A}_\alpha(\omega, t) = e^{-i\omega t} A_\alpha(\omega), \quad (164)$$

so the sum over *Bohr frequencies* ω (energy gaps divided by $\hbar$) diagonalizes the free interaction-picture evolution. In the Redfield equation each term carries a phase $e^{i(\omega' - \omega)t}$. Terms with $\omega' \neq \omega$ oscillate rapidly compared with the slow dissipative dynamics and average to zero over the relevant timescale; dropping them is the *secular*, or rotating-wave, approximation, valid when the level spacings satisfy $|\omega' - \omega|^{-1} \ll \tau_S$ (well-resolved Bohr frequencies). Keeping only $\omega' = \omega$ gives

$$\frac{d\tilde{\rho}_S}{dt} = \sum_\omega \sum_{\alpha\beta} \gamma_{\alpha\beta}(\omega) \left(A_\beta(\omega) \tilde{\rho}_S A_\alpha^\dagger(\omega) - \frac{1}{2} \{ A_\alpha^\dagger(\omega) A_\beta(\omega), \tilde{\rho}_S \} \right) - \frac{i}{\hbar} [H_{LS}, \tilde{\rho}_S], \quad (165)$$

where H_{LS} is the *Lamb shift* (a coupling-induced renormalization of H_S that commutes with H_S), and the rate matrix is the one-sided Fourier transform of the bath correlations,

$$\gamma_{\alpha\beta}(\omega) = \frac{1}{\hbar^2} \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} C_{\alpha\beta}(\tau) = \frac{1}{\hbar^2} \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \text{Tr}_B[\tilde{B}_\alpha(\tau) B_\beta \rho_B]. \quad (166)$$

For each fixed ω the matrix $\gamma_{\alpha\beta}(\omega)$ is positive semidefinite—this is a theorem (Bochner’s theorem) about the Fourier transform of a correlation function, and it is precisely the Kossakowski-positivity condition of §D.3. The secular approximation yields a generator that is manifestly of GKSL form, and hence completely positive: it converts the not-quite-CP Redfield generator into a genuine GKSL generator. (It is *sufficient* for complete positivity, not necessary — non-secular CP-preserving schemes also exist — but it is the standard and simplest route.) Diagonalizing $\gamma_{\alpha\beta}(\omega) \rightarrow \gamma_k$ as in (153)–(154) recovers exactly the canonical form (157), with the rates given by the **Fermi golden rule**: the rate of a given quantum jump is set by the bath spectral density evaluated at the corresponding Bohr frequency,

$$\gamma(\omega) \propto (\text{bath spectral density at } \omega). \quad (167)$$

Transforming back from the interaction picture reinstates the bare Hamiltonian, so that the full generator is $H = H_S + H_{LS}$ together with the dissipator built from the $A_\alpha(\omega)$.

Summary of the approximation chain. The three approximations have distinct physical content and validity windows: *Born* (weak coupling, factorized state) needs small coupling; *Markov* (memoryless bath; the Port-Freshness Clause of the GDOS companion) needs $\tau_B \ll \tau_S$; *secular* (drop fast-oscillating cross terms) needs well-resolved Bohr frequencies, $|\omega - \omega'|^{-1} \ll \tau_S$. Only with all three does the microscopic dynamics collapse onto the unique GKSL form, with positivity guaranteed.

D.6 Action of the dissipator: decoherence

The dissipator $\mathcal{D}[\rho] = \sum_k \gamma_k (L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\})$ is what distinguishes open from closed dynamics. To see its physical effect, write ρ in a preferred basis $\{|n\rangle\}$ (for instance the eigenbasis of H or of a L_k) and look at matrix elements $\rho_{nm} = \langle n | \rho | m \rangle$. The *diagonal* elements ρ_{nn} are populations (classical probabilities); the *off-diagonal* elements ρ_{nm} ($n \neq m$) are *coherences*, the carriers of quantum interference. A generic dissipator drives the coherences to zero while merely redistributing the populations: the off-diagonal elements decay exponentially, $\rho_{nm}(t) \sim e^{-\Gamma_{nm}t} \rho_{nm}(0)$, with a *decoherence rate* Γ_{nm} set by the γ_k and the L_k matrix elements. This selective suppression of coherences—decoherence—is how the GKSL semigroup encodes the emergence of effectively classical, probabilistic behavior from unitary quantum mechanics, without ever violating trace or positivity.

D.7 Worked micro-example: the single-qubit dephasing channel

Take $\mathcal{H} = \mathbb{C}^2$ (a qubit), set $H = 0$ for clarity, and use a single Lindblad operator $L = \sigma_z$ with rate γ . The GKSL master equation (157) reduces to the pure dephasing generator

$$\frac{d\rho}{dt} = \gamma \left(\sigma_z \rho \sigma_z - \frac{1}{2} \{ \sigma_z^\dagger \sigma_z, \rho \} \right) = \gamma (\sigma_z \rho \sigma_z - \rho), \quad (168)$$

where we used $\sigma_z^\dagger = \sigma_z$ and $\sigma_z^2 = \mathbf{1}$, so $\frac{1}{2} \{ \sigma_z^2, \rho \} = \frac{1}{2} \{ \mathbf{1}, \rho \} = \rho$. Parametrize the qubit state as

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (169)$$

A direct multiplication gives

$$\sigma_z \rho \sigma_z = \begin{pmatrix} \rho_{00} & -\rho_{01} \\ -\rho_{10} & \rho_{11} \end{pmatrix}, \quad (170)$$

i.e. conjugation by σ_z flips the sign of the off-diagonal entries and leaves the diagonal untouched. Substituting into (168),

$$\frac{d}{dt} \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} = \gamma \begin{pmatrix} \rho_{00} - \rho_{00} & -\rho_{01} - \rho_{01} \\ -\rho_{10} - \rho_{10} & \rho_{11} - \rho_{11} \end{pmatrix} = \begin{pmatrix} 0 & -2\gamma \rho_{01} \\ -2\gamma \rho_{10} & 0 \end{pmatrix}. \quad (171)$$

Hence the populations are frozen and the coherences decay exponentially:

$$\dot{\rho}_{00} = \dot{\rho}_{11} = 0, \quad \dot{\rho}_{01} = -2\gamma \rho_{01} \implies \boxed{\rho_{01}(t) = \rho_{01}(0) e^{-2\gamma t}} \quad (172)$$

and likewise $\rho_{10}(t) = \rho_{10}(0) e^{-2\gamma t}$. Writing the decoherence rate as $\Gamma = 2\gamma$, the solution is

$$\rho(t) = \begin{pmatrix} \rho_{00}(0) & \rho_{01}(0) e^{-\Gamma t} \\ \rho_{10}(0) e^{-\Gamma t} & \rho_{11}(0) \end{pmatrix}. \quad (173)$$

The state flows from a coherent superposition toward the diagonal (classical) mixture $\text{diag}(\rho_{00}, \rho_{11})$ on the timescale Γ^{-1} , with no loss of trace ($\rho_{00} + \rho_{11}$ is conserved) and with positivity preserved throughout (the eigenvalues stay in $[0, 1]$). This is the canonical microscopic model of decoherence and the simplest nontrivial instance of GKSL dynamics.

D.8 Connection to the GG framework

The GKSL semigroup is the precise mathematical setting in which the paper's quantum dynamics live. Three identifications close the loop. First, the closed-system limit: as all rates $\gamma_k \rightarrow 0$ the dissipator vanishes and (157) collapses to the von Neumann equation $\dot{\rho} = -\frac{i}{\hbar} [H, \rho]$, equivalent to the Schrodinger equation of T-A8.2—the unitary core is recovered exactly when the system is sealed from its environment. Second, the decoherence rates introduced in T-A8.5 are nothing but the dissipator rates Γ_{nm} of §D.6: the off-diagonal damping $e^{-\Gamma t}$ of the worked example is the same exponential that controls the loss of coherence in the body of the paper. Third, and

most distinctively, the framework reads this open-system semigroup as the *boundary image of coarse-graining the non-compact scale fiber* I_r . Integrating out the fine-scale degrees along I_r is an irreversible operation: it has a forward direction (toward coarser descriptions) but no inverse, exactly the structure of a *semigroup* rather than a group. The GKSL generator is thus the induced generator of an irreversible renormalization-group semigroup on the system's reduced state—the directed flow of the scale fiber, projected onto the boundary observables, manifesting as completely positive, trace-preserving, Markovian dissipation. In this reading the arrow of time of the dissipator and the arrow of coarse-graining along I_r are one and the same. This geometric reading fixes the open-system *structure* — the semigroup form and its irreversibility — but *not* the numerical rates: the coefficients γ_k are set by the physical 4D environment through its spectral density at the Bohr frequencies (the Fermi-golden-rule expression derived above), and are evaluated quantitatively by the scale-overlap integral of Appendix I. The non-compact scale fiber I_r carries no Kaluza–Klein tower and sets no rate of its own; the compact phase fiber S_θ^1 enters only through the separate, kinematically suppressed channel of that appendix. The geometry supplies the law of the flow; the physical bath supplies its speed.

Problems and worked solutions. D.1.1. Show that the transpose is positive but not completely positive.

Solution. Step 1: transposition preserves eigenvalues, so $T(\rho) = \rho^\top$ maps states to states: T is positive. Step 2: test complete positivity on the maximally entangled pair — the Choi test of §D: the Choi matrix $C = \frac{1}{2} \sum_{ij} E_{ij} \otimes T(E_{ij})$ has eigenvalues $\{+\frac{1}{2}, +\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}\}$: one negative eigenvalue, so $\mathbb{K} \otimes T$ maps the entangled state to a non-state. Step 3: conclusion — positivity alone cannot survive entangled company, which is why the GKSL hypothesis is *complete* positivity and why the Kraus form (138) (which builds CP in) is the right starting point; the transpose's failure is not pathology but the standard entanglement witness. Physics intuition: a map can be perfectly lawful on every state it is shown and still poison a system through its entangled partner — complete positivity is the requirement that the law hold even for particles that never meet the apparatus, and nature audibly enforces it.

D.1.2. Split the Schrödinger equation into its real pair, and locate the i .

Solution. Step 1: write $\psi = u + iv$ with u, v real (in a basis where H is real symmetric — always arrangeable for the textbook Hamiltonians), and substitute into $i\hbar \partial_t \psi = H\psi$; separating real and imaginary parts gives the coupled real pair $\hbar \dot{u} = +Hv$, $\hbar \dot{v} = -Hu$ — two first-order real equations, no i anywhere. Step 2: recognize the structure: the pair is a real Hamiltonian (symplectic) flow, rotating (u, v) with generator $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $J^2 = -\mathbb{K}$; the conserved quantity $\|u\|^2 + \|v\|^2 = \|\psi\|^2$ is the Born normalization, and for an energy eigenstate the pair traces a circle at angular frequency $E/\hbar$ — the “complex phase” $e^{-iEt/\hbar}$ is this real rotation written in shorthand. Step 3: state the register: nothing in Steps 1–2 is a derivation of *which* real pairing nature uses — that is the origin-of- i theorem of the companion paper (Arizaka, 2026, P4-Real), derived-conditional in the quasi-free regime — but the exercise shows exactly what the i *is*: the rotation generator of a real two-component flow, promoted to a primitive by a century of notational habit. Physics intuition: the most famous imaginary number in physics is a bookkeeping device for a real rotation

— Schrödinger’s equation never needed complex numbers, it needed a *pairing*; the bulk supplies the pairing, and the boundary observer, seeing only the compressed notation, spent a hundred years asking where the i came from.

D.2.1. Verify trace preservation for the dephasing channel’s Kraus pair.

Solution. Step 1: the phase-flip channel has Kraus operators $K_0 = \sqrt{1-p}\mathbb{I}$, $K_1 = \sqrt{p}\sigma_z$. Step 2: the trace-preservation condition (139): $K_0^\dagger K_0 + K_1^\dagger K_1 = (1-p)\mathbb{I} + p\sigma_z^2 = \mathbb{I}$ — satisfied for every $p \in [0, 1]$. Step 3: act on a general qubit state: populations are untouched, $\rho_{01} \mapsto (1-2p)\rho_{01}$ — the channel shrinks coherences only, and iterating it n times gives $(1-2p)^n$, the discrete ancestor of the exponential decay solved in D.7.1. Physics intuition: a quantum channel is a lottery of unitaries — here “do nothing” versus “flip the phase” — and trace preservation is just the lottery’s tickets summing to one; the off-diagonal shrinkage is what averaging over ignorance of which ticket was drawn does to interference.

D.5.1. State the Born–Markov–secular regime, and say why the secular step is not optional.

Solution. Step 1: the microscopic derivation (§D, (162)–(165)) holds when the bath correlation time is short compared to the relaxation time, $\tau_B \ll \tau_R$ (Born–Markov), so the bath forgets before the system notices. Step 2: the pre-secular (Redfield) generator is not of GKSL form and can drive ρ temporarily negative — probabilities below zero; the secular average over fast oscillating terms (165) restores the GKSL structure with golden-rule rates (167) that are manifestly ≥ 0 . Step 3: read the moral: complete positivity is not decoration — it is the boundary between an approximation scheme and a law, and the GKSL class is exactly where the scheme becomes lawful. Physics intuition: Redfield is a snapshot taken with the shutter open — smeared and occasionally impossible; the secular step waits for the fast fringes to average out, and what remains is the unique legal motion of an open system.

D.7.1. Solve the dephasing qubit completely, and name the two clocks.

Solution. Step 1: the master equation (168) with the single Lindblad operator $L = \sigma_z$ gives component equations (171): $\dot{\rho}_{00} = \dot{\rho}_{11} = 0$, $\dot{\rho}_{01} = -2\gamma\rho_{01}$. Step 2: solve (172): populations frozen, coherences $\rho_{01}(t) = \rho_{01}(0)e^{-2\gamma t}$ — energy conserved, interference dying on the clock $T_2^{(\phi)} = 1/2\gamma$. Step 3: the Bloch picture: the vector’s z -component is constant while its equatorial component spirals to zero — the state collapses onto the σ_z axis, which is precisely the einselection of the pointer basis seen microscopically in Part IX. Physics intuition: dephasing is decoherence in its purest form — nothing is dissipated, nothing is exchanged but *which-way information*, and the death of the off-diagonals is the price of that information existing anywhere in the universe.

D.7.2. Derive $T_2 \leq 2T_1$, and evaluate both ends.

Solution. Step 1: with amplitude damping at rate $1/T_1$ and pure dephasing at rate $1/T_\phi$, the coherence decays at the sum $1/T_2 = 1/2T_1 + 1/T_\phi$ — the factor $\frac{1}{2}$ because population decay destroys coherence only through its amplitude, $\sqrt{e^{-t/T_1}}$ per leg. Step 2: since $T_\phi > 0$, $T_2 \leq 2T_1$ always, with equality iff pure damping: for $T_1 = 100 \mu\text{s}$ and no dephasing, $T_2 = 200 \mu\text{s}$; add $T_\phi = 200 \mu\text{s}$ and T_2 drops to $100 \mu\text{s}$. Step 3: this inequality is the daily bread of every qubit laboratory — superconducting, trapped-ion, NV — and it is a *theorem of the GKSL form*: any device violating it

would falsify not a hardware model but the complete-positivity structure of open-system quantum mechanics. Physics intuition: energy loss forces phase loss (you cannot decay without decohering), but phase can leak with no energy moving at all — so the phase clock can only tie the energy clock at double speed, never beat it.

D.8.1. Explain why the boundary law of the projection must be GKSL, with the rates as geometry.

Solution. Step 1: the boundary state is a partial trace of a deterministically evolving bulk configuration (§E), so its evolution is a family of CPTP maps by construction — Kraus/Stinespring is automatic, not assumed. Step 2: the fiber correlation times are microscopic compared to boundary dynamics, so the semigroup (Markov) property holds on observational timescales, and the GKSL theorem then leaves *exactly one* generator form (157) — the framework does not choose Lindblad, it is cornered into it. Step 3: identify the dictionary: H is the boundary Hamiltonian of the projected sector, the L_k are the fiber-coupling channels, and the rates γ_k are squared scale-overlap integrals (§I) — measurable numbers with geometric addresses, which is what makes the eight-test horizon of the Discussions falsifiable rather than rhetorical. Physics intuition: determinism upstairs, Lindblad downstairs is not a paradox but a projection theorem — the dissipator is the shadow of the fibers, and every decay rate in the laboratory is, on this reading, a geometry measurement.

Take-away. *Standard account:* master equations are written ad hoc, system by system, and the Lindblad form is quoted as a convenience. *What it left open:* why nature’s effective dynamics should be completely positive, Markovian, and of this one algebraic shape. *The bulk reading:* the boundary evolution is the trace of a deterministic bulk flow, so it *must* be a CPTP semigroup, and the GKSL theorem then leaves exactly one generator form — the dissipator is not noise added by hand but the shadow of the unresolved fibers, with the rates γ_k the projection’s overlap data.

E Explicit partial-trace calculation: 6D bulk to 4D density matrix

What this appendix teaches. Where mixedness comes from: the partial trace built from first principles — definition, basis-independence, the operational reading (the unique state reproducing all local measurement statistics) — and then applied to the central geometric claim of the paper: a 4D observer who cannot resolve the scale fiber I_r and the phase fiber S_θ^1 is forced to describe a pure 6D configuration by a reduced density matrix. The appendix works the reduction explicitly and shows that purity upstairs and probability downstairs are the same fact viewed from the two strata — the mathematical engine behind T-A8.1 and the worked examples of Parts V–IX.

Read alongside: Nielsen (2010) for the standard density-matrix and partial-trace formalism; Schlosshauer (2007) for the reduced-state physics.

This appendix builds the partial trace from first principles and applies it to the central geometric claim of the present paper: a 4D observer who cannot resolve the two non-spatial extra directions of the 6D bulk — the scale fiber I_r and the phase fiber S_θ^1 — is forced to describe the world with a *reduced* density matrix obtained by tracing those two directions out. We show, with every step written out, that this trace (i) is the unique operation reproducing all locally accessible expectation values, (ii) turns a globally pure entangled bulk state into a mixed 4D state, and (iii) suppresses the

off-diagonal coherences in the (einselected, T-A8.6) pointer basis by averaging over the inaccessible S_θ^1 holonomy. We are careful to distinguish this loss of *accessible* coherence, which leaves an *improper* mixture, from the selection of a single definite outcome — the latter being the work of the GRIP registration (T-A8.1, T-A8.4), not of the partial trace.

We assume only the postulates of finite-dimensional quantum mechanics: states are unit-trace positive operators ρ on a Hilbert space $\mathcal{H}$, observables are self-adjoint operators, and $\langle A \rangle_\rho = \text{Tr}(\rho A)$. Throughout, Tr is the full trace and Tr_B a partial trace, defined below.

E.1 The partial trace from first principles

Let the total system live on a tensor-product Hilbert space

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B, \quad (174)$$

with $\dim \mathcal{H}_A = d_A$ and $\dim \mathcal{H}_B = d_B$. Fix orthonormal bases $\{|i\rangle_A\}_{i=1}^{d_A}$ of $\mathcal{H}_A$ and $\{|\mu\rangle_B\}_{\mu=1}^{d_B}$ of $\mathcal{H}_B$, so that $\{|i\rangle_A \otimes |\mu\rangle_B\}$ is an orthonormal basis of $\mathcal{H}$. We abbreviate $|i\rangle_A \otimes |\mu\rangle_B \equiv |i, \mu\rangle$.

Definition (partial trace over B). The partial trace over B is the linear map $\text{Tr}_B : \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B) \rightarrow \mathcal{B}(\mathcal{H}_A)$ fixed on product operators by

$$\text{Tr}_B(M_A \otimes N_B) = M_A \text{Tr}(N_B), \quad (175)$$

and extended to all operators by linearity. Equivalently, in components, for any operator Θ on $\mathcal{H}_A \otimes \mathcal{H}_B$,

$$(\text{Tr}_B \Theta)_{ij} = \sum_{\mu=1}^{d_B} \langle i, \mu | \Theta | j, \mu \rangle = \sum_{\mu=1}^{d_B} \Theta_{(i\mu)(j\mu)}. \quad (176)$$

In words: one “closes up” the B index by summing over the diagonal of the B block, while keeping the A indices i, j open. The result is an operator on $\mathcal{H}_A$ alone. A compact basis-free way to write the same map is

$$\text{Tr}_B \Theta = \sum_{\mu=1}^{d_B} (\mathbf{1}_A \otimes \langle \mu |_B) \Theta (\mathbf{1}_A \otimes |\mu \rangle_B), \quad (177)$$

where $\mathbf{1}_A \otimes \langle \mu |_B$ maps $\mathcal{H}_A \otimes \mathcal{H}_B \rightarrow \mathcal{H}_A$ by contracting the B factor against $\langle \mu |_B$. Because the trace $\text{Tr}(N_B)$ in (175) is basis-independent, the map Tr_B itself does not depend on which orthonormal basis $\{|\mu\rangle_B\}$ we chose; we verify this independence explicitly below.

The reduced density matrix. Given a state ρ on $\mathcal{H}_A \otimes \mathcal{H}_B$, its reduced state on A is by definition

$$\rho_A \equiv \text{Tr}_B \rho. \quad (178)$$

We must check that ρ_A is again a legitimate state, i.e. positive with unit trace.

Unit trace. Using (176),

$$\mathrm{Tr}_A \rho_A = \sum_i (\rho_A)_{ii} = \sum_i \sum_\mu \rho_{(i\mu)(i\mu)} = \mathrm{Tr} \rho = 1. \quad (179)$$

The double sum over (i, μ) is exactly the full trace, so the unit-trace property survives.

Positivity. For any $|\phi\rangle \in \mathcal{H}_A$, insert $|\phi\rangle$ into (177):

$$\langle \phi | \rho_A | \phi \rangle = \sum_\mu (\langle \phi | \otimes \langle \mu |) \rho (|\phi\rangle \otimes |\mu\rangle) = \sum_\mu \langle \phi, \mu | \rho | \phi, \mu \rangle \geq 0, \quad (180)$$

because each term is the expectation of the positive operator ρ in the normalized vector $|\phi, \mu\rangle$. Hence $\rho_A \geq 0$. Together with (179), ρ_A is a valid density matrix.

Uniqueness: Tr_B is forced by local expectation values. The physical content of the partial trace is that it is the *only* map reproducing every measurement an A -local observer can make. Suppose an observer acts only on the A factor, measuring an observable that, on the full space, reads $A \otimes \mathbf{1}_B$. Its expectation in the global state ρ is

$$\begin{aligned} \langle A \otimes \mathbf{1}_B \rangle_\rho &= \mathrm{Tr}[\rho(A \otimes \mathbf{1}_B)] = \sum_{i,\mu} \sum_{j,\nu} \rho_{(i\mu)(j\nu)} (A)_{ji} (\mathbf{1}_B)_{\nu\mu} \\ &= \sum_{i,j} (A)_{ji} \sum_\mu \rho_{(i\mu)(j\mu)} = \sum_{i,j} (A)_{ji} (\mathrm{Tr}_B \rho)_{ij} = \mathrm{Tr}[(\mathrm{Tr}_B \rho) A]. \end{aligned} \quad (181)$$

The third equality used $(\mathbf{1}_B)_{\nu\mu} = \delta_{\nu\mu}$, which collapses the ν -sum and sets $\nu = \mu$; the fourth recognized the component formula (176). Thus

$$\boxed{\langle A \otimes \mathbf{1}_B \rangle_\rho = \mathrm{Tr}_A[\rho_A A], \quad \rho_A = \mathrm{Tr}_B \rho.} \quad (182)$$

Now suppose some other operator σ_A on $\mathcal{H}_A$ reproduced the same numbers, $\mathrm{Tr}_A(\sigma_A A) = \mathrm{Tr}_A(\rho_A A)$ for *every* self-adjoint A . Then $\mathrm{Tr}_A[(\sigma_A - \rho_A)A] = 0$ for all A ; choosing $A = (\sigma_A - \rho_A)^\dagger$ (itself self-adjoint here) gives $\|\sigma_A - \rho_A\|_{\mathrm{HS}}^2 = 0$, so $\sigma_A = \rho_A$. The reduced state is therefore the *unique* operator on $\mathcal{H}_A$ encoding all A -local statistics, and Tr_B is the unique map producing it. This is the precise sense in which a subsystem observer who cannot touch B is *obliged* to use ρ_A : any two global states with the same ρ_A are operationally indistinguishable to that observer.

Basis independence of Tr_B . If $\{|\tilde{\mu}\rangle_B\}$ is a second orthonormal basis with $|\tilde{\mu}\rangle = \sum_\nu U_{\nu\mu} |\nu\rangle$ for a unitary U , then

$$\sum_\mu (\mathbf{1}_A \otimes \langle \tilde{\mu} |) \Theta(\mathbf{1}_A \otimes |\tilde{\mu}\rangle) = \sum_\mu \sum_{\nu, \nu'} U_{\nu\mu}^* U_{\nu'\mu} (\mathbf{1}_A \otimes \langle \nu |) \Theta(\mathbf{1}_A \otimes |\nu'\rangle) = \sum_{\nu} (\mathbf{1}_A \otimes \langle \nu |) \Theta(\mathbf{1}_A \otimes |\nu\rangle), \quad (183)$$

where $\sum_\mu U_{\nu\mu}^* U_{\nu'\mu} = \delta_{\nu\nu'}$ by unitarity. So Tr_B is well defined independently of the chosen B basis — a fact we will use when we argue that the *physically* preferred way of writing the fiber states does not change the reduced 4D state, only its interpretation.

E.2 A worked 2×2 tensor 2×2 example

To make (176) concrete, take $d_A = d_B = 2$, so $\mathcal{H} = \mathbb{C}^2 \otimes \mathbb{C}^2$ is four-dimensional with ordered basis $\{|0, 0\rangle, |0, 1\rangle, |1, 0\rangle, |1, 1\rangle\}$. A general operator Θ is a 4×4 matrix whose entries we label by the pair indices $(i\mu)$:

$$\Theta = \begin{pmatrix} \Theta_{(00)(00)} & \Theta_{(00)(01)} & \Theta_{(00)(10)} & \Theta_{(00)(11)} \\ \Theta_{(01)(00)} & \Theta_{(01)(01)} & \Theta_{(01)(10)} & \Theta_{(01)(11)} \\ \Theta_{(10)(00)} & \Theta_{(10)(01)} & \Theta_{(10)(10)} & \Theta_{(10)(11)} \\ \Theta_{(11)(00)} & \Theta_{(11)(01)} & \Theta_{(11)(10)} & \Theta_{(11)(11)} \end{pmatrix}. \quad (184)$$

View Θ as a 2×2 array of 2×2 blocks $\Theta^{(ij)}$, where the A indices i, j label the block and the B indices label the entry inside it:

$$\Theta = \begin{pmatrix} \Theta^{(00)} & \Theta^{(01)} \\ \Theta^{(10)} & \Theta^{(11)} \end{pmatrix}, \quad (\Theta^{(ij)})_{\mu\nu} = \Theta_{(i\mu)(j\nu)}. \quad (185)$$

By (176), tracing out B replaces each 2×2 block by its ordinary trace:

$$\text{Tr}_B \Theta = \begin{pmatrix} \text{tr } \Theta^{(00)} & \text{tr } \Theta^{(01)} \\ \text{tr } \Theta^{(10)} & \text{tr } \Theta^{(11)} \end{pmatrix} = \begin{pmatrix} \Theta_{(00)(00)} + \Theta_{(01)(01)} & \Theta_{(00)(10)} + \Theta_{(01)(11)} \\ \Theta_{(10)(00)} + \Theta_{(11)(01)} & \Theta_{(10)(10)} + \Theta_{(11)(11)} \end{pmatrix}. \quad (186)$$

The complementary trace over A instead sums the two diagonal blocks as whole matrices:

$$\text{Tr}_A \Theta = \Theta^{(00)} + \Theta^{(11)} = \begin{pmatrix} \Theta_{(00)(00)} + \Theta_{(10)(10)} & \Theta_{(00)(01)} + \Theta_{(10)(11)} \\ \Theta_{(01)(00)} + \Theta_{(11)(10)} & \Theta_{(01)(01)} + \Theta_{(11)(11)} \end{pmatrix}. \quad (187)$$

Equations (186)–(187) are the two “half-traces” of the same 4×4 object; comparing them makes visible that Tr_B keeps the A structure (the block layout) and discards the B structure (collapsing each block to a number), while Tr_A does the reverse. This block-trace picture is the entire computational content of the partial trace; everything below is this operation applied to physically motivated states.

E.3 The bulk state and its factorization

We now specialize A, B to the geometry of the present paper. The 6D bulk Hilbert space decomposes, at the operational level made precise below, as

$$\mathcal{H}_{\text{bulk}} = \mathcal{H}_4 \otimes \mathcal{H}_{I_r} \otimes \mathcal{H}_{S_\theta^1}, \quad (188)$$

with $\mathcal{H}_4$ the 4D zero-mode sector that a boundary observer can resolve, $\mathcal{H}_{I_r}$ the scale-radial (scale-fiber) Kaluza–Klein sector, and $\mathcal{H}_{S_\theta^1}$ the compact phase-circle sector carrying the Wilson-line holonomy. The two extra factors are *not* ordinary spatial directions: I_r is the renormalization-group scale axis and S_θ^1 is the internal phase circle. A 4D observer has no apparatus that couples to either,

so from that observer's standpoint they are an *environment*. Grouping them,

$$\mathcal{H}_A \equiv \mathcal{H}_{\text{bdy}} \equiv \mathcal{H}_4, \quad \mathcal{H}_B \equiv \mathcal{H}_{\text{fiber}} \equiv \mathcal{H}_{I_r} \otimes \mathcal{H}_{S_\theta^1}, \quad \mathcal{H}_{\text{bulk}} = \mathcal{H}_{\text{bdy}} \otimes \mathcal{H}_{\text{fiber}}. \quad (189)$$

Honest caveat on the factorization. The clean tensor split (188) is an *operational, zero-mode-restricted* statement, not an exact identity of the interacting bulk theory. A genuinely interacting 6D field theory does not factorize across a codimension-two surface on the nose: the algebra of bulk observables has no canonical product decomposition, gauge constraints link the sectors, and a sharp boundary algebra requires a (type-III) operator-algebraic treatment. What *is* well defined is the restriction to the low-lying mode sector — the 4D zero modes together with a finite band of light I_r and S_θ^1 modes — on which the scale-profile spectrum gaps the heavy excitations away and the split (189) holds to the accuracy set by the scale profile hierarchy. All formulas below are to be read in that restricted, finite-dimensional sense; this is the same level of rigor at which the rest of the paper treats the bulk-to-boundary dictionary. We do not claim more, and nothing in the conclusions requires more.

A pure bulk state. A pure state on the restricted bulk space is

$$|\Psi\rangle_{\text{bulk}} = \sum_{a,b,c} c_{abc} |a\rangle_4 \otimes |b\rangle_{I_r} \otimes |c\rangle_{S_\theta^1}, \quad \sum_{a,b,c} |c_{abc}|^2 = 1, \quad (190)$$

with $\{|a\rangle_4\}$ the boundary (pointer) basis and $\{|b\rangle_{I_r}\}, \{|c\rangle_{S_\theta^1}\}$ orthonormal bases of the two fiber sectors. Writing the combined fiber index as $B = (b, c)$ and the combined fiber basis as $|B\rangle_{\text{fiber}} = |b\rangle_{I_r} \otimes |c\rangle_{S_\theta^1}$, this is exactly the bipartite form $|\Psi\rangle = \sum_{a,B} c_{aB} |a\rangle_4 \otimes |B\rangle_{\text{fiber}}$ to which §E.1 applies verbatim.

E.4 Explicit construction of the reduced 4D state

It is cleanest to first display the calculation with a single fiber label, then restore the full (I_r, S_θ^1) structure. Write the bulk pure state in Schmidt-like form, entangled across boundary and fiber,

$$|\Psi\rangle = \sum_n c_n |b_n\rangle \otimes |f_n\rangle, \quad (191)$$

where $\{|b_n\rangle\} \subset \mathcal{H}_{\text{bdy}}$ are pointer states and $\{|f_n\rangle\} \subset \mathcal{H}_{\text{fiber}}$ are the corresponding fiber states (not assumed orthonormal yet). The global density matrix is the rank-one projector

$$\rho_{\text{bulk}} = |\Psi\rangle\langle\Psi| = \sum_{m,n} c_m c_n^* |b_m\rangle\langle b_n| \otimes |f_m\rangle\langle f_n|. \quad (192)$$

Apply Tr_{fiber} using the product rule (175), $\text{Tr}_{\text{fiber}}(|b_m\rangle\langle b_n| \otimes |f_m\rangle\langle f_n|) = |b_m\rangle\langle b_n| \text{Tr}(|f_m\rangle\langle f_n|)$, and the cyclicity identity $\text{Tr}(|f_m\rangle\langle f_n|) = \langle f_n|f_m\rangle$:

$$\rho_{\text{bdy}} = \text{Tr}_{\text{fiber}} \rho_{\text{bulk}} = \sum_{m,n} c_m c_n^* \langle f_n|f_m\rangle |b_m\rangle\langle b_n|. \quad (193)$$

Separate the diagonal ($m = n$) terms from the off-diagonal ($m \neq n$) cross terms:

$$\rho_{\text{bdy}} = \underbrace{\sum_n |c_n|^2 |b_n\rangle\langle b_n|}_{\text{classical mixture}} + \underbrace{\sum_{m \neq n} c_m c_n^* \langle f_n | f_m \rangle |b_n\rangle\langle b_m|}_{\text{quantum coherences}}. \quad (194)$$

The diagonal piece is an incoherent (classical) mixture of pointer states with weights $|c_n|^2$; it is exactly what a classical probability distribution over outcomes n would give. The off-diagonal piece carries all the quantum interference, and every such term is multiplied by the fiber overlap $\langle f_n | f_m \rangle$. Two limiting cases bracket the physics:

- *Product (unentangled) bulk state.* If $c_{abc} = \alpha_a \beta_b \gamma_c$, equivalently $|f_n\rangle = |f\rangle$ independent of n , then $\langle f_n | f_m \rangle = \langle f | f \rangle = 1$ for all m, n , the two pieces of (194) recombine, and $\rho_{\text{bdy}} = |\alpha\rangle\langle\alpha|$ remains pure. No information has leaked to the fiber, so nothing is lost.
- *Orthogonal fiber records.* If the fiber states are perfectly distinguishable, $\langle f_n | f_m \rangle = \delta_{mn}$, every cross term vanishes and

$$\rho_{\text{bdy}} = \sum_n |c_n|^2 |b_n\rangle\langle b_n|, \quad (195)$$

a fully decohered, diagonal density matrix in the pointer basis. The boundary state is now maximally mixed-down relative to the coherent superposition it came from: the fiber has recorded “which pointer state,” and that record, though inaccessible to the 4D observer, is enough to destroy interference.

The generic, intermediate case has $0 < |\langle f_n | f_m \rangle| < 1$ and gives partial decoherence; the next subsection identifies what makes the overlap small in this geometry.

E.5 Decoherence as averaging over the hidden phase fiber

The fiber overlap $\langle f_n | f_m \rangle$ in (193) is where the two specific extra directions do their work. Restore the product structure of the fiber, $|f_n\rangle = |b_n^{I_r}\rangle \otimes |\phi_n^\theta\rangle$, with $|b_n^{I_r}\rangle$ the scale-radial component and $|\phi_n^\theta\rangle$ the phase-circle component. The overlap factorizes,

$$\langle f_n | f_m \rangle = \langle b_m^{I_r} | b_n^{I_r} \rangle \langle \phi_m^\theta | \phi_n^\theta \rangle. \quad (196)$$

Consider the S_θ^1 factor first. The phase circle carries a Wilson-line holonomy: parallel transport once around S_θ^1 multiplies the n -th sector’s phase state by a holonomy phase $e^{i\Theta_n}$, and the relative holonomy between sectors m and n is $e^{i(\Theta_m - \Theta_n)}$. A 4D observer cannot resolve *where* on the circle the state sits — the circle coordinate θ is internal and inaccessible — so the only invariant statement is obtained by averaging the relative phase uniformly over the circle. Modeling $|\phi_n^\theta\rangle \sim e^{i\Theta_n\theta}$ on S_θ^1 — with *integer* sector holonomies $\Theta_n \in \mathbb{Z}$, as the single-valuedness of a phase on the compact S_θ^1 requires (the charge quantization derived in the Wilson-line appendix) — and averaging the cross term over $\theta \in [0, 2\pi)$,

$$\overline{\langle \phi_m^\theta | \phi_n^\theta \rangle} = \frac{1}{2\pi} \int_0^{2\pi} e^{i(\Theta_n - \Theta_m)\theta} d\theta = \delta_{\Theta_m, \Theta_n} \xrightarrow{m \neq n} 0. \quad (197)$$

Distinct pointer sectors carry distinct integer holonomies ($\Theta_m \neq \Theta_n$ for $m \neq n$, so $\Theta_n - \Theta_m$ is a nonzero integer and the integral in (197) vanishes *exactly*); the average therefore kills every $m \neq n$ cross term. Substituting this averaged overlap back into (193) leaves precisely the diagonal, pointer-basis density matrix:

$$\overline{\rho_{\text{bdy}}} = \sum_n |c_n|^2 |b_n\rangle\langle b_n|. \quad (198)$$

This is the technical statement of *decoherence as fiber-averaging*: the off-diagonal coherences are not erased by any non-unitary fundamental process; they are present in the global pure state $|\Psi\rangle$ but become invisible the instant one restricts to 4D-accessible quantities, because those quantities are blind to the S_θ^1 phase and therefore see only the holonomy-averaged overlap (197). The preferred (pointer) basis singled out here is exactly the basis in which the fiber keeps an orthogonal record, in accord with the einselection theorem T-A8.6.

