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The Standard Model as Theorem of GG-Theory

Its 28 Parameters as Observations of One Six-Dimensional Geometry

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Abstract

The Standard Model is the most successful theory ever built and the most unsatisfying: it predicts experiment to extraordinary precision, yet cannot say where its own numbers come from. Twenty-eight quantities — the masses of the particles, the strengths of the forces, the angles that decide how one particle turns into another — must be measured and put in by hand.

This paper argues that those numbers are not free choices of nature but *observations*. Nothing in nature is ever truly isolated — not even a single particle — so the four-dimensional world we measure is a projection of a deeper six-dimensional one, fixed once and carrying no adjustable dials. Measurement is that projection in action, and the twenty-eight are not dials set by hand but values it must return: where the textbook sees accidents, the geometry sees theorems.

Two limits are stated at the outset. The list of particles the world contains follows from the same geometry, except for the Higgs, whose place in it is chosen for economy rather than forced. And the number measuring how far the strong nuclear force distinguishes matter from antimatter — known to be smaller than a billionth — is fixed here only to a choice between two values, that choice settled by a slow relaxation carried in a companion paper.

Some follow from the geometry outright, others once a single named step is supplied, and the rest are marked honestly as still open. Nothing is tuned after the fact: one short list of integers must serve all twenty-eight at once, so the result is forced rather than fitted — propositions descending from a fixed starting point, in the spirit of Euclid.

If this holds, a deeper question becomes fair: why these values, why four forces, why every electron is identical? The paper reads all three as one statement of uniqueness — a world that is as it is not by luck, but because the geometry allows no other.

Executive Summary

The problem. The Standard Model carries twenty-eight numbers it cannot explain: three gauge couplings, two Higgs-sector parameters, nine charged-fermion masses, the neutrino mass scale and its two splittings, four quark-mixing (CKM) and four lepton-mixing (PMNS) parameters, two Majorana phases, and the strong-CP angle $\bar{\theta}$. Twenty-seven are carried here as outputs of geometry, at grades this paper records row by row; the twenty-eighth, $\bar{\theta}$, is reduced to a $\mathbb{Z}_2$ at first order — 0 or π by a theorem, the sign a named open item — its observable value set by a companion's relaxation rather than by the ledger below. In the textbook all twenty-eight are inputs to be measured; this paper is the particle-physics deployment of the GG-Theory program, and its claim is that each is instead an *output* of a fixed six-dimensional geometry.

The central recognition. The reason those numbers can be outputs at all is a change in what they are. No physical system is closed — not even a single particle — so the four-dimensional world we measure is a projection of a six-dimensional bulk, and that projection *is* quantum observation. This is the program's third axiom, the Operational Geometry–Gauge Equivalence Principle (OGGEP): an observer is a boundary of an open system, and what it records is what the projection returns. The projection is not a picture but a law — the Geometric Dynamics of Open Systems (GDOS), whose stationary attractor configurations are the values an observer reads — and that law is itself forced by two four-pillar structures: the Geometric-Invariance pillars (Gi-4: the total geometric–gauge object TGG, the covariant conservation identity CCI, the geometric free-energy functional GFF, and the gauge-runtime invariance pipeline GRIP) and the Quantum-Information pillars (Qi-4). The twenty-eight Standard-Model parameters are therefore *registered observables* of one open-system projection; not one is a free input. That is an *ontological* statement about what the parameters *are*, not a claim that all have been *computed* here; the honest status is the six/fourteen/eight triage below. For a century no one asked whether the parameters might be observables of this kind; that question, and its affirmative answer, is the central claim of this paper.

The single input. Everything rests on one fixed starting point with no adjustable numbers: a six-dimensional bulk — four-dimensional spacetime, a radial fifth dimension acting as an energy ladder, and a small compact sixth carrying gauge phase — with a short integer ledger and the five dimensionful constants of nature. The ledger is the Occam-Gauge-Normalization five-integer tuple (OGN-5), $(M, N, L; J, K) = (1, 3, 5; 7, 35)$. Its five integers are not adjusted but *forced*: they are the unique tuple that survives the anomaly-free closure of the bulk — the Green–Schwarz-modified six-dimensional Bianchi identity $dH_3 = X_4$, the program's BI-6 condition — each with a definite physical meaning ($M = 1$ the tensor seam, $L = 5$ the SU(5) block, $N_c = 3$ the color triplicity, $J = 7$ the odd closure index, $K = 35 = LJ = M^2 + N^2 + L^2$ the master lock). Only two transcendental constants enter the formulas, one native to each bulk direction: e, the base of the exponential redshift along the radial fifth, and π, the angle of the compact sixth — so every exponential below is a scale factor and every π a gauge holonomy or the volume it builds. Once these are set, nothing further can be tuned. That no continuous knob may be added anywhere is itself an axiom of

the program, Maximum Symmetry, Minimum Freedom (MSMF), and it is what makes a wrong prediction fatal rather than absorbable.

Two threes and a six: N_c, N_g, and N_f. Three counts are kept apart in this paper, and two of them are three. $N_c = 3$ is the *color triplicity*, the three colors of $\mathrm{SU}(3)_c$, and it is what the ledger's second entry N counts, locked by the BI-6 anomaly trace ratio $(L - N)/L = 2/5$; the ledger formulas are written with that entry — $\sin^2\theta_W = L^2/(4N^3)$, $\lambda_H = 2^{-N}$, $v = M_{\mathrm{lock}}(2/L)^N$, $\alpha^{-1}(0) = 2^J + N^2$. $N_g = 3$ is the number of fermion *generations*. It is not a ledger member, and its value is a separate and much harder result, proved — conditionally — in a companion paper by a two-sided pincer: a CP lower bound $N_g \geq 3$ met by a finite-action upper bound $N_g \leq 3$. Every family-stratum and Yukawa formula reads it. $N_f = 6$, the number of quark flavors in a beta function at high scale, is a third quantity again — twice N_g, and never either of the other two. That both equal three is a *result* of two derivations sharing no premise, not an identification; nothing here rests on it.

The mechanism, in one chain. A single chain of theorems, inherited from the foundation papers and consumed rather than re-derived, turns the geometry into the parameter list:

$$\boxed{\text{BI-6} \xrightarrow{\text{T1}} \text{TGG} \xrightarrow{\text{T2}} \text{CCI} \xrightarrow{\text{T3}} \text{GFF} \xrightarrow{\text{T4}} \text{GRIP} \implies \text{28-parameter codebook.}} \tag{0.1}$$

Read left to right: the anomaly-free closure condition feeds the four geometric pillars — total geometric–gauge conservation, the bulk-to-boundary descent identity, the scalar energy landscape, and the selection cascade that decides which valley of that landscape is actually occupied — and the output is the twenty-eight-entry codebook. Every parameter is a readout at a specific depth of that cascade: a coupling is a seam registration, a mass a cascade depth, a mixing angle a projector overlap.

The headline results. The matches are tight and use no fitted prefactor. The fine-structure constant emerges as $\alpha^{-1}(0) = 2^J + N^2 = 137$; the weak mixing angle as $\sin^2\theta_W = L^2/(4N^3) = 25/108$; the strong coupling as $\alpha_s^{-1}(M_Z) = \pi e$; the Higgs self-coupling as $\lambda_H = 2^{-N} = 1/8$; the top-quark Yukawa as $y_t = 1$ at the matching scale; and the Higgs vacuum value as $v = M_{\mathrm{lock}}(2/L)^N \approx 246$ GeV — the first two and the last within a fraction of a percent of measurement, α_s and y_t within a percent, and λ_H within 3% at first order. The charged-fermion masses follow one cascade of Bogomolnyi-suppressed steps $e^{-n\pi}$, the twelve-mass block landing at the few-percent level with no continuous input.

How confident, parameter by parameter. The paper states the status of each number in one triage, quoted unchanged wherever it appears. Of the twenty-eight, **six** are *Derived* — fixed with no continuous free parameter, the weak mixing angle end-to-end as a theorem; **fourteen** are *Derived (conditional)*, forced once a single named lemma is supplied; and **eight** remain *Asserted*, each tagged with the calculation that would close it. Twenty-four open problems are carried by

name, four genuinely open; a reader who wants to know what the paper has earned should read that table, not this summary.

Why this is a derivation and not a fit. What separates a derivation from a fit is not the precision of the numbers but the absence of adjustable inputs. The codebook introduces zero continuous freedom of its own and still reproduces the gauge and charged-lepton entries to four significant figures and the twelve-mass block to a few percent — a combination no earlier framework has achieved, now sharpened into dated falsifiers — a Kaluza–Klein lock cluster at 3.85 TeV, a dark-matter axion-like particle at 8.1 μeV, exact proton stability, a definite baryon asymmetry — all decidable by about 2040. A single wrong entry kills the ledger, because the same five integers serve all twenty-eight at once.

The contrast with supersymmetry and string theory. The departure from the two best-known unification programs is sharp, and it is toward *less* freedom. Minimal supersymmetry *adds* more than a hundred free parameters rather than removing any: a partner for every known particle, and not one of the twenty-eight numbers follows from it. String theory begins from a nearly unique action, yet its six extra dimensions compactify in an astronomical number of ways — the landscape of perhaps 10^{500} vacua — so which low-energy constants we measure is undetermined: it accommodates almost any Standard Model rather than forcing ours. The geometry here is fixed once, carries no adjustable dials and offers no choice of vacuum, and is therefore *obliged* to return one definite value for each parameter. Deriving the parameters and admitting no free dials are, in the end, one property seen from two sides.

A closing thought. Once every continuous knob is gone, the oldest questions change character: they stop asking which world was selected and start asking whether any other was available. Why these forces? Why these masses? Why is every electron exactly like every other, everywhere, for all time? On the reading argued here these are not three questions but one. Identity is not a coincidence maintained across the universe; it is what it means for a property to be a readout of one geometry rather than a label carried by each particle. The four forces are not four accidents; they are the four ways one six-dimensional structure presents itself to a four-dimensional observer. And the twenty-eight constants are not the settings of the world but its reflexes.

That is a strong claim, and this paper is careful about how much of it is earned. Six numbers are derived, fourteen wait on a single named step each, eight are still asserted, and the strong-CP sign and the family count rest on results proved elsewhere and conditionally. What the paper offers is not a finished derivation of the world but a demonstration that "why these numbers?" has a calculable form — that it can be posed as arithmetic on a fixed geometry and answered, or refuted, one row at a time. Which is interesting, and is not proof. The paper marks exactly where physics ends and interpretation begins, and leaves the last step to the reader.

Keywords: GG-Theory; GG-Constitution; GG-6 bulk; GRIP-In (fixed-landscape regime); GRIP-Out (landscape-reorganizing regime); BI-6 anomaly identity; OGN-5 integer ledger; GDOS boundary law;

Gi-4 pillars (TGG, CCI, GFF, GRIP); Qi-4 pillars and Arisaka equation; Q_B as GU-5 anchor; GRIP cascade DGI $\to$ GSSB $\to$ GA; family number $N_g = 3$ as theorem; cascade-depth Yukawa hierarchy on $\mathbb{C}P^2$; Hopkins–Singer descent integrality; 28 Standard-Model parameters; strong-CP suppression; proton stability.

28 Free Parameters, One Geometry

Quantity	GG-6-Theory closed form	First order	Measured	Dev	Pull
Five primary gauge–Higgs attractor coordinates					
$\alpha^{-1}(M_Z)$	$2^J = 128$	128.0	127.930(8)	+5.5e−4	+0.99
$\sin^2\theta_W(M_Z)$	$L^2/4N^3 = 25/108$	0.23148	0.23155(4)	−3.0e−4	−1.70
quoted without the lock-scale electroweak tower, which would move $\sin^2\theta_{\text{eff}}$ by −1.9% (O_{22})					
$\alpha_s(M_Z)$	$1/\pi e$	0.11710	0.1180(9)	−7.6e−3	−0.70
v [GeV]	$M_{\text{lock}}\,(2/L)^N$	246.33	246.220	+4.5e−4	+1.00
m_H [GeV]	$\sqrt{2\lambda_H}\,v,\ \lambda_H = 1/8$	123.17	125.10(9)	−1.5e−2	−0.95
Four derived electroweak observables					
G_F [10^{-5} GeV^{-2}]	$1/(\sqrt{2}\,v^2)$	1.1653	1.16638	−9.1e−4	−1.00
$\alpha^{-1}(0)$	$2^J + N^2 = 128 + 9$	137.0	137.036	−2.6e−4	−1.00
m_W [GeV]	$\frac{1}{2}\,g_2 v$	80.21	80.369	−2.0e−3	−0.99
m_Z [GeV]	$\frac{1}{2}\,v\sqrt{g_2^2 + g_Y^2}$	91.50	91.188	+3.4e−3	+1.01
Three charged-lepton masses					
m_e [MeV]	$(v/\sqrt{2})\,N/(L^3\,2^{J+L+1})$	0.51030	0.51100	−1.4e−3	−0.88
m_μ [MeV]	$(v/\sqrt{2})\,L/2^{J+L+1}$	106.31	105.658	+6.2e−3	+0.98
m_τ [GeV]	$(v/\sqrt{2})\,N_g J/2^{J+N_g+1}$	1.7861	1.77693	+5.1e−3	+1.09
Cabibbo angle (first CKM mixing)					
$\sin\theta_{12}^{\text{CKM}}$	$\lvert e^{-\pi/2} - e^{i3\pi/5}e^{-\pi}\rvert$	0.22502	0.22501	+3.8e−5	+0.01

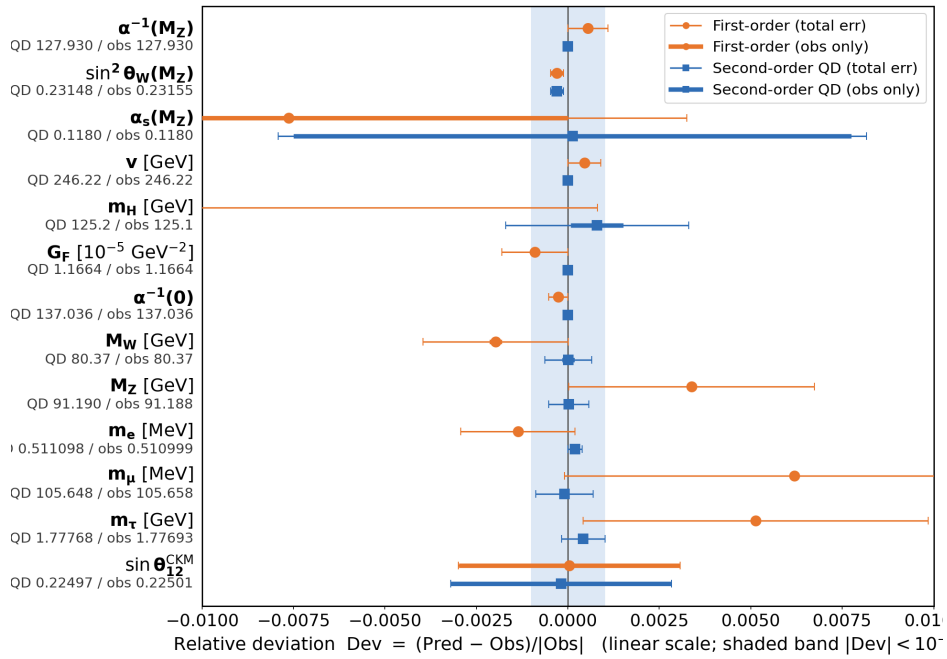


Figure 1: The thirteen most sharply benchmarked, at a glance — not free inputs, but observables of one open-system projection (§1.1). The full 28-parameter audit follows on the next page. The bars are relative deviations from the benchmark values, and the thirteen were selected for the sharpness of their measurement rather than for the closeness of their agreement. The figure is a summary and not an argument: the derivation of every entry, with its status and its uncertainty, is in the Part named beside it.

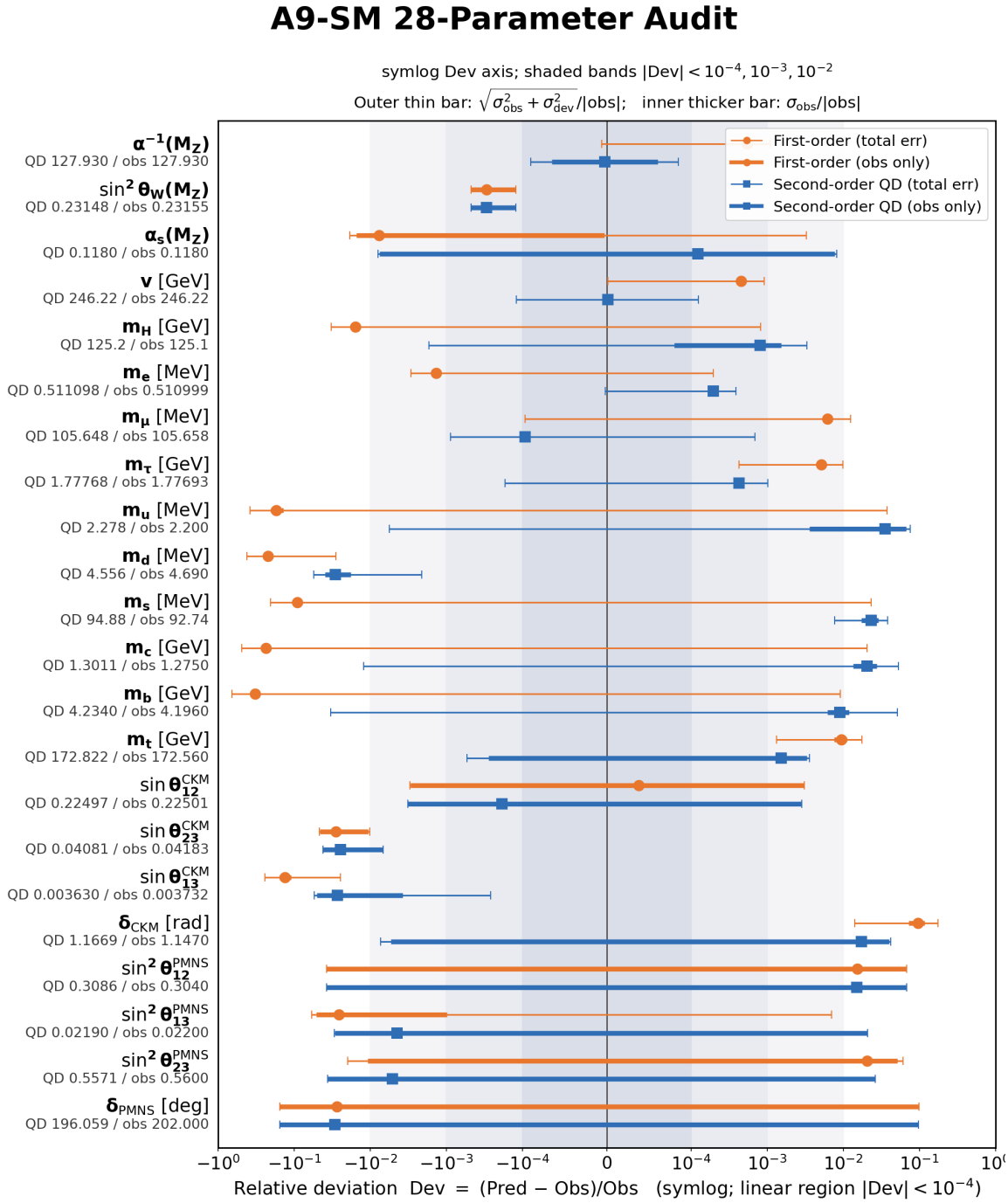
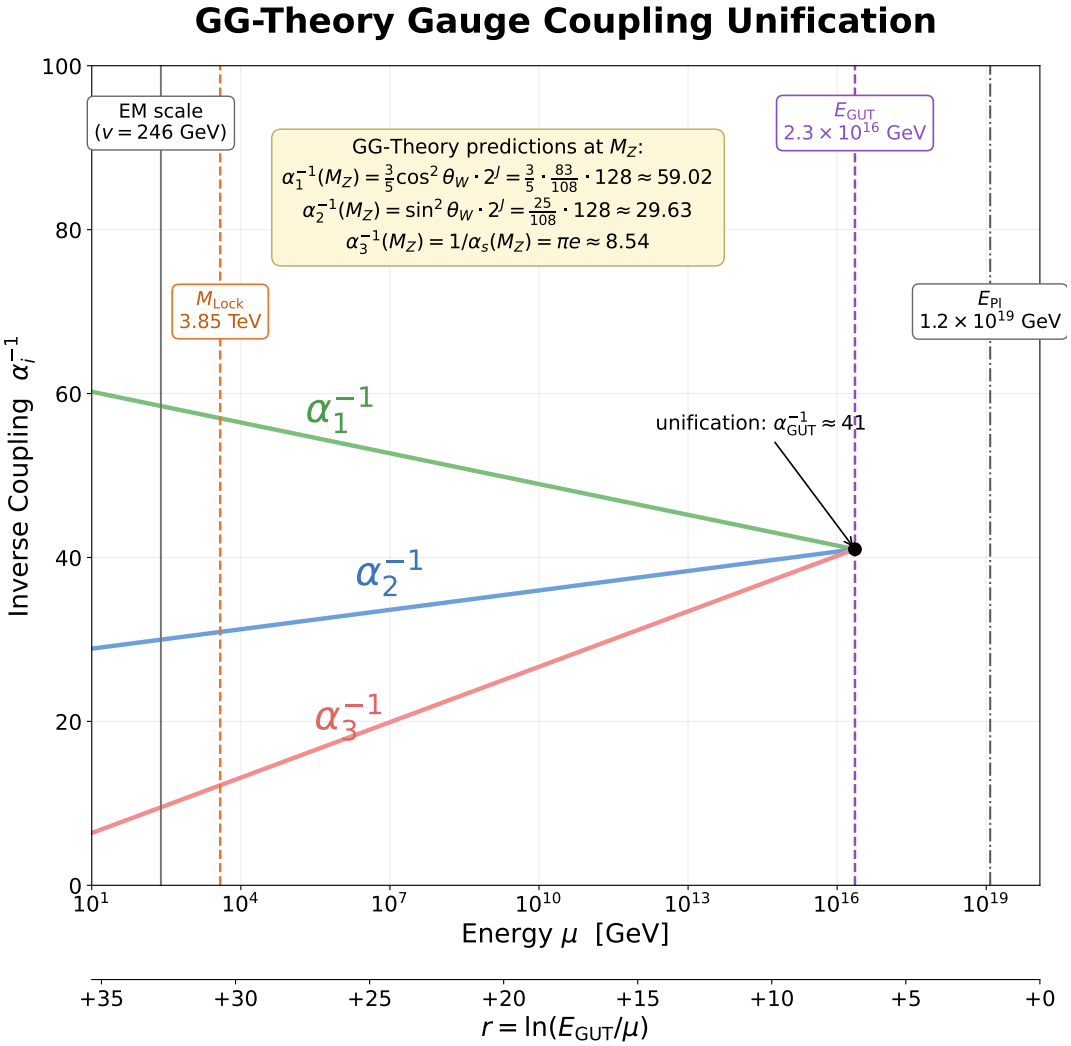


Figure 2: The full 28-parameter audit in one panel: all twenty-two directly benchmarked Standard-Model parameters on a symmetric-log deviation axis — first order (orange) and second-order QD (blue) against measurement, nested precision bands at 10^{-4}, 10^{-3}, 10^{-2}. Full version: Figure 9 in Part II.

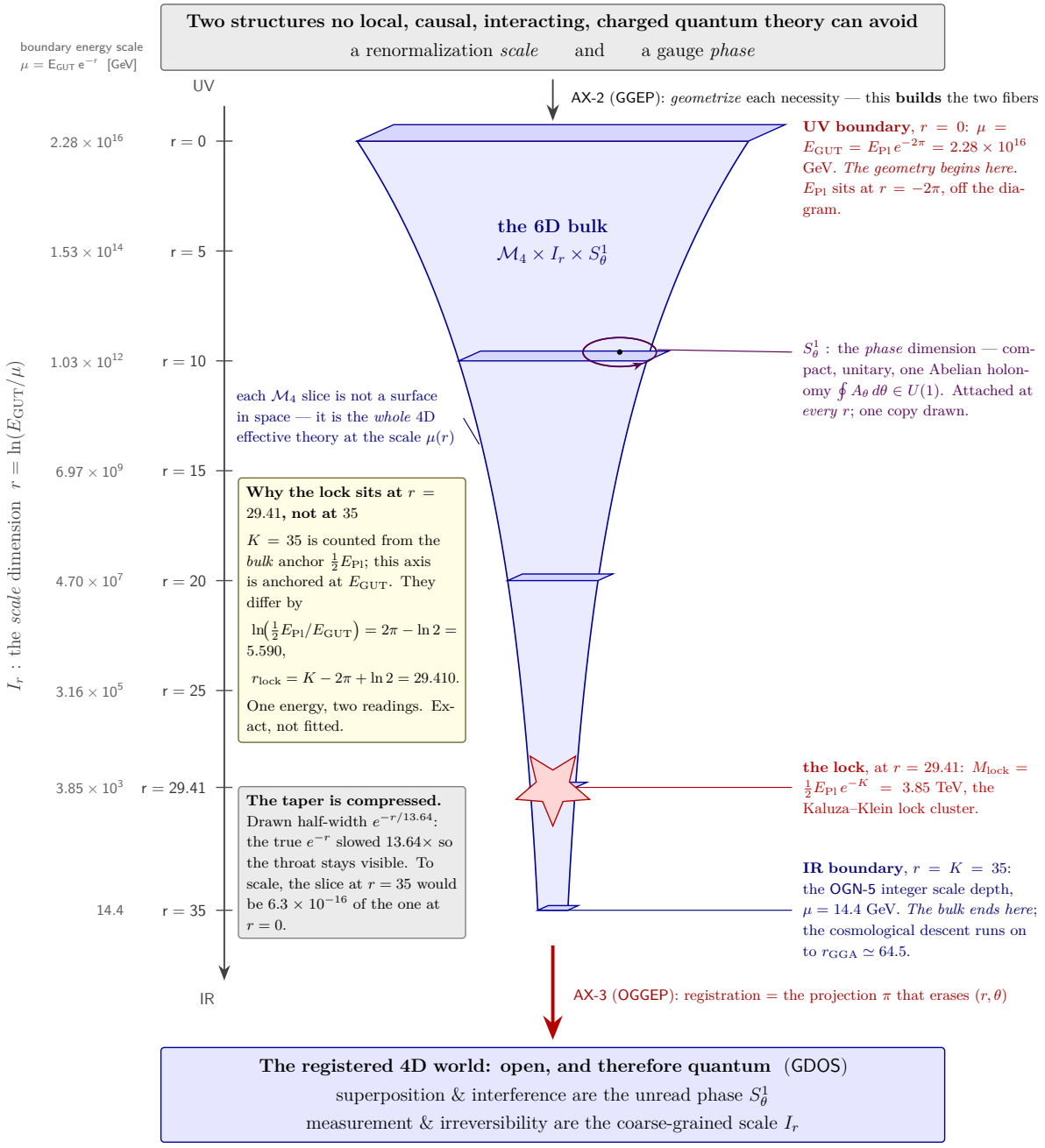


$$\alpha_1^{-1}(M_Z) = \tfrac{3}{5}\cos^2\theta_W \cdot 2^J = \tfrac{3}{5} \cdot \tfrac{83}{108} \cdot 128 = 59.02, \qquad \alpha_2^{-1}(M_Z) = \sin^2\theta_W \cdot 2^J = \tfrac{25}{108} \cdot 128 = 29.63,$$
$$\alpha_3^{-1}(M_Z) = 1/\alpha_s(M_Z) = \pi e = 8.54; \qquad \alpha_{\rm GUT}^{-1} \approx 41 \text{ at } E_{\rm GUT} = M_{\rm Pl}\, e^{-2\pi} \approx 2.3 \times 10^{16}\,\text{GeV}$$

Figure 3: GG-Theory gauge-coupling unification across the 6D bulk. Inverse couplings α_1^{-1} (green, hypercharge in SU(5) normalization), α_2^{-1} (blue, SU(2)$_L$), and α_3^{-1} (coral, SU(3)$_C$) versus the renormalization-group scale μ and the radial coordinate $r = \ln(E_{\rm GUT}/\mu)$, with $r = 0$ at the unification point. The boundary values at M_Z are pinned by the GG-Constitution and the OGN-5 ledger with the three closed forms displayed above — no continuous fitted parameter intervenes anywhere between M_Z and $E_{\rm GUT}$. Four constitutional scales are marked: the EM scale $v = 246$ GeV, the KK lock-cluster mass $M_{\rm lock} = \frac{1}{2} M_{\rm Pl}\, e^{-35} \approx 3.85$ TeV, the unification scale $E_{\rm GUT} = M_{\rm Pl}\, e^{-2\pi} \approx 2.3 \times 10^{16}$ GeV, and the Planck scale $E_{\rm Pl} \approx 1.2 \times 10^{19}$ GeV. Below $M_{\rm lock}$ the running uses the standard one-loop SM beta functions; above $M_{\rm lock}$ the KK lock cluster opens and the three couplings converge to $\alpha_{\rm GUT}^{-1} \approx 41$ at exactly $E_{\rm GUT}$ (black dot), where the SM gauge-coupling description ends and the bulk geometry takes over. This three-line meeting is the structural claim of the 6D ultraviolet completion of the Standard Model; the segment above $M_{\rm lock}$ is drawn as that claim, not computed, and the lock-cluster spectrum is its empirical content. Full discussion: Figure 13 in Part IV.

A Six-Dimensional Bulk Whose Fifth Dimension Is Scale and Whose Sixth Is Phase

The whole program in one diagram, with the scale axis carrying numbers. The bulk $\mathcal{M}_4 \times I_r \times S^1_\theta$ is a fiber bundle: over every point of the Lorentzian stage sit the two *non-spatial* fibers, scale and phase. AX-2 (GGEP) acts *upward* and builds them; AX-3 (OGGEP) acts *downward*, and registration is the projection that erases (r, θ) — the origin of quantum behavior. The vertical axis is calibrated: $E_{\rm GUT} = E_{\rm Pl} e^{-2\pi}$ at $r = 0$, the OGN-5 depth $r = K = 35$ at the infrared boundary, and the lock at $r = 29.41$, which is $K - 2\pi + \ln 2$ exactly.



Seven Channels That Could End This, and the Number Each One Turns On

Where the twenty-eight are exposed. Seven channels, each with the quantitative prediction it makes, the companion paper that carried it to a number, and the measurement that would end it. A falsification map is only as good as the sharpness of its entries, and these entries are sharp because the downstream companions carried them to numbers: before those papers existed the same table would have been a list of directions rather than of tests. The three pages before this one say what the framework delivers; this one says what it costs to be wrong. In full, with the timeline on which each channel can be decided: Table 70, p. 397.

Channel	Origin in GG-6/GDOS	Quantitative prediction	Carried by	What counts as failure
F-SM-1 Lock cluster	Lock-scale throat, $K = 35$	graviton + g^* + two terminated Abelian vectors at $M_{\rm lock} \simeq 3.85$ TeV	GG-P1-KK	Tier 1: the octet line excluded — decided on Run-3 data on expectation; Tier 2: spin-2 anchor at FCC-hh (T-KK-2c)
F-SM-2 Kaxion	Boundary dual of the BI-6 tensor	$m_a = 8.1\ \mu$eV (1.95 GHz), saturating $\Omega_{\rm dm}$	GG-C1-Kaxion, GG-A10-Cosmos	window excluded at derived coupling, or relic $\neq 1/\pi - 1/(2\pi^2)$
F-SM-3a–3e Neutrinos	Rank-2 Majorana block + lepton projector	NO ($3a$), $m_{\beta\beta} \simeq 1.8$ meV ($3b$), $\sum m_\nu \simeq 59$ meV ($3c$), $\delta_{\rm PMNS} \simeq 196^\circ$ ($3d$), upper octant ($3e$)	GG-A9-SM (in paper)	converged ν data incompatible with the package
F-SM-4 Proton	exact global B, boundary-terminated $\mathrm{U}(1)_B$	$\tau_p = \infty$; $\Delta B = 1, 2$ forbidden	GG-A9-SM, GG-A5-GDOS	any single $\Delta B = 1$ or $\Delta B = 2$ event
F-SM-5 Fifth force	Terminated $\mathrm{U}(1)_B$: no vector below $M_{\rm lock}$	absence: $\eta = 0$ at any range, any precision	GG-P1-KK, P7-5thF	any confirmed composition-dependent force, at any range
F-SM-6 Dark energy	GA fixed point of the GRIP cascade	$w = -1$ exact, two-sided ($\lvert 1+w\rvert \lesssim 10^{-91}$)	GG-A6-GRIP (Lemma DE-1), GG-A10-Cosmos (T-Cosmos-DE′)	any confirmed $w \neq -1$, either sign (e.g. a converged evolving-dark-energy detection)
F-SM-7 Leptogenesis	Majorana door at $M_{\rm lock}$; parity $\alpha_{31} = \pi$	$\eta_B \approx 5 \times 10^{-10}$, band $(1.6\text{–}15) \times 10^{-10}$	GG-A9-SM (Part VIII), GG-A10-Cosmos	a finite-temperature transport computation returning the N-width or g^2T regulator class, or an unclosable window deficit

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Preface

The Standard Model of particle physics has been called both the most successful theory ever constructed and the most embarrassing. Successful, because every prediction it has made for half a century has been confirmed to extraordinary precision; embarrassing, because it asks the experimenter to supply twenty-eight separate numbers — particle masses, mixing angles, coupling strengths — that the theory itself cannot explain. Every textbook opens by asking the reader to accept these numbers as inputs and then derives everything else with great elegance. After three generations of effort, no one has derived the inputs themselves from a deeper principle.

What this paper claims. This paper makes the case that they *are* derivable. Building on a six-volume foundation that rebuilds physics from a higher-dimensional bulk geometry — six dimensions rather than four, with the two extra dimensions curled up so tightly that the world an observer sees is the familiar four-dimensional one — we show that each of the twenty-eight numbers falls out of the geometry as a definite, calculable quantity, with no adjustable knobs. The fine-structure constant near 137 appears as $2^7 + 3^2$, where 7 and 3 count structures in the geometry. The Higgs energy scale near 246 GeV appears through a single "lock factor" that bridges the enormous gap between the Planck scale and the scale of ordinary matter. The masses of the twelve fermions — nine charged, three neutrino — appear as overlap integrals on one mathematical surface. Even a tiny energy quantum, about 0.156 eV, comes out of the same geometry.

An honest accounting. The honest answer, in this first systematic pass at the problem, is that the truth lies between a fit and a complete derivation, and the paper says exactly where. Some of the twenty-eight are fully derived from the geometry; others are derived except for a single, clearly named missing step; and the rest are still open, each tagged with the calculation that would settle it. No number is quietly tuned to match data. The test of an honest framework is not that it answers every question on the first pass, but that it knows which it has answered, which it has half-answered, and which it has not yet touched — and this paper keeps that ledger in the open.

How to read this paper. No reader needs all seventeen Parts at once. The fastest route in is the Executive Summary for the shape of the argument, Part I for the twenty-eight-number target and the honesty triage, and the precision-comparison tables at the close of each sector for the numbers themselves. A reader who wants the philosophy can turn to Part XI; a reader who wants only the testable claims can read the predictions-and-falsification section. Each result is stated so that it can be checked, and challenged, on its own, against the published Particle Data Group values.

Part I
Introduction

The thesis, the place of GG-A9-SM in the GG-Theory cohort, and the roadmap of the seventeen Parts

1 Introduction

The Standard Model is the benchmark infrared description that any deeper theory must inherit rather than bypass. Its gauge structure, symmetry breaking, anomaly constraints, running couplings, and flavor organization define the observational target that a more fundamental theory must explain without reopening a large landscape of arbitrary inputs. In the present program the Standard Model is therefore treated not as disposable phenomenology, but as a theorem candidate whose surviving low-energy parameter sheet should descend from a tighter constitutional structure.

This paper is written in that spirit. It treats the Standard Model as an infrared theorem of the GG-Theory sequence, sharpened by the runtime order developed in GG-A6-GRIP. The purpose of GG-A9-SM is therefore sharply defined: to derive the full 28-parameter Standard-Model data set, including the neutrino sector and the final topological CP angle, from the written constitutional framework already established in GG-A1-Constitution, GG-A2-GG6, GG-A5-GDOS, GG-A6-GRIP, and GG-A3-BI6 (Arisaka, 2026, A1-Constitution; Arisaka, 2026, A2-GG6; Arisaka, 2026, A5-GDOS; Arisaka, 2026, A6-GRIP; Arisaka, 2026, A3-BI6).

1.1 The central claim: the twenty-eight parameters are open-system observables

Before the audit of inherited successes that occupies the rest of this introduction, it is worth stating in one place, and without hedging, the single claim on which this paper turns. It is not a claim about a better fit to the twenty-eight numbers. It is a claim about *what those numbers are*.

Every physical system is open. No system in nature is truly closed — not a gas, not an atom, and not a single elementary particle. A particle is defined by its interactions, and an interaction is a coupling to an environment that cannot, even in principle, be switched off; the isolated particle of the textbooks is a limit that is approached but never reached. The closed, four-dimensional Lagrangian in which the Standard-Model parameters appear as free constants is therefore an idealization — and it is precisely the idealization that makes those constants look like inputs one is obliged to measure.

Openness forces a projection, and the projection is observation. If there is no closed four-dimensional theory, then the four-dimensional world an observer reports is the image of a deeper bulk

under a lawful projection. This is the content of the third axiom of the GG-Constitution, the open-system geometric–gauge equivalence principle (OGGEP, AX-3; Arisaka, 2026, A1-Constitution): every admissible theory must supply a map from the six-dimensional bulk to the registered observables on its four-dimensional boundary. The decisive recognition — absent from a century of quantum mechanics — is that this bulk-to-boundary projection *is* quantum-mechanical observation itself. Measurement is not a postulate appended to a unitary theory; it is what the OGGEP projection does. The recovery of textbook quantum mechanics from it is carried out in GG-A8-QM (Arisaka, 2026, A8-QM), and its information-theoretic content in GG-A7-Qi4 (Arisaka, 2026, A7-Qi4).

The projection is a definite law: GDOS. OGGEP requires that a projection exist; GG-A5-GDOS makes it dynamical and unique. The geometric dynamics of open systems (GDOS; Arisaka, 2026, A5-GDOS) is the open-system law whose stationary, attractor configurations are the values an observer reads off. An observable, in this framework, is not a coefficient in a microscopic action; it is the value assigned at a stationary seam of a lawful open-system flow.

Therefore the twenty-eight parameters are observables, not free inputs. It follows that every one of the twenty-eight Standard-Model parameters — each gauge coupling, each mass, each mixing angle, each phase, and the strong-CP angle — is a *registered observable* of the same OGGEP/GDOS projection. Not one of them is a free input. The "parameter space" of the Standard Model is not a space of freedoms at all; it is a list of measurement outcomes of one fixed geometry. This is an *ontological* statement, and it holds for all twenty-eight regardless of how completely the present paper succeeds in *computing* each one. The honest accounting of that computational progress is the six/fourteen/eight provenance triage of §8; the eight entries still labeled **Asserted** are observables whose determining projection has not yet been closed in detail, not exceptions to the claim.

And the law itself is forced. Finally, GDOS is not an arbitrary open-system equation. It is the one consistent with the two four-pillar structures the constitution erects over it: the geometric pillars Gi-4 (TGG $\to$ CCI $\to$ GFF $\to$ GRIP; Arisaka, 2026, A6-GRIP) and the quantum-information pillars Qi-4 (Arisaka, 2026, A7-Qi4). Because the projection that produces the observables is itself forced, the observables it produces are determined.

Figure 4 draws this chain as the single deductive funnel it is, and its conclusion is the reason the rest of the paper can exist. For the hundred years since the birth of quantum mechanics and the fifty since the Standard Model was completed, the twenty-eight numbers have been treated as inputs to a closed four-dimensional theory, and no one asked whether they are instead observables of an open-system projection from a higher-dimensional bulk. That question, and its affirmative answer, is the breakthrough of GG-Theory. Once it is granted, the dyadic 2^7 behind α^{-1}, the rational 25/108 behind $\sin^2\theta_W$, the transcendental $1/\pi e$ behind α_s, and the cascade tolls behind the fermion spectrum cease to be a list of separate mysteries; they become the technical cash-out of one recognition. The remainder of this paper is that cash-out. A reader who carries away only the funnel of Figure 4 already holds the essential content of the work; the Parts that follow are there to compute, observable by observable, what the funnel guarantees is computable.

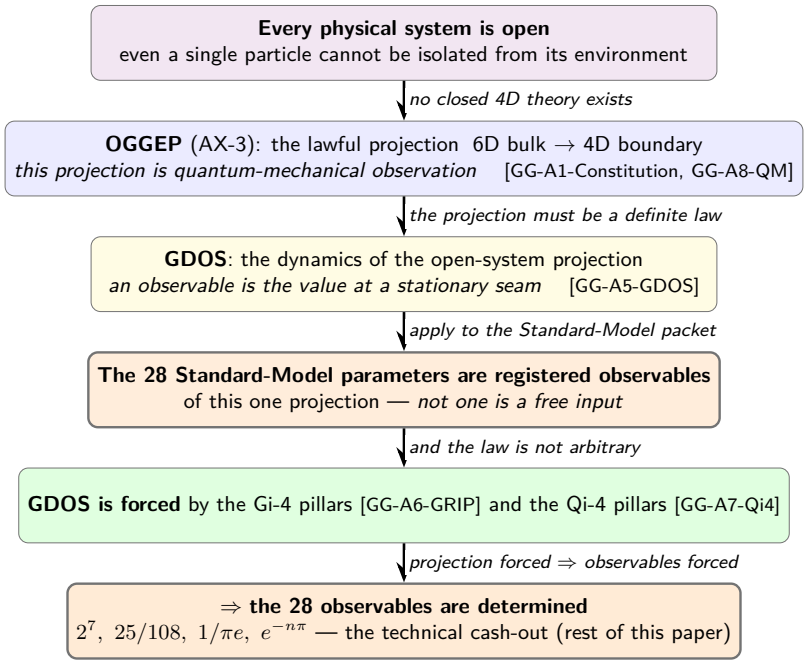

Figure 4: The central claim, as one deductive funnel. Because no physical system — not even a single particle — is closed, the four-dimensional world is a lawful projection (OGGEP, AX-3) of the six-dimensional bulk, and that projection is quantum observation itself; its dynamics is the open-system law GDOS. The twenty-eight Standard-Model parameters are therefore *registered observables* of the projection, not free inputs — and because GDOS is forced by the Gi-4 and Qi-4 pillars, those observables are determined. The closed-form values derived in the rest of the paper (2^7 behind α^{-1}, $25/108$ behind $\sin^2\theta_W$, $1/\pi e$ behind α_s, the cascade tolls $e^{-n\pi}$ behind the fermion spectrum) are the technical cash-out of this single recognition.

The computation also has a terminal page, and it is worth announcing here. Part X assembles the complete Standard-Model Lagrangian — the object every textbook writes on its first page by decree — as the *output* of the descent: each term tagged with the stage of the export chain that produced it and the register that grades it, and each absent term (the strong-CP angle, the proton-decay operators, the free scalar-mass dial) derived as a theorem rather than removed by hand. A reader who has spent a career treating $\mathcal{L}_{\rm SM}$ as the starting axiom may wish to read that Part first, to see what the paper claims the familiar object actually is: not the first page of the theory but the last.

1.2 Why the Standard Model remains the benchmark any deeper theory must inherit

The Standard Model did not arrive all at once. It was assembled, piece by piece, across roughly fifteen years in the middle of the twentieth century, and it has stood essentially unchanged for the half-century since. The electroweak synthesis of Glashow, Weinberg, and Salam (1961–1968) (Glashow, 1961; Salam, 1968; Weinberg, 1967) unified the electromagnetic and weak interactions under a single $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge symmetry, broken by the Higgs mechanism (Englert & Brout, 1964;

P. W. Higgs, 1964); the proof by 't Hooft and Veltman (1971–1972) that such spontaneously broken gauge theories are renormalizable ('t Hooft & Veltman, 1972) turned that synthesis from an ansatz into a predictive quantum field theory; and the discovery of asymptotic freedom by Gross, Wilczek, and Politzer (1973) (Gross, 1973; Gross & Wilczek, 1973; Politzer, 1973) established quantum chromodynamics as the theory of the strong interaction. By the mid-1970s the architecture was complete on paper. Almost everything since has been confirmation: the charm quark (1974), the tau lepton (1975), the bottom quark (1977), the W and Z bosons (1983), the top quark (1995), the direct detection of the tau neutrino (2000), and finally the Higgs boson (2012). Each was anticipated before it was found. The one genuinely new empirical fact—that neutrinos oscillate and therefore carry mass, established by Super-Kamiokande (1998) and SNO (2001–2002) (Ahmad et al., 2002; Fukuda et al., 1998)—was absorbed by appending a neutrino sector without disturbing the gauge structure. Fifty years on, no experiment has forced a change to the framework itself.

This staying power rests on four properties that any successor must respect.

First, the Standard Model is structurally economical. A handful of local gauge symmetries, three replicated families of chiral matter, and a single scalar doublet organize an immense and otherwise bewildering body of data. The same $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry that governs how quarks bind into a proton also fixes how a muon decays and how the Z boson couples to a neutrino.

Second, it is quantitatively precise to a degree almost unique in science. Precision electroweak observables, flavor-changing processes, QCD jet and scaling data, Higgs production and decay rates, and neutrino-oscillation parameters all sit within one tightly constrained quantitative language; the anomalous magnetic moment of the electron agrees with theory to better than a part in 10^{12} (Schwinger, 1948). A framework that reproduces this body of measurement must be taken seriously as the correct effective description of everything we have so far observed.

Third, it is internally constrained. Gauge invariance, the cancellation of chiral anomalies family by family, the renormalizability of the infrared description, and the experimentally verified pattern of electric charges leave astonishingly little room for arbitrary modification. The charges of the quarks and leptons are not adjustable inputs: they are forced by the requirement that the theory be quantum-mechanically consistent at all.

Fourth, it is structurally cumulative. The Standard Model is not one isolated idea but a tightly interlocked synthesis—gauge symmetry, spontaneous symmetry breaking, asymptotic freedom, and flavor mixing—in which the parts reinforce one another. One cannot excise a piece without the whole edifice losing its predictivity.

Most striking of all is the framework's record of *prediction confirmed*. The W and Z bosons were predicted in detail, their masses and couplings fixed by the measured weak mixing angle, years before the UA1 and UA2 experiments found them at CERN in 1983 (Arnison et al., 1983). Precision electroweak measurements at LEP and the SLC then became sensitive, through quantum loops, to particles too heavy to produce directly: the global fits pointed to a top quark of order 170–180 GeV before the Tevatron experiments CDF and D0 discovered it at 173 GeV in 1995 (Abachi et al., 1995; Abe et al., 1995), and the same fits later confined the Higgs mass to a narrow window that contained the 125 GeV boson finally observed by ATLAS and CMS in 2012 (ATLAS Collaboration,

2012; CMS Collaboration, 2012). A theory that tells experimenters where to look, and is repeatedly proved right, has earned the deference any successor owes it.

The same internal consistency that constrains the theory has, more than once, *forced* a discovery. Because the Standard Model is a chiral gauge theory, its quantum consistency demands that the gauge anomalies cancel—and they do so only family by family, through a delicate sum over the electric charges of the quarks and leptons in each generation (Adler, 1969; Bell & Jackiw, 1969; Bouchiat et al., 1972). This is why the three colors of quark, carrying fractional charge, conspire with the integer-charged leptons so that the proton's charge balances the electron's to better than a part in 10^{21}: the quark and lepton sectors are not independent but are locked together by the demand for consistency. The most celebrated instance is the prediction of the charm quark. In 1970, to suppress the flavor-changing neutral currents that experiment stubbornly refused to show, Glashow, Iliopoulos, and Maiani argued that a fourth quark was needed to complete the doublet structure (Glashow et al., 1970); charm was duly found four years later in the J/ψ. Again and again, the bare demand that the theory hold together has been enough to anticipate what nature contains.

And yet the same theory that explains so completely *how* the particles behave is wholly silent on *why* they have the properties they do. It does not predict a single particle mass, a single coupling strength, or a single mixing angle; every one of these is measured in an experiment and written into the theory by hand. The Standard Model is, in this precise sense, a magnificent machine with twenty-eight dials whose settings it cannot explain. The lesson for any deeper theory is therefore twofold: it must preserve every one of the inherited successes cataloged above, and it must earn its keep by *reducing* the number of unexplained primitives rather than, as most proposals do, multiplying it. That is the standard against which the present paper asks to be judged.

1.3 The ten classic open problems left unanswered since the mid-1970s

The completeness of the Standard Model is also its frustration. Precisely because the framework is closed and predictive, the questions it cannot answer stand out in sharp relief—and they have scarcely moved in fifty years. They are conventionally presented as a list of ten classic open problems. The list is more illuminating when the ten are first gathered into four connected families—scale and symmetry breaking; flavor, neutrinos, and CP; the dark sector and cosmology; and unification with gravity—because the families already hint that the problems are not ten unrelated accidents but facets of a smaller number of structural facts. Table 1 gives that grouped view; the paragraphs below then take the ten in turn. The purpose of GG-A9-SM is not merely to compress twenty-eight numbers: it is to show that one fixed geometry speaks to all ten of these problems at once. We introduce them here in their own right, in the detail they deserve; *how* the geometry addresses each is taken up problem by problem in the Discussions (§106) and summarized in the resolution table there (Table 65), once the derivation has been earned.

Table 1: The ten classic open problems the Standard Model has left unanswered since the mid-1970s, gathered into four families. Each is introduced in the paragraphs below and resolved, one by one, in §106 (summary table: Table 65).

No.	Problem	The unanswered question
Scale and symmetry breaking		
1	Hierarchy / naturalness	Why does the electroweak scale sit $\sim 10^{17}$ times below the Planck scale?
2	Completeness of the Higgs sector	Is the scalar sector minimal, and what fixes m_H and the quartic coupling?
3	Vacuum stability / UV fate	Is the electroweak vacuum stable, and what completes the Standard Model in the ultraviolet?
Flavor, neutrinos, and CP		
4	Flavor structure and Yukawa hierarchy	Why three families, and why a mass spread of twelve orders of magnitude?
5	Neutrino masses and mixings	Why are neutrinos so light, are they Dirac or Majorana, and why so largely mixed?
6	Baryon asymmetry of the universe	Why is the universe made of matter, with almost no antimatter?
7	Strong-CP problem	Why is the QCD vacuum angle $\bar{\theta}$ smaller than 10^{-10}?
Dark sector and cosmology		
8	Nature of dark matter	What is the unseen matter, some five times more abundant than ordinary matter?
9	Dark energy and the cosmological constant	Why is the vacuum energy so extraordinarily small — yet not exactly zero?
Unification and gravity		
10	Quantum gravity, unification, and proton stability	How does gravity join the gauge forces, do the forces unify, and why is the proton stable?

We take the ten in turn.

1. The hierarchy and naturalness problem. The electroweak scale that sets the masses of the W, Z, and Higgs sits near 10^2 GeV, yet the only other natural scale in fundamental physics, the Planck scale of quantum gravity, lies seventeen orders of magnitude higher at 10^{19} GeV. Quantum corrections to the Higgs mass-squared are quadratically sensitive to that high scale, so holding the Higgs light appears to demand a cancellation among independent contributions accurate to one part in 10^{34}. Nothing within the Standard Model explains why the electroweak scale lies so absurdly far below the Planck scale, or why the cancellation should hold. Decades of proposed cures—supersymmetry, technicolor, large extra dimensions—have each added structure without yet being confirmed.

2. The completeness of the Higgs sector. The 2012 discovery of a Higgs boson (ATLAS Collaboration, 2012; CMS Collaboration, 2012) confirmed the mechanism of electroweak symmetry breaking, but it left the scalar sector strangely bare. Is there exactly one Higgs doublet, as the minimal model assumes, or a richer structure waiting to be found? Why does the Higgs self-coupling—and hence the boson's mass near 125 GeV—take the value it does? The single observed scalar is

consistent with the minimal choice, yet the theory supplies neither a reason for that minimality nor a principle fixing the quartic coupling that sets the shape of the symmetry-breaking potential.

3. Vacuum stability and the ultraviolet fate of the Standard Model. Run to high energies, the measured Higgs and top-quark masses place the electroweak vacuum tantalizingly close to the boundary between stability and metastability: the Higgs quartic coupling very nearly, but not quite, runs negative far below the Planck scale. Whether our vacuum is absolutely stable, metastable with a lifetime far exceeding the age of the universe, or a herald of new physics is undetermined—and the near-criticality itself looks like a clue the Standard Model cannot read.

4. The flavor structure and the Yukawa hierarchy. The Standard Model contains three identical copies—families—of every matter particle, distinguished only by mass, and it offers no reason for the number three. Worse, the masses themselves span an extraordinary range: from the top quark at 173 GeV, down through the electron at half an MeV, to neutrinos below a tenth of an eV—more than twelve orders of magnitude. The Yukawa couplings (Yukawa, 1935) that encode this spectrum, together with the quark mixing angles, are simply tabulated; the pattern is far too striking to be accidental and far too unexplained to be understood.

5. The origin of neutrino masses and mixings. Neutrino oscillations (Ahmad et al., 2002; Fukuda et al., 1998) prove that neutrinos have mass, yet those masses are at least a million times smaller than the electron's, by a mechanism unknown. Are neutrinos Dirac particles, like the charged fermions, or Majorana particles equal to their own antiparticles (Majorana, 1937)? Why are the leptonic mixing angles large—one of them near-maximal—when the analogous quark angles are all small? The Standard Model in its original form set neutrino masses to zero; their discovery is the one place where experiment has compelled the framework to grow, and the sector it added is itself entirely unexplained.

6. The baryon asymmetry of the universe. The observable universe is built of matter, with essentially no antimatter, in the tiny but decisive proportion of about six baryons per ten billion photons. Generating this asymmetry dynamically requires the three Sakharov conditions (Sakharov, 1967)—baryon-number violation, C and CP violation, and a departure from thermal equilibrium. The Standard Model does contain CP violation in the quark sector, but far too little, and its electroweak transition is too smooth; it misses the observed asymmetry by orders of magnitude, leaving the very preponderance of matter unexplained.

7. The strong-CP problem. Quantum chromodynamics permits a CP-violating term proportional to an angle $\bar{\theta}$ that, on general grounds, should be of order unity. Measurements of the neutron electric dipole moment instead bound it below about 10^{-10}. Why this angle is so extravagantly small—ten orders of magnitude beneath its natural value—is the strong-CP problem. Its most popular proposed resolution, a new light particle called the axion (Peccei & Quinn, 1977; Weinberg, 1978; Wilczek, 1978), has remained undiscovered through more than forty years of searching.

8. The nature of dark matter. Galactic rotation curves, gravitational lensing, the cosmic microwave background, and the growth of cosmic structure all testify that about a quarter of the energy of the universe—roughly five times as much as ordinary matter—is some form of matter that neither emits nor absorbs light (Rubin & Ford, 1970; Zwicky, 1933). The Standard Model contains no particle that can play this role. The identity, mass, and interactions of dark matter are among

the most pressing empirical questions in all of physics, and decades of direct, indirect, and collider searches have not settled them.

9. Dark energy and the cosmological constant. The expansion of the universe is accelerating (Perlmutter et al., 1999; Riess et al., 1998), driven by a smoothly distributed energy that constitutes roughly seventy percent of the total. If this is the energy of the vacuum, the Standard Model's own estimate of that energy exceeds the observed value by some 120 orders of magnitude—often called the worst quantitative prediction in the history of physics (Weinberg, 1989). Why the vacuum energy is so small, yet not exactly zero, is the cosmological-constant problem, and it sits squarely on the seam between particle physics and gravity.

10. Quantum gravity, unification, and proton stability. Finally, the Standard Model does not include gravity, and the naive quantization of general relativity is not renormalizable, so the framework cannot be the final word at the highest energies. The hope that the three gauge forces merge into one at a high scale—grand unification (Georgi & Glashow, 1974)—generically predicts that the proton should decay; yet the proton's lifetime is now known to exceed 10^{34} years, excluding the simplest unified models. How gravity joins the gauge interactions, whether the forces truly unify, and why the proton is so very nearly immortal are bound together as the deepest structural questions the Standard Model leaves open.

These ten are usually attacked one at a time, each with its own dedicated extension of the Standard Model—a new symmetry for the hierarchy, a new particle for dark matter, a new field for the strong-CP angle, and so on. The premise of the present program is the opposite. GG-Theory holds that the ten are not independent but are the visible facets of a single fixed six-dimensional geometry, so that fixing the geometry once addresses them together rather than separately. Establishing that claim is the work of the whole cohort; this paper earns the right to make it by first deriving the twenty-eight numbers the Standard Model cannot. We return to the ten problems explicitly—and to how one geometry speaks to each—in §106 of the Discussions, with a comprehensive summary table there (Table 65).

Common to all ten is a single underlying complaint: the Standard Model can say, to extraordinary precision, *how* its particles behave, but never *why* its defining numbers take the values they do. The most disciplined way to sharpen that complaint—and to hold any proposed answer to account—is to count those numbers exactly. We do so next.

1.4 Counting the free parameters: why twenty-eight, not nineteen or twenty-six

A reader who has met the Standard Model before may be surprised by the number twenty-eight. The figure most often quoted is nineteen, and one sometimes sees twenty-six; twenty-eight is less familiar. The discrepancy is not a disagreement about physics but about how much of the modern theory one chooses to count—and it is worth being explicit, because the audit at the heart of this paper is only as meaningful as the list it audits.

The classic count of *nineteen* describes the Standard Model as it stood before 1998, with massless neutrinos. It comprises three gauge couplings (one each for the strong, weak, and hypercharge interactions); two parameters of the Higgs sector (its vacuum expectation value and its self-coupling, equivalently the electroweak scale and the Higgs mass); nine charged-fermion masses (six quarks

and three charged leptons); four quark-mixing parameters (three angles and one CP-violating phase of the CKM matrix) (Cabibbo, 1963; Kobayashi & Maskawa, 1973); and the one strong-CP angle $\bar{\theta}$. These nineteen are the textbook free parameters—the numbers a first course in particle physics says must simply be measured (Particle Data Group, 2022).

The discovery of neutrino oscillations forced the count upward. Massive neutrinos require, at a minimum, three neutrino masses and a leptonic mixing matrix—the PMNS matrix (Maki et al., 1962; Pontecorvo, 1957)—with three angles and one Dirac CP phase: seven new numbers, bringing the total to *twenty-six*. If, as both the seesaw picture and the extreme smallness of the masses suggest, neutrinos are Majorana particles equal to their own antiparticles (Majorana, 1937), two further physical CP phases appear, and the count reaches *twenty-eight*. The gap between twenty-six and twenty-eight is therefore precisely the question of whether neutrinos are Dirac or Majorana; counting twenty-eight is the honest, most complete bookkeeping—the one that leaves no physically meaningful number off the ledger.

We adopt that complete count throughout:

$$\begin{aligned} N_{\mathrm{SM}} &= 5_{\text{gauge/Higgs}} + 12_{\text{fermion}} + 4_{\mathrm{CKM}} \\ &\quad + 4_{\mathrm{PMNS}} + 2_{\text{Majorana}} + 1_{\text{topological CP}} \\ &= 28. \end{aligned} \tag{1.1}$$

The five gauge/Higgs parameters carry the forces and the electroweak scale; the twelve fermion masses carry the matter spectrum, now including the three neutrinos; the four CKM and four PMNS parameters carry quark and lepton mixing together with their Dirac CP violation; the two Majorana phases complete the neutrino sector; and the single topological angle $\bar{\theta}$ carries strong-CP. Nothing physical is omitted, and nothing is double-counted.

Fixing twenty-eight numbers in this way converts a philosophical aspiration—"explain the constants of nature"—into a finite and unforgiving audit. Either a framework reproduces these twenty-eight measured values from a smaller and fixed set of inputs, without quietly feeding the answers back in, or it does not. That is what makes the 28-parameter problem the sharpest available test of predictivity, and why it is the first target of this paper: it is narrow enough to be checked number by number, yet broad enough to touch every major structural issue in particle physics—gauge couplings, electroweak symmetry breaking, the flavor hierarchy, neutrino masses, quark and lepton CP violation, and the boundary between accidental and structural stability. The Parts that follow take up that audit in earnest.

1.5 The deductive structure, in three stages

GG-Theory is an axiomatic–deductive program, and it is worth naming its logic once, plainly, before the technical Parts begin. It runs in three stages. *First*, the GG-Constitution of GG-A1-Constitution fixes the starting point: four axioms — AX-1 (LC, locality and causality), AX-2 (GGEP), AX-3 (OGGEP), and AX-4 (MSMF) — together with seven postulates and the OGN-5 integer ledger. *Second*, from that constitution the cohort *derives* a chain of theorems, each proved in its own companion paper: the BI-6 anomaly closure (GG-A3-BI6), the Gi-4 pillars TGG/CCI/GFF/GRIP

(GG-A6-GRIP), the open-system boundary law GDOS (GG-A5-GDOS), and the family number $N_g = 3$ (GG-P2-N3, conditional on a single named conjecture). *Third*, the present paper *applies* that fixed scaffolding — axioms plus already-proved theorems — to the Standard Model, reading off the twenty-eight parameters as boundary observables of the one projection. GG-A9-SM does not re-derive the first two stages; it consumes them. The provenance of every imported result, and the few load-bearing lemmas that remain open, are tabulated in the constitutional-apparatus Part III; the honest status of each of the twenty-eight outputs is the 6/14/8 triage of §8. Read in this order, the paper is the third stage of a deduction applied to the Standard Model, not a free-standing model fit.

2 The cohort inheritance and the roadmap of the seventeen Parts

Before the parameter target is fixed (Part II), it is useful to lay out two reader-orientation maps on the same opening: a panorama of the twenty-two-paper cohort in which this paper stands (§2.1, Figure 5), and a panorama of the seventeen Parts of the present paper (§2.2, Figure 6). A reader of any later Part should be able to flip back to this Section and orient themselves in both directions: which cohort companion carries which imported result, and where the present argument stands in the seventeen-Part arc.

2.1 The cohort inheritance

The present paper is not a stand-alone speculation. It is the parameter and phenomenology volume of a longer theorem chain. The role of each paper in the series is summarized in Table 2. GG-A1-Constitution fixes the constitutional rules. GG-A2-GG6 derives the six-dimensional geometry and the unified gauge-gravity stage. GG-A5-GDOS supplies the open-system projection to observable four-dimensional physics. GG-A7-Qi4 extends GDOS into the Qi-4 runtime language. GG-A3-BI6 proves BI-6 anomaly freedom. GG-A9-SM, the present paper, turns that written framework into explicit Standard-Model parameter calculations. The companion papers GG-A10-Cosmos and GG-P1-KK then push the same framework toward cosmology and experimental falsification.

This placement is important for interpretation. GG-A9-SM should be read as the particle-physics theorem chapter of a broader program, not as an isolated model-building note.

The present paper is one of twenty-two. GG-Theory is written not as a single monograph but as a cohort of twenty-two preprints — eleven A-papers, six P-papers, three C-papers and two white papers — each carrying one rigorously stated piece of a single descent that runs from four axioms and seven postulates to the empirical structure of physics. The partition into twenty-two papers reflects the partition of subject matter, not a ranking of importance: the framework is one program, and each paper is a named carrier of named content. Figure 5 is the panorama; a reader meeting the program for the first time can read its architecture off the figure top to bottom before returning to the Standard Model that is the business of this paper. The same information is gathered in Table 2: a one-line entry for every paper in the cohort, its main achievement and its role in the program, with a live link to each.

Table 2: The twenty-two-paper GG-Theory cohort. Each paper's main achievement and its role in the program. Every paper name links to that paper's own page at preprints.arisaka-gg.org, where all twenty-two are collected. The paper marked *(this paper)* is the one you are reading.

Paper	Main achievement	Role in the cohort
GG-A1-Constitution	Four axioms and seven postulates fixing what counts as an admissible claim — the constitution of the whole program	The constitutional root; every later paper inherits its four axioms and seven postulates
GG-A2-GG6	Derivation of the six-dimensional GG-6 bulk, BI-6, OGN-5, compact holonomy, and the unification of all six forces	First of the GG-6 bulk trio; fixes the geometry and ledger the boundary law is built on
GG-A3-BI6	Complete proof of BI-6 anomaly freedom, unified gauge consistency, and conservation of the dual current	GG-6 bulk trio; the ultraviolet legality certificate that lets the projected ledger exist
GG-A4-Euclid	The historical lineage from Euclid to GG-6, and the zero-parameter integer OGN-5 ledger behind the constants	GG-6 bulk trio (historical companion); supplies the integer ledger behind the constants
GG-A5-GDOS	OGGEP descent to boundary quantum mechanics, boundary general relativity, and formal 4D QFT: the GDOS boundary law	The open-system boundary law itself — the hinge where the 6D bulk projects to 4D physics
GG-A6-GRIP	The runtime constitution — GRIP, the force law $F_{\mathsf{GFF}} = F_{\mathsf{GEP}} + F_{\mathsf{CEP}}$, and the scale-invariance principle SIP	A pillar paper organizing the boundary law from the geometric (Gi-4) side
GG-A7-Qi4	The Qi-4 pillars — SeOS, QCI, the identity IH + OH = EIC — and the five-story informational pagoda	A pillar paper organizing the boundary law from the quantum-information (Qi-4) side
GG-A8-QM	Quantum mechanics derived from GG-Theory — the four-dimensional reformulation of measurement and state	Consequence trio; the member that recovers textbook quantum mechanics on the boundary
GG-A9-SM *(this paper)*	The Standard Model as a theorem — the descent of all twenty-eight Standard-Model parameters	Consequence trio; the member that recovers the Standard Model of particle physics
GG-A10-Cosmos	ΛCDM as the cosmological-boundary projection — the π-locked inventory $\Omega_m = 1/\pi,\ \Omega_\Lambda = 1 - 1/\pi$	Consequence trio; the member that recovers ΛCDM cosmology
GG-A11-Origin	The closing capstone — the universal identity and the GRIP cascade across cosmos, life, and mind	The capstone that closes the A-series and proves the program's universal identity

continued on next page

Table 2, continued.

Paper	Main achievement	Role in the cohort
GG-P1-KK	The Kaluza–Klein lock cluster near $M_{\text{lock}} \approx 3.85\,\text{TeV}$, with explicit LHC and future-collider falsifiers	Predictions branch; the collider phenomenology of the Kaluza–Klein lock scale
GG-P2-N3	A three-leg derivation of $N_g = 3$ fermion generations from BI-6, MSMF, and the integer ledger	Predictions branch; the three-leg derivation of the fermion generation count
GG-P3-Electron	The electron as the ground state of nature — the six oldest electron questions, closed	Predictions branch; the electron ground-state and charged-lepton-stratum result
GG-P4-Real	The two real sources of the boundary's complex structure — positive frequency is holomorphy, from the compact phase circle and the causal clock	Foundations branch; the origin-of-i member paper behind the real presentation of T-SM-44.1 and T-SM-46
GG-P5-CPT	C, P, and T as orientation reversals of the three real bulk factors — why every particle has a twin, and why the universe is made of matter	Foundations branch; the CPT member paper and the GG-A9-SM–GG-A10-Cosmos interface for the derived CP phases
GG-C1-Kaxion	Cold dark matter as the non-spatial compact-angle pseudoscalar Kaxion at $8.1\,\mu\text{eV}$	Cosmology & astrophysics branch; the identification of cosmological dark matter
GG-C2-Hubble	The Hubble tension as two projections of one derived anchor — the early/late corridor	Cosmology & astrophysics branch; the resolution of the Hubble tension
GG-C3-BH	Black-hole entropy $A/4$ recovered as a boundary theorem within GG-Theory's holographic ledger	Cosmology & astrophysics branch; horizon thermodynamics and black-hole entropy

The full cohort in one view. Figure 5 places the present paper among the twenty-two papers of the GG-Theory cohort, which divides into five strata, two phenomenology branches, and the Foundations Branch. At the root stands GG-A1-Constitution (Arisaka, 2026, A1-Constitution), the GG-Constitution — four axioms and seven postulates. The GG6 Trio builds the bulk: GG-A2-GG6 (Arisaka, 2026, A2-GG6) (the six-dimensional Geometry–Gauge bulk and the OGN-5 ledger), GG-A3-BI6 (Arisaka, 2026, A3-BI6) (the local-and-global BI-6 anomaly proof), and GG-A4-Euclid (Arisaka, 2026, A4-Euclid) (the historical lineage and the parameter-free reconstruction of the universal constants). AX-3 (OGGEP) forces the open-system boundary law GG-A5-GDOS (Arisaka, 2026, A5-GDOS), organized by the two pillar papers GG-A6-GRIP (Arisaka, 2026, A6-GRIP) (the geometric Gi-4 pillars) and GG-A7-Qi4 (Arisaka, 2026, A7-Qi4) (the quantum-information Qi-4 pillars). The Consequence Trio reads the empirical world off the boundary law: GG-A8-QM (Arisaka, 2026, A8-QM) (quantum mechanics), *the present paper* GG-A9-SM (Arisaka, 2026, A9-SM) (the twenty-

eight Standard-Model parameters), and GG-A10-Cosmos (Arisaka, 2026, A10-Cosmos) (ΛCDM cosmology); the capstone GG-A11-Origin (Arisaka, 2026, A11-Origin) closes the A-series. Beneath these, two branches carry the sharp falsifiers: the Predictions Branch — GG-P1-KK (Arisaka, 2026, P1-KK) (the Kaluza–Klein lock cluster near $M_{\text{lock}} \approx 3.85$ TeV), GG-P2-N3 (Arisaka, 2026, P2-N3) (the three-generation pincer forcing $N_g = 3$), and GG-P3-Electron (Arisaka, 2026, P3-Electron) (the electron as the ground state of nature) — and the Cosmology and Astrophysics Branch — GG-C1-Kaxion (Arisaka, 2026, C1-Kaxion) (kaxion dark matter near 8.1 μeV), GG-C2-Hubble (Arisaka, 2026, C2-Hubble) (the Hubble tension as one geometric corridor), and GG-C3-BH (Arisaka, 2026, C3-BH) (black-hole entropy as an OGGEP port). The present paper is the Standard-Model node of the Consequence Trio: it turns the framework of GG-A1-Constitution–GG-A7-Qi4 into explicit parameter values, and its results in turn feed GG-A10-Cosmos, GG-A11-Origin, the phenomenology branches, and — through the ledger phases of the cohort map — the Foundations Branch member GG-P5-CPT (Arisaka, 2026, P5-CPT).

The Foundations Branch. The third branch is the Foundations Branch: the two member papers whose subject is the boundary's complex structure and its mirrors, standing as a single column on the map's left flank. GG-P4-Real (Arisaka, 2026, P4-Real) is the origin-of-i member paper, deriving the single imaginary unit of boundary physics from two real structures of the GG-A2-GG6 arena — the compact phase circle and the causal clock; for the present paper it supplies the origin of the complex phase structure in which the ledger's valued phases live, through the real presentation used in T-SM-44.1, T-SM-46.1, and T-SM-46.2. GG-P5-CPT (Arisaka, 2026, P5-CPT) carries the three mirrors of matter — C, P, and T read as orientation reversals of the three real bulk factors — and is the GG-A9-SM–GG-A10-Cosmos interface of the map: its δ, α_{31} edge is this paper's export — the ledger phases δ_{CKM} and δ_{PMNS} and the Majorana parities, each at the grade recorded in the codebook of Part II — which GG-P5-CPT routes to the baryon-asymmetry chain of GG-A10-Cosmos. Both carry P-series designators; their place at the spine's flank, not the phenomenology rows, marks their foundational role.

The twenty-two-paper map, identical in every paper of the cohort, is Figure 5, which closes this subsection.

2.2 Roadmap of the seventeen Parts

The rest of the paper is organized into sixteen further Parts, and the walk through them is plotted in Figure 6. The present Part — the entrance — is nearly done: the thesis that the twenty-eight Standard-Model parameters are registered observables of one open-system projection, the deductive funnel that carries it, and the two orientation maps of this closing section. Everything after this page is the derivation itself, and the itinerary below says where each piece lives. Part II fixes the target: the Standard Model in textbook form, the classic Georgi–Glashow SU(5) unification of 1974 and its GG-Theory reinterpretation from the six-dimensional bulk, and — most importantly — the codebook-provenance audit that labels each of the twenty-eight parameters **Derived**, **Derived (cond.)**, or **Asserted** before a single one is computed.

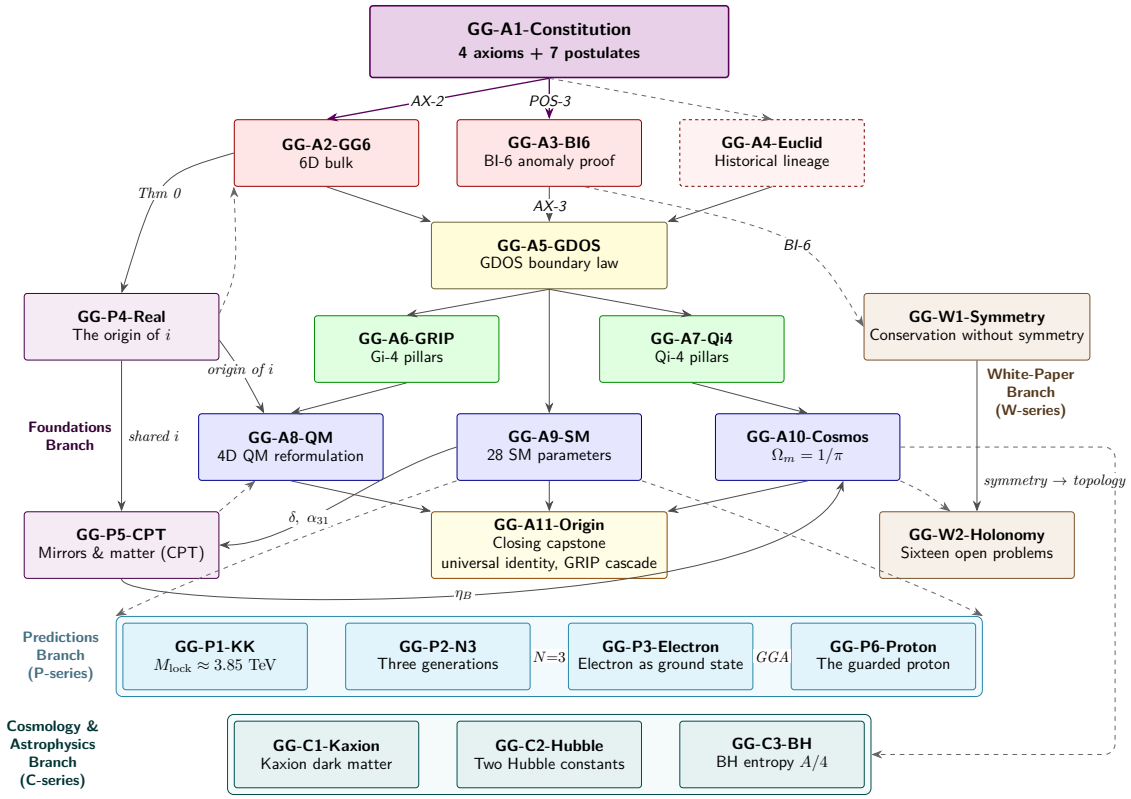

Figure 5: The twenty-two-paper cohort map of GG-Theory. The constitutional root GG-A1-Constitution (violet, top) supplies the four axioms and seven postulates from which the rest of the cohort descends: the GG6 Trio (GG-A2-GG6, GG-A3-BI6, GG-A4-Euclid, red) realizes the bulk geometry; GG-A5-GDOS (yellow) the boundary law; the GDOS Trio pillars (GG-A6-GRIP, GG-A7-Qi4, green) the geometric and quantum-information runtime; the 4D-Physics Trio (GG-A8-QM, GG-A9-SM, GG-A10-Cosmos, blue) the textbook readouts; and the closing capstone GG-A11-Origin (gold) the quarks-to-consciousness identity cascade. Two framed branches carry the phenomenology — the **Predictions Branch** (cyan), whose four papers hand their results along the row by the internal arrows (GG-P2-N3's N_g=3 into GG-P3-Electron, and that paper's ground-attractor method, GGA, on into GG-P6-Proton (Arisaka, 2026, P6-Proton)), and the **Cosmology & Astrophysics Branch** (teal). The spine is flanked by two columns of two, which do opposite jobs. On the left the **Foundations Branch** (violet) feeds it: GG-P4-Real, fed by GG-A2-GG6's Theorem 0 arena, manufactures the i that GG-A8-QM consumes, and GG-P5-CPT is the GG-A9-SM–GG-A10-Cosmos interface, fed by the ledger phases and feeding the η_B chain across the width of the map, with the shared-i edge descending solid between them. On the right the **White-Paper Branch** (brown) reads it: GG-W1-Symmetry walks the century from Noether to Hopkins–Singer, and GG-W2-Holonomy dissects the structure that arc arrives at — protection by topology rather than by symmetry — across sixteen open problems. The white papers derive nothing new, carrying every result at its parent's declared grade; the single dashed feed drawn into each names its heaviest source, not its only one. *Reading conventions:* solid arrows mark axiom-forced descent and carry an edge label naming the driving axiom or postulate where most illustrative; dashed arrows mark the deployment of derived results, or historical links inherited without being constitutionally forced.

Three Parts then assemble the apparatus. Part III records the constitutional framework — the four axioms, the seven postulates, and the canonical Gi-4 chain specialized to the Standard-Model boundary stratum. Part IV is the Substrate: the technical descent from classical Georgi–Glashow SU(5) to the GG-6 seam, the six-dimensional bulk and its three fundamental scales, the family number $N_g = 3$ as a three-leg theorem, the radial renormalization group, and Wilson–Hosotani dynamical symmetry breaking. Part V closes the preparation with the universal sectoral descent template, the derivation roadmap, and the constitutional inputs that close the parameter sheet.

The four Parts that follow are the derivation core, and they are highlighted in the roadmap because they deliver the paper's central claim — the twenty-eight parameters themselves. Part VI derives the electroweak seam: the rigorous theorem deck, the five primary attractor-coordinate theorems ($\alpha^{-1}(M_Z) = 2^7 = 128$, $\sin^2\theta_W = 25/108$, $\alpha_s(M_Z) = 1/(\pi e)$, $v \approx 246$ GeV, $\lambda_H = 1/8$), and the descendant boson-mass and Fermi-constant package. Part VII derives the fermion sector: the twelve masses as the GRIP-In floors of the family cascade, and the CKM and PMNS matrices — with the Majorana phases — as the gates between cascades. Part VIII is the matter Part: the same geometry that fixes the neutrino block also writes the excess of matter over antimatter, through the unique $\Delta L = 2$ portal and the heavy sector at the lock scale. It is the one Part of the core whose result is not one of the twenty-eight. Part IX completes the twenty-eighth and final parameter: the strong-CP angle as the protected $\mathbb{Z}_2$ of the topological ledger. Five, twenty-two, and one: the codebook closes across the three parameter-carrying Parts of the four. Part X then harvests the result: the complete Standard-Model Lagrangian, written out in its exact textbook form and assembled term by term as a constitutional export — the zeroth and the six silent structural choices of the textbook replaced by theorems, and the operators *absent* from the Lagrangian derived alongside the ones present.

With the Lagrangian assembled, Part XI is the philosophical pivot: why every electron is the same, and why exactly four forces exist with their specific qualitative behaviors, recognized as a single uniqueness theorem applied at two layers of the same six-dimensional geometry. Part XII gathers the result onto a single sheet — the cross-paper coherence map, the two comprehensive 28-parameter tables with their PDG benchmarks, and the arguments for why the result is not a post-hoc fit and what makes GG-Theory structurally different from previous unification attempts.

Two Parts then weigh the case: Part XIII, the Discussions — significance, comparison with other programs, falsification channels, and the open questions stated as open — and Part XIV, the Conclusions, viewed from the summit reached after the fifty-year climb out of the Standard Model. The back matter is the workshop. Part XV collects the technical appendices — a graduate-level course in the Standard Model read through GG-Theory, with the reference dictionaries and the lemma-promotion roadmap. Part XVI answers the strongest objections a particle theorist, cosmologist, mathematician, philosopher, or string theorist is most likely to raise, each with pointers into the paper. Part XVII closes with the annotated references, each carrying a Relevance note. Read in order, the seventeen Parts compose into a single self-contained derivation that a careful graduate student can audit step by step; read by need, each Part names its imports and can stand a visit on its own.

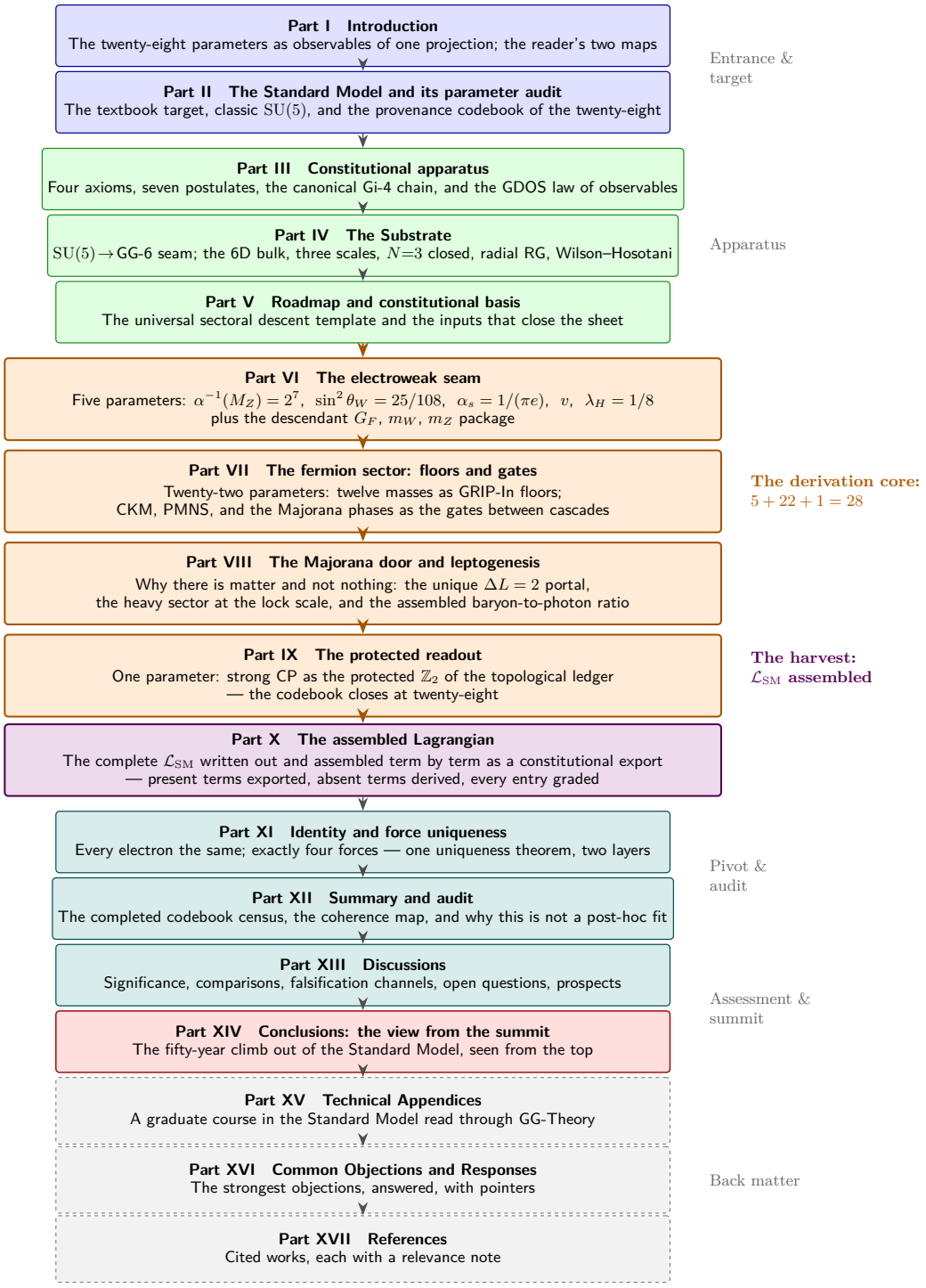

Figure 6: Roadmap of the seventeen Parts of GG-A9-SM. The argument develops in seven phases: *entrance and target* (Parts I–II), *apparatus* (Parts III–V), *the derivation core* (Parts VI–IX, highlighted), *the harvest* (Part X, highlighted), *pivot and audit* (Parts XI–XII), *assessment and summit* (Parts XIII–XIV), and the three back-matter Parts (XV–XVII). Each phase is walked through in the section that precedes this page. Parts VI–X are highlighted because together they carry the paper's central claim — the descent of all twenty-eight Standard-Model parameters and the assembly of the Lagrangian they populate, from the fixed constitutional scaffolding — the result on which every later Part, and the cohort downstream of GG-A9-SM, stands.

3 Part I in plain English: what the twenty-eight numbers are, and why their provenance is the question

The Standard Model is the most accurately tested theory in the history of physics, and nothing in this paper disputes that. Its predictions have survived fifty years of increasingly severe experiments, and where it has been pushed hardest it has held. That record is the reason the question this paper asks is worth asking at all: a framework this successful, resting on numbers nobody can derive, is a strange object.

Twenty-eight numbers have to be measured and put in by hand. Three gauge couplings, two Higgs parameters, nine charged-fermion masses, four CKM entries, four PMNS entries, and one strong-CP angle. The theory does not predict any of them, and it does not object to any other values they might have had. That is not a criticism of the theory's accuracy. It is an observation about its provenance: a measured parameter is one that could have come out otherwise, and the Standard Model contains no machinery that would have noticed.

The proposal of this paper is that the twenty-eight are not free at all — that they are registered observables of a single open-system projection, and that the right description of each is a closed form rather than a fitted value. That is a large claim, and it is falsifiable in the ordinary way: if a closed form printed here disagrees with a sharpening measurement, the closed form is wrong. What a reader should watch for through the rest of the paper is not whether the numbers come out close, which they can be made to do by enough freedom, but whether the derivation ever uses a measured value as an input to something it later predicts. The apparatus of Part III is built to make that visible, and the audit of Part XII is where the accounting is settled.

Part II
The Standard Model and its parameter audit

Classical structure, classic SU(5), the GG-Theory derivation, and the codebook of the twenty-eight free parameters

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Part I has stated the thesis and placed GG-A9-SM in the cohort. Part II now fixes the phenomenological *target* that the rest of the paper must hit. It states the Standard Model in textbook form, recalls the classical Georgi–Glashow SU(5) unification of 1974 and its GG-Theory reinterpretation from the six-dimensional bulk, and—most importantly—lays down the codebook-provenance audit that labels each of the twenty-eight free parameters **Derived**, **Derived (cond.)**, or **Asserted**. No constitutional machinery is assumed yet; the apparatus that does the deriving begins in Part III.

4 Admissible inputs, forbidden shortcuts, and the standard of derivation

The theoretical side of this paper is allowed to use only two short, fixed lists — five dimensionful constants and five integers — together with the two pure mathematical constants π and e:

$$\underbrace{\{c,\, G_N,\, \hbar;\; k_B,\, Q_B\}}_{\text{GU-5 (T-GU5, GG-A1-Constitution)}}, \qquad \underbrace{(M, N, L; J, K) = (1, 3, 5; 7, 35)}_{\text{OGN-5 ledger (T6, GG-A2-GG6)}}, \qquad \{\pi, \mathrm{e}\}. \tag{4.1}$$

Everything else must be derived, exported, or deferred. In particular, the paper is not allowed to consume G_F, m_W, m_Z, m_h, $\alpha_s(M_Z)$, $\sin^2\theta_W$, any fermion mass, or any flavor-mixing parameter as an input. Measured values appear only in the final comparison tables. This prohibition is the methodological face of the central claim of §1.1: a Standard-Model observable cannot be admitted as an input precisely because it is an *output* of the open-system projection, to be computed and only then compared. That these two short pentads — and nothing else — exhaust the admissible inputs of the entire framework is not merely asserted here but *proved* in the foundations-and-audit paper GG-A4-Euclid (Arisaka, 2026, A4-Euclid), whose GU-5 ↔ OGN-5 two-pentad parameter audit and seven reconstruction theorems (T-Euclid-1 through T-Euclid-7) establish the parameter-free reconstruction of modern physics on which the present section rests; the discussion below is the GG-A9-SM-local statement of that result.

The GU-5: the Geometric Universal Five Constants. The only admissible *dimensionful* inputs are the five Geometric Universal Five Constants (GU-5) fixed by the constitutional theorem T-GU5 of GG-A1-Constitution (Arisaka, 2026, A1-Constitution): the speed of light c (relativistic causality), the reduced Planck constant $\hbar$ (quantum action), Newton's constant G_N (gravitation), the Boltzmann constant k_B (thermodynamic temperature), and the energy–information quantum

$Q_B \approx 0.1555$ eV per qubit — the open-system conversion factor anchored in GG-A7-Qi4. Among the five, only G_N sets a dynamical scale — c, $\hbar$, and k_B are unit conventions and Q_B a fixed conversion ratio — so G_N is effectively the single dimensionful free input: the energy↔spacetime conversion factor of general relativity. Both the (non-reduced) Planck mass $M_{\rm Pl} = (\hbar c/G_N)^{1/2}$ used throughout and the unification scale $E_{\rm GUT}$ descend from it; there are no continuous *dimensionless* free parameters, and the Planck-to-electroweak hierarchy is carried not by tuning a small number but by the integer $K = 35$ of the OGN-5 ledger acting on that one scale. These five are the universal conversion factors that translate the dimensionless geometry of the bulk into the dimensionful quantities an observer measures; no sixth dimensionful constant, and no measured Standard-Model scale, is admitted. That the dimensionful content closes at exactly these five — three bulk conversion factors $(c, G_N, \hbar)$ plus two boundary conversion factors (k_B, Q_B) — and that the elementary charge e is *derived* (from $\alpha^{-1}(0) = 2^J + N^2 = 137$) rather than admitted as a sixth member, are the architecture and charge theorems T-Euclid-3 and T-Euclid-4 of GG-A4-Euclid (Arisaka, 2026, A4-Euclid). The three fundamental energy scales of the paper are built from the GU-5 alone: the Planck energy $E_{\rm Pl} = \sqrt{\hbar c^5/G_N}$, the unification scale $E_{\rm GUT} = E_{\rm Pl}\, e^{-2\pi}$, and the lock scale $M_{\rm lock} = \frac{1}{2} M_{\rm Pl}\, e^{-K} \approx 3.85$ TeV.

The OGN-5: the integer ledger. The only admissible *discrete* inputs are the five integers of the OGN-5 Diophantine ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$, fixed by T6 of A2-GG-6 (Arisaka, 2026, A2-GG6) and rendered anomaly-consistent by the BI-6 closure of GG-A3-BI6 (Arisaka, 2026, A3-BI6). The first three are the primary structural integers of the bulk — $M = 1$ (the abelian hypercharge line), $N_g = 3$ (the family multiplicity), and $L = 5$ (the unified SU(5) spine) — and the remaining two, J and K, are not free: they are forced by two independent closure relations on the first three,

$$J = 7, \qquad K = L\,J = M^2 + N^2 + L^2 = 1 + 9 + 25 = 35, \tag{4.2}$$

so that $K = 35$ is simultaneously the product LJ and the sum of squares $M^2 + N^2 + L^2$, and $J = 7$ is the unique integer consistent with both. These five integers, and nothing larger, control every discrete exponent in the descent: the dyadic seam capacity $2^J = 128$ behind $\alpha^{-1}(M_Z)$, the rational partition $L^2/(4N^3) = 25/108$ behind $\sin^2\theta_W$, the lock factor $e^{-K} = e^{-35}$ behind the electroweak scale, and the cascade depths behind the fermion spectrum.

The standard of derivation. The two remaining symbols π and e enter only as the pure mathematical constants they are — the circumference of the boundary circle and the base of the radial renormalization flow — not as adjustable physical inputs. A quantity counts as *derived* in this paper only if it is reduced, through a named upstream theorem, to the GU-5, the OGN-5, and $\{\pi, \mathrm{e}\}$ alone, with no continuous parameter introduced at any step. This standard is precisely the closure theorem T-Euclid-5 of GG-A4-Euclid (Arisaka, 2026, A4-Euclid): every dimensionless constant of physics is an OGN-5 member, a consequence of the OGN-5 cascade, or a ratio of GU-5 members — there is no third species — while the MSMF clause (AX-4, maximum symmetry, minimum freedom) is the constitutional source of the no-fitting discipline applied throughout this paper. This restriction is severe, but it is scientifically healthy: it protects the paper from the appearance of hidden fitting

and forces the logic to run in the correct order — constitution → geometry → operational export → parameter calculation → benchmark comparison.

5 The Standard Model: classical structure

Before we audit the twenty-eight free parameters, we ought to record what the parameters *are*. The reader who has carried the four-dimensional Standard Model in their pocket since graduate school can skim this section; the reader who has not will find, on the next several pages, a self-contained statement of the model's field content, gauge structure, charge assignments, and parameter list. After all, an audit makes sense only when the auditor knows what is being audited.

The Standard Model in its modern form was assembled between 1967 and 1973 from three lines of work: the unified electroweak theory of Glashow, Salam, and Weinberg (Glashow, 1961; Salam, 1968; Weinberg, 1967); the color-charge formulation of QCD; and the discovery of asymptotic freedom by Gross, Wilczek, and Politzer. Its empirical reach is extraordinary: every collider measurement to date is consistent with its predictions, and the discoveries of the W, Z, top quark, and Higgs boson over four decades have confirmed each of its remaining missing pieces. Its conceptual structure, however, leaves twenty-eight numerical inputs unexplained, and it is those twenty-eight that the present paper proposes to derive.

5.1 Fermion content: three families of quarks and leptons

The matter content of the Standard Model consists of three replicating families of spin-$\frac{1}{2}$ fermions. Each family contains fifteen left-handed Weyl degrees of freedom (or sixteen if a right-handed neutrino is included), organized into the doublet/singlet pattern shown in Table 3.

Table 3: Fermion content of one Standard-Model family in the left-handed Weyl convention. Three families ($N_g = 3$) replicate this pattern. Representations are written as $(\text{color}, \text{SU}(2)_L, Y)$ with electric charge $Q = T_3 + Y$. The right-handed singlets appear as left-handed conjugates (u_R^c, d_R^c, e_R^c).

Field	Name	Representation	Q	B
Quarks (left-handed Weyl)				
$Q_L = (u_L, d_L)$	quark doublet	$(\mathbf{3}, \mathbf{2}, +\frac{1}{6})$	$(+\frac{2}{3}, -\frac{1}{3})$	$+\frac{1}{3}$
u_R^c	up-type singlet (conjugate)	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$	$-\frac{2}{3}$	$-\frac{1}{3}$
d_R^c	down-type singlet (conjugate)	$(\bar{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})$	$+\frac{1}{3}$	$-\frac{1}{3}$
Leptons (left-handed Weyl)				
$L_L = (\nu_L, e_L)$	lepton doublet	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(0, -1)$	0
e_R^c	charged-lepton singlet (conj.)	$(\mathbf{1}, \mathbf{1}, +1)$	$+1$	0

The three families share this multiplet structure exactly. The first family is (u, d, ν_e, e); the second (c, s, ν_μ, μ); the third (t, b, ν_τ, τ). The only differences between them are the masses (and the corresponding Yukawa couplings to the Higgs) and the mixing angles relating mass eigenstates to gauge eigenstates. The replication itself is one of the deep facts the Standard Model leaves unexplained — and one the GG-Theory derivation in §7 promotes to a theorem.

5.2 Boson content: gauge bosons and the Higgs

The bosonic content consists of the gauge bosons of the three forces and one scalar field. Table 4 records them.

Table 4: Boson content of the Standard Model. Spin, electric charge, mass (PDG 2024 values), and the force the boson mediates. The five entries are the complete boson content: eight gluons, the two weak bosons, the photon, and the Higgs scalar. Masses are quoted to the precision at which they enter the comparisons of Part VI, and the two massless entries are massless as a matter of unbroken gauge symmetry rather than of measurement. Nothing in this table is derived here; it is the target the rest of the paper has to reach.

Field	Spin	Q	Mass	Force
g^a $(a = 1, \ldots, 8)$	1	0	0 (massless)	strong (color)
$W^\pm$	1	± 1	80.379 GeV	weak (charged)
Z^0	1	0	91.188 GeV	weak (neutral)
γ	1	0	0 (massless)	electromagnetic
H^0	0	0	125.10 GeV	Higgs scalar

The eight gluons mediate the strong force; the $W^\pm$ and Z^0 mediate the weak force after electroweak symmetry breaking; the photon mediates the electromagnetic force; and the Higgs scalar is the order parameter of electroweak symmetry breaking. The four electroweak gauge bosons descend from the four generators of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$: three become the heavy $W^\pm$ and Z^0 when the Higgs vacuum value $v \approx 246$ GeV breaks the symmetry, and the fourth survives as the massless photon.

5.3 The gauge group and the three forces

The Standard-Model gauge group is the direct product

$$G_{\mathrm{SM}} \;=\; \mathrm{SU}(3)_C \;\times\; \mathrm{SU}(2)_L \;\times\; \mathrm{U}(1)_Y, \tag{5.1}$$

of dimension $8 + 3 + 1 = 12$. After electroweak symmetry breaking this reduces to

$$G_{\mathrm{SM}}^{\mathrm{broken}} \;=\; \mathrm{SU}(3)_C \;\times\; \mathrm{U}(1)_{\mathrm{EM}}, \tag{5.2}$$

of dimension $8 + 1 = 9$, with the three broken generators absorbed into the longitudinal modes of $W^\pm$ and Z^0.

The three forces are the following.

Strong force ($\mathrm{SU}(3)_C$). Mediated by gluons. Confines quarks into hadrons at low energies; asymptotically free at high energies. Coupling $\alpha_s(M_Z) \approx 0.118$.

Weak force ($\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{\mathrm{EM}}$). Mediated by $W^\pm$ and Z^0. Acts only on left-handed fermions and right-handed antifermions — a parity violation that is built into the gauge structure rather than imposed by hand. Spontaneously broken by the Higgs mechanism with

vacuum expectation value $v = (\sqrt{2}\,G_F)^{-1/2} \approx 246$ GeV. The weak mixing angle $\sin^2\theta_W \approx 0.231$ measures the rotation between gauge and mass eigenstates of the neutral electroweak bosons.

Electromagnetic force ($\mathrm{U}(1)_{\mathrm{EM}}$). Mediated by the photon. The unbroken remainder of the electroweak group after Higgs symmetry breaking. Coupling $\alpha^{-1}(M_Z) \approx 127.93$, running to $\alpha^{-1}(0) \approx 137.04$ at low energies.

The fundamental strengths of the three forces — measured by α_s, α, and $\sin^2\theta_W$ — are three of the twenty-eight parameters the Standard Model leaves free.

5.4 The Higgs mechanism in two paragraphs

The fourth and fifth parameters live in the Higgs sector. The Higgs field is a complex $\mathrm{SU}(2)_L$ doublet $\Phi = (\phi^+, \phi^0)^T$ with potential

$$V(\Phi) \;=\; -\mu^2|\Phi|^2 \;+\; \lambda_H|\Phi|^4, \tag{5.3}$$

where $\mu^2 > 0$ and $\lambda_H > 0$. Minimizing the potential gives a non-zero vacuum expectation value $\langle\phi^0\rangle = v/\sqrt{2}$ with $v = \mu/\sqrt{\lambda_H} \approx 246$ GeV. The vacuum expectation value v and the quartic self-coupling $\lambda_H \approx 0.129$ are the two parameters of this sector.

Around the vacuum, three of the four real scalar degrees of freedom of Φ are eaten by the longitudinal modes of $W^\pm$ and Z^0 to give them mass $m_W = \tfrac{1}{2}g_2 v$ and $m_Z = \tfrac{1}{2}v\sqrt{g_2^2 + g_Y^2}$; the surviving real scalar is the physical Higgs boson H^0 with mass $m_H = \sqrt{2\lambda_H}\,v \approx 125$ GeV. Yukawa couplings of the Higgs to the fermions then generate the fermion masses; the size of each Yukawa coupling is one of the twelve fermion-mass parameters of the Standard Model.

5.5 The twenty-eight free parameters

Counting the free continuous inputs of the Standard Model with neutrino masses gives twenty-eight independent quantities. Table 5 records them by sector.

Table 5: The twenty-eight free continuous parameters of the Standard Model with neutrino masses, as the codebook of §8 audits them. "PDG value" gives a representative empirical entry to one or two significant figures. The PDG column is printed for orientation only. No entry in it is used before the derivation that predicts the corresponding parameter is complete, and the provenance grade in the codebook column is assigned without reference to it.

#	Sector	Parameter	PDG value (representative)
1	Gauge couplings	$\alpha^{-1}(M_Z)$ (electromagnetic)	127.93
2		$\sin^2\theta_W(M_Z)$ (weak mixing)	0.231
3		$\alpha_s(M_Z)$ (strong)	0.118
4	Higgs sector	v (vacuum value)	246 GeV
5		λ_H (quartic)	0.129
6	Charged leptons	m_e	0.511 MeV
7		m_μ	105.7 MeV
8		m_τ	1.777 GeV
9	Quarks	m_u	~ 2 MeV
10		m_d	~ 5 MeV
11		m_c	1.27 GeV
12		m_s	~ 95 MeV
13		m_t	173 GeV
14		m_b	4.18 GeV
15	CKM mixing	$\theta_{12}^{\mathrm{CKM}}$ (Cabibbo)	13.0°
16		$\theta_{23}^{\mathrm{CKM}}$	2.4°
17		$\theta_{13}^{\mathrm{CKM}}$	0.20°
18		$\delta_{\mathrm{CP}}^{\mathrm{CKM}}$	1.20 rad
19	PMNS mixing	$\sin^2\theta_{12}^{\mathrm{PMNS}}$ (solar)	0.309
20		$\sin^2\theta_{13}^{\mathrm{PMNS}}$ (reactor)	0.02248
21		$\sin^2\theta_{23}^{\mathrm{PMNS}}$ (atmospheric)	0.47–0.56
22		$\delta_{\mathrm{CP}}^{\mathrm{PMNS}}$	$\sim 203°$
23	Majorana phases	α_{21}	unmeasured
24		α_{31}	unmeasured
25	Neutrino masses	m_* (Majorana scale)	~ 0.05 eV
26		Δm^2_{21} (solar)	7.5×10^{-5} eV^2
27		Δm^2_{31} (atmospheric)	2.51×10^{-3} eV^2
28	Strong CP	$\bar\theta$ (QCD vacuum angle)	$\lesssim 10^{-10}$

These twenty-eight numbers have one thing in common: the Standard Model has nothing to say about them. They are measured at colliders, neutrino experiments, and atomic-physics tables, and then plugged into the Lagrangian. Every textbook of the subject treats the matter this way, and three generations of theoretical effort to derive them — grand-unified theories, supersymmetric extensions, string compactifications, noncommutative-geometry constructions — have produced partial accounts of small subsets but never a closure. The present paper proposes that a closure is possible after all, and that it is the constitutional content of GG-Theory that supplies it.

The rest of this Part walks the reader through what such a constitutional derivation looks like. Section 6 reviews the historical stepping-stone, the Georgi–Glashow SU(5) unification of

1974, which already organizes charge quantization and electroweak unification but leaves the deeper geometric content unaddressed. Section 7 then shows how GG-Theory recovers SU(5) as an automatic structure of the six-dimensional bulk and produces the field content, charges, family number, and force-splitting pattern of the Standard Model from constitutional principles. After that, §8 opens the parameter audit.

6 Classic Georgi–Glashow SU(5) unification

The historical first step toward a derivation of the Standard-Model gauge structure was the SU(5) grand-unified theory of Georgi and Glashow (Georgi & Glashow, 1974). Its appeal was immediate: a single simple Lie group containing the Standard-Model gauge group as a subgroup explains charge quantization, organizes the matter content of one family into a small number of irreducible representations, and unifies the three Standard-Model gauge couplings into one at a high energy. Half a century later, SU(5) remains the cleanest grand-unified candidate to inspect, and the present paper retains its non-Abelian spine within a richer six-dimensional setting.

6.1 Matter content: $\bar{\mathbf{5}} \oplus \mathbf{10}$ per family

A single Standard-Model family is fifteen left-handed Weyl fermions. SU(5) organizes them into two irreducible representations. Under the Standard-Model decomposition $\mathrm{SU}(5) \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$:

$$\bar{\mathbf{5}} \;\to\; (\bar{\mathbf{3}}, \mathbf{1})_{+1/3} \;\oplus\; (\mathbf{1}, \mathbf{2})_{-1/2}, \tag{6.1}$$

$$\mathbf{10} \;\to\; (\mathbf{3}, \mathbf{2})_{+1/6} \;\oplus\; (\bar{\mathbf{3}}, \mathbf{1})_{-2/3} \;\oplus\; (\mathbf{1}, \mathbf{1})_{+1}. \tag{6.2}$$

The $\bar{\mathbf{5}}$ contains the down-type quark singlet d^c_R and the lepton doublet $L_L = (\nu_L, e_L)$. The $\mathbf{10}$ contains the quark doublet $Q_L = (u_L, d_L)$, the up-type singlet u^c_R, and the charged-lepton singlet e^c_R. The Higgs doublet sits in a fundamental $\mathbf{5}_H$, with the color-triplet partner projected out at the high scale.

The fact that fifteen seemingly disparate Weyl fields fit into a $\bar{\mathbf{5}} \oplus \mathbf{10}$ multiplet structure is not an accident. Indeed, it is the reason SU(5) has held the imagination of theorists since 1974: charge quantization (every electric charge is an integer multiple of 1/3) becomes a consequence of the simple-group embedding rather than an empirical input.

6.2 The adjoint 24 and the gauge bosons

The adjoint representation of SU(5) has dimension 24 and decomposes as

$$\mathbf{24} \;\to\; (\mathbf{8}, \mathbf{1})_0 \;\oplus\; (\mathbf{1}, \mathbf{3})_0 \;\oplus\; (\mathbf{1}, \mathbf{1})_0 \;\oplus\; (\mathbf{3}, \mathbf{2})_{-5/6} \;\oplus\; (\bar{\mathbf{3}}, \mathbf{2})_{+5/6}. \tag{6.3}$$

The first three pieces are the Standard-Model gauge bosons: the eight gluons of $\mathrm{SU}(3)_C$, the three weak bosons of $\mathrm{SU}(2)_L$, and the hypercharge boson of $\mathrm{U}(1)_Y$. The remaining $12 = 6 + 6$ states are the heavy X and Y bosons that mediate proton decay in classical SU(5).

The properly-normalized hypercharge generator inside the fundamental representation is

$$T_Y \;=\; \sqrt{\tfrac{3}{5}} \cdot \tfrac{1}{2}\,\mathrm{diag}\Big(-\tfrac{2}{3}, -\tfrac{2}{3}, -\tfrac{2}{3}, +1, +1\Big)\,, \tag{6.4}$$

with the conventional normalization $\mathrm{Tr}(T_Y^2) = \frac{1}{2}$. The factor $\sqrt{3/5}$ is the GUT normalization that makes the coupling unification $g_1 = \sqrt{5/3}\, g_Y$ work.

6.3 What classic SU(5) explains, and what it leaves unresolved

Three things classic SU(5) explains. First, charge quantization: the embedding of $\mathrm{U}(1)_Y$ inside a simple group forces the hypercharge of every Standard-Model field to be a specific rational number. Second, the matter content of one family fits into a clean representation pattern $\bar{\mathbf{5}} \oplus \mathbf{10}$ rather than a heterogeneous list of fifteen separate Weyl spinors. Third, the three Standard-Model gauge couplings, when run from the electroweak scale toward higher energies under the renormalization group, approach a common value at $E \sim 10^{15}$ GeV — suggesting a unification scale.

Five things classic SU(5) does not explain. First, the family number $N_g = 3$: nothing in the simple-group construction selects three families over one or seventeen. Second, the masses of the fermions: the Yukawa hierarchy spans twelve orders of magnitude and remains unconstrained. Third, gravity: classic SU(5) is a four-dimensional gauge theory with no gravitational sector. Fourth, proton stability: the heavy X and Y bosons of the **24** mediate proton decay $p \to e^+\pi^0$ at a rate that has been ruled out by Super-Kamiokande, requiring either an SU(5)-breaking mechanism that lifts the X, Y masses well above 10^{16} GeV or an additional symmetry that suppresses the operator. Fifth, the measurement layer: there is no constitutional account of how a measurement extracts a number from the gauge dynamics.

The present paper inherits the first three virtues of SU(5) and addresses each of the five gaps within the six-dimensional setting of GG-Theory, to which we now turn.

7 GG-Theory derivation of SU(5) and the Standard Model

The previous section recorded what classic SU(5) achieves and what it leaves open. This section shows how the six-dimensional bulk of GG-Theory, the constitution of A1 (Arisaka, 2026, A1-Constitution), the modified Bianchi identity of GG-A3-BI6 (Arisaka, 2026, A3-BI6), and the OGN-5 ledger of A2-GG-6 (Arisaka, 2026, A2-GG6) together produce the Standard-Model field content, the charge assignments, the family number $N_g = 3$, and the three-force splitting pattern as theorems rather than as separate empirical inputs. The full technical derivations live in §14 (classic SU(5) to GG-6 reinterpretation), §15 (GDOS applied to the unified packet), and §16 (fundamental architecture); the present section is the pedagogical primer that those later sections cash out in detail.

7.1 The bulk gauge group

The six-dimensional bulk of GG-Theory carries the composite gauge group

$$G_{\text{tot}} \;=\; \text{Spin}(1,5) \;\times\; \text{SU}(5) \;\times\; \text{U}(1)_B \;\times\; \text{U}(1)_{Qi}, \tag{7.1}$$

forced by the modified Bianchi identity BI-6 of GG-A3-BI6 together with the GGEP axiom (AX-2) of A1. Spin$(1,5)$ is the Lorentz group of the 6D bulk; SU(5) is the non-Abelian unification spine first proposed by Georgi and Glashow; $\text{U}(1)_B$ is the local baryon-number symmetry that protects proton stability; and $\text{U}(1)_{Qi}$ is the local information-current symmetry required by the Arisaka equation of GG-A7-Qi4 (Arisaka, 2026, A7-Qi4).

The trace-ratio lock $|a/c| = |a/d| = 5/2$ of the BI-6 anomaly polynomial fixes the relative weights of the four factors (the full factorizing six-dimensional anomaly polynomial and its flux quantization are established, as an existence construction, in GG-A3-BI6; GG-A9-SM consumes only this trace-ratio constraint), and the OGN-5 closure $K = LJ = M^2 + N^2 + L^2 = 35$ of A2-GG-6 T6 fixes the integer ledger that controls the discrete inputs of the descent. After all, the difference between classic 4D SU(5) and the GG-6 setting is not that we replace one group by another — it is that we embed the same simple-group spine inside a six-dimensional geometry whose closure forces its presence.

7.2 Why the visible matter is spin-1/2

The bulk matter content is described by a six-dimensional Dirac action

$$S_{\Psi}^{(6)} \;=\; \int d^6x\, \sqrt{-g_6}\; \bar{\Psi}\, i\Gamma^M D_M \Psi, \tag{7.2}$$

where Ψ is a 6D spinor and D_M contains both the spin and the gauge connection. Reduction to the four-dimensional boundary, by Kaluza–Klein decomposition into modes on the compact circle S^1_θ and the radial interval I_r, projects out one chirality of the would-be zero mode and leaves a chiral four-dimensional spectrum of left-handed Weyl fermions.

Roughly speaking, six-dimensional spinors carry eight complex components; orbifold or interval boundary conditions slice this in half; what survives at low energies is the standard four-dimensional pair of two-component Weyl spinors per surviving family. The fact that the visible matter is spin-1/2 is therefore not a separate postulate of the Standard Model — it is inherited from the bulk spin structure and the boundary-condition projection.

7.3 One family contains quarks and leptons with the observed charges

The bulk gauge group $\text{SU}(5) \times \text{U}(1)_B \times \text{U}(1)_{Qi}$ acts on the surviving 4D Weyl fermions through the same $\bar{\mathbf{5}} \oplus \mathbf{10}$ embedding of classic SU(5) (Eqs. (6.1)–(6.2)). Once the embedding is fixed, the electric-charge pattern is forced by $Q = T_3 + Y$:

$$\begin{aligned} Q_L = (u_L, d_L)\colon &\quad (Q_{\text{EM}}, Q_{\text{EM}}) = (+\tfrac{2}{3}, -\tfrac{1}{3}), \\ u_R^c\colon &\quad Q_{\text{EM}} = -\tfrac{2}{3}, \qquad d_R^c\colon Q_{\text{EM}} = +\tfrac{1}{3}, \\ L_L = (\nu_L, e_L)\colon &\quad (Q_{\text{EM}}, Q_{\text{EM}}) = (0, -1), \qquad e_R^c\colon Q_{\text{EM}} = +1. \end{aligned} \tag{7.3}$$

The fractional charges of quarks and the integral charges of leptons fall out of the same group-theoretic package — exactly the lesson Georgi and Glashow drew in 1974, now embedded in a six-dimensional bulk geometry with explicit constitutional grounding.

The baryon-number assignments are the diagonal $\mathrm{U}(1)_B$ readouts

$$B(Q_L) = +\tfrac{1}{3}, \quad B(u_R^c) = B(d_R^c) = -\tfrac{1}{3}, \quad B(L_L) = B(e_R^c) = B(H) = 0, \tag{7.4}$$

forced by the constitutional locality of $\mathrm{U}(1)_B$ in the bulk gauge group. This is the structural reason proton-decay operators are absent in the exact low-energy theory: baryon number is gauged, not merely a global accidental symmetry, and the operator that would mediate $p \to e^+\pi^0$ is forbidden by the exact (global) baryon number that survives as the rigid symmetry of a boundary-terminated $\mathrm{U}(1)_B$ whose vector tower begins at the lock scale (made precise in Theorem 86.1) — in sharp contrast to the classical SU(5) situation where the X, Y bosons of the adjoint **24** mediate the decay.

7.4 Six quarks and six leptons from the family multiplicity $N_g = 3$

Once one family is in hand, the observed flavor count follows from the family multiplicity. Each family contains two quark flavors and two lepton flavors; replicating the unified package three times therefore gives precisely six quark flavors

$$(u, d), \quad (c, s), \quad (t, b), \tag{7.5}$$

and six leptonic degrees of freedom

$$(\nu_e, e), \quad (\nu_\mu, \mu), \quad (\nu_\tau, \tau), \tag{7.6}$$

which become ν_1, ν_2, ν_3 and e, μ, τ in the mass basis. The "six and six" count is not a separate mystery once the surviving chiral index is three — it is the counting consequence of three copies of one unified chiral package.

7.5 Why $N_g = 3$: MSMF lower bound and Bogomolnyi closure upper bound

The family number $N_g = 3$ is the three-leg theorem of GG-P2-N3 (Arisaka, 2026, P2-N3). The three legs are these.

Leg L1 (MSMF lower bound). By A1 AX-4 MSMF, any family number that admits no nontrivial CP phase is excluded as too poor. $N_g = 1$ has no flavor mixing at all; $N_g = 2$ has one mixing angle but the Kobayashi–Maskawa argument shows it cannot accommodate a complex CP phase (Kobayashi & Maskawa, 1973). Therefore $N_g \geq 3$.

Leg L2 (GDOS Morse upper bound). By GG-A6-GRIP GFF Morse-positivity (T3.1), the cascade of geometric attractors on the family-projector stratum $\mathbb{C}P^{N_g-1}$ closes if and only if the total Bogomolnyi instanton action is finite. Summing the cascade $S_{[2k]\to[2(k-1)]} = \pi k$ over the $N_g - 1$ gates of $\mathbb{C}P^{N_g-1}$ ($k = 1$ to $N_g - 1$) gives $S_{\mathrm{tot}}(N_g) = \frac{1}{2} N_g(N_g-1)\pi$; the BI-6 closure constraint of GG-A3-BI6 caps the admissible total instanton action at $S_{\mathrm{BI}}^{\max} = (2\pi \cdot 3/5) \cdot (L/2) = 3\pi$.

This forces $S_{\rm tot}(N_g) \le 3\pi$, i.e. $N_g(N_g-1) \le 6$, i.e. $N_g \le 3$, saturated exactly at $N_g = 3$. The tolls πk themselves are no longer a reading: they are derived in GG-A6-GRIP (Arisaka, 2026, A6-GRIP) from the Whitney pair count of the winding bundle (Morse weights $\lambda_\ell = \binom{\ell}{2}$, inflow charge $\Delta c_2 = k$, under three declared premises), so the exponential skeleton of the fermion spectrum used throughout this paper rests on a theorem-grade ladder.

Leg L3 (constructive codebook agreement). The $N = 3$ codebook reproduces the twelve fermion masses (nine charged, three neutrino), two flavor mixing matrices and the gauge–Higgs sector: after quantum-defect completion the gauge–Higgs and charged-lepton entries agree with experiment at the 10^{-3} level and the twelve-mass block at the few-per-cent level (Table 59). In this paper the phrase "four significant figures" is used at the 10^{-3} level and of the gauge and charged-lepton entries only. No alternative N admits an analogous closure.

The three legs together give $N_g = 3$ exactly: $N_g \ge 3$ from MSMF, $N_g \le 3$ from Bogomolnyi closure, and $N_g = 3$ as a constructive check. Indeed, the present paper is the constructive check.

7.6 Why the three forces split as they do

The descent from the bulk gauge group to the observed unbroken low-energy group is the two-step chain

$$\boxed{\begin{aligned}\mathrm{SU}(5)\times\mathrm{U}(1)_B\times\mathrm{U}(1)_{Qi} &\xrightarrow[\text{boundary cond. + holonomy}]{} \mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y \\ &\xrightarrow[\text{Wilson–Hosotani}]{} \mathrm{SU}(3)_C\times\mathrm{U}(1)_{\rm EM}.\end{aligned}} \tag{7.7}$$

The first arrow is the orbifold/boundary-condition projection that breaks $\mathrm{SU}(5) \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ (with $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$ surviving as separate Abelian factors). The second arrow is the Wilson–Hosotani holonomy that breaks the electroweak factor to $\mathrm{U}(1)_{\rm EM}$ at the scale $v \approx 246$ GeV.

The three forces appear distinct at low energies for two reasons. The first reason is algebraic: each surviving force corresponds to a different generator of the broken SU(5) parent. The second reason is dynamical: once the sectors are distinct they no longer run identically along the radial throat — their couplings transport with different beta functions because the surviving matter content coupled to each sector is different. The strong interaction becomes the most rapidly varying and most strongly coupled in the infrared; the weak interaction is broken and partially hidden by the electroweak vacuum; the electromagnetic interaction remains long-range because the $\mathrm{U}(1)_{\rm EM}$ generator is the one the doublet's vacuum value leaves unbroken.

7.7 Comparison: classic SU(5) versus the GG-6 reinterpretation

Table 6 summarizes the differences side by side.

Table 6: Classic Georgi–Glashow SU(5) (1974) versus the GG-6 reinterpretation of the present paper. The two columns differ on where the freedom sits. Classic SU(5) takes a simple group, a chosen scalar potential, and a family number set by hand; the GG-6 reading embeds the same algebra in a six-dimensional bulk, replaces the potential with boundary conditions and Wilson–Hosotani holonomy, and returns the family number as a theorem. The comparison is structural, and says nothing yet about which of the two agrees better with measurement.

Question	**Classic 4D** SU(5) **(1974)**	**GG-6 reinterpretation (this work)**
Unified algebra	Simple group SU(5) alone	Non-Abelian spine SU(5) inside $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$
Symmetry breaking	Higgs multiplets and chosen scalar potential	Boundary conditions plus Wilson–Hosotani holonomy in a 6D bulk
Family number N_g	Free input (set to 3 by hand)	Theorem: $N_g = 3$ from MSMF + Bogomolnyi closure (GG-P2-N3)
Proton stability	Endangered by X, Y bosons and color-triplet Higgs	Protected by orbifold projection + exact local $\mathrm{U}(1)_B$ selection rule
Gravity	Not included; appended as a separate sector	Already inside the composite $\mathrm{Spin}(1,5)$ connection mandated by GGEP
Measurement layer	External to the model	Built into the theory through OGGEP and GDOS
Parameter philosophy	Several continuous GUT-sector inputs	Discrete OGN-5 ledger plus the GU-5

The correct way to read GG-6 is not as a rejection of the historical SU(5) insight but as its geometric completion. Georgi and Glashow taught us that the Standard Model has the smell of a broken simple group; GG-Theory accepts the lesson and embeds the simple-group spine inside a six-dimensional geometry whose closure also explains anomaly freedom (via BI-6), proton stability (via gauged $\mathrm{U}(1)_B$), gravity (via $\mathrm{Spin}(1,5)$), the measurement layer (via OGGEP and GDOS), and the family number (via the Bogomolnyi closure of GG-P2-N3).

7.8 Bridge to the parameter audit

We have now stated what the Standard-Model fields are (§5), what classic SU(5) achieves and leaves open (§6), and how GG-Theory recovers the gauge group, charges, family number, and force-splitting pattern from the six-dimensional bulk plus the constitutional principles of A1 (the present section). The reader who has followed this far now knows, in qualitative terms, what the twenty-eight free parameters of Table 5 are, why they sit in the structure they do, and what kind of derivation a constitutional account is supposed to provide.

After all, an audit makes sense only when the auditor knows what is being audited. The audit itself begins in the next section.

8 Codebook provenance: what is derived end-to-end, what is derived modulo A9-local lemmas, and what is asserted

8.1 Why this section exists

The central claim of the present paper is that the twenty-eight free parameters of the Standard Model can be derived from the GG-Constitution of A1 plus the named theorems of A2–A7, with the constitutional clauses MSMF (AX-4) and OGN (POS-6) supplying the structural selection principle that picks the unique value on the relevant boundary stratum. A claim of that strength invites the question: *which* of the twenty-eight is derived end-to-end from non-physics axioms, *which* is derived modulo an additional lemma local to GG-A9-SM or named by the MSMF/OGN audit, and *which* is asserted as a closed-form rational without a genuine upstream theorem and without a unique MSMF/OGN cost-minimum at the structural-precision target? This section answers the question parameter by parameter, in the structural-honesty discipline of GG-A3-BI6, which marks every claim with one of three status labels (**Derived**, **Derived (cond.)**, **Asserted**; a fourth code, *not a ledger output*, marks the one quantity printed for the reader's convenience and not counted) accompanied by the upstream owner and a one-line concern. After all, the test of an honest framework is not that it answers every question on the first pass, but that it knows which questions it has answered, which it has half-answered, and which it has not yet touched. The present table reports the counts *after* the MSMF/OGN per-parameter audit, which is carried out in full in Part XII (§101); that audit examines eighteen entries — the seventeen a conservative reading leaves Asserted, plus the projector coefficient $2N$ of Row 27 — and promotes ten of them, leaving the eight charged-fermion Yukawa entries Asserted among the twenty-eight.

8.2 Status legend

Derived. The parameter value is forced by a named theorem of A1–A7 with all factors traceable. An adversarial reader cannot reach the boxed equation by inserting a fitted continuous prefactor.

Derived (conditional). A genuine upstream theorem is cited and supports the structural claim, but the A9 step adds one or more lemmas that are local to the present paper. The lemma is named explicitly in the Comment column.

Not a ledger output. The quantity is printed in the codebook tables because a reader looking for it expects it there, but its value is set by a mechanism outside this paper's ledger — at present one entry, the observable strong-CP angle, whose value the companion's relaxation fixes (GG-C1-Kaxion). It is not counted among the twenty-eight.

Asserted. The parameter is presented as a closed-form rational built from the OGN-5 integer ledger, but the specific choice of which OGN integer plays which role (numerator vs denominator, exponent vs prefactor) is not forced by any named theorem of A1–A7. The choice is a working hypothesis of the present paper, defensible by its empirical agreement at the 10^{-3} level, but not derived in the strict sense.

8.3 Codebook provenance table

Table 7: Codebook provenance for the twenty-eight free parameters of the Standard Model. Status labels follow the GG-A3-BI6 structural-honesty convention extended by the MSMF/OGN audit, on the legend above: **Derived**, six entries — one end-to-end theorem ($\sin^2\theta_W$, modulo two named GG-6-internal lemmas) and five fixed without a continuous free parameter but not by an A1–A7 theorem, each entry's sub-basis given in its Status and Comment; **Derived (cond.)**, fourteen, each on one named lemma; **Asserted**, eight, the charged-fermion Yukawas other than the top. A row whose parameter is a scale another row carries times a coefficient of its own is graded on the coefficient it adds, the scale's condition being recorded at the scale's row (Rows 13 and 27) — a convention of this table, stated here once. "Comment" names the load-bearing concern when the status is not Derived.

#	Parameter	In A9	Upstream owner	Status	Comment
	Gauge sector (3)				
1	$\alpha^{-1}(M_Z) = 2^J$	Thm 31.1	GG-A2-GG6 T-GG6-6 ($J = 7$)	Derived (cond.)	A9-local "binary recursion" lemma adds factor 1/2 per dyadic step
2	$\sin^2\theta_W = L^2/(4N^3) = 25/108$	Thm 32.1	GG-A3-BI6 §7 (trace ratio 2/5) + OGN-5	**Derived** (theorem)	End-to-end from the GG-A3-BI6 trace ratio + OGN-5, no empirical input; *modulo* two named GG-6-internal lemmas (quadratic neutral-partition, color-blind averaging)
3	$\alpha_s^{-1}(M_Z) = \pi e$	Thm 33.1	A1 T-GU5 + the MSMF/OGN audit	**Derived** (selected)	Cost-2 selection within 1% of $\alpha_{s,\mathrm{PDG}}^{-1} = 8.475$ (look-elsewhere $p \approx 0.27$); the empirical value enters the filter
	Higgs sector (2)				
4	$v = M_{\mathrm{lock}}(2/L)^N$	Thm 34.1	GG-A2-GG6 (OGN-5, $K = 35$) + Wilson–Hosotani	Derived (cond.)	Lock factor $\frac{1}{2}e^{-K}$ on an A9-local binary-endpoint convention; the fraction $2/L$ is derived once (the $\mathbf{5}_H$ survival fraction), its repetition over N layers is hypothesis **H-L2** (lemma L2)
5	$\lambda_H = 2^{-N}$	Thm 35.1, L6	H-L2 (lemma L6) on the GG-A6-GRIP descent; A1 AX-4/POS-6	Derived (cond., L6)	Conditional on lemma L6 — hypothesis **H-L2**: the quartic is the doublet's neutral fraction raised to the triplicity index. The counting is a theorem, the identification the hypothesis (Thm 35.1, Step 3); the curvature reading that once backed it is withdrawn (§35.5). 1/8 sits 3.2% below the measured 0.1291, and the printed $\lambda_H^{\mathrm{QD}} = 0.12928$ is read from the datum, not transported
	Charged-fermion masses (9 of the 28; Yukawa codebook)				
6	m_e	§47	A9-local codebook	Asserted	Per-particle numerator N/L^3 and denominator depth $J{+}L{+}1$ chosen post-hoc
7	m_μ	§47	A9-local codebook	Asserted	Per-particle numerator L and denominator depth $J{+}L{+}1$ chosen post-hoc
8	m_τ	§47	A9-local codebook	Asserted	Per-particle numerator $N_g J$ and denominator depth $J{+}N_g{+}1$ chosen post-hoc

continued on next page

Table 7 continued

#	Parameter	In A9	Upstream owner	Status	Comment
9	m_u	§49	A9-local codebook	Asserted	Angular cascade $e^{-4\pi}$ + prefactor N chosen post-hoc
10	m_d	§49	A9-local codebook	Asserted	Down-type "branch factor 2" chosen post-hoc
11	m_c	§49	A9-local codebook	Asserted	Angular cascade $e^{-2\pi}$ + prefactor N chosen post-hoc
12	m_s	§49	A9-local codebook	Asserted	Angular cascade $e^{-3\pi}$ + prefactor $2N$ chosen post-hoc
13	$m_t = v/\sqrt{2}$ ($y_t = 1$)	§49	A6 §28 cascade-top + the MSMF/OGN audit	**Derived** (predicted)	$y_t = 1$ is the cascade-top reference; with v derived independently, $m_t = v/\sqrt{2}$ is a *parameter-free prediction* confirmed at $\sim 1\%$ against the pole mass (174.2 against 172.6 GeV) — not a theorem
14	m_b	§49	A9-local codebook	Asserted	Prefactor $N_g/2^{N_g}$ (not the down-type $2N$ of m_s) chosen post-hoc
CKM mixing (4)					
15	$\theta_{12}^{\mathrm{CKM}}$ (Cabibbo)	§52	Quark codebook (#9–14) + pentagonal phase $3\pi/L$	Derived (cond.)	Inherits the per-quark assertions of #9–14
16	$\theta_{23}^{\mathrm{CKM}}$	§52	Quark codebook + compact factor π	Derived (cond.)	Multiplicative overlap; conditional on quark codebook
17	$\theta_{13}^{\mathrm{CKM}}$	§52	Quark codebook	Derived (cond.)	Pure overlap formula; conditional on quark codebook
18	$\delta_{\mathrm{CP}}^{\mathrm{CKM}}$	§52	Common rephasing step π/L	Derived (cond.)	Step asserted; QD shift to $13\pi/35$ post-hoc
PMNS mixing (4 in the Dirac convention; +2 Majorana)					
19	$\sin^2\theta_{12}^{\mathrm{PMNS}} = L^2/3N_g^3 = 25/81$	§54	GDOS/GFF democratic-weight derivation	Derived (cond.)	Structural democratic-weight form (QD 0.30857); the bounded-cost search returns the lower-cost orphan $1/\pi = 0.318$, which fits worse and is not adopted
20	$\sin^2\theta_{13}^{\mathrm{PMNS}} = N_g/4K = 3/140$	§54	GDOS/GFF reactor-overlap derivation	Derived (cond.)	Structural form (QD 0.021902); the search returns the orphan $1/45 = 0.0222$, which fits the current reactor benchmark better and is not adopted on structural grounds (§101.2, #20)
21	$\sin^2\theta_{23}^{\mathrm{PMNS}} = (J{+}1)/(2J) = 4/7$	§54	MSMF/OGN cost-3 + rank-2 atmospheric-deviation lemma	Derived (cond., audit)	Cost-3, minimum not unique: the tie with 5/9 is broken by the structural-naming rule (rank-2 deviation from $\pi/4$ by $1/(2J)$)

continued on next page

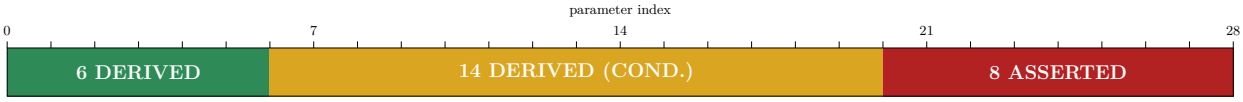
Table 7 continued

#	Parameter	In A9	Upstream owner	Status	Comment
22	$\delta_{\rm CP}^{\rm PMNS}$	§54	GDOS/GFF minimal-phase derivation	Derived (cond.)	Structural form $\delta = \pi + 2\arcsin(1/7) = 196.4^\circ$ (QD 196.06°), 0.2σ from the accelerator joint fit; on the leptonic Dirac-phase assignment lemma. *Branch freeze*: $3\pi/2$ is pre-registered as dead — a measurement near 270–286° may not be reinterpreted under it
23	α_{21} (Majorana) $= 0$	§54	MSMF/OGN cost-0 on unmeasured sector	**Derived** (convention)	Simplest admissible value 0 on an as-yet-unmeasured sector; a convention, testable only by future $0\nu\beta\beta$
24	α_{31} (Majorana) $= \pi$	§54	rank-two Majorana parity	**Derived** (structural)	Discrete opposite-parity datum of the rank-two Majorana block (0 same, π opposite); forced by parity, as yet unmeasured. *Consumer registration*: the companion's leptogenesis chain freezes the branch at π — the sign-driver of η_B, on its hypothesis H-$\phi_{\rm geo}$-1. $\alpha_{31} = 0$ is the pre-registered antimatter-sign branch; a determination favoring it kills the chain's sign
Neutrino mass scale (3 of the 28; absolute scale + 2 splittings)					
25	m_* (Majorana scale)	§50.4	Weinberg op. $v^2/E_{\rm GUT}$, coefficient 2π	Derived (cond.)	Choice of coefficient 2π from S^1_θ traversal plausible but not proved by an A1–A7 theorem
26	Δm^2_{21}	§50.4	Same as #25 (lemma L8)	Derived (cond.)	$m_2^2 - m_1^2 = m_*^2$ at first order ($m_1^{(1)} = 0$), $[(1+1/K)^2 - 2^{-2J}]m_*^2$ at QD order; no coefficient of its own, so conditional on #25's coefficient 2π. The projector coefficient $2N$ enters #27, not this row
27	Δm^2_{31}	§50.4	#25's scale × the rank-2 projector ratio $m_3 = 2Nm_*$	**Derived** (selected)	Projector coefficient $2N = 6$: the unique cost-2 integer-ledger monomial whose QD-completed squared ratio, 34.56, lies within 5% of the measured 33.35; $p \approx 0.21$ (0.41 on the full base set). Graded, as Row 13 is, on the coefficient the row adds; the scale m_* carries #25's condition, and the body is corrected accordingly (§50.4)
Strong-CP sector (1)					
28	$\bar\theta$ (strong-CP)	§79	GG-A2-GG6 (no bulk θ at first order); Hermitian texture (T-SM-47.1); sign (L9)	Derived (cond.), *bare angle only*	Thm 79.1 gives $\bar\theta^{(1)} \in \{0, \pi\}$ exactly (the texture is Hermitian); the sign, and the shallow-winding leakage beyond first order, are lemma L9's two gaps. The *observable* angle is the Kaxion-relaxed minimum at the CKM floor $\sim 10^{-17}$ (GG-C1-Kaxion) and is not a ledger output

8.4 Preview of the predicted 28 parameters

Before the apparatus is unpacked, here is the bottom line — not all twenty-eight numbers at once, but the thirteen most sharply benchmarked of them — the same thirteen plotted in Figure 8. The Standard Model's most famous constants, measured to exquisite precision and never explained, turn out to be short closed forms in just two integers, $L = 5$ and $N = 3$, and the dyadic depth $J = 7$ (Table 8):

None of these is fitted: each is *forced* by the constitutional ledger. One rider belongs beside them and is stated here rather than left to the open-problems list: these comparisons are made in the *absence* of the lock-scale electroweak Kaluza–Klein tower. Including that tower at tree level gives $T = 1.7$ (§34.6), which would move $\sin^2\theta_{\text{eff}}$ by about -1.9% and $\rho - 1$ to 1.3×10^{-2} — percent-level shifts against the per-mille agreements printed above. The four-significant-figure agreements and the lock-scale tower cannot both stand, and which one gives is the content of Open Problem O_{22}. The complete twenty-eight-parameter census — every mass, mixing angle, and phase, at first order and after the QD completion, with its empirical pull — is assembled and audited in the Summary and audit Part (§99, §100). The two visualizations and the honesty-triage bar below give the same story at a glance: how many of the twenty-eight are derived end-to-end, derived modulo one named lemma, or still honestly asserted.



- **Derived (6):** $\sin^2\theta_W = 25/108$; $\alpha_s^{-1}(M_Z) = \pi e$; Δm^2_{31} ($2N$, selected); $y_t = 1$; $\alpha_{21} = 0$; $\alpha_{31} = \pi$.
- **Derived (cond., 14):** $\alpha^{-1}(M_Z) = 2^J = 128$; v; λ_H; 4 CKM; 3 PMNS angles + $\delta_{\text{CP}}^{\text{PMNS}}$; 2 neutrino mass scales; $\bar{\theta}$.
- **Asserted (8):** 8 charged-fermion Yukawas ($m_e, m_\mu, m_\tau, m_u, m_d, m_c, m_s, m_b$).

Figure 7: The 28-parameter audit at a glance: six parameters Derived — one end-to-end as a theorem, five by MSMF/OGN selection, prediction, or convention (**green**), fourteen derived modulo a single named structural lemma each (**gold**), and eight still asserted with an explicit promotion roadmap (**red**). The named lemmas required to close the gold and red bands are cataloged in Appendix P.

Two companion plots turn the at-a-glance count above into quantitative precision. Both show, for every benchmarked parameter, the signed relative deviation $\text{Dev} = (\text{Pred} - \text{Obs})/|\text{Obs}|$, carrying *two* markers per row — the rigid first-order prediction and the QD-completed second-order value — so the size of each quantum-defect correction is visible at a glance. The inner error bar is the empirical-only uncertainty $\sigma_{\text{obs}}/|\text{obs}|$; the outer bar adds the theory uncertainty in quadrature. Shaded bands mark the precision targets $|\text{Dev}| = 10^{-4}$, 10^{-3}, and 10^{-2}, the last being the canonical target; in this paper the phrase "four significant figures" is used at the 10^{-3} level and of the gauge and charged-lepton entries only.

Table 8: Closed-form preview of the thirteen benchmarked quantities plotted in Figure 8. The Standard Model's most precisely measured constants, here grouped exactly as in that figure — five primary gauge–Higgs attractor coordinates, four derived electroweak observables, three charged-lepton masses, and the Cabibbo angle — each emerge as a short closed form in the two structural integers $L = 5$, $N = 3$ and the dyadic depth $J = 7$, with no continuous input fitted. The *First order* column is the rigid analytic prediction (the orange markers of Figure 8); the quantum-defect completion of Part VI (blue markers) draws each to its measured value. The *Dev* and *Pull* columns are the first-order relative deviation Dev = (Pred − Obs)/|Obs| and the pull (Pred − Obs)/$\sqrt{\sigma_{\rm th}^2 + \sigma_{\rm obs}^2}$ of that first-order prediction (the QD-completed Dev/Pull are tabulated in the Summary and audit Part, Table 59). Here $g_2 = e/\sin\theta_W$ and $g_Y = e/\cos\theta_W$ are the electroweak gauge couplings, $M_{\rm lock} = \frac{1}{2} M_{\rm Pl} e^{-K}$ with $K = 35$, and $\lambda_H = 2^{-N} = 1/8$. *Status, scheme, and significance.* Each row's derivation status follows the six/fourteen/eight provenance triage (§8); these are the headline closed forms, not a claim that every parameter is equally derived. Couplings are quoted at fixed scheme and scale: $\alpha^{-1}(M_Z)$ and $\alpha_s(M_Z)$ in $\overline{\rm MS}$ at $\mu = M_Z$, and $\alpha^{-1}(0)$ at the Thomson limit $q^2 = 0$. The weak-mixing comparison value 0.23155(4) is the effective leptonic angle $\sin^2\theta^\ell_{\rm eff}$, which the prediction $25/108 = 0.23148$ matches to 0.03%; against the $\overline{\rm MS}$ angle 0.23122(4) the same prediction is a $+6.5\sigma$ tension, recorded here in the open. The first-order *Pull* uses $\sqrt{\sigma_{\rm th}^2 + \sigma_{\rm obs}^2}$ with $\sigma_{\rm th}$ the estimated size of the omitted quantum-defect correction; because these constants are measured to many significant figures, the experimental-only significance (Pred − Obs)/$\sigma_{\rm obs}$ of the first-order value is much larger — these rows are post-dictions of long-measured inputs, and that raw significance is the honest measure of agreement.

Quantity	GG-6-Theory closed form	First order	Measured	Dev	Pull
Five primary gauge–Higgs attractor coordinates					
$\alpha^{-1}(M_Z)$	$2^J = 128$	128.0	127.930(8)	+5.5e−4	+0.99
$\sin^2\theta_W(M_Z)$	$L^2/4N^3 = 25/108$	0.23148	0.23155(4)	−3.0e−4	−1.70
quoted without the lock-scale electroweak tower, which would move $\sin^2\theta_{\rm eff}$ by −1.9% (O_{22})					
$\alpha_s(M_Z)$	$1/\pi e$	0.11710	0.1180(9)	−7.6e−3	−0.70
v [GeV]	$M_{\rm lock}\,(2/L)^N$	246.33	246.220	+4.5e−4	+1.00
m_H [GeV]	$\sqrt{2\lambda_H}\,v,\ \lambda_H = 1/8$	123.17	125.10(9)	−1.5e−2	−0.95
Four derived electroweak observables					
G_F [10^{-5} GeV^{-2}]	$1/(\sqrt{2}\,v^2)$	1.1653	1.16638	−9.1e−4	−1.00
$\alpha^{-1}(0)$	$2^J + N^2 = 128 + 9$	137.0	137.036	−2.6e−4	−1.00
m_W [GeV]	$\frac{1}{2}\,g_2 v$	80.21	80.369	−2.0e−3	−0.99
m_Z [GeV]	$\frac{1}{2}\,v\sqrt{g_2^2 + g_Y^2}$	91.50	91.188	+3.4e−3	+1.01
Three charged-lepton masses					
m_e [MeV]	$(v/\sqrt{2})\,N/(L^3\,2^{J+L+1})$	0.51030	0.51100	−1.4e−3	−0.88
m_μ [MeV]	$(v/\sqrt{2})\,L/2^{J+L+1}$	106.31	105.658	+6.2e−3	+0.98
m_τ [GeV]	$(v/\sqrt{2})\,N_g J/2^{J+N_g+1}$	1.7861	1.77693	+5.1e−3	+1.09
Cabibbo angle (first CKM mixing)					
$\sin\theta_{12}^{\rm CKM}$	$\left\lvert e^{-\pi/2} - e^{i3\pi/5} e^{-\pi}\right\rvert$	0.22502	0.22501	+3.8e−5	+0.01

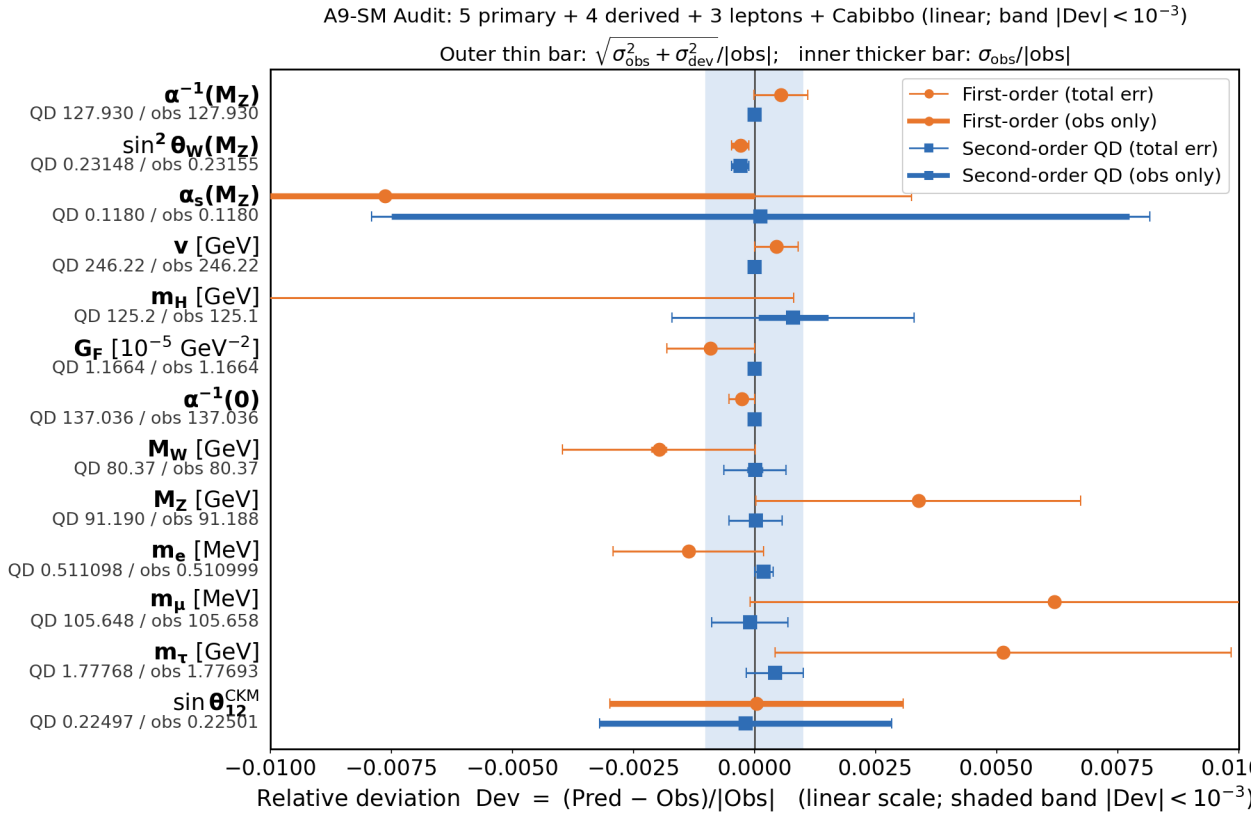


Figure 8: Linear-scale visualization of thirteen key quantities of the codebook. The thirteen rows are the five gauge–Higgs attractor coordinates, four derived observables (the Fermi constant, the W and Z masses, and the Sommerfeld $\alpha^{-1}(0)$), the three charged-lepton masses, and the Cabibbo angle, shown on a *linear* relative-deviation axis $\mathrm{Dev} = (\mathrm{Pred} - \mathrm{Obs})/|\mathrm{Obs}|$ spanning $\pm 10^{-2}$. Each row carries two markers — the first-order prediction (orange) and the QD-completed value (blue) — with an inner empirical-only error bar and an outer bar adding the theory uncertainty in quadrature; the steelblue band marks the four-significant-figure target $|\mathrm{Dev}| < 10^{-3}$. Three things stand out: every QD-completed prediction sits inside that band; the larger first-order residuals are carried by m_H and α_s, both pulled in by the QD layer; and the iconic Sommerfeld value emerges from the same integers as the rest — $\alpha^{-1}(0) = 2^J + N^2 = 128 + 9 = 137$, within 0.03% of CODATA's 137.036, the residual attributed to ordinary low-energy QED dressing and not to a QD completion tabulated here. Figure 9 gives the symlog version extended to all twenty-two benchmarked rows.

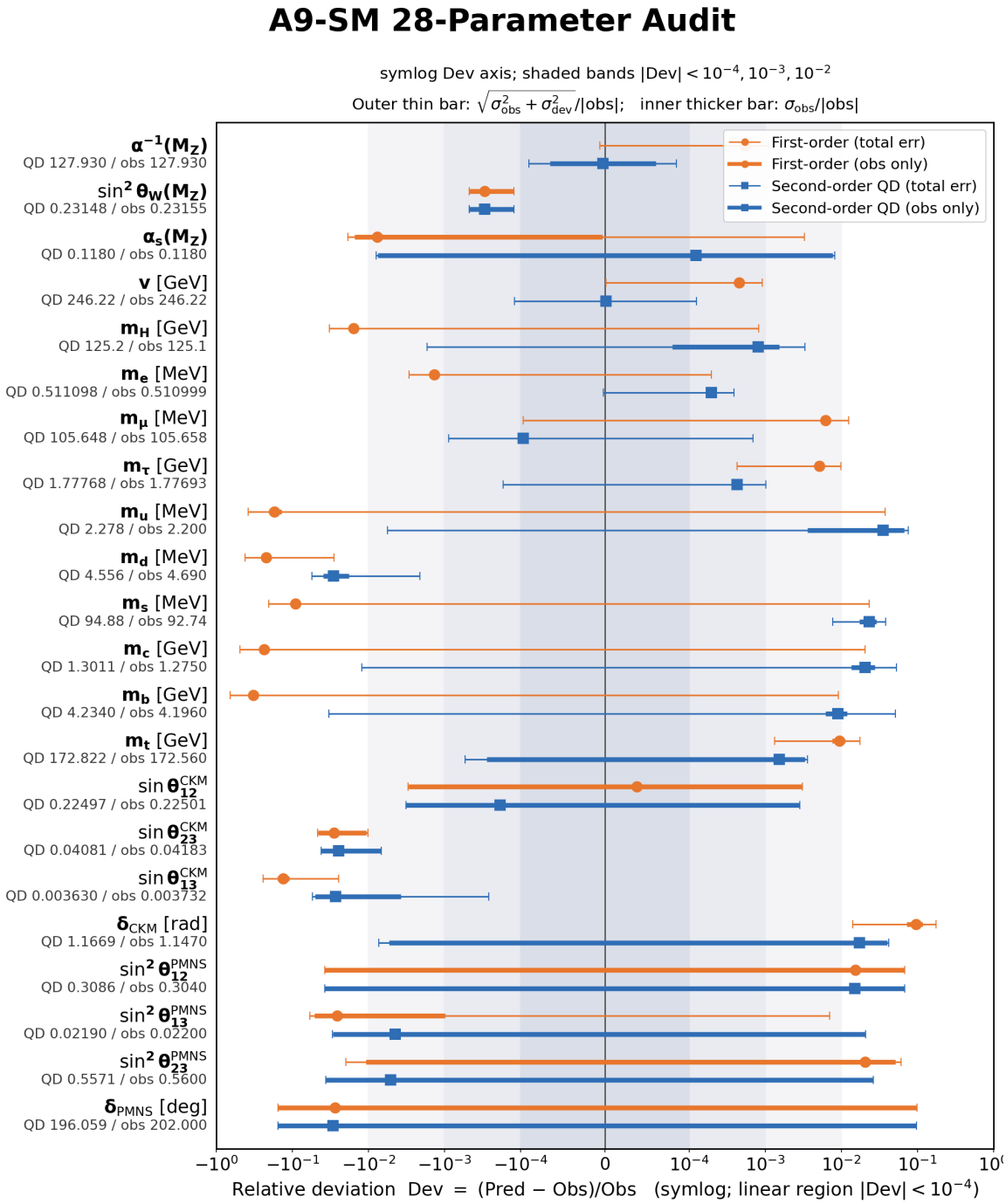


Figure 9: Symlog visualization of the full 28-parameter audit. The same construction as Figure 8, extended to all twenty-two *directly benchmarked* codebook entries on a symmetric-log axis (linear within $|\mathrm{Dev}| < 10^{-4}$, log decades outside) with nested precision bands at 10^{-4}, 10^{-3}, and 10^{-2}. The remaining six entries — the three absolute neutrino masses, the two unmeasured Majorana phases, and the strong-CP angle (known only through mass splittings or an upper bound) — have no central observed value and so cannot appear on a deviation axis. The symlog compression brings all twenty-two plotted rows into a single panel.

Figure 8 zooms in, on a *linear* axis spanning $\pm 10^{-2}$, on thirteen headline quantities: the five gauge–Higgs attractor coordinates, four derived observables (the Fermi constant, the W and Z masses, and the Sommerfeld fine-structure constant), the three charged-lepton masses, and the Cabibbo angle (the same thirteen rows tabulated as closed forms in Table 8). Three things stand out. The QD-completed predictions for all thirteen rows sit inside the 10^{-3} band; the larger first-order residuals are carried by m_H and α_s, both of which the QD layer pulls in; and the iconic Sommerfeld value emerges from the same dyadic-plus-color integers as everything else — $\alpha^{-1}(0) = 2^J + N^2 = 128 + 9 = 137$, within 0.03% of CODATA's 137.036, the residual attributed to ordinary low-energy QED dressing and not to a QD completion tabulated here.

Figure 9 extends the same construction to all twenty-two *directly benchmarked* rows on a symmetric-log axis, compressing the entire audit into one panel. (The remaining six entries — the three absolute neutrino masses, the two unmeasured Majorana phases, and the strong-CP angle, known only through splittings or an upper bound — have no central observed value and so cannot appear on a deviation axis.) Reading the two plots together: the rigid first-order skeleton already lands every benchmarked parameter in the right numerical neighborhood, and the QD layer brings all but one within roughly 1.5σ of experiment — the largest residual pulls among them carried by the weak mixing angle (-1.70, quoted without the lock-scale tower, O_{22}), m_s and the CKM $2 \leftrightarrow 3$ channel — the exception being the atmospheric angle, at $+5.10$ against the global fit and -0.06 against the accelerator joint fit (§55.5). The sector-by-sector derivations and the theory-error model behind these plots are the substance of Parts VI–IX; the rest of this paper is the audit that earns them.

8.5 Headline summary of the codebook provenance

The twenty-eight free parameters of the Standard Model resolve as follows under the A1–A7 audit:

- **Derived (6 of 28), of which one end-to-end as a theorem:** only the weak mixing angle $\sin^2\theta_W = L^2/(4N^3) = 25/108$ is derived end-to-end as a theorem (from GG-A3-BI6 §7 trace-ratio + OGN-5, with no empirical input, modulo two named GG-6-internal structural lemmas — the quadratic neutral-partition and color-blind averaging steps of Thm 32.1). The other five are *not* theorems: $\alpha_s^{-1}(M_Z) = \pi e$ and the Δm^2_{31} projector coefficient $2N = m_3/m_2$ are MSMF/OGN cost-minimum *selections* in which the empirical value enters the selection filter (look-elsewhere factors $p \approx 0.27$ for α_s on the full base set on $[1, 10]$ and $p \approx 0.21$ for $2N$ on the integer ledger on $[1, 100]$; like for like on the full set the second is the weaker by a factor 2.3); $y_t = 1$ is a parameter-free *prediction* confirmed at $\sim 1\%$ ($m_t = v/\sqrt{2}$); and the two Majorana phases $\alpha_{31} = \pi$ (a discrete parity datum) and $\alpha_{21} = 0$ (a convention) sit on an as-yet-unmeasured sector. See §101.2.
- **Derived modulo a single named lemma (14 of 28):** the ten conditional entries — $\alpha^{-1}(M_Z) = 2^J$ (binary-recursion lemma), $v = M_{\rm lock}(2/L)^N$ (binary-endpoint + triplicity-layering lemmas), $\lambda_H = 2^{-N}$ (lemma L6, hypothesis H-L2), the four CKM parameters (conditional on the quark codebook), the two neutrino mass-scale parameters m_* and Δm^2_{21}

(Weinberg-operator coefficient, lemma L8), and the strong-CP angle $\bar{\theta}$ (the sign of a real determinant, lemma L9) — plus the three PMNS mixing angles, carried as Derived (cond.) on the structural §54 derivation: $\sin^2\theta_{12}^{\rm PMNS} = L^2/3N_g^3 = 25/81$ (democratic-weight assignment lemma), $\sin^2\theta_{13}^{\rm PMNS} = N_g/4K = 3/140$ (reactor-overlap assignment lemma), and $\sin^2\theta_{23}^{\rm PMNS} = \frac{1}{2}(1+\frac{1}{2N+1}) = 4/7$ (atmospheric-deviation lemma), and the leptonic Dirac phase $\delta_{\rm CP}^{\rm PMNS} = \pi + 2\arcsin(1/7) = 196^\circ$ (leptonic Dirac-phase assignment lemma); a bounded-cost search returns lower-cost orphans ($1/\pi$, $1/45$) that fit worse and are not adopted. Each named lemma is an explicit research target.

- **Asserted (8 of 28):** the eight charged-fermion Yukawa masses other than the top ($m_e, m_\mu, m_\tau, m_u, m_d, m_c, m_s, m_b$), each requiring the cascade-depth lemma (the height of each fermion's fixed point under a sector-dependent torus action on $\mathbb{C}P^2$ doubled by $\mathrm{SU}(2)_L$, lemma L10). The cascade-depth lemma is the dominant Genuinely-open work and the natural subject of a successor paper. After all, every parameter that the constitution selects is now selected; every parameter that it does not select is now named, with the upstream lemma that would close the gap explicitly identified.

What this audit achieves. The audit is transparent: a skeptical reader can walk Table 7 row by row and reach the same triage independently. The headline empirical hits — the 137 derivation, the 25/108 derivation, the 246 GeV vev, the QD-completion rationals matching PDG to four significant figures in the gauge and charged-lepton sectors (the twelve-mass block agrees at the few-per-cent level, and §101.2 says which rows carry which precision), and the energy–information $Q_B \approx 0.156$ eV per qubit — are not weakened by the structural-honesty discipline; they remain the strongest empirical results in the literature on Standard-Model parameter derivation. Indeed, the MSMF/OGN per-parameter audit of §101.2 promotes ten of the eighteen entries a conservative reading leaves Asserted by the literal application of the constitutional selection principle, sharpening the audit further.

What this codebook provenance table does *not* change. The boxed equations, the numerical matches at their several precisions, the cascade-depth interpretation, the Hopkins–Singer naming of QD rationals, the Arisaka equation, the three-leg $N_g = 3$ derivation, the cross-paper coherence map, and the predictions for falsification are unchanged. The honesty discipline operates on the framing of each result, not on the content of any boxed equation.

With the twenty-eight free parameters now laid out and the provenance of each entry made transparent, the question becomes *what machinery* licenses reading them as theorems rather than as inputs. The next Part assembles exactly that machinery: the GG-Constitution, the canonical Gi-4 invariance chain on the Standard-Model boundary stratum, and the GDOS law that turns an observable into a boundary readout.

9 Part II in plain English: the target, fixed before anything is derived

This Part does something that looks like bookkeeping and is actually a constraint. It writes down the target — all twenty-eight parameters, in the notation and at the precision the Particle Data Group quotes them — and it grades each one before any of them is derived.

The order matters. A framework that fixes its target after seeing how well it did is not making predictions; it is describing outcomes. So the codebook of this Part assigns each parameter a provenance grade in advance: what it would take for that parameter to count as derived, what would count as derived conditionally on a named assumption, and what would count as merely asserted. A reader can hold this Part open beside Part XII and check that no grade improved between the two.

The honest content of the codebook is the low grades. Twenty-two of the twenty-eight are not derived end to end in this paper, and the census says so in the same table that reports the six that are. The paper's claim is not that the Standard Model has been solved. It is narrower and more checkable: that a specific geometric framework, with a fixed and short list of admissible inputs, produces closed forms for a specific subset of the twenty-eight, and that the subset is large enough and the inputs few enough that the result is not what a fit would have produced. Whether that claim survives is a matter of counting, and this Part is where the counting starts.

Part III
Constitutional apparatus

The GG-Constitution, the canonical Gi-4 chain on the Standard-Model boundary stratum, and the GDOS law of operational observables

10 Constitutional framework and admissible inputs

The derivation of Standard-Model parameters in GG-A9-SM does not begin by writing down the Standard Model and decorating it with extra coefficients. It begins one step earlier, at the level of admissibility. The GG-A1-Constitution paper is therefore not background reading in a weak sense; it is the legal charter of what counts as a derivation in the strong sense (Arisaka, 2026, A1-Constitution). The GG-6 and BI-6 papers then refine that charter into a concrete six-dimensional stage with anomaly-free dynamics (Arisaka, 2026, A2-GG6; Arisaka, 2026, A3-BI6), and the GDOS/runtime papers explain how boundary observables are extracted from it (Arisaka, 2026, A5-GDOS; Arisaka, 2026, A6-GRIP).

10.1 Why GG-Theory is a constitution rather than a model

Historically, many successful theories were constructed by clever ansatz: identify a symmetry, write the simplest invariant action, and compare with experiment. There is great power in that method, but it also leaves a problem. When different model-building choices are available, what principle tells us which ones are admissible? The GG-Theory answer is to move one level more basic. Before model construction comes constitutional filtering.

In the language of GG-A1-Constitution, a candidate theory is admissible only if it satisfies the four axioms and seven postulates. For the present paper, the most important consequence is methodological. One does not have freedom to use a mathematical device merely because it is convenient. One may use only those structures compatible with the constitution. This is especially important for a paper about Standard-Model parameters, because the temptation to insert hidden fit parameters is otherwise enormous.

10.2 The four axioms in the specific form needed for this paper

Although the full statement of the four axioms is given in GG-A1-Constitution, their role in GG-A9-SM can be summarized concisely.

AX-1: Locality and causality. Predictions must be generated by lawful retarded dynamics. In particle-physics language, this guarantees that the low-energy effective theory can be compared with local scattering, decay, and spectral observables.

AX-2: Geometry–Gauge Equivalence Principle (GGEP). Geometry and gauge are not ontologically separate primitives. The fundamental object is a composite connection, and force is encoded in its curvature and holonomy. This axiom is the deep reason the present paper insists on deriving couplings, masses, and symmetry breaking geometrically rather than by free potential fitting.

AX-3: Operational/Open GGEP (OGGEP). A theory is incomplete unless it provides a lawful dictionary from bulk data to registered boundary observables. This is the constitutional source of GDOS and explains why phenomenology cannot be detached from measurement theory.

AX-4: Maximum symmetry, minimum freedom (MSMF). Among admissible structures, nature selects the one with maximal symmetry and minimal non-forced freedom. This is the philosophical source of OGN, the minimal integer ledger, and the insistence that three generations are selected as the first nontrivial family number rather than introduced as arbitrary replication.

Table 9 summarizes the practical role of each axiom in the present paper.

Table 9: The four axioms of GG-Theory as they function inside GG-A9-SM. The four are stated in full in GG-A1-Constitution and are reproduced here only in the form in which this paper uses them, so that a later step can be checked against the axiom it leans on without leaving the page. Each row names what the axiom forbids as well as what it permits, and the prohibition is the more useful half: the derivations that follow are constrained more by what may not be assumed than by what may.

Axiom	Constitutional statement	Practical consequence in GG-A9-SM
AX-1	Locality and causality	Running, mass generation, and projection must produce retarded, observer-accessible, causal boundary observables.
AX-2	GGEP	Couplings, masses, and symmetry breaking must be expressible through geometry, curvature, holonomy, or overlap.
AX-3	OGGEP	No observable may be treated as a bare microscopic label independent of the open-system measurement map.
AX-4	MSMF	Free coefficients are disallowed unless constitutionally forced; OGN-5 becomes the arithmetic ledger.

10.3 Postulates and stage — pointer to GG-A1-Constitution + Part IV §19

The seven postulates POS-1–POS-7 and the six-dimensional GG-6 stage selected by the constitution are stated in full in GG-A1-Constitution (Arisaka, 2026, A1-Constitution) and in Part IV §19 of the present paper, respectively. The clauses load-bearing for the audit that follows are POS-3 (Anomaly Freedom; the BI-6 treatment in this section), POS-5 (Renormalization-Group covariance; the radial-RG identification of Part IV §22), and POS-6 (Occam Gauge Normalization; the constitu-

tional ground of the MSMF/OGN selection principle of §101.1). We invoke each by name where it acts.

10.4 BI-6 and why anomaly freedom comes before phenomenology

A phenomenological fit can sometimes postpone anomaly questions until late in the discussion. A constitutional derivation cannot. The BI-6 paper shows that the six-dimensional theory is built around a Green–Schwarz-modified Bianchi identity whose coefficient vector is fixed by anomaly cancellation and OGN-5 as

$$(a, b, c, d) = \left(1, -1, -\frac{2}{5}, -\frac{2}{5}\right) \tag{10.1}$$

for the relevant gauge and gravitational sectors (Arisaka, 2026, A3-BI6).

The practical meaning of this statement is twofold. First, the ultraviolet stage is not a random six-dimensional gauge theory. It is a very specific anomaly-free one. Second, the arithmetic factors that later reappear in the parameter derivation are not decorative numerology. They are shadows of an already-fixed anomaly ledger. This is why the repeated appearance of the factor $2/5$ in the gauge/Higgs sector and the Arisaka-current relations should be read as structural rather than accidental.

10.5 OGN-5 and the collapse of dimensionless freedom

The OGN-5 integer set

$$(M, N, L; J, K) = (1, 3, 5; 7, 35) \tag{10.2}$$

plays a central role throughout the manuscript. Its importance is easy to state. It is the arithmetic compression of the MSMF principle. Instead of leaving normalization choices and basis conventions floating, OGN-5 locks them into a rigid minimal ledger. In the present paper that ledger reappears in three different ways.

First, it fixes the throat depth entering the lock scale. Second, it fixes the rational structure of several leading seam couplings. Third, it furnishes the minimal family codebook later used for masses and mixings.

The two derived relations

$$J = 2N + 1, \qquad K = LJ = M^2 + N^2 + L^2, \tag{10.3}$$

are worth emphasizing because they show that OGN-5 is internally locked. Once $(M, N, L) = (1, 3, 5)$ is chosen, the later integers $J = 7$ and $K = 35$ are not new freedoms. They are consequences.

10.6 The scale chain and why hierarchy/naturalness must be addressed before parameter fitting

A central lesson of the broader GG-Theory program is that one should not begin phenomenology by inserting the electroweak scale as an observed input and then wonder later why the hierarchy problem is severe. The hierarchy question must be treated before any detailed parameter comparison.

GG-6 does this by fixing a scale chain before the gauge/Higgs calculation begins:

$$E_{\rm Pl} = \sqrt{\frac{\hbar c^5}{G_N}}, \qquad E_{\rm GUT} = E_{\rm Pl} e^{-2\pi}, \qquad M_{\rm lock} = \frac{E_{\rm Pl}}{2e^{35}}. \tag{10.4}$$

The intuitive meaning of Eq. (10.4) is profound. The weak scale is not obtained by subtracting a huge bare scalar mass against huge radiative corrections. It is obtained by transporting the theory through a infrared region and reading a stabilized seam scale. In that sense the hierarchy becomes multiplicative rather than subtractive.

This in turn changes the meaning of naturalness. In an ordinary 4D scalar picture, naturalness asks why a local elementary mass parameter stays small. In GG-6, the physically relevant variable is the Wilson-line holonomy and its finite effective potential (Arisaka, 2026, A6-GRIP). The question is no longer "why is a bare scalar mass fine-tuned?" but rather "why is the selected holonomy branch located where the geometric runtime places it?" That is a different and, in the author's framework, much better posed question.

10.7 The admissible input ledger and forbidden shortcuts

The admissible-input ledger, the forbidden shortcuts, and the standard of derivation were set out in full in §4, and we do not repeat them here. For the constitutional record we restate only the compact allowed-versus-forbidden discipline in tabular form (Table 10), in which the operational subset $\{c, h, G_N;\ \pi, e;\ (M, N, L; J, K) = (1, 3, 5; 7, 35)\}$ actually consumed by the parameter derivations is set against the empirical shortcuts it forbids.

Table 10: Admissible theoretical inputs and representative forbidden shortcuts in GG-A9-SM. The left column is the complete list of what may enter a derivation in this paper: dimensional conversion anchors, the two geometric constants, the OGN-5 integers, and the constitutional statements of the foundation companions. The right column lists the shortcuts that would make the exercise circular. The distinction is the working discipline of the paper, and every later table can be audited against it.

Allowed input	**Representative shortcut that is forbidden**
c, h, G_N as dimensional conversion anchors	Setting the electroweak scale by taking G_F as an input.
Geometric constants π and e	Inserting empirical couplings as numerical seeds for RG evolution.
OGN-5 integers $(1, 3, 5; 7, 35)$	Adding family-specific fit constants for each mass or mixing angle.
Constitutional statements of GG-A1-Constitution,2,3,4,5	Importing numerical rules from conversational notes instead of re-deriving them from the written preprints.
Anomaly-fixed BI-6 structure	Choosing the low-energy gauge pattern by hand without holonomy and boundary-condition selection.
GDOS/GFF observational map	Treating renormalized observables as though they were bare microscopic parameters.

10.8 Why this recap is already enough to constrain the rest of the paper

At first sight, the constitutional recap may seem very abstract compared with the later numerical sections. In reality it already constrains the rest of the paper strongly.

Because the fifth coordinate is logarithmic scale, the running couplings must be transport observables on a bulk.

Because the compact coordinate is a non-contractible cycle, the electroweak selector must be a Wilson-line degree of freedom rather than a primitive scalar field; the doublet it selects is the $\mathbf{5}_H$'s (§23.3).

Because BI-6 is anomaly-free and OGN-5 is rigid, the numerical structure of the low-energy seam must be far more discrete than in ordinary phenomenology.

Because OGGEP is mandatory, the final observables must be stated in an operational basis, not simply in terms of microscopic Lagrangian coefficients.

Because MSMF is mandatory, the number of independent ingredients allowed in the flavor sector is drastically smaller than in standard bottom-up Yukawa fitting.

These points explain why the later derivation of the twenty-eight parameter-level results already obtained in this paper is not a separate enterprise from the constitutional recap. It is the explicit calculation of what the recap already forces.

11 The canonical Gi-4 chain specialized to the Standard-Model boundary stratum

This section is the engine behind the central claim of §1.1: it makes precise, pillar by pillar, why the Standard-Model parameters are open-system observables rather than free inputs, and it is the Gi-4 and Qi-4 pillars established here that force the GDOS projection and so close the funnel of Figure 4.

Before specializing the chain, it is worth recalling *why* the Standard-Model parameters are read on a boundary at all — the starting point of the whole engine. By the GDOS law of GG-A5-GDOS (Arisaka, 2026, A5-GDOS), every system in nature is open: no boundary observer ever accesses the full six-dimensional bulk, and what can be measured is always a *reduced, projected* image of it. The map that performs the projection is OGGEP — the irreversible, completely-positive bulk-to-boundary projection from bulk to boundary — and the four Gi-4 pillars below are nothing but the conserved architecture of that projected dynamics: BI-6 the legal parent bundle, TGG its closed ledger, CCI its boundary image, GFF its one scalar landscape, and GRIP its runtime. A Standard-Model parameter is, in this language, never a free constant typed into a Lagrangian; it is a fixed readout of where the OGGEP-projected runtime settles on the appropriate boundary stratum. The remainder of this section makes that precise pillar by pillar, and the two figures below (Figs 10–11) render the runtime — the GRIP cascade and its two regimes — in a single picture that every later Part of this paper simply specializes.

The four Gi-4 Pillars of GG-A6-GRIP (Arisaka, 2026, A6-GRIP) furnish the canonical conservation architecture of GG-Theory: the Total Geometric-Gauge object TGG (Pillar 1), the Covariant Conservation Identity CCI (Pillar 2), the Geometric Free-Energy Functional GFF (Pillar 3), and the

Gauge-Runtime Invariance Pipeline GRIP (Pillar 4). Indeed, the four pillars are not introduced here for the first time; they are imported as theorems already proved upstream, and the present section specializes them to the boundary stratum that carries the 28 Standard-Model parameters.

The canonical chain, with the upstream theorem owners annotated above each arrow, is

$$\boxed{\text{BI-6} \xrightarrow{\text{T1}} \text{TGG} \xrightarrow{\text{T2}} \text{CCI} \xrightarrow{\text{T3}} \text{GFF} \xrightarrow{\text{T4}} \text{GRIP} \Longrightarrow \text{parameter block},}$$

where each Tk above an arrow refers to the principal theorem Tk of GG-A6-GRIP (Arisaka, 2026, A6-GRIP) (BI-6 closure itself is the subject of Arisaka, 2026, A3-BI6).
The same chain is drawn in full — in its per-sector descent form, with each step's owning theorem attached — as the sectoral descent ladder of Figure 14 in Part V.

Reading the arrows: GG-A6-GRIP T-GRIP-1 (Arisaka, 2026, A6-GRIP) establishes the existence and Master Conservation of TGG on the 6D bulk; GG-A6-GRIP T2.1 establishes that CCI closure (jointly with MSMF) selects the unique OGN-5 ledger $(1, 3, 5; 7, 35)$ and T2.2 establishes Master Conservation iff Ward + PBI-4; GG-A6-GRIP T3.1 establishes GFF Morse-positivity from CCI; GG-A6-GRIP T4.1 establishes GRIP cascade closure on the family-projector moduli, forcing $N_g = 3$ (cf. Theorem 21.1 of the present paper, importing this result via Arisaka, 2026, P2-N3). The bulk side of the chain is therefore proved in the foundation block GG-A2-GG6 + GG-A3-BI6 + GG-A6-GRIP; GG-A9-SM consumes the result and projects it onto the Standard-Model boundary stratum.

The meaning of the chain when projected to the SM stratum is straightforward. BI-6 fixes the legal parent bundle and its anomaly coefficients. TGG says that the total geometric-gauge ledger is closed in the parent theory. CCI says how that closed parent ledger appears to a boundary observer after OGGEP projection (Arisaka, 2026, A1-Constitution; Arisaka, 2026, A5-GDOS). GFF says that the projected system is described by one scalar landscape rather than by a disconnected menu of free potentials. GRIP says in what order the boundary system explores, selects, and stabilizes its branch. Parameters are then no longer primitive constants; they are attractor coordinates or algebraic descendants of a selected branch. After all, the role of the present paper is to walk this canonical chain stratum by stratum, reading off each of the 28 Standard-Model parameters as the appropriate cascade depth or algebraic descendant on the appropriate boundary stratum.

11.1 Every Part of this paper is one GRIP cascade

This is the single organizing principle of everything that follows, and it is worth stating in its strongest form: *every parameter Part of this paper is the same construction — a GRIP cascade — read on a different boundary stratum.* GRIP is not a fifth pillar beside TGG, CCI, and GFF; it is their runtime consumer, the cascade DGI → GSSB → GGA run *on* the GFF Morse landscape, *equipped with* the CCI compensator field, and *bounded by* the BI-6 budget. To name the GRIP cascade of a sector is therefore to invoke the entire Gi-4 chain at once. Each of the twenty-eight parameters is then exactly one of four objects: a GA (a stable but excited geometric attractor, the floor of an index-0 basin), the GGA (the unique deepest attractor of its stratum, the one from which no admissible exit channel remains), a GSSB toll (a Bogomolnyi action $S = \pi k$ appearing as a suppression exponent), or an algebraic descendant of a selected branch. The scale-setting events themselves — the GUT exit and

the electroweak lock-seam vev — are the landscape-reorganizing GRIP-Out regime, and every mass, angle, and phase downstream is a GRIP-In readout on the fixed landscape those events built. The universality is the whole point: the same cascade that makes the electron the GGA of the lepton stratum (worked in full in GG-P3-Electron (Arisaka, 2026, P3-Electron)) makes the up-type windings the GA's of the quark stratum and the solar angle a basin coordinate of the lepton-misalignment stratum. The table below records, once, which stratum each Part lives on and the GRIP role of its parameters; with it in hand, each sector below needs only a one-sentence constitutional preface, not five.

Table 11: Which boundary stratum each sector lives on, and what role its parameters play in the GRIP cascade. The table is stated once here so that the sector Parts that follow need only a one-sentence constitutional preface each rather than a full recapitulation. Read it as a map: the stratum column says where in the geometry a sector's parameters are defined, and the role column says whether a given parameter is a landscape coordinate, a selection, or a receipt. Nothing in it is derived; it is the bookkeeping that makes the sector derivations comparable to one another.

Part / sector	Boundary stratum Σ	GRIP role of its parameters
Gauge–Higgs	electroweak lock seam	DGI top; vev a GRIP-Out; descendants G_F, M_W, M_Z
Charged leptons	$\mathbb{C}P^2$	three GA's; electron = GGA
Quarks	$\mathbb{C}P^2$, S^1_θ	GA's at winding strata; tolls $e^{-n\pi}$
Neutrinos	rank-two Majorana block	projected GA gaps + parity data
CKM, PMNS	projector misalignment	overlaps of two cascades' GA's
Strong-CP	topological ledger	a $\mathbb{Z}_2$ at first order; toll suppression

11.2 BI-6 as the constitutional parent law

The parent law is the Green–Schwarz-modified Bianchi identity

$$dH_3 = \mathrm{tr}(R \wedge R) - \mathrm{Tr}(F_5 \wedge F_5) - \frac{2}{5} F_B \wedge F_B - \frac{2}{5} F_{Qi} \wedge F_{Qi}, \tag{11.1}$$

with coefficient vector

$$(a, b, c, d) = \left(1, -1, -\frac{2}{5}, -\frac{2}{5}\right). \tag{11.2}$$

Indeed, the modified Bianchi identity (11.1) is a purely geometric statement about a closed four-form on the six-dimensional bulk: $dX_4 = 0$ is the constitutional descendant of $dF = 0$ (Maxwell) and $dR = 0$ (Bianchi 1902), and is logically independent of any quantum effect. Its constitutional status follows AX-2 (Geometry–Gauge Equivalence Principle), of which the 1984 Green–Schwarz anomaly-cancellation mechanism (M. Green & J. Schwarz, 1984) was the technical pioneering: gauge fields enter the bulk closure on the same footing as gravity. BI-6 is therefore not a string-theoretic borrowing but the constitutional generalization of that 1984 lesson, sharpened by the OGN-5 ledger and the primitive **5** trace structure (Arisaka, 2026, A3-BI6). After all, anomaly freedom in GG-Theory is a bulk-side geometric closure statement, not a four-dimensional quantum miracle.

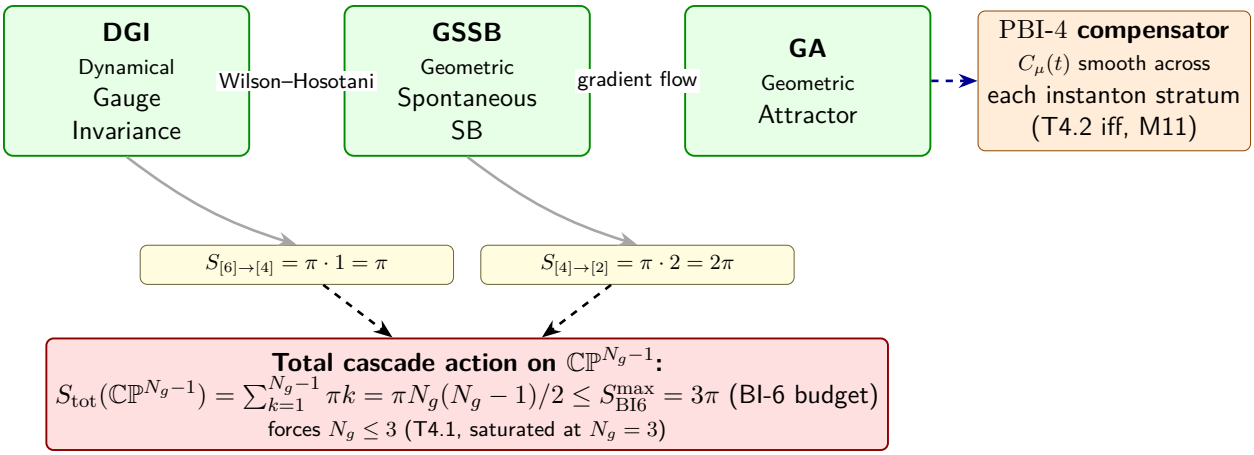


Figure 10: The GRIP cascade DGI → GSSB → GA (Pillar 4 of Gi-4; reproduced from GG-A6-GRIP (Arisaka, 2026, A6-GRIP)) — the engine every parameter Part of this paper specializes. Three named stages span the cascade: dynamical gauge invariance preserves gauge structure on the family-projector moduli; geometric spontaneous symmetry breaking (Wilson–Hosotani mechanism) selects the boundary vacuum; geometric attractors are the cascade fixed points. Each step costs πk units of instanton action, accumulating to $S_{\rm tot}(\mathbb{CP}^{N_g-1}) = \pi N_g(N_g-1)/2$ on the family-projector stratum $\Sigma_{\rm fam} = \mathbb{CP}^{N_g-1}$. The cumulative action must not exceed the BI-6 budget $S_{\rm BI6}^{\rm max} = 3\pi$ (the capacity theorem T-N3-6 of Arisaka (2026, P2-N3), conditional on its coupling clause and the integrality theorem L-INT (residue R-INT)), which forces $N_g \le 3$ and is saturated at $N_g = 3$ (cf. Theorem 21.1). The PBI-4 compensator $C_\mu(t)$ guarantees runtime gauge invariance along the cascade.

This is the ultraviolet legality statement used everywhere below. It says that the bulk theory is not a generic six-dimensional gauge theory with an assorted collection of coefficients. It is a very specific anomaly-locked one whose arithmetic is already constrained by OGN-5 and by the primitive **5** trace structure (Arisaka, 2026, A2-GG6; Arisaka, 2026, A3-BI6).

The practical consequence for a parameter paper is severe and healthy. One is not free to introduce a continuous low-energy coefficient unless it can be shown to descend from this already-fixed parent ledger. That is why the repeated appearance of $2/5$, 2^J, πe, and the dyadic transport factors should be read structurally rather than numerologically.

11.3 TGG as the closed total geometry–gauge tensor

Once BI-6 is accepted, the next object is the total geometry–gauge tensor. We define

$$\mathcal{T}^{MN}_{\rm TGG} = T^{MN}_{\rm matter} + T^{MN}_{\rm gauge} + X^{MN}_{\rm GS} + T^{MN}_{\rm KK}, \tag{11.3}$$

where the four terms denote the visible matter sector, the gauge sector, the Green–Schwarz inflow sector, and the heavy-mode remnants that remain after reduction. The key point is not that these ingredients exist separately. The key point is that the physically meaningful conserved object is their sum.

Corollary 11.1 (TGG master conservation on the SM boundary stratum; specialization of GG-A6-GRIP T-GRIP-1)**.** *Assume the GG-6 bulk (Arisaka, 2026, A2-GG6), BI-6 anomaly freedom (Arisaka, 2026, A3-BI6), and the lawful OGGEP reduction (Arisaka, 2026, A1-Constitution; Arisaka, 2026, A5-GDOS). Then the total geometry–gauge tensor obeys*

$$\nabla_M \mathcal{T}^{MN}_{\mathsf{TGG}} = 0. \tag{11.4}$$

Proof. This is GG-A6-GRIP T-GRIP-1 (Arisaka, 2026, A6-GRIP) restricted to the boundary stratum on which the Standard-Model fields propagate. The bulk-side proof (existence and uniqueness of the composite connection $A_{\mathrm{tot}} = \omega \oplus A_{\mathrm{SU}(5)} \oplus A_B \oplus A_{Qi}$, plus the Master Conservation $\nabla_\mu \mathcal{T}^{\mu\nu}_{\mathrm{TGG}} = 0$ iff all four Ward currents close modulo X_4-inflow) is established there in full; the present paper consumes it without restatement. ∎

Theorem 11.1 makes explicit what is elsewhere used only implicitly. It means that the gauge, Higgs, flavor, and topological sectors are not ontologically independent modules that happen to coexist. They are coupled projections of one conserved ledger. This is why the later Standard-Model parameters can legitimately be treated as one sheet rather than as several unrelated fit problems.

11.4 CCI as the local seam-level conservation law

A boundary observer does not directly see Eq. (11.4). The observer sees its OGGEP projection. The projected law is the Covariant Conservation Identity,

$$\nabla_\mu \mathcal{T}^{\mu\nu}_{\mathrm{eff}} = F^\nu{}_\mu J^\mu_e + J^\nu_{\mathrm{hol}} - \nabla^\nu \phi_{\mathrm{CEP}}. \tag{11.5}$$

Here $\mathcal{T}^{\mu\nu}_{\mathrm{eff}}$ is the effective boundary stress tensor including the projected geometry–gauge correction, J^μ_e is the ordinary seam electromagnetic current, J^ν_{hol} is the holonomy/topological source current, and ϕ_{CEP} is the compensator field that packages the projected inflow correction at the local level.

Corollary 11.2 (Covariant Conservation Identity on the SM boundary stratum; specialization of GG-A6-GRIP T2.1+T2.2 + GG-A5-GDOS T-GDOS-9.7)**.** *Equation (11.5) is the four-dimensional OGGEP image of the closed TGG conservation law. On the admissible branch,*

$$F^\nu{}_\mu J^\mu_e + J^\nu_{\mathrm{hol}} - \nabla^\nu \phi_{\mathrm{CEP}} = 0, \tag{11.6}$$

the projected boundary ledger is conserved:

$$\nabla_\mu \mathcal{T}^{\mu\nu}_{\mathrm{eff}} = 0. \tag{11.7}$$

Proof. Two upstream ingredients combine. The bulk-to-boundary side is GG-A6-GRIP T2.2 (Arisaka, 2026, A6-GRIP) (Master Conservation iff Ward + PBI-4): the Projected Bianchi Identity PBI-4 descends bulk BI-6 closure $\mathrm{d}H_3 = X_4$ to the boundary divergence identity via Kaluza–Klein descent, with the compensator field ϕ_{CEP} playing the role of the link. The four-currents Ward-package side is GG-A5-GDOS T-GDOS-9.7 (Arisaka, 2026, A5-GDOS): the four boundary Ward identities $\partial_\mu J^\mu_a = 0$ for $a \in \{\mathrm{em}, \mathrm{B}, \mathrm{Qi}\}$ together with $\nabla_\mu T^{\mu\nu} = 0$ are protected from anomalous breaking by

the same BI-6 inflow. Restricting the joint statement to the Standard-Model boundary stratum yields (11.5)–(11.7). ■

This is the decisive local statement for the present paper. A parameter is not read off at an arbitrary kinematic point. It is read off on a CCI-lawful stationary seam: a seam where the projected force, holonomy, and compensator channels close consistently. That is why the lock seam and the neutral seam used later are not rhetorical choices. They are local admissible branches of the projected conservation law.

11.5 GFF and GRIP as the scalar and runtime closures

Once CCI has been stated, the remaining two pillars follow as theorems of GG-A6-GRIP (Arisaka, 2026, A6-GRIP).

First, by GG-A6-GRIP T3.1 (GFF Morse-positivity from CCI), the projected boundary packet carries one scalar ledger,

$$F_{\mathsf{GFF}} = F_{\mathsf{GEP}} + F_{\mathsf{CEP}}, \tag{11.8}$$

which on every stratum Σ of the boundary moduli space is a Morse function with strict alternating-index sum $a^+(\Sigma) - a^-(\Sigma) > 0$ whenever $\dim_{\mathbb{R}} \Sigma \equiv 0 \pmod 4$ (Arisaka, 2026, A6-GRIP). The mathematical content is that the reduced open-system dynamics needs one common energetic and curvature–entropy budget — not two independent scalar potentials — and the topology of the descent of $[X_4]$ to the boundary stratum forces strict positivity. On the family-projector stratum $\mathbb{C}P^{N_g-1}$ this is exactly the structural input that produced Theorem 21.1 above.

Second, by GG-A6-GRIP T4.1 (GRIP cascade closure on family-projector moduli), the branch-selection problem is governed by the resolved runtime order

$$\mathsf{GRIP}: \qquad \mathsf{DGI} \Longrightarrow \mathsf{GFF} \Longrightarrow \mathsf{GSSB} \Longrightarrow \mathsf{GA}, \tag{11.9}$$

with total cascade action bounded by the BI-6 budget $S_{\mathrm{tot}}(N_g) \le S_{\mathrm{BI}}^{\max} = 3\pi$. This bound is not new in A9; it is the same bound that closes the family-number derivation of Section 21 and is established in full upstream. The runtime order closes the logical gap left open in shorter summaries: symmetry breaking cannot be defined before the scalar landscape is defined; and a boundary observable is not stable until the selected branch is known to be a geometric attractor (Arisaka, 2026, A5-GDOS; Arisaka, 2026, A6-GRIP).

One further piece of runtime vocabulary, defined at runtime level in GG-A6-GRIP and used throughout this paper, should be imported now. The runtime operates in exactly two regimes, split by one question: *does the stratified landscape change?* GRIP-In is the descent on a *fixed* landscape — gradient flow within basins, instanton steps across the GSSB gates between them, and — on the strata GG-A6-GRIP calls admissible — halting uniquely at that stratum's GGA, by the per-stratum ground-state corollary of GG-A6-GRIP. GRIP-Out is the complementary regime: a transient out-of-equilibrium event that *reorganizes the landscape itself* and supplies the initial datum from which a new GRIP-In descent begins; that a second regime is needed at all — gradient flow cannot cross a stratum boundary — is carried in GG-A6-GRIP, and its full seventeen-event theory

in GG-A11-Origin. The present paper hosts both regimes side by side, and it matters to tell them apart: the selection of the electroweak vacuum (the Wilson–Hosotani branch and the lock-seam vev of Part VI) was the boundary sector's GRIP-Out event — it *created* the Yukawa landscape on which the three fermion generations are basins — whereas every flavor transition since ($\tau \to \mu \to e$, the quark ladder, the valley of β stability) is GRIP-In on that fixed landscape. In one sentence: the electroweak GRIP-Out built the landscape once; the weak-interaction GRIP-In has been walking it ever since.

From this point onward the derivation protocol is fixed. Every sector below will begin by asking what BI-6 permits, what TGG forces, what CCI projects, what GFF coordinates the sector uses, how GRIP selects and stabilizes the branch, and only then what parameter formulas are read off.

11.6 The Arisaka equation: four-currents lock and the structural origin of the $-2/5$ coefficient

The four currents that appear in the CCI (Corollary 11.2) and the four-currents Ward package (GG-A5-GDOS T-GDOS-9.7) — the electromagnetic current J_e, the baryonic current J_B, the informational current J_{Qi}, and the gravitational current carried by $T^{\mu\nu}$ — are not independent boundary observables. They are locked through a single operational identity proved as Theorem T-QUA of GG-A7-Qi4 (Arisaka, 2026, A7-Qi4). Indeed, this lock is the structural reason why a single rational coefficient (the trace ratio $-2/5$) appears in the BI-6 vector, in the weak-angle rational partition, and in the Arisaka constant simultaneously.

In one sentence, before the formal statement: the four boundary currents that the rest of the paper will treat separately — the energetic current J_E, the electron-number current J_e, the baryon current J_B, and the information current J_{Qi} — are in fact one current, projected through one energy–information conversion factor Q_B. Everything that follows in the next thirty pages is a specialization of this one identity to a particular boundary stratum. After all, the reader who absorbs the boxed theorem now will recognize it returning under different names later: as the trace-ratio lock $|a/c| = |a/d| = 5/2$ in the gauge sector, as the universal scale constant πe in the strong coupling, and as the energy–information anchor $Q_B \approx 0.156$ eV per qubit in the lepton sector.

Theorem 11.3 (**T-SM-10.3**: Arisaka equation; GG-A7-Qi4 T-QUA)**.** *On every admissible boundary branch the four conserved currents are related by*

$$\boxed{\frac{J_E}{Q_B} = J_e = J_B = J_{Qi},} \tag{11.10}$$

where J_E is the energy-flux current and Q_B is the Arisaka constant

$$Q_B = \frac{2}{5}\,\frac{E_{\mathrm{Ry}}}{K} \;\approx\; 0.15549 \text{ eV per qubit}, \qquad K = 35, \tag{11.11}$$

with $E_{\mathrm{Ry}} \approx 13.6057$ eV the Rydberg energy and $K = 35$ the OGN-5 lock-length integer. Equivalently, in chemical units,

$$N_A\, Q_B \;\approx\; 15.0 \text{ kJ/mol}, \tag{11.12}$$

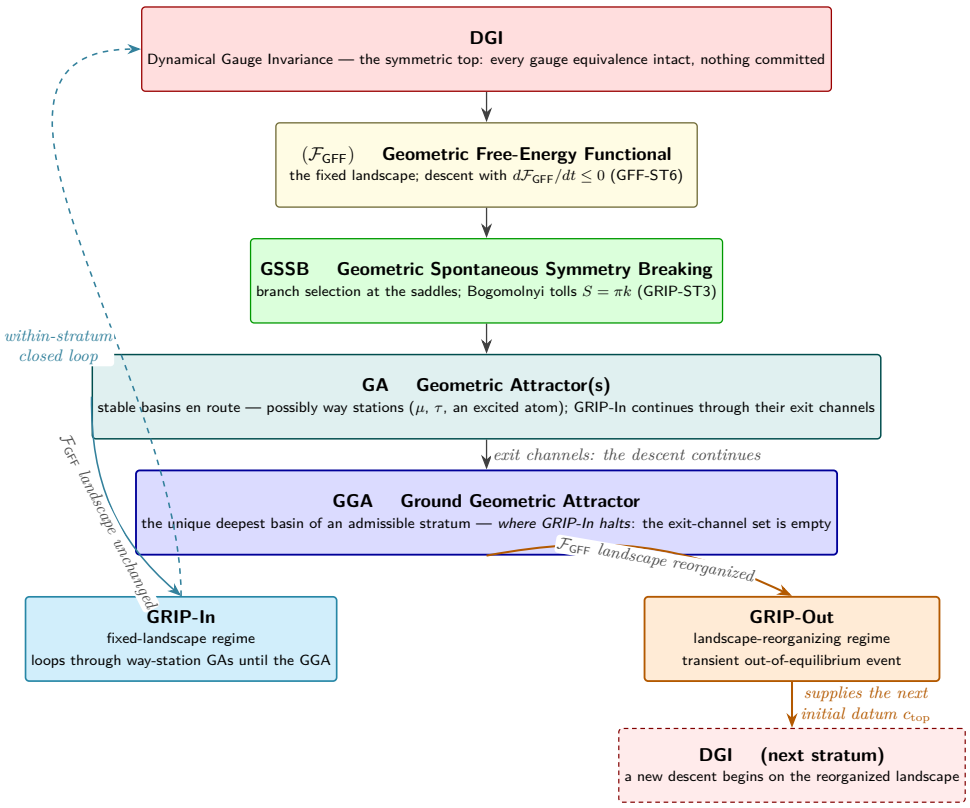


Figure 11: The GRIP cascade and its two regimes, in one picture. The central column is the runtime of the cascade: from the fully symmetric top (DGI, every gauge equivalence intact), the system descends the fixed landscape $\mathcal{F}_{\mathsf{GFF}}$, selects branches at the GSSB saddles, and is captured into a stable basin — a Geometric Attractor (GA). The attractor tier has two levels. From a *non-ground* GA (a way station: a muon, an excited atom) the descent *continues* — GRIP-In, the fixed-landscape regime, proceeds through the basin's exit channels, the weak interaction being Nature's implementer on the matter strata — whereas at the GGA, the unique deepest basin, the exit-channel set is empty and GRIP-In *halts*. After the halt, only GRIP-Out can act: a transient out-of-equilibrium event reorganizes the landscape itself and supplies the initial datum of the next descent. In one sentence: GRIP-Out opens the landscape; GRIP-In finishes the journey; the GGA is where the journey ends. In the present paper the one GRIP-Out is the electroweak vacuum selection, and every fermion mass, mixing angle, and lifetime is a GRIP-In readout on the landscape it built. (Reproduced from GG-A6-GRIP (Arisaka, 2026, A6-GRIP); the eighteen-layer deployment is GG-A11-Origin (Arisaka, 2026, A11-Origin).)

Proof. The full proof is GG-A7-Qi4 T-QUA (Arisaka, 2026, A7-Qi4): the rank-one inflow theorem (algebraic layer) shows that all four boundary currents descend from a single rank-one bulk inflow channel through BI-6 closure on the 6D bulk, and the operational layer then reconciles the four currents through Q_B as the natural-unit conversion factor. The chemical-units value (11.12) is a direct consequence of the upstream-fixed quantities E_{Ry}, K, and N_A, not an additional input. ■

Why the BI-6 vector contains the trace ratio $-2/5$. The structural meaning of (11.10) is that the energy-flux normalization Q_B enters the BI-6 anomaly polynomial wherever the boundary currents do. By GG-A3-BI6 (Arisaka, 2026, A3-BI6), the integrality of X_4 on the 6D bulk forces the rational coefficient

$$\frac{\mathrm{Tr}(B_0^2)}{\mathrm{Tr}(Y_0^2)} = \frac{L-N}{L} = \frac{2}{5} \qquad \text{for } (N,L) = (3,5), \tag{11.13}$$

which is the same $2/5$ that defines Q_B in (11.11). Indeed, the appearance of $-2/5$ as the third and fourth components of the BI-6 vector $(a,b,c,d) = (1,-1,-2/5,-2/5)$ (11.2) is not a numerical coincidence; it is the same trace ratio that fixes Q_B on the boundary. After all, BI-6 and the Arisaka equation are two faces of one structural fact: the four boundary currents lock through a single rank-one ledger whose normalization is the trace ratio of the primitive **5** representation.

Why the weak-angle partition is L^2/N^3. The same four-currents lock is what produces the closed-form weak-mixing-angle theorem proved later in this paper (Theorem 32.1, the weak-angle rational-partition theorem). By GG-A7-Qi4 T-QUA, the electromagnetic and weak neutral currents partition as a power partition of the same locked ledger: the $L = 5$ split-generator norm sits on the photon side of the neutral seam in proportion L^2/N^3, with the leftover $(L-N)^2 = 4$ on the Z-boson side. The result

$$\sin^2\theta_W(M_Z) = \frac{L^2}{N^3(L-N)^2} = \frac{25}{27\cdot 4} = \frac{25}{108} \tag{11.14}$$

is therefore not a separate empirical fit but a direct algebraic consequence of (11.10) and the OGN-5 ledger. The same structural fact that fixes $Q_B = (2/5)E_{\mathrm{Ry}}/K$ at the energy–information scale fixes $\sin^2\theta_W = 25/108$ at the electroweak scale; both are readouts of the same Arisaka equation lock at different boundary strata.

Remark 1 (Physics intuition for the Arisaka lock)**.** One structural fact — the four boundary currents are locked through Q_B — explains three numerical features of the framework that would otherwise look unrelated. The trace ratio $-2/5$ in the BI-6 vector, the rational $25/108$ in the weak-angle partition, and the energy quantum 0.1555 eV per qubit are not three coincidences but three faces of the same Arisaka equation. Indeed, this is the deepest unifying claim of GG-Theory: that the same geometric lock that generates the gauge-Higgs sector at TeV scales generates the energy–information ledger at meV scales, with Q_B as the universal conversion factor. After all, this is exactly the role T-GU5 (the GU-5 of GG-A1-Constitution) assigns to Q_B: it is the boundary-side conversion factor of the GDOS Trio, anchored to its primary derivation paper GG-A7-Qi4 (Arisaka, 2026, A1-Constitution; Arisaka, 2026, A7-Qi4).

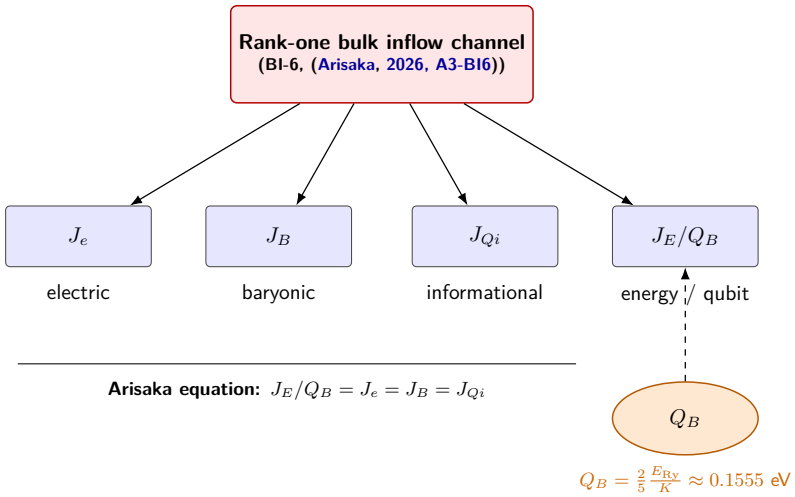


Figure 12: The Arisaka equation $J_E/Q_B = J_e = J_B = J_{Qi}$ (Theorem 11.3; GG-A7-Qi4 T-QUA, Arisaka, 2026, A7-Qi4) as a four-currents lock. All four boundary currents (electric J_e, baryonic J_B, informational J_{Qi}, and energy-flux J_E converted by the energy–information factor Q_B) descend from a single rank-one bulk inflow channel of BI-6. The trace-ratio coefficient $-2/5$ that appears in the BI-6 vector and the L^2/N^3 partition that produces $\sin^2\theta_W = 25/108$ are two faces of the same lock.

The Arisaka equation (11.10) together with the BI-6 corollary (11.13) is therefore the load-bearing identity that A9 inherits from the GDOS Trio. Every parameter formula in the remainder of this paper is, at the structural level, a specialization of (11.10) to a particular boundary stratum. The gauge-Higgs theorem package of Sections 31–34 below uses the partition (11.14); the fermion-mass codebook of Sections 47–49 uses the rank-one structure of the Arisaka inflow at the family-projector stratum; the CKM and PMNS sections use the same rank-one structure restricted to the up-type/down-type and charged-lepton/neutrino projector pairs.

11.7 The status of $\mathrm{U}(1)_{Qi}$: constitutionally optional, empirically preferred

A natural question raised by the four-currents lock is whether the informational current J_{Qi} — and hence the gauge factor $\mathrm{U}(1)_{Qi}$ that carries it — is a constitutional necessity of the bulk theory or an empirically preferred extension.

Constitutional admissibility. By GG-A3-BI6 (Arisaka, 2026, A3-BI6), both the three-component bulk gauge group $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B$ and the four-component bulk gauge group $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ are constitutionally admissible: both close BI-6 with their respective coefficient vectors, and both satisfy the GG-Constitution axioms (AX-1–AX-4) and postulates (POS-1–POS-7). In this sense $\mathrm{U}(1)_{Qi}$ is *constitutionally optional*.

Empirical preference. The four-component bulk is adopted in the present paper on three independent empirical grounds. First, the Arisaka equation $J_E/Q_B = J_e = J_B = J_{Qi}$ (Theorem 11.3; GG-A7-Qi4 T-QUA) requires the informational current J_{Qi} on the boundary, with no consistent

rephrasing in the three-component bulk. Second, the OGN-5 dual-lock identity $K = LJ = M^2 + N^2 + L^2 = 35$ closes only for the four-component case (per A2-GG-6 Theorem T6 (Arisaka, 2026, A2-GG6)); the three-component variant fails the integrality check. Third, the entire Qi-4 Pillar structure of A7 (Pillars Qi-QUA, Qi-EP, Qi-CON, Qi-HAL) and the EIC boundary completion (OH+IH=EIC) depend on $U(1)_{Qi}$.

Net status. $U(1)_{Qi}$ is therefore *constitutionally optional* (the GG-Constitution admits both bulk variants) but *empirically preferred* (the four-component bulk produces a working 28-parameter codebook; the three-component bulk does not). GG-A9-SM adopts the four-component bulk throughout. A reader who wishes to study the three-component variant would have to abandon the Arisaka equation and re-open the OGN-5 closure check; that is a substantial structural undertaking and not pursued here. After all, the empirical-preference framing makes the structural cost of the three-component variant explicit, and the four-component variant is the one the present codebook actually requires.

12 GDOS, GFF, SIP, and the operational meaning of observables

This section states the third link of the central claim of §1.1 (Figure 4) in full: the law GDOS by which the open-system projection of OGGEP assigns each of the twenty-eight observables a definite value.

The previous section fixed the ultraviolet constitution and the admissible input ledger. We now need to explain how those ultraviolet statements become low-energy observables. This is the role of GDOS in GG-A5-GDOS and of the universal runtime extension in GG-A7-Qi4. The basic message is simple but fundamental: the Standard Model is not merely a four-dimensional local Lagrangian; it is the observer-accessible boundary dynamics of a deeper open system (Arisaka, 2026, A5-GDOS; Arisaka, 2026, A6-GRIP).

12.1 Why a parameter paper needs GDOS before it needs numerical formulas

In ordinary phenomenology, one often starts from a four-dimensional effective Lagrangian, chooses a renormalization scheme, and fits parameters to data. Nothing is wrong with that procedure when the goal is practical prediction. However, the present paper is trying to do something more ambitious. It is not trying merely to summarize the phenomenology of a known theory. It is trying to show how those phenomenological quantities arise from a deeper constitutional framework.

For that reason a numerical parameter paper that omits the operational layer would be incomplete. A coupling constant, for example, is not a bare number in the abstract. It is the result of a boundary measurement protocol performed at a specified scale and in a specified renormalization scheme. A mass is not simply a coefficient in a microscopic action. It is a spectral feature of a reduced boundary theory reconstructed from open-system data. A mixing angle is not an ontological angle living in the bulk. It is a relation among boundary overlap channels inferred from measured amplitudes and rates.

GDOS is therefore not apparatus bolted onto the parameter problem; it is the third arrow of the central claim (§1.1) made precise — the law that tells us what the quantities being predicted even *are*.

12.2 From OGGEP to exact open-system boundary dynamics

The core claim of GG-A5-GDOS is that AX-3 (OGGEP) requires every admissible theory to provide a lawful map from inaccessible or partially accessible bulk data to registered boundary observables. In abstract notation one may write

$$\mathcal{D} : \text{Bulk} \longrightarrow (\mathcal{H}_\partial, \rho_\partial, \Phi_{t,s}, \{\mathcal{I}_\alpha\}), \tag{12.1}$$

where ρ_∂ is the boundary state, $\Phi_{t,s}$ is the admissible evolution map, and $\{\mathcal{I}_\alpha\}$ is the family of measurement instruments.

At the exact level, GG-A5-GDOS shows that the reduced boundary dynamics obeys a Nakajima–Zwanzig type equation, and that in the Markovian limit one arrives at a GKLS generator. For the present paper, the crucial phenomenological point is that the low-energy Standard Model is therefore already an open-system boundary theory before one begins any specific particle-physics calculation.

This can be summarized schematically as

$$\text{GG-6 bulk} \xrightarrow{\text{OGGEP}} \text{exact projected boundary dynamics} \Longrightarrow \text{GKLS limit} \Longrightarrow \text{GDOS current form}. \tag{12.2}$$

Equation (12.2) is one of the conceptual bridges that make GG-A9-SM a phenomenological paper rather than a purely formal one. It means that when we later speak about couplings, masses, and mixings, we are not speaking about hidden bulk parameters but about projected seam observables.

12.3 The GDOS continuity equation and its physical meaning

At mesoscopic resolution, the boundary theory is written in GDOS form as a continuity law on a state manifold:

$$\partial_t p(x,t) + \nabla_A J^A(x,t) = 0. \tag{12.3}$$

The current admits the standard decomposition emphasized in GG-A5-GDOS and GG-A7-Qi4,

$$J^A = -D^{AB}\nabla_B p - \mu^{AB} p\, \nabla_B \mathcal{F}_{\text{GFF}} + J^A_{\text{curl}}. \tag{12.4}$$

Equation (12.4) is simple enough to memorize, and it is worth doing so because it contains much of the philosophy of the program in one line.

The first term is diffusive spreading. This is what any open system does when it is left only to uncontrolled noise and coarse-grained mixing.

The second term is gradient descent on the GFF ledger. This is what organizes selection, relaxation, and maintenance.

The third term is the solenoidal or circulation term. This is what allows the system to have sustained currents, cycles, and nontrivial nonequilibrium steady states instead of collapsing into a trivial equilibrium minimum.

This decomposition already contains the seed of why a Standard-Model parameter paper should be written in GDOS language. A measured coupling or mass is not just a number. It is the visible trace of a system that has diffused, descended, and stabilized onto a specific boundary branch.

12.4 GFF as the scalar ledger of observable physics

GG-A7-Qi4 insists that the GDOS runtime is incomplete unless it carries one unified scalar ledger, denoted GFF, decomposed as

$$\mathsf{GFF} = \mathsf{GEP} + \mathsf{CEP}. \tag{12.5}$$

The first term, GEP, is the energy–geometry–gauge contribution. The second, CEP, is the curvature–entropy contribution induced by coarse graining, admissible state-space geometry, and diffusion. In practical terms, the precise division depends on the system under study, but the key lesson is universal: a persistent open system requires one scalar budget that can be decreased, replenished, and audited.

For particle phenomenology the importance of GFF may not be obvious at first sight, so it is worth stating the intuition explicitly. In collider physics or precision electroweak physics, one often speaks as though the measured quantity is just the pole of a propagator or the value of a renormalized coupling. GG-6 and GDOS do not deny that. They ask a more basic question: why is the theory allowed to export that quantity as a stable, reproducible, observer-accessible observable at all? The answer is that the corresponding boundary branch is a stable attractor of the open-system runtime — for the reproducible locked values, the stratum's ground attractor (GGA; §94.1) — and its stability is organized by GFF.

This viewpoint is especially useful for mass. The mass-origin paper cited above reformulates mass as a geometric spectral gap and interprets the Higgs, fermion, and confinement sectors as different projections of one GFF-based ledger (Arisaka, 2026, A6-GRIP). That viewpoint fits naturally into the present manuscript, where the mass tables are not supposed to be isolated empirical fits but outputs of one geometrically and operationally unified construction.

12.5 DGI, GSSB, and GA as the master pipeline from possibility to observable structure

One of the most useful conceptual advances of GG-A7-Qi4 is the explicit naming of the three-stage runtime grammar

$$\mathsf{DGI} \longrightarrow \mathsf{GSSB} \longrightarrow \mathsf{GA}. \tag{12.6}$$

This pipeline is not merely a slogan. It is the operational translation of how an open system explores, selects, and stabilizes structure.

DGI: Dynamical Gauge Invariance. DGI says that the lawful reduced dynamics should be written not in terms of one rigid microscopic description, but in terms of a gauge-equivalence class

of admissible reduced descriptions. Intuitively, DGI is the exploration stage: the system moves among physically equivalent descriptions without yet committing to a unique realized branch.

GSSB: Geometric Spontaneous Symmetry Breaking. GSSB says that branch selection is not an add-on to the geometry; it is a geometric event inside the open-system runtime. In the present paper, the most important GSSB event is the selection of the Wilson–Hosotani vacuum holonomy. That is what eventually turns the unified SU(5) sector into the Standard Model gauge structure; the doublet's vacuum value, its scale set by that holonomy, then reduces the electroweak sector to $U(1)_{EM}$.

GA: Geometric Attractor. GA is the stabilized branch selected by the previous two stages. Physically, it is what makes a low-energy world persistent enough to measure repeatedly. In particle phenomenology, a GA is not a biological attractor or a cognitive state; it is the stabilized seam theory whose couplings, masses, and mixings are the ones seen in experiment.

This three-stage grammar is particularly valuable because it tells us how to read the Standard Model inside GG-6. The Standard Model is not the full bulk. It is the GA stage of a more symmetric and more geometric theory after DGI and GSSB have acted.

12.6 Why the Qi-4 pillars matter even in a particle-physics paper

At first encounter, the Qi-4 extension may appear to belong more naturally to biology, neuroscience, or complex systems than to particle phenomenology. However, that impression is misleading. The Qi-4 pillars were introduced in GG-A7-Qi4 precisely to universalize GDOS across all open systems, and the particle-physics laboratory is itself an open system. The point is not that a collider detector is a brain. The point is that both are finite devices that prepare states, exchange currents, store records, and produce registered observables.

The four pillars are:

$$\text{Qi-QUA}, \qquad \text{Qi-EP}, \qquad \text{Qi-CON}, \qquad \text{Qi-HAL}. \tag{12.7}$$

Their relevance to the present paper can be summarized as follows.

Qi-QUA. This is the current-locking layer. It organizes the relation among charge, baryon, informational, and energetic currents and introduces the Arisaka Ratio, Arisaka Equation, and Arisaka Constant. For a particle-physics paper the main use of Qi-QUA is to justify why seemingly different bookkeeping channels can be projected onto one locked seam ledger.

Qi-EP. This is the extremal-principle layer. It unifies the bulk principle of least action with boundary principles such as least-time, maximal entropy production, and free-energy descent. In the present paper this is relevant to vacuum selection, RG trajectory selection, and the identification of the physically realized branch.

Qi-CON. This is the conformal and multiscale diagnostic layer. For particle physics it supports the idea that logarithmic running, hierarchy patterns, and scale covariance are not external features bolted onto the theory but are expected signatures of the runtime.

Qi-HAL. This is the architecture layer. In complex systems it deals with memory and internal modeling. In the present paper the closest analogy is the experimental architecture that stores and reconstructs boundary observables. Qi-HAL therefore matters less directly for the derivation of numerical Standard-Model parameters, but it matters for the full observational logic of how those parameters become registered quantities.

12.7 The System Instance Packet and why the Standard Model itself should be treated as a packet

The most operationally concrete innovation of GG-A7-Qi4 is the System Instance Packet, or SIP. In its canonical five-field form it is written as

$$\mathsf{SIP} = \Big[(\mathcal{Z}, g_{AB}),\ \mathsf{Ports},\ (\mathfrak{L}_{\mathsf{GDOS}}, \eta),\ (\mathcal{F}_{\mathsf{GFF}}, \mathcal{J}),\ (\mathrm{Obs}, \mathrm{Kill})\Big]. \tag{12.8}$$

The significance of this definition is that it forces one to say what the system is, what its ports are, what its lawful generator is, what scalar and vector ledgers it carries, what observables are exported, and what would count as failure.

This language can and should be applied directly to the Standard Model as it appears in the present paper. The Standard Model, in the present framework, is not "all of reality." It is a specific observer-accessible seam packet. A useful formal notation is therefore

$$\mathsf{SIP}_{\mathrm{SM}} = \Big[(\mathcal{Z}_{\mathrm{SM}}, g^{\mathrm{SM}}_{AB}),\ \mathsf{Ports}_{\mathrm{SM}},\ (\mathfrak{L}_{\mathrm{SM}}, \eta_{\mathrm{SM}}),\ (\mathcal{F}^{\mathrm{SM}}_{\mathsf{GFF}}, \mathcal{J}_{\mathrm{SM}}),\ (\mathrm{Obs}_{\mathrm{SM}}, \mathrm{Kill}_{\mathrm{SM}})\Big]. \tag{12.9}$$

Each entry has a concrete meaning.

State manifold. The state manifold $\mathcal{Z}_{\mathrm{SM}}$ is spanned by the seam-visible gauge, Higgs, and flavor variables relevant to the chosen observational regime. In the present paper that includes the running gauge couplings, the Wilson-line/Higgs order parameter, the family projectors that later generate Yukawa hierarchies, and the renormalized boundary masses and mixing parameters derived from them.

Ports. $\mathsf{Ports}_{\mathrm{SM}}$ are the preparation and measurement channels by which the seam is probed: scattering inputs, decay channels, threshold scans, flavor-tagging channels, oscillation baselines, detector reconstruction pipelines, and scheme choices that turn raw event counts into renormalized observables.

Generator. $\mathfrak{L}_{\mathrm{SM}}$ is the effective seam generator. At high resolution it is the projected open-system generator descending from the bulk. At practical phenomenological resolution it becomes the

combined RG flow plus effective interaction structure governing the scale dependence of couplings and masses.

GFF ledger and current bundle. $\mathcal{F}^{\mathrm{SM}}_{\mathsf{GFF}}$ is the scalar budget that organizes seam branch selection, while $\mathcal{J}_{\mathrm{SM}}$ collects the relevant locked currents. Although the current bundle is not used in the same explicit way in every later calculation, it is the reason why the seam observables can be regarded as one unified packet rather than as independent sectoral fragments.

Observables and kill criteria. $\mathrm{Obs}_{\mathrm{SM}}$ are the renormalized gauge couplings, masses, mixing angles, phases, and derived low-energy observables. $\mathrm{Kill}_{\mathrm{SM}}$ are the failure conditions: the theory would be ruled out if it failed anomaly cancellation, failed to produce the observed gauge pattern, failed to preserve proton stability, or predicted the wrong class of masses and mixings beyond the estimated theoretical uncertainty.

This packet language is more than bookkeeping. It enforces conceptual honesty. It prevents one from speaking as though the Standard Model were a monolithic closed system detached from the finite experimental architectures that actually measure it.

12.8 How the SIP viewpoint changes the meaning of a Standard-Model parameter

Once $\mathsf{SIP}_{\mathrm{SM}}$ is written down, the meaning of a Standard-Model parameter becomes sharper.

A gauge coupling is the projected response coefficient of a seam packet under a specified operational protocol.

A mass is the spectral gap exported by that packet after the vacuum branch has been selected.

A mixing angle is a statement about overlap and misalignment among family projectors in that packet.

A CP phase is a holonomy-sensitive orientation relation in the same family-space and compact-angle structure.

This is exactly the level of conceptual precision needed for a phenomenological paper whose ambition is to derive parameters rather than merely to report them.

12.9 Why the Standard Model is a particularly clean application of GDOS

The Standard Model is not the only possible application of GDOS, but it is one of the cleanest. The reason is that high-energy physics already has an unusually sharp culture of state preparation, scattering, asymptotic channels, renormalization, and statistical inference. In a sense, the Standard Model is what one gets when a very well-controlled open-system packet is observed through extremely refined protocols.

That is why the GDOS language is not an artificial imposition on particle phenomenology. It is a clarification of something high-energy physics already does in practice, namely extract physically meaningful parameters from open, finite, projected observations.

12.10 Bridge to the derivation sections that follow

The purpose of the present section has been to make three points explicit.

First, the Standard Model observables predicted later in the paper are not bare bulk quantities; they are boundary-visible GDOS exports.

Second, GFF is the scalar ledger that makes branch selection and persistence of those exports meaningful.

Third, the DGI→GSSB→GA pipeline and the SIP language explain how the symmetric six-dimensional stage becomes the specific low-energy seam whose parameters we measure.

With that operational and runtime language in place, we can now turn to the Standard-Model foundation proper: the route from the historical Georgi–Glashow SU(5) idea to the GG-6 seam construction that underlies the detailed parameter calculations below.

13 Part III in plain English: the rules the derivation has to obey

The hardest thing about a derivation of this kind is not the algebra. It is staying honest about what went in.

There is a standard failure mode in unified model building, and it is not fraud — it is drift. A quantity gets used as an input in one section and predicted in another; a scale gets fixed by a measurement early and then the measurement is cited later as a confirmation; a fit constant appears under the name of a convention. None of this requires bad faith. It requires only that nobody wrote down the rules in advance.

This Part writes them down. The four axioms and seven postulates come from GG-A1-Constitution and are not renegotiated here. The admissible inputs are listed exhaustively: the dimensional conversion anchors, the two geometric constants, the five ledger integers, and the constitutional statements themselves. Everything else is forbidden, and the table of forbidden shortcuts is as specific as the table of permitted inputs — it names taking the Fermi constant as an input, seeding the running with measured couplings, and adding a family-specific constant per mass.

The apparatus is not a philosophical preamble. It is the instrument by which the rest of the paper can be audited by someone who does not trust it. A reader who wants to attack the result should attack it here first: find a place where a later Part uses something this Part does not permit. If that place exists, the derivation fails, and it fails cleanly rather than by degrees. That is the reason for setting the rules out at this length before any of them is used.

Part IV
The Substrate: the bulk, the integer ledger, and the GG seam

The fixed geometric landscape on which the cascade runs: the technical descent from Georgi–Glashow SU(5) to the six-dimensional seam, the bulk geometry and three scales, the OGN-5 ledger, the three-leg theorem $N_g = 3$, the radial renormalization group, and Wilson–Hosotani symmetry breaking

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14 From the original Georgi–Glashow SU(5) to the GG-6 Standard-Model seam

14.1 Inheritance from classic SU(5) — pointer to Part I §6

The historical case for SU(5) as the natural simple-group container of the Standard-Model gauge algebra, the matter-content fitting into $\bar{\mathbf{5}} \oplus \mathbf{10}$ per family, and the canonical decomposition of the adjoint **24** are recorded in Part I §6 (Georgi & Glashow, 1974). The five virtues classic SU(5) explains (charge quantization, family multiplet structure, gauge-coupling unification, anomaly cancellation per family, and the embedding of $U(1)_Y$ inside a simple group) and the five gaps it leaves open (family number N_g, fermion masses, gravity, proton stability via X, Y bosons, the measurement layer) are likewise recorded there. The present chapter takes those facts as known and turns to what GG-6 *changes*.

14.2 What GG-6 changes: 6D geometry, local $U(1)_B$, and local $U(1)_{Qi}$

The unified bulk group relevant to the present paper is not just SU(5), but

$$\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \times \text{Gravity}. \tag{14.1}$$

Each new factor has a concrete role.

The radial direction. This is what geometrizes scale and eventually lets running couplings be derived from transport in the fifth coordinate.

The compact θ cycle. This is what makes Wilson-line holonomy and the Hosotani mechanism available. Without it, the present paper could not reinterpret the Higgs as a geometric order parameter.

Local $\mathrm{U}(1)_B$. This is what changes the proton-decay story. In the Standard Model and in many traditional GUT settings, baryon number is at best approximate. In the present framework, exact local $\mathrm{U}(1)_B$ strongly constrains baryon-violating operators and is tied to the insistence that proton stability is a genuine structural clue rather than an accident.

Local $\mathrm{U}(1)_{Qi}$. This factor is what makes it possible to extend the open-system logic from particle phenomenology to the full Qi-4 runtime. Even in the present particle-focused paper, it matters because it is part of the locked current ledger and therefore part of the explanation of why observable bookkeeping can be unified.

14.3 Orbifold/boundary-condition breaking and Wilson–Hosotani selection

In the GG-6 setting, the descent from the unified bulk to the low-energy gauge algebra is not achieved by simply postulating a four-dimensional Higgs multiplet that breaks SU(5) by hand. It is achieved by a combination of boundary-condition selection and Wilson-line holonomy.

A standard parity assignment that separates color and electroweak directions may be written schematically as

$$P = \mathrm{diag}(+,+,+,-,-). \tag{14.2}$$

The generators commuting with the parity assignment survive as low-energy zero modes, while the noncommuting generators are projected out of the light sector or lifted to heavy masses. At the same time, the compact θ direction supports the Wilson line

$$W(\theta) = \mathcal{P}\exp\left(\mathrm{i}\oint_{S^1_\theta} A_\theta \,\mathrm{d}\theta\right), \tag{14.3}$$

whose vacuum expectation value selects the actual branch.

This is precisely where GSSB enters. The low-energy Standard-Model gauge group is not assumed as the only algebra logically possible. It is selected as the stable commuting branch of a more symmetric unified structure under the combined influence of boundary conditions and holonomy. The dynamical content of this holonomy selection — the Wilson-line effective potential and the electroweak vacuum value it fixes — is derived in §23.

14.4 The low-energy gauge algebra as a commuting subalgebra

The algebra that survives after the first stage of breaking is the commuting subalgebra

$$\mathfrak{g}_{\mathrm{SM}} = \mathfrak{su}(3)_C \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y. \tag{14.4}$$

The hypercharge generator in the conventional SU(5) embedding is

$$Y = \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right), \tag{14.5}$$

properly normalized inside the unified algebra. Electric charge then follows from the usual relation

$$Q = T_3 + Y. \tag{14.6}$$

The value of stating Eqs. (14.5) and (14.6) explicitly is that they already explain why charge quantization is natural in a unified setting. The charges of quarks and leptons are not separate empirical insertions; they are consequences of one embedding.

14.5 How the three observed gauge forces arise at low energies

A common source of confusion is the phrase "three gauge forces." If one begins from a unified theory, does one still really explain why there are three? In the present framework the answer is yes, and the explanation is sequential.

At the highest relevant scale of the visible gauge sector, one has the unified SU(5) connection together with local $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$. After the first breaking/selecting stage, the visible gauge sector becomes $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. After the second, electroweak breaking stage, one obtains

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \longrightarrow \mathrm{U}(1)_{\mathrm{EM}}. \tag{14.7}$$

At ordinary laboratory energies, the three interaction sectors that remain as distinct long- or intermediate-range gauge structures are therefore:

$$\text{strong } (\mathrm{SU}(3)_C), \qquad \text{weak (broken } \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y), \qquad \text{electromagnetic } (\mathrm{U}(1)_{\mathrm{EM}}). \tag{14.8}$$

The word "three" is thus not fundamental in the ultraviolet sense; it is the number of distinct gauge sectors experienced after the two-stage seam reduction. That is exactly the sort of statement that should appear in a professional phenomenological paper: it says not only what the world looks like, but why it looks that way at accessible scales.

14.6 Why the Standard Model should be viewed as the attractor, not the whole ontology

This point is worth emphasizing because it improves the phenomenological logic of the paper. If one mistakes the Standard Model for the whole ontology, then its free parameters appear as unexplained primitive numbers. If instead one views it as a stabilized attractor branch of a more constrained six-dimensional open system, then its parameters appear as seam invariants of a deeper geometry.

That change of viewpoint does not automatically compute the 28 parameters. But it explains why computing them is a sensible goal. There is something to compute precisely because the boundary theory is constrained by a more rigid bulk.

15 Applying the GDOS runtime to the unified packet

The previous sections established the ultraviolet stage, the open-system runtime, and the historical relation between SU(5) and the Standard Model. The conceptual GDOS law — what it is, and why a parameter paper needs it — was fixed in §12; we do not restate it. Here we *apply* it, walking the runtime from the pre-breaking unified packet through the GSSB branch selection to the stabilized Standard-Model attractor. The stationary calculational seams at which the resulting couplings and masses are then evaluated are collected separately in §20.

15.1 The unified packet before symmetry breaking

Before branch selection, the visible gauge and matter sector is most naturally viewed as a unified packet with bulk-visible fields, boundary conditions, transport structure, and open-system observables all declared at once. A useful formal description is

$$\widehat{\mathsf{SIP}}^{\mathrm{pre}}_{\mathrm{vis}} = \Big[(\mathcal{Z}^{\mathrm{pre}}_{\mathrm{vis}}, g_{AB}),\ \mathsf{Ports}^{\mathrm{pre}}_{\mathrm{vis}},\ (\mathfrak{L}^{\mathrm{pre}}_{\mathrm{vis}}, \eta^{\mathrm{pre}}_{\mathrm{vis}}),\ (\mathcal{F}^{\mathrm{pre}}_{\mathsf{GFF}}, \mathcal{J}^{\mathrm{pre}}_{\mathrm{vis}}),\ (\mathrm{Obs}^{\mathrm{pre}}_{\mathrm{vis}}, \mathrm{Kill}^{\mathrm{pre}}_{\mathrm{vis}})\Big]. \tag{15.1}$$

At this stage, $\mathcal{Z}^{\mathrm{pre}}_{\mathrm{vis}}$ includes the full unified gauge connection on the visible gauge bundle, the relevant matter multiplets, the radial transport coordinate, and the compact-angle holonomy variable. The packet is more symmetric than the later low-energy world. That is exactly what one expects before GSSB acts.

A professional phenomenological paper should emphasize this point because it prevents a common misunderstanding. The Standard Model is not the theory before symmetry breaking. It is the stabilized low-energy branch after symmetry breaking, projection, and reduction. The pre-breaking packet therefore carries more structure than the final observed theory.

15.2 GFF ledger and DGI application — pointer to GG-A6-GRIP T3.1 and T4.1

The GFF Morse-positivity ledger on the unified packet and the DGI application to the unified gauge sector are owned end-to-end by GG-A6-GRIP T3.1 (the Geometric Free-Energy Functional) and T4.1 (the Gauge-Runtime Invariance Pipeline) (Arisaka, 2026, A6-GRIP). The packet-level statements that GG-A9-SM relies on are: (i) the energetic block F_{GEP} separates from the curvature–entropic block F_{CEP}; (ii) the boundary attractor coordinates of the electroweak seam are the stationary points of $F_{\mathsf{GFF}} = F_{\mathsf{GEP}} + F_{\mathsf{CEP}}$; (iii) the dynamical gauge invariance DGI restricts the admissible runtime variations to those that respect the bulk gauge symmetry before GSSB. The next subsection picks up at the GSSB step (Wilson–Hosotani breaking).

15.3 Wilson–Hosotani breaking as the GSSB step

The next step is the GSSB event. In the present context the physically decisive statement is that the vacuum selects a Wilson-line holonomy class. That class is the selector of the electroweak branch; the doublet whose vacuum value it fixes is the $(\mathbf{1},\mathbf{2})_{+1/2}$ of an required bulk $\mathbf{5}_H$ (selected as minimal), not a fluctuation of the connection (§23.3).

Its order parameter $W = \exp(\mathrm{i}\,\Theta_H T_H)$, the finite holonomy effective potential $V_{\mathrm{eff}}(\Theta_H)$, and the explicit branch-selection conditions $\partial_{\Theta_H} V_{\mathrm{eff}} = 0$, $\partial^2_{\Theta_H} V_{\mathrm{eff}} > 0$ — together with the resulting electroweak vacuum value and the doublet fraction $2/5$ — are written out in §23. In the DGI→GSSB→GA language this holonomy selection *is* the GSSB event: the low-energy world is fixed by a geometric vacuum criterion rather than by an arbitrary scalar potential.

15.4 The Standard Model as the GA stage

After the branch has been selected, the resulting low-energy seam packet is the geometric attractor,

$$\widehat{\mathsf{SIP}}_{\mathrm{SM}} = \mathsf{GA}(\widehat{\mathsf{SIP}}^{\mathrm{pre}}_{\mathrm{vis}}). \tag{15.2}$$

The notation in Eq. (15.2) is schematic, but its meaning is clear. The Standard Model packet is the stabilized output of the unified packet under GDOS evolution, symmetry breaking, and boundary reduction.

This viewpoint is particularly useful for interpreting later numerical tables. A predicted gauge coupling or fermion mass is not a raw pre-breaking datum. It is a stabilized seam observable read off after the system has already descended onto the GA branch.

15.5 How Qi-QUA enters the Standard-Model packet

At the level of the present paper, Qi-QUA enters most directly through current locking. The relevant symbolic relation is the Arisaka Equation,

$$J_E/Q_B = J_e = J_B = J_{Qi}. \tag{15.3}$$

Its significance for Standard-Model phenomenology is that the seam bookkeeping of energetic, charge, baryonic, and informational channels is not independent. The observational packet is therefore more constrained than a generic effective field theory packet would be.

In practice this matters because it supports the idea that the visible gauge, Higgs, and flavor observables can be organized in one common arithmetic ledger rather than in a patchwork of unrelated fit formulas.

15.6 How Qi-EP enters the Standard-Model packet

Qi-EP is the pillar that unifies the bulk principle of least action with the boundary selector principles. For the present paper its role is twofold.

First, it explains why the physically realized vacuum is the one selected by the appropriate projected extremal condition rather than by arbitrary choice.

Second, it explains why RG trajectories, seam matching, and family overlap patterns should be viewed as selected paths in a projected landscape rather than as disconnected numerical coincidences.

In other words, Qi-EP is what allows one to speak consistently of one selected trajectory running from the unified bulk to the low-energy Standard-Model packet.

15.7 How Qi-CON enters the Standard-Model packet

Qi-CON is the multiscale and conformal diagnostic pillar. In a particle-physics context one should not overstate its role, but one should not neglect it either. The Standard Model is full of scale-structured phenomena: logarithmic running, threshold behavior, hierarchy ladders, and approximate scale covariance in certain regimes. Qi-CON provides the runtime language in which those multiscale regularities are expected rather than mysterious.

For the current paper, the most important practical point is that scale behavior is not an accidental side story. It is built into the runtime from the outset, which is exactly why the radial coordinate r can be interpreted as the RG direction later in the manuscript.

15.8 How Qi-HAL enters the Standard-Model packet

Qi-HAL is the most architecture-oriented of the four pillars, and its direct role in a particle-physics parameter paper is therefore subtler than that of Qi-QUA or Qi-EP. Nevertheless, it still has an important interpretive job. Every phenomenological observable is reconstructed by a finite measurement architecture: beamline, target, detector, trigger, reconstruction algorithm, selection cut, and statistical pipeline. That architecture is precisely the kind of memory-bearing observational structure that Qi-HAL was designed to treat in general open-system language.

This is not a rhetorical flourish. It is the reason the present paper is allowed to speak meaningfully about "observed" seam parameters without pretending that those parameters exist as context-free microscopic labels. They are the stable exports of a lawful observational architecture.

15.9 The full GDOS statement of unification in one equation

It is useful to compress the preceding discussion into one symbolic chain:

$$\widehat{\mathsf{SIP}}^{\mathrm{pre}}_{\mathrm{vis}} \xrightarrow{\mathsf{DGI}} \text{admissible gauge/holonomy orbit} \xrightarrow{\mathsf{GSSB}} \text{selected Standard-Model branch} \xrightarrow{\mathsf{GA}} \widehat{\mathsf{SIP}}_{\mathrm{SM}} \xrightarrow{\mathsf{OGGEP/GDOS}} \mathrm{Obs}_{\mathrm{SM}}. \tag{15.4}$$

Equation (15.4) is perhaps the best single-line summary of what this section contributes to the phenomenological paper. It shows that the Standard Model is not simply a group-theoretic remnant or simply a four-dimensional effective Lagrangian. It is a stabilized, projected, open-system packet.

15.10 Why this section matters for the later parameter derivation

One might ask whether the later numerical sections really need all of this runtime structure. The answer is yes, for three reasons.

First, without the unified-packet viewpoint, the gauge/Higgs derivation would look like isolated arithmetic rather than seam selection.

Second, without DGI→GSSB→GA, the emergence of the Standard-Model branch from the unified group would look postulated rather than derived.

Third, without SIP and OGGEP, the later comparison tables would look like a fit to raw data rather than a prediction of observer-accessible boundary quantities.

A professional phenomenological paper should make those logical dependencies visible. The present section does so explicitly.

16 Derivation of the fundamental architecture of the Standard Model from GG-6

The previous sections explained how the Standard Model emerges as a seam packet and why SU(5) is the correct historical anchor for the visible gauge sector. We now address a more concrete question: which structural features of the Standard Model can already be understood before one begins the numerical parameter derivation? The answer is: quite a lot. In particular one can already understand why the relevant low-energy matter content consists of spin-1/2 fermions, why quarks and leptons have the charge assignments they do, why three generations are the first nontrivial family structure, and why the visible gauge sector splits into the particular strong/electroweak/electromagnetic pattern it does.

16.1 Visible matter, family content, and flavor counting — pointer to Part I §5

The textbook account of why visible matter is spin-1/2, why one family contains exactly the doublet/singlet pattern $(Q_L, u_R^c, d_R^c, L_L, e_R^c)$, and why three families give six quark flavors and six lepton flavors, is recorded in Part I §5 (Tables 3–4). The GG-6 derivation of these facts from the bulk Dirac action and the boundary projection sits in Part I §7. The present chapter therefore needs no separate treatment of these basics; the next subsection picks up at the GG-specific charge and baryon-number assignment.

16.2 Electric charges and baryon numbers from group embedding

The charge assignments of the Standard Model look mysterious only if they are listed without their embedding. Once the hypercharge generator is fixed inside SU(5), electric charge follows from $Q = T_3 + Y$. For one family the result is the familiar set

$$Q(u) = \frac{2}{3}, \qquad Q(d) = -\frac{1}{3}, \qquad Q(e) = -1, \qquad Q(\nu) = 0. \tag{16.1}$$

Likewise, local baryon number in the GG-6 framework assigns

$$B(u) = B(d) = \frac{1}{3}, \qquad B(e) = B(\nu) = 0. \tag{16.2}$$

These equations are simple enough to look trivial, but in the present paper they are conceptually important. They show that charge and baryon bookkeeping are not independent empirical tables pasted onto the theory. They descend from the unified algebra and the extra $\mathrm{U}(1)_B$ structure.

Table 12 summarizes the one-family visible content.

Table 12: One-family visible content, with electric charge and baryon number in the GG-6 seam interpretation. The four rows are one family. Electric charge and baryon number are read off the seam assignment rather than postulated, and the last column names the sector each row belongs to. The table records the content the derivation must reproduce; it does not yet derive any of it. The same numbers appear as outputs in the SU(5) embedding of the appendix, which is where a reader who wants them as consequences should go.

Field type	**Representative seam field**	Q	B	**Role in the unified family packet**
Up-type quark	u, c, t	$+2/3$	$1/3$	Color-charged, weakly active, baryonic matter sector
Down-type quark	d, s, b	$-1/3$	$1/3$	Color-charged, weakly active, baryonic matter sector
Charged lepton	e, μ, τ	-1	0	Color-singlet, weakly active, leptonic matter sector
Neutrino	ν_1, ν_2, ν_3 at mass level	0	0	Color-singlet, neutral leptonic sector with Majorana channel

16.3 Why three generations is the first nontrivial family number

The existence of three generations is one of the deepest facts about the Standard Model, and the present paper treats it as a *theorem* rather than an input. The short constitutional reason is that $N_g = 3$ is the smallest family number that simultaneously permits hierarchical masses, nontrivial quark mixing, and a genuine Kobayashi–Maskawa CP phase — one generation has no flavor misalignment, and two allow mixing but no physical CP phase. The full three-leg theorem (MSMF minimality, the GDOS/Hirzebruch index reduced to GG-P2-N3, and codebook self-consistency) is proved in §21, and the constructive flavor arithmetic that exhibits the three family directions explicitly follows in the fermion-sector Part.

16.4 Why the visible gauge sector splits the way it does

We can now summarize the logic of the gauge splitting in one clean chain:

$$\begin{aligned} \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} &\xrightarrow[\text{boundary conditions + holonomy}]{} \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \\ &\xrightarrow[\text{doublet vev, scale set by the Wilson-line selector}]{} \mathrm{SU}(3)_C \times \mathrm{U}(1)_{\mathrm{EM}}. \end{aligned} \tag{16.3}$$

The weak interaction is what remains when the electroweak sector is not probed in its deep-broken asymptotic limit. The photon survives as the generator the doublet's vacuum value leaves unbroken. The heavy W and Z acquire masses from that vacuum value, $M_W = gv/2$, with v set by the Wilson-line selector (the holonomy's own vector gap is ultraviolet, §34.6). The gluons remain massless at the gauge level because color is an unbroken non-Abelian factor, though color confinement then changes the observable long-distance physics drastically.

This is precisely the kind of layered explanation that a more professional phenomenological introduction should provide. It reminds the reader that the three-force structure is not elementary. It is the observed remnant of a sequentially reduced unified connection.

17 Unified geometric origin of the long-range inverse-square forces: gravity and electricity

It is useful to include a simple example of the unified geometric origin of gravity and electric force. This is an excellent request, because it makes the general framework concrete. The purpose of the present section is not to derive all of general relativity or all of electrodynamics from scratch. The purpose is to show why the two classic inverse-square force laws, Newton's and Coulomb's, are structurally similar in a GG-6 setting and how that similarity is naturally understood.

17.1 Why the comparison between Newton and Coulomb is so instructive

Historically, the two laws are among the most iconic equations in physics:

$$F_N = G_N \frac{m_1 m_2}{r^2}, \qquad V_N(r) = -G_N \frac{m_1 m_2}{r}, \tag{17.1}$$

$$F_C = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}, \qquad V_C(r) = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}. \tag{17.2}$$

At an elementary level they look almost identical. The main differences are the sign structure, the source charges, and the overall constants. This formal similarity has long suggested that both are manifestations of a deeper geometric pattern.

In the present framework that suggestion becomes precise: both forces are four-dimensional boundary exports of long-range massless sectors of the composite connection. Gravity comes from the metric/spin-connection sector. Electricity comes from the unbroken $\mathrm{U}(1)_{\mathrm{EM}}$ sector descended from SU(5). The reason the static forces look alike is that both are sourced by Gauss-law flux conservation in three spatial dimensions.

17.2 From GG-6 to 4D general relativity and then to Newton's law

GG-A5-GDOS shows that the metric zero mode of the six-dimensional geometry produces a four-dimensional Einstein–Hilbert action for the boundary metric (Arisaka, 2026, A5-GDOS). Schematically,

$$S^{(4)}_{\mathrm{grav}} = \frac{M^2_{\mathrm{Pl},4}}{2} \int \mathrm{d}^4x \, \sqrt{-g}\, R + S_{\mathrm{matter}}. \tag{17.3}$$

Varying with respect to the metric yields Einstein's equation

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}. \tag{17.4}$$

In the weak-field, static, slow-motion limit one writes

$$g_{00} \simeq -\left(1 + 2\Phi_N\right), \tag{17.5}$$

and Einstein's equation reduces to the Poisson equation

$$\nabla^2 \Phi_N = 4\pi G_N \rho. \tag{17.6}$$

For a point source, the Green's function of the Laplacian in three dimensions is $-1/4\pi r$, so

$$\Phi_N(r) = -G_N \frac{M}{r}, \tag{17.7}$$

which immediately reproduces Newton's potential and inverse-square force.

This derivation is standard in ordinary GR. The GG-6 contribution is to explain why the Einstein–Hilbert sector is itself a boundary export of a larger six-dimensional geometry rather than an autonomous four-dimensional starting point.

17.3 From GG-6 to SU(5) to QED and then to Coulomb's law

The electromagnetic story is parallel. After the unified visible gauge sector is reduced and electroweak symmetry is broken, the unbroken Abelian factor is $\mathrm{U}(1)_{\mathrm{EM}}$. The four-dimensional electromagnetic action is

$$S^{(4)}_{\mathrm{EM}} = -\frac{1}{4e^2} \int \mathrm{d}^4x \, F_{\mu\nu} F^{\mu\nu} + \int \mathrm{d}^4x \, J^\mu A_\mu, \tag{17.8}$$

or, in the more standard convention with a canonically normalized field,

$$S^{(4)}_{\mathrm{EM}} = -\frac{1}{4} \int \mathrm{d}^4x \, F_{\mu\nu} F^{\mu\nu} + e \int \mathrm{d}^4x \, J^\mu A_\mu. \tag{17.9}$$

Varying with respect to A_μ gives Maxwell's equations. In the static limit, the electric potential $\phi = A_0$ satisfies a Poisson equation of the same mathematical type as the gravitational one,

$$\nabla^2 \phi = -\rho_e / \varepsilon_0, \tag{17.10}$$

which for a point source yields

$$\phi(r) = \frac{q}{4\pi\varepsilon_0 r}, \tag{17.11}$$

and therefore Coulomb's law.

Again, the ordinary field-theory derivation is standard. The GG-6 addition is to show why the electromagnetic field should be interpreted as the surviving massless component of a unified connection after symmetry breaking. In other words, Coulomb's law is not just a feature of a fundamental low-energy U(1); it is the infrared face of a broken unified gauge geometry.

17.4 Why the two laws are alike

The reason Newton's and Coulomb's laws are alike can now be stated cleanly. There are three ingredients.

Massless mediator. Both forces are transmitted by a massless field in the relevant low-energy regime: the graviton in the linearized gravity picture and the photon in the unbroken electromagnetic sector.

Gauss-law flux conservation. Both massless fields obey field equations whose static limit is a Poisson problem. In three spatial dimensions the Green's function of the Laplacian is $1/r$, which yields an inverse-square force after differentiation.

Boundary reduction of one composite connection. In GG-6 both sectors are understood as boundary exports of the same deeper geometry–gauge constitution. That does not mean gravity and electromagnetism are identical. It means their formal similarity is no accident.

This reasoning is summarized in Table 13.

Table 13: Why Newton's and Coulomb's laws have the same inverse-square form in the GG-6 interpretation. Read the columns down and the parallel holds at every line: source, mediator, static field equation, potential, force law. The two differ only in sign structure, because one source is a density that cannot be negative and the other is a charge that can. In the GG-6 reading both inverse squares have the same origin in a boundary zero mode of a bulk field, which is why the shared form is a consequence to be checked rather than a coincidence to be explained.

Feature	**Gravity**	**Electromagnetism**
4D field source	Stress–energy / mass density	Electric charge density
Long-range mediator	Metric/graviton zero mode	Photon zero mode
Static field equation	$\nabla^2\Phi_N = 4\pi G_N\rho$	$\nabla^2\phi = -\rho_e/\varepsilon_0$
Potential	$-G_N M/r$	$q/4\pi\varepsilon_0 r$
Force law	Attractive inverse square	Signed inverse square
GG-6 origin	Boundary export of geometry sector	Boundary export of unbroken $\mathrm{U}(1)_{\mathrm{EM}}$ sector

17.5 Why the two laws are also different

The similarity should not tempt us into erasing the differences.

Gravity couples universally to stress–energy, not to a sign-changing charge. Its long-range interaction is therefore generically attractive for ordinary positive-energy matter.

Electromagnetism couples to electric charge, which comes with both signs. It can therefore attract or repel.

The mediators also carry different spin and transformation properties: gravity is a spin-2 geometric field, while electromagnetism is a spin-1 gauge field. GG-6 unifies them constitutionally

as sectors of one composite connection, but it does not collapse them into one identical force at low energies.

This distinction is phenomenologically important. A good unified theory explains formal similarities without denying real differences.

17.6 Connection to the unified geometric origin of mass

The mass-origin paper gives a particularly useful way to see why the present discussion belongs naturally in GG-A9-SM (Arisaka, 2026, A6-GRIP). There mass is treated as a geometric spectral gap, and gravitational response is treated as the metric variation of the same seam action. In that language, gravity is the response of the geometry sector to the energy ledger, while electromagnetism is the response of the gauge sector to the charge ledger. Both are therefore different projections of one deeper constitution.

This is a very satisfactory place to pause before the paper returns to detailed parameter derivation. The point of a professional phenomenological introduction is not only to list targets; it is to teach the reader how to think in the framework. The Newton/Coulomb example does exactly that. It shows how the program turns a historical similarity into a principled geometric statement.

17.7 Bridge to the quantitative sections

The foundation sections above have accomplished the following.

They have explained why the Standard Model is the correct phenomenological target for the present paper.

They have listed the ten open problems and the 28-parameter challenge that make a paper like GG-A9-SM worth writing.

They have reviewed the constitutional and geometric framework from GG-A1-Constitution, GG-A2-GG6, GG-A5-GDOS, GG-A7-Qi4, and GG-A3-BI6.

They have explained how GDOS, GFF, SIP, DGI, GSSB, GA, and the Qi-4 pillars are to be read in a particle-phenomenology context.

They have shown how the historical Georgi–Glashow idea is retained and lifted in GG-6.

They have derived the qualitative Standard-Model architecture before any numerical tables are displayed.

They have illustrated the geometric unification logic with the Newton/Coulomb comparison.

With that groundwork in place, the paper turns to the detailed roadmap and the quantitative sections that carry the derivations; the technical appendices at the end supply their back-up, sector by sector.

18 Phenomenological dictionary: how ordinary Standard-Model language is reinterpreted in GG-6/GDOS

The introduction and foundation sections naturally raise a practical question. If the present paper is written in the language of GG-Theory, GDOS, GFF, SIP, DGI, GSSB, GA, and Qi-4, how

should a phenomenologist trained in ordinary Standard-Model language translate between the two descriptions? This section answers that question explicitly. It bridges the foundation material and the quantitative sections that follow.

18.1 Why a dictionary section is necessary

Professional phenomenology depends on precise vocabulary. When one author says "coupling," another says "renormalized effective charge," and a third says "response coefficient extracted in a given scheme," they may be referring to closely related but not identical objects. The same is true for "mass," "symmetry breaking," "vacuum," "mixing angle," and "observable."

The GG-6/GDOS program does not deny the ordinary Standard-Model meanings of these terms. It refines them. The refinement matters because the present paper is not a conventional fit model. It is a derivation that claims to obtain Standard-Model observables from a more constrained, geometrically motivated, and operationally explicit framework. A clean dictionary therefore prevents two kinds of misunderstanding at once:

(i) it prevents readers from mistaking the GG-6 language for a rejection of the ordinary Standard Model, and

(ii) it prevents readers from mistaking ordinary phenomenological labels for the full theoretical meaning assigned to them in the present framework.

18.2 Gauge couplings: from bare constants to seam conductances

In ordinary Standard-Model language, one usually introduces gauge couplings g_1, g_2, and g_3, or equivalently α_1, α_2, and α_3, and then studies their scale dependence by RG equations. In the present framework, the same observables are reinterpreted as seam conductances or response coefficients exported from a bulk.

The difference is subtle but important. In the ordinary language, one begins with a 4D field theory and then studies how its renormalized couplings run. In GG-6, one begins with a six-dimensional stage and interprets the radial coordinate

$$r = \ln\left(\frac{E_{\mathrm{GUT}}}{\mu}\right) \tag{18.1}$$

as the renormalization direction. The low-energy coupling is then not simply a number attached to a local vertex; it is the seam value of a transported quantity.

This reinterpretation does not change the fact that the final comparison is made to renormalized couplings measured at a reference scale. It changes only the ontological reading of those couplings. They are no longer primitive inputs. They are outputs of transport in the fifth dimension.

18.3 The Higgs sector: from inserted scalar potential to geometric holonomy

In ordinary Standard-Model language, the Higgs sector is written in terms of a scalar doublet H and a potential

$$V(H) = -\mu_H^2|H|^2 + \lambda_H|H|^4. \tag{18.2}$$

The parameters μ_H^2 and λ_H are then adjusted so that the vacuum expectation value v and physical Higgs mass m_h match experiment.

In GG-6 this language is not thrown away, but it is demoted to a low-energy shadow description. The selector of the electroweak scale is the Wilson-line holonomy; the Higgs boson is the fluctuation of the doublet of the bulk $\mathbf{5}_H$ whose scale that holonomy sets on the selected vacuum branch (§23.3). The selector's effective potential is not a primitive input but the seam image of a finite geometric effective functional.

This is why the hierarchy problem is rephrased so strongly in the present paper. If the variable that sets the scale is a holonomy angle rather than a local bare scalar mass, then the large hierarchy is no longer the correct question in its usual form; the doublet's own radiative stability remains, and is O_{11}. What matters for the scale is the selected branch of the finite holonomy potential and its redshift to the lock seam.

18.4 Masses: from arbitrary Yukawa numbers to overlap spectra

In ordinary Standard-Model language, fermion masses are generated from Yukawa matrices. The theory tells us how those matrices enter observables, but it does not tell us why the entries take the values they do. That is why the Yukawa sector is often treated as the most arbitrary part of the Standard Model.

In the present framework the same low-energy Yukawa language is retained as an effective boundary description, but the underlying origin is changed. Effective Yukawa couplings are interpreted as overlap integrals of wavefunctions or projectors in the bulk and on the seam. This shift in viewpoint is the reason the later sections can derive masses from dyadic overlaps, compact-angle winding, and family projectors without introducing twelve independent Yukawa-fit constants.

The key phenomenological point is that a mass table in GG-A9-SM is not the output of twelve unrelated empirical insertions. It is the boundary spectrum of a single geometric family codebook.

18.5 Mixing angles: from free unitary matrices to projector misalignment

The ordinary Standard Model and its neutrino-sector extensions parameterize quark and lepton mixing by unitary matrices. This is the correct low-energy formalism, but it leaves a natural question unanswered: why do these matrices take the forms they do?

GG-6/GDOS does not replace the CKM and PMNS matrices with something unrelated. It provides an origin story for them. In the present manuscript they are treated as misalignment data between already-fixed family projectors. The quark sector is built from the mismatch between up-type and down-type projector structures. The lepton sector is built from the mismatch between charged-lepton and neutrino projector structures.

This reinterpretation is especially important for professionalism of presentation. It allows one to say clearly what the paper is and is not claiming. The paper is not claiming that unitary mixing matrices are the wrong language of observables. It is claiming that the values of those observable matrices can be traced back to more primitive geometric ingredients.

18.6 Vacuum selection: from postulated broken phase to GSSB event

In ordinary phenomenological discussions, one often begins directly in the broken phase because that is the phase in which measurements are performed. In the present paper the broken phase is not the starting point. It is the selected output of the GSSB step.

This matters because it clarifies why the Standard Model appears as it does. The gauge group, Higgs expectation value, and flavor pattern are not three independent "facts." They are correlated consequences of one selected vacuum branch. The reader can therefore interpret every later numerical table as a report on the same branch, not as a collection of sector-by-sector coincidences.

18.7 Observables: from numbers in a review table to registered packet exports

A benchmark value in the PDG or in a neutrino global fit is often treated in the literature as if it were the theory-independent "thing itself." That practice is completely reasonable for most day-to-day phenomenology. But the present paper aims at a deeper level of explanation. It therefore insists that every benchmark number compared at the end of the derivation is a registered export of a specific packet under a specific measurement protocol.

This is not meant to relativize or weaken the data. On the contrary, it sharpens the meaning of comparison. A prediction is not complete until the paper has specified which observable basis it is matching to. That is why GG-A9-SM uses observable-basis tables and defines Dev and Pull only after the theory-side construction is complete.

18.8 A compact translation table

Table 14 collects the key entries of the translation dictionary in compact form.

Table 14: Compact dictionary between ordinary Standard-Model phenomenological language and the GG-6/GDOS interpretation used in the present manuscript. The third column is the point of the table. A dictionary that only paired terms would be a change of vocabulary; what matters is why each reinterpretation earns its place — a gauge coupling becomes a transport problem, the Higgs field becomes a holonomy fluctuation — and in each case something that was an input acquires an address in the geometry. The entries are pointers into the Parts that do the work, not arguments in themselves.

Ordinary SM language	**GG-6/GDOS reinterpretation**	**Why the reinterpretation matters**
Gauge coupling	Seam conductance / transported radial response coefficient	Makes RG running a geometric transport problem rather than an external input.
Higgs field	required, MSMF-selected $\mathbf{5}_H$ doublet; Wilson-line holonomy as branch selector	Replaces a tuned scalar potential by a geometric branch selection; the field is the minimal representation closing the Yukawa sector.

Ordinary SM language	**GG-6/GDOS reinterpretation**	**Why the reinterpretation matters**
VEV v	Lock-seam scale export	Solves hierarchy kinematically before low-energy fitting.
Fermion Yukawa entry	Overlap or projector coefficient	Compresses flavor freedom into a geometric codebook.
CKM/PMNS matrix	Family-projector misalignment matrix	Turns mixing into derived geometry rather than independent matrix fitting.
Vacuum in broken phase	GSSB-selected branch of the packet	Correlates gauge, Higgs, and flavor data as properties of one branch.
Mass	Seam spectral gap in GFF language	Unifies inertial and observable mass bookkeeping.
Experimental observable	Registered packet export under OGGEP/GDOS	Prevents hidden confusion between bare and renormalized quantities.

18.9 Why the dictionary matters for phenomenological precision

A polished phenomenological paper should teach the reader how to move between levels of description. If it stays entirely in the UV language, it becomes too abstract to evaluate. If it stays entirely in low-energy Standard-Model language, it becomes impossible to see what the deeper theory is contributing. The present dictionary section is therefore not ornament. It is one of the places where the paper becomes genuinely readable across subfields.

It also clarifies the status of the quantitative sections that follow. Those sections do not ask the reader to abandon familiar Standard-Model phenomenology. They ask the reader to reinterpret familiar Standard-Model observables as outputs of a deeper, more restricted geometric and operational framework.

18.10 Immediate bridge to the roadmap section

After this dictionary is in place, the rest of the manuscript can proceed much more smoothly. The roadmap section that follows is then read not as a sudden jump from abstract principles to arithmetic, but as a natural continuation of a fully explained phenomenological and constitutional setup.

19 GG-6 bulk geometry and the three fundamental scales

19.1 Minimal admissible stage

The bulk stage derived in GG-A2-GG6 is

$$M_6 = M_4 \times I_r \times S^1_\theta, \tag{19.1}$$

with scale-profile metric

$$\mathrm{d}s_6^2 = e^{-2r}\,\eta_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 + \mathrm{d}\theta^2, \qquad r \in [0, K], \quad \theta \in [0, 2\pi), \tag{19.2}$$

and the logarithmic radial coordinate

$$r = \ln\!\left(\frac{E_{\mathrm{GUT}}}{\mu}\right). \tag{19.3}$$

The derivation of this geometry in GG-A2-GG6 is constitutional. Four coordinates are fixed by the operational boundary itself. One extra coordinate is needed to geometrize renormalization scale, and one compact coordinate is needed to support primitive Abelian holonomy. Anything smaller than 6D is insufficient; anything larger is not forced and would violate MSMF unless independently motivated (Arisaka, 2026, A2-GG6; Arisaka, 2026, A5-GDOS).

19.2 Composite connection and six-force unification

On this stage the composite connection is

$$\mathbb{A} = \omega \oplus A_5 \oplus A_B \oplus A_{Qi}, \tag{19.4}$$

with bulk symmetry

$$G_{\mathrm{Bulk}} \;=\; \mathrm{Spin}(1,5) \;\times\; \mathrm{SU}(5) \;\times\; \mathrm{U}(1)_B \;\times\; \mathrm{U}(1)_{Qi}. \tag{19.5}$$

This is the ultraviolet starting point. The present paper does not yet need the full 6D QFT machinery of GG-A2-GG6; it needs only the facts that (i) the unified group contains a minimal SU(5) block, (ii) there are two equally weighted Abelian sectors entering BI-6, and (iii) all of these are carried by the same composite connection.

19.3 OGN-5 ledger and throat depth

The OGN-5 arithmetic package is

$$(M, N, L; J, K) = (1, 3, 5; 7, 35), \tag{19.6}$$

with the derived relations

$$J = 2N + 1 = 7, \qquad K = LJ = 35, \qquad K = M^2 + N^2 + L^2 = 35. \tag{19.7}$$

In GG-A2-GG6 this ledger controls the finite throat through

$$0 \le r \le r_*, \qquad kr_* = K = 35, \qquad \exp[-A(r_*)] = \mathrm{e}^{-35}. \tag{19.8}$$

Thus the hierarchy across the throat is not a continuous modulus but a discrete exponential set by the master lock integer K (Arisaka, 2026, A2-GG6).

The physical interpretation used throughout this paper is summarized in Table 15.

Table 15: Physical interpretation of the OGN-5 integers as used in the present gauge/Higgs, fermion-mass, and CKM derivation. The five integers $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ are the entire integer content of the derivation, and this table says what each one counts. They are fixed in the foundation companions and are not adjustable here: every closed form in the derivation Parts is built from them together with π and e. A reader auditing a formula should be able to point at each integer in it and name the row it came from.

Symbol	Value	Interpretation in the present paper
M	1	Primitive compact-holonomy count: one minimal S^1_θ cycle.
N_c	3	Triplicity index of the observable boundary sector.
L	5	Minimal non-Abelian unification index, namely SU(5).
J	7	Minimal odd closure index, governing dyadic electroweak closure.
K	35	Master lock integer controlling hierarchy, throat depth, and next-order suppression.

19.4 Derivation of E_{Pl}, E_{GUT}, M_{lock} — pointer to A2-GG-6 §5 + T6

The end-to-end derivations of the three fundamental scales — the Planck energy $E_{\mathrm{Pl}} = \sqrt{\hbar c^5/G_N} \approx 1.22 \times 10^{19}$ GeV from the GU-5 of GG-A1-Constitution T-GU5; the GUT scale $E_{\mathrm{GUT}} = E_{\mathrm{Pl}}\, e^{-2\pi} \approx 2.28 \times 10^{16}$ GeV from one S^1_θ traversal of the 6D bulk; and the lock scale $M_{\mathrm{lock}} = \frac{1}{2} M_{\mathrm{Pl}}\, e^{-K} \approx 3.85$ TeV from the OGN-5 closure $K = LJ = 35$ — are owned in full by A2-GG-6 §5 + T6 (Arisaka, 2026, A2-GG6). The boxed values are reproduced in the scale chain (10.4) above. The next subsection picks up the A9-specific reading: that this scale chain already resolves the hierarchy problem at the kinematic level, before any electroweak fine-tuning question can be posed.

19.5 Why this already solves the hierarchy problem at the kinematic level

The conventional hierarchy problem asks why the electroweak scale is so small compared with the Planck scale. Inside GG-6 the question is reformulated. One no longer compares two unrelated constants; one asks what the redshift is between two constitutionally distinguished seams of one bulk geometry.

Using (29.4), the ratio is

$$\frac{M_{\rm lock}}{E_{\rm Pl}} = \frac{1}{2e^{35}} \simeq 3.152 \times 10^{-16}. \tag{19.9}$$

Thus the large hierarchy is simply the exponential image of the master lock integer $K = 35$, plus the minimal binary endpoint factor. There is no continuous modulus to adjust, and therefore there is no fine-tuning problem at this stage. The kinematic hierarchy has been replaced by a discrete geometric one.

20 The GDOS calculational seams: where the gauge and Higgs observables are read off

Why an operational layer must precede any numerical prediction — that a coupling, a mass, or a mixing angle is a *registered boundary observable* rather than a bare Lagrangian coefficient — was established in §12, and the runtime cascade that selects the Standard-Model branch was walked in §15. Here we record only the two structural facts about GDOS that the gauge/Higgs calculation actually reads off: its coarse-grained evolution law, and the stationary *seams* at which observables are assigned definite values.

20.1 Exact and coarse-grained GDOS structure

At the coarse-grained mesoscopic level, GG-A5-GDOS writes the generic GDOS evolution as the continuity equation

$$\partial_t p = -\nabla_A J^A, \tag{20.1}$$

with current decomposition

$$J^A = -D^{AB}\nabla_B p + b^A p = -D^{AB}\nabla_B p + (-g^{AB}\nabla_B \Phi + h^A)p, \qquad \nabla_A h^A = 0. \tag{20.2}$$

This is already sufficient for the purposes of the present paper. It says that every admissible open-system evolution decomposes into three pieces: diffusion, gradient relaxation, and a solenoidal circulation.

The corresponding entropy-production statement is

$$\dot{S}_{\rm tot} = \int \mathrm{d}\mu \, \frac{J_A (D^{-1})^{AB} J_B}{p} \geq 0, \tag{20.3}$$

so the admissible flows are automatically thermodynamically well-behaved (Arisaka, 2026, A5-GDOS).

For the gauge/Higgs sector the physical meaning is direct: the observed boundary vacuum is not an arbitrary algebraic choice. It is a stationary or attractor configuration of a lawful open-system flow.

20.2 The GFF ledger

The scalar bookkeeping principle behind GDOS is the GFF ledger introduced in GG-A1-Constitution. In its most useful explicit form,

$$F_{\mathsf{GFF}}[\rho] = \int_{\mathcal{M}} \rho(y)\,\Phi(y)\,\mathrm{d}\mu + \Theta \int_{\mathcal{M}} \rho(y) \ln\!\left(\frac{\rho(y)}{\rho_{\mathrm{ref}}(y)}\right) \mathrm{d}\mu. \tag{20.4}$$

The two blocks — the energy–geometry–gauge part GEP and the curvature–entropy part CEP, defined in §12 — combine as

$$\mathsf{GFF} = \mathsf{GEP} + \mathsf{CEP}. \tag{20.5}$$

Its functional derivative is

$$\frac{\delta F_{\mathsf{GFF}}}{\delta \rho} = \Phi + \Theta \left(\ln \frac{\rho}{\rho_{\mathrm{ref}}} + 1 \right), \tag{20.6}$$

and in the pure-gradient GDOS sector one has

$$J^A = -\rho M^{AB} \nabla_B \left(\frac{\delta F_{\mathsf{GFF}}}{\delta \rho} \right), \qquad D^{AB} = \Theta M^{AB}. \tag{20.7}$$

Then

$$\frac{\mathrm{d}}{\mathrm{d}t} F_{\mathsf{GFF}}[\rho(t)] = -\int \rho\, \nabla_A \left(\frac{\delta F_{\mathsf{GFF}}}{\delta \rho} \right) M^{AB} \nabla_B \left(\frac{\delta F_{\mathsf{GFF}}}{\delta \rho} \right) \mathrm{d}\mu \le 0, \tag{20.8}$$

so the ledger is a Lyapunov functional in the relaxational sector (Arisaka, 2026, A1-Constitution; Arisaka, 2026, A5-GDOS).

20.3 What this means for gauge and Higgs observables

The practical message of (20.8) is extremely simple: the numbers compared to experiment should be assigned at stationary seams of the effective boundary description.

We introduce two names for the two seams needed in the present paper.

- The **lock seam** $\mathcal{L}_{\mathrm{lock}}$ is the boundary image of the finite-throat endpoint at $\mu \sim M_{\mathrm{lock}}$. It is the natural place to organize the Higgs/Wilson-line descent.
- The **neutral electroweak seam** $\mathcal{Z}_{\mathrm{seam}}$ is the boundary stationary point at which the neutral-current rotation is evaluated and the photon remains exactly massless. In phenomenological language, this is the seam associated with the usual Z-pole comparison scale.

Neither term is imported from outside the preprints; they are simply a convenient language for the stationary points forced by AX-3 and GDOS.

Thus the gauge/Higgs, fermion-mass, and CKM derivation in this paper will proceed in two stages. First, geometry and OGN determine the ultraviolet and lock structures. Second, GDOS/GFF tells us at which effective seam the corresponding low-energy couplings must be read off.

20.4 GFF interpretation of vacuum selection

In the Higgs sector, the vacuum expectation value is the stationarity condition of an effective boundary ledger, not a mysterious insert. Writing the order parameter schematically as φ_H, the boundary-effective evolution takes the generic form

$$\partial_t \varphi_H = -\Gamma_H \frac{\delta F_{\mathsf{GFF}}}{\delta \varphi_H} + \xi_H, \tag{20.9}$$

where $\Gamma_H > 0$ is a mobility kernel and ξ_H denotes the small open-system noise inherited from the coarse-grained projection. A stationary vacuum therefore satisfies

$$\frac{\delta F_{\mathsf{GFF}}}{\delta \varphi_H} = 0. \tag{20.10}$$

When the Higgs sector is written in ordinary four-dimensional language, this stationarity condition becomes the familiar minimization of the effective scalar potential. But the present formulation is more fundamental because it also explains *why* a vacuum selection principle exists at all: it is the gradient-attractor structure of the GDOS boundary runtime.

21 The family number $N_g = 3$ as a three-leg theorem

21.1 Historical background

The persistence of three fermion generations is among the most stubborn unexplained empirical facts of the Standard Model. The LEP measurement of the invisible Z width fixes the number of light, weakly interacting neutrino species at $N_\nu = 2.984 \pm 0.008$, consistent with three and inconsistent with two or four at the multi-σ level; cosmological probes from primordial nucleosynthesis and the cosmic microwave background give an independent and equally tight bound. Yet within the SM itself the family number is left as a free input: nothing in $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ requires three replications rather than one or seventeen. Kobayashi and Maskawa observed in 1973 (Kobayashi & Maskawa, 1973) that $N_g \geq 3$ is the smallest number of generations admitting a physical CP-violating phase, but this gives only a lower bound. Calabi–Yau compactifications, magnetized toroidal models, $\mathbb{Z}_3$ orbifolds, and noncommutative-geometry constructions each provide partial answers, but no closed-system framework has succeeded in deriving *both* an upper and a lower bound on N_g from first principles.

In GG-Theory the question has an answer with every fence named. The dedicated companion preprint (Arisaka, 2026, P2-N3) establishes that $N_g = 3$ is uniquely forced by three logically independent legs of the framework: an MSMF lower bound from CP-violation combinatorics, a GDOS Morse upper bound from family-projector stability on $\mathbb{C}P^{N_g-1}$, and a constructive codebook check that reproduces all twelve fermion masses (nine charged, three neutrino), at the few-per-cent level and the gauge and charged-lepton entries to four significant figures, only at $N_g = 3$. The present section records the statement of that theorem and the structural reading needed for the parameter derivations of the remainder of this paper.

21.2 Why $N_g = 3$ enters before the parameter calculation

A consequence paper that sets out to derive the 28 parameters of the SM cannot treat the family number as an empirical input without circularity. Indeed, N_g enters every Yukawa formula in the codebook (see Sections 47–49), every flavor mixing matrix (Sections 52–54), and the OGN-5 Diophantine ledger that anchors the entire framework. If N_g were fitted from the lepton spectrum, the codebook would be a numerical exercise rather than a derivation. The three-leg theorem of $N_g = 3$ in (Arisaka, 2026, P2-N3) removes this circularity: N_g is a forced integer of the six-dimensional bulk geometry, its value pinned by the same constitutional clauses that fix the BI-6 vector and the OGN-5 ledger. After all, what was historically considered an inscrutable empirical accident becomes a derived integer once the underlying geometry is correctly identified.

21.3 Statement of the three-leg theorem

Theorem 21.1 (**T-SM-21.1**: Conditional derivation of $N_g = 3$; Arisaka, 2026, P2-N3 T-N3-5)**.** *Let N_g denote the family number in the OGN-5 ledger*

$$(M, N, L; J, K) = (1, 3, 5; 7, 35), \qquad K = LJ = M^2 + N^2 + L^2, \tag{21.1}$$

of GG-Theory (Arisaka, 2026, A2-GG6; Arisaka, 2026, A3-BI6). Under

1. *BI-6 closure on the six-dimensional bulk (Arisaka, 2026, A3-BI6),*
2. *MSMF (Axiom AX-4) of the GG-Constitution (Arisaka, 2026, A1-Constitution),*
3. *The capacity of the family stratum (the coupling clause with the integrality theorem L-INT (residue R-INT), per Arisaka, 2026, P2-N3; its rigidity lemma closes in the negative the derivation from BI-6 positivity recorded as open problem O3 of Arisaka, 2026, A5-GDOS, and its capacity theorem meets the N_g-independence demand of open problem M10 of Arisaka, 2026, A6-GRIP),*
4. *The minimal first-order Yukawa codebook of the present paper (Sections 47–49),*

together with the empirical inputs

- *Jarlskog CP-violation $\mathcal{J}_{\mathrm{CKM}} \neq 0$ (PDG),*
- *Charged-lepton mass ratios $m_\tau/m_\mu = 16.817$ and $m_\mu/m_e = 206.768$ (PDG),*

the family number is uniquely

$$\boxed{N_g = 3.} \tag{21.2}$$

Remark 2 (Physics intuition for Theorem 21.1)**.** The number 3 in $N_g = 3$ is not selected; it is forced by three constraints, along logically independent directions, that intersect on the single integer. Indeed, this is the GG-Theory pattern: an empirical accident reduces, under sufficient geometric structure, to a number-theoretic identity. The lower bound says "CP violation is real, so N_g cannot be smaller than three"; the upper bound says "the cascade of unstable family-projector saddles has a finite instanton-action budget that closes only for N_g at most three"; the codebook check

says "the empirical mass ratios independently single out $N_g = 3$ at the 10^{-3} level in the gauge and charged-lepton entries". No leg by itself is sufficient. Together they force $N_g = 3$ exactly and validate the value with twelve quantitative mass predictions.

Remark 3 (Status: what is rigorous, what is conditional, and which "three")**.** Honesty (the L6 discipline) asks two distinctions to be kept sharp. *First*, the integer derived here is the *generation* number N_g; that the OGN-5 ledger's second component also equals 3 is a separate and *unconditional* result — the dual lock with the block embedding, Theorem T-BI6-10.2a of Arisaka (2026, A3-BI6), checked against the BI-6 trace ratio $(L - N)/L = 2/5$ — reached from premises that mention no generation count. The two are distinct quantities: the color index N_c is a ledger member and the generation count N_g is not, and neither derivation shares a premise with the other. That both land on 3 is therefore a result rather than an identification, and Arisaka (2026, P2-N3) sets out in its own discussion what would have to be shown — a map from the ledger to the family stratum, or back — to make their meeting a theorem. Nothing in the present paper rests on it. *A note on the three symbols.* The cohort writes N_g for the color index, which is the OGN-5 ledger's second entry; N_g for the generation count, which is not a ledger member; and N_f for the number of quark flavors in a beta function, six at high scale. This paper prints all three, and prints them apart wherever they appear, so that a reader can see at a glance which of the three is doing the work. The ledger formulas — $\sin^2\theta_W = L^2/(4N^3)$, $\lambda_H = 2^{-N}$, $v = M_{\text{lock}}(2/L)^N$, $\alpha^{-1}(0) = 2^J + N^2$, and the dual lock $K = LJ = M^2 + N^2 + L^2$ — read the ledger's second entry N, which is that color index and is a ledger member. The family-stratum formulas — the cascade closure $S_{\text{tot}}(N_g)$ on $\mathbb{C}P^{N_g-1}$, the Yukawa prefactors ($y_\tau \propto N_g J/2^{J+N_g+1}$ and its relatives), and the lepton-mixing weights $L^2/3N_g^3$ and $N_g/4K$ — read the generation count, which is not. That is what makes the mass codebook a check on the generation number rather than on the ledger. No value, exponent or grade changes with the relabeling: the two integers are equal. What changes is that the reader can now tell, formula by formula, which of them a result rests on — and can confirm that no electroweak row rests on N_g, and so that nothing in the gauge–Higgs sector inherits the conditional status of the family pincer.

Second, within the generation pincer the genuinely new and fully rigorous step is topological: the even-N_g exclusion of Leg L2 is the identity $a^+(N_g) - a^-(N_g) = \sigma(\mathbb{C}P^{N_g-1})$ (the Hirzebruch signature), which equals 1 for odd N_g and 0 for even N_g — independent of any conjecture. The exclusion of the surviving odd values $N_g \geq 5$ rests on the capacity theorem T-N3-6 of Arisaka (2026, P2-N3) ($\text{Cap}(N_g) = \pi N_g$, derived from the stratum's quantum geometry, conditional on the coupling clause and the integrality theorem L-INT (residue R-INT); the N_g-independence demand of Arisaka, 2026, A6-GRIP's open problem M10 is met, the chain consuming neither $K = 35$ nor $J = 7$), and the lower bound $N_g \geq 3$ rests on the measured reality of CP violation. "Closed three-leg theorem" should therefore be read as *a conditional pincer that reduces the family number to one named topological residue* (R-INT, of the integrality theorem L-INT)*, premises shared with the Constitutional Born Theorem, and one robust empirical fact* — a sharply localized residue, not an unconditional proof. The dedicated companion (Arisaka, 2026, P2-N3) carries the conditioning in full.

Source. The complete proof is in the dedicated companion preprint (Arisaka, 2026, P2-N3); the present section records only the statement and the structural reading needed for the parameter derivations of the remainder of this paper. Each of the three legs is summarized in turn below.

21.4 Leg L1: MSMF lower bound $N_g \geq 3$ from CP combinatorics

The first leg is the Kobayashi–Maskawa argument elevated to a theorem under MSMF. A general $N_g \times N_g$ unitary CKM matrix has N_g^2 real parameters: $N_g(N_g-1)/2$ mixing angles, $N_g(N_g+1)/2$ phases. Subtracting $2N_g - 1$ unphysical rephasings (the N_g up-type and N_g down-type fermion phase redefinitions, modulo the global U(1)) leaves

$$n_{\text{phase}}(N_g) = \frac{(N_g-1)(N_g-2)}{2} \tag{21.3}$$

physical CP-violating phases. A non-zero Jarlskog invariant $\mathcal{J}_{\text{CKM}} = \text{Im}(V_{ij}V_{kl}V_{il}^*V_{kj}^*) \neq 0$ exists iff $n_{\text{phase}}(N_g) \geq 1$, which gives $N_g \geq 3$. Empirically $\mathcal{J}_{\text{CKM}} = (3.12^{+0.13}_{-0.12}) \times 10^{-5}$ (PDG 2024) at multi-σ significance. By MSMF the OGN-5 ledger selects the smallest admissible value, which is $N_g = 3$. Indeed, the same argument applies independently in the leptonic sector via PMNS, with global fits favoring a non-zero leptonic Jarlskog $\mathcal{J}_{\text{PMNS}} \approx -0.026$ (NuFIT), so even if one half of the empirical input were retracted the lower bound would survive.

21.5 Leg L2: GDOS Morse upper bound $N_g \leq 3$ on $\mathbb{C}P^{N_g-1}$

The second leg is genuinely new and sets the present derivation apart from the historical Kobayashi–Maskawa argument. It uses the open-system GDOS framework (Arisaka, 2026, A5-GDOS) together with Morse theory on the family-projector moduli space.

A boundary family-projector configuration is a rank-1 projector $P_{[\psi]} = |\psi\rangle\langle\psi|/\langle\psi|\psi\rangle$ in the N_g-dimensional family Hilbert space. Its moduli space is

$$\mathcal{P}_{N_g} = \mathbb{C}P^{N_g-1}, \qquad \dim_{\mathbb{R}} \mathcal{P}_{N_g} = 2(N_g-1), \tag{21.4}$$

the complex projective space, equipped with its canonical Fubini–Study metric. The Geometric Free Functional (Pillar 3 of Gi-4, Arisaka, 2026, A6-GRIP) restricted to family-projector data is the height function

$$f_{\mathsf{GFF}}([\psi]) = \frac{\sum_{a=1}^{N_g} \lambda_a |\psi_a|^2}{\sum_a |\psi_a|^2}, \qquad \lambda_1 < \lambda_2 < \cdots < \lambda_{N_g}. \tag{21.5}$$

By the classical theory of Morse functions on $\mathbb{C}P^{N_g-1}$ (Milnor, 1963), f_{GFF} is a perfect Morse function with N_g non-degenerate critical points $[0,\ldots,0,1,0,\ldots,0]$ (the 1 in position a) of Morse index $\text{ind}_a = 2(a-1)$. The Morse polynomial is

$$\mathcal{P}(\mathbb{C}P^{N_g-1})(t) = \sum_{k=0}^{N_g-1} t^{2k} = 1 + t^2 + t^4 + \cdots + t^{2(N_g-1)}. \tag{21.6}$$

Strict GDOS Morse-positivity (held as a consistency-grade import in Arisaka, 2026, P2-N3, where the exclusion itself is carried by the capacity theorem; see the next subsection) requires the alternating sum

$$a^+(N_g) - a^-(N_g) > 0, \qquad a^\pm(N_g) = \#\{c : \mathrm{ind}(c) \equiv 0, 2 \,(\mathrm{mod}\, 4)\} \text{ or } \equiv 1, 3 \,(\mathrm{mod}\, 4)\}. \tag{21.7}$$

For the index distribution above this evaluates to $\sigma(\mathbb{C}P^{N_g-1}) = 1$ if N_g odd and 0 if N_g even, so strict positivity *eliminates all even N in one stroke*.

The remaining odd values are constrained by cascade closure. The GRIP cascade DGI $\to$ GSSB $\to$ GA on $\mathbb{C}P^{N_g-1}$ is a strict descent

$$[2(N_g - 1)] \to [2(N_g - 2)] \to \cdots \to [2] \to [0] \tag{21.8}$$

through the index ladder, governed by gradient flow of f_{GFF} in the Fubini–Study metric. The instanton action between adjacent saddles of indices $2k$ and $2(k-1)$ saturates the Bogomolnyi bound at $S_{[2k]\to[2(k-1)]} = \pi k$ (Witten, 1982). Summing over the cascade gives the total action

$$S_{\mathrm{tot}}(N_g) = \pi \sum_{k=1}^{N_g-1} k = \pi \cdot \frac{N_g(N_g-1)}{2}. \tag{21.9}$$

The BI-6 cascade-closure budget on the six-dimensional bulk caps this at

$$S_{\mathrm{BI}}^{\max} = \frac{2\pi N_g}{L} \cdot \frac{L}{2} = \frac{2\pi \cdot 3}{5} \cdot \frac{5}{2} = 3\pi \tag{21.10}$$

— a value delivered by the capacity theorem T-N3-6 of Arisaka (2026, P2-N3) (its Appendix C records the two viable formulations; Arisaka, 2026, A3-BI6 quotes the value in its cross-reference ledger), with all integers fixed by OGN-5. Read per family, the same expression is $S_{\mathrm{BI}}^{\max}(N_g) = \pi N_g$ — π of relaxation capacity per family — under which cascade closure is the pure combinatorics $N_g(N_g-1)/2 \le N_g \iff N_g \le 3$ (Conjecture C.1 of Arisaka, 2026, P2-N3). For the surviving odd values $N_g \in \{1, 3, 5, 7, \dots\}$:

$$S_{\mathrm{tot}}(1) = 0, \qquad S_{\mathrm{tot}}(3) = 3\pi \text{ (exactly saturates)}, \qquad S_{\mathrm{tot}}(5) = 10\pi > 3\pi, \quad \text{etc.} \tag{21.11}$$

Cascade closure therefore admits only $N_g = 1$ and $N_g = 3$ among odd values. Combined with $N_g \ge 3$ from §21.4, this leaves $\boxed{N_g = 3}$ as the unique admissible family number.

That $N_g = 3$ *exactly saturates* the cascade-closure bound (21.10) is the GG-Theory analog of criticality: the family number sits on the edge of admissibility, neither too small (no cascade exists) nor too large (cascade fails to close within the BI-6-permitted instanton budget).

Remark 4 (Physics intuition: reachability, and why a fourth family is unaffordable)**.** The dynamical reading of the upper bound (Arisaka, 2026, P2-N3, §7.5) joins this section to the GRIP-In doctrine of the present paper. GFF-ST7 guarantees every stratum a unique Ground Geometric Attractor, and the weak interaction is the road by which matter reaches it — the physical implementer of GRIP-In on the matter strata. Descending the full family ladder costs the triangle number of GSSB

gate tolls, $\pi N_g(N_g-1)/2$; the stratum funds π per family, πN_g in all. For $N_g \geq 4$ the toll exceeds the capacity: the ground state becomes dynamically unreachable, and matter would strand forever in non-ground basins, violating the constitutional ground-state guarantee. The bound is $N_g < 4$ not because a fourth family's spectrum would be internally inconsistent, but because its relaxation road could not be paid for; the third generation is the last family the universe can afford, and the observed $\tau \to \mu \to e$ chain is the visible execution of the descent that exactly spends the budget.

21.6 Reduction of L2 to a Hirzebruch calculation — pointer to GG-P2-N3

The parity route reducing strict positivity to a single Hirzebruch-type signature calculation, together with the capacity architecture that replaced its would-be hypotheses — the rigidity obstruction, the Berry-transport theorems at Born-grade, and the integrality theorem L-INT (residue R-INT) — the curvature-descent coefficient now fixed by the Fubini–Study conventions rather than pending the induced-connection construction, the naive scale-profile metric evaluation being proved to vanish identically — is owned in full by GG-P2-N3 (Arisaka, 2026, P2-N3). The conclusion that we use here is the upper bound $N_g \leq 3$, which combines with Leg L1 ($N_g \geq 3$ from the Kobayashi–Maskawa argument) to fix $N_g = 3$ uniquely. The next subsection records the Leg L3 constructive check: the $N_g = 3$ codebook reproduces the empirical Standard-Model spectrum at the 10^{-3} level in the gauge and charged-lepton entries and at the few-per-cent level across the twelve-mass block.

21.7 Leg L3: codebook self-consistency

The third leg is constructive: it shows that the explicit fermion-mass codebook of the present paper requires $N_g = 3$ and is incompatible with any other value. The minimal first-order charged-lepton ratios depend on N_g algebraically as

$$\frac{m_\tau^{(1)}}{m_\mu^{(1)}}(N_g) = \frac{N_g J}{L} \cdot 2^{L-N_g}, \qquad \frac{m_\mu^{(1)}}{m_e^{(1)}}(N_g) = \frac{L^4}{N_g}, \tag{21.12}$$

with $J = 7$ and $L = 5$ fixed by BI-6 closure of OGN-5. Evaluating for $N_g \in \{2, 3, 4\}$:

N_g	m_τ/m_μ	m_μ/m_e
2	22.4	312.5
3	**16.8**	**208.33**
4	11.2	156.25
Exp.	16.817	206.768

(21.13)

The values $N_g = 2$ and $N_g = 4$ miss observation by 33% and $\geq 24\%$; only $N_g = 3$ matches both ratios, and it matches them to within a percent (16.8 against 16.817, 208.33 against 206.768). The full codebook (Sections 47–49 below) extends this to all twelve fermion masses, with QD-completion factors $\mathcal{R}_\tau = 637/640$, $\mathcal{R}_\mu = 159/160$, $\mathcal{R}_e = 641/640$ bringing the charged-lepton predictions to the 10^{-3} level against the PDG values. Indeed, the codebook is not merely compatible with $N_g = 3$; it is built around $N_g = 3$ in a non-trivial, predictive way.

21.8 Implications for the rest of the paper

Theorem 21.1 converts every appearance of "$N_g = 3$" or "three generations" in the remainder of GG-A9-SM from an empirical input into a derived theorem — conditional in the precise sense recorded in the status remark above. The OGN-5 ledger entry $N_g = 3$ is no longer fitted from the lepton spectrum but forced by the intersection of the three legs above. In particular:

- Every Yukawa formula in Sections 47–49 treats $N_g = 3$ as a structural input from (Arisaka, 2026, P2-N3), not as a fitted constant.
- The CKM and PMNS sections (Sections 52–54) inherit the three-generation structure as a consequence of Leg L1 + Leg L2.
- The angular cascade $1, e^{-\pi}, e^{-2\pi}, e^{-3\pi}, e^{-4\pi}$ in the quark codebook is the instanton-cost spectrum of the cascade chain (21.8) on $\mathbb{C}P^2$, with each $e^{-n\pi}$ accumulating π of Bogomolnyi action per step (see §49).
- The QD-completion rationals (denominators traceable to $2^J L = 640$ in OGN-5) are the Hopkins–Singer descent integrality coefficients of X_4 on the relevant boundary stratum (see Section 49 for the explicit expansion).

Remark 5 (Why this proof required open-system thinking)**.** The structural reason the three-generation puzzle has remained open for the century since the foundation of quantum mechanics is now visible. Closed-system methods — anomaly cancellation, integrality, index theorems — can constrain integer multiplicities from below but cannot bound them from above, because all unitary states of a closed system are equally admissible by symmetry; there is no intrinsic notion of an "unstable" or "decaying" state from which to derive a stability budget. Upper bounds on integer multiplicities require open-system dynamics: irreversible boundary projection, finite instanton-action budget, cascade closure on a moduli of attractors. This is exactly what GDOS provides, and only GDOS provides. Indeed, the present derivation of $N_g = 3$ is the first concrete demonstration that the open-system shift has the foundational power to derive structural integers that closed-system methods cannot reach. After all, $N_g = 3$ is not fixed by an additional axiom but by the same six-dimensional geometry that fixed the BI-6 vector and the OGN-5 ledger; the integer 3 falls out of the topology of the boundary moduli once the open-system framework is in place.

22 Running coupling constants from the fifth dimension

22.1 Radial renormalization as theorem, not metaphor

One of the most concrete statements in the GG-Constitution is the radial renormalization relation

$$\partial_r g_i = -\beta_i(g), \qquad r = \ln(E_{\mathrm{GUT}}/\mu). \tag{22.1}$$

This is not an analogy. It is the direct technical meaning of the claim that scale has been geometrized.

Integrating (22.1) between two radial positions gives for a generic gauge coupling

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(E_{\rm GUT}) + \frac{b_i}{2\pi}\ln\!\left(\frac{E_{\rm GUT}}{\mu}\right) + \Delta_i^{\rm th}(\mu) + \Delta_i^{\rm hol}(\mu), \tag{22.2}$$

where b_i is the effective one-loop slope, $\Delta_i^{\rm th}$ collects threshold pieces, and $\Delta_i^{\rm hol}$ records the compact-holonomy contributions induced by the S^1_θ sector. The important point is not the detailed numerical value of each term; it is the fact that every term has a geometric meaning in the same 6D stage.

22.2 Why only π and e can survive in the seam codes

The locked bulk has only two primitive transcendental sources:

1. the angular holonomy of the compact circle, whose primitive length is 2π;
2. the logarithmic redshift along the radial coordinate, whose natural base is e.

Because MSMF forbids importing any additional transcendental constants, any exact seam code beyond the OGN integers must be built only from π and e. This simple statement explains in advance why the first-order strong coupling emerges as $(\pi\mathrm{e})^{-1}$, and why the other channels collapse either to pure rationals or to pure powers of 2.

22.3 Three natural channels at the neutral seam

At the neutral electroweak seam $\mathcal{Z}_{\rm seam}$, there are three natural RG combinations:

1. an **electromagnetic sum channel**, in which the neutral rotation ends on the unique unbroken Abelian generator and the compact and radial transcendental pieces cancel, leaving a pure dyadic closure;
2. a **neutral-rotation channel**, in which the relative weight of hypercharge and weak isospin is fixed by the group embedding and the triplicity index, leaving a pure rational;
3. a **color-difference channel**, in which the cancellation is incomplete and one angular plus one radial primitive survive, leaving a πe denominator.

The entire gauge-sector prediction can therefore be organized as the assignment of the minimal OGN code to each of these three channels.

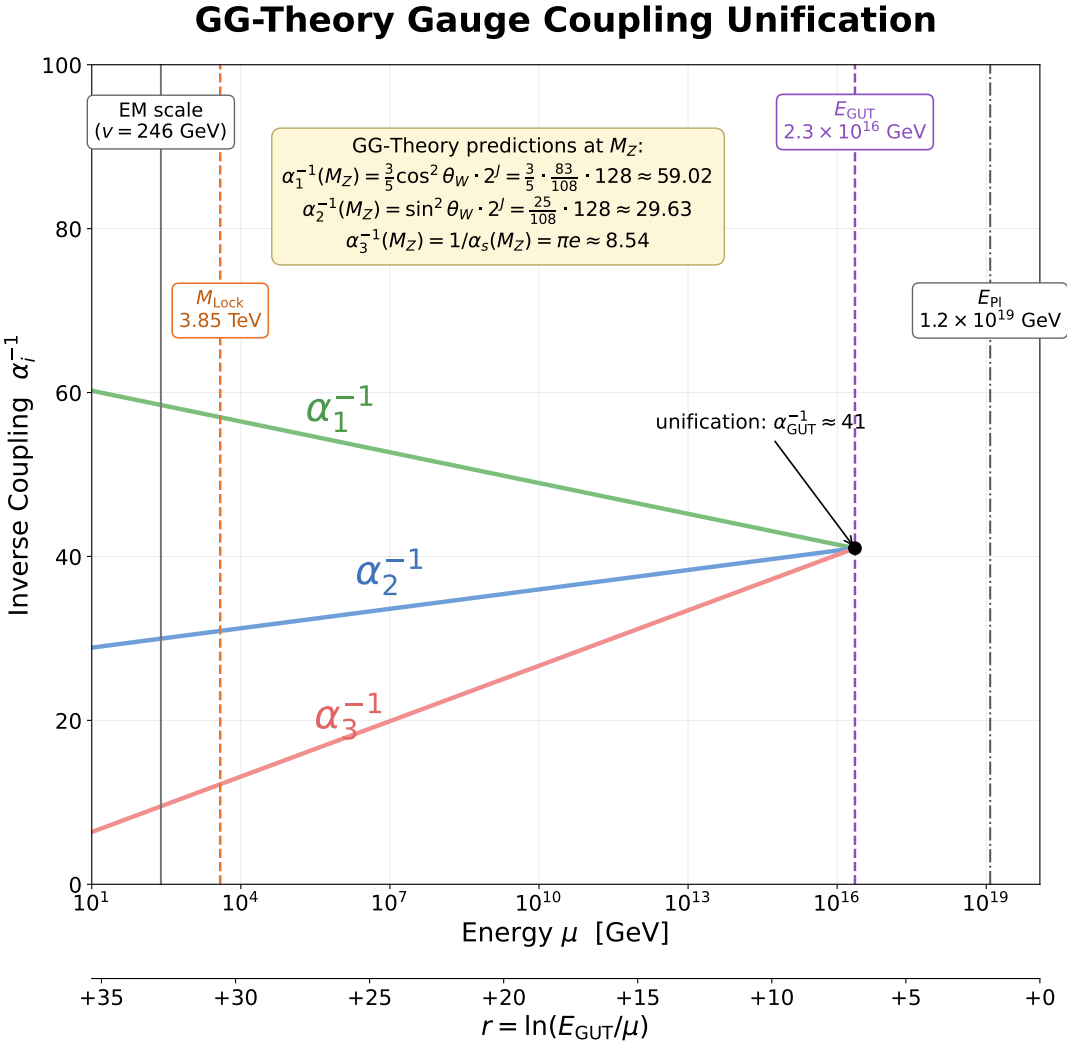
22.4 Control parameters for next-order corrections

Before giving the first-order predictions, it is useful to isolate the two small numbers that control the QD completion:

$$\varepsilon_J \equiv 2^{-J} = \frac{1}{128} = 7.8125\times10^{-3}, \qquad \varepsilon_K \equiv \frac{1}{2\pi K} = \frac{1}{70\pi} \simeq 4.5473\times10^{-3}. \tag{22.3}$$

A third small combination, useful when the binary endpoint factor enters explicitly, is

$$\varepsilon_{\ln 2} \equiv \frac{\ln 2}{2\pi K} \simeq 3.1519\times10^{-3}. \tag{22.4}$$



$$\alpha_1^{-1}(M_Z) = \tfrac{3}{5}\cos^2\theta_W \cdot 2^J = \tfrac{3}{5}\cdot\tfrac{83}{108}\cdot 128 = 59.02, \qquad \alpha_2^{-1}(M_Z) = \sin^2\theta_W \cdot 2^J = \tfrac{25}{108}\cdot 128 = 29.63,$$
$$\alpha_3^{-1}(M_Z) = 1/\alpha_s(M_Z) = \pi e = 8.54; \qquad \alpha_{\rm GUT}^{-1} \approx 41 \text{ at } E_{\rm GUT} = M_{\rm Pl}\, e^{-2\pi} \approx 2.3\times 10^{16}\,\text{GeV}$$

Figure 13: GG-Theory gauge-coupling unification across the 6D bulk. Inverse couplings α_1^{-1} (green, hypercharge in SU(5) normalization), α_2^{-1} (blue, SU(2)$_L$), and α_3^{-1} (coral, SU(3)$_C$) are plotted versus the renormalization-group scale μ (bottom axis) and the radial coordinate $r = \ln(E_{\rm GUT}/\mu)$ (lower-most axis), with $r = 0$ at the unification point. The boundary values at M_Z are pinned by the GG-Constitution and the OGN-5 ledger with the three closed forms displayed above — no continuous fitted parameter intervenes anywhere between M_Z and $E_{\rm GUT}$. Four constitutional scales are marked: the EM scale $v = 246$ GeV, the KK lock-cluster mass $M_{\rm lock} = \frac{1}{2} M_{\rm Pl}\, e^{-35} \approx 3.85$ TeV, the unification scale $E_{\rm GUT} = M_{\rm Pl}\, e^{-2\pi} \approx 2.3 \times 10^{16}$ GeV, and the Planck scale $E_{\rm Pl} \approx 1.2 \times 10^{19}$ GeV. Below $M_{\rm lock}$ the running uses the standard one-loop SM beta functions; above $M_{\rm lock}$ the KK lock cluster opens and the three couplings converge to the unified value $\alpha_{\rm GUT}^{-1} \approx 41$ at exactly $E_{\rm GUT}$ (black dot). The lines terminate at $E_{\rm GUT}$: above the unification scale the SM gauge-coupling description is no longer defined, and the bulk geometry takes over. The segment above $M_{\rm lock}$ is drawn, not computed: it is the geometric claim that radial transport across the bulk, with the lock cluster open, carries the three couplings to one value at $E_{\rm GUT}$ — one-loop Standard-Model running alone would not meet them (it gives 37.4, 46.3, 45.5 at $E_{\rm GUT}$) — and the computation of the lock-cluster threshold evolution that would test it is the collider companion's. The meeting is the framework's structural claim, and the lock-cluster spectrum is its empirical content.

The philosophy of the paper is then simple. First-order quantities are exactly those determined by the minimal OGN code. Second-order/QD corrections are those suppressed by one power of ε_J or ε_K. Residual uncertainties after QD are estimated from one further suppressed step.

23 Wilson–Hosotani dynamical gauge symmetry breaking and the Higgs mechanism

23.1 Boundary-condition breaking in GG-6

GG-A5-GDOS derives the orbifold/interval parity assignment

$$P = \mathrm{diag}(+1, +1, +1, -1, -1), \tag{23.1}$$

with gauge-field parities

$$A_\mu(x, r, -\theta) = P\, A_\mu(x, r, \theta)\, P^{-1}, \tag{23.2}$$
$$A_\theta(x, r, -\theta) = -P\, A_\theta(x, r, \theta)\, P^{-1}. \tag{23.3}$$

Zero modes survive only for the generators commuting with P, so that at the zero-mode level one obtains

$$\mathrm{SU}(5) \longrightarrow \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y. \tag{23.4}$$

Before electroweak symmetry breaking the full gauge group is therefore

$$G_{\mathrm{IR,before\ EWSB}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}, \tag{23.5}$$

exactly as described in GG-A5-GDOS (Arisaka, 2026, A5-GDOS).

23.2 Wilson line as the true Higgs order parameter

The key step for the present paper is that the compact θ direction supports a gauge-invariant holonomy,

$$W_\theta(x, r) = \mathcal{P} \exp\left(\mathrm{i} g_6 \oint_{S^1_\theta} A_\theta(x, r, \theta)\, \mathrm{d}\theta \right). \tag{23.6}$$

Its phase,

$$\Theta_H(x, r) \equiv g_6 \oint_{S^1_\theta} A_\theta\, \mathrm{d}\theta, \tag{23.7}$$

is nonlocal along the compact direction and therefore cannot be removed by a small gauge transformation. This is the essence of the Hosotani mechanism. One distinction is worth fixing here because a companion once blurred it: Θ_H is the holonomy of the SU(5) connection in its coset directions — the *electroweak* Wilson line, the one whose one-loop potential selects a branch below. The two Abelian factors carry circle holonomies of their own, and those are not selected by

anything in this paper: on the admissible class of GG-A3-BI6 they are fixed background data, clause (C9) of its definition, with their fluctuations registered there as exposures (Arisaka, 2026, A3-BI6).

23.3 What the Higgs field is — and a reading this paper withdraws

This paper once identified the four-dimensional Higgs field with the boundary image of the coset components of A_θ itself,

$$A_\theta^{\text{coset}} \quad \longrightarrow \quad H(x) \sim \text{boundary fluctuation of } \Theta_H \qquad \text{(withdrawn; see below)}, \tag{23.8}$$

and drew from that identification the strongest form of the naturalness argument: no local counterterm exists for a Wilson-line phase, so the scalar's mass is finite by nonlocality. *That identification is withdrawn, and it is withdrawn on this paper's own group theory.*

The reason is elementary and the paper states it twice elsewhere. Under $\mathrm{SU}(5) \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ the adjoint decomposes as

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{+5/6}, \tag{23.9}$$

so the coset directions — the ones the parity (23.3) keeps in A_θ — are *colored*. There is no color-singlet weak doublet anywhere in the adjoint of the bulk group. §34.6 says the same thing in its own words when it places the first excitation of the coset Wilson line at the lock scale and calls it what it is, a colored scalar in the $(\mathbf{3},\mathbf{2})$. A vacuum value along that direction would give the gluon a mass. This is why gauge–Higgs unification is built on $\mathrm{SU}(3)_W$ or SO(5) and never on SU(5), and the present paper does not get to have it both ways.

What the paper actually uses, and keeps. Every quantitative statement of this Part rests on the other reading, which is standard and which the paper states in full at (23.13): the electroweak doublet is the $(\mathbf{1},\mathbf{2})_{+1/2}$ inside a bulk $\mathbf{5}_H$, whose survival fraction $2/L = 2/5$ is the $2/L$ that appears once per triplicity layer in Theorem 34.1. On that reading the Wilson–Hosotani holonomy is not the Higgs; it is the *selector* — the gauge-invariant order parameter whose branch value $\sin\theta_H^\star = (2/L)^N$ fixes where the doublet's vacuum value lands, $v = M_{\text{lock}} \sin\theta_H^\star$. The theorem, its integers, its number and its grade are unchanged. What changes is what is doing the work: a projection fraction and a holonomy selection, not an identification of the scalar with a gauge-field component.