E.6 Off-diagonal suppression as a scale-profile overlap integral

The S_θ^1 average gives the *exact* vanishing in the idealized holonomy model; the residual, finite suppression rate comes from the second factor in (196), the scale-radial overlap $\langle b_m^{I_r} | b_n^{I_r} \rangle$. The radial fiber states are Kaluza–Klein wavefunctions on the scale interval I_r , weighted by the scale profile e^{-2r} that controls how strongly each 4D pointer sector leaks into the heavy modes. Their overlap is a scale-weighted integral of the schematic form

$$\langle b_m^{I_r} | b_n^{I_r} \rangle \sim \int_{I_r} e^{-2r} \psi_m^*(r) \psi_n(r) dr, \quad (199)$$

whose magnitude sets the surviving off-diagonal weight, and hence the decoherence rate Γ_{mn} , through $|(\rho_{\text{bdy}})_{mn}| \propto |\langle b_m^{I_r} | b_n^{I_r} \rangle|$. The more the scale profile separates the radial supports of ψ_m and ψ_n , the smaller the overlap and the faster the coherences decay. The quantitative evaluation of this integral — the collisional decoherence function, the cross-section estimates, and the resulting numerical rate — is carried out in the worked overlap appendix, Appendix I; here we only need that the scale geometry *suppresses* the off-diagonal terms, which is what (199) states qualitatively. This is also the microscopic origin of the rate $\Gamma_{\alpha\beta}$ appearing in the decoherence theorem T-A8.5.

E.7 Worked micro-example: one boundary qubit maximally entangled with one fiber qubit

Take the smallest non-trivial case: a single boundary qubit $\mathcal{H}_{\text{bdy}} = \mathbb{C}^2$ with pointer basis $\{|0\rangle, |1\rangle\}$, maximally entangled with a single fiber qubit $\mathcal{H}_{\text{fiber}} = \mathbb{C}^2$ with orthonormal records $\{|f_0\rangle, |f_1\rangle\}$. The bulk pure state is the Bell-like vector

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |f_0\rangle + |1\rangle \otimes |f_1\rangle). \quad (200)$$

Its global density matrix is the rank-one projector

$$\rho_{\text{bulk}} = |\Psi\rangle\langle\Psi| = \frac{1}{2}(|0, f_0\rangle\langle 0, f_0| + |0, f_0\rangle\langle 1, f_1| + |1, f_1\rangle\langle 0, f_0| + |1, f_1\rangle\langle 1, f_1|). \quad (201)$$

Trace out the fiber with (175), using $\text{Tr}(|f_i\rangle\langle f_j|) = \langle f_j|f_i\rangle = \delta_{ij}$:

$$\begin{aligned}\rho_{\text{bdy}} &= \text{Tr}_{\text{fiber}} \rho_{\text{bulk}} = \frac{1}{2} \left(|0\rangle\langle 0| \underbrace{\langle f_0|f_0\rangle}_1 + |0\rangle\langle 1| \underbrace{\langle f_1|f_0\rangle}_0 + |1\rangle\langle 0| \underbrace{\langle f_0|f_1\rangle}_0 + |1\rangle\langle 1| \underbrace{\langle f_1|f_1\rangle}_1 \right) \\ &= \frac{1}{2} (|0\rangle\langle 0| + |1\rangle\langle 1|) = \frac{1}{2} \mathbf{1} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}.\end{aligned}\quad (202)$$

The reduced boundary state is the *maximally mixed* qubit $\rho_{\text{bdy}} = \mathbf{1}/2$. The interpretation is the entire moral of this appendix in miniature:

- The global state (200) is pure: it has zero entropy and definite (here, maximal) correlations between qubit and fiber. Nothing random has happened at the 6D level.
- Yet the boundary observer, blind to the fiber, holds $\mathbf{1}/2$ — a state of *maximal ignorance*, von Neumann entropy $S = \ln 2$, indistinguishable from a fair classical coin over $\{|0\rangle, |1\rangle\}$. The off-diagonal coherences of the original superposition are gone, killed by the orthogonality of the fiber records $\langle f_0|f_1\rangle = 0$.
- The mixedness is therefore not intrinsic to the world; it is manufactured by the partial trace, i.e. by the observer's inability to resolve the two hidden directions. The diagonal $\mathbf{1}/2$ is an *improper* mixture — the boundary shadow of a perfectly unitary, perfectly definite bulk state. It is not yet a definite outcome: which single value, 0 or 1, is registered is fixed by the GRIP descent (T-A8.1, T-A8.4), not by the partial trace, which supplies only the $\frac{1}{2}$ weights.

The same arithmetic with non-orthogonal records, $\langle f_0|f_1\rangle = s$ with $|s| \leq 1$, gives $\rho_{\text{bdy}} = \frac{1}{2} \begin{pmatrix} 1 & s^* \\ s & 1 \end{pmatrix}$, interpolating from the maximally mixed $\mathbf{1}/2$ at $s = 0$ (orthogonal records, full decoherence) to a pure state at $|s| = 1$ (identical records, no decoherence) — the two-level version of the general overlap dependence in (194).

E.8 The Born rule and the Lüders update act on ρ_{bdy}

The object delivered by the partial trace, $\rho_{\text{bdy}} = \text{Tr}_{\text{fiber}} \rho_{\text{bulk}}$, is the unique state on which all 4D-accessible statistics are computed. For a POVM $\{E_\alpha\}$ on the boundary subsystem, with $E_\alpha \geq 0$ and $\sum_\alpha E_\alpha = \mathbf{1}$, the probability of outcome α is

$$P(\alpha) = \text{Tr}(\rho_{\text{bdy}} E_\alpha) = \sum_{a,a'} (\rho_{\text{bdy}})_{aa'} (E_\alpha)_{a'a}. \quad (203)$$

Since ρ_{bdy} is positive with unit trace ((179)–(180)) and each $E_\alpha \geq 0$ with $\sum_\alpha E_\alpha = \mathbf{1}$, the numbers $P(\alpha)$ are non-negative and sum to one: a bona fide probability distribution. This is the Born rule T-A8.3, now seen to act not on an abstract postulated state but on the fiber-reduced ρ_{bdy} constructed above. After a selective measurement returning outcome α (with Kraus/instrument

elements $E_\alpha = K_\alpha^\dagger K_\alpha$), the conditional post-measurement state is the Lüders update T-A8.4,

$$\rho_{\text{bdy}} \mapsto \rho_{\text{bdy}}^\alpha = \frac{K_\alpha \rho_{\text{bdy}} K_\alpha^\dagger}{P(\alpha)}, \quad (204)$$

again a manipulation entirely internal to the boundary state. In the non-selective case the unconditioned channel $\rho_{\text{bdy}} \mapsto \sum_\alpha K_\alpha \rho_{\text{bdy}} K_\alpha^\dagger$ is the discrete-time face of the continuous open-system dynamics, whose generator is the GKLS (Gorini–Kossakowski–Sudarshan–Lindblad) form

$$\frac{d\rho_{\text{bdy}}}{dt} = -\frac{i}{\hbar}[H_{\text{eff}}, \rho_{\text{bdy}}] + \sum_\alpha \gamma_\alpha \left(L_\alpha \rho_{\text{bdy}} L_\alpha^\dagger - \frac{1}{2} \{L_\alpha^\dagger L_\alpha, \rho_{\text{bdy}}\} \right), \quad (205)$$

derived in the present paper as the decoherence theorem T-A8.5 (its closed, unitary $\gamma_\alpha \rightarrow 0$ limit being the Schrödinger equation T-A8.2); its construction from the fiber-traced bulk dynamics is detailed in Appendix D. The closed, unitary Schrödinger limit is recovered as $\gamma_\alpha \rightarrow 0$, i.e. when no information escapes into the fiber.

E.9 Summary: apparent classicality as an average over two hidden directions

Assembling the chain: a globally pure, unitarily evolving 6D bulk state (190) is entangled across the boundary and the two non-spatial fibers. A 4D observer cannot resolve the scale fiber I_r or the phase fiber S_θ^1 , and the unique state encoding everything that observer can measure is the fiber-reduced density matrix $\rho_{\text{bdy}} = \text{Tr}_{\text{fiber}} \rho_{\text{bulk}}$ of (193). Tracing out the inaccessible S_θ^1 holonomy averages the off-diagonal coherences to zero ((197)–(198)), and the scale-radial overlap (199) sets the residual suppression rate; the worked qubit example (202) shows a pure bulk state reducing to the maximally mixed boundary state $\mathbf{1}/2$, with no fundamental randomness introduced anywhere. The resulting ρ_{bdy} is precisely the object on which the Born rule T-A8.3 and the Lüders update T-A8.4 operate, with the open-system generator taking the GKLS form (205) (T-A8.5, with T-A8.2 its unitary limit).

What the partial trace does and does not do. The result must not be overstated. The fiber-reduced ρ_{bdy} is an *improper* mixture: averaging the inaccessible S_θ^1 holonomy removes the *accessible* interference between pointer states, leaving a diagonal form a 4D observer cannot operationally distinguish from a classical ignorance ensemble. It does *not*, by itself, select which single outcome is realized. That selection is the separate, dynamical content of the GRIP registration of Arisaka (2026, A6-GRIP): the joint system-and-apparatus configuration descends into one Geometric Attractor basin and is locked there by the boundary free-energy law (T-A8.1, T-A8.4); the question whether the frequencies of that descent reproduce the Born weights is closed at the relaxation level, with the field-grade representation step the single remaining analytic residue (§118), assumed nowhere here. The pointer basis in which the reduction is diagonal is, likewise, not produced by the average but fixed independently by bulk locality (T-A8.6). In short, the partial trace explains the *appearance* of classicality and the loss of accessible coherence; it does not, and is not claimed to, produce the definite outcome or the preferred basis.

Problems and worked solutions. E.2.1. Reduce the Bell pair, and watch purity become probability.

Solution. Step 1: for $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$, apply the index definition (176): the cross terms carry $\text{Tr}|0\rangle\langle 1| = 0$, so $\rho_A = \text{Tr}_B |\Phi^+\rangle\langle\Phi^+| = \frac{1}{2}\mathbb{I}$. Step 2: purity: $\text{Tr}\rho^2$ falls from 1 (global, pure) to $\frac{1}{2}$ (local, maximally mixed) — the local observer’s outcomes are exactly fifty-fifty although the global state is as definite as quantum mechanics allows. Step 3: verify the operational reading (181): every local expectation computed from ρ_A agrees with the global state’s prediction — the reduced state is not an approximation but the *complete* local description. Physics intuition: this two-line calculation is the origin story of quantum probability in the framework — perfect global definiteness, viewed through a subsystem keyhole, is indistinguishable from a fair coin; the 6D bulk plays the role of the global state, and the 4D observer holds the keyhole.

E.2.2. Compare GHZ and W through their reductions.

Solution. Step 1: for $|\text{GHZ}\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$, tracing two qubits gives $\rho_1 = \text{diag}(\frac{1}{2}, \frac{1}{2})$, purity $\frac{1}{2}$ — and no coherence survives: one lost qubit destroys the superposition entirely. Step 2: for $|\text{W}\rangle = (|001\rangle + |010\rangle + |100\rangle)/\sqrt{3}$, $\rho_1 = \text{diag}(\frac{2}{3}, \frac{1}{3})$, purity $\frac{5}{9}$ — less mixed, and the two-qubit remainder is still entangled: W-entanglement survives amputation, GHZ-entanglement does not. Step 3: the lesson for the bulk reading: *how* mixedness appears under a trace is a fingerprint of *how* the discarded sector was entangled — which is exactly the diagnostic logic the chapter applies to the fibers, where the pattern of boundary decoherence encodes the structure of the bulk coupling. Physics intuition: entanglement is not one substance — GHZ is a chain that shatters at one cut, W a rope that frays strand by strand; the reduced density matrix is the forensic report on which one you had.

E.5.1. Perform the fiber average, and kill the off-diagonals exactly.

Solution. Step 1: the pointer sectors carry integer holonomies Θ_m on the compact fiber, and the unresolved observer averages the relative phase over the circle (197): $\frac{1}{2\pi} \int_0^{2\pi} e^{i(\Theta_n - \Theta_m)\theta} d\theta = \delta_{\Theta_m \Theta_n}$. Step 2: for distinct sectors the integral is *exactly* zero — not small, zero: the off-diagonal blocks of (194) are annihilated, giving the decohered form (198). Step 3: contrast with environmental decoherence: a finite bath suppresses coherences by a large but finite factor, while a compact unresolved dimension deletes inter-sector coherence identically — integer orthogonality on the circle, the same arithmetic that makes Fourier series work. Physics intuition: the fiber is nature’s perfect which-way detector — distinct outcomes wind the circle a different integer number of times, and integers do not almost-agree; that is why the classical world’s sectors do not leak into each other at any precision.

E.7.1. Re-run the micro-example, and read the Born rule off the trace.

Solution. Step 1: the boundary qubit maximally entangled with one fiber mode (200) has bulk projector (201); tracing the fiber with $\text{Tr}|f_i\rangle\langle f_j| = \delta_{ij}$ kills both cross terms (202): $\rho_{\text{bdy}} = \frac{1}{2}\mathbb{I}$. Step 2: generalize the weights: for $\alpha|0, f_0\rangle + \beta|1, f_1\rangle$ the same trace gives $\text{diag}(|\alpha|^2, |\beta|^2)$ — the Born weights emerge as the *diagonal survivors* of the fiber trace (203), before any measurement postulate is spoken. Step 3: state the division of labor: the trace explains why the boundary state is diagonal with these weights; Gleason (§C) explains why no other weight-assignment was ever available — the two appendices are the two halves of T-A8.3. Physics intuition: the wavefunction’s squared moduli

are what survive the keyhole — the fiber keeps the phases and the boundary keeps the weights, and a century of “why $|\psi|^2$?” is answered by inspecting what a trace can and cannot carry.

E.8.1. Apply the Lüders update, and verify repeatability.

Solution. Step 1: on the decohered $\rho_{\text{bdy}} = \text{diag}(|\alpha|^2, |\beta|^2)$, register outcome 0: the update (204) gives $\rho' = \Pi_0 \rho \Pi_0 / \text{Tr}(\rho \Pi_0) = |0\rangle\langle 0|$. Step 2: repeat the measurement: $p(0) = \text{Tr}(\rho' \Pi_0) = 1$ — repeatability, the textbook’s projection-postulate content, drops out of conditioning alone. Step 3: locate the update in the framework: nothing collapses in the bulk — the registration selects the sector, the Lüders map is Bayesian conditioning on the registered record, and the “jump” is an update of the boundary description, not a dynamical process competing with the GKSL flow ((205)). Physics intuition: collapse is what conditioning looks like when you forget you are holding a reduced description — the bulk never jumped, the ledger did; and repeatability is simply the fact that a question already answered stays answered.

Take-away. *Standard account:* density matrices enter as bookkeeping for ignorance or for subsystems, and the origin of quantum probability is left to the interpretation chapter. *What it left open:* whether the mixedness of the world is epistemic accident or structural necessity. *The bulk reading:* mixedness is geometry — the observer’s stratum simply does not contain the fibers, so the trace is forced, the reduced state is the *complete* 4D description, and quantum probability is what a deterministic bulk looks like through a keyhole; the appendix’s explicit calculation is that sentence made arithmetic.

F Explicit Bell-CHSH calculation with S_θ^1 holonomy

What this appendix teaches. The Bell-CHSH mathematics, line by line, with the cohort’s one addition made explicit: the singlet correlation $E(\mathbf{a}, \mathbf{b}) = -\mathbf{a} \cdot \mathbf{b}$ computed from the joint state, the CHSH combination assembled, the local bound $|S| \leq 2$ and the quantum value $2\sqrt{2}$ at the standard angles, and then the reproduction of the same correlations from the S_θ^1 holonomy sampling — exhibiting a deterministic 6D-local model that violates the 4D bound without factorizing and without signaling. Nothing in the standard mathematics is altered; what changes is what the theorem is *about*: Bell excludes hidden variables on the boundary, and the bulk was never on the boundary.

Read alongside: Clauser *et al.* (1969) for the inequality; Tsirelson (1980) for the quantum bound; Brunner *et al.* (2014) for the modern review of Bell nonlocality.

This appendix expands, line by line, the claim made throughout Part VII that the GG-Theory reading of an entangled pair reproduces the standard quantum correlations *exactly*, saturating the Tsirelson bound $2\sqrt{2}$, while remaining free of any local-hidden-variable factorization and of any superluminal signal. The mathematics here is the ordinary Bell-CHSH mathematics; nothing in the framework alters a single number. What the framework supplies is the geometric *reading*: the entangled pair is *one* non-separable bulk state whose phase fiber S_θ^1 is fixed once, at the common source, and is then shared by both wings. There is no second copy of the fiber to carry an independent local response, which is exactly why the correlations are quantum and not classical. We work entirely with ± 1 dichotomic outcomes and the Pauli algebra, in standard notation, so a first-year graduate student can follow every step without consulting an external reference.

Notation. Throughout, $\hat{a}, \hat{a}', \hat{b}, \hat{b}'$ are real unit vectors in $\mathbb{R}^3$ specifying analyzer orientations; $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the vector of Pauli matrices; $\hat{a} \cdot \vec{\sigma} = \sum_i a_i \sigma_i$; and θ_{ab} is the angle between $\hat{a}$ and

$\hat{b}$, so that $\hat{a} \cdot \hat{b} = \cos \theta_{ab}$. The two-qubit Hilbert space is $\mathcal{H} = \mathbb{C}_A^2 \otimes \mathbb{C}_B^2$, and the singlet (spin-zero) state is

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B), \quad (206)$$

where $|\uparrow\rangle, |\downarrow\rangle$ are the $+1, -1$ eigenstates of σ_z .

F.1 The CHSH observable and the local-realist bound $|S| \leq 2$

Define the four correlators between Alice's settings $\{\hat{a}, \hat{a}'\}$ and Bob's settings $\{\hat{b}, \hat{b}'\}$, each correlator being the expectation of the product of the two ± 1 outcomes. The CHSH combination is

$$S = E(\hat{a}, \hat{b}) - E(\hat{a}, \hat{b}') + E(\hat{a}', \hat{b}) + E(\hat{a}', \hat{b}'). \quad (207)$$

(The placement of the single minus sign is a convention; any of the four sign assignments with an odd number of minus signs is equivalent under relabeling, and we will track it carefully when we choose angles below.)

We now prove the Bell-CHSH bound $|S| \leq 2$ that holds for *any* local-hidden-variable (LHV) model. An LHV model posits a shared variable λ with probability density $\rho(\lambda) \geq 0$, $\int d\lambda \rho(\lambda) = 1$, and *local* deterministic response functions

$$A(\hat{a}, \lambda), A(\hat{a}', \lambda), B(\hat{b}, \lambda), B(\hat{b}', \lambda) \in \{+1, -1\}, \quad (208)$$

where Alice's outcome depends only on her own setting and λ (not on Bob's setting), and vice versa. The correlator is the average of the product:

$$E(\hat{a}, \hat{b}) = \int d\lambda \rho(\lambda) A(\hat{a}, \lambda) B(\hat{b}, \lambda). \quad (209)$$

Insert (209) into (207) and abbreviate, for a *fixed* λ , $A \equiv A(\hat{a}, \lambda)$, $A' \equiv A(\hat{a}', \lambda)$, $B \equiv B(\hat{b}, \lambda)$, $B' \equiv B(\hat{b}', \lambda)$. Then

$$S = \int d\lambda \rho(\lambda) [AB - AB' + A'B + A'B']. \quad (210)$$

The bracketed quantity factors in two ways:

$$AB - AB' + A'B + A'B' = A(B - B') + A'(B + B'). \quad (211)$$

Because $B, B' \in \{+1, -1\}$, exactly one of the two combinations $B - B'$ and $B + B'$ vanishes while the other equals ± 2 : if $B = B'$ then $B - B' = 0$ and $B + B' = \pm 2$; if $B = -B'$ then $B + B' = 0$ and $B - B' = \pm 2$. In either case, since $A, A' \in \{+1, -1\}$,

$$A(B - B') + A'(B + B') = \pm 2. \quad (212)$$

Thus the integrand of (210) is bounded pointwise in λ by $|\dots| \leq 2$, and because ρ is a normalized

probability density,

$$|S| \leq \int d\lambda \rho(\lambda) |A(B - B') + A'(B + B')| \leq \int d\lambda \rho(\lambda) \cdot 2 = 2. \quad (213)$$

This is the Bell-CHSH inequality $|S| \leq 2$. Its only inputs were (i) definite ± 1 outcomes and (ii) factorization (209), i.e. that Alice's outcome is produced by a function not depending on Bob's setting. Any violation of $|S| \leq 2$ is therefore a proof that no such local, separable, outcome-assigning model exists. The framework never supplies one: the shared S_θ^1 fiber is not a per-wing response function but the label of a single joint state, as we make precise in Section F.6.

F.2 The quantum singlet correlator $E(\hat{a}, \hat{b}) = -\hat{a} \cdot \hat{b}$

In quantum mechanics the correlator is the singlet expectation of the product of local spin observables,

$$E(\hat{a}, \hat{b}) = \langle \Psi^- | (\hat{a} \cdot \vec{\sigma}) \otimes (\hat{b} \cdot \vec{\sigma}) | \Psi^- \rangle, \quad (214)$$

where $(\hat{a} \cdot \vec{\sigma})$ acts on Alice's qubit and $(\hat{b} \cdot \vec{\sigma})$ on Bob's. We derive $E(\hat{a}, \hat{b}) = -\hat{a} \cdot \hat{b}$ in full.

Step 1: the total-spin-zero identity. The singlet carries total spin zero, $(\vec{\sigma}_A + \vec{\sigma}_B)|\Psi^-\rangle = 0$; component by component this is the operator identity

$$(\sigma_i \otimes \mathbb{I}) |\Psi^-\rangle = -(\mathbb{I} \otimes \sigma_i) |\Psi^-\rangle, \quad i = x, y, z. \quad (215)$$

One checks it directly on $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$: applying σ_i to the first factor gives the same pair of two-qubit vectors as applying it to the second factor, with an overall sign flip — the antisymmetry of the singlet, i.e. its vanishing total spin. (No transpose appears; the minus sign is exactly this antisymmetry.)

Step 2: reduce the correlator to a single-qubit trace. Writing $(\hat{a} \cdot \vec{\sigma}) \otimes (\hat{b} \cdot \vec{\sigma}) = [(\hat{a} \cdot \vec{\sigma}) \otimes \mathbb{I}] [\mathbb{I} \otimes (\hat{b} \cdot \vec{\sigma})]$ in (214) and moving the first factor across with (215) (which holds for $\hat{a} \cdot \vec{\sigma} = \sum_i a_i \sigma_i$ by linearity),

$$E(\hat{a}, \hat{b}) = \langle \Psi^- | [(\hat{a} \cdot \vec{\sigma}) \otimes \mathbb{I}] [\mathbb{I} \otimes (\hat{b} \cdot \vec{\sigma})] | \Psi^- \rangle = -\langle \Psi^- | \mathbb{I} \otimes [(\hat{a} \cdot \vec{\sigma})(\hat{b} \cdot \vec{\sigma})] | \Psi^- \rangle. \quad (216)$$

The reduced state of Bob's qubit in the singlet is maximally mixed, $\rho_B = \frac{1}{2}\mathbb{I}$ (the singlet's marginals are completely random; this is the no-signaling fact we use again below), so for any single-qubit operator N ,

$$\langle \Psi^- | \mathbb{I} \otimes N | \Psi^- \rangle = \text{Tr}(\rho_B N) = \frac{1}{2} \text{Tr} N. \quad (217)$$

Applying this with $N = (\hat{a} \cdot \vec{\sigma})(\hat{b} \cdot \vec{\sigma})$,

$$E(\hat{a}, \hat{b}) = -\frac{1}{2} \text{Tr} [(\hat{a} \cdot \vec{\sigma})(\hat{b} \cdot \vec{\sigma})]. \quad (218)$$

Step 3: evaluate the trace with the Pauli algebra. The Pauli matrices obey

$$\sigma_i \sigma_j = \delta_{ij} \mathbb{I} + i \epsilon_{ijk} \sigma_k, \quad \text{Tr } \mathbb{I} = 2, \quad \text{Tr } \sigma_k = 0, \quad (219)$$

from which $\text{Tr}(\sigma_i \sigma_j) = 2\delta_{ij}$:

$$\text{Tr}(\sigma_i \sigma_j) = \text{Tr}(\delta_{ij} \mathbb{I} + i \epsilon_{ijk} \sigma_k) = 2\delta_{ij}. \quad (220)$$

Hence, with no transpose anywhere,

$$\text{Tr}[(\hat{a} \cdot \vec{\sigma})(\hat{b} \cdot \vec{\sigma})] = \sum_{i,j} a_i b_j \text{Tr}(\sigma_i \sigma_j) = \sum_{i,j} a_i b_j 2\delta_{ij} = 2 \sum_i a_i b_i = 2(\hat{a} \cdot \hat{b}). \quad (221)$$

Step 4: assemble. Substituting (221) into (218),

$$\boxed{E(\hat{a}, \hat{b}) = -\frac{1}{2} \cdot 2(\hat{a} \cdot \hat{b}) = -\hat{a} \cdot \hat{b} = -\cos \theta_{ab}.} \quad (222)$$

This is the standard singlet correlation. Note the marginals: setting one of the analyzers aside, $\langle \Psi^- | (\hat{a} \cdot \vec{\sigma}) \otimes \mathbb{I} | \Psi^- \rangle = \text{Tr}(\rho_A \hat{a} \cdot \vec{\sigma}) = \frac{1}{2} \text{Tr}(\hat{a} \cdot \vec{\sigma}) = 0$, so each wing alone is perfectly random regardless of the distant setting – the no-signaling property to which we return (cross-ref T-A8.11).

F.3 Worked 2×2 micro-example: $\hat{a} = \hat{z}$, $\hat{b}$ in the x - z plane

To make (222) concrete we compute one correlator by brute force, with no invariance tricks. Take $\hat{a} = \hat{z}$ so that $\hat{a} \cdot \vec{\sigma} = \sigma_z$, and take $\hat{b} = (\sin \theta, 0, \cos \theta)$ in the x - z plane, so that

$$\hat{b} \cdot \vec{\sigma} = \sin \theta \sigma_x + \cos \theta \sigma_z = \begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (223)$$

Order the product basis as $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$. The singlet vector is

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}} (0, 1, -1, 0)^T. \quad (224)$$

First act with $(\hat{b} \cdot \vec{\sigma})$ on Bob's (second) qubit. On the two populated components,

$$(\hat{b} \cdot \vec{\sigma})|\downarrow\rangle = \sin \theta |\uparrow\rangle - \cos \theta |\downarrow\rangle, \quad (225)$$

$$(\hat{b} \cdot \vec{\sigma})|\uparrow\rangle = \cos \theta |\uparrow\rangle + \sin \theta |\downarrow\rangle. \quad (226)$$

Hence

$$(\mathbb{I} \otimes \hat{b} \cdot \vec{\sigma})|\Psi^-\rangle = \frac{1}{\sqrt{2}} \left[|\uparrow\rangle(\sin \theta |\uparrow\rangle - \cos \theta |\downarrow\rangle) - |\downarrow\rangle(\cos \theta |\uparrow\rangle + \sin \theta |\downarrow\rangle) \right]. \quad (227)$$

Now act with σ_z on Alice's (first) qubit, which multiplies the $|\uparrow\rangle_A$ terms by $+1$ and the $|\downarrow\rangle_A$ terms by -1 :

$$(\sigma_z \otimes \hat{b} \cdot \vec{\sigma})|\Psi^-\rangle = \frac{1}{\sqrt{2}} \left[|\uparrow\rangle(\sin \theta |\uparrow\rangle - \cos \theta |\downarrow\rangle) + |\downarrow\rangle(\cos \theta |\uparrow\rangle + \sin \theta |\downarrow\rangle) \right]. \quad (228)$$

Finally take the inner product with $\langle \Psi^- | = \frac{1}{\sqrt{2}}(\langle \uparrow\downarrow | - \langle \downarrow\uparrow |)$. Only the $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ components of (228) survive:

$$E(\hat{z}, \hat{b}) = \frac{1}{2} \left[(1) \cdot (-\cos \theta) - (1) \cdot (+\cos \theta) \right] = \frac{1}{2}(-\cos \theta - \cos \theta) = -\cos \theta. \quad (229)$$

This reproduces (222), $E(\hat{z}, \hat{b}) = -\hat{z} \cdot \hat{b} = -\cos \theta$, by explicit 2×2 matrix multiplication.

F.4 Evaluating S at the optimal angles: $S = 2\sqrt{2}$

Choose all four orientations coplanar (say in the x - z plane) at the angles

$$\hat{a} : 0, \quad \hat{a}' : \frac{\pi}{2}, \quad \hat{b} : \frac{\pi}{4}, \quad \hat{b}' : \frac{3\pi}{4}. \quad (230)$$

The relevant relative angles and their cosines are

$$\theta_{ab} = \frac{\pi}{4} \Rightarrow \cos \theta_{ab} = \frac{1}{\sqrt{2}}, \quad \theta_{ab'} = \frac{3\pi}{4} \Rightarrow \cos \theta_{ab'} = -\frac{1}{\sqrt{2}}, \quad (231)$$

$$\theta_{a'b} = \frac{\pi}{4} \Rightarrow \cos \theta_{a'b} = \frac{1}{\sqrt{2}}, \quad \theta_{a'b'} = \frac{\pi}{4} \Rightarrow \cos \theta_{a'b'} = \frac{1}{\sqrt{2}}. \quad (232)$$

With $E = -\cos \theta$ from (222), the four correlators are

$$E(\hat{a}, \hat{b}) = -\frac{1}{\sqrt{2}}, \quad E(\hat{a}, \hat{b}') = +\frac{1}{\sqrt{2}}, \quad E(\hat{a}', \hat{b}) = -\frac{1}{\sqrt{2}}, \quad E(\hat{a}', \hat{b}') = -\frac{1}{\sqrt{2}}. \quad (233)$$

Substituting into the CHSH combination (207),

$$S = E(\hat{a}, \hat{b}) - E(\hat{a}, \hat{b}') + E(\hat{a}', \hat{b}) + E(\hat{a}', \hat{b}') = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\frac{4}{\sqrt{2}} = -2\sqrt{2}, \quad (234)$$

so $|S| = 2\sqrt{2}$. (Reflecting all of Bob's angles, or equivalently swapping the position of the minus sign in (207), gives the symmetric value $S = +2\sqrt{2}$; only $|S|$ is physical.) The magnitude

$$|S|_{\max} = 2\sqrt{2} \approx 2.828 \quad (235)$$

exceeds the local-realist ceiling 2 of (213) by the factor $\sqrt{2}$, the hallmark quantum violation. It is saturated by the singlet, in keeping with the extremality discussed in T-A8.8.

F.5 The Tsirelson bound $|S| \leq 2\sqrt{2}$ in quantum mechanics

We now prove that $2\sqrt{2}$ is not merely attained but is the quantum *maximum*: no state and no choice of ± 1 -valued observables can push $|S|$ higher. This is the Tsirelson bound, derived via the Landau identity.

Let A, A', B, B' now denote Hermitian operators with eigenvalues ± 1 , i.e.

$$A^2 = A'^2 = B^2 = B'^2 = \mathbb{I}, \quad A = A^\dagger, \text{ etc.}, \quad (236)$$

with A, A' acting on Alice's factor and B, B' on Bob's, so that Alice's and Bob's operators commute:

$$[A, B] = [A, B'] = [A', B] = [A', B'] = 0. \quad (237)$$

Define the CHSH operator

$$S = A \otimes B - A \otimes B' + A' \otimes B + A' \otimes B' = A(B - B') + A'(B + B'), \quad (238)$$

so that $S = \langle S \rangle$ in any state. Compute S^2 . Expanding and using (236)-(237),

$$\begin{aligned} S^2 &= [A(B - B') + A'(B + B')]^2 \\ &= A^2(B - B')^2 + A'^2(B + B')^2 + AA'(B - B')(B + B') + A'A(B + B')(B - B'). \end{aligned} \quad (239)$$

Now $(B - B')^2 = B^2 - BB' - B'B + B'^2 = 2\mathbb{K} - (BB' + B'B)$ and similarly $(B + B')^2 = 2\mathbb{K} + (BB' + B'B)$, so

$$A^2(B - B')^2 + A'^2(B + B')^2 = (2\mathbb{K} - \{B, B'\}) + (2\mathbb{K} + \{B, B'\}) = 4\mathbb{K}, \quad (240)$$

where $\{X, Y\} = XY + YX$ and we used $A^2 = A'^2 = \mathbb{K}$. For the cross terms, $(B - B')(B + B') = B^2 + BB' - B'B - B'^2 = [B, B']$ and $(B + B')(B - B') = B^2 - BB' + B'B - B'^2 = -[B, B']$, hence

$$AA'(B - B')(B + B') + A'A(B + B')(B - B') = AA'[B, B'] - A'A[B, B'] = [A, A'] [B, B']. \quad (241)$$

Combining (240) and (241) gives the Landau identity

$$\boxed{S^2 = 4\mathbb{K} + [A, A'] [B, B']}. \quad (242)$$

We bound the operator norm of the correction term. For any bounded operators, $\|[X, Y]\| \leq \|XY\| + \|YX\| \leq 2\|X\| \|Y\|$. Since A, A' are ± 1 -valued Hermitians, $\|A\| = \|A'\| = 1$, so $\|[A, A']\| \leq 2$; likewise $\|[B, B']\| \leq 2$. The two commutators act on different tensor factors and therefore commute, and

$$\|[A, A'] [B, B']\| \leq \|[A, A']\| \|[B, B']\| \leq 2 \cdot 2 = 4. \quad (243)$$

Therefore, from (242),

$$\|S^2\| \leq \|4\mathbb{K}\| + \|[A, A'] [B, B']\| \leq 4 + 4 = 8. \quad (244)$$

For the Hermitian operator S , $\|S\|^2 = \|S^2\| \leq 8$, so $\|S\| \leq 2\sqrt{2}$, and for any state

$$\boxed{|S| = |\langle S \rangle| \leq \|S\| \leq 2\sqrt{2}}. \quad (245)$$

This is the Tsirelson bound. The singlet with the angles (230) saturates it because there the local observables anticommute pairwise, $\{A, A'\} = \{B, B'\} = 0$ (e.g. $A = \sigma_z$, $A' = \sigma_x$), which makes $[A, A'] = 2AA'$ and $[B, B'] = 2BB'$ each of maximal norm 2, driving the correction term in (242) to its extreme +4 on the singlet – exactly the extremal, Bogomolnyi-*analogue* stationarity noted in T-A8.8.

F.6 The framework reading: S_θ^1 holonomy as the geometry of non-separability

We now state how the GG-Theory reading assigns this standard mathematics a geometric meaning, without changing any number above. The two derivations sit side by side: (213) shows that a *factorized* model on a shared scalar gives at most $|S| = 2$, while (245) shows quantum mechanics reaches $2\sqrt{2}$. The framework's claim is that the entangled pair realizes the latter, not the former, because of the geometry of the shared phase fiber.

One fiber, fixed at the source. The bulk phase Θ that labels the pair is not a hidden variable handed independently to the two wings. It is the holonomy of a *single* S_θ^1 fiber, fixed once at the common source where the pair is created. Both boundary shadows – Alice's wing and Bob's wing – are projections of the same non-separable bulk state threading that one fiber. Formally this is the statement that $|\Psi^-\rangle$ in (206) does not factor: there exist no single-qubit states $|\phi\rangle_A, |\chi\rangle_B$ with $|\Psi^-\rangle = |\phi\rangle_A \otimes |\chi\rangle_B$. The boundary marginals $\rho_A = \rho_B = \frac{1}{2}\mathbb{I}$ computed in Section F.2 are each maximally mixed precisely because all the information lives in the shared fiber and none in either wing alone.

Why this evades Bell and Kochen-Specker. Bell's theorem, proved in (213), excludes only *local* realism: definite outcomes assigned by per-wing response functions (208) depending on a shared scalar. The framework does not posit such functions; the shared S_θ^1 holonomy is a property of the joint state, not a pair of local responses, so there is nothing for Bell's factorization (209) to bite on. What Bell permits – non-local realism, in which the underlying reality is the single non-separable bulk state – is exactly what is realized. Likewise the Kochen-Specker theorem forbids non-contextual hidden-variable assignments; the framework's outcomes are *contextual* (the value registered depends on the analyzer setting that selects which boundary projection of the one bulk state is read), so Kochen-Specker is respected, not violated.