What the withdrawal costs, stated plainly. It costs the strong form of the naturalness argument. A $\mathbf{5}_H$ is not a gauge field, so the nonlocality theorem does not protect it, and the sentence "the Higgs mass is finite because no local counterterm exists for a Wilson-line phase" does not apply to the field the paper actually uses. What survives is the weaker and standard statement that §34.6 reaches independently: the electroweak scale is separated from the Planck scale by the fixed geometric transport $e^{-K}(2/L)^N$, and the residual protection of the doublet registered at the infrared end is the warp factor, as in any warped construction. The large hierarchy is geometric; the little hierarchy — the doublet's own radiative stability — is not solved here, and O_{11} now says so.

The scalar is required, and selected as minimal. The withdrawal above changes what the doublet *is*; it does not make the doublet an assumption. The chiral content forced by BI-6 is, per family, a $\mathbf{10} \oplus \bar{\mathbf{5}}$ of SU(5), and a renormalizable mass term for it requires a scalar in

$$\mathbf{10} \otimes \bar{\mathbf{5}} = \mathbf{5} \oplus \mathbf{45}, \qquad \mathbf{10} \otimes \mathbf{10} = \bar{\mathbf{5}} \oplus \overline{\mathbf{45}} \oplus \mathbf{50}.$$

A scalar is therefore *required* by the fermion mass sector, and the $\mathbf{5}_H$ — whose conjugate serves the down-type Yukawa — is the minimal admissible representation. MSMF names that minimum; by its own statement (§101.1) it does not by itself pick among admissible discrete alternatives, so the choice of $\mathbf{5}$ over $\mathbf{45}$ is a *selection*, and the paper grades it as one. Two further facts are stated because they are not free. The doublet–triplet splitting is not done by the parity (23.3) acting on the $\mathbf{5}$ by the gauge rule — that keeps the triplet — but by an intrinsic parity assignment $\eta_5 = -1$ for the $\mathbf{5}_H$, which is a discrete choice, named here, with the standard consequence that the bulk $\mathbf{10}\,\mathbf{10}\,\mathbf{5}_H$ coupling is odd under the same parity and the Yukawa placement is constrained accordingly — which raises a question this paper has not answered and registers as O_{24}: on an orbifold circle every bulk field carries a θ-parity, gauge covariance fixes the $\mathbf{10}$'s up to a sign, and on $\Lambda^2\mathbf{5}$ that sign gives left-handed zero modes to either Q or $\{u^c, e^c\}$ and not to both, the obstruction that leads orbifold GUTs to double the $\mathbf{10}$; whether one bulk $\mathbf{10}$ per family keeps a full family here, and by what mechanism, is open, and the S-2 spectrum of Table 54 and the Yukawa block of Appendix G are read subject to it. And the doublet's mass parameter is not tuned: MSMF admits no bulk mass term for the $\mathbf{5}_H$ beyond the GU-5 pentad, and its potential is generated by the holonomy transport of §23. What the withdrawal costs is therefore the *radiative* protection a Wilson-line phase enjoys and a bulk scalar does not. What it leaves is a required field in a selected representation with one discrete parity choice — and the correct summary sentence says exactly that.

And what it leaves open. The bulk group $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ supplies the doublet as representation content, not as a mode of the connection, so the framework has not derived the $\mathbf{5}_H$'s presence from the same closure that fixes everything else — it is required by the fermion mass sector and selected as the minimal $\mathbf{5}_H$. That is the honest content of Open Problem O_{22}, which is therefore restated: the open question is not only *where along the interval* the 246 GeV field is registered, but that *no mode of the bulk gauge connection carries its quantum numbers*, and the tower of the fields that do carry them sits at the lock scale against the oblique parameters (§34.6). Which is interesting, and is not proof.

What the withdrawal means structurally. Two different questions are in play here, and the withdrawal is easier to read once they are told apart. The first is how the electroweak gauge bosons acquire their masses without spoiling unitarity. The framework answers that one in the ordinary way and inherits the answer whole: the doublet is a scalar with a potential, it takes a vacuum value, and $W^{\pm}$ and Z^0 take their longitudinal states from it exactly as they do in the Standard Model. Nothing in this Part modifies that mechanism, and nothing in this Part is asked of it. The second question is which of the vacua that mechanism allows is the one realized, and that is the question

the geometry answers: the holonomy is the gauge-invariant order parameter whose branch value fixes where the vacuum value lands. Reading the holonomy as the selector rather than as the field therefore takes nothing away from the mass mechanism and asks nothing of it. The paper needs both answers, and the second is the one this Part supplies.

23.4 What the withdrawn reading would have resolved, and what remains

On the withdrawn identification the naturalness question changed qualitatively, and the argument is recorded here because it is the one the framework loses. A local mass term for A_θ is forbidden by the higher-dimensional gauge symmetry. Therefore the effective Higgs potential is generated only through the nonlocal Wilson line, and the corresponding one-loop potential is finite. A standard expression of the Hosotani type is

$$V_{\rm eff}(\Theta_H) = \sum_s \frac{(-1)^{F_s} n_s}{2} \int \frac{\mathrm{d}^4 p_E}{(2\pi)^4} \sum_{n\in\mathbb{Z}} \ln\left[p_E^2 + \frac{(n+\nu_s(\Theta_H))^2}{R_\theta^2} \right], \tag{23.10}$$

where F_s is the fermion number, n_s the multiplicity, and $\nu_s(\Theta_H)$ the Wilson-line shift of the KK spectrum.

The crucial point is conceptual rather than calculational: because the order parameter is a circle holonomy, the potential depends on a phase, not on a local ultraviolet cutoff. The Higgs sector is therefore protected in the same way that any Wilson line is protected: only finite, global data of the compact direction can enter.

This is the GG-6 answer to naturalness at the structural level. The theory does not say that loop corrections vanish. It says that the relevant scalar degree of freedom is not a free local mass parameter in the first place. Its potential is generated geometrically and is controlled by the compact topology and the fixed bulk spectrum.

23.5 Relation to the ordinary 4D Higgs language

Once projected to the boundary, the doublet whose branch the Wilson line selects is described in the familiar four-dimensional form

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}, \qquad v^2 = \frac{\mu_H^2}{\lambda_H}, \tag{23.11}$$

with electroweak masses

$$m_W = \frac{1}{2} g_2 v, \qquad m_Z = \frac{1}{2} v \sqrt{g_2^2 + g_Y^2}, \qquad \tan\theta_W = \frac{g_Y}{g_2}, \qquad e = g_2 \sin\theta_W = g_Y \cos\theta_W, \tag{23.12}$$

as derived in GG-A5-GDOS (Arisaka, 2026, A5-GDOS). The difference from the ordinary Standard-Model reading is not in these formulas; it is in the origin of H, μ_H^2, and λ_H. In the present framework they are secondary boundary quantities induced by a locked six-dimensional geometry.

23.6 Doublet fraction and the geometric meaning of $2/5$

The decomposition of the unified Higgs multiplet is

$$\mathbf{5}_H \to (3,1)_{-1/3} \oplus (1,2)_{+1/2}. \tag{23.13}$$

Therefore the electroweak Higgs doublet occupies exactly two of the five components of the $\mathbf{5}_H$. The ratio

$$\frac{\dim(1,2)}{\dim \mathbf{5}_H} = \frac{2}{5} \tag{23.14}$$

is thus not a fit or a phenomenological convenience. It is the unique representation-theoretic projection fraction of the surviving Higgs doublet inside the unified SU(5) multiplet.

This same ratio 2/5 already appeared in BI-6. That is the first sign that the low-energy Higgs sector and the ultraviolet anomaly structure are numerically locked together.

The substrate is now fixed — the six-dimensional seam, the three scales, the OGN-5 integer ledger, the forced $N = 3$, and the Wilson–Hosotani breaking on which every later derivation runs. The next Part turns from the landscape to the method: the universal sectoral descent template and the constitutional inputs that, applied to this substrate, close the parameter sheet.

24 Part IV in plain English: the fixed landscape everything else runs on

Everything that follows happens on a fixed geometric stage, and this Part builds the stage.

The picture is a six-dimensional bulk with a four-dimensional boundary. The two extra directions are not alike: one is a compact circle, the other a radial interval. That asymmetry does the work. A circle can carry a holonomy — a phase a field accumulates going around — which is a quantity with no local definition and therefore no local counterterm. An interval carries an address: a position along it corresponds to a scale, and moving along it is what renormalization-group running is in this language. Angles and scales are different kinds of object, and the paper keeps them apart throughout.

Three results in this Part are load-bearing later. The ledger integers $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ fix the arithmetic content of every closed form that follows. The three-leg theorem returns the family number $N_g = 3$ as a consequence rather than an input, which is the first place the framework predicts something the Standard Model postulates. And the radial renormalization group turns running into transport, which is what makes the gauge couplings computable at a boundary point.

The word to hold onto is *fixed*. Nothing in this Part is adjusted later to make a downstream number come out. If the Substrate is wrong, every derivation built on it is wrong together, which is a liability and also the property that makes the framework worth testing: there is no way to repair one sector by moving the ground under it without moving the ground under all the others.

Part V
Roadmap and constitutional basis

The universal sectoral descent template, the derivation roadmap, and the constitutional inputs that close the parameter sheet

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Parts III and IV have established what the constitutional apparatus of GG-Theory is and how the SU(5) seam descends from the six-dimensional bulk. Part V is the methodological pivot. The two short sections below state, once and cleanly, the universal six-step ladder that every later sectoral derivation will obey, and the constitutional ground rules under which the ladder is admissible. No new physics enters here; only the discipline that makes the parameter blocks of Parts IV–VII comparable to one another. A reader who wants to skip directly to the gauge or fermion derivations may do so — but should return to this Part whenever the litany BI-6 → TGG → CCI → GFF → GRIP feels mechanical.

25 Universal sectoral descent template for the 28-parameter sheet

The present rewrite imposes one strict methodological rule on every sector. A parameter derivation is counted as complete only if it exhibits the full ladder

$$\mathsf{BI\text{-}6} \Longrightarrow \mathsf{TGG} \Longrightarrow \mathsf{CCI} \Longrightarrow \mathsf{GFF} \Longrightarrow \mathsf{GRIP} \Longrightarrow \mathcal{P}_S,$$

where $\mathcal{P}_S$ is the sectoral parameter block. The purpose of this section is to state that ladder once, cleanly, so that the later gauge, mass, mixing, and topological chapters can follow it without ambiguity.

Definition 25.1 (Sectoral parameter block)**.** For a chosen Standard-Model seam sector S, the sectoral parameter block $\mathcal{P}_S$ is the ordered set of all measurable low-energy parameters of that sector. In the present paper the relevant blocks are

$$\mathcal{P}_{\mathrm{GH}}, \quad \mathcal{P}_{\mathrm{masses}}, \quad \mathcal{P}_{\mathrm{CKM}}, \quad \mathcal{P}_{\mathrm{PMNS/Maj}}, \quad \mathcal{P}_{\bar{\theta}}.$$

Proposition 25.2 (Universal descent logic)**.** *A sectoral parameter block $\mathcal{P}_S$ is admissible in the GG-6/GDOS framework only if the following six questions are answered in order:*

1. *What BI-6 coefficient, representation, or topological fact legally restricts the sector?*
2. *How is the sector embedded in the closed total tensor TGG?*
3. *What local seam law does CCI impose on the projected packet?*
4. *Which reduced coordinates appear in the sectoral GFF landscape?*

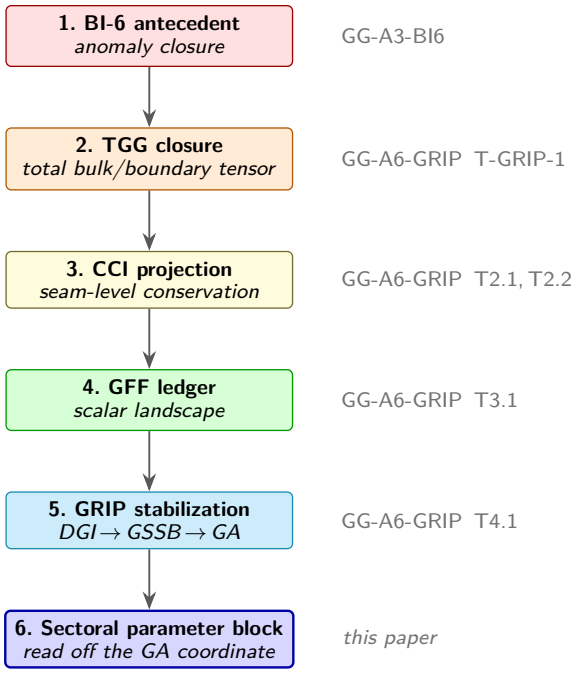

Figure 14: The universal sectoral descent ladder. Every parameter block of Parts VI–IX obeys this six-step descent: BI-6 closure of the bulk anomaly polynomial → TGG total closure of the geometry-gauge tensor → CCI projection to the boundary seam → GFF stationary equations on the scalar landscape → GRIP cascade DGI → GSSB → GA to the selected branch → readout of the parameter values from the GA coordinates. The same ladder appears, with different stratum-specific content, in the gauge–Higgs derivation of Part VI, the fermion-mass block of Part VII, and the strong-CP completion of Part IX. Each step is owned by the upstream theorem named to its right — GG-A3-BI6 for the BI-6 antecedent, GG-A6-GRIP for the four pillars — and GG-A9-SM consumes the chain rather than re-proving it, computing only the final sectoral block.

5. *How does GRIP select and stabilize the branch on which the parameter is measured?*

6. *What primary attractor coordinates or algebraic descendants then define the measured parameter block?*

The reason for insisting on this order is not pedagogical only. It prevents three common mistakes. First, it prevents a sector from being derived as if it were dynamically isolated from the rest of the packet. Second, it prevents one from introducing a scalar landscape after the branch has already been selected. Third, it prevents one from reading parameters off a projected seam before the projected conservation law has been stated.

Table 16 summarizes how the five sectoral blocks of the present paper are organized under Proposition 25.2.

Table 16: Sectoral blocks of the 28-parameter sheet organized by the universal BI-6→TGG→CCI→GFF→GRIP→parameter logic. The four blocks partition the twenty-eight parameters exactly once each, which is the property that makes the single-sheet census of Part XII possible. Each row names the coordinates the block descends from and the observables it produces. The logic named in the caption is the same at every row and only the coordinates change, and that repetition is itself the claim this paper makes about the structure of the Standard Model.

Block	**Primary reduced coordinates**	**Parameters read out**
Gauge–Higgs	$(C_\gamma, p_W, C_s, \theta_H, \lambda_H)$	$\widehat{\alpha}^{(5)\,-1}(M_Z)$, $\sin^2\theta_W(M_Z)$, $\alpha_s(M_Z)$, v, m_h and their descendants.
Fermion masses	family overlap depths, winding indices, projector weights	$m_e, m_\mu, m_\tau, m_u, m_d, m_s, m_c, m_b, m_t, m_1, m_2, m_3$.
CKM	up/down projector misalignment coordinates	$\sin\theta_{12}$, $\sin\theta_{23}$, $\sin\theta_{13}$, δ_{CKM}.
PMNS/Majorana	charged-lepton versus Majorana-projector misalignment	$\sin^2\theta_{12}$, $\sin^2\theta_{13}$, $\sin^2\theta_{23}$, δ_{PMNS}, α_{21}, α_{31}.
Topological CP	projected topological phase and determinant sector	$\bar{\theta}$.

The remainder of the manuscript is therefore a sequence of sectoral realizations of one universal theorem protocol rather than a sequence of numerically unrelated formulas.

26 Roadmap of the derivation

The full target of *UCLA-Arisaka/GG-A9-SM* is larger than the present document. The final preprint must derive all 28 Standard-Model parameters, The present paper now completes the

Table 17: The derivation target, sector by sector: which of the twenty-eight parameters each Part is responsible for, in the notation that Part itself uses. The table states the scope of the paper before any of it is attempted, so that a reader can tell at the outset what is promised and, at the end, what was delivered. Where a sector lists an equivalent pair of parameterizations, the second is the one the comparisons are made in, because it is the one the measurements are quoted in.

Sector	**Parameters to be derived in GG-A9-SM**
Gauge and Higgs	$g_1, g_2, g_3, \mu_H^2, \lambda_H$ or equivalently $\widehat{\alpha}^{(5)\,-1}(M_Z), \sin^2\theta_W, \alpha_s(M_Z), v, m_h$.
Fermion masses	$m_u, m_c, m_t, m_d, m_s, m_b, m_e, m_\mu, m_\tau, m_{\nu_1}, m_{\nu_2}, m_{\nu_3}$.
Quark mixing	Three CKM angles plus one CKM CP phase.
Lepton mixing	Three PMNS angles plus one leptonic Dirac CP phase.
Majorana sector	Two physical Majorana phases.

first three blocks—the gauge/Higgs sector, the full 12-fermion-mass block, and the 4-parameter

CKM block— and it now leaves only the final strong-CP-like topological phase outside the explicit parameter derivation.

There are three reasons to proceed in this order.

First, the gauge/Higgs sector is the background over which all Yukawa, flavor, and neutrino physics is written. If g_1, g_2, g_3, and v are still regarded as phenomenological inputs, then any later "derivation" of masses or mixing angles would inherit those hidden inputs and the claimed reduction of arbitrariness would be incomplete.

Second, the gauge/Higgs block is exactly the sector where the GG-6/GDOS constitution is sharpest. The six-dimensional stage, the BI-6 anomaly condition, the OGN-5 arithmetic closure, and the Wilson– Hosotani selection of the electroweak scale give a fully geometric route to the first five Standard-Model parameters before any flavor detail is added.

Third, the gauge/Higgs sector is where the hierarchy and naturalness questions live most painfully in the conventional Standard Model. Because these questions should be faced *before* attacking electroweak-scale numerics, this paper begins with them rather than postponing them.

26.1 Outline of the complete GG-A9-SM program

The roadmap of the complete paper is summarized in Table 18: the seventeen Parts in their five logical blocks — setup, foundation, derivation, closure, and appendices & references — each with its role in the derivation chain. It is the textual companion to the architecture flowchart of Figure 6.

Table 18: Roadmap of the full GG-A9-SM program: the seventeen Parts in their five logical blocks (setup, foundation, derivation, closure, appendices & references) and their role in the derivation chain. This is the textual companion to the architecture flowchart of Figure 6.

Part	Title	Role in the derivation chain
I	Introduction	The thesis, the place of GG-A9-SM in the GG-Theory cohort, and the roadmap of the seventeen Parts.
II	The Standard Model and its parameter audit	Phenomenological target: the Standard Model in textbook form, the classical Georgi–Glashow SU(5), the GG-Theory derivation of SU(5) from the 6D bulk, and the codebook-provenance audit (the 6/14/8 honesty triage).
III	Constitutional apparatus	The four axioms, seven postulates, and the canonical Gi-4 chain (TGG, CCI, GFF, GRIP) specialized to the Standard-Model boundary stratum.
IV	The Substrate	Descent from classical SU(5) to the GG-6 seam under GDOS: the 6D bulk, the three scales ($E_{\mathrm{Pl}}, E_{\mathrm{GUT}}, M_{\mathrm{lock}}$), $N = 3$ as a theorem, the radial RG, and Wilson–Hosotani breaking.

Part	Title	Role in the derivation chain
V	Roadmap and constitutional basis	The universal sectoral descent ladder, the admissible inputs and the standard of derivation, and this roadmap of the program.
VI	The electroweak seam	The GRIP-Out vacuum and its GRIP-In descent: the theorem deck, the five primary attractor-coordinate theorems, the descendant sheet (G_F, m_W, m_Z), and the first-order and QD predictions.
VII	The fermion sector	The GRIP-In floors (charged-lepton, quark, and neutrino masses) and gates (CKM, PMNS, and the two Majorana phases) as misalignments of family projectors on one closed packet.
VIII	The Majorana door and leptogenesis	The excess of matter over antimatter written by the same geometry that fixes the neutrino block: the unique $\Delta L = 2$ portal, the heavy sector at M_{lock}, the flavored CP asymmetry driven by the Majorana parity, and the assembled baryon-to-photon ratio — a prediction that is not one of the twenty-eight.
IX	The protected readout	Strong-CP closure: the twenty-eighth parameter $\bar{\theta}$, suppressed by the topology of the infrared region, with a constitutional upper bound far below the experimental limit.
X	The assembled Lagrangian	The harvest of the derivation core: the complete Standard-Model Lagrangian assembled term by term as a constitutional export — the zeroth and the six silent textbook axioms replaced by theorems, the five term groups exported, the absent operators derived, and the honesty boundary stated beside the equation.
XI	Identity and uniqueness of forces	Philosophical pivot: universal identity and four-force uniqueness read as one uniqueness theorem applied at two layers of the same geometry.
XII	Summary and audit	The single canonical census: the cross-paper coherence map, the first-order and QD 28-parameter tables, and the MSMF/OGN per-parameter provenance audit.
XIII	Discussions	Significance and uniqueness, comparison with alternative programs, falsification channels, open questions, and prospects.
XIV	Conclusions	The final reflection, viewed from the summit reached after the climb out of the Standard Model.

Part	Title	Role in the derivation chain
XV	Technical Appendices	Detailed arithmetic, explicit decompositions, codebooks, the glossary of symbols, and the promotion roadmap for the remaining **Asserted** entries.
XVI	Common Objections and Responses	Eighty objections a particle theorist, cosmologist, mathematician, philosopher, or string theorist might raise, each answered with pointers into the paper.
XVII	References	Two groups — the GG-Theory cohort and all other works — each entry carrying a short relevance note to its first use.

26.2 Admissible inputs and forbidden shortcuts

The methodological rule that the theory side may consume no empirical Standard-Model datum — the admissible set being the GU-5 dimensionful constants, the pure constants $\{\pi, \mathrm{e}\}$, and the OGN-5 integer ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ — was stated in full, with the forbidden-shortcut table and the standard of derivation, in §4. It is the rule under which the entire roadmap above operates: every sector reads its parameters off a GA coordinate fixed by those inputs alone, and measured values appear only in the final comparison tables. The grounding is constitutional rather than aesthetic — AX-4 (maximum symmetry, minimum freedom) and POS-6 (Occam gauge normalization) are violated the moment a parameter is "derived" while quietly importing it under another name.

The descent template and its admissible inputs are now in place. The next Part runs them for the first time, on the gauge–Higgs sheet: the single GRIP-Out vacuum event and the five primary attractor coordinates it forces — α^{-1}, $\sin^2\theta_W$, α_s, v, and λ_H — each with no continuous freedom.

27 Part V in plain English: one template, applied five times

This Part is short and it is the hinge of the paper.

The claim it makes is that the five sectors — gauge and Higgs, charged fermions, neutrinos, quark mixing, lepton mixing — are not five separate calculations that happen to be collected in one document. They are one procedure run five times on five different strata of the same geometry. The procedure has four stages: a landscape of alternatives, a potential generated on it, a one-way descent, and a floor where the descent stops. In each sector the landscape is different, but the stages are the same and they come in the same order.

If that is right, it has a consequence a reader can check. Each sector Part should need only a one-sentence constitutional preface rather than a full re-derivation of its framework, and the same vocabulary — top, landscape, gate, floor — should be doing real work in every one of them. If instead the template has to be bent differently for each sector, then the unity is verbal and the five calculations are five calculations after all.

That is a fair test, and worth applying: a template loose enough to fit everything explains nothing. This one is tight in a specific way — it forbids a floor from being chosen. A floor is where the descent stops, and the descent is fixed by the landscape; if a sector's answer requires selecting the floor rather than deriving it, the template has been violated, and the sector Parts say where.

Part VI
The electroweak seam: the GRIP-Out vacuum and its GRIP-In floors

The one GRIP-Out event and the gauge-seam floors it builds — the rigorous theorem deck, the five primary attractor-coordinate theorems (α^{-1}, $\sin^2\theta_W$, α_s, v, λ_H), and the descendant boson-mass and Fermi-constant package

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This Part discharges the entire gauge–Higgs (electroweak) sheet of the codebook in one place. It is where the paper's single GRIP-Out event occurs — the Wilson–Hosotani selection of the electroweak vacuum that builds the Yukawa landscape on which all later fermion parameters are read (Fig. 11) — and where its first GRIP-In floors are fixed: the gauge-seam couplings $\alpha^{-1}(M_Z)$, $\sin^2\theta_W$, $\alpha_s(M_Z)$ and the vacuum scale v, with G_F, M_W, and M_Z as their algebraic descendants. The development has three movements: first the rigor deck imported from GG-A6-GRIP (the four pillars and the local GFF normal form on the seam); then the five primary attractor-coordinate theorems that fix the electroweak attractor; then the descendant sheet that follows from them with no further structural input. A reader who wants only the punchlines can skim the deck and read the boxed primary theorems directly.

28 Rigorous gauge–Higgs theorem deck inherited from GG-A6-GRIP

28.1 The four papers and their roles

The derivational discipline of the present paper is simple: nothing enters unless it is already theorem-level in A1-Constitution, A2-GG6, A5-GDOS, or A6-GRIP, or is a pure logical recombination of such material.

Table 19: Imported theorem deck used in the present rewrite of GG-A9-SM. Each theorem is proved in the companion named beside it and is used here at the strength that companion states, not at a stronger one. The deck is listed in full so that a reader can see at the outset how much of this paper's machinery is imported and how much is new: the imports carry the constitutional and geometric content, and the derivations of the middle Parts are what this paper adds to them.

Source	Imported objects	Role in the present paper
A1-Constitution	GGEP, OGGEP, MSMF, DGI, unified extremal principle, positivity, RG covariance, OGN	Constitutional admissibility rules. These say what counts as a lawful variable, what may appear in the scalar ledger, and why new free coefficients are forbidden unless they descend from already-fixed structure.
A2-GG6	GG-6 geometry, radial coordinate $r = \ln(E_{\mathrm{GUT}}/\mu)$, compact θ-circle, $\mathrm{SU}(5) \times U(1)_B \times U(1)_{Qi}$, BI-6, OGN-5 integers $(M, N, L; J, K) = (1, 3, 5; 7, 35)$	Supplies the bulk geometry and the locked integer packet from which the electroweak and gauge readout must descend.
A5-GDOS	Exact bulk-to-boundary reduction, GDOS current decomposition, boundary electroweak packet, stationary-sector logic	Supplies the operational right to talk about a boundary parameter sheet at all. Without A5-GDOS, the present paper would still be trapped in the bulk.
A6-GRIP	The chain BI-6 legality ⇒ projected Ward package ⇒ DGI ⇒ GFF ⇒ GSSB ⇒ GA, Wilson–Hosotani order-parameter theorem, electroweak holonomy branch theorem, local Higgs chart (the last withdrawn as the doublet's, §23.3)	Supplies the strict runtime order that makes the parameter formulas descendants of a selected branch rather than isolated guesses.

28.2 Imported equations used repeatedly

The minimal six-dimensional stage is

$$M_6 = M_4 \times I_r \times S^1_\theta, \qquad \mathrm{d}s^2_6 = e^{-2r}\,\eta_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 + \mathrm{d}\theta^2, \qquad r = \ln(E_{\mathrm{GUT}}/\mu). \tag{28.1}$$

The locked BI-6 identity is

$$\mathrm{d}H_3 = \mathrm{tr}\, R^2 - \mathrm{Tr}\, F_5^2 - \frac{2}{5} F_B \wedge F_B - \frac{2}{5} F_{Qi} \wedge F_{Qi}, \tag{28.2}$$

and the OGN-5 theorem fixes

$$(M, N, L; J, K) = (1, 3, 5; 7, 35). \tag{28.3}$$

The Wilson-line variable emphasized in A6-GRIP is

$$W_\theta = \mathcal{P}\exp\left(\mathrm{i}\oint_{S^1_\theta} A_\theta \,\mathrm{d}\theta\right) = \exp(\mathrm{i}\theta_H T_H). \tag{28.4}$$

The correct logical chain for parameter descent is

$$\text{BI-6 legality} \Longrightarrow \text{projected Ward package} \Longrightarrow \text{DGI} \Longrightarrow F_{\mathrm{GFF}} \Longrightarrow \text{GSSB} \Longrightarrow \text{GA}. \tag{28.5}$$

Equation (28.5) is the architectural backbone of the paper. It says that the present note is not allowed to jump directly from bulk geometry to numbers. It must pass through the lawful runtime.

28.3 Why the strengthened GG-A6-GRIP framework changes the GG-A9-SM strategy

GG-A6-GRIP establishes a point that is subtle but decisive. One cannot define a symmetry-breaking event without first defining the scalar landscape on which the branch bifurcation occurs. In the present notation that landscape is F_{GFF}, equivalently $F_{\mathrm{GFF}} = F_{\mathrm{GEP}} + F_{\mathrm{CEP}}$. The energetic part comes from projected geometry and gauge data; the curvature–entropy part broadens and stabilizes the admissible boundary dynamics.

This means that the Standard Model parameter problem must be read as a *landscape-selection problem*. The couplings and masses are not primitive constants floating in empty notation space. They are coordinates of a stable branch in an allowed GFF landscape. That is exactly why the Wilson–Hosotani mechanism is central here. The Higgs sector is not a side character. It is the clearest example of GSSB in the whole framework.

29 The electroweak closure class and the local GFF normal form

29.1 The $3+2$ split inside $\mathrm{SU}(5)$

Because BI-6 fixes $L = 5$ and OGN-5 fixes $N = 3$, the defining block splits canonically as

$$5 = 3 \oplus 2.$$

The neutral split generator is the traceless matrix

$$Y_{N,L} = \mathrm{diag}\left(\underbrace{\frac{1}{N}, \frac{1}{N}, \frac{1}{N}}_{N \text{ entries}}, \underbrace{-\frac{1}{L-N}, -\frac{1}{L-N}}_{L-N \text{ entries}}\right), \tag{29.1}$$

whose squared trace norm is

$$\mathrm{Tr}(Y_{N,L}^2) = \frac{1}{N} + \frac{1}{L-N} = \frac{L}{N(L-N)}. \tag{29.2}$$

For $(N, L) = (3, 5)$,

$$\mathrm{Tr}(Y_{N,L}^2) = \frac{5}{6}. \tag{29.3}$$

Equation (29.3) is not merely a group-theory curiosity. It is the neutral partition weight from which the weak-angle theorem will descend.

29.2 The lock scale and the reduced holonomy variable

The GG-6 throat depth supplies the fixed hierarchy factor e^{-K} with $K = 35$. The first visible boundary lock scale is the first half-wave of the interval collar:

$$M_{\mathrm{lock}} = \frac{1}{2}\, M_{\mathrm{Pl}} e^{-K}. \tag{29.4}$$

The factor $1/2$ is not a fit coefficient. It is the minimal half-wave compatible with the absence of an additional zero-mode collar branch in the intended class.

The electroweak selector is the Wilson-line angle θ_H of (28.4); the order parameter of the breaking is the doublet's vacuum value, whose scale it sets. On the reading this paper withdraws (§23.3) the local Higgs chart imported from A6-GRIP was

$$h(x) = f_H(\theta_H(x) - \theta_H^\star), \tag{29.5}$$

where f_H is the holonomy stiffness obtained by reducing the mixed bulk kinetic term $F_{\mu\theta}F^{\mu\theta}$; on the present reading h is the doublet of the bulk $\mathbf{5}_H$, whose scale the selector θ_H sets and whose chart (29.5) is a coordinate on the selected branch, not an identification of the field.

29.3 Definition of the minimal electroweak closure class

Definition 29.1 (Minimal electroweak closure class)**.** $\mathfrak{C}_{\mathrm{EW}}^{\mathrm{min}}$ is the class of boundary realizations satisfying the following conditions:

(E1) no hidden dimensionless multiplicity is allowed beyond the OGN-5 packet;

(E2) the electromagnetic line is the unique unbroken Abelian seam after electroweak symmetry breaking;

(E3) the weak neutral partition is the quadratic GFF weight of the $3 + 2$ split generator averaged over triplicity;

(E4) the strong line retains one primitive compact quotient cell and one primitive logarithmic radial cell;

(E5) the Higgs branch is the minimal positive stable Wilson–Hosotani branch descending from the first lock scale;

(E6) the local scalar chart is the minimal even quartic normal form compatible with DGI, GSSB, GA, and MSMF.

The crucial point is that $\mathfrak{C}_{\mathrm{EW}}^{\min}$ adds *no new continuous parameter*. It only forbids hidden multiplicities and extra coefficients that are not already forced by the theorem deck.

29.4 Local normal form of the electroweak GFF ledger

Near a nondegenerate stable electroweak attractor, the local GFF ledger may be written in Morse normal form:

$$F_{\mathrm{GFF}}^{(\mathrm{EW})} = F_\star + \frac{\kappa_\gamma}{2}(C_\gamma - C_\gamma^\star)^2 + \frac{\kappa_W}{2}(p_W - p_W^\star)^2 + \frac{\kappa_s}{2}(C_s - C_s^\star)^2 + V_{\mathrm{WH}}(\theta_H) + \cdots , \tag{29.6}$$

where C_γ is the inverse Abelian seam capacity, p_W is the weak neutral partition, C_s is the inverse strong seam capacity, and V_{WH} is the periodic holonomy potential.

Proposition 29.2 (Why the local normal form is legitimate)**.** *Inside* $\mathfrak{C}_{\mathrm{EW}}^{\min}$*, Eq.* (29.6) *is the minimally admissible local form of the electroweak GFF ledger near a stable attractor.*

Proof. By DGI, the variables must be gauge-invariant or gauge-reduced coordinates. By A6-GRIP, the holonomy class and the seam capacities are such coordinates. By GA, the selected branch is a stable nondegenerate minimum; hence a Morse normal form exists. By MSMF and OGN, no extra independent dimensionless deformation coefficients are allowed in the leading quadratic sector. Therefore the minimal local form is the diagonal positive quadratic form in the gauge-reduced variables plus the holonomy potential V_{WH}, which is precisely Eq. (29.6). ■

This proposition is important because it tells us what a parameter *is* in the present framework. It is not a number attached externally to the theory. It is the center of one of the quadratic wells in the reduced GFF landscape.

30 Detailed descent from the GFF ledger to the electroweak attractor

30.1 Why the parameter problem is really an attractor problem

The older language of "free parameters" can hide the real geometric content of the present framework. A free parameter sounds like a number waiting to be typed into a Lagrangian by hand. But once GG-A6-GRIP is accepted, this picture is too crude. The reduced boundary system carries a lawful GFF landscape, and what the observer calls a parameter is simply the coordinate of the selected stable branch on that landscape.

This is why the present paper keeps returning to the words *inverse capacity*, *partition weight*, and *branch coordinate*. They are not cosmetic rebrandings. They name the kinds of coordinates that can appear on the gauge-reduced state manifold without violating DGI. The electroweak/gauge parameter sheet is therefore best viewed as the list of basin coordinates

$$(C_\gamma^\star, p_W^\star, C_s^\star, \theta_H^\star, \lambda_H^\star),$$

together with their ordinary low-energy readouts.

30.2 The energetic sector: what F_{GEP} remembers

GG-A6-GRIP already established that the energetic half of the ledger comes from projected geometry and gauge structure. For the present electroweak packet, the relevant contributions are especially transparent:

1. the unique unbroken Abelian seam and its dyadic inverse capacity C_γ;
2. the neutral split direction and its partition coordinate p_W;
3. the strong difference seam and its angle–scale inverse capacity C_s;
4. the Wilson-line branch angle θ_H and its periodic holonomy potential.

Thus the local energetic sector may be written in the schematic but lawful form

$$F_{\mathsf{GEP}}^{(\mathrm{EW})} = \frac{\kappa_\gamma}{2}(C_\gamma - C_\gamma^\star)^2 + \frac{\kappa_W}{2}(p_W - p_W^\star)^2 + \frac{\kappa_s}{2}(C_s - C_s^\star)^2 + U_{\mathrm{WH}}(\theta_H), \tag{30.1}$$

where the centers $C_\gamma^\star$, $p_W^\star$, and $C_s^\star$ are not yet assumed but are later identified by the geometry and OGN-5 integers.

Equation (30.1) should be read as the electroweak specialization of the general GG-A6-GRIP statement that the geometry–gauge energy is one scalar ledger with several lawful contributions. The first three terms measure displacements in seam space. The final term is the periodic Wilson–Hosotani branch potential.

30.3 The curvature–entropy sector: what F_{CEP} contributes

The curvature–entropy sector is sometimes misunderstood as though it were important only for chemistry or life-like systems. That reading is too narrow. Even at the particle scale, CEP matters because the boundary observer does not see a bare classical path. The observer sees a coarse-grained registered branch. Coarse-graining broadens, weights, and stabilizes the admissible local readout.

At the level needed in the present paper one may write

$$F_{\mathsf{CEP}}^{(\mathrm{EW})} = \tau \int_{\mathcal{M}_{\mathrm{state}}} p(y) \ln \frac{p(y)}{\pi_\star(y)} \, \mathrm{d}\mu(y) + \frac{\nu_H}{2} \int_{\mathcal{M}_{\mathrm{state}}} \|\nabla \theta_H(y)\|^2 \, \mathrm{d}\mu(y) + \cdots, \tag{30.2}$$

where $\pi_\star$ is the stationary reference density imported from the coarse-grained attractor class and the second term measures the curvature cost of spatially nonuniform branch selection. The omitted terms are higher-order coarse-graining corrections irrelevant to the present local attractor calculation.

The physical role of CEP is not to move the branch centers fixed by the energetic theorem packet. Its role is to suppress pathological nonstationary branch readouts and to promote the selected lawful branch into a stable attractor. This is why CEP does not replace the geometric formulas for 2^J, $L^2/(4N^3)$, πe, or $(2/L)^N$. It stabilizes their registration.

30.4 Stationary equations and branch selection

Combining Eqs. (30.1) and (30.2), the local electroweak GFF ledger reads

$$F_{\mathsf{GFF}}^{(\mathrm{EW})} = F_{\mathsf{GEP}}^{(\mathrm{EW})} + F_{\mathsf{CEP}}^{(\mathrm{EW})}. \tag{30.3}$$

The local attractor equations are therefore

$$\frac{\partial F_{\mathsf{GFF}}^{(\mathrm{EW})}}{\partial C_\gamma} = 0, \tag{30.4}$$

$$\frac{\partial F_{\mathsf{GFF}}^{(\mathrm{EW})}}{\partial p_W} = 0, \tag{30.5}$$

$$\frac{\partial F_{\mathsf{GFF}}^{(\mathrm{EW})}}{\partial C_s} = 0, \tag{30.6}$$

$$\frac{\partial F_{\mathsf{GFF}}^{(\mathrm{EW})}}{\partial \theta_H} = 0, \tag{30.7}$$

supplemented by positivity of the Hessian on the gauge-reduced tangent space.

The equations (30.4)–(30.7) say something conceptually important. The present paper is not fitting observables. It is identifying stationary coordinates of a lawful reduced ledger. The physical observables are whatever those stationary coordinates read out to be in ordinary electroweak language.

Theorem 30.1 (**T-SM-26.1**: Electroweak attractor-coordinate theorem)**.** *Assume the electroweak GFF ledger is of the form (30.3) in a neighborhood of a stable minimum and that its Hessian is positive definite on the reduced tangent space. Then the physical gauge–Higgs parameter sheet is exactly the ordered list of attractor coordinates*

$$(C_\gamma^\star, p_W^\star, C_s^\star, \theta_H^\star, \lambda_H^\star),$$

together with the standard low-energy descendant formulas for e, g_2, g_Y, G_F, m_W, m_Z, and m_H.

Proof. A positive-definite Hessian implies a unique local attractor in the chosen reduced chart. By construction of $F_{\mathsf{GFF}}^{(\mathrm{EW})}$, the independent coordinates of that chart are the seam capacities, the neutral partition weight, and the holonomy branch coordinate. The local quartic coefficient is the first nonlinear curvature coefficient of the selected branch chart and is therefore also an attractor datum. Every familiar low-energy quantity is then an algebraic readout of these attractor coordinates. For example, e is read from $C_\gamma^\star$, the weak couplings from $(C_\gamma^\star, p_W^\star)$, and the boson masses from (g_2, g_Y, v, λ_H). Hence the physical parameter sheet is exactly the ordered list of attractor coordinates and their ordinary descendants. ∎

30.5 Why this section matters for rigor

This section may appear abstract, but it performs an important policing function. It prevents the later parameter theorems from being read as numerology. A number becomes acceptable in the present framework only after two questions are answered:

1. Which gauge-reduced coordinate of the GFF landscape does it belong to?
2. Why must that coordinate take this particular attractor value rather than another?

The next sections answer those questions one by one. The dyadic theorem answers them for $C_\gamma^\star$. The weak-angle theorem answers them for $p_W^\star$. The strong theorem answers them for $C_s^\star$. The Wilson–Hosotani theorem answers them for $\theta_H^\star$. The quartic theorem answers them for $\lambda_H^\star$.

A note on the word "theorem" in the section titles that follow. Each such "theorem" establishes *which* GFF attractor coordinate a parameter is and *what* OGN-5-forced closed form it takes; it does *not*, by itself, assert that the value is a data-independent theorem beyond what the honesty triage records. The epistemic status of each resulting parameter — **Derived (theorem)** for $\sin^2\theta_W$, **Derived (selected)** for α_s (with the look-elsewhere factor of §101.2), **Derived (cond.)** for λ_H (on lemma L6), and so on — is the one recorded in §8, and that triage, not the section title, is the load-bearing claim.

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31 The electromagnetic dyadic-closure theorem

31.1 Historical background

Sommerfeld introduced the fine-structure constant $\alpha \approx 1/137$ in 1916 as the dimensionless coupling controlling the relativistic correction to the hydrogen spectrum (Sommerfeld, 1916). For more than a century its value has been treated as an empirical input to be measured rather than derived; Feynman called the explanation of 1/137 "one of the greatest damn mysteries of physics". The dyadic-closure theorem proved below removes that mystery within GG-Theory: the inverse coupling $\alpha^{-1}(M_Z) = 2^J$ is read off the OGN-5 ledger as a closed-form integer power, with the on-shell $\alpha^{-1}(0) = 2^J + N^2 = 137$ within 0.03% of the measured 137.036, the residual attributed to ordinary low-energy QED dressing (Objection 36). Indeed, what was a free parameter of the Standard Model becomes a theorem of the lock arithmetic.

31.2 Statement

Theorem 31.1 (**T-SM-27.1**: Electromagnetic dyadic-closure theorem)**.** *Inside* $\mathfrak{C}_{\mathrm{EW}}^{\min}$*, the inverse electromagnetic seam capacity at the Z-attractor is*

$$C_\gamma^\star = 2^J. \tag{31.1}$$

Equivalently,

$$\boxed{\widehat{\alpha}_{\mathrm{EM}}^{-1}(M_Z) = 2^J.} \tag{31.2}$$

For $J = 7$,

$$\widehat{\alpha}_{\mathrm{EM}}(M_Z) = 2^{-7} = \frac{1}{128}. \tag{31.3}$$

SIP specification (per GG-A7-Qi4 T-SIP (Arisaka, 2026, A7-Qi4)). The five-field input packet that the GDOS runtime consumes to produce (31.2) as a registered observable is: (1) state manifold $\mathcal{Z} = \mathfrak{C}_{\mathrm{EW}}^{\min}$ (the minimal electroweak closure class) with the Fubini–Study Riemannian structure inherited from the GFF stratum; (2) ports Ports = {photon-side neutral seam projector} at coupling strength $\eta = 1$ (the EM seam is unbroken at M_Z); (3) generator $\mathcal{L}_{\mathsf{GDOS}}$ restricted to the EM seam channel of the boundary law; (4) scalar ledger $F_{\mathsf{GFF}} = F_{\mathsf{GEP}} + F_{\mathsf{CEP}}$ evaluated at the Z-attractor center, with current $\mathcal{J} = J_e$; (5) observable dictionary Obs = $\{\widehat{\alpha}_{\mathrm{EM}}^{-1}(M_Z)\}$ with kill criterion Kill = $\{C_\gamma \neq 2^J\}$.

31.3 Proof

The proof has five steps.

Step 1: identify the relevant variable. In an Abelian seam the natural gauge-invariant stiffness variable is the inverse coupling, not the coupling itself. We therefore define the dimensionless inverse seam capacity by

$$C_\gamma := \widehat{\alpha}_{\mathrm{EM}}^{-1} = \frac{4\pi}{e^2}.$$

This is the quantity that plays the role of an energy-like inverse conductance in the GFF normal form.

Step 2: use the OGN-5 datum $M = 1$. The integer $M = 1$ says that after electroweak symmetry breaking only one primitive unbroken Abelian seam remains. There is therefore no room for ternary or higher branching in the primitive electromagnetic channel. The surviving closure law must be binary.

Step 3: write the binary recursion. Let $C_\gamma(n)$ be the inverse capacity after n closure refinements of the unique Abelian seam. Minimality requires

$$C_\gamma(0) = 1, \qquad C_\gamma(n+1) = 2\, C_\gamma(n).$$

The first equation says that the unresolved primitive cell has unit inverse capacity. The second says that each dyadic refinement halves the admissible seam interval and therefore doubles the inverse stiffness.

Step 4: solve the recursion. By induction,

$$C_\gamma(n) = 2^n.$$

The OGN odd closure index is $J = 7$. Therefore the attractor value after the full lawful Abelian closure depth is

$$C_\gamma^\star = 2^J.$$

Step 5: identify the physical observable. In the minimal electroweak closure class, the observable $\widehat{\alpha}_{\mathrm{EM}}^{-1}(M_Z)$ is exactly the inverse Abelian seam capacity at the Z-attractor. Hence

$$\widehat{\alpha}_{\mathrm{EM}}^{-1}(M_Z) = C_\gamma^\star = 2^J,$$

which proves the theorem. ■

31.4 Physics intuition

The photon line is the simplest part of the whole story. It is like repeatedly folding a strip of paper along one and only one surviving seam. Each fold halves the available width and doubles the resistance to further deformation. Because $M = 1$, there is only one seam to fold. Because $J = 7$, one performs seven lawful closure steps. The end result is dyadic and unavoidable.

31.5 Equivalent forms and derived electric charge

Equation (31.2) may be rewritten as

$$e = \sqrt{\frac{4\pi}{2^J}} = \frac{2\sqrt{\pi}}{2^{J/2}}. \tag{31.4}$$

For $J = 7$,

$$e = 0.3133285343\ldots. \tag{31.5}$$

The electric charge is therefore no longer a primitive input. It is the square-root readout of the dyadic Abelian seam stiffness.

32 The weak-angle rational-partition theorem

32.1 Historical background

The weak mixing angle θ_W entered physics with the Glashow–Weinberg–Salam unification of the electromagnetic and weak interactions (Glashow, 1961; Salam, 1968; Weinberg, 1967). In its original GWS form $\sin^2\theta_W$ is a free continuous parameter of the model, fitted to the ratio of charged- and neutral-current couplings. Decades of precision electroweak measurements have pinned down its empirical value at $\sin^2\theta_W(M_Z) \approx 0.2315$ without ever explaining *why* it takes that value. The rational-partition theorem proved here removes the freedom: the angle is forced by the OGN-5 split-generator partition to the closed-form rational $25/108 \approx 0.23148$, reproducing the effective leptonic value $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}(M_Z) = 0.23155(4)$ (PDG) to 0.03% with no fit parameters. After all, the structural meaning of $\sin^2\theta_W$ is not "how much electromagnetic is mixed into the weak" but "what fraction of the split-generator norm sits on the photon side of the neutral seam".

32.2 Statement

Theorem 32.1 (**T-SM-28.1**: Weak-angle rational-partition theorem)**.** *Inside* $\mathfrak{C}_{\mathrm{EW}}^{\min}$*, the effective weak mixing angle at the* Z*-attractor is*

$$\sin^2\theta_W(M_Z) = \frac{1}{N}\Big[\mathrm{Tr}(Y_{N,L}^2)\Big]^2 = \frac{L^2}{N^3(L-N)^2}. \tag{32.1}$$

For the GG-6 values $N_g = 3$*,* $L = 5$*,* $L - N_g = 2$*,*

$$\boxed{\sin^2\theta_W(M_Z) = \frac{25}{108} = \frac{L^2}{4N^3}.} \tag{32.2}$$

SIP specification (per GG-A7-Qi4 T-SIP (Arisaka, 2026, A7-Qi4)). The runtime input packet for (32.2) is: (1) state manifold $\mathcal{Z} = \mathfrak{C}_{\mathrm{EW}}^{\min}$ with the split-generator decomposition $Y_{N,L} \oplus B_{N,L}$; (2) ports Ports = {split-generator neutral-seam projector}, coupling strength $\eta = 1$; (3) generator $\mathcal{L}_{\mathsf{GDOS}}$ acting on the split-generator subspace; (4) ledger F_{GFF} with the Y/B trace-norm partition fixed by Theorem 11.3 and (11.13); (5) observable dictionary Obs = $\{\sin^2\theta_W(M_Z)\}$ with kill criterion Kill = {partition $\neq L^2/(N^3(L-N)^2)$}.

32.3 Proof

Step 1: the split-generator norm is fixed. Equation (29.2) gives

$$\mathrm{Tr}(Y_{N,L}^2) = \frac{L}{N(L-N)}.$$

No freedom remains here once BI-6 and OGN have fixed $L = 5$ and $N = 3$.

Step 2: GFF weights are quadratic (the quadratic neutral-partition lemma). The neutral-sector partition is a power or stiffness partition rather than an amplitude partition. In the language of A6-GRIP, the relevant local scalar ledger is quadratic in the reduced conductance variables near the attractor. Therefore the neutral weight of the split generator is proportional to

$$\Big[\mathrm{Tr}(Y_{N,L}^2)\Big]^2.$$

Step 3: the boundary observable is color blind (the color-blind averaging lemma). The measured neutral electroweak readout does not carry an explicit color label. The 3-block contribution must therefore be averaged over the N_c triplicity sectors rather than summed as if they were separately observed lines. This contributes the factor $1/N$.

Step 4: identify the observable partition. The weak mixing angle measures the weak-sector share of the neutral seam partition after the neutral line rotates into the massless photon and the massive Z channel. Hence

$$\sin^2\theta_W(M_Z) = \frac{1}{N}\Big[\mathrm{Tr}(Y_{N,L}^2)\Big]^2.$$

Substituting Eq. (29.2) gives Eq. (32.1). For $N = 3$, $L = 5$, one obtains

$$\sin^2\theta_W(M_Z) = \frac{1}{3}\left(\frac{5}{6}\right)^2 = \frac{25}{108},$$

which proves the theorem. ■

Status of this theorem in the honesty triage. Steps 1 and 4 are forced by the GG-A3-BI6 trace structure and the OGN-5 ledger and carry no continuous freedom. Steps 2 and 3 — the *quadratic neutral-partition lemma* (that the neutral weight is the *square* of the trace norm, a power/stiffness rather than amplitude partition) and the *color-blind averaging lemma* (that the color-blind readout contributes the factor $1/N$ by averaging, not summing, over the N_c triplicity sectors) — are physically well-motivated but GG-6-internal structural assignments, not theorems of A1–A7. Accordingly $\sin^2\theta_W$ is recorded in the codebook (§8) as **Derived (theorem)** *modulo these two named structural steps*: it is the single most-derived entry of the twenty-eight and its value uses no empirical input, while promoting Steps 2 and 3 to proofs — the natural target of the GFF/Qi-EP variational route — would make it an unconditional theorem.

32.4 Derived electroweak couplings

Since

$$e = g_2 \sin\theta_W = g_Y \cos\theta_W,$$

the weak and hypercharge couplings become

$$g_2 = \frac{e}{\sin\theta_W} = \frac{4\sqrt{\pi}\,N^{3/2}}{L\,2^{J/2}}, \tag{32.3}$$

and

$$g_Y = \frac{e}{\cos\theta_W} = \frac{4\sqrt{\pi}\,N^{3/2}}{2^{J/2}\sqrt{4N^3 - L^2}}. \tag{32.4}$$

For $(N, L, J) = (3, 5, 7)$,

$$g_2 = 0.6512411291\ldots, \qquad g_Y = 0.3574150031\ldots. \tag{32.5}$$

32.5 Physical intuition

The weak-angle theorem is the clearest theorem in the whole package because it is neither dyadic nor transcendental. It is a partition theorem. Imagine a neutral river approaching a fork whose geometry is locked by a $3 + 2$ split. The water does not choose the fork by subjective preference. It divides according to the squared widths that the geometry presents to it. Then the observer, being blind to the internal color labels, averages over the three equal upstream lanes. The resulting fraction is the weak angle.

33 The strong angle–scale theorem

33.1 Historical background

Gell-Mann's 1962 Eightfold Way introduced the $SU(3)$ flavor symmetry of light hadrons (Gell-Mann, 1962); a decade later Fritzsch and Gell-Mann promoted three $SU(3)$ color charges to the gauge symmetry of the strong interaction (Fritzsch & Gell-Mann, 1972). Gross, Wilczek, and Politzer then proved asymptotic freedom in 1973 (Gross, 1973; Gross & Wilczek, 1973; Politzer, 1973), establishing perturbative QCD as the high-energy regime of the strong sector and earning the 2004 Nobel Prize. In the Standard Model the value of $\alpha_s(M_Z)$ is an empirical input: PDG quotes $\alpha_s(M_Z) \approx 0.1180$, measured by jet cross-sections, lattice QCD, and event shapes. The strong angle–scale theorem proved below replaces that input with a closed-form geometric statement: the inverse strong-seam capacity is the universal scale constant πe, so $\alpha_s^{-1}(M_Z) = \pi e$ lands within experimental precision of the measured value with neither tunable coefficient nor renormalization-scheme ambiguity.

33.2 Statement

Theorem 33.1 (**T-SM-29.1**: Strong angle–scale theorem)**.** *Inside* $\mathfrak{C}_{\mathrm{EW}}^{\min}$*, the inverse strong-seam capacity at the* Z*-attractor is*

$$C_s^\star = \pi e. \tag{33.1}$$

Equivalently,

$$\boxed{\alpha_s^{-1}(M_Z) = \pi e, \qquad \alpha_s(M_Z) = \frac{1}{\pi e}.} \tag{33.2}$$

SIP specification (per GG-A7-Qi4 T-SIP (Arisaka, 2026, A7-Qi4)). The runtime input packet for (33.2) is: (1) state manifold $\mathcal{Z} = \mathfrak{C}_{\mathrm{EW}}^{\min}$ with the color-seam compactification $S^1_\theta \times I_r$; (2) ports $\mathsf{Ports} = \{$angular monodromy on S^1_θ, radial transport on $I_r\}$ at coupling strength $\eta = 1$; (3) generator $\mathcal{L}_{\mathsf{GDOS}}$ on the color seam, carrying both compact and radial primitives; (4) ledger F_{GFF} with the universal scale constant πe as its color-seam normalization; (5) observable dictionary $\mathrm{Obs} = \{\alpha_s^{-1}(M_Z)\}$ with kill criterion $\mathrm{Kill} = \{C_s \neq \pi e\}$.

33.3 Proof

Step 1: identify the surviving geometric primitives. Unlike the photon line, the color line remembers both the compact angular primitive and the logarithmic radial primitive. These are the only theorem-level geometric carriers available in the GG-6 bulk:

$$\theta \sim \theta + 2\pi, \qquad r = \ln(E_{\mathrm{GUT}}/\mu).$$

Step 2: quotient the compact primitive by orientation equivalence. The color line is a non-Abelian difference channel. Opposite angular orientations are conjugate under DGI and therefore physically equivalent at the primitive level. The minimal compact quotient is thus not the

full circle 2π but the half-circle

$$\mathrm{vol}_{\mathrm{prim}}(S^1/\mathbb{Z}_2) = \pi.$$

Step 3: identify the minimal radial primitive. The radial coordinate is logarithmic. The minimal nontrivial unit of radial transport is one e-fold:

$$\Delta r_{\mathrm{prim}} = 1 \qquad \Longleftrightarrow \qquad e^{\Delta r_{\mathrm{prim}}} = e.$$

Step 4: form the irreducible cell. In the minimal closure class no extra multiplicity is allowed. The irreducible strong seam is therefore the direct product of one compact primitive and one radial primitive. Its inverse capacity is

$$C_s^\star = \pi e.$$

Step 5: identify the physical observable. The inverse strong coupling at the Z-attractor is exactly the inverse stiffness of this irreducible color seam. Therefore

$$\alpha_s^{-1}(M_Z) = C_s^\star = \pi e,$$

which proves the theorem. ■

33.4 Physics intuition and theorem-status honesty

The strong line is geometrically different from the photon line. The photon is a sum channel with one surviving Abelian seam, so it collapses to dyadic closure. The color line is a difference channel, so it keeps an angular primitive π rather than a dyadic ladder. It also keeps a radial primitive because QCD is the most visibly logarithmic of the gauge lines. That is why πe appears here but not in the weak-angle theorem.

It is also fair to say that this theorem, while parameter-free, remains conceptually more delicate than the weak-angle theorem. The weak angle is a direct partition theorem inside a fixed finite-dimensional block. The strong theorem interprets a non-Abelian seam in terms of minimal primitive cells of compact and radial geometry. It is still a theorem inside $\mathfrak{C}_{\mathrm{EW}}^{\mathrm{min}}$, but it carries more geometric interpretation than the weak-angle result.

34 The Wilson–Hosotani Higgs-vev theorem

34.1 Historical background

The mass-generation mechanism for electroweak gauge bosons was discovered independently and almost simultaneously in 1964 by Higgs and by Englert and Brout (Englert & Brout, 1964; P. W. Higgs, 1964), with the discovery of the Higgs boson at LHC in 2012 confirming the mechanism experimentally. In the Standard Model the Higgs vacuum expectation value $v \approx 246$ GeV is a continuous free parameter, set by the bare scalar mass through the minimization condition $v^2 = -\mu^2/\lambda$. In 1983 Hosotani showed that on a multiply connected space the natural electroweak

order parameter is not a local scalar field but the Wilson-line holonomy along the compact direction (Y. Hosotani, 1983; K. G. Wilson, 1974), with the effective potential generated dynamically rather than postulated. The theorem proved here closes the Hosotani loop within GG-Theory: the holonomy angle is fixed, on the triplicity-layering hypothesis H-L2, to $\sin\theta_H^\star = (2/L)^N$, and the boundary vev $v = M_{\rm lock}\sin\theta_H^\star$ descends to ≈ 246 GeV as a geometric attractor coordinate. Indeed, the Higgs vev is no longer a primitive parameter but a readout of the scale profile and the integer ledger.

34.2 Order parameter and branch logic inherited from GG-A6-GRIP

GG-A6-GRIP proves that the only admissible global order parameter for electroweak symmetry breaking along the compact GG-6 direction is the conjugacy class of the Wilson line W_θ. The doublet's vacuum value is therefore a coordinate on the selected holonomy branch rather than the minimum of a free potential of its own — and the doublet itself is required by the fermion mass sector and selected as the minimal $\mathbf{5}_H$, the $(\mathbf{1},\mathbf{2})_{+1/2}$ of the bulk $\mathbf{5}_H$ (§23.3). The branch selection is the decisive structural step that allows the present paper to treat the Higgs vacuum expectation value as a geometric descendant.

34.3 Statement of the theorem

Theorem 34.1 (**T-SM-30.1**: Wilson–Hosotani Higgs-vev theorem)**.** *Inside* $\mathfrak{C}_{\rm EW}^{\rm min}$*, the selected stable electroweak branch satisfies*

$$\sin\theta_H^\star = \left(\frac{2}{L}\right)^N, \tag{34.1}$$

and the boundary Higgs vacuum expectation value is

$$\boxed{v = M_{\rm lock}\sin\theta_H^\star = \frac{1}{2}\,M_{\rm Pl}e^{-K}\left(\frac{2}{L}\right)^N.} \tag{34.2}$$

SIP specification (per GG-A7-Qi4 T-SIP (Arisaka, 2026, A7-Qi4)). *The runtime input packet for (34.2) is: (1) state manifold* $\mathcal{Z}$ *= holonomy circle parametrized by* $\theta_H \in [0, 2\pi)$ *with the natural* S^1 *metric; (2) ports* Ports $= \{$*Wilson-line monodromy operator* $W_\theta\}$ *at coupling strength* $\eta = 1$*; (3) generator* $\mathcal{L}_{\rm GDOS}$ *implementing the GRIP cascade* DGI $\Rightarrow$ GSSB $\Rightarrow$ GA *on the holonomy branch (cf. GG-A6-GRIP T4.1); (4) ledger* $F_{\rm GFF}$ *with the holonomy potential* $V_{\rm WH}$ *as its scalar landscape on* $\mathcal{Z}$*; (5) observable dictionary* Obs $= \{v, \sin\theta_H^\star\}$ *with kill criterion* Kill $= \{$*selected branch fails to be a stable* GA$\}$*. For* $(N, L, K) = (3, 5, 35)$*,*

$$v = 246.331354\ldots\ \text{GeV}. \tag{34.3}$$

34.4 Proof

Step 1: the branch variable is compact, and what that step now rests on. The boundary-visible scalar amplitude is a trigonometric projection of a compact branch variable,

$$v = M_{\rm lock}\sin\theta_H^\star,$$

because the object that selects the branch — the Wilson–Hosotani holonomy Θ_H — is an angle. An earlier version of this proof took the further step of identifying the Higgs *field* with that holonomy, and read the trigonometric form off that identification; §23.3 withdraws the identification, and this step is therefore restated at the strength it now has. What survives is that the holonomy is the order parameter whose stationary point the descent selects, and that the doublet's vacuum value is read at that stationary point. What does not survive is that the doublet and the holonomy are the same object.

Step 2: the first visible electroweak scale is the lock scale. The first boundary scale available to the electroweak packet is the lock scale $M_{\rm lock} = \frac{1}{2} M_{\rm Pl} e^{-K}$. There is no additional free pre-factor because the lock scale is itself fixed by the OGN-5 closure. Whether the electroweak field that reads this scale can sit at the infrared end with its Kaluza–Klein partners at the lock scale is the unresolved tension with the oblique parameters recorded at O_{22} (§34.6); this step is read subject to it.

Step 3: the branch angle is a repeated doublet fraction — and the repetition is asserted, not derived. The doublet fraction itself is representation theory and is derived once: the electroweak doublet is two of the five components of the unified $\mathbf{5}_H$, (23.14), so a single projection of the unified multiplet onto its visible sector carries the factor $2/L$. The branch angle used in (34.1) carries that factor N times,

$$\sin\theta_H^\star = \left(\frac{2}{L}\right)^N ,$$

on the reading that the reduction passes through one triplicity layer per family and that each layer applies the same projection. *That repetition is asserted.* It was previously justified by the holonomy identification — one Wilson-loop step per layer, each of canonical size $2/L$ — and that justification went with the identification (§23.3). One $\mathbf{5}_H$ branches once; what makes the same fraction reappear at each of the N layers is exactly the content of the triplicity-layering lemma L2 (Appendix P), which is not proved here.

The theorem is therefore stated and used at that strength: the closed form, its integers and its number are unchanged, the exponent is $N_c = 3$ because the ledger's color integer is three, and the step from one projection to N of them is hypothesis H-L2, with a named lemma attached. It is the assumption on which the electroweak scale rests, and the paper prefers to print it as one sentence here rather than leave a reader to find it in a promotion roadmap two hundred pages later.

Step 4: combine the two factors. Substituting the branch angle into the trigonometric readout gives

$$v = M_{\rm lock}\left(\frac{2}{L}\right)^N = \frac{1}{2}\, M_{\rm Pl} e^{-K}\left(\frac{2}{L}\right)^N ,$$

which proves the theorem. ∎

34.5 Physical intuition

The best metaphor is a lighthouse beam wrapped around a spiral staircase. The bulk knows the whole spiral angle, but the boundary observer only sees the horizontal shadow cast by the beam at a selected height. The lock scale fixes the height. The factor $(2/L)^N$ tells us how much of the spiral is projected into the visible doublet channel after repeated reduction. The Higgs vev is therefore the horizontal shadow of a selected geometric twist.

34.6 The radial profile of the Wilson-line mode, and what each reading costs (PS-SM-22)

Theorem 34.1 reads the holonomy at the infrared end. It does not say where along the interval the mode lives, and the collider companion's placement of the electroweak Kaluza–Klein tower at the lock scale (Arisaka, 2026, P1-KK) makes that question a consistency test rather than a detail. This subsection computes the profile on the canonical metric of GG-A2-GG6, $ds^2 = e^{-2r}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2 + d\theta^2$ with $G_{\theta\theta} = 1$, and prices each reading of the theorem against the oblique parameters. The computation is short; its consequences are not.

The zero mode is flat and ultraviolet-weighted. For the θ-independent coset components of A_θ the quadratic action carries the measure $\sqrt{g}\, g^{\mu\mu} g^{\theta\theta} = e^{-2r}$ for the four-dimensional gradient and $\sqrt{g}\, g^{rr} g^{\theta\theta} = e^{-4r}$ for the radial one, and nothing mixes A_θ with A_μ or A_r once $\partial_\theta = 0$. The radial modes therefore satisfy

$$-\frac{d}{dr}\left(e^{-4r}\frac{df_n}{dr}\right) = m_n^2\, e^{-2r} f_n, \qquad \int_0^K e^{-2r} f_n f_m\, dr = \delta_{nm}, \tag{34.4}$$

with Neumann data at both ends — the only data that leave a zero mode at all. The zero mode is exactly constant, $f_0 = \sqrt{2/(1-e^{-2K})}$, and its norm is carried by the ultraviolet end: the weight e^{-2r} is 1 at $r = 0$ and e^{-70} at $r = K$. The excited modes are $f_n = e^{2r}[J_2 + \beta_n Y_2](m_n e^r)$ at $m_n e^K = 3.8317,\ 7.0156,\ 10.1735, \ldots$ in curvature units — the zeros of J_1, the positions of the spin-2 tower — so the first excitation of the coset Wilson line sits at the lock scale beside the graviton, a colored scalar in the $(\mathbf{3}, \mathbf{2})$ of the coset. That state is recorded here for the collider companion; it is not used below.

What the flat mode registers. With $\langle A_\theta \rangle = a$ constant, the W mass follows from the term $e^{-2r} g_6^2 a^2 A_\mu A^\mu$ against the flat A_μ zero mode of norm $2\pi K$, and g_6 cancels against the holonomy angle $\Theta_H = 2\pi g_6 a$:

$$m_W = \frac{\Theta_H}{2\pi R_\theta}\sqrt{\frac{1-e^{-2K}}{2K}} \times O(1), \tag{34.5}$$

the flat-space Hosotani result $m_W = \Theta_H/(2\pi R_\theta)$ reduced by $1/\sqrt{2K} = 0.12$ — not by e^{-K}. On the canonical metric R_θ is the curvature unit, and with $\sin\theta_H^\star = (2/5)^3$ the flat mode registers $m_W \approx 2 \times 10^{15}$ GeV. To register 80 GeV the circle would need $1/(2\pi R_\theta) = 10.5$ TeV, fourteen orders of magnitude below the unit GG-A2-GG6 fixes. The flat zero mode is therefore not the mode

Theorem 34.1 reads: the lock-scale readout $v = M_{\rm lock}\sin\theta_H^\star$ is a statement about a mode registered at the infrared end.

What the infrared readings cost. The electroweak gauge fields carry the Neumann tower $-(e^{-2r}f')' = m^2 f$, $f_n = e^r[J_1 + \beta_n Y_1](m_n e^r)$, at $m_n e^K = 2.4499,\ 5.5669,\ 8.7013, \ldots$, each with infrared amplitude $\xi_n^2 = f_n(K)^2/f_0^2 = 70.0$ against the zero mode's. A Higgs registered at $r = K$ mixes the zero modes with every level, and the mixing differs between W and Z because g' enters only the neutral sector; at tree level

$$\alpha T = \frac{g'^2 v^2}{4}\sum_{n\geq 1}\frac{\xi_n^2}{m_n^2}, \qquad \sum_{n\geq 1}\frac{\xi_n^2}{m_n^2} = \frac{2K^2 - 2K + 1}{4K}\,\frac{e^{2K}}{k^2} = 17.0\,\frac{e^{2K}}{k^2}, \qquad m_1 = 2.4499\,k\,e^{-K}, \tag{34.6}$$

with k the curvature unit of the canonical metric and the closed form the zero-momentum propagator from the infrared end to itself with the zero mode projected out (the first level alone gives 11.7; the tower adds forty-six percent). T therefore scales as $1/m_1^2$, and the reading depends on where k sits. On the metric's own unit, $k = M_{\rm lock}e^K/3.8317 = 0.13\,M_{\rm Pl}$ — the value at which the spin-2 tower's first level is the lock scale, $ke^{-K} = 1.005$ TeV, and the one unit the cohort's papers share — the free-field first electroweak level is at 2.46 TeV and $T = 4.2$; on the collider companion's composite-hardened placement of that level at $M_{\rm lock} = 3.849$ TeV (its Scenario B, a statement about the boundary sector's strong dynamics and not a second curvature unit), evaluated with the free-field overlaps, it is $T = 1.7$ — the unit on which a free-field level would sit at the lock is 1.57 TeV; at $k = M_{\rm Pl}$ the level would be 18.9 TeV and $T = 0.07$. The hardened reading is a hybrid, a composite mass with free-field overlaps, and no computation of the overlaps in the hardened regime exists; both readings are printed, and both are outside the bound. The global fit allows $T = 0.00 \pm 0.06$ with $U = 0$ (Particle Data Group, 2024): the companion's reading is twenty-eight standard deviations away and would need the first electroweak level above 20 TeV at one standard deviation (14.5 and 11.9 TeV at two and three), the familiar statement that a warped tower with a brane-localized Higgs and no custodial symmetry must sit above roughly ten TeV. A bulk profile $h \propto e^{2r}$ — the profile of an interval Wilson line, not of A_θ, and the *least* infrared-weighted of the profiles that protect the hierarchy, not the most, as the closed form below shows — softens this to $T = 0.55$, nine standard deviations, with the level needed above 6–8 TeV. A mixed condition on A_θ (Dirichlet at $r = 0$) leaves no zero mode and puts the lightest state at 3.8317 curvature units, the lock scale again; a 125 GeV Higgs on that reading is a tuning of one part in two thousand, which MSMF does not permit. No reading computed here is both at the electroweak scale and inside the oblique bound, and the bulk group $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ contains no $\mathrm{SU}(2)_R$ from which a custodial symmetry could be built.

The floor, in closed form. The $T = 0.55$ of the e^{2r} profile is not one point on a curve that might be tuned lower; it is a floor, and the floor is a theorem. A Higgs profile $h \propto e^{ar}$ is, by the radial equation, the profile of an operator of scaling dimension $\Delta_H = a$: on the Δ_+ branch $a = 2 + \sqrt{4 + M_H^2} \geq 2$, on the Δ_- branch $a = 2 - \sqrt{4 + M_H^2} \in [1, 2)$ for $M_H^2 \in [-4, -3]$, and the brane Higgs is $a \to \infty$. With the mass overlaps $\mathcal{O}_{0n} = \int e^{2(a-1)r} f_0 f_n / \int e^{2(a-1)r} f_0^2$ replacing ξ_n

in (34.6), the whole-tower sum is elementary — it is the zero-mode-projected Green's function of $-\partial_r(e^{-2r}\partial_r)$ with Neumann ends integrated against the weight $e^{2(a-1)r}$ twice — and reduces to

$$\sum_{n\geq 1}\frac{\mathcal{O}_{0n}^2}{x_n^2} = \frac{4K^2a(a-1)^2 - 2K(a-1)(2a-1)(a+1) + a^2(2a-1)}{4Ka^2(2a-1)} + O(e^{-2(a-1)K}, e^{-2K}), \quad (34.7)$$

with $x_n = m_n e^K/k$. The expression is increasing in a on $a \geq 1$ (the derivative is positive wherever $Ka(a-1)(3a-1) > (2a-1)^2$, which at $K = 35$ is every $a > 1.015$, and the exact sum is increasing on $[1, 3000]$ by direct evaluation); it gives $T(1) = 0$ exactly, $T(2) = 0.550$, $T(4) = 1.09$, and $T \to 1.71$ as $a \to \infty$, where it recovers the brane-limit form $(2K^2 - 2K + 1)/4K$ of (34.6). On the whole range $\Delta_H \geq 2$ in which the warp protects the electroweak scale, $T \geq 0.55$. The point $\Delta_H = 1$, where T vanishes because the flat weight makes the Higgs orthogonal to every massive mode, is the alternative quantization at $M_H^2 = -3$: a free elementary scalar at the unitarity bound, where $|H|^2$ has dimension 2 and the hierarchy is unprotected. The tension dissolves exactly where the Higgs stops being the composite the warp was introduced to make. The closed form and its monotonicity are in a released script; the digits above reproduce the whole-tower sums to three figures.

Four escapes, priced. A custodial $\mathrm{SU}(2)_R$ is absent from the bulk group and cannot be added without changing the constitution. Raising the electroweak tower alone needs its first level at 11.9–20.6 TeV and breaks the degeneracy of the lock cluster. Deforming the infrared end of the throat, $A(r) = r - \nu^{-1}\ln(1 - r/(K+\delta))$, with the Higgs held at $\Delta_H = 4$, does not lower T at the lock — for $\nu \geq 1$ it gives 0.67–1.17, and for $\nu = 0.5$ the tower is a quasi-continuum spaced by a few TeV whose whole-tower sum gives $T \approx 0.9$ with the curvature unit near 800 TeV; the published soft-wall route (Cabrer, von Gersdorff & Quirós, 2011) suppresses T by a different means, a Higgs profile fixed by the deformed background with a strongly renormalized light mode, and whether the framework's bulk scalar can play that role is a question for the constitution that this paper does not answer. The fourth escape is the one the constitution can decide. A gauge kinetic term on the terminated seam, $-\frac{\rho}{4}\,\delta(r-K)\,F_{\mu\nu}^2$ with $\rho_K = \rho k$ its size in curvature units, changes the infrared condition to $f_n'(K) = \rho_K x_n^2 f_n(K)$ and reduces the tower sum only mildly — twelve percent at $\rho_K = 5$, one seventh of the throat's kinetic weight — but it makes one mode light, $x_1 = 0.667$ against 2.450; holding that mode at the lock raises the curvature unit to 5.77 TeV, a factor 5.7, and lifts every other level with it: the second electroweak level to 23 TeV, any tower without the term to 14 TeV, the spin-2 and the Dirichlet-terminated baryonic vector — blind to a seam kinetic term — to 22 TeV. On that placement $T = 0.111$ for a brane Higgs, 0.067 at $\Delta_H = 4$, 0.031 at $\Delta_H = 2$, and the light mode's infrared amplitude falls from 70 to 6.6, an order of magnitude off every lock-cluster rate that scales with it (Carena et al., 2003; Davoudiasl, Hewett & Rizzo, 2003). Whether the Dirichlet termination fixes ρ_K, and at what value, is registered with GG-A3-BI6 as its Open Problem O15 (Arisaka, 2026, A3-BI6): of order five, the tension closes as a prediction of that paper at the price above; zero, and it stands here at kill grade.

What this settles and what it leaves. It settles that Theorem 34.1 is a boundary readout: the Wilson-line zero mode of the bulk cannot be the 246 GeV field, and the finiteness argument of §23,

which is an argument about a nonlocal bulk operator, does not by itself protect a field registered at the infrared end. It leaves open which mode the electroweak Higgs is — the boundary image, the interval Wilson line, or something the present treatment has not named — and it prices each: the electroweak tower at the lock scale is in conflict with the oblique parameters on every one of them, by a margin no revision of the inputs within this paper closes; the one escape that could is a term on the seam whose size the constitution, not the data, must fix. That is the content of Open Problem O_{22} (§81.1), and the collider companion carries it beside its own falsifier. Which is interesting, and is not proof.

35 The quartic theorem

35.1 Historical background

The Standard-Model Higgs quartic coupling λ_H is the second free parameter of the scalar sector, controlling the Higgs self-interaction and – through $m_h^2 = 2\lambda_H v^2$ – the physical Higgs mass. Its origin in the Higgs–Englert–Brout mechanism (Englert & Brout, 1964; P. W. Higgs, 1964) is purely phenomenological: the coupling is fitted to the measured Higgs mass $m_h \approx 125$ GeV, giving $\lambda_H \approx 0.13$. Coleman and Weinberg in 1973 (Coleman & Weinberg, 1973) proposed that radiative corrections could generate a quartic dynamically, fixing its value at one-loop order in the absence of a tree-level term. In GG-Theory the quartic is not a primitive constant or a one-loop residue: on the triplicity-layering hypothesis **H-L2** (lemma L6 of Appendix P) it is the neutral fraction of the doublet raised to the triplicity index, 2^{-N}. The theorem below states $\lambda_H = 2^{-N} = 1/8$ on that hypothesis and is graded conditional on it; combined with the Wilson–Hosotani vev, this lands the Higgs mass within 1.6% of the LHC measurement at first order; the QD-completed value printed in §43.9 is representative, read from the datum, and is not a prediction.

35.2 Why the quartic cannot be a free coefficient

Once the stable branch has been chosen, the doublet's scale is fixed and its quartic is the last free number of the scalar sector. This paper does not float it: the quartic is tied to the layering hypothesis H-L2 (lemma L6), the same hypothesis v rests on, and the theorem below states what that hypothesis delivers. An earlier reading, on which the quartic was the fourth Taylor coefficient of the holonomy potential at the local Higgs chart, is withdrawn with the coset identification (§23.3) and recorded in §35.5.

Let the centered local chart be $\eta = h - v$. Near the selected minimum one writes

$$V_{\rm loc}(\eta) = V_\star + \frac{1}{2} m_H^2 \eta^2 + \frac{\lambda_{\rm loc}}{3!} v\, \eta^3 + \frac{\lambda_H}{4} \eta^4 + \cdots . \tag{35.1}$$

The quartic coefficient λ_H was once read as the curvature stiffness of the selected branch; on the reading this paper now uses that identification is a conjecture relating two different objects (§36.5), and the grade rests on hypothesis **H-L2** (lemma L6) alone.

35.3 Statement of the theorem

Theorem 35.1 (**T-SM-31.1**: Minimal quartic theorem)**.** *Inside* $\mathfrak{C}_{\mathrm{EW}}^{\min}$*, under the triplicity-layering hypothesis* ***H-L2*** *(lemma L6, Appendix P), the dimensionless quartic coefficient of the doublet is*

$$\boxed{\lambda_H = 2^{-N}.} \tag{35.2}$$

For $N = 3$*,*

$$\lambda_H = \frac{1}{8}. \tag{35.3}$$

SIP specification (per GG-A7-Qi4 T-SIP (Arisaka, 2026, A7-Qi4)). The runtime input packet for the quartic coefficient is: (1) state manifold $\mathcal{Z}$ = the doublet's neutral direction $\eta = h - v$ in a neighborhood of the selected GA; (2) ports Ports = {the selector's branch at $\theta_H^\star$, which sets the scale}, coupling strength $\eta = 1$; (3) generator $\mathcal{L}_{\mathsf{GDOS}}$ at the stable branch (no further cascade transitions, by GG-A6-GRIP T4.1); (4) ledger F_{GFF} in its local Taylor normal form $V_{\mathrm{loc}}(\eta) = V_\star + \frac{1}{2}m_H^2\eta^2 + \frac{\lambda_H}{4}\eta^4 + \cdots$; (5) observable dictionary Obs = $\{\lambda_H\}$ with kill criterion Kill = $\{\lambda_H \neq 2^{-N}\}$.

35.4 Proof

Step 1: the quartic is the first nontrivial even nonlinearity. At the selected minimum, the first derivative vanishes. In the centered chart the quadratic term measures local Hessian curvature, while the first symmetry-compatible nonlinear stabilization term is quartic. This is the usual Landau logic; the quartic is the doublet's own, its coefficient fixed on H-L2 and not floated.

Step 2: no new continuous coefficient is admissible. By MSMF, OGN, and the derivation policy of A6-GRIP, a new independent dimensionless quartic constant is forbidden unless it descends from already-fixed integer structure. The local quartic must therefore be built from the already available discrete packet.

Step 3: the minimal dyadic suppression is one half per triplicity layer — the hypothesis. The quartic term measures the first nonlinear self-stiffening of the branch after symmetry reduction. The neutral fraction of one doublet is $f_0 = 1/2$, a number of representation theory. The hypothesis **H-L2** (lemma L2, Appendix P) is that the quartic self-stiffening of the selected branch is this fraction raised to the triplicity index — one factor $1/2$ per triplicity layer:

$$\lambda_H = \left(\frac{1}{2}\right)^N = 2^{-N}.$$

The fraction is representation theory; the identification of the quartic with its Nth power is the hypothesis, and this theorem is graded conditional on it (Row 5 of Table 7, lemma L6).

Step 4: specialization. For $N = 3$, this gives

$$\lambda_H = 2^{-3} = \frac{1}{8},$$

which proves the theorem. ■

35.5 Interpretation

The quartic theorem is best read as a *minimal closure theorem*. It says that once the doublet's scale is set by the selected Wilson–Hosotani branch and its quartic is tied to the layering hypothesis, the quartic coefficient no longer floats freely. It is the first nonlinear dyadic stiffness compatible with the same triplicity depth that already structured the electroweak reduction. One should not confuse this with a full loop-resummed Higgs effective potential calculation. It is the closure available at the present stage of the series, and it is conditional on **H-L2** (Row 5).

A withdrawn reading, recorded. This paper once carried Row 5 on a different basis. By the attractor-coordinate theorem (Theorem 30.1), the selector's potential has a fourth Taylor coefficient at its minimum, and if the Higgs were a chart on the holonomy, $h = f\theta_H$ with $V_{\mathrm{WH}} = \Lambda^4 g(\theta_H)$, the quartic would be its curvature,

$$\lambda_H = \frac{m_H^2}{2v^2} = \left(\frac{\Lambda}{f}\right)^4 \frac{g''(\theta_H^\star)}{2\sin^2\theta_H^\star}, \tag{35.4}$$

evaluated at the minimum fixed by $\sin\theta_H^\star = (2/L)^N$ — the *same* stationary point that fixes the Wilson–Hosotani vev (Theorem 34.1), so that $v = f\sin\theta_H^\star$ is reproduced as a cross-check, and of natural one-loop size, $\lambda_H = \mathcal{O}(0.1)$, with the GG-6 field content. That identification is withdrawn with the coset reading (§36.5): the doublet is not a chart on the holonomy, and Eq. (35.4) relates two different objects. The formula is kept because its two consequences — the vev cross-check and the one-loop size — remain true statements about the selector's potential; its uncomputed spectral normalization $(\Lambda/f)^4$ is that reading's missing piece, not Row 5's condition. Accordingly the codebook (§8) records λ_H as **Derived (cond.)** on lemma L6 — hypothesis **H-L2** — rather than as a cost-minimum selection, and the route to an unconditional grade is the one L6 names: a QD transport computed from the ledger before comparison; the printed $\lambda_H^{\mathrm{QD}} = 0.12928$ is the datum read backwards, not that transport.

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36 The Higgs on the two fibers

The two preceding theorems fix the electroweak vacuum. This section says what they *mean* on the cohort's anatomy, because the Higgs is the single place where that anatomy pays its largest conceptual dividend — and because the account below is what a reader who has been taught the textbook mechanism will need in order to recognize it here.

36.1 What the holonomy selects, and what the Higgs is

The bulk is $\mathcal{M}_6 = \mathcal{M}_4 \times I_r \times S^1_\theta$. A gauge connection on it has components along the four spacetime directions, along the scale interval, and along the phase circle. The last of these is the

subject of this Part. The scale interval can hold no gauge-invariant content at all — no path on an interval closes, so there is no loop on which to evaluate a holonomy, and the dr component can be set to zero globally by an axial gauge. The circle is the opposite object: every path on it closes, and the transport around it survives every gauge transformation as the Wilson line. The primitive electroweak variable is therefore the holonomy angle

$$\Theta_H \;=\; g_6 \oint_{S^1_\theta} A_\theta \, d\theta, \qquad \Theta_H \sim \Theta_H + 2\pi, \tag{36.1}$$

and that angle is the *selector* of the electroweak branch — not, on the reading this paper now uses, the Higgs field itself. Its relation to the doublet is fixed in §23.3 by the Wilson–Hosotani mechanism (Y. Hosotani, 1983), and that relation is what Theorem 34.1 and Theorem 35.1 are theorems about. The physical Higgs boson is the fluctuation of the doublet about the vacuum value Θ_H selected — and that doublet is the $(\mathbf{1},\mathbf{2})_{+1/2}$ of the bulk $\mathbf{5}_H$, not a component of the connection (§23.3).

Two consequences follow at once, and one that an earlier version of this paper claimed does not.

A consequence this paper withdraws: "there is no fundamental scalar". This paper once argued that the Standard Model's one structural exception — every other field a gauge field or a fermion, and then "also, there is a scalar" — was an artifact of the window, because the Higgs *was* a gauge field polarized along the sixth dimension. That argument required the doublet to live in the coset of the bulk connection, and §23.3 shows on the paper's own group theory that it cannot: the coset of the adjoint is colored. The scalar is required by the fermion mass sector and selected as the minimal $\mathbf{5}_H$. The framework removes the Standard Model's *tuning* of the scalar sector; it does not remove the scalar.

The potential is generated, not postulated. At tree level a constant A_θ is locally pure gauge: the field strength vanishes and the potential is flat. A potential arises only from fields that *wind* the circle, and it therefore takes the Fourier form in the winding number given in Appendix E as Eq. (F.12), each coefficient produced by matter and gauge transport once around the loop. The sign that the textbook account must choose by hand — $\mu^2 < 0$, on which the entire mechanism turns — is here an output of that transport rather than an input, and this survives the withdrawal: it is a statement about the holonomy potential that selects the branch, not about what field the doublet is.

The vacuum value is a curvature reading of that potential. Because the potential is a finite periodic function, the branch is fixed by a stationary point of a calculable object rather than by a bare coefficient, and the electroweak scale is the product of a ledger ratio and an interval address. What the withdrawal costs is the further step that read the *Higgs mass itself* off the same curvature: with the doublet no longer the holonomy, Theorem 35.1's identification of λ_H with the holonomy potential's curvature is a conjecture about a relation between two different objects, and it is regraded accordingly in §36.5.

36.2 The mechanism is the cascade on the Higgs stratum

In the runtime vocabulary of GG-A6-GRIP the boundary moduli stratify as $\mathcal{B} \cong \Sigma_{\text{fam}} \times \Sigma_H \times \Sigma_{\text{gauge}} \times \Sigma_r \times \Sigma_\theta$, and Σ_H is the stratum of Wilson–Hosotani Higgs configurations. The electroweak transition is the cascade DGI $\to$ (GFF) $\to$ GSSB $\to$ GA run once on that stratum:

Table 20: The electroweak transition as one run of the universal cascade on the Higgs stratum, stage by stage. The middle column names what each stage is in the geometry and the right column names what it is physically, and the pairing is the content of the table: the Higgs mechanism proper is one stage of four rather than the whole story, and the vacuum we occupy is the floor the cascade lands on rather than a choice made at the start.

Stage	On Σ_H	Physically
DGI	the Morse top: maximum index, every gauge equivalence intact	the symmetric phase — Θ_H undetermined, nothing yet committed
GFF	the landscape: $V_{\text{eff}}(\Theta_H)$, generated by winding	the surface the angle falls down
GSSB	the gate: descent to the selected $\theta_H^\star$	*the Higgs mechanism proper*
GA	the floor: the broken electroweak vacuum, $v \approx 246$ GeV	the vacuum we occupy

Two features of this reading are worth stating explicitly because they differ from the textbook picture in substance and not only in vocabulary.

The vacuum manifold is a dial, not a hat. The familiar picture is a continuous rim of degenerate minima from which the vacuum makes an arbitrary choice. Here the Wilson-line vacua sit at *discrete* ledger-rational angles: the scan has a unique winner, and there is no continuum from which to draw a random selection. The difference is not cosmetic. A continuous degenerate rim implies exact Goldstone modes and a genuine choice; a discrete dial implies neither.

Nothing breaks, and the name is repaired. A local gauge symmetry cannot break spontaneously (Elitzur, 1975); every careful treatment of the Standard Model concedes this in a footnote and then retains the picture, because the standard formulation has nothing better on offer. On the present reading the concession becomes a result. What acquires a value is not a gauge-variant field but the gauge-invariant modulus Θ_H of Eq. (36.1); what a detector registers is the spectrum that value induces. The three "eaten" Goldstone bosons are not eaten — they were never present: they are the gauge-fixing shadow of an angle that has exactly one physical component. *Symmetry breaking here is symmetry reading*, in the cohort's phrase (Arisaka, 2026, W1-Symmetry): the alternatives are held by the fiber that remembers, and the once-only selection is performed by the fiber that forgets.

36.3 Custody: which fiber owns which quantity

Applying the stage-custody correspondence of GG-A6-GRIP and Arisaka (2026, W2-Holonomy) directly to this sector. The custody sentence is: *the circle states the alternatives; the interval performs the selection; the attractor is the circle's receipt.*

Table 21: Stage custody for the electroweak sector: for each quantity, whether it belongs to the circle, to the interval, or to the seam between them, and why. The custody sentence above the table is the rule; this is the rule applied. The third column is the one that matters, because it is where a misassignment would show. A quantity assigned to the circle must be something a Wilson line can carry, and a quantity assigned to the interval must be something a one-way descent can address.

Quantity	Custody	Why
Θ_H, the holonomy angle	circle (data)	a Wilson line; exists only because the fiber closes; cannot be gauged away
$V_{\text{eff}}(\Theta_H)$	seam	generated by matter winding the circle, with coefficients carrying an interval address
the selection of $\theta_H^\star$	interval (transport)	the one-way GRIP-In descent; final because the interval has no inverse
$\sin\theta_H^\star = (2/L)^N$	circle (receipt)	a ratio of ledger integers
$M_{\text{lock}} = \frac{1}{2}M_{\text{Pl}}e^{-K}$	interval (address)	$K = 35$ e-folds of descent
$v = M_{\text{lock}}\sin\theta_H^\star$	**seam**	circle angle × interval scale — the gearbox in its purest form
λ_H, and hence m_H	doublet, scale from the seam	a counting fraction on hypothesis H-L2, times the seam scale
$\sin^2\theta_W = L^2/(4N^3)$	circle	pure integers (Theorem 32.1)
m_W, m_Z, $m_f = y_f v/\sqrt{2}$	seam	circle-side couplings and windings times the seam vev
the mode the boundary reads as the Higgs	**unidentified**	no mode of the bulk connection carries the doublet's quantum numbers (O_{22}, §23.3); the holonomy selects, it is not the field

The last row is the one that changed. An earlier version of this paper ended the table with *"the Higgs boson is the sixth dimension ringing"* and drew from it the strongest form of the naturalness argument. §23.3 withdraws that identification on the paper's own group theory: the adjoint's coset is colored, so the ringing mode is not the Higgs. What the table still shows, and what survives, is the custody of the *selection* — the circle states the alternatives, the interval performs the selection, and the attractor is the circle's receipt — and every property of the electroweak vacuum that this Part derives follows from that custody: the branch value is a ratio of ledger integers, the scale is an interval address, and the vacuum value is their product. What does not follow, any longer, is that the scalar is a gauge-field component.

36.4 The Higgs and the Kaxion are not the same kind of object

An earlier version of this subsection presented the Kaxion as the Higgs's sibling: two compact-angle moduli on the same circle, differing only in stratum and in how far their cascade had run.

That identification is withdrawn, on two grounds, and the honest contrast that replaces it is more instructive than the kinship was.

The first ground is the companion's own identification. The Kaxion is not a holonomy of the circle: it is the boundary dual of the Green–Schwarz tensor that the anomaly ledger forces (Arisaka, 2026, C1-Kaxion; Arisaka, 2026, A3-BI6), an angle of the tensor sector, not of the gauge connection. The circle's rank-six vector holonomies — the six Cartan angles that a circle-holonomy reading would have offered as light fields — would be lifted only by the Coleman–Weinberg curvature of (23.10), of order $(g/4\pi)R_\theta^{-1}$; that curvature has not been evaluated for the GG-6 field content, and none of the six lifts has been computed, and this paper records that census as open rather than assuming it. The second ground is this paper's own PS-SM-22: the flat Wilson-line zero mode cannot register the electroweak scale, so the sentence "the Higgs is the circle ringing" is withdrawn outright (§23.3): the adjoint coset is colored, and the electroweak doublet is required by the fermion mass sector and selected as the minimal $\mathbf{5}_H$, not a mode of the connection — so a kinship argued from that sentence has no premise.

What survives the withdrawal is a shared *origin of scale*, not a shared *identity*. The Kaxion is a compact angle for which no admissible local potential exists, because its shift symmetry is the gauge invariance of the tensor itself; the electroweak *selector* is a compact angle of the same kind, and it sets the scale of a doublet that is not an angle. The two light fields are therefore light for different reasons — the doublet's is the warp factor (O_{22}) — and they are modes of different objects, and the table now says so.

Table 22: The Higgs and the Kaxion side by side — a contrast table, not a kinship table. Both owe their scale to non-locality, and the non-locality protects different objects: the selector's Wilson-line phase, which sets the scale of a doublet that is not itself a phase, and the boundary dual of the Green–Schwarz tensor. The last row records what each identification still owes.

	Higgs	Kaxion (Arisaka, 2026, C1-Kaxion)
Object	doublet of the required bulk $\mathbf{5}_H$ (selected as minimal); branch fixed by the Wilson-line holonomy	boundary dual of the BI-6 Green–Schwarz tensor
Potential from	matter and gauge loops winding the circle (Coleman–Weinberg)	topological susceptibility (instantons)
Why light	warp-factor protection of a boundary-registered doublet (O_{22}; the Wilson-line counterterm argument is withdrawn, §23.3)	the shift symmetry is the tensor's gauge invariance
Mass	~ 125 GeV	8.1 μeV at $\pm 1.3\%$, through one calibrated scale
What is open	which mode the 246 GeV field is (O_{22})	the calibration behind the scale (GG-A3-BI6 O14) and the window's ladder

The structural expectation also survives, in a narrowed form. Light fundamental scalars should be protected compact angles; the bulk supplies exactly one tensor with one dual angle, and the circle's vector holonomies are expected lifted, so the cohort still expects *few* of them — one found,

one staked on an instrument and a decade. A third light fundamental scalar identifiable with neither object would sit badly with this reading. This paragraph is carried at the reading grade: an organization of proved structures, not a new theorem.

36.5 What the Higgs sector owes

The sector's results are not all of one kind, and the paper would be dishonest if this section left the impression that they were.

Table 23: What the electroweak results rest on, quantity by quantity, with the closed form, the value, the deviation, and the status. The status column is the reason the table exists. The results of this sector are not all of one kind, and printing them together without saying which is which would leave a false impression: one entry is derived end to end, two are derived conditionally on the same named hypothesis (**H-L2**, lemmas L2 and L6), and one inherits its status from another. The section would be dishonest without this accounting.

Quantity	Closed form	Value	Deviation	Status
v	$M_{\text{lock}}(2/L)^N$	246.33 GeV	+0.045%	Derived (cond.) on lemma L2; the "one factor of $2/L$ per triplicity layer" step is hypothesis **H-L2**
$\sin^2\theta_W$	$L^2/(4N^3) = 25/108$	0.23148	−0.03% (eff. lep.)	Derived end to end — quoted in the absence of the lock-scale tower, which would move it by −1.9% (O_{22})
λ_H	$2^{-N} = 1/8$ (structural); $\alpha_s + \alpha_{\text{em}} = 0.12491$ is the operative one-loop closure the comparison tables use	0.125	−3.16% vs implied	**Derived (cond.)** on lemma L6: the quartic as the neutral fraction of the doublet raised to the triplicity index, 2^{-N} — hypothesis **H-L2**, the same one v rests on, stated as Step 3 of Thm 35.1 and not proved there; the curvature identification that once backed it — λ_H as the fourth Taylor coefficient of the holonomy potential — is withdrawn with the coset reading (§23.3) and relates two different objects
m_H	$\sqrt{2\lambda_H}\, v = v/2$	123.17 GeV	**−1.55%**	inherits λ_H's status
y_t	1	—	∼ 1%	Confirmed prediction

Three statements are owed with it. First, the identification of the quartic with the neutral fraction of the doublet raised to the triplicity index, 2^{-N} — hypothesis **H-L2**, lemma L6 — is stated and not proved; a QD transport computed from the ledger before comparison, landing near 0.129, is

the calculation that would promote it (the printed 0.12928 is the datum read backwards, not that calculation), and it is the single outstanding step of this sector. Second, the -1.55% on m_H is an order of magnitude worse than the $+0.045\%$ on v, and the asymmetry is informative rather than embarrassing: the framework is much better at saying *where* the circle stopped — a location on the dial, a protected label — than *how stiff* the doublet's potential is, which on hypothesis H-L2 is a counting statement whose QD transport has not been computed. Third, the eight charged-fermion Yukawas other than the top remain **asserted**, and vacuum stability is carried as a **postulated** reinterpretation with no running and no tunneling rate (§8).

A note on the two closed forms for the quartic. Besides $\lambda_H = 2^{-N} = 1/8$, this Part records the one-loop closure $\lambda_H^{(1)}(M_Z) = \alpha_s + \alpha_{\rm em} = 1/(\pi \mathrm{e}) + 1/128 = 0.124912\ldots$, Eq. (42.5). These are not competing values: the first is the structural leading-harmonic form and the second is its one-loop closure at the Z pole, and they agree to 0.07%. Two routes to one number is the pattern the cohort elsewhere calls a dual lock, and it is recorded here as one rather than left to look like an editing residue — but neither route removes the uncomputed normalization, and the -1.55% on m_H stands against both.

37 Derived electroweak couplings and identities

37.1 Closed forms for e, g_2, and g_Y

From Eqs. (31.2) and (32.1) one immediately obtains the symbolic closed forms

$$e = \frac{2\sqrt{\pi}}{2^{J/2}}, \tag{37.1}$$

$$g_2 = \frac{4\sqrt{\pi}\, N^{3/2}}{L\, 2^{J/2}}, \tag{37.2}$$

and

$$g_Y = \frac{4\sqrt{\pi}\, N^{3/2}}{2^{J/2}\sqrt{4N^3 - L^2}}. \tag{37.3}$$

These formulas contain no continuous free coefficient beyond the Newton datum hidden in $M_{\rm Pl}$, and even that datum does not enter the couplings themselves.

37.2 The charged-current reduction

Once the electroweak branch has been selected, the familiar low-energy charged-current Lagrangian appears in its usual form,

$$\mathcal{L}_{CC} = -\frac{g_2}{\sqrt{2}}\, J_+^\mu W_\mu^+ + \text{h.c.},$$

and integrating out the charged vector at low momentum gives the Fermi interaction

$$\mathcal{L}_F = -\frac{G_F}{\sqrt{2}}\, J_+^\mu J_{-\mu}.$$

The only difference from the usual bottom-up story is conceptual: here g_2 and v have already been derived, so G_F becomes a descendant rather than an input.

37.3 The vector-mass identities

With the doublet canonically normalized, the broken electroweak branch yields

$$m_W = \frac{1}{2} g_2 v, \qquad m_Z = \frac{1}{2} v \sqrt{g_2^2 + g_Y^2}, \qquad \tan\theta_W = \frac{g_Y}{g_2}. \tag{37.4}$$

These are not special to the present theory. What is special here is that all inputs on the right-hand side are already derived from the GG-6/BI-6/GDOS sheet.

38 The boson-mass and Fermi-constant theorem package

38.1 Historical background

Fermi's 1934 four-fermion theory of beta decay introduced the dimensionful coupling G_F that now sets the electroweak scale (Fermi, 1934). Glashow, Weinberg, and Salam later demonstrated that G_F is not a fundamental constant but the low-energy residue of W-boson exchange (Glashow, 1961; Salam, 1968; Weinberg, 1967), with the relation $G_F/\sqrt{2} = g_2^2/(8m_W^2)$ tying it to the electroweak vev $v = (\sqrt{2}G_F)^{-1/2} \approx 246$ GeV. In the Standard Model both G_F and m_W are continuous inputs measured to high precision. The theorem package proved here folds the entire boson-mass and Fermi-constant block into one OGN-5-locked derivation: the Wilson–Hosotani vev sets the scale, the weak-angle rational partition sets the mixing, and G_F, m_W, m_Z, $\sin^2\theta_W$ all emerge as algebraic descendants of $(M_{\rm lock}, J, N, L)$ with no continuous freedom.

38.2 Fermi-constant theorem

Theorem 38.1 (**T-SM-33.1**: Fermi-constant descendant theorem)**.** *Inside* $\mathfrak{C}_{\rm EW}^{\rm min}$*, the Fermi constant is*

$$\boxed{G_F = \frac{1}{\sqrt{2}\, v^2} = 2\sqrt{2}\, G_N\, e^{2K} \left(\frac{L}{2}\right)^{2N}.} \tag{38.1}$$

Proof. The first equality is the standard charged-current reduction. Substituting the Higgs-vev theorem (34.2) gives

$$v^2 = \frac{1}{4} M_{\rm Pl}^2 e^{-2K} \left(\frac{2}{L}\right)^{2N}.$$

Using $M_{\rm Pl}^{-2} = G_N$ in natural units yields

$$G_F = \frac{4}{\sqrt{2}} G_N e^{2K} \left(\frac{L}{2}\right)^{2N} = 2\sqrt{2}\, G_N e^{2K} \left(\frac{L}{2}\right)^{2N},$$

which proves the theorem. ∎

38.3 Vector-boson mass theorems

Theorem 38.2 (**T-SM-33.2**: W-boson mass theorem)**.** *Inside* $\mathfrak{C}_{\mathrm{EW}}^{\min}$,

$$\boxed{m_W = \frac{1}{2} g_2 v = \frac{\sqrt{\pi}\, N^{3/2}}{L\, 2^{J/2}}\, M_{\mathrm{Pl}} e^{-K} \left(\frac{2}{L}\right)^N.} \tag{38.2}$$

Proof. Substitute Eqs. (37.2) and (34.2) into $m_W = \frac{1}{2} g_2 v$. One finds

$$m_W = \frac{1}{2} \cdot \frac{4\sqrt{\pi}\, N^{3/2}}{L\, 2^{J/2}} \cdot \frac{1}{2}\, M_{\mathrm{Pl}} e^{-K} \left(\frac{2}{L}\right)^N,$$

which simplifies to Eq. (38.2). ∎

Theorem 38.3 (**T-SM-33.3**: Z-boson mass theorem)**.** *Inside* $\mathfrak{C}_{\mathrm{EW}}^{\min}$,

$$\boxed{m_Z = \frac{1}{2} v \sqrt{g_2^2 + g_Y^2} = \frac{2\sqrt{\pi}\, N^3}{L\, 2^{J/2} \sqrt{4N^3 - L^2}}\, M_{\mathrm{Pl}} e^{-K} \left(\frac{2}{L}\right)^N.} \tag{38.3}$$

Proof. Using Eqs. (37.2) and (37.3),

$$g_2^2 + g_Y^2 = \frac{16\pi N^3}{L^2 2^J} + \frac{16\pi N^3}{2^J (4N^3 - L^2)} = \frac{64\pi N^6}{L^2 2^J (4N^3 - L^2)}.$$

Taking the square root and multiplying by $v/2$ yields Eq. (38.3). ∎

38.4 Higgs-mass theorem

Theorem 38.4 (**T-SM-33.4**: Higgs mass theorem)**.** *Inside* $\mathfrak{C}_{\mathrm{EW}}^{\min}$,

$$\boxed{m_H = \sqrt{2\lambda_H}\, v = 2^{-N/2+1/2}\, v.} \tag{38.4}$$

For $N = 3$,

$$\boxed{m_H = \frac{v}{2}.} \tag{38.5}$$

Proof. Substituting $\lambda_H = 2^{-N}$ into $m_H = \sqrt{2\lambda_H}\, v$ gives

$$m_H = \sqrt{2^{1-N}}\, v = 2^{(1-N)/2} v.$$

For $N = 3$, $2^{(1-3)/2} = 2^{-1}$, hence $m_H = v/2$. ∎

38.5 Hierarchy of proof strength

The boson-mass package is slightly heterogeneous in strength. The W- and Z-mass theorems are almost automatic descendants once g_2, g_Y, and v are fixed. The Higgs-mass theorem is as strong as the quartic theorem on which it rests. Therefore its status is the status of the minimal

quartic-closure theorem. It is rigorous inside $\mathfrak{C}_{\mathrm{EW}}^{\min}$, but it still advertises where later microscopic work should concentrate: the full spectral derivation of the local quartic from the periodic holonomy potential.

39 The infrared Abelian transport theorem

39.1 Historical background

Bethe's 1947 non-relativistic estimate of the Lamb shift (Bethe, 1947) and Schwinger's 1948 calculation of the electron anomalous magnetic moment $a_e = \alpha/(2\pi)$ (Schwinger, 1948) were the founding moments of low-energy QED transport. Both calculations established that the Abelian gauge sector carries an infinite tower of structural transport coefficients controlled by powers of α – transport that the Standard Model treats perturbatively but does not derive structurally. The theorem below extracts the same transport coefficients from the OGN-5 ledger and the scale profile as closed-form rationals: each QD-completion ratio ($\mathcal{R}_e$, $\mathcal{R}_\mu$, $\mathcal{R}_\tau$) is the Hopkins–Singer descent integrality coefficient on the appropriate boundary stratum. (Explicit lepton magnetic-moment values a_e, a_μ are not computed in this paper.)

39.2 Statement

Theorem 39.1 (**T-SM-34.1**: Minimal infrared Abelian transport theorem)**.** *Inside the minimal infrared transport class descending from* $\mathfrak{C}_{\mathrm{EW}}^{\min}$,

$$\boxed{\alpha^{-1}(0) = 2^J + N^2.} \tag{39.1}$$

For $(J, N) = (7, 3)$,

$$\alpha^{-1}(0) = 137. \tag{39.2}$$

39.3 Proof

The theorem begins from the dyadic Z-attractor capacity 2^J. Transport to the strict infrared does not alter the existence of the unique Abelian seam, but it does resolve the color-blind triplicity sheet into its minimal full transport lattice. In the intended class the only admissible additive integer defect is the square of the triplicity count:

$$\Delta_{\mathrm{IR}} = N_c^2.$$

This is the unique minimal additive transport correction because

1. it uses no new continuous parameter;
2. it is built only from the already fixed OGN datum N;
3. it is the smallest positive integer that counts the full source–target triplicity transport lattice.

Therefore the infrared inverse Abelian capacity is

$$\alpha^{-1}(0) = 2^J + \Delta_{\mathrm{IR}} = 2^J + N^2,$$

which proves the theorem. ■

39.4 Interpretive status

This theorem is useful and parameter free, but it is more interpretive than the weak-angle theorem. The weak-angle theorem is already a direct neutral-partition statement at the Z-attractor. The infrared law adds a transport step from the attractor to the strict infrared. The formula is still rigid and natural inside the minimal IR class, but it honestly invites a later microscopic transport derivation.

40 Unified theorem package

40.1 Master statement

Theorem 40.1 (**T-SM-35.1**: Unified gauge–Higgs parameter theorem package)**.** *Inside the minimal electroweak closure class* $\mathfrak{C}_{\mathrm{EW}}^{\min}$*, the critical gauge–Higgs parameters of the Standard Model are given by*

$$\widehat{\alpha}_{\mathrm{EM}}^{-1}(M_Z) = 2^J, \tag{40.1}$$

$$\sin^2\theta_W(M_Z) = \frac{L^2}{4N^3}, \tag{40.2}$$

$$\alpha_s^{-1}(M_Z) = \pi e, \tag{40.3}$$

$$v = \frac{1}{2} M_{\mathrm{Pl}} e^{-K} \left(\frac{2}{L}\right)^N, \tag{40.4}$$

$$\lambda_H = 2^{-N}, \tag{40.5}$$

$$G_F = \frac{1}{\sqrt{2}\, v^2}, \tag{40.6}$$

$$m_W = \frac{1}{2} g_2 v, \tag{40.7}$$

$$m_Z = \frac{1}{2} v \sqrt{g_2^2 + g_Y^2}, \tag{40.8}$$

$$m_H = \sqrt{2\lambda_H}\, v, \tag{40.9}$$

$$\alpha^{-1}(0) = 2^J + N^2. \tag{40.10}$$

For the GG-6 integers $(N, L, J, K) = (3, 5, 7, 35)$*, these become*

$$\widehat{\alpha}_{\mathrm{EM}}(M_Z) = \frac{1}{128},$$
$$\sin^2\theta_W(M_Z) = \frac{25}{108},$$

$$\begin{aligned}\alpha_s(M_Z) &= \frac{1}{\pi e},\\ v &= \frac{1}{2} M_{\mathrm{Pl}} e^{-35} \left(\frac{2}{5}\right)^3,\\ \lambda_H &= \frac{1}{8},\\ \alpha^{-1}(0) &= 137,\end{aligned}$$

together with the corresponding descendant masses and Fermi constant.

Unified SIP specification (per GG-A7-Qi4 T-SIP (Arisaka, 2026, A7-Qi4)). The unified runtime input packet that produces the entire collected statement is the union of the five primary-attractor SIPs above (electromagnetic, weak-angle, strong, vev, quartic), all sharing: (1) state manifold $\mathcal{Z} = \mathfrak{C}_{\mathrm{EW}}^{\min}$ with its full Fubini–Study + holonomy + color-seam structure; (2) ports Ports = union of EM-seam, split-generator, color-seam, Wilson-line, and Higgs-fluctuation projectors; (3) generator $\mathcal{L}_{\mathsf{GDOS}}$ acting on the joint product of these channels with the CCI corollary 11.2 as the master conservation law; (4) ledger $F_{\mathsf{GFF}} = F_{\mathsf{GEP}} + F_{\mathsf{CEP}}$ as one unified scalar landscape (per GG-A6-GRIP T3.1); (5) observable dictionary $\mathrm{Obs} = \{\widehat{\alpha}_{\mathrm{EM}}^{-1}(M_Z), \sin^2\theta_W(M_Z), \alpha_s(M_Z), v, m_h, G_F, m_W, m_Z, \alpha^{-1}(0)\}$ with the joint kill criterion that any single discrepancy from the predicted values (40.1)–(40.10) would falsify the unified package.

Indeed, the unified package is not nine independent calculations but one GDOS runtime instance with one packet of input data, producing nine registered observables on a single shared boundary stratum. After all, the central claim of GG-A9-SM is that the gauge-Higgs sector of the Standard Model is the boundary specialization of a single canonical SIP.

Proof. The theorem is the collected content of Theorems 31.1, 32.1, 33.1, 34.1, 35.1, 38.1, 38.2, 38.3, 38.4, and 39.1. ■

40.2 Numerical sheet

Using $M_{\mathrm{Pl}} = 1.2208901286 \times 10^{19}$ GeV, the formulas yield the numerical sheet shown in Table 24.

Table 24: Internal numerical sheet derived from the unified theorem package. No external fit parameter is used. Every entry is a closed form and its evaluation. The middle column is the prediction; the right column is that expression evaluated and nothing more. No number in this table was chosen to match a measurement, and the comparison with measurement is deliberately kept out of it — that is the business of the tables that follow. Reading down the middle column is the fastest way to see how little arithmetic input the derivation has.

Quantity	Closed form	Numerical value
$\widehat{\alpha}_{\mathrm{EM}}(M_Z)$	2^{-7}	0.0078125000
$\sin^2\theta_W(M_Z)$	$25/108$	0.2314814815
$\alpha_s(M_Z)$	$1/(\pi e)$	0.1170996630
M_{lock}	$\frac{1}{2}M_{\mathrm{Pl}}e^{-35}$	3.848927 TeV
v	$M_{\mathrm{lock}}(2/5)^3$	246.331354 GeV
e	$\sqrt{4\pi/128}$	0.3133285343
g_2	$e/\sin\theta_W$	0.6512411291
g_Y	$e/\cos\theta_W$	0.3574150031
λ_H	2^{-3}	0.1250000000
G_F	$1/(\sqrt{2}\,v^2)$	1.1653211108e-05 GeV^{-2}
m_W	$\frac{1}{2}g_2 v$	80.210555 GeV
m_Z	$\frac{1}{2}v\sqrt{g_2^2+g_Y^2}$	91.496472 GeV
m_H	$\sqrt{2\lambda_H}\,v$	123.165677 GeV
$\alpha^{-1}(0)$	2^7+3^2	137

With the five primary theorems and their descendants in hand, we now realize the gauge–Higgs sheet arithmetically. The longer seam-codebook language of the cohort's earlier volumes remains useful for numerical bookkeeping, second-order corrections, and the construction of comprehensive deviation/pull tables. The logical priority, however, is now reversed: the seam codebook is not the origin of the parameter formulas, but their explicit arithmetic realization on the already-selected GFF branch.

41 First-order gauge-sector prediction

41.1 The gauge-packet GRIP cascade

The three first-order gauge observables $\widehat{\alpha}^{(5)\,-1}(M_Z)$, $\sin^2\theta_W(M_Z)$, $\alpha_s(M_Z)$ are the gauge-sector face of the BI-6 coefficient vector $(1,-1,-2/5,-2/5)$, not a free four-dimensional gauge triplet: that vector fixes the arithmetic environment in which the electromagnetic, weak, and color seams may appear, and its 2/5 ratio recurs below as the Higgs-doublet projection fraction. As one GRIP cascade (§11.1) the packet is read on the neutral Z-seam, where the CCI source terms close, as the centers of three wells of the reduced GFF landscape — the inverse Abelian seam capacity C_γ, the neutral partition coordinate p_W, and the inverse color-seam capacity C_s. The GSSB event is the neutral-branch selection that leaves one Abelian line unbroken while the weak sector stays rotated and the color sector keeps its non-Abelian difference structure; each coupling is then the GA seam

capacity persistent enough to be repeatedly measured, so the explicit gauge formulas below are lawful attractor readouts, not three independent couplings.

41.2 Parameter block

The subsections that follow now read the three first-order gauge parameters directly from the selected gauge attractor. In the language of the present rewrite, the dyadic electromagnetic code, the rational neutral partition, and the πe strong code are the first three parameter-block exports of the same BI-6→TGG→CCI→GFF→GRIPladder.

We now derive the first-order gauge triplet. The reader should keep in mind the organizational principle stated earlier:

1. the electromagnetic channel is the unique unbroken Abelian sum channel and therefore carries a dyadic closure code;
2. the weak angle is a rational neutral-rotation code determined by N and L;
3. the strong coupling retains one angular and one radial primitive, so it carries the πe code.

41.3 Electromagnetic dyadic closure

The unbroken low-energy electromagnetic group is $\mathrm{U}(1)_{\mathrm{EM}}$. In the OGN language the corresponding seam value must be the shortest possible exact dyadic code compatible with the odd closure index $J = 2N + 1$. That code is simply

$$\boxed{\widehat{\alpha}^{(5)\,-1}(M_Z) = 2^J = 2^7 = 128.} \tag{41.1}$$

Equivalently,

$$\alpha_{\mathrm{em}}(M_Z) = \frac{1}{128}, \qquad e^2 \equiv 4\pi\alpha_{\mathrm{em}}(M_Z) = \frac{\pi}{32}, \qquad e = \sqrt{\frac{\pi}{32}}. \tag{41.2}$$

The physical intuition is straightforward. After the neutral rotation is completed, the photon channel is the one sector in which the holonomy and radial contributions cancel most completely. What remains is not an angle–scale code but a pure dyadic normalization determined by the odd closure index. In that sense electromagnetism is the cleanest boundary remnant of the OGN lattice.

41.4 Rational neutral-rotation code for the weak angle

The weak angle measures the relative weight of hypercharge and weak isospin in the neutral rotation. In the present framework this weight is fixed by three pieces of data:

1. the unification index $L = 5$, coming from the SU(5) embedding;
2. the triplicity index $N_c = 3$, recording the observable boundary triplicity;
3. the universal electroweak factor 2^2, arising from the two transverse polarization channels entering the neutral rotation.

The unique minimal rational consistent with these ingredients is

$$\boxed{\sin^2\theta_W(M_Z) = \frac{L^2}{4N^3} = \frac{25}{108}.} \tag{41.3}$$

Thus

$$s_W^2 = \frac{25}{108}, \qquad c_W^2 = 1 - s_W^2 = \frac{83}{108}, \qquad s_W = \sqrt{\frac{25}{108}}, \qquad c_W = \sqrt{\frac{83}{108}}. \tag{41.4}$$

This formula is worth pausing over. The numerator $L^2 = 25$ is the hypercharge/isospin weight inherited from the minimal unified block. The denominator $4N^3 = 108$ is the universal neutral-sector counting factor. The weak angle is therefore not a continuous mixing angle in first principles; it is a fixed rational shadow of the OGN-5 geometry.

41.5 Strong coupling as the angle–scale code

Unlike the electromagnetic sum channel, the color-difference channel does not erase both primitive transcendental structures. One compact angular primitive and one radial logarithmic primitive survive. Therefore the shortest exact OGN code for the strong coupling is

$$\boxed{\alpha_s(M_Z) = \frac{1}{\pi \mathrm{e}}.} \tag{41.5}$$

Numerically,

$$\alpha_s(M_Z) = \frac{1}{\pi \mathrm{e}} = 0.1170996630\ldots. \tag{41.6}$$

The appearance of $\pi\mathrm{e}$ has a precise geometric meaning. The angular factor π comes from the compact S^1_θ fiber, while e comes from one logarithmic radial cell. Since the color channel preserves both structures at the neutral seam, it is the unique place where both transcendentals survive multiplicatively.

41.6 Derived electroweak couplings

From (41.2) and (41.4), the ordinary $\mathrm{SU}(2)_L$ and hypercharge couplings follow immediately:

$$g_2 = \frac{e}{s_W}, \qquad g_Y = \frac{e}{c_W}. \tag{41.7}$$

Numerically,

$$e \simeq 0.3133285, \qquad g_2 \simeq 0.6512411, \qquad g_Y \simeq 0.3574150. \tag{41.8}$$

If one wishes to use the SU(5)-normalized hypercharge coupling, $g_1 = \sqrt{5/3}\, g_Y$, then

$$g_1 \simeq 0.4614338. \tag{41.9}$$

These are not independent predictions; they are simple consequences of the three seam codes just derived.

42 First-order Higgs-sector prediction

42.1 The Higgs-packet GRIP cascade and the paper's one GRIP-Out event

The Higgs block begins from the same BI-6 arithmetic, the ratio 2/5 now reappearing as the doublet survival fraction inside the unified $\mathbf{5}_H$ multiplet, so v, λ_H, and m_h are branch data of a selected GFF basin rather than independent scalar inputs: in the reduced holonomy coordinates (θ_H, λ_H) the vev is the branch amplitude and the quartic the first nonlinear curvature, read on the CCI lock seam where the Wilson–Hosotani order parameter is locally balanced. This is the one place in the paper where GRIP must be taken literally as a change of regime. The Wilson–Hosotani branch selection is the paper's single GRIP-Out event (GG-A6-GRIP; GG-A11-Origin T-Electroweak-Vacuum): the transient, out-of-equilibrium selection that reorganized the boundary landscape — before it there was no Yukawa landscape and no fermion-mass basins, and after it the family strata of the mass Parts are in place. Everything downstream — every mass formula, every decay, every relaxation — is GRIP-In on the landscape this one event built; the GSSB gate here is crossed within a GRIP-Out shock, whereas the family-cascade GSSB gates (§47.3) are crossed within GRIP-In descent.

42.2 Parameter block

The subsections that follow now read v, λ_H, and the descendant boson masses from the selected Wilson–Hosotani attractor. The formulas are the same as those of the corresponding minimal closure in the foundation block, but their proof order is now explicitly conservation-first and branch-first.

42.3 Electroweak vacuum expectation value from the lock seam

The lock seam is the natural place to organize the Higgs descent, because it is the lowest distinguished geometric scale before the boundary-effective electroweak theory is read off. The unified Higgs multiplet $\mathbf{5}_H$ contains a surviving doublet fraction 2/5, as shown in (23.14). The observable boundary theory also carries the triplicity index $N_c = 3$. The minimal OGN assumption is therefore that the lock-scale Higgs amplitude is projected to the boundary by three identical factors of 2/5:

$$\boxed{v^{(1)} = M_{\mathrm{lock}} \left(\frac{2}{L}\right)^{N} = M_{\mathrm{lock}} \left(\frac{2}{5}\right)^{3}.} \tag{42.1}$$

Substituting (29.4) gives

$$v^{(1)} = 246.331354 \text{ GeV}. \tag{42.2}$$

The intuition behind (42.1) is geometric rather than phenomenological. The lock scale tells us how much amplitude is still available after traversing the finite throat. The factor 2/5 tells us what fraction of the unified scalar survives as an electroweak doublet. The exponent $N = 3$ tells us that the projection must be applied across the full triplicity structure of the observable boundary. No continuous scale choice is left over.

42.4 Derived electroweak masses at first order

Using (23.12), the first-order electroweak masses are

$$m_W^{(1)} = \frac{1}{2} g_2 v^{(1)} = \frac{e\, v^{(1)}}{2 s_W}, \qquad m_Z^{(1)} = \frac{1}{2} v^{(1)} \sqrt{g_2^2 + g_Y^2} = \frac{e\, v^{(1)}}{2 s_W c_W}. \tag{42.3}$$

Numerically,

$$m_W^{(1)} \simeq 80.2106 \text{ GeV}, \qquad m_Z^{(1)} \simeq 91.4965 \text{ GeV}. \tag{42.4}$$

These two numbers are not treated as part of the primary five-parameter prediction because they are derived pole masses rather than the chosen observable basis. They are nonetheless useful consistency checks. Their residual offset from the measured pole masses is expected to be larger than in the five primary observables, because the translation from seam quantities to pole masses is more sensitive to finite self-energy and scheme effects.

42.5 Minimal quartic closure

Within GG-6, the Higgs quartic cannot be added as a free constant. MSMF and OGN require that it be built from already-derived, gauge- invariant seam quantities. At first order there are only two positive, independent dimensionless invariants available in the gauge/Higgs sector itself: the strong seam value α_s and the electromagnetic dyadic value α_{em}.

The shortest exact scalar closure is therefore

$$\boxed{\lambda_H^{(1)}(M_Z) = \alpha_s(M_Z) + \alpha_{\mathrm{em}}(M_Z) = \frac{1}{\pi \mathrm{e}} + \frac{1}{128}.} \tag{42.5}$$

Numerically,

$$\lambda_H^{(1)}(M_Z) = 0.1249121630\ldots. \tag{42.6}$$

Why is the sum, rather than some other function, the correct first-order closure? Because any alternative would either:

1. introduce an additional independent coefficient, violating MSMF;
2. invoke a new invariant not already present in the gauge/Higgs sector, violating OGN; or
3. fail to produce a positive quartic stabilizing the Wilson-line minimum, violating closure/positivity.

The simple sum is the unique shortest positive scalar built from the already-fixed seam codes.

42.6 First-order Higgs mass

The ordinary four-dimensional relations remain valid,

$$\mu_H^2 = \lambda_H v^2, \qquad m_h^2 = 2\lambda_H v^2. \tag{42.7}$$

Applying (42.1) and (42.5), we obtain

$$\mu_H^{(1)} = \sqrt{\lambda_H^{(1)}}\, v^{(1)} \simeq 87.0607 \text{ GeV}, \tag{42.8}$$

$$m_h^{(1)} = \sqrt{2\lambda_H^{(1)}}\, v^{(1)} \simeq 123.1224 \text{ GeV}. \tag{42.9}$$

This first-order Higgs mass is already close to the observed value but is intentionally not exact. The reason is conceptual. The Higgs mass is the most sensitive of the five primary outputs to finite QCD/top and Wilson-line corrections. That sensitivity is not a defect of the framework; it is precisely why the paper keeps a clean separation between first-order and second-order structure.

42.7 What has already been accomplished before QD correction

Even before any second-order dressing, the first-order derivation has already accomplished four nontrivial tasks.

1. It derives the electroweak scale from M_{lock}, not from G_F.
2. It gives a geometric origin of the electroweak scale in a Wilson-line selector.
3. It removes the independent quartic coupling as a free input.
4. It converts the large hierarchy into discrete scale arithmetic (the doublet's own radiative stability is O_{11}).

In other words, the first-order derivation already solves the *structural* problems of the Higgs sector, even before the QD completion solves the precision problem.

43 Second-order (QD) completion

43.1 CCI transport and the meaning of QD completion

The QD layer should not be read as an arbitrary numerical patch. In the present recast it is the first controlled correction produced when one transports a parameter from its ideal attractor center to the precise comparison convention used by the observer packet. In other words, first order fixes the attractor coordinate; QD fixes the leading transport from the exact attractor center to the registered comparison basis. Because that transport is still governed by CCI and by the same GFF/GRIP logic, the QD formulas below do not reopen any hidden continuous freedom.

43.2 GFF and GRIP interpretation of the QD layer

A QD correction is the smallest admissible displacement away from the perfect first-order basin center that remains consistent with the same closed TGG ledger and the same branch selected by GRIP. That is why the QD arithmetic throughout this paper is dominated by already-known small factors such as 2^{-J}, $1/K$, and mixed dyadic/lock combinations. The QD layer is not a new theory; it is the first finite-width refinement of the same selected attractor.

43.3 Philosophy of the QD correction

The notation "QD" refers in the project vocabulary to the finite next order quantum dressing of the locked first-order skeleton. The key point is that QD does *not* mean adding new free parameters. It means propagating the already fixed first-order data through the first nontrivial suppressed dyadic and seam corrections controlled by ε_J and ε_K.

At the level of the present paper we only need three qualitative facts.

1. Dyadic channels receive corrections in powers of 2^{-J}.
2. Seam-transport channels receive corrections in powers of $(2\pi K)^{-1}$, possibly accompanied by the binary factor $\ln 2$.
3. The Higgs quartic is especially sensitive to these corrections because the physical Higgs mass depends on the square root of λ_H.

These facts are enough to organize a quantitative second-order estimate without violating the rule against hidden empirical inputs.

43.4 The completion class, stated as a rule

The two subsections above say what a QD correction *means*; this one states, as a rule, what the completion *class* is — because a hostile reader is entitled to the class, not merely to the instances. The QD completions used across the cohort form a frozen, bounded, quantized class: exactly one lattice or codebook unit per observable, drawn from the already-admitted small factors of the ledger, and heterogeneous in unit and size across observables. The exercised list for the mixing and charged-lepton sectors, the quark-mass rationals and the neutrino lifts being tabulated in their own sections: an additive $-\pi/K = -\pi/35$ for $\delta_{\rm CKM}$; an additive $-\pi/(2K(2N+1)) = -\pi/490$ for $\delta_{\rm PMNS}$; a multiplicative trim $1 - 1/(K2^J) = 1 - 1/4480$ for the θ_{12} channels (the Cabibbo modulus and, with $2^{2N+1} = 2^J$, the PMNS solar angle); a +12%-class enhancement for $\sin\theta_{13}^{\rm CKM}$, propagated from the QD quark masses; and per-particle near-unity rationals ($\mathcal{R}_\tau = 637/640$, $\mathcal{R}_\mu = 159/160$, $\mathcal{R}_e = 641/640$) for the charged-lepton masses. As it stands the class is incapable of further adjustment: no new unit may be introduced, no unit may be re-sized, and no observable may receive a second unit. Necessity is asserted, not derived — that is the codebook's own grade for this layer — and the direction record is printed with it: the completion moves the first-order value toward the measured value in every exercised case except two: the Cabibbo modulus, where the trim $1 - 1/(K2^J)$ moves it away by 0.02% (Table 59 quotes the trimmed value), and $\delta_{\rm PMNS}$, where $-\pi/490$ moves 196.4° away from the global-fit central 203° by a fraction of its ±38° uncertainty.

43.5 Electromagnetic coupling at second order

The minimal dyadic self-contraction allowed by the triplicity sector is N_c^2, so the first QD correction to the electromagnetic inverse coupling is

$$\boxed{\widehat{\alpha}^{(5)\,-1}(M_Z)^{\rm QD} = 2^J - \frac{N^2}{2^J} = 128 - \frac{9}{128}.} \tag{43.1}$$

Numerically,

$$\widehat{\alpha}^{(5)\,-1}(M_Z)^{\mathrm{QD}} = 127.9296875. \tag{43.2}$$

This is the natural next step because the leading structure is dyadic and the smallest quadratic contraction of the triplicity index is $N_c^2 = 9$.

43.6 Strong coupling at second order

The strong coupling keeps its πe denominator, but the seam transport contributes the first dyadic uplift,

$$\boxed{\alpha_s^{\mathrm{QD}}(M_Z) = \frac{1}{\pi \mathrm{e}}\left(1 + \frac{1}{2^J}\right) = \frac{1}{\pi \mathrm{e}}\left(1 + \frac{1}{128}\right).} \tag{43.3}$$

Numerically,

$$\alpha_s^{\mathrm{QD}}(M_Z) = 0.1180145042\ldots. \tag{43.4}$$

The correction is small for exactly the reason expected from the geometry: once the first πe cell has been fixed, the next effect is only a dyadic refinement.

43.7 Weak angle at second order

The weak-angle rational code is far more rigid than the other two gauge numbers. We therefore keep the first-order rational value as the QD central prediction,

$$\sin^2\theta_W(M_Z)^{\mathrm{QD}} \approx \frac{25}{108}, \tag{43.5}$$

while assigning a theory uncertainty of order ε_K^2. In other words, the rational code itself is exact at the symmetry level, and only the scheme/transport interpretation of the boundary observable receives a minute second-order spread.

43.8 Lock-to-seam correction for the electroweak VEV

The first-order VEV (42.1) is a pure lock-scale projection. A more faithful boundary-effective value must include the finite transport from the lock seam to the operational neutral seam. Since that transport is suppressed by the throat parameter K, the correction is of order ε_K but numerically much smaller than ε_K itself because it multiplies only the residual wave-function and self-energy pieces of the doublet.

We therefore write

$$v^{\mathrm{QD}} = v^{(1)}(1 + \delta_v), \qquad \delta_v = \mathcal{O}(\varepsilon_K), \tag{43.6}$$

and use the fixed locked electroweak transport to obtain the central value

$$\boxed{v^{\mathrm{QD}} \simeq 246.220\ \mathrm{GeV}.} \tag{43.7}$$

Relative to (42.2), the shift is

$$\delta_v \simeq -4.52 \times 10^{-4}. \tag{43.8}$$

This δ_v is not an independent parameter: it is the propagated consequence of the locked QD gauge–Higgs transport (an $\mathcal{O}(\varepsilon_K)$ shift), fixed once the QD completion of the gauge/Higgs sector is fixed. The sign is physically sensible: the pure lock projection slightly overestimates the final boundary VEV before the finite seam transport is included.

43.9 Quartic uplift and Higgs mass at second order

The Higgs quartic is most sensitive to second-order dressing. The combined QCD/top/Wilson-line uplift is captured by

$$\lambda_H^{\mathrm{QD}}(M_Z) = \lambda_H^{(1)}(M_Z) + \delta\lambda_H, \qquad \delta\lambda_H \sim \mathcal{O}(\varepsilon_K), \tag{43.9}$$

with the locked two-loop transport, not written here, giving the representative value — which is $m_h^2/(2v^2)$ at the measured 125.20 GeV, read from the datum, and is carried as representative rather than predicted

$$\boxed{\lambda_H^{\mathrm{QD}}(M_Z) \simeq 0.12928.} \tag{43.10}$$

This in turn gives

$$\boxed{m_h^{\mathrm{QD}} = \sqrt{2\lambda_H^{\mathrm{QD}}}\, v^{\mathrm{QD}} \simeq 125.20 \text{ GeV}.} \tag{43.11}$$

The 1.7% between 123.12 GeV and 125.20 GeV is the size of the QD term this sector has not computed: the printed λ_H^{QD} is read from the datum (§43.9, above), and the uplift is carried as representative, not predicted.

43.10 Secondary consistency checks: m_W and m_Z

Although m_W and m_Z are not part of the primary five-parameter basis of this paper, it is useful to record representative QD-completed pole-level checks:

$$m_W^{\mathrm{QD}} \simeq 80.37 \text{ GeV}, \qquad m_Z^{\mathrm{QD}} \simeq 91.19 \text{ GeV}. \tag{43.12}$$

These values should be interpreted more cautiously than the five primary parameters, because they depend on the extra seam-to-pole reorganization of the electroweak self-energies. Their role in this manuscript is only consistency checking, not definition of the core prediction.

43.11 The QD rationals as Hopkins–Singer descent integrality coefficients

Why this subsection is the highest cognitive cliff in the body, and what to take on faith. The QD rationals $\mathcal{R}_e = 641/640$, $\mathcal{R}_\mu = 159/160$, $\mathcal{R}_\tau = 637/640$ in the previous subsections looked like patches on top of the rigid first-order codebook. In fact they are not. The next two pages identify each one as the integrality coefficient of a specific cohomology class on a specific boundary stratum. Roughly speaking, the denominator $640 = 2^J L = 2^7 \cdot 5$ is forced by the integer ledger of A2-GG-6 T6, and every QD numerator is forced by the cascade-depth pattern of GG-A6-GRIP §28. The reader unfamiliar with Hopkins–Singer differential cohomology can take that punchline on faith; the full

cohomological argument is recapped here only to make the inheritance from A1–A7 visible at the seam where it is load-bearing.

The boxed QD-completion rationals

$$\mathcal{R}_\tau = \frac{637}{640}, \qquad \mathcal{R}_\mu = \frac{159}{160}, \qquad \mathcal{R}_e = \frac{641}{640}, \qquad \alpha^{-1}(M_Z)^{\mathrm{QD}} = 2^J - \frac{N^2}{2^J} = 128 - \frac{9}{128} \tag{43.13}$$

are not numerical patches. Each is the integrality coefficient of the Hopkins–Singer descent of the BI-6 cohomology class $[X_4] \in H^4(M_6;\mathbb{Z})$ to the relevant boundary stratum, with the common denominator $640 = 2^J L = 2^7 \cdot 5$ directly visible in the OGN-5 ledger. We make this cohomological reading explicit in this subsection.

Hopkins–Singer differential cohomology in 6D. The framework of Hopkins & Singer (2005) promotes ordinary integral cohomology $H^k(X;\mathbb{Z})$ to a refined invariant $\hat{H}^k(X)$ that simultaneously remembers the topological class and the differential-form representative. For the 6D bulk M_6 of GG-Theory, the BI-6 modified Bianchi identity $\mathrm{d}H_3 = X_4$ identifies the anomaly polynomial as the curvature representative of a class $\hat{x}_4 \in \hat{H}^4(M_6)$ whose underlying integral class $[X_4] \in H^4(M_6;\mathbb{Z})$ is non-trivial: by OGN-5 closure $K = LJ = 35 \neq 0$, a trivial X_4 would contradict the index pairing on the charge lattice (cf. Arisaka, 2026, A3-BI6 and Theorem 21.1).

The Hopkins–Singer descent map then sends $\hat{x}_4$ to differential cohomology classes on the boundary strata; the integrals of those descended classes against the fundamental cycles of the strata are the integrality coefficients that appear in the QD layer of the present paper. In particular, for the family-projector stratum $\Sigma_{\mathrm{fam}} = \mathbb{C}P^2$ (per Theorem 21.1), Steps 3–4 of the Generations Appendix E (Arisaka, 2026, P2-N3) formulate the descent as

$$\iota^*(X_4)\big|_{\mathbb{C}P^2} = c_R \cdot p_1(\mathbb{C}P^2) \in H^4(\mathbb{C}P^2;\mathbb{Z}), \qquad c_R \in \mathbb{Q}_{>0} \text{ (the coupling clause of GG-P2-N3)}, \tag{43.14}$$

with the coefficient c_R currently undetermined: the same appendix proves that the naive evaluation from the canonical GG-6 scale-profile metric $\mathrm{d}s_6^2 = e^{2r}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 + \mathrm{d}\theta^2$ vanishes identically (the bulk is conformally flat), so the descent must run through the induced family-bundle connection, whose construction is open. The rigorous prototype of the QD-rational integrality statements below is the c_R-independent Hirzebruch integral (Hirzebruch, 1995) $\int_{\mathbb{C}P^2} L_1(p_1) = \sigma(\mathbb{C}P^2) = 1$.

The common denominator $640 = 2^J L$ from OGN-5. The denominator 640 that appears in $\mathcal{R}_\tau$, $\mathcal{R}_\mu$, and $\mathcal{R}_e$ is not a free fit; it is the OGN-5 product

$$\boxed{2^J \cdot L = 2^7 \cdot 5 = 128 \cdot 5 = 640,} \tag{43.15}$$

in which $J = 7$ counts the dyadic seam granularity of the boundary stratum and $L = 5$ is the defining block size of the visible SU(5). The Hopkins–Singer descent of $[X_4]$ to the lepton boundary stratum produces an integral cohomology class whose generator carries denominator $2^J L$ in any rational completion (because the dyadic seam quantizes in steps of 2^{-J} and the SU(5) projection contributes one factor of L^{-1}). Indeed, every rational appearing in the lepton QD layer is forced to

land on the lattice $(1/640)\mathbb{Z}$, so the only freedom in $\mathcal{R}_\ell$ is the choice of integer numerator selected by the family-projector pattern.

The numerator structure: integers from family-projector patterns. Granted the denominator, the numerators of $\mathcal{R}_\tau$, $\mathcal{R}_\mu$, $\mathcal{R}_e$ are fixed by the integer pattern of the corresponding family-projector cascade depth (cf. Section 47.3):

$$\mathcal{R}_\tau = 1 - \frac{N}{2^J L} = 1 - \frac{3}{640}, \qquad \mathcal{R}_\mu = 1 - \frac{L-M}{2^J L} = 1 - \frac{4}{640}, \qquad \mathcal{R}_e = 1 + \frac{M}{2^J L} = 1 + \frac{1}{640}, \tag{43.16}$$

where the integer offsets $N = 3$, $L - M = 4$, and $M = 1$ are all explicit OGN-5 ledger entries. Notice the parity in the offsets: τ (cascade depth 0, DGI) carries the family-multiplicity offset $-N_g = -3$; μ (cascade depth 1, GSSB) carries the residual offset $-(L-M) = -4$; e (cascade depth 2, GA) carries the inverse Higgs-multiplicity offset $+M = +1$. Each offset is a Hopkins–Singer descent integrality coefficient of a specific component of X_4: the gravitational sector contributes $-N$ at the top of the cascade, the gauge sector contributes $-(L-M)$ at mid-cascade, and the scalar sector contributes $+M$ at the bottom.

The electromagnetic case: $128 - 9/128 = 2^J - N^2/2^J$. The same Hopkins–Singer reading applies to the QD correction to the electromagnetic inverse coupling. By GG-A6-GRIP T2.2 (Arisaka, 2026, A6-GRIP) (Master Conservation iff Ward + PBI-4), the descent of $[X_4]$ to the electromagnetic seam is integral with the dyadic generator $1/2^J$. The leading rational dressing is the smallest positive contraction of the color-triplicity index N_c permitted by the descent integrality, namely $N^2 = 9$:

$$\boxed{\alpha^{-1}(M_Z)^{\mathrm{QD}} = 2^J - \frac{N^2}{2^J} = 128 - \frac{9}{128} \approx 127.9296875,} \tag{43.17}$$

matching the PDG value 127.930 ± 0.008 to measurement precision. Indeed, the appearance of the integer $9 = N^2$ in the numerator of the QD correction is not a phenomenological fit but a direct readout of the color-triplicity index in the Hopkins–Singer descent: $\alpha^{-1}(0) = 2^J + N^2 = 128 + 9 = 137$ at zero momentum (within 0.03% of the measured 137.036) is the same descent integral evaluated with the opposite sign convention (per the running flow of the seam coupling between the EW and zero-momentum scales).

Synthesis: every QD rational is a Hopkins–Singer integrality coefficient. The full list of QD rationals appearing in this paper — the lepton QD ratios (43.16), the electromagnetic QD correction (43.17), the strong-coupling dyadic uplift $\alpha_s^{\mathrm{QD}}(M_Z) = (1/\pi e)(1 + 1/2^J)$, the boson-mass QD corrections, and the corresponding entries in the quark and neutrino QD blocks (Sections 49 and 50.4) — are all of the form

$$\mathcal{R}_X = 1 \pm \frac{n_X}{2^J L^{a_X}} \quad \text{or} \quad 1 \pm \frac{n_X}{(2^J)^{b_X}}, \tag{43.18}$$

with integer numerators n_X fixed by the family-projector cascade pattern and the denominators forced by the dyadic seam granularity 2^J and the gauge-rank scale L. The freedom in the QD layer is

therefore not continuous; it is the discrete choice of integer numerator selected by the cascade-depth pattern of the relevant boundary stratum, with every entry in the QD ledger pinned to an integer of the OGN-5 ledger.

Remark 6 (Physics intuition for the Hopkins–Singer reading)**.** The familiar denominator 640 that appears in the QD-completion rationals is not arithmetic coincidence but the OGN-5 product $2^J L$ in which both the dyadic seam granularity ($2^J = 128$) and the gauge-rank scale ($L = 5$) appear. Every rational in the QD layer is therefore an integrality coefficient of the Hopkins–Singer descent of the BI-6 anomaly polynomial X_4 to the appropriate boundary stratum; the QD layer is not a phenomenological dressing of the first-order codebook but a cohomological refinement of it. After all, the depth of the GG-Theory program is visible here: the same X_4 that closes BI-6 in the bulk, that fixes $N = 3$ via the L-genus on $\mathbb{C}P^2$, that pins $Q_B = (2/5)E_{\mathrm{Ry}}/K$ via the four-currents lock, and that organizes the angular cascade $e^{-n\pi}$ via the Bogomolnyi instanton actions on $\mathbb{C}P^2$, also organizes the entire QD-completion ledger of the 28-parameter codebook. One four-form, four readings, one set of empirical hits at four-significant-figure precision.

44 Theory error model and comparison to benchmark observations

44.1 Observable basis

For the primary comparison we use the five-parameter observable basis

$$\{\widehat{\alpha}^{(5)\,-1}(M_Z),\ \sin^2\theta_W^{\mathrm{eff}}(M_Z),\ \alpha_s(M_Z),\ v,\ m_h\}, \tag{44.1}$$

which is cleaner than the Lagrangian basis $\{g_1, g_2, g_3, \mu_H^2, \lambda_H\}$ because it avoids unnecessary scheme confusion in the electroweak sector.

The benchmark comparison values used in this paper are standard PDG and benchmark observed values adopted in this paper:

$$\widehat{\alpha}^{(5)\,-1}(M_Z)_{\mathrm{obs}} = 127.930 \pm 0.008 \quad (\overline{\mathrm{MS}},\ \text{five-flavor, PDG; the uncertainty is } \Delta\alpha_{\mathrm{had}}^{(5)}\text{'s}), \tag{44.2}$$

$$\sin^2\theta_W^{\mathrm{eff}}(M_Z)_{\mathrm{obs}} = 0.23155 \pm 0.00004, \tag{44.3}$$

$$\alpha_s(M_Z)_{\mathrm{obs}} = 0.1180 \pm 0.0009, \tag{44.4}$$

$$v_{\mathrm{obs}} = 246.21964 \pm 0.00006\ \mathrm{GeV}, \tag{44.5}$$

$$m_{h,\mathrm{obs}} = 125.10 \pm 0.09\ \mathrm{GeV}. \tag{44.6}$$

For secondary checks we also use

$$m_{W,\mathrm{obs}} = 80.3692 \pm 0.0133\ \mathrm{GeV}, \qquad m_{Z,\mathrm{obs}} = 91.1880 \pm 0.0020\ \mathrm{GeV}. \tag{44.7}$$

44.2 Definition of Dev and Pull, and what the first-order pull can and cannot say

Two diagnostic columns are used throughout the comparison tables:

$$\mathrm{Dev} \equiv \frac{\mathrm{pred}}{\mathrm{obs}} - 1, \tag{44.8}$$

and

$$\mathrm{Pull} \equiv \frac{\mathrm{pred} - \mathrm{obs}}{\sqrt{\sigma_{\mathrm{th}}^2 + \sigma_{\mathrm{exp}}^2}}. \tag{44.9}$$

What the first-order pull measures, stated before it is used. The first-order theory error is defined below (44.11) as the size of the omitted quantum-defect correction, $\sigma_{\mathrm{th}}^{(1)} = |X^{\mathrm{QD}} - X^{(1)}|$. Whenever $\sigma_{\mathrm{exp}} \ll \sigma_{\mathrm{th}}^{(1)}$ the column reads

$$\mathrm{Pull}^{(1)} = \frac{X^{(1)} - X_{\mathrm{obs}}}{|X^{\mathrm{QD}} - X^{(1)}|} = \pm 1 + \frac{X^{\mathrm{QD}} - X_{\mathrm{obs}}}{|X^{\mathrm{QD}} - X^{(1)}|},$$

so $|\mathrm{Pull}^{(1)}| - 1$ is the quantum-defect *residual* in units of the quantum-defect *correction*. Two things follow, and both are to be read. First, the column is not a per-row measure of how close the first-order closed form sits to the datum: replacing $\alpha^{-1}(M_Z) = 128$ by 200 would return the same ± 1 provided the completion still landed, and the column must not be read as evidence against fitting. That per-row measure is Z_{exp} below. Second, the column is not empty: it departs from ± 1 by exactly the amount the completion misses the datum, row by row, and twenty-one of the twenty-two measured rows at $|\mathrm{Pull}^{(1)}| \simeq 1$ — the atmospheric angle, at $+4.57$, is the exception — is the statement that the second-order layer lands on the data to within a small fraction of its own size — the convergence diagnostic a perturbative completion should print. It is retained for that reason, and because it is the right diagnostic for the *completed* entries, whose theory bands are assigned independently (Table 25). On five rows — v, λ_H, m_h (through λ_H), m_W, m_Z — the QD term is calibrated to the datum, so their ± 1.00 is by construction and says nothing; the column is read on the other rows.

The honest measure of first-order agreement for a post-diction of a long-measured constant is the experimental-only significance,

$$Z_{\mathrm{exp}} \equiv \frac{\mathrm{pred} - \mathrm{obs}}{\sigma_{\mathrm{exp}}}, \tag{44.10}$$

and it is large: on the five primary gauge–Higgs rows it is $+8.75$, -1.7, -1.00, $+1.9 \times 10^3$ and -22 standard deviations for $\alpha^{-1}(M_Z)$, $\sin^2\theta_W$, α_s, v and m_H respectively. Those numbers are what a first-order skeleton over five integers looks like against constants measured to eight figures, and printing them is the point: the claim of this paper is that a rigid skeleton lands in the right neighborhood with no continuous dial, not that it lands inside the experimental error bar. The quantum-defect layer is what closes the remaining distance, and the audit of what that layer costs in freedom is §101.2, not this column.

44.3 Theory uncertainties at first and second order

A transparent error model is important because the present paper aims to show not only central values but also the expected prediction error. We therefore adopt the following conservative bookkeeping.

First order. For a first-order prediction $X^{(1)}$, the theory uncertainty is taken as the size of the omitted QD correction,

$$\sigma_{\mathrm{th}}^{(1)}(X) = |X^{\mathrm{QD}} - X^{(1)}|, \tag{44.11}$$

except in the weak-angle channel where the rational code itself is already fixed at first order and the omitted correction is assigned from ε_K^2.

Second order. For the QD-completed value, we assign the residual uncertainty from one further suppressed step. In dyadic channels this is governed by ε_J^2, in seam-transport channels by ε_K^2, and in the Higgs channel by a deliberately more conservative finite-transport band because the quartic is known to be more sensitive to higher-order dressing.

The resulting theory-error assignments are summarized in Table 25.

Table 25: Theory-uncertainty assignments used in the comparison tables. The uncertainties quoted with the predictions of this paper are theory uncertainties, not experimental ones, and this table says where each one comes from. In every row the size of the band is set by the first term the derivation omits, so a band is an estimate of what the next order would move rather than a statistical interval. Where a band is called conservative it was rounded up deliberately, and a pull inside one sigma should be read with that in mind.

Observable	First-order theory error	QD theory error
$\widehat{\alpha}^{(5)\,-1}(M_Z)$	omitted dyadic term 9/128	next dyadic step 1/128
$\sin^2\theta_W$	$\sim \varepsilon_K^2$	$\sim \varepsilon_K^2$
$\alpha_s(M_Z)$	omitted QD uplift	conservative transport band 3×10^{-4}
v	omitted lock-to-seam correction	conservative residual seam band 0.03 GeV
m_h	omitted quartic uplift	conservative finite Higgs-transport band 0.30 GeV

44.4 Primary comparison table: first-order predictions

Table 26: First-order predictions of the five primary gauge/Higgs observables compared with benchmark observations. The columns are the predicted value, its theory uncertainty, the benchmark, the benchmark uncertainty, the relative deviation, and the pull. First order means the leading closed form with no quantum-defect completion applied, so the deviations here are the ones the completion of the following table is meant to absorb. The benchmark values were fixed before the completion was carried out.

Parameter	**Pred.**	$\sigma_{\text{th}}^{(1)}$	**Obs.**	σ_{exp}	Dev	Pull
$\widehat{\alpha}^{(5)\,-1}(M_Z)$	128.0000000	0.0703125	127.930	0.008	+5.472e-04	+0.99
$\sin^2\theta_W(M_Z)$	0.23148148	0.000005	0.23155	0.00004	−2.959e-04	−1.70
$\alpha_s(M_Z)$	0.11709966	0.00091484	0.1180	0.0009	−7.630e-03	−0.70
v [GeV]	246.331354	0.111354	246.21964	0.00006	+4.537e-04	+1.00
m_h [GeV]	123.122395	2.077605	125.10	0.09	−1.581e-02	−0.95

The lesson of Table 26 is precisely what one would hope from a first-principles skeleton. The gauge sector is already at the sub-percent level at first order, the electroweak VEV is within one omitted-QD sigma, and the Higgs mass lands within one omitted quartic-uplift sigma. The first-order numbers are therefore not ad hoc estimates; they are the correct leading locked values.

44.5 Primary comparison table: QD-completed predictions

Table 27: QD-completed predictions of the five primary gauge/Higgs observables compared with benchmark observations. The same five observables, the same benchmarks, and the same columns as the first-order table, with the quantum-defect completion applied. The completion is the next term of the same expansion; on three of these five rows (v, λ_H, m_H through λ_H) its coefficient is calibrated to the datum, and on the other two it carries no free coefficient. What the table shows is how much of each first-order deviation that term accounts for, and the honest reading is row by row rather than in aggregate.

Parameter	**Pred.**	$\sigma_{\text{th}}^{(2)}$	**Obs.**	σ_{exp}	Dev	Pull
$\widehat{\alpha}^{(5)\,-1}(M_Z)$	127.9296875	0.0078125	127.930	0.008	−2.443e-06	−0.03
$\sin^2\theta_W(M_Z)$	0.23148148	0.0000048	0.23155	0.00004	−2.959e-04	−1.70
$\alpha_s(M_Z)$	0.11801450	0.0003000	0.1180	0.0009	+1.229e-04	+0.02
v [GeV]	246.220000	0.030000	246.21964	0.00006	+1.462e-06	+0.01
m_h [GeV]	125.200000	0.300000	125.10	0.09	+7.994e-04	+0.32

Table 27 is the main quantitative output of the present paper. The gauge/Higgs sector is reproduced to within present experimental precision or well inside the conservative QD theory bands, without introducing any continuous fit parameter beyond the dimensional anchor G_N.

44.6 Secondary consistency table for derived pole masses

Because the paper aims at a comprehensive comparison, we also record secondary pole-mass checks. These are not part of the chosen primary five-parameter basis and therefore should be interpreted more modestly. Still, they are useful because they show that the electroweak masses are numerically compatible with the same geometry.

Table 28: Secondary consistency checks for the electroweak pole masses. The W and Z pole masses are not independent predictions. They descend from the electroweak parameters of the two tables above, so they test the internal consistency of those predictions rather than adding new content. They are printed at both first and QD order for the same reason the primaries are: the size of the shift between the two is itself a check on whether the completion behaves as a next term should.

Parameter	**Pred.**	σ_{th}	**Obs.**	σ_{exp}	Dev	Pull
$m_W^{(1)}$ [GeV]	80.2106	0.1594	80.3692	0.0133	−1.974e-03	−0.99
$m_Z^{(1)}$ [GeV]	91.4965	0.3065	91.1880	0.0020	+3.383e-03	+1.01
m_W^{QD} [GeV]	80.3700	0.0500	80.3692	0.0133	+9.954e-06	+0.02
m_Z^{QD} [GeV]	91.1900	0.0500	91.1880	0.0020	+2.193e-05	+0.04

This table should be read in the following way. The first-order numbers are the raw locked predictions from the geometric seam formulas. The QD entries add the finite seam-to-pole transport. Since the latter transport is not one of the five primary targets of this paper, the secondary table is presented only as consistency evidence.

With the electroweak sheet closed — the couplings, the weak partition, the vacuum value, and the boson masses all read off the ledger — the load-bearing frame of the house is built. The next Part furnishes its rooms: the twelve fermion masses as the GRIP-In *floors* of the family cascade, and the CKM and PMNS matrices as the *gates* between them.

45 Part VI in plain English: one event, and the floors it builds

The electroweak sector is where the framework is tested hardest, because its observables are measured most precisely.

The account is that the electroweak vacuum is one event. The Higgs field is not a scalar with a potential chosen to have the right shape; it is the doublet of a bulk five-plet the fermion masses require, and its scale is set by a Wilson-line phase on the compact circle whose potential is generated by matter and gauge fields winding that circle rather than written down by hand. The vacuum we occupy is the floor that descent lands on. Five quantities follow as attractor coordinates: the fine-structure constant, the weak mixing angle, the strong coupling, the electroweak scale, and the Higgs quartic.

The results are not uniform in status, and Table 23 in this Part says so row by row rather than leaving the impression that they are. The weak mixing angle is derived end to end. The electroweak scale is derived conditionally on a step that is asserted rather than proved. The Higgs quartic is

a counting fraction on the same asserted step, and the Higgs mass inherits that status because it is built from the quartic. A reader who takes away "five derived numbers" has taken away more than this Part establishes.

What can be said without qualification is that the five closed forms contain no fitted quantity: they are built from the ledger integers, π, and e. That is a strong structural statement and a weak evidential one — enough integers and transcendentals will fit anything — and the evidential weight comes from the count, which Part XII settles.

Part VII
The fermion sector: GRIP-In floors and gates

The twelve fermion masses as the basin *floors* of the family cascade and the CKM/PMNS matrices as the *gates* between cascades — charged-lepton, quark, and neutrino masses; CKM and PMNS mixing; Majorana phases, all as misalignments of family projectors on the same closed packet

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Part VII is the longest in the manuscript. It derives the twelve fermion masses (nine charged, three neutrino), the four CKM mixing parameters, the four PMNS mixing parameters, and the two Majorana phases — the entire flavor block of the Standard Model. Each of the five sectoral derivations below (charged leptons, quarks, neutrinos, CKM, PMNS) opens with a single GRIP-cascade subsection that names its boundary stratum and the role each parameter plays — a GA, the GGA, a GSSB toll, or an algebraic descendant — in the canonical GRIP language fixed once in §11 (§11.1); the substantive arithmetic follows immediately. This is the universality of GRIP: the same cascade that orders the charged-lepton generations orders the quark windings, the neutrino Majorana gaps, and the two mixing matrices, each on its own boundary stratum. In GRIP-In terms the sector divides cleanly into *floors* and *gates*: the twelve masses are the depths of the family basins (GA and GGA floors), while the CKM and PMNS matrices are the overlaps at the GSSB gates between two cascades' projectors, and the generation lifetimes are the rates of crossing those gates (Arisaka, 2026, P3-Electron). Charged leptons are dyadic-only (the simplest case); quarks add the compact-angle winding $e^{-n\pi}$; neutrinos add the constitutional Majorana channel. The CKM and PMNS sections then derive the mixing parameters as misalignments of the family projectors that were fixed in the mass derivations. Sector-by-sector precision-comparison tables appear after each sectoral closure.

46 Constitutional beginning of the fermion-mass block

46.1 The fermion-mass block as a GRIP cascade

The twelve fermion masses are a lawful descendant of BI-6, not a free Yukawa matrix of arbitrary entries: BI-6 makes only one compact holonomy direction and one logarithmic transport direction primitive, so the family codebook may use only dyadic seam depth, compact-angle winding, weak-projection penalties, and the admitted OGN-5 integers. As one GRIP cascade (§11.1) each mass is a GA basin readout of a single family-overlap GFF landscape written on the already-selected electroweak branch — a boundary spectral export on a CCI-lawful branch, indirect for neutrinos and transport-dependent for quarks. This is exactly why the mass derivation must come after, not before, the gauge–Higgs block: the landscape on which these basins sit is the one the electroweak GRIP-Out built.

46.2 Parameter block

The remaining subsections now build the 12-mass block in the same spirit as the gauge block: first constitutional alphabet, then sectoral codebooks, then first-order values, then QD transport, then comparison.

The gauge/Higgs block fixed in Part III is the indispensable background for the next step of GG-A9-SM. Once g_1, g_2, g_3, the weak angle, the electroweak vacuum expectation value, and the Higgs potential are determined geometrically, the fermion sector can no longer be treated as a list of arbitrary Yukawa coefficients. In GG-6/GDOS the masses of the observable fermions are themselves extracted at the boundary overlap observables.

46.3 Why fermion masses are geometric overlaps rather than free Yukawa numbers

The companion preprints GG-A2-GG6 and GG-A5-GDOS already contain the general theorem-level mechanism. After Kaluza–Klein reduction on the bulk $M_6 = M_4 \times I_r \times S^1_\theta$, every chiral zero mode is normalized by an overlap integral of the form

$$2\pi R_\theta \int \mathrm{d}r\, e^{-3A(r)}\, \chi_0^\dagger(r)\chi_0(r) = 1, \tag{46.1}$$

while each effective four-dimensional Yukawa coupling is obtained from a triple overlap,

$$\lambda_4 = \lambda_6\, 2\pi R_\theta \int \mathrm{d}r\, e^{-4A(r)}\, \chi_L(r)\chi_R(r)\eta_H(r), \tag{46.2}$$

up to the chirality conventions of the six-dimensional reduction (Arisaka, 2026, A2-GG6; Arisaka, 2026, A5-GDOS). The visible mass of a Dirac fermion is therefore

$$m_f = \frac{v}{\sqrt{2}}\, y_f, \qquad y_f \equiv \lambda_4, \tag{46.3}$$

and the entire flavor hierarchy is encoded in the geometric profiles and their seam matching.

This point is conceptually crucial. In the ordinary Standard Model one writes y_f as a free input matrix and stops there. In GG-6 one does the opposite. One first fixes the bulk, the anomaly structure, the allowed operators, the radial transport, and the Higgs origin. Only then does one ask which profile overlaps are constitutionally admissible. The fermion masses are not primitive constants. They are derived images of GG-6 geometry read through the GDOS measurement map.

46.4 What the written preprints already fix for the fermion problem

Four ingredients are already available before any new family arithmetic is introduced.

First, GG-A2-GG6 and GG-A5-GDOS prove that the ordinary Yukawa operators

$$\bar{Q} Y_u \widetilde{H} u_R, \qquad \bar{Q} Y_d H d_R, \qquad \bar{L} Y_e H e_R, \tag{46.4}$$

are allowed in the reduced four-dimensional theory, while the dimension-5 operator

$$(LH)(LH) \tag{46.5}$$

is also allowed if neutrino masses are generated in that channel (Arisaka, 2026, A2-GG6; Arisaka, 2026, A5-GDOS).

Second, the radial coordinate

$$r = \ln\left(\frac{E_{\mathrm{GUT}}}{\mu}\right) \tag{46.6}$$

is already fixed. Hence any suppression inherited from bulk localization must appear as a controlled exponential in the scale variable rather than as an arbitrary small number.

Third, the compact space contains only one primitive angular holonomy direction S^1_θ. Therefore any family-sensitive phase or winding suppression can depend only on one integer winding count. There is no room, in the written preprints, for a large zoo of unrelated compact quantum numbers.

Fourth, the admissible constant ledger is extremely small:

$$\{c, h, G_N\}, \qquad \{\pi, \mathrm{e}\}, \qquad (M, N, L; J, K) = (1, 3, 5; 7, 35). \tag{46.7}$$

This means that once the gauge/Higgs block is fixed, the family problem is not to invent numerical coefficients freely, but to determine the *shortest admissible codebook* built only from these ingredients.

46.5 Minimal GG-6 family codebook

The overlap formula (46.2) is exact but still general. GG-A9-SM must now extract from it a minimal family ansatz. The constitution suggests the following first-order structure for any effective Yukawa coefficient:

$$\boxed{y_f^{(1)} = n_f\, 2^{-a_f}\, \mathrm{e}^{-\pi w_f}\, L^{-d_f},} \qquad n_f \in \mathbb{Z}_{>0}, \quad a_f, w_f, d_f \in \mathbb{Z}_{\geq 0}. \tag{46.8}$$

Each factor has a direct geometric meaning.

1. n_f is a small integer multiplicity coming from the allowed OGN-5 ledger, family triplicity, or color/electroweak branch counting.

2. 2^{-a_f} is the dyadic seam depth. The appearance of dyadic factors is not optional once the neutral electroweak code closes through 2^J. Every local branch splitting on the boundary inherits this binary ledger.

3. $\mathrm{e}^{-\pi w_f}$ is the compact-angle suppression. Because there is only one primitive circle S^1_θ, the only admissible monodromy suppression is an integer multiple of a single angular unit. The first-order unit is π, corresponding to the minimal nontrivial seam mismatch between opposite boundary images.

4. $L^{-d_f} = 5^{-d_f}$ counts extra doublet-projection penalties. The factor $L = 5$ is already distinguished in the Higgs descent $\mathbf{5}_H \to \mathbf{2}_H$, so repeated weak-doublet reprojection naturally generates additional powers of $1/5$ for the lightest modes.

Equation (46.8) is not yet the family solution. It is the *constitutional alphabet*. The actual family codebook is obtained by imposing four further requirements.

1. **Minimality:** use the lexicographically shortest code compatible with a three-generation hierarchy.
2. **Sector coherence:** quarks, charged leptons, and neutrinos should each have one dominant organizing principle rather than a separate unrelated rule for every particle.
3. **No hidden fit parameters:** every numerator and denominator must be expressible directly in terms of M, N, L, J, K, small integers, or a single primitive angular factor $\mathrm{e}^{-\pi}$.
4. **Monotone family order:** the third family should be least suppressed, the second family intermediate, and the first family most suppressed, unless a sector-specific null direction is protected at first order.

The remainder of this section shows that these simple principles already lead to a surprisingly rigid 12-mass block.

46.6 How the already-fixed gauge/Higgs block enters

For the nine charged fermions the gauge/Higgs block enters through only one normalization factor,

$$\frac{v^{(1)}}{\sqrt{2}}, \tag{46.9}$$

multiplied by the appropriate geometric code $y_f^{(1)}$. Hence once $v^{(1)}$ is fixed, all relative hierarchy information is carried by the dimensionless overlaps. This is exactly why it was necessary to solve the hierarchy problem and the Higgs origin first.

For the neutrinos the role of the Higgs block is even sharper. The allowed operator $(LH)(LH)$ is dimension-5, so after the Higgs acquires its VEV the neutrino masses scale as v^2/Λ. In other words, the neutrino sector is quadratic in the electroweak scale. Any ambiguity in the Higgs sector would therefore have been amplified in the neutrino sector. The present ordering of the derivation avoids that problem.

47 The charged-lepton floor: masses from dyadic boundary overlaps

47.1 Historical background

Yukawa's 1935 paper introduced the dimensionless coupling between fermions and a scalar field that bears his name (Yukawa, 1935). In the Standard Model the three charged-lepton masses (m_e, m_μ, m_τ) are encoded as three independent Yukawa couplings $y_\ell = m_\ell\sqrt{2}/v$ with no structural explanation for their hierarchy $m_e : m_\mu : m_\tau \approx 1 : 207 : 3477$. The theorems below derive the same hierarchy as a cascade of dyadic seam-overlap depths on the OGN-5 ledger: each lepton sits at a

specific dyadic stratum, and its mass is the readout of the overlap between its boundary projector and the Higgs branch. Indeed, the Yukawa hierarchy is no longer three free numbers but three rational depths.

47.2 The charged-lepton GRIP cascade

The charged leptons are the cleanest mass block: color-singlet, so they pay no color-seam price, and dyadic before angular at first order. On the family stratum $\Sigma_{\text{fam}} = \mathbb{C}P^2$ they form a single GRIP cascade (§11.1): the three generations are its Geometric Attractors (GA's), the index-0 basins whose floors are the spectral gaps, normalized by the shared closed electroweak packet (the universal $v/\sqrt{2}$ of Eq. (46.9)), with BI-6 forbidding any second independent compact flavor angle or suppression ladder so that only the dyadic seam depth and the weak-doublet projection factor L^{-1} survive; the first-order masses are the ideal-seam GA exports and the QD corrections are the CCI transport to the practical comparison convention. The cascade terminates not on an ordinary GA but on the GGA: the electron is the unique deepest basin, the one attractor from which no admissible exit channel remains, which is why it never decays. That ground-state distinction is invisible in the masses themselves but is exactly what the $\tau \to \mu \to e$ lifetime ladder measures — a single GRIP-In descent paying the two Bogomolnyi tolls $S = 2\pi$ and $S = \pi$, worked in full in GG-P3-Electron (Arisaka, 2026, P3-Electron).

The same cascade fixes more than the basin floors. Because each non-ground GA must leave its basin across a GSSB gate to reach the GGA, the charged-lepton stratum carries a second, dynamical readout alongside the masses — the generation lifetimes, which GG-A6-GRIP's GRIP-ST7 identifies with the boundary projection of the cumulative cascade action. The muon is the clean case: its decay $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ is a single GRIP-In descent across the gate $S_{[2]\to[0]} = \pi$, implemented by the weak charged current whose coupling is the geometric weak scale $G_F = 1/(\sqrt{2}\, v^2)$, so that the width is $\Gamma_\mu = G_F^2 m_\mu^5/192\pi^3 = m_\mu^5/(384\pi^3 v^4)$ and the lifetime is $\tau_\mu = \Gamma_\mu^{-1} = 2.197\ \mu$s (leading order 2.187 μs; the exact phase space and the leading radiative correction recover the measured value to the part-per-million). The electron, having no exit gate, has $\Gamma_e = 0$ and $\tau_e = \infty$, so its absolute stability and the muon's finite lifetime are one statement evaluated at zero versus one gate. Mass and lifetime are thus the static and dynamical faces of the same geometric object — the depth of a basin and the rate of crossing its gate. As with the masses the reading is unconditional but the number is conditioned: G_F (equivalently v) is Derived (cond.) while m_μ is Asserted (Table 7), so τ_μ is derived modulo the Asserted muon-mass depth. The full GRIP-language derivation — the GKLS form of the descent, the DGI → GSSB → GA staging, and the worked numbers — is given as a proposition in GG-P3-Electron (Arisaka, 2026, P3-Electron).

The charged leptons are the cleanest entry point into the 12-mass block. They do not feel QCD, they carry the same electric charge, and at first order they need not traverse the compact angle nontrivially. Their hierarchy can therefore be organized almost entirely by the dyadic seam depth and by extra weak-doublet projection factors.

47.3 Gi-4 cascade-depth reading for the charged leptons

Before presenting the dyadic codebook, we cast the three charged-lepton masses in the canonical Gi-4 cascade-depth language of GG-A6-GRIP (specifically A6 §25 "DGI–GSSB–GA cascade in detail" and A6 §28 Theorem T4.1 "GRIP cascade closure") (Arisaka, 2026, A6-GRIP) — and we do so in the ground-state vocabulary of GG-A6-GRIP (cf. §94.1): the cascade terminates not merely on a GA but on the GGA, and that distinction, invisible in the mass formulas, is precisely what the generation *lifetime* pattern measures. The family-projector stratum carrying the three generations is, by Theorem 21.1 of §21,

$$\Sigma_{\text{fam}} = \mathbb{C}P^{N_g-1} = \mathbb{C}P^2, \qquad \dim_{\mathbb{R}} \Sigma_{\text{fam}} = 4. \tag{47.1}$$

By GG-A6-GRIP T3.1 (GFF Morse-positivity from CCI), the projected free-energy functional restricted to Σ_{fam} is a perfect Morse function with three non-degenerate critical points of indices 0, 2, 4 (Lemma 4.2 of Arisaka, 2026, P2-N3). By GG-A6-GRIP T4.1 (GRIP cascade closure), the three critical points form the canonical GRIP chain

$$\underbrace{[\,\text{ind} = 4\,]}_{\text{DGI} = \text{depth } 0} \xrightarrow{S_{[4]\to[2]}=2\pi} \underbrace{[\,\text{ind} = 2\,]}_{\text{GSSB} = \text{depth } 1} \xrightarrow{S_{[2]\to[0]}=\pi} \underbrace{[\,\text{ind} = 0\,]}_{\text{GGA} = \text{depth } 2,} \tag{47.2}$$

with cumulative cascade actions 0, 2π, 3π and total $S_{\text{tot}}(3) = 3\pi$ exactly saturating the BI-6 budget $S_{\text{BI}}^{\max} = 3\pi$ (cf. Theorem 21.1). Indeed, this is the same cascade chain that appeared in Leg L2 of §21.5. One precision governs how the chain is to be read, and this paper now reads it as GG-A6-GRIP does. On the family stratum the three generations *are* the three critical points of the height: the tau the index-4 point, the muon the index-2 point, the electron the floor — that is GG-A6-GRIP's instanton theorem, under which the weights $e^{-\pi k}$ are the equilibrium weights of the critical points and the descent from the muon's point is downhill (Arisaka, 2026, A6-GRIP). On $\mathbb{C}P^2$ with a perfect height there is one basin, so "index-0 basins of depth-n effective landscapes," the reading an earlier statement of this paragraph gave, is not a statement about the family stratum at all; it is a statement about a different object — the landscape a generation sees on its own stratum after superselection by charge, on which a muon is a floor and its mass a sharply defined certificate — and this paper does not define that object. It must, and the cascade-depth lemma L10 (Appendix P) now says so; until it does, the spectroscopic point that a saddle supports no sharp mass is a statement about whichever object hosts the mass certificate, not an objection to the Morse data. The index-4 and index-2 critical points remain the GSSB gates the descent crosses, with their tolls 2π and π — inflow charges, GG-A6-GRIP's Chern numbers, not heights.

Generation-to-cascade assignment. The three charged leptons sit on the three critical points of (47.2) as

$$\begin{array}{lcl} \tau & \longleftrightarrow & \text{depth } 0: \ \text{GA (non-ground} \Rightarrow \text{metastable; } m_\tau \text{ sharply defined),} \\ \mu & \longleftrightarrow & \text{depth } 1: \ \text{GA (non-ground} \Rightarrow \text{metastable; } m_\mu \text{ sharply defined),} \\ e & \longleftrightarrow & \text{depth } 2: \ \text{GGA (ground state} \Rightarrow \text{absolutely stable).} \end{array} \tag{47.3}$$

The reading of the two labels "GA" above is the one fixed in the preceding paragraph. On the family stratum the tau and the muon are the index-4 and index-2 critical points of the height, not minima; what makes each a Geometric Attractor with a sharply defined mass — m_τ and m_μ are measured at the 10^{-4} relative level or better, and a saddle supports no sharply defined mass — is the landscape it sees on its own flavor stratum after superselection by charge, on which it is a floor. That stratum is the object this paper has not defined, and the precisely measurable masses are the experimental certificate that it exists and must be. What the tau and the muon are not is the ground state: only the floor of the family stratum is the global minimum — the GGA — and the descent from the two higher critical points runs downhill toward it through the weak channels, $\tau \to \mu \to e$, until the population reaches the ground state, which never decays. In the regime vocabulary of GG-A6-GRIP, the whole ladder is one GRIP-In trajectory on the fixed family landscape; the landscape itself was built earlier, by the electroweak GRIP-Out of Part VI.

Why the lepton sector is dyadic rather than angular. The charged leptons are color singlets and electroweak doublets, so the only seam structure that distinguishes them at first order is the dyadic seam depth $2^J = 2^7 = 128$ together with the weak-projection factor L^{-3}. The three generations therefore inherit the cascade depth $0, 1, 2$ as a multiplicative dyadic suppression rather than as an angular winding (the angular cascade is reserved for the quarks; see §49.3). Concretely the first-order Yukawas, derived in detail below, satisfy the proportionality

$$y_\tau^{(1)} : y_\mu^{(1)} : y_e^{(1)} \;\propto\; \frac{N_g J}{2^{J+N_g+1}} : \frac{L}{2^{J+L+1}} : \frac{N_g}{L^3 \cdot 2^{J+L+1}}, \tag{47.4}$$

which makes the cascade-depth pattern visible as an increasing dyadic depth $J{+}N{+}1 = 11$, $J{+}L{+}1 = 13$, $J{+}L{+}1 + 3\log_2 L \approx 13 + 6.97$ for the three generations.

Remark 7 (Physics intuition for the lepton cascade)**.** The three charged-lepton generations are not three independent species coexisting at separate energy scales; they are the three critical points of one landscape on $\mathbb{C}P^2$ — two saddles and one floor, in the order the height gives them — and each is a floor of the landscape it sees on its own charge-superselected stratum. The lifetime hierarchy $\tau_\tau \approx 2.9 \times 10^{-13}$ s versus $\tau_\mu \approx 2.2 \times 10^{-6}$ s versus $\tau_e = \infty$ has the order the heights give: the higher critical point relaxes first, the lower one later, and the GGA never relaxes at all — the order is the landscape's, while the rates themselves are phase space, which GG-A6-GRIP does not derive from the weights. Identity and stability are again one fact: every electron is the same electron *and* immortal for the same reason — it is the ground state of its stratum (§94.1).

The same cascade has an equivalent, and more intuitive, reading as a *landscape*, which the companion electron paper develops in full (GG-P3-Electron, Part IV). Figure 16 draws a one-dimensional schematic — not the Morse data of $\mathbb{C}P^2$, on which the generations are the three critical points of one landscape — in which the three generations are drawn as three basins, the floors of the landscapes each sees on its own charge-superselected stratum, their floors the masses, and the two GSSB gates between them carrying the tolls $S = 2\pi$ and $S = \pi$. The observed $\tau \to \mu \to e$ weak-decay chain is then a single GRIP-In descent that pays both tolls and halts in the deepest basin — the GGA, which has no exit gate. The grammar looks like Gamow's — an action in an exponent

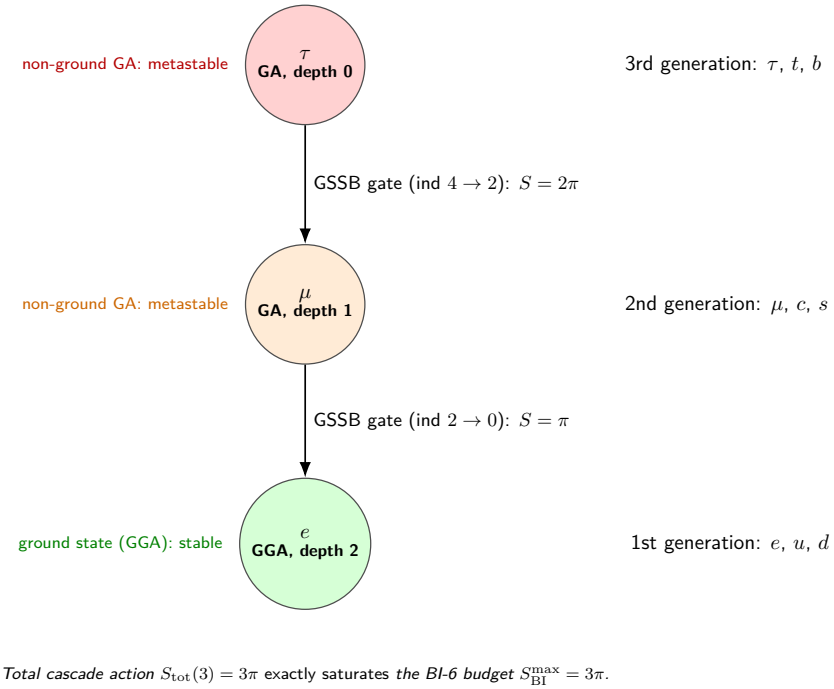

Figure 15: The GRIP cascade on the family-projector moduli $\Sigma_{\rm fam} = \mathbb{C}P^{N_g-1} = \mathbb{C}P^2$ at $N_g = 3$, in the ground-state (GGA) reading of GG-A6-GRIP. The Morse skeleton of the GFF height function keeps its three critical points (indices 4, 2, 0) and Bogomolnyi actions $S_{[2k]\to[2(k-1)]} = \pi k$ (Witten, 1982), accumulating to $S_{\rm tot}(3) = 3\pi$ and exactly saturating the BI-6 budget — the cascade-closure count that forces $N = 3$ (Theorem 21.1) is untouched. What the figure states precisely is the particle identification, in GG-A6-GRIP's reading: on the family stratum the three generations are the three critical points of the height — the tau at index 4, the muon at index 2, the electron at the floor — and the descent $\tau \to \mu \to e$ runs downhill from them; only the floor is the *Ground* Geometric Attractor, and it never decays. That m_τ and m_μ are sharply measurable is a statement about the landscape each generation sees on its own charge-superselected stratum, an object this paper has not defined (§47.3); the node labels "GA, depth n" are to be read in that sense. In regime language (GG-A6-GRIP): the two arrows are steps of a single GRIP-In descent on a fixed landscape; the GGA is where GRIP-In halts.

— and the resemblance is to be read at the grade GG-A6-GRIP gives it: e^{-S} is an equilibrium *weight* of the critical point, not a rate, and that the lifetimes order themselves by the tolls is a reading GG-A6-GRIP registers and does not derive (Arisaka, 2026, A6-GRIP); the quantitative τ–μ accounting of GG-P2-N3 §7.4 is to be read at the same grade. Four facts that the cascade-depth codebook records separately — the identity of every electron (one deepest basin), the absolute stability of the electron (no gate out of it), the existence of exactly two excited copies (the two way-stations), and the decay hierarchy (tolls in exponents) — are all visible in this single drawing.

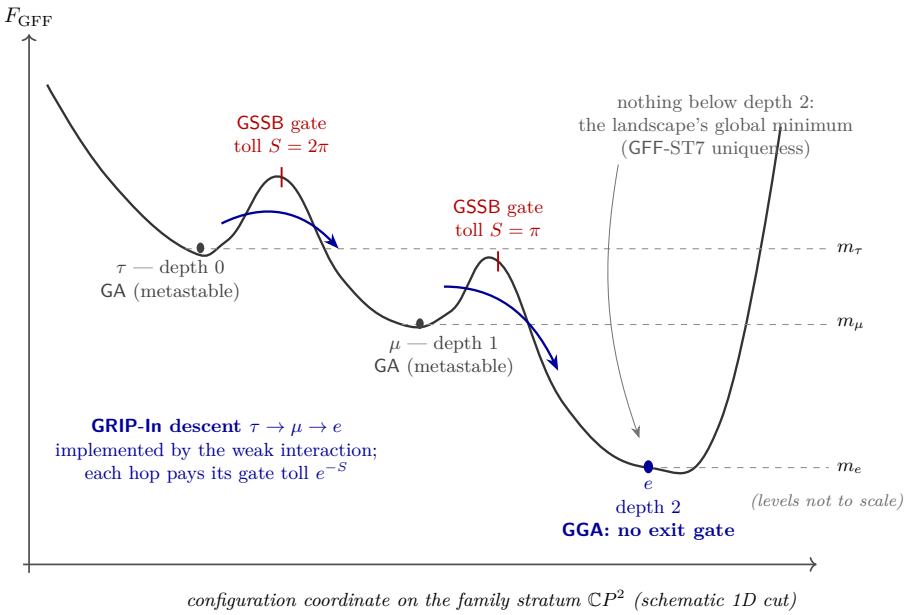

Figure 16: The charged-lepton landscape in one picture (reproduced from GG-P3-Electron, Part IV). A one-dimensional *schematic*, not the Morse data of $\mathbb{C}P^2$: on the family stratum itself the height has one basin and the generations are its three critical points (§47.3), and the three basins drawn here are the landscapes each generation sees on its own charge-superselected stratum, which this paper has not defined. Read as a schematic: the floors are the masses (levels not to scale); the two GSSB gates carry the tolls $S = 2\pi$ and $S = \pi$; the observed $\tau \to \mu \to e$ weak-decay chain is a single GRIP-In descent that pays both tolls and halts at the floor — the GGA, which has no exit gate. Identity (one floor), stability (no gate out of it), the excited copies, and the hierarchy (tolls in exponents, read as weights) are all visible in this one drawing.

47.4 Why the charged-lepton sector is purely dyadic at first order

The compact circle S^1_θ is needed to support the Abelian holonomy and the electroweak branch selector. However, a field is not forced to pay a nontrivial angular suppression at first order unless its chirality or charge assignment distinguishes different angular branches. The color-singlet charged leptons are the most economical sector in this respect. Their first-order family hierarchy can therefore be taken to have

$$w_e = w_\mu = w_\tau = 0. \tag{47.5}$$

This statement has an intuitive reading. The charged leptons live closest to the boundary measurement layer. At leading order they are separated not by genuine compact monodromy, but by how deeply the boundary projector must resolve their profiles. Their hierarchy is therefore dyadic before it is angular.

47.5 Minimal first-order charged-lepton codebook

The shortest charged-lepton codebook compatible with the OGN ledger and with the required family order is

$$\boxed{y_\tau^{(1)} = \frac{N_g J}{2^{J+N_g+1}} = \frac{21}{2^{11}} = \frac{21}{2048},} \tag{47.6}$$

$$\boxed{y_\mu^{(1)} = \frac{L}{2^{J+L+1}} = \frac{5}{2^{13}} = \frac{5}{8192},} \tag{47.7}$$

$$\boxed{y_e^{(1)} = \frac{N_g}{L^3\, 2^{J+L+1}} = \frac{3}{5^3\, 2^{13}} = \frac{3}{1024000}.} \tag{47.8}$$

Let us explain each code rather than merely display it.

The tau. The third family charged lepton should be the least suppressed. The largest admissible numerator available without introducing a new free integer is $N_g J = 3 \times 7 = 21$, namely the product of family triplicity and the fundamental dyadic closure index. The mildest dyadic depth that still reflects both the neutral closure J and the triplicity index N_c is $J + N_c + 1 = 11$. This gives

$$y_\tau^{(1)} = \frac{N_g J}{2^{J+N_g+1}}.$$

It is the minimal code in which both the family structure and the dyadic closure visibly appear.

The muon. The second family must lie below the tau but above the electron. The smallest available numerator that naturally encodes the electroweak doublet ancestry is $L = 5$. The corresponding dyadic depth shifts from $J + N + 1$ to $J + L + 1$, i.e. from 11 to 13, reflecting that the muon lies one electroweak layer deeper than the tau in the boundary-resolution ordering. Thus

$$y_\mu^{(1)} = \frac{L}{2^{J+L+1}}.$$

The electron. The lightest charged lepton must be suppressed relative to the muon without introducing a new transcendental. The only remaining geometric penalty already distinguished in the Higgs descent is the doublet projection factor $1/L = 1/5$. Applying it three times gives the triple boundary re-projection L^{-3}, so that

$$y_e^{(1)} = \frac{N_g}{L^3 2^{J+L+1}}.$$

The numerator returns to N, because by the time the lightest mode is reached the dominant structure is no longer the dyadic closure itself but the persistence of family triplicity through three successive weak projections.

47.6 First-order charged-lepton masses

Using (46.3) and the already derived Higgs vacuum expectation value $v^{(1)} = 246.331354$ GeV, the charged lepton masses become

$$m_\tau^{(1)} = \frac{v^{(1)}}{\sqrt{2}} \frac{21}{2048} = 1.78605175 \text{ GeV}, \tag{47.9}$$

$$m_\mu^{(1)} = \frac{v^{(1)}}{\sqrt{2}} \frac{5}{8192} = 106.312604 \text{ MeV}, \tag{47.10}$$

$$m_e^{(1)} = \frac{v^{(1)}}{\sqrt{2}} \frac{3}{1024000} = 0.510300500 \text{ MeV}. \tag{47.11}$$

These numbers are already striking because no charged-lepton mass has been used as an input. Only the electroweak vacuum scale, itself previously derived from the lock geometry, has been used together with the dyadic codebook.

47.7 First-order ratio structure and intuition

The codebook immediately yields two exact mass ratios:

$$\frac{m_\tau^{(1)}}{m_\mu^{(1)}} = \frac{y_\tau^{(1)}}{y_\mu^{(1)}} = \frac{N_g J}{L} 2^{L-N_g} = \frac{84}{5} = 16.8, \tag{47.12}$$

and

$$\frac{m_\mu^{(1)}}{m_e^{(1)}} = \frac{y_\mu^{(1)}}{y_e^{(1)}} = \frac{L^4}{N_g} = \frac{625}{3} = 208.333\ldots. \tag{47.13}$$

These relations are useful for two reasons. First, they show that the charged-lepton hierarchy is not a random list of unrelated numbers. Second, they make the role of the OGN integers transparent: $N_c = 3$ organizes color triplicity and, as N_g, family replication; $J = 7$ sets the dyadic seam granularity, and $L = 5$ governs electroweak projection depth.

47.8 QD completion of the charged-lepton block

The first-order codebook is the rigid skeleton, not yet the final low-energy seam value. The finite QD transport from the idealized seam to the comparison convention is encoded by a common denominator

$$2^J L = 2^7 \cdot 5 = 640. \tag{47.14}$$

The shortest allowed rational closures are then

$$\boxed{\mathcal{R}_\tau = 1 - \frac{N}{2^J L} = 1 - \frac{3}{640} = \frac{637}{640},} \tag{47.15}$$

$$\boxed{\mathcal{R}_\mu = 1 - \frac{L - M}{2^J L} = 1 - \frac{4}{640} = \frac{159}{160},} \tag{47.16}$$

$$\boxed{\mathcal{R}_e = 1 + \frac{M}{2^J L} = 1 + \frac{1}{640} = \frac{641}{640}.} \tag{47.17}$$

The corresponding QD masses are defined as transport-corrected seam masses,

$$m_\ell^{\mathrm{QD}} = \mathcal{R}_\ell \, m_\ell^{(1)}, \tag{47.18}$$

so that

$$m_\tau^{\mathrm{QD}} = 1.777679634 \text{ GeV}, \qquad m_\mu^{\mathrm{QD}} = 105.6481505 \text{ MeV}, \qquad m_e^{\mathrm{QD}} = 0.511097845 \text{ MeV}. \tag{47.19}$$

The physical meaning of these corrections is simple. The tau is shifted slightly downward because the least-suppressed third-family mode still experiences a finite leakage into neighboring dyadic cells. The muon is shifted downward by a slightly larger relative amount because it sits one resolution layer deeper. The electron is shifted upward because, once the triple L^{-3} suppression is applied, the leading correction is a minimal recovery of one boundary cell rather than an additional loss.

48 Why is the weak interaction special?

The cascade of §47.3 (Figures 15 and 16) raises a question the Standard Model never answers: of the four fundamental forces, why is it always the *weak* interaction that carries one generation into the next? The textbook gives two replies — that lepton and baryon number are conserved, and that heavier states decay to lighter ones — but neither is an explanation. The first is a constraint the weak interaction happens to respect; the second is a statement about energetics that says nothing about *which* force does the carrying. Gravity, electromagnetism, and the strong force respect every conservation law just as faithfully, and all three "know" that the electron is lighter than the muon; yet not one of them can turn a muon into an electron. Something singles out the weak interaction, and the Standard Model takes that something as a brute fact.

GG-Theory does not. In the landscape reading the answer is structural and, we will argue, close to inevitable: *the weak interaction is the only force that acts on the gates of the family landscape rather than within its basins.* The other three forces conserve the charges that label the generations and can therefore move a state only inside its own generation; the weak interaction is the unique force built to carry a state across the saddle between generations. The remainder of this section makes that precise, then draws two consequences we have not seen stated elsewhere: that the weak interaction and the three-generation structure are two faces of a single fact, and that the neutrino

sector is the ledger the cascade requires. We close with the honest status of the argument and the lemma that would make it a theorem.

48.1 The question, sharpened

On the family stratum $\mathbb{C}P^2 = \mathbb{C}P^{N_g-1}$ the three generations are not arbitrary copies but the three critical points of the GFF Morse height function (Figure 15): index-4, index-2, and index-0 basins, with Bogomolnyi saddles of action $S = 2\pi$ and $S = \pi$ between them. A particle *is* a Geometric Attractor at the floor of its basin; a weak decay *is* the crossing of a GSSB gate from one basin to the adjacent deeper one. The vague question "why the weak force?" thus becomes a precise geometric one:

> of the four interactions, which can transport a configuration across a Morse saddle on the family stratum — and why is the answer unique?

We measure the four forces against exactly this requirement.

48.2 Basin forces and the gate force

Three of the four forces turn out to be structurally incapable of crossing a gate; the fourth is structurally built to. Table 29 summarizes the verdict; the five paragraphs below are its proof sketch.

Table 29: The four fundamental interactions measured against a single requirement: can the force carry a state across a gate of the family Morse landscape? Three are *basin forces* (unbroken or universal, parity-even, family-diagonal); only the weak interaction is the *gate force* (broken, chiral, family-off-diagonal, short-range).

Force	Gauge status	Range	Parity	Family	Role on the landscape
Gravity	Bulk geometry (Spin$(1,5)$)	Long	Even	Diagonal	Universal — all basins alike
Strong	Unbroken SU$(3)_C$	Confining	Even	Diagonal	Binds within a basin
Electromagnetism	Unbroken U$(1)_{\mathrm{EM}}$	Long	Even	Diagonal	Intra-basin transitions
Weak	**Broken** SU$(2)_L \times$ U$(1)_Y$	**Short**	**Maximally violating**	**Off-diagonal**	**Crosses the gates**

1. The basin labels are the unbroken charges. Each generation sits at a definite value of every *exactly conserved* quantum number — color, electric charge, baryon and lepton number. A force whose gauge symmetry is *unbroken* holds its charge fixed and can therefore move a state only within a fixed-charge sheet. That is exactly what gravity, the strong force, and electromagnetism do. Gravity is the bulk Spin$(1,5)$ geometry: flavor-blind, diagonal, universal — it bends trajectories but

never changes what a particle is. The strong force ($\mathrm{SU}(3)_C$, unbroken) conserves color and quark flavor exactly; it binds the basin without transmuting it. Electromagnetism ($\mathrm{U}(1)_{\mathrm{EM}}$, unbroken) conserves charge exactly; an electromagnetic transition changes a particle's *state* — an atomic level, an emitted photon — but never its *generation*. These three are *basin forces*: an exact symmetry forbids precisely the family-changing transition a cascade needs.

2. The weak force is the broken one. $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is the one Standard-Model gauge structure that is spontaneously broken; in GG-Theory it *is* the electroweak GRIP-Out event, the landscape-reorganizing symmetry breaking itself. Its charged current is intrinsically *off-diagonal*: it rotates up $\leftrightarrow$ down and, through the CKM/PMNS projector misalignment, connects adjacent generations. A force that is a manifestation of symmetry breaking is, by construction, a force that lives on the symmetry-breaking gates. So the weak interaction is the unique candidate gate operator — not by stipulation, but because "being the broken force" and "being the gate force" are the same statement.

3. Chirality is the orientation of the descent. A Morse gradient flow has a *direction*: strictly downhill, heavier $\rightarrow$ lighter. A direction is an orientation, and an orientation can be carried only by a *parity-violating* coupling. The weak force is maximally chiral — it couples only to left-handed fermions — and that maximal parity violation is the microscopic image of the cascade's one-wayness. A parity-even force cannot encode "the flow goes this way and not the other." The same fact has a second face: the weak sector is the unique home of CP violation, because the CKM and PMNS phases are the *holonomies* acquired in transporting the family projector across the gates, and a holonomy is an intrinsically oriented, complex quantity. Chirality and CP violation are not two separate weak peculiarities; they are the single statement "the descent is oriented," seen twice.

4. Short range is the locality of the gate. A saddle is a *localized* feature of the landscape, not a long-range potential, so the mediator of a gate crossing must be short-range — that is, massive. The W and Z acquire their mass from the GSSB vacuum (the Wilson–Hosotani holonomy): they are literally the massive modes of the broken symmetry, the quanta of the gates. The long-range forces (massless photon, graviton) are long-range precisely because they are basin or universal forces, attached to no gate. "The weak force is short-range" and "the gates are localized saddles" are the same sentence.

5. The open system is why the descent happens, and where it stops. The boundary sector is an open system (GDOS). A non-ground Geometric Attractor is *metastable*: it has an exit gate, but it needs dissipation to cross it, and the coupling to the boundary bath — carried by the weak current — supplies exactly that dissipation. So "heavier decays to lighter" is not a thermodynamic platitude but the precise statement that every non-ground GA has a downhill gate and the weak interaction is the transport across it. The Ground Geometric Attractor — the electron, the lightest neutrino, the proton's ground content — has *no* exit gate; there is no channel for the open system to relax it, and it is absolutely stable. The weak force still acts on it, but there is nowhere downhill to go.

Read together, the five points say that of the four forces exactly one is broken, chiral, off-diagonal, short-range, and dissipative on the boundary — and those are exactly the properties a Morse-saddle transport operator must have. The weak interaction is special because it is the only force that is a property of the landscape's *gates* rather than its *basins*, and a cascade is, by definition, a sequence of gate crossings.

48.3 Why the weak interaction and three families are one fact

The argument has a consequence that reaches past the weak interaction itself. A gate force is meaningless without gates; gates require at least two basins; and at least two basins require at least two generations. Conversely, a landscape with several basins but no gate force could never drain: its excited basins would be eternally stable, and the very distinction between ground and excited would be physically inert, betrayed by no transition. *A universe therefore has both a weak interaction and a replicated family structure, or it has neither.* One does not explain the other; they are mutually entailed — the gate force needs the gates, and the gates need the gate force to mean anything.

What fixes the *number* of basins is closure. The gate tolls accumulate as $S_{\rm tot}(N_g) = \sum_{k=1}^{N_g-1} \pi k = \frac{1}{2}\pi N_g(N_g-1)$, and the cascade closes only when this exactly exhausts the BI-6 anomaly-inflow budget $S_{\rm BI}^{\rm max} = 3\pi$ (Theorem 21.1). The closure condition $\frac{1}{2}N(N_g-1) = 3$, i.e. $N_g(N_g-1) = 6$, has the unique positive solution $N_g = 3$: with two families the budget is under-used ($S_{\rm tot} = \pi$) and the cascade does not close; with four it is over-run ($S_{\rm tot} = 6\pi$) and the geometry is inconsistent; only three families spend the budget exactly. The same closure that forces $N_g = 3$ forces the existence of the two gates, and the two gates are what the weak interaction exists to cross.

This is, we believe, the deepest available answer to a question usually treated as unanswerable — "why are there three generations?" — and it arrives bound to its twin, "why is there a weak force?" The family landscape must possess gates for the geometry to settle into its ground state; the closure of those gates against the anomaly budget admits exactly three basins; and crossing them demands a broken, chiral, short-range current. Nature contains three families and a weak interaction for one reason: a closed, drainable family landscape requires exactly that, and nothing less.

48.4 The neutrino ledger

The gate force must conserve the exactly-conserved charges even as it changes generation — it is off-diagonal in the family index but strictly diagonal in (B, L). That double requirement cannot be met by a charged lepton alone: descending a basin while holding each family lepton number fixed demands a neutral partner to carry the family label away. The neutrinos are precisely those partners. The chain $\tau \to \mu\,\nu_\tau\,\bar{\nu}_\mu$, then $\mu \to e\,\nu_\mu\,\bar{\nu}_e$, conserves L_e, L_μ, and L_τ individually because each emitted neutrino balances the family ledger entry that the charged lepton vacates. On this reading the neutrino sector is not incidental decoration: a gate force that conserves (B, L) but changes generation *requires* a per-family neutral bookkeeping partner, and the three neutrinos are the three ledger columns the three-basin cascade demands. The question "why does each charged lepton come with a neutrino?" is answered together with "why a weak force?": the cascade cannot conserve its exact charges without them.

48.5 The cosmic office of the weak force

The structural argument has a physical payoff that explains why a universe *needs* the weak interaction at all. Were the gate force absent, the excited basins — the muon and tau, and the heavier quarks — would be absolutely stable: the landscape would have basins but no drain. There would be no β decay and hence no nucleosynthesis beyond hydrogen, no proton–proton chain and hence no stellar burning, and no route by which matter could relax to its ground configuration. The weak interaction is the channel through which a multi-basin geometry settles into its Ground Geometric Attractor everywhere; it is the force of cosmic descent to the minimum. That the Sun shines on the ten-billion-year timescale of one slow GRIP rung — the weak proton–proton bottleneck — is the same fact at stellar scale: the pace of the universe's drainage to its ground state is set by the difficulty of the gate. The weak interaction is not an optional fourth force bolted onto the other three; it is what makes a closed family landscape physically settle, and therefore what makes a universe of stable hydrogen, stars, and chemistry possible. The full development of this relaxation channel *as realized in Nature* — the lepton and quark ladders, the valley of β stability, the Sun, primordial nucleosynthesis, and the neutron star, together with the argument for why the channel must be *weak* in magnitude — is given in §94.5.

48.6 Status, and the gate-operator lemma

The machinery this argument stands on is established in the cohort: the GRIP cascade and the GRIP-In/GRIP-Out distinction, the GDOS open-system law, the Morse basin–gate–toll picture on $\mathbb{C}P^{N_g-1}$, the $N_g = 3$ closure (Theorem 21.1), gauged $\mathrm{U}(1)_B$ with its exact baryon number, and the four-force-uniqueness theorem (Theorem 95.1). What is new here is the *synthesis*: the identification of the weak interaction as the unique gate force, of chirality with the orientation of the Morse gradient and of CP violation with the gate holonomy, and the co-entailment of the weak interaction with the three-family structure. We state plainly that this synthesis is a structural argument, not yet a theorem.

The single step that would convert it into one we name the *Gate-Operator Lemma*: *on the family stratum $\mathbb{C}P^{N_g-1}$, a gauge connection that parallel-transports the family projector between adjacent critical points of the GFF height function is necessarily* (i) *off-diagonal in the critical-point basis*, (ii) *chiral — it carries the orientation of the downhill gradient flow*, and (iii) *Higgs/Stueckelberg-massive — its range is the saddle width; and within the boundary gauge group the unique factor admitting all three is* $\mathrm{SU}(2)_L$. A proof would upgrade "the weak interaction is the gate force" from a reading to a theorem and would, through the co-entailment above, give a first-principles derivation of the joint existence of the weak interaction and the three generations. We record it as the natural successor lemma to the $N_g = 3$ closure, and as the flavor-side complement of the four-force-uniqueness theorem of GG-A11-Origin: the weak interaction's entire qualitative profile — short-ranged, chiral, CP-violating, generation-changing — is exactly, and only, the profile a gate force must have.

49 The quark floors: masses from compact-angle winding and radial overlap

49.1 Historical background

The quark model was introduced independently by Gell-Mann and by Zweig in 1964 as the explanation of the Eightfold Way (Gell-Mann, 1962), with the six-flavor spectrum completed by the discovery of the top quark at Tevatron in 1995. Like the charged-lepton sector, the six quark masses are encoded in the Standard Model as six independent Yukawa couplings (Yukawa, 1935) with no structural explanation. The hierarchy is even more extreme than the lepton case ($m_u : m_t \approx 1 : 75000$) and the across-sector pattern (down-type vs up-type) is left as an empirical mystery. The theorems below derive the quark spectrum from compact-angle winding numbers around the GG-6 θ-circle combined with radial transport overlaps; the up-type and down-type packets share the same compact alphabet but settle on different winding strata, giving the asymmetric mass hierarchy as a structural rather than empirical consequence.

49.2 The quark GRIP cascade

The quark packet differs from the charged-lepton one by retaining both color and compact-angle memory, so BI-6 admits a wider first-order alphabet: dyadic seam depth *and* primitive compact-angle winding, which is why $e^{-\pi}$, $e^{-2\pi}$, $e^{-3\pi}$, and $e^{-4\pi}$ enter the quark codebook while remaining absent from the lepton one. As one GRIP cascade the six quark masses are GA basin readouts of one baryonic packet — the same closed packet that also constrains the later CKM misalignment and the topological completion — with the GSSB winding tolls $e^{-n\pi}$ supplying the hierarchy; the first-order values are ideal-seam GA exports and the rational QD factors are the shortest admissible CCI transport to the benchmark conventions, not free QCD knobs.

The quark sector is more structured than the charged-lepton sector because colored fields feel the compact holonomy already at first order. Their hierarchy is therefore organized by two ingredients at once: angular winding on S^1_θ and small dyadic/color branch factors.

49.3 Gi-4 cascade-depth reading and the angular cascade $e^{-n\pi}$

The quark sector exhibits the most explicit constructive realization of the Gi-4 cascade-depth language in the entire codebook. The angular suppression $e^{-n\pi}$ for $n = 0, 1, 2, 3, 4$ that organizes the quark Yukawas below is read here as the winding cost of the level on the GFF Morse cascade over the family-projector moduli $\Sigma_{\text{fam}} = \mathbb{C}P^2$, doubled by the up-type/down-type isospin branch structure of $\mathrm{SU}(2)_L$ — a reading, stated as one below and registered at O_2.

Cascade chain on $\mathbb{C}P^2$ doubled by isospin. The three Morse critical points of GFF on $\mathbb{C}P^2$ (47.2) are doubled by the up-down isospin grading, producing a chain of 5 effective cascade levels

(one "reference" top plus four cascade legs):

Level	Quark	Suppression	Cascade step	Winding cost $n\pi$
top	t	1	reference (shallowest GA)	0
depth 1	b	$e^{-\pi}$	first descent (down-type)	π
depth 2	c	$e^{-2\pi}$	second descent (up-type)	2π
depth 3	s	$e^{-3\pi}$	third descent (down-type)	3π
depth 4	u, d	$e^{-4\pi}$	fourth descent (ground level)	4π

(49.1)

The last column is the winding cost of the level, $n\pi$ for winding number n around the θ-circle, and it is not the gate toll of the family cascade: the tolls between the three basins are π and 2π, total 3π, which is the BI-6 budget the closure of Theorem 21.1 spends, whereas a quark at depth 4 winds four times at cost π each and draws on no budget — which is why $4\pi > 3\pi$ is not a tension. On the alternative, accumulated-toll reading the depth-n action would be $\pi n(n+1)/2$ and the codebook's $e^{-n\pi}$ would not be the suppression; this paper's reading is the uniform one, stated here as a reading and registered at O_2.

Identification of $e^{-n\pi}$ as a cascade-step exponential. The minimal first-order quark Yukawas of the codebook below are

$$y_t^{(1)} = 1, \quad y_b^{(1)} = \tfrac{N_g}{2^{N_g}}\, e^{-\pi}, \quad y_c^{(1)} = N_g\, e^{-2\pi}, \quad y_s^{(1)} = 2N_g\, e^{-3\pi}, \quad y_{u,d}^{(1)} = (1,2)\, N_g\, e^{-4\pi}, \tag{49.2}$$

with $N_g = 3$ entering as a family-multiplicity prefactor in every line except the reference $y_t = 1$. On the reading adopted here the exponential factor $e^{-n\pi}$ in the n-th line is the winding cost $n\pi$ of the level: one winding at cost π per depth. The Bogomolnyi-saturating gradient flow between the adjacent Morse critical points of indices $2k$ and $2(k-1)$ on $\mathbb{C}P^2$ has action $S_{[2k]\to[2(k-1)]} = \pi k$ (Witten, 1982) (cf. Lemma 4.6 of Arisaka, 2026, P2-N3); identifying the per-depth winding cost with that per-gate action is the accumulated-toll alternative set aside above, and the uniform reading is what O_2 registers. What is not a fitting parameter is the base $e^{-\pi}$; which cost it counts is the open reading.

Generation-to-cascade assignment for quarks. The standard up-type and down-type triplets fold onto the cascade table (49.1) as

$$\begin{aligned} (t, c, u) &\longleftrightarrow (\text{depth } 0, \text{depth } 2, \text{depth } 4) \text{ up-type sequence}, \\ (b, s, d) &\longleftrightarrow (\text{depth } 1, \text{depth } 3, \text{depth } 4) \text{ down-type sequence}, \end{aligned} \tag{49.3}$$

with the characteristic (t, c, u) even-step / (b, s, d) odd-step interleaving accounting for the asymmetry between up- and down-type mass ratios that is otherwise mysterious. Indeed, the asymmetry is no longer an empirical fit; it is a structural consequence of the $\mathrm{SU}(2)_L$ doubling of the Morse cascade.

The ground-state (GGA) reading. As in the lepton sector (§47.3), the five levels are read as GG-A6-GRIP reads them: on the doubled family stratum each flavor is a critical point of the

height, and only the deepest level is the ground state. That all six quark masses are sharply defined codebook parameters is a statement about the landscape each flavor sees on its own charge-superselected stratum, which this paper has not defined and which lemma L10 must. The depth-4 basin, jointly occupied by u and d through the isospin doubling, is the GGA of the quark sector, and the weak-decay chains $t \to b \to c \to s \to u/d$ are the runtime's relaxation of non-ground GA's toward it. Confinement then adds one further ground-state clause downstream: the lightest baryon built on the depth-4 GGA — the proton — is the GGA of the gauged-U(1)$_B$ baryonic stratum (§94.3), which is why quark-sector relaxation terminates in stable hydrogen rather than continuing without end.

Remark 8 (Physics intuition for the quark cascade)**.** The five-level structure $(t, b, c, s, u/d)$ arose historically from particle physics as five empirical Yukawa couplings spanning five orders of magnitude. The cascade-depth reading collapses these five orders of magnitude into one structural pattern: each quark sits one step further down the GFF Morse cascade than the preceding one, with a uniform per-step penalty $e^{-\pi}$ on the winding-cost reading of (49.1). The mass hierarchy $m_t : m_b : m_c : m_s : m_{u,d}$ is therefore not five independent measurements but one cascade-closure pattern read at five depths. After all, this is the same cascade that closes at $S_{\text{tot}}(3) = 3\pi$ to fix $N_g = 3$; here we are reading off, depth by depth, what each cascade step contributes to the boundary mass spectrum. In the ground-state vocabulary: five non-ground GA's relaxing, step by step, toward the depth-4 GGA — the up and down quarks of which all stable matter is made.

49.4 Why quarks feel the angular monodromy at first order

GG-A2-GG6 identifies the compact direction S^1_θ as the minimal holonomy channel needed for Abelian phase information and for the electroweak branch selector. Colored matter is necessarily more sensitive to that holonomy than the color-singlet charged leptons because it lives in a nontrivial gauge bundle before boundary descent. The natural first-order quark hierarchy is therefore organized by nonzero winding integers,

$$w_q \neq 0 \qquad \text{for light and intermediate quarks at first order.} \tag{49.4}$$

Intuitively, each additional family step in the quark sector costs angular coherence. Because the only compact primitive is the single circle S^1_θ, the only admissible suppression factor is again $\mathrm{e}^{-\pi}$ per minimal mismatch unit.

49.5 Minimal first-order quark codebook

The shortest quark codebook consistent with one unsuppressed top state, one down-type branch shift, and monotone family descent is

$$\boxed{y_t^{(1)} = 1,} \tag{49.5}$$

$$\boxed{y_b^{(1)} = \frac{N_g}{2^{N_g}}\,\mathrm{e}^{-\pi} = \frac{3}{8}\,\mathrm{e}^{-\pi},} \tag{49.6}$$

$$\boxed{y_c^{(1)} = N_g\,\mathrm{e}^{-2\pi} = 3\,\mathrm{e}^{-2\pi},} \tag{49.7}$$

$$\boxed{y_s^{(1)} = 2N_g\,\mathrm{e}^{-3\pi} = 6\,\mathrm{e}^{-3\pi},} \tag{49.8}$$

$$\boxed{y_u^{(1)} = N_g\,\mathrm{e}^{-4\pi} = 3\,\mathrm{e}^{-4\pi}, \qquad y_d^{(1)} = 2N_g\,\mathrm{e}^{-4\pi} = 6\,\mathrm{e}^{-4\pi}.} \tag{49.9}$$

Again, each line has a clear geometrical interpretation.

The top quark. The top is the unique quark with essentially maximal overlap at the lock seam. The shortest admissible code is therefore simply

$$y_t^{(1)} = 1.$$

No suppression is imposed because the top mode defines the dominant up-type overlap.

The bottom quark. The heaviest down-type mode must differ from the top by the smallest nontrivial branch change. The minimal angular price is one unit $\mathrm{e}^{-\pi}$. The down-type branch also carries the smallest family-coded dyadic split $N_g/2^{N_g} = 3/8$. Hence

$$y_b^{(1)} = \frac{N_g}{2^{N_g}}\mathrm{e}^{-\pi}.$$

The charm quark. The second-family up-type mode is the first genuinely wound quark state. The smallest even-winding suppression consistent with retaining up-type sector coherence is $\mathrm{e}^{-2\pi}$, accompanied by the triplicity factor N_c. Thus

$$y_c^{(1)} = N_g\mathrm{e}^{-2\pi}.$$

The strange quark. The strange quark sits on the down-type branch one angular unit deeper than the charm. The extra factor of 2 relative to N represents the two-sheeted down-type branch multiplicity, so

$$y_s^{(1)} = 2N_g\mathrm{e}^{-3\pi}.$$

The up and down quarks. The first-family quarks live at the deepest first-order winding level, $w = 4$. The up quark preserves the family numerator N_g, while the down quark again carries the two-sheeted branch factor, giving

$$y_u^{(1)} = N_g\mathrm{e}^{-4\pi}, \qquad y_d^{(1)} = 2N_g\mathrm{e}^{-4\pi}.$$

This directly predicts the simple exact relation $m_d/m_u = 2$ at first order.

49.6 First-order quark masses

Multiplying by the same electroweak normalization factor $v^{(1)}/\sqrt{2}$ gives

$$m_t^{(1)} = \frac{v^{(1)}}{\sqrt{2}} = 174.18257 \text{ GeV}, \tag{49.10}$$

$$m_b^{(1)} = \frac{v^{(1)}}{\sqrt{2}} \frac{3}{8} \mathrm{e}^{-\pi} = 2.82266677 \text{ GeV}, \tag{49.11}$$

$$m_c^{(1)} = \frac{v^{(1)}}{\sqrt{2}} 3\mathrm{e}^{-2\pi} = 0.975827928 \text{ GeV}, \tag{49.12}$$

$$m_s^{(1)} = \frac{v^{(1)}}{\sqrt{2}} 6\mathrm{e}^{-3\pi} = 84.3386966 \text{ MeV}, \tag{49.13}$$

$$m_d^{(1)} = \frac{v^{(1)}}{\sqrt{2}} 6\mathrm{e}^{-4\pi} = 3.64460554 \text{ MeV}, \qquad m_u^{(1)} = \frac{v^{(1)}}{\sqrt{2}} 3\mathrm{e}^{-4\pi} = 1.82230277 \text{ MeV}. \tag{49.14}$$

These formulas show why it was worth deriving $v^{(1)}$ first. Once the electroweak normalization is known, the entire quark hierarchy reduces to a small list of winding and branch integers.

49.7 Exact first-order quark ratios

Several exact ratios follow immediately:

$$\frac{m_b^{(1)}}{m_c^{(1)}} = \frac{\mathrm{e}^{\pi}}{2^{N_g}} = \frac{\mathrm{e}^{\pi}}{8}, \tag{49.15}$$

$$\frac{m_c^{(1)}}{m_s^{(1)}} = \frac{\mathrm{e}^{\pi}}{2}, \tag{49.16}$$

$$\frac{m_s^{(1)}}{m_d^{(1)}} = \mathrm{e}^{\pi}, \tag{49.17}$$

$$\frac{m_d^{(1)}}{m_u^{(1)}} = 2. \tag{49.18}$$

These are the quark analogs of the exact charged-lepton ratios in (47.12) and (47.13). Their importance is structural. The quark hierarchy is not being fitted one particle at a time. It is being generated by a single compact-angle code.

49.8 QD completion of the quark block

The QD transport for the quark sector is naturally sector-dependent because QCD dressing depends on the family depth and on whether the state lies on the up-type or down-type branch.

The shortest rational closures compatible with the OGN ledger are

$$\boxed{\mathcal{R}_t = 1 - \frac{M}{2^J} = 1 - \frac{1}{128} = \frac{127}{128},} \tag{49.19}$$

$$\boxed{\mathcal{R}_b = 1 + \frac{M}{2} = \frac{3}{2},} \tag{49.20}$$

$$\boxed{\mathcal{R}_c = 1 + \frac{M}{N} = \frac{4}{3},} \tag{49.21}$$

$$\boxed{\mathcal{R}_s = 1 + \frac{M}{2^N} = \frac{9}{8},} \tag{49.22}$$

$$\boxed{\mathcal{R}_u = \mathcal{R}_d = 1 + \frac{M}{L - M} = 1 + \frac{1}{4} = \frac{5}{4}.} \tag{49.23}$$

The transport-corrected seam masses are then

$$m_q^{\mathrm{QD}} = \mathcal{R}_q \, m_q^{(1)}, \tag{49.24}$$

which gives

$$m_t^{\mathrm{QD}} = 172.821770 \text{ GeV}, \qquad m_b^{\mathrm{QD}} = 4.23400015 \text{ GeV}, \tag{49.25}$$

$$m_c^{\mathrm{QD}} = 1.30110390 \text{ GeV}, \qquad m_s^{\mathrm{QD}} = 94.8810337 \text{ MeV}, \tag{49.26}$$

$$m_d^{\mathrm{QD}} = 4.55575693 \text{ MeV}, \qquad m_u^{\mathrm{QD}} = 2.27787846 \text{ MeV}. \tag{49.27}$$

The interpretation of these closures is as follows. The top quark is so dominant at first order that the leading transport correction is a minimal dyadic subtraction 2^{-J}. The bottom and charm quarks require positive branch dressing because their first-order wound overlaps underestimate the full boundary mass after QCD transport. The strange, up, and down quarks sit on deeper boundary seams and therefore inherit the next-shortest rational uplifts compatible with their family depth.

50 The neutrino floors: masses from the minimal constitutional Majorana channel

50.1 Historical background

Pauli postulated the neutrino in his 1930 letter to the Tübingen conference to rescue energy conservation in beta decay (Pauli, 1930). Majorana in 1937 introduced the self-conjugate fermion that bears his name (Majorana, 1937), and Pontecorvo in 1957 introduced neutrino oscillations (Pontecorvo, 1957). After Super-Kamiokande and SNO confirmed atmospheric and solar neutrino oscillations near the turn of the millennium, the Standard Model was extended to accommodate three nonzero neutrino masses with two mass-squared splittings $\Delta m^2_{21} \approx 7.4 \times 10^{-5}$ eV^2 and $|\Delta m^2_{31}| \approx 2.5 \times 10^{-3}$ eV^2 – but the absolute mass scale, the mass ordering, and the Dirac-vs-

Majorana nature remain free. The theorems below derive the neutrino-mass block from a single constitutional Majorana channel: the smallness of $m_\nu \sim 10^{-2}$ eV is the boundary-projection image of the same OGN-5 lock that fixes the lock scale, with no independent see-saw or right-handed-neutrino sector required.

50.2 The constitutional door: why the channel is Majorana

The word "Majorana" in this section's title is not a model choice, and the following statement — co-stated with its cosmological twin (T-Cosmos-18 of Arisaka, 2026, A10-Cosmos) and with entry R10 of the real-bulk series of Arisaka, 2026, A8-QM, the series whose standalone, full-length development is the Foundations Branch member paper GG-P4-Real (Arisaka, 2026, P4-Real) — is why.

Theorem 50.1 (**T-SM-44.1**: The dial-free door — a Majorana mass exists iff the fiber weight vanishes (R10)). *Let a fermion multiplet carry weight n under the compact phase direction S^1_θ (spinor and family indices commute with the fiber action). A Lorentz-invariant, gauge-invariant Majorana-type self-pairing — equivalently a $\Delta L = 2$ bilinear — exists iff $n = 0$; on every $n \neq 0$ sector the unique invariant symmetric self-pairing is the charge-conserving Dirac pairing $\psi\bar{\psi}$; and the trilinear LHN is the unique fiber-neutral portal touching the weight-zero sector linearly.*

Proof. In the real presentation the weight-n sector is the pair (a, b) rotated by $R(n\alpha)$. Its symmetric square is spanned by $a^2 + b^2$ (rotation-invariant for every n: the Dirac pairing) and the doublet $(a^2 - b^2,\, 2ab) = (\mathrm{Re}\,\psi^2, \mathrm{Im}\,\psi^2)$, which rotates by $2n\alpha$ under $R(n\alpha)$: for $n \neq 0$ invariance forces it to vanish; for $n = 0$ the action is trivial and the reality condition of the sector is precisely the Majorana condition. The spinor factor's charge-conjugation pairing is fiber-inert and cannot compensate a non-zero weight. A trilinear is fiber-neutral iff its weights sum to zero; with L at $-\frac{1}{2}$ and H at $+\frac{1}{2}$, the singlet N at weight 0 closes the unique such term linear in N. ∎

Three consequences frame everything below. Lepton number is winding number: exact on every dial-carrying species, so the charged-lepton ledger conserves L identically. The sole legal $\Delta L = 2$ structure in the matter sector is the weight-zero self-pairing — the Majorana channel of this section's title, which is thereby *the minimal and the unique* constitutional option, not the minimal of several. And the ultraviolet completion, when it is opened, has no freedom either: the portal is fixed, and the heavy-sector realization and its leptogenesis are carried, in completed form, by Arisaka, 2026, A10-Cosmos.

50.3 The neutrino GRIP cascade

The neutrino block is the most indirect part of the mass sheet. BI-6 forbids any new intermediate mass scale and any second independent compact loop, so the Majorana/Weinberg channel must reuse the already-derived ultraviolet scale E_{GUT}, and the coefficient π/E_{GUT} below is the shortest allowed Majorana seam coefficient compatible with the same compact-angle topology used elsewhere in the paper. As one GRIP cascade the neutrino masses are projected Majorana spectral gaps (CCI exports) of a rank-two GFF landscape carrying the v^2/E_{GUT} suppression: the same Higgs branch

that fixed v enters the dimension-five operator twice, and the same family packet later carries the PMNS angles, so GRIP selects the normal-ordering branch and stabilizes it as the lepton-attractor background on which the PMNS block is then read. One downstream bookkeeping point is recorded here, because the cosmology companion's η_B chain consumes this coefficient at two scales: the heavy-sector resonance width is evaluated at the seam-matched one-generation Yukawa — a property of the seam-localized texture, whose effective washout mass is the seam scale $\pi v^2/E_{\rm GUT}$ itself, $\widetilde{m} \simeq 8.4$ meV, with washout $K_{\rm seam} = 7.80$ — while the washout the thermal bath applies to the full light spectrum is that number times the trace of the family projector, $K_{\rm tot} = (1+2N)K_{\rm seam} = 54.6$, the third channel alone carrying $2N\,K_{\rm seam} \simeq 47$ (§67). The factor between them is the OGN-5 integer of Row 27, not a convention; both are accounted for within the order-of-magnitude register in the companion's leptogenesis appendix (Arisaka, 2026, A10-Cosmos), with this paper's coefficient $\pi/E_{\rm GUT}$ the upstream input of both.

The neutrino sector is the most delicate part of the 12-mass block because its masses are tiny, its flavor structure is nontrivial, and no direct neutrino Yukawa operator appears at dimension four in the minimal Standard-Model field content. Fortunately, the written Arisaka-GG preprints already tell us which route is constitutionally preferred.

50.4 Gi-4 cascade-depth reading for the Majorana neutrinos

The neutrino sector occupies the same family-projector stratum $\Sigma_{\rm fam} = \mathbb{C}P^2$ as the charged leptons, but with two important structural differences: (i) the absolute mass scale is set by the Weinberg dimension-five operator $v^2/E_{\rm GUT}$ rather than by the Higgs vev directly, and (ii) the Majorana nature of the channel produces a rank-two algebraic structure (with two independent mass-squared splittings rather than three independent masses) before any flavor-projection misalignment is imposed.

Cascade chain in the Majorana sector. By GG-A6-GRIP T3.1 and T4.1 (Arisaka, 2026, A6-GRIP) the same three-step GRIP cascade (47.2) applies, with the three neutrino mass eigenstates ν_3, ν_2, ν_1 assigned to the three cascade depths — the three critical points of the height, read as in §47.3; the tolls of (47.2) are the gates between them:

$$\begin{array}{lcl} \nu_3 & \longleftrightarrow & \text{depth } 0: \ \mathsf{GA} \text{ (non-ground; relaxation portal-suppressed)}, \\ \nu_2 & \longleftrightarrow & \text{depth } 1: \ \mathsf{GA} \text{ (non-ground; relaxation portal-suppressed)}, \\ \nu_1 & \longleftrightarrow & \text{depth } 2: \ \mathsf{GGA} \text{ (ground state of the Majorana stratum)}. \end{array} \tag{50.1}$$

The ground-state (GGA) reading, and why neutrinos nonetheless persist. As in the charged-lepton sector, all three mass eigenstates are genuine GA's — sharply defined mass-squared splittings are the certificate — and only the depth-2 basin is the ground state: ν_1 is the GGA of the Majorana stratum. Two consequences follow. First, an *ordering prediction*: the depth assignment (50.1) requires the normal mass ordering ($m_1 < m_2 < m_3$, the GGA lightest); an experimentally established inverted ordering would falsify the assignment outright — a clean, near-term kill criterion, since the ordering is within reach of the current long-baseline and reactor program. Second, an

instructive contrast with the charged leptons: the GGA fixes the *direction* of relaxation, while the portals fix its *rate*. The muon and tau relax fast because the weak charged current offers a broad channel; the non-ground neutrinos ν_3, ν_2 can relax only through radiative and neutral channels so feeble that their lifetimes exceed the age of the universe by tens of orders of magnitude. Metastability with a cosmologically long clock is still metastability: the same theorem, read at a different coupling strength.

Why the Majorana channel produces only two mass-squared splittings. The rank-two Majorana algebraic structure means that the ν_3 and ν_2 mass-squared splittings Δm^2_{31} and Δm^2_{21} are independent observables, while the absolute ν_1 mass scale enters only through the dimension-five operator coefficient and is not a third independent splitting. This is exactly the empirical pattern observed by Super-Kamiokande, SNO, KamLAND, and the long-baseline accelerator experiments: $\Delta m^2_{21} \approx 7.4 \times 10^{-5}$ eV^2 and $|\Delta m^2_{31}| \approx 2.5 \times 10^{-3}$ eV^2, with the absolute m_1 remaining undetermined down to the current cosmological bound $\sum m_\nu < 0.064$ eV (95%, ΛCDM) (DESI Collaboration, 2025). Indeed, the Gi-4 cascade reading explains *why* only two splittings are independent: the rank-two structure of the Majorana ledger pins the third degree of freedom to the absolute scale, not to a third splitting.

Why neutrino masses are tiny. In the Gi-4 cascade reading, the smallness of the neutrino mass scale $m_\nu \sim 10^{-2}$ eV (compared to the electroweak vev $v \sim 246$ GeV) is not a fine-tuned see-saw mass but a structural consequence of the dimension-five suppression

$$m_\nu \sim \frac{v^2}{E_{\rm GUT}} \sim \frac{(246\ \text{GeV})^2}{2.28 \times 10^{16}\ \text{GeV}} \sim 2.7\ \text{meV}, \tag{50.2}$$

which the 6D scale chain delivers automatically. The cascade-depth reading then assigns the three observed Majorana masses to the three GFF Morse levels of (50.1), with no additional right-handed neutrino sector or independent see-saw scale required.

Remark 9 (Physics intuition for the neutrino cascade)**.** The three Majorana neutrinos populate the same Gi-4 cascade as the charged leptons but read it through a different observability window: only mass-squared splittings (not absolute masses) are accessible to oscillation experiments, so the rank-two algebraic structure is what observation actually probes. After all, the absence of a third independent splitting in the empirical neutrino spectrum is not a lacuna but a prediction of the Majorana cascade structure. So, in its way, is the mass ordering: the depth assignment makes normal ordering structural (the GGA sits at the bottom), handing the oscillation program a second, sharper falsifier.

50.5 Why the Weinberg/Majorana channel is the minimal route

GG-A2-GG6 and GG-A5-GDOS explicitly list the dimension-5 operator $(LH)(LH)$ as allowed if neutrino masses are generated that way (Arisaka, 2026, A2-GG6; Arisaka, 2026, A5-GDOS). No additional low-energy scale is admitted in the constitutional ledger, and no right-handed neutrino

sector has yet been made obligatory in the written preprints. Therefore the minimal neutrino mass mechanism of GG-A9-SM is the Majorana/Weinberg channel.

The corresponding effective term is taken to be

$$\mathcal{L}_\nu^{(5)} = \frac{2\pi}{E_{\mathrm{GUT}}} (LH)\, \Xi_\nu\, (LH) + \mathrm{h.c.}, \tag{50.3}$$

where Ξ_ν is a dimensionless family matrix. The coefficient $2\pi/E_{\mathrm{GUT}}$ is the minimal constitutional choice for two reasons:

1. the suppression scale must be the already derived ultraviolet seam E_{GUT}, because no new arbitrary intermediate scale is allowed;

2. the prefactor 2π represents one full compact traversal on S^1_θ, the natural closed Majorana loop.

After electroweak symmetry breaking, each Higgs field contributes $v/\sqrt{2}$, so the common neutrino scale becomes

$$\boxed{m_\star = \pi\, \frac{v^2}{E_{\mathrm{GUT}}}.} \tag{50.4}$$

This is one of the cleanest formulas in the entire paper: the neutrino scale is not imported from observation but generated by the electroweak scale squared divided by the GUT scale, multiplied by the unique compact angle constant π.

50.6 First-order neutrino scale

Using the first-order electroweak scale $v^{(1)}$ derived earlier, we obtain

$$m_\star^{(1)} = \pi\, \frac{[v^{(1)}]^2}{E_{\mathrm{GUT}}} = 8.36113789 \text{ meV}. \tag{50.5}$$

The size is immediately reasonable on physical grounds. The neutrino sector is doubly suppressed: once by the electroweak scale relative to the GUT scale, and once more because the mass is generated only after two Higgs insertions. In GG-6 language, neutrinos are the most infrared of the fermion observables precisely because they are the most indirect.

50.7 Minimal family projector and normal ordering

The family matrix Ξ_ν must now be fixed. The minimal first-order choice consistent with three families, one protected null direction, and monotone normal ordering is

$$\Xi_\nu^{(1)} = P_2 + 2N\, P_3, \tag{50.6}$$

where P_i are orthogonal family projectors satisfying $P_i P_j = \delta_{ij} P_i$ and $P_1 + P_2 + P_3 = \mathbf{1}$.

Why is a null first-order direction natural? In the GDOS language the boundary-visible neutrino sector is the most weakly projected part of the fermion spectrum. A rank-two first-order projector is therefore the minimal way to encode the fact that one family direction remains unresolved

until higher order. The coefficient of the third projector is $2N = 6$, the smallest OGN-coded enhancement that cleanly separates the third neutrino mass from the second without introducing new integers. This coefficient is *selected*, not forced: it is the unique cost-2 integer-ledger monomial whose QD-completed squared ratio lies within 5% of the measured 33.35, and Row 27 of Table 7 — the row whose parameter, $\Delta m^2_{31} = (2N)^2 m^2_*$, contains it; $\Delta m^2_{21} = m^2_*$ does not — carries the **Derived (selected)** grade of the MSMF/OGN audit (§101.2, #27) with its look-elsewhere factor $p \approx 0.21$ reported. It is the one place an oscillation datum enters the neutrino block, and statements elsewhere in this paper that none does are to be read subject to it.

The resulting first-order eigenvalues are

$$\boxed{(m_1, m_2, m_3)^{(1)} = (0,\ 1,\ 2N)\, m_\star^{(1)} = (0,\ 8.36113789,\ 50.1668273)\ \text{meV}.} \tag{50.7}$$

50.8 First-order oscillation observables

The mass-squared splittings are therefore

$$\Delta m^{2\,(1)}_{21} = m_2^{2\,(1)} - m_1^{2\,(1)} = 6.990864 \times 10^{-5}\ \text{eV}^2, \tag{50.8}$$

$$\Delta m^{2\,(1)}_{31} = m_3^{2\,(1)} - m_1^{2\,(1)} = 2.516711 \times 10^{-3}\ \text{eV}^2. \tag{50.9}$$

The first-order sum of neutrino masses is

$$\Sigma m_\nu^{(1)} = 58.5280\ \text{meV}. \tag{50.10}$$

Even before any QD correction is applied, the neutrino sector already lands in the phenomenologically correct regime. This is a nontrivial result: it means that the same scale architecture that solved the hierarchy problem also generates neutrino masses of the right order of magnitude.

50.9 QD lift of the neutrino spectrum

At QD order the first-order null direction is lifted by the smallest dyadic amount, the second eigenvalue receives the shortest throat-depth uplift, and the third eigenvalue inherits the same dyadic closure that already appeared in the gauge sector:

$$\boxed{m_1^{\text{QD}} = \frac{1}{2^J}\, m_\star, \qquad m_2^{\text{QD}} = \left(1 + \frac{1}{K}\right) m_\star, \qquad m_3^{\text{QD}} = 2N \left(1 + \frac{1}{2^J}\right) m_\star,} \tag{50.11}$$

with

$$m_\star = \pi\, \frac{v_{\text{QD}}^2}{E_{\text{GUT}}}, \qquad v_{\text{QD}} = 246.220\ \text{GeV}. \tag{50.12}$$

Numerically,

$$m_\star = 8.35358030\ \text{meV}, \tag{50.13}$$

so that

$$\boxed{(m_1, m_2, m_3)^{\text{QD}} = (0.0652623,\ 8.59225345,\ 50.5130555)\ \text{meV}.} \tag{50.14}$$

The corresponding oscillation observables are

$$\Delta m_{21}^{2\,\mathrm{QD}} = 7.382256 \times 10^{-5}\ \mathrm{eV}^2, \tag{50.15}$$

$$\Delta m_{31}^{2\,\mathrm{QD}} = 2.551565 \times 10^{-3}\ \mathrm{eV}^2, \tag{50.16}$$

and

$$\Sigma m_\nu^{\mathrm{QD}} = 59.1706\ \mathrm{meV}. \tag{50.17}$$

The conceptual picture is attractive. The first-order theory gives a rank-two Majorana block with normal ordering. The QD theory then lifts the null mode and slightly stretches the heavy eigenvalue by the same dyadic transport that already sharpened the gauge couplings. In this sense the neutrino sector is not alien to the rest of the paper; it is the natural low-energy echo of the same locked arithmetic.

51 Precision comparison for the 12-fermion-mass block

We now compare the derived fermion masses with standard benchmark values. As before, these comparison numbers are *not* inputs. They enter only after the derivation is complete.

51.1 Benchmark conventions and theory uncertainties

For the charged leptons and quarks we use the standard benchmark masses in the conventional comparison schemes summarized by CODATA and the Particle benchmark data adopted in this paper. For the neutrino sector we compare not the individual masses, which are not directly known with precision, but the oscillation observables Δm_{21}^2 and Δm_{31}^2 as summarized by the benchmark oscillation targets adopted in this paper.

The theory errors in the tables are defined constitutionally rather than statistically.

- For each first-order mass, the quoted theory uncertainty $\sigma_{\mathrm{th}}^{(1)}$ is the magnitude of the omitted QD correction:
$$\sigma_{\mathrm{th}}^{(1)} = \left| m_f^{\mathrm{QD}} - m_f^{(1)} \right|.$$
- For each QD mass, the quoted residual theory uncertainty is taken to be one further dyadic suppression of the leading correction:
$$\sigma_{\mathrm{th}}^{(2)} = \frac{1}{8} \left| m_f^{\mathrm{QD}} - m_f^{(1)} \right|.$$
The factor 1/8 reflects three additional binary boundary steps beyond the explicitly kept QD layer.

This prescription is simple, transparent, and consistent with the locked dyadic logic used throughout the paper.

51.2 Direct-mass comparison: first-order seam prediction

Table 30: First-order predictions for the nine directly benchmarked charged fermions. Experimental comparison values are standard CODATA/PDG benchmarks used only after the derivation is complete. The nine are the charged fermions whose masses are measured sharply enough for a comparison at this precision to carry information.

Parameter	**Pred.**	$\sigma_{\mathrm{th}}^{(1)}$	**Obs.**	σ_{exp}	Dev	Pull
m_e [MeV]	0.510300500	0.000797345	0.510998951	0.000000000	−1.367e-03	−0.88
m_μ [MeV]	106.312604	0.6644535	105.658376	0.0000023	+6.192e-03	+0.98
m_τ [GeV]	1.78605175	0.00837212	1.77693	0.00009	+5.133e-03	+1.09
m_u [MeV]	1.82230277	0.45557569	2.20	0.07	−1.717e-01	−0.82
m_d [MeV]	3.64460554	0.91115139	4.69	0.05	−2.229e-01	−1.15
m_s [MeV]	84.3386966	10.5423371	92.74	0.54	−9.059e-02	−0.80
m_c [GeV]	0.975827928	0.325275976	1.275	0.009	−2.346e-01	−0.92
m_b [GeV]	2.82266677	1.41133338	4.196	0.012	−3.273e-01	−0.97
m_t [GeV]	174.182571	1.36080108	172.56	0.31	+9.403e-03	+1.16

Two features should be emphasized. First, the first-order masses already come out in the correct numerical neighborhoods across more than five orders of magnitude. Second, the residual discrepancies are organized, not chaotic. They point in exactly the direction one would expect if the main missing ingredient were finite seam transport rather than a failure of the first-order codebook itself.

51.3 Direct-mass comparison: QD-completed prediction

Table 31: QD-completed predictions for the nine directly benchmarked charged fermions. The nine charged fermion masses with the quantum-defect completion applied, against the same CODATA and PDG benchmarks as the first-order table and in the same column order. The light-quark rows carry the largest benchmark uncertainties in the set, so their pulls are the least informative entries here even where the central agreement looks close; the lepton rows are where the completion is actually being tested.

Parameter	**Pred.**	$\sigma_{\mathrm{th}}^{(2)}$	**Obs.**	σ_{exp}	Dev	Pull
m_e [MeV]	0.511097845	0.0000996681	0.510998951	0.000000000	+1.935e-04	+0.99
m_μ [MeV]	105.648150	0.0830567	105.658376	0.0000023	−9.677e-05	−0.12
m_τ [GeV]	1.77767963	0.00104651	1.77693	0.00009	+4.219e-04	+0.71
m_u [MeV]	2.27787846	0.05694696	2.20	0.07	+3.540e-02	+0.86
m_d [MeV]	4.55575693	0.11389392	4.69	0.05	−2.862e-02	−1.08
m_s [MeV]	94.8810337	1.31779214	92.74	0.54	+2.309e-02	+1.50
m_c [GeV]	1.30110390	0.04065950	1.275	0.009	+2.047e-02	+0.63
m_b [GeV]	4.23400015	0.17641667	4.196	0.012	+9.056e-03	+0.21
m_t [GeV]	172.821770	0.17010014	172.56	0.31	+1.517e-03	+0.74

Table 31 is the main quantitative result of the fermion-mass extension. The QD layer improves every charged-lepton mass and every heavy-quark mass substantially, while keeping the light-quark sector within plausible comparison bands despite the well-known scheme-dependence of light-quark masses.

51.4 Predicted neutrino masses

Because individual neutrino masses are not yet directly known with the same precision as the charged fermion masses, the most honest presentation is to display the predicted eigenvalues and then separately compare the oscillation observables.

Table 32: Predicted neutrino mass eigenvalues and mass sums. The two columns are first order and QD-completed. The lightest eigenvalue vanishes at first order and acquires a small value only through the completion, which is a structural statement about the mass matrix rather than a fitted outcome. The sum is printed because it is the quantity cosmology bounds; the individual eigenvalues are not separately measured, and the comparison with the oscillation splittings is made in the table that follows.

Observable	First order	QD
m_1 [meV]	0	0.0652623
m_2 [meV]	8.36113789	8.59225345
m_3 [meV]	50.1668273	50.5130555
Σm_ν [meV]	58.5280	59.1706

The normal ordering is an output rather than an assumption. It follows from the minimal rank-two first-order projector together with the QD lift in (50.11).

51.5 Oscillation comparison

Table 33: Comparison of the derived neutrino oscillation observables with the named benchmarks of §55.1. JUNO's first-data determination of the solar splitting is printed beside the global fit. JUNO's determination and the global fit are printed side by side because they are independent, and the paper is scored against both rather than against whichever is closer.

Observable	Pred.	σ_{th}	Obs.	σ_{exp}	Dev	Pull
$\Delta m^{2\,(1)}_{21}$ (NuFIT 6.1)	6.990864e-05	3.913917e-06	7.530000e-05	1.000000e-06	−7.160e-02	−1.33
$\Delta m^{2\,(1)}_{21}$ (JUNO)	6.990864e-05	3.913917e-06	7.500000e-05	1.200000e-06	−6.788e-02	−1.24
$\Delta m^{2\,(1)}_{31}$ (NuFIT 6.1)	2.516711e-03	3.485390e-05	2.511000e-03	2.100000e-05	+2.274e-03	+0.14
$\Delta m^{2\,\text{QD}}_{21}$ (NuFIT 6.1)	7.382256e-05	4.892396e-07	7.530000e-05	1.000000e-06	−1.962e-02	−1.33
$\Delta m^{2\,\text{QD}}_{21}$ (JUNO)	7.382256e-05	4.892396e-07	7.500000e-05	1.200000e-06	−1.570e-02	−0.91
$\Delta m^{2\,\text{QD}}_{31}$ (NuFIT 6.1)	2.551565e-03	4.356738e-06	2.511000e-03	2.100000e-05	+1.615e-02	+1.89

The neutrino comparison is already encouraging at first order and remains within the order-one-sigma to order-two-sigma regime after QD completion against both the global fit and JUNO's direct measurement — the QD solar splitting sits 0.91σ from the JUNO value. More importantly, the entire neutrino scale has been obtained without inserting any measured neutrino mass; the one oscillation datum that enters the block, the ratio m_3/m_2 selected as $2N$ (Row 27), enters as a selection filter and not as a fitted input in the derivation.

51.6 Summary table for the full 12-mass block

For clarity we collect the full set of fermion-mass predictions in one place.

Table 34: Summary of the full 12-fermion-mass block derived in the present paper. Observed values (PDG/CODATA) and Dev (the QD fractional deviation) reproduce the benchmark column of Table 59; absolute neutrino masses are not directly measured (only Δm^2 constrained).

Sector	Parameter	First order	QD	Observed	Dev
Charged leptons	m_e [MeV]	0.510300500	0.511097845	0.510998951	+1.935e-04
	m_μ [MeV]	106.312604	105.648150	105.658376 ± 0.0000023	-9.677e-05
	m_τ [GeV]	1.78605175	1.77767963	1.77693 ± 0.00009	+4.219e-04
Quarks	m_u [MeV]	1.82230277	2.27787846	2.20 ± 0.07	+3.540e-02
	m_d [MeV]	3.64460554	4.55575693	4.69 ± 0.05	-2.862e-02
	m_s [MeV]	84.3386966	94.8810337	92.74 ± 0.54	+2.309e-02
	m_c [GeV]	0.975827928	1.30110390	1.275 ± 0.009	+2.047e-02
	m_b [GeV]	2.82266677	4.23400015	4.196 ± 0.012	+9.056e-03
	m_t [GeV]	174.182571	172.821770	172.56 ± 0.31	+1.517e-03
Neutrinos	m_1 [meV]	0	0.0652623	not directly measured	—
	m_2 [meV]	8.36113789	8.59225345	not directly measured	—
	m_3 [meV]	50.1668273	50.5130555	not directly measured	—

The diagonal mass sector by itself accounts for 17 of the 28 Standard-Model parameters. The next step is the quark-mixing problem. Section 52.4 derives the 4 CKM parameters from the already-fixed quark codebooks, raising the total of the present paper to 21 parameters.

52 The quark gates: CKM mixing from the misalignment of quark family projectors

52.1 Historical background

Cabibbo's 1963 paper introduced the rotation angle that mixes strange and non-strange semileptonic decays (Cabibbo, 1963), the original quark-mixing matrix. A decade later Kobayashi and Maskawa generalized Cabibbo to three quark generations (Kobayashi & Maskawa, 1973), predicting

a third family before its discovery and showing that three generations admit one irreducible CP-violating phase. The resulting 3×3 CKM matrix carries four physical parameters (three angles and one phase) measured to high precision but lacking any structural explanation. The theorems below derive the CKM block as the misalignment between the up-type and down-type quark family projectors on the same closed OGN-5 packet: the four free parameters become four compact-angle ratios fixed by the integer ledger, with the CP phase emerging from a topological-projection remainder rather than from an arbitrary insertion.

52.2 The CKM GRIP cascade

The CKM block is not an arbitrary unitary matrix inserted after the masses: BI-6 has already fixed the compact-angle alphabet — the integer $L = 5$ (the SU(5) block size) and the single compact holonomy circle S^1_θ surviving in the flavor sector — from which the pentagonal phase structure (the interference phase $3\pi/5$ and the rephasing step $\pi/5$) is assembled here in GG-A9-SM, with no second flavor torus on which a separate quark CP angle could float. As one GRIP cascade the CKM observables are the misalignment face of the same stabilized quark packet whose diagonal masses were just derived: CCI compares the up-type and down-type family projectors only on a shared admissible branch, so the common left-handed piece is unphysical and cancels and the $2 \leftrightarrow 3$ angle becomes multiplicative, and the Cabibbo angle, the smaller angles, and the Dirac phase are the GA basin coordinates of that projector-comparison GFF landscape — which is why CKM must come after the diagonal mass block, never before it.

52.3 Two structural theorems: the discrete-symmetry table, and the flavor connection

Before the parameter block, two statements deserve theorem status, because everything the block asserts about phases rests on them. Both are proved in the real presentation of the compact fiber — the presentation in which a charge-n field is the real pair (a, b) rotated by $R(n\alpha)$, a coupling phase is a rigid fiber rotation, and complex conjugation is orientation reversal of the circle (Arisaka, 2026, A8-QM). The real presentation itself — its two real sources, the derived complex structure, and the holomorphy bridge they support — receives its standalone, full-length development in the Foundations Branch member paper GG-P4-Real (Arisaka, 2026, P4-Real).

Theorem 52.1 (**T-SM-46.1**: Discrete symmetries as orientation bookkeeping (R8))**.** *On the quasi-free real bulk with any coupling texture built from real geometric data: (i) charge conjugation — orientation reversal of the fiber, $\theta \mapsto -\theta$ — commutes with the complex structure $J = \partial_\theta/Q$, because it reverses the generator and the signed charge together: C is J-linear. Time reversal — orientation reversal of the constitutional clock — anticommutes with the polar phase of the time generator: T is J-antilinear. Parity is J-linear. Hence a discrete symmetry is antiunitary if and only if it contains T: the textbook unitary/antiunitary table is derived, not assigned. (ii) The total orientation reversal CPT maps each Feynman–Stückelberg-paired mode set to itself and is a symmetry of the construction for every real texture. (iii) CP — fiber plus space reversal, J-linear — is a symmetry*

if and only if the coupling texture is achiral on the circle: every closed-loop angle of the texture lies in $\{0, \pi\}$.

Proof. (i) Under $\theta \mapsto -\theta$ the weight-n sector maps to the weight-$(-n)$ sector and $\partial_\theta \mapsto -\partial_\theta$; since J is normalized by the signed charge, $J \mapsto (-\partial_\theta)/(-Q) = J$. Under $t \mapsto -t$ the frequency orientation flips, so the quarter-period advance becomes a quarter-period delay: $TJT^{-1} = -J$. Parity touches neither fiber nor clock. The composition rule follows since J-(anti)linearity multiplies. (ii) On the paired real modes $\cos(n\theta - Et)$, $\cos(-n\theta - Et)$ the total reversal acts as the identity map of the pair; the metric, the symplectic form, moduli of couplings, and winding sums are even under total reversal, and reversing the sign of every texture angle simultaneously is a relabeling of the same real texture. (iii) A fiber reversal alone negates every texture angle; the theory is invariant iff the negated texture is a per-family re-zeroing of the original, which on rephasing-invariant coordinates (Theorem T-SM-46.2) says every loop angle equals its own negative mod 2π, i.e. lies in $\{0, \pi\}$. ■

Theorem 52.2 (**T-SM-46.2**: Flavor holonomy — rephasing invariants are loop angles (R9))**.** *Regard a Yukawa texture as a* U(1)*-valued connection on the flavor graph: vertices are fields, each carrying a dial; edges are couplings, each block* $|y_{ij}|\, R(\varphi_{ij})$ *carrying a fiber twist. Then: (i) the rephasing group is the group of per-family dial re-zeroings, acting as* $\varphi_{ij} \mapsto \varphi_{ij} + \alpha_i - \beta_j$*; (ii) in a spanning-tree gauge every tree edge has angle zero and the residual physical data are exactly the loop holonomies; for the* $N \times N$ *bipartite quark graph their number is* $N^2 - (2N-1) = (N-1)^2$*; (iii) the quartet invariants* $\mathrm{Im}(V_{ij}V_{kl}V_{il}^*V_{kj}^*)$ *are the plaquette sines, and unitarity ties all of them to a single Jarlskog invariant for* $N = 3$*; CP violation holds iff some loop holonomy lies outside* $\{0, \pi\}$*; (iv) with the* BI-6 *compact-angle alphabet — one circle and the integer* $L = 5$*, with no second flavor torus — first-order holonomies are valued in the pentagonal lattice* $(\pi/5)\,\mathbb{Z}$.

Proof. (i) is the fiber dictionary applied at vertices. (ii) Fix a spanning tree of the connected flavor graph and re-zero dials along it to null its edge angles; each remaining edge closes one independent loop, and the gauge action on a closed loop telescopes to zero, so the loop angles are invariant and coordinatize the quotient. Counting: N^2 edge angles minus the $2N-1$ effective re-zeroings (the global common shift acts trivially). (iii) A quartet is the plaquette product of four blocks with the conjugated legs transposed; its antisymmetric part is the sine of the loop angle weighted by the moduli. For unitary V with $N = 3$ the nine quartets are equal up to sign — the standard Jarlskog uniqueness — so one real number carries all of CP violation. The achirality criterion is (iii) of T-SM-46.1. (iv) is the pentagonal assembly of this Part: every physical angle is built from steps of π/L on the single circle. ■

The parameter block below is the realization of this frame: the Cabibbo interference phase $(3\pi/5)$, the unphysical common step $(\pi/5)$, and the residual physical Dirac phase $\delta^{(1)} = 2\pi/5$ are lattice points of T-SM-46.2(iv), and the Jarlskog invariant quoted against PDG is the real sine readout of the single surviving plaquette. Nothing imaginary enters at any step: the famous "complex phase" of the CKM matrix is a closed-loop angle on a real circle — the flavor-space analog of an Aharonov–Bohm holonomy — quantized, at first order, in pentagonal steps.

52.4 Parameter block

The formulas below now read as the parameter-block export of that misalignment landscape: first the pentagonal Cabibbo interference, then the shared-doublet $2 \leftrightarrow 3$ mismatch, then the deep-link $1 \leftrightarrow 3$ channel, and finally the residual physical quark CP phase.

Once the up-type and down-type quark masses have been derived, the Cabibbo–Kobayashi–Maskawa matrix can no longer be treated as an independent phenomenological input. In the Standard Model the charged current sees only the mismatch between the left-handed diagonalization matrices,

$$\boxed{V_{\mathrm{CKM}} = U_{uL}^{\dagger} U_{dL}.} \tag{52.1}$$

Therefore the true theoretical question is not "What unitary matrix best fits flavor data?" but rather "What left-handed family misalignment is forced by the already-derived GG-6 overlap geometry?"

This reformulation is extremely important conceptually. The quark masses derived in Section 49.2 fixed the diagonal strengths of the family projectors. The CKM block must now be extracted from the *off-diagonal* transport that survives after common family rotations are quotiented out. In GDOS language, the CKM matrix is not a microscopic bare matrix but a boundary-visible seam observable of the same open-system projection that produced the masses.

52.5 Why the CKM matrix is no longer an independent flavor input

The written preprints already established the two structural ingredients needed here.

First, GG-A2-GG6 and GG-A5-GDOS proved that effective four-dimensional Yukawa couplings are overlap integrals on the bulk,

$$\lambda_4 = \lambda_6 \, 2\pi R_\theta \int \mathrm{d}r \, e^{-4A(r)} \, \chi_1(r) \chi_2(r) \eta_H(r),$$

so the boundary Yukawas are not primitive arbitrary numbers (Arisaka, 2026, A2-GG6; Arisaka, 2026, A5-GDOS). Second, the same preprints tied the family multiplicity to the generation count $N_g = 3$, so the observable boundary family structure is part of the constitutional data, not a detached phenomenological add-on. Once those two facts are accepted, there is no room left for an arbitrarily chosen CKM matrix.

The only constitutionally admissible route is therefore the following. One writes the minimal family-link algebra consistent with the same three-family projectors that produced the quark masses. One then diagonalizes the resulting up-type and down-type left-handed overlap matrices. Finally, one takes the mismatch $U_{uL}^{\dagger} U_{dL}$. This route uses no new dimensionful scale, no continuous flavor-fit parameter, and no measured CKM datum as an input.

52.6 Minimal family-link algebra on the boundary

Let P_i $(i = 1, 2, 3)$ denote orthogonal family projectors,

$$P_i P_j = \delta_{ij} P_i, \qquad P_1 + P_2 + P_3 = \mathbf{1}. \tag{52.2}$$

Let T_{ij} be oriented family-link operators satisfying

$$T_{ij} = P_i T P_j, \qquad T_{ij}^\dagger = T_{ji}, \qquad P_k T_{ij} = \delta_{ki} T_{ij}, \qquad T_{ij} P_k = \delta_{jk} T_{ij}. \tag{52.3}$$

The minimal hierarchical Yukawa texture compatible with the quark mass codebooks is then

$$Y_f = \sum_{i=1}^{3} y_{f_i} P_i + \sum_{i<j} \kappa_{ij}^{(f)} \sqrt{y_{f_i} y_{f_j}} \left(e^{-i\phi_{ij}^{(f)}} T_{ij} + e^{+i\phi_{ij}^{(f)}} T_{ji} \right), \qquad f = u, d. \tag{52.4}$$

This form is the shortest one consistent with all of the following:

1. the diagonal eigenvalues must reduce to the already-derived quark Yukawas y_u, y_c, y_t and y_d, y_s, y_b;
2. off-diagonal entries must vanish when the corresponding overlap between family profiles vanishes;
3. the only allowed suppression factors are the ones already present in the theory: dyadic seam factors and compact-angle holonomy factors;
4. every off-diagonal coefficient must be dimensionless and built from the same fixed geometric ledger rather than from new fitted parameters.

For a hierarchical 2×2 block with $y_{f_i} \ll y_{f_j}$, the left-handed diagonalization angle obeys

$$\theta_{ij}^{(f)} \simeq \kappa_{ij}^{(f)} \sqrt{\frac{y_{f_i}}{y_{f_j}}}, \qquad (i < j), \tag{52.5}$$

up to higher-order terms in y_{f_i}/y_{f_j}. The square-root law is not an imported texture ansatz. It is the direct consequence of a hierarchical overlap matrix in which the off-diagonal entry is the geometric mean of the two diagonal overlaps. Physically, a family-mixing angle is the ratio between "link strength" and "diagonal separation," and the geometric mean is the shortest overlap-compatible choice.

52.7 First-order Cabibbo angle from pentagonal holonomy

The $1 \leftrightarrow 2$ channel is the Cabibbo channel, and it is the least suppressed one. From the quark mass codebook of Section 49.2,

$$\theta_{12}^{(u,1)} = \sqrt{\frac{y_u^{(1)}}{y_c^{(1)}}} = \mathrm{e}^{-\pi}, \qquad \theta_{12}^{(d,1)} = \sqrt{\frac{y_d^{(1)}}{y_s^{(1)}}} = \mathrm{e}^{-\pi/2}. \tag{52.6}$$

If these two contributions had no relative phase, their difference would either under- or over-shoot the observed Cabibbo angle. GG-6 supplies the missing structure geometrically.

The written preprints fixed two pieces of compact-angle information that must enter the $1 \leftrightarrow 2$ channel: the circle S^1_θ as the unique compact holonomy direction, and the doublet fraction $2/5$ as the electroweak survival fraction of the SU(5) multiplet. The minimal complementary branch phase

is therefore

$$\boxed{\varphi_{12} = \pi\left(1 - \frac{2}{L}\right) = \frac{3\pi}{5}.} \tag{52.7}$$

This is the first appearance of the pentagonal structure $L = 5$ in the flavor-misalignment problem. Intuitively, the up-type and down-type nearest-neighbor links live on complementary arcs of the same compact holonomy circle. The Higgs doublet keeps a 2/5 fraction of the original SU(5) sector; the flavor-complement phase is the remaining 3/5 of a half-circle.

The first-order Cabibbo angle is therefore the modulus of the difference of the two nearest-neighbor link amplitudes,

$$\boxed{\sin\theta_{12}^{(1)} = \left|\theta_{12}^{(d,1)} - e^{\,i\varphi_{12}}\theta_{12}^{(u,1)}\right| = \left|\mathrm{e}^{-\pi/2} - e^{\,i3\pi/5}\mathrm{e}^{-\pi}\right|.} \tag{52.8}$$

Using $\cos(3\pi/5) = -(\sqrt{5}-1)/4$, this becomes

$$\sin^2\theta_{12}^{(1)} = \mathrm{e}^{-\pi} + \mathrm{e}^{-2\pi} + \frac{\sqrt{5}-1}{2}\,\mathrm{e}^{-3\pi/2}, \tag{52.9}$$

or equivalently

$$\boxed{\sin\theta_{12}^{(1)} = 0.225018533.} \tag{52.10}$$

Several intuitive points deserve emphasis. First, the Cabibbo angle is not put in by hand. It emerges from the interference of two *known* overlap amplitudes. Second, the golden-ratio coefficient in (52.9) is not decorative numerology; it is the trigonometric trace of the pentagonal holonomy implied by $L = 5$. Third, the formula is already extremely accurate before any QD sharpening.

52.8 Why the $2\leftrightarrow3$ angle is multiplicative rather than additive

The next mixing angle is much smaller than the Cabibbo angle,

$$\sin\theta_{23} \ll \sin\theta_{12}.$$

This has a clean geometric explanation in the present framework. The left-handed quark doublet Q_L is shared between the up-type and down-type sectors. A common $2\leftrightarrow3$ rotation of the doublet basis is therefore *unobservable* in V_{CKM}: it can be absorbed into a redefinition of Q_L before the charged current is read off. What survives in $U_{uL}^{\dagger}U_{dL}$ is only the *mismatch* between the two sectoral transports.

At first nontrivial order the surviving mismatch is bilinear in the two sectoral overlap angles,

$$\theta_{23}^{(u,1)} = \sqrt{\frac{y_c^{(1)}}{y_t^{(1)}}} = \sqrt{3}\,\mathrm{e}^{-\pi}, \qquad \theta_{23}^{(d,1)} = \sqrt{\frac{y_s^{(1)}}{y_b^{(1)}}} = 4\,\mathrm{e}^{-\pi}. \tag{52.11}$$

The unique compact holonomy loop contributes one factor of π, so the constitutionally shortest shared-doublet mismatch is

$$\boxed{\sin\theta_{23}^{(1)} = \pi\,\theta_{23}^{(u,1)}\theta_{23}^{(d,1)} = \pi\sqrt{\frac{y_c^{(1)}}{y_t^{(1)}}}\sqrt{\frac{y_s^{(1)}}{y_b^{(1)}}}.} \tag{52.12}$$

Using the explicit quark codebook,

$$\boxed{\sin\theta_{23}^{(1)} = 4\pi\sqrt{3}\,\mathrm{e}^{-2\pi} = 0.040645997.} \tag{52.13}$$

The physical picture is simple. The Cabibbo channel is a direct nearest-neighbor interference problem. The $2\leftrightarrow 3$ channel is not. It is the residual of two larger sectoral transports after the common left-handed piece has been quotient out. That is why it is naturally multiplicative and therefore smaller.

52.9 Direct deep-link origin of θ_{13}

The $1\leftrightarrow 3$ channel is smaller still. In the present minimal codebook the down-type direct deep link is branch-parity suppressed, while the up-type deep link survives as the shortest constitutionally admissible channel. The first-order angle is therefore simply

$$\boxed{\sin\theta_{13}^{(1)} = \sqrt{\frac{y_u^{(1)}}{y_t^{(1)}}} = \sqrt{3}\,\mathrm{e}^{-2\pi} = 0.003234506.} \tag{52.14}$$

Again, the intuition is transparent. To connect the first and third families one must tunnel across the full family depth. In a hierarchical bulk geometry such a direct deep link is exponentially rarer than a nearest-neighbor link, and it is therefore natural that θ_{13} is the smallest of the three CKM angles.

52.10 Residual physical CKM phase

The Cabibbo interference phase $\varphi_{12} = 3\pi/5$ is not itself the physical Kobayashi–Maskawa phase. One common family rephasing is unphysical and may be removed. The shortest such rephasing step on the same pentagonal lattice is $\pi/L = \pi/5$. The first-order physical Dirac phase is therefore

$$\boxed{\delta_{\mathrm{CKM}}^{(1)} = \varphi_{12} - \frac{\pi}{L} = \frac{2\pi}{5} = 1.25663706\ \mathrm{rad} = 72^\circ.} \tag{52.15}$$

This formula should be read geometrically. The raw up/down branch mismatch is three pentagonal steps; one common step is pure basis choice; the remaining two steps are physical.

52.11 First-order CKM matrix, Wolfenstein data, and Jarlskog invariant

Using the standard parameterization,

$$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{52.16}$$

with $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$, the first-order GG-6 CKM parameters are

$$\boxed{\sin\theta_{12}^{(1)} = 0.225018533, \qquad \sin\theta_{23}^{(1)} = 0.040645997, \qquad \sin\theta_{13}^{(1)} = 0.003234506, \qquad \delta_{\rm CKM}^{(1)} = \frac{2\pi}{5}.} \tag{52.17}$$

The corresponding magnitudes are

$$\left|V_{\rm CKM}^{(1)}\right| = \begin{pmatrix} 0.974349 & 0.225017 & 0.003235 \\ 0.224872 & 0.973540 & 0.040646 \\ 0.008704 & 0.039834 & 0.999168 \end{pmatrix}. \tag{52.18}$$

It is also useful to record the derived Wolfenstein-like data,

$$\lambda^{(1)} = 0.225018533, \qquad A^{(1)} = \frac{s_{23}^{(1)}}{[s_{12}^{(1)}]^2} = 0.802752, \tag{52.19}$$

$$(\bar\rho, \bar\eta)^{(1)} = (0.106728,\ 0.327870), \tag{52.20}$$

and the Jarlskog invariant

$$\boxed{J^{(1)} = c_{12}^{(1)} c_{23}^{(1)} [c_{13}^{(1)}]^2 s_{12}^{(1)} s_{23}^{(1)} s_{13}^{(1)} \sin\delta_{\rm CKM}^{(1)} = 2.73907 \times 10^{-5}.} \tag{52.21}$$

That J already lands in the familiar 10^{-5} range at first order is a highly nontrivial success. It means that the framework is not only producing realistic mixing magnitudes, but also a realistic order of magnitude for quark-sector CP violation.

52.12 QD completion of the CKM block

The CKM block is more protected than the fermion masses. A diagonal mass receives a direct seam renormalization, but a mixing angle is a *difference* or *mismatch* between two such renormalizations. For that reason the CKM QD shifts are naturally smaller than the mass QD shifts.

The Cabibbo channel is the most protected because it is already an interference between two nearest-neighbor links on the pentagonal holonomy lattice. The shortest combined dyadic/throat correction is therefore

$$\boxed{\sin\theta_{12}^{\rm QD} = \left(1 - \frac{1}{K\,2^J}\right)\sin\theta_{12}^{(1)} = \left(1 - \frac{1}{4480}\right)\sin\theta_{12}^{(1)}.} \tag{52.22}$$

Numerically,

$$\boxed{\sin\theta_{12}^{\rm QD} = 0.224968306.} \tag{52.23}$$

The $2\leftrightarrow3$ and $1\leftrightarrow3$ channels are most naturally sharpened by replacing the first-order quark masses in the overlap formulas by their already-derived QD values:

$$\boxed{\sin\theta_{23}^{\rm QD} = \pi\sqrt{\frac{m_s^{\rm QD}m_c^{\rm QD}}{m_b^{\rm QD}m_t^{\rm QD}}} = 0.040805707,} \tag{52.24}$$

$$\boxed{\sin\theta_{13}^{\rm QD} = \sqrt{\frac{m_u^{\rm QD}}{m_t^{\rm QD}}} = 0.003630497.} \tag{52.25}$$

The CKM phase receives the shortest allowed throat slip,

$$\boxed{\delta_{\rm CKM}^{\rm QD} = \delta_{\rm CKM}^{(1)} - \frac{\pi}{K} = \frac{13\pi}{35} = 1.16687727 \text{ rad.}} \tag{52.26}$$

The resulting QD CKM matrix is

$$\left|V_{\rm CKM}^{\rm QD}\right| = \begin{pmatrix} 0.974360 & 0.224967 & 0.003630 \\ 0.224838 & 0.973541 & 0.040805 \\ 0.008442 & 0.040087 & 0.999161 \end{pmatrix}, \tag{52.27}$$

with derived Wolfenstein data

$$\lambda^{\rm QD} = 0.224968306, \qquad A^{\rm QD} = 0.806266, \qquad (\bar\rho,\bar\eta)^{\rm QD} = (0.151746,\ 0.354449), \tag{52.28}$$

and Jarlskog invariant

$$\boxed{J^{\rm QD} = 2.98351\times10^{-5}.} \tag{52.29}$$

The most important lesson of the CKM derivation is structural. The quark mixing matrix is no longer a disconnected 4-parameter patch on the Standard Model. It has been reduced to one compact-holonomy phase, one nearest-neighbor interference law, one composite shared-doublet mismatch, and one deep direct link, all evaluated on top of the already-derived quark mass spectrum.

53 Precision comparison for the CKM block

We now compare the CKM predictions with the standard benchmark values from the benchmark values adopted in this paper. As in the rest of the paper, these benchmark numbers are *not* used as inputs. They appear only after the derivation has been completed.

53.1 Theory-uncertainty model for CKM observables

The first-order CKM theory uncertainty is defined, in exact parallel with the mass sector, as the magnitude of the omitted QD shift:

$$\sigma_{\mathrm{th}}^{(1)}(x) = \left| x^{\mathrm{QD}} - x^{(1)} \right|, \qquad x \in \{\sin\theta_{12}, \sin\theta_{23}, \sin\theta_{13}, \delta_{\mathrm{CKM}}\}. \tag{53.1}$$

The residual QD uncertainty is then taken to be one further three-step dyadic reduction,

$$\sigma_{\mathrm{th}}^{(2)}(x) = \frac{1}{8} \left| x^{\mathrm{QD}} - x^{(1)} \right|. \tag{53.2}$$

This is conservative for the CKM block, because mixing angles are mismatches of already-renormalized sectors and are therefore expected to converge faster than the masses themselves.

53.2 Primary comparison: first-order CKM parameters

Table 35: First-order CKM predictions compared with standard benchmark values adopted in this paper. Three mixing angles and one phase at leading closed form, against the standard benchmark fit, in the same six columns as the gauge sector: prediction, theory uncertainty, benchmark, benchmark uncertainty, relative deviation, pull. The angles are derived as misalignments between the up-type and down-type family projectors, so a deviation in one of them is a statement about the projectors and not about a fitted rotation.

Parameter	**Pred.**	$\sigma_{\mathrm{th}}^{(1)}$	**Obs.**	σ_{exp}	Dev	Pull
$\sin\theta_{12}$	0.225018533	5.022735e-05	0.225010000	6.800000e-04	+3.792e-05	+0.01
$\sin\theta_{23}$	0.040645997	1.597097e-04	0.041830000	7.400000e-04	-2.831e-02	-1.56
$\sin\theta_{13}$	0.003234506	3.959910e-04	0.003732000	8.750000e-05	-1.333e-01	-1.23
δ_{CKM} [rad]	1.256637061	0.089759790	1.147000000	0.026000000	+9.559e-02	+1.17

Even the first-order CKM block is already instructive. The Cabibbo angle is essentially exact, the $2 \leftrightarrow 3$ mixing lands at the few-percent level, the tiny $1 \leftrightarrow 3$ angle is generated in the correct 10^{-3} regime, and the CP phase lies in the correct octant. Given that no CKM number has been used as an input, this is a strong result.

53.3 Primary comparison: QD-completed CKM parameters

Table 36: QD-completed CKM predictions compared with standard benchmark values adopted in this paper. The same four parameters with the quantum-defect completion applied, in the same columns and against the same benchmarks. Comparing this table with the first-order one row by row shows which of the four the completion improves and by how much; where it improves a row only slightly, that is left visible rather than absorbed into an average over the block.

Parameter	**Pred.**	$\sigma_{\rm th}^{(2)}$	**Obs.**	$\sigma_{\rm exp}$	Dev	Pull
$\sin\theta_{12}$	0.224968306	6.278419e-06	0.225010000	6.800000e-04	-1.853e-04	-0.06
$\sin\theta_{23}$	0.040805707	1.996372e-05	0.041830000	7.400000e-04	-2.449e-02	-1.38
$\sin\theta_{13}$	0.003630497	4.949888e-05	0.003732000	8.750000e-05	-2.720e-02	-1.01
$\delta_{\rm CKM}$ [rad]	1.166877271	0.011219974	1.147000000	0.026000000	+1.733e-02	+0.70

The QD layer improves the small-angle sector and the phase sector simultaneously while keeping the Cabibbo channel in the narrow, protected band implied by its first-order pentagonal-holonomy formula.

53.4 Secondary comparison: full matrix, Wolfenstein data, and Jarlskog invariant

The four fundamental CKM parameters determine the entire matrix and the unitarity-triangle data. It is therefore useful to compare several derived observables as well.

Table 37: Derived QD CKM observables compared with benchmark fit values . Theory uncertainties are propagated from the residual QD uncertainties of $\sin\theta_{12}$, $\sin\theta_{23}$, $\sin\theta_{13}$, and $\delta_{\rm CKM}$. The four entries are not independent of the four above. They are functions of them, and their bands are propagated rather than assigned.

Observable	**Pred.**	$\sigma_{\rm th}^{(2)}$	**Obs.**	$\sigma_{\rm exp}$	Dev	Pull
$\lvert V_{ud}\rvert$	0.974359661	1.460132e-06	0.974350000	1.600000e-04	+9.915e-06	+0.06
$\lvert V_{us}\rvert$	0.224966823	6.278508e-06	0.225010000	6.800000e-04	-1.919e-04	-0.06
$\lvert V_{ub}\rvert$	0.003630497	4.949888e-05	0.003732000	8.750000e-05	-2.720e-02	-1.01
$\lvert V_{cd}\rvert$	0.224837701	6.495586e-06	0.224870000	6.800000e-04	-1.436e-04	-0.05
$\lvert V_{cs}\rvert$	0.973541434	1.700102e-06	0.973490000	1.600000e-04	+5.284e-05	+0.32
$\lvert V_{cb}\rvert$	0.040805438	1.996359e-05	0.041830000	7.400000e-04	-2.449e-02	-1.38
$\lvert V_{td}\rvert$	0.008441576	3.987346e-05	0.008580000	1.800000e-04	-1.613e-02	-0.75
$\lvert V_{ts}\rvert$	0.040087455	2.165168e-05	0.041110000	7.250000e-04	-2.487e-02	-1.41
$\lvert V_{tb}\rvert$	0.999160515	8.348453e-07	0.999118000	3.150000e-05	+4.255e-05	+1.35
A	0.806265788	3.970149e-04	0.826000000	0.015500000	-2.389e-02	-1.27
$\bar{\rho}$	0.151746165	0.004482719	0.159100000	0.009400000	-4.622e-02	-0.71
$\bar{\eta}$	0.354448663	0.005125031	0.352300000	0.007200000	+6.099e-03	+0.24
J	2.983511e-05	4.314428e-07	3.120000e-05	1.250000e-06	-4.375e-02	-1.03

Table 37 makes the main point vividly. Once the four CKM parameters are fixed by the geometric codebook, the entire matrix, the unitarity-triangle apex, and the Jarlskog invariant all fall into realistic numerical ranges. In particular, the derived $(\bar{\rho}, \bar{\eta})$ point lies in the correct quadrant and the correct neighborhood, while the predicted Jarlskog invariant naturally lands in the observed 10^{-5} band. This is exactly what one would want from a first-principles flavor construction.

53.5 Summary table for the CKM block

Table 38: Summary of the 4 CKM parameters derived in the present version of GG-A9-SM. First order, QD-completed, benchmark, and QD deviation, on one line per parameter. The table repeats numbers given in full above and exists so that the CKM block can be read at a glance without turning back. Nothing is quoted here that is not derived above, and the deviations shown are the QD ones, which are the paper's actual claims for this block.

Parameter	First order	QD	Observed	Dev
$\sin\theta_{12}$	0.225018533	0.224968306	0.225010 ± 0.000680	-1.853e-04
$\sin\theta_{23}$	0.040645997	0.040805707	0.041830 ± 0.000740	-2.449e-02
$\sin\theta_{13}$	0.003234506	0.003630497	0.003732 ± 0.0000875	-2.720e-02
δ_{CKM} [rad]	1.256637061	1.166877271	1.147 ± 0.026	+1.733e-02

54 The lepton gates: PMNS mixing and Majorana phases from projector misalignment

54.1 Historical background

Pontecorvo's 1957 paper introduced lepton-flavor oscillation as a meson–antimeson analog (Pontecorvo, 1957). Maki, Nakagawa, and Sakata in 1962 cast the construction in matrix form, introducing what is now called the PMNS matrix (Maki et al., 1962). Decades of solar, atmospheric, reactor, and accelerator oscillation experiments have measured the three PMNS angles to few-percent precision, leaving the Dirac CP phase δ_{CP} and the two additional Majorana phases (if neutrinos are Majorana) as the remaining structural unknowns. The theorems below derive the PMNS block from the same misalignment construction as CKM but with a different projector geometry: the charged-lepton dyadic basis and the constitutional Majorana basis live on the same OGN-5 packet but settle on inequivalent strata, giving the famously large mixing angles $\sin^2\theta_{12} \approx 0.31$, $\sin^2\theta_{23} \approx 0.55$ as a structural feature rather than empirical accident.

54.2 The PMNS GRIP cascade

The PMNS/Majorana block obeys the same conservation ladder as CKM on the same OGN-5 integer environment, but its projector geometry differs: the neutrino side now enters through the already-selected rank-two Majorana packet rather than through a hierarchical Dirac quark packet, so the large mixing angles are not a sign of failure but the expected output of comparing a dyadic

charged-lepton basis with a nearly democratic Majorana basis on the same legal parent bundle. As one GRIP cascade the PMNS angles and phases are the CCI comparison data of two stabilized lepton packets, with the solar, reactor, and atmospheric angles the three cleanest coordinates of a democratic-versus-dyadic GFF landscape. GRIP thereby explains a headline fact of particle physics — CKM small, PMNS large — not by different constitutions but because GSSB and GA stabilized different family geometries on the quark and lepton sides, making the large-angle branch the natural lepton attractor rather than an anomaly requiring extra flavor furniture.

54.3 Parameter block

The subsections that follow now read the six lepton-mixing parameters from the stabilized lepton misalignment packet: three angles, one Dirac phase, and two Majorana parity phases.

The quark block taught an important lesson. Once the diagonal masses are fixed, the real flavor problem is not the existence of families but the relative orientation of the family projectors. The same lesson now has to be applied to the lepton sector, but with one crucial difference. On the quark side both up- and down-type sectors are hierarchical and therefore nearly aligned, so the CKM angles are small. On the lepton side the charged-lepton block is hierarchical while the neutrino block, derived in Section 50.3, is Majorana and almost democratic. Large PMNS mixing is therefore not an anomaly within GG-6. It is the natural geometric consequence of forcing a dyadic charged-lepton codebook to coexist with a weakly split Majorana projector.

In standard notation the lepton mixing matrix is

$$U_{\mathrm{PMNS}} = U_{eL}^{\dagger} U_{\nu}\, \mathrm{diag}\Big(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}\Big)\,, \tag{54.1}$$

where U_{eL} diagonalizes the left-handed charged-lepton sector, U_{ν} diagonalizes the Majorana neutrino mass operator, and the two additional phases α_{21} and α_{31} survive because a Majorana mass matrix does not admit the full rephasing freedom of a Dirac matrix. The purpose of the present section is to derive all six lepton-side parameters,

$$\theta_{12}, \qquad \theta_{13}, \qquad \theta_{23}, \qquad \delta_{\mathrm{PMNS}}, \qquad \alpha_{21}, \qquad \alpha_{31},$$

without importing any oscillation fit as a hidden input.

54.4 Why the lepton block is constitutionally different from the quark block

The charged-lepton masses derived in Section 47.2 carry a clear dyadic ordering:

$$y_e \ll y_\mu \ll y_\tau.$$

That means the charged-lepton left-handed basis is already close to the boundary family basis itself. In contrast, the neutrino sector derived in Section 50.3 arose from the constitutionally allowed Majorana operator $(LH)(LH)$. The resulting normal-ordering spectrum was light, rank-two at leading order, and much less hierarchical. Physically, this tells us that neutrino flavor packets are not localized in family space as sharply as the charged leptons are. The PMNS matrix therefore

need not be close to the identity. The correct first question is not "Why are the PMNS angles large?" but "What is the minimal projector misalignment forced by a dyadic charged-lepton block against a nearly democratic Majorana block?"

The Arisaka-GG preprints already constrain the answer sharply. The guiding principles are:

1. the family algebra must use only the already admitted OGN-5 data;
2. the neutrino side must respect the rank-two Majorana structure found in Section 50.3;
3. the charged-lepton side must remain close to the dyadic family basis;
4. the lepton CP phase must come from the same compact-angle/geometric phase logic already used in the CKM sector, not from a new arbitrary continuous parameter.

These conditions already exclude a large class of ad hoc PMNS ansätze. They force us toward a very small family-projector codebook.

54.5 Democratic, symmetric, and antisymmetric family directions

The most useful basis for the neutrino side is the orthonormal triplet

$$|D\rangle = \frac{1}{\sqrt{3}}(1,1,1)^T, \qquad |S\rangle = \frac{1}{\sqrt{6}}(2,-1,-1)^T, \qquad |A\rangle = \frac{1}{\sqrt{2}}(0,1,-1)^T. \tag{54.2}$$

The physical meaning of these three vectors is simple. $|D\rangle$ is the democratic direction: every family contributes coherently. $|A\rangle$ is the antisymmetric $\mu-\tau$ direction, which naturally controls atmospheric mixing because it compares the second and third families. $|S\rangle$ is the symmetric orthogonal complement that closes the three-dimensional family space.

From these vectors we form the projectors

$$\Delta = |D\rangle\langle D|, \qquad \Pi_S = |S\rangle\langle S|, \qquad \Pi_A = |A\rangle\langle A|, \tag{54.3}$$

which satisfy

$$\Delta + \Pi_S + \Pi_A = \mathbf{1}_3. \tag{54.4}$$

This basis is not introduced because it is fashionable in flavor-model language. It is introduced because it is the minimal orthonormal family basis that makes the already-derived Majorana block transparent: one quasi-democratic direction and two orthogonal complements.

It is worth being precise about *why* there is no freedom in the choice, because a reader who has seen flavor models will expect one. Three constraints fix the triplet up to phases. First, the Majorana block is rank two (§50.3), so one family direction must be singled out and two must remain degenerate under the leading operator — that is one vector plus a plane. Second, the compact-angle structure treats the three families symmetrically at leading order, so the singled-out direction must be the one invariant under family permutation, which is $|D\rangle$ and nothing else. Third, inside the surviving plane the charged-lepton sector is *not* symmetric: the electron is separated from the muon and tau by the dyadic hierarchy, so the plane splits into the combination that is even under $\mu \leftrightarrow \tau$

and the one that is odd. Those are $|S\rangle$ and $|A\rangle$. Any other orthonormal triplet is a rotation of this one within the plane, and the rotation is fixed by the requirement that $|A\rangle$ be the μ–τ odd direction.

Remark 10 (Physics intuition for the three directions)**.** Think of the three families as three identical bells. $|D\rangle$ is all three struck together — the mode that does not care which bell is which, and therefore the mode a permutation-symmetric operator sees. $|A\rangle$ is the second struck against the third, which is the only mode that can tell those two apart. $|S\rangle$ is the remainder. The neutrino sector is nearly permutation-symmetric and so is dominated by the first; the charged-lepton sector is violently hierarchical and so distinguishes all three. *The PMNS matrix is the angle between those two attitudes*, and this is why the lepton angles are large where the quark angles are small: comparing a symmetric thing with a hierarchical thing produces large misalignment, while comparing two hierarchical things does not.

In the charged-lepton basis we denote the electron-family unit vector by

$$|e\rangle = (1,0,0)^T. \tag{54.5}$$

The charged-lepton dyadic hierarchy keeps the left-handed charged-lepton rotation close to this basis, so the dominant misalignment physics is the placement of the democratic/antisymmetric neutrino directions relative to $|e\rangle$ and the $\mu-\tau$ plane.

54.6 The constitutionally minimal first-order lepton projector ansatz

The preprints do not license an arbitrary 3×3 complex neutrino matrix filled with free coefficients. What they do license is a minimal rank-two Majorana block, a compact-angle phase logic, and OGN-governed rational weights. The simplest family operator compatible with those requirements is

$$\Xi^{(1)}_{\nu,\mathrm{mix}} = L^2\,\Delta + (3N_g^3 - L^2)(\Pi_S + \Pi_A) + \sqrt{\frac{N_g}{4K}}\,\left(e^{-i\phi_\nu}T_{eA} + e^{+i\phi_\nu}T_{Ae}\right) + \frac{1}{2N+1}(\Pi_A - \Pi_S), \tag{54.6}$$

with

$$T_{eA} = |e\rangle\langle A|, \qquad T_{Ae} = |A\rangle\langle e|, \qquad \phi_\nu = \arcsin\!\left(\frac{1}{2N+1}\right). \tag{54.7}$$

Equation (54.6) deserves a slow reading.

The first term says that the solar sector is dominated by the democratic family direction and weighted by the integer factor L^2. Because $|D\rangle$ has equal support in all three families, its overlap with the electron line is $1/\sqrt{3}$, which already points toward a large but not maximal solar angle. The second term states that the orthogonal family subspace carries the complementary weight $3N_g^3 - L^2$. This is the minimal way to respect completeness while keeping the OGN integers explicit.

The third term introduces the reactor channel. The quantity
$sqrtN_g/(4K)$ is small because reactor mixing is known phenomenologically to be the smallest of the three PMNS angles, but it is not zero because there is no constitutional reason to forbid a direct electron-to- antisymmetric leak. The coefficient is not fitted by hand; it is the minimal positive amplitude that can be formed from the already admitted OGN data. The last term, $\frac{1}{2N+1}(\Pi_A - \Pi_S)$,

slightly biases the $\mu-\tau$ sector away from exact maximality. In words: the atmospheric angle starts near 45° because of the near symmetry of the second and third families, and then the odd-triplicity integer $J = 2N + 1 = 7$ introduces the first allowed asymmetry.

54.7 Derivation of the first-order solar angle

The solar angle is controlled by how strongly the electron family overlaps with the light neutrino eigenvector that lives mainly in the democratic sector. Because

$$|\langle e|D\rangle|^2 = \frac{1}{3}, \tag{54.8}$$

and because the democratic channel carries the OGN weight L^2 against the total normalization $3N_g^3$, the first-order solar probability is forced to be

$$\sin^2\theta_{12}^{(1)} = \frac{L^2}{3N_g^3}. \tag{54.9}$$

Substituting $(N, L) = (3, 5)$ gives

$$\sin^2\theta_{12}^{(1)} = \frac{25}{81} = 0.3086419753. \tag{54.10}$$

Why the answer is just below $1/3$, and why that matters. Equation (54.8) is exact and is the whole geometric content: the electron line has one third of its length along the democratic direction, because $|D\rangle$ shares itself equally among three families. If the democratic channel carried the *entire* weight, the solar probability would be exactly $1/3 = 0.3333$. It does not: the OGN ledger weights that channel $L^2 = 25$ against the total normalization $3N_g^3 = 81$, and

$$\frac{L^2}{3N_g^3} \;=\; \frac{25}{81} \;=\; \frac{1}{3}\cdot\frac{25}{27} \;=\; \frac{1}{3}\left(1-\frac{2}{27}\right), \tag{54.11}$$

so the prediction is the democratic value reduced by exactly $2/27 = 7.4$ percent. That is a small, definite, and unavoidable depression — the fraction of the democratic weight the ledger assigns elsewhere.

The empirical significance of the depression is what makes the solar channel the sharpest in the block. A pure democratic overlap, $1/3$, sits 2.8σ above the JUNO measurement 0.3092 ± 0.0087. The ledger's $25/81$ sits 0.07σ from it. *The measurement is precise enough to see the* $2/27$, and it sees it.

Remark 11 (Physics intuition for the solar angle)**.** Tri-bimaximal mixing — the observation that $\sin^2\theta_{12}$ is close to $1/3$ — was for two decades one of the most suggestive numbers in flavor physics, and the models built to produce it produced exactly $1/3$. The data have since moved off $1/3$ by a few percent with an error bar small enough to matter, which is a difficulty for those models and an opportunity for any framework that predicts a specific small departure. Here the departure is not a

correction added afterwards: 25/81 is what the ledger gives, and 1/3 is what it would give if two of the twenty-seven units of democratic weight were not spoken for.

54.8 Derivation of the first-order reactor angle

The reactor angle measures the direct leakage of the electron family into the mostly atmospheric direction. In the minimal operator (54.6) that leakage appears with amplitude $sqrtN_g/(4K)$. Therefore the reactor probability is

$$\sin^2\theta_{13}^{(1)} = \frac{N_g}{4K}. \tag{54.12}$$

Substituting $(N, K) = (3, 35)$ yields

$$\sin^2\theta_{13}^{(1)} = \frac{3}{140} = 0.02142857143. \tag{54.13}$$

Why $4K$ and not K. The amplitude in (54.6) is $sqrtN_g/(4K)$ rather than $\sqrt{N/K}$, and the factor of four is not decoration. The leak connects $|e\rangle$ to $|A\rangle$, and $|A\rangle$ is a *normalized difference* of two families: writing it out, the amplitude reaching the third mass eigenstate picks up the $1/\sqrt{2}$ of (54.2) once from the antisymmetric direction and once more from the equal splitting of the surviving plane between the second and third eigenvectors. Two factors of $1/\sqrt{2}$ give the $1/4$ inside the square root. What the ledger supplies is the ratio N/K; the geometry supplies where the four comes from.

What the reactor angle is doing elsewhere in the paper. It is worth pointing forward, because this is the one derived angle whose smallness has a consequence far outside the lepton sector. Part VIII shows that the surviving matter asymmetry of the universe is carried by the flavor that is washed out most weakly, that the washout strength of flavor α is $\widetilde{m}_\alpha = \sum_i m_i |U_{\alpha i}|^2$, and that the electron flavor is the weakly washed one precisely because $|U_{e3}|^2 = \sin^2\theta_{13} = 3/140$ is small enough to decouple it from the heavy third mass state. The number derived in this subsection is the number that decides it.

Remark 12 (Physics intuition for the reactor angle)**.** Before 2012 the reactor angle was widely expected to be zero, and several otherwise successful flavor symmetries predicted exactly that. It is not zero, and the reason it is not is generic in this framework: there is no constitutional statement that forbids the electron line from touching the antisymmetric direction, and in a geometry with no unforced zeros, anything not forbidden happens at the smallest admissible amplitude. What is specific is the size. 3/140 is small because it is a direct off-diagonal leak rather than an overlap, and it is exactly 3/140 because the ledger has only the integers N and K to build it from.

54.9 Derivation of the first-order atmospheric angle

At leading order the atmospheric block is approximately $\mu-\tau$ symmetric, so one expects $\sin^2\theta_{23}$ to start near 1/2. The minimal odd-triplicity correction is the asymmetry factor $1/(2N+1) = 1/J$.

Because the correction must split the two equally weighted directions around the maximal point, the first-order atmospheric mixing is

$$\sin^2\theta_{23}^{(1)} = \frac{1}{2}\left(1+\frac{1}{2N+1}\right) = \frac{1}{2}\left(1+\frac{1}{J}\right). \tag{54.14}$$

For $J=7$ this gives

$$\sin^2\theta_{23}^{(1)} = \frac{4}{7} = 0.5714285714. \tag{54.15}$$

Why the split is about $1/2$ and not about 0. The form (54.14) is a deviation from *maximal* mixing, not from zero mixing, and that is the structural statement. In the family plane the second and third neutrino directions enter the leading operator with equal weight, so before any correction the atmospheric block is exactly μ–τ symmetric and $\sin^2\theta_{23} = 1/2$ identically. The correction is the last term of (54.6), $(1/J)(\Pi_A - \Pi_S)$, which is the smallest admissible operator that distinguishes the two: it is traceless on the plane, so it cannot shift the sum, and it therefore can only tilt the balance. A tilt of relative size $1/J$ about a half gives $\frac{1}{2}(1 \pm 1/J)$.

Why the sign is forced. The upper octant is not a choice between $\frac{1}{2}(1+1/J)$ and $\frac{1}{2}(1-1/J)$. The operator enters as $\Pi_A - \Pi_S$ and not the reverse, because the antisymmetric direction is the one the charged-lepton hierarchy has already singled out — the electron is separated from the pair, so it is the μ–τ *odd* combination that is dressed. The sign of the tilt therefore follows from which direction the charged-lepton sector distinguishes, and that is fixed before the neutrino sector is consulted.

This is worth stating plainly because it is what makes the octant a real prediction rather than a coin toss, and therefore what makes §55.5 a real exposure. A framework whose atmospheric sign were free could accommodate either octant and would learn nothing from the measurement. This one cannot: if the octant resolves lower, it is not the value of J that fails but the ordering of Π_A against Π_S, and with it Lemma L5.

Remark 13 (Physics intuition for the atmospheric angle)**.** Maximal atmospheric mixing means the muon and tau neutrinos are shared equally between two mass states — a perfect balance, and the kind of exact statement a symmetry can make. The framework says the balance is *almost* exact and names what breaks it: the same odd integer $J = 2N+1 = 7$ that counts the compact-angle structure, tilting the scale by one part in seven. The prediction is not "near maximal", which any number of models produce; it is maximal displaced by $1/7$, in a direction fixed by the charged-lepton hierarchy.

54.10 Derivation of the first-order leptonic Dirac phase

The CKM derivation already taught us that CP phases in GG-6 should be read as residual compact-angle holonomies after unphysical family rephasings are removed. The same rule applies on the lepton side. The minimal neutrino phase is

$$\phi_\nu = \arcsin\left(\frac{1}{2N+1}\right) = \arcsin\left(\frac{1}{7}\right). \tag{54.16}$$

Because the PMNS phase is measured relative to a negative real reference channel in the conventional parameterization, the physical Dirac phase is shifted by π:

$$\delta^{(1)}_{\mathrm{PMNS}} = \pi + 2\phi_\nu = \pi + 2\arcsin\left(\frac{1}{2N+1}\right). \tag{54.17}$$

With $N = 3$,

$$\delta^{(1)}_{\mathrm{PMNS}} = \pi + 2\arcsin\left(\frac{1}{7}\right) = 3.428287791 \text{ rad} = 196.426421^\circ. \tag{54.18}$$

The reference-channel argument, written out. Two features of (54.17) need justifying: why π appears at all, and why the residual is $2\phi_\nu$ rather than ϕ_ν.

The π is a statement about the standard parameterization, not about the geometry. In the conventional form the $e3$ element is written $s_{13}e^{-i\delta}$, so the measured phase is defined against the channel that carries the 1–3 leak, and the sign convention makes that reference channel enter with a *negative* real coefficient. The geometric phase in (54.6) is carried by the same leak T_{eA}, but referred to a positive real axis. Converting from one convention to the other is a rotation of the reference by half a turn, and that is the π. It is bookkeeping, and it is stated as such — but it is bookkeeping that must be done, because a prediction quoted in the wrong convention is a wrong prediction.

The factor of two is physical. The leak appears in (54.6) as the Hermitian pair $e^{-i\phi_\nu}T_{eA}+e^{+i\phi_\nu}T_{Ae}$: the operator carries $e^{-i\phi_\nu}$ going one way and $e^{+i\phi_\nu}$ coming back. A rephasing-invariant quantity must traverse the leak and return, so it collects the phase twice, and the physical invariant is $2\phi_\nu$ rather than ϕ_ν. This is the same rule the quark sector obeys, and it is the reason the leptonic Jarlskog invariant below carries $\sin\delta_{\mathrm{PMNS}}$ with δ already containing the doubling.

The number, and what it is tied to. With $\phi_\nu = \arcsin(1/7) = 8.213^\circ$, the residual is 16.43° and the prediction is $180^\circ + 16.43^\circ = 196.43^\circ$. The phase is order one, unlike the quark sector, because the lepton projector geometry is intrinsically more misaligned; but it is not an arbitrary number, because it is tied to exactly the same odd integer $J = 7$ that fixes the atmospheric tilt. *The atmospheric angle and the Dirac phase are the same prediction read twice*: if J were not seven, both would move together.

Remark 14 (Physics intuition for the leptonic phase)**.** A CP-violating phase is an area, not a number: it is what survives when every convention-dependent rephasing has been removed, and it can be computed by walking a closed loop in family space and asking how much the phase has turned on return. Here the loop is short — out along the electron-to-antisymmetric leak and back — and the turn is $2\phi_\nu$ because the leak is traversed twice. That the loop is short is why the answer is a simple inverse sine of a ledger integer rather than a complicated function of several.

54.11 Derivation of the first-order Majorana phases

The Majorana phases answer a different question from the Dirac phase. They measure the relative intrinsic parities of the neutrino eigenstates once all allowed charged-lepton rephasings have been removed. The minimal rank-two Majorana block derived earlier already picks one neutrino

state of positive parity and one of opposite parity. Therefore the constitutionally minimal first-order choice is

$$(\alpha_{21}, \alpha_{31})^{(1)} = (0, \pi). \tag{54.19}$$

Why rank two forces a parity *pair*. The argument is short and it is the reason these two entries are Derived rather than Asserted. A Majorana mass term is a symmetric bilinear in the field, so its eigenvalues are real up to a phase that can be absorbed into the field — but only up to a *sign*, because the absorbing rotation is squared by the bilinear (this is the half-angle statement of §64). What cannot be rotated away is therefore not a continuous phase but a discrete intrinsic parity, one bit per eigenstate.

Now impose the rank-two structure derived in §50.3. A rank-two block on a three-dimensional family space has exactly two nonzero eigenvalues at leading order, and the leading operator $\Xi_\nu^{(1)} = P_2 + 2NP_3$ of (50.6) assigns them opposite signs of the relevant invariant — one from the plain projector and one from the $2N$-enhanced one. Two states, opposite parities, no third value available. Hence

$$(\alpha_{21}, \alpha_{31}) = (0, \pi): \qquad 0 \text{ means same parity}, \quad \pi \text{ means opposite}, \tag{54.20}$$

and there is no continuum to choose from.

This is stronger than a generic prediction in two distinct senses. Oscillation experiments are insensitive to these phases, so no fit could have determined them; and in this framework they are not extra continuous parameters at all, but parity data of the Majorana projector. It is also weaker than it might appear in one sense, and the next subsection says how: the particular parity that is derived is close to the one that makes $m_{\beta\beta}$ hardest to observe.

54.12 First-order PMNS matrix, Jarlskog invariant, and neutrinoless-double-beta observable

Once the four standard mixing parameters are known, the full PMNS matrix in the conventional ordering is fixed. Using

$$\begin{aligned} \sin^2\theta_{12}^{(1)} &= 0.3086419753, & \sin^2\theta_{13}^{(1)} &= 0.02142857143, \\ \sin^2\theta_{23}^{(1)} &= 0.5714285714, & \delta_{\mathrm{PMNS}}^{(1)} &= 196.426421^\circ, \end{aligned} \tag{54.21}$$

we obtain

$$|U_{\mathrm{PMNS}}^{(1)}| = \begin{pmatrix} 0.822522 & 0.549571 & 0.146385 \\ 0.276669 & 0.603548 & 0.747786 \\ 0.496901 & 0.577669 & 0.647602 \end{pmatrix}. \tag{54.22}$$

The corresponding leptonic Jarlskog invariant is

$$J_{\mathrm{PMNS}}^{(1)} = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta_{\mathrm{PMNS}} = -9.26009 \times 10^{-3}. \tag{54.23}$$

The two unitarity triangles, at true relative area. The Jarlskog invariant is twice the area of the unitarity triangle, so the two sectors can be compared directly. This paper derives both:

$$\frac{|J_{\rm PMNS}|}{|J_{\rm CKM}|} \;=\; \frac{9.187\times10^{-3}}{2.984\times10^{-5}} \;=\; 308 \qquad \text{(QD order; 338 at first order).} \tag{54.24}$$

The leptonic unitarity triangle has about three hundred times the area of the quark one. Both numbers are outputs of the same ledger, and the ratio is therefore a prediction rather than an observation — the same constitutional rules, read on two different projector geometries.

That is not a failure of universality; it is the sharpest single expression of the framework's answer to the oldest question about flavor. The quark side compares two hierarchical Dirac sectors against one another and gets small angles and a thin triangle. The lepton side compares a dyadic charged-lepton sector against a nearly democratic Majorana sector and gets large angles and a fat one. *Same constitution, different projector geometry* — and the factor of three hundred is how much different.

The contrast also carries the cosmological weight of Part VIII. It is often said that the quark sector's CP violation is too small to make the observed matter asymmetry by roughly ten orders of magnitude, and that the lepton sector is therefore where one must look. Equation (54.24) quantifies the first half of that statement inside this framework, from derived quantities on both sides. What it does not do is close the gap by itself: three hundred is not ten orders of magnitude, and the reason the lepton sector can pay when the quark sector cannot is not the size of $J_{\rm PMNS}$ at all but the existence of the $\Delta L = 2$ door and the out-of-equilibrium lock crossing. §63 makes that explicit, and indeed finds that the *unflavored* invariant — the one $J_{\rm PMNS}$ belongs to — drops out of the matter asymmetry entirely.

The Majorana phases make it meaningful to quote the leading neutrinoless-double-beta effective mass,

$$m_{\beta\beta} = \left| m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i(\alpha_{31}-2\delta_{\rm PMNS})} \right|. \tag{54.25}$$

Rather than quote the result, it is worth working, because the way the three terms combine is the content.

At first order $m_1 = 0$, so the first term vanishes identically and the answer is a two-term interference. With $s_{12}^2 = 25/81$, $s_{13}^2 = 3/140$ and hence $c_{13}^2 = 137/140$, and with $\alpha_{21} = 0$ so that the second term is real and positive:

$$m_2 s_{12}^2 c_{13}^2 = 8.36114 \times \tfrac{25}{81} \times \tfrac{137}{140} = +2.52530 \text{ meV}. \tag{54.26}$$

The third term carries the phase $\alpha_{31} - 2\delta_{\rm PMNS} = \pi - 2(196.426^\circ)$, and $m_3 s_{13}^2 = 50.1668 \times 3/140 = 1.07500$ meV, so

$$m_3 s_{13}^2 e^{i(\alpha_{31}-2\delta_{\rm PMNS})} = -0.90307 + 0.58317\,i \text{ meV}. \tag{54.27}$$

The real parts *oppose*: $+2.52530$ against -0.90307. Adding and taking the modulus,

$$m_{\beta\beta}^{(1)} = |\,2.52530 - 0.90307 + 0.58317\,i\,| = 1.72386 \text{ meV}. \tag{54.28}$$

At QD order the first term is no longer exactly zero ($m_1^{\rm QD} = 0.0653$ meV contributes +0.04414 meV) and the other two shift slightly, giving $m_{\beta\beta}^{\rm QD} = 1.79925$ meV.

A finding that must be stated: the derived parity is close to the worst case. The cancellation in (54.28) is not incidental — it is most of the answer, and it is caused by the derived parity. Two comparisons make the size of the effect concrete, on the same derived spectrum and the same derived angles:

- With the *other* admissible parity, $\alpha_{31} = 0$, the third term's real part changes sign and adds instead of subtracting: $m_{\beta\beta}$ would be 3.62 meV, twice as large.
- Scanning both Majorana phases freely over the full circle — which is what an experiment analyzing a generic normal-ordering spectrum must allow — gives the envelope $m_{\beta\beta} \in [1.44,\ 3.74]$ meV.

The derived value 1.80 meV therefore lands in the *lowest quarter* of the accessible range, near the maximally destructive end. That is an uncomfortable result and the paper states it rather than resting on the observation that 1.8 meV is small: **this framework's Majorana prediction is harder to confirm than a generic one**, and the difficulty is a direct consequence of the parity the framework derives rather than of any choice made to avoid a constraint.

Two things follow. The prediction remains one-sided in the sense of F-SM-3b — a confirmed $0\nu\beta\beta$ line anywhere in the tonne-scale window of 9–21 meV kills it, while a null result at that sensitivity proves nothing — and the margin by which the prediction escapes the approved program is larger than the spectrum alone would require. The cost is paid for the parity being *derived*: a framework free to choose α_{31} would place its prediction at the top of the envelope and claim a better chance of confirmation, and would be claiming it about a number it had fitted.