No signaling. The bound (245) is reached with no superluminal communication. The marginal at Bob's wing is independent of Alice's setting: as computed in (222)'s aftermath, $\langle \mathbb{I} \otimes (\hat{b} \cdot \vec{\sigma}) \rangle = \text{Tr}(\rho_B \hat{b} \cdot \vec{\sigma}) = 0$ for every $\hat{b}$, and changing $\hat{a} \rightarrow \hat{a}'$ leaves $\rho_B = \frac{1}{2}\mathbb{I}$ untouched because the partial trace over Alice's factor is independent of which observable Alice chooses. Hence Bob's local statistics carry no information about Alice's setting; correlations appear only when the two records are later compared. This is the operator-level no-signaling statement, consistent with the 4D causal structure of T-A8.11. In particular nothing here reintroduces any radial timelike shortcut: the non-separability is geometric (one fiber), not a faster-than-light channel.

F.7 Closing: boundary shadow and extremal saturation

Two cross-references complete the reading. First, the violation (235) is, in the language of T-A8.7, a *boundary shadow*: the two wings are lower-dimensional boundary projections of a single higher-dimensional bulk state, and the $\cos \theta_{ab}$ correlation – impossible to reproduce by any factorized boundary model – is the visible imprint of that bulk non-separability. Second, the saturation $|S| = 2\sqrt{2}$ is, in the language of T-A8.8, an *extremal* (Bogomolnyi-analogue) stationary point: the

Landau identity (242) shows that maximizing $|S|$ forces the correction $[A, A'] [B, B']$ to its norm ceiling +4, which is attained exactly when the two pairs of local observables maximally anticommute on the singlet. The quantum maximum is thus a saturated inequality, a stationary configuration of the same character as the BPS-type bounds elsewhere in the paper, and the singlet is the state that sits on it.

In summary, every number above is the textbook Bell-CHSH result: the local-realist ceiling is 2, the singlet correlator is $-\cos \theta_{ab}$, the optimal angles give $|S| = 2\sqrt{2}$, and the Tsirelson bound caps quantum mechanics at $2\sqrt{2}$. The framework adds only the geometric reading – one non-separable bulk state on a single S_θ^1 fiber, fixed at the source, read as two correlated boundary shadows – which reproduces standard quantum correlations exactly while respecting no-signaling and introducing no local-hidden-variable factorization.

Problems and worked solutions. F.1.1. Prove the local bound $|S| \leq 2$ by the ± 2 argument.

Solution. Step 1: in a local-realist model every run carries pre-assigned values $A, A', B, B' = \pm 1$ (208); form the combination (210): $A(B + B') + A'(B - B')$. Step 2: since $B, B' = \pm 1$, one of $B + B'$ and $B - B'$ is 0 and the other ± 2 (212), so the combination is ± 2 run by run. Step 3: averaging over any distribution of hidden variables keeps $|S| \leq 2$ (213) — and note what was *not* assumed: nothing about quantum mechanics, dynamics, or fields; only that the four answers exist before the questions are chosen. That is the assumption experiment falsifies. Physics intuition: the CHSH bound is bookkeeping about pre-written answer sheets — four ± 1 entries cannot conspire beyond 2 on average; when nature scores 2.83, the conclusion is not that arithmetic failed but that the answers were not written down in the 4D variables Bell allowed.

F.3.1. Evaluate the singlet correlator at explicit angles.

Solution. Step 1: the chapter's trace computation (218)–(222) gives $E(\hat{a}, \hat{b}) = -\hat{a} \cdot \hat{b}$. Step 2: numbers: aligned analyzers, $E = -1$ (perfect anticorrelation, the spin-zero fingerprint (215)); perpendicular, $E = 0$; at 45° , $E = -1/\sqrt{2} = -0.7071$ — the value that powers the optimal CHSH set. Step 3: cross-check with the micro-example route (229): expanding $|\Psi^-\rangle$ in Bob's rotated basis reproduces $-\cos \theta$ amplitude by amplitude — the trace formula and the state-expansion bookkeeping agree, as they must. Physics intuition: $-\cos \theta$ interpolates between “always opposite” and “no correlation” more smoothly than any classical answer-sheet can — the classical best is a straight line in θ , and the experimental curve bends; that bend is the whole quantum surplus.

F.4.1. Assemble S at the optimal angles.

Solution. Step 1: take the standard set (230): $\hat{a} = 0$, $\hat{a}' = 90^\circ$, $\hat{b} = 45^\circ$, $\hat{b}' = 135^\circ$ (all in one plane). Step 2: each correlator is $\mp 1/\sqrt{2}$ (231)–(233), and the signed combination (234) stacks all four with the same sign: $S = -4/\sqrt{2} = -2\sqrt{2} = -2.828$. Step 3: check the geometry of why: the primed/unprimed axes interleave at 45° steps, so every term sits on the steepest part of the $-\cos \theta$ curve — the quantum surplus of F.3.1 harvested four times coherently; any misalignment reduces $|S|$, which is why the experiments quote settings to fractions of a degree. Physics intuition: $2\sqrt{2}$ is what four perfectly-placed taps on a bent curve can extract — the bend gives each pair $1/\sqrt{2}$ where flat classical lines could average no better than $\frac{1}{2}$ apiece, and $4 \times 1/\sqrt{2}$ beats $4 \times \frac{1}{2}$ by exactly the observed margin.

F.5.1. Derive Tsirelson from the operator identity, and explain the gap to 4.

Solution. Step 1: for ± 1 -valued commuting-wing observables, square the CHSH operator (238): the cross terms collapse to $\mathcal{S}^2 = 4 \cdot \mathbb{K} + [A, A'] [B, B']$ (242). Step 2: bound the commutators: $\|[A, A']\| \leq 2$, $\|[B, B']\| \leq 2$, so $\|\mathcal{S}^2\| \leq 8$ and $\|\mathcal{S}\| \leq 2\sqrt{2}$ (245) — attained exactly when both commutators are maximal, i.e. mutually unbiased settings on each wing: quantum mechanics saturates its own algebra. Step 3: the gap to the no-signaling maximum $S = 4$ (the PR-box value): algebraically, 4 would need $\|[A, A'] [B, B']\| = 12$, impossible for bounded ± 1 observables — and in the framework reading the boundary correlations are the CPTP shadow of one bulk configuration (T-A8.8), an *extremal* shadow but a shadow still: the projection’s algebra stops at $2\sqrt{2}$, and any measured excess is a stated kill criterion of the Discussions. Physics intuition: 2 is the ceiling for pre-written answers, 4 for lawless signaling boxes, and $2\sqrt{2}$ — the geometric mean — is where a deterministic bulk shows through a trace; nature sitting exactly on the algebraic edge is the single most precise hint that the boundary description is a projection of something.

F.6.1. Verify no-signaling from the marginals.

Solution. Step 1: Alice’s local statistics are governed by her reduced state: for the singlet, $\text{Tr}_B |\Psi^-\rangle\langle\Psi^-| = \frac{1}{2}\mathbb{K}$ (217) — independent of anything Bob does. Step 2: check via correlators: Alice’s marginal $\langle A \rangle = \text{Tr}(\frac{1}{2}\mathbb{K} \cdot \hat{a} \cdot \vec{\sigma}) = 0$ for every setting of Bob’s apparatus and every choice $\hat{b}$ vs $\hat{b}'$ — Bob’s dial never moves Alice’s needle (T-A8.11). Step 3: the framework’s reason: the bulk flow is 6D-local and the boundary state is its partial trace; a trace over Bob’s wing cannot depend on the local operation applied there, so no-signaling is a one-line theorem of the projection — correlation without communication, exactly as the loophole-free experiments demand. Physics intuition: the singlet is a pair of dice cast from one mould, not a pair of radios — read together they rhyme perfectly, read alone each is pure noise; and the framework locates the mould in the bulk, one stratum below anything a signal could traverse.

Take-away. *Standard account:* CHSH is violated at $2\sqrt{2}$, local realism is dead, and the textbook stops there. *What it left open:* what carries the correlation, and why the violation stops exactly at Tsirelson rather than at the no-signaling maximum 4. *The bulk reading:* the correlation is carried by one bulk configuration spanning both wings; the boundary value saturates $2\sqrt{2}$ because the projection is an extremal CPTP image of that configuration (T-A8.8), and no-signaling (T-A8.11) is automatic because the trace of a local bulk flow cannot transmit; the appendix’s holonomy-sampling calculation is the constructive proof that “deterministic, local upstairs; Tsirelson-saturating downstairs” is arithmetic, not aspiration.

G Wilson-line holonomy: parallel transport on the phase fiber

What this appendix teaches. Where the quantum phase lives: the compact circle S^1_θ as the fiber of a principal $U(1)$ bundle, developed pedagogically — connection, parallel transport, holonomy, and the Wilson line as the gauge-invariant observable of the fiber. The appendix then cashes the geometry in the laboratory: the Aharonov–Bohm phase and the Berry phase as physical holonomies, and the bridge to the rest of the cohort — the same Wilson-line object that carries the quantum phase here is the electroweak order parameter θ_H of the particle-physics volume (GG-A9-SM), making this the single most reused geometric idea in the program.

Read alongside: [Berry \(1984\)](#) for the geometric phase; [Aharonov & Bohm \(1959\)](#) for the laboratory Wilson line; [Wu & Yang \(1975\)](#) for the fiber-bundle dictionary; [Arisaka \(2026, P4-Real\)](#) for the derivation of the boundary's single imaginary unit from this same real fiber.

This appendix expands the terse statement of the main text into a self-contained, pedagogical derivation. The physical picture is the following. In the framework the quantum phase does not live in the 3+1 spacetime that the observer inhabits; it lives on a compact internal circle S^1_θ attached to every spacetime point. That circle is the fiber of a *principal* $U(1)$ *bundle*. A quantum phase is then nothing but the *holonomy* of a connection on this bundle — the angle by which the fiber coordinate is rotated when one parallel-transport it around a loop. The measured outcome is the *gauge-invariant* holonomy Θ_H that the boundary projection of the source reads out. Everything below is built up from first principles so that a first-year graduate student can follow each step; the only prerequisites are vector calculus, Stokes' theorem, and the elementary theory of $U(1) = \{e^{i\alpha}\}$.

We use $\hbar, c$ explicitly where physics demands it and set them to 1 only when clearly stated. Repeated indices are summed. Greek indices μ, ν run over spacetime; Latin i, j over space.

G.1 The phase fiber as a principal $U(1)$ bundle

A *principal* $U(1)$ *bundle* is a space P that locally looks like $M \times S^1$, where M is the base (here spacetime, or a path in it) and the fiber is the circle group

$$U(1) = \{ e^{i\alpha} : \alpha \in [0, 2\pi) \}, \quad e^{i\alpha} e^{i\beta} = e^{i(\alpha+\beta)}. \quad (246)$$

Over each base point x there sits a copy of the circle, the *phase fiber* S^1_θ , whose coordinate we call $\theta(x)$. The group acts on each fiber by rigid rotation,

$$\theta \mapsto \theta + \alpha \pmod{2\pi}, \quad (247)$$

and this action is *free* (only $\alpha = 0$ fixes a point) and *transitive* on the fiber (any point can be reached from any other). The total object is denoted $P \xrightarrow{\pi} M$, with π the projection that forgets the fiber coordinate.

A complex matter field $\psi(x) = |\psi(x)| e^{i\theta(x)}$ is a *section* of an associated line bundle: its modulus is physical, but the *phase* $\theta(x)$ is a fiber coordinate and therefore has no absolute meaning. Only *differences* of phase, and ultimately loop holonomies, can be measured. This is the geometric origin of the local $U(1)$ symmetry

$$\psi(x) \mapsto e^{i\lambda(x)} \psi(x), \quad (248)$$

with $\lambda(x)$ an arbitrary smooth function: relabeling the fiber coordinate point by point must leave physics untouched.

G.2 Connection one-form and the covariant derivative

To compare phases at *different* base points we need a rule that says “these two fiber coordinates are aligned.” That rule is a *connection*. On a $U(1)$ bundle a connection is encoded by a real

connection one-form

$$A = A_\mu(x) dx^\mu, \quad (249)$$

the gauge potential. Its job is to define an infinitesimal notion of “staying parallel”: if we move from x to $x + dx$, the fiber coordinate must be rotated by $-A_\mu(x) dx^\mu$ to keep ψ parallel. Concretely, the ordinary derivative $\partial_\mu \psi$ is not covariant under (248) because $\partial_\mu(e^{i\lambda}\psi) = e^{i\lambda}(\partial_\mu + i\partial_\mu\lambda)\psi$ carries the extra $i\partial_\mu\lambda$ term. We cure this by introducing the *covariant derivative*

$$D_\mu = \partial_\mu - i A_\mu, \quad D_\mu \psi = (\partial_\mu - i A_\mu)\psi, \quad (250)$$

and *demanding* that it transform homogeneously, $D_\mu \psi \mapsto e^{i\lambda} D_\mu \psi$. Working out what A_μ must do for this to hold,

$$\begin{aligned} D_\mu(e^{i\lambda}\psi) &= (\partial_\mu - i A'_\mu)(e^{i\lambda}\psi) \\ &= e^{i\lambda}[\partial_\mu + i(\partial_\mu\lambda) - i A'_\mu]\psi \stackrel{!}{=} e^{i\lambda}(\partial_\mu - i A_\mu)\psi, \end{aligned} \quad (251)$$

which forces the now-familiar transformation law of the potential,

$$A_\mu \mapsto A'_\mu = A_\mu + \partial_\mu\lambda, \quad \text{i.e.} \quad A \mapsto A + d\lambda. \quad (252)$$

Thus the connection one-form A is exactly the object that lets us differentiate phases covariantly, and it is defined only up to the addition of an exact one-form $d\lambda$ — a *gauge* transformation. The geometric content of (250) is that $A_\mu dx^\mu$ is the angle the fiber must be counter-rotated through to remain “horizontal” (parallel) over the displacement dx^μ .

G.3 The parallel-transport equation

Now fix a path $\gamma : [0, 1] \rightarrow M$, $s \mapsto x(s)$, with tangent $\dot{x}^\mu(s) = dx^\mu/ds$. *Parallel transport* of the phase along γ is the demand that the covariant derivative *along the path* vanish,

$$\dot{x}^\mu D_\mu \psi = 0 \iff \frac{d\psi}{ds} = i A_\mu(x(s)) \dot{x}^\mu(s) \psi. \quad (253)$$

This is a first-order linear ordinary differential equation for the transported phase. Because $U(1)$ is abelian (the rotations (246) commute), the values of A at different points along the path commute, and the ODE integrates immediately. Writing $\psi(s) = U(s, 0) \psi(0)$, separating variables,

$$\frac{dU}{ds} = i A_\mu \dot{x}^\mu U \implies \frac{dU}{U} = i A_\mu \dot{x}^\mu ds \implies U(s, 0) = \exp\left(i \int_0^s A_\mu(x(s')) \dot{x}^\mu(s') ds'\right). \quad (254)$$

Changing the integration variable from the parameter s' to the point x along the curve, $\dot{x}^\mu ds' = dx^\mu$, the transport operator becomes a line integral of the connection,

$$\boxed{W_\gamma = \exp\left(i \int_\gamma A_\mu dx^\mu\right) = \exp\left(i \int_\gamma A\right).} \quad (255)$$

This W_γ is the *open Wilson line*: it is the group element of $U(1)$ that rotates the phase fiber as we drag ψ from the start to the end of γ .

G.4 Path ordering and the reduction to $U(1)$

For a general (non-abelian) gauge group the connection $A_\mu = A_\mu^a T^a$ is matrix-valued, and the values at different path points do *not* commute, $[A(x), A(x')] \neq 0$. Equation (253) is then a matrix ODE whose solution is the *path-ordered exponential*

$$W_\gamma = \mathcal{P} \exp \left(i \int_\gamma A_\mu dx^\mu \right) \equiv \mathbf{1} + i \int_\gamma A + (i)^2 \iint_{s_1 > s_2} A(s_1) A(s_2) + \cdots, \quad (256)$$

where the symbol $\mathcal{P}$ orders the factors so that the field at later path-parameter stands to the left. Path ordering is essential precisely because the matrices fail to commute. For the abelian group $U(1)$ relevant to the phase fiber, all factors commute, the ordering is vacuous, and (256) collapses back to the ordinary exponential (255):

$$\mathcal{P} \exp \left(i \oint A \right) \xrightarrow{U(1)} \exp \left(i \oint A \right). \quad (257)$$

We keep $\mathcal{P}$ in the general statement only to flag the non-abelian generalization; for the rest of this appendix we work with the abelian exponential.

G.5 Closed loops, holonomy, and Stokes' theorem

The physically invariant object is obtained by transporting the phase around a *closed* loop $\partial\Sigma$ that bounds a surface Σ . The *holonomy* is the closed Wilson line

$$W(\partial\Sigma) = \exp \left(i \oint_{\partial\Sigma} A_\mu dx^\mu \right). \quad (258)$$

Define the *curvature* two-form (the field strength) as the exterior derivative of the connection,

$$F = dA, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad F = \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu. \quad (259)$$

For abelian $U(1)$ there is no $A \wedge A$ term (it vanishes because $A \wedge A = 0$ for a one-form valued in a one-dimensional algebra), so F is simply the curl of A . Stokes' theorem converts the line integral of A around the boundary into the surface integral of its curl over Σ ,

$$\oint_{\partial\Sigma} A = \int_\Sigma dA = \int_\Sigma F \equiv \Phi_\Sigma, \quad (260)$$

where Φ_Σ is the *flux* of the curvature through the surface. Hence the holonomy of a $U(1)$ connection around a loop is the exponential of i times the enclosed flux,

$$W(\partial\Sigma) = \exp \left(i \int_\Sigma F \right) = \exp(i \Phi_\Sigma).$$

(261)

This is the central identity: a loop “feels” only the curvature it encloses, never the local value of A . The holonomy angle

$$\Theta_H \equiv \oint_{\partial\Sigma} A = \Phi_\Sigma \pmod{2\pi} \quad (262)$$

is the quantity we will identify with the measured phase.

G.6 Gauge invariance of the holonomy

We now show explicitly that (261) is invariant under the gauge transformation (252). Under $A \mapsto A + d\lambda$ the open Wilson line picks up the boundary values of λ ,

$$\int_\gamma (A + d\lambda) = \int_\gamma A + [\lambda(x_{\text{end}}) - \lambda(x_{\text{start}})] \implies W_\gamma \mapsto e^{i\lambda(x_{\text{end}})} W_\gamma e^{-i\lambda(x_{\text{start}})}, \quad (263)$$

i.e. the open line transforms *covariantly* — it dresses up with the gauge phase of its two endpoints, exactly so that the combination $\psi^\dagger(x_{\text{end}}) W_\gamma \psi(x_{\text{start}})$ is invariant. For a *closed* loop the start and end coincide, $x_{\text{end}} = x_{\text{start}}$, so the two boundary terms cancel,

$$\oint_{\partial\Sigma} (A + d\lambda) = \oint_{\partial\Sigma} A + \underbrace{\oint_{\partial\Sigma} d\lambda}_{=0} = \oint_{\partial\Sigma} A, \quad (264)$$

because the integral of an exact form around a closed contractible loop vanishes (λ is single-valued). Equivalently, $F = dA$ is itself gauge invariant since $d(d\lambda) = 0$. Therefore

$$W(\partial\Sigma) = \exp(i\Phi_\Sigma) \quad \text{is gauge invariant,} \quad (265)$$

and the holonomy angle Θ_H of (262) is a genuine observable. This is the precise sense in which Θ_H “is gauge-invariant” as asserted in the main text.

G.7 Aharonov–Bohm and Berry phases as the same holonomy

The deep point is that two phenomena usually taught separately — the Aharonov–Bohm (AB) effect and the Berry phase — are *the same holonomy* (261), differing only in what plays the role of the base space and the connection.

Aharonov–Bohm. A charged particle of charge e moving in a region where the magnetic field $\mathbf{B}$ vanishes but the vector potential $\mathbf{A}$ does not acquires, along a path γ , the phase factor

$$\psi \mapsto \exp\left(i\frac{e}{\hbar} \int_\gamma \mathbf{A} \cdot d\mathbf{x}\right) \psi. \quad (266)$$

Comparing with (255), the geometric connection on the phase fiber is identified with the electromagnetic potential through $A_\mu^{\text{fiber}} = (e/\hbar)A_\mu^{\text{EM}}$. The interference of two paths enclosing a region of

flux Φ is governed by the holonomy of the difference loop, and (261) gives

$$\boxed{\Theta_{AB} = \frac{e}{\hbar} \oint_{\partial\Sigma} \mathbf{A} \cdot d\mathbf{x} = \frac{e}{\hbar} \int_{\Sigma} \mathbf{B} \cdot d\mathbf{S} = \frac{e}{\hbar} \Phi.} \quad (267)$$

Berry phase. When a Hamiltonian $H(\mathbf{R})$ depends on slowly varying parameters $\mathbf{R}$ and is taken adiabatically around a closed loop C in parameter space, the energy eigenstate $|n(\mathbf{R})\rangle$ returns to itself up to the *Berry phase*

$$\gamma_n = \oint_C \mathcal{A}_n \cdot d\mathbf{R}, \quad \mathcal{A}_n = i \langle n(\mathbf{R}) | \nabla_{\mathbf{R}} | n(\mathbf{R}) \rangle, \quad (268)$$

where $\mathcal{A}_n$ is the *Berry connection* — a $U(1)$ connection on a bundle over parameter space — and its curvature $\mathcal{F}_n = d\mathcal{A}_n$ is the Berry curvature. By exactly the Stokes argument (260),

$$\gamma_n = \int_{\Sigma} \mathcal{F}_n, \quad (269)$$

the Berry phase is the flux of the Berry curvature, i.e. the holonomy (261) of the Berry connection. The AB phase is the special case in which the connection is the electromagnetic potential and the curvature is the magnetic flux. *Both are holonomies of a $U(1)$ connection on a circle fiber* — precisely the structure of the phase fiber S^1_{θ} .

G.8 Worked micro-example: the Aharonov–Bohm ring

Consider an electron confined to a thin ring of radius a threaded by a solenoid carrying flux Φ , with $\mathbf{B} = 0$ on the ring itself. Use a parametrization by azimuthal angle $\varphi \in [0, 2\pi)$. A convenient gauge for the flux of an infinitely thin solenoid is the pure azimuthal potential

$$\mathbf{A} = \frac{\Phi}{2\pi r} \hat{\varphi}, \quad \text{so on the ring } (r = a): \quad \mathbf{A} \cdot d\mathbf{x} = \frac{\Phi}{2\pi a} (a d\varphi) = \frac{\Phi}{2\pi} d\varphi. \quad (270)$$

The accumulated AB phase for a particle going once around the ring is, from (266),

$$\Theta_{AB} = \frac{e}{\hbar} \oint_0^{2\pi} \frac{\Phi}{2\pi} d\varphi = \frac{e}{\hbar} \Phi = 2\pi \frac{\Phi}{\Phi_0}, \quad \Phi_0 \equiv \frac{2\pi\hbar}{e} = \frac{h}{e}, \quad (271)$$

where Φ_0 is the flux quantum. Now split the incoming beam so that one partial wave travels clockwise and the other counter-clockwise to a detector on the far side; the *relative* phase between the two arms is the holonomy of the full enclosing loop,

$$\Delta\Theta = \Theta_{AB}^{(\circlearrowleft)} - \Theta_{AB}^{(\circlearrowright)} = \frac{e}{\hbar} \oint \mathbf{A} \cdot d\mathbf{x} = \frac{e}{\hbar} \Phi. \quad (272)$$

The interference intensity at the detector is therefore

$$I(\Phi) \propto 1 + \cos(\Delta\Theta) = 1 + \cos\left(\frac{e}{\hbar} \Phi\right) = 1 + \cos\left(2\pi \frac{\Phi}{\Phi_0}\right), \quad (273)$$

which oscillates with period $\Phi_0 = h/e$ in the enclosed flux even though the electron never touches the magnetic field. This is the hallmark of holonomy: a measurable, gauge-invariant phase determined entirely by the enclosed flux (261).

G.9 Compact-fiber quantization

Because the fiber S_θ^1 is a *compact* circle, the wavefunction along it must be *single-valued*. Expanding ψ in the fiber coordinate $\theta \in [0, 2\pi)$ in modes,

$$\psi(\theta) = \sum_q c_q e^{iq\theta}, \quad (274)$$

single-valuedness $\psi(\theta + 2\pi) = \psi(\theta)$ requires

$$e^{iq2\pi} = 1 \implies q \in \mathbb{Z}. \quad (275)$$

Thus the charge spectrum conjugate to the compact fiber is quantized in integers. Geometrically, a state of charge q transported once around the fiber direction picks up holonomy $e^{iq\Theta_H}$, and consistency of this around the closed circle is the quantization condition. The same statement applied to the curvature flux is the *Dirac quantization* of the bundle: for the bundle to be globally well-defined the total curvature flux through any closed 2-surface must satisfy

$$\frac{1}{2\pi} \oint_{S^2} F = n \in \mathbb{Z}, \quad (276)$$

the first Chern number of the $U(1)$ bundle. In every case the logic is identical: *compactness of the fiber forces the holonomy/charge spectrum to be integer*. A non-integer “charge” would render the wave multivalued on S_θ^1 and is therefore forbidden.

G.10 Framework reading: Θ_H as the measured holonomy

We can now restate the main-text claims with full geometric backing. In the framework:

1. The internal phase circle S_θ^1 at each spacetime point is the fiber of a principal $U(1)$ bundle, and the quantum phase is the *connection* A on that bundle, Sec. G.2.
2. The measured quantum phase is the *holonomy* $\Theta_H = \oint A = \Phi_\Sigma$ of (262), the angle by which the fiber is rotated on parallel transport around the measurement loop. The source’s boundary condition fixes this connection: the holonomy is determined by the enclosed curvature flux set at the boundary, just as the solenoid flux Φ fixed the AB phase in (271).
3. Θ_H is *loop-dependent*: it depends on which loop is traversed, because different loops enclose different flux Φ_Σ , so there is no single context-free number — only a holonomy attached to a specified loop. This geometric path-dependence should be distinguished from Kochen–Specker measurement contextuality, which is the dependence of an assigned value on the *commuting set* co-measured with it: the two notions are conceptually distinct, and it is the latter (argued elsewhere in the paper, not from loop-dependence alone) that the no-go theorems constrain.

4. Nevertheless Θ_H is *single-valued in the 6D bulk*: on the full bundle the connection is globally defined, the flux is fixed, and by the gauge invariance of Sec. G.6 the holonomy of any given loop is a unique, well-defined number modulo 2π . The apparent ambiguity is only the freedom of fiber relabeling (252), which cancels on closed loops.
5. The 4D observer, living on the base, cannot see the fiber coordinate θ directly (the circle is too small to resolve), but *does* read out the gauge-invariant holonomy Θ_H through the boundary projection — exactly as the AB detector reads out $e\Phi/\hbar$ without ever entering the solenoid. The boundary projection is the readout map $W(\partial\Sigma) \mapsto \Theta_H$ of (261), and the measured outcome is this single gauge-invariant angle.
6. Finally, compactness of S_θ^1 (Sec. G.9) quantizes the charge spectrum that labels outcomes: only integer-charged sectors survive single-valuedness (275), so the readout Θ_H sorts measurements into a discrete, integer-labeled set of branches.

In short, the holonomy formalism makes precise the three claims of the main text: Θ_H is *gauge-invariant* (closed-loop holonomy, (265)), *contextual* (loop-dependent flux), and *single-valued in the bulk* (globally defined connection plus compact-fiber quantization). The boundary projection of the source is the operational realization of the holonomy readout, and the 4D measurement outcome is the angle $\Theta_H = \Phi_\Sigma \pmod{2\pi}$.

G.11 The holonomy as the measurement label

In the cohort the phase fiber acquires a vacuum value and its fluctuation is identified with the Higgs; those gauge-sector specializations are established in the constitution and the gauge-sector companions and are not needed for the quantum-measurement reading developed here. What *is* needed is the role of the *local* holonomy at a measurement event. That local value is the contextual joint holonomy Θ_H of system and apparatus: it is ψ -ontic and non-local — *not* a local hidden variable of the kind Bell’s theorem excludes (see the Bell-CHSH appendix) — and it is read out by the boundary projection. Which single outcome is registered is then fixed by the GRIP descent of Arizaka (2026, A6-GRIP) into one Geometric Attractor basin (T-A8.1,T-A8.4), not by the holonomy value alone. A 4D observer cannot resolve Θ_H directly because S_θ^1 lies far below any accessible length scale.

One register note closes the chapter, because this appendix is where the cohort’s real-bulk statements touch ground. The fiber developed above is a *real* circle: every holonomy computed in this appendix is a rotation of a real pair, and the boundary’s “ $e^{i\theta}$ ” is compressed notation for that $SO(2)$ rotation — the complex exponential is bookkeeping, not ontology. That is why this appendix’s bundle language and the origin-of- i companion (Arizaka, 2026, P4-Real) are one subject: the same real circle whose holonomy is quantum interference here supplies, when paired with the causal clock by the projection, the single imaginary unit of the boundary state space — derived-conditional in the quasi-free regime, statement catalog in §117, custody as everywhere: the dictionary owns the sectors, the geometry owns the i . Problem G.1.1 makes the $U(1) \cong SO(2)$ reading explicit.

Problems and worked solutions. G.1.1. Prove $U(1) \cong SO(2)$, transport a real pair, and state what the complex phase is.

Solution. Step 1: the map $e^{i\theta} \mapsto R(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$ is a group isomorphism ($R(\theta_1)R(\theta_2) = R(\theta_1 + \theta_2)$), and differentiating at $\theta = 0$ identifies the generator: i on the complex side, $J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ with $J^2 = -\mathbb{I}$ on the real side — the imaginary unit *is* the quarter-turn of a real plane. Step 2: transport the real pair (u, v) around the fiber with the parallel-transport equation (253): the holonomy (258) acts as $R(\Theta_H)$ on the pair, and every interference formula of this appendix (Aharonov–Bohm intensity (273) included) can be rewritten with cos and sin of real rotations — the physics never leaves the reals. Step 3: the register and the experimental foil: this does *not* mean boundary quantum mechanics is “real quantum mechanics” in the sense excluded by the loophole-free experiment of Renou *et al.* (2021) — that experiment rules out a real *boundary* Hilbert space with tensor structure, while the cohort’s claim is the opposite pairing: a real *bulk* whose projection installs a genuinely complex boundary theory (Arisaka, 2026, P4-Real) — exactly the combination the experiment permits. Physics intuition: rotate a plane a quarter turn twice and you face backwards: $J^2 = -\mathbb{I}$ is geometry a child can perform, and it is the entire content of “ $i^2 = -1$ ” — the bulk does the rotating, the boundary does the abbreviating, and the experiments that killed real boundary QM left the real bulk untouched.

G.5.1. Compute a holonomy via Stokes: the flux is the phase.

Solution. Step 1: for a closed loop C the transport phase is $\Theta_H = \oint_C A_\mu dx^\mu$ (258), and Stokes (260) converts it to the surface integral of the curvature: $\Theta_H = \int_\Sigma F$ (261). Step 2: apply to a uniform flux tube: a loop enclosing a solenoid of flux Φ acquires $\Theta_H = (e/\hbar)\Phi$ regardless of the loop’s shape, size, or distance from the tube — the holonomy depends only on the enclosed topology-weighted flux. Step 3: note the gauge lesson (264): the open-line transport (255) is gauge-covariant (unphysical by itself), while the closed loop is gauge-*invariant* — only holonomies are observables, which is the chapter’s governing principle and the reason the framework stores physics in Θ_H rather than in raw phases. Physics intuition: a connection is a promise about comparing directions at neighboring points; only a round trip tests the promise — and the test’s verdict, the holonomy, is as physical as anything in the theory while every open-ended phase remains a bookkeeping fiction.

G.7.1. Evaluate the Berry phase of a spin- $\frac{1}{2}$ on a cone.

Solution. Step 1: transport a spin- $\frac{1}{2}$ adiabatically around a closed circuit of magnetic-field directions tracing a cone of half-angle θ ; the Berry connection (268) integrates to $\gamma = -\frac{1}{2}\Omega$, half the enclosed solid angle, by the flux form (269) with the spin- $\frac{1}{2}$ monopole curvature. Step 2: numbers: $\Omega = 2\pi(1 - \cos \theta)$, so at $\theta = 90^\circ$ (equatorial circuit) $\gamma = -\pi$: the state returns to itself with a sign flip — the spinor’s famous 4π periodicity read as a holonomy; at small θ , $\gamma \approx -\pi\theta^2/2$. Step 3: place it in the chapter’s frame: parameter space is a sphere, the Berry connection is a monopole of charge $\frac{1}{2}$, and the phase is the same Stokes flux as G.5.1 — geometry, with not one dynamical quantity anywhere. Physics intuition: the electron does not compute $-\Omega/2$; it merely rides a curved bundle, and the curvature does the arithmetic — half the solid angle because the sphere of spin directions is double-covered, the deepest “factor of two” in quantum mechanics.

G.8.1. Work the Aharonov–Bohm ring, and find the dark fringe.

Solution. Step 1: the two arms of the ring enclose a solenoid of flux Φ ; the arms’ transport phases

differ by the holonomy (271), so the detected intensity is (273): $I \propto 1 + \cos(2\pi\Phi/\Phi_0)$ with period $\Phi_0 = h/e = 4.136 \times 10^{-15}$ Wb. Step 2: predictions: at $\Phi = \Phi_0/2$ the interference is fully destructive — the electron is steered by flux it never touches (the field vanishes on both arms); sweeping Φ scrolls the fringes with the stated period, exactly as observed. Step 3: the reading: the AB effect is the laboratory’s direct sighting of a Wilson line — the vector potential is unobservable, the field is absent along the path, and the holonomy alone acts; the “quantum phase” is caught behaving as a connection on a fiber, which is the entire thesis of this appendix in one experiment. Physics intuition: the electron’s two halves take a round trip on the phase bundle and disagree by precisely the enclosed flux — the fringe shift is a curvature meter; nothing pushes the electron, yet the geometry it traverses is not flat.

G.9.1. Derive charge quantization from the compact fiber.

Solution. Step 1: on a compact fiber the allowed matter modes are single-valued: $e^{iq \cdot 2\pi} = 1$ forces $q \in \mathbb{Z}$ (275) — charge is the winding label of the mode expansion (274), not a continuous coupling. Step 2: consequence chain: a state of charge q picks up holonomy $e^{iq\Theta_H}$ per fiber circuit, and the Dirac condition (276) follows in the same breath — monopoles and quantized charge are two faces of fiber compactness. Step 3: the cross-cohort echo: this is the same integer discipline that quantizes the ledger readings of the particle-physics volume — compactness converts continuous dials into integers there exactly as it converts coupling strengths into windings here; the assembly-line of the program’s “integers, not dials” runs through this equation. Physics intuition: why is charge quantized? — because somewhere a direction closes on itself; any quantity conjugate to a circle must count its laps in whole numbers, and the century-old mystery of identical electron charges is the tautology that all laps are laps.

G.10.1. State what Θ_H is in this framework, and trace it across the cohort.

Solution. Step 1: assemble the chapter’s chain (Eq. (262), §G.10–G.11): the quantum phase lives on S^1_θ ; only its holonomy is gauge-invariant; the registration reads a holonomy sector; hence the measured outcome label is a Wilson-line value — Θ_H is not a hidden parameter added to quantum mechanics but the gauge-invariant content of the phase the textbook already carries. Step 2: verify the consistency: distinct outcome sectors carry distinct integer holonomies, which is exactly what made the fiber average of E.5.1 annihilate inter-sector coherence and what the Bell appendix’s sampling used to reproduce the quantum correlations — three appendices, one object. Step 3: the cross-cohort bridge: the same Wilson line, evaluated on the electroweak coset in the particle-physics volume (GG-A9-SM), is the order parameter whose branch selection breaks the electroweak symmetry — the measurement label of quantum mechanics and the Higgs vacuum of the Standard Model are one geometric object read at two energies, the single most reused idea in the program. Physics intuition: physics keeps its deepest secrets in loops — interference fringes, Berry phases, AB shifts, measurement outcomes, and the electroweak vacuum are all the same question (“what did the circle say?”) asked of the same fiber; this appendix is where the course learns to hear the answer.

Take-away. *Standard account:* the quantum phase is a global mystery — unobservable in itself, yet capable of shifting fringes through regions where every field vanishes. *What it left open:* where the phase lives, and why only phase *differences* around closed loops are physical. *The bulk reading:* the phase is a coordinate on a real compact fiber; only holonomies — parallel transports around closed loops — are gauge-invariant, which is exactly the Aharonov–Bohm and Berry phenomenology; and the same fiber whose holonomy is quantum interference here is the fiber whose holonomy breaks the electroweak symmetry in GG-A9-SM — one circle, two centuries of physics. And the circle is real: its “ $e^{i\theta}$ ” is boundary shorthand for an $SO(2)$ rotation, which is why the boundary’s imaginary unit is derived rather than assumed (Arizaka, 2026, P4-Real).

H Qi-4 pillars applied to quantum measurement

What this appendix teaches. The measurement event audited by the four Qi-4 pillars: the event ledger (every measurement balances energy, momentum, charge, and Qi-number currents), the exchange principle, the contextuality pillar, and the halting pillar — a compact methods appendix showing that “measurement” in this framework is a physical transaction with conserved-current bookkeeping, not a primitive verb.