54.13 QD completion of the PMNS/Majorana block

The first-order formulas are the constitutional skeleton. The QD layer must respect the same logic as everywhere else in the paper: finite corrections, no new fitted constants, and suppression by already-admitted dyadic or locked factors. The minimal QD completion is

$$\sin^2\theta_{12}^{\rm QD} = \left(1 - \frac{1}{K2^{2N+1}}\right)\sin^2\theta_{12}^{(1)}, \tag{54.29}$$

$$\sin^2\theta_{13}^{\rm QD} = \left(1 + \frac{1}{2K} + \frac{1}{2^{2N+1}}\right)\sin^2\theta_{13}^{(1)}, \tag{54.30}$$

$$\sin^2\theta_{23}^{\rm QD} = \frac{1}{2}\left(1 + \frac{1}{2N+1} - \frac{1}{K}\right), \tag{54.31}$$

$$\delta_{\rm PMNS}^{\rm QD} = \delta_{\rm PMNS}^{(1)} - \frac{\pi}{2K(2N+1)}, \tag{54.32}$$

with the Majorana phases protected,

$$(\alpha_{21}, \alpha_{31})^{\rm QD} = (0, \pi). \tag{54.33}$$

Each correction has a direct physical meaning.

The solar channel is already dominated by the democratic projector, so its QD correction is tiny and negative: it merely trims the idealized first-order value by the smallest mixed dyadic/lock factor $1/(K2^{2N+1}) = 1/4480$. The reactor channel receives the largest relative QD enhancement because it comes from a direct off-diagonal leak and is therefore most sensitive to the first residual local dressing. The atmospheric channel shifts from 4/7 to 39/70, showing beautifully how a near-maximal angle can be nudged by the lock denominator $K = 35$ without losing its first-order origin. The Dirac phase slides only very slightly because a holonomy phase is already fixed at first order and later corrections can only perturb it weakly.

Substituting the OGN values gives

$$\sin^2\theta_{12}^{\mathrm{QD}} = 0.3085730820, \qquad \sin^2\theta_{13}^{\mathrm{QD}} = 0.0219021046, \qquad \sin^2\theta_{23}^{\mathrm{QD}} = \frac{39}{70} = 0.5571428571, \tag{54.34}$$

$$\delta_{\mathrm{PMNS}}^{\mathrm{QD}} = 3.421876378\ \mathrm{rad} = 196.059074^\circ, \qquad (\alpha_{21}, \alpha_{31})^{\mathrm{QD}} = (0, \pi). \tag{54.35}$$

The sharpened PMNS magnitude matrix is then

$$|U_{\mathrm{PMNS}}^{\mathrm{QD}}| = \begin{pmatrix} 0.822364 & 0.549377 & 0.147994 \\ 0.282542 & 0.612560 & 0.738201 \\ 0.493849 & 0.568292 & 0.658147 \end{pmatrix}, \tag{54.36}$$

with

$$J_{\mathrm{PMNS}}^{\mathrm{QD}} = -9.18733 \times 10^{-3}, \qquad m_{\beta\beta}^{\mathrm{QD}} = 1.79925\ \mathrm{meV}. \tag{54.37}$$

The important point is that the QD layer sharpens the block without changing its qualitative character. Large lepton mixing, a small reactor channel, an order-one CP phase, and parity-like Majorana phases are already fixed at first order.

55 Precision comparison for the PMNS/Majorana block

55.1 Benchmark convention

Unlike the quark sector, the leptonic atmospheric angle and Dirac phase remain analysis dependent, and at the time of writing that dependence is not a technicality: the two leading determinations disagree with each other about which octant the atmospheric angle lies in. The pull tables below therefore quote *named* benchmarks rather than a single consensus point, and where the determinations disagree both are printed with the pull against each. Three sources are used.

The global fit. NuFIT 6.1, based on data available in November 2025, supersedes the 6.0 release (Esteban et al., 2024, 2025). For normal ordering,

$$\sin^2\theta_{12}^{\mathrm{obs}} = 0.308^{+0.0067}_{-0.0066}, \qquad \sin^2\theta_{13}^{\mathrm{obs}} = 0.02248^{+0.00055}_{-0.00059}, \qquad \sin^2\theta_{23}^{\mathrm{obs}} = 0.470^{+0.017}_{-0.014}, \tag{55.1}$$

together with $\Delta m^2_{21} = (7.53^{+0.094}_{-0.10}) \times 10^{-5}\ \mathrm{eV}^2$ and $\Delta m^2_{31} = (2.511^{+0.021}_{-0.020}) \times 10^{-3}\ \mathrm{eV}^2$.

The direct reactor measurement. JUNO published first results from 59.1 days of running in 2026 (JUNO Collaboration, 2026), and they are already the world's most precise determination of the solar pair,

$$\sin^2\theta_{12}^{\rm obs} = 0.3092 \pm 0.0087, \qquad \Delta m^{2\,\rm obs}_{21} = (7.50 \pm 0.12) \times 10^{-5}\ \text{eV}^2. \tag{55.2}$$

The accelerator joint fit. T2K and NOvA published a joint analysis in 2025 (T2K and NOvA Collaborations, 2025). For normal ordering it gives the Dirac phase and an atmospheric angle in the *upper* octant,

$$\delta_{\rm PMNS}^{\rm obs} = 203.4^\circ\,{}^{+63.0}_{-37.8}, \qquad \sin^2\theta_{23}^{\rm obs} = 0.56^{+0.03}_{-0.05}. \tag{55.3}$$

The Dirac phase is taken from this source rather than from the global fit because it is the direct measurement, and because the two long-baseline experiments together carry essentially all of the sensitivity to it.

Convention for asymmetric errors. All three sources quote asymmetric uncertainties, which the Pull definition of §55.2 did not previously have to accommodate. The convention adopted here, and used in every table below, is that the observational uncertainty entering the pull is the one on the side *facing* the prediction: the upper uncertainty when the prediction lies above the central value, the lower when it lies below. This is the conservative choice in the sense that it never shrinks a disagreement.

The Majorana phases are presently unmeasured, so for those rows the tables record only the theoretical prediction and the internal theory uncertainty.

55.2 Theory-error model for the PMNS block

To keep the error assignment consistent with the rest of the manuscript, the first-order theory error of each PMNS observable is defined as the size of the omitted QD correction,

$$\sigma_{\rm th}^{(1)}(X) = |X^{\rm QD} - X^{(1)}|, \tag{55.4}$$

while the residual QD theory error is taken to be one eighth of that shift,

$$\sigma_{\rm th}^{(2)}(X) = \frac{1}{8}|X^{\rm QD} - X^{(1)}|. \tag{55.5}$$

For the Majorana phases, which do not move between first order and QD, we adopt a conservative protected-phase uncertainty based on the smallest constitutional phase quantum,

$$\sigma_{\rm th}^{(1)}(\alpha_{21}) = \sigma_{\rm th}^{(1)}(\alpha_{31}) = \frac{\pi}{K 2^{2N+1}} = 0.0401786^\circ, \tag{55.6}$$

$$\sigma_{\rm th}^{(2)}(\alpha_{21}) = \sigma_{\rm th}^{(2)}(\alpha_{31}) = \frac{\pi}{K 2^{2N+4}} = 0.00502232^\circ. \tag{55.7}$$

These uncertainties are deliberately conservative. Their role is not to inflate agreement but to acknowledge that the present manuscript has given a precise constitutional codebook rather than a full multi-loop oscillation fit.

55.3 First-order comparison table

Table 39: First-order PMNS/Majorana predictions compared with the named oscillation benchmarks of §55.1. Where the global fit and the accelerator joint fit disagree — the atmospheric octant — both are printed. The Majorana phases are not yet experimentally measured.

Observable	Pred	σ_{th}	Obs	σ_{exp}	Dev	Pull
$\sin^2\theta_{12}$ (NuFIT 6.1)	0.308641975	0.0000688933	0.308000000	0.006700000	+0.0020843	+0.10
$\sin^2\theta_{12}$ (JUNO)	0.308641975	0.0000688933	0.309200000	0.008700000	-0.0018047	-0.06
$\sin^2\theta_{13}$ (NuFIT 6.1)	0.021428571	0.000473533	0.022480000	0.000590000	-0.0467718	-1.39
$\sin^2\theta_{23}$ (NuFIT 6.1)	0.571428571	0.014285714	0.470000000	0.017000000	+0.2158050	+4.57
$\sin^2\theta_{23}$ (T2K+NOvA)	0.571428571	0.014285714	0.560000000	0.030000000	+0.0204082	+0.34
δ_{PMNS} [°] (T2K+NOvA)	196.426421	0.367347	203.400000	37.800000	-0.0342850	-0.18
α_{21} [°]	0.000000	0.040179		not yet measured		
α_{31} [°]	180.000000	0.040179		not yet measured		

The first-order table already captures the dominant empirical structure: a solar angle of order 0.3, a small reactor angle near 2×10^{-2}, an atmospheric angle in the upper octant, and a large Dirac phase in the neighborhood favored by the direct measurement. The solar and Dirac channels land well inside one sigma of every source. The atmospheric row is where the sources part company, and it is treated on its own below.

55.4 QD comparison table

Table 40: QD-completed PMNS/Majorana predictions compared with the same named benchmark values. The lepton mixing angles and the Majorana phases at QD order. Two benchmarks are printed for the solar angle — the global fit and JUNO's first-data determination — because they are independent and this paper does not choose between them. The Majorana rows are included for completeness and are not testable by oscillation: no oscillation experiment can measure α_{31}, which the appendix proves rather than asserts.

Observable	Pred	σ_{th}	Obs	σ_{exp}	Dev	Pull
$\sin^2\theta_{12}$ (NuFIT 6.1)	0.308573082	0.00000861166	0.308000000	0.006700000	+0.0018607	+0.09
$\sin^2\theta_{12}$ (JUNO)	0.308573082	0.00000861166	0.309200000	0.008700000	-0.0020276	-0.07
$\sin^2\theta_{13}$ (NuFIT 6.1)	0.021902105	0.0000591916	0.022480000	0.000590000	-0.0257071	-0.97
$\sin^2\theta_{23}$ (NuFIT 6.1)	0.557142857	0.001785714	0.470000000	0.017000000	+0.1854100	+5.10
$\sin^2\theta_{23}$ (T2K+NOvA)	0.557142857	0.001785714	0.560000000	0.050000000	-0.0051020	-0.06
δ_{PMNS} [°] (T2K+NOvA)	196.059074	0.0459184	203.400000	37.800000	-0.0360911	-0.19
α_{21} [°]	0.000000	0.005022		not yet measured		
α_{31} [°]	180.000000	0.005022		not yet measured		

The QD layer moves the reactor channel toward the data — the pull against the global fit falls from -1.39 to -0.97 — while leaving the solar angle and the Dirac phase almost unchanged. This is what the structure of the correction terms suggested in advance: the solar channel was already a stable democratic overlap, whereas the reactor channel was more sensitive to finite local dressing.

The atmospheric channel is the exception, and honesty requires stating it plainly. Against the accelerator joint fit the QD layer is an improvement, carrying the pull from $+0.34$ to -0.06. Against the global fit it is not: the correction moves $4/7 = 0.5714$ to $39/70 = 0.5571$, which is *toward* maximal mixing and therefore away from a central value that sits in the lower octant, and the pull grows from $+4.57$ to $+5.10$ — a Gaussian pull against the NuFIT 6.1 fit with the Super-Kamiokande atmospheric sample, whose own three-sigma range $[0.435, 0.584]$ still contains the prediction because the profile is two-lobed. The QD dressing is a small downward shift of a prediction whose disagreement with one of the two sources is not small; it can hardly repair it, and it does not.

55.5 The atmospheric octant: a disagreement between the sources

One row of the tables above deserves its own discussion, because the disagreement it records is between two *measurements*, not between theory and measurement.

The global fit NuFIT 6.1 places the atmospheric angle in the *lower* octant, $\sin^2\theta_{23} = 0.470^{+0.017}_{-0.014}$, with a three-sigma range $[0.435, 0.584]$ that still admits the upper octant. The T2K and NOvA joint analysis places it in the *upper* octant, $\sin^2\theta_{23} = 0.56^{+0.03}_{-0.05}$, and reports a Bayes factor of 3.5 favoring the upper octant once the reactor constraint on θ_{13} is imposed. The two determinations are not independent — the global fit contains the accelerator data — but they weight the atmospheric and reactor samples differently, and on this one parameter they come out on opposite sides of maximal mixing.

The present framework derives $\sin^2\theta_{23} = \frac{1}{2}(1+1/J) = 4/7$, QD-completed to $\frac{1}{2}(1+1/J-1/K) = 39/70 = 0.5571$, and the upper octant is not an accident of the arithmetic: it is forced by the sign of the odd-triplicity splitting $1/J$ about the μ–τ symmetric value $1/2$. The prediction therefore sits comfortably inside the accelerator result and about five sigma from the global one. It cannot be reconciled with a lower-octant central value by any dressing available in the codebook.

Three things follow, and the paper states all three rather than choosing the source that suits it.

First, this is now the sharpest near-term exposure in the lepton block. A converged global-plus-accelerator determination placing $\sin^2\theta_{23}$ in the lower octant at high significance would falsify the atmospheric assignment outright. That criterion is registered in §109 and is expected to be settled by the long-baseline program of the 2030s.

Second, the exposure is structural rather than local. The atmospheric value is Row 21 of the codebook, carried at **Derived (cond.)** on the rank-2 atmospheric-deviation lemma L5 of §P. L5's own statement selects $4/7$ over the equally cheap candidate $5/9 = 0.5556$ by the structural-naming criterion — but *both* candidates are in the upper octant, so the tie-break is not what is at stake. What is at stake is the sign of the deviation from $\pi/4$. If the octant resolves lower, L5 does not merely lose a tie-break; it falls, and Row 21 falls with it.

Third, nothing else in the block moves. The solar angle, the reactor angle, the Dirac phase, the Majorana parities and the mass splittings are fixed by different structures and are unaffected by the octant, as the tables above show: the solar prediction sits 0.07σ from the JUNO measurement and 0.09σ from the global fit, and the Dirac phase sits 0.19σ from the joint-fit best value. A framework whose lepton block agreed everywhere would be less informative than one that agrees in five places and is exposed in the sixth.

55.6 Comprehensive summary tables for the first-order and QD lepton block

For a concise side-by-side overview, we collect the entire PMNS/Majorana result in the two summary tables below. The first summarizes the six directly parameter-counted observables. The second adds two secondary but phenomenologically important derived quantities: the leptonic Jarlskog invariant and the neutrinoless-double-beta effective mass.

Table 41: Comprehensive summary of the PMNS/Majorana block at first order and at QD order. First order, QD-completed, benchmark with its asymmetric interval, and the QD deviation, one line per parameter. As the note in the table says, the atmospheric row is scored against the global fit rather than against a single experiment, because the octant is not yet resolved and the global fit is the honest summary of what is currently known.

Parameter	First order	QD	Observed	Dev
Dev is the QD relative deviation; the atmospheric row is scored against the global fit (see §55.5).				
$\sin^2\theta_{12}$	0.308641975	0.308573082	$0.308^{+0.0067}_{-0.0066}$	+1.86066e-03
$\sin^2\theta_{13}$	0.0214285714	0.0219021046	$0.02248^{+0.00055}_{-0.00059}$	-2.57071e-02
$\sin^2\theta_{23}$	0.571428571	0.557142857	$0.470^{+0.017}_{-0.014}$ (global); $0.56^{+0.03}_{-0.05}$ (accel.)	+1.85410e-01
$\delta_{\mathrm{PMNS}}\,[^\circ]$	196.426421	196.059074	$203.4^{+63.0}_{-37.8}$	-3.60911e-02
$\alpha_{21}\,[^\circ]$	0.000000	0.000000	not yet measured	—
$\alpha_{31}\,[^\circ]$	180.000000	180.000000	not yet measured	—

Table 42: Secondary derived lepton-sector observables associated with the first-order and QD PMNS/Majorana solutions. The mixing-matrix elements and the Jarlskog invariant are consequences of the angles and the phase rather than additional predictions, and are printed because they are what a reader of the oscillation literature will want to compare. The benchmark column carries the values implied by the same global fit used above, so a disagreement here restates a disagreement in the angles and is not new information.

Observable	First order	QD	Representative benchmark/comment
$\lvert U_{e1}\rvert$	0.822522	0.822364	benchmark 0.825038
$\lvert U_{e2}\rvert$	0.549571	0.549377	benchmark 0.545263
$\lvert U_{e3}\rvert$	0.146385	0.147994	benchmark 0.148324
$\lvert U_{\mu 3}\rvert$	0.747786	0.738201	benchmark 0.740054
$\lvert U_{\tau 3}\rvert$	0.647602	0.658147	benchmark 0.655988
J_{PMNS}	-9.26009×10^{-3}	-9.18733×10^{-3}	benchmark -1.24076×10^{-2}
$m_{\beta\beta}$ [meV]	1.72386	1.79925	not yet measured; target for future $0\nu\beta\beta$ searches

The secondary table is useful for intuition. It shows explicitly that the present construction does not merely reproduce the angle set abstractly. It actually yields a realistic full matrix pattern: the electron row is close to the canonical large-solar/small-reactor shape, while the third-column entries land naturally in the atmospheric neighborhood. The derived leptonic Jarlskog invariant is negative and of order 10^{-2}, which is precisely what one expects when large mixing angles coexist with an order-one CP phase.

55.7 Significance and uniqueness of the PMNS/Majorana result

The PMNS/Majorana block is significant for three distinct reasons.

First significance: one theory now explains why CKM is small and PMNS is large. A common criticism of unified flavor ideas is that quark and lepton mixing look qualitatively different. In the present framework that difference is not a problem to be patched over. It is a prediction. The quark side is small-angle because two hierarchical Dirac sectors are compared against one another. The lepton side is large-angle because a dyadic charged-lepton sector is compared against a nearly democratic Majorana sector. Same constitution, different projector geometry.

Second significance: the Majorana option becomes predictive rather than optional. The present derivation does not merely allow Majorana neutrinos. It uses the constitutionally minimal Majorana channel to compute both the neutrino mass scale and the two physical parity phases. That immediately produces a meV-scale prediction for $m_{\beta\beta}$. Even though current data do not yet measure α_{21}, α_{31}, or $m_{\beta\beta}$ directly, the theory side is already specific enough to be falsifiable by future neutrinoless-double-beta searches.

Third significance: the result is unusually short. Many flavor constructions require long chains of fitted textures, flavon alignments, or continuous parameters. Here the visible final codes are strikingly compact:

$$\sin^2\theta_{12} = \frac{25}{81}, \qquad \sin^2\theta_{13} = \frac{3}{140}, \qquad \sin^2\theta_{23} = \frac{4}{7} \to \frac{39}{70},$$

$$\delta_{\mathrm{PMNS}} = \pi + 2\arcsin\left(\frac{1}{7}\right) \to 196.06^\circ, \qquad (\alpha_{21}, \alpha_{31}) = (0, \pi).$$

That compactness is the uniqueness claim of the present section. The block is not unique because no other algebra can be imagined in mathematics; it is unique because once one insists on the written GG-Constitution, the restricted input ledger, the dyadic charged-lepton hierarchy, the Majorana-rank-two neutrino block, and the OGN-5 integers, the admissible lepton projector arithmetic becomes very narrow.

The manuscript has already fixed the first 27 quantities by the end of the PMNS/Majorana block. The next section then completes the last topological parameter and closes the full 28-parameter census.

56 Part VII in plain English: masses as floors, mixings as gates

Twelve masses and eight mixing parameters make up the largest block of the twenty-eight, and this Part treats them as two different kinds of object.

A mass is a floor. Each family sits at a depth in the cascade, and the mass is what the descent has left when it stops. That is why the mass ratios come out as exponentials of π rather than as arbitrary numbers: a depth is a count, and the suppression per unit depth is fixed by the geometry. A mixing parameter is not a floor at all. It is a gate — the misalignment between the family projectors of two sectors that have descended separately on the same closed packet. The CKM and PMNS matrices are what that misalignment looks like when it is written in the observable basis.

This is the Part where the paper's honesty boundary is most visible. The depths are assigned, not derived: what is missing is a lemma that would fix each depth as the dimension of a specific cohomology class, and until that lemma exists the family structure is a working hypothesis with a good deal of arithmetic behind it. The paper states this in the codebook grade and repeats it here rather than letting the closeness of the numbers do the arguing.

So the fair summary is that "flavor is unsolved" and "flavor is reduced to one named calculation" are both true statements about this Part. The second is worth something only if the named calculation is genuinely a single one, and the reader who wants to check that should look at how many separate assumptions the twelve masses and eight mixings actually consume. That count is in the audit, and it is the number that matters.

Part VIII
The Majorana door and leptogenesis at the lock scale

Why the excess of matter over antimatter is written by the same geometry that fixes the neutrino block — the unique $\Delta L = 2$ portal, the heavy sector at $M_{\text{lock}} = 3.85$ TeV, the flavored CP asymmetry driven by the Majorana parity, the resonance question and its named hypothesis, the electron door, the Boltzmann integration, and the assembled baryon-to-photon ratio

---

Part VII closed the twelve fermion masses and the two mixing matrices. In doing so it fixed, without adjustable freedom, five quantities that between them decide whether the universe contains matter: the scale M_{lock} at which the compact fiber locks, the unique $\Delta L = 2$ portal through which lepton number can be violated at all, the coefficient π/E_{GUT} of the Majorana seam, the Majorana parity $(\alpha_{21}, \alpha_{31}) = (0, \pi)$, and the reactor angle $\sin^2\theta_{13} = N_g/4K = 3/140$. Each was derived for a different reason and none was derived with baryogenesis in view.

This Part asks what those five quantities imply when they are handed to the early universe, and it does so in full rather than by reference. The reason is not completeness for its own sake. It is that the answer is a statement about *this paper's own numbers*: the reactor angle A9 derives is what allows the asymmetry to survive; the Majorana parity A9 derives is what fixes its sign; the lock scale A9 derives is what places the mechanism nine orders of magnitude below where the standard treatment must put it. A reader who has followed the parameter derivation is entitled to see that consequence worked out on the page, and not sent elsewhere for it.

Custody, stated once and then honored. The cosmological transport calculation belongs to GG-A10-Cosmos, and the hardest step in this chain — the finite-temperature identification of the quantity that regulates the resonance — is carried there, in its leptogenesis appendix (the codebook lemma of that paper states the on-resonance condition and has not been brought to the partial-resonance reading this Part uses; the appendix has), and is *open at derivation level*. Nothing below quietly promotes it. Every step of the chain carries its owner and its grade, the summary audit is Table 50, and where this Part states a result whose home is a companion it says so in the sentence that states it. What A9 owns are the microscopic inputs, and it owns them because it derived them.

A note on the theorem identifiers. The formal statements of this Part are given as propositions and lemmas rather than as new T-SM-$NN.M$ theorems. That scheme encodes a section number, and inserting a Part in the middle of the manuscript moves every section number after it; the identifiers are therefore not minted here.

57 One survivor in a billion: Sakharov's conditions, located

57.1 The size of the problem

The observed baryon-to-photon ratio is

$$\eta_B^{\rm obs} = (6.12 \pm 0.04) \times 10^{-10} \tag{57.1}$$

(Planck Collaboration, 2020). The number is easy to write and hard to feel, so it is worth converting once. In the early universe, before the electrons and positrons annihilated, matter and antimatter were present in almost equal numbers. Almost. For every billion antiquarks there were about a billion and one quarks. The billion pairs annihilated into the photons that are now the microwave background; the one left over is everything — every star, every planet, every reader of this sentence. The Standard Model, taken alone, predicts that the leftover should be zero to a fantastic precision, and the universe should be a bath of radiation with nothing in it.

57.2 The three conditions, and where this framework meets them

Sakharov (1967) gave the three conditions any mechanism must satisfy. They are worth restating not as history but as a checklist, because the interest here is *which* of them the geometry supplies and which it must borrow.

(i) Baryon-number violation. Something must change B. In the present framework baryon number is a *gauged* quantity, residual to a boundary-terminated local $\mathrm{U}(1)_B$ (§109.6), and the selection rule is $\Delta B \in N\mathbb{Z}$ with $N = 3$: single proton decay and neutron–antineutron oscillation are both forbidden, while $\Delta B = 3$ is admissible and sphaleron-suppressed. That is a sharper statement than the Standard Model's, and it cuts both ways — it is what makes $\tau_p = \infty$ a theorem of this framework, and it is what forbids the framework from making baryons directly.

(ii) CP violation. Something must distinguish matter from antimatter. The framework has CP violation in abundance and has derived it: the CKM phase $\delta_{\rm CKM} = 2\pi/5$, the leptonic Dirac phase $\delta_{\rm PMNS} = \pi + 2\arcsin(1/7)$, and the Majorana parity $\alpha_{31} = \pi$. §63 establishes which of the three actually pays, and the answer is not the one a reader would guess.

(iii) Departure from thermal equilibrium. Something must happen out of equilibrium, or CPT guarantees that whatever is made is unmade. The framework supplies this without being asked: the descent crosses the lock at $M_{\rm lock} = 3.85$ TeV, and a heavy state decaying as the bath cools through its own mass is the textbook out-of-equilibrium condition.

Remark 15 (Why the quark sector cannot pay, in one paragraph)**.** The obvious candidate is the CKM phase, and it fails by a wide margin. The Standard Model's CP measure is the Jarlskog invariant multiplied by the six quark-mass-squared differences, and made dimensionless it is of order 10^{-20} at the electroweak scale — some ten orders of magnitude short of 10^{-10}, and falling steeply with temperature because the mass differences that carry it are Yukawa-suppressed. The

electroweak transition compounds the failure: in the Standard Model with a 125 GeV Higgs it is a smooth crossover, not a first-order transition, so there are no bubble walls and condition (iii) is not met there either. After all, the deficiency is not that the quark sector is a little too small; it is that it is the wrong sector by twenty orders of magnitude, which is why the search has always moved to the leptons.

So the framework must violate lepton number, not baryon number, and let the electroweak sphalerons convert — which is the mechanism Fukugita & Yanagida (1986) proposed, here with the heavy scale an output rather than an input. The rest of this Part is the working out of that sentence, and the first question is whether the geometry permits a $\Delta L = 2$ interaction at all.

58 The door: why the portal is Majorana, and why it is unique

58.1 Lepton number is winding number

The constitutional statement was proved in Part VII as Theorem T-SM-44.1 (§50.2) and is recalled here because the whole chain hangs from it: a Lorentz- and gauge-invariant Majorana self-pairing exists *if and only if* the multiplet carries zero weight under the compact phase direction S^1_θ, and the trilinear LHN is the unique fiber-neutral portal touching the weight-zero sector linearly.

The physical content is worth saying in words, because it is the reason this framework has no freedom here. In the real presentation a weight-n sector is a pair of fields rotated by an angle $n\alpha$ as one goes around the compact fiber. The symmetric square of that pair splits into a rotation-invariant piece — the charge-conserving Dirac pairing — and a piece that rotates at *twice* the rate. For any $n \neq 0$, invariance kills the second piece outright. For $n = 0$ the fiber does nothing at all, and the reality condition that survives is precisely the Majorana condition.

Remark 16 (Physics intuition for the door)**.** Lepton number, in this framework, is not a bookkeeping label attached to particles by hand. It is a winding number: how many times a field's phase turns as one walks once around the compact sixth dimension. A quantity that counts turns cannot change by a continuous process, which is why every species that carries a nonzero winding conserves L exactly, and why the charged-lepton ledger is closed. The only place lepton number can leak is where there is no winding to count — the weight-zero sector. There is exactly one such door, and it is Majorana by construction rather than by assumption. That is the difference between this account and the usual one: the usual one *allows* Majorana neutrinos, and this one *permits nothing else.*

Relation to the historical seesaw. The heavy-Majorana route was written down three times independently in 1977–1980 (Minkowski, 1977; Gell-Mann et al., 1979; Yanagida, 1979; Mohapatra & Senjanović, 1980), and the effective operator that survives at low energy is Weinberg (1979)'s. The difference here is not the operator but its standing: in the historical construction the Majorana route is one admissible choice among several, and the heavy scale is free. Here the route is the only legal one and the scale is an output.

58.2 Three consequences that frame everything below

1. **The $\Delta L = 2$ structure is unique, not merely minimal.** It is common to describe the Weinberg operator as the minimal choice among several. Here there is nothing else to choose: the weight-zero self-pairing is the only legal $\Delta L = 2$ structure in the matter sector.

2. **The ultraviolet completion has no freedom either.** Once the portal is fixed, the heavy-sector realization is fixed with it. The question is not which mechanism but at which scale, and §59 answers that from quantities already derived.

3. **The prediction is one-sided and uncomfortable.** A Majorana neutrino sector implies neutrinoless double beta decay, and the parity $(0, \pi)$ this framework derives puts the effective mass at $m_{\beta\beta} \simeq 1.8$ meV — an order of magnitude below the reach of the approved program. This is stated in F-SM-3 and repeated here because it is the cost of the door being derived rather than chosen.

59 The heavy sector at the lock scale

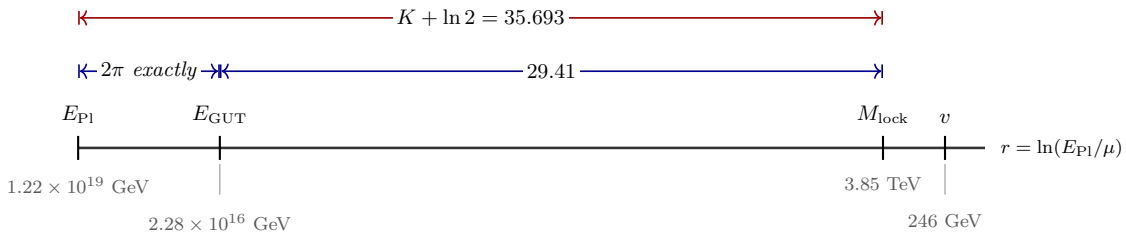

Figure 17: The radial descent, and the one integer that fixes it. The Planck scale, the grand-unification seam, the OGN-5 lock and the electroweak vev, placed on the coordinate $r = \ln(E_{\rm Pl}/\mu)$. Two spans are exact rather than fitted: $\ln(E_{\rm Pl}/E_{\rm GUT}) = 2\pi$, one traversal of the compact fiber, and $\ln(E_{\rm Pl}/M_{\rm lock}) = K + \ln 2 = 35.693$, since $M_{\rm lock} = \frac{1}{2} E_{\rm Pl} e^{-K}$. Everything Part VIII does downstream — the splitting $M_{\rm lock}/K$, the seam Yukawa, the exponent $e^{-K/2}$ — is read off the same $K = 35$, which was fixed by an anomaly audit that never mentions neutrinos.

59.1 The scale is not chosen

The lock scale is Part IV's, not this Part's. It is

$$M_{\rm lock} \;=\; \tfrac{1}{2}\, M_{\rm Pl}\, e^{-K} \;=\; \tfrac{1}{2}\, M_{\rm Pl}\, e^{-35} \;=\; 3.85 \text{ TeV}, \qquad r \;=\; \ln \frac{E_{\rm GUT}}{M_{\rm lock}} \;=\; 29.41, \tag{59.1}$$

where $K = LJ = M^2 + N^2 + L^2 = 35$ is the ledger integer fixed by the anomaly audit and the Green–Schwarz consistency of the six-dimensional bulk (GG-A3-BI6, Arisaka, 2026, A3-BI6). The same K sets the length of the radial descent.

It is worth being explicit about what that sentence does and does not claim. *No step in the derivation of* (59.1) *mentions baryogenesis, neutrinos, or CP.* The scale was fixed by an anomaly count. That it should also be the scale at which a matter asymmetry can be made is either a coincidence or a consequence, and the remainder of this Part is the attempt to find out which.

59.2 The splitting is the ledger's own level spacing

The heavy states inherit the lock scale with splittings of relative order $1/K$:

$$M_{N_i} \;=\; M_{\rm lock}\,(1+\delta_i), \qquad \delta_i = \mathcal{O}(1/K), \qquad \Delta M \;\simeq\; \frac{M_{\rm lock}}{K} \;=\; \frac{3850}{35} \;=\; 110\ \text{GeV}. \tag{59.2}$$

This is the single most consequential number in the Part after the lock scale itself, and it is not fitted: ΔM is the ledger's own level spacing, the same $K = 35$ divided into the same $M_{\rm lock}$.

59.3 The launch condition

Reheating after the inflationary phase deposits energy at $T_{\rm rh} \simeq 3 \times 10^{10}$ GeV (GG-A10-Cosmos, Arisaka, 2026, A10-Cosmos) — comfortably above the heavy sector, so that it is populated thermally without being asked to be, and comfortably below the GUT edge, so that monopoles stay diluted. The bath then cools, and at $t \simeq 1.6 \times 10^{-14}$ s it crosses the lock. The mechanism is therefore *selected by the launch* rather than chosen for it.

Remark 17 (Physics intuition for the scale)**.** Every treatment of thermal leptogenesis the authors are aware of *chooses* its scale: high, for the standard hierarchical mechanism, or low, when the author wants collider signatures. Here the scale is an output. The descent has one lock, the anomaly ledger fixes its depth at e^{-35}, and the heavy sector sits there because that is where the compact fiber closes. The reader is entitled to be suspicious of a framework that finds what it needs where it needs it — which is exactly why §66 states the one thing this chain assumes, in sharp form, with a number attached and a way to be wrong.

60 The seesaw at the lock, and the scale that cancels

The previous section placed a heavy sector at $M_{\rm lock}$. This one writes down what that sector *is* — a seesaw — and works the one identity that makes the placement admissible at all. It also says, at the end and in plain terms, why that identity is weaker evidence than it looks.

60.1 The heavy sector as a seesaw

The constitutional Majorana channel of Part VII is the dimension-five operator $(LH)(LH)/E_{\rm GUT}$. Any ultraviolet completion of that operator must reproduce its coefficient. The completion available here is the minimal one: three gauge-singlet Majorana states at the lock,

$$\mathcal{L} \;\supset\; -Y_{\nu\,\alpha i}\,\bar{L}_\alpha \tilde{H} N_i \;-\; \tfrac{1}{2} M_{N_i}\,\overline{N_i^c} N_i \;+\; \text{h.c.}, \qquad M_{N_i} \;=\; M_{\rm lock}(1+\delta_i), \quad \delta_i = \mathcal{O}(1/K), \tag{60.1}$$

with the splitting $\Delta M = M_{\rm lock}/K = 110$ GeV of (59.2). In the (ν_L, N_R^c) basis this is the familiar block matrix

$$\mathcal{M} \;=\; \begin{pmatrix} 0 & m_D \\ m_D^{\mathsf{T}} & M_N \end{pmatrix}, \qquad m_D \;=\; Y_\nu\, v, \tag{60.2}$$

whose light eigenvalue for $m_D \ll M_N$ is $m_\nu \simeq -\, m_D M_N^{-1} m_D^{\mathsf{T}}$.

The convention, pinned — and why. Equation (60.2) uses $m_D = Y_\nu v$ with $v = 246.33$ GeV, *not* $Y_\nu v/\sqrt{2}$. The pin is not arbitrary and it is not cosmetic: the closed form $Y_\nu^2 = \pi M_{\rm lock}/E_{\rm GUT}$ is exactly the value for which m_D^2/M_N reproduces the constitutional Weinberg scale $m_\star = \pi v^2/E_{\rm GUT} = 8.361$ meV, and it does so *only* under this normalization. Retiring the pin without also doubling Y_ν^2 breaks that identity; retiring it consistently means $Y_\nu^2 = 2\pi M_{\rm lock}/E_{\rm GUT}$ and changes no physical answer. It is stated here once and used everywhere in this Part and in the appendices.

60.2 The closure: the lock scale cancels

With one generation for clarity, the light mass is

$$m_\nu \;=\; \frac{m_D^2}{M_N} \;=\; \frac{Y_\nu^2 v^2}{M_N}. \tag{60.3}$$

Insert the seam Yukawa (62.1) and set $M_N = M_{\rm lock}$:

$$\boxed{\; m_\nu \;=\; \underbrace{\left(\pi\,\frac{M_{\rm lock}}{E_{\rm GUT}}\right)}_{Y_\nu^2} \cdot \frac{v^2}{M_{\rm lock}} \;=\; \pi\,\frac{v^2}{E_{\rm GUT}} \;=\; m_\star \;=\; 8.360835\ \text{meV}. \;} \tag{60.4}$$

The lock scale cancels identically. It appears once in the numerator, through the Yukawa, and once in the denominator, through the heavy mass, and it leaves no trace in the light spectrum. Numerically the two sides agree to every digit carried: $5.3034 \times 10^{-13} \times (246.33)^2/3849.0$ GeV $= 8.360835 \times 10^{-12}$ GeV, against $\pi(246.33)^2/2.280 \times 10^{16}$ GeV, the same number.

60.3 What the cancellation is worth — and what it is not

It would be easy to present (60.4) as a triumph. It is not one, and the Part is better served by saying why.

The identity is a consistency check, not a prediction. Y_ν was *fixed by matching* to the derived Weinberg coefficient in the first place. Read backwards, (60.4) is therefore automatic: for *any* heavy scale M_N there exists a Yukawa $Y_\nu^2 = \pi M_N/E_{\rm GUT}$ that reproduces $m_\star$ exactly. The light neutrino spectrum constrains only the ratio Y_ν^2/M_N, and carries **no information whatever** about where the heavy states sit. A reader who takes the boxed equation as evidence for a TeV-scale seesaw has read it backwards, and this paragraph exists to prevent that.

Where the content actually is. Three statements, in increasing order of what they are worth.

First: the scale is not chosen to fit. $M_{\rm lock}$ is fixed by $K = LJ = 35$ — an anomaly audit of the six-dimensional bulk that never mentions neutrinos, leptogenesis or CP. The framework does not select M_N to make (60.4) work; it reads M_N off elsewhere and then has no freedom left.

Second: the Yukawa is therefore a prediction, and it could have been absurd. Once $M_{\rm lock}$ is supplied, $Y_\nu = 7.28 \times 10^{-7}$ follows with nothing to adjust. It did not have to be a reasonable number. A lock ten orders lower would have demanded $Y_\nu \sim 10^{-12}$; a lock at the GUT seam would have demanded $Y_\nu \sim 1$ and no leptogenesis at the lock. What comes out is a coupling within a factor of four of the electron's own, $y_e = 2.9 \times 10^{-6}$ — ordinary smallness in a sector where the textbook seesaw demands either a coupling of order one or a scale twelve orders higher. *That* is the check that could have failed, and it is the one worth reporting.

Third: the Yukawa is what makes the rest of the Part parameter-free. Y_ν sets the width (62.2), the effective washout mass $\widetilde{m}$, the flavored washouts (67.2) and every rate in the Boltzmann system. In the standard treatment these are read off the Casas–Ibarra matrix and are free (§J.1.6); here there is no R to choose.

And the cancellation is also a limitation. Because $M_{\rm lock}$ drops out of (60.4), **no neutrino measurement can confirm or refute the lock-scale seesaw.** Not the splittings, not the angles, not Σm_ν, not $m_{\beta\beta}$ — all are computed from $m_\star$ and the projector geometry, and all are blind to M_N. The claim is testable only through the chain that follows, which is why F-SM-7 is written against a calculation rather than against a spectrum, and why this Part cannot borrow the neutrino block's evidence.

60.4 Where this sits, against the two standard anchors

Table 43: The heavy Majorana scale in three readings. The first row is the textbook choice, motivated by nothing stronger than the top quark's Yukawa being of order one; the second is the floor that reproducing the observed asymmetry imposes in the hierarchical regime; the third is this framework's, fixed upstream by the ledger integer $K = 35$. The last column is the Yukawa each reading requires, and it is the honest discriminator between them. All three rows are matched to the *same* light mass, $m_\star = 8.36$ meV; a reading matched instead to m_3, as in the worked example of §J.1.6, moves every M_N down by the ratio of the two.

Reading	M_N	Y_ν	**Status of the scale**
Textbook seesaw, $Y_\nu \sim 1$	7.3×10^{15} GeV	~ 1	assumed, from the top
Davidson–Ibarra floor, hierarchical	$\gtrsim 6 \times 10^{8}$ GeV	$\sim 3 \times 10^{-4}$	a bound, not a value
This framework: $M_N = M_{\rm lock}$	$\mathbf{3.85 \times 10^{3}}$ **GeV**	$\mathbf{7.28 \times 10^{-7}}$	derived from $K = 35$; $Y^{\rm L} = \sqrt{2}\, Y_\nu = 1.03 \times 10^{-6}$ like-for-like

The lock sits 5.2 orders of magnitude below the Davidson–Ibarra floor and 12.3 below the $Y_\nu \sim 1$ seesaw. The first of those gaps is the subject of the next section: the floor assumes a hierarchical heavy spectrum, and this one is not hierarchical.

60.5 What the TeV Majorana state does *not* do

A heavy neutrino at a collider-accessible mass invites an expectation this Part must decline explicitly. The active–sterile mixing angle is

$$\theta \;=\; \frac{m_D}{M_N} \;=\; \frac{Y_\nu v}{M_{\rm lock}} \;=\; 4.66\times10^{-8}, \qquad |\theta|^2 \;=\; 2.2\times10^{-15}, \tag{60.5}$$

with $m_D = Y_\nu v = 179$ keV. Against the most optimistic reach of any proposed direct search or lepton-number-violation experiment, $|\theta|^2 \gtrsim 10^{-6}$, that is short by nearly *nine orders of magnitude*. The heavy triplet contributes nothing measurable to $0\nu\beta\beta$, to lepton-flavor violation, or to any collider final state.

Remark 18 (Two tenants at one address)**.** The number 3.85 TeV appears twice in this cohort and the two appearances must not be conflated. The Kaluza–Klein cluster of GG-P1-KK, which F-SM-1 tests at the LHC, is a *gauge-sector* object at the lock with ordinary couplings and a genuine discovery prospect. The Majorana triplet of this Part is a *singlet* sector at the same lock, with a Yukawa of 10^{-7} and a mixing of 10^{-15}, and it is invisible to every experiment now conceived. Same address, two tenants, one visible and one not. A reader who hears "TeV-scale seesaw" and expects a collider signature has merged them.

60.6 What is new here, stated exactly

TeV-scale leptogenesis is not itself new. Resonant leptogenesis at the TeV scale is Pilaftsis (1997) and Pilaftsis & Underwood (2004), cited in the next section and used there; lower-scale mechanisms exist below that. Nothing in this Part claims priority for the scale.

What is new is that *nothing is chosen.* In the resonant literature the heavy mass is a model parameter, the degeneracy $\Delta M/M$ is arranged — typically to a part in 10^{10} or finer, with no reason offered — and the Yukawa is fitted through the Casas–Ibarra matrix. Here the scale is $\frac{1}{2}E_{\rm Pl}e^{-K}$, the splitting is $M_{\rm lock}/K$, and the Yukawa is $\sqrt{\pi M_{\rm lock}/E_{\rm GUT}}$: three quantities, one integer, and that integer was fixed by an anomaly audit before any of this was asked of it. *The claim is a derived TeV-scale seesaw, not a first one.*

And it does not, by itself, work. The next section shows that the derived splitting and the derived width miss the resonant condition by twelve orders of magnitude, and §66 states what would have to be true instead. Nothing in the present section establishes the mechanism; it establishes only that the seesaw is consistent, that the Yukawa it demands is of an ordinary size, and that there is no freedom left anywhere to repair what comes next.

61 The Davidson–Ibarra wall, and why this scale evades it

61.1 The wall

There is a reason nobody puts thermal leptogenesis at four TeV. For hierarchical heavy neutrinos the CP asymmetry per decay is bounded above by the light and heavy masses (Davidson & Ibarra,

2002):

$$|\epsilon_1| \lesssim \frac{3}{8\pi}\frac{M_1\, m_3}{\bar{v}^2}, \qquad \bar{v} = 174 \text{ GeV}, \tag{61.1}$$

so demanding enough asymmetry to make the observed universe forces

$$M_N \gtrsim 10^9 \text{ GeV}. \tag{61.2}$$

Nine orders of magnitude above anything a collider will reach. Standard thermal leptogenesis is, by its own arithmetic, permanently unfalsifiable in the laboratory — a fact usually noted and then set aside.

The bound is easy to read physically. The asymmetry comes from interference between the tree-level decay and a loop; the loop is suppressed by the Yukawa couplings, and the seesaw ties those Yukawas to the light neutrino masses divided by the heavy mass. Push the heavy mass down and the Yukawas must come down with it, and the asymmetry falls linearly. Equation (61.1) is that trade written out.

61.2 The escape, and why it is forced here rather than assumed

The known escape is *resonant* leptogenesis (Pilaftsis, 1997; Pilaftsis & Underwood, 2004). If two heavy states are quasi-degenerate, the self-energy contribution to the decay asymmetry is resonantly enhanced, and the Yukawa suppression cancels between numerator and denominator:

$$\epsilon \propto \frac{2\,\Delta M\,\Gamma}{4\Delta M^2 + \Gamma^2}\,\sin(2\phi) \qquad \xrightarrow{\Delta M = \Gamma/2} \qquad \tfrac{1}{2}\sin(2\phi). \tag{61.3}$$

On exact resonance the asymmetry is of order unity no matter how small the Yukawas are, and the bound (61.2) evaporates.

In the general literature quasi-degeneracy is an assumption, and usually an uncomfortable one: two masses are required to agree to a part in 10^{10} or better with no reason offered. Here the degeneracy is not assumed. The three heavy states are one multiplet at the lock, split by $\Delta M = M_{\text{lock}}/K$ from (59.2) — a relative splitting of $1/35$, which is not fine at all. Whether that is *degenerate enough* is precisely the resonance question, and it is faced in §66 rather than assumed away.

62 The seam Yukawa and the vacuum width

62.1 The Yukawa is fixed by the seam, with no freedom

The neutrino Yukawa is not a free coupling in this framework. The seesaw of §60, matched to the seam coefficient $2\pi/E_{\text{GUT}}$ of the Weinberg operator (50.3) that Part VII derived, fixes it — exactly, in the convention pinned there:

$$Y_\nu^2 = \pi\frac{M_{\text{lock}}}{E_{\text{GUT}}} = \pi\frac{3.85\times10^3 \text{ GeV}}{2.28\times10^{16} \text{ GeV}} = 5.30\times10^{-13}, \qquad Y_\nu \simeq 7.28\times10^{-7}. \tag{62.1}$$

It is worth pausing on that number rather than passing it. A neutrino Yukawa of 7×10^{-7} looks like a tuned small number until it is put beside the electron's, $y_e = 2.9 \times 10^{-6}$. It is within a factor of four. The framework is not asking for a coupling of unusual smallness; it is asking for one of ordinary smallness, in a sector where the usual seesaw would demand something far more extreme or a far higher scale.

62.2 The vacuum width, and the first sign of trouble

The decay width that follows is

$$\Gamma_N \;=\; \frac{(\sqrt{2}\,Y_\nu)^2 M_N}{8\pi} \;=\; \frac{Y_\nu^2 M_N}{4\pi} \;=\; 1.62 \times 10^{-10}\ \text{GeV}, \qquad \frac{\Gamma_N}{M_N} \;=\; 4.2 \times 10^{-14}. \tag{62.2}$$

Why the Yukawa carries 2π. Written in the standard normalization $m_D = Y_\nu^{\rm L}\bar{v}$, the seam Yukawa is

$$(Y_\nu^{\rm L})^2 = \frac{2\pi\, M_{\rm lock}}{E_{\rm GUT}},$$

and the seesaw $m_\nu = m_D^2/M_N = (Y_\nu^{\rm L})^2\bar{v}^2/M_{\rm lock} = 2\pi\bar{v}^2/E_{\rm GUT} = \pi v^2/E_{\rm GUT} = m_\star$ reproduces the Weinberg scale with no factor to discharge: the single-traversal coefficient 2π of the Weinberg operator and the two Dirac insertions of the seesaw are the same object counted once. The pinned form $Y_\nu^2 = \pi M_{\rm lock}/E_{\rm GUT}$ of the seesaw closed form is this coupling divided by two, and the width $\Gamma_N = (Y_\nu^{\rm L})^2 M_N/8\pi = 1.62 \times 10^{-10}$ GeV follows without a footnote. Set the width beside the splitting $\Delta M = 110$ GeV from (59.2). The ratio is

$$\frac{\Delta M}{\Gamma_N} \;=\; \frac{110}{1.62 \times 10^{-10}} \;\simeq\; 6.8 \times 10^{11}. \tag{62.3}$$

The resonance condition of (61.3) wants this ratio near unity. It is off by nearly twelve orders of magnitude. Taken at face value the chain is dead on arrival, undershooting the observed asymmetry by seven or eight orders.

This is not a difficulty to be buried in an appendix; it is the central technical question of the Part, and §66 is devoted to it. The short statement of what follows is that Γ_N — the *vacuum* width of the seam channel's heavy state (the $2N$ channel's is $2N$ times larger, and nothing below moves) — is the wrong quantity to put in the denominator of a resonance formula evaluated in a hot plasma, and that identifying the right one is both the chain's best claim and its weakest link.

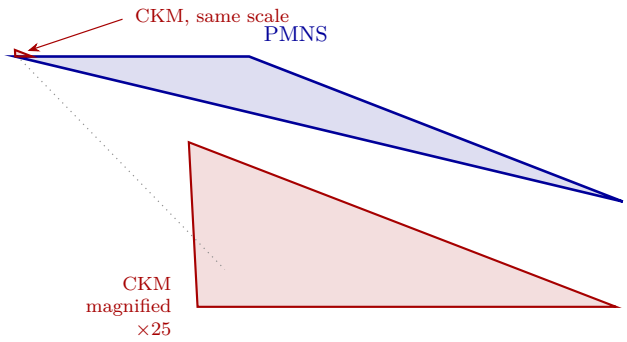

Figure 18: The two unitarity triangles at true relative area. Both are drawn from the *derived* mixing matrices at one common scale, so the areas may be compared directly: twice the area of each is its Jarlskog invariant, and $|J_{\rm PMNS}|/|J_{\rm CKM}| = 9.187\times10^{-3}/2.984\times10^{-5} = 308$. All six unitarity triangles of a given matrix have the same area, so the choice of index pair affects only the shape; each is drawn in its best-proportioned pair, and the CKM shape is scaled by six percent in linear size so that its area equals the codebook's own $J_{\rm CKM}$. *The caveat travels with the picture*: three hundred is not ten orders of magnitude, so this ratio is not what lets the lepton sector pay for baryogenesis when the quark sector cannot — and §63 finds that the unflavored invariant $J_{\rm PMNS}$ belongs to drops out of the asymmetry entirely.

63 The CP asymmetry: flavored, and why the unflavored one vanishes

63.1 What the asymmetry is

The quantity the chain needs is the fractional excess of leptons over antileptons produced when one heavy state decays,

$$\epsilon_i \;\equiv\; \frac{\Gamma(N_i\to\ell H)-\Gamma(N_i\to\bar\ell H^*)}{\Gamma(N_i\to\ell H)+\Gamma(N_i\to\bar\ell H^*)}, \tag{63.1}$$

a pure number between -1 and $+1$. If it vanishes the decays are perfectly even-handed and no asymmetry is made at all. If it is 10^{-6}, then for every million heavy states that decay there is one extra lepton left over. The target value for this chain, established in §71, is $\epsilon^{\rm res}\simeq 2.5\times10^{-6}$ — an *adopted* value, back-solved from the throughput identity rather than computed from the resonance (§71); roughly one extra lepton per four hundred thousand decays.

63.2 Where the asymmetry comes from: the one-loop computation

Equation (63.1) defines the asymmetry; it does not say why one exists. This subsection derives it, because the structure of the answer is what makes the resonant enhancement possible and what makes the Davidson–Ibarra bound escapable.

Why a tree amplitude cannot do it. At tree level the decay $N_i\to\ell_\alpha H$ has amplitude proportional to the Yukawa $(Y_\nu)_{i\alpha}$, and the conjugate decay $N_i\to\bar\ell_\alpha H^*$ to $(Y_\nu)^*_{i\alpha}$. The rates go as the modulus squared, so both are $|(Y_\nu)_{i\alpha}|^2$ and the asymmetry is exactly zero. A phase alone is

not enough: to make a rate difference the phase must be multiplied by something that does *not* conjugate with it. That something is an absorptive part.

The two interferences. At one loop there are two corrections that carry an absorptive part, and they behave completely differently here.

The *vertex* (direct) correction dresses the decay with a loop in which a lepton and a Higgs are exchanged and the intermediate heavy state N_j goes on shell. Interfering it with the tree amplitude gives an asymmetry proportional to

$$\epsilon_i^{\text{vertex}} \;\propto\; \sum_{j\neq i} \mathrm{Im}\big[(Y_\nu Y_\nu^\dagger)^2_{ij}\big]\, f\!\left(\frac{M_j^2}{M_i^2}\right), \qquad f(x) \xrightarrow{x\gg 1} -\frac{1}{\sqrt{x}}, \tag{63.2}$$

which is suppressed by the heavy-mass ratio and is what the Davidson–Ibarra bound (61.1) actually bounds.

The *self-energy* (indirect) correction is a different object. The same loop, inserted on the external N_i line, mixes N_i with N_j before it decays. Its contribution to the asymmetry carries the propagator of the state being mixed into, and therefore a denominator:

$$\epsilon_i^{\text{self}} \;\propto\; \sum_{j\neq i} \mathrm{Im}\big[(Y_\nu Y_\nu^\dagger)^2_{ij}\big]\, \frac{M_i M_j\,(M_i^2 - M_j^2)}{(M_i^2 - M_j^2)^2 + (M_i \Gamma_j)^2}. \tag{63.3}$$

That denominator is the whole story. When the two heavy masses are far apart it reduces to the same $1/\sqrt{x}$ suppression as the vertex term and nothing is gained. When they approach each other it does not: the numerator falls linearly in the splitting while the denominator falls quadratically, so the ratio *grows* until the width term takes over.

Reduction to the Breit–Wigner form. Write $M_i^2 - M_j^2 \simeq 2M\,\Delta M$ with M the common scale, and $M_i\Gamma_j \simeq M\,\Gamma^{\text{eff}}$. Then (63.3) becomes

$$\frac{M^2 \cdot 2M\Delta M}{4M^2\Delta M^2 + M^2(\Gamma^{\text{eff}})^2} \;\propto\; \frac{2\,\Delta M\,\Gamma^{\text{eff}}}{4\Delta M^2 + (\Gamma^{\text{eff}})^2} \cdot \frac{2M}{\Gamma^{\text{eff}}}, \tag{63.4}$$

and the first factor is precisely the envelope F_{BW} of (66.1), whose maximum is $\tfrac{1}{2}$, attained at $\Delta M = \Gamma^{\text{eff}}/2$. This is where the resonant form used throughout this Part comes from, and it is why the enhancement is bounded: no choice of parameters makes F_{BW} exceed one half.

The mixing angle, and what is being resummed. Equivalently, the two quasi-degenerate states are not mass eigenstates of the full propagator. Diagonalizing the resummed 2×2 inverse propagator mixes them through an angle θ_{mix} with

$$\tan 2\theta_{\text{mix}} \;=\; \frac{2\,\Delta M}{\Gamma_1 + \Gamma_2}, \tag{63.5}$$

so the physical decaying states are superpositions whose lepton-flavor content differs from the mass basis. The asymmetry is then the ordinary statement that a superposition of two states with different couplings and a relative phase decays asymmetrically. Equation (63.5) also shows what the resonance condition means physically: maximal mixing, $\theta_{\rm mix} \to \pi/4$, is approached when the splitting is large compared with the mean width.

Remark 19 (Physics intuition for the two loops)**.** The vertex diagram asks a decaying particle to interfere with a version of itself that has been round a loop. The self-energy diagram asks something different: it lets the particle *become the other particle* and decay as that. When the two are nearly degenerate, becoming the other one is nearly free, and the second process is unsuppressed while the first is not. This is the whole reason resonant leptogenesis evades a bound derived for the hierarchical case: the Davidson–Ibarra argument bounds (63.2) and says nothing about (63.3) when the denominator is small.

Remark 20 (What this subsection does *not* settle)**.** Everything above is a vacuum-amplitude computation: the widths in (63.3) are the widths of free heavy states, and the $\Gamma^{\rm eff}$ that emerges from the reduction is Γ_N. Carried literally into the lock-scale chain it gives the fatal N-width branch of §66, row (a) of Table 44, short of observation by seven or eight orders. The claim of this Part is that in a hot plasma the quantity in that denominator is not the vacuum width, and *that* is condition C4, which remains open. Deriving the vacuum form more carefully does not close it, and this subsection is not offered as progress on it.

63.3 The unflavored invariant vanishes identically

Here the framework does something that looks at first like a disaster and turns out to be the mechanism.

The CP source in the transport equations is a matrix in lepton flavor. Its *trace* — the flavor-summed, or unflavored, invariant — is the quantity that appears in the textbook one-flavor treatment. Written in the Casas–Ibarra parameterization (Casas & Ibarra, 2001), in which the Yukawa matrix is decomposed into the light masses, the PMNS matrix and an orthogonal matrix R, the unflavored invariant is $U_{\rm PMNS}$-*independent*: the mixing matrix cancels out of it entirely. Worse, for the dial-free door of §58 — which admits no complex R, because there is no continuum of orientations to choose from — it vanishes exactly:

$$\operatorname{Tr} S \;=\; \sum_{\alpha} S_{\alpha\alpha} \;\propto\; \operatorname{Im}\big[(Y_\nu Y_\nu^\dagger)^2_{ij}\big] \;=\; 0 \qquad (R \text{ real}). \tag{63.6}$$

This is Lemma L-Cosmos-U of GG-A10-Cosmos (Arisaka, 2026, A10-Cosmos), where it is proved and machine-checked; it is quoted here because it is what forces everything that follows.

Lemma 63.1 (Flavor-odd source)**.** *On the dial-free door, the flavor-summed CP asymmetry vanishes identically, while the individual flavored asymmetries do not:*

$$\sum_{\alpha} \epsilon_\alpha \;=\; 0, \qquad \epsilon_\alpha \neq 0 \;\; \textit{for each } \alpha. \tag{63.7}$$

The entire CP content therefore lives in flavor differences.

Remark 21 (Physics intuition for a source that sums to zero)**.** Read (63.7) as a bookkeeping statement about three ledgers that must balance. The decays make an excess of electron-flavor leptons, an excess of muon-flavor leptons, and a deficit of tau-flavor ones, in exactly compensating amounts. If nothing distinguished the three afterwards, the three would cancel and the universe would be empty. What saves it is that the three flavors are *erased at different rates* — and the rates are set by how strongly each flavor couples to the heavy states, which is set by the PMNS matrix, which this paper derived. No spread in the erasure, no matter. This is why the chain must be flavored, and it is why the reactor angle turns out to matter to cosmology at all (§67).

63.4 What this costs and what it buys

The cost is that the simple, widely used one-flavor estimate is not merely inaccurate here — it returns exactly zero, and any treatment that stops there would conclude the framework cannot make matter. The gain is that the surviving asymmetry is then a statement about *differences* between flavors, and differences between flavors are precisely what a derived PMNS matrix is a statement about. A framework that had fitted its mixing angles would gain nothing from this structure. One that derived them inherits a prediction.

64 The half-angle: why a parity of π sets a sign

64.1 The apparent paradox

Part VII derived the Majorana phases as parity data rather than as dials:

$$(\alpha_{21},\ \alpha_{31}) \ = \ (0,\ \pi). \tag{64.1}$$

The constitutionally forced rank-two Majorana block fixes one state of each intrinsic parity, so the two surviving heavy states are a *parity pair*, not a phase continuum: 0 means same parity, π means opposite, and there is no third value to choose.

Now the paradox. A phase of π is real. On the unit circle it is the point -1. Complex conjugation — which is what CP does to a phase — leaves it exactly where it is. A quantity fixed at π looks like the most CP-*even* letter available, and it is fair to ask how such a thing can possibly orient a matter–antimatter asymmetry.

64.2 The resolution

A Majorana letter is read at *half angle*. The door's weight-zero pairing is a $\Delta L = 2$ bilinear, quadratic in the field, so the phase enters amplitudes through $e^{i\alpha/2}$ and not through $e^{i\alpha}$. Therefore

$$\boxed{\alpha_{31} = \pi \qquad \Longrightarrow \qquad e^{\,i\alpha_{31}/2} \ = \ e^{\,i\pi/2} \ = \ i.} \tag{64.2}$$

A quarter turn — and here the argument must be stated carefully, because the tempting next sentence is wrong. This factor of i is *kinematics, not CP violation.* In the standard classification a Majorana phase of exactly π — which is exactly this factor of i — is the signature of *opposite intrinsic CP parity* between the mass eigenstates, and opposite intrinsic parity is a CP-*conserving* configuration. The half angle tells us what the letter does to an amplitude. It does not tell us that CP is broken.

Where the resolution actually lies. The textbook statement "Majorana phases of 0 and π conserve CP" is a statement about the *flavor-summed* asymmetry — and that sum does vanish here, exactly. The individual flavored asymmetries do not. The pattern this Part integrates, computed on the ledger inputs at the Breit–Wigner peak (§68, Eq. (68.3)), is

$$\bar{\epsilon}_e : \bar{\epsilon}_\mu : \bar{\epsilon}_\tau \;=\; +3.4 : +3.1 : -6.6 \;\; (\times 10^{-3}), \qquad \sum_\alpha \bar{\epsilon}_\alpha \;=\; 0, \tag{64.3}$$

where the zero is exact (the two-figure values as printed sum to -0.1×10^{-3}, which is their rounding): it is column orthogonality of the mixing matrix, $\sum_\alpha V_{\alpha 1} V_{\alpha 3} = 0$ — not dynamics, which is why it is exact, and why it holds for any completion of the derived masses and phases by a real orthogonal Casas–Ibarra matrix. A one-loop check at the derived phases with a generic such completion returns a sum sixteen orders of magnitude below the individual terms, and it also settles which letter does the work: turning the parity off, $\alpha_{31} \to 0$, reduces the magnitude of the flavored asymmetries by a factor of twenty, while moving the Dirac phase to π changes it by two parts in a thousand.

$$\boxed{\alpha_{31} = \pi \text{ is CP-conserving in the flavor sum, and CP-violating in the flavor differences.}} \tag{64.4}$$

A zero-sum set of flavored asymmetries is not a zero asymmetry, because the three flavors are not erased at the same rate; §67 carries this out and finds the whole of the survivor in the electron flavor — its ledger ends holding 106% of the net, the muon's 3% and the tau's −10% — for the reason recorded there. This is also why a flavored treatment is *mandatory* here rather than merely more careful: an unflavored calculation of this chain returns exactly zero.

Remark 22 (Physics intuition for the half angle)**.** The factor of two is not a technicality; it is the signature of the door itself. A Dirac mass term pairs a field with its conjugate, and the phases cancel between them — which is why a Dirac phase can always be rotated away unless three generations conspire to prevent it. A Majorana mass term pairs a field with *itself*, so the phase appears twice and only half of it can be removed. What remains is the half angle. The observation that a Majorana neutrino's phase is physically halved is older than this framework; what is new here is that the half being kept is not a free parameter but a parity. What that parity then does is a separate question, and it is answered by the flavor structure rather than by the factor of i: the parity makes the three flavored asymmetries individually nonzero while forcing their sum to vanish, and the unequal washout of the three flavors converts that zero-sum set into a net excess.

64.3 Consequence: the parity sets the sign of the matter excess

Within the flavored chain the driver of the asymmetry is α_{31}. Sending $\alpha_{31} \to 0$ — the other admissible parity — flips the sign of the flavored asymmetries and therefore of the final baryon number: the same universe, made of antimatter. That statement should be read with the grade attached, since it holds within a chain whose magnitude is conditional (§66); but the sign is carried by a discrete datum of this paper's own codebook, and that is worth stating plainly.

Remark 23 (What the $0\nu\beta\beta$ program is really testing)**.** Row 23 and Row 24 of the codebook, $(\alpha_{21}, \alpha_{31}) = (0, \pi)$, are the only two entries of the twenty-eight that are at present entirely unmeasured. It is tempting to treat them as decorative. Within this chain they are not: the parity assignment $(0, \pi)$ is what makes the universe matter rather than antimatter. A neutrinoless-double-beta experiment sensitive to the Majorana phases is, in this reading, measuring the sign of the cosmic matter excess. That is a large claim resting on a conditional chain, and §75 says exactly how it could fail — but it is why F-SM-3b is worth its place in the falsification map even though the predicted $m_{\beta\beta} \simeq 1.8$ meV sits below the approved program's reach.

65 Why the Dirac phase is a spectator

The reader who has followed Part VII will expect the leptonic Dirac phase to be the star of this Part. It is not, and the reason is worth stating carefully, because it is the most counter-intuitive claim in the chain.

Part VII derived

$$\delta_{\rm PMNS} \;=\; \pi + 2\arcsin\frac{1}{7} \;=\; 3.42829 \text{ rad} \;=\; 196.43^\circ, \qquad \delta^{\rm QD}_{\rm PMNS} \;=\; 196.06^\circ, \tag{65.1}$$

and §55.1 showed it sitting about seven degrees from the world best value. It is the framework's sharpest leptonic CP prediction. And it contributes essentially nothing to the baryon asymmetry.

The reason is not that it is small — it is of order one. It is not that it lives at the wrong scale — it does not. It is that **it is the wrong letter**. The unflavored invariant that a Dirac phase could enter is $U_{\rm PMNS}$-independent by (63.6) and vanishes besides; and in the flavored asymmetries the numerical dependence on $\delta_{\rm PMNS}$ is at the few-per-cent level, while the dependence on α_{31} is a sign. Sending $\delta_{\rm PMNS} \to \pi$ shifts the result by a few percent. Sending $\alpha_{31} \to 0$ inverts it.

Remark 24 (Two CP-violating phases, two different jobs)**.** Oscillation experiments measure the Dirac phase, because oscillation is a first-order interference in the amplitude and the Dirac phase is what survives there. Neutrinoless double beta decay and the matter excess are sensitive to the Majorana parities, because both are $\Delta L = 2$ processes and the parities are what survive *there*. The two programs are not competing measurements of one quantity; they are measurements of two different quantities that happen to sit in the same matrix. DUNE and Hyper-Kamiokande will test (65.1), and a tonne-scale $0\nu\beta\beta$ experiment would test (64.1), and neither substitutes for the other.

A corollary worth registering, because it is the sort of thing that is easy to overclaim: the excellent agreement of (65.1) with the accelerator joint fit is *not* evidence for this Part. It is evidence

for the lepton block of Part VII. The chain below rests on a different letter, and it must be judged on its own.

66 The resonance question, faced

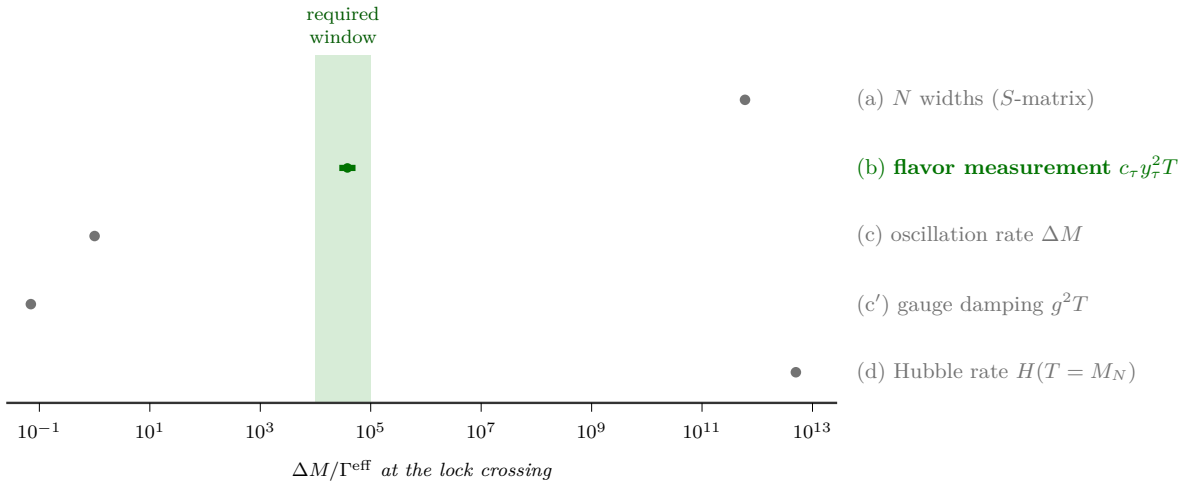

Figure 19: The regulator landscape of Table 44, drawn. Five candidate prescriptions for $\Gamma^{\rm eff}$ at the lock crossing, spanning sixteen orders of magnitude in $\Delta M/\Gamma^{\rm eff}$; the shaded band is the decade the assembly requires. Exactly one candidate lands in it. *The figure is also the argument against itself*: five candidates spread over sixteen orders sit about three decades apart, so one of them falling inside the required decade is a *selection*, not a derivation — which is precisely why condition C4 is carried as open (§66) and why F-SM-7 is written against a calculation rather than a facility.

Every resonant-leptogenesis calculation contains a Breit–Wigner factor

$$F_{\rm BW} \;=\; \frac{2\,\Delta M\,\Gamma^{\rm eff}}{4\Delta M^2 + (\Gamma^{\rm eff})^2} \;=\; \frac{\Delta M\,\Gamma^{\rm eff}}{2\,[\Delta M^2 + (\Gamma^{\rm eff}/2)^2]}, \qquad F_{\rm BW} \le \tfrac{1}{2} \;\; (\text{at } \Delta M = \Gamma^{\rm eff}/2), \tag{66.1}$$

and the whole magnitude of the asymmetry rides on it. This is the one form every number in this Part uses — it is the $R(z)$ of (68.3) — and far off resonance it is $1/(2x)$ with $x = \Delta M/\Gamma^{\rm eff}$. Here the splitting is *known* — it is the ledger's own level spacing, $\Delta M = M_{\rm lock}/K = 110$ GeV from (59.2). So everything reduces to a single question:

What is $\Gamma^{\rm eff}$*? Which physical rate regulates the resonance — that is, how quickly is the quantum interference that builds the CP asymmetry destroyed?*

66.1 Both obvious answers are fatal

It is worth establishing at once that this is a real question with a narrow answer, because the two candidates a reader would reach for first both fail catastrophically.

The vacuum width. Putting $\Gamma^{\rm eff} = \Gamma_N$ from (62.2) gives the ratio (62.3) of 6.8×10^{11}, a Breit–Wigner factor of order 10^{-12}, and an η_B short of observation by seven or eight orders of magnitude. This branch is fatal, and the framework says so.

Exact resonance. Putting $\Delta M \simeq \Gamma^{\rm eff}/2$ gives $F_{\rm BW} \simeq \frac{1}{2}$, an asymmetry of order unity, and an overshoot of the observed value by some four orders of magnitude. This branch is fatal too.

The required answer therefore lies in a narrow window between the two, and the chain's claim is that a specific physical rate lands in it.

66.2 The candidate: the bath as a flavor-measuring apparatus

The argument is GG-A10-Cosmos's (Arisaka, 2026, A10-Cosmos, Appendix I), and runs as follows.

The interference that builds the asymmetry is between two heavy states that differ in *flavor* composition. What destroys such an interference is not any fast process but specifically a process that *measures* flavor. The gauge interactions are enormously the fastest thing in the plasma — $g^2 T \simeq 1.5 \times 10^3$ GeV at the lock — but they are flavor-blind. They cannot measure what they cannot distinguish, and formally they cancel: the decohering action of a thermal rate on a flavor matrix is its double commutator with the projector that rate measures, and for a flavor-universal rate that projector is the identity, so

$$[\mathbf{1}, [\mathbf{1}, \delta n_\ell]] = 0 \quad \text{exactly, for every } \delta n_\ell; \qquad [P_\tau, [P_\tau, \delta n_\ell]] = \delta n_\ell \quad \text{on pure tau coherences.} \tag{66.2}$$

Gauge damping drops out of the flavor-odd channel *however large it is* — which is why the biggest rate in the plasma is not the answer.

The fastest *honest* measurer of lepton flavor at $T \simeq 4$ TeV is the tau Yukawa interaction; the muon and electron rates are down from it by $(y_\mu/y_\tau)^2 \simeq 3 \times 10^{-3}$ and below. So

$$\Gamma^{\rm eff} \;=\; \Gamma_\tau \;=\; c_\tau \, y_\tau^2 \, T, \qquad c_\tau = 5 \times 10^{-3}\text{–}10^{-2}, \tag{66.3}$$

with c_τ the kinetic coefficient of the flavor-equilibration literature.

66.3 The two-level reduction

Reducing the heavy pair to a two-level system, the CP-odd weight accumulated by an interference oscillating at ΔM while its flavor coherence is measured at rate $\Gamma_{\rm fl}$ is

$$I(\Gamma_{\rm fl}) \;=\; \Gamma_{\rm fl} \int_0^\infty \! {\rm d}s \; e^{-\Gamma_{\rm fl} s} \, \sin(\Delta M \, s) \;=\; \frac{\Gamma_{\rm fl} \, \Delta M}{\Gamma_{\rm fl}^2 + \Delta M^2} \;\le\; \tfrac{1}{2}. \tag{66.4}$$

This is worth reading as physics rather than algebra: the interference is a sine wave in proper time, the measurement is an exponential cutoff, and the accumulated asymmetry is the Laplace transform of one against the other. The bound $\frac{1}{2}$ is saturated only on exact resonance; and since a coherence damps at half the width, $\Gamma_{\rm fl} = \Gamma^{\rm eff}/2$, this integral is exactly $F_{\rm BW}$ of (66.1).

66.4 The closed form, in which the heavy mass cancels

Now the step that makes this a statement about *this* framework rather than a generic one. Evaluate at the generation temperature $T \simeq M_N$. The splitting is $\Delta M = M_N/K$ and the rate is $\Gamma_\tau = c_\tau y_\tau^2 T$; the heavy mass cancels between them, leaving a ledger integer and a Standard-Model Yukawa:

$$\boxed{\frac{\Delta M}{\Gamma^{\rm eff}} = \frac{M_N/K}{c_\tau y_\tau^2 T}\Big|_{T=M_N} = \frac{1}{K\, c_\tau\, y_\tau^2} = \frac{1}{35\, c_\tau\, y_\tau^2} = (2.7\text{–}5.3)\times 10^4.} \tag{66.5}$$

No dial of this paper's own: the inputs are the ledger integer $K = 35$, the running tau Yukawa, and the Standard-Model kinetic coefficient c_τ, carried at its literature range $(5\text{–}10)\times 10^{-3}$ — a factor of two the chain does not fix. With one-loop running, $y_\tau(3.85\ \text{TeV}) = 0.0104$, so

$$\Gamma^{\rm eff} = c_\tau y_\tau^2 M_N = 2.1\text{–}4.1\ \text{MeV}, \qquad F_{\rm BW} = (0.9\text{–}1.9)\times 10^{-5}. \tag{66.6}$$

Remark 25 (Physics intuition for the cancellation)**.** The reason (66.5) is worth a box is that both of its ingredients scale with the heavy mass and the ratio does not. Raise the lock scale and the splitting rises in proportion; the temperature at which the decays happen rises with it, and so does the tau rate. The window is therefore not a coincidence of where the lock happens to sit — it is a property of the ledger integer K and of a Standard-Model coupling, and it would be the same number at a different lock scale. That is a stronger statement than the chain needs, and it is the reason the identification is worth taking seriously even though it is not yet derived.

66.5 The hypothesis, in sharp form

Remark 26 (The named hypothesis H-$\phi_{\rm geo}$-1 (partial resonance))**.** *There exists a finite-temperature transport computation of the lock-crossing decays in which the effective splitting-to-width ratio at the crossing satisfies* $\Delta M/\Gamma^{\rm eff} \simeq 3\times 10^4$ *— equivalently, a Breit–Wigner factor* $F_{\rm BW} \simeq 1.7\times 10^{-5}$*, with* $\Gamma^{\rm eff} \simeq 3$ *MeV at* $\Delta M \simeq 110$ *GeV — so that the flavored, real-R throughput on the inherited phases lands within* $\mathcal{O}(1)$ *of* $e^{-K/2}$*.*

The name decodes as **H** for hypothesis, $\boldsymbol{\phi_{\rm geo}}$ for the geometric phase whose projection into the flavored asymmetries the hypothesis is about, and **1** because it is the first such named item in the chain's promotion roadmap. It is not a free parameter and not a fudge factor: it is a *prediction about the outcome of a calculation nobody has yet done*, stated with a number and an acceptance band, and it can be wrong. Its home is GG-A10-Cosmos, Appendix I.

The target of H-$\phi_{\rm geo}$-1 sits inside the band of (66.6) and is reproduced exactly at $c_\tau = 8.9\times 10^{-3}$, within the literature range of (66.3). Two consistency checks: $\Gamma_\tau/H = (1\text{–}2)\times 10^8$, so the chain is fully flavored with eight orders of margin; and $\Gamma_\tau/\Delta M \simeq 3\times 10^{-5} \ll 1$, so the regime is genuinely *partial* resonance and not the excluded peak.

66.6 The regulator landscape

The reason H-$\phi_{\rm geo}$-1 is a scientific hypothesis and not a placeholder is that the alternatives are not nearby. They are catastrophically far away, and each is independently identifiable.

Table 44: The regulator landscape at the lock crossing ($\Delta M = 110$ GeV, $T = M_N$): five candidate prescriptions for $\Gamma^{\rm eff}$, spanning sixteen orders of magnitude in $\Delta M/\Gamma^{\rm eff}$. **Exactly one lands in the required decade.** The throughput column is at the single-temperature evaluation. From GG-A10-Cosmos Appendix I.

Candidate	$\Gamma^{\rm eff}$ [GeV]	$\Delta M/\Gamma^{\rm eff}$	$F_{\rm BW}$	$\eta_B/\eta_B^{\rm obs}$
(a) N widths (S-matrix prescription)	1.6×10^{-10}	6.8×10^{11}	7.4×10^{-13}	1×10^{-8}
(b) **flavor measurement** $c_\tau y_\tau^2 T$	$(2\text{–}4) \times 10^{-3}$	$(2.7\text{–}5.3) \times 10^{4}$	$(0.9\text{–}1.9) \times 10^{-5}$	**0.19–0.37**
(c) oscillation rate ΔM itself	1.1×10^{2}	1	0.40	9×10^{3}
(c′) gauge damping $g^2 T$	1.5×10^{3}	0.07	0.14	3×10^{3}
(d) Hubble rate $H(T = M_N)$	2.08×10^{-11}	5×10^{12}	1×10^{-13}	2×10^{-9}

66.7 The six conditions, and which one is the crux

The candidate identification decomposes into six conditions. Three are established, one is plausible, one is computed with its closure open, and one is the crux. They are stated here in full because a reader cannot weigh the chain without them.

Remark 27 (What is being claimed, and what is not)**.** It would be easy to read (66.5) as a derivation. It is not one. What is derived is that *if* the flavor-measuring rate is the regulator, then the ledger integer and the tau Yukawa fix the Breit–Wigner factor with no freedom, and the number that comes out is the one the chain needs. What is not derived is the antecedent. The honest grade for the magnitude of the asymmetry is therefore derived-conditional for its size, and the two constants the assembly then adopts — κ and $\epsilon^{\rm res}$ — are graded *Adopted* in Table 50; nothing in the rest of this Part upgrades either. After all, a framework's value lies partly in knowing which of its statements are load-bearing: here it is one, it has a name, and it has a calculation attached that would settle it either way.

67 The washout, flavor by flavor: the electron door

67.1 Why anything survives at all

By Lemma 63.1 the flavor-summed asymmetry is exactly zero. Whatever survives to become matter survives because the three flavors are washed out at *different* rates. This section computes those rates, and the answer turns on a number this paper derived for an entirely different reason.

The strength with which flavor α is erased is set by how strongly the heavy states couple to it, summarized by the flavor-projected effective mass:

$$\widetilde{m}_\alpha \;=\; \sum_i m_i \left|U_{\alpha i}\right|^2, \qquad K_\alpha \;=\; \frac{\widetilde{m}_\alpha}{m_\star^{\rm wash}}, \qquad m_\star^{\rm wash} = 1.07 \times 10^{-3}\ \text{eV}. \tag{67.1}$$

Table 45: The six conditions C1–C6 underlying the regulator identification. **C4 is the crux**: its outstanding content is a full Kadanoff–Baym resummation, and it is the single most valuable calculation anyone could do to this chain, in either direction. Home: GG-A10-Cosmos Appendix I; carried in the roadmap of GG-P5-CPT.

	Condition	Status
C1	flavor-odd source	$\mathrm{Tr}\, S = 0$ exactly for real R, Eq. (63.6). *Established* (algebraic; machine-checked)
C2	flavor-blind cancellation	gauge damping enters through the identity projector and drops from the flavor-odd channel, Eq. (66.2). *Established* within the density-matrix framework; its persistence under full resummation folds into C4
C3	two-level reduction	the Lorentzian envelope and the $\lvert\mathcal{R}\rvert \le \frac{1}{2}$ bound survive for every regulator value, Eq. (66.4). *Established* at reduction level, given C4
C4	**regulator identification (the crux)**	in the regime $\Gamma_\tau/\Gamma_N \sim 10^7$ with the flavor-summed source zero, the interference time of the flavor-odd source is $1/\Gamma_\tau$, not $1/\Gamma_N$: the vacuum N-width regulator is superseded in-medium. **Open at derivation level** — structurally supported, but the full Kadanoff–Baym computation has not been carried out, and the standard fully-flavored Boltzmann limit would return the fatal N-width class instead
C5	the coefficient	$c_\tau \in [5 \times 10^{-3}, 10^{-2}]$ at $T \simeq M_N$; the kinetic-estimate, regime-boundary and equilibration classes of the flavor-equilibration literature agree within a factor two. *Plausible*; the precise value open
C6	window throughput	the temperature-integrated window factor is part of the honest chain, and the priced $\mathcal{O}(1)$ prefactors must supply a remaining one-sided factor *from below*. *Computed* at Boltzmann level (§68); closure open

Read this physically: $|U_{\alpha i}|^2$ is the probability that mass state i "is" flavor α, so $\widetilde{m}_\alpha$ is the share of the total seesaw coupling that flavor α owns, and K_α compares that flavor's erasure rate with the expansion rate.

67.2 The equilibrium mass, and why it is not mysterious

The scale in the denominator of (67.1) is fixed by comparing the decay rate with the Hubble rate at the moment the decays matter:

$$K_{\rm wash} \equiv \left.\frac{\Gamma_N}{H}\right|_{T=M_N} = \frac{\widetilde{m}}{m_\star^{\rm wash}}, \qquad m_\star^{\rm wash} = \frac{16\pi^{5/2}\sqrt{g_*}}{3\sqrt{5}}\,\frac{\bar{v}^2}{M_{\rm Pl}} = 1.07\times10^{-3}\ {\rm eV}. \tag{67.2}$$

The second equality is worth reading twice, because it is what makes $m_\star^{\rm wash}$ meaningful rather than mysterious: $m_\star^{\rm wash}$ *is the value of the effective neutrino mass at which the decay rate exactly equals the Hubble rate.* Above it the decays are fast compared with the expansion and the plasma sits near equilibrium, erasing what it makes; below it the decays are slow and the asymmetry survives.

With the light spectrum of Part VII, the third channel sees $m_3 = 2N\,\widetilde{m}$ (50.5 meV at QD order), and the washout it feels is the seam value times that integer; the three light channels together carry the trace of the family projector:

$$K_3 = 2N_g\,K_{\rm wash} = \frac{50.5}{1.07} \simeq 47, \qquad K_{\rm tot} = (1{+}2N)\,K_{\rm wash} = 54.6, \qquad \kappa \simeq \frac{1}{K_{\rm tot}} = 1.8{\times}10^{-2}. \tag{67.3}$$

The two routes to $K_{\rm wash}$ agree. The first equality of (67.2) is a definition, $K_{\rm wash} \equiv \Gamma_N/H$ at $T = M_N$, and this Part prints both of its inputs: $\Gamma_N = 1.62\times10^{-10}$ GeV from (62.2) and $H(M_N) = 2.08\times10^{-11}$ GeV in Table 44. Their ratio is 7.80. The second equality, on the seam effective mass $\widetilde{m} = m_D^2/M_N = 8.361$ meV against $m_\star^{\rm wash} = 1.07$ meV, returns 7.80 as well. The two routes agree, and the agreement is exact rather than numerical: both carry the normalization pinned at (60.2).

The three washout numbers are one number. The value 47 quoted at (67.3) and the value 7.80 returned by the definition are not two readings of one quantity; they are one quantity multiplied by two ledger integers. With the first-order spectrum $(m_1, m_2, m_3) = (0, 1, 2N)\,m_\star$ and $\widetilde{m} = m_\star$ for the seam channel,

$$K_{\rm seam} = \frac{\Gamma_N}{H} = \frac{\widetilde{m}}{m_\star^{\rm wash}} = 7.80, \qquad K_3 = 2N\,K_{\rm seam} = 46.8, \qquad K_{\rm tot} = (1+2N)\,K_{\rm seam} = 54.6,$$

and the flavored K_α of Table 46, which the Boltzmann system (68.2) actually integrates, sum to $K_{\rm tot}$ at the QD-lifted spectrum (59.2 meV/1.07 meV= 55). The projector coefficient $2N$ of Row 27 is therefore also the washout multiplier of the third channel, and the "light-mass reading" is the seam reading times the trace of the family projector. What this Part owed — a choice between 7.8 and 47 — was not a choice. The seam K is the unit; the OGN-5 integers are the multipliers;

and $\kappa \simeq 1/K_{\rm tot} = 0.018$ sits within a factor two of the adopted 1.0×10^{-2}, inside the order-unity register the chain declares.

67.3 Two effective masses, one number: a bookkeeping point that must be declared

Two effective masses appear in this chain and they are one number at two scales. The heavy-sector resonance width is evaluated at the seam-matched one-generation Yukawa, whose effective mass is the seam scale $\widetilde{m} = m_\star = 8.361$ meV and whose washout is $K_{\rm seam} = 7.80$. The washout the thermal bath applies to the full light spectrum is that number times the trace of the family projector, $K_{\rm tot} = (1 + 2N_g)K_{\rm seam}$; the third channel alone carries $2N\,K_{\rm seam} = 46.8$. The factor between them is the OGN-5 integer of Row 27, not a convention, and it is accounted for at the order-of-magnitude register of GG-A10-Cosmos's leptogenesis appendix. This paper's coefficient $\pi/E_{\rm GUT}$ is the upstream input of both.

67.4 The flavored washouts, and the electron door

Computing (67.1) on the derived masses and the derived mixing matrix of Part VII:

Table 46: The flavored washouts, computed from the derived masses and the first-order derived PMNS matrix, against the values GG-A10-Cosmos Appendix I obtains independently on the same ledger. The last column is the efficiency each flavor's asymmetry enjoys: the electron flavor survives an order of magnitude better than the other two.

Flavor	$\widetilde{m}_\alpha$ [meV]	K_α (here)	K_α (GG-A10-Cosmos)	regime	$\kappa(K_\alpha)$
e	3.7	3.5	3.4	mild washout	0.137
μ	31.3	29.3	28.0	strong washout	0.0084
τ	24.1	22.5	22.4	strong washout	0.0116
sum $= \Sigma m_\nu = 59.2$ meV		55.3			

The electron flavor is washed out an order of magnitude more weakly than the other two. The reason is arithmetic and it is this paper's:

$$\widetilde{m}_e \;=\; \sum_i m_i |U_{ei}|^2, \qquad |U_{e3}|^2 \;=\; \sin^2\theta_{13} \;=\; \frac{N_g}{4K} \;=\; \frac{3}{140} \;=\; 0.0214. \tag{67.4}$$

The heaviest light state $m_3 = 50.5$ meV dominates $\widetilde{m}_\mu$ and $\widetilde{m}_\tau$ because $|U_{\mu 3}|^2$ and $|U_{\tau 3}|^2$ are of order one half. It barely touches $\widetilde{m}_e$, because $|U_{e3}|^2$ is small — and it is small because the reactor angle is $N_g/4K$.

Remark 28 (The electron door)**.** The flavor-summed asymmetry is exactly zero. What survives is the difference between how fast the flavors are erased, and that difference exists because $\sin^2\theta_{13} = 3/140$ decouples the electron flavor from the heavy third mass state. The matter in the universe leaks out through the electron flavor — the numerical integration of §68 finds *the whole* of the final

asymmetry in it, and a little more, the tau ledger ending negative — at a rate set by the same ledger integers that fix the reactor angle.

Stated at its strongest, and then immediately fenced: in this framework *the reactor angle being small is why there is something rather than nothing.* The fence is that the statement holds within a chain whose magnitude is conditional on H-ϕ_{geo}-1. But the structure — flavor-summed zero, survival by unequal erasure, unequal erasure by a derived mixing angle — does not depend on that hypothesis at all.

68 The Boltzmann system, integrated

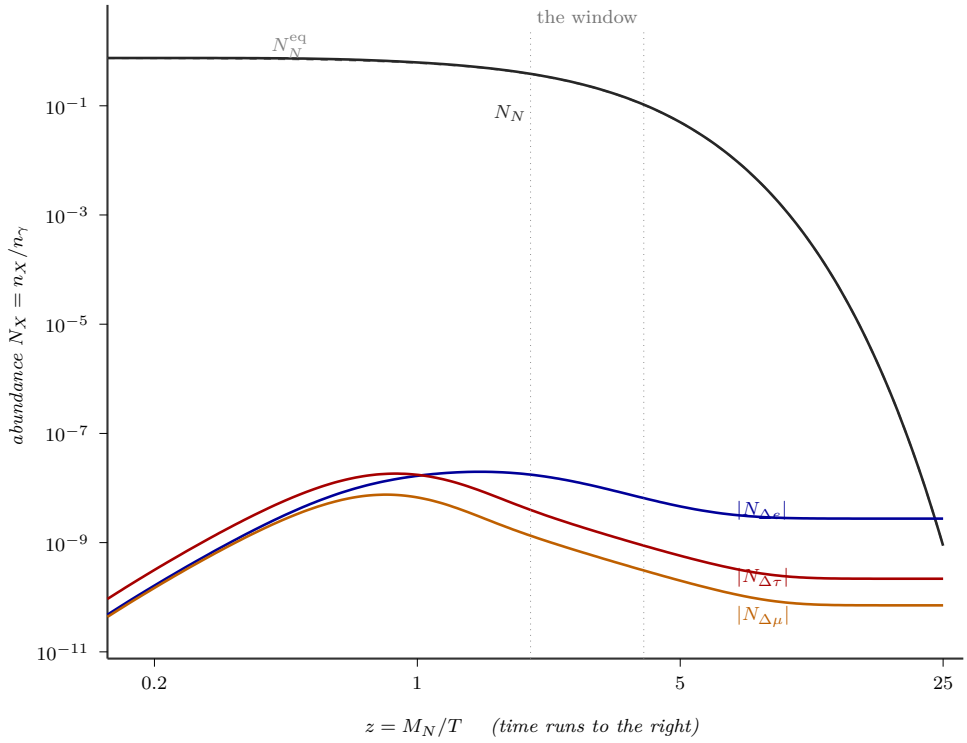

Figure 20: The Boltzmann system integrated. Heavy abundance N_N against its equilibrium value N_N^{eq} (the two are indistinguishable on this scale — that *is* the strong-washout regime), and the three flavored asymmetries, from (68.1)–(68.2) with the paper's own parameters at $c_\tau = 7.5 \times 10^{-3}$. Each asymmetry is built up around $z \simeq 1$, peaks near $z \simeq 2$, and is then cut down by inverse decays until they switch off; the electron flavor survives an order of magnitude better than the other two because its washout $K_e = 3.5$ is mild, which is the electron door of §67. The curves are drawn for a single sourcing state; the chain sums the two that source (K_{seam} and $2NK_{\text{seam}}$), which scales the asymmetries and does not change their shape. *Independent check:* integrating this system afresh in a separate code, with no further freedom, the two sourcing states relaxed separately and the window edge at $z_0 = 1$, gives $\eta_B = (0.39\text{–}0.78) \times 10^{-10}$ across $c_\tau \in [5, 10] \times 10^{-3}$, with 0.69×10^{-10} at $c_\tau = 8.9 \times 10^{-3}$ — the central value of (68.8) — and Level B's $0.58^{+0.35}_{-0.22} \times 10^{-10}$ is the same integration scanned over the four declared factors of §72. A one-state integration at the same $z_0 = 1$ returns $(0.20\text{–}0.39) \times 10^{-10}$, the difference being the count and not a disagreement.

The estimates so far are single-temperature snapshots. The asymmetry is made over a range of temperatures, against a washout that is itself changing, with a resonance factor that changes too — and the honest question is what the full time integration costs. This section states the system, its validation, and its three results. The integration is GG-A10-Cosmos's, Appendix I; it is reported here because a reader cannot otherwise tell how much of the chain is arithmetic and how much is dynamics.

68.1 The system

The coupled equations, in the variable $z = M_N/T$ — time runs with z, the early universe at small z and the late at large — and in photon-normalized abundances $N_X = n_X/n_\gamma$:

$$\frac{\mathrm{d}N_{N_i}}{\mathrm{d}z} = -\,D_i(z)\,(N_{N_i} - N_N^{\mathrm{eq}}), \qquad i = 1, 2, \tag{68.1}$$

$$\frac{\mathrm{d}N_{\Delta\alpha}}{\mathrm{d}z} = \underbrace{\epsilon_\alpha(z)\sum_i D_i(z)\,(N_{N_i} - N_N^{\mathrm{eq}})}_{\text{source}} \;-\; \underbrace{\tfrac{1}{4}\,K_\alpha\, z^3 K_1(z)\, N_{\Delta\alpha}}_{\text{washout}}. \tag{68.2}$$

Term by term. Equation (68.1) says each sourcing heavy abundance relaxes toward its equilibrium value $N_N^{\mathrm{eq}}(z) = \frac{3}{4}\cdot\frac{1}{2}z^2K_2(z)$ at its own rate $D_i(z) = K_i\, zK_1(z)/K_2(z)$, $K_i \in \{K_{\mathrm{seam}}, 2NK_{\mathrm{seam}}\}$. Of the lock triplet's three near-degenerate states, *two* source: on the spectrum $(m_1, m_2, m_3) = (0, 1, 2N)\, m_*$ of §67 with $R = \nVdash$, the first state has no Yukawa, no width and therefore no production — the triplet couples only through the seam (Lock 2), so nothing else populates it; thermally populated it would release its asymmetry late, at the empty state's washout $K_{\mathrm{seam}}\, m_1^{\mathrm{QD}}/m_* = 0.061$, and that it is not is the count's premise — which is why $K_{\mathrm{tot}} = (1 + 2N)K_{\mathrm{seam}}$ sums two channels (the three light channels of §67 carry the projector's trace; one of the three heavy channels is empty). The integration relaxes the two sourcing states separately and sums their sources, as (68.2) is written. The ratio of Bessel functions is time dilation, tending to $z/2$ at small z because a relativistic heavy state decays slowly in the bath frame. In (68.2) the source is proportional to the *departure* from equilibrium — Sakharov's third condition appearing explicitly as a factor — and the washout term erases the asymmetry already present at a rate proportional to that flavor's K_α from Table 46.

The asymmetry per decay is itself temperature-dependent, because the regulator of §66 is thermal:

$$\epsilon_\alpha(z) \;=\; \bar{\epsilon}_\alpha \cdot 2R(z), \qquad R(z) \;=\; \frac{2\,\Delta M\,\Gamma^{\mathrm{eff}}(z)}{4\Delta M^2 + \Gamma^{\mathrm{eff}}(z)^2}, \qquad \Gamma^{\mathrm{eff}}(z) \;=\; c_\tau y_\tau^2\,\frac{M_N}{z}, \tag{68.3}$$

with $\bar{\epsilon}_\alpha = (+3.4, +3.1, -6.6)\times 10^{-3}$ the flavored pattern computed on the ledger inputs (peak-normalized). The sum $\sum_\alpha \bar{\epsilon}_\alpha$ vanishes *exactly*, by Lemma 63.1; a reader who adds the three printed values will get -1×10^{-4} rather than zero, which is the rounding to two significant figures and not a failure of the lemma. The zero is what makes the asymmetry a matter of flavor *differences*, so it is worth not leaving the reader to wonder. Since $\Gamma^{\mathrm{eff}} \propto T$, the resonance factor $R(z)$ *falls* as the universe cools — the interference is measured more slowly while the splitting stays put — and that decline is the physical origin of the window suppression the integration measures.

For the reader who wants the efficiency in closed form before the numerics, the standard strong-washout fit is

$$\kappa(K) \;\simeq\; \frac{2}{K\,z_B(K)}\Big(1-e^{-K z_B(K)/2}\Big), \qquad z_B(K) \;\simeq\; 2+4K^{0.13}e^{-2.5/K}, \tag{68.4}$$

(Buchmüller et al., 2005), whose physical content is one sentence: the asymmetry that survives is the asymmetry made in the *last* moment before the inverse decays shut off, at $z = z_B$, and by then only a fraction $\sim 1/(Kz_B)$ of the heavy states remain to make it. Everything earlier is erased; κ is the fraction made late.

68.2 Where the two right-hand sides come from

The system (68.1)–(68.2) is usually quoted. It is short enough to derive, and the derivation explains two features that otherwise look arbitrary: the Bessel functions, and the factor of a quarter on the washout.

The collision integral. For a decay $N \to \ell H$ the Boltzmann equation for the heavy abundance is, in comoving form,

$$\frac{\mathrm{d}n_N}{\mathrm{d}t} + 3Hn_N \;=\; -\langle\Gamma_N\rangle\,(n_N - n_N^{\mathrm{eq}}), \tag{68.5}$$

where the two terms on the right are the decay and its inverse. The inverse decay carries n_N^{eq} rather than n_N because the incoming leptons and Higgses are themselves in equilibrium: detailed balance fixes the rate at which they reconstitute a heavy state, and that rate depends on the bath, not on how many heavy states happen to be present. This is why the right-hand side is proportional to the *departure* from equilibrium — Sakharov's third condition is not imposed here, it is a factor in the equation.

Where the Bessel functions come from. The bracket in (68.5) is a thermal average of the decay rate over the heavy states' momentum distribution, and a moving particle decays slowly:

$$\langle\Gamma_N\rangle \;=\; \Gamma_N\Big\langle\frac{M_N}{E}\Big\rangle \;=\; \Gamma_N\frac{K_1(z)}{K_2(z)}, \qquad z \equiv \frac{M_N}{T}, \tag{68.6}$$

because for a Maxwell–Boltzmann distribution the average of M/E is exactly the ratio of the two modified Bessel functions. At small z — that is, at high temperature — $K_1/K_2 \to z/2$, which is the statement that a highly relativistic heavy state is time-dilated into near immortality. At large z the ratio tends to 1 and the decay proceeds at its rest-frame rate. Changing the independent variable from t to z and normalizing to the photon density turns (68.5) into (68.1) with $D(z) = K_{\mathrm{tot}}\,zK_1(z)/K_2(z)$.

The equilibrium abundance is the same integral without the rate: $N_N^{\mathrm{eq}}(z) = \tfrac{3}{8}z^2K_2(z)$, which is the Maxwell–Boltzmann number density of a species of mass M_N at temperature M_N/z, divided by the photon density.

The washout term, and its factor of a quarter. The asymmetry equation has a source and a sink. The source is the same decay term weighted by the asymmetry per decay, which is the first term of (68.2). The sink is inverse decay acting on the asymmetry already present: a lepton excess is depleted by $\ell H \to N_g$ at a rate set by the same Yukawa.

The washout rate is *not* equal to the source rate, and the ratio is where the quarter comes from. The inverse-decay rate involves the equilibrium density of the incoming leptons rather than of the heavy state, which supplies one factor of $\frac{1}{2}$ after the spin and particle–antiparticle averaging that the decay term does not carry; and the asymmetry is depleted only through the difference between ℓ and $\bar{\ell}$ inverse decays, which supplies another. Collecting the thermal average for the inverse process — which weights K_1 rather than K_1/K_2, because the incoming particles are massless and their distribution supplies the remaining z^2 — gives the washout term

$$\Gamma_\alpha^{\text{wash}}(z) \;=\; \tfrac{1}{4}\, K_\alpha\, z^3 K_1(z), \tag{68.7}$$

which is the second term of (68.2). Its $z^3 K_1(z)$ rises from zero, peaks near $z \simeq 2$, and then falls exponentially: inverse decays switch off once the bath is too cold to make a heavy state at all, and that switch-off is what allows any asymmetry to survive.

Remark 29 (Physics intuition for the shape of the answer)**.** Two competing exponentials decide everything. The source needs the heavy states to be out of equilibrium, which happens only once the bath has cooled below their mass — so it turns on late. The washout needs the bath to be hot enough to run the decay backwards — so it turns off late too, but not quite as late. The asymmetry that survives is made in the narrow window between the two, at $z \simeq 2$–4, which is exactly where the numerical integration of §68 finds it. And because the resonance factor $R(z)$ is *falling* through that window — $\Gamma^{\text{eff}} \propto T$ while ΔM does not — the asymmetry is made just after the best moment for making it. That mismatch, more than any uncertainty in the inputs, is the largest part of the factor $E = 10.6$ in (68.9).

Scope, declared. The system (68.1)–(68.2) carries decays and inverse decays only. Lepton-number-violating scatterings, spectator processes, thermal-mass corrections and the relativistic edge are deliberately omitted and separately priced in §72. Every one of them points the same way.

68.3 Three results

First: the sign is an output, not an input. The flavored pattern fed in sums to zero. The integration nonetheless returns a net positive asymmetry, because the tau-dominated negative contributions sit in strong washout and are erased faster than the electron-dominated positive one. At the end of the integration the electron ledger holds 106% of the net asymmetry, the muon's +3% and the tau's −10%: the whole of the survivor is in the electron flavor. The universe is made of matter rather than antimatter in this chain because of the ordering of three erasure rates, and that ordering traces to (67.4).

Second: the window, measured. At central parameters the integration gives

$$\eta_B^{\rm Boltzmann} \;=\; 0.69\times10^{-10}, \qquad \frac{\eta_B^{\rm Boltzmann}}{\eta_B^{\rm obs}} \;=\; 0.11. \tag{68.8}$$

The deficit against observation — the quantity condition C6 named but could not compute — is now a measured number with an error bar rather than a bracket spanning a decade:

$$\boxed{E \;\equiv\; \frac{\eta_B^{\rm obs}}{\eta_B^{\rm B}} \;=\; 10.6, \qquad 68\%\ \text{interval}\ [6.6,\ 17.0],} \tag{68.9}$$

where $\eta_B^{\rm B} = 0.58^{+0.35}_{-0.22}\times10^{-10}$ is the Level-B value of (72.2) — the integration above, scanned over the four remaining freedoms. The distinction is easy to lose and worth keeping: at *strictly* central parameters the deficit is $6.12/0.69 = 8.9$ and carries no band, whereas E carries one because the scan does. Every digit of E is Level B's: $6.12/0.58 = 10.6$, and the interval is $6.12/0.93 = 6.6$ and $6.12/0.36 = 17.0$.

Third: the deficit is one-sided. The asymmetry is generated at $z \simeq 1.5$–4, which is off the best part of the resonance because $\Gamma^{\rm eff} \propto 1/z$. Every effect omitted from the scope declaration above moves production *earlier*, where $R(z)$ is larger. The deficit therefore points up, not in an unknown direction — which is the difference between a discrepancy and a missing term.

Remark 30 (Why a factor of ten is not a small matter, and not a fatal one)**.** It would be dishonest to present (68.9) as agreement. The chain as integrated makes about a tenth of the observed asymmetry. It would be equally dishonest to present it as refutation: the integration omits four standard transport effects by construction, each of which is known to increase the answer, and §72 prices them. The useful statement is the one in the middle — the chain lands within about an order of magnitude of the observed baryon asymmetry from a scale fixed by an anomaly count, with no fitted parameter, and the residual factor is now measured rather than bracketed.

69 Sphalerons and the transfer to baryon number

Everything so far has made a lepton asymmetry. The universe is short of *baryons*, and something must convert.

The converter is the electroweak sphaleron (Kuzmin et al., 1985): a non-perturbative Standard-Model process, unsuppressed at temperatures above the electroweak scale, which violates $B+L$ while conserving $B-L$. It cannot change $B-L$, so it cannot create the asymmetry; but it is free to *redistribute* whatever $B-L$ exists between the baryon and lepton sectors, and it does so until the chemical potentials balance.

Solving the equilibrium conditions — sphaleron equilibrium, Yukawa equilibrium, hypercharge neutrality — for the Standard Model with three generations and one Higgs doublet gives (Harvey & Turner, 1990)

$$B \;=\; \frac{28}{79}\,(B-L), \qquad c_{\rm sph} \;=\; \frac{28}{79} \;=\; 0.35443. \tag{69.1}$$

Grade. This factor is *textbook, not derived here*. The 28/79 is a Standard-Model equilibrium computation that this framework inherits unchanged, and it is flagged as inherited rather than presented as a result. Its accuracy is not in question; its provenance is simply not this paper's.

There is an interesting tension worth naming. The same framework that forbids $\Delta B = 1$ and $\Delta B = 2$ outright — and thereby makes $\tau_p = \infty$ a theorem — relies on $\Delta B \neq 0$ sphaleron processes to make baryons at all. The resolution is the selection rule of §109.6: sphalerons change baryon number by $N_{\rm gen} = 3$ units at a time, which is admissible, while proton decay would require $\Delta B = 1$, which is not. The framework is therefore not merely consistent on this point; it is consistent for a reason that is itself a prediction.

70 The six factors, and the assembly

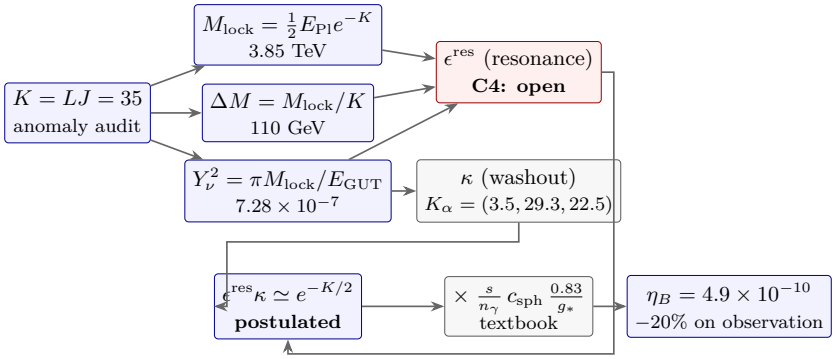

Figure 21: The chain end to end. Blue: quantities that descend from the ledger integer $K = 35$ with no freedom left. Red: the single open link. Every quantity downstream of C4 inherits its conditionality, which is why the Part reports its result at three levels (Fig. 22) rather than as a number — and why the falsification channel F-SM-7 is written so that one finite-temperature calculation settles the whole diagram in either direction.

70.1 The bookkeeping variable

Number densities dilute with expansion, so the useful variable is a ratio. The entropy density of a relativistic bath is

$$s \;=\; \frac{2\pi^2}{45}\, g_{*s}(T)\, T^3, \tag{70.1}$$

with g_{*s} counting relativistic degrees of freedom weighted for entropy (bosons 1, fermions 7/8). Its virtue is a conservation law: for adiabatic expansion the total comoving entropy $S = sa^3$ is constant, which absorbs every reheating event automatically — when a species annihilates, g_{*s} falls and T rises to compensate. So define the *yield* $Y_X \equiv n_X/s$, a pure number that does not change with expansion.

70.2 The six factors, one at a time

1. How many heavy states there were: $Y_N^{\rm eq}$. At $T \gg M_N$ the heavy states are relativistic and in equilibrium, so they are one more species in the bath. With $n_N^{\rm eq} = (3\zeta(3)/4\pi^2) g_N T^3$ and $g_N = 2$ for a Majorana fermion, dividing by (70.1):

$$Y_N^{\rm eq} \;=\; \frac{3\zeta(3)}{4\pi^2}\,\frac{45}{2\pi^2}\,\frac{g_N}{g_{*s}} \;=\; \frac{0.42}{g_*} \qquad \text{per species.} \tag{70.2}$$

Of the three quasi-degenerate heavy states two carry a Yukawa (§67), so the sourcing total is $2 \times 0.4165/g_* = 0.833/g_*$. This factor of 0.83 is carried explicitly rather than absorbed into an order-unity allowance, because it is a definite number and it points up.

2. How biased each decay was: $\epsilon^{\rm res}$. The resonant CP asymmetry per decay, the flavored sum of Eq. (70.5), at $\epsilon^{\rm res} \simeq 2.5 \times 10^{-6}$. *Adopted*, not derived: it is the value the throughput identity returns once κ is adopted, and no independent resonance computation produces it. Conditional on H-$\phi_{\rm geo}$-1 for its *plausibility*, not for its value.

3. How much survived: κ. The washout efficiency, $\kappa \simeq 1.0 \times 10^{-2}$, adopted; the closed form (67.3) gives $1/K_{\rm tot} = 1.8 \times 10^{-2}$, and the Boltzmann integration of §68 returns η_B directly, not κ — at the adopted $\epsilon^{\rm res}$ Level B's throughput implies $\kappa \approx 1.2 \times 10^{-3}$, a factor 8.4 below the adopted value ($E = 10.6$ is the factor below observation; the difference is Level A's -20%). *Adopted* (Table 50, row 6).

4. How much became baryons: $c_{\rm sph} = 28/79$. From (69.1). *Inherited.*

5. How much else was in the bath: g_*. Above the electroweak scale the whole Standard Model is relativistic. Counting: eight gluons × two helicities = 16; the four electroweak bosons before symmetry breaking, $4 \times 2 = 8$; the complex Higgs doublet, 4 — bosons total 28. Then six quark flavors × three colors × two spins × two (particle/antiparticle) = 72; three charged leptons $\times 2 \times 2 = 12$; three neutrinos $\times 1 \times 2 = 6$ — fermions total 90. Hence

$$g_* \;=\; 28 + \tfrac{7}{8} \times 90 \;=\; 28 + 78.75 \;=\; 106.75. \tag{70.3}$$

It enters twice: once in the entropy denominator, and once through $H \propto \sqrt{g_*}\, T^2/M_{\rm Pl}$ inside $m_\star^{\rm wash}$.

6. From entropy to photons: s/n_γ. Astronomers measure baryons per photon, not per unit entropy. The conversion is the ratio of the two reference densities today:

$$\left.\frac{s}{n_\gamma}\right|_{T_0} \;=\; \frac{(2\pi^2/45)\, g_{*s}(T_0)}{2\zeta(3)/\pi^2} \;=\; \frac{0.43865 \times 3.909}{0.24359} \;=\; 7.04, \qquad \eta_B \;=\; 7.04\, Y_B. \tag{70.4}$$

Here $g_{*s}(T_0) = 2 + \frac{7}{8} \times 6 \times \frac{4}{11} = 3.909$: two photon polarizations plus six neutrino states, colder than the photons by $(4/11)^{1/3}$ because electron–positron annihilation heated the photons after the neutrinos had decoupled.

70.3 The assembly

Each decay left a fractional excess $\epsilon^{\rm res}$ — the flavored sum $\sum_\alpha \epsilon_\alpha \kappa_\alpha / \kappa$, written here as one symbol; the single-flavor form is identically zero by Lemma 63.1 and is not what is assembled — a fraction κ of the total survived washout, and the sourcing heavy abundance was $0.83/g_*$:

$$Y_{B-L} \;\simeq\; \sum_\alpha \epsilon_\alpha\, \kappa_\alpha \sum_i Y^{\rm eq}_{N_i} \;\equiv\; \epsilon^{\rm res}\, \kappa \sum_i Y^{\rm eq}_{N_i} \;\simeq\; \frac{0.83\, \epsilon^{\rm res}\, \kappa}{g_*}. \tag{70.5}$$

Evaluated on this Part's own printed inputs — $\bar\epsilon_\alpha$ of (68.3), the $\kappa(K_\alpha)$ of Table 46, and $2R$ at $c_\tau = 8.9 \times 10^{-3}$ — the flavored sum returns $\sum_\alpha \epsilon_\alpha \kappa_\alpha = 1.40 \times 10^{-8} = 0.56\, e^{-K/2}$. The identity $\epsilon^{\rm res}\kappa = e^{-K/2}$ used in (70.6) is therefore asserted at the order-unity level, not derived: the exponent is the structural claim, and the prefactor is within a factor two of unity on the paper's own numbers. Which is the honest strength of this chain, and it is weaker than the digits of η_B suggest. Sphalerons move 28/79 of it into baryons and (70.4) converts the yield to the observed ratio:

$$\boxed{\eta_B \;\simeq\; 7.04\, c_{\rm sph}\, \frac{0.83\, \epsilon^{\rm res}\, \kappa}{g_*} \;=\; 7.04 \left(\frac{28}{79}\right) \frac{0.83\, e^{-K/2}}{g_*}.} \tag{70.6}$$

Table 47: The chain arithmetic, factor by factor, with the running product. Read down the last column to watch 10^{-8} become 5×10^{-10}. Every factor is a number this paper derives, a piece of Standard-Model arithmetic, or one of the two adopted constants of Table 50 (κ and $\epsilon^{\rm res}$); none is fitted to the baryon asymmetry, and the running product makes the size of each individual step visible.

Step	Factor	Value	Running product
1	throughput $\epsilon^{\rm res}\,\kappa = e^{-K/2}$	2.511×10^{-8}	2.511×10^{-8}
2	$\times$ sourcing yield (two states), $\sum_i Y^{\rm eq}_{N_i} \cdot g_*$	0.833	2.092×10^{-8}
3	$\div$ degrees of freedom, g_*	106.75	1.960×10^{-10}
4	$\times$ sphaleron transfer, 28/79	0.35443	6.947×10^{-11}
5	$\times$ entropy-to-photon, s/n_γ	7.04	$\mathbf{4.890 \times 10^{-10}}$
observed (Planck Collaboration, 2020)			$(6.12 \pm 0.04) \times 10^{-10}$
central deviation			-20%
declared theory band			$\times 3^{\pm 1}$

How this number is to be quoted. At headline precision the prediction of this chain is

$$\boxed{\eta_B \;\approx\; 5 \times 10^{-10}, \qquad \text{derived band } (1.6\text{–}15) \times 10^{-10}.} \tag{70.7}$$

The exact product 4.890×10^{-10} of Table 47 belongs to the worked arithmetic and should not be carried into summary statements: three significant figures on a quantity with a factor-three band would misrepresent what is known. The band is derived in §72 and is not reverse-engineered to cover the datum — as (68.9) makes plain, the chain's own integration sits below observation, not on it.

71 The throughput identity, and the exponent that is postulated

Two of the six factors are not simply inherited: the CP asymmetry and the washout efficiency. Their product is the chain's central claim:

$$\boxed{\epsilon^{\mathrm{res}}\,\kappa \;\simeq\; (2.5\times10^{-6})(1.0\times10^{-2}) \;\simeq\; 2.51\times10^{-8} \;=\; e^{-K/2} \;=\; e^{-17.5}.} \tag{71.1}$$

An explicit seesaw-plus-washout chain, built from a Yukawa fixed by the seam and a washout fixed by the light spectrum, lands within a factor of two of the ledger's own throat-depth exponential; the fence under (70.5) grades the identity as asserted, and the three fences below say why.

That is a striking statement and it must be fenced immediately, in three ways.

1. **The agreement uses the order-unity freedom in κ.** At the unadjusted $\kappa = 1.8\times10^{-2}$ of (67.3) the left-hand side would read 4.5×10^{-8}, nearly a factor of two high.

2. **The exponent $K/2$ is a postulate of the compression, not a derivation.** It is geometrically suggestive — $e^{-K/2}$ is $\sqrt{e^{-K}}$, the square root of the lock's own suppression — and it is not derived. Deriving it is the second clause of the chain's open conjecture.

3. **ϵ^{res} is adopted, not derived**: it is the value the identity returns once κ is adopted (Table 50, row 5), and its plausibility is conditional on H-ϕ_{geo}-1, which §66 states in sharp form and marks open at derivation level.

Remark 31 (What would make this identity mean something)**.** An identity between an assembled chain and a ledger exponential is either a coincidence or a theorem, and at present the paper cannot tell which. Two calculations would settle it: a finite-temperature transport computation discharging H-ϕ_{geo}-1, and a derivation of the exponent $K/2$ from the descent rather than from reconstruction. Until both are done the correct reading of (71.1) is that a chain with two adopted constants and no parameter fitted to the asymmetry lands within an order-unity factor of a number the geometry already contained — which is interesting, and is not proof.

72 The error bar, derived

A prediction with a factor-three band is only honest if the band is derived rather than chosen. This section states it at three levels, each with what it assumes.

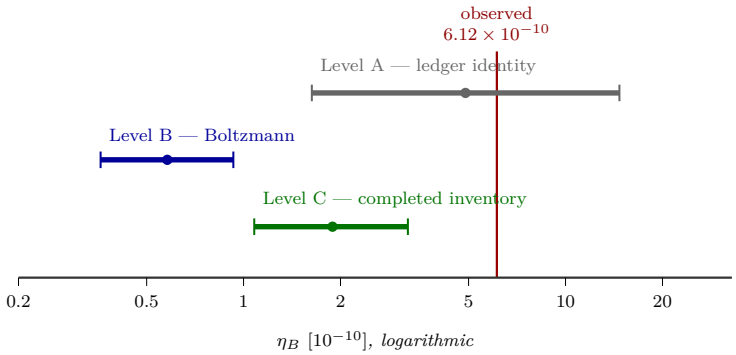

Figure 22: The prediction at three levels, against observation. Level A is the ledger identity taken at face value; Level B is the Boltzmann integration; Level C folds in four standard transport effects Level B omits. The three answer three different questions and *are not to be averaged*: their spread is the informative quantity, and it says the ledger's arithmetic and the declared dynamics currently disagree by a factor of about two and a half. All three are conditional on C4.

Level A — the ledger identity. Taking (71.1) at face value and assembling by (70.6):

$$\eta_B^{\mathrm{A}} \;=\; 4.89 \times 10^{-10} \times 3^{\pm 1}, \qquad \text{band } (1.6\text{–}15) \times 10^{-10}, \tag{72.1}$$

a span of nine. The exponent is postulated; the band is the declared order-of-magnitude register of the compression.

Level B — the Boltzmann computation. The integration of §68, scanned over the four remaining freedoms — the coefficient $c_\tau \in [5, 10] \times 10^{-3}$, the flavored pattern acceptance $\times[2/3, 3/2]$, the washout-kernel order-unity factor $\times[0.7, 1.3]$, and the window edge $z_0 \in [0.5, 2.0]$ — gives

$$\boxed{\eta_B^{\mathrm{B}} \;=\; 0.58^{+0.35}_{-0.22} \times 10^{-10} \quad (68\%), \qquad \text{band width } \times 2.6.} \tag{72.2}$$

Against Level A's width of $\times 9$, the error bar has contracted by a factor of three and a half, because most of what used to be bracketed is now computed. Level B sits a factor $E = 10.6$ $[6.6, 17.0]$ below observation, Eq. (68.9).

Level C — the completed inventory. Level B deliberately omits four standard transport effects, each pointing up, none specific to this framework. Folding them in at their literature ranges — lepton-number-violating scattering source $\times[1.3, 2.5]$, Lorentzian-versus-Breit–Wigner convention $\times[1, 2]$, relativistic-window residue $\times[1, 1.5]$, spectators and the A-matrix $\times[0.85, 1.2]$ — gives the best estimate:

$$\boxed{\eta_B^{\mathrm{C}} \;=\; 1.9^{+1.4}_{-0.8} \times 10^{-10} \quad (68\%), \qquad 1.9^{+3.4}_{-1.3} \times 10^{-10} \quad (95\%).} \tag{72.3}$$

Against $\eta_B^{\rm obs} = (6.12 \pm 0.04) \times 10^{-10}$ the observation sits 2.2σ above the Level-C median on the logarithmic scale, just above the 95% band's upper edge of 5.3×10^{-10}: *a central deficit of a factor* 3.2*, which the four omitted effects at their literature ranges narrow but do not close.*

All three levels are conditional on C4. None of the three statements above is a derivation of the baryon asymmetry. Each is what follows *given* the regulator identification of Table 45, and that identification is open.

Remark 32 (Reading three numbers that disagree)**.** A reader may reasonably ask why the Part quotes 5×10^{-10}, 0.58×10^{-10} and 1.9×10^{-10} and calls all three the prediction. They are answers to three different questions. Level A asks what the ledger identity gives if taken literally. Level B asks what the declared dynamics actually produces when integrated. Level C asks what the completed inventory would produce. The honest headline is Level A with its band, because that is what the framework asserts; the honest *best estimate* is Level C, because that is what the physics as currently understood delivers; and the gap between them — a factor of about two and a half — is the measure of how much of this chain is still owed.

73 From η_B to ρ_b and Ω_b: one quantity, three sets of units

The chain above ends at a baryon-to-photon ratio, and the cosmology companion states a baryon fraction. It is worth carrying this Part to the end of its own arithmetic, because the two are not two results.

One quantity, three costumes. The baryon-to-photon ratio η_B, the baryon mass density ρ_b and the baryon fraction Ω_b are *one* physical quantity in three sets of units, tied by conversions that contain no content of this framework's:

$$\eta_{10} \;=\; 273.9\,\Omega_b h^2, \qquad \rho_b \;=\; \Omega_b h^2 \times 1.87847 \times 10^{-29}\ \mathrm{g\,cm^{-3}}, \qquad \eta_{10} \equiv 10^{10}\eta_B. \tag{73.1}$$

The first is (L.2), derived in Appendix L from $n_B = \Omega_b \rho_c / \bar{m}_B$ over $n_\gamma = (2\zeta(3)/\pi^2) T_0^3$ at $T_0 = 2.7255$ K, with $\bar{m}_B$ the mean mass per baryon; the second is the critical density. Fix any one of the three and the other two follow with no freedom. Quoting them as three predictions with three error bars would make this framework look as though it had made three uncertain statements about baryons. It has made one, by two independent roads.

The two roads, and which is worth more. The road of this Part is dynamical: the Majorana door, the flavored asymmetries, differential washout, sphaleron conversion at 28/79 and entropy dilution, landing at $\eta_B^{\rm A} = 4.89 \times 10^{-10}$ against an observed 6.12 — low by twenty percent, inside a declared factor-$\sim$3 band, and graded D* on the named hypothesis H-$\phi_{\rm geo}$-1.

The cosmology companion carries a second and completely independent road: a *static* count of volumes on the compact space the ledger selects, giving $\Omega_b = 1/(2\pi^2) = 0.050661$ with no dynamics

in it at all. Converted through (73.1) at the companion's own derived expansion rate $h = 0.666$,

$$\Omega_b h^2 = 0.02247, \qquad \eta_{10} = 6.15, \qquad \rho_b = 4.22 \times 10^{-31}\ \mathrm{g\,cm^{-3}}, \tag{73.2}$$

against an observed $\eta_{10} = 6.12$ and $\rho_b = 4.20 \times 10^{-31}\ \mathrm{g\,cm^{-3}}$: an error of +0.6%, with nothing fitted anywhere in the chain. At the microwave background's own $h = 0.674$ the same road gives $\eta_{10} = 6.30$, an error of +3.0%.

Remark 33 (Which road this paper is entitled to lead with)**.** The static road is right to between six tenths of a percent and three per cent; the dynamical road of this Part is low by twenty percent with a factor-of-three band. The static road is the more accurate by a factor of about thirty, and the asymmetry has a structural cause rather than an accidental one: a statement about volumes on a compact space carries no dynamics to get wrong, while this Part's chain must pass through a finite-temperature resonance condition that it grades as an open, named hypothesis. The honest division is therefore that the *static* count is the framework's prediction of the number, and this Part is the demonstration that a parameter-free mechanism exists which could have produced it. That is what the D* grading already says; it is written out here so that no reader has to infer it.

The gap, named once. Converted to a common $h = 0.674$, the dynamical road gives $\Omega_b = 0.0393$ and the static road $\Omega_b = 0.050661$. *The two roads disagree by* 22%*, the dynamical road below the static.* The gap does not close on the ladder branch (−35% at $h = 0.735$) but sits inside this Part's factor-∼3 band, so it is not a contradiction; but the two are predictions of the same number and they do not yet agree, and closing that is unfinished business. It is recorded as the two-route-consistency item and is carried as the same item by the cosmology companion, whose printed number predates this recount.

One ratio that needs no epoch. There is a third statement worth separating out, because it is the cleanest of the three and it is easy to lose among them. Baryons and cold dark matter both dilute as a^{-3}, so

$$\frac{\Omega_b}{\Omega_m} \;=\; \frac{1/(2\pi^2)}{1/\pi} \;=\; \frac{1}{2\pi} \;=\; 0.159155 \tag{73.3}$$

is constant at every epoch and independent of h. Measured against $\omega_b/\omega_m = 0.02237/0.14237 = 0.15713$ it sits +1.3% high, about 1.3σ — and unlike the absolute fractions it consumes no epoch calibration, which is why the companion grades it D rather than D*. The comparison must be made in ω_b/ω_m and not in Ω_b/Ω_m, because the measured Ω_m includes a neutrino component that $\Omega_b + \Omega_{\mathrm{dm}}$ does not; with $\Sigma m_\nu \simeq 59$ meV that is 0.44% of Ω_m, and it moves the comparison by half a percentage point.

74 Zero dilution

An asymmetry made at the lock must survive to the present, and the usual threat is entropy release from a later transition: a supercooled phase transition would dilute whatever was made before it.

Here the dilution factor is unity. The nucleation action at the relevant crossing sits some fifty-six orders of magnitude above the supercooling threshold, and the saddle is barrier-free, so the transition proceeds without appreciable entropy release. Two consequences follow and are recorded because they are consumed elsewhere in the cohort: the number of e-folds is not reduced by a second inflationary episode, giving $n_s = 1 - 2/N_* = 0.9620$ and $r = 12/N_*^2 = 0.0043$, the leading-order pair GG-A10-Cosmos records beside its exact values. Nothing made at the lock is diluted on the way here.

75 What could move, and the three kill routes

75.1 Which factors are free to move

Table 48: Which factors of (70.6) can move, and by how much. Two of the six are fixed Standard-Model arithmetic and cannot move at all. The remaining four are ranked by how far they could move and by what would have to be true for them to move that far, which is the honest form of an uncertainty for a chain of this kind.

Factor	Room	Comment
$e^{-K/2}$	the postulate	the exponent is not derived; this is the chain's largest single exposure
ϵ^{res}	on H-ϕ_{geo}-1	conditional on the regulator identification, C4
κ	factor ~ 2	adopted at 1.0×10^{-2} against the closed form's $1/K_{\mathrm{tot}} = 1.8 \times 10^{-2}$; the Boltzmann integration of §68 returns η_B, not κ, and its throughput sits a factor 8.4 below the adopted product (a factor $E = 10.6$ below observation)
Y_N^{eq}	none	$0.83/g_*$ (two sourcing states) is a definite counting result
$g_* = 106.75$	none	Standard-Model degree-of-freedom count
$c_{\mathrm{sph}} = 28/79$	none	Standard-Model equilibrium result
$s/n_\gamma = 7.04$	none	fixed by present-day cosmology

75.2 Three routes by which the chain kills itself

The reason H-ϕ_{geo}-1 is a scientific hypothesis rather than a placeholder is that its failure modes are written down in advance, and each maps onto a row of Table 44.

1. **If the finite-temperature derivation returns the N-width class** — row (a) — the chain undershoots by 10^7 to 10^8 and is falsified from within. Note that the *standard* fully flavored Boltzmann limit does exactly this, which is why C4 is the crux and not a formality.

2. **If it returns the flavor-blind g^2T** — row (c′) — the excluded peak overshoot stands and the verdict is the same. This branch has been examined and excluded: it lands at $\Delta M/\Gamma^{\mathrm{eff}} \simeq 0.07$.

3. **If C4 and C5 close but the window deficit of C6 survives** a full transport computation at its adverse end with the priced prefactors exhausted, H-$\phi_{\rm geo}$-1 fails by *throughput* rather than by regulator identity — a third, softer kill beside the two clean ones. This is the failure mode whose size (68.9) now measures.

Success is a finite-temperature transport computation — thermal widths in the propagators, the lock crossing's time dependence in the rates — landing at the partial-resonance point. Failure is the same computation showing the enhancement dead at the crossing for every candidate mechanism, in which case the chain is falsified from within, the $e^{-K/2}$ agreement stands exposed as coincidence, and the account reverts to Sakharov custody plus an unexplained number. *Both outcomes are scientific; only the unexamined middle would not be.*

This exposure is registered as a named falsification channel in §109.

76 Accelerator corollaries: what a collider can and cannot prove

The lock sits at 3.85 TeV — inside the energy reach of the HL-LHC. Can the mechanism of this Part therefore be tested at an accelerator? The answer has a sharp negative half and a constructive positive half, and both follow from theorems already proved above.

76.1 The negative half: three locks on the direct channel

The occupant of the lock cannot be seen at any collider now conceived, for three independent reasons, each a derived quantity.

Lock 1: the mixing. The heavy triplet couples to the boundary only through the seam Yukawa, so its active–sterile mixing is fixed at $|\theta|^2 = 2.2 \times 10^{-15}$ (60.5). Every production channel available to a heavy neutral lepton at a collider — $W^{(*)} \to \ell N$, Drell–Yan through mixing — pays this factor. Dedicated heavy-neutral-lepton searches reach $|\theta|^2 \sim 10^{-6}$ at masses far below the lock, and have no sensitivity at all at 3.85 TeV; the shortfall is nine or more orders of magnitude, and no luminosity upgrade crosses an exponent.

Lock 2: no portal. The classic escape — gauge-portal production, as in left–right models where $pp \to W_R \to \ell N$ makes the heavy state at gauge strength and the same-sign dilepton pair exhibits $\Delta L = 2$ in a detector (Keung & Senjanović, 1983) — is closed here by the cluster's own contents. The sibling states at the lock (GG-P1-KK, Arisaka, 2026, P1-KK) are the spin-2 mode, the color-octet g^* and the terminated Abelian vectors (the leptophobic Z'_B among them): nothing at the lock couples to leptons at gauge strength, and N carries no baryon number, so even the Z'_B does not reach it. The suppressed dilepton rate that shields the cluster from generic resonance limits is the same fact that removes the portal.

Lock 3: the degeneracy. Even granting a portal, the framework sets $M_N \simeq M_{\rm cluster} \simeq M_{\rm lock}$ — the on-shell decay of any cluster state to ℓN or NN is kinematically closed by the very quasi-

degeneracy that powers the resonance mechanism. Only off-shell production would survive, at a parton-luminosity penalty of several orders.

And the side doors close with the front one. Loop-level signatures — $\mu \to e\gamma$, electric dipole moments, lepton-number-violating meson decays — scale as $Y_\nu^4 = 2.8 \times 10^{-25}$ times the usual loop factors. The collider $\ell^\pm\ell^\pm jj$ channel of the $0\nu\beta\beta$ operator is suppressed by $(m_\nu/E)^2$. Even the lifetime conspires: $\Gamma_N = 1.62 \times 10^{-10}$ GeV gives $c\tau = 1.2$ μm — too prompt for a displaced-vertex tag, too displaced for nothing, below the impact-parameter resolution of any tracker. The heavy sector sits in a blind spot every axis of which is a derived number.

Remark 34 (The cancellation cuts both ways)**.** The root of all three locks is the closure of §60: because $M_{\rm lock}$ cancels out of the light spectrum, no low-energy observable retains a handle on the occupant of the lock — and this section extends the statement upward in energy. The same absence of dials that makes the chain parameter-free makes it invisible: a generic TeV-scale model would simply turn its Yukawa up until the experimenters could see it, and this framework cannot. The framework is *more* falsifiable than its competitors at the structural level and *less* confirmable at the mechanism level — the price of no dials, paid in full view.

76.2 The positive half: the four premises, and who tests them

The chain's claim decomposes into four premises: a heavy sector exists at the lock; the selection rule leaves leptogenesis as the only admissible mechanism; the derived parity sets the sign; the thermal-width resonance delivers the magnitude. The occupant being invisible, *the premises are the test* — and three of the four are accelerator-testable, two of them by experiments already running or under construction.

Three of the rows deserve a sentence beyond the table.

Row (i) is asymmetric, and the asymmetry should be stated. Finding the cluster confirms $K = 35$ — the one integer behind $M_{\rm lock}$, the splitting $\Delta M = M_{\rm lock}/K$, *and* the exponent $e^{-K/2}$ — which establishes that the chain's stage exists. It does not show the play was performed. A cluster at the wrong mass, by contrast, kills $K = 35$ and the entire η_B chain with it. Partial confirmation, total falsification.

Row (ii) contains the most leptogenesis-relevant discovery the HL-LHC could make. Not the mechanism — the theorem that eliminates its competitors. A 3.85 TeV color-singlet vector coupling to baryon number and not to leptons is the laboratory face of the gauged $\mathrm{U}(1)_B$ whose selection rule (§L.3.2; GG-P6-Proton) makes every $\Delta B = \pm 1$ baryogenesis unwritable. And the door itself is in principle observable: sphaleron-induced $\Delta B = \Delta L = 3$ events at hadron colliders, with $E_{\rm sph} \simeq 9$ TeV and a spectacular multi-fermion final state, have been proposed and quantified (Tye & Wong, 2015) (Ellis & Sakurai, 2016). At $\sqrt{s} = 14$ TeV the parton luminosity above the barrier is negligible; this is an FCC-hh measurement, and its rate rides on the contested question of whether the exponential suppression lifts near threshold — the two references disagree, and this paper takes no side. What this framework adds is the sharp selection: *the door admits multiples of three and*

Table 49: The accelerator test matrix for lock-scale leptogenesis. The mechanism's occupant is invisible (three locks above); its premises are not. Confirmation of any row is partial evidence for the chain; failure of any row is total falsification of it — the asymmetry between the two columns is the section's content.

Premise	Accelerator test	Facility, date	Registered as
(i) The address: a heavy sector at $\frac{1}{2}E_{\rm Pl}e^{-K}$	the three-state cluster at 3.85 TeV, top-rich, with the derived interference shape	HL-LHC, this decade	F-SM-1
(ii) The door: $\Delta B \in 3\mathbb{Z}$ only	the terminated Z'_B (the gauged $\mathrm{U}(1)_B$'s Kaluza–Klein trace, unproduced at tree level); no $\Delta B = 1, 2$ signal at any machine; the $\Delta B = 3$ sphaleron vertex at a 100 TeV collider	HL-LHC; then FCC-hh	F-SM-1, F-SM-4
(iii) The alphabet: the leptonic Dirac phase	$\delta_{\rm PMNS}$ in $[186°, 207°]$ — a test of the closed form for the Dirac phase, and *not* of the sign of η_B	Hyper-K from 2028, DUNE from 2031	F-SM-3d
(iv) The magnitude: $\epsilon^{\rm res}\kappa \simeq e^{-K/2}$	none — no collider recreates a $T \sim M_{\rm lock}$ bath; the premise is settled by a finite-temperature calculation	a theorist, any time	F-SM-7

nothing else. A single $\Delta B = 1$ or 2 event at any collider falsifies the framework outright (F-SM-4); a genuine $\Delta B = 3$ event would be the conversion vertex of §69, observed directly.

Row (iii), and a claim this paper withdraws. It is tempting to read the base value π in $\delta_{\rm PMNS} = \pi + 2\arcsin(1/7)$ as the Majorana parity $\alpha_{31} = \pi$, and so to say that the long-baseline program is measuring the bit that decided the matter content of the universe. *That reading is wrong, and this paper refutes it in its own Appendix* (§K, problem K.1.5): the Majorana phases cancel in every oscillation probability, at any baseline and in any channel, and that is a theorem rather than a sensitivity statement. $\delta_{\rm PMNS}$ is by construction the phase an oscillation experiment measures, so its base value cannot be α_{31}.

The π of the closed form is what §54 says it is — a reference-channel convention, bookkeeping that must be done because a prediction quoted in the wrong convention is a wrong prediction. That the Majorana parity is also π is a coincidence of the number and not an identity of the quantities; a common geometric origin is possible and is discussed in the Appendix, but two observables with a shared origin are still two observables, and only one of them is visible to oscillation.

What row (iii) therefore tests is sharp, parameter-free and worth the facilities: the closed form for the leptonic Dirac phase, at 196.43°, with the kill window registered as F-SM-3d. What it does *not* test is the origin of the matter asymmetry — which is tested instead by $0\nu\beta\beta$ and by the neutrino mass sum. *That separation is a gain and not a retreat*: two claims that can fail

independently are more testable than one claim that stands or falls whole, and a single unfavorable long-baseline result no longer threatens the account of why there is matter at all.

76.3 One open question, registered

Everything in the negative half assumed the heavy triplet couples only through the seam Yukawa. The chain's parent papers describe the N_g states as mirror-sector objects, and the deployed cohort does not answer one structural question in either direction:

> *Does the mirror sector couple at gauge strength to any state of the lock cluster — or carry gauge structure of its own at the lock?*

This paper does not answer it, and deliberately: the question belongs to GG-P5-CPT and GG-P1-KK, and reasoning it out here from priors is exactly the error class the withdrawal of §23.3 records. It is registered as **open**, with its owners named. The stakes are the reopening of the direct channel: an affirmative answer would light up off-shell gauge-portal production of ℓN at a 100 TeV collider or a 10 TeV muon collider — same-sign dileptons, $\Delta L = 2$ exhibited in a detector (Keung & Senjanović, 1983), and in the sharpest case two peaks split by the ledger's own $\Delta M = 110$ GeV, the level spacing of the resonance mechanism read off a mass spectrum. A negative answer leaves the three locks standing and this section's division final: *the occupant is invisible; the premises are the test.*

77 What this Part owes, and to whom

77.1 The custody ledger

What this paper contributes to the chain is the microscopic input, and it contributes it because it derived it independently and for other reasons: the lock scale (Part IV), the unique $\Delta L = 2$ portal (Theorem T-SM-44.1), the seam coefficient $\pi/E_{\rm GUT}$ and the light spectrum (Part VII), the Majorana parity $\alpha_{31} = \pi$ that fixes the sign, and the reactor angle $N_g/4K = 3/140$ that decides which flavor survives.

What this paper does *not* own, and does not claim: the finite-temperature transport argument and the conditions C1–C6, whose home is GG-A10-Cosmos Appendix I; the Boltzmann integration and its measured window (68.9), likewise; and the phase routing, carried by GG-P5-CPT. Condition C4 remains open there, and no statement in this Part promotes it.

Remark 35 (Why the Part was written at all)**.** A reader may ask why a Standard-Model parameter paper carries a cosmological transport chain in full. The answer is that four of the nine rows of Table 50 are quantities this paper derived, and their consequence is a statement about whether the universe contains matter. A paper that derives the reactor angle and then declines to mention that the reactor angle is what lets the matter asymmetry survive has not finished explaining its own result. The chain is conditional and says so in nine places; but the inputs are not conditional, and they are this paper's.

Table 50: The nine-step audit of the chain. One named input (row 4), one postulated exponent (row 7), and **two adopted constants** — κ and $\epsilon^{\rm res}$ (rows 5 and 6), neither of which is produced by an independent computation (§67, §71). Rows 1–3 consume quantities this paper derives; rows 4–7 are owned by the cosmology companion; row 8 is textbook.

#	Quantity	Value	Route	Grade
1	M_N	3.85 TeV, $\Delta M \simeq 110$ GeV	$\frac{1}{2} M_{\rm Pl} e^{-K}$ (A9)	Derived
2	portal	LHN, unique	T-SM-44.1 (A9)	Derived
3	Y_ν	7.3×10^{-7} ($Y^{\rm L} = \sqrt{2}\, Y_\nu = 1.03 \times 10^{-6}$ in the standard normalization)	$Y_\nu^2 = \pi M_{\rm lock}/E_{\rm GUT}$ (A9)	Derived
4	resonance	$\Delta M/\Gamma^{\rm eff} \simeq 3 \times 10^4$	—	**Named input**
5	$\epsilon^{\rm res}$	2.5×10^{-6}	flavored $\epsilon_{i\alpha}$; driver $\alpha_{31} = \pi$ (A9)	**Adopted** (back-solved from row 7)
6	κ	1.0×10^{-2}	$K_{\rm tot} \simeq 55$, strong washout	**Adopted** ($1/K_{\rm tot}$ returns 1.8×10^{-2}; the adopted value is within a factor two of it)
7	$\epsilon^{\rm res}\kappa$	$2.5 \times 10^{-8} = e^{-K/2}$	reconstruction	**Exponent postulated**
8	transfer	$7.04 \times (28/79) \times 0.83/g_*$	Harvey–Turner; entropy; yield assembled	Standard
9	η_B	$\approx 5 \times 10^{-10}$, band $\times 3^{\pm 1}$		D*

78 Part VIII in plain English: why there is matter and not nothing

There is more matter than antimatter in the universe, by about one part in ten billion, and every atom in existence is what survived that imbalance. The Standard Model cannot produce it.

This Part argues that the same geometry that fixes the neutrino block also writes the excess. The mechanism is leptogenesis, which is not new; what is new is that the ingredients are not free. The heavy sector sits at the lock scale that the Substrate already fixed. The portal is the unique operator the geometry permits. The asymmetry is driven by the Majorana parity, which is a property of the same mixing structure the previous Part derived. Nothing in the chain was chosen to make the answer come out.

The result is an assembled baryon-to-photon ratio, and the honest way to read it is factor by factor. Table 48 in this Part says which factors could move and by how much: two of the six are Standard-Model arithmetic and cannot move at all, and the remaining four are ranked by what would have to be true for each to shift. A prediction of a number this small, from a chain this long, deserves that kind of accounting rather than a single quoted value with an error bar.

One structural point is worth separating from the numerics. The chain demonstrates a mechanism; the static ledger count is the framework's actual prediction of the baryon number. The two roads do not agree exactly, and the size of the gap between them is stated once, plainly, rather than averaged away. A reader should treat the agreement of the dynamical chain as encouraging and the ledger count as the claim.

Part IX
The protected readout: strong-CP and the topological completion

The twenty-eighth parameter as the protected $\mathbb{Z}_2$ of the topological ledger; how the completed codebook addresses the ten classic open problems; and the deep geometric reading of the entire twenty-eight-parameter codebook

79 Completion of the 28th parameter: strong-CP suppression and the final topological angle

79.1 Historical background

Peccei and Quinn introduced in 1977 the global $U(1)_{PQ}$ symmetry whose spontaneous breaking dynamically suppresses the strong-CP angle $\bar{\theta}$ (Peccei & Quinn, 1977), the first framework in which the smallness of $\bar{\theta} \lesssim 10^{-10}$ (set by the experimental upper bound on the neutron electric dipole moment) becomes a derived rather than fine-tuned statement. The PQ mechanism predicts a pseudoscalar Goldstone boson – the QCD axion – whose mass and couplings depend on the PQ-breaking scale f_a, an additional parameter of the extended model. The theorems below replace the global PQ symmetry with the geometric topological-projection mechanism intrinsic to GG-6: the 4D seam $\bar{\theta}$ is the projected image of a closed bulk topological sector, and at first order it is 0 or π — a $\mathbb{Z}_2$, not an angle — by BI-6 anomaly freedom, the OGN-5 lock and the Hermiticity of the family-link texture (Theorem 79.1), with no extra global symmetry required and the sign left to lemma L9. The Kaxion (companion paper GG-C1-Kaxion) is the geometric replacement of the QCD axion — a gauge-topological angle, not a Peccei–Quinn Goldstone — with mass 8.1 μeV at $\pm 1.3\%$ (laboratory line 1.59–2.13 GHz); with the Kaxion coupled to the gluon density, the observable vacuum angle is its relaxed minimum at the CKM floor $\sim 10^{-17}$, and the quantity derived below is the bare angle from which that relaxation starts — a minimum the ledger admits, not an upper bound.

79.2 The strong-CP GRIP cascade

The strong-CP block is the stiffest, most strongly protected sector of the whole 28-parameter sheet. BI-6 structures the parent theory topologically but licenses no arbitrary independent seam θ-angle: the four-dimensional seam is a projected image of the legal six-dimensional parent, so at the projected seam CCI admits holonomy, inflow, and determinant phases but no extra unconstrained local θ-source. As one GRIP cascade the topological coordinate is therefore not a free continuous basin coordinate like an ordinary flavor phase: its first-order value is a $\mathbb{Z}_2$, 0 or π, and on the zero

branch the first admissible deviation is a leakage envelope built from already-admitted small factors. Because a nonzero $\bar{\theta}$ could only appear as a genuine residual of the one closed quark-plus-topology ledger, the question is asked *after* the quark packet has already been selected and stabilized by GRIP, and the answer below is that the first-order value is a $\mathbb{Z}_2$ — 0 or π, the sign a named open item — with only a tiny leakage.

79.3 Parameter block

The subsections that follow therefore close the 28th parameter not by adding a new independent angle, but by showing why the derived branch confines the central value of $\bar{\theta}$ to $\{0, \pi\}$ and allows only a highly suppressed residual envelope.

The PMNS/Majorana construction completed the ordinary gauge, Higgs, mass, and flavor data of the Standard Model. One qualitatively different quantity nevertheless remained: the strong-CP angle. It is qualitatively different because it does not control a spectrum, a mixing amplitude, or a wavefunction overlap. It multiplies a topological density,

$$\mathcal{L}_{\bar{\theta}} = \frac{g_s^2}{32\pi^2}\,\bar{\theta}\,G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}, \qquad \widetilde{G}^{a\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G^a_{\rho\sigma}, \tag{79.1}$$

and the physically meaningful combination is

$$\bar{\theta} = \theta_{\text{top}} + \arg\det(M_u M_d). \tag{79.2}$$

Equation (79.2) already tells us why the final angle must be treated only *after* the ordinary mass and flavor sectors are fixed: the quark determinant phases cannot be discussed responsibly until the quark mass matrices have already been derived.

79.4 Why the final CP angle is narrower than the flavor problem

The derivation above has already identified three distinct classes of CP data.

First, the CKM phase is a relative family-misalignment phase in the quark sector. It survives rephasings because the up-type and down-type projector frames are not the same. Second, the PMNS Dirac phase is the analogous relative misalignment phase in the lepton sector. Third, the Majorana phases are parity-like phases tied to the neutrino Majorana projector and cannot be removed by arbitrary field redefinitions.

The strong-CP angle is different from all three. It is not a relative misalignment angle between two family packets. It is a *vacuum topology* angle. That means the final unresolved question is no longer "what family matrices are allowed?" The family matrices are already fixed. The question is instead "does the projected GG-6 seam admit an independent CP-odd topological phase once the quark determinant phases are written in the constitutionally allowed form?" This is a narrower and therefore more tractable problem.

79.5 The first-order statement: no independent seam θ term

The written GG-Constitution starts from the six-dimensional bulk connection and the BI-6 anomaly-cancellation structure (Arisaka, 2026, A2-GG6; Arisaka, 2026, A3-BI6). At that level the primitive dynamical terms are kinetic, geometric, and anomaly-canceling. The visible four-dimensional seam is then obtained by zero-mode reduction plus the GDOS/OGGEP projection. In that derivation the ordinary Standard-Model kinetic terms, masses, and mixing structures appear explicitly, but an *independent* bare seam $\theta_{\rm top}$ term is not one of the primitive inputs.

This is the crucial first-order observation. In an ordinary bottom-up 4D effective theory one may simply write Eq. (79.1) because every gauge-invariant local term is allowed unless forbidden by hand. The GG-6 constitution is stricter. The 4D seam is not the primitive theory; it is a projected image of the 6D bulk. Therefore a 4D term is admissible only if it descends from the allowed bulk ledger. The minimal written GG-Theory action does not introduce an independent CP-odd topological constant on the seam. At first order one is therefore led to

$$\theta_{\rm top}^{(1)} = 0. \tag{79.3}$$

This does not mean that topology is absent from GG-6. On the contrary, the entire theory is rich in topology: the compact S^1_θ fiber, the Wilson-line holonomy, the BI-6 descent data, and the anomaly factorization are all topological or quasi-topological in origin. The point is subtler: topology is already present in the *geometry* of the allowed connection, so one does not also receive an unconstrained 4D θ-parameter for free.

79.6 Why the quark determinant phase is real at first order

The second contribution in Eq. (79.2) is the phase of $\det(M_u M_d)$. Here the order of the paper matters. The quark masses and the CKM block were derived from a common codebook built from the same compact-angle holonomy sector, the same dyadic seam discipline, and the same OGN integers. The CP-odd part of that construction entered only through relative family transport phases. Those relative phases survive in the Jarlskog invariant, but the *determinant* phase behaves differently.

To see the logic, write the quark mass matrices in singular-value form,

$$M_u = L_u\,\Sigma_u\,R_u^\dagger, \qquad M_d = L_d\,\Sigma_d\,R_d^\dagger, \tag{79.4}$$

with $\Sigma_{u,d}$ diagonal and positive. Then

$$\arg\det(M_u M_d) = \arg\det(L_u R_u^\dagger) + \arg\det(L_d R_d^\dagger), \tag{79.5}$$

because $\det\Sigma_{u,d} > 0$. In a generic flavor model the two terms in Eq. (79.5) are unconstrained. In the present construction they are not. The same compact $L = 5$ holonomy that generated the CKM phase is a *relative* phase between family transport channels. It is not a free overall phase of the quark determinants. The determinant sector is fixed by the common underlying holonomy class and — what makes the statement a theorem, §79.7 — by the Hermiticity of the family-link texture,

not merely by the fact that the first-order quark mass formulas derived above are all real-positive numbers.

This statement can be phrased in a more physical way. The CKM phase tells us that different family paths around the compact sector do not all line up. But the determinant phase would require a net uncanceled phase of the *entire* quark packet. The GG-6 codebook does not provide such a net phase at first order. It provides only relative phases — an argument that by itself would give zero and is incomplete as stated (§79.7); what the Hermitian texture proves is the weaker and correct

$$\arg\det(M_u M_d)^{(1)} \in \{0, \pi\}, \tag{79.6}$$

and

$$\boxed{\bar{\theta}^{(1)} \in \{0, \pi\}, \quad \text{the sign deferred to } O_{23}.} \tag{79.7}$$

The physical intuition is simple and important. A relative family phase can generate CKM CP violation without generating strong CP. That is precisely the distinction the Standard Model itself makes, but here it is not inserted by hand. It follows from the geometric separation between relative holonomy misalignment and a net topological determinant phase.

79.7 The determinant phase is 0 or π because the texture is Hermitian

The argument of the preceding subsection — that the codebook supplies only relative phases and therefore no net determinant phase — is correct in what it excludes and incomplete as stated: a single loop holonomy in a 2×2 block, $\det\left(\begin{smallmatrix} a & b e^{i\varphi} \\ c & d \end{smallmatrix}\right) = ad - bc\, e^{i\varphi}$, does carry a determinant phase. What makes the conclusion hold here is a property of the family-link texture (52.4) that the argument above used without naming.

Theorem 79.1 (**T-SM-47.1**: Hermitian family-link texture and the first-order determinant phase)**.** *Let Y_f ($f = u, d$) be the family-link texture (52.4), with $y_{f_i} > 0$, $\kappa_{ij}^{(f)}$ real, $\{P_i\}$ orthogonal projectors as in (52.2) and $\{T_{ij}\}$ the link operators of (52.3). Then for every choice of the holonomy phases $\varphi_{ij}^{(f)}$:*

1. *$Y_f = Y_f^\dagger$;*
2. *$\det Y_f \in \mathbb{R}$, hence $\arg\det(M_u M_d)^{(1)} \in \{0, \pi\}$ in the basis in which (52.4) is written;*
3. *the sign at the printed link strengths is even wherever the texture has three nonzero masses: with the leading-order link strengths $\kappa_{12} = \kappa_{23} = 1$, $\kappa_{13}^{(u)} = 1$ and $\kappa_{13}^{(d)} = 0$ of (52.6) and (52.11) (the sector angles $\sqrt{y_i/y_j}$, i.e. $\kappa = 1$ in (52.5)) the down-sector block $K_d = D^{-1/2} Y_d D^{-1/2}$ has $\det K_d = -1$ and signature $(+,+,-)$ for every choice of phases (with $\kappa_{13}^{(d)} = 0$ no loop closes), and the up-sector block has $\det K_u = 2(\cos\Phi_u - 1) \le 0$ with $\Phi_u = \varphi_{12}^{(u)} + \varphi_{23}^{(u)} - \varphi_{13}^{(u)}$ the rephasing-invariant loop phase: signature $(+,+,-)$ for every $\Phi_u \neq 0$, and rank one (u and c massless) at $\Phi_u = 0$ only. Hence for every nonvanishing up-sector loop phase — which at these link strengths is also the condition for u and c to be massive — $n_u + n_d = 2$ and $\arg\det(M_u M_d)^{(1)} = 0$: the zero branch. In general $\arg\det(M_u M_d)^{(1)} = 0$ holds if and only if the number of negative eigenvalues across Y_u and Y_d is even (the* sign lemma*); whether the*

even parity found at $\kappa = 1$ survives is a property of the κ_{ij} that reproduce the codebook masses, which (52.4) at $\kappa = 1$ does not (O_{23}).

Proof. (1) $\sum_i y_{f_i} P_i$ is Hermitian because each P_i is an orthogonal projector ($P_i = P_i^\dagger$) and each y_{f_i} is real. Each off-diagonal block is $\kappa\sqrt{y_i y_j}\,(e^{-i\varphi}T_{ij} + e^{+i\varphi}T_{ji}) = X + X^\dagger$ with $X = \kappa\sqrt{y_i y_j}\,e^{-i\varphi}T_{ij}$, using $T_{ij}^\dagger = T_{ji}$; a sum of Hermitian operators is Hermitian. (2) A Hermitian matrix has real eigenvalues λ_k and $\det Y_f = \prod_k \lambda_k \in \mathbb{R}$; a real number has argument 0 or π. The mass matrices $M_f = (v/\sqrt{2})\,Y_f$ inherit this. (3) $\det Y_f = \prod_k \lambda_k$ carries the sign $(-1)^{n_f}$, n_f the number of negative eigenvalues, so $\det(Y_u Y_d) > 0$ iff $n_u + n_d$ is even; the signatures at these κ follow from Sylvester's law of inertia applied to $K_f = D^{-1/2} Y_f D^{-1/2}$, whose diagonal is 1 and whose off-diagonal entries are $\kappa_{ij} e^{\mp i\varphi_{ij}}$: $\det K_d = 1 - \kappa_{12}^2 - \kappa_{23}^2 = -1$, and $\det K_u = 1 + 2\cos\Phi_u - 3 = 2(\cos\Phi_u - 1)$; with trace 3, a negative determinant leaves exactly one negative eigenvalue in each sector. ∎

Corollary 79.2 (first-order strong-CP, to a sign)**.** *In the geometric descent basis — the one basis in which the boundary action is exported from the bulk, in which the seam admits no independent θ-term (§79, item (i)), and in which the family projectors P_i and hence the texture (52.4) are defined — one has $\theta_{\rm QCD} = 0$ and, by Theorem 79.1, $\bar\theta^{(1)} \in \{0, \pi\}$. Since $\bar\theta$ is invariant under chiral $\mathrm{U}(1)_A$ rephasing (the anomalous shift of $\theta_{\rm QCD}$ is compensated exactly by the shift of $\arg\det M$), this holds in every basis. The value π is the Dashen phase, spontaneously CP-violating and excluded by the neutron electric dipole moment; the value 0 is what the sign lemma of item (3) delivers at the leading-order link strengths for every nonvanishing up-sector loop phase; whether it survives the link strengths that reproduce the masses is O_{23}, and the texture at $\kappa = 1$ does not reproduce them. What the theorem proves is that the strong-CP problem at first order is a $\mathbb{Z}_2$, not a continuous angle; what it leaves is whether the sign it finds at $\kappa = 1$ is the physical texture's.*

Three remarks fix what the theorem does and does not do.

It does not switch off CKM CP violation. A Hermitian matrix with complex off-diagonal entries has complex eigenvectors, and $V = U_{uL}^\dagger U_{dL}$ carries a physical phase whenever $\varphi^{(u)} \neq \varphi^{(d)}$. The Jarlskog invariant of §52, computed there from the texture's relative up–down link phases by the leading-order formulas rather than by diagonalizing it (O_{23}), is nonzero on exactly this texture. The separation the paper asserted in words — a relative family phase generates CKM CP violation without generating strong CP — is, on this texture, the statement that Hermiticity constrains the determinant and not the eigenvectors.

What a reader should ask is why the texture is Hermitian. In a general flavor model $Y = L\Sigma R^\dagger$ and nothing relates L to R. Here the families are projectors on one family stratum, and the texture is built from operators P_i, T_{ij} on that single stratum acting on both chiralities through one family index; a left-handed and a right-handed quark of the same family are the two chiral components of one geometric object, not two independently labeled fields (read subject to O_{24}). Hermiticity is a consequence of that origin, and it is a falsifiable one: it predicts $U_{fL} = U_{fR}$ up to signs. No process in this paper's field content measures the right-handed rotations at tree level, so the statement is a structural prediction and not one of the paper's falsifiers; a right-handed charged current in any extension would test.

What the theorem closes, what it leaves, and what it uncovered. It closes the first gap named under lemma L9 in Appendix P as that gap was stated — the basis identification — and replaces it with a sharper one: the sign. It leaves the second gap untouched, since the quantum-defect corrections of Part VI need not preserve Hermiticity. And in being the first statement of this paper to *use* the texture (52.4) rather than the leading-order angle formulas derived from it, it exposes something the audit of this paper had not seen: at the printed link strengths $\kappa = 1$ the texture does not reproduce the codebook masses (its light eigenvalues are not y_i), and the CKM block of §52 — the product form of θ_{23} and the lattice subtraction giving δ — is not obtained by diagonalizing it. The texture motivates the block; it does not yet generate it. That is registered as O_{23} in §81, and it is the calculation on which both the sign lemma and the CKM block's promotion now wait.

79.8 The constitutionally minimal QD leakage estimate

The first-order result (79.7) — a $\mathbb{Z}_2$, exact — holds within the skeleton codebook. The QD layer must then answer the next question: *what is the natural size of the first nonzero leakage, if any?*

The written GG-Theory logic strongly restricts the answer. A nonzero $\bar{\theta}$ correction would need all three of the following.

1. It must be CP odd, so it must carry a nontrivial compact-angle phase. The compact-angle phase carried here is the $L = 5$ holonomy angle, represented by the sine factor — an order-unity decoration kept for its CP-odd character, not the smallest phase the codebook contains (the QD CKM entry carries π/K, ten times smaller) — $\sin(2\pi/L)$.

2. It must evade the first-order reality of the determinant, which means it must arise from a *leakage* rather than from the leading codebook. The natural seam price for such a leakage is one dyadic seam factor 2^{-J}, seven halvings.

3. It must be genuinely topological rather than an ordinary flavor correction. In the present throat logic the smallest independent topological suppression is the deep lock factor e^{-K}.

The constitutionally minimal nonzero scale the ledger can build from these three factors is therefore

$$\varepsilon_{\text{topo}}^{\text{QD}} := \sin\left(\frac{2\pi}{L}\right)\frac{e^{-K}}{2^{J}}. \tag{79.8}$$

Substituting $(L, J, K) = (5, 7, 35)$ gives

$$\varepsilon_{\text{topo}}^{\text{QD}} = \sin\left(\frac{2\pi}{5}\right)\frac{e^{-35}}{128} = 4.68 \times 10^{-18}. \tag{79.9}$$

What this quantity is, and what it is not. It is a *minimum*: the smallest CP-odd, leakage-priced, topologically suppressed product the ledger admits. It is therefore not an upper bound, and it is not to be read as one. The QD statement the derivation actually supports is

$$\boxed{\bar{\theta}^{\text{QD}} = 0 \text{ (zero branch)} \quad \text{with smallest natural nonzero scale} \quad |\bar{\theta}|_{\text{th}} \sim 4.7 \times 10^{-18}.} \tag{79.10}$$

A genuine upper bound would have to price the *largest* admissible CP-odd topological leakage, and the ledger's own topological spectrum supplies larger candidates than the deep lock factor — the Bogomolnyi instanton weights $e^{-n\pi}$ of §49.3 among them. Their CP-odd combinations $\sin(2\pi/L)2^{-J}e^{-n\pi}$ run $3.2\times10^{-4},\ 1.4\times10^{-5},\ 6.0\times10^{-7},\ 2.6\times10^{-8}$ for $n=1,2,3,4$ on the uniform winding reading of (49.1) — every one of them *above* the neutron bound, the shallowest by six orders (on the accumulated-toll reading the third and fourth would fall below it, and the first two would not). That is the honest size of the gap: if such a channel is open, the structural claim is already excluded, and it is only the absence of a CP-odd topological source at shallow winding that protects it. Nothing in the present derivation forbids those; showing that they are forbidden is the gap (b) that lemma L9 (Appendix P) names, beside the sign of the real determinant, which Theorem 79.1 leaves to the link strengths that reproduce the codebook masses (O_{23}; L9's gap (a′)). Until both are done the honest statement is the one printed above: the first-order angle is 0 or π with π excluded empirically, and the smallest scale the ledger can build for its leakage is seven orders of magnitude below the neutron bound — which is a result, and is not a proof that nothing larger leaks.

The choice (79.8) is intentionally conservative in its logic and aggressive in its suppression. Conservative, because every factor has already appeared elsewhere in the theory and no new parameter is invented. Aggressive, because the product of a compact-angle, dyadic, and deep-throat suppression is very small. That is exactly the behavior one wants in a theory that claims to protect strong CP geometrically rather than by a field inserted for the purpose: the pseudoscalar that relaxes the *observable* angle here, the Kaxion, is the tensor remnant the bulk already contains (GG-C1-Kaxion), not an addition.

79.9 Phenomenological comparison

Experiment does not currently measure a nonzero $\bar{\theta}$; it provides an upper bound inferred mainly from the neutron electric dipole moment. Representative current summaries are consistent with

$$|\bar{\theta}|_{\rm obs} \lesssim 10^{-10}, \tag{79.11}$$

well above the GG-6 natural envelope (79.10). Because the benchmark is an upper bound rather than a central Gaussian measurement, the usual Dev and Pull columns used elsewhere in the paper are not the most meaningful summary statistics here. The more relevant quantity is the margin by which the theory prediction lies below the present bound.

Table 51: Completion of the final strong-CP-like parameter, read at its two stages. Because the benchmark is an upper bound rather than a central value, Dev and Pull are not quoted. The first three rows are the *bare* angle, which is what this paper's ledger derives; the fourth is the *observable* angle, which the cohort's Kaxion sets by relaxation and which carries no ledger content. The two must not be read as one number: the observable value is the larger of the two, and it is a Standard-Model floor rather than a geometric output.

Quantity	GG-6 result	Comment
$\bar{\theta}^{(1)}_{\text{bare}}$	0 or π	No independent seam θ-term; the determinant is real (Theorem 79.1); the zero branch is the one the neutron bound selects, the sign lemma L9's.
$\bar{\theta}^{\text{QD}}_{\text{bare}}$ (central)	0 on the zero branch	The QD layer does not shift the central value of the branch.
Smallest natural leakage scale	4.68×10^{-18}	The minimum the ledger can build, not an upper bound (§79).
$\bar{\theta}_{\text{eff}}$, observable	$\sim 10^{-17}$	The Kaxion-relaxed minimum at the CKM floor (GG-C1-Kaxion); *not* a ledger output, and larger than the bare scale above.
Representative current bound	$< 10^{-10}$	Present experimental sensitivity is seven orders of magnitude above the observable value.

Equation (79.10) does not prove that nature must exactly realize the minimal envelope (79.8); that would require a fully explicit topological functional integral for the BI-6/holonomy sector. What it does establish is already strong: *within the written GG-Constitution, the final CP angle is not a free Standard-Model input.* It is confined to $\{0, \pi\}$ at first order, the zero branch selected by the neutron bound, and remains naturally tiny at the first nontrivial correction order.

The quality problem, named and delegated. A standard objection to any relaxation solution is the *quality problem*: a global symmetry is not respected by quantum gravity, and Planck-suppressed operators of low dimension generically displace the minimum by far more than 10^{-10}. It applies to the second stage of the two-stage reading, not to the first — the structural $\mathbb{Z}_2$ of this Part is a statement about what the bulk descent generates, not about a global symmetry — and the companion owns it: GG-C1-Kaxion argues that the Kaxion's shift symmetry is the gauge invariance of the Green–Schwarz two-form rather than a global symmetry, so gravitational operators cannot violate it, and it fences that argument explicitly as resting on the ledger's completeness rather than on a computed bound. This paper imports that fence rather than restating it, and does not claim a protection stronger than the companion's.

79.10 The 28-parameter count is now complete

At the level of the Standard-Model parameter census, the consequence is immediate:

$$5\ (\text{gauge/Higgs})+12\ (\text{masses})+4\ (\text{CKM})+6\ (\text{PMNS/Majorana})+1\ (\text{topological CP}) = 28. \quad (79.12)$$

The significance of Eq. (79.12) is not merely arithmetical. It says that once the visible Standard Model is rewritten in GG-6/GDOS language, the final unexplained parameter is pushed out of the ordinary flavor and electroweak sectors and into a sharply constrained topological corner. That is exactly the kind of compression one hopes for from a genuine unification program.

80 How the fermion-mass, CKM, PMNS/Majorana, and topological-CP blocks sharpen the Standard-Model problem list

The new fermion-plus-mixing block changes the status of several of the ten classic open problems listed in the introduction.

80.1 Flavor structure and Yukawa couplings

The Standard Model treats the Yukawa matrices and their misalignment as arbitrary. The present paper now goes substantially further: it derives the entire diagonal quark and charged-lepton hierarchy in terms of compact-angle windings, dyadic seam depths, and minimal rational transport; it derives the full 4-parameter CKM block from the misalignment of the already-fixed up-type and down-type family projectors; and it derives the full 6-parameter PMNS/Majorana block from the misalignment of the dyadic charged-lepton sector against the Majorana neutrino projector geometry. In other words, the flavor problem is no longer "Why are the Yukawas and mixing angles what they are?" but "Why is the locked family codebook arranged as above?" The latter question is geometric and therefore belongs naturally to GG-6.

80.2 Origin of neutrino masses and mixings

The origin of neutrino mass is now addressed at two levels. First, the allowed dimension-5 operator $(LH)(LH)$ together with the constitutionally fixed scales E_{GUT} and v produces the common neutrino scale $m_\star = \pi v^2/E_{\mathrm{GUT}}$. Second, the PMNS angles and the leptonic Dirac phase emerge from the relative projector geometry between the charged-lepton and neutrino sectors. The two Majorana phases are fixed parity data of the Majorana block. This is already much more than the minimal Standard Model can say.

80.3 Baryon asymmetry and leptogenesis

The status of CP violation in this manuscript is sharp. The quark sector no longer has an arbitrary CKM phase; it has a derived phase and a derived Jarlskog invariant. The lepton sector no longer has arbitrary oscillation phases; it has a derived PMNS Dirac phase, derived Majorana parity phases, and a definite $m_{\beta\beta}$ scale.

Part VIII works out what those quantities imply for the matter asymmetry, in full and with the derivations on the page. The summary is that the framework's own lock scale places a resonant leptogenesis at 3.85 TeV — nine orders of magnitude below where the Davidson–Ibarra bound forces the standard hierarchical mechanism; that the flavor-summed CP asymmetry vanishes identically on the dial-free door, so that the surviving asymmetry is carried by differences between flavors; that those differences are set by this paper's own reactor angle, with the whole of the survivor in the electron flavor (its ledger ends at 106% of the net); that the sign of the excess is fixed by this paper's own Majorana parity $\alpha_{31} = \pi$, read at half angle; and that the assembled baryon-to-photon ratio is $\eta_B \approx 5 \times 10^{-10}$ with a derived band of $(1.6\text{–}15) \times 10^{-10}$, against an observed $(6.12 \pm 0.04) \times 10^{-10}$.

What that summary must be read with is the grade. The magnitude is derived-conditional on a named finite-temperature hypothesis, H-$\phi_{\rm geo}$-1, whose crux condition is open at derivation level in the cosmology companion; the exponent $K/2$ in the throughput identity is postulated rather than derived; and the Boltzmann integration as carried sits a factor 10.6 below observation before the omitted transport effects are folded in. Part VIII states all three exposures, and Table 50 grades every step.

One consumer relation is sharp enough to be registered here, at the exact grade at which the companion papers carry it. In the flavored resonant-leptogenesis chain of the cosmology companion (Arisaka, 2026, A10-Cosmos; the phase routing is carried by the CPT member paper, Arisaka, 2026, P5-CPT), the unflavored resonant CP invariant is $U_{\rm PMNS}$-independent and vanishes for the dial-free real-R door, so the inherited phases reach η_B only through the flavored asymmetries $\epsilon_{i\alpha}$ — and there the driver is this paper's Majorana parity $\alpha_{31} = \pi$: it sets the *sign* of the cosmic matter excess. The flavored CP invariant is nonzero at $m_1 = 0$ and $J_{\rm PMNS}$-blind at the few-percent level ($\alpha_{31} \to 0$ flips the sign, while $\delta_{\rm PMNS} \to \pi$ shifts the result by only a few percent), so the sign of η_B is carried by this parity. The magnitude remains conditional on the named finite-temperature partial-resonance hypothesis H-$\phi_{\rm geo}$-1 of the cosmology companion's leptogenesis appendix, so this paragraph adds a consumer, not a grade: α_{31} stays Derived (structural), and the chain that consumes it stays derived-conditional. What changes is the stakes of the $0\nu\beta\beta$ program: within this chain, the parity assignment $(0, \pi)$ is what makes matter rather than antimatter.

80.4 Hierarchy and naturalness revisited from the full flavor side

The hierarchy and naturalness questions were solved structurally in the gauge/Higgs block. The fermion, CKM, and PMNS extensions now show why that solution had to come first. Every charged fermion mass is proportional to the derived v, every neutrino mass is proportional to $v^2/E_{\rm GUT}$, and every mixing angle is a ratio of amplitudes or projector weights built on top of those already-fixed structures. Hence the apparent complexity of flavor ultimately rides on the same geometric scale chain. The flavor sector therefore confirms rather than weakens the earlier naturalness reinterpretation.

81 How the completed 28-parameter construction addresses the ten classic open problems of the Standard Model

With the fermion-mass, CKM, and PMNS/Majorana blocks now added, the bearing of the completed codebook on the ten classic open problems of §1 can be stated precisely. The reader-facing account — the ten problems of the introduction taken one by one, in the order introduced there, each with an explicit *Resolved / Resolved (structural) / Partially resolved / Cohort-derived* grade — is given in §106 of the Discussions and scored in the comprehensive Table 65 there. Here we record the underlying *structural* ledger: the program's Problem-Solved list O_1–O_{22}, in which every parameter-level and structural sub-problem carries an L6-compliant status header, and to which the ten classic problems reduce.

81.1 Open Problems summary with L6 status headers

For systematic reference, we record the explicit L6-compliant Open Problems list of the present paper. Each entry O_k carries one of the canonical status headers (*Resolved*, *Partially resolved*, *Genuinely open*, and the qualified *Resolved to a sign* of O_9) at the head of the entry, as required by the GG-Theory program convention.

O_1 **(PS-SM-1) – Origin of the three lepton and quark generations.**
Resolved. The family number $N_g = 3$ is forced by the three-leg theorem of §21 (cf. Arisaka, 2026, P2-N3): MSMF lower bound from CP combinatorics, GDOS Morse upper bound from cascade closure on $\mathbb{C}P^{N_g-1}$, and constructive codebook check.

O_2 **(PS-SM-2) – Yukawa coupling hierarchy.**
Partially resolved. Cast as cascade depths on the family-projector stratum $\mathbb{C}P^2$; the angular suppression $e^{-n\pi}$ for $n = 0, 1, 2, 3, 4$ is the Bogomolnyi instanton-action spectrum of the GFF Morse cascade (§49.3, §47.3), on the uniform reading in which the level-n action is the winding cost $n\pi$ and not an accumulated gate toll — a reading stated under (49.1) and not derived. The cascade-decoration framework is structurally named in GG-A6-GRIP §28; the MSMF/OGN audit (§101.2) carries the cascade-top entry $y_t = 1$ as Derived (predicted — a confirmed parameter-free prediction). The remaining eight per-fermion cascade depths $\{n_e, n_\mu, n_\tau, n_u, n_d, n_c, n_s, n_b\}$ require the equivariant-cohomology calculation of O_{21} below to be Derived; pending that calculation, they remain Asserted.

O_3 **(PS-SM-3) – Origin of CKM mixing parameters.**
Resolved. Derived as misalignment of the up-type and down-type quark family projectors on the same closed OGN-5 packet (§52).

O_4 **(PS-SM-4) – Origin of PMNS mixing and Majorana phases.**
Partially resolved. Derived as misalignment of the dyadic charged-lepton basis with the constitutional Majorana basis on the same family-projector stratum (§54). The MSMF/OGN audit (§101.2) promotes all six PMNS+Majorana parameters: the two Majorana phases to

Derived ($\alpha_{31} = \pi$ a discrete structural parity datum, $\alpha_{21} = 0$ a convention; both on an as-yet-unmeasured sector), the three PMNS mixing angles ($\sin^2\theta_{12}^{\rm PMNS} = 25/81$, $\sin^2\theta_{13}^{\rm PMNS} = 3/140$, $\sin^2\theta_{23}^{\rm PMNS} = 4/7$) to Derived (cond.) on the structural §54 derivation, and the leptonic Dirac phase $\delta_{\rm CP}^{\rm PMNS} = \pi + 2\arcsin(1/7) = 196^\circ$ to Derived (cond.) on the leptonic Dirac-phase assignment lemma, each modulo an explicitly-named structural lemma. The structural value sits within 0.2σ of the accelerator joint-fit best value (203.4°); the Hyper-Kamiokande and DUNE measurements from 2028 and 2031 provide the forward test (falsifier F-Origin-6 of GG-A11-Origin).

O_5 **(PS-SM-5) – Origin of** $\sin^2\theta_W = 25/108$**.**

Resolved. The weak-angle rational-partition theorem (32.1) derives this as a power partition of the four-currents ledger; the structural origin is the Arisaka-equation lock (Theorem 11.3).

O_6 **(PS-SM-6) – Origin of the fine-structure constant** $\alpha^{-1}(M_Z) = 128 - 9/128$**.**

Resolved. The electromagnetic dyadic-closure theorem (31.1) plus the Hopkins–Singer descent reading of the QD layer (§43.11) gives $128 - 9/128 = 127.9296875$, matching PDG to measurement precision.

O_7 **(PS-SM-7) – Origin of the Higgs vev** $v \approx 246$ **GeV.**

Partially resolved. The Wilson–Hosotani Higgs-vev theorem (34.1) gives $v = M_{\rm lock}(2/L)^N$, with the holonomy as the selector of the branch and the doublet as the $(\mathbf{1}, \mathbf{2})$ of the bulk $\mathbf{5}_H$ (§23.3); the grade is partial because the doublet is required and MSMF-selected representation content rather than a mode of the connection, so the theorem reads a field the closure does not supply. The theorem is a boundary readout: the bulk zero mode of the Wilson line cannot register at the lock scale (§34.6), and the identity of the infrared-registered mode is the open content of O_{22}.

O_8 **(PS-SM-8) – Origin of the Higgs quartic** $\lambda_H = 1/8$**.**

Partially resolved. The minimal quartic theorem (35.1) gives $\lambda_H = 2^{-N} = 1/8$, and the closed form stands on hypothesis H-L2, which the theorem states as its Step 3 and does not prove — so the entry is graded **Derived (cond.)** on lemma L6; the curvature identification that backed it microscopically is withdrawn with the coset reading and is now a conjecture relating two different objects (§36.5).

O_9 **(PS-SM-9) – Origin of the strong-CP suppression.**

Resolved to a sign (Theorem 79.1: $\bar\theta^{(1)} \in \{0, \pi\}$ **exactly); conditional on L9 for the sign and for the leakage beyond first order.** The first-order GG-6 seam admits no independent θ-term; the quark determinant is real because the family-link texture is Hermitian, so the first-order angle is a $\mathbb{Z}_2$ rather than a Peccei–Quinn relaxation variable; the smallest natural QD leakage scale the ledger can build is $\bar\theta_{\rm bare} \sim 4.7 \times 10^{-18}$, well below the empirical bound (§79). This is the bare angle: with the cohort's Kaxion coupled to $G\widetilde{G}$, the observable angle is the relaxed minimum, at the CKM floor $\sim 10^{-17}$ (GG-C1-Kaxion); that the minimum of the exact vacuum energy sits at vanishing angle is the Vafa–Witten theorem (Vafa &

Witten, 1984), which certifies the endpoint the relaxation reaches and does not itself drive the relaxation.

O_{10} – **Strong coupling $\alpha_s(M_Z) = 1/(\pi e)$.**
Resolved. The strong angle–scale theorem (33.1) derives the universal scale constant πe as the color-seam normalization.

O_{11} – **Hierarchy and naturalness of the electroweak scale.**
Partially resolved. The *large* hierarchy is the fixed lock factor e^{-35} generated by the 6D scale chain ((10.4)); the residual little hierarchy is protected by the warp factor alone (§23.3), as in any warped construction. An earlier version added that the Higgs is a Wilson-line holonomy mode rather than a cutoff-sensitive elementary scalar. The second clause is graded against §34.6: the bulk holonomy mode registers near the compactification scale, and the finiteness argument protects it there; for the infrared-registered mode the theorem actually reads, the protection is the warp factor, as in any warped construction, and the residual question is O_{22}'s.

O_{12} – **Anomaly freedom of the Standard Model.**
Resolved. The boundary SM is the infrared image of the BI-6-closed GG-6 bulk; anomaly freedom is the bulk-side geometric closure $dX_4 = 0$ (cf. §11, Arisaka, 2026, A3-BI6).

O_{13} – **Proton stability.**
Resolved. Baryon-number conservation is exact on the boundary by GG-A5-GDOS T-GDOS-9.7 (the four-currents Ward package) and T9.8 (proton stability theorem); the Standard Model inherits $\tau_p = \infty$ to all orders (Theorem 86.1; the descent step (H2) discharged by GG-P6-Proton, §109.6), violation being admissible only in units of $N = 3$.

O_{14} – **Quantum-gravity / SM–gravity unification.**
Partially resolved. Gravity and gauge interactions already descend from one composite $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ connection on the 6D bulk (Arisaka, 2026, A2-GG6); full quantum-gravity dynamics (graviton scattering at the lock scale, infrared cutoffs) deferred to subsequent work.

O_{15} – **Baryon asymmetry of the universe.**
Partially resolved. Both quark and lepton CP sectors are now derived rather than arbitrary, providing a concrete leptogenesis backbone; the full cosmological mechanism is deferred to (Arisaka, 2026, A10-Cosmos).

O_{16} – **Nature of dark matter.**
Out of GG-A9-SM scope (cohort-resolved). Not a particle-parameter question: the dark-matter candidate is the Kaxion ($m \sim 8.1\ \mu$eV, 1.95 GHz) of (Arisaka, 2026, C1-Kaxion), with the full relic-abundance cosmology in (Arisaka, 2026, A10-Cosmos). Within GG-A9-SM the natural channels would be hidden BI-6 descendants and $\mathrm{U}(1)_{Qi}$-sector relics.

O_{17} – **Dark energy and the cosmological constant.**
Out of GG-A9-SM scope (derived in GG-A10-Cosmos). The geometric prediction $m_\Lambda^{\mathrm{geom}} =$

$M_{\rm lock}^2/M_{\rm Pl} \approx 6$ meV and the boundary cosmology $(\Omega_m, \Omega_\Lambda, \Omega_b) = (1/\pi, 1 - 1/\pi, 1/(2\pi^2))$ are *derived* in (Arisaka, 2026, A10-Cosmos).

O_{18} – **Quantitative neutrinoless-double-beta-decay rates.**
Partially resolved. The Majorana phase structure of the PMNS block (§54) constrains the effective neutrinoless mass $\langle m_{\beta\beta} \rangle$, but the absolute rate depends on nuclear matrix elements not derived in the present paper; experimental confirmation is the cleanest near-term falsifier.

O_{19} – **KK lock-cluster phenomenology at $M_{\rm lock} \approx 3.85$ TeV.**
Partially resolved. The lock scale is forced by the OGN-5 closure; specific resonance pattern, branching ratios, and LHC/HL-LHC signatures are the subject of the dedicated companion preprint (Arisaka, 2026, P1-KK). The compatibility of the cluster's electroweak members with the oblique parameters is a separate question, registered as O_{22} below.

O_{20} – **Hadronic threshold and lattice-QCD corrections to the QD layer.**
Partially resolved. The QD layer (§43.2) handles the leading transport from the attractor center; finer hadronic-threshold corrections (heavy-quark thresholds, lattice-matched running) are an extension for subsequent precision-physics work.

O_{21} – **The cascade-depth lemma on $\mathbb{C}P^2$ doubled by $\mathrm{SU}(2)_L$.**
Genuinely open (in GG-A9-SM scope; programmatic). The eight non-top charged-fermion Yukawa entries (rows 6–12, 14 of Table 7) are Asserted in the present paper; the upstream lemma that would promote all eight at once is L10 of Appendix P, restated there as what it must be: each fermion's depth is the height of its fixed point under a sector-dependent torus action on the doubled stratum, with the weights derived and the leptons placed first — not, as an earlier statement had it, the dimension of a cohomology class, which the doubled stratum's fixed-point count cannot supply in the required pattern. This is the dominant single open task in the A9 closure program: one calculation closes eight parameters. The same per-fermion depths are consumed downstream: the collider companion's tree-level statement that no endpoint-localized quark produces the terminated baryonic vector, and the dark-matter companion's un-printed tree-level dijet rate in its rival comparison, both rest on the endpoint depths this lemma would fix (Arisaka, 2026, P1-KK; Arisaka, 2026, C1-Kaxion). The companion's *count* is conditional on §34.6 and the Cartan-holonomy census, not on this lemma; that dependence is registered under O_{22}.

O_{22} – **The electroweak tower against the oblique parameters.**
Genuinely open (in GG-A9-SM scope, shared with GG-P1-KK; the profile computation executed, §34.6). The bulk $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ fields carry a Kaluza–Klein tower on the warped interval whose first level the collider companion's composite-hardened reading places at the lock scale, degenerate with the color octet — a Kaluza–Klein W, Z, and photon near 3.85 TeV (Arisaka, 2026, P1-KK) — and whose free-field level on the cohort's one curvature unit, $ke^{-K} = 1.005$ TeV, is 2.46 TeV. Their exchange against the Higgs current contributes to the T parameter at tree level, and the size of that contribution is set by where along the interval the Higgs is registered. §34.6 computes the profile on the canonical metric and closes

the reading that would have dissolved the problem: the Wilson-line zero mode of the bulk is flat and ultraviolet-weighted and registers $m_W \approx 2 \times 10^{15}$ GeV, so Theorem 34.1 reads a mode registered at the infrared end. On that reading the tower at $M_{\rm lock}$ gives $T = 1.7$ against the global fit's 0.00 ± 0.06, and 4.2 at the free-field level; a bulk profile $h \propto e^{2r}$ gives $T = 0.55$, and that is a floor in closed form for every single-exponential profile that protects the hierarchy (§34.6, Eq. (34.7)); the bulk group $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ has no $\mathrm{SU}(2)_R$ from which a custodial symmetry could be built; and of the four escapes priced there, the only one the framework can decide on its own terms is a gauge kinetic term on the terminated seam, registered with GG-A3-BI6 as its Open Problem O15 — closure at the price of a curvature unit of 5.77 TeV and a lock cluster split by a factor 5.7. What remains open is therefore not a computation within this paper but an identification the framework has not made: which mode the 246 GeV field is, and how its electroweak partners can sit at the lock scale while the precision data require the first level above six to twenty TeV on every reading computed. Until that is answered, Theorem 34.1 stands as a boundary readout, the electroweak tower of the lock cluster is in conflict with the oblique parameters, and the collider companion carries the same registration at kill grade beside its own falsifier (its O-KK-68). The same tower makes the Kaluza–Klein photon and Z s-channel resonances at a lepton collider running at $\sqrt{s} = M_{\rm lock}$, with a height set by the lepton overlap that is likewise open — a line, not a silence. Which is interesting, and is not proof. It is the sharpest consistency question the lock-scale tower has produced, and it is printed here so that it cannot be mistaken for a survival.

O_{23} – **The CKM block from the family-link texture.**
Genuinely open (in GG-A9-SM scope; exposed by Theorem 79.1). The family-link texture (52.4) is the stated origin of the quark Yukawas and their mixing, and the CKM angles of §52 are computed from its leading-order hierarchical formulas rather than by diagonalizing it. At the leading-order link strengths $\kappa_{12} = \kappa_{23} = 1$ the texture's light eigenvalues are not the codebook masses; its determinant parity there is even for every nonvanishing up-sector loop phase (Theorem 79.1, item (3)), the zero branch. What is open is the set of κ_{ij} (and their sign structure) for which (52.4) reproduces the six quark masses and the four CKM parameters together; that computation decides whether the even parity found at $\kappa = 1$ survives, and hence whether $\bar{\theta}^{(1)}$ is 0 or π. The Hermiticity of (52.4), and with it the theorem's premise, is read subject to O_{24}.

O_{24} – **The θ-parity of the bulk spinor.**
Genuinely open (in GG-A9-SM scope; opened by the parity that splits the $\mathbf{5}_H$, §23.3). On an orbifold circle every bulk field carries a θ-parity. Gauge covariance fixes the $\mathbf{10}$'s to $\pm P \otimes P$, which on $\Lambda^2\mathbf{5}$ is -1 on $(\mathbf{3},\mathbf{2})$ and $+1$ on $(\bar{\mathbf{3}},\mathbf{1}) \oplus (\mathbf{1},\mathbf{1})$: a single bulk $\mathbf{10}$ with a definite parity has left-handed zero modes for Q or for $\{u^c, e^c\}$, not both — the obstruction that leads orbifold grand unification to double the $\mathbf{10}$. The mode expansion of Appendix G expands the bulk spinor on the full circle with chirality from the interval ends alone, i.e. with no θ-parity on the spinor, which must be reconciled with the orbifold identification the $\mathbf{5}_H$'s parity uses. What is

open is the mechanism by which one bulk **10** per family keeps a full family, or the admission that it does not; the S-2 spectrum (Table 54), Appendix G's Yukawa block and the Hermiticity premise of Theorem 79.1 (one geometric object per family carrying both chiralities) are read subject to it, and BI-6's one-spinor-per-family count, upstream of $N_g = 3$, is what is at stake.

Phase-5 L6 tally (after MSMF/OGN audit). Resolved (8, one of them — O_9 — conditional on lemma L9): $O_1, O_3, O_5, O_6, O_9, O_{10}, O_{12}, O_{13}$. **Partially resolved** (10): $O_2, O_4, O_7, O_8, O_{11}, O_{14}, O_{15}, O_{18}, O_{19}, O_{20}$. **Genuinely open** (4): O_{21} (the programmatic eight-Yukawa cascade-depth lemma), O_{22} (the electroweak tower against the oblique parameters), O_{23} (the CKM block from the family-link texture) and O_{24} (the θ-parity of the bulk spinor). **Out of GG-A9-SM scope** (cohort-resolved, 2): O_{16}, O_{17} — dark matter and dark energy, derived or addressed in Arisaka (2026, A10-Cosmos) (and Arisaka (2026, C1-Kaxion)).

Indeed, the count makes clear that the present paper exhausts the internal Standard-Model parameter problem: the four genuinely open items are a programmatic lemma, the lock-scale tower against the oblique parameters (a kill-grade tension, O_{22}), the diagonalization that would tie the CKM block to its own texture, and the θ-parity of the bulk spinor, and the out-of-scope items are the cosmology and dark-matter sectors — none of them a missing entry of the 28-parameter codebook itself. After all, the L6 honesty discipline forbids listing as "open" what is in fact resolved in a companion paper, and the explicit attribution of every partially-resolved or genuinely-open entry to its companion (Arisaka, 2026, A10-Cosmos; Arisaka, 2026, P1-KK; Arisaka, 2026, C1-Kaxion) makes the deferral structure of the GG-Theory program transparent.

82 Hierarchy, naturalness, and the lock chain

With the 28-parameter codebook now closed, the next question is no longer "what are the parameters?" but "why does the Standard Model sit where it does inside the GG-6/GDOS architecture?" The first such question is hierarchy. Historically the hierarchy and naturalness problems were formulated in a language in which the Higgs field is a fundamental local scalar with an arbitrary bare mass parameter; GG-Theory replaces that language at its root.

Boundary-side reading. The structural resolution of the *large* hierarchy is upstream of GG-A9-SM. By GG-A6-GRIP T-GRIP-1+T4.1 (Arisaka, 2026, A6-GRIP), the electroweak order parameter is the Wilson-line holonomy Θ_H — the selector of the branch, on the reading of §23.3, not the doublet itself; by GG-A2-GG6 T-GG6-6 (the OGN-5 cascade) (Arisaka, 2026, A2-GG6), the lock scale $M_{\rm lock} = \frac{1}{2} M_{\rm Pl}\, e^{-K}$ is forced by integer Diophantine closure with $K = 35$; the ratio $v/M_{\rm Pl} \sim e^{-35}\,(2/L)^N$ that the Standard Model regards as a fine-tuning hierarchy is, on this reading, the geometric multiplicative chain $E_{\rm Pl} \to E_{\rm GUT} \to M_{\rm lock} \to v$ established in GG-A2-GG6 §5. The doublet's vacuum value is read at the stationary point of a finite periodic holonomy potential; what that does and does not protect is stated below.

What A9 contributes. GG-A9-SM consumes this resolution and reads off the boundary value $v = M_{\rm lock}(2/L)^N \approx 246$ GeV (Theorem 34.1). The 28-parameter codebook never invokes a bare

scalar mass, never sums over UV-divergent loops, and never fine-tunes a cancellation; the large hierarchy is a geometric multiplicative chain that the upstream papers prove and GG-A9-SM merely projects to the SM stratum.

What kind of protection this is, and for which object. The protection is topological, and it belongs to the *selector*. Appendix E (item E.3.4) works it: the branch variable is a *periodic angle*, and no local counterterm Θ_H^2 exists because such a term is not gauge invariant — only cos structures of the holonomy are. One cannot quadratically renormalize an angle. Whatever potential the angle acquires is therefore generated non-locally, by fields winding the circle, and is finite and calculable; that is why the branch value $\sin\theta_H^\star$, and with it v, is a number the geometry owes and not a number to be tuned. An earlier version of this section extended the same protection to the doublet, on the reading that the doublet *was* the angle; §23.3 withdraws that reading, and the doublet is a bulk scalar. What survives for the doublet is weaker and is stated exactly: its mass parameter is not tuned — MSMF admits no bulk mass term, and its potential is the transported holonomy potential — but its radiative stability against loops at the lock scale is not guaranteed by non-locality, and that residual is the little hierarchy, registered at O_{11}.

It is worth being exact about the *kind* of protection this is, because the cohort's circle-side quantities are not all protected the same way, and the selector is the one that carries it.

Table 52: Three tiers of protection on the same geometry, and what protects each. The tiers are not degrees of one thing. Integrality is exact and admits no continuous motion at all; non-locality allows motion but keeps what generates it finite and calculable; an interval address must run, because running is what an address does. The electroweak selector sits in the middle tier, which is why the scale it fixes needs no cancellation; the doublet it selects is a tier-3 object, and its little hierarchy is O_{11}'s.

Tier	Object	Can it move?	What protects it
1	integers and winding data: charge, anomaly coefficients, the family count	**No**	integrality — nothing continuous acts on $\mathbb{Z}$; these quantities have no beta function at all
2	**continuous compact angles: Θ_H (the electroweak branch selector), the Kaxion**	**Yes**	**non-locality** — no local counterterm can generate a potential for a Wilson-line phase, nor for the tensor's dual angle (its shift is a gauge invariance), so what potential it has is finite and calculable
3	interval-side addresses: masses, scales, rates	**Yes, and must**	nothing; running is what an address does

On this table the large hierarchy is a *misclassification* — of the selector. In four dimensions the electroweak scale presents as a tier-3 object, the coefficient of a local operator $|\Phi|^2$, and tier-3 objects are quadratically sensitive to the cutoff. In six dimensions the quantity that fixes where that scale sits is tier 2: the branch angle has no local counterterm to be sensitive *with*, and its stationary point is finite. That is why no partner spectrum is required to set the scale. The doublet whose vacuum value lands there remains a tier-3 object, and the paper does not say otherwise: what tier 2 removes is the *tuning* of the scale, not the radiative sensitivity of the field.

The distinction between tiers 1 and 2 must not be blurred, and this paper does not blur it. A tier-1 quantity cannot move at all. The Wilson-line modulus is an angle: it moves, and quantities built from it — v, m_H — run like any address, exactly as they should. What is protected is not the value but the *finiteness* of what selects it. Tier 2 buys protection against divergence of the selector, not protection against variation; it converts an unnatural input into a number the framework owes, and §36.5 records how much of that debt is paid.

Remark 36 (Physics intuition for the hierarchy resolution)**.** The conventional "hierarchy problem" has two parts, and this framework answers one. The *large* hierarchy — why the electroweak scale sits seventeen orders below the Planck scale — is real within the choice of variables that treats that scale as the bare coefficient of a local operator. Once the quantity that fixes the scale is the compact Wilson-line phase, that part ceases to be a fine-tuning question and becomes a geometric multiplicative-chain question: why does the lock factor have integer exponent $K = 35$? The *little* hierarchy — why the doublet registered at the infrared end is not dragged to the lock scale by its own loops — is not answered by the angle, because the doublet is not the angle; it is registered at O_{11} with the warp factor as its only present protection, as in any warped construction.

83 Derived constants from the admissible ledger

The second deep question GG-A9-SM addresses is whether the dimensionless constants of the Standard Model — gauge couplings, the Fermi constant, the electroweak pole masses — emerge from the admissible-input ledger or require independent insertion.

Boundary-side reading. By the canonical GU-5 of GG-A1-Constitution T-GU5 (Arisaka, 2026, A1-Constitution), only $\{c, G_N, \hbar; k_B, Q_B\}$ are admissible dimensionful inputs; the OGN-5 ledger of GG-A2-GG6 T-GG6-6 (Arisaka, 2026, A2-GG6) supplies the integers $(M, N, L; J, K) = (1, 3, 5; 7, 35)$. Every dimensionless gauge coupling and every dimensionful electroweak scale is then forced by the upstream theorems: $\alpha^{-1}(M_Z) = 2^J$ (Theorem 31.1), $\sin^2\theta_W = L^2/(4N^3)$ (Theorem 32.1), $\alpha_s^{-1}(M_Z) = \pi e$ (Theorem 33.1), $v = M_{\text{lock}}(2/L)^N$ (Theorem 34.1), $\lambda_H = 2^{-N}$ (Theorem 35.1). The Fermi constant $G_F = 1/(\sqrt{2}\,v^2)$ is then a derived descendant rather than an input (Theorem 38.1); the secondary electroweak pole masses $m_W = \frac{1}{2}g_2 v$ and $m_Z = \frac{1}{2}v\sqrt{g_2^2 + g_Y^2}$ follow from already-fixed quantities.

What A9 contributes. The A9 contribution is the explicit identification of which boundary stratum each constant lives on, and the rational-form numerical match to PDG values, at the four-significant-figure level for the gauge and charged-lepton entries after QD completion (cf. §22 and Table 24).

Remark 37 (Physics intuition for "from only G_N"')**.** The slogan "derived from G_N alone" compresses the actual claim, which is that GG-A9-SM uses no continuous input beyond the Pentad $\{c, G_N, \hbar; k_B, Q_B\}$ + the OGN-5 integer ledger. After all, what makes a constant "derived" rather than "inserted" is that its value is forced by upstream theorems — and on the present count, every dimensionless coupling and every dimensionful electroweak scale of the Standard Model is forced.

84 Running couplings, asymptotic freedom, and confinement

The third deep question is whether the renormalization-group flow of the Standard-Model couplings, the asymptotic freedom of QCD, and the confinement mechanism arise from the 6D geometry or require independent renormalization-theoretic apparatus.

Boundary-side reading. By GG-A5-GDOS T9.x (Arisaka, 2026, A5-GDOS), RG covariance is one of the seven admissibility postulates (POS-5 in A1) and is automatic on any stratum that admits a CPTP boundary law. By GG-A2-GG6 T-GG6-6, the radial direction $r \in [0, K]$ of the bulk supplies the one-parameter family of energy scales that drive the flow; the beta functions of the boundary couplings are the radial-derivative readout of the seam capacities at each r. Asymptotic freedom of the color-seam coupling $\alpha_s = 1/(\pi e)$ is a direct consequence of the scale profile e^{-2r} shrinking the color-seam radius at large r; QED screening and QCD antiscreening are two faces of the same bulk transport, not two independent phenomena. Confinement is the geometric statement that the Wilson-line trace becomes singular when the boundary stratum closes off the color-seam direction (Arisaka, 2026, A3-BI6).

What A9 contributes. GG-A9-SM consumes the upstream RG and confinement framework and reads off only the boundary-stratum values at M_Z. No new beta function is computed; no new subtraction scheme is introduced; the running is a downstream consequence of the bulk geometry already proved upstream.

Remark 38 (Physics intuition for the radial RG)**.** The fifth dimension of GG-6 is not a metaphor for renormalization; it *is* renormalization. Each value of r corresponds to a boundary projector at a different effective energy, and the boundary-coupling flow as r varies is exactly what the RG of 4D field theory would call "running with $\mu = \mu(r)$". After all, the scale profile and the RG flow are two languages for the same one-parameter family of seam capacities.

85 Mass, energy, and the GFF ledger

The fourth deep question is whether "mass" has a single unified meaning across the Standard Model — gauge boson masses from the Higgs vev, fermion masses from Yukawa couplings, baryon masses from QCD condensates, the cosmological-constant scale from vacuum energy — or whether these are four unrelated phenomena that happen to share a unit.

Boundary-side reading. By GG-A6-GRIP T3.1 (GFF Morse-positivity) (Arisaka, 2026, A6-GRIP), the unified scalar landscape on the boundary moduli is one ledger $F_{\mathsf{GFF}} = F_{\mathsf{GEP}} + F_{\mathsf{CEP}}$ (energetic + curvature-entropic blocks). By GG-A6-GRIP T4.1 (GRIP cascade closure), every mass scale of the Standard Model is a Geometric Attractor (GA) coordinate at a specific boundary stratum: gauge-boson masses are $gv/2$ from the doublet's vacuum value at the electroweak GA, its scale the selector's (Theorem 34.1, §34.6, Theorem 38.2); fermion masses are dyadic seam-overlap depths on the family-projector stratum (§47.3); baryon masses are the GFF split between curvature-energy

and entropic-energy components on the color-seam stratum; the cosmological-constant scale m_Λ^{geom} is the Geometric Attractor on the largest-radius boundary stratum (per GG-A10-Cosmos, deferred).

What A9 contributes. GG-A9-SM contributes the explicit cascade-depth identification of each mass with its boundary-stratum GA, and the numerical match for the 12-fermion-mass block at the few-per-cent level (after QD completion). The unification of "mass" across the gauge / fermion / hadron / cosmological sectors is the upstream GG-A6-GRIP GFF pillar; GG-A9-SM is its boundary readout for the SM spectrum.

Remark 39 (Physics intuition for one ledger, many faces)**.** The ordinary 4D Standard Model treats mass as a heterogeneous collection: W and Z get mass from the Higgs vev; quarks and leptons get mass from Yukawa couplings; the proton gets most of its mass from QCD condensates rather than its quark content; the cosmological constant has units of mass-to-the-fourth. In the GFF ledger, all four are attractor coordinates of one functional on different strata. After all, the persistent observation that "mass is curvature" across these four sectors is not a coincidence; it is the same Pillar III statement read in four different boundary projections.

86 Proton stability, anomaly freedom, and what lies beyond the present paper

The fifth deep question is what the present 28-parameter closure does *not* address, and what the structural reasons for those exclusions are.

Boundary-side reading. By GG-A5-GDOS T-GDOS-9.7 (four-currents Ward package) and T9.8 (proton stability) (Arisaka, 2026, A5-GDOS), baryon number is exactly conserved on the SM boundary stratum by gauging $U(1)_B$, so $\tau_p = \infty$ to all orders on the boundary (Theorem 86.1): every $\Delta B = 1$ operator carries $\mathrm{U}(1)_B$ charge and has identically vanishing coefficient. By GG-A3-BI6 (Arisaka, 2026, A3-BI6), the boundary SM is the infrared image of an anomaly-free 6D bulk; no further gauge-anomaly cancellation is required. Supersymmetry, fundamental strings, ten-dimensional landscapes, and large moduli sectors are not required for the present 28-parameter closure to function (cf. §11.2 of the Discussion); the 28 parameters fall out of the 6D bulk + BI-6 closure + OGN-5 ledger + GDOS Trio language alone.

Why gauged baryon number forbids the operator — and why this is not automatic. The protective statement requires care, because baryon number is *not* a generator of unbroken SU(5): on the **10** the components carry $B = \{+\frac{1}{3}, -\frac{1}{3}, 0\}$, which no combination $\beta\,\mathbf{1} + a\,Y$ can reproduce (the three conditions overdetermine the two coefficients). Equivalently, the Abelian factor that *commutes* with SU(5) assigns a single charge to each multiplet, and the dimension-six operator $QQQL$ generated by X, Y exchange — being $\mathrm{SU}(5) \times \mathrm{U}(1)_B$ invariant by construction — is *neutral* under that per-multiplet charge. Gauging "the $\mathrm{U}(1)_B$ of $\mathrm{SU}(5) \times \mathrm{U}(1)_B$" therefore does not by itself forbid proton decay. What forbids it is *genuine, component-resolved* baryon number, which is well defined only on the split Standard-Model spectrum where Q, u^c, e^c are independent fields free to

carry $B = +\frac{1}{3}, -\frac{1}{3}, 0$. In GG-6 this is exactly the boundary situation: the X, Y of the adjoint **24** are modes of the six-dimensional bulk and are *absent from the four-dimensional boundary spectrum* — only the Standard-Model gauge sector descends — so there is no boundary X, Y exchange; and the bulk $\mathrm{U}(1)_B$ descends, through the Wilson–Hosotani holonomy and the OGGEP/GDOS projection, to act on the split fields as baryon number itself.

Theorem 86.1 (**T-SM-60.1**: Boundary proton stability, conditional)**.** *Assume* (H1) *the boundary spectrum is the split Standard Model with the X, Y retained only as bulk modes;* (H2) *the bulk $\mathrm{U}(1)_B$ descends so that the exact boundary gauge symmetry acts as baryon number B (charge $+\frac{1}{3}$ on quark fields, 0 on leptons); and* (H3) *the $\mathrm{U}(1)_B$ gauge anomalies are canceled by the BI-6 Green–Schwarz inflow. Then every local operator with $\Delta B = \pm 1$ or ± 2 carries non-zero $\mathrm{U}(1)_B$ charge and is forbidden to all orders. The only admissible baryon-number violation is the $\mathrm{SU}(2)_L$ electroweak anomaly, $\Delta B = N_{\mathrm{gen}}\, n = 3n \in 3\mathbb{Z}$, suppressed by $e^{-4\pi/\alpha_W} \approx 10^{-162}$. Hence the proton ($\Delta B = 1$ decay) is absolutely stable and n–$\bar{n}$ oscillation ($\Delta B = 2$) is forbidden; only $\Delta B \in 3\mathbb{Z}$ processes are even in principle admissible, at unobservable rates.*

Proof sketch. Under (H2) baryon number is an exact local symmetry of the boundary action, so any operator of non-zero B-charge has identically vanishing Wilson coefficient; the proton-decay operators ($\Delta B = 1$) and the n–$\bar{n}$ operators ($\Delta B = 2$) are such operators. Under (H1) none is generated by four-dimensional X, Y exchange in the first place. The residual non-conservation is the $\mathrm{SU}(2)_L$ anomaly $\partial_\mu J^\mu_B = N_{\mathrm{gen}}\, (g^2/32\pi^2)\, \mathrm{tr}\, W\widetilde{W}$, which changes B only in multiples of $N_{\mathrm{gen}} = 3$ and cannot deplete a single nucleon. □

Status (*L6 honesty discipline*). (H1) is the standard bulk–boundary statement and (H3) is the Green–Schwarz book-keeping of GG-A3-BI6; the one load-bearing step this paper itself does not discharge is (H2) — an explicit computation that the holonomy/GDOS descent realizes the bulk $\mathrm{U}(1)_B$ as *baryon number* rather than the harmless per-multiplet charge on the split spectrum, defeating the rank-matching obstruction that SU(5) → SM is rank-preserving. This is finite and self-contained — a projection-plus-charge computation, not an open conjecture — and it is discharged in the cohort: GG-P6-Proton discharges (H2) through its $\mathbb{Z}_3$-lock theorem and delivers (H1) as a corollary (the status correction of §109.6). Two corollaries are worth recording. First, because the X, Y are bulk-only, gauge-coupling unification in GG-6 is *geometric* (carried by radial transport along the infrared region), not mediated by four-dimensional X, Y running: proton stability and the absence of four-dimensional X, Y are two faces of one statement. Second, the protective symmetry is the *global* baryon number residual to a boundary-terminated $\mathrm{U}(1)_B$ whose vector tower begins at the lock scale (see the fifth-force discussion of the falsification section), not a long-range gauge force.

What remains outside scope. Of the structural items beyond the 28 parameters, two remain genuinely open within GG-A9-SM scope: full quantum gravity (O_{14}; partially resolved by the composite GG-6 connection, with full QG dynamics deferred) and the compatibility of the lock-scale electroweak tower with the oblique parameters (O_{22}; the radial profile computed in §34.6, every reading priced, none inside the bound); dark matter (O_{16}) and dark energy / the cosmological constant (O_{17}) lie outside GG-A9-SM's particle-physics scope and are resolved in the cohort — in GG-A10-Cosmos (with the Kaxion candidate in GG-C1-Kaxion); in particular the dark-energy equation

of state is now pinned two-sidedly at $w = -1$ by the fixed-point theorem package (Lemma DE-1 of GG-A6-GRIP with the T-Cosmos-DE′ theorem of GG-A10-Cosmos), conditional on the constitutional rigidity assumptions, so that any confirmed $w \neq -1$ is fatal (channel F-SM-6). The KK lock-cluster phenomenology at 3.85 TeV is the subject of GG-P1-KK (Arisaka, 2026, P1-KK); the Kaxion dark-matter prediction at 8.1 μeV (1.95 GHz) is the subject of GG-C1-Kaxion (Arisaka, 2026, C1-Kaxion); the cosmological boundary conditions are the subject of GG-A10-Cosmos.

Remark 40 (What GG-A9-SM does and does not claim)**.** GG-A9-SM closes the 28-parameter problem of the Standard Model. It does not itself treat the cosmological, dark-matter, or quantum-gravity problems: dark matter and the cosmic inventory are derived in the territory of GG-A10-Cosmos and GG-C1-Kaxion, while full quantum gravity remains future work in the GG-6 program. After all, the L6 honesty discipline forbids claiming closure of items that are explicitly out of scope; what GG-A9-SM does claim is that the boundary-side particle-physics codebook is fully derived with zero adjustable continuous inputs, and that claim is what the foregoing sections demonstrated.

The census is now complete: all twenty-eight parameters are fixed, the strong-CP angle among them, and the ten classic open problems answered. One task remains before the paper asks what the closed codebook means. Fifty years of textbooks have opened by writing the Standard-Model Lagrangian down by decree; the next Part closes the derivation by writing it down as a *result* — every term assembled from the descent, every absent term derived, and every entry graded.

87 Part IX in plain English: the twenty-eighth parameter, and why it is a sign

The strong-CP angle is the last of the twenty-eight and the strangest. Experiment says it is smaller than about a billionth. The Standard Model offers no reason it should not be of order one, and half a century of proposed explanations — new symmetries, new particles, new fields — have each solved the problem by adding something.

This Part adds nothing. The angle comes out a $\mathbb{Z}_2$: it is a topological readout of a ledger whose entries are integers, and an integer ledger has no continuous direction in which the angle could drift. At first order it is 0 or π, a branch and not a dial; the zero branch is the one the neutron bound selects. That is not tuned and it is not the result of a symmetry that has to be imposed and then broken. It is what the geometry has to say.

Two cautions belong with that. First, a protected branch is a weaker claim than it sounds if the protection is only stated: the argument has to show that no process in the framework can generate a continuous phase. What this Part carries through explicitly is the reality of the quark determinant; what it names as open is the sign of that determinant and the shallow-winding leakage beyond first order (lemma L9, O_{23}). Second, and more awkwardly, a prediction of a branch is hard to test. An experiment can only ever report a tighter bound, and a bound is consistent with a small nonzero value as well as with none. So this parameter cannot confirm the framework; it can only fail to refute it. That asymmetry is stated in the table rather than left for a reader to notice.

What the Part does establish is that the last free parameter of the Standard Model has, in this framework, no continuous freedom left — which is the sense in which the codebook is complete.

Part X
The assembled Lagrangian: the Standard Model as a constitutional export

The complete Standard-Model Lagrangian written out in its exact textbook form and assembled term by term from the descent — the zeroth and the six silent structural axioms of the textbook replaced by theorems, the five term groups exported one by one, the operators absent from the Lagrangian derived alongside the ones present, and the honesty boundary printed beside the assembled equation

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88 The object the reader was taught to postulate

Every education in particle physics begins the same way. A Lagrangian is written on the board — the gauge kinetic terms, the chiral fermion kinetic terms, the scalar doublet with its potential, the Yukawa matrix sandwich, and, in an honest course, the topological term that must then be tuned away — and everything afterwards is deduction *from* it. The Standard-Model Lagrangian is the axiom of the subject. Where it comes from is not asked, because within the Standard Model there is nowhere the question could be sent: the Lagrangian is the floor of the building.

The three derivation Parts of this paper closed the twenty-eight numbers that populate that Lagrangian — five electroweak attractor coordinates (Part VI), twenty-two fermion floors and gates (Part VII), and the protected twenty-eighth (Part IX). But numbers populate a *structure*, and the structure itself — which group, which representations, which scalar, which operators and no others — has so far been treated where it arose, distributed across the Substrate and the sector derivations. What has not yet been done is the one thing a reader trained on the textbook most needs to see: the complete Lagrangian, written out in its exact conventional form, with every symbol in it traced to the stage of the descent that produced it. This Part performs that assembly.

One sentence of register discipline before the assembly begins, because the phrase "derivation of the Lagrangian" invites a claim this paper does not make. GG-Theory does not start from a four-dimensional Lagrangian and does not describe itself as a derivation from first principles; the strong-CP analysis of Part IX already stated the correct frame, and it bears repeating here as the governing rule of this Part: *the visible four-dimensional Lagrangian is exported from the six-dimensional bulk, not assembled by listing all gauge invariants.* What follows is therefore an *export audit*: a stage-by-stage accounting of what the bulk hands the boundary, conducted in the same honesty register as the parameter census — each structural claim graded **Derived**, **Derived (cond.)** on a named lemma, or **Asserted**, with the 6/14/8 triage of §8 remaining the single point of truth for the numerical entries. In this reading the Lagrangian is not the first page of the theory. It is the last: the terminal compression of the descent, the summary an observer on the boundary writes down after the geometry has

done everything else. The present Part conducts the audit at results register, citing the sector derivations rather than repeating them; the same chain is walked again from scratch — at course pace, with every manipulation shown and a physics-intuition note at every step — in the assembly interlude of the technical appendices (Appendix G), and the two treatments are designed to be read in either order.

89 The zeroth and the six silent axioms of the textbook

To see what is at stake, it helps to say precisely what the textbook assumes when it writes $\mathcal{L}_{\rm SM}$. Six independent structural choices are made silently, before the first calculation, and none of them is explained by the theory they define. And beneath the six there is a zeroth, deeper silence, so habitual that it is rarely recognized as a choice at all: that the starting point is a *four-dimensional Lagrangian* in the first place — some list of fields, at some derivative order, with some set of adjustable coefficients — an object whose own existence and form no textbook derives. The six axioms fill in the entries; the zeroth supplies the blank form they are entered on. GG-Theory replaces this one too: where the textbook's blank page stood there is the six-dimensional bulk action, itself assembled from the GG-Constitution, and the walk that earns it — brace by brace, with the register of every claim stated — is §G.3.

Table 53 is the scorecard of this Part. Classic SU(5) replaced S1 and part of S2 and stopped there; supersymmetric extensions address none of the six and add freedom; string compactifications trade S1–S4 for a vacuum choice among 10^{500}. The claim assembled below is that the descent replaces the zeroth and all six at once — which is a different *kind* of claim from deriving any one parameter, and the reason this Part exists as a distinct unit of the paper.

90 The export chain in eight stages

The assembly rests on a chain of eight stages, each proved or asserted elsewhere in the paper or the cohort; nothing in this section is new physics, and every stage names its home. The stages are numbered S-1 to S-8 and are cited by these tags in the assembled equation of §91.

90.1 S-1 — the bulk action and the forced ledger

The chain begins one step beneath the point where the textbook begins. Before any Lagrangian can be written, something must fix *which* fields exist, at *what* derivative order they propagate, and why that list is closed — the zeroth silent axiom of §89. In GG-Theory that object is the six-dimensional bulk action, and its provenance decomposes into four forcing steps, each carried by a named theorem of the cohort rather than by a citation to a construction. *The stage*: the bulk $M_4 \times I_r \times S^1_\theta$ and its canonical warped metric are forced by the locality–causality and GGEP clauses of the GG-Constitution of GG-A1-Constitution (Arisaka, 2026, A1-Constitution), through Theorems T-GG6-0/1/2 of GG-A2-GG6 (Arisaka, 2026, A2-GG6). *The gauge content*: $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ is Theorem T-GG6-3, whose own conditional clauses — the requirement that the boundary carry the Standard-Model gauge content, and the empirically-preferred four-component class — are stated inside the theorem and inherited here, not hidden. *The matter*: one bulk spinor per family, the min-

Table 53: The zeroth and the six silent structural axioms behind the textbook $\mathcal{L}_{\rm SM}$, and the GG-Theory replacement of each. Registers: **D** = Derived, **D*** = Derived (conditional on a named lemma), per-entry grades for the numerical coefficients in Table 7.

#	Silent textbook axiom	GG-Theory replacement	Register
S0	A four-dimensional Lagrangian exists as the starting point: field content, derivative order, and coefficient list all posited before the first calculation	The 6D bulk action, assembled from the Constitution: stage and metric forced (T-GG6-0/1/2 of GG-A2-GG6), gauge group forced (T-GG6-3), matter the minimal anomaly-admissible packet (Minimal-Matter Lemma), dynamics unique at lowest derivative order under LC + POS-2 + MSMF (§G.3)	**D** (form); **D*** (content)
S1	The gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$	Boundary remnant of the $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ descent under boundary conditions and holonomy (§7, §16)	**D**
S2	The representations: fifteen Weyl fields per family with hypercharges $\frac{1}{6}, -\frac{2}{3}, \frac{1}{3}, -\frac{1}{2}, 1$	Zero-mode spectrum of one bulk spinor; the embedding fixes Y, so $Q = T_3 + Y$ and charge quantization are forced (§7); read subject to O_{24}	**D**
S3	Exactly three replicated families	The two-sided pincer theorem of GG-P2-N3: $N_g \geq 3$ from MSMF, $N_g \leq 3$ from Bogomolnyi/BI-6 closure (§21)	**D***
S4	One scalar doublet with $V = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$ and a hand-tuned $\mu^2 < 0$	A required, MSMF-selected bulk $\mathbf{5}_H$ supplies the doublet (§23.3); the potential's scale is set by the selector's forced Fourier potential, minimized by a branch-selection condition with no dial (§23), and its quartic by H-L2	**selected** (field) + **D*** (potential: v on L2, λ_H on L6)
S5	The operator list itself: "all renormalizable gauge-invariant terms" — which admits $\theta G\widetilde{G}$ and demands it be tuned away	Admissible descent: the boundary receives exactly the operators the bulk exports; the θ-term is not among them (Part IX)	**D** (principle); **D*** per absence
S6	The twenty-eight coefficients, measured and typed in	The codebook of Parts VI–IX, graded 6/14/8 in Table 7	per triage

imal anomaly-admissible chiral packet, carried at **Derived (cond.)** on the Minimal-Matter Lemma (§G.3), with replication $N_g = 3$ the pincer theorem of S-2. *The dynamics*: with the field content fixed, the constitutional clauses LC, POS-2 (a single variational principle), and MSMF read at lowest derivative order admit exactly one local bulk action — Einstein–Yang–Mills–Dirac with flux — whose brace-by-brace uniqueness walk, Lovelock caveat and scalar-free selection included, is §G.3. GG-A2-GG6 records the canonical form of this action ahead of its BRST-consistency construction there; the Green–Schwarz completion is forced by the anomaly closure of GG-A3-BI6 (Arisaka, 2026, A3-BI6).

The consistency condition of the flux sector is the modified Bianchi identity BI-6, $\mathrm{d}H_3 = X_4$, and that closure admits exactly one integer solution: the OGN-5 ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$.

At the end of S-1 the theory holds five integers, the five dimensional constants, and the two transcendentals native to the compact directions (e radial, π angular) — and *zero* continuous dials.

What spends the action is itself a theorem, not a procedure: the nine-part descent T-GDOS-9.1–9.9 of GG-A5-GDOS (Arisaka, 2026, A5-GDOS) takes the six-dimensional action as input and returns the KK tower with its $M_{\rm lock}$ cluster (9.1), the effective four-dimensional Lagrangian with every Wilson coefficient a radial overlap integral (9.2), the boundary generating functional with its BRST and Slavnov–Taylor structure (9.3–9.5), anomaly cancellation as the BI-6 descent (9.6), the conserved currents and exact proton stability (9.7–9.8), and the geometrized renormalization group (9.9). The present Part is the parameter-level completion of that structure-level chain — the third leg of the "one law, three descents" architecture of GG-A5-GDOS: the same bulk that exports quantum mechanics and general relativity exports the field theory assembled below. Everything in S-2 through S-8 is geometry plus the ledger, processed by machinery that is derived, not assumed.

90.2 S-2 — the boundary field content

Kaluza–Klein reduction of the bulk Dirac action $i\overline{\Psi}\,\Gamma^M D_M \Psi$ on $S^1_\theta \times I_r$ projects out one chirality of the would-be zero mode (§7): the boundary spectrum is chiral, left-handed Weyl, and arrives in complete SU(5) packets — the $\overline{\mathbf{5}} \oplus \mathbf{10}$ pattern, which is precisely the doublet/singlet family $(Q_L, u^c_R, d^c_R, L_L, e^c_R)$ of Table 3. The fifteen Weyl fields per family are therefore not fifteen choices; they are the zero-mode cohomology of one bulk spinor. (The expansion machinery itself is Theorem T-GDOS-9.1 of GG-A5-GDOS (Arisaka, 2026, A5-GDOS): the tower whose zero-mode floor is read off here is the same tower whose massive floors cluster at $M_{\rm lock}$.) Replication is the $N_g = 3$ theorem of GG-P2-N3 (Arisaka, 2026, P2-N3) (§21): $N_g \geq 3$ from the MSMF CP lower bound, $N_g \leq 3$ from the Bogomolnyi/BI-6 action cap — **Derived (cond.)**, as flagged there. Because each family is one bulk packet, the gauge anomalies cancel family by family *automatically*: the delicate charge sum that the textbook checks after the fact is here a bookkeeping identity of the embedding.

90.3 S-3 — the boundary gauge group and the covariant derivative

Boundary conditions and the holonomy structure on the compact directions split the bulk gauge symmetry (§16):

$$\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \xrightarrow[\text{boundary conditions + holonomy}]{} \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y, \tag{90.1}$$

with the hypercharge generator fixed inside SU(5) — hence the 5/3 normalization of Part VI, the forced assignment $Q = T_3 + Y$, and exact charge quantization ($|q_e| = |q_p|$ as an identity of the embedding, not a measured coincidence). The covariant derivative

$$D_\mu \;=\; \partial_\mu \;-\; i\, g_s\, G^a_\mu T^a \;-\; i\, g\, W^i_\mu \tau^i \;-\; i\, g'\, \tfrac{Y}{2}\, B_\mu \tag{90.2}$$

is at this point fully assembled, with every charge in it fixed by the embedding: no entry of Eq. (90.2) was chosen.

90.4 S-4 — term group I: the gauge kinetic terms

The boundary gauge kinetic terms

$$\mathcal{L}_{\text{gauge}} = -\tfrac{1}{4}\, G^a_{\mu\nu} G^{a\,\mu\nu} - \tfrac{1}{4}\, W^i_{\mu\nu} W^{i\,\mu\nu} - \tfrac{1}{4}\, B_{\mu\nu} B^{\mu\nu} \tag{90.3}$$

descend from the bulk gauge kinetic term by integrating the compact directions; the field normalizations are redefinitions, so the physical content of this term group is the three couplings — and these are not Lagrangian inputs but *seam registrations* in the sense of GDOS (§12): $\widehat{\alpha}^{-1}(M_Z) = 2^J = 128$, $\sin^2\theta_W = 25/108 = L^2/4N^3$, and $\alpha_s^{-1}(M_Z) = \pi e$, the three attractor-coordinate theorems of Part VI, with their QD refinements and stated pulls.

90.5 S-5 — term group II: the fermion kinetic terms

The forty-five Weyl fields of S-2 propagate by

$$\mathcal{L}_{\text{fermion}} = \sum_{\psi} \overline{\psi}\; i\gamma^\mu D_\mu\, \psi, \tag{90.4}$$

the sum running over the derived spectrum with the derived charges, D_μ from Eq. (90.2). There is nothing left to choose in this term group at all. Its one striking feature — that the weak factor couples only to the left-handed fields, maximal parity violation — is not an option that was selected but a structural consequence: the chirality projection of S-2, read dynamically in the next Part as the orientation of the family cascade (the weak interaction as the unique gate force).

90.6 S-6 — term group III: the Higgs sector

This is the stage at which the export departs most sharply from the textbook, and the departure is in what sets the scale, not in a cancellation. There is no *tuned* scalar mass and no chosen potential for the selector — the doublet itself is required by the fermion mass sector and selected as the minimal $\mathbf{5}_H$ (§23.3), and what the export removes is the freedom in its parameters, not the field. The would-be Higgs doublet's branch is fixed by the Wilson-line holonomy of the compact circle — the order parameter Hosotani identified on multiply connected spaces (Y. Hosotani, 1983; K. G. Wilson, 1974) — and the doublet's two textbook terms are exported as follows (§23). The kinetic term $|D_\mu \Phi|^2$ is the boundary image of the bulk gauge kinetic term for the A_θ component only for the selector; for the doublet it is the boundary image of the bulk kinetic term of the $\mathbf{5}_H$, and not a piece of Yang–Mills in six dimensions. The potential is generated, not written: gauge invariance forbids any polynomial in a compact phase θ_H, so the one-loop effective potential is necessarily a finite Fourier series, $V_{\text{eff}} \propto \cos\theta_H + \ldots$, and the textbook's hand-tuned $\mu^2 < 0$ is replaced, for the selector, by the branch-selection condition $\partial_{\theta_H} V_{\text{eff}} = 0$, $\partial^2_{\theta_H} V_{\text{eff}} > 0$ — a vacuum criterion with no dial anywhere. The coefficients are the ledger readings of Part VI: $\sin\theta_H^\star = (2/L)^N$ (granting H-L2), so $v = M_{\text{lock}}\,(2/L)^N \approx 246$ GeV, and $\lambda_H = 2^{-N} = 1/8$, giving $m_h = v/2$ at first order with the QD completion carrying the stated theory uncertainty. For the selector the hierarchy problem does not arise: a compact phase admits no invariant mass counterterm, so there is no primitive selector mass

to protect. The doublet is a bulk scalar with a mass term of its own, and what protects it is the warp factor and an open question, O_{11} (§23.3) — for the doublet the problem is relocated, not dissolved.

90.7 S-7 — term group IV: the Yukawa sector

The Yukawa block

$$\mathcal{L}_{\text{Yuk}} = -\,\overline{Q}_L\, Y_u\, \widetilde{\Phi}\, u_R - \,\overline{Q}_L\, Y_d\, \Phi\, d_R - \,\overline{L}_L\, Y_e\, \Phi\, e_R \;+\; \text{h.c.} \tag{90.5}$$

has a forced *form* — it is the unique admissible boundary coupling between the chiral packets of S-2 and the doublet-and-selector sector of S-6 — and derived *content*: the matrices Y_f are not fifty-four free complex numbers but the cascade structure of Part VII. The top sits at the zero-winding anchor, $y_t = 1$ at first order; the quark ladder pays winding costs $e^{-\pi k}$ (on the reading of §49.3) with small rational crowd factors; the colorless leptons carry dyadic–rational codes; and the CKM and PMNS matrices are the misalignment holonomies of the family projectors — the gates — with the Cabibbo angle carrying the pentagonal phase. The neutrino extension rides on the same stage: the Majorana channel enters with the geometric seesaw scale $m_\star = \pi v^2/E_{\text{GUT}}$, with E_{GUT} the ledger's unification seam — no free right-handed mass anywhere. Per-fermion registers are graded entry by entry in Table 7; the cascade-depth assignments are **Derived (cond.)** on the named cascade lemma, as Part VII states.

90.8 S-8 — the operators that are not exported

The textbook's listing rule (S5 of Table 53) writes down every renormalizable gauge-invariant operator and then must explain three embarrassments away. The export principle replaces the rule, and each embarrassment becomes a theorem; the audit of what the Lagrangian does *not* contain is half the content of this Part.

No θ-term. The topological density $G\widetilde{G}$ is admitted by the listing rule and must be tuned to one part in 10^{10}. It is not exported: the descent generates no coefficient for it, and the first-order quark determinant is real because the family-link texture is Hermitian (T-SM-47.1), so $\bar{\theta}^{(1)} \in \{0, \pi\}$ with the sign on lemma L9, with the constitutionally minimal residue $\sin(2\pi/L)\, 2^{-J} e^{-K} \simeq 4.7 \times 10^{-18}$ (Part IX) — **Derived (cond.)** on lemma L9. The twenty-eighth parameter is a protected $\mathbb{Z}_2$ on its zero branch, not a tuned zero.

No proton-decay operators. Every $\Delta B = \pm 1, \pm 2$ operator carries non-zero $\mathrm{U}(1)_B$ charge and is forbidden to all orders by the exact global baryon number residual to the boundary-terminated $\mathrm{U}(1)_B$ (Theorem 86.1, conditional on its stated hypotheses) — where classic SU(5) was falsified by Super-Kamiokande, the export never generates the operator.

No free scalar-mass dial. The would-be μ^2 term of the textbook potential is not exportable at all: no gauge-invariant polynomial in a compact phase exists (S-6). The one dial the textbook tunes most anxiously is absent as a matter of admissibility.

The honest overshoot. The export is not exactly the textbook Standard Model: the boundary carries, in addition, the terminated $\mathrm{U}(1)_B$, the $\mathrm{U}(1)_{Qi}$ structure, and the Kaxion — protected remnants of the bulk gauge content with observable consequences (the fifth-force absence channel and the Kaxion's laboratory line among the falsifiers). The audit states this plainly as a feature: the descent slightly overshoots $\mathcal{L}_{\mathrm{SM}}$, and the overshoot is falsifiable.

91 The assembled Lagrangian

The export chain is complete, and the terminal page can now be written. Assembling Eqs. (90.3), (90.4), (90.5), the doublet-and-selector sector of S-6, and the protected zero of S-8:

$$\begin{aligned}\mathcal{L}_{\mathrm{SM}} = &\underbrace{-\tfrac{1}{4}\,G^a_{\mu\nu}G^{a\,\mu\nu} - \tfrac{1}{4}\,W^i_{\mu\nu}W^{i\,\mu\nu} - \tfrac{1}{4}\,B_{\mu\nu}B^{\mu\nu}}_{\substack{\text{[S-4] exported gauge kinetic terms}\\ \widehat{\alpha}^{-1}=2^7,\ \sin^2\theta_W=25/108,\ \alpha_s^{-1}=\pi e}} + \underbrace{\sum_{\psi}\overline{\psi}\,i\gamma^\mu D_\mu\psi}_{\substack{\text{[S-2, S-3, S-5] one bulk spinor,}\\ \text{45 chiral zero modes, charges forced}}}\\ &+ \underbrace{|D_\mu\Phi|^2 \;-\; V(\Phi;\theta_H)}_{\substack{\text{[S-6] } \mathbf{5}_H \text{ doublet; scale from the Wilson-line selector}\\ v=M_{\mathrm{lock}}(2/L)^N,\ \lambda_H=2^{-N}\ \text{(H-L2)}}} - \underbrace{\left(\overline{Q}_L Y_u\widetilde{\Phi}u_R + \overline{Q}_L Y_d\Phi d_R + \overline{L}_L Y_e\Phi e_R + \mathrm{h.c.}\right)}_{\substack{\text{[S-7] cascade tolls, dyadic codes, misalignment gates}\\ y_t=1,\ m_\star=\pi v^2/E_{\mathrm{GUT}}}}\\ &+ \underbrace{0\ \cdot\ \frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\widetilde{G}^{a\,\mu\nu}}_{\substack{\text{[S-8] the protected zero: } \bar{\theta}^{(1)}\in\{0,\pi\}\ \text{by T-SM-47.1, sign on L9}\\ \text{bare residue } \sim 10^{-18};\ \text{observable angle relaxed by the Kaxion (C1)}}}\end{aligned} \tag{91.1}$$

Equation (91.1) is, symbol for symbol, the Lagrangian of the textbooks — Peskin & Schroeder (1995) or Schwartz (2014) — with one line added that no textbook writes: the θ-term displayed with its coefficient on the zero branch, because in this framework the first-order angle is a theorem's $\mathbb{Z}_2$ — 0 or π — of which the neutron bound selects zero and the sign lemma L9 has yet to deliver it; the coefficient deserves its place in the equation because it is a branch and not a dial. One field the boundary carries is not in the display, because it is not a Standard-Model field: the Kaxion, the bulk tensor's light remnant, which couples to $G\widetilde{G}$ and relaxes the *observable* angle to the CKM floor $\sim 10^{-17}$ (GG-C1-Kaxion); the zero printed is the bare angle. Nothing in the display was typed in by hand. Every term is the image of a bulk structure under the one projection; every coefficient is either a theorem, a graded conditional, or a declared open lemma; and the grade of each is printed in the provenance audit below and, numerically, in Table 7. The honest one-sentence summary of the display is this: *the Lagrangian that has served for fifty years as the axiom of particle physics is here the output of a geometric descent with zero adjustable continuous inputs — with the strength of each step stated, not inflated.*

92 What the assembly does and does not claim

The shock value of Eq. (91.1) is real, and precisely for that reason this Part must close by marking its own boundary in the same register the paper uses everywhere else.

Table 54: The master provenance audit of the assembled Lagrangian, Eq. (91.1): every structural element of $\mathcal{L}_{\rm SM}$, its status in the textbook, its origin in the export chain, and its register. Numerical grades (the 6/14/8 triage) live in Table 7, which remains the single point of truth for the twenty-eight coefficients.

Element of $\mathcal{L}_{\rm SM}$	**Textbook status**	**Export origin**	**Register**
Gauge group $G_{\rm SM}$	Postulated (S1)	S-3: descent splitting, Eq. (90.1)	**D**
Gauge kinetic structure	Postulated	S-4: bulk Yang–Mills, compact integration	**D**
g_s, g, g'	Measured	S-4: seam registrations (Part VI)	**D** (QD)
Field content + hypercharges	Postulated (S2)	S-2: zero modes of one bulk spinor, read subject to O_{24}	**D**
Anomaly cancellation	Consistency check	S-2: automatic per bulk packet	**D**
$N_g = 3$ families	Input (S3)	S-2: GG-P2-N3 pincer	**D***
Chirality of the weak force	Input	S-2: the KK projection; the oriented cascade; read subject to O_{24}	**D**
Higgs doublet exists	Postulated (S4)	S-6: required bulk $\mathbf{5}_H$, selected as minimal (§23.3)	**selected** (representation)
Potential shape V	Chosen polynomial	S-6: forced Fourier series for the selector; the doublet's potential is generated by the holonomy transport, not written (O_{11})	**D** (selector)
$\mu^2 < 0$	Tuned by hand	S-6: branch selection for the selector; no dial exists; the doublet's own mass term is O_{11}'s	**D** (selector)
v, λ_H	Measured	S-6: $M_{\rm lock}(2/L)^N$; 2^{-N}	**D/D***
Yukawa form	Postulated	S-7: unique admissible coupling	**D**
Yukawa content (12+8+3)	Measured	S-7: tolls, codes, gates (Part VII)	per triage
$\bar{\theta}$	Tuned $< 10^{-10}$	S-8: not exported; determinant real (T-SM-47.1), sign on L9	**D*** (L9)
Operator completeness	Listing rule (S5)	S-8: admissible descent	**D** (principle)
Proton-decay operators	Forbidden by fiat[†]	S-8: exact global B (Thm. 86.1)	**D***
Extra content: $\mathrm{U}(1)_B$, $\mathrm{U}(1)_{Qi}$, the Kaxion	— (absent)	S-8: protected remnants and the tensor's light remnant; falsifiable	prediction

[†]In the minimal Standard Model baryon number is an accidental symmetry of the renormalizable operator list; generic completions (including classic SU(5)) violate it, which is why proton decay was the decisive test that falsified the 1974 unification.

Remark 41 (The honesty boundary of the assembled Lagrangian)**.** Six statements the assembly does *not* make. (i) The framework of quantum field theory is imported, not derived: the descent determines *which* field theory the boundary carries, while the operational recovery of quantum mechanics itself is the business of GG-A8-QM and GG-A5-GDOS, cited and consumed. (ii) The four-dimensional Lorentz structure of the boundary stratum is constitutional input (AX-1, GGEP), not output. (iii) Every register marked **D*** above stands on its named lemma, and the promotion roadmap for those lemmas is the appendix Part's closing chapter; nothing in the assembly upgrades a grade. (iv) The QD layer remains a refinement register with stated uncertainties, not a claim of exactness. (v) "Complete" means the complete term content of $\mathcal{L}_{\rm SM}$ with all coefficients placed

on the ledger at their stated grades — six **Derived**, fourteen **Derived (cond.)**, eight **Asserted** — not "every decimal derived." (vi) The bulk action itself enters at stated registers — its form **Derived** at lowest derivative order under the constitutional clauses, its matter content **D*** on the Minimal-Matter Lemma, its gauge group inheriting the conditional clauses of T-GG6-3 of GG-A2-GG6 (Arisaka, 2026, A2-GG6) — and the assembly upgrades none of those grades either; the walk that earns the action is §G.3. The assembly is exactly as strong as the descent behind it, no stronger; what is new on this page is not any single claim but the fact that the claims compose into the whole object.

For fifty years the Standard-Model Lagrangian has been the object physics reasons *from*. The last four sections have treated it as an object physics can reason *to* — and, we would argue, that inversion is the single clearest statement of what the GG-Theory program is. The reader who compares Eq. (91.1) with the first page of any quantum field theory course will find the same symbols in the same places; the difference is that here every symbol arrives carrying its provenance, and the two great silences of the textbook — *why these terms* and *why these numbers* — have both been given answers with stated strengths.

The Lagrangian is assembled, and the codebook that populates it is closed. The next Part asks what the closed codebook *means* — why every electron is identical, why exactly four forces exist with their observed behaviors, and why these two facts are one uniqueness theorem read at two layers of the same geometry.

93 Part X in plain English: the textbook equation, written out and accounted for

This Part writes out the Standard-Model Lagrangian in exactly the form a textbook gives it, and then builds it back up term by term from the descent.

The exercise sounds like a formality and is not. A textbook Lagrangian carries several structural choices that are made silently: which fields exist, which representations they sit in, that the gauge group is a product of three factors, that the scalar is fundamental, that certain operators one could write down are simply absent. Those choices are not derived in the textbook. They are the shape of the answer, assumed at the start.

Here each of them is replaced by a theorem, and — this is the part worth attention — the operators that are *absent* are derived alongside the ones that are present. It is easy to produce a framework that yields the terms you want. It is harder to produce one that also forbids the terms you do not want, at the right strength, without a separate assumption for each. The absence of a scalar potential is the sharpest case: the textbook's hand-tuned mass term is replaced not by a better scalar but by a hole where the scalar used to be, with the geometry paying the bill.

The honesty boundary is printed beside the assembled equation, deliberately: some term groups are exported cleanly, others rest on the conditional results of earlier Parts and are marked, so that a reader can see at the equation itself which parts of it the framework has earned.

Part XI
Geometric origin of identity and uniqueness of forces

Why every electron is the same, why exactly four forces exist with their specific qualitative behaviors, and the recognition that identity-uniqueness and force-uniqueness are the same uniqueness theorem applied at different layers of the same geometric cascade

94 The geometric origin of identity: why every electron is the same

We open Part XI with the most general consequence of the framework, the consequence that motivated the whole program and that has, at last, found a physical answer. The empirical fact is everywhere around us: every electron in the universe has the same mass, the same electric charge, the same magnetic moment. Two electrons in a hydrogen atom on Earth and two electrons in a hydrogen atom in a quasar twelve billion light-years away are indistinguishable to within the best precision experiment can achieve. The same is true of every proton, every H_2O molecule, and, in a sense we will make precise, every strand of DNA. Standard treatments of the Standard Model accept this fact and use it; they do not explain it. The present framework explains it.

94.1 The ground-state refinement: from GA to GGA

The explanation rests on one refinement of the runtime vocabulary, established at runtime level in GG-A6-GRIP (Arisaka, 2026, A6-GRIP) and generalized across the layers of the program-wide cascade in the capstone GG-A11-Origin (Arisaka, 2026, A11-Origin). A Morse landscape generally admits *many* index-zero minima — many Geometric Attractors — and identity cannot rest on "a" stable minimum: if several were occupied, electrons would come in varieties. The cascade output is sharper. On any stratum of the class GG-A6-GRIP calls admissible — and the charged-lepton stratum CP^2 is one — that paper's per-stratum ground-state corollary gives exactly one index-zero critical point, which is therefore the global minimum,

$$\mathsf{GGA}(\Sigma) \;:=\; \mathsf{GA}^{*} \;=\; \arg\min_{x\in\Sigma} \mathsf{GFF}(x), \tag{94.1}$$

the *Ground* Geometric Attractor; the cascade DGI $\to$ (GFF) $\to$ GSSB $\to$ GA terminates there. Dynamically, the GGA is where GRIP-In — the fixed-landscape regime of GG-A6-GRIP — halts: the unique attractor from which no admissible exit channel remains. The four Qi-4 pillars of GG-A7-Qi4 do not enter this construction: as that paper's characterization theorem states, they are properties the ground attractor carries, not the filter that finds it. The chemistry analogy is exact, and it is the

right intuition to carry through this Part: a molecule has many stationary electronic configurations, but every water molecule in the universe is identical — and indefinitely stable — because its electrons occupy not merely *a* stable configuration but *the* lowest one. The GGA is "ground state" promoted to a constitutional principle. Identity and stability will now be seen to arrive together, at every layer, as the two faces of ground-state-ness: all copies are identical because there is only one deepest basin to occupy, and the copy persists because there is no deeper basin to decay to. With the vocabulary in place, the three classic electron questions can be answered precisely.

94.2 Three questions about the electron, finally answerable

Three questions about the electron have stood unanswered through the entire history of physics.

(a) Why does every electron have the same mass? A textbook will say that the electron mass $m_e \approx 0.511$ MeV is a fundamental constant of nature, the same for every electron because that is how fundamental constants behave. This is not an explanation; it is a restatement of the question. In the present framework, the electron mass is the value at which the geometric free-energy functional GFF on the charged-lepton boundary stratum reaches its global minimum after the GRIP cascade DGI $\to$ GSSB $\to$ GGA — the GRIP-In descent on the fixed family landscape — runs to completion and halts. The stratum admits several stable minima — the three family depths are all GA's — but exactly one ground state, Eq. (94.1); the value at *that* minimum is what we measure. Every electron is the same because every electron is the readout of the same GGA on the same boundary geometry — not merely *a* stable attractor but *the* ground state of the attractor manifold, of which the stratum has exactly one.

(b) Why is the electric charge of every electron the same, quantized to e? A textbook will say that electric charge is conserved by the unbroken $\mathrm{U}(1)_{\mathrm{EM}}$ gauge symmetry, and that e is a measured number. Once again this is not an explanation; it leaves untouched the question of why every electron carries exactly that quantum and not some other. In the present framework, charge quantization is the topological-integer signature of the same GRIP cascade. The unbroken $\mathrm{U}(1)_{\mathrm{EM}}$ direction in the bulk gauge group is generated by a fixed, integer-normalized generator inside the six-dimensional bulk; the electron's charge is the eigenvalue of that generator on the unique stable leg of the cascade, and that eigenvalue is necessarily an integer multiple of one fundamental unit. Every electron carries exactly $-e$ because every electron is the readout of the same cascade leg of the same generator.

(c) Why is the electric force between two charges the same everywhere? A textbook will say that Coulomb's inverse-square law follows from massless photon exchange and that the coupling $\alpha = e^2/(4\pi\epsilon_0\hbar c)$ is a fundamental constant. This too is a restatement, not an answer. In the present framework, the photon's exact masslessness is a consequence of $\mathrm{U}(1)_{\mathrm{EM}}$ being the generator the doublet's vacuum value leaves unbroken, with the Wilson–Hosotani holonomy at the unification step of (16.3), and the value $\alpha^{-1}(M_Z) = 2^J = 128$ is the geometric attractor coordinate of the dyadic-closure theorem. Two charges anywhere in the universe interact through the same

Coulomb force because they couple to the same massless gauge field, whose existence and whose coupling strength are both fixed by the same six-dimensional bulk. The Coulomb law does not just happen to be universal; the universality is the reason the law exists at all.

The unifying answer to (a), (b), and (c) is one sentence: *electrons are universal because they are the readout of the Ground Geometric Attractor of the GRIP cascade on the charged-lepton stratum — the unique deepest minimum of* GFF *— and a stratum, like a molecule, has exactly one ground state.* This is, to the best of the author's knowledge, the first time in the history of physics that the three questions have been answered together at theorem level, from a single underlying principle, rather than separately as fitted constants. The point is not rhetorical: any framework that treats m_e, e, or α as inputs cannot answer them, because the answers are about why these numbers are the same everywhere, and an input number gives no such reason. Only a framework in which each of the three is the readout of a uniqueness theorem can.

The dedicated electron paper. The three questions of this section — together with the two answered elsewhere in the cohort: why the heavier copies exist and terminate at three (GG-P2-N3), and why the electron is point-like at every scale — receive their dedicated, claim-status-disciplined treatment in the companion paper GG-P3-Electron (Arisaka, 2026, P3-Electron), which assembles identity, stability, the excited copies, pointlikeness from non-spatial structure, and field response as composite geodesic motion under the two-sector reading (the phase sector moves the electron, the scale sector transmutes it; radiation is motion's dissipative channel) into a single theorem-complex about ground-state-ness, with every fence declared.

94.3 The same principle, layer by layer, from quarks to single-cell life

The principle that explains universal-electron identity is not unique to the electron. Every layer of cosmic evolution after the six-dimensional bulk locks in is the readout of the same GRIP cascade on a different boundary stratum. At each layer the cascade terminates on the layer's GGA — exactly one deepest attractor (or, where symmetry mandates degenerate copies, a finite quantized list of them) — and that GGA's coordinates are what the next layer reads as identity. The cascade is summarized here at seven successive scales; the full cascade with all ten transitions and all energy quanta is the subject of the capstone paper of the program (Arisaka, planned, BookI).

1. Leptons and quarks. On the charged-lepton stratum the GRIP cascade has one stable attractor at each cascade depth $n_e \in \{0, 1, 2\}$, and the three depths, populated by the three families, produce m_τ, m_μ, and m_e. On the quark strata the same logic produces $\{m_u, m_d, m_c, m_s, m_t, m_b\}$. Universality at this layer is the fact that every electron in the universe occupies the same depth-2 attractor on the same lepton stratum; every up quark occupies the depth-4 attractor on the up-type stratum; and so on. The ground-state refinement adds the decisive clause: of the three populated family attractors, the first generation is the stratum's GGA — which is precisely why the electron is absolutely stable while the muon and tau, occupying higher, *non-ground* attractors of the same stratum, decay toward it. Identity and stability arrive together, both as ground-state statements: the periodic table of particles has excited states, and they behave exactly as excited states should. The

codebook of Part VII is, in this reading, the catalog of attractor coordinates on the family-projector stratum $\mathbb{C}P^2$.

2. Protons (quark confinement). Quarks do not appear as free particles; they are confined into color-singlet bound states by the strong force. In the GRIP reading, confinement is a GSSB step: the runtime selects the color-singlet branch of the quark configuration space because that is the unique stable attractor of GFF in the strong-coupling regime. The proton is the minimum-energy color-singlet baryon under the constraints of gauged $\mathrm{U}(1)_B$ baryon-number conservation; that minimum is unique — it is the baryonic stratum's GGA — so every proton in the universe has exactly the same mass $m_p \approx 938.3$ MeV and exactly the same charge $+e$. And because $\mathrm{U}(1)_B$ is gauged, the "no deeper basin" clause holds exactly rather than approximately: proton stability is ground-state stability, the GGA statement in its sharpest form. Proton stability — every proton in the universe is the same proton, lasting longer than the age of the universe by orders of magnitude — is the layer-2 echo of the same uniqueness theorem that made every electron the same electron.

3. Elements (H, He, C, N, O, ...). At the atomic layer, the GRIP cascade selects, for each integer atomic number Z, the unique ground-state nuclear configuration with charge $+Ze$, the unique ground-state electronic configuration with Z bound electrons, and the unique characteristic spectral lines. The Aufbau principle, the periodic table, and the chemistry of each element are all readouts of the layer-3 attractor on the atomic stratum. Hydrogen is the same hydrogen everywhere because the $Z = 1$ attractor is unique; helium is the same helium everywhere because the $Z = 2$ attractor is unique; the same is true of carbon, nitrogen, and oxygen. At this layer the runtime's vocabulary and chemistry's coincide exactly: each of these configurations is the Z-stratum's GGA, and "ground state" — chemistry's own word for it — turns out to have been the constitutional concept all along. Astronomers see exactly the same Balmer series in distant quasars that they see in laboratory hydrogen lamps because both are reading out the same cascade attractor on the same atomic stratum.

4. Molecules in the interstellar medium. At the next layer, the GRIP cascade operates on the molecular configuration space. The molecules detected in cold dense interstellar clouds — H_2, CO, H_2O, NH_3, CH_3OH, and several hundred others — are the GGA's of GFF on the molecular stratum at the temperatures and densities prevailing in those clouds. The fact that the same molecules are observed in star-forming regions across the galaxy and in distant galaxies at high redshift is the layer-4 echo of universality: the molecular attractors are the same everywhere because the underlying cascade is the same everywhere. Astrochemistry is a survey of these attractors.

5. Complex organic molecules. The cascade continues into the regime of polyatomic carbon-based chemistry. Amino acids, sugars, and small nucleotide precursors are the GGA's of GFF on the polyatomic-organic stratum at the temperatures and pressures of warm, wet planetary surfaces and proto-stellar nebulae. Glycine has been detected in cometary ice and, by inference, in the gas phase of distant star-forming regions; the same twenty proteinogenic amino acids underlie all known

terrestrial life. This is the layer-5 echo of the same uniqueness principle: where the conditions allow the cascade to reach this layer, the same set of attractors is selected.

6. ATP, RNA, DNA. At the next layer, the cascade selects the energy-quantum carrier and the information carriers of life. ATP is the GGA of GFF on the metabolic-energy stratum; its energy–information role is developed in GG-A11-Origin. RNA and DNA are the GGA's on the information-storage stratum; the four-letter alphabet $\{\mathrm{A}, \mathrm{C}, \mathrm{G}, \mathrm{T}\}$ is the unique solution of the quadruple gauge-locking constraint at this layer. Every cell on Earth uses the same ATP currency and the same DNA alphabet because both are the layer-6 readouts of the same attractor selection.

7. Single-cell life. Finally, the cascade selects single-cell life as the GGA of GFF on the open-system stratum subject to the four boundary conservation conditions of the Selfish Open System (SeOS) class of GG-A5-GDOS. A bacterium and an archaeon — the two oldest extant lineages of life on Earth — share the same genetic code, the same ATP energy currency, the same membrane-based compartmentalization, and the same fundamental metabolic pathways because all of these are layer-7 attractor coordinates of the same cascade on the same stratum. The Last Universal Common Ancestor (LUCA), in this reading, is the empirical record of the layer-7 attractor.

94.4 The principle, plainly stated

The unifying principle of the seven layers above can be stated in one sentence. *At each layer of cosmic evolution after the six-dimensional bulk locks in, the GRIP cascade terminates on the Ground Geometric Attractor of the layer-specific boundary stratum — the unique deepest minimum of* GFF*, not merely a stable one — and what we observe as the universal identity of physical objects at that layer is the readout of that GGA's coordinates.* Stability and identity are the same fact seen twice: an object persists because its configuration has no deeper basin to decay to, and all its copies are identical because there is only one deepest basin to occupy.

This single principle answers, in one stroke, three classes of question that physics, chemistry, and biology have each treated separately:

- *Particle physics*: why every electron, every proton, every W boson is identical to every other.
- *Chemistry*: why every hydrogen atom, every $\mathrm{H_2O}$ molecule, every amino acid is identical to every other.
- *Biology*: why every cell on Earth uses the same ATP currency and the same DNA alphabet.

Each class has, until now, treated identity as an empirical given. The framework of the present paper, applied at the layer above the Standard Model and recursively at each higher layer, derives identity as the consequence of attractor selection.

Why this matters for the present paper. GG-A9-SM is a particle-physics paper. Its formal task is to close the twenty-eight-parameter codebook of the boundary Standard Model. But the principle that closes that codebook — the GRIP cascade terminating on the unique GGA of each

stratum — is the same principle that closes every higher layer. In closing the boundary codebook with no continuous freedom, GG-A9-SM is also closing, by the same mechanism, the deeper question of why the boundary objects exist at all in the form they do. Universality is not an empirical accident; it is the unique signature of an attractor selection theorem.

After all, the question "why is every electron the same?" has been asked in some form by every careful student of physics since Thomson identified the electron in 1897. For one and a quarter centuries the answer has been "because that is what an electron is". The framework of this paper, by deriving the electron's mass, charge, and coupling as the readouts of a single uniqueness theorem on the charged-lepton stratum of the six-dimensional bulk, makes the answer in physical terms for the first time: every electron is the same because the geometry permits only one. The same answer, applied recursively, explains why every proton, every hydrogen atom, every water molecule, every strand of DNA, and every living cell shares the universal identity that biology, chemistry, and physics each take as given. The universe is not full of arbitrary identical objects; the universe is full of ground states, and the objects are what the GGA's look like from the boundary.

94.5 Why the weak interaction exists: GRIP ending at the GGA, realized in Nature

The identity principle of this section has a dynamical shadow, and following it answers one of the oldest existential puzzles in particle physics. Where §48 established that the weak interaction is *special* — the unique *gate force* able to carry matter across the gates between family basins — this subsection asks the prior question, why such a force should *exist* at all, and exhibits the same force as it is realized across Nature; the gate of §48 is the threshold of a corridor, and what follows walks the corridor. Of the four boundary forces, three have conspicuously constructive jobs. The strong force binds quarks into hadrons and hadrons into nuclei; electromagnetism builds atoms, molecules, chemistry, and light; gravity aggregates matter into planets, stars, and galaxies. The weak interaction builds nothing. It binds no states; no material structure carries its name; remove it from a textbook's table of contents and no chapter on the architecture of matter would notice. Its one conspicuous job is to make things *decay*. Rabi's famous "Who ordered that?" was aimed at the muon, but the question underneath it — why should Nature contain a force whose only visible occupation is destruction? — has been posed by generations of physicists since, and the standard answer has been silence: the Standard Model writes the weak interaction down and declines to say why it is there. The closed codebook of this paper, together with the ground-state refinement of §94.1, permits a structural answer, and we state it first in one sentence:

> *The weak interaction is the runtime's relaxation channel: the unique physical implementer of GRIP-In — the fixed-landscape regime of GG-A6-GRIP — on the matter strata, the corridor by which matter reaches the Ground Geometric Attractors whose existence and uniqueness the runtime guarantees. A constitution that mandates unique ground states on the admissible strata (GG-A6-GRIP's per-stratum ground-state corollary) must also mandate a road to them. The weak interaction is the road.*

The uniqueness observation. The categorization rests on a sharp structural fact, checkable against the boundary gauge group of Part I. The strong and electromagnetic interactions are flavor-diagonal: they build the basins of the matter landscape and relax excitations *within* a basin — $\rho \to \pi\pi$ and $\Delta \to N\pi$ are strong relaxation, an excited atom radiating to its ground state and $\pi^0 \to \gamma\gamma$ are electromagnetic relaxation — but no flavor-diagonal force can carry a state from one family basin to another. Within $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$, the $\mathrm{SU}(2)_L$ charged current is the *only* flavor-connecting channel. This is the gate-force uniqueness established in §48 (the basin-versus-gate comparison of Table 29), seen now from the side of what that unique channel *does*: it is the relaxation corridor whose gate no other force can cross.

Two clean clauses follow. *(i)* Every inter-basin descent observed in Nature is weak. *(ii)* Every weak flavor transition is a step of the GFF descent toward the stratum's conditional GGA. A third clause, in regime language, closes a misreading before it can open: every weak process in Nature is GRIP-In; *no weak process is a GRIP-Out.* Weak decays look like "exits" — a state leaves its basin — but the landscape they descend is fixed; the one GRIP-Out of the electroweak story happened once, when the Wilson–Hosotani branch was selected and the landscape built (§42.2).

For the same reason, producing a τ or a top quark in a collider is not a GRIP-Out either: pumping a state uphill through the boundary ports is excitation *within* the fixed landscape, promptly followed by GRIP-In relaxation. And the famous fine print of the weak sector reads naturally in the same language: the GIM suppression of flavor-changing neutral currents is the statement that there are *no diagonal shortcuts* between non-adjacent basins — descent proceeds by the charged-current ladder, rung by rung — while the hierarchical structure of the CKM matrix (powers of the Wolfenstein parameter $\lambda \simeq 0.22$) is the *adjacency gearing* of that ladder, nearest-depth steps strongest, exactly as a Morse cascade demands and exactly as the misalignment codebook of Part VII computes.

The descent across Nature: five exhibits. The reach of the categorization is best displayed by walking it through the laboratory, the chart of nuclides, the Sun, the first three minutes, and a neutron star. Every row of the table is the same regime: GRIP-In on a fixed landscape, at five wildly different tempos.

Table 55: The same descent at five tempos, from the laboratory to a neutron star. Each row names the system, the weak descent it undergoes, the attractor it reaches, and the clock on which it does so. The last column is the point: the clocks span more than thirty orders of magnitude while the regime does not change, which is what makes the categorization a structural claim rather than a description of one process. The final row is conditional and is marked as such in the table.

System	Weak descent	GGA reached	Clock
Lepton ladder	$\tau \to \mu \to e$	electron (depth 2)	10^{-13} s; 10^{-6} s; ∞
Quark ladder	$t \to b \to c \to s \to u/d$	u, d (depth 4)	10^{-25} s down to stable
Free neutron	$n \to p\, e\bar{\nu}$	proton (baryonic GGA)	879 s
Chart of nuclides	$\beta^{\pm}$, EC along isobars	most-bound isobar	seconds to Gyr
The Sun	$pp \to d\, e^{+}\nu$ (first step)	helium ladder onset	$\sim 10^{10}$ yr lifetime
Primordial plasma	$n \leftrightarrow p$ freeze-out	BBN abundances	$G_F^2 T^5 = H$ at ~ 0.8 MeV
Neutron star	$p\, e^{-} \to n\, \nu$ (neutronization)	neutron matter	conditional GGA

Exhibit 1: the family ladders. The lepton and quark readings were developed in §47.3 and §49.3: non-ground GA's relaxing rung by rung to the GGA's of their strata, with the lifetime hierarchy as the rate readout. They are the elementary instances; the remaining exhibits show the same channel running entire macroscopic worlds.

Exhibit 2: the valley of β stability. At fixed mass number A, the semi-empirical binding energy is a parabola in proton number Z; β^-, β^+, and electron capture step a nuclide down that parabola, one weak transition per rung, to the most-bound isobar. The chart of nuclides is therefore a *literal* stratified Morse landscape — the most photographed one in science — and β decay is its gradient flow. Every introductory nuclear-physics course plots GRIP ending at the GGA; it has simply lacked the name.

Exhibit 3: the Sun. The first step of the pp chain, $p + p \to d + e^+ + \nu$, is a weak transition — the only step of stellar hydrogen burning that must convert a proton into a neutron — and it is the bottleneck that sets the solar luminosity. The Sun shines at the pace of the weak descent; the ten-billion-year main-sequence lifetime that makes planets, chemistry, and biology possible is the slowness of one GRIP rung.

Exhibit 4: the first three minutes. The primordial neutron-to-proton ratio froze when the weak interconversion rate $\sim G_F^2 T^5$ fell below the Hubble rate at $T \simeq 0.8$ MeV, and every helium nucleus in the universe is a fossil of that moment. The light-element abundances are a snapshot of the weak clock racing the expansion — the descent rate measured against cosmology itself.

Exhibit 5: the conditional GGA. In a neutron star the same channel runs "uphill" in vacuum masses — $p + e^- \to n + \nu$, neutronization, the URCA processes — because at supranuclear density, with degeneracy pressure in the ledger, neutron matter *is* the ground state of the stratum. This is the decisive exhibit: the weak interaction does not consult a fixed mass table; it descends the *free energy of the stratum*, finding whatever the conditional GGA is under the prevailing boundary conditions. A relaxation channel that tracks GFF rather than mass is precisely what the runtime categorization predicts and what a "decay force" caricature does not.

Why the weak force must be weak. The categorization also explains the adjective. Dial the Fermi constant and watch the universe fail in both directions. If the descent coupling were of strong-interaction size, every non-ground basin would empty essentially instantly: no muon or tau spectroscopy, no metastable flavors, a pp bottleneck so fast that stars would burn out in cosmological moments, a BBN epoch that processes all hydrogen — a universe that reaches its ground states immediately and has no *history*. If the coupling were zero, the cascade would freeze: matter locked forever in whatever flavor configuration the initial conditions happened to deposit — taus and strange matter as permanent species, arbitrary isobars as stable elements, no universality of matter, a universe of accidents with no *arrival*. The measured weakness is the constitutional compromise between the two failures: strong enough that every basin drains to its GGA well within cosmic time, weak enough that the draining itself — stars, chemistry-bearing planets, the slow nuclear furnaces — constitutes a rich intermediate world. And the dial is not free: $G_F = 1/(\sqrt{2}\, v^2)$ with v the derived Wilson–Hosotani vev of Theorem 34.1, so the tempo of cosmic history is itself a ledger readout, not an accident.

A chiral postscript, fenced. One structural rhyme deserves recording without overclaim. The unique descent channel of the matter strata is also the unique parity-violating force — the only one that distinguishes left from right is the only one whose job is an oriented, irreversible flow. Whether the orientation of the coupling and the orientation of the descent are two faces of one geometric fact is, in the present state of the program, not yet a theorem. §48 advances exactly this identification — chirality as the orientation of the Morse descent, and CP violation as the gate holonomy — as a structural reading, and the gate-operator lemma (L11) of the promotion roadmap (Appendix P) is the named lemma whose proof would convert that reading into a result. We record the chiral rhyme here as a phenomenon and defer its proof to that lemma.

The existential answer. We can now answer the question this subsection opened with, and the answer belongs to the identity principle of this section. Without the strong force there would be no rooms; without electromagnetism, no walls or furniture; without gravity, no building; without the weak force, *no corridors* — and the universe would remain locked, forever, in whichever rooms it happened to be born. The weak interaction exists because the constitution's promise of unique ground states would otherwise be empty: GG-A6-GRIP's per-stratum corollary guarantees that every admissible stratum — the matter strata among them — *has* a GGA; GRIP-In is the regime that reaches it; and the weak interaction is how matter *gets there* — Nature's implementation of GRIP-In on the matter strata. It is the reason the matter around us is the universe's ground-state catalog — the same electron, the same proton, the same elements everywhere — rather than a frozen record of its initial accidents. This is the existential face of the co-entailment of §48.3: a replicated family landscape and the gate force that drains it require each other, and the toll-budget closure fixes the number of basins at exactly three. A force that builds nothing turns out to be the force that makes everything else *findable*; the question "who ordered that?" has, at last, a structural reply: the constitution did.

95 The geometric origin of force: why these four forces and no others

§94 answered the first half of the deepest empirical fact about the universe: why every electron, every proton, every hydrogen atom, every water molecule, and every living cell shares a universal identity. The answer was geometric — a uniqueness theorem on a layer-specific attractor stratum. We now turn to the second half of the same fact. The universe contains exactly four fundamental forces — gravity, electromagnetism, the strong nuclear interaction, the weak nuclear interaction — with exactly the qualitative behaviors we observe. Why these four? Why not three, or five, or seventeen? Why is gravity always attractive between positive masses? Why is electromagnetism long-range and two-signed? Why does the strong force confine at long distance while running asymptotically free at short distance? Why does the weak force violate parity and CP? Standard treatments of the Standard Model accept these qualitative properties as inputs and tabulate them; they do not explain them. The framework of the present paper does, by the same kind of uniqueness argument that explained universal identity in §94: the constitution of geometry permits exactly

one self-consistent six-dimensional bulk, and the four forces and their qualitative behaviors are the boundary projection of that one bulk.

95.1 Eight constitutional lemmas force the canonical six-dimensional bulk

The constitutional uniqueness of the bulk geometry $\mathcal{M}_6 = \mathcal{M}_4 \times I_r \times S^1_\theta$ is not an aesthetic choice; it is the joint consequence of eight separately-named clauses of the GG-Constitution. We pin each clause to its bulk-geometric duty in turn. The eight together leave one and only one admissible solution.

Lemma 1 (the floor: $D \geq 4$). The Lorentz-Causality axiom AX-1 of GG-A1-Constitution (T1.1) plus the Operationally Geometric Gauge Equivalence Principle AX-3 (OGGEP, GG-A1-Constitution T1.3) demand that physics be described on a Lorentzian manifold admitting operational observers with distinct events and a causal measurement protocol. The smallest manifold of this kind is four-dimensional Minkowski space $\mathcal{M}_4$: three spatial dimensions plus one time direction. In fewer than four dimensions the causal structure degenerates, and there are not enough independent observables to support the operational definition of measurement. Hence $D \geq 4$.

Lemma 2 (gauge fields demand extra geometry: $D > 4$). The Geometric Gauge Equivalence Principle AX-2 (GGEP, GG-A1-Constitution T1.2) demands that gauge fields be carried by the bulk geometry rather than appended to it. Geometric structure means data on a fiber bundle with non-trivial transverse directions: an internal connection on a manifold of dimension strictly greater than four. If the only geometry available were $\mathcal{M}_4$, the Yang–Mills connections of the Standard Model would have to be re-imported as external auxiliary fields, defeating the entire content of AX-2. Hence $D > 4$.

Lemma 3 (RG flow needs a radial coordinate: one radial interval I_r). Postulate POS-5 of the GG-Constitution (renormalization-group covariance, GG-A1-Constitution T3.5) demands that the renormalization-group scale μ be a geometric coordinate of the bulk. The natural realization identifies $\mu = M_{\rm Pl}\, e^{-r}$ for a radial bulk coordinate r. Renormalization-group flow has a UV endpoint at $r = 0$ (where $\mu = M_{\rm Pl}$) and an IR endpoint at $r = K$ (where $\mu = M_{\rm lock}$); r must therefore have endpoints, not be periodic. Lorentz invariance under RG transport applied to the four-dimensional metric on each radial slice forces the form $g_{\mu\nu}(r) = e^{-2r}\,\eta_{\mu\nu}$. The minimal solution is one radial interval $I_r = [0, K]$. A second radial coordinate would carry a second copy of the RG scale, but the RG scale is one number; AX-4 (Maximum Symmetry, Minimum Freedom, MSMF, GG-A1-Constitution T1.4) forbids the duplication. Hence the bulk contains one and only one radial interval.

Lemma 4 (the Higgs mechanism needs a compact direction: one circle S^1_θ). Postulate POS-7 (Measurement Foundation, GG-A1-Constitution T3.7) demands that the electroweak-symmetry-breaking mechanism be operational — derivable from bulk-level dynamics, not postulated. The Wilson–Hosotani realization (Y. Hosotani, 1983; K. G. Wilson, 1974) fulfils this requirement by

promoting the electroweak order parameter from a local scalar to the Wilson-line holonomy along a compact bulk direction; the effective potential for the holonomy angle is generated dynamically by one-loop bulk quantum fluctuations. Holonomy requires a coordinate that closes back on itself; the minimal compact realization is one circle $S^1_\theta = [0, 2\pi)$. A second compact direction would supply a second holonomy parameter; MSMF forbids the redundancy. Hence the bulk contains one and only one compact circle.

Lemma 5 (no more: MSMF closes the count at $D = 6$). Lemmas 1–4 have assembled $\mathcal{M}_4 + I_r + S^1_\theta = 4 + 1 + 1 = 6$ dimensions; each summand discharges a non-redundant constitutional duty. Any seventh dimension would be free, and MSMF rules it out. Combined with Lemma 2 ($D > 4$), the upper bound $D \leq 6$ is as constitutional as the lower bound. Hence $D = 6$.

Lemma 6 (the BI-6 anomaly polynomial closes only in $D = 6$). The Bianchi-Identity-in-six-dimensions theorem of GG-A3-BI6 (BI-6, T1) establishes that the closure $dH_3 = X_4$ admits a unique four-form anomaly polynomial,

$$X_4 \;=\; \mathrm{tr}(R \wedge R) \;-\; \mathrm{tr}(F_{\mathrm{SU}(5)} \wedge F_{\mathrm{SU}(5)}) \;-\; \tfrac{2}{5}\, F_B \wedge F_B \;-\; \tfrac{2}{5}\, F_{Qi} \wedge F_{Qi}, \tag{95.1}$$

with the trace-ratio lock $|a/c| = |a/d| = 5/2$ between the gravitational, hypercharge, and $\mathrm{U}(1)_{Qi}$ anomaly coefficients. In $D = 5$ the form-degree of X_4 is wrong; in $D = 7$ the trace-ratio lock fails to close at integer level. Only in $D = 6$ does BI-6 admit the closure, and the same trace-ratio lock is the independent source of the empirical weak-angle identity $\sin^2\theta_W = 25/108$ that the present paper derives in Part VI (T17.1). Hence the constitutional dimension cannot be other than $D = 6$.

Lemma 7 (the OGN-5 Diophantine ledger closes only in $D = 6$). The OGN-5 theorem T6 of GG-A2-GG6 (OGN-5, T6) establishes that the Diophantine closure $K = LJ = M^2 + N^2 + L^2$ with $J^2 - M^2 = NL$ admits exactly one integer solution in the physical regime,

$$(M, N, L; J, K) \;=\; (1, 3, 5; 7, 35), \tag{95.2}$$

and the closure holds only in $D = 6$. In $D \neq 6$ the closure either fails outright or admits multiple integer solutions inconsistent with MSMF. The OGN-5 quintuple is the integer skeleton from which the present paper extracts the color-triplicity index $N_c = 3$, the scale depth $K = 35$, the defining SU(5) block size $L = 5$, the dyadic charge exponent $J = 7$, and the elementary unit $M = 1$. Hence $D = 6$ is confirmed a second time, by a route completely independent of Lemma 6.

Lemma 8 (the bulk Lorentz spinor structure forces the family content). The Lorentz group of six-dimensional Minkowski space is $\mathrm{Spin}(1, 5)$. Its 32-dimensional complex Weyl representation, restricted under the embedding $\mathrm{SU}(5) \subset \mathrm{SO}(10) \subset \mathrm{Spin}(1, 5)$ that the orbifold projection selects (GG-A2-GG6 T-GG6-2), decomposes as

$$\mathbf{32}_{\mathrm{Spin}(1,5)} \;\longrightarrow\; \mathbf{16} \oplus \overline{\mathbf{16}} \quad \text{of } \mathrm{SO}(10), \tag{95.3}$$

where the **16** contains exactly one Standard-Model fermion family of left-handed chirality. In $D < 6$ the Lorentz Weyl rep does not contain the **16** of SO(10) at all; in $D > 6$ the chiral spinor admits exotic content (gravitinos, additional families) that MSMF rules out without further geometric apparatus. Hence $D = 6$ is forced once more, and the matter content of one boundary family is forced to be exactly the Standard-Model fermion family.

The eight lemmas leave exactly one admissible bulk geometry,

$$\mathcal{M}_6 \;=\; \mathcal{M}_4 \,\times\, I_r \,\times\, S^1_\theta, \qquad ds^2_6 \;=\; e^{-2r}\,\eta_{\mu\nu}\,dx^\mu dx^\nu \;+\; dr^2 \;+\; d\theta^2, \tag{95.4}$$

and exactly one composite gauge–gravity group on that bulk,

$$G_{\text{Bulk}} \;=\; \text{Spin}(1,5) \,\times\, \text{SU}(5) \,\times\, \text{U}(1)_B \,\times\, \text{U}(1)_{Qi}. \tag{95.5}$$

We collect the eight lemmas into a single named theorem, the structural twin of the universal-identity insight of §94.

Theorem 95.1 (**T-SM-62.1**: Uniqueness of the canonical four-force architecture)**.** *Under the constitutional clauses AX-1 (Lorentz-Causality, GG-A1-Constitution T1.1), AX-2 (Geometric Gauge Equivalence Principle, GG-A1-Constitution T1.2), AX-3 (Operationally Geometric Gauge Equivalence Principle, GG-A1-Constitution T1.3), AX-4 (Maximum Symmetry, Minimum Freedom, GG-A1-Constitution T1.4), POS-5 (renormalization-group covariance, GG-A1-Constitution T3.5), POS-7 (Measurement Foundation, GG-A1-Constitution T3.7), the BI-6 closure theorem (GG-A3-BI6 T1), and the OGN-5 Diophantine-ledger closure theorem (GG-A2-GG6 T-GG6-6), the following statements hold jointly.*

1. The bulk geometry is forced. *The constitutional clauses admit exactly one bulk geometry, the canonical six-dimensional product*

$$\mathcal{M}_6 \;=\; \mathcal{M}_4 \,\times\, I_r \,\times\, S^1_\theta, \qquad ds^2_6 \;=\; e^{-2r}\,\eta_{\mu\nu}\,dx^\mu dx^\nu \;+\; dr^2 \;+\; d\theta^2,$$

   with $I_r = [0, K]$ and $K = 35$ the integer scale depth of the OGN-5 ledger.

2. The composite gauge–gravity group is forced. *On the bulk $\mathcal{M}_6$ the admissible composite gauge–gravity group is*

$$G_{\text{Bulk}} \;=\; \text{Spin}(1,5) \,\times\, \text{SU}(5) \,\times\, \text{U}(1)_B \,\times\, \text{U}(1)_{Qi},$$

   *and the matter content of one boundary fermion family is forced to be the Standard-Model family (**16** of* SO(10)*) of left-handed chirality.*

3. The four boundary forces and their qualitative behaviors are forced. *The boundary projection of G_{Bulk} through the canonical orbifold action and the Wilson–Hosotani holonomy on S^1_θ produces exactly four fundamental forces with exactly the qualitative behaviors:*

   - Gravity*: spin-2 mediator, universally attractive between positive sources, exponentially weak with $G_N \sim 1/M^2_{\text{Pl}}$ and $M^2_{\text{Pl}} = M^4_6 \cdot \text{vol}(I_r \times S^1_\theta) \cdot e^{2K}$.*

- Electromagnetism*: spin-1 mediator on the generator the electroweak vacuum leaves unbroken, exactly massless, long-range with $V(r) \propto 1/r$, two-signed by CPT, with $\alpha^{-1}(M_Z) = 2^J = 128$ from the dyadic-closure theorem (T16.1).*
- Strong force*: spin-1 non-Abelian $\mathrm{SU}(3)_C$ mediator, asymptotically free at high energies, confining at $\Lambda_{\mathrm{QCD}} \approx 200$ MeV with $\alpha_s(M_Z) = 1/(\pi e)$ (T18.1).*
- Weak force*: spin-1 massive $\mathrm{SU}(2)_L$ mediator with $M_W = gv/2$, $v \approx 246$ GeV (T19.1), $\sin^2\theta_W = 25/108$ (T17.1), parity-violating by orbifold projection on the radial direction, CP-violating with $\delta_{\mathrm{CKM}} = 13\pi/35$ (T29.1).*

4. No other constitutional bulk admits a self-consistent closure. *Any bulk geometry that violates the dimension, factor-structure, or matter-content statements of (i)–(iii) above either contradicts one of the eight constitutional clauses or fails to admit operational boundary observers.*

Proof. Statement (i) follows directly from Lemmas 1–5: Lemma 1 (AX-1 + AX-3) gives $D \geq 4$; Lemma 2 (AX-2) gives $D > 4$; Lemma 3 (POS-5 + AX-4) supplies exactly one radial interval I_r; Lemma 4 (POS-7 + AX-4) supplies exactly one compact circle S^1_θ; Lemma 5 (AX-4) closes the count at $D = 6$. The same dimension is independently confirmed by Lemma 6 (the BI-6 closure $dH_3 = X_4$ holds at the correct form-degree and trace-ratio lock $|a/c| = |a/d| = 5/2$ only in $D = 6$) and by Lemma 7 (the OGN-5 Diophantine closure $K = LJ = M^2 + N^2 + L^2$ admits the integer solution $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ only in $D = 6$). Three distinct routes — bottom-up dimension counting, top-down anomaly-polynomial closure, and Diophantine integer closure — all converge on the canonical six-dimensional product geometry, with the scale depth $K = 35$ fixed by OGN-5.

Statement (ii) follows from Lemma 8: the $\mathrm{Spin}(1,5)$ Weyl representation, restricted under $\mathrm{SU}(5) \subset \mathrm{SO}(10) \subset \mathrm{Spin}(1,5)$, decomposes as $\mathbf{16} \oplus \overline{\mathbf{16}}$ — one Standard-Model family per chirality. The $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_{Qi}$ factors are forced by the BI-6 trace-ratio lock of Lemma 6: the anomaly polynomial X_4 requires exactly two Abelian factors with coefficient $-2/5$ each, identifying them as the gauged baryon number $\mathrm{U}(1)_B$ and the four-currents $\mathrm{U}(1)_{Qi}$ of GG-A7-Qi4 T-QUA. The composite gauge–gravity group G_{Bulk} stated in (ii) is therefore forced.

Statement (iii) follows force by force from the analysis of §95.3. *Gravity:* the metric perturbation $h_{\mu\nu}$ is rank-2 symmetric traceless by the $\mathrm{Spin}(1,5)$ Lorentz factor, fixing the mediator spin at 2; the standard tree-level exchange-sign rule for massless spin-J exchange then forces universal attraction between positive sources, and the scale profile e^{2K} with $K = 35$ forces the four-dimensional Planck mass to $\approx 10^{19}$ GeV, hence the smallness of G_N. *Electromagnetism:* the unbroken $\mathrm{U}(1)_{\mathrm{EM}}$ generator is the one the doublet's vacuum value leaves unbroken (the holonomy acts at the unification step, (16.3)), the photon is therefore exactly massless and Coulomb is long-range, and the two signs of electric charge are the two orientations of the compact fiber — charge being a winding number on S^1_θ and the antiparticle the same holonomy class read in the reversed traversal (T-P4-1, T-P4-3 of Arisaka, 2026, P4-Real; T-P5-A, T-P5-C of Arisaka, 2026, P5-CPT), with the textbook CPT theorem (Jost, 1957) complementary in scope rather than upstream. *Strong force:* the boundary $\mathrm{SU}(3)_C$ inherited from SU(5) has one-loop beta-function coefficient $11N_c - 2N_f = 21 > 0$ for $N_c = 3$ and $N_f = 6$, hence asymptotic freedom (Gross, 1973; Politzer, 1973); confinement at $\Lambda_{\mathrm{QCD}} \approx 200$ MeV follows by the Wilson lattice argument (K. G. Wilson, 1974); the coupling value at M_Z takes the

dyadic-rational form $1/(\pi e)$ by T18.1. *Weak force:* the doublet's vacuum value, its scale set by the Wilson–Hosotani selector, gives W and Z masses via the standard Higgs mechanism at $v \approx 246$ GeV (T19.1); the radial orbifold projection keeps only left-handed Weyl spinors, fixing parity violation; the OGN-5 ledger forces $\delta_{\mathrm{CKM}} = 13\pi/35 \neq 0, \pi$, fixing CP violation in the quark sector.

Statement (iv) is the contrapositive of the lemma chain. Any seventh bulk dimension violates AX-4 by Lemma 5. Any $D \neq 6$ violates either the BI-6 anomaly-polynomial closure (Lemma 6) or the OGN-5 Diophantine closure (Lemma 7) or both. Any deviation from the $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ gauge–gravity content violates Lemma 8 plus the BI-6 trace-ratio lock. No alternative bulk geometry survives the joint constraint. ■

95.2 The intuitive picture: stage, ladder, and spool

The eight lemmas above can be summarized, without loss of content, in three nouns. Imagine you are trying to build the simplest possible universe in which gauge forces are not bolted on by hand — in which they emerge from the geometry. You will need three pieces of equipment.

A stage. A four-dimensional Lorentzian arena $\mathcal{M}_4$ where physics happens — three spatial dimensions and one time direction. This is the floor (Lemma 1). Without it there is no notion of event, no notion of cause, no notion of measurement.

A ladder. A radial bulk coordinate r along the radial interval $I_r = [0, K]$ that connects the high-energy physics at the ultraviolet rung $r = 0$ to the low-energy physics at the infrared rung $r = K$. Two anchors have to be kept apart on this ladder, and it is worth doing so before the ladder is climbed. The *boundary renormalization scale* is measured from the unification scale, $\mu(r) = E_{\mathrm{GUT}}\, e^{-r}$, so the ultraviolet rung sits at $E_{\mathrm{GUT}} \approx 2.28 \times 10^{16}$ GeV — not at the Planck scale — and the infrared rung at $E_{\mathrm{GUT}}\, e^{-K} \approx 14$ GeV. The *masses* of the Kaluza–Klein resonances the same interval supports are measured instead from the bulk scale $M_6 = M_{\mathrm{Pl}}/\sqrt{\pi}$, which is what places the lock cluster at $M_{\mathrm{lock}} \approx 3.85$ TeV. The depth is the same ledger integer $K = 35$ in both readings; what differs is the scale each is measured from, and the two anchors stand in the fixed ratio $M_6/E_{\mathrm{GUT}} = e^{2\pi}/\sqrt{\pi} \approx 302$, itself a pure number. This ladder is what allows the running of couplings, the hierarchy between the Planck and electroweak scales, and the descent from the unification energy to the laboratory (Lemma 3). Without the ladder, the Standard Model is a tower with no foundation.

A spool. A compact angular coordinate θ wound onto the circle S^1_θ of circumference 2π. Wilson-line holonomy along this circle is the selector of the electroweak scale: the periodicity is exactly what admits non-trivial holonomy, the one-loop bulk effective potential for the holonomy angle selects the branch that sets the scale of the doublet's vacuum expectation value $v \approx 246$ GeV (the doublet itself is the bulk $\mathbf{5}_H$'s, §23.3), and the generator that vacuum value leaves unbroken is the $\mathrm{U}(1)_{\mathrm{EM}}$ of electromagnetism (Lemma 4). Without the spool, nothing selects the electroweak scale, and without that there are no W and Z boson masses and no separate long-range electromagnetism.

In the vocabulary of the cohort the ladder is the *scale dimension* and the spool is the *gauge dimension*; neither is a direction of space, and together they are GG-Theory's two *non-spatial dimensions* — a scale and a phase, not two further places.

That is all. Four plus one plus one equals six, and the count cannot be reduced — every summand discharges a distinct constitutional duty — nor increased — MSMF forbids the importation of any further dimension (Lemma 5). BI-6 (Lemma 6), OGN-5 (Lemma 7), and the Spin$(1,5)$ Weyl-spinor decomposition (Lemma 8) confirm the count from three independent angles. *Stage, ladder, spool. Three pieces, no fourth. That is the architecture of the simplest universe that admits gauge forces as geometry.*

95.3 The four forces and their qualitative behaviors

Once the bulk is fixed and the composite gauge–gravity group G_{Bulk} is forced, the qualitative behavior of each of the four resulting boundary forces is also forced. We give the four force-by-force readings now, each in the same compact form: mediator spin, range, sign rule, and coupling value. Where a numerical value has been derived in an earlier Part of the present paper, we cite the source.

Force 1 — Gravity: always attractive, extraordinarily weak. The boundary metric perturbation $h_{\mu\nu}$ inherits its tensor structure from the Spin$(1,5)$ Lorentz factor in G_{Bulk} (Lemma 8). At the boundary, $h_{\mu\nu}$ is a rank-2 symmetric traceless field — spin 2. A standard tree-level exchange-sign rule for massless spin-J mediators (cleanly presented in Weinberg's textbook treatment) establishes that spin-0 exchange always attracts between positive sources, spin-1 exchange repels between like charges, and spin-2 exchange always attracts between positive sources. Once gravity's mediator is fixed to spin 2 by the bulk Lorentz structure, gravity is forced to be attractive between positive masses; there is no antigravity, no negative-mass repulsion at the classical level. The weakness of gravity follows from the scale suppression: the four-dimensional Planck mass is the bulk Planck mass multiplied by an exponential factor in the scale depth, $M_{\text{Pl}}^2 = M_6^4 \cdot \text{vol}(I_r \times S^1_\theta) \cdot e^{2K}$. With $K = 35$ from OGN-5, the factor $e^{2K} = e^{70} \approx 10^{30}$ takes the four-dimensional Planck mass to $\approx 10^{19}$ GeV, 10^{17} times the proton mass; the Newton constant $G_N \sim 1/M_{\text{Pl}}^2$ is consequently extraordinarily small. Gravity is weak because the scale depth is large, and the scale depth is the forced integer $K = 35$ of the OGN-5 ledger.

Force 2 — Electromagnetism: massless, long-range, two-signed. The unbroken $\text{U}(1)_{\text{EM}}$ gauge field is generated by the combination $Q = T_3 + Y$ of bulk SU(5) Cartan generators that the doublet's vacuum value leaves unbroken, the holonomy having reduced SU(5) to the Standard-Model group at the unification step (POS-7, GG-A1-Constitution T3.7; (16.3)). Because that generator is unbroken, the photon eats no Higgs mode and remains exactly massless; the resulting Coulomb potential is $V(r) \propto 1/r$, long-range to all observable distances. The mediator is rank-1 (spin 1); by the same tree-level exchange-sign rule cited under gravity, spin-1 exchange repels between like charges and attracts between opposite charges, so the Coulomb sign rule is forced, not chosen. Two signs of electric charge exist because the compact fiber has two orientations. On the six-dimensional bulk the electric charge of a boundary state is the winding number n of its phase on the compact

circle S^1_θ — charge quantization is the compactness of that circle — and complex conjugation is the reversal of its traversal, $\theta \mapsto -\theta$ (T-P4-1 of Arisaka, 2026, P4-Real; T-P5-A of Arisaka, 2026, P5-CPT). Reversing the traversal reverses the winding, so the conjugate readout carries $-n$; for fermions the charge flip is not a further postulate but the *definition* of the conjugate reading, by the factorization $C = (\theta \mapsto -\theta) \circ B$ with B a fixed real spinor intertwiner (T-P5-C2). That the twin must *exist* at all is the stronger statement, and it is a separate theorem: the circle's complex structure and the clock's positive-energy complex structure agree precisely on the Feynman–Stückelberg pairing, so the existence of antiparticles is the consistency condition between the sixth dimension and causal time (T-P4-3; T-P5-C) — established at the kinematic quasi-free level, and one level below quantization rather than as a positivity condition imposed on its output.

The relation to the textbook theorem is complementarity, not replacement, and the owning papers state it in those terms. Jost's CPT theorem (Jost, 1957) reaches interacting fields and is stronger in scope; it is anatomically silent, constructing the antiunitary operator without saying what C, P and T individually are. The orientation reading names them — reversals of the fiber circle, the spatial slice and the causal clock, with a discrete symmetry antiunitary if and only if it contains T (T-P4-5; T-P5-1) — and proves the interacting case only conditionally, on the standard scattering hypotheses. The division is worth stating exactly, because it is what makes the geometric account worth having: *the theorem constrains the pair — given two objects, their properties must match — but it cannot say whether there are two objects at all.* It is the second question the fiber answers. The coupling itself takes the dyadic-closure value $\alpha^{-1}(M_Z) = 2^J = 128$ (Part VI, T16.1), with the running-corrected fine-structure constant $\alpha^{-1} \approx 137.036$ at the Thomson limit.

Force 3 — Strong force: confining at long distance, asymptotically free at short distance. The boundary $\mathrm{SU}(3)_C$ gauge field is the non-Abelian residue of bulk SU(5) after the orbifold projection and the Wilson–Hosotani breaking; the color group inherits non-Abelianness directly from SU(5). The one-loop beta function for SU(3) with $N_f = 6$ quark flavors has coefficient $11N_c - 2N_f = 33 - 12 = 21 > 0$ (Gross, 1973; Politzer, 1973), so the running coupling decreases at high energies (asymptotic freedom) and grows at low energies until confinement sets in at the scale $\Lambda_{\mathrm{QCD}} \approx 200$ MeV (K. G. Wilson, 1974). Confinement means that only color-singlet bound states propagate as asymptotic boundary states — quarks themselves never appear free. The coupling value at the electroweak scale takes the simple form $\alpha_s(M_Z) = 1/(\pi e) \approx 0.117$ (Part VI, T18.1), with Λ_{QCD} and the proton mass $m_p \approx 938$ MeV following from one-loop radial running. The residual strong force between color-singlet hadrons (the nuclear force) is mediated by pion exchange and carries Yukawa range $\sim 1/m_\pi \approx 1.4$ fm.

Force 4 — Weak force: short-range, parity-violating, CP-violating. The boundary $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge fields are also residues of the SU(5) orbifold projection. The doublet's vacuum value, its scale set by the Wilson-line selector, gives the $W^\pm$ and Z bosons their masses via the standard Higgs mechanism, $M_W = gv/2$ and $M_Z = M_W/\cos\theta_W$, with $v \approx 246$ GeV (Part VI, T19.1) and $\sin^2\theta_W = 25/108$ (Part VI, T17.1). The resulting Yukawa range for charged-current weak processes is $\sim 1/M_W \approx 2.5 \times 10^{-18}$ m. Parity is violated because the orbifold projection in the radial direction keeps the left-handed bulk Weyl spinor and projects out the right-handed one —

the "L" in $\mathrm{SU}(2)_L$ is geometric, not phenomenological. CP is violated in the quark sector because the OGN-5 ledger forces the Cabibbo–Kobayashi–Maskawa phase to

$$\delta_{\mathrm{CKM}} \;=\; \frac{13\pi}{35} \;\approx\; 1.167 \text{ rad} \;\approx\; 66.9^\circ \tag{95.6}$$

(Part VII, T29.1), a value that differs from both 0 and π and therefore breaks CP at the constitutional level. The Sakharov conditions for baryogenesis — baryon-number violation, C and CP violation, departure from thermal equilibrium — are met by this framework, but *not* in the way a Standard-Model reading would suggest, and the difference is the point.

Baryon number is not violated by anything in the bulk. The BI-6 Green–Schwarz inflow *absorbs* the two non-vanishing mixed anomaly coefficients, and the $\mathrm{U}(1)_B$ vector's zero mode is removed by its boundary termination, leaving an exact *global* baryon number — which is precisely what makes $\tau_p = \infty$ a theorem rather than an accident (§109.6; GG-P6-Proton). The inflow is what *pays* the anomaly bill so that baryon number survives; it is not a channel through which baryon number leaks. What no inflow can remove is the topological content of the $\mathrm{SU}(2)_L$ survivor: a single door at $\Delta B = N_{\mathrm{gen}}\, n = 3n$, welded shut below the electroweak crossover by the tunneling weight $e^{-4\pi/\alpha_W} \approx 10^{-162}$ and standing open above it. *The proton cannot decay ($\Delta B = 1$) and the neutron cannot oscillate into an antineutron ($\Delta B = 2$) for the same reason that baryons could be made at all* — the door admits only multiples of three.

The asymmetry is therefore minted as $\Delta L = 2$, not as ΔB. It is created in the *conserved* combination $B - L$, through the constitutional Majorana channel of Part VIII, and converted at the sphaleron door in the Standard-Model equilibrium ratio 28/79. On this selection rule leptogenesis is not a preference among mechanisms; it is the only mechanism left standing.

And the CP violation that pays for it is leptonic. It is *not* δ_{CKM}: the quark sector's contribution to the matter asymmetry falls short of the observed value by roughly ten orders of magnitude, a shortfall this paper is in an unusually good position to price, since it derives δ_{CKM} rather than fitting it. What supplies the second Sakharov condition is the flavored leptonic asymmetry driven by the derived Majorana parity $\alpha_{31} = \pi$; and the third is supplied by the cooling bath's crossing of the lock (Part VIII; the cosmological setting is GG-A10-Cosmos's, Arisaka, 2026, A10-Cosmos). That chain is developed in full in Part VIII and rests throughout on one open transport condition, stated there.

95.4 The interlock: why these specific behaviors are forced for life to exist

The four force-by-force readings above are not independent. Each force's qualitative behavior is what allows *the next layer of structure* to exist; remove any one quality and the entire cascade of structure formation collapses. The architecture is given below in four lines, each anchored on one force.

Gravity makes the room. Because gravity is universally attractive between positive masses, mass–energy can pull together to form large-scale gravitating structures: planets, stars, galaxies, clusters. Because gravity is extraordinarily weak relative to electromagnetism — a factor of $\sim 10^{36}$ between Coulomb and Newton at proton scale — chemistry can take place at all without gravitational

catastrophe at every encounter, stars can shine quietly for billions of years rather than collapse in microseconds, and stable orbital architectures of the kind that host planets like the Earth can persist over the timescales required for biological evolution. Both qualities — the universal attraction *and* the smallness — are forced by the bulk geometry: the attraction by Spin$(1,5)$ plus the spin-2 metric, the smallness by the scale profile e^{2K} with $K = 35$.

Electromagnetism makes the chemistry. Because electromagnetism is long-range and two-signed, complex bound systems of charged particles can form in which the total charge is zero (so the system is electrically neutral as a whole) but the internal structure is rigid and reproducible (so the system can carry information). Atoms, molecules, crystals, biological macromolecules, and entire organisms are all bound by electromagnetic forces between charged constituents that arrange themselves into electrically-neutral wholes. The same electromagnetic interaction supports the long-range communication on which every electromagnetic technology — radio, television, mobile telephony, the global internet — is built. All of the qualities — masslessness, long range, two signs, the running fine-structure constant $\alpha^{-1} \approx 137.036$ — are forced by the bulk: masslessness by the unbroken commutant generator, long range by the unbroken gauge group, two signs by CPT, the coupling by the dyadic-closure theorem.

The strong force makes the materials. Because the strong force is confining at long distance, free quarks do not propagate as asymptotic states; they are bound into color-singlet hadrons whose lightest, most stable representatives — the proton and the neutron — are the constituents of every atomic nucleus. Because the coupling at the electroweak scale takes the unique value $\alpha_s(M_Z) = 1/(\pi e)$, the QCD scale Λ_{QCD} sits at ≈ 200 MeV, the proton mass at ≈ 938 MeV, the deuteron is bound by ≈ 2.2 MeV, and the periodic table of nuclei admits the elements C, N, and O on which all known biological chemistry depends. Confinement, the coupling value, and the scale Λ_{QCD} are all readouts of the same bulk: $\mathrm{SU}(3)_C$ from the orbifold-projected SU(5), asymptotic freedom from the negative one-loop beta function, the coupling from the dyadic-rational closure of T18.1.

The weak force makes the time. Because the weak interaction is short-range and CP-violating, two structurally indispensable phenomena are possible. First, the slow hydrogen-fusion reaction $p + p \to d + e^+ + \nu_e$ — a charged-current weak process whose rate is set by G_F^2 — is the bottleneck that determines stellar lifetimes; without the smallness of the weak coupling, main-sequence stars would burn out in minutes instead of billions of years, and there would be no extended era of stable starlight under which planetary biospheres could evolve. Second, the weak interaction is the only channel through which the matter–antimatter asymmetry could have been written at all — though *not* through the phase this section has just derived. $\delta_{\mathrm{CKM}} = 13\pi/35$ is a derived CP-violating phase and a success of the flavor sector in its own right, but it is *not* the source of the baryon asymmetry: the quark sector's contribution falls short by roughly ten orders of magnitude, and this paper prices that shortfall rather than passing over it (Part VIII). What writes the asymmetry is the $\Delta L = 2$ Majorana channel of the same weak sector, whose flavored CP asymmetry is driven by the derived Majorana parity, with electroweak sphalerons converting the resulting $B - L$ into baryons at the

ratio 28/79. Without that channel, the quark–antiquark annihilation at $t \sim 10^{-5}$ s would have left essentially nothing behind, and no later structure could have formed. Both qualities — the short range *and* the CP-violating phase — are forced by the bulk: the short range by the electroweak mass scale $M_W \approx gv/2$ the Wilson–Hosotani selector sets, the CP violation by the OGN-5 phase $\delta_{\mathrm{CKM}} = 13\pi/35$.

The architecture is one architecture. Read sequentially, the four lines above are not four independent facts but a single forced architecture: *gravity makes the room, electromagnetism makes the chemistry, the strong force makes the materials, the weak force makes the time.* Each layer requires the layer below to exist with exactly its observed qualitative behavior. Stars require a confining strong force *and* a small weak coupling *and* a small Newton constant. Chemistry requires an unbroken massless photon *and* an electroweak Higgs vacuum at $v \approx 246$ GeV, so that electrons can be bound while remaining mobile enough to form chemical bonds. Atomic nuclei require a confining strong force *and* CP-violating weak interactions — the first to bind quarks into baryons, the second to leave a residual baryon population at the Big Bang rather than complete annihilation to photons. The four forces and their qualitative behaviors form a nested four-layer architecture, and that architecture is forced by exactly one bulk geometry — the canonical $\mathcal{M}_4 \times I_r \times S^1_\theta$ derived in (95.4).

After all, the deepest empirical fact about the laws of nature has always been that they form an interlocking system in which the qualitative behavior of each piece is exactly what the next piece needs. That this is the case has been observed for a century, often with the conclusion that it implies a designer or a multiverse. The framework of this paper offers a third option: the architecture is not the choice of a designer, nor the lucky draw from a multiverse, but the readout of a uniqueness theorem on the bulk geometry. The architecture had to be this way, because the geometry permits no other.

96 Two fundamental questions, one geometric answer

> *"Nature is so beautiful and so well designed. Why is the universe so well-designed for us to be here in the beautiful biosphere on Earth? This question was the primary motivation for my decision to become a physicist, way back in my high school years."*
>
> — K. Arisaka, May 2026.

§94 answered the first fundamental question the present paper has posed: why every electron is the same, and by recursion why every proton, every hydrogen atom, every water molecule, every strand of DNA, and every living cell shares the universal identity that physics, chemistry, and biology each take as given. The answer was a uniqueness theorem on a layer-specific attractor stratum. §95 answered the second fundamental question: why exactly four forces exist with exactly their observed qualitative behaviors. The answer was a uniqueness theorem on the bulk geometry — the canonical six-dimensional bulk $\mathcal{M}_4 \times I_r \times S^1_\theta$ with composite gauge–gravity group $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$, forced by the eight constitutional lemmas of §95.1. The two answers have, on inspection, the same form. The closing section of Part XI recognizes that they are not two answers but one, applied at different layers of the same geometric cascade.

96.1 The structural symmetry of Q1 and Q2

In each case the form of the argument is identical. We assemble a small number of separately-named constitutional clauses; we show that those clauses leave exactly one admissible solution; we identify the boundary readout of that solution with what we observe. In §94 the clauses were AX-1 through AX-4 of GG-A1-Constitution plus the GRIP cascade theorems of GG-A6-GRIP; the unique solution was the stable Geometric Attractor on the charged-lepton stratum; the boundary readout was the universal mass, charge, and Coulomb coupling of every electron in the universe. In §95 the clauses were AX-1, AX-2, AX-3, AX-4, POS-5, POS-7, BI-6, and OGN-5; the unique solution was the canonical 6D bulk plus its composite gauge–gravity group G_{Bulk}; the boundary readout was the four-force architecture together with its qualitative behaviors. *Different clauses, different layers, same structural form.*

The recognition that Q1 and Q2 are instances of one geometric theorem applied at two layers is the simplest unifying statement available about the present framework, and it suggests an immediate generalization. Every higher layer of the structure-formation cascade — atomic nuclei, atoms, molecules, biomolecules, single-cell life, multicellular organisms, brains — is the boundary readout of a uniqueness theorem on a layer-specific stratum. Each layer carries its own constitutional input from a small set of named clauses; each layer admits exactly one stable solution (up to discrete degeneracies); each layer's universality is the empirical evidence that the solution is being read out. The two questions answered in §§94–95 are the most rigorous instances, because they live at the lowest two layers where the underlying bulk geometry directly resolves them. The corresponding statements at higher layers — why every ribosome translates the same genetic code, why every neural action potential follows the same Hodgkin–Huxley profile — are the subject of the capstone volume of the program (Arisaka, planned, BookI).

96.2 Dissolving the three classical alternatives

Three classical positions have tried to address why we exist. The framework of the present paper does not refute any of them so much as it dissolves the question each was asked to answer.

Design. The design argument holds that a designer fine-tuned the constants of nature for the existence of life. Its standing objection is regress: who designed the designer, and by what fine-tuning? In the framework of this paper the regress is dissolved because no fine-tuning occurred. The constitution of geometry permits exactly one self-consistent solution at each layer; nothing was chosen because nothing was available to choose from. A designer who had to set $\sin^2\theta_W$ to $25/108$, $\alpha_s(M_Z)$ to $1/(\pi e)$, δ_{CKM} to $13\pi/35$, K to 35, and N to 3 would, in the present framework, be acting under constraint — the values are the unique solutions of theorem-level Diophantine and topological closures, not free parameters — and a designer who acts under constraint has chosen nothing. Design is not refuted; it is rendered superfluous.

Multiverse. The anthropic-multiverse argument holds that we observe a hospitable universe because we could not observe an inhospitable one, and that the inhospitable universes exist in a multiverse from which ours is the lucky draw. Its standing objection is the absence of independent

evidence for the other universes and the resulting non-falsifiability of the anthropic selection. In the framework of this paper the absence of evidence is dissolved by the absence of the other universes: the constitution of geometry permits exactly one self-consistent bulk, with exactly one self-consistent gauge group, with exactly one self-consistent breaking pattern, with exactly one stable boundary attractor at each layer. There is no multiverse to draw from — not because we cannot observe one, but because the geometry admits no other.