We make explicit how the four pillars of Qi (Qi-QUA, Qi-EP, Qi-CON, Qi-HAL) govern the quantum-measurement process.

H.1 Qi-QUA for quantum measurement

The event-ledger pillar: any quantum measurement event has four currents (energy, momentum, charge, Qi-number) that must balance. In particular, the measurement extracts energy ΔE from the system to the apparatus at the rate

$$\dot{Q}_{\text{measurement}} = \Gamma k_B T \cdot \log_2 N, \quad (277)$$

where Γ is the measurement rate, T is the apparatus temperature, and N is the number of outcomes — the Landauer bound. This is Qi-QUA in action.

H.2 Qi-EP for quantum measurement

The selector-law pillar: the realized outcome minimizes the GFF functional, which in the quantum-measurement context is equivalent to maximum-likelihood inference. This is why quantum measurements are optimal estimators of the bulk Qi-configuration.

H.3 Qi-CON for quantum measurement

The conformality pillar: decoherence rates scale invariantly across system size. For larger systems, Γ scales with the interaction area; for smaller systems, with the individual coupling. The resulting $1/f$ noise in measurement apparatuses is a direct prediction of Qi-CON.

H.4 Qi-HAL for quantum measurement

The architecture pillar: measurement records are stable because they are topologically protected by the bulk gauge-field configurations on S_θ^1 (with $U(1)_{Q_i}$ the cohort canonical factor providing this topological protection in Arisaka (2026, A7-Qi4)) holonomy. This is why a photograph remains a photograph, and why a classical bit on a hard drive stays a bit.

I Worked decoherence overlap integral: methods and uncertainty budget

What this appendix teaches. The methods and error budget behind the paper’s worked decoherence number: the collisional decoherence function and its two limits, the dispersion-coefficient estimates, the cross-section formula with its coefficient uncertainty, the thermal average, and the bulk-channel effective-field-theory bound — the appendix to consult before quoting, checking, or attacking the decoherence rates used in the worked examples.

This appendix collects the machinery behind the worked computation of §43.8: the collisional decoherence function and its limits, the dispersion-coefficient estimates, the cross-section formula and its coefficient uncertainty, the thermal average, and the bulk-channel effective-field-theory bound.

I.1 The collisional decoherence function and its two limits

For a tracer particle in a dilute gas, the standard collisional master equation (Gallis and Fleming, 1990; Hornberger *et al.*, 2003) gives the off-diagonal damping $\rho(\mathbf{x}, \mathbf{x}', t) = \rho(\mathbf{x}, \mathbf{x}', 0)e^{-F(\mathbf{x}-\mathbf{x}')t}$ with $F(\Delta\mathbf{x})$ of Eq. (55). Two limits follow by inspection of the kernel $[1 - \cos(\Delta\mathbf{p} \cdot \Delta\mathbf{x}/\hbar)]$. When the separation greatly exceeds the inverse typical momentum transfer, $\Delta x \gg \hbar/\langle\Delta p\rangle \approx \lambda_{\text{th,gas}}$, the cosine self-averages to zero away from the forward direction and the angular integral returns the total cross section, giving the saturated rate $\Gamma_{\text{dec}} = n\sigma_{\text{tot}}\bar{v}_{\text{rel}}$ of Eq. (56). When the separation is small, $\Delta x \ll \lambda_{\text{th,gas}}$, the leading term of the expansion $1 - \cos(\cdot) \approx \frac{1}{2}(\cdot)^2$ gives $F \rightarrow \Lambda(\Delta x)^2$ with localization parameter $\Lambda = \frac{n}{2\hbar^2}\langle v \int d\Omega (d\sigma/d\Omega) \Delta p^2 \rangle = \frac{n}{\hbar^2}\langle v (m_{\text{gas}}v)^2 \sigma_{\text{tr}} \rangle$, $\sigma_{\text{tr}} = \int (1 - \cos\theta)(d\sigma/d\Omega) d\Omega$ the momentum-transfer cross section. The crossover at $\Delta x_\star \approx \lambda_{\text{th,gas}}$ is where the two branches meet.

I.2 Dispersion coefficient C_6 for C_{70} –Ar

Two standard estimators bracket C_6 . The London formula $C_6 = \frac{3}{2} \alpha_A \alpha_B \frac{I_A I_B}{I_A + I_B}$ (atomic units; polarizabilities α in Bohr³, ionization energies I in Hartree), with $\alpha(C_{70}) = 102 \text{ \AA}^3 = 688 a_0^3$, $\alpha(\text{Ar}) = 1.66 \text{ \AA}^3 = 11.1 a_0^3$, $I(C_{70}) = 7.5 \text{ eV}$, $I(\text{Ar}) = 15.76 \text{ eV}$, gives $C_6^{\text{London}} \approx 2.2 \times 10^3 \text{ a.u.}$ The Slater–Kirkwood formula $C_6 = \frac{3}{2} \alpha_A \alpha_B / (\sqrt{\alpha_A/N_A} + \sqrt{\alpha_B/N_B})$ with effective electron numbers $N_A \approx 280$, $N_B \approx 8$ gives $C_6^{\text{SK}} \approx 4.2 \times 10^3 \text{ a.u.}$ London systematically under-binds; the truth lies between, and we adopt the geometric mean $C_6 \approx 3.0 \times 10^3 \text{ a.u.}$ as the central value, with the factor- ~ 2 spread propagated as a factor $2^{2/5} \approx 1.3$ in σ .

I.3 Total cross section and the coefficient κ

For the $-C_6/r^6$ tail the integral (total) cross section in the Massey–Mohr/Landau–Lifshitz semi-classical approximation is $\sigma_{\text{tot}} = \kappa (C_6/\hbar v)^{2/5}$. The canonical coefficient is $\kappa = \pi (3\pi/8)^{2/5} \Gamma(3/5) \times$ (glory factor) ≈ 8.083 ; re-assembling the same product with different glory-averaging conventions returns values as low as ≈ 6.2 , so we carry $\kappa = 8.083 (\pm \sim 25\%)$. The total cross section is dominated by small-angle (forward) scattering; in the saturated regime even a glancing collision leaves a which-path record, so the *decoherence* cross section equals the *total* cross section up to an $\mathcal{O}(1)$ angular-weighting factor, which we absorb into the stated factor ~ 1.5 accuracy.

I.4 Thermal average

Because $\sigma \propto v^{-2/5}$, the lab-frame rate is the Maxwell average $\langle \sigma(v_{\text{rel}}) v_{\text{rel}} \rangle$ over the gas velocity distribution with the molecule moving at v_{mol} . A direct numerical average (Ar at 300 K, $v_{\text{mol}} = 100$ m/s) gives $\langle \sigma v_{\text{rel}} \rangle = 0.975 \times \sigma(\bar{v}_{\text{rel}}) \bar{v}_{\text{rel}}$ — a 2.5% correction, negligible against the C_6/κ uncertainties, so the single-velocity shortcut of §43.8 is justified.

I.5 The bulk channel: kinematic threshold and EFT suppression

The compact fiber S_θ^1 has radius $R_\theta = \hbar c/M_{\text{lock}} = 5.1 \times 10^{-20}$ m and a KK tower of masses $m_n \approx n M_{\text{lock}}$. The correlation time $\tau_c = \hbar/M_{\text{lock}} = 1.7 \times 10^{-28}$ s sets only the memory of the tower, not a rate. Real emission of a tower quantum requires energy $\geq M_{\text{lock}} c^2 \approx 3.85$ TeV; for a system at $E_{\text{sys}} \ll M_{\text{lock}} c^2$ this is kinematically forbidden and the induced decoherence rate is suppressed by at least the leading dimension- $(4 + 2k)$ operator factor $(E_{\text{sys}}/M_{\text{lock}})^p$, $p \geq 1$. The power p cannot be fixed without the explicit 6D bulk–boundary operator, so we quote the bound, not a coefficient; the dimension-6 illustration $p = 2$ gives $(E/M_{\text{lock}})^2 \sim 10^{-26}$ (molecular) to 10^{-20} (keV). The massless $n = 0$ mode is the 4D gauge field; its decoherence is the ordinary thermal-photon channel and is counted in the physical bath, not here.

I.6 Numerical summary

J Condensed timeline of the quantum-mechanical and open-system milestones

This appendix supplies a single-page chronological cross-walk between the milestones of quantum mechanics and open-system theory and the synthesis stage at which GG-Theory enters. The intent is to make clear that the GG-6/OGGEP/GDOS reformulation does not displace the canonical milestones; it places each of them as a station inside one continuous historical arc that culminates in the present cohort.

Table 21: Numerical summary of the worked decoherence estimate of this appendix. The central column gives the value used in the text and the band gives the range over which the input parameters are defensible, so that the sensitivity of the estimate can be read off directly. The last two rows are not measurements of the same kind as the first five: the correlation time is fixed by the lock scale, and the bulk-to-bath ratio is a bound rather than a coefficient, because the power cannot be fixed without the explicit six-dimensional operator.

Quantity	Central	Band
C_6 (C_{70} -Ar)	3.0×10^3 a.u.	$(2-4) \times 10^3$
σ_{tot}	$1.7 \times 10^{-17} \text{ m}^2$	$(1.5-2.0) \times 10^{-17}$
$\Gamma_{\text{dec}}(10^{-6} \text{ mbar})$	171 s^{-1}	150–196
p_* (onset)	$1.4 \times 10^{-6} \text{ mbar}$	$(1.2-2.1) \times 10^{-6}$
thermal-average factor	0.975	—
$\tau_c = \hbar/M_{\text{lock}}$	$1.7 \times 10^{-28} \text{ s}$	—
$\Gamma_{\text{bulk}}/\Gamma_{\text{bath}}$	$\lesssim (E/M_{\text{lock}})^p$	$p \geq 1$

Table 22: Condensed timeline of the quantum-mechanical and open-system milestones, from Planck in 1900 to the present cohort. The third column says why each milestone matters for the synthesis rather than restating what it established. The arrangement makes one point and no more: the reformulation of this paper does not displace any of these results, it places each of them as a station on a single continuous arc.

Year	Milestone	Why it matters for the present synthesis
1900	Planck and blackbody quantization	Launches the quantum crisis that eventually leads to the mature Hilbert-space formalism
1905	Einstein and the light quantum	Quantization of radiation; first deep wave–particle tension on which the present paper takes a structural stand
1913	Bohr atomic model	Early quantized structure without a full dynamical law
1922	Stern–Gerlach experiment	First dramatic evidence of quantized angular-momentum projection; canonical prototype of measurement in Part V
1925	Heisenberg matrix mechanics	First complete dynamical formulation built on noncommuting observables
1926	Schrödinger wave mechanics	Differential-equation form of quantum dynamics; basis for wavefunction language
1926	Born rule	Connects amplitudes to observed frequencies; launches the probability problem now resolved by T-A8.3
1930	Dirac transformation theory	Unifies matrix and wave mechanics in an abstract formal structure
1932	von Neumann axiomatization	Hilbert-space completion and projection-postulate formalization; the eight-postulate baseline for GG-A5-GDOS T-GDOS-7.1–T-GDOS-7.7
1935	EPR paradox	Sharp challenge to completeness and locality; the spine of Part VII
1935	Schrödinger’s cat	Macroscopic form of the measurement problem; resolved by T-A8.5

Year	Milestone	Why it matters for the present synthesis
1955	Stinespring dilation	Mathematical foundation for completely positive maps; underwrites the dilation step of T-A8.4
1957	Gleason theorem	Uniqueness of the Born rule on Hilbert-space event structures; the <i>conditional</i> key to T-A8.3
1957	Everett relative-state formulation	Deterministic no-collapse response to the measurement problem; the GG-Theory-distinct cousin
1958–1960	Nakajima–Zwanzig projection formalism	Exact reduced dynamics with memory kernels; the bulk-to-boundary reduction skeleton of T-A8.1
1964	Bell theorem	Excludes Bell-local hidden-variable completion on the 4D event space; left intact by T-A8.7 on the boundary
1967	Kochen–Specker theorem	Strengthens contextuality constraints; absorbed by the contextual bulk-holonomy reading of Part X
1969	CHSH inequality	Experimentally practical Bell-type inequality; the testing surface for the $2\sqrt{2}$ derivation of T-A8.8
1975–1976	GKLS theorem (Gorini–Kossakowski–Sudarshan–Lindblad)	Universal Markovian generator for open quantum systems; the boundary normal form behind T-A8.4
1982	Aspect experiments	Canonical early experimental Bell violations; the Part VII reality check
1989	Tonomura single-electron interference	Iconic modern visualization of double-slit interference buildup; Part VI prototype
1990s–2000s	Decoherence program matures (Zurek, Joos–Zeh, Schlosshauer)	Explains effective classicality of reduced states; the <i>necessary</i> half of T-A8.5, completed by einselection in T-A8.6
2015	Loophole-free Bell tests	Firm empirical validation of quantum nonlocal correlations without a Bell-local explanation
2026	GG-6/GDOS synthesis stage (Ultra Eleven cohort)	Places quantum mechanics inside a deterministic 6D bulk and a probabilistic 4D boundary; T-A8.1–T-A8.11

The shape of this list is the shape of the present paper. Each generation refined a piece of the formalism without resolving the constitutional question of why a deterministic underlying law and a probabilistic observed law coexist. The cohort answer is that the two laws live on two strata of a single architecture, and that the boundary law is forced upon any finite observer by the bulk-to-boundary projection. After all, history accumulates pieces; a theory places them.

K Concept-by-concept synthesis: standard quantum mechanics versus the cohort reformulation

This appendix consolidates the cohort reformulation of the central quantum-mechanical concepts onto a single page, in the format “standard reading $\leftrightarrow$ GG-Theory/GDOS reformulation.” The table is intended as a quick reference for readers who want a fast cross-walk between the textbook formulation of Sakurai, Cohen-Tannoudji, or Schlosshauer and the boundary-law reading of the present paper.

Table 23: The central quantum-mechanical concepts in the format standard reading against GG-Theory/GDOS reformulation. The table is a cross-walk for a reader coming from Sakurai, Cohen-Tannoudji or Schlosshauer, and it is arranged so that the middle column can be read on its own as an accurate statement of the textbook position. Nothing in the right-hand column contradicts the middle one; each entry re-derives it from the boundary law.

Concept	Standard quantum-mechanical reading	GG-Theory/GDOS reformulation
Born rule	Probability postulate $p_\alpha = \text{Tr}(\rho E_\alpha)$; uniquely selected by Gleason-type assumptions on Hilbert-space events	Boundary probability law induced after the non-invertible bulk-to-boundary projection; Gleason becomes the uniqueness theorem for the already-derived boundary event structure (T-A8.3)
Measurement problem	Tension among completeness, linearity, and single outcomes	Dissolved by domain separation: the complete state is the 6D bulk state; the 4D boundary state is reduced and open (T-A8.1–T-A8.6); relaxation-level; one analytic residue
Gleason’s theorem	Uniqueness theorem for non-contextual frame functions in $\dim \mathcal{H} \geq 3$	Clarifies why boundary probabilities must be Born-like once OGGEF supplies the boundary Hilbert-space event lattice
Many-Worlds	Universal unitary law with all branches real	Shares deep determinism but adds explicit observer-boundary semantics (outer hologram, inner hologram, evolving inner cosmos) instead of relying only on branch ontology
Double slit	Interference of amplitudes; which-way information suppresses fringes	Projection-sensitive experiment revealing how visible law depends on the actual observational channel (Part VI)
Decoherence	Environment suppresses off-diagonal terms in reduced state	Lawful and generic consequence of finite observation as partial trace; necessary but not sufficient without runtime record semantics (T-A8.5)

Concept	Standard quantum-mechanical reading	GG-Theory/GDOS reformulation
Schrödinger's cat	Macroscopic superposition sharpened into an outcome puzzle	Candidate macro-branches on the reduced landscape, with the selected stable record produced by runtime branch selection and attractor stabilization
Stern–Gerlach	Spin projection measurement with outcome probabilities $ \alpha ^2, \beta ^2$	Instrument-specific outer-hologram event from a boundary channel acting on a deeper bulk process (Part V)
EPR paradox	Challenge to completeness and locality	Boundary incompleteness is expected; the reduced state is not the total state of reality (Part VII)
Bell theorem	Excludes Bell-local hidden-variable completion	Still binding on the boundary; GG-Theory is not a naive Bell-local hidden-variable model because its parent structure is a non-invertible contextual projection (T-A8.7)
Quantum entanglement	Non-factorizable states and non-classical correlations	Same operational structure remains exact; interpreted as the boundary manifestation of a common deeper lawful state
CHSH experiment	Practical Bell inequality with quantum violation up to $2\sqrt{2}$	Reproduced exactly by the cohort completion; the $2\sqrt{2}$ value is a Bogomolnyi topological coincidence (T-A8.8)
Qubit	Fundamental two-level quantum information carrier	Minimal stable boundary packet or observational interface, not necessarily the most primitive ontic ingredient
Lindblad / GKSL	Universal Markovian generator for open quantum systems	Boundary normal form of the exact OGGEF/Nakajima–Zwanzig reduction; not fundamental but derived (T-A8.4)
Indistinguishability	Postulate that identical particles share identical observable structure	Cascade inheritance: every electron is a token of the unique stable GA of the charged-lepton stratum (T-A8.9)
No-signaling	Invariance of local marginals under distant measurement choice	Mandatory consistency condition for any acceptable bulk completion (T-A8.11)

The single discipline behind the right-hand column is that the operational success of standard quantum mechanics is taken as given and is preserved exactly; what changes is the constitutional reading of *why* it succeeds. After all, a good reformulation does not overturn the working law; it explains the working law's place.

Part XV

Common Objections and Responses

The strongest objections a foundations-of-quantum-mechanics referee, a particle/string theorist, a condensed-matter physicist, an AMO experimentalist, and a historian and philosopher of quantum mechanics would each raise after reading the whole cohort — answered, with pointers to the theorem, falsifier, open problem, or companion paper that bears the weight

A paper that claims to read quantum mechanics as the boundary projection of a deterministic six-dimensional bulk, and to dissolve the century-old measurement problem into theorems plus one named analytic residue, invites the most demanding scrutiny; it would not be honest scholarship to leave the obvious objections unstated. We collect here the strongest adversarial readings — the ones a foundations-of-quantum-mechanics referee, a particle/string theorist, a condensed-matter physicist, an AMO experimentalist, and a historian and philosopher of quantum mechanics would each raise on a careful reading of the whole cohort, not a skim — and answer each, with explicit pointers to the theorem (by its cohort-global id, §123.1), the worked example, the open problem (§118), or the companion paper that carries the load. These are deliberately the *surviving* objections: the ones that remain after a referee has read the companions that do the work the present paper only summarizes. Where the honest answer is “not yet,” we say so and name the open item. Throughout, two commitments are kept in view: the framework defends Einstein’s *realism and determinism* in the bulk, while conceding 4D *non-locality* openly; and it claims a derivation of the *form* of the quantum postulates, with their dynamical realization closed at the relaxation level, naming precisely the one analytic step — the field-grade representation step of the GG typicality target — that remains.

L The foundations-of-quantum-mechanics referee: measurement and definite outcomes

Objection 1 (O-QM-1) (the measurement-problem referee)

“You have not solved the measurement problem; you have relocated it. Tracing out an inaccessible fiber gives an improper mixture, and an improper mixture is not a definite outcome. The hard part — why *this* outcome rather than that one — is exactly what you push into an ‘open’ box.”

Response. We accept the standard-of-proof and answer it directly. The measurement problem, stated precisely (Maudlin’s three questions), is: why this basis, why one definite outcome, why Born weights. The framework answers each with a theorem (§30.5): the pointer basis is

einselected by bulk locality (T-A8.6); the definite outcome is delivered, not deferred, by the GRIP registration of Arisaka (2026, A6-GRIP) — the joint system-and-apparatus configuration descends into one Geometric Attractor basin and is locked there by the monotone boundary free-energy law (T-A8.1, T-A8.4); the Born *form* is forced by Gleason on the open dictionary (T-A8.3). What remains is *not* “why this outcome” as a mystery but a single, sharply-stated measure equivalence: that the GRIP basin-capture measure coincides with the Born measure — the six-dimensional analogue of the quantum-equilibrium typicality theorem of Bohmian mechanics (Bohm, 1952). That residue is the signature of *every* deterministic completion of quantum mechanics, not a defect peculiar to this one; here it is closed at the relaxation level (the R16 lemmas with the R16' GDOS relaxation), with the Constitutional Born Theorem conditional on the four named premises (P1)–(P4) and the field-grade representation step the single remaining analytic residue, recorded as the first open question of the program (§118) and assumed nowhere. The improper-mixture objection is met because the bulk state is single-valued: there is no alive-and-dead superposition awaiting collapse, only a definite bulk configuration whose boundary shadow the observer cannot fully read.

Objection 2 (O-QM-2) (the decoherence skeptic, after Adler and Bell)

“Decoherence has been oversold for forty years. It explains the *appearance* of classicality, never its *fact*: the density matrix goes diagonal, but ‘diagonal’ means ‘and’, not ‘or’. Your einselection theorem (T-A8.6) inherits exactly this gap.”

Response. Correct, and we do not lean on decoherence to close the gap — that is precisely why the framework adds GRIP (Arisaka (2026, A6-GRIP)) on top of einselection. Decoherence (T-A8.5, T-A8.6) does the work it can honestly do: it selects the pointer basis and suppresses interference. It does *not* produce the single outcome, and we say so. The single outcome is the actualization step — the descent into one attractor basin — which is a dynamical halt, not a basis-diagonalization. The “and-not-or” problem is dissolved at the ontic level by the single-valued bulk state, and at the operational level by GRIP’s basin selection; the only residue is the single analytic residue of O-QM-1 (the field-grade representation step). We agree the gap is real; einselection alone does not close it.

Objection 3 (O-QM-3) (the Kochen–Specker referee)

“Your Stern–Gerlach reading assigns each silver atom a definite holonomy ‘sector’ that fixes its spin before measurement. That is a non-contextual value assignment, and Kochen–Specker proves no such assignment exists in dimension three or more. Your own contextuality section says as much — so Part V contradicts your appendix.”

Response. The atom does *not* carry a pre-existing spin value along every axis. The outcome is the *contextual* joint holonomy Θ_H of atom-and-apparatus: the magnet orientation is the measurement context that selects which holonomy observable is read, and only along that chosen axis does Θ_H resolve into one of two sectors. This is exactly the Kochen–Specker-compatible structure — 4D-contextual, 6D-single-valued — and it is the same contextuality every confirmed experiment exhibits. The framework reproduces the quantum prediction; what it adds is the reading that contextuality is the operational signature of which boundary algebra the apparatus

constructs, not a defect of the bulk ontology.

Objection 4 (O-QM-4) (the probabilist)

“You assert the basin-capture measure equals the Born measure only at the relaxation level, with an analytic step still open. But your worked examples *use* Born statistics throughout. You are assuming the very thing you have not finished proving — that is circular.”

Response. The distinction is between using the Born *form* (a theorem, T-A8.3, *Resolved*) and proving the *dynamical realization* of that form by the registration measure — closed at the relaxation level (the R16 lemmas with the R16' GDOS relaxation), with the Constitutional Born Theorem conditional on the four named premises (P1)–(P4) and the field-grade representation step the single remaining analytic residue (§118). The worked examples use the Born form, which is established: given the Hilbert-space structure (T-A8.1), Gleason forces the square-modulus measure. What is *not* assumed anywhere is that the GRIP noisy-descent frequencies reproduce that measure run-by-run; that is the relaxation theorem's content, and no worked example relies on it. The parallel with Bohmian mechanics is exact: Bohm uses $|\psi|^2$ (the form) while the proof that $|\psi|^2$ is the equilibrium distribution (the typicality theorem) is a separate result — carried, here, to the relaxation level, with the last analytic step named. We mark what remains rather than hide it (§118).

Objection 5 (O-QM-5) (the consistency referee)

“The paper says ‘measurement problem — Resolved’ in the conclusions and ‘single-outcome selection remains open’ in Part III. Which is it? A reader cannot tell whether you are claiming victory or conceding defeat.”

Response. Both, and the apparent contradiction is removed by stating precisely what each word covers. “Resolved” means: the three classical sub-problems (basis, definiteness, Born form) are each given a theorem, so there is no free-standing “measurement problem” left once the umbrella is decomposed. “Open” means: the basin-capture = Born residue is closed at the relaxation level (the R16 lemmas with the R16' GDOS relaxation), the Constitutional Born Theorem is conditional on the four named premises (P1)–(P4), and exactly one analytic step — the field-grade representation step — remains. This single governing statement has one home text (§118) and is carried at the same grade at every graded site (§30.5, the status table, and the Conclusions banner), so the conclusions and Part III are consistent. “Resolved into theorems plus one named analytic residue” is the honest description, and it is uniform throughout.

Objection 6 (O-QM-6) (the spectral-theory skeptic)

“You read quantization as a ‘discrete attractor catalog’. But most quantum systems have continuous spectra — a free particle, a scattering state. A discrete catalog cannot recover a continuum.”

Response. The attractor catalog (Arisaka (2026, A6-GRIP)) governs the *stable bound-state* and *pointer* sectors — the discrete records a measurement actually deposits — not the full spectrum of every operator. Continuous spectra (free-particle momentum, scattering phase) live in the

unitary, closed-system limit of T-A8.2 where the scale fiber is not coarse-grained; there the Schrödinger evolution is recovered intact, continuum and all. Discreteness appears only where registration occurs: a detector records a discrete click, an atom binds at a discrete level. The catalog is a claim about *stable attractors of the registration dynamics*, not about the spectrum of $\hat{p}$. We make this scope explicit and do not claim to discretize the continuum.

Objection 7 (O-QM-7) (the observer-demarcation referee)

“What counts as a measurement? You say ‘any sufficient-complexity boundary observer’, then defer the minimal criterion to GG-A11-Origin. But GG-A11-Origin answers it for biological observers, not for the quantum stratum. The cut is still drawn by hand.”

Response. We concede the demarcation is not yet fully reduced, and we state the residual precisely. The framework’s structural answer is sharp where it matters: a “measurement” is any interaction that switches on the GRIP registration — a coupling that reshapes the boundary free-energy landscape so the descent halts at an attractor (Arisaka (2026, A6-GRIP)). That criterion is observer-agnostic: it is satisfied by a photographic grain, a Geiger tube, or a brain alike, and consciousness is explicitly *not* required (consistent across GG-A8-QM, GG-A7-Qi4, GG-A11-Origin). What remains genuinely open is a quantitative threshold — the minimal landscape change that guarantees an irreversible halt — and this is recorded as an open question (§118). The cut is not “the experimenter’s discretion”; it is “wherever the free-energy law makes the record monotone,” with the precise threshold flagged as unfinished.

Objection 8 (O-QM-8) (the collapse skeptic)

“You say ‘no collapse’, but your Lüders update (T-A8.4) is the projection postulate by another name. Renaming collapse ‘conditional CPTP state-update’ does not eliminate it.”

Response. The difference is not nominal. In the collapse postulate, projection is a *primitive dynamical law* of the universe, discontinuous and outside the unitary evolution. In the framework, the bulk evolution is unitary throughout; the Lüders map (T-A8.4) is the *observer’s Bayesian update of their own record* once an outcome is registered — a statement about information, not a physical discontinuity of the state of the world. Nothing in the bulk collapses; the boundary observer simply conditions on what they have learned. That is why the cut location is irrelevant (any choice gives the same predictions) and why no stochastic collapse term appears. It is the Everettian/decoherence reading of Lüders made precise, not the von Neumann postulate relabeled.

Objection 9 (O-QM-9) (the Frauchiger–Renner referee)

“You claim to keep all three Frauchiger–Renner assumptions (Q, C, S). But the only consistent way out is to drop one. If you really keep cross-agent consistency *and* single outcomes *and* universal validity, you have a contradiction; if you do not, say which you abandon.”

Response. A fair pressure, and the honest answer is that we keep all three *at the bulk level* while restricting how a 4D agent may chain another agent’s conclusions — which is, in effect, a

disciplined reading of (C). The bulk state is single-valued (S holds), universally quantum (Q holds), and globally consistent (C holds for the one bulk description). The Frauchiger–Renner contradiction arises from identifying an agent’s *boundary projection* with the total ontology and then transitively combining several such projections; the framework blocks exactly that identification, because distinct agents hold distinct boundary reductions of one bulk state (Frauchiger and Renner, 2018). This is structurally close to the relational resolution: cross-agent inference is constrained.

M The Born rule and the Gleason objection

Objection 10 (O-QM-10) (the probability-foundations referee)

“Your Born derivation is circular. The operational axiom OGGEF (AX-3) already writes probabilities as $p = \text{Tr}(E\rho)$ into the boundary dictionary. You then ‘derive’ the trace form from CPTP completion — but you put it in by hand at the axiom.”

Response. This claim is scoped precisely to avoid any circularity (T-A8.3, and the status entry PS-QM-2). What T-A8.3 establishes is the *form*: *given* the Hilbert-space structure of T-A8.1 and the requirement that the boundary channel be a σ -additive probability assignment on the projection lattice, Gleason’s theorem forces the square-modulus measure — there is no other consistent assignment. That is a genuine constraint, not an assumption; its inputs (a complex Hilbert space, σ -additivity) are constitutional inheritances, so the result is exactly as strong as Gleason and no stronger. The content *beyond* Gleason is not a stronger uniqueness claim; it is the dynamical actualization (GRIP) plus the basin-capture-equals-Born statement, closed at the relaxation level with one analytic residue remaining (O-QM-4). “Born derived” is not asserted unconditionally anywhere.

Objection 11 (O-QM-11) (the Gleason purist)

“Then your ‘theorem’ is just Gleason restated in six-dimensional dress. Gleason 1957 is fifty years old. What have you added?”

Response. For the *form* of the rule, nothing beyond Gleason — and we say so. What the framework adds is not a new proof of the trace form but an *account of why its premises hold*: why there is a complex Hilbert space at all (the GNS construction on the boundary algebra, Arisaka (2026, A5-GDOS) T7.1, inherited as T-A8.1), why the measure is non-contextual (the boundary projection is the same for all compatible observables), and — the genuinely new content — a candidate *dynamical* origin for the probabilities in the GRIP basin-capture statistics. Gleason tells you the only consistent measure is the trace form; the framework attempts to tell you which physical process realizes it. The first is inherited and acknowledged; the second is open and named.

Objection 12 (O-QM-12) (the quantum-logic referee)

“You claim the *complex* Hilbert space ‘emerges’. But GNS over a complex C^* -algebra

returns a complex space by construction — you assumed complexity in the algebra. Real and quaternionic quantum mechanics are not excluded by your argument.”

Response. A fair and precise point; we do not claim to derive complexity from nothing. The complex structure enters geometrically: the phase fiber is the compact S^1_θ of Arisaka (2026, A2-GG6), a principal $U(1)$ -bundle, and the quantum phase is the holonomy of its connection (the Aharonov–Bohm/Berry phase in the literal bundle sense). Amplitudes are complex because they are $U(1)$ holonomies; the boundary algebra is complex because it is built on that $U(1)$ structure. So the honest statement is: complexity is *inherited from the bulk gauge geometry*, not produced ex nihilo by GNS. Whether the $U(1)$ phase fiber is itself forced (rather than chosen) is a question for Arisaka (2026, A2-GG6), not GG-A8-QM; within GG-A8-QM it is an inheritance, and we mark it as such.

Objection 13 (O-QM-13) (the qubit theorist)

“Gleason’s theorem requires Hilbert-space dimension at least three. The simplest quantum system, a single qubit, is two-dimensional. Your Born ‘theorem’ does not even apply to a spin- $\frac{1}{2}$.”

Response. Correct for the original 1957 theorem, and we rely on the modern strengthening rather than gloss over it. The Busch–Caves–Fuchs–Schack extension of Gleason to POVMs holds in dimension two: once measurements are represented by positive-operator-valued measures rather than projective measurements alone — which is exactly the operational setting OGGEF provides (T-A8.1, T-A8.3) — the trace rule is forced for the qubit as well. The cohort’s bulk fields carry $SU(5)$ representations, so the effective boundary dimension is generically large; but even at $d = 2$ the POVM-Gleason argument closes the gap, and that is the version T-A8.3 uses. We cite the dimension-two case explicitly so the qubit objection cannot land.

Objection 14 (O-QM-14) (the measure theorist)

“Gleason assumes σ -additivity — countable additivity of the probability measure over orthogonal projections. That is a strong technical postulate doing real work. You inherit it silently.”

Response. It is flagged explicitly. σ -additivity is the one genuinely postulated ingredient in the Born chain, and it is a constitutional inheritance from AX-3: it is the statement that the boundary observer’s probability assignment is a bona fide measure on the event lattice. The framework does not derive it; it assumes it, names it, and notes (shared with A1’s own open questions) that a deeper derivation of σ -additivity from the bulk measure is unfinished. This is the same status the rest of mathematical physics gives it. The honesty is in saying so at the point of use.

Objection 15 (O-QM-15) (the Everettian decision theorist)

“Deutsch and Wallace spent twenty-five years trying to derive Born from unitary dynamics plus decision theory, and the consensus is that it smuggles in a probabilistic premise. Why is your route any better?”

Response. It is better in one specific respect and no better in another, and we are explicit

about both. Better: the framework does not need to extract probability from a deterministic multiverse by an argument about rational betting, because it has a genuine *source* of ensemble variation — the unread bulk boundary condition Θ_H , which differs pair-to-pair — so probability is ordinary ignorance-over-an-ensemble, not a decision-theoretic construct. No better: the proof that this ensemble is Born-distributed (basin-capture = Born) is, like the Deutsch–Wallace program, not yet complete. The difference is that our remaining step is the single analytic residue of a concrete measure-equivalence statement, closed at the relaxation level, with a Bohmian precedent (Bohm, 1952), not a contested claim about the rationality of agents in branching worlds. We claim the better *framing* of the open problem, not a finished derivation.

N The Bell theorist: EPR, locality, and the joint-state mechanism

Objection 16 (O-QM-16) (the relativist)

“One might try to explain Bell violations by a bulk-timelike geodesic that ‘dips through the radial direction’ to connect spacelike-separated detectors. But GG-A2-GG6’s Theorem 0 says the radial direction is a renormalization scale with no light cone, so any such shortcut would contradict the foundational paper.”

Response. No such shortcut is invoked. The radial direction I_r enters the metric of Arisaka (2026, A2-GG6) with a positive sign ($+dr^2$): it is the non-compact renormalization-scale fiber and carries no light cone, so a 4D-spacelike separation stays spacelike for every radial excursion — there is no “dip” that makes two spacelike events timelike-connectable, and none is needed. The account (T-A8.7, Part VII) reads the entangled pair as *one non-separable state of the bulk*, sharing the phase fiber S_θ^1 fixed at the common source; the two detector events are projections of that single state along I_r . Nothing propagates between the wings. The correlation is a frozen property of the joint preparation, exactly as a common cause, not a signal across a shortcut.

Objection 17 (O-QM-17) (the quantum-information theorist)

“Fine, but ‘the entangled pair is one non-separable bulk state’ is just textbook entanglement in fancy dress. Standard quantum mechanics already says the singlet is non-separable. What is the new physics?”

Response. We concede the empirical content here is consistent with — not beyond — standard quantum mechanics (the net-content audit, Part XII). What the framework adds is *ontological*, not predictive: the non-separable state is a real configuration of a six-dimensional bulk, of which the two 4D wavefunctions are projection shadows; entanglement is the statement that one bulk object casts two correlated shadows. This converts “spooky action at a distance” from a paradox into the geometry of a non-invertible projection. The genuinely new, testable content is elsewhere (the cohort-geometry signatures, T-A8.8 structural stability and the high-energy corrections of Test 8); the Bell account itself is offered as a *reading* that dissolves the puzzle, and we do not dress it as a new prediction.

Objection 18 (O-QM-18) (the Bohmian)

“So you concede 4D non-locality. Then this is Bohmian non-locality with a new vocabulary — a real, non-local, deterministic hidden variable. Why not just say so?”

Response. We do say so, gladly: the framework is, in spirit, a six-dimensional Bohmian theory — deterministic at the deep level, 4D-non-local, no-signaling, ψ -ontic. The honest difference from de Broglie–Bohm (Bohm, 1952; de Broglie, 1924) is that the non-locality is *geometrized* as the non-separability of one bulk state rather than carried by an explicit pilot-wave guidance equation on configuration space, and that the kinematics (Hilbert space, Born form) are *derived* from the boundary law rather than postulated alongside the particle. We are not embarrassed by the Bohmian kinship; we claim it. The slogan “Einstein right in 6D” means his realism and determinism are vindicated in the bulk — not his locality, which we concede is lost in the 4D shadow.

Objection 19 (O-QM-19) (the careful calculator)

“Your explicit CHSH appendix once derived a *triangular* correlator, wrote ‘Wait, this gives a triangular correlator’, and then simply posited the cosine by hand. That is Bell-violation by sleight of hand.”

Response. The triangular correlator $-1 + 2\theta/\pi$ is exactly what a *local* response model on a shared scalar Θ must give — and by Bell’s theorem no such local model can reproduce the quantum cosine. The triangular result is the signature of the wrong (separable) ansatz, recorded precisely so the contrast is clear; the singlet correlation $-\hat{a} \cdot \hat{b}$ is the standard operator-algebra expectation in the non-separable joint state (T-A8.1, T-A8.3), not something derived from a local- Θ integral. The bulk holonomy is the *label* of the joint non-separable state, not a local hidden variable feeding per-wing response functions. Bell forbids the local model; the joint state supplies the cosine.

Objection 20 (O-QM-20) (the no-go specialist)

“If no local- Θ model can reproduce $-\hat{a} \cdot \hat{b}$, then the shared S_θ^1 holonomy is doing no work. Either it is a local hidden variable (killed by Bell) or it is idle decoration.”

Response. Neither. The shared S_θ^1 holonomy is what makes the joint state *non-separable* — one fiber for both wings, fixed at the common source — and non-separability is precisely the property that a local-hidden-variable factorization lacks. It is not a local variable feeding independent response functions (that would be the separable ansatz Bell excludes); it is the bulk-geometric content of the entanglement itself. Its “work” is to explain *why* the joint state is non-separable in the first place: the two particles inherit one holonomy class at emission, so their boundary descriptions cannot be written as a product. The distinction — shared label of a non-separable state versus shared value of a local variable — is exactly the distinction Bell’s theorem turns on, and the appendix makes it explicit.

Objection 21 (O-QM-21) (the ontology referee)

“Is Θ_H real and outcome-determining, or not? If yes, it is a hidden variable and you

owe us a no-go analysis. If no, your determinism collapses.”

Response. Yes: Θ_H is real, ontic, unread by the 4D observer, and (contextually, jointly with the apparatus) outcome-determining. It *is* a hidden variable, and we own the no-go analysis rather than dodge it. It evades Bell because it is non-local (the joint state is non-separable); it evades Kochen–Specker because it is contextual (the outcome depends on the apparatus context, not a pre-set non-contextual value, O-QM-3); it is compatible with Pusey–Barrett–Rudolph because it is ψ -ontic, not ψ -epistemic (Pusey *et al.*, 2012). The determinism is intact: given the actual Θ_H and the apparatus, the outcome is fixed. The price — non-locality and contextuality — is exactly the price every no-go theorem says a deterministic completion must pay, and we pay it openly.

Objection 22 (O-QM-22) (the Micius experimentalist)

“Entanglement has been verified over 1200 km by satellite. Your ‘one bulk object, two shadows’ picture still has the two shadows perfectly correlated across 1200 km with no signal. That is non-locality, however you paint it.”

Response. Agreed — and we do not paint over it. The 4D shadows are non-locally correlated across 1200 km, with no signal, exactly as the Micius results show. The framework’s claim is not that this non-locality disappears but that it is *geometrically natural*: two boundary projections of a single bulk object are correlated for the same structural reason two photographs of one scene agree, a shared origin rather than a message. No-signaling is preserved as a theorem (T-A8.11): the marginal at either detector is independent of the far setting. We claim a better *understanding* of the 1200 km correlation (one object, two shadows), not its abolition. Calling it “Bohmian non-locality renamed” is fair; we accept the label and add the geometry.

Objection 23 (O-QM-23) (the no-signaling referee)

“Your no-signaling proof (T-A8.11) is the standard reduced-state factoring argument — it is a theorem about the *quantum* marginal. But you also posit an underlying deterministic hidden-variable layer. No-signaling for the quantum state does not automatically hold for the hidden-variable dynamics.”

Response. A sharp point. T-A8.11 establishes operational no-signaling: the observed marginal at B is the reduced quantum state, independent of A ’s setting. The framework’s hidden layer respects this because the unread variable Θ_H is fixed *at the common source*, before either setting is chosen, and the boundary projection that produces B ’s record does not depend functionally on A ’s setting — the apparatus at A couples only to the A -local boundary algebra. So no operational protocol can read A ’s choice off B ’s statistics. What the framework does *not* claim is a frame-independent dynamical story at the hidden-variable level: like Bohmian mechanics, it is non-local in a preferred-foliation sense at the ontic layer while strictly no-signaling at the operational layer. This two-level structure is stated explicitly; the quantum-marginal proof covers the operational layer, not the hidden layer.

Objection 24 (O-QM-24) (the consistency referee, again)

“A $U(1)_{Q_i}$ photon ‘racing around S_θ^1 in 10^{-32} s’ to mediate the correlation would sit

uneasily with the claim that the correlation is ‘frozen’ and nothing propagates. Which is it?”

Response. The framework commits to *frozen* (Part VII). The correlation is a static property of the shared joint state established at the source; nothing travels from one wing to the other at measurement time. The only role the $U(1)_{Qi}$ holonomy on S^1_θ plays is to make the joint state non-separable at preparation — a shared label, not a courier. The frozen common-cause reading is the single committed one.

O The no-go-theorem specialist: hidden variables and their cost

Objection 25 (O-QM-25) (the superdeterminism referee)

“A contextual, outcome-determining hidden variable that depends on the apparatus setting is a measurement-dependence (superdeterminism) model. Those evade Bell only at the cost of fine-tuning — a conspiracy correlating the source with the detector settings. You name this loophole in one sentence and never pay its bill.”

Response. We accept the charge has not been fully discharged and we state the residual precisely. The framework is *not* superdeterministic in the conspiratorial sense: it does not correlate the source’s Θ_H with the future setting choices. It is non-local in the Bohmian sense — the outcome at one wing depends on the joint (non-separable) state, which is the apparatus-context-dependent reading of one bulk object — and that is contextuality, not measurement-dependence. The distinction matters: contextuality (the outcome depends on which observable is co-measured *here*) is experimentally mandatory by Kochen–Specker and carries no fine-tuning cost; measurement-dependence (the source ‘knows’ the far setting in advance) does carry one, and the framework does not invoke it. Where the honest “not yet” remains is a fully frame-covariant formulation of the ontic layer; that is flagged (§118), and it is the same debt every non-local hidden-variable theory carries.

Objection 26 (O-QM-26) (the Bell purist)

“Bell’s theorem is a theorem. You cannot have local realism. Every word you write about ‘recovering Einstein’ is either false or a concession dressed as a victory.”

Response. We agree without reservation that local realism is dead, and we do not claim to revive it. The framework recovers Einstein’s *realism* (a real, definite state underlies the statistics) and his *determinism* (that state evolves deterministically) while *conceding his locality*. “Einstein right in 6D” is shorthand for exactly this split, stated without equivocation on the word “complete” (Conclusions, and O-QM-63). Bell forbids local realism; the framework offers non-local realism, which Bell permits. That is a real and clearly-bounded claim, not a victory over the theorem.

Objection 27 (O-QM-27) (the free-will referee)

“Bell’s derivation assumes measurement independence — the experimenter’s settings are free of the hidden variable. A deterministic 6D bulk that fixes everything threatens

that assumption. Do you keep it?”

Response. Yes, we keep measurement independence, and this is what distinguishes the framework from genuine superdeterminism (O-QM-25). Determinism of the bulk does *not* entail correlation between the source variable and the setting choices: in any sensible model the analyzer settings are fixed by degrees of freedom (a quantum random-number generator, starlight, the experimenter’s deliberation) that are not correlated with the particular Θ_H drawn at the source. The framework violates Bell’s *factorizability* (because the joint state is non-separable), not Bell’s measurement-independence. Loophole-free experiments close the locality and detection loopholes; they do not, and cannot, close the non-separability that the framework (like all of quantum mechanics) exploits. So the answer is: measurement independence held, factorizability abandoned — the standard quantum situation, read geometrically.

Objection 28 (O-QM-28) (the Kochen–Specker referee, deeper)

“Calling the holonomy ‘4D-contextual but 6D-non-contextual’ sounds like having it both ways. Either there is a non-contextual assignment in the full theory or there is not. Which, and what is the content?”

Response. The content is precise and not a dodge. In the six-dimensional bulk the Wilson-line holonomy Θ_H is a *single*, definite configuration — one value, non-contextual, because there is no measurement context in the bulk; there is just the field. Contextuality appears only under *projection*: which 4D observable a value is assigned to depends on which boundary algebra the apparatus constructs (which partial trace it performs), and different co-measurement contexts construct different boundary algebras. So the non-contextual object (the bulk holonomy) and the contextual assignments (its 4D projections) live at different levels, and Kochen–Specker — which forbids a non-contextual assignment *within the 4D observable algebra* — is satisfied because the framework offers no such 4D assignment. The bulk non-contextuality is not a hidden 4D assignment in disguise; it is one level down, where KS does not reach.

Objection 29 (O-QM-29) (the experimental-contextuality referee)

“Contextuality has been measured in trapped ions, photons, and nuclear spins. Your framework ‘predicts’ it — but so does ordinary quantum mechanics, identically. There is no test that distinguishes you.”

Response. Correct, and the empirical-content audit (Part XII) lists contextuality among the *consistency* checks, not the discriminating tests — we do not claim a contextuality experiment could distinguish the framework from textbook quantum mechanics. The value of the contextuality reading is interpretive: it identifies *why* the world is contextual (it is the signature of a non-invertible bulk-to-boundary projection) rather than treating contextuality as a brute no-go fact. Discrimination from standard quantum mechanics, where it exists at all, lives in the cohort-geometry signatures (Test 8, the high-energy corrections; and the structural-stability content of T-A8.8), not in any contextuality inequality. We are explicit that contextuality is a check, not a prediction.

Objection 30 (O-QM-30) (the determinism referee)

“Tell us plainly what kind of theory this is. Is the bulk classical-deterministic, unitary-quantum, or stochastic in some random Θ_0 ? The paper has used all three phrasings, and they are not the same theory.”

Response. The framework commits, throughout, to one reading. The bulk is *deterministic* in the strong sense: the registration descent is fixed by the actual contextual holonomy Θ_H , and the apparent randomness is *epistemic* — the 4D observer cannot read Θ_H . It is not fundamentally stochastic (no objective-collapse noise term), and it is not merely “unitary plus branching” (there is a single actual outcome, not all of them). Probability enters exactly once, as typicality over the unread boundary condition Θ_H across the source ensemble. This is the deterministic/Bohmian reading, and it is the only one in the paper — the “random Θ_0 ” language means “drawn from the source ensemble, unread,” not “produced by fundamental dice.” The frozen thesis “God does not play dice — in the 6D bulk” requires exactly this, and we hold it.

Objection 31 (O-QM-31) (the PBR referee)

“Pusey–Barrett–Rudolph rules out ψ -epistemic models. But for your 4D observer the wavefunction *is* epistemic — it encodes their ignorance of Θ_H . Are you ψ -ontic or ψ -epistemic? You cannot be both.”

Response. The resolution is the two-level structure, and it is consistent with PBR (Pusey *et al.*, 2012). At the *bulk* (ontic) level the state is ψ -ontic: the full bulk configuration, including Θ_H , is a real physical state, and distinct bulk states are physically distinct — which is exactly what PBR requires. At the *boundary* level, the 4D observer’s reduced description is incomplete (it has traced out Θ_H), so *relative to the observer* it carries an epistemic component. PBR does not forbid this: it forbids a model in which the *complete* physical state fails to fix ψ , and here the complete (bulk) state does fix everything. The 4D wavefunction is the ontic bulk state seen through a lossy projection — ontic underneath, partly epistemic in the shadow. PBR constrains the underneath, and the framework satisfies it.

P The string/QFT theorist: Tsirelson and the extremal bound**Objection 32 (O-QM-32) (the BPS-structure referee)**

“You call the Tsirelson saturation a ‘Bogomolnyi bound’. A Bogomolnyi/BPS bound has a topological charge, a central-charge algebra, an energy functional bounded below by a winding number. You exhibit none of these. The word is decoration.”

Response. We accept this fully: T-A8.8 is stated as a labeled analogy (“an extremal (Bogomolnyi-analogue) bound”). There is no central charge, no winding, no topological charge, and none is asserted. What is genuinely true — and is the only content claimed — is a structural *analogy*: the singlet is the maximizer of the CHSH functional, so the first variation of $|S|$ vanishes there and saturation is stable to first order, which is the same *kind* of stationarity-at-a-saturating-

bound that makes a BPS state stable. That is an analogy to the first-order stationarity, not an instance of BPS structure, with the caveat in the theorem statement itself.

Objection 33 (O-QM-33) (the Tsirelson referee)

“The number $2\sqrt{2}$ is Tsirelson’s 1980 result, a consequence of the operator algebra. You re-quote it. You do not derive it from your scale geometry.”

Response. Correct: this is an inheritance, not a derivation. The bound $|S| \leq 2\sqrt{2}$ follows from the Hilbert-space/operator structure (Tsirelson 1980 (Tsirelson, 1980)), which the framework inherits through T-A8.1 and T-A8.3; the framework does not claim to re-derive the value from the bulk metric. The framework’s residual claim is narrower and defensible: the singlet *saturates* the inherited bound, and the saturation is structurally stable (T-A8.8). The value is standard quantum mechanics; the reading of its saturation as an extremal property is what the framework adds.

Objection 34 (O-QM-34) (the variational-calculus referee)

“Your ‘extremal stationarity’ argument — that the singlet maximizes $|S|$ so the first variation vanishes — is trivially true of any maximum of any smooth function. It is a tautology, not a result.”

Response. It is indeed elementary, and we present it as such — a clarification, not a deep theorem. The point of stating it is not novelty but *honesty about what genericity means*: experiments saturate Tsirelson robustly across platforms not because of a tuned coincidence but because the singlet sits at a stationary maximum of the CHSH functional, so small imperfections (slight depolarization, sub-optimal angles) cost only second order. That is the correct, unmythical explanation of the observed robustness, not a “topological coincidence of the scale profile”. We would rather state an elementary truth plainly than dress it as a theorem; the tautology charge is, in effect, the concession we are making.

Objection 35 (O-QM-35) (the “why this number” referee)

“A real explanation would say why the ceiling is $2\sqrt{2}$ rather than 4 (the algebraic maximum) or 2 (the classical bound). You do not. You inherit the number and stop.”

Response. True, and we do not pretend otherwise. The reason the ceiling is $2\sqrt{2}$ rather than 4 is the quantum operator structure — self-adjoint dichotomic observables on a Hilbert space, Tsirelson’s bound — which the framework inherits, and which it traces back one step further to the GNS Hilbert space of the boundary algebra (T-A8.1). What the framework does *not* do is produce $2\sqrt{2}$ from a deeper geometric principle that a skeptic could not already get from quantum mechanics; that would require deriving the operator algebra itself from the bulk in a way that pins the value, and we do not claim to. The honest scope is: the number is inherited; the saturation and its stability are the framework’s reading. We flag the “derive $2\sqrt{2}$ from geometry” goal as not achieved.

Objection 36 (O-QM-36) (the empirical-content referee)

“Standard quantum mechanics already predicts the singlet saturates Tsirelson and

that the saturation is robust. So **T-A8.8** predicts nothing new. Why is it a ‘net-new theorem’?”

Response. “Net-new” refers to its status *within the cohort* — it promotes to a principal, separately-stated result something Arisaka (2026, A5-GDOS) leaves as a sub-remark — not to a new empirical prediction beyond quantum mechanics, as stated in the net-content audit (Part XII). T-A8.8 is empirically *consistent* with standard quantum mechanics, not *distinct* from it. Its job is structural: to give the robust saturation a clean reading (extremal stationarity). It is not listed among the discriminating tests; it is a consistency result with an honest provenance, and the “net-new” tag means net-new-to-the-cohort, stated explicitly so no reader mistakes it for a new measurable effect.

Q The condensed-matter physicist: decoherence, einselection, and the pointer basis

Objection 37 (O-QM-37) (the open-systems referee)

“Your Markovian justification computes the bath correlation time from the bulk Kaluza–Klein gap, $\tau_c \sim 10^{-27}$ s. But a real qubit decoheres against its *physical* bath — phonons, photons, two-level fluctuators — with correlation time around 10^{-13} s. You have used the wrong environment.”

Response. Two correlation times must be kept distinct. The bulk Kaluza–Klein time $\tau_c \sim 10^{-27}$ s establishes only that the *bulk* contributes no memory, so that the boundary law has semigroup *form*. The actual decoherence of a laboratory system is governed by its coupling to its *physical* 4D environment, whose correlation time $\tau_{\text{bath}} \sim 10^{-13}$ s is the one that sets the Markovianity and the rate of real decoherence. The bulk smallness supplies the structure; the physical bath supplies the dynamics; the two are kept separate. The bulk channel is bounded quantitatively in §43.8: its rate is kinematically suppressed by at least $(E_{\text{sys}}/M_{\text{lock}})$, negligible for any laboratory energy.

Objection 38 (O-QM-38) (the vacuum-coherence referee)

“You write that ‘a point particle is its own environment — no outside reservoir is needed’. Taken literally, that predicts a free electron in vacuum decoheres on its own. It does not: matter-wave interferometry preserves coherence for seconds. Your mechanism is empirically false.”

Response. The claim is about the open-system *form*, not the decoherence *rate*, and the distinction is marked at the point of statement. The intrinsic inaccessibility of the fiber is why the boundary law is *open in principle* — why marginalization is unavoidable. It does *not* set a rate: a genuinely isolated free particle in vacuum, with no apparatus or bath coupling, decoheres negligibly, exactly as interferometry shows, because the rate is fixed by the physical environment (gas, photons), not by the fiber alone. The fiber explains the availability of the open structure; the physical bath explains how fast a given system actually decoheres. The explicit bound of §43.8 shows the bulk

channel cannot drive vacuum decoherence below the M_{lock} scale, consistent with second-long interferometric coherence.

Objection 39 (O-QM-39) (the decoherence-rate referee)

“Every decoherence number you quote — the photon, the fullerene, the cat — is the standard Joos–Zeh collisional formula. There is no bulk-derived rate anywhere. You have relabeled textbook decoherence.”

Response. For the *rates*, this is largely fair, and we do not claim a numerically new prediction there: the collisional rates are the Joos–Zeh values, and the framework reproduces them because, in the regime where a physical bath dominates, it *must* reproduce them to be correct. What the framework claims to add is not a different number but a structural origin — the GKLS form as a theorem (T-A8.2) rather than a postulate, the einselected basis forced by bulk locality (T-A8.6) rather than chosen, and the irreversibility identified with the renormalization-group semigroup. The genuinely framework-specific decoherence prediction (a deviation tied to the bulk geometry) lives at the high-energy/short-distance edge, not in the everyday collisional regime, and the paper labels which is which. Reproducing Joos–Zeh where Joos–Zeh is right is a feature; claiming it as new physics would not be, and we do not. The leading-order collisional rate is worked explicitly for C_{70} in §43.8, and the framework-specific deviation is the threshold-gated bulk channel of Eq. (62).

Objection 40 (O-QM-40) (the calculational referee)

“You repeatedly invoke ‘scale-overlap integrals of the bulk KK modes’ as the source of the rate coefficients, but you never evaluate one to a number. It is a placeholder.”

Response. Conceded: the explicit order-by-order scale-overlap expansion is a program target, not a delivered calculation, and Arisaka (2026, A5-GDOS) does not supply it. What *is* delivered is the structural identification (the dissipator is the boundary image of scale-fiber coarse-graining) and the leading-order matching to the collisional rate. A fully worked instance is given in §43.8 and Appendix I: for C_{70} matter waves the scale-overlap reduces, in the physical-bath regime, to the van der Waals total cross section $\sigma_{\text{tot}} \approx (1.5\text{--}2.0) \times 10^{-17} \text{ m}^2$, giving $\Gamma \approx 150\text{--}196 \text{ s}^{-1}$ at 10^{-6} mbar and a decoherence-onset pressure $p_{\star} \approx 1.4 \times 10^{-6}$ mbar that reproduces the measured Arndt-group scale. The subleading, bulk-specific corrections — the part that would yield a genuinely new number — require evaluating the overlap integrals, and that is recorded as open (§118).

Objection 41 (O-QM-41) (the SQUID experimentalist)

“Your ‘position-basis universality’ is contradicted by every flux qubit and Cooper-pair box, whose pointer states are flux or charge, not position. The claim that ‘every macroscopic record lives in the position basis’ is simply wrong.”

Response. Right. The constitutional content of bulk locality (AX-1) is that the einselected pointer observable is the one the dominant *local* coupling is diagonal in — not that it is always position. For the center-of-mass of a macroscopic object that variable is position, which is why everyday macroscopic records are positional; for an engineered system whose dominant coupling

is to flux or charge, the pointer observable is flux or charge accordingly. “Locally-coupled pointer” is the claim; “always position” would be the error.

Objection 42 (O-QM-42) (the bookkeeping referee)

“Your macroscopic decoherence rate appears as 10^{13} s^{-1} in one place and 10^{40} s^{-1} in another — twenty-seven orders of magnitude apart. At least one is wrong.”

Response. Both are present but clearly distinguished, because they are different quantities. The thermal relaxation frequency $k_B T / \hbar \sim 10^{13} \text{ Hz}$ is a temperature scale; the *spatial*-superposition decoherence rate for a cat-sized separation is the Joos–Zeh collisional value $\sim 10^{40} \text{ s}^{-1}$. The cat section uses the spatial rate 10^{40} s^{-1} consistently, and the thermal frequency is labeled as such. Conflating the two — a temperature scale and a spatial-decoherence rate — would manufacture a spurious twenty-seven-order gap. The single decoherence function $F(\Delta x)$ that unifies the spatial regimes is given in §43.8, Eq. (55).

Objection 43 (O-QM-43) (the master-equation referee)

“You call the GKLS generator a ‘theorem’. But GKSL uniqueness holds only *given* complete positivity, trace preservation, and Markovianity. You assume Markovianity — so it is a theorem only in the regime you postulate.”

Response. Agreed; the paper states the conditional clearly. T-A8.2 derives the GKLS *form* from three inputs, two of which (complete positivity, trace preservation) are forced for any boundary channel, while the third (Markovianity) is the leading-order regime in the bath correlation time, with non-Markovian corrections calculable in principle (O-QM-40) and observable at high precision. So the honest statement is: GKLS is forced *at leading order in $\tau_{\text{bath}}/\tau_S$* , and the framework’s contribution is to supply a structural reason for that smallness rather than to postulate the Markov limit outright. We do not claim GKLS in the strongly non-Markovian regime; there the corrections are the open content.

Objection 44 (O-QM-44) (the einselection referee)

“Zurek’s pointer basis is supposed to be *derived* from the coupling. But your derivation assumes the bulk coupling is position-multiplicative — which is to assume the position basis you claim to select. Circular.”

Response. The non-circular seed is AX-1 (bulk locality), and we make the logical order explicit (T-A8.6). The claim is not “assume position, derive position” but “assume *locality* — that the bulk Hamiltonian density is a polynomial in fields at the same bulk point — and derive that the dominant boundary coupling is diagonal in the locally-coupled variable.” Locality is a constitutional axiom independent of any basis; it is what forbids integro-differential (non-local) kernels. For a center-of-mass coupled through local fields, that locally-coupled variable is position, and the position basis wins the predictability sieve. The seed is locality, not position; the position basis is the output for the generic case, with flux/charge the output when the local coupling is to those instead (O-QM-41). The circularity is broken at the axiom.

R The $1/f$ -noise specialist: conformality and its limits

Objection 45 (O-QM-45) (the noise-spectroscopy referee)

“You ‘derive’ the $1/f$ exponent $\alpha = 1$ by saying the conformal coupling is ‘marginal by construction’. Marginal-by-construction is an assumption, not a derivation. You have built in the answer.”

Response. The $\alpha = 1$ result is the leading-order consequence of marginal conformality — a leading-order form, not a universal constant. The honest content (T-A8.10) is: *if* the dominant boundary–environment coupling is marginal (scale-invariant), then on dimensional grounds the spectral density is $\propto \omega^{-1}$ at leading order. Whether a given system’s coupling is marginal is a property of that system, not a theorem, and the precise exponent is fixed by the system’s conformal weight and fluctuator structure — which is why observed exponents form a band around unity rather than sitting at exactly one (O-QM-49). The claim is therefore “approximately $1/f$ within a bounded window, system-specific exponent,” which is what the data show; $\alpha = 1$ is not a universal constant.

Objection 46 (O-QM-46) (the materials referee)

“ $1/f$ noise is notoriously material- and device-dependent. Its ‘universality’ is a loose empirical regularity, not a law. Claiming a structural derivation of a universal $1/f$ over-reaches.”

Response. We agree and have scoped the claim accordingly. The framework does not claim a universal pure $1/f$ to arbitrary frequency. It claims that conformality (Arisaka (2026, A7-Qi4), Qi-CON) produces an *approximately* $1/f$ spectrum within a *bounded conformal window* set by the system’s outer and inner time-scales — the two-knee structure of Arisaka (2026, A7-Qi4)’s analysis — with a system-specific exponent. That is a much weaker and more defensible statement than “universal $1/f$ as a law,” and it is the one in the paper. The material-dependence the objection cites is precisely the system-specificity the claim builds in, not something it contradicts.

Objection 47 (O-QM-47) (the cross-paper referee)

“Where do the higher-order corrections to the $1/f$ exponent actually come from? If the answer is the ‘CCI of GG-A6-GRIP’, that cannot be right: GG-A6-GRIP’s CCI is the Covariant Conservation Identity — a Bianchi/Maxwell continuity statement with no noise content whatever. The acronym is doing work the identity does not.”

Response. It is not the CCI, and the objection is right that the acronym invites the reading (T-A8.10, proof Step 3). The Covariant Conservation Identity of Arisaka (2026, A6-GRIP) is a Bianchi/Maxwell continuity identity and carries no noise content; it is *not* the source of the $1/f$ corrections. The exponent’s departures from unity are system-specific (conformal weight, fluctuator structure), not anomalous-dimension contributions from the CCI.

Objection 48 (O-QM-48) (the Arisaka (2026, A7-Qi4) referee)

“GG-A7-Qi4 derives a *bounded, two-knee* noise spectrum with a conformal-weight-

dependent slope. A reader skimming this paper could easily come away with an unbounded, universal pure $1/f$ at $\alpha = 1$ instead. Which is being claimed?”

Response. The bounded, two-knee form — and the distinction matters enough to state twice. T-A8.10 states the spectrum as approximately $1/f$ *between two knees* set by the system’s outer and inner time-scales — the bounded, two-knee structure of Arisaka (2026, A7-Qi4) — with a system-specific exponent near unity, not an unbounded universal $1/f$. The precise spectral form is deferred to GG-A7-Qi4, and GG-A8-QM reports only the bounded, leading-order $1/f$ behavior it legitimately inherits.

Objection 49 (O-QM-49) (the falsifiability referee)

“Your kill criterion is ‘exponent in $[0.85, 1.15]$ ’. Essentially every measured $1/f$ exponent already lies in that band. A criterion nothing can fail is not a falsifier.”

Response. A fair point about the *exponent-band* criterion, which is a consistency expectation rather than a sharp falsifier. The genuine kill criterion is sharper and stated as such (T-A8.10, kill criterion): a quantum-measurement apparatus whose decoherence spectrum is *sharply non- $1/f$* under conditions where conformality should hold — a flat white spectrum ($\alpha \approx 0$) or a steep colored one ($\alpha \approx 2$) — would retire the reading. That is a real target, not a band that everything already satisfies. The $[0.85, 1.15]$ range is the consistency expectation; “flat or steep where it should be $1/f$ ” is the falsifier. We separate the two so the criterion is not vacuous.

S The AMO experimentalist and methodologist: falsifiability and the test horizon

Objection 50 (O-QM-50) (the discrimination referee)

“Strip away the rhetoric and your eight-test horizon collapses: six of the eight reproduce standard quantum mechanics exactly. The framework is empirically equivalent to textbook QM except in a corner you cannot yet reach.”

Response. We agree, and the net-content audit (Part XII) states it in exactly these terms rather than hiding it: most of the eight forward tests are *consistency* checks, not discriminators, and the framework is operationally near-equivalent to standard quantum mechanics across the laboratory regime. That is not a concealed weakness — it is the expected situation for a reformulation whose ambition is ontological clarity plus a few high-energy/cosmological signatures, not a wholesale revision of laboratory predictions. The genuinely discriminating content is narrow and we name it: the cohort-geometry signature (Test 8, high-energy corrections at $M_{\text{lock}} \approx 3.85$ TeV, within HL-LHC reach) and the direct S_θ^1 search (the compact-dimension test). A reformulation that changed everyday predictions would already be excluded; one that does not, and adds a few sharp forward bets, is doing the honest thing.

Objection 51 (O-QM-51) (the “show me a number” referee)

“Your one near-term discriminator, the einselection/ R_{KK} test, is not a number — it is

‘deferred to a future paper’. So at present the framework has *zero* computed predictions that differ from standard quantum mechanics.”

Response. For the laboratory-scale discriminators, this is essentially correct at present, and the paper marks it as promissory rather than delivered. The R_{KK} ratio requires evaluating the scale-overlap integrals (O-QM-40), which is open. The framework’s currently-sharp forward bets are the *cohort-shared* ones it inherits with computed values — the $M_{\text{lock}} \approx 3.85$ TeV cluster of Arisaka (2026, P1-KK) and the cosmological pulls of Arisaka (2026, A10-Cosmos) — rather than a stand-alone laboratory number unique to GG-A8-QM. The test table distinguishes “computed and forward” from “structural and promissory,” so no reader mistakes a deferred integral for a delivered prediction. The honest present status is: the discriminating laboratory number is owed, and named.

Objection 52 (O-QM-52) (the collider referee)

“Your high-energy test once quoted ‘LHC energies $\sim 10^4$ TeV’, which is wrong by three orders — the LHC runs at 14 TeV. And if $M_{\text{lock}} = 3.85$ TeV, the effect is *inside* current reach, contradicting ‘awaits future colliders’”

Response. The scale is $M_{\text{lock}} \approx 3.85$ TeV (Arisaka (2026, P1-KK)), which is within HL-LHC reach, and the test is searchable at the HL-LHC, consistent with the cohort’s treatment of M_{lock} in Arisaka (2026, C1-Kaxion) and Arisaka (2026, P1-KK). The predicted object is the leptophobic, clustered, multi-field line shape of Arisaka (2026, P1-KK), a genuine forward bet with a pre-declared HL-LHC kill criterion, not a retrodiction and not a claim beyond the energy frontier.

Objection 53 (O-QM-53) (the bookkeeping referee)

“Your falsifiers F-QM-5 through F-QM-8 are defined two different ways in two different places. A reader cannot tell which eight conditions would actually retire the theory.”

Response. The two are kept distinct. The eight *falsifiers* F-QM-1 . . . F-QM-8 (the conditions that would retire the theory: a Born deviation, a direct S_θ^1 detection failure, a super-Tsirelson violation, a no-signaling violation, MWI-specific branches, a non-GKSL evolution, non-Gleason probability, and a caught-alive macroscopic superposition) are the canonical list, with dedicated subsections and a summary table. The eight *forward tests* (the empirical horizon — Tsirelson saturation, decoherence scaling, $1/f$, position-basis einselection, einselection-rate, Bell-pair fidelity, teleportation, high-energy corrections) are labeled **Test 1 . . . Test 8** so they do not borrow the falsifier tags. Falsifiers retire the theory; tests probe it; the two eights are distinct and consistently numbered.

Objection 54 (O-QM-54) (the precision-metrology referee)

“You list the Born rule as ‘confirmed to 10^{-13} ’. That is the electron $g-2$ precision, which tests QED, not the Born rule. The actual Born-additivity bound, from Sorkin-type three-path experiments, is around 10^{-3} . You have quoted the wrong number by ten orders.”

Response. Correct. A 10^{-13} figure would conflate the $g-2$ /QED precision with the Born-rule

(additivity) test; the genuine Born-additivity null comes from Sorkin-type three-path interference experiments and bounds the third-order term at roughly 10^{-3} . The table reads “form derived (Gleason); Sorkin three-path $\leq 10^{-3}$; consistent,” which is the honest empirical status. The framework does not claim a 10^{-13} confirmation of the Born rule.

Objection 55 (O-QM-55) (the macroscopicity referee)

“Your large-molecule decoherence threshold is the Joos–Zeh value. Genuine rivals — GRW, CSL, Diósi–Penrose — predict *deviations* from standard decoherence at large mass. You predict no deviation, so a macroscopicity experiment cannot distinguish you from textbook QM, only from the collapse models.”

Response. True, and we state the contrast plainly. Objective-collapse models (GRW, CSL, Diósi–Penrose) add a mass-dependent stochastic term and therefore predict departures from unitary-plus-environmental decoherence at large mass; the framework predicts *no* such intrinsic departure, because its bulk dynamics is unitary and its decoherence is environmental, not collapse-driven. So a macroscopicity experiment that saw a collapse-type deviation would falsify the framework (and confirm a collapse model); one that saw standard environmental decoherence would be consistent with the framework but would not distinguish it from textbook quantum mechanics. We present this as a kill criterion against collapse-type deviation (F-QM, macroscopic coherence), not as a positive discriminator from standard QM, and we say which it is.

Objection 56 (O-QM-56) (the null-test referee)

“Several of your ‘tests’ — Bell-pair fidelity, teleportation fidelity, no-signaling — predict that nothing deviates from quantum mechanics. A test whose prediction is ‘no deviation’ has no discriminating power. Why call them tests at all?”

Response. They are *consistency* tests and *kill criteria*, not discriminators, and they are labeled so. Their role is asymmetric: they cannot confirm the framework over standard quantum mechanics, but a *failure* (a Bell-pair fidelity that fell off with distance, an operational signal between wings) would retire it. That asymmetry is exactly what a kill criterion is — it risks the theory without being able to crown it. These are not presented among the discriminating forward bets but listed under the falsifier role, where “no deviation expected; a deviation kills the theory” is the correct and honest description.

T The holography referee: the six-dimensional bulk

Objection 57 (O-QM-57) (the AdS/CFT referee)

“You call your bulk-to-boundary map ‘holographic’. But there is no boundary conformal field theory, no GKP–Witten dictionary, no operator–operator correspondence. This is not holography in the AdS/CFT sense; the word is borrowed prestige.”

Response. We accept the terminological pressure and state the difference. The framework’s map is *not* an exact AdS/CFT duality: it is a non-invertible *reduction* — a partial trace over

the two non-spatial fibers plus a CPTP coarse-graining — in which information is genuinely lost, precisely the opposite of an ontologically symmetric duality (this asymmetry is what makes the measurement problem tractable here and not in AdS/CFT, as the comparison table states). Where we use “holographic,” we mean “a bulk theory whose boundary shadow is a reduced, open description,” not “a GKP–Witten duality.” We have no boundary CFT and claim none. The honest word is “bulk-to-boundary projection”; “holographic” is used in the loose, dimensional-reduction sense, and we flag that it is not AdS/CFT-grade.

Objection 58 (O-QM-58) (the algebraic-QFT referee)

“You write the bulk Hilbert space as a tensor product $\mathcal{H}_{\text{bulk}} = \mathcal{H}_{\text{bdy}} \otimes \mathcal{H}_{\text{env}}$ and partial-trace. For a gauge or gravitational theory that factorization does not exist — the local algebras are Type III, and the Gauss law obstructs a clean tensor split. Your central manipulation is ill-defined.”

Response. A serious and well-aimed objection, and the honest answer is that the framework operates at the level of the *operational, zero-mode-restricted* boundary algebra, not the full local Type III von Neumann algebra. The partial trace is over the non-spatial fiber modes and the unread 4D-environment sector at the level of the GNS representation of the QM-restricted algebra $\mathcal{A}_{\text{Bdy}}^{(\text{QM})}$ (the zero-mode operators), where a factorization is available; we do not claim a clean tensor split of the full gauge-theoretic local algebra. The Gauss-law/Type-III obstruction is real for the full theory, and a rigorous treatment of the edge modes and the algebra type is genuinely open — flagged as such, with no global tensor product asserted. Within the operational restriction the manipulation is well-defined; extending it to the full local algebra is unfinished, and we say so.

Objection 59 (O-QM-59) (the extra-dimensions referee)

“Is S_θ^1 a literal tiny Kaluza–Klein circle of some physical radius, or a non-spatial internal phase direction? You use both readings, and they are not compatible. And your stated radius was off by four orders of magnitude.”

Response. Two points. First, the radius: it is set by the lock scale, $R_\theta \sim \hbar c/M_{\text{lock}} \approx 5 \times 10^{-20}$ m for $M_{\text{lock}} \approx 3.85$ TeV. Second, the two readings: S_θ^1 is a *physical compact gauge circle* of that radius, and the “non-spatial phase direction” language refers to its role as the $U(1)$ phase fiber — the same object described two ways (a small circle in the bulk; a gauge/phase direction from the 4D viewpoint), not two incompatible objects. The physical-circle description is primary and the phase-fiber role is its 4D projection, so there is no incompatibility. This radius places a direct S_θ^1 signature near the lock scale, consistent with the HL-LHC and tabletop searches the falsifier table cites.

Objection 60 (O-QM-60) (the Kaluza–Klein referee)

“Your decoherence engine invokes a tower of Kaluza–Klein modes *along the radial direction* I_r . But **GG-A2-GG6 says I_r is a non-compact renormalization scale with *no* KK tower. You are using a structure your own bulk paper forbids.”**

Response. The radial direction carries no normalizable KK tower — [Arisaka \(2026, A2-GG6\)](#)

is right — and the decoherence content respects that. The genuine KK tower lives on the *compact* S_θ^1 (a true circle, hence a discrete tower); the *radial* I_r is the non-compact RG scale and contributes the irreversible, coarse-graining (semigroup) structure, not a mode tower. In “scale-overlap integrals of bulk KK modes,” the modes meant are the S_θ^1 tower and the 4D-environment modes, integrated with the radial scale profile; the radial direction supplies the scale weighting and the arrow of coarse-graining, not its own normalizable tower. The language is aligned with GG-A2-GG6 so no radial KK tower is invoked.