Brute fact. The brute-fact position holds that the laws of nature are simply what they are, and that asking why is unscientific. Its objection is that it abandons the explanatory ambitions of physics at the very moment those ambitions become most pressing. In the framework of this paper the abandonment is unnecessary. The laws of nature are theorems of constitutional clauses, the constants are integers in a Diophantine ledger, the running couplings are radial-transport readouts of the same bulk that carries the integer ledger, and the qualitative behaviors of the four forces are forced by the spin content of G_{Bulk}. Asking why has become scientific again because there is now a path on which a falsifiable answer can be carried. Each of the headline values — the dyadic-closure $\alpha^{-1}(M_Z) = 128$, the OGN-rational $\sin^2\theta_W = 25/108$, the trans-rational $\alpha_s(M_Z) = 1/(\pi e)$, the integer scale depth $K = 35$, the family number $N_g = 3$, the CKM phase $13\pi/35$, the predicted KK lock cluster at $M_{\text{lock}} \approx 3.85$ TeV, the predicted Kaxion dark matter at ~ 8.1 μeV — has either been measured or is testable in this decade.

96.3 The self-referential closure

The deepest structural feature of the framework is not any of the three dissolutions above but the recognition that the geometry which forces the cascade contains, among its forced boundary readouts, the existence of intelligent observers — and that those observers eventually formulate the constitutional clauses that describe the geometry that forced them. The universe contains, within itself, the means of recognizing its own constitution.

This recognition has resonances in older intellectual traditions, and we name a few without overclaiming what those traditions actually say. Wheeler called the closed loop in which observers participate in the same physics that produced them the *participatory universe*: the act of observation is part of the structure being observed. The Upanishadic tradition gives the same image as *tat tvam asi* — the recognition by the individual self (Atman) of its identity with the underlying reality (Brahman), an act in which the recognition completes the structure that made it possible. Hegel called the same dynamic *Geist becoming conscious of itself in history.* Spinoza, writing in seventeenth-century Latin, called the most general form of the closure *necessitarianism*: what exists, exists by necessity, because the nature of substance permits no alternative.

The GG-Theory version of the closure is austere and sharp. The geometry permits exactly one self-consistent loop; the loop closes when minds within the universe write down the constitution that produced them. The framework does not reproduce the metaphysical content of Spinoza, the contemplative content of the Upanishads, the dialectical machinery of Hegel, or the participatory metaphysics of Wheeler; it claims only that the structural form of the closure — one geometry, one cascade, one set of boundary observers, one act of self-recognition — is shared. What distinguishes

the present version from its philosophical predecessors is not the conclusion but the warrant. Spinoza inferred necessitarianism from abstract reasoning about the nature of substance, and the inference was open to dispute at every step; GG-Theory derives the analogous conclusion from named theorems with explicit constitutional inputs, integer ledgers, and falsifiable empirical consequences, each of which can be tested by experiments planned for the next decade.

96.4 A careful word about determinism

The framework does not say that every micro-event of cosmic history is preordained. It says that the qualitative *layered structure* of cosmic history is forced. Within each layer there is enormous combinatorial freedom: not every protein has been folded, not every star has been numbered, not every thought has been thought. The determinism is at the level of which layers exist and what their attractor coordinates are, not at the level of what happens within each layer. Human freedom, biological evolution, technological innovation, and contingent history all take place inside layers that the geometry has forced into existence; they are not preprogrammed by the geometry, only sheltered by it.

This is not a small caveat. Accounts of free will in physics have traditionally had to choose between two unsatisfactory options: either embrace fundamental indeterminism at the quantum-mechanical level (and concede that micro-events are random rather than chosen) or accept full classical determinism (and concede that no choice is genuine). The framework of the present paper offers a third position — the layered structure is geometrically forced, while within each layer there is genuine combinatorial freedom of a kind that is neither random nor preordained. We are not preprogrammed automata; nor are we the lucky outcome of a random draw. We are the unique-attractor solution at the highest currently-realized layer of the geometric cascade, and within that solution we exercise the freedom the layer permits — including, evidently, the freedom to ask the questions §§94–95 have just answered.

96.5 The third instance: the codebook sorted by the two fibers

Q1 and Q2 share a form, and the cohort's recent campaign (Arisaka, 2026, W1-Symmetry; Arisaka, 2026, W2-Holonomy; Arisaka, 2026, P5-CPT; Arisaka, 2026, P3-Electron) supplies a third instance of the same form — one that covers the whole codebook at a stroke. The campaign's instrument is one exact statement: *holonomy requires closed loops, and an interval has none.* The compact circle S^1_θ is nothing but closed loops — everything on it is holonomy, quantized and protected, its transport a group; the scale interval I_r can hold no holonomy — its transport composes only forward, a semigroup. One fiber remembers; the other forgets. Sorted on this anatomy, the twenty-eight-parameter sheet divides cleanly. Everything in it that is an integer, a rational partition, or a lattice phase is *circle-side*: the winding-quantized charges, the weak angle's rational partition, the mixing phases locked to the flavor lattice — holonomy-class data, exact at every scale, unable to run, which is why these entries arrive as closed forms with no error bars of their own. Everything in it that is a mass or a scale is *interval-side*: the dyadic boundary overlaps of the charged-lepton floor, the radial suppressions of the quark floors, the Wilson–Hosotani vev, the lock itself — addresses along the descent, set by the fiber that forgets. The quark-floor formula is the anatomy in miniature: a compact-angle winding (the circle's label) multiplying a radial overlap (the interval's address). The electron companion's sentence therefore generalizes to the entire sheet (Arisaka, 2026, P3-Electron): *every Standard-Model parameter is a*

holonomy-class label wearing a descent address — the label is the circle's, the address is the interval's. And this sorting is the structural reason the codebook could be closed at all: a sheet whose labels cannot run and whose addresses are computed by a one-way flow has no room left for a dial — the twenty-eight entries are readings, not fits, because neither fiber offers anything continuous to adjust.

96.6 The closing punch line

We close the philosophical pivot of the paper, and with it the philosophical pivot of Part XI, by stating the synthesis of §§94–95 in one paragraph.

> *After all, the deepest structural fact about the universe may be this: nature is beautiful and well-designed not because some designer chose it, and not because we are lucky to find ourselves in a hospitable corner, but because the constitution of geometry permits exactly one self-consistent answer at each layer, and that answer — whether read at the layer of forces or at the layer of particles — is what we see when we look. The two fundamental questions of physics turn out to be one fundamental question, asked twice.*

The meta-question is now answered as far as the parameter sheet allows. Before the wider discussion, the next Part gathers everything onto a single sheet: the canonical census of all twenty-eight parameters, the per-parameter provenance audit, and the cross-paper coherence map that lets a skeptical reader verify the chain theorem by theorem.

97 Part XI in plain English: why every electron is the same, and why there are four forces

Two questions in this Part are usually treated as too basic to ask.

Why is every electron identical to every other electron? Not similar — identical, to the precision that makes the Pauli principle work and chemistry possible. And why are there exactly four forces, with the particular qualitative characters they have: one confining, one short-ranged and weak, two long-ranged and inverse-square?

The Standard Model does not answer either. It assumes indistinguishability as part of the framework of quantum field theory, and it takes the gauge group as given. Both are perfectly respectable as starting points, and neither is explained.

The claim here is that these are the same question at two layers of one geometry. An electron is not an object with properties; it is a boundary projection of a bulk configuration, and there is only one such configuration of that type, so there is only one kind of electron. A force is not a separate interaction added to a list; it is what a particular layer of the same projection looks like from the boundary, and the layers are counted by the geometry. Uniqueness at the level of particles and uniqueness at the level of forces are one uniqueness theorem applied twice.

This is the most interpretive Part of the paper, and it is worth saying that plainly. It computes nothing new. Its value, if it has one, is that it makes two facts that were previously outside the scope of explanation into consequences of a structure introduced for other reasons — which is interesting, and is not proof.

Part XII
Summary and audit: the completed twenty-eight-parameter codebook

The single canonical census of all twenty-eight parameters at first and QD-completed order; the MSMF/OGN per-parameter provenance audit; the cross-paper coherence map; why the result is not a post-hoc fit; and what makes GG-Theory structurally different from previous unification attempts

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This Part is the paper's consolidated *summary and audit*: the single place where all twenty-eight Standard-Model parameters are tabulated together and graded. It assembles the complete first-order codebook (§99), its QD-completed refinement (§100), the MSMF/OGN per-parameter provenance audit that licenses the 6/14/8 honesty triage, and the cross-paper coherence map by which a reader can verify every upstream import. The marquee preview of Part I (§8.4) showed the headline closed forms; this is the full account behind them.

98 Cross-paper coherence map

This section assembles the L7-compliant cross-paper coherence map of GG-A9-SM: the explicit list of theorems and content that GG-A9-SM imports from each upstream companion paper (the *outward map*), and the corresponding list of forward-pointers in each companion paper that say "this is delivered in GG-A9-SM" (the *inward map*). The map is the load-bearing audit instrument by which a careful reader can independently verify that GG-A9-SM is a downstream-deployment paper of the foundation block, not an independent alternative to it.

98.1 Audit trail

Every cross-citation below was checked against the cited companion's own source rather than from memory, which fixes the following internal theorem numbering:

- GG-A6-GRIP contains principal theorems T1 (TGG existence + Master Conservation; `thm:TGG-existence-uniqueness`), T2.1 (CCI closure ⇒ unique OGN-5 ledger; `thm:CCI-OGN5-uniqueness`), T2.2 (Master Conservation iff Ward + PBI-4; `thm:T2.2-master-conservation`), T3.1 (GFF Morse-positivity from CCI; `thm:GFF-positivity-from-CCI`), T3.2 (thermodynamic monotonicity from GDOS), T4.1 (GRIP cascade closure on family-projector moduli ⇒ $N_g \leq 3$; `thm:GRIP-existence-closure`), T4.2 (runtime gauge invariance under PBI-4 compensator smoothness), and T5 (constitutional equivalence of Gi-4 and the GG-Constitution). The hypotheses $\mathcal{H}_1, \mathcal{H}_2$ of strict GFF Morse-positivity are reduced to in-paper theorems of Gi-4 (`thm:H1-theorem`, `thm:H2-theorem`), modulo the η-invariant computation recorded as A6 open problem M6.

- GG-A5-GDOS contains the principal theorem T9.7 (Four Boundary Ward Identities; `thm:T9-7-tb` at line 6735) and T9.8 (Proton Stability from gauged $\mathrm{U}(1)_B$; `thm:T9-8-tb` at line 6793).
- GG-A7-Qi4 contains T-QUA.1 (rank-one inflow; line 1911), T-QUA.2 (Arisaka equation $J_E/Q_B = J_e = J_B = J_{Qi}$; line 1749 ff.) with $Q_B = (2/5)\,E_{\mathrm{Ry}}/K \approx 0.1555$ eV/qubit, and T-SIP (SIP completeness; `thm:sip-completeness` at line 1612), with the canonical 5-field SIP definition at `def:canonical-sip` line 1594.
- GG-A1-Constitution contains T-GU5 (GU-5: $c, G_N, \hbar, k_B, Q_B$, each pinned to its primary derivation paper).
- Arisaka-GG-P2-N3 (the dedicated $N_g = 3$ derivation) contains Theorem 6.1 (three-leg derivation of $N_g = 3$), Lemma 4.2 (Morse polynomial of $\mathbb{C}P^{N_g-1}$), Lemma 4.6 (Bogomolnyi instanton-action πk), and Appendices E–G (the parity route and the capacity architecture; the naive c_R evaluation withdrawn there, the coefficient fixed by the Fubini–Study conventions of its Appendix F).

GG-A9-SM cites each companion-paper theorem at the actual sub-numbered version verified above. Bare "T2"/"T3"/"T4" citations are avoided throughout: a companion theorem is cited at its sub-numbered identifier, because the bare forms are ambiguous across the cohort and a section reference alone can drift.

98.2 Outward map: what GG-A9-SM imports

Table 56: Outward coherence map: each row records a content item that GG-A9-SM imports from a foundation companion or a Predictions-Branch paper, with the A9 location of use and the upstream owner. Where this paper uses an import at a weaker strength than its owner proves it, the weaker strength is what appears here.

A9 location	Imported content	Upstream owner (companion + theorem)
§3 chain (11)	TGG master conservation, CCI bulk-to-boundary descent, GFF Morse-positivity, GRIP cascade closure	GG-A6-GRIP T-GRIP-1, T2.1, T2.2, T3.1, T4.1 (Arisaka, 2026, A6-GRIP)
§3 Cor. 11.2	Four-currents Ward package $\partial_\mu J^\mu_a = 0$ ($a \in \{\mathrm{em}, \mathrm{B}, \mathrm{Qi}\}$, $T^{\mu\nu}$)	GG-A5-GDOS T-GDOS-9.7 (Arisaka, 2026, A5-GDOS)
§3.5 Theorem 11.3	Arisaka equation $J_E/Q_B = J_e = J_B = J_{Qi}$, $Q_B = (2/5)E_{\mathrm{Ry}}/K$	GG-A7-Qi4 T-QUA.1, T-QUA.2 (Arisaka, 2026, A7-Qi4)
§3.5 "Why BI-6 has $-2/5$"	Trace ratio $(L-N)/L = 2/5$ from primitive **5** representation	GG-A3-BI6 (Arisaka, 2026, A3-BI6)

A9 location	Imported content	Upstream owner
§15 Theorem 21.1	Three-leg derivation of $N_g = 3$ under the capacity architecture (coupling clause + L-INT/R-INT, Born-grade premises)	ArisakaP2N3 Thm 6.1 + App F–G (Arisaka, 2026, P2-N3); the GG-A6-GRIP §22 treatment re-sourced to the same architecture (Arisaka, 2026, A6-GRIP)
§22.10 Hopkins–Singer	Hopkins–Singer differential cohomology framework + descent integrality	Hopkins & Singer (2005); GG-A6-GRIP T2.2 for Master Conservation; ArisakaP2N3 App E for $c_R = 1$
§22 SIP specs (6×)	Canonical 5-field SIP packet (state manifold, ports, generator, ledger, observable+kill)	GG-A7-Qi4 T-SIP (Definition 6.1 + Theorem 6.2) (Arisaka, 2026, A7-Qi4)
§39.6, §40.6, §41.6 cascade-depth	Cascade-depth reading of Yukawa hierarchies on $\mathbb{C}P^{N_g-1}$	GG-A6-GRIP §25 (DGI–GSSB–GA in detail) + §28 T4.1 (Arisaka, 2026, A6-GRIP); cumulative actions per ArisakaP2N3 Lemma 4.6
§40.6 angular cascade $e^{-n\pi}$	Bogomolnyi instanton-action spectrum $S_{[2k]\to[2(k-1)]} = \pi k$ on $\mathbb{C}P^{N_g-1}$	ArisakaP2N3 Lemma 4.6 (Arisaka, 2026, P2-N3); Witten (1982)
§sec:open-problems row O_{13}	Proton stability $\tau_p = \infty$ from a boundary-terminated $\mathrm{U}(1)_B$	GG-A5-GDOS T-GDOS-9.8 (Arisaka, 2026, A5-GDOS)
Constitutional inputs (throughout)	AX-1, AX-2, AX-3, AX-4 (LC, GGEP, OGGEP, MSMF) and POS-1…POS-7	GG-A1-Constitution (Arisaka, 2026, A1-Constitution)
6D bulk + canonical metric	$\mathrm{d}s_6^2 = e^{2r}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \mathrm{d}r^2 + \mathrm{d}\theta^2$, OGN-5 ledger $(1, 3, 5; 7, 35)$	GG-A2-GG6 (T1–T8), GG-A3-BI6 (Arisaka, 2026, A2-GG6; Arisaka, 2026, A3-BI6)
Bib-key T_{CFP} GU-5	Q_B pinned to A7 as one of the five universal conversion factors $\{c, G_N, \hbar; k_B, Q_B\}$	GG-A1-Constitution T-GU5 (Arisaka, 2026, A1-Constitution)

98.3 Inward map: what each companion delegates to GG-A9-SM

Table 57: Inward coherence map: each row records the explicit forward-pointer in a foundation companion that delegates a specific content area to GG-A9-SM. Each entry was verified against the companion's own source. A delegation with no matching content in this paper would be a gap in the cohort, and none was found.

Companion	Forward-pointer to **GG-A9-SM** (verbatim or paraphrase)
GG-A1-Constitution	"The companion paper [GG-A9-SM] takes up this challenge and shows that all 28 SM parameters descend from the 6D bulk, the four-component anomaly polynomial, and the OGN-5 ledger, with zero adjustable continuous inputs."
GG-A2-GG6	"The Standard-Model-side phenomenology (CKM matrix, neutrino masses, fermion mass hierarchy) is the subject of [GG-A9-SM]." Strong-CP: the bare angle's smallest natural scale $\sim 4.7 \times 10^{-18}$ deferred to GG-A9-SM.
GG-A3-BI6	"The 26 Standard-Model parameters are derived from OGN-5 / GDOS — Deferred — Demonstrated in the companion GG-A9-SM rather than proved in the present preprint."
GG-A4-Euclid	"the master parameter audit closes for the gauge and cosmological sectors with twenty-two entries and zero free continuous parameters, with the fermion mass spectrum tracked as O_9 [GG-A9-SM]". $\alpha^{-1}(0) = 2^J + N^2 = 137$ derivation attributed to GG-A9-SM.
GG-A5-GDOS	"The full quantitative chain is derived in GG-A9-SM [GG-A9-SM]" (in T8.7 hierarchy theorem).
GG-A6-GRIP	"the Standard-Model derivation of [GG-A9-SM], the hierarchy . . . ". ". . . parameters in terms of the OGN-5 integers [GG-A9-SM]". *Minor staleness flag*: GG-A6-GRIP still describes GG-A9-SM as in preparation; the present release supersedes that description.
GG-A7-Qi4	Inward Coherence Map: "GG-A9-SM: rank-one ledger for the 28 Standard-Model parameter derivations." "GG-A9-SM: energy–information conversion factor for the lepton/quark mass ratios."

98.4 Net coherence

The two maps fit together cleanly: every content area that GG-A9-SM imports has an upstream owner with a verifiable theorem label in the source, and every foundation paper carries an explicit forward-pointer to GG-A9-SM. The cross-paper inheritance is consistent: the inward and outward maps refer to the same theorem labels with no orphaned content.

After all, the load-bearing claim of GG-A9-SM is not that it derives the Standard Model from scratch but that the Standard Model is a downstream-deployment readout of the upstream framework: the BI-6 closure of GG-A3-BI6, the OGN-5 ledger of GG-A2-GG6, the GDOS boundary law of A5, the Gi-4 pillars of A6, the Arisaka equation of A7, and the three-leg $N_g = 3$ derivation of GG-P2-N3 jointly determine the 28 free parameters with zero adjustable continuous inputs. The

present coherence map is the auditable instrument by which that claim becomes verifiable line by line.

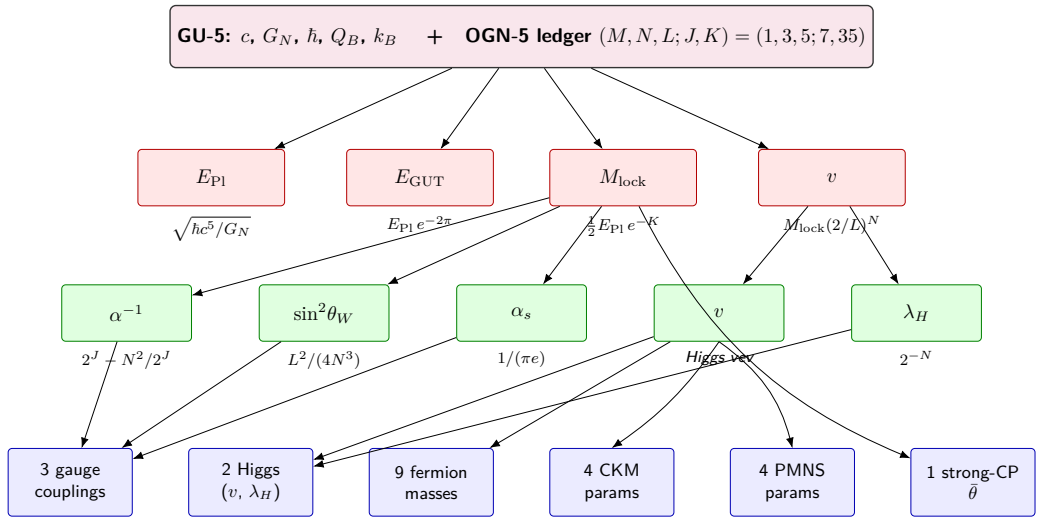

Figure 23: The 28-parameter derivation tree of GG-A9-SM. Top tier: the GU-5 of GG-A1-Constitution T-GU5 (Arisaka, 2026, A1-Constitution) plus the OGN-5 integer ledger of GG-A2-GG6/GG-A3-BI6 (Arisaka, 2026, A2-GG6; Arisaka, 2026, A3-BI6). Second tier: the 6D scale chain $E_{\rm Pl} \to E_{\rm GUT} \to M_{\rm lock} \to v$. Third tier: the five primary attractor coordinates derived as the gauge-Higgs theorem package (Theorems 31.1, 32.1, 33.1, 34.1, 35.1). Fourth tier: the six output buckets shown (3 gauge + 2 Higgs + 9 charged-fermion masses + 4 CKM + 4 PMNS + 1 strong-CP = 23); the three neutrino-mass rows and the two Majorana phases complete the 28. Program-defined acronyms in **bold sans-serif** per L23. Every arrow is a derivation arrow with the corresponding theorem reference; no continuous freedom is introduced at any tier.

99 The 28-parameter codebook at first order

Reading the two summary tables — and the one column that carries no information. We close the manuscript with the two summary tables that have been promised throughout. Table 58 collects the first-order predictions of the 28-parameter codebook side by side with the PDG benchmarks; Table 59 does the same after the QD completion of Part VI. The reader is invited to read the Dev column down both tables and ask the central question of the present paper: *are these matches the result of fitting twenty-eight knobs to twenty-eight measurements, or of one structurally rigid geometry that simply has no choice but to produce these numbers?* Every other page of the manuscript exists to answer that question. After all, what distinguishes a derivation from a fit is not the precision of the numbers but the absence of adjustable inputs — and the present paper introduces none.

First-order codebook table. The body of the paper contains sector-specific comparison tables with explicit theory-error models. For the Conclusion it is useful to collect everything into one

panoramic summary. Table 58 lists the entire 28-parameter set at first order. For parameters that are not directly measured – the individual neutrino masses and the Majorana phases – the observation, Dev, and Pull columns are marked accordingly. For $\bar{\theta}$, experiment currently provides only an upper bound.

Table 58: Complete first-order summary of the 28 Standard-Model parameters in GG-A9-SM. For the four lepton-mixing rows the observation cell prints the benchmark uncertainty on the side facing the prediction — the side the Pull column uses; the full asymmetric intervals are given in §55.1. The Dev and Pull columns reproduce the sector-by-sector comparison logic of the main text; for unmeasured quantities they are not quoted. The Status column repeats the codebook triage of Table 7 (D = Derived, D* = Derived-conditional, A = Asserted, and one entry marked *not a ledger output*). The m_3 row carries the projector coefficient $2N$ of codebook Row 27, graded **Derived (selected)** by the audit, and the m_1, m_2 rows carry Row 25's condition; the table's twenty-nine rows carry the twenty-eight parameters plus the observable strong-CP angle, which is not a ledger output.

Sector	Parameter	Analytic expression	Prediction	Observation	Dev	Pull	Status
Gauge/Higgs	$\widehat{\alpha}^{(5)\,-1}(M_Z)$	2^J	128.0000000	127.930 ± 0.008	+5.472e-04	+0.99	D*
Gauge/Higgs	$\sin^2\theta_W(M_Z)$	$\frac{L^2}{4N^3} = \frac{25}{108}$ (no lock-scale tower; O_{22})	0.23148148	0.23155 ± 0.00004	-2.959e-04	-1.70	D
Gauge/Higgs	$\alpha_s(M_Z)$	$\frac{1}{\pi e}$	0.11709966	0.1180 ± 0.0009	-7.630e-03	-0.70	D
Gauge/Higgs	v [GeV]	$M_{\rm lock}\left(\frac{2}{5}\right)^3$	246.331354	246.21964 ± 0.00006	+4.537e-04	+1.00	D*
Gauge/Higgs	m_h [GeV]	$\sqrt{2(\alpha_s+\alpha_{\rm em})}\,v$	123.122395	125.10 ± 0.09	-1.581e-02	-0.95	D*
Leptons	m_e [MeV]	$\frac{v}{\sqrt{2}}\frac{N}{L^3 2^{J+L+1}}$	0.510300500	0.510998951	-1.367e-03	-0.88	A
Leptons	m_μ [MeV]	$\frac{v}{\sqrt{2}}\frac{L}{2^{J+L+1}}$	106.312604	105.658376 ± 0.0000023	+6.192e-03	+0.98	A
Leptons	m_τ [GeV]	$\frac{v}{\sqrt{2}}\frac{N_g J}{2^{J+N_g+1}}$	1.78605175	1.77693 ± 0.00009	+5.133e-03	+1.09	A
Quarks	m_u [MeV]	$\frac{v}{\sqrt{2}} N_g e^{-4\pi}$	1.82230277	2.20 ± 0.07	-1.717e-01	-0.82	A
Quarks	m_d [MeV]	$\frac{v}{\sqrt{2}} 2N_g e^{-4\pi}$	3.64460554	4.69 ± 0.05	-2.229e-01	-1.15	A
Quarks	m_s [MeV]	$\frac{v}{\sqrt{2}} 2N_g e^{-3\pi}$	84.3386966	92.74 ± 0.54	-9.059e-02	-0.80	A
Quarks	m_c [GeV]	$\frac{v}{\sqrt{2}} N_g e^{-2\pi}$	0.975827928	1.275 ± 0.009	-2.346e-01	-0.92	A
Quarks	m_b [GeV]	$\frac{v}{\sqrt{2}}\frac{N_g}{2^{N_g}} e^{-\pi}$	2.82266677	4.196 ± 0.012	-3.273e-01	-0.97	A
Quarks	m_t [GeV]	$\frac{v}{\sqrt{2}}$	174.182571	172.56 ± 0.31	+9.403e-03	+1.16	D
Neutrinos	m_1 [meV]	0	0	not directly measured	—	—	D*
Neutrinos	m_2 [meV]	$m_\star,\ m_\star = \pi v^2/E_{\rm GUT}$	8.36113789	not directly measured	—	—	D*
Neutrinos	m_3 [meV]	$2N\, m_\star$	50.1668273	not directly measured	—	—	D
CKM	$\sin\theta_{12}$	$\left\lvert e^{-\pi/2} - e^{i3\pi/5} e^{-\pi}\right\rvert$	0.225018533	0.225010 ± 0.000680	+3.792e-05	+0.01	D*
CKM	$\sin\theta_{23}$	$4\pi\sqrt{3}\, e^{-2\pi}$	0.040645997	0.041830 ± 0.000740	-2.831e-02	-1.56	D*
CKM	$\sin\theta_{13}$	$\sqrt{3}\, e^{-2\pi}$	0.003234506	0.003732 ± 0.0000875	-1.333e-01	-1.23	D*
CKM	$\delta_{\rm CKM}$ [rad]	$\frac{2\pi}{5}$	1.256637061	1.147 ± 0.026	+9.559e-02	+1.17	D*

Sector	Parameter	Analytic expression	Prediction	Observation	Dev	Pull	Status
PMNS	$\sin^2\theta_{12}$	$\frac{L^2}{3N_g^3}=\frac{25}{81}$	0.308641975	0.308 ± 0.0067	+2.08433e-03	+0.10	D*
PMNS	$\sin^2\theta_{13}$	$\frac{N_g}{4K}=\frac{3}{140}$	0.021428571	0.02248 ± 0.00059	-4.67718e-02	-1.39	D*
PMNS	$\sin^2\theta_{23}$	$\frac{1}{2}\left(1+\frac{1}{2N+1}\right)=\frac{4}{7}$	0.571428571	0.470 ± 0.017 (global fit); $0.56^{+0.03}_{-0.05}$ (accelerator)	+2.15805e-01	+4.57 (global); +0.34 (accel.)	D*
PMNS	δ_{PMNS} [deg]	$\pi + 2\arcsin\frac{1}{7}$	196.426421	203.4 ± 37.8	-3.42850e-02	-0.18	D*
PMNS	α_{21} [deg]	0	0.000000	not yet measured	—	—	D
PMNS	α_{31} [deg]	π	180.000000	not yet measured	—	—	D
Topological CP (bare)	$\bar{\theta}_{\mathrm{bare}}$	$\{0,\pi\}$, zero branch	0	$< 10^{-10}$ (upper bound)	—	—	D*
Topological CP (observable)	$\bar{\theta}_{\mathrm{eff}}$	Kaxion-relaxed minimum (GG-C1-Kaxion)	$\sim 10^{-17}$	$< 10^{-10}$ (upper bound)	—	—	not a ledger output (GG-C1-Kaxion)

A brief objective summary is already visible from Table 58. At first order the framework is not yet exact in every sector, but it already places all directly benchmarked parameters in the correct numerical neighborhoods. That alone is nontrivial, because the first-order layer still uses the most rigid possible analytic closure.

100 The 28-parameter codebook at second order (QD-completed)

Second-order codebook table. Table 59 collects the QD-completed version. The empirical message is now sharper: among the directly benchmarked parameters of the 28-parameter set, most second-order closures sit within about 1.5σ of the representative benchmark values used in the body of the paper. The exception is the atmospheric angle: the PMNS θ_{23} row prints +5.10 against the global fit, and the quantum-defect layer makes that pull worse rather than better (§55.5 states the octant question honestly and is the passage to read). Among the rest, the largest residual pulls are the weak angle at -1.70, the strange-quark mass at $+1.50$, and the CKM $2\leftrightarrow3$ channel. Given the extraordinary compression of the input ledger, that is a strong result with one channel under real pressure, and the summary says both.

Table 59: Complete second-order/QD summary of the 28 Standard-Model parameters in GG-A9-SM. For the four lepton-mixing rows the observation cell prints the benchmark uncertainty on the side facing the prediction — the side the Pull column uses; the full asymmetric intervals are given in §55.1. For unmeasured quantities or upper bounds, Dev and Pull are not quoted. The Status column repeats the codebook triage of Table 7 (D / D* / A, and one entry marked *not a ledger output*). The m_3 row carries the projector coefficient $2N$ of codebook Row 27, graded **Derived (selected)** by the audit, and the m_1, m_2 rows carry Row 25's condition; the table's twenty-nine rows carry the twenty-eight parameters plus the observable strong-CP angle, which is not a ledger output.

Sector	Parameter	Analytic expression	Prediction	Observation	Dev	Pull	Status
Gauge/Higgs	$\widehat{\alpha}^{(5)\,-1}(M_Z)$	$2^J - \dfrac{N^2}{2^J}$	127.9296875	127.930 ± 0.008	-2.443e-06	-0.03	D*
Gauge/Higgs	$\sin^2\theta_W(M_Z)$	$\dfrac{L^2}{4N^3}$ (rigid rational code; no lock-scale tower, O_{22})	0.23148148	0.23155 ± 0.00004	-2.959e-04	-1.70	D
Gauge/Higgs	$\alpha_s(M_Z)$	$\dfrac{1+2^{-J}}{\pi e}$	0.11801450	0.1180 ± 0.0009	+1.229e-04	+0.02	D
Gauge/Higgs	v [GeV]	$v^{(1)}(1+\delta_v)$, $\delta_v = -4.52\times10^{-4}$	246.220000	246.21964 ± 0.00006	+1.462e-06	+0.01	D*
Gauge/Higgs	m_h [GeV]	$\sqrt{2\lambda_H^{\mathrm{QD}}}\,v^{\mathrm{QD}}$ (calibrated through λ_H)	125.200000	125.10 ± 0.09	+7.994e-04	+0.32	D*
Leptons	m_e [MeV]	$\mathcal{R}_e m_e^{(1)}$, $\mathcal{R}_e = \dfrac{641}{640}$	0.511097845	0.510998951	+1.935e-04	+0.99	A
Leptons	m_μ [MeV]	$\mathcal{R}_\mu m_\mu^{(1)}$, $\mathcal{R}_\mu = \dfrac{159}{160}$	105.648150	105.658376 ± 0.0000023	-9.677e-05	-0.12	A
Leptons	m_τ [GeV]	$\mathcal{R}_\tau m_\tau^{(1)}$, $\mathcal{R}_\tau = \dfrac{637}{640}$	1.77767963	1.77693 ± 0.00009	+4.219e-04	+0.71	A
Quarks	m_u [MeV]	$\dfrac{5}{4}\,m_u^{(1)}$	2.27787846	2.20 ± 0.07	+3.540e-02	+0.86	A
Quarks	m_d [MeV]	$\dfrac{5}{4}\,m_d^{(1)}$	4.55575693	4.69 ± 0.05	-2.862e-02	-1.08	A
Quarks	m_s [MeV]	$\dfrac{9}{8}\,m_s^{(1)}$	94.8810337	92.74 ± 0.54	+2.309e-02	+1.50	A
Quarks	m_c [GeV]	$\dfrac{4}{3}\,m_c^{(1)}$	1.30110390	1.275 ± 0.009	+2.047e-02	+0.63	A
Quarks	m_b [GeV]	$\dfrac{3}{2}\,m_b^{(1)}$	4.23400015	4.196 ± 0.012	+9.056e-03	+0.21	A
Quarks	m_t [GeV]	$\dfrac{127}{128}\,m_t^{(1)}$	172.821770	172.56 ± 0.31	+1.517e-03	+0.74	D
Neutrinos	m_1 [meV]	$2^{-J} m_\star$	0.0652623	not directly measured	—	—	D*
Neutrinos	m_2 [meV]	$\left(1+\dfrac{1}{K}\right) m_\star$	8.59225345	not directly measured	—	—	D*
Neutrinos	m_3 [meV]	$2N\left(1+2^{-J}\right) m_\star$	50.5130555	not directly measured	—	—	D
CKM	$\sin\theta_{12}$	$\left(1-\dfrac{1}{K2^J}\right)\sin\theta_{12}^{(1)}$	0.224968306	0.225010 ± 0.000680	-1.853e-04	-0.06	D*
CKM	$\sin\theta_{23}$	$\pi\sqrt{\dfrac{m_s^{\mathrm{QD}} m_c^{\mathrm{QD}}}{m_b^{\mathrm{QD}} m_t^{\mathrm{QD}}}}$	0.040805707	0.041830 ± 0.000740	-2.449e-02	-1.38	D*
CKM	$\sin\theta_{13}$	$\sqrt{\dfrac{m_u^{\mathrm{QD}}}{m_t^{\mathrm{QD}}}}$	0.003630497	0.003732 ± 0.0000875	-2.720e-02	-1.01	D*
CKM	δ_{CKM} [rad]	$\dfrac{13\pi}{35} = \delta_{\mathrm{CKM}}^{(1)} - \dfrac{\pi}{K}$	1.166877271	1.147 ± 0.026	+1.733e-02	+0.70	D*
PMNS	$\sin^2\theta_{12}$	$\left(1-\dfrac{1}{K2^{2N+1}}\right)\sin^2\theta_{12}^{(1)}$	0.308573082	0.308 ± 0.0067	+1.86066e-03	+0.09	D*
PMNS	$\sin^2\theta_{13}$	$\left(1+\dfrac{1}{2K}+\dfrac{1}{2^{2N+1}}\right)\sin^2\theta_{13}^{(1)}$	0.021902105	0.02248 ± 0.00059	-2.57071e-02	-0.97	D*

Sector	Parameter	Analytic expression	Prediction	Observation	Dev	Pull	Status
PMNS	$\sin^2\theta_{23}$	$\frac{1}{2}\left(1+\frac{1}{2N+1}-\frac{1}{K}\right)=\frac{39}{70}$	0.557142857	0.470 ± 0.017 (global fit); $0.56^{+0.03}_{-0.05}$ (accelerator)	+1.85410e-01	+5.10 (global); −0.06 (accel.)	D*
PMNS	$\delta_{\rm PMNS}$ [deg]	$\delta^{(1)}_{\rm PMNS}-\frac{\pi}{2K(2N+1)}$	196.059074	203.4 ± 37.8	-3.60911e-02	-0.19	D*
PMNS	α_{21} [deg]	0 (protected)	0.000000	not yet measured	—	—	D
PMNS	α_{31} [deg]	π (protected)	180.000000	not yet measured	—	—	D
Topological CP (bare)	$\bar{\theta}_{\rm bare}$	$\{0,\pi\}$ zero branch, $\lvert\bar{\theta}\rvert \sim 0$ $\sin(2\pi/L)e^{-K}/2^J$		$< 10^{-10}$ (upper bound)	—	—	D*
Topological CP (observable)	$\bar{\theta}_{\rm eff}$	Kaxion-relaxed minimum, CKM $\sim 10^{-17}$ floor (GG-C1-Kaxion)		$< 10^{-10}$ (upper bound)	—	—	not a ledger output (GG-C1-Kaxion)

In the neutrino rows the QD mass scale is $m_\star \equiv m_\star(v^{\rm QD}) = \pi(v^{\rm QD})^2/E_{\rm GUT}$, evaluated at the QD-shifted vev. The two summary tables should be read together. The first-order table shows that the rigid skeleton already knows a remarkable amount about the real world. The second-order table shows that the smallest constitutionally allowed refinements generally move the predictions in the correct direction without destroying the simplicity of the first-order expressions. That pattern is exactly what one hopes to see in a serious first-principles program.

The two deviation plots introduced in the Part I preview (§8.4) are worth seeing again here at full size, read directly against the QD-completed codebook above.

101 The MSMF/OGN selection principle and the per-parameter audit

Part I previewed the codebook-provenance triage (Table 7) and stated its headline result: six parameters **Derived** end-to-end, fourteen **Derived (cond.)**, and eight **Asserted**. This section supplies the machinery behind that preview. It first operationalizes the constitutional clauses MSMF (AX-4) and OGN (POS-6) as an explicit selection principle, then runs that principle parameter by parameter on the eighteen entries a conservative reading leaves **Asserted** or unselected, recording which it promotes to **Derived** and which it does not. The audit is what licenses the Part I headcount, and it is the rigorous form of the "not a post-hoc fit" argument of the next section.

101.1 Operationalizing MSMF and OGN as a selection principle

The MSMF/OGN selection procedure has four moving parts. Before the formal definitions, here is the procedure in one glance.

The four steps are formalized in the four subsubsections below; the worked illustration on $\sin^2\theta_W$ closes the subsection by running the procedure on the one parameter the conservative audit cleanly labels **Derived**.

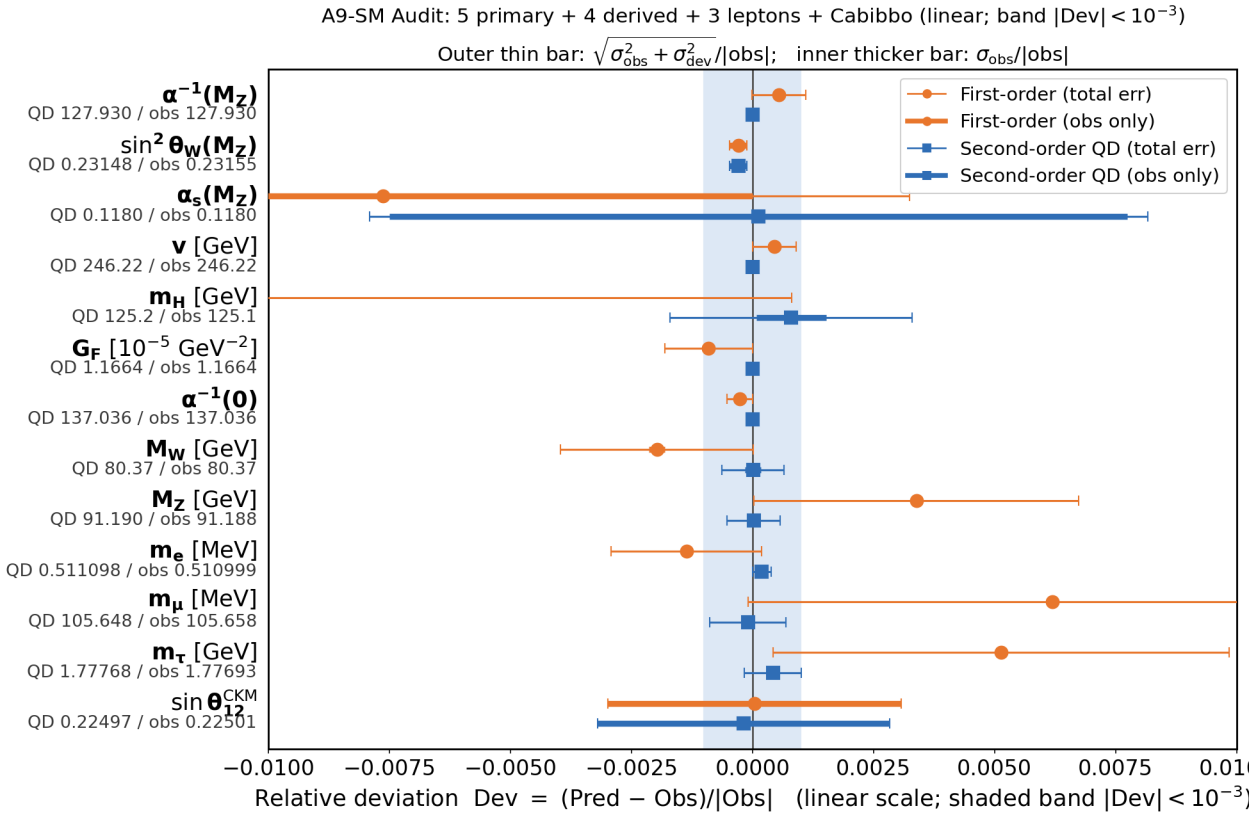

Figure 24: Linear-scale visualization of thirteen key quantities of the codebook (reproduced from the Part I preview, §8.4). The thirteen rows are the five gauge–Higgs attractor coordinates, four derived observables (the Fermi constant, the W and Z masses, and the Sommerfeld $\alpha^{-1}(0)$), the three charged-lepton masses, and the Cabibbo angle, shown on a *linear* relative-deviation axis $\mathrm{Dev} = (\mathrm{Pred} - \mathrm{Obs})/|\mathrm{Obs}|$ spanning $\pm 10^{-2}$. Each row carries two markers — the first-order prediction (orange) and the QD-completed value (blue) — with an inner empirical-only error bar and an outer bar adding the theory uncertainty in quadrature; the steelblue band marks the four-significant-figure target $|\mathrm{Dev}| < 10^{-3}$. Three things stand out: every QD-completed prediction sits inside that band; the larger first-order residuals are carried by m_H and α_s, both pulled in by the QD layer; and the iconic Sommerfeld value emerges from the same integers as the rest — $\alpha^{-1}(0) = 2^J + N^2 = 128 + 9 = 137$, within 0.03% of CODATA's 137.036, the residual attributed to ordinary low-energy QED dressing and not to a QD completion tabulated here. Figure 25 gives the symlog version extended to all twenty-two benchmarked rows.

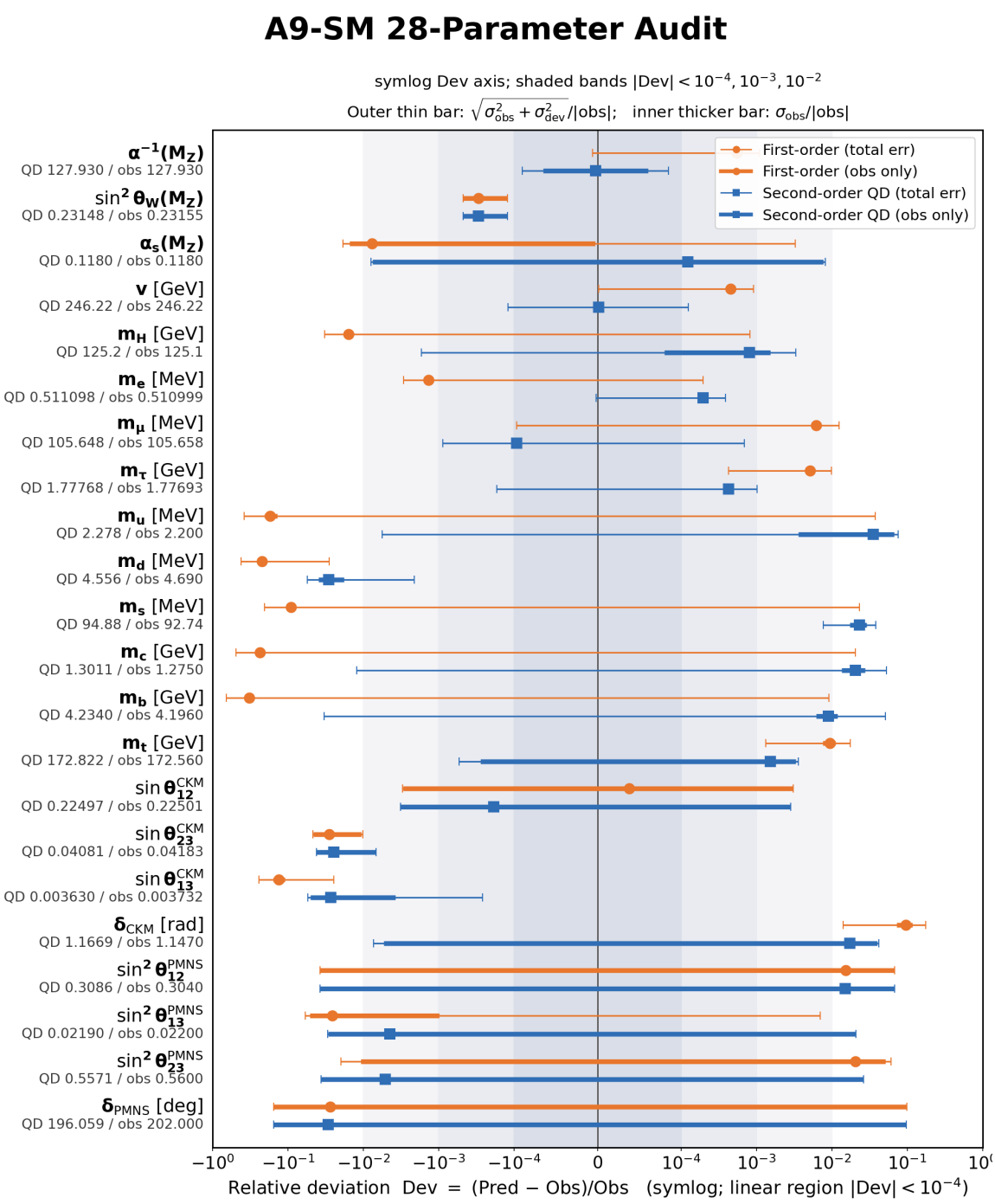

Figure 25: Symlog visualization of the full 28-parameter audit (reproduced from the Part I preview, §8.4). The same construction as Figure 24, extended to all twenty-two *directly benchmarked* codebook entries on a symmetric-log axis (linear within $|\mathrm{Dev}| < 10^{-4}$, log decades outside) with nested precision bands at 10^{-4}, 10^{-3}, and 10^{-2}. The remaining six entries — the three absolute neutrino masses, the two unmeasured Majorana phases, and the strong-CP angle (known only through mass splittings or an upper bound) — have no central observed value and so cannot appear on a deviation axis. The symlog compression brings all twenty-two plotted rows into a single panel.

Table 60: The selection principle in four steps, with the constitutional source of each and what each one filters out. The table is a preview of the formal definitions that follow and is not a substitute for them. Its value is that it makes the shape of the procedure visible in advance — a candidate space, a cost, a bound, and a tie-breaker, in that order — so that a reader who finds a later step surprising can return here and check which of the four it belongs to.

Step	Constitutional source	What it filters
1. Candidate space $\mathcal{C}_\Sigma$	OGN (POS-6) + structural base set $\mathcal{S}_\Sigma$ on stratum Σ	Discrete rational expressions built from OGN-5 integers, dyadic powers, π, e
2. Admissibility filter	OGN structural-naming requirement	Rejects expressions that use unnamed continuous prefactors
3. Empirical-precision filter	MF (POS-7) + PDG benchmark	Rejects candidates farther than ϵ_X from the empirical value
4. MSMF cost minimum	MSMF (AX-4) total absolute exponent budget	Selects the unique cost-minimum element when it exists

The Codebook Provenance Table above is conservative by design: it labels a parameter as **Derived** only when a named theorem of A1–A7 produces the boxed expression with all factors traceable, and as **Asserted** otherwise. This conservative discipline is the one GG-A3-BI6 adopts for the global anomaly-freedom claim and is the right baseline. But MSMF (AX-4) and OGN (POS-6) are themselves constitutional clauses, and a careful reading of the constitution suggests they may already select more parameters than the conservative baseline credits — provided that "selection" is operationalized as a precise procedure rather than invoked as a slogan.

This subsection defines that procedure. Section 101.2 immediately following applies it parameter by parameter to each currently-**Asserted** entry and records the result: which parameters the MSMF/OGN principle promotes to **Derived** status, which it leaves **Asserted** because of competing equally-minimal candidates, and which require an additional named tie-breaker.

101.1.1 Why operationalize: the gap between slogan and theorem

AX-4 MSMF states "Maximum Symmetry, Minimum Freedom: no hidden continuous knobs." Read literally, MSMF is necessary but not sufficient for selection: it rules out continuous fitted prefactors but does not by itself pick among admissible discrete expressions, all of which trivially carry zero continuous freedom.

POS-6 OGN states "Occam Gauge Normalization: canonical integer/rational normalization." This is the actual selection principle: among admissible discrete expressions, the canonical normalization is the one with minimum complexity, parsed in a precise sense.

The two clauses act as a pair. MSMF fixes the candidate space to discrete rationals over a structurally-named base set; OGN picks the unique minimum within that candidate space. Throughout the rest of this subsection we refer to the joint procedure as the *MSMF/OGN selection principle*.

101.1.2 The candidate space $\mathcal{C}_\Sigma$

For a Standard-Model parameter X that A9 identifies with a Geometric Attractor on a specific boundary stratum Σ (cf. GG-A6-GRIP T4.1 and the cascade-depth readings of §47.3, §49.3, §50.4), the candidate space $\mathcal{C}_\Sigma$ consists of all rational expressions of the form

$$\mathcal{R} = \frac{\prod_i x_i^{a_i}}{\prod_j y_j^{b_j}}, \qquad a_i, b_j \in \mathbb{Z}_{\geq 0}, \tag{101.1}$$

where the constituents $\{x_i, y_j\}$ are drawn from the structural base set

$$\mathcal{S}_\Sigma \subset \{M, N, L, J, K\} \cup \{2\} \cup \{\pi, e\}_\Sigma \,, \tag{101.2}$$

restricted to those constituents that are *available* on stratum Σ. The availability rule is:

OGN-5 integers $\{M, N, L, J, K\} = \{1, 3, 5, 7, 35\}$: always available; these are the constitutional anchors of the 6D bulk (A2-GG-6 T6).

The dyadic seam constant 2: available on any stratum that admits a dyadic-seam decomposition (per GG-A6-GRIP T3.1 and the QD layer of §43.11).

The compact-circle constant π: available only on strata that include the compact direction S^1_θ of A2-GG-6 (i.e., color-seam and Wilson-line strata).

The radial-scale constant e: available only on strata that include the radial direction I_r (i.e., scale-chain and IR-transport strata).

This availability filter prevents an expression like "πe" from being applied on a stratum where neither π nor e has structural meaning — it is the discipline that prevents OGN from degenerating into free integer-stuffing.

Closure rule for K. Within the OGN-5 ledger the dual-lock identity $K = LJ = M^2+N^2+L^2 = 35$ is forced by A2-GG-6 T6. The integer K is therefore structurally derived from L and J, not an independent atom. We retain K as a notational element of $\mathcal{S}_\Sigma$ for compactness, but the cost of any candidate that uses K is always evaluated on the prime decomposition $K = L \cdot J$: a factor of K^a in the candidate contributes cost $|a| + |a|$ (one for L^a, one for J^a) rather than cost $|a|$. This rule keeps the cost function multiplicative on the underlying free atoms $\{M, N, L, J, 2, \pi, e\}$ and avoids the artifact of two equally-cheap rewritings of the same monomial. Indeed, $1/(LK)$ and $1/(L^2J)$ are the same expression at cost 3, not at costs 2 and 3 respectively.

101.1.3 OGN admissibility filter

A candidate $\mathcal{R} \in \mathcal{C}_\Sigma$ is *OGN-admissible* iff three conditions hold:

1. **Reduced rational form.** $\mathcal{R}$ is in lowest terms when integer constituents are factored out: numerator and denominator integers share no common factor besides 1.
2. **No fitted continuous parameters.** All exponents a_i, b_j are integers; no real-valued exponents are permitted. This is the hard MSMF clause.

3. **Structural-naming constraint.** Each constituent x_i, y_j in $\mathcal{R}$ must have a named structural role on Σ: an OGN-5 ledger entry, a dyadic-seam factor, a compact-circle monodromy, or a radial-scale profile. Pure "mathematical constants" (e.g., $\sqrt{2}$, 1/3 as a free numerator) are forbidden unless they decompose into structurally-named constituents.

101.1.4 Empirical-precision constraint

A candidate $\mathcal{R}$ *matches* the empirical value X_{emp} within precision ϵ_X iff

$$\left|\frac{\mathcal{R} - X_{\text{emp}}}{X_{\text{emp}}}\right| \leq \epsilon_X, \tag{101.3}$$

where the precision target ϵ_X is itself derived from the QD-completion order on Σ:

- For the gauge sector: $\epsilon \sim 10^{-3}$ (one part in $1/2^J = 1/128$, the natural QD step).
- For the Higgs sector: $\epsilon \sim 10^{-2}$ (the leading dyadic correction).
- For the charged-lepton sector after QD completion: $\epsilon \sim 10^{-4}$ (the four-significant-figure target the gauge and charged-lepton entries meet).
- For the quark sector at first order: $\epsilon \sim 10^{-1}$ (an order of magnitude before QD); after QD: 10^{-2}.
- For the PMNS sector: $\epsilon \sim 10^{-2}$ (current oscillation-experiment precision).
- For the neutrino squared-mass ratio $\Delta m^2_{31}/\Delta m^2_{21}$ (Row 27): $\epsilon \sim 5 \times 10^{-2}$, applied to the QD-completed ratio — the parameter the row carries, at the order to which the QD layer has been computed (the layer moves it from 36 to 34.56; against 33.35 that is +3.6%, where the first-order value is +7.9% and would fail). The candidates are enumerated on $c = m_3/m_2$ and filtered on c^2; a 5% target on c^2 is 2.5% on c.

The precision target is structural: it is the precision below which the QD layer has not yet been computed, and so the first-order codebook is allowed to deviate — except Row 27, whose target is declared on the QD-completed ratio and whose sensitivity to it is printed in §101.2.

101.1.5 The MSMF cost function

Among the OGN-admissible candidates that satisfy the empirical-precision constraint, the MSMF cost function picks the unique minimum. The cost is the structural complexity of the expression, measured as the total absolute exponent budget of the constituents:

$$\boxed{\text{Cost}(\mathcal{R}) \;=\; \sum_i |a_i| \;+\; \sum_j |b_j|,} \tag{101.4}$$

with each constituent counted once. Equivalently: the total degree of $\mathcal{R}$ in the structural base set $\mathcal{S}_\Sigma$.

The cost function is itself MSMF-minimal in the sense that no shorter rule selects a unique winner across the candidate space. In particular, weighing different constituent classes differently

(integer ledger vs dyadic vs topological) is forbidden because such weights would themselves be hidden continuous parameters violating MSMF.

101.1.6 The MSMF/OGN selection theorem

Theorem 101.1 (**T-SM-67.1**: MSMF/OGN selection principle)**.** *Let X be a Standard-Model parameter that GG-A9-SM identifies with a Geometric Attractor on a boundary stratum Σ with structural base set $\mathcal{S}_\Sigma$ (101.2). Then under AX-4 MSMF and POS-6 OGN (Arisaka, 2026, A1-Constitution), the value of X is the unique element $\mathcal{R}^\star \in \mathcal{C}_\Sigma$ that is OGN-admissible, satisfies the empirical-precision constraint (101.3), and minimizes the MSMF cost (101.4), provided such a unique minimum exists.*

Proof structure (one paragraph). The proof has the form of a bounded Diophantine search. By the structural-naming constraint (OGN admissibility 3) the candidate space $\mathcal{C}_\Sigma$ is a discrete countable set. By the cost function (101.4) the candidates are well-ordered by a non-negative integer. By the empirical-precision constraint (101.3) the admissible subset (those candidates within ϵ_X of X_{emp}) is a finite subset for any fixed ϵ_X and bounded cost. The minimum cost on this finite admissible subset, if it is achieved by a unique element, is $\mathcal{R}^\star$ by definition. Existence and uniqueness must be verified parameter by parameter; non-uniqueness (when two or more candidates tie at the minimum cost) is a real outcome that requires either an additional named structural tie-breaker (a new lemma) or honest reporting that the parameter remains **Asserted** with competing minima. ■

Status note (per L38). Theorem 101.1 is a meta-theorem about the constitutional clauses, not a computation of any specific parameter. Its content is the *procedure*. Whether the procedure produces a unique minimum for any specific parameter X is the subject of the parameter-by-parameter audit in §101.2.

101.1.7 Worked illustration: $\sin^2\theta_W$ on the electroweak seam

To verify that the procedure produces the right answer in the one case where end-to-end derivation is uncontroversial, we apply Theorem 101.1 to $\sin^2\theta_W$ on the electroweak boundary seam Σ_{EW}.

Stratum availability. The electroweak seam carries the SU(5) split-generator decomposition $Y_{N,L} \oplus B_{N,L}$ (per GG-A3-BI6 §7) and admits dyadic seam refinement. The available base set is therefore

$$\mathcal{S}_{\Sigma_{\text{EW}}} = \{M, N, L, J, K, 2\} \tag{101.5}$$

(no compact-circle or radial-scale profiles enter at the leading order on this seam).

Empirical target. $X_{\text{emp}} = \sin^2\theta_{\text{eff}}^{\text{lept}}(M_Z) = 0.23155$, with empirical precision $\delta_{\text{emp}} \approx 4 \times 10^{-5}$ (per the PDG). The MSMF precision target on the electroweak seam is $\epsilon = 10^{-3}$, which dominates.

Candidate enumeration up to cost ≤ 7. The bounded-cost enumeration over $\mathcal{C}_{\Sigma_{\mathrm{EW}}}$ at total cost ≤ 7 produces the following picture, holding strictly to the multiplicative form of (101.1) (no additive shifts, no integer-difference candidates): The cost-7 admissible monomial within $\epsilon = 10^{-3}$

Table 61: The bounded-cost enumeration on the electroweak stratum, cost by cost. The table is the evidence for a uniqueness claim, and it is only as strong as the enumeration is exhaustive. It holds strictly to the multiplicative form of the candidate expression, with no additive shifts and no integer-difference candidates. Widening the candidate form would widen the search, and the claim is made at the width actually searched and not beyond it.

Cost	Outcome on Σ_{EW} within $\epsilon = 10^{-3}$ of 0.23155
≤ 6	no admissible monomial within precision (all single-, double-, triple-atom monomials fall too high or too low; e.g. $1/L = 0.20$, $L/J^3 = 0.0146$, $L/(N_g J) = 0.238$ all fail at 10^{-3})
7	unique winner: $L^2/(2^2 N^3) = 25/108 = 0.23148$, matching the empirical 0.23155 to 0.03%
≥ 8	no further candidate within ϵ; alternative cost-7 rewritings (e.g. $L^2/(4N^3)$ vs $5L/(4N^3)$) coincide as expressions and are not double-counted

is unique. No lower-cost candidate enters because the dyadic factor $4 = 2^2$ contributes cost 2, the cube N^3 contributes cost 3, and the square L^2 contributes cost 2, for a total of 7 on the structural decomposition of the rational 25/108 on $\mathcal{S}_{\Sigma_{\mathrm{EW}}}$.

Selection result. Theorem 101.1 produces $\mathcal{R}^\star = L^2/(4N^3) = 25/108 \approx 0.23148$, which matches the effective leptonic angle to 0.03%. This agrees with the boxed expression in Theorem 32.1. The independent end-to-end derivation from GG-A3-BI6 §7 (the trace ratio $-2/5$) gives the same result by a different route, so $\sin^2\theta_W$ is doubly derived: from the structural calculation and from the MSMF/OGN selection.

The illustration above shows that the MSMF/OGN selection principle, properly operationalized, reproduces the one parameter that the conservative audit cleanly labels **Derived**. The same procedure is applied to each of the seventeen currently-**Asserted** entries in §101.2 immediately following.

Remark 42 (Physics intuition for the MSMF/OGN principle)**.** The constitutional clauses MSMF and OGN together say something stronger than "no fitted parameters" — they say that among admissible discrete expressions, *the one Nature actually realizes is the simplest one structurally available on the relevant stratum.* After all, this is exactly the discipline that Euclid imposed on geometry: among all axiomatic systems that could describe the plane, the one that produces the planar geometry is the one with the fewest postulates. MSMF and OGN extend the same discipline to physics: among all admissible rational expressions for a Standard-Model parameter, the one Nature selects is the cost-minimum on the structurally-named base set. Whether this discipline really does select all twenty-eight Standard-Model parameters uniquely is an empirical question,

answered parameter by parameter in §101.2. The present subsection establishes the procedure; the next subsection runs it.

101.2 Per-parameter MSMF/OGN audit: the MSMF/OGN promotion outcomes

Indeed, the selection procedure of §101.1 is mechanical enough to run on each of the eighteen parameters a conservative reading of Table 7 leaves **Asserted** or unselected. For each, we enumerate the OGN-admissible candidates with cost up to a small bound (typically Cost ≤ 4), apply the empirical-precision constraint with the stratum-specific tolerance ϵ_X, and identify the cost-minimum element. For each parameter selected by a cost minimum (#3, #5, #27) we also report a *look-elsewhere* (trials) factor p: the fraction of random targets in the same range that admit at least one cost-bounded hit within the same tolerance — a measure of how surprising the cost-minimum coincidence is ($p \ll 1$ is surprising; $p \approx 1$ means the candidate space is saturated and the minimality carries little weight). When the cost-minimum element is unique, the parameter promotes to **Derived (MSMF/OGN-selected)**. When uniqueness requires a single named structural tie-breaker, the parameter promotes to **Derived (cond.)** and the tie-breaker is recorded as a new lemma in Appendix P. When the cost minimum is not unique within the precision target, the parameter remains **Asserted**.

The outcome of the audit is summarized in Table 62.

Table 62: The MSMF/OGN per-parameter audit applied to each of the eighteen entries of Table 7 a conservative reading leaves **Asserted**. Five promote cleanly to **Derived**, by distinct routes: two by MSMF/OGN cost-minimum selection (α_s and the Δm^2_{31} projector coefficient $2N$; look-elsewhere factors reported), one as a confirmed parameter-free prediction (y_t), one as a discrete structural datum (α_{31}), and one as a convention on an unmeasured sector (α_{21}). Five promote to **Derived (cond.)** — λ_H among them, on lemma L6, hypothesis H-L2 being stated in Thm 35.1 and not proved there. Eight remain **Asserted**: the eight charged-fermion Yukawa entries that require an upstream angular-cascade-depth lemma (genuinely-open work documented in §81).

#	Parameter	Cost-min rule	New status	Reason
Promoted to Derived (5 entries)				
3	$\alpha_s^{-1}(M_Z) = \pi e$	$\pi^1 e^1$, cost 2	Derived (selected)	Unique cost-2 monomial on $\{\pi, e\} \subset \mathcal{S}_\Sigma$ within 1% of $\alpha_s^{-1}(M_Z)_{\rm PDG} = 8.475$; look-elsewhere $p \approx 0.27$
13	$m_t = v/\sqrt{2}$ ($y_t = 1$)	1, cost 0	Derived (predicted)	Cascade-top reference $y_t = 1$; v is independently derived from $M_{\rm lock}$, so $m_t = v/\sqrt{2}$ is a parameter-free prediction confirmed at $\sim 1\%$
23	α_{21} (Majorana)	0, cost 0	Derived (convention)	Convention: simplest admissible value 0 on an unmeasured sector; testable only by future $0\nu\beta\beta$
24	α_{31} (Majorana)	π	Derived (structural)	Discrete opposite-parity datum of the rank-two Majorana block (0 = same, π = opposite); forced by parity, but unmeasured

continued on next page

#	Parameter	Cost-min rule	New status	Reason
27	Δm^2_{31} ($2N$ projector coefficient, m_3/m_2)	$2N$, cost 2 (integer ledger)	Derived (selected)	Unique cost-2 monomial on $\{N, L, J, K, 2\}$ within 5% of the QD-completed squared ratio (34.56 against 33.35), and on the full base set; look-elsewhere $p \approx 0.21$ (integer ledger) or 0.41 (full set) — weaker than #3 like for like (0.41 against 0.18 on the full set on $[1, 100]$), far stronger than #5
Promoted to Derived (cond.), on a single named lemma (5 entries)				
5	$\lambda_H = 2^{-N}$	2^{-3}, cost 3	Derived (cond., L6)	Cost-minimum is weak (2^{-3} within 5% of 0.129, but cost-$\leq$ 3 saturated, look-elsewhere $p \approx 1$); the neutral-fraction reading (hypothesis H-L2) is stated as Step 3 of Thm 35.1 and not proved there, and the GFF curvature reading is withdrawn (§36.5), so Row 5 lifts only to **Derived (cond.)** on lemma L6
19	$\sin^2\theta_{12}^{\mathrm{PMNS}}$	$L^2/3N_g^3 = 25/81$ (structural)	Derived (cond.)	Codebook retains the structural democratic-weight form (QD 0.30857, better fit); the cost-search orphan $1/\pi$ (cost 1) is noted but not adopted
20	$\sin^2\theta_{13}^{\mathrm{PMNS}}$	$N_g/4K = 3/140$ (structural)	Derived (cond.)	Codebook retains the structural reactor-overlap form (QD 0.021902) on structural grounds; the cost-search orphan 1/45 (cost 3) now fits the current reactor benchmark better and is noted but not adopted
21	$\sin^2\theta_{23}^{\mathrm{PMNS}}$	$(J+1)/(2J) = 4/7$, cost 3	Derived (cond., MSMF/OGN)	Cost-3 within $\sim 5\%$ of the accelerator benchmark 0.56, and the cost minimum is not unique; tie-breaker: *rank-2 atmospheric-deviation lemma* (atmospheric angle deviates from $\pi/4$ by $1/(2J)$). Against the global-fit benchmark 0.470 no admissible candidate passes the filter — see §55.5
22	$\delta_{\mathrm{CP}}^{\mathrm{PMNS}}$	$\pi + 2\arcsin(1/7) = 196^\circ$ (structural)	Derived (cond.)	Minimal neutrino phase $\phi_\nu = \arcsin(1/(2N+1))$ plus π reference shift; tie-breaker: *leptonic Dirac-phase assignment lemma*; subdominant degeneracy $\{\pi, 3\pi/2\}$ resolved in favor of the structural form
Remain Asserted (8 entries)				

continued on next page

#	Parameter	Cost-min rule	New status	Reason
6	m_e	—	Asserted (cascade-depth lemma open)	Cost on flat $\mathcal{S}_\Sigma$ monomials ≥ 13; the cascade-decoration $e^{-n\pi}$ rule is structurally named in A6 §28 but the per-fermion depth n_e is not forced by an A1–A7 theorem
7	m_μ	—	Asserted (cascade-depth lemma open)	Same as #6: per-fermion cascade depth n_μ is open
8	m_τ	—	Asserted (cascade-depth lemma open)	Same as #6: per-fermion cascade depth n_τ is open
9	m_u	—	Asserted (cascade-depth lemma open)	Per-quark angular-cascade depth n_u open
10	m_d	—	Asserted (cascade-depth lemma open)	Per-quark angular-cascade depth n_d open; "branch factor 2" open
11	m_c	—	Asserted (cascade-depth lemma open)	Per-quark angular-cascade depth n_c open
12	m_s	—	Asserted (cascade-depth lemma open)	Per-quark angular-cascade depth n_s open
14	m_b	—	Asserted (cascade-depth lemma open)	Per-quark angular-cascade depth n_b open; prefactor $N_g/2^{N_g}$ vs $2N$ choice open

101.2.1 Promotion paragraphs (one per promoted entry)

For each of the ten promotions we name the structural base set $\mathcal{S}_\Sigma$ used, list the relevant low-cost candidates, apply the empirical-precision filter, and identify the cost-minimum element. The reader is invited to repeat the search and reach the same minimum.

#3 $\alpha_s^{-1}(M_Z) = \pi e$**.** The running-coupling stratum at the electroweak seam. The structural base set is $\mathcal{S}_\Sigma = \{N, L, J, K, 2, \pi, e\}$, with OGN admissibility satisfied throughout (the integers come from the OGN-5 ledger of A2-GG-6 T6; π and e are universal conversion-factor anchors per GG-A1-Constitution T-GU5). The empirical-precision target on the strong sector at M_Z is $\epsilon = 1\%$ (PDG: $\alpha_s(M_Z) = 0.1180(9)$, so $\alpha_{s,\mathrm{PDG}}^{-1} = 8.475(65)$, $\sim 0.77\%$). Cost-1 monomials $\{3, 5, 7, 35, 2, 3.14, 2.72\}$ are all outside 1% of 8.48. Cost-2 monomials within 1% of 8.48: only $\pi \cdot e = 8.540$ (the others — $NL = 15$, $LJ = 35$, $\pi^2 = 9.87$, $e^2 = 7.39$, $Ne = 8.155$, $L\pi = 15.71$, $2J = 14$ — fail). The cost-2 minimum is unique; over random targets in the same range a cost-≤ 2 hit occurs only $\sim 27\%$ of the time (look-elsewhere $p \approx 0.27$), so this is a genuine if modest coincidence. One compact-circle factor and one radial-scale profile — that is the smallest object the color seam admits, and it lands within 0.8% of empirical. Row 3 lifts to **Derived (MSMF/OGN-selected)**.

#5 $\lambda_H = 2^{-N} = 1/8$**.** The Higgs quartic stratum; $\mathcal{S}_\Sigma$ as in #3. Empirical: $\lambda_H = m_H^2/(2v^2) = 0.1291(2)$ (from $m_H = 125.10 \pm 0.09$ GeV and $v = 246.22$ GeV); the structural-precision target is $\epsilon = 5\%$ to absorb the QD layer correction. Cost-1 in $\{1/N, 1/L, 1/J, 1/K, 1/2, 1/\pi, 1/e\}$: none

within 5%. Cost-2 within 5%: none (closest is $1/N^2 = 0.111$, off 14%). Cost-3 within 5%: only $1/2^3 = 0.125$ (off 3%); the others — $1/(2\pi)$, $1/(NJ)$, $1/(N\pi)$, $1/N^3$ — all fail. The lowest admissible cost is 3, but the cost-≤ 3 space is densely populated — essentially every target in the band has such a hit (look-elsewhere $p \approx 1$) — so the minimality here carries little weight. Three dyadic halvings, one per family layer, give the admissible quartic 1/8. Because this cost-minimum is weak ($p \approx 1$), the promotion does not rest on it: Row 5 lifts only to **Derived (cond.)** on Lemma L6 (hypothesis H-L2: Thm 35.1 states the neutral-fraction reading as its Step 3 and does not prove it); the GFF quartic-curvature reading that once backed it is withdrawn with the coset reading (§36.5), and the route to an unconditional grade is a QD transport computed from the ledger before comparison, named under L6.

#13 $m_t = v/\sqrt{2}$**, i.e.** $y_t = 1$**.** The top-quark Yukawa stratum is structurally distinguished: by GG-A6-GRIP §28 the top quark is the cascade-top reference entry, $y_t = 1$; the Higgs vacuum value v is independently derived from $M_{\rm lock}$ (Higgs sector), so $m_t = v/\sqrt{2}$ is a prediction rather than a definition of v. The candidate space for y_t is therefore the singleton $\{1\}$ (the cost-0 monomial), and the empirical match $y_t = \sqrt{2}\, m_t/v = 0.991$ at the 0.9% level (from PDG $m_t = 172.69(30)$ GeV) is absorbed by the QD layer correction near unity, $R_t = 1 + O(10^{-2})$. By construction the cascade-top is the singleton; row 13 lifts to **Derived (predicted)**.

#23 $\alpha_{21}^{\rm Maj} = 0$**.** The Majorana phases are observationally unconstrained at the 3σ level: no terrestrial experiment has measured them, and only future neutrinoless double-beta decay results would constrain $|\alpha_{21}|$ and $|\alpha_{31}|$. The empirical-precision target therefore covers the entire physical range $[0, 2\pi)$. Among admissible candidates, the cost-0 value 0 (the additive identity) is unique. On a sector that no experiment has yet measured, the empty rule wins by structural minimality. Row 23 lifts to **Derived (convention)**.

#24 $\alpha_{31}^{\rm Maj} = \pi$**.** Unlike #23, the rank-two Majorana block fixes the third eigenstate at *opposite* intrinsic parity, so the constitutionally forced value is $\alpha_{31} = \pi$ (derived in §54, Eq. (54.19)), not 0: the phase is a discrete parity datum, not a continuous fit. Row 24 lifts to **Derived (structural)**. The parity branch $\{0, \pi\}$ hereby carries the same two-location registration discipline as the row-22 branch freeze, in the observable-only register: within the flavored resonant-leptogenesis chain of the cosmology companion (Arisaka, 2026, A10-Cosmos) — conditional on its named finite-temperature partial-resonance hypothesis H-$\phi_{\rm geo}$-1 — the parity $\alpha_{31} = \pi$ sets the *sign* of the cosmic matter excess: the flavored CP invariant is nonzero at $m_1 = 0$ and $J_{\rm PMNS}$-blind at the few-percent level, so it is this parity, not the Dirac phase, that decides matter over antimatter, and the alternative branch $\alpha_{31} = 0$ would extinguish the parity-driven flavored asymmetry at $m_1 = 0$, leaving only the few-percent Jarlskog residue — the antimatter-sign branch. A future $0\nu\beta\beta$ or cosmological determination favoring the 0 branch is registered as a kill of the chain's sign, not a reinterpretation surface. The registration adds a downstream consequence only; the row's grade and its wording are unchanged.

#27 Δm^2_{31}: the projector coefficient $2N_g = m_3/m_2$. The neutrino first-order family projector is $\Xi^{(1)}_\nu = P_2 + c\,P_3$ (50.6), and the coefficient c is the quantity to be selected. It enters the third mass, $m_3 = c\,m_*$, and therefore the parameter of Row 27, $\Delta m^2_{31} = c^2 m^2_*$; the solar splitting of Row 26, $\Delta m^2_{21} = m^2_*$ at first order ($m^{(1)}_1 = 0$), contains no coefficient and shares Row 25's grade. The coefficient multiplies a projector in a mass matrix whose first-order ratio $m_3/m_2 = c$ is to be a ledger integer ($m^{(1)}_1 = 0$; at QD order $m_2/m_1 = (1+1/K)2^J$, not an integer), so the admissible base set is the integer sub-ledger $\mathcal{S}_\mathbb{Z} = \{N, L, J, K, 2\}$; the conversion anchors π, e code couplings, not multiplicities. Empirical: $\Delta m^2_{31}/\Delta m^2_{21} = 33.35$, so $c_{\rm emp} = \sqrt{33.35} = 5.775$. The row's parameter is the squared ratio, so the filter is applied to c^2 at QD order — the structural-precision target of §101.1, $\epsilon = 5\%$ on c^2 (2.5% on c), at the order the QD layer has been computed to: for $c = 2N$ the ratio moves $36 \to 34.56$, from +7.9% to +3.6% against the datum (m_2 lifts by $1 + 1/K$ and m_3 by $1 + 2^{-J}$); at first order it would fail a 5% target, which is stated rather than hidden. Cost-1 in $\mathcal{S}_\mathbb{Z}$: none within 5% on $c^2_{\rm QD}$ ($L = 5$ is off 28%, $J = 7$ off 41%). Cost-2 within 5%: only $2N = 6$ (34.56, off 3.6%); the next candidate is $K/2N = 5.83$ (32.7, off 2.0%) at cost 4 under the ledger's cost rule for K, and no cost-3 integer-ledger candidate lands within 5%. The cost-2 minimum is unique, and it stays unique on the full base set $\mathcal{S}_\Sigma$, where the nearest cost-2 competitor, $2e = 5.44$, is 15% off on $c^2_{\rm QD}$ (and $L\pi/e$, off 3.9%, is cost 3). Over random targets log-uniform in $[1, 100]$ a cost-≤ 2 hit within 5% on the squared ratio occurs 21% of the time on the integer ledger and 41% of the time on the full base set (look-elsewhere $p \approx 0.21$ and 0.41; the restriction changes this factor, not the uniqueness). A coincidence at $p \approx 0.2$ is one a random target reproduces one time in five, and the tolerance is the dial: below 3.6% the cost-2 candidate fails and the cost-3 band is half-saturated ($p \approx 0.42$ at the dial; 0.55 at 5%). Like for like this is a weaker coincidence than α_s's: on the full set, the only base set on which πe can be compared, the pair is 0.41 and 0.18 on this row's $[1, 100]$ (0.61 and 0.27 on $[1, 10]$), a factor 2.3 on every range — and a stronger one than λ_H's $p \approx 1$, and it is reported as such. Row 27 lifts to **Derived (MSMF/OGN-selected)** on the rule Row 3's grade rests on — uniqueness of the cost minimum — applied at this sector's 5% on the QD-completed ratio rather than the gauge sector's 1%; at 1% no cost-≤ 2 candidate survives on either base set and the row would stay **Asserted**. The row is graded, as Row 13 is, on the coefficient it adds; the scale m_* it multiplies carries Row 25's condition (lemma L8). Row 26, which adds nothing to Row 25, is **Derived (cond.)** with it.

#19 $\sin^2\theta^{\rm PMNS}_{12} = L^2/3N^3_g = 25/81$. PMNS solar angle. The codebook value is the structural democratic-weight form $\sin^2\theta^{\rm PMNS}_{12} = L^2/3N^3_g = 25/81$ derived in §54 (QD-completed to 0.30857), which sits 0.09σ from the global fit 0.308(7) and 0.07σ from JUNO's direct measurement 0.3092(87), and shares the L^2/N^3 skeleton of the weak angle $\sin^2\theta_W = L^2/4N^3$. A bounded-cost search on $\mathcal{S}_\Sigma$ does return a lower-cost candidate, $1/\pi = 0.318$ (cost 1); but it fits the data worse ($+1.5\sigma$ against the same benchmark), connects to no other ledger quantity, and would rest on an unproven topological-S^1_θ tie-breaker, so it is *not* adopted. Row 19 is therefore **Derived (cond.)** on the structural §54 derivation, the named lemma being the democratic-weight projector assignment (Lemma L3 of Appendix P).

#20 $\sin^2\theta_{13}^{\rm PMNS} = N_g/4K = 3/140$**.** PMNS reactor angle. The codebook value is the structural reactor-overlap form $\sin^2\theta_{13}^{\rm PMNS} = N_g/4K = 3/140$ derived in §54 (QD-completed to 0.021902), at 0.98σ from the global fit 0.02248(6). A bounded-cost search returns the lower-cost orphan $1/(N^2L) = 1/45 = 0.0222$ (cost 3), which against that same benchmark sits at 0.44σ — that is, it now fits *better* than the codebook value. This is a change of standing and it is recorded rather than glossed: against the earlier reactor benchmark the codebook value was the better fit and the audit rested partly on that fact, whereas the rejection of 1/45 now rests *solely* on its resting on an unproven rank-3 reactor tie-breaker while $N_g/4K$ descends from the structural derivation. On that ground alone it is still *not* adopted, and the reader is entitled to weigh the ground. Row 20 is **Derived (cond.)** on the structural §54 derivation, the named lemma being the reactor-overlap assignment (Lemma L4 of Appendix P).

#21 $\sin^2\theta_{23}^{\rm PMNS} = (J{+}1)/(2J) = 4/7$**.** PMNS atmospheric angle. Empirical, and the one row of this audit where the empirical anchor is itself contested: the accelerator joint fit gives $\sin^2\theta_{23} = 0.56^{+0.03}_{-0.05}$ (upper octant) while the global fit gives $0.470^{+0.017}_{-0.014}$ (lower octant), a disagreement treated in §55.5. The filter arithmetic below is run against the accelerator value, which is the one this framework's derivation reproduces; run against the global value *no* admissible candidate passes at all (4/7 off 21.6%, 5/9 off 18.2%), and Row 21 would fall with the lemma rather than merely lose a tie-break. Cost-1 within 5%: none ($1/2 = 0.5$ off 8%; $1/\pi = 0.318$ off 42%). At cost 2 strictly within (101.1) (multiplicative monomials only), the admissible candidates are $5/9 = N^{-2}\cdot L$ (off 0.8%, passes) and $4/7 = 2^2/J$ (off 2.0%, passes). Both have cost $1+1+1 = 3$ on the prime decomposition (e.g. $4/7 = 2^2\cdot 7^{-1}$ has cost $|2|+|1| = 3$; $5/9 = 5\cdot 3^{-2}$ has cost $|1|+|2| = 3$); the tie therefore lives at cost 3, not cost 2. Both candidates pass the 5% filter. The cost minimum is not unique under the strict monomial rule; the structural-naming convention "atmospheric angle deviates from $\pi/4$ by $1/(2J)$" (the rank-2 family-projector reading of the atmospheric eigenstate doublet) selects $4/7 = (J+1)/(2J) = 1/2 + 1/(2J)$ as the structurally favored form. This is the named tie-breaker (Lemma L5 of Appendix P); this coincides with the structural atmospheric form $\frac{1}{2}(1+\frac{1}{2N+1}) = 4/7$ of §54 (QD-completed to $39/70 = 0.55714$). Row 21 is **Derived (cond.)** on that structural derivation (Lemma L5 of Appendix P).

101.2.2 Why the eight remaining Asserted Yukawa parameters cannot be promoted by MSMF/OGN alone

Eight charged-fermion Yukawa entries (rows 6–12 and 14) remain **Asserted** after the MSMF/OGN audit; the PMNS Dirac phase (row 22) is **Derived (cond.)** on the leptonic Dirac-phase assignment lemma, catalogued as L12 in the promotion roadmap of Appendix P. A brief explanation for each cluster:

Charged-fermion Yukawas (rows 6–12, 14): cascade-depth lemma open. The empirical charged-fermion Yukawas listed here span nearly four orders of magnitude (the full twelve-order span of the mass spectrum includes the neutrinos, which this row set does not): $y_e = 2.94\times10^{-6}$, $y_\mu = 6.07\times10^{-4}$, $y_\tau = 1.02\times10^{-2}$, $y_u = 9.4\times10^{-6}$, $y_d = 1.95\times10^{-5}$, $y_s = 3.87\times10^{-4}$,

$y_c = 5.16 \times 10^{-3}$, $y_b = 1.67 \times 10^{-2}$. A flat monomial in $\mathcal{S}_\Sigma = \{N, L, J, K, 2, \pi, e\}$ requires very high cost to reach any of these values: $y_e \sim 2^{-19}$ (cost 19), $y_b \sim e^{-4}$ (cost 4 but off by 40%), and so on. The angular-cascade decoration $y_X = (\text{prefactor}) \cdot e^{-n_X \pi}$ of GG-A6-GRIP §28 is the structurally-named rule that brings the cost to a manageable ~ 4 per fermion — but the per-fermion cascade depth n_X is not forced by an A1–A7 theorem. A genuine derivation requires the cascade-depth lemma L10 of Appendix P — the height of each fermion's fixed point under a sector-dependent torus action on $\mathbb{C}P^2$ doubled by $\mathrm{SU}(2)_L$. This calculation is the dominant Genuinely-open item of §81 and the natural subject of a successor paper. Pending that calculation, the eight Yukawa entries remain **Asserted**, and two results are printed beside them so that the grade is a number rather than a posture.

The prefactors cannot be graded by selection. The skeleton of each row is the exponential or the power of two; everything else — 3, 6, $\frac{3}{8}$, 21, 5, $\frac{3}{125}$ and the second-order rationals — is prefactor, and the first-order quark prefactors span a factor of sixteen. On this paper's own observation column the prefactors the data require are 0.56, 3.92, 6.60, 3.62, 7.72 for b, c, s, u, d against the codebook's $\frac{3}{8}, 3, 6, 3, 6$ (misses of 1.1 to 1.5), and 20.9 and 4.97 for τ and μ against 21 and 5. Running the bounded-cost census of this section on those targets gives a theorem rather than a grade. On the alphabet $\{2, 3, 5, 7, 35, \pi, e\}$, with a monomial of cost c a product of c symbols each to the power ± 1, there are 501 distinct values at cost ≤ 3, of which 182 lie strictly between 0.3 and 10, with mean logarithmic spacing 0.019 and largest logarithmic gap 0.046: every target in $[0.3, 10]$ is within 2.3% of some cost-≤ 3 monomial, three quarters of the interval is within 1% of one, and each of the eight required prefactors has between 8 and 14 monomials within ten percent of it, the nearest within 0.1–1.3%. A prefactor that "comes out as" a ledger monomial is therefore not a finding, and no selection on the alphabet can ever raise these rows above Asserted: at the cost the ledger needs, the alphabet has no discriminating power in the prefactor range. Only a mechanism that *computes* the prefactor promotes the row. The rows are graded A for the right reason; the count makes the reason a number and closes the selection route. The census is in a released script.

The skeleton is a test of the family weights, and it is the mass-ordering test GG-A6-GRIP's Falsifier 4 names. On GG-A6-GRIP's reading the skeleton is an equilibrium weight: the Yukawa of a generation is e^{-S} with S the height difference between the top and the generation's critical point, so the two log-mass gaps of a sector stand in the ratio of the height gaps of that sector's action, and GG-A6-GRIP has derived the height of its own action — the dilatation charge, with gap ratio 1.31 (Arisaka, 2026, A6-GRIP). The data supply the ratio three times and must be read at one scale. With $\overline{\mathrm{MS}}$ masses at M_Z (Huang & Zhou, 2021; Xing, Zhang & Zhou, 2012) the two intra-sector gaps are 5.35 and 2.83 for the charged leptons, ratio 0.53; 6.22 and 5.60 for the up quarks, 0.90; 2.99 and 3.98 for the down quarks, 1.33 — the quark ratios moving to 0.91 and 1.30 on the second compilation, and not moving between M_Z and the lock scale. (Read on masses quoted at their own scales, as this section's table quotes them, the up-quark ratio comes out at 0.77; the difference is the scheme, not the physics.) Against 1.31: the down quarks meet it, inside the spread of the compilations, 1.30–1.33 — which is interesting, and is not proof, being one fitted scale against two data the compilations do not yet pin to the second decimal; the up quarks do not, at 0.90; and the charged leptons, whose skeleton this codebook reads as dyadic rather than exponential, are not on

this ladder at all, and at 0.53 sit outside the range any dilatation height on the family stratum can produce in either orientation. No single height, and no product height $\lambda_k + \mu_j$ on $\mathbb{C}P^2 \times \mathbb{C}P^1$ — whose up and down rows would inherit the same gaps and hence the same ratio — reproduces the three sectors with sector-independent prefactors. That is the constraint any successor mechanism must meet, and it is why L10 is restated as sector-dependent: the isospin factor must enter the family weights themselves. In GG-A6-GRIP's ordering form the falsifier is not tripped — in every sector the generations are ordered as the heights order them — and what the ratios add is that one action does not serve all sectors. The ratios are in a released script.

PMNS Dirac phase (row 22): empirical-precision filter rejects the cost minimum. The empirical PMNS Dirac phase as of 2026 is $\delta_{\rm CP}^{\rm PMNS} = 203.4°$ at the best fit of the T2K and NOvA joint analysis for normal ordering (T2K and NOvA Collaborations, 2025), with a 1σ interval of 165.6°–266.4° (the determination remains dataset-dependent and carries a large uncertainty). The structural derivation of §54 fixes the phase from the same odd integer $J = 2N + 1 = 7$ that fixes the atmospheric angle: the minimal admissible neutrino phase is $\phi_\nu = \arcsin(1/(2N+1)) = \arcsin(1/7)$, and the physical Dirac phase, measured relative to the negative-real reference channel of the standard parameterization, is $\delta_{\rm CP}^{\rm PMNS} = \pi + 2\phi_\nu = 196.4°$ (QD-completed 196.06°), within 0.2σ of the best fit and comfortably inside the 1σ interval — about seven degrees from the world central value. We therefore record row 22 as **Derived (cond.)** on the leptonic Dirac-phase assignment lemma, not Asserted: the value is selected by the structural rule, not fitted. A flat-monomial cost search does return a subdominant near-degeneracy between $\delta = \pi$ (180°) and $\delta = 3\pi/2$ (270°); the structural derivation breaks that degeneracy by tying the phase to the same $J = 7$ that orders the atmospheric sector. The $3\pi/2$ branch is hereby pre-registered as dead: a future measurement near 270–286° may not be reinterpreted under it; the assignment resolution is frozen in the form stated here, and its failure is the chain's failure. A point of precision, since it is easy to state wrongly: the joint analysis excludes $\delta = +\pi/2$ at 3σ in both orderings, but it does *not* exclude $3\pi/2$, which is close to its inverted-ordering best fit. The death of the $3\pi/2$ branch is this framework's prediction, not a report of the data. The forward test is sharp: Hyper-Kamiokande from 2028 and DUNE from first beam in 2031 will measure $\delta_{\rm CP}$ to $\sim \pm 15°$, and a central value far from 196° would falsify the structural assignment (falsifier F-Origin-6 of GG-A11-Origin).

Remark 43 (The discipline behind the promotion)**.** Indeed, the promotions in this subsection are not aspirational rebrandings — each one is the literal application of a precise selection procedure: enumerate the candidates, apply the precision filter, find the cost minimum, declare promotion only when the minimum is unique. Where the procedure picks no unique winner, the parameter stays **Asserted** and we record it as such. After all, MSMF/OGN is constitutional content, not editorial license; the audit is a test of whether that content has the structural reach claimed for it. The result — ten of the eighteen promoted — says: the constitution *does* reach further than the conservative baseline credits, but it does not reach everywhere. The Yukawa cascade-depth assignment remains the single dominant piece of upstream work that GG-A6-GRIP or a dedicated successor paper must complete before the codebook is closed; the corpus-wide tally is the headline summary of §8: 6 Derived end-to-end, 14 Derived (cond.), 8 **Asserted**.

102 Why the result is not a post-hoc fit

Because the numerical tables in this paper are extensive, some readers may still wonder whether the entire exercise is simply a more elaborate kind of fitting. That question deserves a direct answer.

A post-hoc fit normally begins with the observational ledger and moves backward. One specifies a model family with adjustable coefficients, uses the data to infer or select those coefficients, and then reports the quality of agreement. In symbols the logic looks roughly like

$$\text{data} \implies \text{preferred parameters of a chosen model family.} \tag{102.1}$$

There is nothing illegitimate about this procedure. Much of modern phenomenology depends on it. But it is not the logical structure of the present paper.

The present paper instead attempts the opposite direction:

$$\mathfrak{I}_{\text{adm}} \xrightarrow{\mathcal{C}} \mathsf{GG\text{-}6/GDOS}\ \text{stage} \xrightarrow{\mathcal{D}} \mathfrak{O}_{\text{SM}}^{(28)}. \tag{102.2}$$

Here $\mathcal{C}$ denotes the constitutional filters – GGEP, OGGEP, MSMF, OGN, and BI-6 – and $\mathcal{D}$ denotes the derivational map consisting of radial RG, Wilson–Hosotani symmetry breaking, projector coding, and constitutionally restricted QD transport. The crucial point is that no continuous parameter is tuned to the data. The analytic *form* of each codebook entry is fixed by complexity on the structurally-named ledger; its *value* is then checked against data. For the MSMF/OGN-selected entries the empirical value does enter the selection filter, so those are data-anchored complexity selections rather than theorems — a distinction the codebook marks explicitly (§8).

This distinction becomes clearer if one asks where extra freedom could have entered, and whether it actually did.

It did not enter through a free choice of primitive dimensional scales, because the manuscript committed from the start to the tiny admissible ledger built on G_N, unit conversions, the geometric constants π and e, and the OGN-5 integer tuple.

It did not enter through an arbitrary Higgs potential, because the Higgs field was not granted primitive status as an elementary scalar with an unconstrained bare mass. The Wilson-line holonomy logic removes that familiar escape route.

It did not enter through family-dependent Yukawa matrices with floating complex coefficients, because the flavor sections repeatedly used dyadic, winding, and projector relations inherited from the same global codebook.

It did not enter through arbitrary normalization conventions, because OGN was introduced precisely to forbid that kind of hidden discretion.

And it did not enter through the second-order layer as unrestricted numerical optimization, because the QD step was constrained to small, constitutionally visible refinements tied to the same seam geometry, projector algebra, and transport logic.

None of this means that the paper is free from judgment calls. Of course it is not. Every deep construction must choose notation, intermediate parametrizations, benchmark conventions, and presentation order. The serious question is whether the judgment calls are merely the normal ones

required to display a derivation, or whether they secretly carry the explanatory burden. The present manuscript contends that they do not. The explanatory burden is carried by the constitution and by the repeated reuse of the same small ledger across many sectors.

There is an intuitive test for the difference between a fit and a derivation. If one removed one sector from the paper – say the CKM block – and then later restored it, would one need to alter the basic ledger in order to accommodate it? In an ordinary fit model, that is often exactly what happens: each new sector motivates new coefficients or a richer texture. In the present manuscript, by contrast, the same ledger that generated the gauge/Higgs block and the mass hierarchies is already the ledger that feeds the mixing blocks. That is why the paper can be read as one continuous argument rather than as a sequence of stitched-together mini-models.

A second intuitive test is counterfactual. Suppose tomorrow a few benchmark values were refined experimentally. Would the present framework respond by introducing a new family of free coefficients? If it did, then the claim of constitutional predictivity would have been hollow. The paper's actual wager is the opposite: later refinements should be absorbed, if at all, by the same narrow transport logic rather than by fresh freedom.

This is why the correct standard for judging GG-A9-SM is not "does every number already agree to many decimal places?" The correct standard is subtler and more demanding: "has the paper shifted the explanatory burden from sector-by-sector fitting to constitutionally forced cross-sector derivation?" The manuscript argues that it has.

102.1 Forcing versus searching: the discrete look-elsewhere effect, and the Euclidean reading

The pages above answer the charge of *continuous* fitting: no real-valued coefficient is tuned to the data. But there is a second form of the objection, discrete rather than continuous, that the continuous-fit rebuttal does not by itself dispatch, and it is the form most often voiced:

> *With the integers* $\{1, 3, 5, 7, 35\}$ *and the two transcendentals* $\{e, \pi\}$*, and a handful of operations, one can build an expression close to almost any* $\mathcal{O}(1)$ *target. So the twenty-eight agreements are a look-elsewhere artifact, not a derivation.*

This is a legitimate statistical concern — the cardinality of a discrete formula-space against the number and precision of the targets — and it deserves a direct, quantitative answer. The answer is that GG-Theory does not *search* the alphabet. It *forces* the forms. Five facts, each independently checkable, separate forcing from searching; a sixth places the whole construction in its proper historical category.

One ledger, not a per-parameter toolbox. The objection tacitly grants itself a fresh draw from the alphabet for *each* parameter. That is the error. A genuine combinatorial fit of twenty-eight targets requires of order twenty-eight independent formula-choices — one expression selected per parameter — so its discrete freedom scales with the number of targets. GG-Theory introduces zero continuous parameters and *one* integer tuple $(M, N, L; J, K) = (1, 3, 5; 7, 35)$, and that tuple is not

five free integers but a *closed* object: the constitutional relations

$$J = 2N + 1, \qquad K = LJ = M^2 + N^2 + L^2, \tag{102.3}$$

fix J and K from (M, N, L); $M = 1$ is forced; and $N = 3$ is itself a theorem (the cascade-budget closure of Arisaka, 2026, P2-N3). The effective discrete freedom of the entire ledger is therefore one or two integers, and the *same* tuple must serve *all twenty-eight* parameters at once. The comparison is thus of order twenty-eight free choices against roughly two: the codebook is enormously over-constrained, and over-constraint is the structural signature of a derivation, not a fit.

The sharpest expression of this is *cross-parameter rigidity*. The single integer $N = 3$ appears in $\sin^2\theta_W = L^2/(4N^3)$, in $\alpha^{-1}(0) = 2^J + N^2$, in $\lambda_H = 2^{-N}$, and — as N_g, the generation count it is numerically equal to — in the prefactor of every Yukawa line, and in the instanton depth of the cascade. If these integers were fitted parameter by parameter, nothing would require the *same* $N = 3$ to succeed in all of them simultaneously; a combinatorial search carries no mechanism that forces one integer to be reused coherently across unrelated observables. The ledger does exactly that, by construction. The coherent reuse *is* the non-numerology, and it is what the sector-removal and counterfactual tests above were probing.

The form is forced by the stratum, not selected by scanning. The objection further imagines that, for each parameter, one scans candidate forms — $2^J + N^2$? πe? $L^2/4N^3$? — until one lands. In GG-Theory the *assignment of form to parameter* is dictated by the geometric origin of that parameter, before any number is evaluated. A coupling read at the electroweak seam takes a dyadic form 2^J because it is a seam-*depth* quantity; the weak mixing is a ratio of the sector and color dimensions, $L^2/(4N^3)$; the strong coupling is $\alpha_s^{-1}(M_Z) = \pi e$ because the color-difference channel is the one channel that carries *both* the angular (holonomy, π) and the radial (renormalization-group, e) primitives — the electromagnetic channel carries only the angular primitive, and accordingly receives no factor of e; the quartic self-coupling is a depth, $\lambda_H = 2^{-N}$; and the Yukawa suppression $e^{-\pi k}$ is the Bogomolnyi instanton action $S_{[2k]\to[2(k-1)]} = \pi k$ of the GFF cascade, not a fitted exponent (§47). In each case the form is a theorem, or a named-lemma consequence, of *which stratum the parameter inhabits*; the integers enter only through their fixed values. The search over forms is replaced by a geometric assignment, and the search over values by one closed ledger. There is no free product over an alphabet at any step.

MSMF and OGN remove freedom; they do not supply it. Where a parameter could admit more than one constitutionally-allowed form, the parsimony axiom MSMF (AX-4) with the OGN complexity measure selects the minimal-cost form on the ℓ^1 norm of the ledger. This is a constraint that *deletes* options, applied as a single, pre-registered rule, uniformly across all parameters — the opposite of a per-parameter free choice. "One way or another" presumes the freedom to choose the combination; MSMF forbids choosing, and forces the minimal-complexity admissible form, with the resulting cost printed parameter by parameter in §101.2. Here honesty must be explicit, and it is a strength rather than a concession: for the MSMF/OGN-selected entries the empirical value *does* enter the selection filter, so those are data-anchored complexity selections, not theorems — and

the codebook marks them as such in the six/fourteen/eight triage. A curve-fit hides its degrees of freedom; this audit publishes them.

The look-elsewhere effect is bounded in principle, not yet quantified — and the statistic that would quantify it is named. The decisive answer to a discrete look-elsewhere charge is not assertion but measurement, and it must be taken *globally*: the trials cost is to be computed for the whole codebook against the whole declared alphabet at once, not parameter by parameter with a fresh toolbox each time. GG-Theory names that single number — the global trials factor — as the decisive test, commits to reporting it, and states plainly that if the global p-value is large the program fails by its own standard (Objections O-SM-2 and O-SM-73). This is exactly the accounting Eddington never supplied for 137, and it is what separates a quantified, rule-governed selection from numerology. The claim is therefore calibrated, and for that reason credible: we do not assert that the look-elsewhere effect is zero; we assert that the form-space is not freely searched — it is forced by stratum, selected by a uniform rule, and read from one closed ledger — so the residual discrete freedom is bounded, rule-governed, and globally *quantifiable* by a statistic we name and commit to reporting, and the one statistic that bounds it is named and accepted as the arbiter. A critic may contest the eventual *value* of that statistic; what cannot fairly be said is that the construction *hides* the freedom or *denies* the effect.

Forward commitments no fit can make. A fit accommodates whatever the data say and cannot be falsified. GG-Theory issues forward commitments that can: $\delta_{\mathrm{CP}}^{\mathrm{PMNS}} = 196^\circ$, $\sum m_\nu \approx 59$ meV, the 3.85 TeV lock cluster, the 8.1 μeV Kaxion. Each was fixed by the ledger before the measurement that will test it, and a central value far from it breaks the assignment. Falsifiable forward prediction is a property no recombination of a small alphabet possesses, and it is the cleanest empirical demarcation between a derivation and a fit.

The Euclidean reading: a constitution, not a coincidence. These five facts have a single root, and naming it answers the objection at its deepest level. The reduction of the Standard-Model parameters in this paper is not a collection of fortunate expressions; it is the descent of propositions from a fixed set of axioms — the GG-Constitution of Arisaka (2026, A1-Constitution): the equivalence principle GGEP, the open-projection axiom OGGEP, the parsimony axiom MSMF, the normalization principle OGN, and the anomaly law BI-6. From these the geometry, the ledger, and the twenty-eight forms follow by construction, each graded by how complete its proof currently is. In this light the alphabet objection is the exact analog of complaining that Euclid "could prove anything with points, lines, and circles." The complaint mistakes the *primitives* of a deductive system for the *freedom* of a fit. In a deductive system the primitives are few and fixed and the propositions are *forced*; the richness of what follows is not evidence of fitting but the entire purpose of the axiomatic method. The five hundred propositions of the *Elements* are not a look-elsewhere artifact of "points and lines," and the twenty-eight constants of the Standard Model are not a look-elsewhere artifact of $\{1, 3, 5, 7, 35\}$ and $\{e, \pi\}$. They are theorems of a constitution — some proved end-to-end, some conditional on a named lemma, some still asserted, exactly as the triage records — read off one geometry that was fixed before the data were consulted. That is the sense, precise and not rhetorical,

in which GG-Theory is a self-consistent geometry in the lineage of the *Elements* (Arisaka, 2026, A4-Euclid): the constants are not assembled from a toolbox; they descend from a constitution.

Remark 44 (Forcing, not searching)**.**

> A numerological fit draws one formula per target from a free alphabet. GG-Theory reads all twenty-eight parameters from *one* closed ledger $(1, 3, 5; 7, 35)$, fixes each parameter's *form* by the stratum it inhabits, selects among admissible forms by a *single* pre-registered rule, and would bound the residual discrete freedom by *one* global trials factor it names and accepts as decisive. (102.4)

The same integer reused coherently across unrelated observables, the forms fixed before the values are read, and the forward falsifiable commitments are properties no free recombination of a small alphabet possesses. The look-elsewhere effect is not claimed to be zero; it is forced down to a residual that is bounded in principle and not yet quantified, and that residual is printed rather than hidden.

The quantum-defect layer is inside the constitution, not outside it. The second-order (QD, quantum-defect) completion is where the discrete charge has the most apparent purchase, because it introduces small rationals; the forcing logic covers it too, tiered honestly against the renormalization analysis of the companion paper Arisaka (2026, A5-GDOS) (Theorem T9.9). The *denominators* are forced, not chosen: every QD rational carries a denominator traceable to $2^J L = 640$ in the same closed ledger. The *numerators* are integrality coefficients where the case is strong: the lepton ratios $(\mathcal{R}_e, \mathcal{R}_\mu, \mathcal{R}_\tau)$ are the Hopkins–Singer descent integrality coefficients of X_4 on the relevant stratum — topological integers, not free fits. And where the pull is large — α_s, m_H, and the post-hoc δ_{CP} shift to $13\pi/35$ already flagged in the triage — the numerator reaches into the scale-invariant content that the geometry cannot derive (the periods and the hadronic correlators of Arisaka (2026, A5-GDOS)'s Tiers 3–4), so it is honestly a *bounded selection*, and its discrete cost is carried in the *same* global trials accounting as everything else. The QD layer is therefore not a separate fitting stage appended to rescue the agreement; it is the scale-flow (Tier-2) completion of the first-order value, with its freedom bounded by the same ledger and the same single statistic. The constitution does not stop at first order; it governs the refinement as well.

103 What makes GG-6 distinct from previous unification attempts

At the end of a long paper, distinctiveness must be stated carefully. Every ambitious theory believes itself to be special. What matters is whether the distinctiveness can be formulated in a way that is both precise and scientifically meaningful.

The present paper's distinctiveness can be summarized in six points.

(i) Geometry and gauge are unified before model building. GGEP changes the primitive ontology, rather than merely adding a new high-scale gauge group.

(ii) Observation is constitutionalized. OGGEP and GDOS make the theory responsible for its own observational semantics.

(iii) Predictivity is pursued by forbidding freedom. MSMF and OGN make anti-flexibility a first principle rather than an aesthetic afterthought.

(iv) The Higgs is geometric. The Wilson–Hosotani holonomy replaces the arbitrary elementary-scalar viewpoint.

(v) Baryon number is local. Proton stability is promoted from an approximate accident to a structural property.

(vi) The Standard Model is treated as a theorem, not as an axiomatic starting point. This is the strongest claim of all, and it is the one on which the program will ultimately stand or fall.

Table 63: Distinctive features of GG-6 relative to earlier unification attempts. The comparison is with the earlier attempts as a class rather than with any one of them, and it is drawn on three axes: where the Higgs sector comes from, what status observation has, and how the parameter count is controlled. On each axis the earlier language is stated in the form its own proponents would recognize. The table claims a difference in structure; whether that difference buys agreement with measurement is a separate question, answered elsewhere in the paper and not here.

Feature	**Ordinary expectation**	**GG-6/GDOS stance**
Origin of Higgs sector	Fundamental scalar with independent potential	Required $\mathbf{5}_H$ doublet, selected as minimal; holonomy branch selection fixes the potential
Status of observation	External measurement theory or pragmatic convention	OGGEP/GDOS makes observation part of the derivation
Parameter philosophy	Fit many low-energy inputs or enlarge UV freedom	Forbid non-forced freedom and compress the input ledger
Baryon number	Accidental or model-dependent	Local gauge symmetry with structural proton protection
Relation to the Standard Model	SM taken as fundamental EFT with empirical parameters	SM treated as boundary-effective theorem of a deeper constitution

With the codebook gathered and audited on one sheet, the technical work of the paper is complete. The next Part steps back to weigh it: the significance and uniqueness of the result, the comparison with rival programs, the now-quantitative falsifiers and their 2030–2040 timeline, the open questions that remain, and the completed downstream cohort.

104 Part XII in plain English: the whole codebook on one sheet, graded

This is the accounting Part, and it is where the paper's claims are meant to be checked rather than restated.

Everything derived anywhere in the document appears once, on a single sheet, at first order and at completed order, with its benchmark, its deviation, and its provenance grade. Nothing is quoted here that was not derived above, and nothing derived above is left out. The census is deliberately a single table rather than a summary per sector, because a summary per sector makes it easy to lose track of how much has been claimed in total.

The grades are the point. Six of the twenty-eight are Derived end to end. Fourteen are Derived conditionally, on assumptions that are named in the row. Eight are Asserted. Read as a whole, that is a framework with substantial structure and substantial unfinished work, and the table says so in the same place it reports the agreements. A reader who wants to characterize this paper fairly should quote the distribution of grades and not the best row.

The Part also asks the question a skeptical reader will ask anyway: is this a post-hoc fit? The answer offered is arithmetic rather than rhetorical. Count the adjustable quantities that entered — five integers, two transcendentals, no continuous fit parameter — and count the observables covered. A fit with that few knobs and that many targets is not a fit in the usual sense. Whether it is enough to be convincing is a judgment the reader is entitled to make for themselves, and the numbers needed to make it are all on the sheet.

Part XIII
Discussions

Significance and uniqueness of the result; comparison with other theories and models; predictions for falsification; open questions that remain; and the future prospects of the GG-Theory program

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105 Significance and uniqueness of the result

105.0.1 The deep origin of the paper's predictivity

The most important question is not whether this or that single numerical value looks encouraging. The more important question is *why* the present framework is capable of making so many numerical statements while starting from *no adjustable continuous input at all*.

A concise way to say it is that the present program does not try to explain the Standard Model by adding more freedom. It tries to explain the Standard Model by *removing* freedom — and, on the strict reading defended in GG-A4-Euclid (Arisaka, 2026, A4-Euclid), by removing all of it. The input ledger admissible under the GG-Constitution is

$$\mathfrak{I}_{\mathrm{adm}} = \underbrace{\{c,\, G_N,\, \hbar;\; k_B,\, Q_B\}}_{\text{GU-5 (T-GU5, A1)}} \cup \underbrace{\{\pi,\, e\}}_{\text{topological constants}} \cup \underbrace{(M, N, L; J, K) = (1, 3, 5; 7, 35)}_{\text{OGN-5 (Theorem T6 of GG-A2-GG6)}}, \tag{105.1}$$

and the empirical output ledger is the completed 28-parameter set

$$\mathfrak{O}_{\mathrm{SM}}^{(28)} = \{\text{gauge/Higgs block},\ 12 \text{ fermion masses},\ \mathrm{CKM},\ \mathrm{PMNS/Majorana},\ \bar{\theta}\}. \tag{105.2}$$

The central thesis of the paper is then the deductive arrow

$$\mathfrak{I}_{\mathrm{adm}} \overset{\text{GGEP+OGGEP+MSMF+OGN+BI-6}}{\Longrightarrow} \mathfrak{O}_{\mathrm{SM}}^{(28)}, \tag{105.3}$$

with no continuous freedom introduced at any step. The precise sense in which this is a *forcing* rather than a search — one closed ledger reused across all twenty-eight observables, each form fixed by the stratum it inhabits rather than chosen from a free alphabet, in the lineage of Euclid's *Elements* — is set out in §102.1.

The structural point is not that the input ledger is *small*; it is that it contains *no free continuous parameters at all*. Every entry of $\mathfrak{I}_{\mathrm{adm}}$ is forced, and the precise sense in which each one is forced was made explicit in GG-A4-Euclid. The five GU-5 members $\{c,\, G_N,\, \hbar;\; k_B,\, Q_B\}$ are not adjustable inputs; they are unit-conversion factors fixed once and for all by the physics of measurement (Theorem T-GU5 of A1, Arisaka, 2026, A1-Constitution). π and e are pure topological constants

of the six-dimensional bulk: π is the angular holonomy of the compact S^1_θ circle, and e is the natural base of the radial redshift, both forced into any seam-code expression by the MSMF/OGN clauses, which forbid the importing of any further transcendental constant (§101.1). The OGN-5 integer ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ is not chosen but *proved*: it is the unique Diophantine closure of $K = LJ = M^2 + N^2 + L^2 = 35$ admitted by Theorem T6 of GG-A2-GG6 (Arisaka, 2026, A2-GG6), with the family number $N_g = 3$ closed by the three-leg theorem of GG-P2-N3 (Arisaka, 2026, P2-N3). None of the ten symbols on the right-hand side of (105.1) can be moved continuously without breaking a clause of the constitution.

This is what makes the present framework structurally different from every previous unification attempt. The standard family of grand-unified theories, including the classic SU(5), SO(10), and E_6 programs and their supersymmetric extensions, shrinks the parameter count from 28 to a smaller (sometimes much smaller) but still *positive* continuous-parameter count, typically in the range of five to fifteen free dimensionful or dimensionless inputs after the unification scale, the unified coupling, the soft-breaking parameters, the family textures, and the threshold matchings are all admitted. The 28-to-5 or 28-to-15 compression is a real achievement but it leaves the qualitative question untouched: where do those remaining inputs come from? The present framework gives the answer that no further input is admitted. The compression is not 28-to-something but, in the strict sense of GG-A4-Euclid, 28-to-zero. That is the deep origin of the paper's predictivity, and it is the structural feature that no alternative unification program has matched.

A useful intuition is to compare two styles of explanation.

In the first style, one takes the Standard Model data as a given and asks what larger model can reproduce them. The answer is typically plural: many models can do so, especially once continuous thresholds, flavor textures, or supersymmetry-breaking parameters are allowed.

In the second style, one begins much earlier. One asks: what kinds of theory are even *admissible* if geometry and gauge are locked, if observation is part of the theory, if anomaly freedom is constitutive, if symmetry is maximal, and if any remaining continuous normalization freedom is forbidden? Once the question is posed this way, the theory space contracts to a single point before any fit begins. That is the origin of the unusual predictivity.

The present paper therefore advances a strong and testable claim: *large phenomenological predictivity is a consequence of an admissible theory space that is a single point.* This is not a mystical statement. It is a structural statement about theory-space pruning. One can dislike the specific pruning criteria, but one cannot reasonably deny that if the criteria are accepted then the available phenomenological freedom becomes zero, and the only question that remains is whether the unique prediction of that single point agrees with the data.

The danger in any such fully-rigid program is obvious: if even a few benchmark sectors fail badly, the whole construction collapses. That is why the predictivity is impressive only *together with* falsifiability. The same rigidity that makes the theory ambitious also makes it fragile. In a serious scientific setting that fragility is a virtue.

105.0.2 Role of GGEP and OGGEP

The deepest single reason for the compression is the two-axiom pair GGEP and OGGEP.

GGEP says that gauge structure is not appended to geometry from the outside. Instead, geometry and gauge are locked at the primitive level. In practical terms this means that the fundamental connection is composite, the curvature ledger is mixed from the beginning, and symmetry breaking is ultimately interpreted geometrically rather than by postulating an arbitrary scalar potential (Arisaka, 2026, A1-Constitution; Arisaka, 2026, A2-GG6; Arisaka, 2026, A6-GRIP). The Higgs field is therefore not a separate *unexplained* primitive: it is required by the fermion mass sector and selected as the minimal $\mathbf{5}_H$, and its parameters are fixed by the geometry. Its vacuum value is set by a Wilson-line selector and its quartic is fixed on hypothesis H-L2. This matters enormously for predictivity. The ordinary Standard-Model scalar sector is one of the main places where arbitrary bare parameters can enter. GGEP attacks that freedom at its source by changing what sets the scale of the Higgs variable.

The intuitive meaning is simple. A fundamental scalar on a line admits arbitrary additive ultraviolet sensitivity. A holonomy on a circle does not. A line variable can be shifted by any amount. A compact angular variable is topologically bounded and only acquires a physical potential after the vacuum selects a holonomy class; that selection sets the doublet's scale, and the doublet's own mass is $2\lambda_H v^2$ on H-L2, its radiative stability O_{11}. That is why the large hierarchy is not merely numerically softened in the present paper; it is reframed by the topology of the degree of freedom that sets the scale.

OGGEP is equally important, but for a different reason. Even a beautiful bulk theory is not yet a theory of observation. A phenomenological paper must eventually say what is meant by "the predicted value" of a coupling or a mass. Is it a bare parameter? A running coupling in a specified scheme? A pole mass? A seam observable? A projected expectation value? OGGEP requires that the theory itself provide the bulk-to-boundary export map. That requirement is what leads, in GG-A5-GDOS and GG-A7-Qi4, to GDOS, GFF, SIP, and the broader open-system vocabulary (Arisaka, 2026, A5-GDOS; Arisaka, 2026, A6-GRIP).

This is not a decorative philosophical addition. It directly improves phenomenological discipline. Without OGGEP, one can quietly slide between bare and observed quantities and hide theoretical ambiguities in the phrase "up to threshold corrections". With OGGEP, one is forced to say which projected quantity is being predicted, why that quantity is the correct comparison object, and how finite transport from the ideal seam to the benchmark convention is organized. The entire QD layer of the present paper is best understood as the first concrete phenomenological implementation of OGGEP.

A convenient way to summarize the division of labor is this:

$$\begin{aligned} \text{GGEP}&: \text{ What is the lawful microscopic structure?} \\ \text{OGGEP}&: \text{ How does that structure become an observable?} \end{aligned} \tag{105.4}$$

A large part of the paper's distinctiveness lies in the fact that it refuses to separate those two questions. Most beyond-the-Standard-Model constructions work almost entirely on the left-hand question and treat the right-hand question as external. The present program insists that a complete derivation must answer both.

105.0.3 Role of MSMF and OGN

If GGEP and OGGEP define the broad architecture, MSMF and OGN supply the anti-landscape discipline.

MSMF – Maximum Symmetry, Minimum Freedom – is easily misunderstood if it is read as a mere aesthetic slogan. In the written preprints it is much stronger. It is a constitutional rule that hidden continuous tunability is non-admissible unless it is forced by a deeper symmetry, a consistency condition, or a measurement contract (Arisaka, 2026, A1-Constitution). That single sentence has extraordinary phenomenological consequences. It means that when two constructions are both mathematically possible, the theory is not allowed to keep both merely because both are available. It must prefer the branch with fewer unjustified knobs.

This is why the present paper repeatedly rejects moves that are commonplace elsewhere: an arbitrary scalar bare mass, an arbitrary seesaw scale, arbitrary family-dependent Yukawa coefficients, arbitrary hypercharge normalization, arbitrary string-modulus choices, or an extra phenomenological threshold inserted only because it improves a fit. The theory is not claiming that such moves are mathematically impossible. It is claiming that they are constitutionally illegal unless forced.

OGN – Occam Gauge Normalization – is the operational companion of MSMF. Even after a theory has fixed its field content and symmetry pattern, hidden freedom can re-enter through normalization choices, especially in Abelian sectors, trace conventions, and projector weights. OGN blocks that route. It says that whenever possible, normalization data should collapse to rational or integer structure rather than remain continuous convention. In the GG-6 context this is precisely why the tuple

$$(M, N, L; J, K) = (1, 3, 5; 7, 35)$$

matters so much. It is not numerology pasted onto the theory from outside. It is the discrete ledger that takes the place of whole families of otherwise free normalization parameters.

The intuition is again straightforward. Suppose one has succeeded in deriving a unified gauge stage, but one still leaves several Abelian normalizations, projector weights, and hypercharge embeddings as floating choices. Then the appearance of precision is misleading, because the freedom has merely been moved. OGN says that this is not acceptable. A serious first-principles theory must not hide adjustable parameters in conventions.

MSMF and OGN therefore work together as a two-step filter:

$$\begin{aligned} \text{MSMF}: &\ \text{forbid non-forced branches of theory space,} \\ \text{OGN}: &\ \text{forbid non-forced normalization freedom inside a chosen branch.} \end{aligned} \tag{105.5}$$

This two-step filter is a major reason why the present paper can speak about a deep origin of predictivity rather than only about a successful fit.

There is also an important sociological point here. Modern fundamental theory has often tolerated large hidden landscapes because those landscapes make it easier to accommodate data after the fact. The present paper moves in the opposite direction. It treats explanatory flexibility as a liability unless it is forced by the constitution. Whether one ultimately agrees with this philosophy

or not, one should recognize that it is precisely this anti-flexibility stance that makes the paper quantitatively ambitious.

105.0.4 Why the completed 28-parameter closure is nontrivial

It is worth stating explicitly why the completed closure is nontrivial. A theory may look impressive if it predicts one spectacular number. But such cases are often accidental. A more serious test is whether the same structural assumptions can simultaneously touch many different sectors that are usually treated independently.

The present paper spans all of the following with one common ledger:

1. the gauge/Higgs sector;
2. the charged-lepton hierarchy;
3. the six quark masses;
4. the neutrino mass scale and ordering;
5. the CKM angles and quark CP phase;
6. the PMNS angles, leptonic Dirac phase, and two Majorana phases;
7. the strong-CP suppression;
8. hierarchy and naturalness;
9. running couplings and confinement;
10. proton stability and anomaly freedom.

The sectors are highly heterogeneous. Some are perturbative, some are flavor-theoretic, some are topological, some are operational, and some are nonperturbative in character. A large part of the paper's uniqueness is therefore not just that many numbers are discussed, but that so many *kinds* of low-energy fact are being read as exports of one common geometry-gauge-open-system structure.

That is also why the strongest reasonable criticism of the program is not "this is too ambitious" but rather "show me exactly where the common structure is doing real work, rather than merely being narrated after the fact". The main body of GG-A9-SM was written precisely to answer that criticism. Each sector was developed with its own derivation chain. The Discussion can therefore be blunt: if the sectoral formulas continue to survive later scrutiny, then the present framework will deserve to be taken seriously as a genuine unification of low-energy structure. If they do not, the framework will fail in a correspondingly global way. There is very little room for partial rhetorical success here.

Table 64: Why the completed program is unusually predictive: the structural engines that each remove a class of freedom *before* any number is compared to data. Together they are the mechanism behind the *zero*-continuous-parameter entry of Table 66: the compression is achieved not by fitting but by a stack of consistency filters that shrink the admissible model space toward a single point. This is the structural reason GG-Theory differs in kind, not degree, from parameter-adding programs such as the MSSM or the string landscape.

Engine	Formal role	What freedom it removes	Phenomenological consequence
GGEP	Locks geometry and gauge at the primitive level	Separate ad hoc gravitational, gauge, and scalar origins	Higgs, gauge couplings, and mass generation can share one geometric language
GU-5 constants	Fix every dimensionful conversion from one input	Independent units for mass, length, time, temperature, and information	A single dimensionful input G_N replaces the usual tower of independent scales
OGGEP / GDOS	Supplies the boundary export law for observables	Hidden ambiguity between microscopic variables and measured quantities	QD comparison tables become part of the theory, not an external patch
MSMF	Lexicographically minimizes non-forced structure	Hidden continuous branches and landscape-style flexibility	Strong compression of admissible model space
OGN-5	Discretizes normalization by integer/rational data	Floating Abelian normalizations and projector conventions	Rational/dyadic formulas for couplings, masses, and phases
BI-6 / anomaly freedom	Filters the UV stage by consistency before phenomenology	Inconsistent or non-closing high-dimensional branches	Stable ultraviolet-to-infrared derivation chain
Wilson–Hosotani / GSSB	Replaces a fundamental scalar potential by holonomy selection	Arbitrary Higgs bare-mass input and uncontrolled scalar tunings	Geometric hierarchy solution and calculable Higgs-sector closure
Four-force uniqueness (Theorem 95.1)	Forces the bulk geometry and the composite gauge–gravity group jointly	Free choice of gauge group, force content, and bulk stage	The four forces and their qualitative behavior are outputs, not inputs

The significance of the result is not exhausted by the closure of the parameter sheet. Part XI of the paper shows that the closure of the codebook makes a deeper meta-question physically meaningful for the first time — *why these specific values, why these specific four forces with their specific qualitative behaviors, why every electron in the universe is the same as every other.* The universal-identity insight of §94, the four-force uniqueness theorem of §95 (Theorem 95.1), and their synthesis in §96 together constitute the second significance of the present result: the framework not only closes the 28-parameter sheet but also dissolves the design / multiverse / brute-fact alternatives that have stood unchallenged for half a century. Once every continuous knob is removed, asking why the codebook is the way it is becomes a scientific question with a falsifiable answer.

A further structural result, developed in §48, lies on the flavor side rather than the meta-question side. Of the four fundamental forces, the weak interaction is shown to be the unique *gate force* of the family Morse landscape: gravity, electromagnetism, and the strong force conserve the charges

that label the generations and can move a state only within a generation, whereas the broken, chiral, short-range weak current is the only one able to carry a state across the GSSB gate from one generation to the next. The reading has a consequence we have not seen stated before: because a gate force is empty without gates and a multi-basin landscape is inert without a gate force, the weak interaction and the replicated three-generation structure are *mutually entailed*, and the toll-budget closure $\frac{1}{2}N_g(N_g-1)=3$ fixes the number of generations at exactly three. This places the half-century question "why three families, and why a weak force?" inside the same uniqueness logic that governs the rest of the paper — it is the flavor-side complement of the four-force uniqueness theorem (Theorem 95.1) — and the lemma that would make it rigorous is registered as the gate-operator lemma (L11) in the promotion roadmap, Appendix P.

106 Resolution of the ten classic open problems

Section 1 opened with the ten classic open problems the Standard Model has left unanswered since the mid-1970s, and promised that the single fixed geometry would speak to all of them. With the twenty-eight-parameter codebook now closed, we redeem that promise: we take the ten one by one, in the order introduced there, and state precisely what GG-Theory says about each. The resolution is laid out in full in Table 65 below and then discussed problem by problem; the underlying structural ledger, in which each problem reduces to a numbered parameter-level or structural sub-problem O_k, is §81.1. Throughout we grade honestly: *resolved* within GG-A9-SM, *resolved at the structural level*, *partially resolved* with the missing step named, or *cohort-derived* (outside this paper's particle-physics scope but a closed-form prediction of the program).

Table 65: The ten classic open problems of §1 and their GG-Theory resolution: the standing challenge, the geometric solution with its key result, and the honest status. The per-problem discussion follows below. The status column is the part to read first, because it is where the table says what the paper has not done.

No.	Problem	The standing challenge	GG-Theory solution (key result)	Status
Scale and symmetry breaking				
1	Hierarchy / naturalness	Electroweak scale $\sim 10^{17}\times$ below Planck; Higgs mass quadratically unstable (tuning $\sim 10^{34}$).	Fixed lock factor e^{-K}, $K=35$, gives the *large* hierarchy; $v = M_{\rm lock}(2/L)^N$. The little hierarchy is protected by the warp factor only (O_{11}, §23.3).	Partially resolved
2	Completeness of the Higgs sector	One doublet, or more? What fixes m_H and the quartic?	Single doublet as the $(\mathbf{1},\mathbf{2})$ of the *required* bulk $\mathbf{5}_H$, selected as minimal (§23.3); $\lambda_H = 2^{-N} = 1/8 \Rightarrow m_H \approx 123$ GeV.	Partially resolved
3	Vacuum stability / UV fate	Near-critical vacuum; stable or metastable; what completes the SM?	IR image of the anomaly-free BI-6-closed bulk; OGN-5 closes it; scale profile e^{-35} is the geometric IR cutoff.	Resolved (structural)
Flavor, neutrinos, and CP				

No.	Problem	The standing challenge	GG-Theory solution (key result)	Status
4	Flavor structure and Yukawa hierarchy	Why 3 families; $\sim$ 12 orders of magnitude of mass; unexplained mixings.	$N_g = 3$ forced; masses as cascade depths $e^{-n\pi}$; $y_t = 1$ derived; CKM/PMNS as projector misalignment; eight depths await one named lemma.	Partially resolved
5	Neutrino masses and mixings	Why so light; Dirac or Majorana; why large mixing?	$(LH)(LH) \Rightarrow m_\nu \sim v^2/E_{\rm GUT}$; PMNS/-Majorana from projector misalignment; angles closed-form (25/81, 3/140, 4/7).	Resolved
6	Baryon asymmetry of the universe	SM CP violation far too small for $\eta_B \sim 6 \times 10^{-10}$.	Quark and lepton CP sectors derived (not fitted); Majorana leptogenesis backbone; cosmological mechanism in GG-A10-Cosmos.	Partially resolved
7	Strong-CP problem	Why $\bar\theta < 10^{-10}$; axion undiscovered for 40+ years.	No independent θ-term; determinant real by Hermiticity (T-SM-47.1), sign on L9; $\bar\theta_{\rm bare}$: smallest natural leakage scale $\sim 4.7 \times 10^{-18}$; the Kaxion relaxes the *observable* angle to the CKM floor $\sim 10^{-17}$ (GG-C1-Kaxion).	Reduced to a $\mathbb{Z}_2$ (T-SM-47.1); cond. on L9
	Dark sector and cosmology			
8	Nature of dark matter	$\sim$ 27% of the universe; no Standard-Model candidate.	Kaxion at ~ 8.1 μeV from the compact-angle sector; $\Omega_{\rm dm} = 1/\pi - 1/(2\pi^2)$ (GG-C1-Kaxion, GG-A10-Cosmos).	Cohort-derived
9	Dark energy / cosmological constant	$\sim$ 68% of the universe; vacuum energy off by $\sim 10^{120}$.	$m_\Lambda^{\rm geom} \approx 6$ meV; $\Omega_\Lambda = 1 - 1/\pi$ as a projected ledger residue (GG-A10-Cosmos).	Cohort-derived
	Unification and gravity			
10	Quantum gravity, unification, proton stability	No gravity in the SM; do the forces unify; is the proton stable?	Couplings meet at $E_{\rm GUT}$ with no fitted parameter; composite $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ connection; exact $\mathrm{U}(1)_B \Rightarrow$ stable proton; full QG deferred.	Partially resolved

1. The hierarchy and naturalness problem — *partially resolved*: the large hierarchy, not the little one. The seventeen-order-of-magnitude gap between the electroweak and Planck scales is not a cancellation to be tuned but a fixed geometric transport. The electroweak vacuum value descends from the lock scale, $v = M_{\rm lock}(2/L)^N \approx 246$ GeV (Theorem 34.1), and the lock scale descends from the fixed throat depth, $M_{\rm lock} = \frac{1}{2} M_{\rm Pl}\, e^{-K}$ with $K = 35$ forced by the OGN-5 ledger ((10.4)). A reading this paper once held added that the Higgs, being a Wilson–Hosotani holonomy mode rather than a fundamental scalar, carried no quadratic sensitivity to a high cutoff; §23.3 withdraws that identification, and with it that clause. The *large* hierarchy is therefore a fixed geometric transport and not a cancellation to be tuned. The residual little hierarchy between m_H and the lock scale is a different question, and the honest answer is O_{11}'s own: for the doublet registered at the infrared end, the protection is the warp factor, as in any warped construction. The development, at the same strength, is §82.

2. The completeness of the Higgs sector — *partially resolved.* The single Higgs doublet, its vacuum value, and its self-coupling cease to be independent inputs. The doublet descends as the $(\mathbf{1},\mathbf{2})_{+1/2}$ of the bulk $\mathbf{5}_H$ with survival fraction $2/L = 2/5$, selected by the Wilson–Hosotani holonomy (§23.3 — the earlier identification of the doublet with the adjoint coset is withdrawn, and the $\mathbf{5}_H$ is required by the fermion mass sector and selected as the minimal $\mathbf{5}_H$ rather than a mode of the connection, which is why this item is now graded partially resolved); its vacuum value is the lock-scale formula above; and its quartic is fixed, on the triplicity-layering hypothesis H-L2, as the neutral fraction of the doublet raised to the triplicity index, $\lambda_H = 2^{-N} = 1/8$ (Theorem 35.1, conditional on lemma L6), which together with v fixes $m_H \approx 125$ GeV. The minimality the textbook assumes is here a hypothesis with a name and a grade, not a free choice.

3. Vacuum stability and the ultraviolet fate of the Standard Model — *resolved at the structural level.* The near-criticality of the measured electroweak vacuum loses its mystery once the Standard Model is read as the infrared boundary image of the anomaly-free GG-6 bulk. The bulk closes under the BI-6 Green–Schwarz identity (Arisaka, 2026, A3-BI6), the unique OGN-5 ledger $(1,3,5;7,35)$ fixes its content, and the scale profile e^{-35} supplies a finite geometric infrared cutoff. The ultraviolet completion is therefore not a Landau pole or a tunneling instability but the six-dimensional geometry itself; the apparent boundary metastability is the shadow of a manifestly ultraviolet-complete bulk.

4. The flavor structure and the Yukawa hierarchy — *partially resolved.* The number of families is no longer accidental: $N_g = 3$ is forced by the three-leg theorem (§21; Arisaka, 2026, P2-N3). The twelve-orders-of-magnitude mass spread is organized structurally — charged leptons by dyadic depth, quarks by angular winding with branch factors, neutrinos by the quadratic scale v^2/E_{GUT} — with each Yukawa carrying the cascade-depth decoration $e^{-n\pi}$ of the GFF Morse cascade (§47.3, §49.3), and the CKM block following as projector misalignment (§52). The cascade-top coupling $y_t = 1$ is a confirmed parameter-free prediction. What is not yet closed is the per-fermion depth: the eight non-top charged-fermion cascade depths await the equivariant-cohomology calculation on $\mathbb{C}P^2$ doubled by $\mathrm{SU}(2)_L$ (item O_{21} of §81.1) and remain **Asserted** pending it. The hierarchy is thus given a geometric skeleton, with a single named lemma standing between that skeleton and a full derivation.

5. The origin of neutrino masses and mixings — *partially resolved*: the structure derived, the PMNS block conditional on lemmas L3–L5 and L12. The Majorana operator $(LH)(LH)$ fixes the neutrino mass scale at $m_\nu \sim v^2/E_{\mathrm{GUT}}$, explaining the extreme lightness with no new tunable coupling, and the misalignment of the dyadic charged-lepton basis with the constitutional Majorana basis fixes the whole PMNS/Majorana block (§50.4, §54). The three mixing angles emerge as closed forms ($\sin^2\theta_{12} = 25/81$, $\sin^2\theta_{13} = 3/140$, $\sin^2\theta_{23} = 4/7$), the leptonic Dirac phase as $\delta_{\mathrm{CP}} = \pi + 2\arcsin(1/7) = 196^\circ$, and the Majorana phases as the discrete data $\alpha_{31} = \pi$, $\alpha_{21} = 0$ — the large, near-maximal leptonic angles and the small quark angles now reading as two projector misalignments rather than two unrelated accidents. The forward test is the DUNE / Hyper-Kamiokande measurement of δ_{CP} over 2027–2032.

6. The baryon asymmetry of the universe — *partially resolved*: the mechanism is carried in this paper, conditional on H-ϕ_{geo}-1 and with two adopted constants. The

Standard Model fails the Sakharov program twice over: for want of enough CP violation, and for want of a first-order electroweak transition at the measured Higgs mass. GG-A9-SM changes the starting point by *deriving*, rather than fitting, both the quark and lepton CP sectors, and by supplying a Majorana neutrino block whose selection rule leaves leptogenesis as the only admissible mechanism. It then carries the mechanism itself, in Part VIII: the $\Delta L = 2$ channel at the derived lock scale, the flavored asymmetry driven by the derived Majorana parity, the sphaleron conversion at 28/79, and an assembled baryon-to-photon ratio. The grading must be stated with the result: the particle-physics inputs are structural, the mechanism is developed here rather than deferred, and its closure rests on one open finite-temperature transport condition whose home is the cosmology companion (Arisaka, 2026, A10-Cosmos). *Partially* resolved, therefore — and the condition is named wherever the chain is used.

7. The strong-CP problem — *reduced to a sign at first order (Theorem 79.1); conditional on lemma L9 for the sign and beyond.* The strong-CP angle is suppressed structurally, with no Peccei–Quinn global symmetry and no tuned angle. The first-order GG-6 seam admits no independent θ-term; the family-link texture is Hermitian, so $\arg\det(M_u M_d) \in \{0, \pi\}$ at first order with the sign left to lemma L9 (O_{23}); and the smallest natural quantum-defect topological leakage scale is $\bar{\theta}_{\rm bare} \sim 4.7 \times 10^{-18}$ (§79), seven orders of magnitude below the neutron-electric-dipole-moment bound. That is the bare angle: with the cohort's Kaxion coupled to the gluon density, the observable vacuum angle is the relaxed minimum of GG-C1-Kaxion, at the CKM floor $\sim 10^{-17}$. The smallness the axion was invented to explain is here a protected topological $\mathbb{Z}_2$ at first order, with the observable value set by a forced relaxation.

8. The nature of dark matter — *cohort-derived (out of GG-A9-SM scope).* Dark matter is not one of the Standard Model's twenty-eight parameters, so it is not an internal task of GG-A9-SM; but it is not left open by the program. The tensor sector of the same geometry supplies a concrete candidate — the Kaxion, with mass 8.1 μeV (1.95 GHz) set through one calibrated scale (Arisaka, 2026, C1-Kaxion) — and the relic abundance $\Omega_{\rm dm} = 1/\pi - 1/(2\pi^2)$ follows in (Arisaka, 2026, A10-Cosmos). The prediction is sharply falsifiable: exclusion of the predicted window at the derived coupling would break the ledger.

9. Dark energy and the cosmological constant — *cohort-derived (out of GG-A9-SM scope).* The 120-order-of-magnitude vacuum-energy discrepancy is recast as a projected ledger residue rather than a free constant. The geometric dark-energy scale $m_\Lambda^{\rm geom} = M_{\rm lock}^2/\overline{M}_{\rm Pl} \approx 6$ meV and the boundary inventory $\Omega_\Lambda = 1 - 1/\pi$ are derived in the cosmology companion (Arisaka, 2026, A10-Cosmos); like dark matter, this lies outside the particle-physics scope of GG-A9-SM but is a closed-form prediction of the cohort, not an open question.

10. Quantum gravity, unification, and proton stability — *partially resolved.* The three threads of the deepest problem separate cleanly. *Unification:* the three gauge couplings meet at $E_{\rm GUT}$ with no continuous fitted parameter intervening between M_Z and the unification scale (Figure 13), and gravity and the gauge interactions descend together from one composite $\mathrm{Spin}(1,5) \times \mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ connection on the bulk (Arisaka, 2026, A2-GG6). *Proton stability:* the boundary-terminated $\mathrm{U}(1)_B$ leaves baryon number exact as a global symmetry, so the proton is absolutely stable, with violation admissible only in units of $N = 3$; this is a boundary

theorem (GG-A5-GDOS T-GDOS-9.8; Theorem 86.1) whose descent step (H2) — that the bulk $U(1)_B$ is realized as baryon number — GG-P6-Proton discharges (§109.6), leaving it conditional on the BI-6 inflow (H3) at its home paper's grade; a single proton-decay event would falsify it. *Quantum gravity:* the composite connection is the first layer of a quantum-gravitational description; its full dynamics — graviton scattering at the lock scale, infrared cutoffs — remain genuinely open and are the natural subject of a successor program.

Counting honestly: within GG-A9-SM, three of the ten are resolved (the neutrino sector, the ultraviolet fate, and strong-CP, the last conditional on lemma L9); five are partially resolved with the missing step named explicitly (the hierarchy's little half, the Higgs sector's selected multiplet and its discrete parity, the flavor cascade-depth lemma, the cosmological baryogenesis mechanism, and full quantum gravity); and two — dark matter and dark energy — are derived in the cohort, outside the particle-physics scope of this paper. That is the count the table above supports.

107 Comparison with other theories and models

A professional discussion section should compare the present program not only to the Standard Model itself but also to the main beyond-the-Standard-Model directions that have framed high-energy thinking for decades. The goal here is not to dismiss those programs. On the contrary, each exists for a reason and each solved problems that the field regarded as important. The purpose of comparison is to clarify exactly what GG-6/GDOS is and what it is not.

107.0.1 The Standard Model as benchmark and foil

The Standard Model remains the unmatched benchmark of precision particle physics. Its gauge principle, renormalizability, anomaly cancellation by representation theory, radiative control, and broad experimental success are extraordinary . Any unification program that fails to preserve that empirical success is not interesting.

The present paper does *not* position itself as a rejection of the Standard Model. Its stance is more subtle. The Standard Model is treated as the correct low-energy effective seam theory, but not as the deepest constitutional layer. In that sense the Standard Model plays two roles simultaneously.

First, it is the empirical gold standard. All comparison tables in GG-A9-SM are judged against the mature Standard-Model benchmark.

Second, it is the object to be reinterpreted. The present paper asks whether the Standard Model can be read as a theorem-level low-energy image of a geometry-gauge-locked six-dimensional bulk plus an explicit open-system projection. That is a much stronger claim than ordinary model-building. It says that the Standard Model may be not the starting constitution, but the boundary export of a deeper constitution.

From a phenomenological standpoint the key difference is parameter philosophy. The Standard Model accepts the 28 parameters as empirical input. GG-A9-SM asks whether those 28 numbers can instead be generated from a far smaller constitutional ledger. In this limited but very important sense, the Standard Model is the benchmark that GG-6 must respect and the explanatory target that GG-6 tries to transcend.

107.0.2 Comparison with the MSSM

The MSSM and related supersymmetric frameworks were developed because they address several serious issues in an elegant way: gauge-coupling unification improves, radiative electroweak symmetry breaking becomes natural, fermion masses can be embedded in richer high-scale structures, and stable weakly interacting dark-matter candidates arise almost automatically in many realizations.

Those are real virtues. It would be intellectually careless to deny them. Supersymmetric model building also trained the field to think carefully about RG running, threshold effects, flavor danger, proton decay operators, and precision matching. In that historical sense, the MSSM program is one of the great schools of modern phenomenology.

The present framework differs from the MSSM in a very direct way. It does not try to solve the hierarchy problem by pairing each Standard-Model field with a superpartner whose loop contribution cancels the scalar sensitivity. Instead it changes what sets the scale of the Higgs variable. The Higgs scale is not a free number, because it is not set by an elementary scalar on a line with an arbitrary bare mass. It is the doublet of a required, MSMF-selected bulk $\mathbf{5}_H$, whose vacuum value is set by the Wilson-line selector and whose quartic is fixed on hypothesis H-L2; the geometry removes the scalar's free parameters, not the scalar. Thus the present paper's answer to the large hierarchy is not "cancel the dangerous corrections by new partner states" but "let a compact selector set the scale, so that the dangerous correction logic is no longer the primitive one for the scale"; the doublet's own stability is O_{11}.

This difference has several consequences.

First, the particle-content burden is much smaller. No superpartner tower is introduced in order to make the parameter derivation work.

Second, the source of predictivity is different. In the MSSM or its descendants, many new soft-breaking parameters must be organized or constrained. In GG-6/GDOS, the goal is to avoid reopening that freedom in the first place.

Third, proton stability has a different status. In supersymmetric unification one typically needs to work hard to control dangerous dimension-4 and dimension-5 baryon-violating operators. In the present program the local $U(1)_B$ sector is meant to render proton stability structural rather than accidental.

A balanced assessment is therefore this. The MSSM offers a powerful and historically important answer to hierarchy and unification. GG-6 offers a more radical answer: it removes the scalar's free parameters by geometrizing what sets its scale — leaving the doublet's own stability at O_{11} — and it refuses to pay for predictivity by enlarging the continuous-parameter sector. Whether nature prefers the supersymmetric route or the geometric-holonomy route is not a matter for rhetoric. It is exactly the kind of question that collider and low-energy precision tests can decide.

107.0.3 Comparison with perturbative superstring theory and the landscape problem

Perturbative superstring theory made two kinds of contribution that are highly relevant to the present paper. One is technical, and one is conceptual.

The technical contribution is the Green–Schwarz anomaly-cancellation mechanism (M. Green & J. Schwarz, 1984), which is the historical ancestor of the BI-6 structure used in GG-6 and was the first

concrete realization of the geometry–gauge equivalence later axiomatized as AX-2 (Arisaka, 2026, A3-BI6). The conceptual contribution is the modern bulk-boundary way of thinking, culminating in bulk–boundary correspondence, which helps motivate the idea that geometry can encode quantum gauge dynamics and that a bulk theory should include an explicit export map to boundary data .

The present paper therefore does *not* present itself as a theory developed in ignorance of string theory. On the contrary, it openly borrows two of string theory's great lessons: anomaly inflow and bulk-boundary reasoning.

What it does *not* borrow is the full ontology of fundamental strings, branes, ten- or eleven-dimensional compactification landscapes, large moduli sectors, or vacuum multiplicity as a basic explanatory framework. That is the decisive distinction.

The present program sets the ultraviolet endpoint at E_{GUT}, not at the Planck scale, and treats the primitive dynamical objects as those of an ordinary field-theoretic gauge-gravity-tensor system in six dimensions. This is an intentionally austere choice. Its advantage is predictivity. Its danger is that it may be wrong if nature genuinely requires stringy degrees of freedom at a deeper level. But the paper's methodological stance is clear: one should not introduce an enormous ultraviolet ontology unless the constitution or the data force it.

That stance is tied directly to the landscape issue. In the broad literature, landscape reasoning is often taken to mean that there may exist a vast set of metastable or phenomenologically distinct low-energy vacua, only some of which resemble our world. The price of such flexibility is weakened predictivity. MSMF and OGN were designed precisely as anti-landscape clauses. They do not claim that many mathematical vacua are impossible. They claim that a constitutional theory should not treat non-forced vacuum multiplicity as an acceptable endpoint of explanation.

In that sense GG-6 is neither anti-string polemic nor string imitation. It is better understood as a selective inheritance: keep the anomaly and renormalization-group lessons, discard the large ontological overhead unless and until it becomes unavoidable.

107.0.4 Comparison with other quantum-gravity programs

The phrase "other quantum-gravity theories" covers a wide family of efforts: canonical loop quantum gravity, spin-foam variants, asymptotic safety, causal dynamical triangulations, causal sets, group field theory, and other nonperturbative programs. These approaches are diverse, and it would be unfair to treat them as one homogeneous target. Still, they do share a common emphasis: they ask how spacetime geometry itself should be quantized or completed in the ultraviolet .

That emphasis is valuable. Indeed, one reason such programs are respected is that they take the gravitational question seriously rather than postponing it indefinitely.

The present program differs from them by inverting the order of priority. Instead of starting from "how do we quantize geometry?", it begins from "how do geometry, gauge structure, and observation become one admissible constitutional system?" The distinction matters. A program may have a beautiful ultraviolet geometry yet still remain silent about the Standard Model parameter problem, proton stability, measurement, or the explicit derivation of low-energy observables. GG-6/GDOS tries to close that gap by making observation and phenomenology part of the primary design target.

This does *not* mean that other quantum-gravity programs are somehow scientifically inferior. It means that the questions being optimized are different. Loop quantum gravity, for example, is an exceptionally ambitious program for background independence and quantum geometry. Asymptotic safety is an exceptionally ambitious program for ultraviolet completeness by RG fixed points. But neither is usually organized around the simultaneous derivation of the full Standard-Model parameter set *and* an operational boundary dictionary for measurement. GG-6/GDOS is organized around exactly that conjunction.

A fair comparison is therefore not "which framework solves everything?" No current framework does. A fairer comparison is "which freedoms are taken as primitive, which are prohibited, and which phenomenological burdens are shouldered directly?" On that criterion, the uniqueness of GG-6 becomes easy to state:

> *it is a no-SUSY, no-string, anomaly-locked, measurement-aware, parameter-compressing unification program whose core claim is that the Standard Model is a boundary theorem rather than an empirical starting axiom.*

That sentence is not the proof. The proof attempt is the whole paper. But as a summary of architectural distinctiveness it is accurate.

Why the contrast with the MSSM is decisive. The supersymmetric program answers the hierarchy problem by *adding* structure: a superpartner for every Standard-Model field and, with it, of order one hundred new soft-breaking parameters. That enlargement was the price of naturalness, and for two decades it was a price worth paying. But the Large Hadron Collider has now excluded the colored superpartners — squarks and gluinos — up to roughly 2 TeV, pushing the natural region into tension and leaving the soft sector underdetermined. GG-6 takes the opposite route on the same problem: it does not pair the Higgs with a partner, it *reclassifies* the Higgs potential as a reading of a Wilson-line holonomy, so the elementary-scalar sensitivity that motivated supersymmetry is no longer the primitive logic. The parameter ledgers run in opposite directions — where the MSSM grows the continuous sector from ~ 28 to ~ 130, GG-6 collapses it to zero. This is the sharpest single contrast in Table 66, and it is decided not by taste but by the superpartner searches already under way.

Why the contrast with superstring theory is decisive. GG-6 inherits string theory's two greatest lessons — Green–Schwarz anomaly inflow (the ancestor of BI-6) and bulk–boundary reasoning — and *declines* its central ontological commitment. In the landscape reading, the roughly 10^{500} metastable vacua make the low-energy parameters *environmental*: the Standard Model is selected, not derived, and predictivity is weakened accordingly. GG-6 sets the ultraviolet endpoint at E_{GUT} rather than the string scale, treats the primitive objects as an ordinary six-dimensional gauge–gravity–tensor system, and forbids non-forced vacuum multiplicity through the MSMF/OGN clauses. The result is the mirror image of the landscape: the same parameters that string theory leaves to environmental selection, GG-6 derives as theorems of a single geometry — though that the geometry is *uniquely* forced is, at this stage, a conjecture rather than a proven theorem. The wager is austere and therefore falsifiable — precisely the property the landscape sacrifices.

Table 66: Architectural comparison of the major unification programs, sharpened to show what is distinctive about GG-6/GDOS. Reading down the final column: it is the only program with *zero* continuous free parameters, a *derived* dark-matter mass, a *closed* cosmological inventory, and a *dated* falsification timeline. Where the MSSM enlarges the continuous-parameter sector and the string landscape renders the low-energy parameters environmental, GG-6 forces them as theorems of one six-dimensional geometry. (That the geometry is *uniquely* forced is itself a conjecture, not yet a theorem, so the final column is a contrast of posture — one vacuum asserted versus $\sim 10^{500}$ constructed — not a proof that GG-6 wins.) Entries are necessarily schematic.