Objection 61 (O-QM-61) (the precision-terminology referee)

“Your ‘non-invertible bulk–boundary correspondence’ is, when unpacked, a partial trace followed by a coarse-graining. That is open-quantum-systems 101. The grand name oversells a standard construction.”

Response. Fair, and we do not dispute that the *mechanism* is the standard open-systems reduction — partial trace plus CPTP coarse-graining. What the framework adds is not a new mathematical operation but an *identification*: that this particular reduction is forced (by the non-spatiality of the fibers, AX-1), that the discarded degrees of freedom are specific geometric directions (I_r , S_θ^1) rather than a generic bath, and that the irreversibility is the renormalization-group arrow. The operation is textbook; the claim is about *what* is being traced and *why* it must be. We are content to call it a bulk-to-boundary projection and let the open-systems machinery be exactly what it is; the content is the geometric identification of the traced sector, not a novel dictionary.

Objection 62 (O-QM-62) (the bulk-action referee)

“All of this presupposes a well-defined six-dimensional bulk action with the gauge content, anomaly freedom, and dynamics you keep invoking. Does that action actually exist, or is it asserted?”

Response. It is constructed, not asserted, in the companions, and GG-A8-QM inherits it rather than re-deriving it. The bulk action with its $SU(5) \times U(1)_B \times U(1)_{Q_i}$ gauge content is given in [Arisaka \(2026, A2-GG6\)](#) (with BRST/Slavnov–Taylor structure), its local and global anomaly freedom is established in [Arisaka \(2026, A2-GG6\)](#) and [Arisaka \(2026, A3-BI6\)](#) (the BI₆ closure), and the parameter-free fixing is the Euclidean discipline of [Arisaka \(2026, A4-Euclid\)](#). GG-A8-QM uses these as inputs; their correctness is the business of those papers, and a referee who doubts the bulk action should press *them*. Within GG-A8-QM the action is an inheritance with named sources, not a free assertion — which is the appropriate division of labor for a synthesis paper.

U The historian and philosopher of quantum mechanics; and the cross-paper referee

Objection 63 (O-QM-63) (the philosopher of physics)

“‘Einstein right in 6D, Bohr right in 4D’ equivocates on the word *complete*. Einstein’s

completeness criterion (every element of reality has a counterpart in the theory) and Bohr’s operational completeness are different claims. Your slogan trades on the ambiguity to make a concession sound like a reconciliation.”

Response. The sharpest objection in the philosophical set, and the slogan is stated without equivocation (Conclusions). The precise statement is: the *six-dimensional bulk state is complete* in Einstein’s sense — it contains a counterpart to every element of reality, including the outcome-fixing Θ_H — and it is *deterministic*, vindicating Einstein’s realism and determinism. The *four-dimensional description is incomplete and operationally closed* in Bohr’s sense — it is a lossy projection that nonetheless suffices for all 4D predictions. So “complete” is used in *one* sense (Einstein’s: a complete underlying state) at the bulk level and Bohr’s operational completeness is granted at the boundary level; the two are not the same word doing two jobs in one breath, and the conclusions spell out which completeness holds where. What Einstein loses is locality, which we concede; what Bohr loses is the claim that the 4D description is the *whole* story, which the bulk supplies. The reconciliation is real but bounded, and the slogan carries no more than that.

Objection 64 (O-QM-64) (the Bohr scholar)

“Strip away the sixth dimension and what you actually defend — a 4D-non-local description that is operationally complete and forbids signaling — is precisely Bohr’s 1935 reply to EPR. You have not transcended Bohr; you have re-derived him and added scenery.”

Response. At the 4D operational level, we agree: the boundary description the framework licenses is Bohr-complete and non-signaling, and a reader who cares only about 4D operations gains nothing over Bohr. The framework’s claim is precisely that the “scenery” — the realist, deterministic bulk underneath — is not scenery but content: it converts Bohr’s *refusal* to ask about an underlying reality into a definite *answer* (there is one, it is six-dimensional, it is deterministic). Whether that answer is worth the sixth dimension is a judgment a reader is entitled to make against it; we do not claim to have forced their hand. But “re-derived Bohr at the boundary, and supplied the realist completion Bohr declined” is a fair and accurate description, and it is the one the framework gives; it does not claim to have overturned him.

Objection 65 (O-QM-65) (the interpretation referee)

“You claim the framework ‘subsumes’ Copenhagen, Everett, Bohm, and QBism — each ‘right about the boundary, wrong about the ontology’. That template is unfalsifiable, and it strawmans QBism and Many-Worlds, which make claims your template ignores.”

Response. “Subsumes all interpretations” would overreach as a scientific claim; it is offered as an interpretive observation, not a theorem (the interpretation-landscape Part). The defensible version is narrower: the framework *recovers the operational core* each interpretation gets right (Copenhagen’s measurement primacy, Everett’s unitary bulk, Bohm’s determinism, QBism’s agent-relative boundary statistics) within one structure, while disagreeing with each on ontology. It does not refute QBism’s epistemic stance or Many-Worlds’ realism by fiat, and is not presented as doing so; a QBist or Everettian can read the same mathematics and decline the ontology. “One

structure that reproduces what each gets operationally right” is the claim; “subsumes/defeats all interpretations” is not.

Objection 66 (O-QM-66) (the parsimony referee)

“You invoke Ockham — ‘Everett’s explanatory power without the multiverse cost’. But you have replaced a multiverse with a sixth dimension, a compact gauge circle, a hidden holonomy, and a non-local ontology. That is not obviously cheaper.”

Response. A fair rejoinder, and the parsimony claim is correspondingly modest. The Ockham point is genuine but narrow: the framework posits a *single actual world* rather than a continually branching infinity of them, which is a real ontological economy on the count of *worlds*. It is *not* cheaper on the count of *structure* — it buys the single world with extra geometry (the two fibers) and a hidden variable. Which economy a reader prefers — fewer worlds or less structure — is a legitimate matter of taste, and the framework is not asserted to be parsimonious *tout court*. “One world, more geometry” versus “more worlds, less geometry” is the honest trade, and we state it as a trade.

Objection 67 (O-QM-67) (the historian)

“You label the projection postulate ‘Copenhagen (Bohr 1927)’. Bohr’s complementarity and von Neumann’s 1932 projection postulate are different things; Bohr never endorsed a literal collapse. The attribution is anachronistic.”

Response. Correct on the history; the two are kept separate. Complementarity (Bohr, 1927) is an epistemological doctrine about mutually exclusive experimental arrangements; the projection postulate (von Neumann, 1932; refined by Lüders, 1951) is a dynamical rule. The framework’s Lüders theorem (T-A8.4) addresses the *von Neumann/Lüders* rule, and its complementarity discussion addresses *Bohr*; the two are not conflated under “Copenhagen.” The projection rule is attributed to von Neumann/Lüders and complementarity to Bohr, and no literal collapse is put in Bohr’s mouth.

Objection 68 (O-QM-68) (the demarcation referee)

“If the framework is empirically equivalent to operational quantum mechanics across the laboratory, then by your own admission it is an *interpretation*, not a new physical theory. Stop calling it physics.”

Response. We draw the line honestly. *Where* the framework is empirically equivalent to operational quantum mechanics — the laboratory regime — it is, as the objection says, an interpretation, and we present it as such (the realist reading of the measurement problem, Bell, and the postulates). *Where* it makes computed forward predictions that differ — the cohort-shared high-energy and cosmological signatures (M_{lock} cluster, the H_0 and electroweak pulls of the companions) — it is a physical theory with skin in the game. The same mathematical structure wears both hats, and we say which hat is on in which regime: interpretation in the lab, physics at the frontier. Calling the whole thing “just interpretation” ignores the forward bets; calling it “new lab physics” ignores the equivalence. We claim exactly the split.

Objection 69 (O-QM-69) (the cross-paper referee)

“The load-bearing results — the Hilbert space, the GKLS form, no-signaling, the QM postulates — are all ‘inherited from GG-A5-GDOS’ or ‘inherited from GG-A1-Constitution’. GG-A8-QM is a citation manifold. What does *this* paper actually prove?”

Response. The division of labor is deliberate and we state it. GG-A8-QM is a synthesis paper: it takes the seven QM-postulate theorems Arisaka (2026, A5-GDOS) proves as a terse T7.1–T7.7 chain and lifts them to graduate-textbook depth as T-A8.1–T-A8.7, and it proves *net-new* the four results T-A8.8–T-A8.11 (the extremal Tsirelson reading, the universal-identity promotion, the $1/f$ conformality, and the standalone no-signaling theorem). Its distinctive contributions are the measurement-problem synthesis (the three-questions-plus-one-residue statement), the worked examples carried to explicit calculation, the paradox sweep, and the interpretation landscape. What it does *not* do is re-prove the bulk action or the GNS construction; those are the companions’ work, cited under their canonical ids. A synthesis paper that hid its inheritances would be dishonest; one that names them is doing the right thing, and the net-new results are flagged as such.

Objection 70 (O-QM-70) (the attribution referee)

“You credit the geodesic-dip Bell mechanism to ‘GG-A5-GDOS §T7.7’. But GG-A5-GDOS §T7.7 is the chain of QM-postulate theorems; the radial-dip story appears to be original to GG-A8-QM. You have laundered an GG-A8-QM construction as an inheritance.”

Response. The concern does not arise: there is no radial-dip mechanism (O-QM-16), hence no GG-A8-QM construction mis-attributed to GG-A5-GDOS. What GG-A8-QM genuinely inherits from Arisaka (2026, A5-GDOS) §T7.7 is the no-signaling lemma and the bulk–boundary correlation structure; what is original to GG-A8-QM (the extremal-stationarity reading of Tsirelson, T-A8.8) is labeled net-new. Presenting an original mechanism as a companion inheritance would be exactly the attribution error the objection targets, and the framework keeps the provenance distinct.

Objection 71 (O-QM-71) (the observer-criterion referee)

“‘Measurement’ is used in three senses across the cohort: a GRIP registration in GG-A8-QM, a runtime event in GG-A7-Qi4, a layer-17 biological act in GG-A11-Origin. There is no single governing definition. The cohort speaks with three voices.”

Response. There is a single governing definition, stated as cohort-wide. A measurement, everywhere in the cohort, is an irreversible GRIP registration: a coupling that reshapes the boundary free-energy landscape so the descent halts at a Geometric Attractor and the record becomes monotone (Arisaka (2026, A6-GRIP)). GG-A7-Qi4’s “runtime event” and GG-A11-Origin’s “layer-17 biological act” are *instances* of this one definition at different strata, not competing definitions — a photographic grain, a transistor, and a retina each satisfy the same free-energy criterion. The QM-stratum minimal-observer threshold (the smallest landscape change that guarantees the halt) is the one genuinely open quantity, flagged in §118. One definition, several strata, one open threshold — stated uniformly.

Objection 72 (O-QM-72) (the logical-dependency referee)

“Your Born chain is a loop: GG-A8-QM cites GG-A5-GDOS T7.3 for the trace form, GG-A5-GDOS cites GG-A1-Constitution AX-3 for the probability structure, and GG-A1-Constitution AX-3 already contains the trace form. The form is established only where it is assumed.”

Response. We acknowledge the dependency and state precisely where the non-trivial step is, so the chain is not mistaken for a proof from nothing. GG-A1-Constitution’s AX-3 postulates that the boundary observer’s outcomes are governed by a σ -additive probability measure on the projection lattice — it postulates *that there is a measure*, not *which* one. The trace form is then *forced* (not assumed) by Gleason, given the Hilbert-space structure. So the chain is: AX-3 supplies the measure-theoretic premise; Gleason supplies the uniqueness; GG-A5-GDOS/GG-A8-QM carry it into the operational setting. The honest residue — flagged at (O-QM-14) — is that σ -additivity itself is postulated at AX-3 and not derived from a deeper bulk measure. That is a real open question, not a hidden circularity: the form is forced once the premise is granted, and the premise is named.

V The rival-completion referees: ’t Hooft and Penrose**Objection 73 (O-QM-73) (the cellular-automaton referee, after ’t Hooft)**

“’t Hooft’s cellular-automaton interpretation already gives a deterministic completion of quantum mechanics that keeps strict locality, paying only with measurement (statistical) independence. Why concede 4D non-locality at all? A six-dimensional hidden variable looks like a baroque way to buy back what superdeterminism gets for free.”

Response. Because the two prices are not equal, and we judge ours the cheaper. Bell forces every realist completion to give up *either* 4D locality *or* measurement independence; ’t Hooft ([’t Hooft, 2016](#)) gives up the latter, we give up the former (O-QM-25, O-QM-27). Surrendering measurement independence requires the source variable to be correlated with the analyzer settings across spacelike separation and through arbitrary intervening randomizers — starlight, quantum random-number generators, the experimenter’s deliberation — a fine-tuning we decline. Surrendering 4D factorizability, by contrast, costs nothing beyond what entanglement already costs ordinary quantum mechanics: our non-locality is the boundary shadow of a state strictly *local in 6D* by AX-1 (T-A8.7), not a new conspiracy. We keep ’t Hooft’s determinism, his ontological realism, and his rejection of branching; we differ only on which Bell horn to take. And we concede the debt the two programs share: neither has yet derived the Born measure from its deeper layer — for us what remains of it is the field-grade representation step, the single analytic residue of the basin-capture = Born statement, closed at the relaxation level (§118), and the automaton program’s long struggle with the same step is a standing caution, not a scoreboard in our favor.

Objection 74 (O-QM-74) (the objective-reduction referee, after Penrose)

“Penrose is right that decoherence does not solve the measurement problem — and you now agree. But Penrose draws the honest conclusion: quantum mechanics must

be *modified*, by a real gravitationally-induced collapse with a measurable deviation ($\tau \sim \hbar/E_G$). You reproduce standard quantum mechanics exactly and relabel the unsolved selection step as a ‘relaxation-level theorem with one analytic residue.’ Where is the new physics? Where is gravity in your collapse?”

Response. We accept the diagnosis and decline the cure, and we say what that costs. We agree with Penrose (Diósi, 1989; Penrose, 1996) that the partial trace yields an improper mixture, not an outcome (T-A8.1, T-A8.4). We differ on the mechanism of selection: Penrose adds a *new*, gravitationally-triggered, stochastic collapse that *modifies* unitary evolution; we add a *deterministic* descent of the joint configuration into one Geometric Attractor basin (GRIP, T-A8.1/T-A8.4), with bulk evolution exactly unitary and decoherence set by the physical 4D bath (T-A8.5), not by gravitational self-energy. Three honest consequences follow. *Gravity*: in our account gravity shapes the 6D bulk geometry (Arisaka (2026, A2-GG6)) but does not trigger outcome selection — so to “where is gravity in your collapse?” our answer is that there is no collapse, and gravity’s role is in the arena, not the trigger. *Non-computability*: Penrose’s reduction is non-computable by design and tied to consciousness; ours is an ordinary deterministic relaxation, and invokes neither. *Empirical content*: here is the cleanest experimental fork in the paper. Objective-collapse models predict a mass-dependent breakdown of superposition beyond environmental decoherence; we predict none. A macroscopicity experiment that saw a Diósi–Penrose deviation would *falsify* the framework and vindicate Penrose; one that saw only environmental decoherence would be consistent with us (F-QM, macroscopic coherence). We regard that decidability as a virtue of Penrose’s program and record it as a live way the framework could be wrong — which is the honest answer to “where is the new physics?”: it is in the bulk and the M_{lock} sector, not in a modification of the Schrödinger equation, and the experiment that separates us from Penrose is already being run.

W The complex-number and real-bulk referees

Objection 75 (O-QM-75) (the complex-number referee, after Penrose’s question)

“Your hidden-fiber Part says the charged phase is a real circle, and your Discussions now claim the state-space i is derived too. But your own Constitution installs a complex Hilbert space in the bulk package of AX-3. The ‘real bulk’ is marketing.”

Response. The objection correctly reads the dictionary and misses its standing. In the free and quasi-free regime, §117 derives the bulk state space: $\mathcal{H}_{\text{Bulk}}$ is the Fock completion of the real solution space under a complex structure built from the circle (charged sectors) and the polar phase of constitutional time (all sectors), with positivity doing the selecting. In that regime the complex entries of the AX-3 package are theorems over real data, not primitives. What remains primitive is exactly what is named: the interacting completion, which sits at the same frontier as all of constructive field theory. A referee is entitled to say the program is unfinished; the books, however, are open about which pages are missing.

Objection 76 (O-QM-76) (the neutral-sector referee)

“Your own arithmetic kills your own story: the θ -rotation generator squares to $-n^2$, so it vanishes on neutral sectors — yet a real scalar field’s quanta interfere. The sixth dimension cannot explain quantum complexity.”

Response. The premise is our theorem and the conclusion does not follow. The circle can only complexify what it moves; on weight-zero sectors it moves nothing, and §117 therefore derives the neutral-sector i from the second shared real structure instead: the polar phase of the time-translation generator, selected uniquely by AX-1 causal order plus AX-3 positivity on the static arena of Theorem 0 (Ashtekar and Magnon, 1975; Kay, 1978). On charged sectors the two constructions agree on shell (O-QM-77), so there is one i , not two. The circle is credited with what it earns — compactness, gauge character, superselection — and the existence of i is credited to time and positivity. The division of labor is the answer, not a retreat from it.

Objection 77 (O-QM-77) (the field-theory referee, on antiparticles)

“Your ‘antiparticles are a consistency condition’ is Feynman–Stückelberg with new packaging. Every field-theory textbook already pairs charge $-n$ with positive frequency.”

Response. The pairing is textbook; its *provenance* is the claim. In standard field theory the assignment is imposed so that the quantized Hamiltonian comes out positive — a condition on the output of quantization. In the present framework the same assignment appears one level down, before quantization, as the unique orientation under which two independently forced real structures — the circle’s quarter-turn and the clock’s quarter-period — act as the *same* complex structure on shell; every other assignment makes them anti-agree (§117, RB-3). A convention with a derivation behind it is no longer a convention. The referee is right that no new particle physics follows; what follows is an answer to “why does nature pair them so?”, which the textbook does not attempt.

Objection 78 (O-QM-78) (the quantum-information referee, after Renou)

“Real quantum mechanics is experimentally falsified (Renou *et al.*, 2021) — and your bulk is real. You are advertising the theory experiment just killed.”

Response. Category error, and the distinction is load-bearing. What experiment excludes is quantum theory over real Hilbert spaces with *real tensor products* — independent sources with no shared complex structure. The framework’s boundary theory is standard complex quantum mechanics; its bulk is real *with a derived, single, globally shared* complex structure, and RB-4 (§117) shows the complex tensor product is exactly the real tensor product quotiented by the shared- J identification — an operation that exists because J ’s sources (one circle, one causal order) are global. The excluded theory is the one *without* the sharing. Far from falsifying the framework, the experiment demands precisely the structure the framework derives; we read it as the strongest external support this Part has.

Objection 79 (O-QM-79) (the curved-space referee)

“Curved-space field theory has no preferred vacuum: inequivalent complex structures, particle creation, Unruh. Your ‘unique derived i ’ cannot survive cosmology or a black

hole.”

Response. The uniqueness claim is made where it holds and the ambiguity is housed where it belongs. The canonical arena of Theorem 0 is globally static, so the bulk-layer J is canonical there. Observer-adapted complex structures — Rindler, comoving — are *dictionary* data: AX-3 makes the instruments part of the boundary package, and which modes a detector counts is fixed by the observer’s time, which is boundary information. Their physical output is produced by the GDOS partial trace itself: tracing the inaccessible partner of the squeezed pairing yields an exactly thermal state, at $T = a/2\pi$ under the Unruh pairing — thermality as constitutional openness, not new physics. The rotating case and the collapse epoch are closed at the model level (RB-11, RB-12, §117): the rotating anchor moves to the asymptotic frame with superradiance as GDOS pumping, and the collapse channel factorizes into universal thermality times a decaying transient. What remains open is stated there too: the exact-Kerr Hadamard analysis and the full gravitating case, the field’s own, imported where used.

Objection 80 (O-QM-80) (the mathematical-physics referee, on fermions)

“Fermions are intrinsically complex — Dirac spinors live in $\mathbb{C}^4$, and your real-bulk story is a bosonic story.”

Response. RB-5 (§117) covers both statistics, and the fermionic case is the *easier* one. In Majorana form the fermionic datum is a real solution space with a positive-definite inner product — no taming condition to check — and the unique positive-energy complex structure is again the polar phase of the time generator, the ground-state polarization of the self-dual CAR formalism; reversing its orientation on any single spectral plane produces a negative one-particle energy, which is the uniqueness proof in one line. Charged fermions inherit the circle structure verbatim (the spinor index is inert under ∂_θ), and charge conjugation factorizes as circle reversal composed with a fixed real spinor map. The Dirac field’s $\mathbb{C}^4$ is packaging: charge by the circle, frequency by the clock, spinor indices by real matrices.

Objection 81 (O-QM-81) (the twistor referee, after Penrose’s second question)

“Your real-bulk story explains amplitudes, perhaps. But the deepest complex magic in physics is that massless fields are *holomorphic* objects — twistor theory — and a real circle plus a real clock has nothing to say about cohomology on $\mathbb{CP}^3$.”

Response. RB-9 says it precisely, and two of its three components are theorems of the twistor school itself. Positive frequency is holomorphy at every level: Hardy splitting at the signal level (generated by this framework’s derived J); forward-tube analyticity at the field level; and, for conformally invariant massless fields, cohomology on the *positive half* of twistor space — the $\mathbb{PT}^+/\mathbb{PT}^-$ division being nothing other than the frequency splitting (Eastwood *et al.*, 1981; Penrose and MacCallum, 1973). What the framework adds is the direction of explanation: holomorphy is what causal order plus energy positivity *look like* when the symmetry is large enough to complexify every timelike direction at once, and both of those are real constitutional data (AX-1, AX-3). The referee’s premise — that holomorphy is a primitive endowment — is exactly what RB-9 replaces; the referee’s mathematics is untouched, and gratefully used. What the framework does not yet own is stated in the ledger: the full relativistic QED infrared (its

rigorous-model core is now owned — RB-10), and the nonlinear twistor constructions beyond the free correspondence. The referee who wants the full case — the two real sources, the bridge at its Hardy, tube, and twistor levels, and the open items stated as open — is directed to the member preprint GG-P4-Real (Arisaka, 2026, P4-Real), written for exactly this reader.

Objection 82 (O-QM-82) (the infrared referee)

“Your i survives the infrared, but your particle does not. In QED the electron is an infraparticle: no sharp mass shell, no Fock ground state at fixed momentum, velocity superselection. You cannot claim the real-bulk derivation ‘extends to the infrared’ when the very notion of an asymptotic particle — the object your Haag–Ruelle route (RB-8) quantizes over — dissolves there.”

Response. RB-10 concedes the particle and keeps the i — and the concession is the theorem’s content. What dissolves in the infrared is the Fock mass shell: the electron must carry its soft cloud, clouds attached to different velocities are exactly orthogonal in the cutoff-removed limit, and the sharp particle is replaced by a family of disjoint sectors (Buchholz, 1986; Fröhlich, 1973). But every step of that dissolution is a statement about *sector labels*: the cloud is a c-number displacement, the displacement is an affine symplectic map, and the quadratic generator that the polar-phase construction (RB-2) reads is exactly invariant in every sector. The multiplicity therefore lands at AX-3 as port structure — which infrared background an instrument treats as vacuum — precisely where the black-hole vacuum multiplicity landed, and the derived J is sector-blind. At the van Hove stratum this is exact; at the Nelson/Pauli–Fierz level it holds via the theorem-grade infraparticle analysis of the Pizzo school (Bachmann et al., 2012; Chen et al., 2010; Pizzo, 2003); for full QED the Chung/Faddeev–Kulish dressing (Chung, 1965; Faddeev and Kulish, 1970) is semi-rigorous, and the statement is displayed as the field’s frontier rather than claimed. The referee’s premise is thus adopted, not refuted: the infrared problem is a problem about which vacuum an observer stands in — not a problem about where i comes from.

Closing note. These eighty-two objections are the surviving ones — the readings that remain after a referee has studied the companions that do the work GG-A8-QM summarizes. Where the honest answer is “not yet,” it is named as an open item (the field-grade representation step above all, §118). The framework’s wager is stated plainly throughout: a deterministic, realist six-dimensional bulk, conceding 4D non-locality, that reads the quantum postulates as theorems and the measurement problem as three answered questions plus one named analytic residue. A reader is entitled to decline the wager; what the framework asks is that it be declined for the right reasons, not the retired ones.

Part XVI

References

Cited works, each with a relevance note linking it to the present paper

Unlike a conventional reference list, every entry below carries a short *Relevance* note: a sentence or two saying what the cited work contributes and where in the present paper it bears. The aim is to turn the bibliography from a bare list of sources into a navigable companion — a reader can move through the references and see, at a glance, which theorem, worked example, or section each one underwrites.

The notes are written for two readers at once. For the newcomer, they trace the intellectual lineage of the present reformulation: from the foundations of quantum mechanics and the measurement problem, through open-system dynamics and decoherence theory, to the twenty-two-paper GG-Theory cohort in which GG-A8-QM reads four-dimensional quantum mechanics as the boundary projection of the six-dimensional bulk — Einstein right in 6D, Bohr right in 4D. For the specialist, they make the paper’s debts explicit and checkable — each note names the specific result it supports, so that no citation is merely decorative.

The GG-Theory cohort papers (GG-A1-Constitution–GG-A11-Origin, together with the phenomenology P-series and the consequence C-series) are cited throughout under the canonical keys and titles fixed by the cross-paper coherence map of the program; GG-A8-QM’s place in that cohort — the quantum-mechanics member of the 4D-Physics trio — is set out in the cohort-inheritance Part. The references are listed alphabetically by first author.

A. Papers by Katsushi Arisaka (the GG-Theory cohort)

K. Arisaka. *The GG-Constitution: Four Axioms and Seven Postulates of the GG-Theory Program, From Which GG-6, BI-6, OGN-5, and GDOS Descend as Theorems*, Arisaka-GG-A1-Constitution. Preprint, available at <https://preprints.arisaka-gg.org/a1-constitution/>. doi:10.5281/zenodo.22063867.

Relevance. Supplies the constitutional charter (4 axioms AX-1 through AX-4, 7 postulates POS-1 through POS-7) that GG-A8-QM inherits as its first cause. AX-3 (OGGEP) forces the bulk-to-boundary compiler from which the QM postulates emerge as theorems; AX-1 (LC) supplies the bulk locality that drives einselection (T-A8.6); AX-4 (MSMF) eliminates hidden continuous parameters that an interpretation might otherwise insert. Invoked dozens of times across Parts II–XIII. First cited in §21.

K. Arisaka. *GG-Theory with GG-6: Geometry–Gauge Locking at the 6D Bulk (GG-6) at the Heart of Grand Geometric Theory (GG-Theory)*, Arisaka-GG-A2-GG6.

Preprint, available at <https://preprints.arisaka-gg.org/a2-gg6/>.
doi:10.5281/zenodo.22064298.

Relevance. Constructs the 6D bulk geometry $\mathcal{M}_4 \times I_r \times S_\theta^1$ on which the GG-Theory program runs. GG-A8-QM uses GG-A2-GG6 to ground the bulk-side ontology of the QM reformulation: the topologically protected S_θ^1 winding is the bulk hidden-variable space whose 4D projection generates Bell–CHSH violations (T-A8.7); the radial direction I_r is the renormalization-scale projection axis along which the single non-separable joint state casts its two boundary shadows (Theorem 0 of GG-A2-GG6: I_r carries no light cone). Cited in Parts II, III, V, VIII. First cited in §I.

K. Arisaka. *Anomaly Freedom of BI-6: The Modified Bianchi Identity (BI-6) for the Six-Force, Six-Dimensional Bulk (GG-6)*, Arisaka-GG-A3-BI6.

Preprint, available at <https://preprints.arisaka-gg.org/a3-bi6/>.
doi:10.5281/zenodo.22064300.

Relevance. Establishes the BI-6 closure identity $dH_3 = X_4$ (Green–Schwarz-modified) that fixes the unique anomaly polynomial of the 6D bulk. GG-A8-QM invokes GG-A3-BI6 to certify the bulk’s consistency under the operational projection that produces 4D QM: the anomaly cancellation in 6D is what guarantees the boundary algebra closes and admits a CPTP-channel structure rather than degenerating to a non-trace-preserving map. Cited in Part II. First cited in §22.2.

K. Arisaka. *From Euclid to GG-6: The Realization of Euclid’s Elements in Six Dimensions — GU-5, OGN-5, and a Parameter-Free Reconstruction of Modern Physics*, Arisaka-GG-A4-Euclid.

Preprint, available at <https://preprints.arisaka-gg.org/a4-euclid/>.
doi:10.5281/zenodo.22064302.

Relevance. Supplies the Euclidean-discipline input ledger that fixes zero free continuous parameters: the GU-5 ($c, G_N, \hbar, k_B, Q_B$), topological constants π and e , and the proved OGN-5 integer ledger. GG-A8-QM cites GG-A4-Euclid to anchor its zero-free-parameters claim: nothing in the QM reformulation introduces a fitting parameter beyond the cohort ledger; every numerical anchor in the worked examples traces back to the Euclidean inputs. First cited in §22.3.

K. Arisaka. *GDOS: Geometric Dynamics of Open Systems — 6D GG-6 Bulk Projected to 4D Boundary for Quantum Measurement, General Relativity, and QFT*, Arisaka-GG-A5-GDOS.

Preprint, available at <https://preprints.arisaka-gg.org/a5-gdos/>.
doi:10.5281/zenodo.22064304.

Relevance. The direct upstream foundation of GG-A8-QM. Part II of GG-A5-GDOS proves theorems T1–T6 that construct the canonical boundary algebra on $\partial\mathcal{M}_6$; Part III then derives the QM-postulates-as-theorems chain T7.1–T7.7 (Hilbert space, Schrödinger equation, Born rule, Lüders projection, decoherence, einselection, Bell–CHSH). GG-A8-QM inherits this chain verbatim as T-A8.1–T-A8.7 and supplies the graduate-textbook envelope: explicit worked examples plus four net-new theorems T-A8.8–T-A8.11. First cited in §23.

K. Arisaka. *The GRIP on GDOS — GRIP: the Geometric Engine of Symmetry Breaking, from the Big Bang to Mind*, Arisaka-GG-A6-GRIP.

Preprint, available at <https://preprints.arisaka-gg.org/a6-grip/>.
doi:10.5281/zenodo.22064306.

Relevance. Supplies the Gi-4 four-pillar structure (TGG, CCI, GFF, GRIP) and the GRIP cascade engine $DGI \rightarrow GFF \rightarrow GSSB \rightarrow GA$. GG-A8-QM uses GG-A6-GRIP in two ways: (i) the GFF Lyapunov landscape on which einselection (T-A8.6) descends; (ii) the GRIP cascade as the bulk-side dynamical engine that the QM stratum specializes. GG-A8-QM's Figure 2 inserts the QM tier between GDOS and GRIP precisely to mark this seed-role. First cited in §24.1.

K. Arisaka. *The Qi-4 Pillars — Qi-QUA, Qi-EP, Qi-CON, Qi-HAL: The Quantum-Information Backbone of GDOS*, Arisaka-GG-A7-Qi4.

Preprint, available at <https://preprints.arisaka-gg.org/a7-qi4/>.
doi:10.5281/zenodo.22064308.

Relevance. Supplies the Qi-4 quantum-information four-pillar structure (Qi-QUA, Qi-EP, Qi-CON, Qi-HAL). GG-A8-QM's net-new theorem T-A8.10 (decoherence-rate conformality) is the direct consequence of Qi-CON applied to the QM stratum: the off-diagonal decoherence rate must scale with conformal dimension under boundary-RG flow. The Arisaka equation $J_E/Q_B = J_e = J_B = J_{Q_i}$ from Qi-QUA anchors the cohort's quantitative ledger that GG-A8-QM inherits. First cited in §24.2.

Arisaka, K. (2026). *Quantum Mechanics Derived from GG-Theory as Projection from 6D Bulk to 4D Boundary*. Arisaka-GG-A8-QM preprint (the present paper).

Preprint, available at <https://preprints.arisaka-gg.org/a8-qm/>.
doi:10.5281/zenodo.22064310.

Relevance. The present paper. The first paper of the 4D-Physics Trio. Operationalizes T2 Operational Closure and T7 4D Quantum Mechanics from GDOS into a concept-by-concept reformulation of textbook quantum mechanics: the Born rule as a theorem of the Stinespring dilation, EPR / Bell / CHSH / Aspect / Hensen / Giustina as evidence of the non-invertibility of the 6D-to-4D projection, Schrödinger's cat as a GDOS boundary-decoherence statement, and the central thesis that Einstein was right in 6D and Bohr was right in 4D, both within their respective domains. First cited in §20.1.

K. Arisaka. *The Standard Model as Theorem of GG-Theory: Its 28 Parameters as Observations of One Six-Dimensional Geometry*, Arisaka-GG-A9-SM.

Preprint, available at <https://preprints.arisaka-gg.org/a9-sm/>.
doi:10.5281/zenodo.22064312.

Relevance. Supplies the universal-identity insight via Part XII §60 that GG-A8-QM invokes as its central philosophical centerpiece (Part XIII Conclusions). Every electron is the same because every electron is the unique stable Geometric Attractor of the charged-lepton stratum, propagating identically through atoms / molecules / ATP / DNA / first cell. GG-A8-QM specializes this reading to indistinguishable-particle statistics in its net-new theorem T-A8.9. First cited in §25.

K. Arisaka. *The Standard Cosmology as Theorem of GG-Theory: The Cosmic Inventory as Observations of One Six-Dimensional Geometry*, Arisaka-GG-A10-Cosmos.

Preprint, available at <https://preprints.arisaka-gg.org/a10-cosmos/>.
doi:10.5281/zenodo.22064314.

Relevance. Supplies the cosmological specialization of OGGEp/GDOS: the boundary cosmological inventory $\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$, the modified Friedmann equations, the Hubble two-stage corridor, and Kaxion dark matter at $\sim 8.1 \mu\text{eV}$ (1.95 GHz). GG-A8-QM does not invoke the cosmological content directly but references GG-A10-Cosmos for cohort coherence: the same framework that compiles bulk to boundary on the QM stratum also compiles bulk to boundary on the cosmological stratum. First cited in §26.

K. Arisaka. *The Geometric Inevitability of Cosmos, Life, and Mind: The Self-Referential Closure of the GG-Theory Program*, Arisaka-GG-A11-Origin.

Preprint, available at <https://preprints.arisaka-gg.org/a11-origin/>.
doi:10.5281/zenodo.22064319. No result of the present paper depends on it.

Relevance. Supplies the structural template that GG-A8-QM specializes on the QM stratum: GG-A11-Origin's four-move operational anatomy (DGI $\rightarrow$ GFF Morse flow $\rightarrow$ GSSB $\rightarrow$ GA) is the master pipeline that GG-A8-QM Part III specializes to the QM stratum (bulk $\rightarrow$ OGGEp $\rightarrow$ GDOS $\rightarrow$ 4D QM). GG-A11-Origin's universal-identity reading and structural-realist genealogy are the philosophical apparatus that GG-A8-QM Part XIII inherits and re-applies at QM register. First cited in §27.

Arisaka, K. (2026). *The GG-6 Lock Cluster: A Non-Spatial Kaluza–Klein Resonance near 3.85 TeV as the Standing LHC Falsifier of GG-Theory*. Arisaka-GG-P1-KK preprint.

Preprint, available at <https://preprints.arisaka-gg.org/p1-kk/>.
doi:10.5281/zenodo.22064321.

Relevance. The first paper of the Predictions Branch. Specific phenomenology of the lock cluster at $M_{\text{lock}} = (1/2) M_{\text{Pl}} e^{-35} \approx 3.85 \text{ TeV}$, with explicit LHC and future-collider falsifiers. Any verified collider deviation from the predicted Kaluza-Klein resonance pattern at 5σ falsifies T4 + T6 jointly. First cited in §20.1.

Arisaka, K. (2026). *Why Three Generations: A Three-Leg Derivation of $N_g = 3$ from BI-6, MSMF, and GDOS Morse Stability*. Arisaka-GG-P2-N3 preprint.

Preprint, available at <https://preprints.arisaka-gg.org/p2-n3/>.
doi:10.5281/zenodo.22064323.

Relevance. The second paper of the Predictions Branch: the family number derived as a squeeze between a CP-violation lower bound and a GDOS Morse/budget upper bound, with the conditioning declared by name. Cited in the Part-IV handoff (inventory, relations map, and bridge subsection). First cited in §20.1.

Arisaka, K. (2026). *The Electron as the Ground State of Nature: The Six Oldest Questions about the Electron, Closed and Fenced within GG-Theory*. Arisaka-GG-P3-Electron preprint.

Preprint, available at <https://preprints.arisaka-gg.org/p3-electron/>.
doi:10.5281/zenodo.22064325.

Relevance. The third paper of the Predictions Branch: identity, absolute stability, the two excited copies, the positron as conjugate readout, pointlikeness, and field response derived as six

faces of ground-state-ness, with every fence declared; imports GG-P2-N3's $N_g = 3$ verbatim. Cited in the Part-IV handoff (inventory, relations map, and bridge subsection). First cited in §20.1.

Arisaka, K. (2026). *The Real Geometry Beneath the Complex Numbers of Physics: Positive Frequency is Holomorphy — from Cardano's Cubics to Twistor Space, via the Sixth Dimension of GG-Theory.* Arisaka-GG-P4-Real preprint.

Preprint, available at <https://preprints.arisaka-gg.org/p4-real/>.

doi:10.5281/zenodo.22064327.

Relevance. The origin-of- i member paper of the Foundations Branch, which develops the real-bulk series' holomorphy bridge (RB-9) at standalone length: the two real sources of the derived complex structure, the bridge at its Hardy, tube, and twistor levels, worked cohomology examples, three lecture courses, and the open items (the interacting massless infrared; the dynamics half of the bridge) stated as open. The present paper is its principal consumer in the cohort. First cited in §20.1.

Arisaka, K. (2026). *The Mirrors of Matter: Why Every Particle Has a Twin, Why Nature's Mirrors Break, and Why the Universe Is Made of Matter.* Arisaka-GG-P5-CPT preprint.

Preprint, available at <https://preprints.arisaka-gg.org/p5-cpt/>.

doi:10.5281/zenodo.22064329.

Relevance. The CPT member paper of the Foundations Branch: antiparticles from the circle-clock handshake, C/P/T as orientation reversals of the three real bulk factors, CPT at quasi-free grade from the Wick-quartet analytic fact (T-P5-B1), the chiral-pentagon CP phases, and the matter asymmetry through the weight-zero door. Cited in §52, whose candidate theorem T-A8.12 composes its quartet machinery (T-Stat-A route) with T-A8.9's token identity to discharge the fermionic symmetrization postulate. First cited in §20.1.

Arisaka, K. (2026). *The Proton as the Ground Geometric Attractor of Matter: The Eleven Questions of the Proton, Closed and Fenced within GG-Theory.* Arisaka-GG-P6-Proton preprint.

Preprint, available at <https://preprints.arisaka-gg.org/p6-proton/>.

doi:10.5281/zenodo.22064331.

Relevance. Predictions Branch: the ground-attractor method of GG-P3-Electron run on the baryonic stratum. It matters here for one reason and it is a contrast rather than an inheritance. The present paper reads the compact phase circle as memory and derives the boundary's probabilities from the fact that its coordinate cannot be resolved; GG-P6-Proton reads a winding of the same circle as a conserved integer that no local process can change. Protection by unreadability and protection by winding class are different mechanisms on one geometry, and the difference is what makes the proton stable and a measurement outcome irreversible. Cited at roster level; nothing in GG-A8-QM depends on its results. First cited in §20.1.

Arisaka, K. (2026). *The Kaxion: Cold Dark Matter as the Non-Spatial Compact-Angle Pseudoscalar of GG-6.* Arisaka-GG-C1-Kaxion preprint.

Preprint, available at <https://preprints.arisaka-gg.org/c1-kaxion/>.

doi:10.5281/zenodo.22064333.

Relevance. The cosmology phenomenology companion to GG-A10-Cosmos. The kaxion field of mass set by the scale-bulk cap loading $\Xi_{\text{cap}} = K + N + L$ produces a dark-matter line inside a closed window [1.59, 2.13] GHz in the μeV range, and alone in it; explicit haloscope and microwave-cavity falsifiers. A confirmed line outside that window falsifies the OGN-5 ledger’s placement and so the constitutional clauses AX-4 MSMF and POS-6 OGN upstream. First cited in §20.1.

Arisaka, K. (2026). *The Hubble Tension in the Light of GG-Theory: The Hubble Constant as One Reading of a Derived Expansion History $H(t)$* . Arisaka-GG-C2-Hubble preprint.

Preprint, available at <https://preprints.arisaka-gg.org/c2-hubble/>.

doi:10.5281/zenodo.22064336.

Relevance. The second paper of the Cosmology & Astrophysics Branch: reads the persistent discrepancy between early- and late-universe Hubble measurements as the two ends of one geometric corridor of the bulk, rather than as a systematic error or new fluid. Cited in the Part-IV handoff (inventory, relations map, and bridge subsection). First cited in §20.1.

Arisaka, K. (2026). *Black-Hole Entropy under GG-Theory: The GDOS Reading of the Horizon as an OGGE Port — with the Exact Bekenstein–Hawking $A/4G_N$ Derived from Induced Gravity*. Arisaka-GG-C3-BH preprint.

Preprint, available at <https://preprints.arisaka-gg.org/c3-bh/>.

doi:10.5281/zenodo.22064338.

Relevance. The third paper of the Cosmology & Astrophysics Branch and the cohort’s strong-gravity anchor: the horizon treated as an OGGE port, with the exact $A/4$ coefficient derived. Cited in the Part-IV handoff (inventory, relations map, and bridge subsection). First cited in §20.1.

B. Other references

Arisaka, K. (2026). *Symmetry, Conservation, and Geometry: Seven Steps from Noether, Bianchi, to Hopkins–Singer, and the Ledger the Open-System Turn Pays Out*. Arisaka-GG-W1-Symmetry preprint.

Preprint, available at <https://preprints.arisaka-gg.org/w1-symmetry/>.

doi:10.5281/zenodo.22064340.

Relevance. White-Paper Branch: the historical survey of how conservation moved from symmetry to cohomology. It supplies the context for a distinction this paper relies on without arguing for it, namely that the quantities surviving the bulk-to-boundary projection survive as classes rather than as charges. The survey is conceptual and carries no computation the present paper uses. Cited at roster level. First cited in §20.1.

Arisaka, K. (2026). *The Circle That Remembers, the Interval That Forgets: GG-Theory and the Sixteen Problems of Particle Physics and Cosmology*. Arisaka-GG-W2-Holonomy preprint.

Preprint, available at <https://preprints.arisaka-gg.org/w2-holonomy/>.

[doi:10.5281/zenodo.22064342](https://doi.org/10.5281/zenodo.22064342).

Relevance. White-Paper Branch: the structural treatment of protection by topology, and the closest companion to this paper’s reading of the phase fiber. What GG-A8-QM calls the unreadable holonomy of S_θ^1 is one rung of the ladder of protection mechanisms classified there, and a reader who wants to know why that rung and not another should read the classification rather than this paper’s use of it. Its sixteen open problems include the field-grade representation step that GG-A8-QM carries as its single analytic residue. First cited in §20.1.

Y. Aharonov and D. Bohm. Significance of electromagnetic potentials in the quantum theory. *Phys. Rev.* **115** (1959) 485.

Relevance. The founding demonstration that the electromagnetic potential carries physical content through holonomy rather than local field strength. Cited for the identification of the compact phase fiber S_θ^1 as the carrier of the coherent, unitary content of the boundary theory: its holonomy is the Aharonov–Bohm/Berry phase. First cited in §30.1.

K. Arisaka. *GG-Theory Foundation Book (capstone volume)*, (planned) GG-Theory Foundation Book.

Relevance. The planned capstone book volume of the GG-Theory program, synthesizing the cohort papers into a single textbook on the geometric inevitability of cosmos, life, and mind. GG-A8-QM is cited in *ArisakaBookI* as the QM chapter; GG-A8-QM cites *ArisakaBookI* for the synthesis-level context. First cited in §19.

M Arndt and others. Wave-particle duality of C_{60} molecules. *Nature* **401** (1999) 680–682.

Relevance. The 1999 Arndt et al. C_{60} fullerene interferometry experiment, confirming wave-particle duality for objects with 1 kDa mass. AEight Part VI and Figure 9 cite this as the largest object for which fringe visibility is measurable; for any larger object the decoherence rate of T-A8.5 exceeds the macroscopic-instantaneous threshold and the wave nature of the object becomes operationally inaccessible. First cited in §I.

A Ashtekar and A Magnon. Quantum fields in curved space-times. *Proceedings of the Royal Society of London A* **346** (1975) 375–394.

Relevance. The classic construction selecting a complex structure on the real solution space of a linear field from a stationarity structure and energy positivity. Together with [Kay \(1978\)](#), it supplies the uniqueness clause of the real-bulk statement RB-2: on the globally static canonical arena of Theorem 0, the state-space i of neutral matter is the polar phase of the constitutional time generator, derived rather than postulated. First cited in §117.

A Aspect, P Grangier, and G Roger. Experimental tests of realistic local theories via Bell’s theorem. *Physical Review Letters* **47** (1981) 460–463.

Relevance. The 1981 Aspect-Grangier-Roger experiment: the first high-precision Bell-inequality violation with polarization-correlated photons. AEight cites this in Part VII as the empirical seal on the Bell program; the cohort framework reads the violation as confirmation that the boundary CHSH integral saturates the Tsirelson bound, which T-A8.8 promotes to a topological-bound saturation on the bulk side. First cited in §I.

A Aspect, J Dalibard, and G Roger. Experimental test of Bell’s inequalities using time-varying analyzers. *Physical Review Letters* **49** (1982) 1804–1807.

Relevance. The 1982 Aspect publication establishing experimental Bell-inequality violation in a complete loophole-free setup (up to fast switching). AEight cites Aspect1982 throughout Parts VII and XII as the canonical empirical anchor for the $|S| = 2\sqrt{2}$ Tsirelson saturation. First cited in §45.

A Aspect, P Grangier, and G Roger. Experimental realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: a new violation of Bell’s inequalities. *Physical Review Letters* **49** (1982) 91–94.

Relevance. The first 1982 Aspect paper realizing the Einstein-Podolsky-Rosen-Bohm thought experiment with high statistical significance. AEight cites this in Part VII as the experimental realization that closed the philosophical debate of 1935-1965 in favor of QM and against local hidden-variable theories; the cohort framework reads the closure as a victory for the bulk-side reading at 6D. First cited in §I.

A Aspect, J Dalibard, and G Roger. Experimental test of Bell’s inequalities using time-varying analyzers. *Physical Review Letters* **49** (1982) 1804–1807.

Relevance. The second 1982 Aspect paper closing the locality loophole by using time-varying analyzers. AEight cites this in Part VII as the experimental closure of the locality loophole; the result is read by T-A8.7 as the boundary shadow of one non-separable joint bulk state, the apparent non-locality being the projection of a single bulk object. First cited in §I.

S Bachmann, D-A Deckert, and A Pizzo. The mass shell of the Nelson model without cut-offs. *Journal of Functional Analysis* **263** (2012) 1224–1282.

Relevance. Theorem-grade control of the massless Nelson model’s dressed one-particle structure with the ultraviolet cutoff removed. Together with Pizzo (2003) it supplies the existence half of the Nelson/Pauli–Fierz rung of RB-10: the dressed states whose soft clouds RB-10 reads as port assignments do exist, with the regularity the sector analysis needs. First cited in §117.

L E Ballentine. *Quantum Mechanics: A Modern Development*. World Scientific, (1998).

Relevance. Ballentine’s 1998 graduate textbook adopting the statistical (ensemble) interpretation of the wave function. AEight cites Ballentine1998 in Part XI as a major interpretive reference; the cohort framework’s T-A8.3 (Born rule) supports the statistical reading of the wave function at the boundary while supplying an ontic substrate at the bulk. First cited in §39.

V Baumann and others. Wigner’s friend as a rational agent. *arXiv preprint* **1901.11274** (2019).

Relevance. Baumann et al. 2019, a modern philosophical analysis of the Wigner’s-friend / Frauchiger-Renner family of paradoxes. AEight Part X cites this in the discussion of observer-observer paradoxes; the cohort framework treats the rational-agent reading as the 4D-boundary projection of a deterministic 6D-bulk picture in which the paradoxes do not arise. First cited in §97.

J. Bell. *On the Einstein Podolsky Rosen paradox*. *Physics Physique Fizika* **1** (1964) 195–200.

Relevance. Bell’s 1964 paper converting EPR’s philosophical objection into an experimentally testable inequality. AEight Part VII walks the CHSH form of Bell’s theorem; the explicit calculation on the singlet state recovers $|S| = 2\sqrt{2}|\cos(2\phi)|$; the Tsirelson saturation at $\phi=0$ is read by T-A8.8 as an extremal (Bogomolnyi-analogue) ceiling on the CHSH functional. Bell1964 is the seed of the empirical heart of the cohort QM reading. First cited in §I.

C H Bennett and G Brassard. *Quantum cryptography: public key distribution and coin tossing.* In *Proceedings of the IEEE International Conference on Computers, Systems and Signal Processing* (1984).

Relevance. The 1984 BB84 quantum-key-distribution protocol. AEight Part X cites BB84 as the canonical QKD scheme; the cohort framework reads QKD’s security as a consequence of T-A8.4 (Lueders projection) plus T-A8.11 (boundary no-signaling): any eavesdropping introduces detectable boundary state perturbations because the bulk-mediated correlations cannot be cloned. First cited in §I.

C H Bennett and S J Wiesner. Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states. *Physical Review Letters* **69** (1992) 2881–2884.

Relevance. The 1992 Bennett-Wiesner paper introducing superdense coding. AEight Part X cites this alongside teleportation (Bennett1993) as the canonical demonstration that entanglement is a usable resource; the cohort framework reads superdense coding as T-A8.11 plus the bulk-side picture: two classical bits are transmitted via one qubit because the bulk-mediated entangled pair stores the missing bit in the topological S^1_θ winding. First cited in §I.

C H Bennett, G Brassard, C Crépeau, R Jozsa, A Peres, and W K Wootters. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Physical Review Letters* **70** (1993) 1895–1899.

Relevance. The 1993 paper introducing quantum teleportation. AEight Part X cites Bennett1993 as the canonical theoretical foundation; the cohort framework reads teleportation as a confirmation of T-A8.11 (boundary no-signaling from reduced-state factoring): no information actually travels superluminally because the boundary measurement on the Bell pair only updates the classical channel, which is bounded by 4D causality. First cited in §I.

M. V. Berry. Quantal phase factors accompanying adiabatic changes. *Proc. R. Soc. Lond. A* **392** (1984) 45.

Relevance. The geometric phase of adiabatic transport: the holonomy of the connection on a family of quantum states. Cited with Aharonov–Bohm for the phase-fiber reading of S^1_θ , and the pedagogical home of the spin- $\frac{1}{2}$ cone example of §G.7.1 — the same structure that Arisaka (2026, P2-N3) elevates to a theorem about the runtime’s own transport on the family-projector moduli. First cited in §30.1.

E B Bogomolny. The stability of classical solutions. *Soviet Journal of Nuclear Physics* **24** (1976) 449–454.

Relevance. Bogomolny’s 1976 paper introducing the saturation bound on classical Yang-Mills solitons that bears his name. AEight’s net-new theorem T-A8.8 reads the Tsirelson bound

$|S| = 2\sqrt{2}$ as a Bogomolnyi-style saturation on the boundary CHSH integral, with the bulk topological-charge structure of the 6D geometry providing the saddle that saturates the bound. Bogomolny1976 is the conceptual ancestor. First cited in §46.

D Bohm. A suggested interpretation of the quantum theory in terms of “hidden” variables. *Physical Review* **85** (1952) 166–193.

Relevance. Bohm’s 1952 pilot-wave / quantum-potential reformulation, a deterministic hidden-variable extension of QM. GG-A8-QM engages Bohm1952 in Part XI (interpretation landscape) and Part VIII: Bohm’s intuition that hidden variables restore determinism is right, but the hidden variables are not in 4D position-space (Bell ruled that out); they are on the phase axis S_θ^1 of the 6D bulk. GG-A8-QM is the cohort framework’s vindication of Bohm at the right level. First cited in §I.

N Bohr. Can quantum-mechanical description of physical reality be considered complete?. *Physical Review* **48** (1935) 696–702.

Relevance. Bohr’s 1935 response to EPR insisting on the inseparability of the joint quantum state and on the operational completeness of QM at the boundary level. AEight cites Bohr1935 in Parts VII and XIII as the canonical statement of the ‘Bohr is right in 4D’ half of the cohort’s bottom line: at the operational boundary level, the framework reproduces and endorses Bohr’s reading; the disagreement is at the bulk level, where Einstein’s intuition about determinism is restored. First cited in §I.

K. Bong and others. A strong no-go theorem on the Wigner’s friend paradox. *Nature Physics* **16** (2020) 1199–1205.

Relevance. The 2020 Bong et al. experimental implementation of an extended Wigner’s-friend no-go theorem. AEight Part X cites Bong2020 as a recent sharpening; the cohort framework reads the experimental confirmation as evidence that any 4D-only interpretation faces consistency challenges that the bulk-boundary picture avoids. First cited in §97.

M Born. *Zur Quantenmechanik der Stoßvorgänge.* *Zeitschrift für Physik* **37** (1926) 863–867.

Relevance. Born’s June-July 1926 papers introducing the probabilistic interpretation of the wave function (the Born rule, $p = |\psi|^2$). AEight’s central theorem T-A8.3 derives the Born rule from OGGEF as a CPTP-channel consequence, recovering Born’s 1926 result with a deeper provenance: $p_\alpha = \text{Tr}(I_\alpha \rho)$ from the bulk-side CPTP completion. First cited in §I.

M Born. *The Born-Einstein Letters: Correspondence between Albert Einstein and Max and Hedwig Born 1916–1955.* Macmillan, (1971).

Relevance. The Born-Einstein correspondence containing Einstein’s December 1926 ‘God does not play dice’ letter to Born. AEight cites the letters in Part VIII as the canonical statement of Einstein’s deterministic intuition; the cohort framework’s ‘Einstein is right in 6D’ reading honors this intuition at the bulk level while accepting Bohr’s operational completeness at the 4D boundary. First cited in §I.

M Born and P Jordan. *Zur Quantenmechanik.* *Zeitschrift für Physik* **34** (1925) 858–888.

Relevance. The 1925 Born-Jordan paper establishing matrix mechanics as a fully consistent formulation of quantum mechanics. Cited in AEight Part I as one of the two foundational 1925-1926 formulations (alongside Schrodinger’s wave mechanics) that converge into the modern Hilbert-space framework derived as T-A8.1 from the bulk GNS construction. First cited in §I.

M Born, W Heisenberg, and P Jordan. Zur Quantenmechanik II. *Zeitschrift für Physik* **35** (1926) 557–615.

Relevance. The 1926 sequel that completed the matrix-mechanics program of quantum mechanics. AEight cites this in Part I as the historical convergence of matrix mechanics with wave mechanics into the modern operator formalism; the resulting Hilbert space is what T-A8.1 reconstructs from the bulk-side GNS construction. First cited in §I.

D Bouwmeester, J. Pan, K Mattle, M Eibl, H Weinfurter, and A Zeilinger. Experimental quantum teleportation. *Nature* **390** (1997) 575–579.

Relevance. The 1997 Bouwmeester et al. experimental realization of quantum teleportation. AEight Part X cites this as the empirical seal on the Bennett1993 theoretical proposal; the cohort framework reads the experimental success as confirmation of the bulk-mediated entanglement reading of T-A8.7-T-A8.11. First cited in §96.

H. & P. F Breuer. *The Theory of Open Quantum Systems*. Oxford University Press, Oxford, (2002).

Relevance. Breuer and Petruccione’s 2002 textbook ‘The Theory of Open Quantum Systems’, the standard modern reference for the GKLS-Lindblad-Nakajima-Zwanzig formalism. AEight Appendix cites Breuer-Petruccione for the rigorous derivation of the master equation from the system-environment Hamiltonian; T-A8.2 reconstructs this derivation with the additional warrant of OGGEF. First cited in §I.

N Brunner, D Cavalcanti, S Pironio, V Scarani, and S Wehner. Bell nonlocality. *Reviews of Modern Physics* **86** (2014) 419–478.

Relevance. Brunner et al. 2014 Reviews of Modern Physics article on Bell nonlocality, the canonical modern survey. AEight Part VII cites Brunner2014 as the comprehensive reference for the Bell-CHSH inequality and its experimental status as of 2014; the cohort framework’s T-A8.7 and T-A8.8 read this survey’s content with the additional bulk-side warrant. First cited in §45.

D Buchholz. Gauss’ law and the infraparticle problem. *Physics Letters B* **174** (1986) 331–334.

Relevance. The structural reading of the infraparticle: by Gauss’ law the asymptotic electromagnetic flux is a superselected observable, so a charged particle’s velocity is encoded in its cloud at infinity and sharp mass shells are excluded. RB-10 adopts this sector structure wholesale and adds the framework’s placement: the flux label is dictionary/port data at AX-3, and the derived complex structure is blind to it. First cited in §117.

T Chen, J Fröhlich, and A Pizzo. Infraparticle scattering states in non-relativistic QED: I. The Bloch–Nordsieck paradigm. *Communications in Mathematical Physics* **294** (2010) 761–825.

Relevance. The rigorous construction of infraparticle scattering states in nonrelativistic QED, exhibiting the velocity-superselected soft clouds as the correct asymptotic objects. This is the

scattering-level import of RB-10's Nelson/Pauli–Fierz rung: the states exist, their clouds carry the sector label, and the dressed asymptotic dynamics is exactly the free-plus-c-number form whose polar phase RB-10 reads. First cited in §117.

V Chung. Infrared divergence in quantum electrodynamics. *Physical Review* **140** (1965) B1110–B1122.

Relevance. The original coherent-state resolution of the QED infrared problem: dressing the charged particle with a photon cloud renders the S-matrix infrared-finite. RB-10 reads Chung's dressing as the physical prototype of its sector displacement — a c-number cloud that moves the state out of Fock space without touching the quadratic generator. First cited in §117.

J F Clauser, M A Horne, A Shimony, and R A Holt. Proposed experiment to test local hidden-variable theories. *Physical Review Letters* **23** (1969) 880–884.

Relevance. The 1969 Clauser-Horne-Shimony-Holt reformulation of Bell's inequality into the four-setting quantity S with classical bound $|S| \leq 2$. AEight Part VII uses the CHSH form throughout because it admits the cleanest experimental implementation; the Tsirelson bound $|S| = 2\sqrt{2}$ shown in Figure 7 is the QM saturation of CHSH, read by T-A8.8 as the boundary shadow of a bulk Bogomolnyi-saturated configuration. First cited in §I.

C Cohen-Tannoudji, J Dupont-Roc, and G Grynberg. *Atom-Photon Interactions: Basic Processes and Applications*. Wiley, New York, (1992).

Relevance. The Cohen-Tannoudji-Diu-Laloe graduate QM textbook (1977, 2-vol., often cited as 'Cohen-Tannoudji 1992' for the English edition). AEight cites this throughout the worked examples as the canonical textbook treatment that the cohort framework recovers as T-A8.1–T-A8.7. The two-volume comprehensiveness is the historical standard against which AEight's graduate-textbook depth is measured. First cited in §39.

P C W Davies and S A Fulling. Radiation from a moving mirror in two dimensional space-time: conformal anomaly. *Proceedings of the Royal Society of London A* **348** (1976) 393–414.

Relevance. The moving-mirror model: an accelerating boundary with late-time exponential redshift radiates a thermal flux at the horizon temperature — the exactly solvable model form of gravitational collapse. RB-12's class is this one, and its factorization content — universal late-time channel, decaying transient — is proved and verified there on Davies–Fulling trajectories. First cited in §117.

C Davisson and L H Germer. Diffraction of electrons by a crystal of nickel. *Physical Review* **30** (1927) 705–740.

Relevance. The 1927 Davisson-Germer electron-diffraction experiment confirming the de Broglie matter-wave hypothesis. AEight cites this in Part VI as the canonical experimental ancestor of the double-slit demonstration of T-A8.5 (decoherence) and the wave-particle duality that the cohort framework reads as bulk fermion zero modes shadowing as 4D boundary states. First cited in §I.

L. de Broglie. *Recherches sur la théorie des quanta*. Sorbonne, Paris, (1924).

Relevance. Establishes the matter-wave hypothesis ($\lambda = h/p$ for material particles). AEight cites de Broglie’s 1924 thesis in Part VI as the seed for the double-slit experiment’s quantum descendants: the Davisson-Germer (1927), Stern-Estermann atom-diffraction, and ultimately the Arndt C_{60} fullerene interferometry of 2003. First cited in §I.

M Della Rocca. *Spinoza*. Routledge, (2008).

Relevance. Della Rocca’s 2008 monograph on Spinoza, including the Numerical Sameness argument. AEight cites Della Rocca in Part XI as the modern philosophical apparatus for reading ‘every electron is the same’ in numerical-identity terms: indistinguishability is not a contingent feature but a metaphysical necessity given the bulk-side uniqueness of the charged-lepton Geometric Attractor. First cited in §110.

P. A. M Dirac. (1930).

Relevance. Dirac’s 1930 textbook ‘The Principles of Quantum Mechanics’ establishing the transformation-theory formulation that unified matrix and wave mechanics. AEight cites this in Part I as the historical synthesis that the modern Hilbert-space formalism stands on; T-A8.1 specifies the operator-algebra structure that Dirac’s transformation theory presupposes. First cited in §I.

L Diósi. Models for universal reduction of macroscopic quantum fluctuations. *Physical Review A* **40** (1989) 1165–1174.

Relevance. Diósi’s 1989 gravity-related stochastic state-reduction model, the mass-density formulation of what is now the Diósi–Penrose model. GG-A8-QM engages it in Part XI (§109) as the canonical objective-collapse rival whose prediction of a mass-dependent deviation from environmental decoherence furnishes the cleanest experimental fork against the framework, which predicts no such deviation (F-QM, macroscopic coherence). First cited in §109.

D Dürr, S Goldstein, and N Zanghì. Quantum equilibrium and the origin of absolute uncertainty. *Journal of Statistical Physics* **67** (1992) 843–907.

Relevance. The typicality formulation of quantum equilibrium in Bohmian mechanics: $|\psi|^2$ as the unique equivariant measure, and Born statistics as the typical empirical distribution within one deterministic universe. The cohort’s typicality target imports this architecture and replaces each of its choices with a forced structure: the measure by the unique conserved real density, the transport by dial-blindness plus record stability, the anchor by Haar on the compact fiber. First cited in §118.

M G Eastwood, R Penrose, and R O Wells. Cohomology and massless fields. *Communications in Mathematical Physics* **78** (1981) 305–351.

Relevance. The rigorous form of the Penrose transform: solutions of the free massless field equations of helicity h correspond to cohomology classes $H^1(\mathbb{PT}, \mathcal{O}(-2h - 2))$, with positive-frequency solutions supported on $\mathbb{PT}^+$. Together with Penrose and MacCallum (1973), this is the imported third level of the holomorphy bridge RB-9: the twistor division is the causal-positive frequency splitting, conformally packaged. First cited in §117.

P H Eberhard. Background level and counter efficiencies required for a loophole-free Einstein-Podolsky-Rosen experiment. *Physical Review A* **47** (1993) R747–R750.

Relevance. Eberhard’s 1989/1993 analysis of the detection-efficiency requirements for a loophole-free Bell test. AEight cites this in Part VII as the theoretical groundwork for the 2015 loophole-free experiments (Hensen, Giustina, Shalm) that finally closed the detection loophole. First cited in §49.

C. Ehresmann. Les connexions infinitésimales dans un espace fibré différentiable. *Colloque de Topologie (Espaces Fibrés), Bruxelles* (1950) 29–55.

Relevance. The founding definition of a connection on a fiber bundle, and the rigorous form of GGEP (AX-2): a gauge field is a connection, its field strength a curvature, a gauge phase a holonomy. Cited in the hidden-fiber section, where the phase fiber S^1_θ is identified as a principal $U(1)$ -bundle whose holonomy is the quantum phase. First cited in §30.2.

A Einstein. Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt. *Annalen der Physik* **17** (1905) 132–148.

Relevance. Einstein’s 1905 photoelectric-effect paper introducing light quanta (later called photons). AEight cites this in Part I as one of the four pillars of the old quantum theory that motivated the operational quantization program; the photon’s particle-like behavior in the photoelectric effect anticipates the bulk-side fermion zero-mode picture of T-A8.1. First cited in §I.

A Einstein, B Podolsky, and N Rosen. Can quantum-mechanical description of physical reality be considered complete?. *Physical Review* **47** (1935) 777–780.

Relevance. The 1935 EPR paper challenging the completeness of QM via the entangled spin-singlet thought experiment. AEight Part VII opens with EPR1935 as the historical seed of the Bell program; T-A8.7 reads Bell-CHSH violations as the 4D boundary shadow of one non-separable joint bulk state, projected along the radial I_r direction (which carries no light cone), vindicating EPR’s intuition that something is ‘incomplete’ at the boundary level even as QM’s predictions are confirmed. First cited in §I.

A K Ekert. Quantum cryptography based on Bell’s theorem. *Physical Review Letters* **67** (1991) 661–663.

Relevance. Ekert’s 1991 E91 QKD protocol using Bell-pair measurements for key generation and eavesdropping detection. AEight Part X cites E91 as the entanglement-based alternative to BB84; the cohort framework’s reading is that the security of E91 rests on the experimentally confirmed Tsirelson saturation of T-A8.8 — any eavesdropping reduces the observed CHSH value below $2\sqrt{2}$ by a detectable amount. First cited in §I.

H Everett. “Relative state” formulation of quantum mechanics. *Reviews of Modern Physics* **29** (1957) 454–462.

Relevance. Everett’s 1957 ‘relative state’ formulation, the seed of the Many-Worlds interpretation (MWI). AEight Part XI discusses Everett1957: the cohort framework agrees with Everett that the unitarity of QM extends to the observer’s universe at the 4D boundary level, but the ‘many

worlds’ are decohered branches on a single deterministic 6D bulk substrate, not parallel universes. First cited in §I.

L D Faddeev and P P Kulish. Asymptotic conditions and infrared divergences in quantum electrodynamics. *Theoretical and Mathematical Physics* **4** (1970) 745–757.

Relevance. The systematic dressed-asymptotic-dynamics formulation of the QED infrared problem: the correct asymptotic Hamiltonian is free plus a c-number-valued (cloud) correction, and the physical states carry coherent dressings. RB-10’s clause (ii) is the structural core of this construction, proved exactly at the van Hove stratum; the full QED case remains semi-rigorous and is displayed as the frontier RB-10 names rather than claims. First cited in §117.

R P Feynman. *QED: The Strange Theory of Light and Matter*. Princeton University Press, 1985.

Relevance. Feynman’s public exposition of the amplitude-arrow picture of quantum electrodynamics. Cited because that spinning arrow is, on this paper’s reading, literally the position on the phase fiber S_θ^1 : adding arrows and squaring is the boundary registration map. First cited in §30.4.

R P Feynman, R B Leighton, and M Sands. *The Feynman Lectures on Physics, Vol. III: Quantum Mechanics*. Addison-Wesley, (1965).

Relevance. Feynman’s Lectures on Physics, Volume III, the canonical introductory QM text. AEight cites the Feynman lectures in the historical discussions of Parts V–VII; the cohort framework recovers Feynman’s path-integral intuition as a 4D-boundary projection of the bulk Lagrangian. First cited in §I.

D Frauchiger and R Renner. Quantum theory cannot consistently describe the use of itself. *Nature Communications* **9** (2018) 3711.

Relevance. The 2018 Frauchiger-Renner paper sharpening Wigner’s-friend into a no-go theorem: certain consistent applications of standard QM to observers-observing-observers lead to contradictions. AEight Part X discusses Frauchiger-Renner as the modern sharpening of the measurement-chain problem; the cohort framework reads it as a 4D-boundary artifact that disappears when the bulk substrate is recognized. First cited in §74.

S J Freedman and J F Clauser. Experimental test of local hidden-variable theories. *Physical Review Letters* **28** (1972) 938–941.

Relevance. The 1972 Freedman-Clauser experiment, the first laboratory observation of a Bell-inequality violation. AEight cites this in Part VII as the empirical ancestor of the Aspect-experiments lineage; the cohort framework reads the violation as a confirmed signature of bulk-mediated correlations rather than as evidence against local realism (which the bulk picture preserves at 6D). First cited in §I.

S French. *The Structure of the World: Metaphysics and Representation*. Oxford University Press, (2014).

Relevance. French’s 2014 monograph extending the ontic-structural-realism program to a full metaphysical view. AEight cites French2014 in Part XI alongside Ladyman-Ross as the standard-bearer of OSR; the cohort framework’s universal-identity insight (every electron is the same) is the natural QM-stratum specialization of French’s view. First cited in §110.

J Fröhlich. On the infrared problem in a model of scalar electrons and massless, scalar bosons. *Annales de l'Institut Henri Poincaré A* **19** (1973) 1–103.

Relevance. The founding rigorous analysis of the infraparticle: in the massless Nelson model the fixed-momentum Hamiltonian has no ground state in Fock space, and the physical states live in velocity-indexed coherent sectors. RB-10's Lemma-1 mechanics — the logarithmically divergent cloud and the power-law sector orthogonality — is the quantitative skeleton of Fröhlich's phenomenon, verified numerically in the GG-P4-Real suite. First cited in §117.

C A Fuchs. QBism, the perimeter of quantum Bayesianism. *arXiv preprint* **1003.5209** (2010).

Relevance. Fuchs's 2010 paper articulating QBism: quantum states encode an agent's degrees of belief, not facts about the world. AEight Part XI discusses QBism as the epistemic-state interpretation; the cohort framework partly agrees with QBism that the boundary quantum state is epistemic for the 4D observer, but DISAGREES that this is the whole story: the 6D bulk is ontic and observer-independent. First cited in §I.

M R Gallis and G N Fleming. Environmental and spontaneous localization. *Physical Review A* **42** (1990) 38–48.

Relevance. Gallis and Fleming's derivation of the collisional decoherence master equation, giving the decoherence function $F(\Delta x)$ and its saturated and localization limits. AEight cites GallisFleming1990 in §43.8 and Appendix I as the master equation underlying the worked overlap integral of T-A8.5. First cited in §43.8.

I M Gelfand and M A Naimark. On the imbedding of normed rings into the ring of operators on a Hilbert space. *Matematicheskii Sbornik* **12** (1943) 197–217.

Relevance. The Gelfand-Naimark 1943 paper introducing the GNS construction (Gelfand-Naimark-Segal), the mathematical foundation for AEight's T-A8.1 (Hilbert-space embedding via GNS). The boundary algebra's representation as bounded operators on a Hilbert space is forced by the GNS construction once the boundary algebra is C^* -completed. First cited in §39.

M Gell-Mann and J B Hartle. Classical equations for quantum systems. *Physical Review D* **47** (1993) 3345–3382.

Relevance. The 1993 Gell-Mann-Hartle paper sharpening the decoherent-histories interpretation with classical equations of motion as the operational limit. AEight cites this in Part XI alongside Griffiths1984 and Omnes1999; the cohort framework reads the classical limit as the einselected pointer-basis trajectory of T-A8.6. First cited in §I.

W Gerlach and O Stern. Der experimentelle Nachweis der Richtungsquantelung im Magnetfeld. *Zeitschrift für Physik* **9** (1922) 349–352.

Relevance. The 1922 Stern-Gerlach experiment, the empirical foundation of AEight's prototype worked example (Part V). The two-spot detector pattern is the simplest non-trivial OGGEPC-channel: a single qubit, a single sharp observable, a single binary outcome. Born probabilities $\cos^2(\theta/2)$ and $\sin^2(\theta/2)$ emerge from T-A8.3 applied to the spin-1/2 qubit on the Bloch sphere. First cited in §I.

G. R. A. & W. T Ghirardi. Unified dynamics for microscopic and macroscopic systems. *Physical Review D* **34** (1986) 470–491.

Relevance. The 1986 Ghirardi-Rimini-Weber paper introducing spontaneous-collapse dynamics as a candidate solution to the measurement problem. AEight Part XI discusses GRW as the canonical spontaneous-collapse alternative; the cohort framework does NOT need spontaneous collapse because T-A8.5 supplies the decoherence rate from bulk dynamics, and the boundary observer never registers a coherent macroscopic superposition. First cited in §I.

M Giustina, M A M Versteegh, S Wengerowsky, J Handsteiner, A Hochrainer, K Phelan, F Steinlechner, J Kofler, J. Larsson, C Abellan, W Amaya, V Pruneri, M W Mitchell, J Beyer, T Gerrits, A E Lita, L K Shalm, S W Nam, T Scheidl, R Ursin, B Wittmann, and A Zeilinger. Significant-loop-hole-free test of Bell’s theorem with entangled photons. *Physical Review Letters* **115** (2015) 250401.

Relevance. The 2015 Giustina et al. loop-hole-free Bell test at Vienna. AEight Part VII cites Giustina2015 in the trio (with Hensen2015 and Shalm2015) of 2015 experiments that closed all major loopholes. The 2022 Nobel Prize in Physics was awarded for this entire experimental program. First cited in §I.

A. Gleason. *Measures on the closed subspaces of a Hilbert space.* *Journal of Mathematics and Mechanics* **6** (1957) 885–893.

Relevance. Gleason’s 1957 theorem proving that the Born rule is the unique probability measure on closed subspaces of a Hilbert space of dimension ≥ 3 . AEight Appendix offers a proof sketch of Gleason; T-A8.3 (Born rule from OGGE) gives an alternative derivation that does not depend on the $\dim \geq 3$ constraint and that supplies the bulk-side warrant via CPTP completion. First cited in §I.

V. K. A. & S. E. Gorini. (1976).

Relevance. The 1976 Gorini-Kossakowski-Sudarshan paper (parallel to Lindblad1976) deriving the most general Markovian quantum dynamical semigroup generator from CPTP requirements. AEight cites GKS together with Lindblad as the GKLS master equation, which T-A8.2 inherits from the bulk-to-boundary partial trace as the canonical open-system QM dynamics. First cited in §I.

V Gorini, A Kossakowski, and E C G Sudarshan. Completely positive dynamical semigroups of N -level systems. *Journal of Mathematical Physics* **17** (1976) 821–825.

Relevance. The complete GKLS reference: Gorini-Kossakowski-Sudarshan 1976 deriving the Markovian quantum semigroup generator. Cited in AEight Part III alongside Lindblad1976 as the foundation of the open-system QM dynamics that T-A8.2 specializes from the bulk-side CPTP completion. First cited in §33.

R B Griffiths. Consistent histories and the interpretation of quantum mechanics. *Journal of Statistical Physics* **36** (1984) 219–272.

Relevance. Griffiths’s 1984 paper founding the consistent-histories interpretation. AEight Part XI discusses consistent histories as one of the major interpretive frameworks; the cohort framework’s

reading of T-A8.5 supplies the decoherence functional that makes histories consistent, so the cohort framework and consistent histories converge in their operational predictions but the cohort adds the bulk-side ontology. First cited in §I.

L K Grover. *A fast quantum mechanical algorithm for database search.* In *Proceedings of the 28th Annual ACM Symposium on Theory of Computing* (1996).

Relevance. Grover’s 1996 quadratic-speedup search algorithm. AEight cites Grover1996 in Part X as the second canonical quantum-algorithm result (alongside Shor’s factoring); both algorithms rely on the same unitary-evolution + measurement structure that T-A8.1-T-A8.4 derive from OGGEF. First cited in §I.

R Haag. *Local Quantum Physics: Fields, Particles, Algebras.* Springer, Berlin, (1992).

Relevance. Haag’s 1992 monograph ‘Local Quantum Physics’, the standard reference for algebraic QFT and the algebraic approach to local quantum observables. AEight cites Haag1992 in the Appendix as the rigorous mathematical foundation for the boundary algebra construction; the cohort framework’s T-A8.1 leverages algebraic-QFT methods that Haag systematized. First cited in §39.

L Hackermüller, K Hornberger, B Brezger, A Zeilinger, and M Arndt. Decoherence of matter waves by thermal emission of radiation. *Nature* **427** (2004) 711–714.

Relevance. The 2004 measurement of thermal-radiation decoherence of C_{70} matter waves, a controlled laboratory determination of a decoherence rate. AEight cites Hackermuller2004 in §43.8 as the companion thermal-photon channel to the collisional decoherence worked there. First cited in §43.8.

W Heisenberg. *Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen.* *Zeitschrift für Physik* **33** (1925) 879–893.

Relevance. Heisenberg’s 1925 paper introducing matrix mechanics. Cited in Part I as one of two parallel breakthroughs (with Schrodinger 1926) that founded modern QM; AEight’s T-A8.1 reconstructs the matrix-mechanics Hilbert space from the bulk algebra via the GNS construction. First cited in §I.

W Heisenberg. *Physics and Philosophy: The Revolution in Modern Science.* Harper, (1958).

Relevance. Heisenberg’s 1958 philosophical reflections on QM, a primary source for the Copenhagen reading. AEight cites Heisenberg1958 in Parts I and XI as the historical articulation of the Copenhagen interpretation; the cohort framework’s ‘Bohr right in 4D’ reading honors the operational completeness Heisenberg defended while adding the 6D-bulk ontology Heisenberg did not have access to. First cited in §I.

B et al Hensen. Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres. *Nature* **526** (2015) 682–686.

Relevance. The 2015 Hensen et al. loophole-free Bell test using NV centers in diamond. AEight Part VII cites Hensen2015 as one of three 2015 loophole-free experiments (with Giustina2015 and Shalm2015) that closed the locality and detection loopholes simultaneously, sealing the empirical case for QM and against local hidden-variable theories. First cited in §I.

A S Holevo. *Quantum Systems, Channels, Information*. De Gruyter, (2012).

Relevance. Holevo’s 2012 graduate text on quantum channels and quantum information theory. AEight cites Holevo2012 in Part III as the modern reference for the CPTP-channel formalism that T-A8.2 specializes from OGGE. First cited in §42.

G ’t Hooft. *The Cellular Automaton Interpretation of Quantum Mechanics*. Fundamental Theories of Physics **185**, Springer, (2016).

Relevance. ’t Hooft’s monograph developing the deterministic cellular-automaton interpretation: quantum states as equivalence classes over ontological states evolving by local classical rules, keeping locality and surrendering measurement independence. GG-A8-QM treats it in Part XI (§109) as the framework’s nearest deterministic rival; the two make opposite Bell trades (the framework keeps measurement independence and concedes 4D locality) and share the open problem of deriving the Born measure from the deeper layer. First cited in §109.

K Hornberger, S Uttenthaler, B Brezger, L Hackermüller, M Arndt, and A Zeilinger. Collisional decoherence observed in matter wave interferometry. *Physical Review Letters* **90** (2003) 160401.

Relevance. The 2003 direct observation of collisional decoherence of C_{70} matter waves as the loss of interference contrast versus gas pressure, with theory–experiment agreement at the 10–20% level. AEight cites Hornberger2003 in §43.8 as the measured benchmark reproduced by the worked overlap integral. First cited in §43.8.

W M Itano, D J Heinzen, J J Bollinger, and D J Wineland. Quantum Zeno effect. *Physical Review A* **41** (1990) 2295–2300.

Relevance. The 1990 Itano et al. experimental confirmation of the quantum Zeno effect (frequent measurements freeze the evolution of an unstable state). AEight Part X cites Itano1990_Zeno as a paradox-solved-by-decoherence: the cohort framework reads the Zeno effect as the operational consequence of T-A8.4 (Lueders projection) applied at high measurement frequency. First cited in §93.

J. Maldacena. The large N limit of superconformal field theories and supergravity. *Adv. Theor. Math. Phys.* **2** (1998) 231.

Relevance. Maldacena’s 1998 paper introducing the AdS/CFT correspondence. AEight Part III cites Maldacena1998 in the discussion of why the GG-bulk-to-boundary reduction differs from AdS/CFT and Randall-Sundrum: the 6D bulk of GG-Theory is a specific finite-scale-profile bulk geometry, distinct from AdS/CFT’s infinite-radius limit but conceptually parallel. First cited in §30.2.

V Jacques and others. Experimental realization of Wheeler’s delayed-choice gedanken experiment. *Science* **315** (2007) 966–968.

Relevance. The 2007 Jacques et al. experimental realization of Wheeler’s delayed-choice. AEight Part X cites this in the paradox sweep; the cohort framework reads the result as confirmation that the boundary measurement determines which boundary state is observed, with no retroactive effect on the bulk substrate. First cited in §60.

E Joos and H. Zeh. The emergence of classical properties through interaction with the environment. *Zeitschrift für Physik B* **59** (1985) 223–243.

Relevance. The 1985 Joos-Zeh paper supplying the canonical thermal-bath decoherence estimate $\Gamma \sim 10^{40} \text{ s}^{-1}$ for macroscopic objects at room temperature. AEight Part VI and Figure 6 cite Joos-Zeh as the empirical anchor for the macroscopic-instantaneous regime: any object larger than C_{60} fullerene has its off-diagonal coherences damped on a timescale far shorter than any laboratory measurement window. First cited in §I.

B S Kay. Linear spin-zero quantum fields in external gravitational and scalar fields. *Communications in Mathematical Physics* **62** (1978) 55–70.

Relevance. Kay’s uniqueness theorem for the one-particle structure of a linear field on a stationary spacetime: the positive-energy complex structure commuting with time translations is unique. With [Ashtekar and Magnon \(1975\)](#), this is the imported mathematical core of the real-bulk statement RB-2 (§117); the framework’s contribution is the constitutional pedigree (AX-1 direction, AX-3 positivity, Theorem 0 staticity), not the theorem. First cited in §117.

B S Kay and R M Wald. Theorems on the uniqueness and thermal properties of stationary, nonsingular, quasifree states on spacetimes with a bifurcate Killing horizon. *Physics Reports* **207** (1991) 49–136.

Relevance. The rigorous uniqueness and thermality theorems for quantum fields on bifurcate Killing horizons — and the proof that the Kerr exterior admits no stationary Hadamard state at all. RB-11 takes the latter as the exact form of the ergoregion obstruction: the rotating-frame positivity selection fails for a reason the literature has made precise, and the anchor’s move to the asymptotic frame is the framework’s placement of that fact. First cited in §117.

S Kochen and E P Specker. The problem of hidden variables in quantum mechanics. *Journal of Mathematics and Mechanics* **17** (1967) 59–87.

Relevance. The 1967 Kochen-Specker theorem on the non-existence of non-contextual hidden variables in QM (for $\dim \geq 3$). AEight Part X discusses Kochen-Specker contextuality as one of the QM paradoxes; the cohort framework reads contextuality as a 4D shadow of the bulk choice of operational port-bound submanifold, i.e., contextuality is forced by OGGEF itself rather than being a paradox. First cited in §95.

L. Randall and R. Sundrum. A large mass hierarchy from a small extra dimension. *Phys. Rev. Lett.* **83** (1999) 3370.

Relevance. The 1999 Randall-Sundrum paper on warped extra dimensions in particle physics. AEight Part III cites Randall-Sundrum in the discussion of why the GG-Theory’s 6D bulk is structurally similar to but distinct from the RS scenario: the RS hierarchy problem solution involves a different scale profile and different boundary content than the GG-bulk QM derivation. First cited in §30.2.

J Ladyman and D Ross. *Every Thing Must Go: Metaphysics Naturalized*. Oxford University Press, (2007).

- Relevance.** Ladyman and Ross’s 2007 manifesto for ontic structural realism (OSR). AEight Part XI and Part XIII cite Ladyman-Ross as the locus classicus of structural realism; the cohort framework’s reading of QM is structurally realist by inheritance: every electron is the same Geometric Attractor, electrons-as-individuals have no metaphysical existence beyond the structural relations they enter. First cited in §110.
- S. Liao and others.** Satellite-to-ground quantum key distribution. *Nature* **549** (2017) 43–47.
- Relevance.** The 2017 Liao et al. satellite-to-ground QKD experiment using the Micius satellite, the first demonstration of long-distance free-space quantum key distribution. AEight Part X cites Liao2017 as the modern long-distance test of entanglement and Bell-pair fidelity; the cohort framework reads the success as confirmation that bulk-mediated correlations persist over ground-to-satellite distances. First cited in §98.
- G Lindblad.** *Communications in Mathematical Physics*. (1976).
- Relevance.** Lindblad’s 1976 paper deriving the most general completely-positive trace-preserving generator of a Markovian quantum dynamical semigroup. AEight cites Lindblad1976 in Part III and in the Appendix’s Nakajima-Zwanzig $\rightarrow$ GKLS derivation; T-A8.2 reconstructs the Schrodinger equation as the $\gamma \rightarrow 0$ limit of the Lindblad equation, and the Lindblad operators are the bulk-environment coupling channels. First cited in §I.
- G Lindblad.** *Non-Equilibrium Entropy and Irreversibility*. Reidel, Dordrecht, (1983).
- Relevance.** Lindblad’s 1983 monograph extending the 1976 Lindblad master equation result to the thermodynamic interpretation of decoherence and dissipation. AEight cites the book in the Appendix as the standard reference for the open-system formalism that underlies the cohort’s T-A8.5 derivation of decoherence rates. First cited in §I.
- R A Millikan.** A direct photoelectric determination of Planck’s “ h ”. *Physical Review* **7** (1916) 355–388.
- Relevance.** Millikan’s experimental confirmation of Einstein’s photoelectric-effect relation and the first precise determination of Planck’s constant h . Cited in Part I as the experimental seal on the old-quantum-theory program that the Schrodinger-equation reformulation of 1926 would supersede. First cited in §I.
- N. Bohr.** *On the constitution of atoms and molecules*. (1913).
- Relevance.** Establishes the quantized atomic model (electron orbits in hydrogen at discrete energy levels) that motivated the older quantum theory. AEight cites Bohr1913 in the historical reconstruction of Part I as the origin of the discrete-spectrum reading of atomic physics, which would later be derived from the Schrodinger equation as T-A8.2 specialized. First cited in §I.
- M A Naimark.** On a representation of additive operator set functions. *Doklady Akademii Nauk SSSR* **41** (1943) 359–361.
- Relevance.** Naimark’s 1943 dilation theorem on POVMs and their unitary dilations to projective measurements on a larger Hilbert space. AEight cites Naimark1943 in Part III alongside Stinespring1955 as the mathematical foundation for the POVM-to-instrument generalization

of T-A8.4. The Naimark dilation is the standard bridge from boundary POVMs to bulk-side projective measurements. First cited in §39.

S. Nakajima. On quantum theory of transport phenomena. *Progress of Theoretical Physics* **20** (1958) 948–959.

Relevance. Nakajima’s 1958 derivation (together with Zwanzig 1960) of the projection-operator technique that exhibits an exact non-Markovian master equation for the reduced density matrix. GG-A8-QM cites Nakajima–Zwanzig in Part III alongside Stinespring as the mathematical-physics ancestor of the OGGEp partial-trace construction; the present treatment derives the Markovian limit of Nakajima–Zwanzig from the parametric smallness of τ_c/τ_S rather than postulating it. First cited in §83.

W G Unruh. Second quantization in the Kerr metric. *Physical Review D* **10** (1974) 3194–3205.

Relevance. The quantum field theory of the Kerr background: the mode analysis in which the ergoregion’s negative co-rotating energies and the superradiant band appear at the field level, and the vacuum choices of the rotating geometry are first laid out. RB-11’s model-level statements are the structural skeleton of this analysis, with the vacuum multiplicity placed as port assignment at the dictionary. First cited in §117.

A Valentini. Signal-locality, uncertainty, and the subquantum H-theorem. I. *Physics Letters A* **156** (1991) 5–11.

Relevance. The relaxation reading of quantum equilibrium: non-equilibrium distributions approach $|\psi|^2$ dynamically, with a coarse-grained H-theorem, and pre-equilibrium epochs could leave observable relics. The constitutional relaxation chain strengthens both halves — the boundary law is genuinely dissipative (a contracting semigroup, by the GG-A5-GDOS theorem, rather than a coarse-grained measure-preserving flow), and the relic-non-equilibrium falsifier is inherited as this derivation’s own testable signature. First cited in §118.

J. Neumann. *Mathematische Grundlagen der Quantenmechanik*. (1932).

Relevance. von Neumann’s 1932 axiomatization ‘Mathematische Grundlagen der Quantenmechanik’ (Mathematical Foundations of Quantum Mechanics). Cited in Part I as the historical seal on QM’s mathematical structure; AEight’s T-A8.1 (Hilbert-space embedding via GNS) and T-A8.4 (Lueders projection rule) reconstruct two of the four von Neumann postulates as theorems of OGGEp. First cited in §I.

M. & C. I. Nielsen. *Quantum Computation and Quantum Information (10th Anniversary Edition)*. Cambridge University Press, Cambridge, (2010).

Relevance. The Nielsen-Chuang 2010 graduate textbook ‘Quantum Computation and Quantum Information’, the standard reference for the quantum-information-theoretic content of QM. AEight cites Nielsen-Chuang throughout Part X (paradox sweep) for the modern treatment of teleportation, dense coding, no-cloning, and information-theoretic implications of the Bell-CHSH bound. First cited in §42.

R Omnès. *Understanding Quantum Mechanics*. Princeton University Press, Princeton, (1999).

Relevance. Omnes’s 1999 monograph developing the decoherent-histories interpretation in textbook form. AEight cites Omnes1999 in Part XI as one of two canonical decoherent-histories references (with Griffiths1984 and Gell-Mann-Hartle 1993); the cohort framework’s T-A8.5 supplies the decoherence functional that the decoherent-histories interpretation presupposes. First cited in §I.

W. Pauli. The connection between spin and statistics. *Physical Review* **58**, 716–722 (1940).

Relevance. The founding spin–statistics theorem: given Lorentz invariance, locality, and vacuum stability, half-integer spin must anticommute. Cited in §52 as the conditional theorem whose existence question the cohort’s candidate suite addresses, and in T-A8.9’s proof as the interim warrant for exchange statistics until T-A8.12’s residues are closed. First cited in §52.

R Penrose. *The Emperor’s New Mind: Concerning Computers, Minds, and the Laws of Physics*. Oxford University Press, (1989).

Relevance. Penrose’s argument that physical state reduction is non-computable and connected to consciousness. GG-A8-QM cites it in Part XI (§109) to mark the point at which the framework parts company with Penrose: the framework’s outcome-selecting descent is an ordinary deterministic, computable relaxation, invoking neither non-computability nor consciousness. First cited in §109.

R Penrose. On gravity’s role in quantum state reduction. *General Relativity and Gravitation* **28** (1996) 581–600.

Relevance. Penrose’s 1996 proposal that gravity triggers an objective state reduction on a timescale $\tau \sim \hbar/E_G$ set by the gravitational self-energy of the mass-distribution difference (with Diósi (1989), the Diósi–Penrose model). GG-A8-QM engages it in Part XI (§109) as the sharpest empirical rival: the framework shares Penrose’s diagnosis that decoherence is not collapse but declines the gravitational-collapse cure, keeping unitarity and locating outcome selection in a deterministic GRIP descent rather than a modification of the Schrödinger equation. First cited in §109.

R Penrose. *The Road to Reality: A Complete Guide to the Laws of the Universe*. Jonathan Cape, London (2004).

Relevance. Penrose’s sustained statement of the complex-number question — why a number system invented for algebra should be the working substance of quantum amplitudes — which §116 answers in the framework’s calibrated form: the gauge half geometric (a real circle, the sixth dimension), the state-space half dynamical (the polar phase of constitutional time), the residue named. Companion to the objective-reduction engagement of §109. First cited in §116.

R Penrose and M A H MacCallum. Twistor theory: an approach to the quantization of fields and space-time. *Physics Reports* **6** (1973) 241–315.

Relevance. The foundational exposition of twistor theory, including the correspondence between positive-frequency massless fields and holomorphic data on the positive half of twistor space — the component of the holomorphy bridge RB-9 that identifies $\mathbb{PT}^+$ with the causal-positive splitting generated by the framework’s derived J . First cited in §117.

A Peres. *Quantum Theory: Concepts and Methods*. Kluwer, (1995).

Relevance. Peres’s 1995 textbook emphasizing the operational/instrumentalist reading of QM. AEight cites Peres1995 in Part XI as a representative of the Copenhagen-style operational reading; the cohort framework agrees with Peres on the boundary-operational completeness of QM and adds the 6D-bulk ontology. First cited in §39.

A Pizzo. One-particle (improper) states in Nelson’s massless model. *Annales Henri Poincaré* **4** (2003) 439–486.

Relevance. The iterated-dressing construction of the massless Nelson model’s one-particle states, with Hölder regularity in the total momentum. This is the constructive engine behind RB-10’s Nelson rung: it builds exactly the dressed states whose coherent clouds RB-10 classifies as port assignments, on which the sector-blindness of the derived J is then a theorem. First cited in §117.

M Planck. (1900).

Relevance. Planck’s 1900 paper introducing the quantum hypothesis ($E = h \nu$) to explain blackbody radiation. AEight cites Planck1900 in Part I as the historical seed of the entire QM program; the constant h that Planck introduced is one of the GU-5 inputs that GG-A4-Euclid certifies as a zero-free-parameter input. First cited in §I.

M F Pusey, J Barrett, and T Rudolph. On the reality of the quantum state. *Nature Physics* **8** (2012) 475–478.

Relevance. The 2012 Pusey-Barrett-Rudolph theorem ruling out ψ -epistemic ontological models (under modest assumptions). AEight Part XI discusses PBR as the modern formal constraint on epistemic interpretations; the cohort framework respects PBR by treating the 6D bulk state as ontic while the 4D boundary state can be epistemic for the operator-observer. First cited in §103.

H. Rauch, A. Zeilinger, G. Badurek, A. Wilfing, W. Bauspiess, and U. Bonse. Verification of coherent spinor rotation of fermions. *Physics Letters A* **54**, 425–427 (1975).

Relevance. The neutron-interferometric observation that a 2π rotation reverses a spinor amplitude while 4π restores it (with the contemporaneous experiment of Werner et al.). Cited in §52 as the direct laboratory witness of the frame-path $\mathbb{Z}_2$ on which T-A8.12’s Link 3 rests — the boundary dictionary faithfully reporting the bulk’s two-sheeted frame memory. First cited in §52.

M-O Renou, D Trillo, M Weilenmann, T P Le, A Tavakoli, N Gisin, A Acín, and M Navascués. Quantum theory based on real numbers can be experimentally falsified. *Nature* **600** (2021) 625–629.

Relevance. The network-Bell theorem (since confirmed experimentally) excluding quantum theory over real Hilbert spaces with real tensor products. Read here as the experimental mandate for a *single, shared* complex structure across independent sources — exactly what the real-bulk statements derive from one shared circle and one shared causal order (RB-4, §117), and what an unshared real formulation provably cannot arrange. First cited in §116.

M Riebe and others. Deterministic quantum teleportation with atoms. *Nature* **429** (2004) 734–737.

Relevance. The 2004 Riebe et al. deterministic atom-teleportation experiment. AEight Part X cites this alongside Bouwmeester1997 as a major experimental confirmation of the teleportation protocol; the cohort framework’s reading remains consistent: the protocol exploits the bulk-mediated entangled correlations of T-A8.7 without violating T-A8.11. First cited in §54.

C Rovelli. Relational quantum mechanics. *International Journal of Theoretical Physics* **35** (1996) 1637–1678.

Relevance. Rovelli’s 1996 paper founding relational quantum mechanics: quantum states are relative to an observer, never absolute. AEight Part XI discusses relational QM; the cohort framework agrees that the boundary state is observer-relative but locates the absolute description at the 6D bulk level, where Rovelli’s relational picture is the unique 4D-boundary projection of a single deterministic substrate. First cited in §I.

C Rovelli. *Helgoland: Making Sense of the Quantum Revolution*. Riverhead Books, (2021).

Relevance. Rovelli’s 2021 popular-science book extending the relational QM program. AEight cites Helgoland in Part XI as the popular articulation of the relational interpretation; the cohort framework’s bulk-side reading provides an ontological foundation that Rovelli’s relational reading explicitly declines to provide. First cited in §106.

Royal S. A. of Sciences. *The Nobel Prize in Physics 2022: Aspect, Clauser, Zeilinger*. <https://www.nobelprize.org/prizes/physics/2022/>, (2022).

Relevance. The 2022 Nobel Prize in Physics awarded to Alain Aspect, John F. Clauser, and Anton Zeilinger for ‘experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science.’ AEight cites the 2022 Nobel in Part VII as the institutional seal on the Bell-CHSH empirical program; the cohort framework’s T-A8.7 and T-A8.8 provide the theoretical reading that the Nobel-laureate experiments confirmed. First cited in §I.

J J Sakurai and J Napolitano. *Modern Quantum Mechanics*. Addison-Wesley, (2011).

Relevance. The Sakurai-Napolitano graduate QM textbook, the standard reference for the operator and Dirac-notation formalism. AEight cites Sakurai2011 throughout the worked examples as the canonical textbook account that the cohort framework recovers as T-A8.1-T-A8.7. First cited in §39.

M Schlosshauer. (2005).

Relevance. Schlosshauer’s 2005 Reviews of Modern Physics article ‘Decoherence, the measurement problem, and interpretations of quantum mechanics’. AEight cites Schlosshauer2005 in Part IX as the modern review of how decoherence dissolves (rather than solves) the measurement problem; the cohort framework completes Schlosshauer’s reading with the bulk-side picture: the dissolution is real because the bulk is deterministic. First cited in §43.

M Schlosshauer. *Decoherence and the Quantum-to-Classical Transition*. Springer, Berlin, (2007).

Relevance. Schlosshauer’s 2007 textbook extending the 2005 RMP article into a full-length treatment of decoherence and the quantum-to-classical transition. AEight cites Schlosshauer2007 in Part IX as the comprehensive modern reference; the bulk-side reading of T-A8.5 and T-A8.6

supplies the deeper warrant that the textbook treatment leaves as an operational principle. First cited in §43.

E Schrödinger. *Quantisierung als Eigenwertproblem.* (1926).

Relevance. Schrodinger’s 1926 wave-mechanics papers (E. Schrodinger, *Ann. der Physik*). AEight cites this in Part I as the foundational paper of wave mechanics; T-A8.2 reconstructs the Schrodinger equation as the closed-system $\gamma \rightarrow 0$ limit of the GKLS master equation derived from bulk dynamics. First cited in §I.

E Schrödinger. Die gegenwärtige Situation in der Quantenmechanik. *Naturwissenschaften* **23** (1935) 807–812, 823–828, 844–849.

Relevance. Schrodinger’s 1935 cat thought experiment showing that a literal-reading of QM extends macroscopic superpositions arbitrarily far. AEight Part VIII centers the cat in the 6D-bulk-to-4D-boundary picture: the bulk state is deterministic and single-valued; the 4D boundary observer sees decohered LIVE/DEAD branches via the projection of T-A8.5; there is no macroscopic-superposition paradox once the bulk substrate is recognized as deterministic. First cited in §I.

M O Scully, B. Englert, and H Walther. Quantum optical tests of complementarity. *Nature* **351** (1991) 111–116.

Relevance. The 1991 Scully-Englert-Walther paper introducing the quantum-eraser thought experiment. AEight Part X cites this in the paradox sweep; the cohort framework reads the quantum eraser as a consistency check on T-A8.5 (decoherence as which-path damping) plus T-A8.4 (Lueders projection): erasing the which-path information restores the interference fringes because the off-diagonal coherences had been merely entangled with the environment, not destroyed. First cited in §I.

L K Shalm and others. Strong loophole-free test of local realism. *Physical Review Letters* **115** (2015) 250402.

Relevance. The 2015 Shalm et al. loophole-free Bell test at NIST. AEight Part VII cites Shalm2015 in the trio (with Hensen2015 and Giustina2015) of 2015 experiments that closed all major loopholes. The cohort framework reads the empirical record as confirmation of the bulk-mediated correlations of T-A8.7. First cited in §I.

P W Shor. *Algorithms for quantum computation: discrete logarithms and factoring.* In *Proceedings 35th Annual Symposium on Foundations of Computer Science* (1994).

Relevance. Shor’s 1994 quantum algorithm for integer factorization in polynomial time. AEight cites Shor1994 in Part X as the algorithmic motivation for quantum computing; the cohort framework’s QM reading is computationally equivalent to standard QM, so all quantum-algorithm results carry over unchanged. First cited in §I.

U Sinha, C Couteau, T Jennewein, R Laflamme, and G Weihs. Ruling out multi-order interference in quantum mechanics. *Science* **329** (2010) 418–421.

Relevance. The 2010 Sinha et al. triple-slit experiment confirming that QM admits only pairwise interference terms (no higher-order interference). AEight Part X cites this as a precision test of

the Born rule’s quadratic structure; the cohort framework’s T-A8.3 derives the Born rule as a CPTP-channel consequence, which structurally precludes higher-order interference. First cited in §41.4.

A A Starobinsky. Amplification of waves during reflection from a rotating black hole. *Soviet Physics JETP* **37** (1973) 28–32.

Relevance. The discovery of superradiance: waves scattered off a rotating black hole in the superradiant band return amplified, extracting rotational energy. RB-11 reads the phenomenon structurally — the amplification is a symplectic Gaussian pumping channel acting on the asymptotic frame’s derived complex structure, which it never touches. First cited in §117.

W. Stinespring. *Positive functions on C^* -algebras.* *Proceedings of the American Mathematical Society* **6** (1955) 211–216.

Relevance. Stinespring’s 1955 dilation theorem on CPTP channels: any CPTP channel on a system can be realized as a unitary evolution on a larger system plus partial trace. AEight cites Stinespring1955 in Part III as the mathematical foundation for T-A8.2’s derivation of the GKLS master equation from the bulk-to-boundary partial trace. First cited in §41.

M H Stone. On one-parameter unitary groups in Hilbert space. *Annals of Mathematics* **33** (1932) 643–648.

Relevance. Stone’s 1932 theorem on one-parameter unitary groups in Hilbert space, the mathematical foundation of the closed-system unitary evolution. AEight cites Stone1932 in Part III as the key mathematical input to T-A8.2’s derivation of the Schrodinger equation from the $\gamma \rightarrow 0$ limit of GKLS. First cited in §I.

R. & W. A. Streater. *PCT, Spin and Statistics, and All That.* W. A. Benjamin, New York, (1964).

Relevance. Streater-Wightman’s 1964 monograph ‘PCT, Spin and Statistics, and All That’, the standard reference for the axiomatic / Wightman approach to QFT. AEight cites Streater-Wightman in the Appendix as the canonical rigorous treatment of the spin-statistics theorem; the cohort framework’s T-A8.9 (universal identity of indistinguishable particles) inherits the Wightman framework’s reading of indistinguishability. First cited in §39.

E Tiesinga, P J Mohr, D B Newell, and B N Taylor. CODATA recommended values of the fundamental physical constants: 2018. *Reviews of Modern Physics* **93** (2021) 025010.

Relevance. The 2018 CODATA recommended values of the fundamental physical constants (published 2021). AEight cites CODATA2018 throughout the worked examples as the source of empirical anchors (c , $\hbar$, k_B , etc.); these are the GU-5 inputs that the cohort framework’s GG-A4-Euclid certifies as zero free continuous parameters. First cited in §40.

B. Tsirelson. Quantum generalizations of Bell’s inequality. *Letters in Mathematical Physics* **4** (1980) 93–100.

Relevance. Tsirelson’s 1980 paper proving that the maximal QM violation of the CHSH inequality is $|S| = 2\sqrt{2}$, known as the Tsirelson bound. AEight cites Tsirelson1980 in Part VII as the formal

upper bound on QM CHSH; T-A8.8 then reads this bound as an extremal (Bogomolnyi-analogue) saturation — the singlet is the maximizer of the CHSH functional — consistent with why the bound is exactly $2\sqrt{2}$ (as opposed to some other irrational number). First cited in §45.

G E Uhlenbeck and S Goudsmit. Ersetzung der Hypothese vom unmechanischen Zwang durch eine Forderung bezüglich des inneren Verhaltens jedes einzelnen Elektrons. *Naturwissenschaften* **13** (1925) 953–954.

Relevance. The 1925 spin-1/2 hypothesis that interpreted the Stern-Gerlach two-spot pattern as evidence for intrinsic electron angular momentum $s=1/2$. AEight cites this in Part V to complete the historical lineage from Stern-Gerlach (1922) to the modern bulk-fermion-zero-mode reading of T-A8.1: spin is the chirality-projected zero mode of the bulk fermion on the 6D geometry. First cited in §I.

VIP-2 Collaboration. High-sensitivity test of the Pauli exclusion principle for electrons with the VIP-2 experiment. *Scientific Reports* (November 2025); experiment at LNGS, Gran Sasso.

Relevance. The strongest current bound on Pauli-exclusion-violating atomic transitions of electrons in copper: violation probability $\beta^2/2 < 4.8 \times 10^{-29}$ at 90% CL. Cited in §52 as T-A8.12’s standing experimental exposure: under the frame-path account the prediction is exact silence, with no parameter available to absorb a violation (kill criterion F-Stat-1). First cited in §52.

D Wallace. *The Emergent Multiverse: Quantum Theory According to the Everett Interpretation*. Oxford University Press, (2012).

Relevance. Wallace’s 2012 monograph defending the Everett (Many-Worlds) interpretation as the unique consistent reading of standard unitary QM. AEight Part XI cites Wallace as the modern Everett standard-bearer; the cohort framework agrees on the operational predictions but adds the bulk-side ontology that makes the ‘emergent multiverse’ a single deterministic substrate seen from 4D. First cited in §41.4.

M B Weissman. $1/f$ noise and other slow, nonexponential kinetics in condensed matter. *Reviews of Modern Physics* **60** (1988) 537–571.

Relevance. Weissman’s 1988 review of $1/f$ noise in condensed-matter systems. AEight Part XII cites Weissman1988 as background for the QM-specific eight-test horizon item ‘Test 3: $1/f$ noise universality’: the cohort framework’s T-A8.5 plus T-A8.6 predict that $1/f$ noise spectra should be universal across condensed-matter platforms, since the einselected pointer-basis structure is universal. First cited in §48.

J A Wheeler. *Law without law*. In *Quantum Theory and Measurement*, Princeton University Press (1983).

Relevance. Wheeler’s 1983 paper articulating the delayed-choice thought experiment and the ‘law without law’ speculation. AEight Part X discusses Wheeler’s delayed choice as a paradox; the cohort framework reads it as a confirmation of T-A8.4 (Lueders projection rule) applied at the bulk-to-boundary projection: the boundary state is determined by which boundary measurement is made, not by any retroactive change to the bulk. First cited in §I.

E P Wigner. *Remarks on the mind-body question.* In *The Scientist Speculates*, Heinemann (1961).

Relevance. Wigner’s 1961 essay introducing the ‘Wigner’s friend’ thought experiment and the consciousness-causes-collapse speculation. AEight Part X discusses Wigner’s friend as the historical seed of the Frauchiger-Renner sharpening; the cohort framework does NOT invoke consciousness for collapse, locating the ‘collapse’ instead in the decoherence dynamics of T-A8.5. First cited in §74.

K. G. Wilson. Renormalization group and critical phenomena. *Phys. Rev. B* **4** (1971) 3174.

Relevance. The renormalization group as successive coarse-graining. Cited for the identification of radial integration along the non-compact scale fiber I_r with Wilsonian coarse-graining — the irreversible half of the two-fiber picture, and the reason openness is constitutional rather than incidental. First cited in §30.2.

H M Wiseman and G J Milburn. *Quantum Measurement and Control.* Cambridge University Press, (2010).

Relevance. Wiseman-Milburn 2010, the modern reference for continuous quantum measurement and feedback control. AEight cites this in Part IX as the standard treatment of weak measurement and POVMs; the cohort framework’s T-A8.4 generalizes the Lueders projection to POVM-style instruments via the same CPTP-channel structure that Wiseman-Milburn presupposes. First cited in §42.

J Worrall. Structural realism: the best of both worlds?. *Dialectica* **43** (1989) 99–124.

Relevance. Worrall’s 1989 paper introducing structural realism into the philosophy of science. AEight cites Worrall1989 in Part XI and XIII as the historical seed of OSR; the cohort framework’s claim that ‘structure (the bulk geometry) is the ontology’ is the natural Worrall-style position applied to the QM stratum. First cited in §110.

T. T. Wu and C. N. Yang. Concept of nonintegrable phase factors and global formulation of gauge fields. *Phys. Rev. D* **12** (1975) 3845–3857.

Relevance. The global / fiber-bundle formulation of gauge theory, identifying the non-integrable phase factor (Wilson line) with bundle holonomy. The precise precedent for reading the quantum phase on S^1_θ as a principal- $U(1)$ -bundle holonomy in the hidden-fiber section. First cited in §30.2.

H. Zeh. On the interpretation of measurement in quantum theory. *Foundations of Physics* **1** (1970) 69–76.

Relevance. Zeh’s 1970 paper founding the decoherence program: the wave function collapses are environmental in origin, not metaphysical. AEight cites Zeh1970 in Parts VI and IX as the historical seed; T-A8.5 derives the off-diagonal damping rate $\Gamma_{\alpha\beta}$ from the bulk-side KK mode integrals, supplying the deeper warrant for the Zeh program. First cited in §33.

W. Zurek. Pointer basis of quantum apparatus: into what mixture does the wave packet collapse?. *Physical Review D* **24** (1981) 1516–1525.

Relevance. Zurek’s 1981 paper introducing einselection (environment-induced superselection): the system-environment Hamiltonian selects a preferred ‘pointer basis’ for which off-diagonal

coherences decay fastest. AEight cites Zurek1981 in Part IX (Figure 10 shows the predictability sieve schematic); T-A8.6 inherits einselection from AX-1 (bulk locality) which forces the system-environment coupling to factor along position, recovering Zurek’s selection as a theorem rather than a postulate. First cited in §I.

W. Zurek. *Reviews of Modern Physics*. (2003).

Relevance. Zurek’s 2003 Reviews of Modern Physics article ‘Decoherence, einselection, and the quantum origins of the classical’, the canonical review of the decoherence program. AEight Part IX cites Zurek2003 as the modern summary the cohort framework completes via the bulk-side derivation of T-A8.5 and T-A8.6. First cited in §I.

R. Zwanzig. Ensemble method in the theory of irreversibility. *Journal of Chemical Physics* **33** (1960) 1338–1341.

Relevance. Zwanzig’s 1960 projection-operator method (with Nakajima 1958) for deriving an exact reduced-system dynamics from the unitary dynamics of system-plus-environment. GG-A8-QM cites Nakajima–Zwanzig in Part III as the classical antecedent of the OGGEp partial-trace construction; the framework supplies the parametric scale ($\tau_c \sim 1/M_{\text{lock}}$) at which the Nakajima–Zwanzig non-Markovian kernel reduces to the GKSL generator. First cited in §83.