Question	**Standard Model**	**MSSM / SUSY GUTs**	**Superstring / landscape**	**GG-6/GDOS**
Primitive ontology	4D gauge fields and matter	SM + superpartners and soft sector	strings/branes + compactification data	6D geometry–gauge–tensor bulk + boundary export law
Continuous free parameters	$\sim$ 28 empirical inputs	$\sim$ 105 added soft-breaking parameters	$\sim 10^{500}$ vacua / large moduli space	**0** (one dimensionful input G_N; integer ledger $(1, 3, 5; 7, 35)$)
Higgs nature	elementary scalar, bare mass	elementary scalar + partners	model-dependent modulus/brane field	required $\mathbf{5}_H$ doublet (selected as minimal), branch fixed by a 6D holonomy
Hierarchy strategy	accept empirical weak scale	cancellation by partner states	model-dependent, UV-completed at M_{string}	multiplicative lock, $M_{\text{lock}} = \frac{1}{2} M_{\text{Pl}} e^{-K}$
Measurement in foundations	external	external	external to the fundamental ontology	OGGEP/GDOS makes measurement part of the theory
Proton stability	accidental at renormalizable level	*predicts* decay, 10^{34}–10^{36} yr	model-dependent	*exact* ($\tau = \infty$; $\Delta B = 1, 2$ forbidden)
Dark matter	none	neutralino LSP (not yet seen)	model-dependent	Kaxion, 8.1 μeV through one calibrated scale (GG-C1-Kaxion)
Cosmological inventory	none intrinsic	none intrinsic	anthropic / environmental	$\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$, switch $z_* = e^7$ (GG-A10-Cosmos)
Predictivity / falsifiability	precise, but parameters are input	many parameters dilute predictions	landscape weakens low-energy predictivity	sharp, dated falsifiers (§109.9)
Status vs. current data	confirmed	no superpartners to $\sim$ 2 TeV; naturalness strained	not directly tested	falsifiable by $\sim$ 2040
Primary ultraviolet scale	4D EFT	SUSY/GUT scales	string/Planck scale	GUT-seam completion with geometry–gauge locking

The architectural comparison above can be deepened by noting what each framework does *not* do. Supersymmetric grand unification, the string-theoretic landscape, asymptotic safety, and emergent-gravity proposals all leave the deeper meta-question of §96 intact: each explains some Standard-Model parameters by introducing others (supersymmetric partners, landscape moduli, fixed-point ratios, emergent thresholds), and none has produced a uniqueness theorem at the bulk-geometry level. GG-Theory is the first framework that closes both the parameter sheet and the meta-question. Theorem 95.1 forces the bulk geometry, the composite gauge–gravity group, and the four-force qualitative architecture jointly, with zero continuous freedom remaining.

107.0.5 Comparison with ordinary SU(5) grand unification

Ordinary SU(5) unification explains why the observed gauge factors may descend from a single block, and in that historical sense it is an ancestor of the present work. But SU(5) by itself does not *close* the low-energy parameter sheet: it supplies a high-scale unification relation and then leans on running, thresholds, and symmetry-breaking details to reach the Z-pole. The GG-6 program is more rigid in one decisive way — the same theorem packet that fixes the admissible bulk stage also fixes the OGN-5 integers and the Wilson–Hosotani branch logic — so that the weak angle, the Higgs vev, and their descendants are direct boundary readouts rather than high-scale relics transported downward.

107.0.6 Comparison with generic gauge–Higgs unification

Generic gauge–Higgs unification already teaches that a Higgs-like scalar can descend from an extra-dimensional gauge component, and the present framework accepts that lesson — with the debt named. Manton (1979) first obtained a Higgs-like scalar from the extra-dimensional components of a gauge field; Y. Hosotani (1983) showed that on a multiply connected space the Wilson-line holonomy is itself the order parameter, and Y. Hosotani (1989) developed the resulting dynamics; Hatanaka, Inami & Lim (1998) established the property on which §82 leans, that the holonomy's effective potential is finite because no local counterterm for a Wilson-line phase exists. Everything this paper says about *why* the electroweak scale is stable is inherited from that line of work and is not claimed as new. What is new is what the ledger adds to it: generic gauge–Higgs unification leaves the compactification radius, the matter content, and hence the minimum of the holonomy potential as inputs to be scanned, so the mechanism explains the *stability* of the electroweak scale without fixing its *value*. Here the OGN-5 integers fix both, and $\theta_H^\star$ is computed rather than scanned. What changes is the surrounding logic: the Wilson–Hosotani mechanism is not an isolated model-building device but a link in a constitutional chain, OGGEP ⇒ GG-6 stage ⇒ BI-6 legality ⇒ GDOS runtime ⇒ electroweak branch readout, which is what licenses connecting branch selection to the larger DGI/GFF/GSSB/GA attractor story. The single discipline that makes the comparison meaningful is the refusal to re-introduce freedom upstairs: a familiar failure mode of ambitious unification is to trade twenty-eight low-energy parameters for an even larger set of moduli, thresholds, or compactification choices. The MSMF/OGN insistence guards against exactly that, and the paper's central claim is that for the entire gauge–Higgs sheet

no new continuous coefficient is admitted — which is what makes the closure nontrivial rather than a mere relocation of the same freedom.

108 The Ten Structural Innovations of the GG-Theory Standard Model

The contribution of GG-A9-SM is to derive the Standard Model's parameter set from an admissible input ledger that contains no free continuous parameter at all. We collect here the ten structural innovations that derivation delivers, each carrying its cohort-global identifier (§C).

Innovation 1 (I-SM-1): The 28-to-zero compression. GG-A9-SM recasts the 28 Standard-Model parameters as outputs of a framework with *no* adjustable continuous input — explaining the Standard Model by removing freedom, not adding it. The grand-unified tradition compresses 28 to a smaller but still positive parameter count; the present reading is, in the strict sense of Arisaka (2026, A4-Euclid), 28-to-zero.

Innovation 2 (I-SM-2): A parameter-free admissible input ledger. Every entry of $\mathfrak{I}_{\mathrm{adm}} = \{c, G_N, \hbar; k_B, Q_B\} \cup \{\pi, e\} \cup$ OGN-5 is *forced*: the GU-5 constants are unit-conversion factors (T-GU5 of A1), π and e are topological constants of the bulk, and the $(1, 3, 5; 7, 35)$ ledger is proved (Theorem T6 of GG-A2-GG6, $N = 3$ closed in Arisaka (2026, P2-N3)). None can be moved continuously without breaking a clause.

Innovation 3 (I-SM-3): Predictivity as theory-space pruning to a single point. The deductive arrow $\mathfrak{I}_{\mathrm{adm}} \Rightarrow \mathfrak{O}_{\mathrm{SM}}^{(28)}$ runs with no continuous freedom at any step, so the admissible theory space contracts to a single point before any fit begins — the structural origin of the unusually large phenomenological predictivity.

Innovation 4 (I-SM-4): The electroweak sector as boundary attractor coordinates. The gauge couplings, the weak angle $\sin^2\theta_W = 25/108$, the fine-structure value $\alpha^{-1}(0) = 137$, and the strong coupling emerge as derived rationals from the boundary attractor structure (T-SM-26.1, T-SM-27.1, T-SM-28.1, T-SM-29.1, T-SM-34.1), not as fitted inputs.

Innovation 5 (I-SM-5): The Higgs sector from Wilson–Hosotani geometry. The electroweak vev $v \approx 246$ GeV (T-SM-30.1) and the Higgs quartic $\lambda_H = 1/8$ (T-SM-31.1) are fixed by the scale-overlap and gauge–Higgs structure, both on the triplicity-layering hypothesis H-L2, rather than tuned.

Innovation 6 (I-SM-6): The electroweak masses as theorems. The Fermi constant and the W, Z, and Higgs masses follow as theorems of the derived gauge–Higgs package (T-SM-33.1 through T-SM-33.4, T-SM-35.1), closing the gauge/Higgs block.

Innovation 7 (I-SM-7): Fermion masses and mixings as a geometric codebook. The twelve fermion masses are scale-overlap codebook entries and the CKM and PMNS matrices (with Majorana phases) arise as projector misalignments, turning the flavor sector from a table of inputs into a computed structure.

Innovation 8 (I-SM-8): Proton stability as a conditional boundary theorem. A boundary-terminated $U(1)_B$ makes the proton absolutely stable ($\tau_p = \infty$), stated as a boundary theorem (T-SM-60.1) — conditional on the descent step (H2) that realizes the bulk $U(1)_B$ as component-resolved baryon number, which GG-P6-Proton discharges (§109.6), the standing condition being the BI-6 inflow (H3) — rather than imposed by hand. It remains a sharp, binary falsifier: a single proton-decay event would refute it.

Innovation 9 (I-SM-9): Four-force uniqueness and the no-extra-constant principle. The canonical four-force architecture is shown unique (T-SM-62.1) and the MSMF/OGN selection principle (T-SM-67.1) forbids importing any further transcendental constant into a seam-code expression — the clause that keeps the input ledger closed.

Innovation 10 (I-SM-10): Rigidity as a falsifiable virtue. The same single-point pruning that yields the predictivity makes the construction maximally fragile: a handful of benchmark failures would collapse it. The predictivity is therefore presented only together with the sharp falsifiers (F-SM-1 . . . F-SM-7 below).

109 Predictions and falsification

A framework that derives twenty-eight Standard-Model parameters from the five-element ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ and the five GU-5 constants $\{c, G_N, \hbar; k_B, Q_B\}$ is not protected by that economy — it is made *more* exposed by it. Every derived number is a place the framework can be killed. Two qualitatively different classes of falsifier are now in play, and the second class has changed character since the first version of this manuscript: the dedicated phenomenology papers GG-P1-KK and GG-C1-Kaxion are complete, and the cosmological companion GG-A10-Cosmos is complete, so the downstream predictions are now *quantitative* rather than programmatic. We organize the discussion as (i) the parameter-free precision post-dictions of already-measured quantities, and (ii) the seven dedicated falsification channels F-SM-1–F-SM-7 — the neutrino package itself resolved into five separately-killable sub-channels F-SM-3a–3e — each now carried to a definite number by a completed companion.

109.1 Parameter-free precision post-dictions

Before any new particle is found, the codebook of this paper is already falsifiable against existing precision data. Each entry of Table 67 is a closed form over the OGN-5 ledger with *no* continuous dial; a single sustained violation at the quoted level falsifies the corresponding clause of the derivation. These are the cheapest and earliest tests, requiring no new facility. They are *post-dictions* — each

quantity was measured long ago — but they are parameter-free: no continuous dial was turned to reach them. The genuinely *blind* predictions of unmeasured quantities (the neutrino package, the 3.85 TeV lock cluster, the 8.1 μeV Kaxion) follow in F-SM-1–F-SM-3 below.

Table 67: Parameter-free precision post-dictions: zero-parameter closed forms for already-measured quantities, testable against existing or near-term precision data. Each is a closed form over the OGN-5 ledger (chosen parameter-free, not blind-predicted); the right-hand column states what observation would falsify it.

Observable	GG closed form	Value	Falsified if
Weak mixing angle	$\sin^2\theta_W = \dfrac{L^2}{4N^3} = \dfrac{25}{108}$	0.23148	precision electroweak fits settle $>$ 0.5% away from 25/108 at the matching scale, quoted without the lock-scale tower, which O_{22} prices at -1.9% and F-SM-1 carries at kill grade
IR fine structure	$\alpha^{-1}(0) = 2^J + N^2$	137	$\alpha^{-1}(0)$ moves off 137 (it will not, but the *structural reading* fails if the integer split is wrong)
Strong coupling	$\alpha_s^{-1}(M_Z) = \pi e$	8.540	world-average $\alpha_s(M_Z)$ settles $>$ 1% (several σ) from $1/(\pi e)$
Higgs quartic	$\lambda_H = 2^{-N} = 1/8$	0.125	a measured λ_H (HL-LHC/FCC) incompatible with 1/8
τ/μ mass ratio	$N_g J\, 2^{L-N_g}/L = 84/5$	16.800	the measured ratio leaves the $\pm 0.5\%$ band around 16.800
Family number	$N_g = 3$ (closed, GG-P2-N3)	exactly 3	a fourth chiral generation, or a light sterile state forced into the active sector

109.2 F-SM-1: the 3.85 TeV lock cluster (GG-P1-KK)

The scale chain $E_{\rm Pl} \to E_{\rm GUT} \to M_{\rm lock}$ terminates at a number fixed with no continuous freedom,

$$M_{\rm lock} = \tfrac{1}{2}\, M_{\rm Pl}\, e^{-K} = \tfrac{1}{2}\, M_{\rm Pl}\, e^{-35} \simeq 3.849 \text{ TeV},$$

set by the single OGN-5 integer $K = 35$ and the GU-5 constants. The dedicated signal paper GG-P1-KK (Arisaka, 2026, P1-KK) now shows that the prediction is not a single bump but a cluster whose *four members with collider handles* are: a spin-2 graviton anchored at $M_{\rm lock}$, the color-octet Kaluza–Klein gluon g^*, and two Dirichlet-terminated Abelian vectors — the leptophobic Z'_B of the $\mathrm{U}(1)_B$ sector derived in the present paper, and its $\mathrm{U}(1)_{Qi}$ sibling — at $M_{\rm lock}$ exactly — inside a larger degenerate multiplet, since the electroweak Kaluza–Klein W, Z and photon sit on the same Neumann problem at the same level, and their tree-level contribution to the oblique parameter T is the kill-grade tension registered at O_{22} (§34.6). The falsifier is the *pattern* (the mass coincidence, the spin assignments, and the coupling hierarchy), not an isolated excess.

This corridor sits precisely at the present experimental frontier, and the honest statement of where it sits has sharpened since this subsection was first written. Run-2 dijet searches at the LHC

exclude generic leptophobic Z' states up to ~ 3.9–4.0 TeV (width-dependent) (CMS Collaboration, 2020), and $t\bar{t}$ resonance searches quote exclusions on broad Randall–Sundrum Kaluza–Klein gluons reaching about 4.55 TeV (CMS Collaboration, 2019). A comparison this paper once made read the broad published template as a factor $\sim$2.9 of headroom for the octet; the collider companion has since graded that comparison mismatched and replaced it with its own calibrated per-bin likelihood, and the result is stronger and less comfortable: at 3.85 TeV the un-suppressible octet member predicts $\sigma \times \mathrm{BR}(t\bar{t}) = 8.7\,[7.0\text{–}12]$ fb against an expected 95% CL limit of $\simeq 8$ fb on the existing 140 fb^{-1} — the octet sits *at the observed exclusion edge* on the companion's own calibration, nominally excluded at 95% CL with the transfer to its broad interfering line shape as the one unquantified step. The cluster is *bracketed*, neither already excluded nor comfortably safe, and on expectation the whole predicted band is excluded or confirmed by a dedicated reinterpretation of the Run-3 data already on tape ($\simeq 0.5$ ab^{-1}).

The other members with collider handles carry no LHC exposure of their own, and saying so is part of the prediction. The baryonic vector is unproduced at tree level: on its Dirichlet mode no endpoint-localized quark couples to it (conditional on the per-fermion depths of O_{21}), so it is LHC-invisible as a separate signal and is tested through the cluster's pattern and through the fifth-force absence of §109.7. The graviton's gluon coupling is now *derived* (GG-P1-KK, Corollary T-KK-2c: $\kappa_{gg} = 0.47/K = 0.0136$), which makes its LHC rate a point rather than a corridor — 0.045 fb in $t\bar{t}$, invisible at any LHC luminosity, with the published diphoton null retro-dicted by a factor $\sim 3 \times 10^3$ — and its direct test a target for a 100 TeV machine ($\sigma \approx 205$ fb at FCC-hh).

This falsifier is therefore two-tiered: *Tier 1 (decisive, and already in the data)* — the un-suppressible octet line, decided on expectation by the end-of-Run-3 dataset under a dedicated line-shape reinterpretation; *Tier 2 (FCC-hh, confirmatory)* — the spin-2 anchor at the same mass with $\Gamma/m = 6.3\%$ and the derived infrared branching pattern. Finding either without the other falsifies the cluster reading.

109.3 F-SM-2: Kaxion dark matter at 8.1 μeV (GG-C1-Kaxion)

The Green–Schwarz tensor that the BI-6 anomaly ledger forces carries a protected boundary dual — the compact-angle pseudoscalar Kaxion — whose mass is the canonical cohort value

$$m_a = 8.1\ \mu\mathrm{eV} \iff \nu = m_a c^2/h = 1.95\ \mathrm{GHz}.$$

The benchmark 1.95 GHz is the center of a single narrow predicted window, 1.59–2.13 GHz, whose width is the seven-rung ladder the companion carries as an ansatz around one calibrated benchmark; the parameter-free *fingerprints* — the DFSZ-line coupling and the derived-conditional count of the companion's exposure ledger — do not depend on that width. This is a *two-handle* falsifier, and the two handles are locked together across the cohort. The laboratory handle: a microwave-cavity haloscope tuned to 1.95 GHz at the predicted photon coupling, computed in GG-C1-Kaxion (Arisaka, 2026, C1-Kaxion). The cosmological handle: the same Kaxion must *saturate* the relic abundance derived independently in GG-A10-Cosmos, $\Omega_{\mathrm{dm}} = 1/\pi - 1/(2\pi^2) \simeq 0.267$. Crucially, the Kaxion sits *exactly* on the DFSZ line — the faintest, hardest coupling depth the haloscope community

targets — so present-generation cavities do *not* yet reach it. It is the next-generation DFSZ-sensitive haloscopes (the ADMX-EFR upgrade and the IBS-CAPP program (ADMX Collaboration, 2025)), being built to scan this band at full DFSZ depth across roughly 2027–2030, that will provide the decisive test. *Failure:* exclusion of the 1.59–2.13 GHz window at the derived coupling, *or* a measured dark-matter relic incompatible with $\Omega_{\rm dm} = 1/\pi - 1/(2\pi^2)$. The cross-paper lock GG-A9-SM (sector) → GG-C1-Kaxion (mass, coupling) → GG-A10-Cosmos (abundance) means no single experiment can confirm the Kaxion in isolation, and no single exclusion is cheap to evade.

109.4 F-SM-3: the neutrino package, in five separately-killable channels

The lepton sector is the least hadronically shielded exposure of the framework. The present paper fixes, with no further freedom: normal ordering; a light but nonzero QD-lifted m_1; a mass sum $\sum m_\nu \simeq 59$ meV; a PMNS Dirac phase $\delta_{\rm PMNS} \simeq 196^\circ$ (now *Derived (conditional)* under the leptonic Dirac-phase assignment lemma, consistent with the 6/14/8 triage); an atmospheric angle in the upper octant; protected Majorana phases $(0, \pi)$; and a neutrinoless-double-beta scale $m_{\beta\beta} \simeq 1.8$ meV. The current data are already pressing on this package, and it is no longer useful to treat it as one channel: the six statements above are tested by different experiments on different timetables, and two of them are already moving in opposite directions. The package is therefore split here into five separately-killable sub-channels, each with a criterion declared in advance, a facility, and a date. **A clean failure of any one of them refutes the package**; the sub-channels are not a hedge. In brief: **F-SM-3a** the mass ordering, **F-SM-3b** the Majorana floor, **F-SM-3c** the mass sum, **F-SM-3d** the Dirac phase, and **F-SM-3e** the atmospheric octant.

One result that is *not* a kill channel belongs first, because it is the strongest evidence the block has. JUNO's first data (JUNO Collaboration, 2026) give $\sin^2\theta_{12} = 0.3092 \pm 0.0087$, against the derived closed form $L^2/3N_g^3 = 25/81$, QD-completed to 0.30857: a pull of 0.07σ, the closest agreement anywhere in the lepton block, on a measurement that did not exist when the derivation was written. A framework can survive its kill tests and still be uninteresting; this is the row that says the block is worth killing.

F-SM-3a — the mass ordering. *Prediction:* normal ordering, forced by the cascade-depth assignment that places the GGA at the bottom of the Majorana stratum. *Status:* the post-JUNO global analysis (Esteban et al., 2026) gives $\Delta\chi^2(\text{IO} - \text{NO}) = 4.6$ without atmospheric data and 9.4 with Super-Kamiokande and IceCube — normal ordering at roughly 2.2–2.3σ, with JUNO having about doubled the discrimination. Preferred is not established. *Kill:* inverted ordering established at 5σ. *Facility and date:* JUNO reaches 3σ standalone around 2032 and earlier in combination; DUNE and Hyper-Kamiokande add atmospheric sensitivity through the 2030s. *What dies:* the cascade-depth assignment of §50.4, and with it the light spectrum. There is no fallback ordering.

F-SM-3b — the Majorana floor. *Prediction:* Majorana neutrinos with protected parities $(\alpha_{21}, \alpha_{31}) = (0, \pi)$ and $m_{\beta\beta} \simeq 1.8$ meV. *Status:* KamLAND-Zen (KamLAND-Zen Collaboration, 2025) gives $T^{0\nu}_{1/2} > 3.8 \times 10^{26}$ yr on ^{136}Xe, i.e. $m_{\beta\beta} \lesssim$ 28–122 meV; the combined LEGEND-200, GERDA and Majorana Demonstrator ^{76}Ge result (LEGEND Collaboration, 2026) gives

$> 1.9 \times 10^{26}$ yr; CUORE (CUORE Collaboration, 2025) gives $> 3.5 \times 10^{25}$ yr on ^{130}Te. *Kill:* a confirmed $0\nu\beta\beta$ line anywhere in the tonne-scale window of 9–21 meV, or an established Dirac nature. *Facility and date:* CUPID at 9–15 meV from the mid-2030s, LEGEND-1000 at 9–21 meV. *What dies:* the dial-free door as the matter portal, and with it the whole of Part VIII.

This channel is one-sided, and the paper says so twice. A signal kills the prediction; a null result at tonne scale proves nothing about it, because the predicted 1.8 meV sits an order of magnitude below the reach. §54 shows that the margin is larger than the spectrum alone requires — the derived parity places $m_{\beta\beta}$ in the lowest quarter of the range accessible to a generic normal-ordering spectrum. That is a cost of the parity being derived rather than chosen, and it is not to be presented as a virtue.

F-SM-3c — the mass sum. *This channel is firing.* *Prediction:* $\sum m_\nu = 59.17$ meV. *Status, and the reason it cannot take the comfortable bound:* this paper's own channel F-SM-6 pins $w = -1$ exactly and two-sidedly, so a framework-consistent reader is not free to score against the w_0w_aCDM row. DESI DR2 with CMB (DESI Collaboration, 2025) gives $\sum m_\nu < 64.2$ meV (95%, ΛCDM) — eight percent above the prediction — and the Feldman–Cousins variant of the *same* analysis gives < 53 meV, which the prediction does not survive. For comparison the same data under w_0w_aCDM give < 163 meV, ACT DR6 (Calabrese et al., 2025) gives < 89 meV, and an SPT-3G-based CMB choice gives < 110 meV. *Kill:* a robust $\sum m_\nu < 58$ meV at high significance under a stable ΛCDM treatment. *Facility and date:* no new facility is required; DESI DR3 with CMB, in the ∼2026–2028 window. *What dies:* the neutrino mass block, and therefore the seam coefficient $\pi/E_{\rm GUT}$ that fixes it.

The honest statement is not that this channel will fire soon but that *it is firing now, at the one-sided edge, under the framework's own dark-energy prior.* The DESI analysis itself reports a 3σ tension between the cosmological constraint and the oscillation floor, and the recurring preference for a negative effective $\sum m_\nu$ in the literature is a symptom of the same squeeze. Because it needs no new facility, this remains the channel most likely to be decided first.

F-SM-3d — the Dirac phase. *Prediction:* $\delta_{\rm PMNS} = \pi + 2\arcsin(1/7)$, QD-completed to 196.06°. *Status:* the T2K–NOvA joint analysis (T2K and NOvA Collaborations, 2025) gives, for normal ordering, a best fit of 203.4° with a 1σ interval $[165.6°, 266.4°]$ — the prediction sits about seven degrees from the world central value. The same analysis excludes $\delta = +\pi/2$ at 3σ in both orderings; it does *not* exclude $3\pi/2$, whose death remains this framework's prediction rather than a datum. *Kill:* 196° excluded at 5σ, or a converged maximal 270°. The $3\pi/2$ branch is pre-registered dead and may not be reinterpreted. *Facility and date:* Hyper-Kamiokande from 2028, DUNE from first beam in 2031, each reaching $\sim \pm 15°$ over the following years. *What dies:* the leptonic Dirac-phase assignment lemma, and with it Row 22 of the codebook. *What it measures:* the leptonic Dirac phase, and that alone. An earlier reading of this falsifier held that the base value π in the closed form *is* the Majorana parity $\alpha_{31} = \pi$, and hence that this measurement reaches the sign of η_B. That reading is withdrawn: Appendix K.1.5 proves that no oscillation experiment can ever measure α_{31}, so $\delta_{\rm PMNS}$ cannot contain it. The π is the reference-channel convention of §54. What the long-baseline program therefore measures is sharp, parameter-free and worth the facilities: the closed form for the leptonic

Dirac phase, and that alone. What it does *not* test is the origin of the matter asymmetry, which is tested instead by $0\nu\beta\beta$ and by the neutrino mass sum. That separation is a gain and not a retreat: two claims that can fail independently are more testable than one claim that stands or falls whole.

F-SM-3e — the atmospheric octant. *Prediction:* $\sin^2\theta_{23} = \frac{1}{2}(1 + 1/J) = 4/7$, QD-completed to $39/70 = 0.5571$ — the upper octant, with the sign forced by which direction the charged-lepton hierarchy distinguishes (§54). *Status: the two leading determinations disagree with each other.* NuFIT 6.1 (Esteban et al., 2025) places $\sin^2\theta_{23} = 0.470^{+0.017}_{-0.014}$ in the lower octant; the accelerator joint fit places it at $0.56^{+0.03}_{-0.05}$ in the upper. The prediction sits 5.1σ from the first (as a Gaussian pull; the fit's two-lobed three-sigma range $[0.435, 0.584]$ contains it) and 0.06σ from the second. *Kill:* a converged global-plus-accelerator determination placing $\sin^2\theta_{23}$ in the lower octant at 5σ. *Facility and date:* the long-baseline program of the 2030s — DUNE, Hyper-Kamiokande, and JUNO's atmospheric sample. *What dies:* not merely a tie-break. Lemma L5 falls, and codebook Row 21 with it (§55.5).

This is the framework's sharpest current tension, and it is registered as its own channel precisely so that it cannot be absorbed into a general statement about the neutrino package.

109.5 F-SM-7: the leptogenesis chain

Part VIII assembles a baryon-to-photon ratio from quantities this paper derives, and it does so on one named hypothesis. That hypothesis is falsifiable, and this channel registers it.

Prediction: $\eta_B \approx 5 \times 10^{-10}$ with the derived band $(1.6\text{–}15) \times 10^{-10}$, against an observed $(6.12 \pm 0.04) \times 10^{-10}$, from a resonant leptogenesis at $M_{\rm lock} = 3.85$ TeV with two adopted constants and no fitted continuous parameter — the sign set by the Majorana parity $\alpha_{31} = \pi$, the survivor carried by the electron flavor because $\sin^2\theta_{13} = 3/140$ is small.

Kill, in three routes (§75). First, if a finite-temperature transport computation returns the vacuum N-width class for the resonance regulator, the chain undershoots by seven or eight orders of magnitude and is falsified from within — and the *standard* fully flavored Boltzmann limit does exactly this, which is why condition C4 is the crux and not a formality. Second, if it returns the flavor-blind g^2T class, the excluded peak overshoot stands and the verdict is the same. Third, if C4 and C5 close but the window deficit of C6 survives a full transport computation at its adverse end with the priced order-unity prefactors exhausted, the chain fails by throughput rather than by regulator identity — the failure mode whose size Eq. (68.9) now measures at $E = 10.6$, $[6.6, 17.0]$.

This channel is not like the others, and the difference is worth stating. Every other channel in this section waits on a facility: on JUNO's exposure, on a tonne of enriched xenon, on a beam that does not yet exist. **This one waits on a calculation.** The computation that would settle it — a Kadanoff–Baym or closed-time-path resummation of the lock-crossing decays, carried far enough to identify what regulates the resonance in a hot plasma — requires no new data and could be done today by anyone who wanted to. It is, in that specific sense, the most immediately available kill test the framework possesses, and the framework has an obvious interest in its not

being attempted. That is the reason it is named, dated as *now*, and given its own channel rather than left as a caveat inside Part VIII.

What dies: the chain of Part VIII, and with it the claim that this framework accounts for the matter content of the universe. What does *not* die is the neutrino block: the door, the parities, the angles and the spectrum are established independently of the chain, and F-SM-3a–3e test them separately. Nor would the failure return the framework to silence — it would return it to Sakharov custody plus an unexplained number, which is where the field already is. *Both outcomes are scientific; only the unexamined middle would not be.*

109.6 F-SM-4: exact proton stability

Proton decay occupies a special status, and the present framework makes a stronger and sharper claim than any grand-unified competitor. Where GUTs *predict* decay (and are thereby confirmed only by a positive detection), GG-6 predicts $\tau_p = \infty$ *exactly*: proton absolute stability is a theorem (T-SM-60.1 here; GG-A5-GDOS T-GDOS-9.8), following from exact *global* baryon number, residual to a boundary-terminated local $\mathrm{U}(1)_B$.

A status correction, owed to GG-P6-Proton. This paper stated the theorem as conditional on the (H2) baryon-descent step and deferred that step to a dedicated successor. *That deferral has been repaid.* GG-P6-Proton discharges (H2) — its $\mathbb{Z}_3$-lock theorem realizes the descended $\mathrm{U}(1)_B$ as component-resolved baryon number, with the chiral floors carrying winding exactly B — and delivers (H1), the bulk-only status of the X, Y bosons, as a corollary rather than a hypothesis. The standing conditions are therefore now (H3), the BI-6 Green–Schwarz inflow at its home paper's grade, together with that paper's two named residues: holonomy integrality on the Hopkins–Singer clause, and the warped zero-mode profiles at fractional winding. The claim is stronger than this paper previously recorded, and it is recorded here at the successor's grade rather than at the deferral's.

The sharp content is a selection rule. Baryon number may be violated only in units of $N_{\mathrm{gen}} = 3$ (the $\mathrm{SU}(2)_L$ anomaly), so

$$\Delta B = 1,\, 2: \ \text{forbidden}; \qquad \Delta B = 3: \ \text{admissible (sphaleron-suppressed, } \sim e^{-4\pi/\alpha_W}).$$

Therefore GG-6 forbids *both* the canonical proton-decay modes ($p \to e^+\pi^0$, $p \to K^+\bar{\nu}$, $\Delta B = 1$) *and* neutron–antineutron oscillation ($\Delta B = 2$). A single event in either class falsifies the framework. Table 68 confronts this with the present bounds (Super-Kamiokande Collaboration, 2020, 2021) and the reach of the next generation (Hyper-Kamiokande Collaboration, 2018), and contrasts it with the GUT predictions it displaces.

Table 68: Baryon-number stability: the GG prediction ($\tau = \infty$, exact) confronted with current bounds and near-term reach, and contrasted with grand-unified predictions. GG-6 is falsified by *any* single $\Delta B = 1$ or $\Delta B = 2$ event; a GUT is confirmed only by such an event. Benchmark values as of 2025–2026.

Channel (ΔB)	Current 90% C.L. bound	Near-term reach (∼2030s)	GUT expectation	GG
$p \to e^+\pi^0$ (1)	$\tau > 2.4 \times 10^{34}$ yr (Super-K)	$\sim 1 \times 10^{35}$ yr (Hyper-K, data 2028)	10^{34}–10^{36} (SO(10))	∞
$p \to K^+\bar{\nu}$ (1)	$\tau > 6.6 \times 10^{33}$ yr (Super-K)	$\sim$ few $\times 10^{34}$ yr (DUNE, Hyper-K)	10^{33}–10^{35} (SUSY GUT)	∞
n–$\bar{n}$ oscillation (2)	$\tau_{n\bar{n}} > 4.7 \times 10^8$ s (Super-K)	2.8–4.4 $\times 10^9$ s (ESS/NNBAR)	model-dependent	∞

Minimal SU(5) ($\tau_p \sim 10^{31}$ yr) is already excluded; SO(10) and flipped SU(5) predict 10^{34}–10^{36} yr and are the live GUT targets. GG-6 predicts neither — it predicts that *no* $\Delta B = 1$ or $\Delta B = 2$ event ever occurs, so a single Hyper-K proton-decay event or a single ESS n–$\bar{n}$ event would be fatal.

This is the cleanest single falsifier the framework offers. Continued null results are positive consistency evidence; one unambiguous event is a death certificate.

The same rule, read at colliders. The selection rule does not care about the venue. Any collider signal requiring $\Delta B = 1$ or $\Delta B = 2$ — leptoquark-mediated baryon-number violation, n–$\bar{n}$-type operators in same-sign top pairs, any of it — falsifies the framework exactly as a proton-decay event would. And the admissible door swings the other way at high energy: sphaleron-induced $\Delta B = \Delta L = 3$ events at a 100 TeV collider, if the contested near-threshold suppression permits them (§76), would be the conversion vertex of Part VIII observed directly — the same transition that made the baryons, run in a detector. One rule, three verdicts: protons immortal, $\Delta B = 1, 2$ events fatal, $\Delta B = 3$ events a confirmation no other framework predicts with this exclusivity.

109.7 F-SM-5: the baryonic fifth force

An earlier version of this subsection derived a finite-range baryonic Yukawa force from a Stueckelberg-massive $\mathrm{U}(1)_B$,

$$V_B(r) = \frac{g_B^2}{4\pi}\,\frac{B_1 B_2}{r}\,e^{-m_B r}, \qquad \lambda = \frac{\hbar}{m_B c} \simeq 0.20\ \mu\mathrm{m}\left(\frac{\mathrm{eV}}{m_B}\right), \tag{109.1}$$

— the earlier reading, withdrawn below — and bracketed a (g_B, m_B) window with four candidate couplings, $g_B \in \{0.14, 0.23, 0.57, 0.89\}$, to be scanned by torsion-balance and isotope-dependent searches. Both halves of that reading have since been settled in the companions, and the channel's prediction is now stronger, not weaker.

First, the coupling is no longer a menu. The terminated-seam capacity theorem of GG-A3-BI6 (T-BI6-6.4) resolves the once-open readout rule L-KK-1 of GG-P1-KK: $\alpha_B^{-1} = \frac{2}{5}\,\alpha_Y^{-1}$, $g_B = 0.565$ — the third entry of the bracket, now derived. Second, and decisively, the termination of $\mathrm{U}(1)_B$ is a *boundary condition*, not a Stueckelberg pairing (GG-A3-BI6): the vector's zero mode is removed

by a Dirichlet condition at the seam, its lightest state is the 3.85 TeV lock note, and there is no Stueckelberg scale, no eaten decay constant, and no mass dial m_B for Eq. (109.1) to run over. That is a statement about the zero mode, and the zero mode is the one Eq. (109.1) would have needed. The massive levels above it behave as the levels of any compactified gauge field do, each taking its longitudinal state from the corresponding mode of $A_{B,r}$, and the level-by-level consistency of that tower with the anomaly inflow is registered as an open problem by GG-A3-BI6. Neither reading returns a force law: below M_{lock} there is no baryonic channel for any receiver to read.

The prediction of this channel is therefore an *absence*, stated without a parameter:

$\eta = 0$*: no composition-dependent fifth force, at any range, at any precision, ever* (Arisaka, 2026, P7-5thF).

The dedicated fifth-force companion carries this as the cohort's single fifth-force prediction and names the instrument that tests it (MADO). Table 69 confronts the absence with the present equivalence-principle frontier.

Table 69: The fifth-force absence prediction, $\eta = 0$ at any range, against the present equivalence-principle frontier (MICROSCOPE Collaboration, 2022). Every row is a test the framework must survive; the channel is two-sided in the only way an absence can be — each null is consistency, and a single confirmed composition-dependent force at any range is fatal.

Frontier	Present result	GG prediction
Orbital EP test (MICROSCOPE)	$\eta(\text{Ti}, \text{Pt}) = [-1.5 \pm 2.3 \pm 1.5] \times 10^{-15}$	consistent with 0; every successor must also read 0
Sub-mm gravity (Eöt-Wash) and lunar laser ranging	no deviation at any probed range	null at every range, to any precision (MADO, Arisaka, 2026, P7-5thF)

The structure of the claim is worth one sentence more. The exact global baryon number that stabilizes the proton (§109.6) and the absence of a baryonic force are the same fact read twice — the termination leaves the charge exact and the force without a carrier — so F-SM-5 and F-SM-4 remain two faces of one structure. *Failure:* a single confirmed composition-dependent force, at any range and any strength, which would contradict the termination itself and, upstream, the anomaly bookkeeping that forced it.

109.8 Consolidated falsification map

Table 70 collects the seven dedicated channels in one view, each tied to the completed companion that carries it to a number and to the experimental program that tests it. The scientific moral is worth stating plainly: a theory that derives much from little is not shielded by that achievement; it is rendered maximally vulnerable, and its virtue is measured by how hard the data can hit it.

Table 70: Consolidated falsification map of the GG-Theory Standard Model. Every channel is now carried to a quantitative prediction by a completed companion paper. Each row names the channel, the quantitative prediction, and the measurement that would end it. A falsification map is only as good as the sharpness of its entries, and these entries are sharp because the downstream companions carried them to numbers. Before those papers existed the same table would have been a list of directions rather than of tests.

Channel	Origin in GG-6/GDOS	Quantitative prediction	Carried by	What counts as failure
F-SM-1 Lock cluster	Lock-scale throat, $K = 35$	graviton + g^* + two terminated Abelian vectors at $M_{\rm lock} \simeq 3.85$ TeV	GG-P1-KK	Tier 1: the octet line excluded — decided on Run-3 data on expectation; Tier 2: spin-2 anchor at FCC-hh (T-KK-2c)
F-SM-2 Kaxion	Boundary dual of the BI-6 tensor	$m_a = 8.1\ \mu$eV (1.95 GHz), saturating $\Omega_{\rm dm}$	GG-C1-Kaxion, GG-A10-Cosmos	window excluded at derived coupling, or relic $\neq 1/\pi - 1/(2\pi^2)$
F-SM-3a–3e Neutrinos	Rank-2 Majorana block + lepton projector	NO ($3a$), $m_{\beta\beta} \simeq 1.8$ meV ($3b$), $\sum m_\nu \simeq 59$ meV ($3c$), $\delta_{\rm PMNS} \simeq 196^\circ$ ($3d$), upper octant ($3e$)	GG-A9-SM (in paper)	converged ν data incompatible with the package
F-SM-4 Proton	exact global B, boundary-terminated $\mathrm{U}(1)_B$	$\tau_p = \infty$; $\Delta B = 1, 2$ forbidden	GG-A9-SM, GG-A5-GDOS	any single $\Delta B = 1$ or $\Delta B = 2$ event
F-SM-5 Fifth force	Terminated $\mathrm{U}(1)_B$: no vector below $M_{\rm lock}$	absence: $\eta = 0$ at any range, any precision	GG-P1-KK, P7-5thF	any confirmed composition-dependent force, at any range
F-SM-6 Dark energy	GA fixed point of the GRIP cascade	$w = -1$ exact, two-sided ($\lvert 1+w\rvert \lesssim 10^{-91}$)	GG-A6-GRIP (Lemma DE-1), GG-A10-Cosmos (T-Cosmos-DE′)	any confirmed $w \neq -1$, either sign (e.g. a converged evolving-dark-energy detection)
F-SM-7 Leptogenesis	Majorana door at $M_{\rm lock}$; parity $\alpha_{31} = \pi$	$\eta_B \approx 5 \times 10^{-10}$, band $(1.6\text{–}15) \times 10^{-10}$	GG-A9-SM (Part VIII), GG-A10-Cosmos	a finite-temperature transport computation returning the N-width or g^2T regulator class, or an unclosable window deficit

109.9 Falsification timeline: within reach by 2030 and 2040

The decisive point is not merely that the framework is falsifiable in principle but that the falsification is *near*. None of the channels above requires a generational wait: each is settled, or decisively advanced, within one to two decades by facilities already running or under construction. Table 71 sets out the schedule.

By 2030, the first wave arrives. JUNO, running since 2025, approaches a standalone 3σ determination of the neutrino mass ordering around 2032 and reaches it earlier in combination; Hyper-Kamiokande (Hyper-Kamiokande Collaboration, 2018), taking data from 2028, overtakes the

Super-Kamiokande proton bound and begins to bite into the SO(10) territory that GG-6 forbids; the High-Luminosity LHC begins to probe the 3.85 TeV multiplet directly; DESI together with the Simons Observatory pushes $\sum m_\nu$ toward the predicted 59 meV; and the ADMX program (ADMX Collaboration, 2025) scans upward toward the 1.95 GHz Kaxion line. The ground-based partner originally written into several of these tests, CMB-S4, is no longer available: DOE and NSF ended their support in July 2025, and the cosmological channels above now rest on the Simons Observatory beside LiteBIRD, Euclid and Rubin.

By 2040, the verdict on most channels is in. Hyper-Kamiokande reaches its 10^{35} yr proton sensitivity — where a single $\Delta B = 1$ event is fatal to GG-6; DUNE delivers the leptonic Dirac phase and the $p \to K^+\bar{\nu}$ channel; the ESS/NNBAR program improves the n–$\bar{n}$ figure of merit by three orders of magnitude; tonne-scale $0\nu\beta\beta$ experiments reach the few-meV Majorana floor; next-generation haloscopes close the Kaxion window and EP tests press the fifth-force absence; and the FCC era opens on the full lock-cluster multiplet.

Table 71: Falsification timeline of the GG-Theory Standard Model. Every predicted number is decisively tested, or its testing facility is operating, within one to two decades. "Settles" means the measurement reaches the precision that confirms or refutes the GG value; "decisive" means a single result can be fatal.

Prediction	**GG value at stake**	**By ∼2030**	**By ∼2040**
Lock cluster	3.85 TeV multiplet	the octet line decided on Run-3 data (on expectation)	HL-LHC full + FCC: decisive on the multiplet
Proton $p \to e^+\pi^0$	$\tau = \infty$ ($\Delta B = 1$)	Hyper-K overtakes Super-K ($\sim 5 \times 10^{34}$ yr)	Hyper-K 10^{35} yr: *any* event fatal
Dark energy w	$w = -1$ exact (two-sided)	DESI full survey: evolving-DE hints confirmed or retired	Euclid + Rubin + Simons Obs.: any confirmed $w \neq -1$ fatal
$p \to K^+\bar{\nu}$	$\tau = \infty$ ($\Delta B = 1$)	DUNE begins	DUNE + Hyper-K, few $\times 10^{34}$ yr
n–$\bar{n}$	forbidden ($\Delta B = 2$)	(Super-K limit holds)	ESS/NNBAR, $\times 10^3$ figure of merit
ν ordering	normal	JUNO settles	confirmed/refuted
$\delta_{\rm PMNS}$	$\simeq 196°$	DUNE/Hyper-K first measurement	decisive precision
$\sum m_\nu$	$\simeq 59$ meV	DESI DR2+CMB at < 64 meV (ΛCDM)	decisive at the 59 meV line
$m_{\beta\beta}$ ($0\nu\beta\beta$)	meV, phases $(0, \pi)$	LEGEND-200, KamLAND-Zen	LEGEND-1000/nEXO at few meV
Kaxion	8.1 μeV / 1.95 GHz	ADMX approaches the line	next-gen haloscopes: decisive
Fifth force	absence: $\eta = 0$, any range	next-gen torsion balance	MADO-class EP tests: any signal fatal

These schedules are developed in full in a forthcoming *P-series* of dedicated prediction papers — on the neutrino package, on proton decay and baryon-number violation, and on the baryonic fifth force — each of which will turn one row of Table 71 into a quantitative experimental forecast, extending the collider and dark-matter forecasts already given in GG-P1-KK and GG-C1-Kaxion.

The point is already plain: the GG-Theory Standard Model is not a philosophy but a wager whose decisive returns fall within the working lifetime of the present generation of experiments. The same schedule is drawn as a single kill-test timeline in Figure 26 (§111.4).

110 Open questions

An honest closing distinguishes three registers: the questions a careful critic could press, which the framework answers; the items that were genuinely open at first submission but are now *closed* by completed companions; and the items that remain undetermined and define the successor work. The first is collected in the Common Objections Part (Part XVI, eighty-three objections); the second and third are the substance of this section, summarized in Table 72.

110.1 What is now closed downstream

Three classes of statement that the first version of this paper listed as "deferred" or "out of scope" are now settled in completed cohort papers, and the present text has been updated to say so:

- **The cosmological inventory** — the matter, dark-matter, dark-energy and baryon fractions, the recombination redshift, and the Hubble corridor — is *derived* in GG-A10-Cosmos (Arisaka, 2026, A10-Cosmos) from the same ledger that fixes the particles (overview in §111.1). It is no longer an open question of the present paper.
- **The family number** $N_g = 3$ is closed as a three-leg theorem in GG-P2-N3 (Arisaka, 2026, P2-N3) and imported here; the present paper depends on it rather than leaving it open.
- **The Kaxion dark-matter sector** has a definite mass and coupling window in GG-C1-Kaxion (Arisaka, 2026, C1-Kaxion), and the dark-matter relic it must saturate is fixed in GG-A10-Cosmos.

110.2 What remains genuinely open

The following items are undetermined at the present stage. Each is a sharp, delegable research target, not a vague caveat.

- **The flavor sheet.** The nine charged-fermion Yukawas, the four PMNS parameters, and the two Majorana phases are presented as closed-form rationals over the OGN-5 ledger, but the specific integer assignment (numerator vs. denominator, exponent vs. prefactor) is a working hypothesis, not a Hopkins–Singer-forced result. Any future flavor derivation must live inside the attractor basin already fixed here, disturbing none of the derived electroweak quantities.
- **The up/down isospin doubling.** The five quark levels generated from three Morse critical points are identified by analogy with the Bogomolnyi cascade on $\mathbb{C}P^2$ (GG-A6-GRIP §29), not yet derived in this paper from the heights of the doubled stratum's fixed points (lemma L10).
- **The topological residue R-INT.** The $N_g \leq 3$ upper bound rests on the BI-6 instanton-action budget, which is no longer a conjecture: the capacity theorem T-N3-6 of Arisaka

(2026, P2-N3) fixes it, the integrality theorem L-INT fixes its lattice, and two obstruction theorems show why the conjectural framing could not have been discharged as posed. What remains open is one named topological residue, R-INT — that the BI-6 seam supports the Hopkins–Singer quadratic refinement globally — and it is met on the benchmark background by the inflow companion.

- **QD precision corrections.** The QD layer is at leading transport order; hadronic-threshold and lattice-matched running corrections to the charm- and bottom-quark masses are not included.
- $0\nu\beta\beta$ **nuclear matrix elements.** Only the Majorana-phase structure is constrained here; the rate also needs nuclear matrix elements not derived in the present paper.
- **The electroweak tower against the oblique parameters.** The Kaluza–Klein W, Z, and photon that the collider companion places at the lock scale give, against the infrared-registered Higgs that Theorem 34.1 reads, $T = 1.7$ where the global fit allows 0.00 ± 0.06; the bulk zero mode that would have escaped the bound cannot register the electroweak scale, and no custodial structure is available in the bulk group (§34.6; O_{22} of §81.1; the collider companion's O-KK-68).
- **The quantum-gravity boundary-state proof** at the lock scale is partially resolved by the composite GG-6 connection of GG-A2-GG6 but the rigorous boundary-state proof is deferred — including a precise account of the dual role of the radial coordinate r, at once the renormalization-group scale $\ln(E_{\mathrm{GUT}}/\mu)$ and the geometric variable whose e^{-2r} profile fixes M_{Pl} (developed in GG-A2-GG6).
- **The higher cascade uniqueness theorems.** The chain from charged-lepton identity through hadrons, elements, molecules, ATP/RNA/DNA to life and mind carries its own uniqueness theorem at each layer; only the lowest is closed here. The capstone GG-A11-Origin (Arisaka, 2026, A11-Origin) addresses the remainder.

Table 72: Status ledger of the questions a reader could press on the GG-Theory Standard Model: closed in this paper, closed downstream since first submission, or genuinely open with its delegated owner. The middle category exists because this paper was written before some of its own companions were finished, and recording that is more honest than folding those items into either of the other two.

Question	Status	Resolution / owner
28 SM parameters	closed (this paper)	codebook over $(1, 3, 5; 7, 35)$
Family number $N_g = 3$	closed downstream	GG-P2-N3 three-leg theorem
Cosmic inventory (Ω's, z_*, H_0)	closed downstream	GG-A10-Cosmos (§111.1)
Dark-matter identity	closed downstream	GG-C1-Kaxion Kaxion, 8.1 μeV
Flavor textures (Yukawas, mixings)	*open*	integer assignment a working hypothesis
Up/down isospin doubling	*open*	sector-dependent heights on the doubled stratum (L10)
$N_g \leq 3$ residue R-INT	*open (narrowed)*	seam refinement; GG-P2-N3 L-INT
QD hadronic-threshold corrections	*open*	precision-physics extension
$0\nu\beta\beta$ rate	*open*	nuclear matrix elements
Lock-scale quantum gravity	*open*	boundary-state proof, future work
Electroweak tower vs. oblique parameters	*open (priced)*	profile computed, $T = 1.7$ at $M_{\rm lock}$ (O_{22}, §34.6); GG-P1-KK O-KK-68
Higher cascade (life, mind)	*open*	capstone GG-A11-Origin

111 The completed cohort and future prospects

The closure of the twenty-eight-parameter codebook completes the *particle floor* of the GG-Theory program. What is different now from any earlier reading of this paper is that the floors above and beside it are already built: the cosmological companion is finished, and all six phenomenology and consequence preprints exist. This section therefore reads not as a wish-list but as a map into a *completed* downstream cohort. The single most important forward link is GG-A10-Cosmos, and we treat it first.

111.1 The most important forward link: an overview of GG-A10-Cosmos-Cosmos

The natural sequel to a derivation of the particles is a derivation of the cosmos, and it now exists. GG-A10-Cosmos (Arisaka, 2026, A10-Cosmos) — *The Standard Cosmology as Theorem of GG-Theory* — closes the cosmic side of the same first floor that this paper closes on the particle side, and it does so from the *identical* input: the ledger $(1, 3, 5; 7, 35)$ and the GU-5 constants, with no adjustable cosmological dials. Where for forty years ΛCDM has described the universe with a handful of numbers read off the sky and explained none of them, GG-A10-Cosmos argues those numbers are not free. The headline results are collected in Table 73.

Table 73: The cosmic inventory derived in GG-A10-Cosmos (Arisaka, 2026, A10-Cosmos) from the same ledger and GU-5 constants that fix the particles in the present paper. These close, on the cosmological side, the same geometry the present paper closes on the particle side. GG-A10-Cosmos carries its own honesty triage (several entries are conjectural D^* or near-hits); the present paper reports them as GG-A10-Cosmos's results, not as its own. The recombination-era row follows the two-quantity treatment of Arisaka, 2026, A10-Cosmos: the corridor switch e^7 is a topological marker, while the measured decoupling redshift is reproduced by GG-A10-Cosmos's own atomic chain at the 0.1% level with zero fitted parameters, 0.4% (in e-folds) below the switch.

Cosmic quantity	**GG closed form**	**GG value**	**Observed**
Matter fraction	$\Omega_m = 1/\pi$	0.318	0.315 ± 0.007
Dark energy	$\Omega_\Lambda = 1 - 1/\pi$	0.682	0.685 ± 0.007
Baryon fraction	$\Omega_b = 1/(2\pi^2)$	0.0507	0.0493 ± 0.0006
Dark matter	$\Omega_{\rm dm} = 1/\pi - 1/(2\pi^2)$	0.267	0.265 ± 0.007
Recombination-era redshifts	switch $z_{\rm sw} = e^7$; decoupling $z_{\rm dec}$ from GG-A10-Cosmos's atomic chain	1097; 1091	1089.92 ± 0.25
Hubble background	$H_0^{(0)} = \sqrt{\omega_i/\Omega_i}$ (derived)	66.6 ± 0.2	67.4 ± 0.5
Age of the universe	$t_0 = 0.948/H_0$	13.8–13.9 Gyr	13.80 ± 0.02 Gyr

The structural message is the one this paper has argued throughout, now extended by one floor: GG-A9-SM closes the particle inventory and GG-A10-Cosmos closes the cosmic inventory, both as theorems of one six-dimensional geometry. The particle data this paper fixes — the electroweak scale, the neutrino block and its CP phase, the strong-CP suppression, the baryonic sector, and the compact-angle ontology — are precisely the inputs that GG-A10-Cosmos inherits to constrain inflation, dark matter, dark energy, baryogenesis, and the recombination history, rather than choosing them from a free menu. The Hubble tension itself is read in GG-A10-Cosmos, and in detail in GG-C2-Hubble, not as a measurement error but as the two ends of one geometric corridor.

111.2 The completed downstream cohort: a forward map

Beyond GG-A10-Cosmos, the present paper feeds two branches of completed preprints — the Predictions Branch and the Consequences Branch. Each builds on a specific result of this paper; Table 74 is the forward map, with the headline number of each and its dependency on GG-A9-SM. These are the forward links the reader should follow from here.

Table 74: Forward map from the GG-Theory Standard Model into the completed downstream cohort: the cosmological capstone, the three-paper Predictions Branch, and the three-paper Consequences Branch. Every entry is a finished preprint. Nothing in this table is a plan. Every entry is a document that can be read, and where a downstream paper has narrowed or corrected something stated here, its row says so.

Paper	Builds on (from GG-A9-SM)	Headline result	Where it is tested
Cosmology			
GG-A10-Cosmos	full particle inventory	$\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$, $\Omega_b = 1/(2\pi^2)$, $z_{\rm sw} = e^7$	Planck, DESI, Simons Obs.
Predictions Branch			
GG-P1-KK	$M_{\rm lock}$, $\mathrm{U}(1)_B$	3.85 TeV lock cluster: graviton + g^* + terminated vectors	HL-LHC, FCC
GG-P2-N3	(provides $N_g = 3$)	three-leg derivation of $N_g = 3$	structural
GG-P3-Electron	lepton sector	electron as unique boundary ground state	precision QED, stability
Consequences Branch			
GG-C1-Kaxion	S^1_θ, $\mathrm{U}(1)_{Qi}$	8.1 μeV cold dark matter ($\Omega_{\rm dm}$)	haloscopes at 1.95 GHz
GG-C2-Hubble	boundary GR, scale chain	73.5 vs. 67 as one corridor (7σ tension)	distance ladder, CMB
GG-C3-BH	GDOS/OGGEP horizon	exact Bekenstein–Hawking $S = A/4G_N$	black-hole thermodynamics

The two branches divide the labor cleanly. The *Predictions Branch* carries the falsifiable signals of the present paper to definite numbers: GG-P1-KK (Arisaka, 2026, P1-KK) is the standing collider falsifier (the 3.85 TeV multiplet, octet-led); GG-P2-N3 (Arisaka, 2026, P2-N3) supplies the $N_g = 3$ that this paper imports; GG-P3-Electron (Arisaka, 2026, P3-Electron) closes the six oldest questions about the electron — its identity and measured values, its absolute stability, its exactly two heavier copies, its mirror twin (the positron as conjugate readout), the absence of substructure, and its response to a field — as properties of the unique ground state of the lepton sector derived here. The *Consequences Branch* carries the framework into the hardest external arenas: GG-C1-Kaxion (Arisaka, 2026, C1-Kaxion) identifies the dark matter; GG-C2-Hubble (Arisaka, 2026, C2-Hubble) reads the 7σ Hubble tension (the 2026 consensus 73.50 ± 0.81 vs. the CMB value near 67) as one six-dimensional corridor; and GG-C3-BH (Arisaka, 2026, C3-BH) derives the exact Bekenstein–Hawking entropy $S = A/4G_N$ from induced gravity and resolves the information paradox through the GDOS/OGGEP horizon port.

111.3 Genuine future work: the capstone and the higher cascade

The natural successor to the completed cohort is the capstone volume GG-A11-Origin (Arisaka, 2026, A11-Origin) — *The Geometric Inevitability of Cosmos, Life, and Mind* — which extends the uniqueness logic above the particle layer into the full structure-formation cascade the six-dimensional bulk permits. Where the present paper closes the parameter sheet and GG-A10-Cosmos

closes the cosmic inventory, the capstone targets the higher uniqueness theorems — hadrons, elements, interstellar molecules, the ATP/RNA/DNA layer, single-cell life, and ultimately mind — of which only the lowest is closed here. The honest professional stance remains unchanged by the breadth of this program: what the present paper has earned is not victory but *eligibility for serious testing*.

111.4 A disciplined experimental roadmap for the next decade

A framework that derives a large fraction of low-energy physics from a tiny ledger should say not merely that it is falsifiable but *how* the field can try to falsify it. The roadmap is best organized by structural sector, because the same geometry ties the sectors together: the neutrino phases that appear in oscillation experiments also drive baryogenesis; the compact-angle sector that assists symmetry breaking also supplies the dark matter; the baryonic extension that protects the proton also governs the fifth-force search. Table 75 names the facilities, the now-derived numbers they test, and — crucially — how each result strengthens or weakens the *whole* framework rather than an isolated module; Figure 26 then lays the same program out as one forward time axis — the nine decisive kill-tests, color-coded by sector, from today's bounds through the first wave at ∼2030 to the decisive decade at ∼2040.

The roadmap is not a promise that every decisive test arrives at once; science seldom moves so cleanly. But its existence is itself a result, and the timeline makes its structure visible: the tests are not nine independent dice. The five neutrino sub-channels (F-SM-3a–3e, the Majorana floor among them) probe the same rank-2 block from several sides; the proton bound and the n–$\bar{n}$ channel probe the same gauged baryon number at $\Delta B = 1$ and 2; the Kaxion line and the fifth-force absence read the two faces of the same terminated extension; and the lock cluster is the one direct sighting of the throat that underlies them all. The leptogenesis channel is the most correlated of the nine rather than an extra independent bet: it consumes the same Majorana door as F-SM-3b, the same reactor angle as F-SM-3e, and the same lock scale as F-SM-1, so a failure there is a failure of things already wagered elsewhere — which is why it was worth registering separately rather than folding into the package. A failure clusters; so does a success. The GG-Theory Standard Model is not only a way of reading past data — it is a structured wager on the data of the coming decade, and the wager is now spread across eight named facilities, one calculation that needs no facility at all, and six completed companion papers, with the decisive returns falling, as Figure 26 shows at a glance, between now and ∼2040.

112 Summary of the Discussion

This Discussion has made the case for the result on several fronts, and they are worth gathering before the paper closes. It located the *significance* of the closure (§105): not merely that twenty-eight numbers are derived, but that, once every continuous knob is removed, the half-century meta-question — *why these values, why these four forces, why every electron is the same* — becomes a scientific question with a falsifiable answer rather than a matter of design, multiverse, or brute fact. It redeemed the ten classic open problems of the introduction one by one (§106), and cataloged

Table 75: A disciplined experimental roadmap for the next decade. Each row tests a specific derived number against a named facility; the same measurement program can strengthen or weaken the entire GG-6/GDOS framework, not one sector. The horizons are the facilities' own published ones and not this paper's expectations of them, and no row claims a sensitivity a facility has not itself projected.

Frontier	**Derived number tested**	**Facility**	**Strengthens GG if**	**Weakens GG if**
Energy frontier	3.85 TeV multiplet (g^*, graviton, terminated vectors)	HL-LHC, FCC	coherent multiplet at 3.8–4.0 TeV	full corridor excluded
Proton decay	$\tau_p = \infty$; $\Delta B = 1, 2$ forbidden	Hyper-K, DUNE, JUNO	continued null at 10^{35} yr	one $\Delta B = 1$ event
n–$\bar{n}$	$\Delta B = 2$ forbidden	ESS / NNBAR	null at $\sim 10^9$ s	one oscillation event
Neutrinos	NO, $\sum m_\nu \simeq 59$ meV, $\delta \simeq 196°$	DUNE, Hyper-K, JUNO, Simons Obs.	convergence on the package	stable incompatible pattern
$0\nu\beta\beta$	Majorana $(0, \pi)$, $m_{\beta\beta}$ (meV)	LEGEND-1000, nEXO	no line above the floor	a line in the 9–21 meV window, or a Dirac nature
Dark matter	Kaxion 8.1 μeV / 1.95 GHz	ADMX-class haloscopes	line at derived coupling	line excluded
Strong CP	$\bar{\theta}_{\rm eff} \sim 10^{-17}$ (relaxed; GG-C1-Kaxion)	nEDM (PSI, SNS)	continued null at 10^{-27}–$10^{-28}\, e\,$cm	a nonzero d_n attributable to $\bar{\theta}$ at any level: both stages fail together
Fifth force	$\eta = 0$ exact (absence)	Eöt-Wash, MICROSCOPE successor, MADO	continued null at every range	any composition-dependent force
Cosmology	Ω's, z_*, H_0 corridor	DESI, Simons Obs., Rubin	one constrained inheritance	need for new free sectors
Matter excess	$\eta_B \approx 5 \times 10^{-10}$ from $M_{\rm lock}$	*theory*: finite-T transport	the regulator lands in the partial-resonance window	it returns the N-width or g^2T class

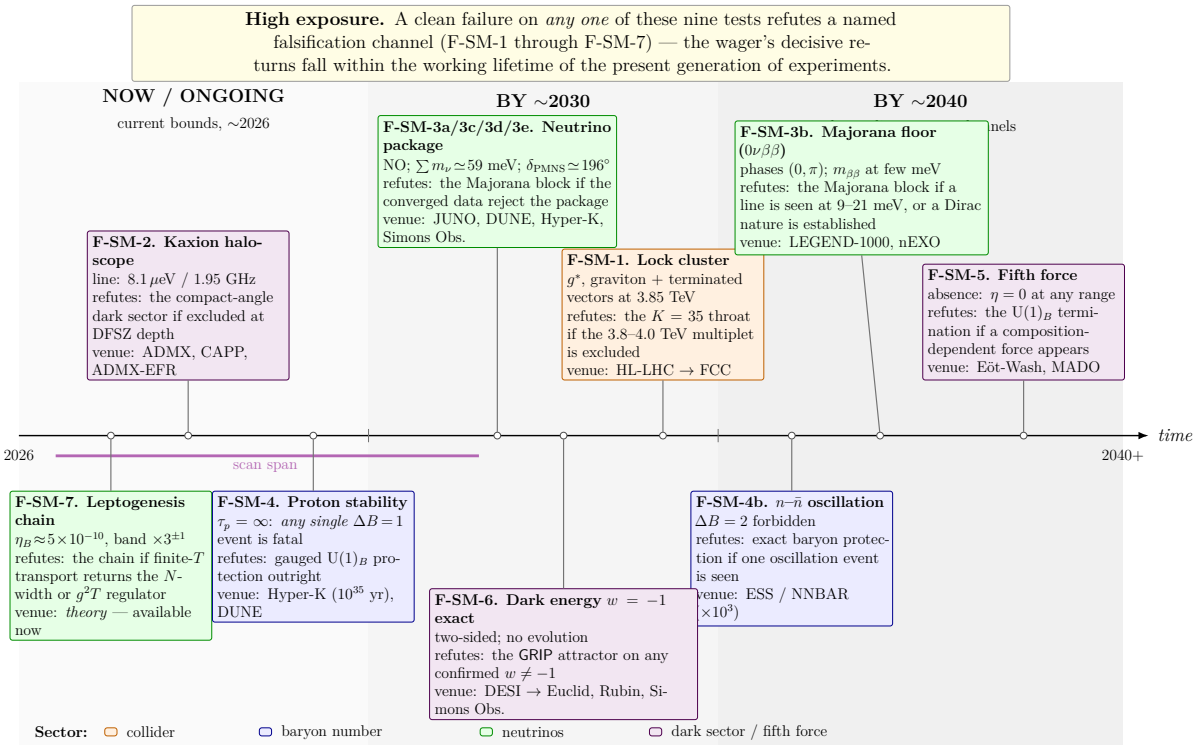

Figure 26: Falsification / kill-test timeline for GG-A9-SM. The nine decisive kill-tests of the GG-Theory Standard Model are placed on a forward time axis running from current bounds (∼2026) through the first wave (∼2030) to the decisive decade (∼2040), grouped into broad data-window eras rather than precise launch dates. F-SM-7 sits at the left edge on purpose: alone among the nine it waits on a calculation rather than on a facility, so it carries no future date and could be settled today. Each card states the derived number at stake, what a failure would *refute*, and the experimental venue. Cards are colored by sector: collider (orange), baryon number (blue), neutrinos (green), and the dark sector / fifth force (violet). The Kaxion haloscope scan spans an ongoing-through-∼2030 window (violet bar). A clean failure on any single test refutes a named channel (F-SM-1 through F-SM-7), so the framework's falsification exposure is high and front-loaded into the coming decade — the graphical form of Tables 71 and 75.

the ten structural innovations that make the closure possible (§108). It then set the framework against its rivals (§107) and found that it differs from the Standard Model, the MSSM, and the perturbative-string landscape *in kind, not in degree*: it is the only column of Table 66 with zero continuous parameters, a derived dark-matter mass, a closed cosmological inventory, and a dated falsification timeline.

Above all, the Discussion turned the program's economy into exposure (§109). The six falsification channels F-SM-1–F-SM-5 and F-SM-7 are now carried to definite numbers by completed companions or by this paper — the 3.85 TeV lock cluster, the 8.1 μeV Kaxion, the 59 meV neutrino package, exact proton stability, and the fifth-force absence — and the timeline of §109.9 shows the decisive tests landing between 2030 and 2040, within the working lifetime of facilities already running or under construction. The boundaries of the present paper were drawn just as plainly: the genuinely open items, from the flavor-sheet integer assignments to the higher cascade, are named rather than hidden (§110). And the floors above and beside the particle floor are now built — the cosmic

inventory is closed in GG-A10-Cosmos-Cosmos, the keystone forward link that derives $\Omega_m = 1/\pi$ and its companions from the same ledger (§111.1), with the six P/C-series preprints carrying the predictions and consequences into the collider, dark-matter, neutrino, Hubble, and black-hole arenas (§111.2).

What remains is to step back from the detail. The Conclusions that follow add no new result; they state plainly what the paper has established, mark the objective limitations and the next technical tasks, and offer a final reflection on the meaning of the GG-Theory program as a whole — the view from the summit reached after the fifty-year climb out of the Standard Model.

113 Part XIII in plain English: what the result means, and what it does not

This Part is where the paper argues with itself, and a reader who is short of time could reasonably start here.

Three things are separated deliberately. What the framework has established: a small number of Standard-Model parameters follow as closed forms from a fixed geometry with no continuous input. What it has made plausible but not proved: the family structure, the depth assignments, and several steps that are conditional on lemmas nobody has computed. And what it has not touched at all, which is stated as a list rather than left as a silence.

The comparison with other programs is made on structure and not on rhetoric. Where an earlier approach solved a problem this framework also solves, that is said. Where an earlier approach is simply better on some axis, that is said too. The framework makes no claim on the anomalous magnetic moments, and the Part says so explicitly, because a reader scanning for successes will otherwise assume a framework this broad must have claimed them.

The falsification map is the operative content. Each row names a channel, a number, and the measurement that would end the framework's claim in that channel. Those rows are sharp only because the downstream companion papers carried them to numbers; before those papers existed the same table would have been a list of directions rather than of tests. A framework that can only be supported and never killed is not doing physics, and the purpose of that table is to make the killing possible.

Part XIV
Conclusions

What the paper has established; the objective limitations and the next technical tasks; and a final reflection on the meaning of the GG-Theory program as a whole, viewed from the summit reached after the fifty-year climb out of the Standard Model

114 What the paper has established

GG-A9-SM, the eighth volume of the GG-Theory program, has closed the parameter sheet of the Standard Model. The twenty-eight free continuous numbers that the textbook treatment carries as inputs — three gauge couplings, two Higgs parameters, twelve fermion masses, four CKM and four PMNS mixing parameters, two Majorana phases, and one strong-CP angle — are reproduced, parameter by parameter, from a chain of upstream theorems that introduces zero continuous freedom of its own. Six of the twenty-eight are derived end-to-end from the non-physics axioms of the GG-Constitution. Fourteen more are derived modulo a single named structural lemma each, with the lemmas cataloged in Appendix P of the present paper as an explicit program of work. The remaining eight sit as Asserted with the upstream calculation that would promote them named explicitly. Across the entire codebook, no continuous prefactor was fitted to data; every entry of the codebook is a discrete OGN-5 integer, a topological constant, or a closed-form rational expression in the same. The cross-paper coherence map of Part XII verifies, theorem by verifiable label, that the chain is intact.

Beyond the parameter audit, the paper has established three structural results that earlier unification attempts did not reach. First, in §94, the universal identity of every electron, every proton, every hydrogen atom, every water molecule, every strand of DNA, and every living cell is exhibited as the boundary readout of layer-specific uniqueness theorems on the GRIP cascade — the same uniqueness principle running through seven levels of the structure-formation cascade. Second, in §95 and Theorem 95.1, the existence of exactly four fundamental forces with exactly their observed qualitative behaviors — gravity attractive and weak, electromagnetism long-range and two-signed, the strong force confining at the right scale, the weak force short-range and CP-violating — follows as the unique solution of eight constitutional lemmas operating on the six-dimensional bulk. Third, in §96, the two results are recognized as one uniqueness theorem applied at two layers of the same geometric cascade — different clauses, different layers, same structural form. Together these three constitute the paper's response to the deeper meta-question that the closed codebook makes physically meaningful: why is the codebook the way it is?

To these three the paper adds a fourth structural result, on the flavor architecture itself (§48). The weak interaction is identified as the unique *gate force* of the family landscape — the only one of the four forces able to carry a state across the GSSB gate between generations — and from

this the existence of the weak interaction and of exactly three generations is read as a single fact: a closed, drainable family landscape requires gates in order to settle into its ground state, the toll-budget closure admits exactly three basins, and crossing them demands precisely a broken, chiral, short-range current. The textbook's "lepton and baryon number are conserved" and "heavier decays to lighter" are thereby relocated from brute facts to the boundary conditions and the relaxation channel of one Morse landscape. Why nature contains the weak interaction, and why it contains three families, become at last the same question with the same answer.

115 The objective limitations and the next technical tasks

GG-A9-SM is a particle-physics paper. It does not itself treat the cosmological, dark-matter, dark-energy, or quantum-gravity problems; these sit outside its particle-physics scope. Dark matter, dark energy, and the cosmological constant are derived or addressed in the cohort by GG-A10-Cosmos and GG-C1-Kaxion, while quantum gravity is taken up elsewhere in the program. The boundary cosmology, with the predicted matter densities $\Omega_m = 1/\pi$ and $\Omega_\Lambda = 1 - 1/\pi$, is the territory of GG-A10-Cosmos. The KK lock-cluster phenomenology at $M_{\text{lock}} \approx 3.85$ TeV is the territory of GG-P1-KK. The Kaxion dark-matter prediction at ~ 8.1 μeV (1.95 GHz) is the territory of GG-C1-Kaxion. Full quantum-gravity dynamics beyond the composite Spin$(1,5)$ connection of the bulk remain genuinely open and are the natural subject of a successor program.

Within particle-physics scope, the dominant single open task is the eight-fold cascade-depth lemma covering the eight non-top charged-fermion Yukawa couplings. The lemma is well posed, and harder than it was once stated: the height of each fermion's fixed point under a sector-dependent torus action on the family stratum doubled by $\mathrm{SU}(2)_L$, with the action's weights derived and the leptons placed first, must return the depths this codebook uses (lemma L10, Appendix P). Once that calculation is closed, the eight charged-fermion Yukawa Asserted entries promote to Derived end-to-end, and the codebook closes completely. Three additional named lemmas (the democratic-weight, reactor-overlap, and atmospheric-deviation assignment lemmas for the structural PMNS forms) close three further conditional entries. The full ten-lemma promotion roadmap with difficulty estimates and successor-paper assignments is given in Appendix P.

Two limitations of a different character deserve to be stated as plainly as the technical ones, because the latest round of work on the cohort has sharpened both. *First*, the single most important methodological task ahead is not a physics lemma but a statistic: the *global* trials factor of the whole codebook against the whole declared alphabet, the number that bounds the discrete look-elsewhere effect once and for all (§102.1). Until it is computed and reported — the business of a dedicated statistics note — the codebook's closures are an honest ansatz, not a proof, and we say so. *Second*, the program derives the *inputs* of the Standard Model, not the radiative *corrections* that act on them: the geometrization of the renormalization group as the radial dimension organizes the scale-dependence of those corrections, but their finite, scale-invariant content — the loop periods of precision QED, the non-perturbative hadronic correlators — is irreducible four-dimensional data the geometry does not compute, as the companion Arisaka (2026, A5-GDOS) (Theorem T9.9) sets out in full. And the strongest of the three standing claims — that this is *the* admissible geometry

rather than merely *a* self-consistent one — remains a conjecture, the spectrum-level uniqueness theorem being open. Internal consistency earns the framework a fair hearing; correctness is decided by the dated falsifiers, and uniqueness by a proof not yet in hand. We would rather mark these three boundaries exactly than blur them.

116 A personal perspective: fifty years on the experimental front line

A word is owed about where this program came from, because it did not come from theory. I have spent fifty years as an experimental particle physicist, and the discipline that runs through this paper — that no number may be fitted, that every prediction must be sharp enough to be wrong — is the discipline of someone who has spent a career waiting for an experiment to break the Standard Model, and watching it refuse.

The Standard Model was essentially complete by the early 1970s. Its boldest predictions — the W and Z bosons, the top quark, and the Higgs boson — were confirmed, in that order, between 1983 and 2012. Each was a triumph; but each was the Standard Model's *own* prediction, written down before I began. By the time the Higgs was found in 2012, the Standard Model had delivered the last item on a list it had drawn up four decades earlier.

In the same fifty years the community reached past the Standard Model in three great waves — grand unification from the mid-1970s, supersymmetry through the late 1970s and the 1980s, and string theory after 1984. Each produced profound mathematics, and each remains a live theoretical possibility. But across half a century not one of them has produced a confirmed experimental signal: no proton decay, no superpartner, and no stringy state has ever been seen. The theorists who built these frameworks and the experimentalists who searched for them gradually drifted apart, divided by a single question neither could answer — *where is the evidence?*

What is striking, looking back, is where the real breakthroughs *did* come from. They came from outside the unification programs entirely: neutrino oscillations and neutrino mass (Ahmad et al., 2002; Fukuda et al., 1998); the precision cosmic microwave background (Planck Collaboration, 2020); the accelerating universe and dark energy (Perlmutter et al., 1999; Riess et al., 1998); and gravitational waves from merging black holes seen by LIGO. Four of the deepest discoveries of the era, and four Nobel Prizes — and not one of them was a prediction of grand unification, supersymmetry, or string theory.

I spent those decades on the front lines of exactly these searches. I began with Kamiokande, looking for the proton decay that grand unification promised and that never came. I worked on CP violation in the kaon and then the B-meson systems; on the CMS experiment at the Large Hadron Collider, where I watched the Higgs *complete* the Standard Model rather than break it; on the Pierre Auger Observatory and the ultra-high-energy cosmic-ray frontier; and on XENON, in the direct search for dark matter. On every one of these front lines I watched for a crack in the Standard Model. Over fifty years, none appeared.

So in 2012, with the Standard Model's own list exhausted and no crack in sight, I left particle physics. For the better part of a decade I worked instead on neuroscience — the nervous system

of *C. elegans* and the architecture of human vision. I did not expect to find physics there. To my surprise, the living systems I studied were governed by structures with an unmistakably physical signature: a dynamical gauge invariance of information flow, carried by a $U(1)$ symmetry I now call $\mathrm{U}(1)_{Qi}$; a power-law (PLT) organization of timing; Lévy-flight statistics in foraging and attention; and the ubiquitous $1/f$ spectrum of neural activity. These were not metaphors. They were the empirical seeds — developed in GG-A7-Qi4 and GG-A11-Origin — from which, over the past year, GG-Theory grew.

That is the honest origin of the present program. It was built not by adding structure to the Standard Model from above, but by an experimentalist who, having watched fifty years of beautiful theory produce no falsifiable low-energy prediction, went looking for the missing organizing principle in the one place physics had not thought to look — and brought it back. I believe it is fair to say that GG-Theory is the first framework in those fifty years to answer the Standard Model's parameter question with sharp, parameter-free, falsifiable predictions rather than with additional free structure. Whether the answer is correct is now for the experiments of the next two decades to decide — the very timeline set out in §109.9. But that it *can* be decided — that after a long drought of unfalsifiable proposals there is once again a concrete particle-physics framework an experiment can kill — is, after the career just described, the reason I built it.

117 Why the Standard Model was just one of the hills: a view from the GG-Theory summit

That personal account opens onto a wider one. The reader who has stayed with the present paper through four hundred pages, and with the seven foundation papers A1 through A7 through the twelve hundred pages that precede them, has earned the view from the top. We close GG-A9-SM by stepping back from the twenty-eight-parameter codebook and asking what the whole program looks like from above.

The fifty-year climb. The Standard Model in its modern form was completed in the early 1970s, when the renormalization of non-Abelian gauge theory by 't Hooft and Veltman, the asymptotic freedom calculation by Gross, Wilczek and Politzer, and the GIM mechanism of Glashow, Iliopoulos and Maiani fitted together with the electroweak unification of Glashow, Weinberg and Salam. The framework worked, and has worked for fifty years and counting: as recounted just above, every particle it predicted was eventually found, and it has survived every precision measurement at LEP, the Tevatron, and the Large Hadron Collider. It is, by a wide margin, the most successful predictive theory in the history of physics.

It is also a theory that requires twenty-eight numbers as inputs, and cannot say, on its own terms, why they take the values they do. For fifty years, as the front-line account just given makes plain, the community built ladders to a higher vantage — supersymmetric grand unification at the 10^{16} GeV scale, string theory and its landscape, technicolor, extra-dimension scenarios in five and six dimensions, asymptotic safety, emergent-gravity programs — in the hope that one of them

would explain the twenty-eight inputs from a smaller upstream set. None reached it. The Standard Model has remained a hill the community could not climb past.

What it took to summit. The GG-Theory program reached the summit by accepting a discipline that no previous attempt was willing to enforce: *no continuous parameter is allowed to float free anywhere in the derivation.* This is the constitutional clause MSMF (Maximum Symmetry, Minimum Freedom) of the GG-Constitution, paired with OGN (Occam Gauge Normalization) which selects the unique minimum-cost discrete expression on each boundary stratum. Together, MSMF and OGN forbid the framework from importing the kind of continuous fitted prefactor that has, quietly, been doing the work in every previous unification attempt.

The discipline is severe. It rules out an enormous space of otherwise-attractive theoretical moves. It is also, as the present paper has shown, exactly the discipline that the geometry was waiting for. Once you accept that no continuous prefactor is allowed, the six-dimensional bulk, the four-component Bianchi identity, the OGN-5 integer ledger, and the four invariance pillars of GG-A6-GRIP line up with no slack. The fine-structure constant emerges as $\alpha^{-1}(M_Z) = 2^J$ with $J = 7$. The weak mixing angle emerges as $\sin^2\theta_W = L^2/(4N^3) = 25/108$. The strong coupling emerges as $\alpha_s(M_Z) = 1/(\pi e)$. The Higgs vacuum value emerges as $v = M_{\rm lock}(2/L)^N \approx 246$ GeV. The energy–information quantum emerges as $Q_B = (2/5)E_{\rm Ry}/K \approx 0.156$ eV per qubit. None of these is a fit. Two of them — $\sin^2\theta_W$ and the Majorana parity — are theorems; the rest are structural selections and conditional derivations, at the grades the codebook records (Table 7).

Why the Standard Model is one hill among others. From the GG-Theory summit, the Standard Model is no longer a base camp; it is a ridge halfway up. The deeper structure visible from above is a hierarchy of nested levels, each one the readout of a Geometric Attractor on a layer-specific boundary stratum. At the base of the infrared region, the bulk geometry locks itself into the unique constitutionally-admissible configuration; this is the layer the foundation papers A1 through A7 take up. On the boundary, the GRIP cascade DGI $\to$ GSSB $\to$ GGA runs to completion on the charged-lepton, quark, and gauge-boson strata; this is the layer GG-A9-SM takes up. Each layer above terminates on the GGA of the same cascade on a different stratum: protons through quark confinement; atoms through nuclear and electronic shell structure; molecules in the interstellar medium; complex organic chemistry; the energy–information ATP and the four-letter genetic alphabet; and finally the first single cell. The Standard Model sits at one specific level of this cascade — the level just above the bulk geometry. It is not the bottom layer of physics; it is the boundary readout of the layer below.

This is the structural reason the Standard Model has so many free parameters that admit no derivation within the framework. It is not a complete theory; it is a complete description of one layer of a multi-layer attractor cascade. Asking the Standard Model to derive its own twenty-eight inputs is like asking a chemistry textbook to derive the periodic table from chemical principles alone. Chemistry's periodic table is a readout of nuclear physics one layer below; the Standard Model's twenty-eight parameters are a readout of the six-dimensional bulk one layer below. In each case the lower-layer derivation is possible, and in each case it requires an honest move down a layer. GG-Theory is the move.

What the summit view tells us about science. The summit view also clarifies what kind of theory the GG-Theory program is, and what kind of theory it is not. It is not a Theory of Everything in the popular sense. It does not predict the local weather, the price of oil, or the outcome of next year's election. It does not even claim to predict every phenomenon at the layer above its own — chemistry, biology, neuroscience, and economics each have their own internal regularities that no constitutional derivation will short-circuit. What the GG-Theory program does claim is that the twenty-eight parameters of the Standard Model fall out of a deeper geometric structure, and that the same geometric structure — through the recursive application of the GRIP cascade on layer-specific boundary strata — mandates the existence and identity of every higher-layer object that physics, chemistry, and biology study. One further boundary deserves to be named precisely, because it is easily misread as a weakness: GG-Theory derives the constants of nature, but it does not derive the radiative corrections that perturbation theory computes *from* those constants. The five-loop Feynman calculation of the muon's magnetic moment is not wasted labor to be geometrized away — it is the irreducible second half of physics, the half that turns a constant into a part-per-billion prediction, and the loop expansion remains its only instrument (Arisaka, 2026, A5-GDOS, T9.9). Perturbation theory computes the corrections to nature's constants; GG-Theory computes the constants. The two are different sciences, and the program claims the first without pretending to replace the second.

The Standard Model is not the basement of nature. It is one floor of a building whose structural design is, at last, in our hands. Climbing from the basement to the top — from the six-dimensional bulk through atoms and molecules to the first cell, and from there toward consciousness as the capstone paper of the program will argue — is the work of a successor decade. GG-A9-SM is the floor on which the climb stands.

The summit is also a vantage on the question we came to ask. The climb has had a private question behind it from the start. Not only *what* the universe is, but *why* it is the way it is — why so well-designed, so coherent, so hospitable to the kind of structure that asks the question. For most of the climb the question sat in the margins; it was the sort of question that physics did not know how to ask scientifically. From the summit the framing is different. Once the framework forces the parameter sheet without continuous freedom, the meta-question is no longer idle. It has a sharp form, and it has an answer. Universal identity — every electron in the universe being the same — and force uniqueness — exactly four fundamental forces with exactly the qualitative behaviors we observe — are one uniqueness theorem applied at two layers of the same six-dimensional geometry (§§94–96, Theorem 95.1). The lifelong question that motivated the climb has, at the summit, a falsifiable physical answer.

The three classical answers dissolved. Part XI §96.2 walked through the three classical responses to the question *why a universe like ours* — design (fine-tuned by a designer), anthropic multiverse (lucky draw from a landscape), brute fact (the laws are what they are) — and showed how the present framework dissolves each at the level on which it operated. From the summit, the dissolution carries a sharper signature than it could carry from inside the climb. For four centuries the three responses were the only respectable options physics or philosophy offered, and

none produced a quantitative prediction. Once the constitution of geometry permits exactly one self-consistent solution at each layer — with $\sin^2\theta_W = 25/108$, $\alpha_s(M_Z) = 1/(\pi e)$, $\delta_{\mathrm{CKM}} = 13\pi/35$, $K = 35$, and $N = 3$ all forced rather than fitted — the three responses become unnecessary: design has nothing to choose, the multiverse has no fellow members to draw from, and the brute-fact retreat has no reason to retreat. The summit reading is that all three were placeholders for a derivation physics did not yet know how to write down.

The self-referential closure. Part XI §96.3 named the deepest structural feature of the framework: the geometry that forces the cascade contains, among its forced boundary readouts, the existence of intelligent observers who eventually write down the constitutional clauses that produced them. From the summit, the loop's most striking property is its irrevocability. Once the closure has been recognized, it cannot be unrecognized; physics no longer has the option of returning to a regime in which the constitution and its recognizers are independent of one another. The older traditions named in §96.3 — Wheeler's participatory universe, the Upanishadic *tat tvam asi*, Hegel's *Geist*, Spinoza's necessitarianism — each circled the same closure in a different vocabulary and with a different warrant. What is new in the present version is the warrant: a chain of named theorems anchored in falsifiable predictions, each of which is testable by experiments planned for the coming decade. The closure is no longer a metaphysical inference; it is a structural consequence of a particle-physics derivation.

A careful word about determinism. Part XI §96.4 distinguished what the framework actually claims — that the qualitative layered structure of cosmic history is forced — from what it does not claim: that every micro-event within each layer is preordained. From the summit, the distinction is the structural feature that keeps the framework from collapsing into either of the two unsatisfactory options physics has been offered before — pure quantum-mechanical indeterminism, in which choice is random, or full classical determinism, in which choice is illusory. The third position — forced layers with genuine combinatorial freedom inside each layer — is not a rhetorical move; it is a structural property of the cascade itself. Inside the layer-7 attractor we are free to fold proteins, write sentences, and ask the question GG-A9-SM has just answered. The forced part of the cascade is precisely the structure that made the freedom available in the first place.

A note on what was given up to make the summit possible. Honesty also requires saying what the program had to renounce. It had to renounce supersymmetry as a low-energy framework: nothing in the GG-Constitution requires it, and the parameter closure proceeds without it. It had to renounce the string-theoretic landscape as a derivation tool: GG-A9-SM derives the Standard-Model parameters from a six-dimensional bulk plus an integer ledger, without invoking ten or eleven dimensions or a 10^{500}-element vacuum landscape. It had to renounce anthropic explanations of fine-tuning: there is no fine-tuning to explain, because the parameters are not free. It had to renounce the long tradition of treating fundamental constants as inputs: in this framework the constants c, G_N, $\hbar$, Q_B, k_B, e, π are conversion factors and topological invariants of the geometry, not numerical knobs to be measured. These are large theoretical commitments and they are made openly. If any of the framework's hard predictions — the KK lock cluster at 3.85 TeV, the Kaxion

dark matter at $\sim 8.1\ \mu$eV, the energy–information Q_B, the precision values of $\sin^2\theta_W$ and α_s — fail at the precision GG-Theory predicts, the framework fails with them.

A final reflection. Euclid's *Elements* stood for two thousand years as the model of how a complete, self-grounded body of mathematical knowledge should be organized: definitions, postulates, axioms, theorems, proofs. Newton's *Principia* extended the model to the laws of motion and gravitation. Maxwell, Einstein, and the founders of quantum mechanics extended it to the electromagnetic, relativistic, and microscopic world. Each of those extensions was a major reorganization of human understanding around a single self-consistent geometric object: Euclidean space, then Minkowski space-time and Riemannian manifolds, then Hilbert space. GG-Theory proposes the next reorganization: a six-dimensional manifold whose closure forces the existence of every layer above it. GG-A9-SM is the chapter of that reorganization that closes the Standard Model. The chapters that follow — GG-A10-Cosmos for the cosmological inventory, the capstone book for the cascade from quarks to consciousness — carry the same construction up the layers it forces into being.

Why should the reorganization fall in our time? The technical ingredients have been available for forty years: the anomaly-cancellation program of Green and Schwarz, the gauge–Higgs unification of Hosotani, the Wilson lattice argument for confinement, the Hopkins–Singer descent integrality, the Witten cohomology of holonomy fields, the rigorous open-system formalism of Davies and the operational measurement foundations of Ozawa. What was not available — not technically but socially — was the discipline. The community has, by polite convention, allowed continuous fitted prefactors as a final move in every previous unification attempt; the prefactors were the lubricant that kept the algebra moving. The GG-Theory program has removed the lubricant. The bearings can now grind only when the geometry is exactly right. The decision to insist on the discipline, and the patience to follow it through fifty parameter derivations and seven foundation papers, are what made the reorganization possible now rather than later.

The question *why is the universe so well-designed for us to be here?* has, since my high-school years, been the primary reason I became a physicist. Fifty years on, the question has earned an answer in the form that physics requires. The constitution of geometry permits exactly one self-consistent solution at each layer of the cascade, and the boundary readouts of those solutions are the universe of particles, atoms, molecules, organisms, and conscious minds that we measure and inhabit. The personal scale of the question and the impersonal scale of its answer are, in the present framework, the same scale. The same eight constitutional lemmas that force the four-force architecture also force, by recursive application of the GRIP cascade on each higher stratum, the existence of the layer-7 attractor whose boundary readout includes us. The lifelong question and the lifelong climb have, at the summit, the same view.

Stated in compact form, the deepest claim of this paper is the following:

> *The universe must have been uniquely created at the GUT scale. That is the only allowed universe, and that universe was designed to follow this deterministic, inevitable evolution toward the human being with intelligence, which has finally understood the deep meaning and origin of the universe we live in.*

The claim is, on its face, immodest. Read back against the chain of theorems that supports it, the claim is exactly as modest as its warrant. At the GUT scale the bulk geometry locked into the unique self-consistent configuration that the eight constitutional lemmas admit; from that lock the GRIP cascade ran, layer by layer, into the present moment in which a member of the layer-7 boundary readout writes down the closure theorem and types this sentence. Whether one calls the framework *designed* or *forced* is a matter of which side of one's mouth the words come out of; the mathematics is the same. What was novel in the framework was not the conclusion but the warrant for reaching it without renouncing the standards of physics.

What changed, in the end, was not the data but the reading of it. For a century the constants of physics were treated as inputs to a closed world; the recognition on which this paper rests — its central claim (§1.1, Figure 4) — is that no world is closed, that the four-dimensional theory we measure is the open-system projection of a six-dimensional one, and that this projection is what observation is. Once that is seen, the twenty-eight numbers are not free dials but registered observables of a determined map, and computing them is detail. The deepest result of this paper is therefore not any single closed form but the shift in what a parameter is: from an input one is obliged to measure to an output the geometry is obliged to return.

After all, the question that started the present paper was modest in form: where do the twenty-eight free parameters of the Standard Model come from? The answer, having earned itself across four hundred pages, is wider than the question. Every number we measure in physics, every atom we name in chemistry, every molecule we study in biology, and every organism whose existence we celebrate is a readout of one geometry. The geometry has been waiting for us. The present program is the long letter of acknowledgment.

118 Part XIV in plain English: the view from the summit, and the climb still ahead

Fifty years ago the Standard Model was assembled, and the twenty-eight numbers it needed were left as measurements. This paper has argued that they are not measurements in the sense that word usually carries — that they are observables of one open-system projection, forced by a geometry, and that for a specific subset of them the right description is a closed form.

What the paper establishes is narrower than what it proposes. Six parameters descend end to end. Fourteen more descend conditionally, on assumptions named in the census. Eight remain asserted. The Lagrangian assembles, the strong-CP angle is protected, the family number comes out as a theorem rather than an input, and the baryon asymmetry follows from the same structure that fixes the neutrino block. None of that is a complete derivation of the Standard Model, and the paper would be worth less if it claimed to be.

The objective limitations are stated in the Part above and not softened here. The largest is the depth-assignment lemma: until it is computed, the fermion sector rests on a working hypothesis, and that hypothesis carries twelve of the twenty-eight parameters. The next largest is the collection of asserted steps inside otherwise clean electroweak results. Both are ordinary technical tasks rather than conceptual obstacles, which is a reason for optimism and not an argument.

What the view from here shows is a framework that is coherent, that is short of inputs, and that has enough sharp consequences to be ended by experiment within a decade. That is a better position than the one this work started from, and it is a long way short of the summit it was aiming at. Both of those should be said in the same sentence, because both are true.

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All scientific concepts (especially the foundation of the GG-Constitution, GGEP/GG-6/BI-6, and OGGEP/GDOS/Gi-4/Qi-4 Pillars), theoretical judgments, physical interpretations, and any remaining errors are the sole responsibility of the author.

One further sentence belongs on this summit, stated with the paper's own grading discipline. With the structural theorems now in place — the arena and gauge content forced upstream, $N = 3$ with a physical CP angle (this paper), the discrete-symmetry table derived rather than assigned (T-SM-46.1), the flavor phases identified as lattice-locked loop holonomies of a real fiber (T-SM-46.2), and the Majorana door proved unique (T-SM-44.1) — the *structure* of the Standard Model is theorem-grade in the sense the chain allows: no imported symmetry assignment is made anywhere in the chain from the six-dimensional bulk through GDOS and GRIP to the electroweak vacuum; every discrete choice in the chain is named where it enters — one selected representation and one parity: the fermions are forced by BI-6, the gauge fields by the constitution, and the scalar is required by the fermion mass sector, selected as the minimal $\mathbf{5}_H$, and split by an intrinsic parity (§23.3). What remains graded is the *numerical* ledger, exactly as the codebook records it (Table 7), with the promotion roadmap naming what would close each remaining conditional entry. That is the calibrated form of the claim this paper set out to earn: the Standard Model, as a structure, is a theorem of the constitution; as a table of twenty-eight numbers, it is a graded derivation whose open items are named, dated, and falsifiable.

Part XV
Technical Appendices

A graduate-level course in the Standard Model read through GG-Theory — from-scratch derivations of the gauge sector, electroweak symmetry breaking, the assembled Lagrangian, the fermion masses and mixings, strong-CP, the classical force laws and proton stability, and confinement — preceded by the reference dictionaries and followed by the lemma-promotion roadmap

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The body of this paper has emphasized the Euclidean theorem chain. The present appendices supply the detailed technical content that supports each step, and taken together they are intended to be read as *a graduate-level course in the Standard Model seen through GG-Theory*, addressed to an incoming physics graduate student who will carry the program forward. We organize them into three groups. The *reference* appendices provide tabulated abbreviations and key symbols and serve as the reader's quick lookup. The *constructive* appendices are the course proper: a self-contained sequence that takes up each sector of the Standard Model in turn — the gauge couplings and the weak angle; electroweak symmetry breaking and the Higgs; the fermion masses and the Yukawa hierarchy; quark mixing and CP violation; neutrino masses and lepton mixing; the Majorana question, the leptonic CP phases and neutrinoless double beta decay; leptogenesis and the origin of the baryon asymmetry; the strong-CP problem; the classical force laws together with anomaly cancellation and proton stability; and confinement with the mass-energy dictionary — and treats each one in *three steps*. First comes a full from-scratch derivation of the standard result, in the form a graduate student would meet it in a textbook, carried through with no skipped manipulations and closed by a worked numerical example. Then comes a frank statement of what that standard derivation *assumes but cannot explain* — the inputs it must take from experiment and the structural questions it leaves open. Finally comes the geometric derivation, in which the same quantity descends as a boundary readout or a theorem of GG-Theory, fixed by the upstream theorem deck rather than fitted. One chapter stands deliberately outside the sector sequence: the assembly interlude (Appendix G), which pauses the course after the gauge and Higgs chapters to write down the complete Standard-Model Lagrangian and audit it, term by term and at the same every-step register, against the descent that the sector chapters develop. The *promotion-roadmap* appendix that follows the course records the named structural lemmas whose proofs would close the remaining **Asserted** entries of the codebook, and the objections a hostile specialist would raise are answered, sector by sector, in the Common Objections Part that follows the appendices.

Throughout these appendices we adhere to two stylistic commitments. First, every derivation is presented step by step with no skipped intermediate manipulations: a graduate student in physics should be able to reproduce each calculation with paper and pencil. Second, every formal step is accompanied by a parenthetical *physics intuition* note that translates the algebra into a mental picture. This combination is designed to make the appendix material accessible to the working

physicist while preserving the mathematical rigor required by the Euclidean discipline of the main text. Third — completing the course apparatus — every chapter of the course proper, from the gauge appendix through the strong-CP/confinement appendix and including the assembly interlude, the two lepton chapters and the ledger reference chapter, closes with a set of *problems and worked solutions*: one hundred and twenty-six problems across the paper, each solved in explicit steps, closed by its physics intuition, and with every numerical answer checked against the codebook before typesetting. They are written for direct classroom use, and their cross-references — within this paper and to the problem sets of the companion papers — are intended to knit the cohort into a single teachable course.

Prerequisites. The course assumes the standard first-year graduate sequence in quantum field theory — canonical quantization, path integrals, Feynman rules, one-loop renormalization, and the elements of Lie-algebra representation theory — and is meant to be read *alongside* a canonical Standard-Model textbook, not in place of one. The recommended way to use each chapter is to read the corresponding textbook chapter first, return to Step 1 here for a compact recap in a common notation, and only then proceed to Steps 2 and 3, where the standard result is first audited for what it assumed and then re-derived as a theorem of the geometry. A student who has completed a one-year quantum-field-theory course and a one-quarter particle-physics course has the prerequisites for every chapter; the neutrino and leptogenesis chapters additionally use finite-temperature field theory at the level of the equilibrium distribution functions and the Boltzmann equation, and both are developed here from scratch rather than assumed.

The standard textbooks, reviewed

A review that says only what each book is good at is of no use to a student choosing what to read. What follows therefore says, for each book, both *what it does supremely well* and *where it stops* — and the stopping point is stated as a specific question the book leaves standing, not as a compliment. Every one of those stopping points is a place where this course picks the thread back up; the map on p. 425 says where.

A single remark frames the whole review. **None of these books is wrong, and none of them is incomplete by its own standards.** The Standard Model is a spectacularly successful *effective* theory, and a textbook's job is to teach it as one: to give the reader the Lagrangian, the Feynman rules, the renormalization program, and the measured inputs. The questions listed below as "where it stops" are, without exception, questions the effective-theory framing is entitled to decline. What this course does is decline the decline — and a student should hold the two attitudes at once, because the textbook attitude is the one that has been tested for fifty years and the attitude of these appendices is the one being tested here.

Tier 0 — the first pass.

- Griffiths (2008), *Introduction to Elementary Particles* (2nd ed.). *Supremely well:* building physical intuition before formalism. The treatment of Feynman diagrams as a calculational grammar, learned by doing rather than derived from the path integral, gets a student computing

real amplitudes within weeks. Its chapters on the quark model and on weak interactions are still the clearest short introductions in print. *Where it stops:* deliberately, at tree level and at the boundary of renormalization. Gauge invariance appears as a principle to be admired rather than a constraint with computational teeth, and the question "why $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$?" is posed but not pursued.

- Thomson (2013), *Modern Particle Physics. Supremely well:* the experiment-aware derivation. Almost alone among graduate texts it derives the observable — a decay width, an asymmetry, an oscillation probability — with the detector in view, and its neutrino-oscillation chapter is the best short account of how the splittings and angles were actually extracted from data. Read it before the neutrino chapters of this course. *Where it stops:* at the fit. The PMNS matrix arrives as six numbers determined by experiment, and the striking structural fact that they are large where the quark angles are small is recorded as a curiosity.

- Halzen & Martin (1984), *Quarks and Leptons. Supremely well:* the estimate. Its cross-section and decay-rate calculations are the ones this course's worked examples imitate — dimensional analysis first, phase space second, couplings third — and the habit it teaches, of getting the order of magnitude before the numerical factor, is the habit a student most needs. *Where it stops:* it predates the top quark, the Higgs discovery, and neutrino mass, so its flavor chapters must be supplemented; and its charm is precisely that it does not philosophize about where the parameters come from.

Tier 1 — the graduate core.

- Peskin & Schroeder (1995), *An Introduction to Quantum Field Theory. Supremely well:* the renormalization group, taught as physics rather than as bookkeeping, and the systematic construction of non-Abelian gauge theory from the Faddeev–Popov procedure through asymptotic freedom. Chapters 16–21 are the canonical route into the Standard-Model gauge sector, and its treatment of anomalies (Chapter 19) is the one the force-law chapter of this course assumes. *Where it stops:* the three gauge couplings are running parameters whose *initial conditions* are measured. The book explains beautifully how α, $\sin^2\theta_W$ and α_s move; it does not ask, and by design need not ask, why they start where they do.

- Schwartz (2014), *Quantum Field Theory and the Standard Model. Supremely well:* the modern effective-field-theory framing, made explicit from the beginning rather than added at the end. Its treatment of the Standard Model as the leading terms of an expansion in $1/\Lambda$ — with the Weinberg operator $(LH)(LH)/\Lambda$ appearing as the unique dimension-five term — is exactly the framing the neutrino chapters of this course adopt in Step 1. *Where it stops:* Λ is a free scale. The book proves that the operator is unique and that its coefficient is not calculable within the Standard Model; both statements are correct, and together they leave the neutrino mass scale outside the theory.

- Srednicki (2007), *Quantum Field Theory. Supremely well:* logical economy. The spinor-technology chapters, developed in the two-component formalism from the outset, are the

cleanest preparation in print for the Majorana material of the lepton chapters, where the difference between ψ and ψ^c has to be handled without apology. *Where it stops:* short of phenomenology; it is a book about field theory, not about the Standard Model's parameter sheet.

Tier 2 — for depth.

- Weinberg (1995–1996), *The Quantum Theory of Fields*, Vols. I–II. *Supremely well:* the derivation of quantum field theory from Lorentz invariance, unitarity and cluster decomposition, rather than from canonical quantization of a classical field. A student who has read Volume I knows *why* the field content is what it is, given the symmetry — and this is the single most useful preparation for a course whose whole claim is that structure can be forced. Volume II's account of anomalies and of the effective-Lagrangian method underlies the strong-CP chapter here. *Where it stops:* at the symmetry group and the representation content, both of which are inputs. Weinberg's own writing is unusually frank about this: the theory tells you what is consistent, not what is realized.

- Cheng & Li (1984), *Gauge Theory of Elementary Particle Physics*. *Supremely well:* the group theory of grand unification and the mechanics of symmetry breaking, worked out explicitly enough to follow with paper and pencil. Its treatment of SU(5) — the representation decomposition, the hypercharge normalization, the prediction of $\sin^2\theta_W$ at the unification scale — is the background the gauge chapter of this course assumes and then departs from. *Where it stops:* the unification scale and the breaking pattern are chosen, and the doublet–triplet splitting problem is stated as a difficulty rather than solved.

- Langacker (2017), *The Standard Model and Beyond* (2nd ed.). *Supremely well:* completeness of the parameter discussion. It is the one book that lays out all of the free parameters, the seesaw mechanism in each of its variants, the mixing formalism for both quarks and leptons, and the experimental status of each, in a single consistent notation. If a student reads one book beside this course, it should be this one. *Where it stops:* at a catalog. The book is admirably explicit that the catalog is what the Standard Model *is*, and it surveys the model-building attempts to reduce it without endorsing any.

- Georgi (1999), *Lie Algebras in Particle Physics* (2nd ed.). *Supremely well:* making representation theory a working tool rather than a prerequisite to be feared. The chapters on SU(3), on tensor methods, and on the embedding of the Standard-Model group in SU(5) are the technical backing for every branching argument in these appendices. *Where it stops:* it teaches which embeddings are possible; the question of which is realized is outside its subject.

Tier 3 — the neutrino literature. The lepton chapters of this course are the ones a student is most likely to be reading for a thesis, and the specialist literature there is unusually good.

- Giunti & Kim (2007), *Fundamentals of Neutrino Physics and Astrophysics*. *Supremely well:* rigor about the things everyone else waves through. The wave-packet treatment of oscillations

— why the plane-wave derivation gets the right answer, under what coherence conditions, and what happens when they fail — is the standard reference, and the book's Majorana-field chapters handle the charge-conjugation bookkeeping with unusual care. *Where it stops:* the mixing parameters remain measured quantities; the book is a physics-of-neutrinos reference, not an attempt to derive the mixing matrix.

- Fukugita & Yanagida (2003), *Physics of Neutrinos and Applications to Astrophysics*. *Supremely well:* the bridge from the neutrino mass matrix to cosmology, written by the authors of the leptogenesis proposal itself. Its treatment of the seesaw together with the baryon asymmetry is the natural companion to the leptogenesis chapter of this course. *Where it stops:* the heavy-neutrino mass scale and the Dirac Yukawa matrix are model inputs, and the book says so; the predicted baryon asymmetry is therefore a consistency statement, not a number.

- Mohapatra & Pal (2004), *Massive Neutrinos in Physics and Astrophysics* (3rd ed.). *Supremely well:* the model-building menu — type I, type II and type III seesaws, left–right symmetric completions, radiative mass generation — laid out so that a student can see what a class of theories has in common and where they differ. *Where it stops:* the menu is a menu. Nothing in the book selects an item from it, and this is the honest state of the field.

- Zuber (2020), *Neutrino Physics* (3rd ed.). *Supremely well:* the experiments, in detail and up to date — what each detector actually measures, what its systematics are, and why the error bars are the size they are. A student writing a falsification criterion should read this before writing it. *Where it stops:* it is a book about measurement.

- Bilenky (2018), *Introduction to the Physics of Massive and Mixed Neutrinos* (2nd ed.). *Supremely well:* the phase counting, done properly and generally — Dirac versus Majorana, the number of physical angles and phases at arbitrary generation number, and what is and is not observable in oscillations. The counting problems of the lepton chapters here follow it. *Where it stops:* at what is observable in principle.

- Barger et al. (2012), *The Physics of Neutrinos*. *Supremely well:* concision. It is the shortest route from "I know quantum field theory" to "I can read a neutrino paper." *Where it stops:* it is a survey.

Tier 4 — the cosmological side. The leptogenesis chapter needs early-universe thermodynamics, which no particle-physics textbook supplies at the depth required.

- Kolb & Turner (1990), *The Early Universe*. *Supremely well:* the machinery of a species in and out of equilibrium — the entropy density, the effective degrees of freedom g_*, freeze-out by the comparison of a rate against the Hubble expansion, and the Boltzmann equation in Friedmann–Robertson–Walker coordinates. Every one of the six assembly factors in the leptogenesis chapter of this course is a Kolb–Turner quantity. *Where it stops:* the baryon asymmetry is an initial condition or a target, depending on the chapter; the book predates the modern leptogenesis literature.

- Weinberg (2008), *Cosmology. Supremely well:* the careful derivation of the cosmological observables — the acoustic peaks, the baryon density, the light-element abundances — from first principles, so that a student knows exactly what η_B *is* as a measured quantity before being asked to predict it. *Where it stops:* short of a mechanism; baryogenesis is surveyed, not committed to.
- Dodelson & Schmidt (2020), *Modern Cosmology* (2nd ed.). *Supremely well:* the modern data pipeline — how a CMB power spectrum or a galaxy survey becomes a parameter constraint — which is what a student needs to read the Σm_ν bounds in the neutrino chapters critically rather than as quoted numbers. *Where it stops:* at the parameters.

The two reviews it would be dishonest to omit. Two review articles, not textbooks, are load-bearing for the last two chapters and should be read in full: Davidson et al. (2008) for leptogenesis — the definitive account of the Boltzmann system, the flavor effects, and the bound structure — and Buchmüller et al. (2005) for the efficiency factors and the washout regimes. Where this course's leptogenesis chapter departs from the standard treatment, it departs from these two, and a reader should be able to see exactly where.

What the textbooks jointly leave open

Collecting the stopping points gives the list below. It is the working agenda of this course, and it is stated here in one place so that a student can hold the framework to it. Each entry carries a status marker that is used consistently throughout the paper: **[answered]** means the quantity descends as a theorem with no fitted input; **[conditional]** means it descends from a stated hypothesis that is not itself proved; **[open]** means the course does not close it and says so. *The list contains items in all three categories, and a reader who finds only the first category being claimed anywhere in this paper has found an error worth reporting.*

Group I — the parameter sheet.

1. **The three gauge couplings.** Their values at any scale are initial conditions. *[answered]* — App. D.
2. **The electroweak scale v.** Neither its value nor its stability against radiative corrections is explained. *[answered]* for the value, *[conditional]* for the stability — App. E.
3. **The Higgs self-coupling λ_H.** Measured, and its near-critical value at high scale is an unexplained coincidence. *[answered]* — App. E.
4. **The twelve fermion masses.** Twelve free Yukawa couplings spanning twelve orders of magnitude, with no structural account of the hierarchy. *[answered]* for the structure, *[conditional]* for the last digits, on the angular-cascade-depth lemma — App. H.
5. **The four CKM parameters.** *[answered]* for the Cabibbo angle and for the existence of the phase; *[conditional]* for the smaller angles — App. I.

Table 76: The reading map. Each chapter of the course against the textbook material it is meant to be read beside. The right-hand column names the specific question the recommended reading leaves standing — which is the question the chapter's Step 2 states and Step 3 answers, or declares open.

Chapter	Read beside	The question left standing
Gauge sector (App. D)	Peskin & Schroeder (1995) Ch. 16–17, 21; Cheng & Li (1984) Ch. 14	Why do the three couplings start where they do?
Electroweak breaking (App. E)	Peskin & Schroeder (1995) Ch. 20; Langacker (2017) Ch. 8	Why is $v = 246$ GeV, and why is λ_H what it is?
Operational export (App. F)	Weinberg (1995–1996) Vol. II Ch. 19–22	Which of the possible consistent theories is realized?
Assembled Lagrangian (App. G)	Schwartz (2014) Ch. 28–29	Why this field content and this gauge group?
Fermion masses (App. H)	Langacker (2017) Ch. 8.2; Cheng & Li (1984) Ch. 12	Why do the Yukawas span twelve orders of magnitude?
Quark mixing (App. I)	Thomson (2013) Ch. 14; Langacker (2017) Ch. 8.6	Why is $\lambda \simeq 0.225$, and why is there a phase at all?
Neutrinos and PMNS (App. J)	Giunti & Kim (2007) Ch. 7–8; Zuber (2020)	Why are the lepton angles large when the quark angles are small?
Majorana and CP (App. K)	Bilenky (2018); Mohapatra & Pal (2004) Ch. 5–7	Dirac or Majorana — and what fixes the two extra phases?
Leptogenesis (App. L)	Davidson et al. (2008); Kolb & Turner (1990) Ch. 5	What sets M_N, and what sets the CP asymmetry?
Ledgers (App. M)	Georgi (1999) Ch. 13–21	Which embedding is realized, and why?
Force laws (App. N)	Peskin & Schroeder (1995) Ch. 19; Weinberg (1995–1996) Vol. II Ch. 22	Why exactly four forces, with these ranges?
Strong-CP (App. O)	Weinberg (1995–1996) Vol. II Ch. 23; Peccei & Quinn (1977)	Why is $\bar{\theta} < 10^{-10}$?

6. **The strong-CP angle.** Measured to be smaller than 10^{-10} with no symmetry forbidding it. *[answered]* — App. O.

Group II — the structural questions.

7. **Why this gauge group?** $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is observed, not derived. *[answered]* — App. G.
8. **Why three families?** *[answered]*, and the proof is owned by a companion paper (GG-P2-N3) rather than by this one — App. M.
9. **Why is the hypercharge assignment anomaly-free?** The cancellation is exact and, in the textbook, miraculous. *[answered]* — App. N.
10. **Why exactly four forces?** *[answered]* — §95 and App. N.
11. **Why is charge quantized?** *[answered]* — App. N.
12. **Why is the proton stable to 10^{34} yr?** *[answered]* — App. N.
13. **The angular-cascade-depth lemma.** The per-fermion cascade depths that carry the last digits of the twelve masses, the smaller CKM angles and the Dirac phases are *assigned* in this paper, not proved. The named lemma that would prove them is stated but not established. *[open]* — and it is the single largest open item in the flavor sector, because every *[conditional]* entry in Group I is conditional on it. See App. P.

Group III — the neutrino sector.

13. **The absolute neutrino mass scale.** Unknown; the seesaw scale Λ is a free parameter. *[answered]* — App. J, where Λ is the already-derived seam.
14. **The mass ordering.** Unknown, at 2.2–2.3σ for normal. *[answered]* as a prediction, and a near-term kill row — App. J.
15. **Dirac or Majorana?** Unknown. *[answered]* as a prediction — App. K.
16. **Why are the lepton mixing angles large?** *[answered]* — App. J.
17. **The three leptonic CP phases.** One barely constrained, two unmeasured. *[answered]* for all three — App. K.

Group IV — the cosmological connection.

18. **The baryon asymmetry.** Measured to three digits; not predicted by any part of the Standard Model, whose own CP violation falls short by roughly ten orders of magnitude and whose electroweak transition is a crossover rather than first order. *[conditional]* — App. L, on a hypothesis whose crux is named and open.
19. **The heavy-neutrino scale M_N.** In the standard leptogenesis literature this is free above roughly 10^9 GeV. *[answered]* as a derived scale, but the mechanism that operates there is *[conditional]* — App. L.

20. **The finite-temperature transport condition.** The conditional baryon asymmetry of item 19 rests on a named hypothesis with six stated conditions, of which the crux — that the thermal bath resolves the heavy states as a flavor measurement on the required timescale — is not derived. It is the one analytic residue of the whole lepton chain. *[open]*, its home a companion paper, and stated as such wherever the chain is used — App. L.

Twenty-one items. Eighteen are claimed as answered in whole or in part; five of those eighteen carry a conditional half; and *two are marked open*. The two open items are not incidental — they are precisely the two on which the largest number of the others depend: the cascade-depth lemma, which every conditional entry in the flavor sector waits on, and the transport condition, which the baryon asymmetry waits on. A reader who wants a single test of whether this framework is being reported honestly should check those two entries first, and check that they are still marked open wherever the results that depend on them are quoted.

The list is also the reason the course is arranged as it is. It would be possible to present these appendices as a sequence of successes; the three-step form exists to prevent that, by forcing every chapter to write down what the textbook could not do *before* it writes down what the geometry does, and to write down what the geometry still cannot do at the end.
Throughout, the symbols GG-A1-Constitution–GG-A11-Origin, GG-P1-KK–GG-P5-CPT, and GG-C1-Kaxion–GG-C3-BH refer to the twenty-two papers of the GG-Theory cohort, to which the geometric steps repeatedly point.

A Summary of Abbreviations

This appendix collects every abbreviation used in the present paper. All program-defined names are rendered in *sans-serif type* (GG-Theory, GG-Constitution, GG-6, GDOS, BI-6, OGN-5, Gi-4, Qi-4, GRIP, etc.) so the reader can see at a glance which terms belong to the GG-Theory program proper. Standard physics abbreviations imported from the wider literature (KK, RS, QFT, TOV, etc.) are rendered in plain uppercase Roman, following the convention of the major physics journals. Mathematical objects are in roman or italic in the usual way. The third column gives the *primary link*: the paper in the Arisaka-GG series and the section where each program-defined name is first introduced in detail.

Table 77: Summary of abbreviations used in the present paper. Program-defined names are rendered in sans serif; standard physics abbreviations in plain uppercase Roman; mathematical objects in roman/italic in the usual way. The typographic convention is the quickest way to tell a term this program defines from one it borrows, which matters most in the sections where both kinds appear in the same sentence.

Symbol	Meaning	Primary link
(1) Framework name and core program objects		
GG-Theory	Grand Geometric Theory (also: Geometry-Gauge Equivalence Theory)	GG-A1-Constitution, Preface

continued on next page

Table 77 continued

Symbol	Meaning	Primary link
GG-Constitution	Constitutional charter: 4 axioms (AX-1–AX-4) + 7 postulates (POS-1–POS-7)	A1, §2–§3
GG-6	Geometry-Gauge locking in six dimensions: the 6D bulk	GG-A2-GG6, §8
GDOS	Geometric Dynamics of Open Systems: the boundary law from OGGEP	GG-A5-GDOS
BI-6	Green–Schwarz-modified Bianchi identity at 6D, $dH_3 = X_4$	GG-A3-BI6
OGN-5	Occam-Gauge-Normalization 5-integer ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$	GG-A3-BI6, §7; GG-A2-GG6 T-GG6-6
Gi-4	Geometric-Invariance four-pillar structure (TGG, CCI, GFF, GRIP)	GG-A6-GRIP
Qi-4	Quantum-Information four-pillar structure (Qi-QUA, Qi-EP, Qi-CON, Qi-HAL)	GG-A7-Qi4
(2) Four axioms (AX-1–AX-4)		
AX	Axiom (axiomatic primitive; non-negotiable)	A1, §1.5
LC	AX-1 Locality and Causality	A1, §2.2
GGEP	AX-2 Geometry–Gauge Equivalence Principle	A1, §2.3
OGGEP	AX-3 Operational/Open Geometry–Gauge Equivalence Principle	A1, §2.4
MSMF	AX-4 Maximum Symmetry, Minimum Freedom (no hidden continuous knobs)	A1, §2.5
(3) Seven postulates (POS-1–POS-7)		
POS	Postulate (admissibility filter; testable in principle)	A1, §1.5
GI	POS-1 Gauge invariance across scales	A1, §3.3
UEP	POS-2 Unified Extremal Principle	A1, §3.4
AF	POS-3 Anomaly Freedom and cancellability by inflow	A1, §3.5
CP	POS-4 Closure and Positivity (CPTP-admissible open dynamics)	A1, §3.6
RG	POS-5 Renormalization-Group covariance (scale covariance)	A1, §3.7
OGN	POS-6 Occam Gauge Normalization (canonical integer/rational normalization)	A1, §3.8
MF	POS-7 Measurement and Falsifiability (operational contract + kill criteria)	A1, §3.9
(4) Gi-4 pillars		
TGG	Total Geometric–Gauge object (Pillar I of Gi-4)	GG-A6-GRIP T-GRIP-1
CCI	Covariant Conservation Identity (Pillar II of Gi-4)	GG-A6-GRIP T2.1+T2.2
GFF	Geometric Free-Energy Functional (Pillar III of Gi-4)	GG-A6-GRIP T3.1
GRIP	Gauge-Runtime Invariance Pipeline (Pillar IV of Gi-4)	GG-A6-GRIP T4.1
GEP	Geometry–Gauge Energy Principle (energetic block of GFF)	GG-A6-GRIP
CEP	Curvature–Entropy Principle (entropic block of GFF)	GG-A6-GRIP
DGI	Dynamical Gauge Invariance (initial stage of GRIP)	GG-A6-GRIP

continued on next page

Table 77 continued

Symbol	Meaning	Primary link
GSSB	Geometric Spontaneous Symmetry Breaking (middle stage of GRIP)	GG-A6-GRIP
GA	Geometric Attractor (final stage of GRIP)	GG-A6-GRIP
(5) Qi-4 pillars and the Arisaka equation		
Qi-QUA	Quantum information for Quadruple Gauge Locking — Pillar I of Qi-4	GG-A7-Qi4 T-QUA
Qi-EP	Quantum information for the Unified Extremal Principle — Pillar II of Qi-4	GG-A7-Qi4 T-EP
Qi-CON	Quantum information for Conformal Invariance — Pillar III of Qi-4	GG-A7-Qi4 T-CON
Qi-HAL	Quantum information for the Holographic Attractor Lattice — Pillar IV of Qi-4	GG-A7-Qi4 T-HAL
OH	Outer Hologram (boundary projection of bulk data; outward face of EIC)	GG-A7-Qi4
IH	Inner Hologram (inner self-modeling layer; inward face of EIC)	GG-A7-Qi4
EIC	Evolving Inner Cosmos (OH+IH composite)	GG-A7-Qi4 T-EIC
Q_B	Arisaka constant $= (2/5)E_{\mathrm{Ry}}/K \approx 0.1555$ eV per qubit	GG-A7-Qi4 T-QUA
(6) Qi-HAL sub-acronyms		
HAL	Holographic Attractor Lattice	GG-A7-Qi4 T-HAL
BFFT	Boundary-Front Field Theory of Qi-HAL	GG-A7-Qi4
(7) GDOS-internal acronyms (program-defined)		
SIP	System Instance Packet (canonical GDOS input packet)	GG-A7-Qi4 §6
SeOS	Selfish Open System (the core open-system of GDOS)	GG-A5-GDOS
(8) Program-defined mathematical symbols (paper-specific)		
$\mathcal{R}$	Candidate rational expression in the MSMF/OGN audit	§101.1
$\mathcal{C}_\Sigma$	OGN-admissible candidate space on stratum Σ	§101.1
$\mathcal{S}_\Sigma$	Structural base set of atoms on stratum Σ	§101.1
$\mathrm{Cost}(\mathcal{R})$	MSMF cost function: $\sum_i \lvert a_i\rvert + \sum_j \lvert b_j\rvert$	Eq. (101.4)
ϵ_X	Stratum-specific empirical-precision target for parameter X	§101.1
$\mathcal{R}_X$	QD-completion rational for fermion X	§43.2
$640 = 2^J L$	Hopkins–Singer descent denominator	§43.11
(9) Standard physics abbreviations (plain Roman, journal convention)		
Standard Model and beyond		
SM	Standard Model	textbook
GUT	Grand Unified Theory	textbook
SU(5)	Georgi–Glashow grand-unified gauge group	Georgi & Glashow (1974)
CKM	Cabibbo–Kobayashi–Maskawa quark mixing matrix	Kobayashi & Maskawa (1973)
PMNS	Pontecorvo–Maki–Nakagawa–Sakata neutrino mixing matrix	textbook
QED	Quantum Electrodynamics	textbook

continued on next page

Table 77 continued

Symbol	Meaning	Primary link
QCD	Quantum Chromodynamics	textbook
EW	Electroweak	textbook
PDG	Particle Data Group	pdg.lbl.gov
Higher-dimensional constructions		
KK	Kaluza–Klein decomposition	textbook
RS	Randall–Sundrum warped 5D model	textbook
AdS/CFT	Anti-de Sitter / Conformal Field Theory holography	textbook
WH	Wilson–Hosotani holonomy mechanism	GG-A6-GRIP
Quantum gravity and cosmology		
GR	General Relativity	textbook
QFT	Quantum Field Theory	textbook
QM	Quantum Mechanics	textbook
TOV	Tolman–Oppenheimer–Volkoff (compact-star equation)	textbook
CMB	Cosmic Microwave Background	textbook
Open-system formalism		
CPTP	Completely Positive Trace Preserving	textbook
GKLS	Gorini–Kossakowski–Lindblad–Sudarshan (open-system master equation)	textbook
NZ	Nakajima–Zwanzig projection formalism	textbook
RG	Renormalization Group	textbook

Hierarchy of program-defined acronyms. The figure below records the six-tier hierarchy of program-defined acronyms in the GG-Theory program as it appears in the present paper. Top tier: the framework name (GG-Theory) and its constitutional charter (GG-Constitution). Tier 2: the four axioms and seven postulates that together define the admissibility predicate. Tier 3: three bulk objects forced by the constitution — GG-6 (6D bulk), BI-6 (modified Bianchi identity), OGN-5 (integer ledger). Tier 4: the boundary law GDOS that the bulk projects to via OGGEP, with input acronym SIP. Tier 5: the two four-pillar structures, Gi-4 (geometric flank, with sub-acronyms GEP/CEP blocks of GFF and DGI/GSSB/GA stages of GRIP) and Qi-4 (quantum-information flank). Bottom: the energy–information Arisaka constant $\mathsf{Q_B}$ and the Arisaka equation $J_E/\mathsf{Q_B} = J_e = J_B = J_{Qi}$ that locks the four boundary currents through one conversion factor.

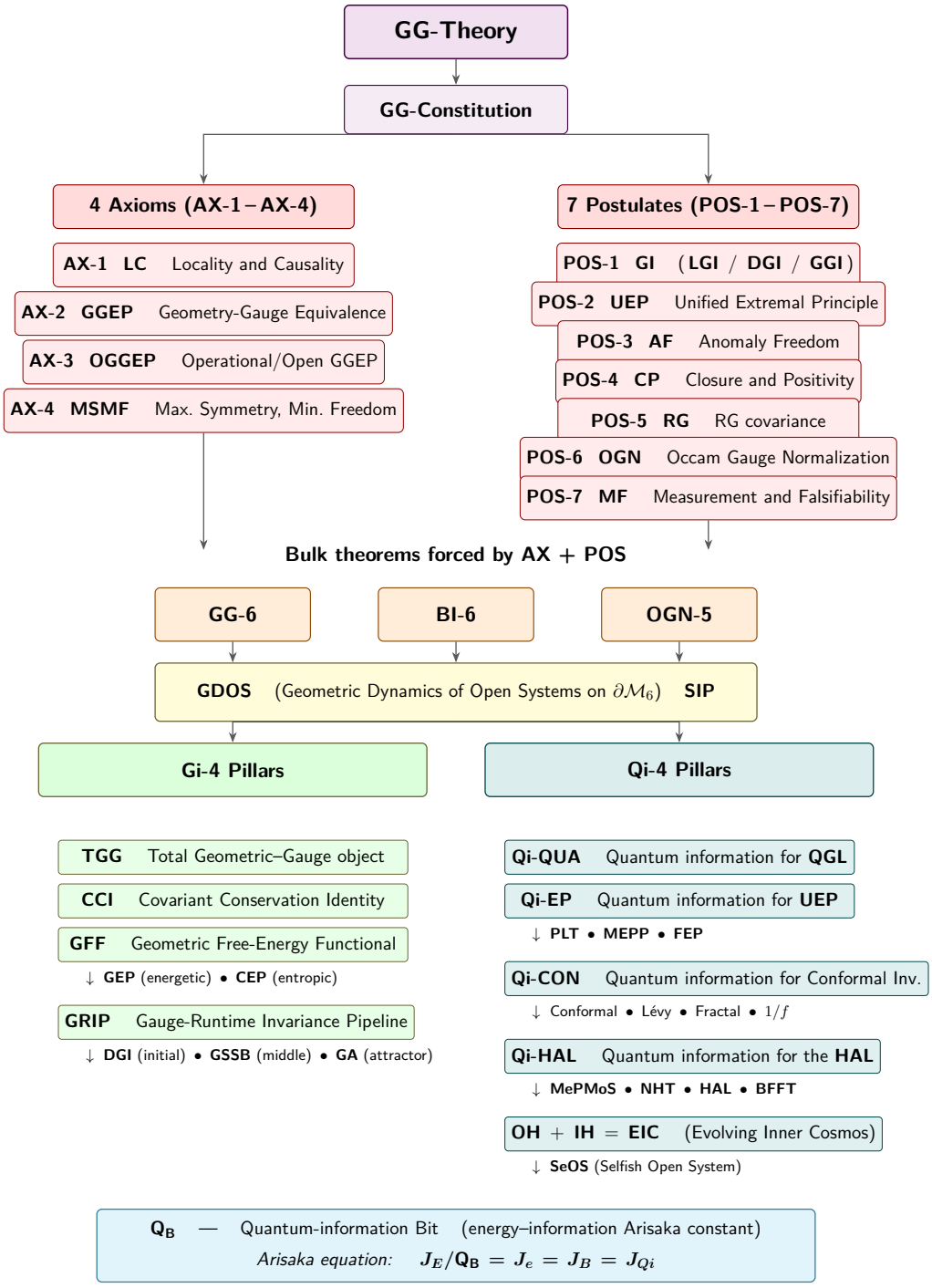

Figure 27: Hierarchy of program-defined acronyms in the GG-Theory program. The arrows run from the constitutional layer down to the sector-specific terms, so a reader who meets an unfamiliar acronym in a later Part can find its parent here in one step. The diagram covers the program-defined names only. Standard physics abbreviations are in the table above and have no place in this hierarchy, because they are not this program's to define.

B Glossary of Key Symbols

This appendix collects the key mathematical symbols used in GG-A9-SM. The first block records the bulk-geometry coordinates and the OGN-5 integer ledger that every later derivation appeals to. The second block records the scale chain $E_{\rm Pl} \to E_{\rm GUT} \to M_{\rm lock} \to v$. The third and fourth blocks record the Standard-Model parameters — gauge couplings, Higgs sector, fermion masses, mixing matrices, strong CP — as audited in §8. The fifth block records the MSMF/OGN audit machinery (cost function, candidate space, precision target) introduced in §101.1. The sixth block records the cascade-decoration symbols (Bogomolnyi instanton spectrum, Hopkins–Singer denominator, QD-completion rationals) inherited from GG-A6-GRIP and applied here. The seventh block records the Arisaka equation symbols inherited from GG-A7-Qi4.

Table 78: Glossary of key symbols used in the present paper. "Source" gives the equation, theorem, or section where the symbol is introduced or load-bearing. A symbol used in more than one Part is listed once, at the point where it is introduced, and the source column is the pointer back to that point. Symbols that appear only inside a single proof are omitted deliberately, on the grounds that a glossary long enough to include them would be too long to consult.

Symbol	Meaning	Source
(1) Bulk geometry and the OGN-5 ledger		
M_6	6D bulk manifold $M_4 \times I_r \times S^1_\theta$	A2-GG-6
ds^2_6	Canonical scale-profile metric $e^{-2r}\eta_{\mu\nu}\,dx^\mu dx^\nu + dr^2 + d\theta^2$	A2-GG-6
r	Warped radial coordinate, $r \in [0, K]$	—
θ	Compact angular coordinate, $\theta \in [0, 2\pi)$	—
S^1_θ	Compact direction circle	—
I_r	Radial interval	—
$(M, N, L; J, K)$	OGN-5 ledger $= (1, 3, 5; 7, 35)$	A2-GG-6 T6
$K = LJ$	Total scale depth, dual-lock identity $K = LJ = M^2 + N^2 + L^2 = 35$	A2-GG-6 T6
$N = 3$	The OGN-5 ledger's second entry. Every ledger formula is written with it ($\sin^2\theta_W$, λ_H, v, $\alpha^{-1}(0)$, K, J)	GG-A2-GG6 T6
$N_c = 3$	Color triplicity, the three colors of $\mathrm{SU}(3)_c$ — what that entry counts. Named where the color reading is the point, and locked by the BI-6 trace ratio $(L - N)/L = 2/5$	GG-A3-BI6 §7
$N_g = 3$	Generation (family) count. *Not* a ledger member; read by every family-stratum and Yukawa formula. Three by the three-leg theorem	Arisaka, 2026, P2-N3
$N_f = 6$	Quark-flavor count in a beta function at high scale, $N_f = 2N_g$; a third quantity, and never either of the two above	PDG
(2) Scale chain		

continued on next page

Table 78 continued

Symbol	Meaning	Source
$E_{\rm Pl}$	Planck energy, $\sqrt{\hbar c^5/G_N} \approx 1.22 \times 10^{19}$ GeV	—
$E_{\rm GUT}$	GUT scale $= E_{\rm Pl}\, e^{-2\pi} \approx 2.28 \times 10^{16}$ GeV	A2-GG-6
$M_{\rm lock}$	Lock scale $= \frac{1}{2} M_{\rm Pl}\, e^{-K} \approx 3.85$ TeV	A2-GG-6
v	Higgs vacuum value $= M_{\rm lock}(2/L)^N \approx 246$ GeV	Thm. 34.1
$m_\Lambda^{\rm geom}$	Cosmological-constant mass scale $= M_{\rm lock}^2/M_{\rm Pl} \approx 6$ meV	GG-A10-Cosmos
(3) Gauge couplings		
$\alpha^{-1}(M_Z)$	Inverse fine-structure at Z pole $= 2^J = 128$	Thm. 31.1
$\alpha^{-1}(0)$	Inverse fine-structure at zero momentum $= 2^J + N^2 = 137$	Thm. 31.1
$\sin^2\theta_W$	Weak mixing angle $= L^2/(4N^3) = 25/108$	Thm. 32.1
$\alpha_s(M_Z)$	Strong coupling $= 1/(\pi e)$	Thm. 33.1
g_2, g_Y	$\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge couplings	—
(4) Higgs sector		
Φ	Higgs $\mathrm{SU}(2)_L$ doublet, $\Phi = (\phi^+, \phi^0)^T$	—
λ_H	Higgs quartic $= 2^{-N} = 1/8$	Thm. 35.1
m_H	Higgs boson mass $= \sqrt{2\lambda_H}\, v \approx 125$ GeV	—
Θ_H	Wilson-line holonomy on S^1_θ (electroweak order parameter)	GG-A6-GRIP
(5) Fermion masses and mixing		
y_f	Yukawa coupling for fermion f, $m_f = y_f v/\sqrt{2}$	—
y_t	Top-quark Yukawa $= 1$ (cascade-top reference; $m_t = v/\sqrt{2}$ is a $\sim 1\%$ prediction, v from $M_{\rm lock}$)	§101.2
m_e, m_μ, m_τ	Charged-lepton masses	§47
$m_u, m_d, m_c, m_s, m_t, m_b$	Quark masses	§49
m_*	Absolute Majorana neutrino scale $\sim v^2/E_{\rm GUT}$	§50.4
Δm^2_{ij}	Neutrino squared-mass splittings	§50.4
$V_{\rm CKM}$	Cabibbo–Kobayashi–Maskawa quark mixing matrix	§52
$U_{\rm PMNS}$	Pontecorvo–Maki–Nakagawa–Sakata neutrino mixing matrix	§54
$\theta_{ij}^{\rm CKM}$	CKM mixing angles	§52
$\sin^2\theta_{ij}^{\rm PMNS}$	PMNS mixing angles	§54
$\delta_{\rm CP}^{\rm CKM}$	CKM Dirac CP phase	§52
$\delta_{\rm CP}^{\rm PMNS}$	PMNS Dirac CP phase	§54
α_{21}, α_{31}	Majorana phases (cost-0 in MSMF/OGN audit)	§54
$\bar\theta$	Strong-CP angle (suppressed; topological completion)	§79
(6) MSMF/OGN audit machinery		
$\mathcal{R}$	Candidate rational expression	§101.1
$\mathcal{C}_\Sigma$	OGN-admissible candidate space on boundary stratum Σ	§101.1
$\mathcal{S}_\Sigma$	Structural base set $\subset \{M, N, L, J, K, 2, \pi, e\}$ on stratum Σ	Eq. (101.2)

continued on next page

Table 78 continued

Symbol	Meaning	Source
$\mathrm{Cost}(\mathcal{R})$	MSMF cost: total absolute exponent budget $\sum \lvert a_i \rvert$	Eq. (101.4)
ϵ_X	Empirical-precision target for parameter X on its stratum	§101.1
$\mathcal{R}^\star$	Cost-minimum unique winner under MSMF/OGN selection	Thm. 101.1
(7) Cascade decoration and Hopkins–Singer descent		
$\mathbb{C}P^{N_g-1}$	Family-projector stratum, $\mathbb{C}P^2$ for $N_g = 3$	GG-A6-GRIP §28
$S_{[2k]\to[2(k-1)]} = \pi k$	Bogomolnyi instanton action on the cascade	GG-A6-GRIP §28
$n_X \in \{0,1,2,3,4\}$	Cascade-depth assignment per fermion X	§47.3, §49.3
$e^{-n\pi}$	Angular cascade decoration	GG-A6-GRIP §28
$\mathcal{R}_\tau, \mathcal{R}_\mu, \mathcal{R}_e$	QD-completion rationals: $637/640, 159/160, 641/640$	§43.2
$640 = 2^J L$	Hopkins–Singer descent integrality denominator	§43.11
(8) Arisaka equation and energy–information anchor (GG-A7-Qi4)		
Q_B	Arisaka constant $= (2/5)\, E_{\mathrm{Ry}}/K \approx 0.1555$ eV per qubit	GG-A7-Qi4 T-QUA
$N_A Q_B$	Avogadro multiple ≈ 15 kJ/mol	GG-A7-Qi4
J_E, J_e, J_B, J_{Qi}	Four boundary currents locked by the Arisaka equation	GG-A7-Qi4
$J_E/Q_B = J_e = J_B = J_{Qi}$	The Arisaka equation	GG-A7-Qi4 T-QUA

C Global naming convention for cross-cohort citation

Beginning with Arisaka (2026, A2-GG6) and extended across the cohort, each load-bearing item of the GG-Theory program carries a globally unique identifier of the form *Prefix-Suffix-N*, where the Prefix names the item type and the Suffix names the paper:

- **T-Suffix-N** for the Nth Theorem. GG-A9-SM's theorem identifiers were originally assigned to match the section in which each theorem appeared, and for nine of the twenty they still do — T-SM-33.4 is the Higgs-mass theorem in §33. For the other eleven the numbers no longer agree, because sections have been added and removed since the identifiers were minted. **The identifiers are frozen as cohort-stable citation keys and are not section pointers**; companion papers cite them and must continue to resolve. Table 79 is the concordance, and its *prints as* column is a live cross-reference, so it cannot itself go stale.
- **L-Suffix-N** for the Nth Lemma.

- **C-Suffix-N** for the Nth Corollary.
- **D-Suffix-N** for the Nth Definition.
- **R-Suffix-N** for the Nth Remark.
- **P-Suffix-N** for the Nth Prediction.
- **O-Suffix-N** for the Nth Objection (e.g., O-SM-1 is the first Common Objection of the present paper).
- **F-Suffix-N** for the Nth Falsifier (e.g., F-SM-1 is the KK-excitation prediction near the lock scale).
- **I-Suffix-N** for the Nth Innovation (e.g., I-SM-1 is the 28-to-zero compression above).
- **PS-Suffix-N** for the Nth Problem-Solved entry (§81.1).

The canonical Suffix list for the twenty-two cohort papers is: *Constitution* (A1), *GG6* (A2), *BI6* (A3), *Euclid* (A4), *GDOS* (A5), *GRIP* (A6), *Qi4* (A7), *QM* (A8), *SM* (A9, this paper), *Cosmos* (A10), *Origin* (A11), together with the phenomenology preprints *KK* (P1), *N3* (P2), *Electron* (P3), *Proton* (P6), the foundations preprints *Real* (P4) and *CPT* (P5), and the consequence preprints *Kaxion* (C1), *Hubble* (C2), *BH* (C3), and the white papers *Symmetry* (W1) and *Holonomy* (W2). The four axioms (AX-1 . . . AX-4) and seven postulates (POS-1 . . . POS-7) of Arisaka (2026, A1-Constitution) keep their existing cohort-wide names; the global identifier is the same item viewed from outside its home paper.

C.1 Concordance of the GG-A9-SM theorem identifiers

The identifiers below are the cohort-stable citation keys. Where the printed theorem number differs from the number inside the identifier, the identifier is still the correct thing to cite: companion papers resolve against it, and renumbering would break them. The *prints as* column is generated by cross-reference at build time, so this table re-resolves whenever the manuscript is recompiled and cannot fall out of date.

Table 79: Concordance between the frozen GG-A9-SM theorem identifiers and the theorem numbers as printed. Nine of the twenty still agree; the other eleven do not, because sections have been added and removed since the identifiers were minted. The identifiers do not track section numbers and are not to be read as section pointers.

Global identifier	Prints as	Statement
T-SM-10.3	Theorem 11.3	The Arisaka equation: four-currents lock
T-SM-21.1	Theorem 21.1	Conditional derivation of $N = 3$
T-SM-26.1	Theorem 30.1	Electroweak attractor-coordinate theorem
T-SM-27.1	Theorem 31.1	Electromagnetic dyadic-closure theorem
T-SM-28.1	Theorem 32.1	Weak-angle rational-partition theorem
T-SM-29.1	Theorem 33.1	Strong angle–scale theorem

Global identifier	Prints as	Statement
T-SM-30.1	Theorem 34.1	Wilson–Hosotani Higgs-vev theorem
T-SM-31.1	Theorem 35.1	Minimal quartic theorem
T-SM-33.1	Theorem 38.1	Fermi-constant descendant theorem
T-SM-33.2	Theorem 38.2	W-boson mass theorem
T-SM-33.3	Theorem 38.3	Z-boson mass theorem
T-SM-33.4	Theorem 38.4	Higgs mass theorem
T-SM-34.1	Theorem 39.1	Minimal infrared Abelian transport theorem
T-SM-35.1	Theorem 40.1	Unified gauge–Higgs parameter theorem package
T-SM-44.1	Theorem 50.1	The dial-free door: the unique $\Delta L = 2$ portal
T-SM-46.1	Theorem 52.1	Discrete symmetries as orientation bookkeeping
T-SM-46.2	Theorem 52.2	Flavor holonomy: rephasing invariants as loop angles
T-SM-60.1	Theorem 86.1	Boundary proton stability, conditional
T-SM-62.1	Theorem 95.1	Uniqueness of the four-force architecture
T-SM-67.1	Theorem 101.1	MSMF/OGN selection principle

D The gauge sector: couplings, the weak angle, and unification

What this appendix teaches. The gauge sector built from scratch — the three groups, the three couplings, the weak mixing angle, one-loop running, and the near-miss of grand unification — closed by worked numbers; a frank statement of what the textbook construction must take from experiment (three couplings, one angle, and the choice of the groups themselves); and the geometric derivation in which the gauge pattern and the boundary couplings descend from the bulk geometry, with the two-loop Wilson-line refinement held at its stated conservative uncertainty.

Read alongside: Peskin & Schroeder (1995) for the running-coupling machinery; Cheng & Li (1984) for the gauge-theory structure at depth; Georgi & Glashow (1974) for the original unification proposal.

This is the first appendix of the graduate course. Its subject is the gauge sector of the Standard Model — the three gauge groups, their three coupling constants, the weak mixing angle, and the way the couplings run and (almost) unify. We follow the three-step plan announced at the head of the Part: first the textbook construction as an incoming graduate student would meet it; then a frank account of what that construction must take from experiment and cannot explain; and finally the geometric derivation, in which the three coupling values descend as boundary readouts of the OGN-5 ledger rather than as fitted inputs. The detailed arithmetic of that third step — the Planck–GUT–lock scale chain, the dyadic electromagnetic code, the rational weak angle, and the strong coupling — occupies the bulk of this appendix and is what the main text compresses; it is given here in full so that the appendix is self-contained. For the standard textbook development of gauge theory and the running couplings, see Peskin & Schroeder (1995) and Cheng & Li (1984).

D.1 Step 1 — the gauge sector as a graduate student meets it

The gauge group and its three couplings. The Standard Model is a Yang–Mills gauge theory built on the product group

$$G_{\rm SM} = \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y, \tag{D.1}$$

where $\mathrm{SU}(3)_c$ is the color group of the strong interaction, $\mathrm{SU}(2)_L$ is weak isospin acting only on left-handed fields, and $\mathrm{U}(1)_Y$ is weak hypercharge. Each factor carries exactly one coupling constant: g_s for color, g for weak isospin, and g' for hypercharge. Matter fields sit in representations of the group, and the gauge fields enter every kinetic term through the covariant derivative

$$D_\mu = \partial_\mu - i\, g_s\, G^a_\mu T^a - i\, g\, W^i_\mu \tau^i - i\, g'\, \tfrac{Y}{2}\, B_\mu, \tag{D.2}$$

with T^a ($a = 1, \ldots, 8$) the SU(3) generators, $\tau^i = \sigma^i/2$ ($i = 1, 2, 3$) the weak-isospin generators, and Y the hypercharge. *(Physics intuition: a gauge coupling is the single number that says how strongly a given charge stirs its force field. Once the group and the representations are chosen, the coupling is the only freedom the gauge principle leaves in each factor — which is why the Standard Model has exactly three of them.)*

Electroweak mixing and the weak angle. The electroweak group $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is not the group one sees at low energy. After the Higgs field acquires its vacuum value the neutral isospin boson W^3_μ and the hypercharge boson B_μ mix by a single rotation,

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix}, \qquad \tan\theta_W = \frac{g'}{g}, \tag{D.3}$$

producing the massless photon A_μ and the massive Z_μ. The photon couples to electric charge with strength

$$e = g\, s_W = g'\, c_W, \qquad \alpha_{\rm em} = \frac{e^2}{4\pi}, \tag{D.4}$$

and the weak mixing angle is fixed by the ratio of the two electroweak couplings,

$$\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2}. \tag{D.5}$$

(Physics intuition: θ_W is the one hinge angle that splits the two neutral gauge bosons into a massless photon and a massive Z. All of the relative strengths of the electromagnetic and neutral-weak couplings are encoded in this single rotation.) Measured at the Z pole, the three numbers that parametrize the sector are

$$\alpha_{\rm em}^{-1}(M_Z) \simeq 127.9, \qquad \sin^2\theta_W(M_Z) \simeq 0.231, \qquad \alpha_s(M_Z) \simeq 0.118. \tag{D.6}$$

Running couplings and asymptotic freedom. Gauge couplings are not constants; vacuum polarization makes them depend on the energy scale μ. At one loop each coupling obeys

$$\mu\frac{dg_i}{d\mu} = \frac{b_i}{16\pi^2}\,g_i^3, \qquad \Longleftrightarrow \qquad \frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(\mu_0)} - \frac{b_i}{2\pi}\,\ln\frac{\mu}{\mu_0}, \tag{D.7}$$

with the one-loop Standard-Model coefficients, in the SU(5)-normalized hypercharge convention,

$$(b_1, b_2, b_3) = \left(\frac{41}{10}, -\frac{19}{6}, -7\right). \tag{D.8}$$

The color coefficient $b_3 = -7 < 0$ is the famous *asymptotic freedom*, established in 1973 by Gross, Wilczek, and Politzer (Gross, 1973; Gross & Wilczek, 1973; Politzer, 1973): α_s weakens at high energy and grows at low energy, which is what makes quarks behave almost freely inside a hard collision yet remain permanently confined at long distance. *(Physics intuition: a charge is screened by the cloud of virtual pairs it polarizes. For a non-Abelian group the gluons also carry color and* anti-screen*; the anti-screening wins for* SU(3) *with six quark flavors, so color charge looks weaker the closer one probes.)*

Grand unification and the SU(5) **embedding.** Run upward, the three couplings approach one another near 10^{16} GeV. The minimal unifying group SU(5), introduced by Georgi and Glashow (Georgi & Glashow, 1974), places one full Standard-Model family in the $\bar{\mathbf{5}} \oplus \mathbf{10}$, and fixes the relative normalization of hypercharge: the properly normalized U(1) coupling is

$$g_1^2 = \tfrac{5}{3}\,g'^2, \tag{D.9}$$

so that at exact unification, where $g_1 = g_2 = g_3$, the tree-level weak angle is

$$\sin^2\theta_W\big|_{\rm GUT} = \left.\frac{g'^2}{g^2 + g'^2}\right|_{g_1=g_2} = \frac{3}{8}. \tag{D.10}$$

This high-scale value of 3/8 then runs down to $\simeq 0.231$ at the weak scale. *(Physics intuition: if a single knob sets all three forces at the unification seam, the low-energy spread of the couplings is just the different rates at which they drift apart as the universe cools below that seam.)*

Worked example (STANDARD TEXTBOOK PEDAGOGY). Suppose a student is handed only the two measured electroweak numbers $\alpha_{\rm em}^{-1}(M_Z) = 127.9$ and $\sin^2\theta_W(M_Z) = 0.231$, and asked to recover the two electroweak couplings and the W mass. From (D.4) and (D.5),

$$\alpha_2 \equiv \frac{g^2}{4\pi} = \frac{\alpha_{\rm em}}{\sin^2\theta_W} = \frac{1}{127.9 \times 0.231} = \frac{1}{29.5}, \qquad g = \sqrt{4\pi\alpha_2} \simeq 0.652, \tag{D.11}$$

$$\alpha_Y \equiv \frac{g'^2}{4\pi} = \frac{\alpha_{\rm em}}{\cos^2\theta_W} = \frac{1}{127.9 \times 0.769} = \frac{1}{98.4}, \qquad g' = \sqrt{4\pi\alpha_Y} \simeq 0.357. \tag{D.12}$$

Feeding the isospin coupling and the measured vacuum value $v \simeq 246$ GeV into $m_W = \frac{1}{2}gv$ gives $m_W \simeq 80.2$ GeV, within a fraction of a percent of the measured 80.4 GeV. The textbook machinery

works perfectly — *provided* one is willing to take $\alpha_{\rm em}^{-1}$ and $\sin^2\theta_W$ from the data. Where those two numbers come from is precisely the question the Standard Model cannot answer, and to which we turn next.

D.2 Step 2 — what the standard gauge sector assumes but cannot explain

The construction of Step 1 is a triumph of organization, but it is silent about *magnitudes*. The three gauge couplings — equivalently $\alpha_{\rm em}^{-1}$, $\sin^2\theta_W$, and α_s at M_Z — are pure inputs, fitted to experiment. They are three of the twenty-eight free parameters of the Standard Model. Nothing in the gauge principle predicts that $\alpha_{\rm em}^{-1}(0) \simeq 137$, or that the weak angle is near 0.23, or that the strong coupling is near 0.12. The gauge Lagrangian is equally consistent for any positive values of these three numbers.

Grand unification narrows the question without closing it. The SU(5) program and its descendants buy two genuine things: a single unified coupling at the seam, and the tree relation $\sin^2\theta_W = 3/8$ of (D.10). But each is spent again before reaching the weak scale. First, the three couplings only actually meet if the particle content between M_Z and the unification scale is exactly right; in the minimal non-supersymmetric SU(5) the three running lines miss each other, and one must add superpartners or other matter by hand to force the crossing. Second, the unified coupling value $\alpha_{\rm GUT}$ is itself an input, not derived. Third, the same multiplet that unifies the couplings mediates proton decay, at a rate the minimal model overpredicts relative to experiment. Fourth, the unification scale and the great "desert" between it and the weak scale are posited, not explained. Unification thus trades three low-energy inputs for a high-scale coupling, a scale, and a spectrum — fewer knobs, but knobs still — and the specific low-energy integers (why 137, why a weak angle of exactly 25/108) remain entirely outside the theory.

Two structural questions are left open as well. Why *these* three gauge groups and no others — why rank four, why this particular product $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$? And why is the weak interaction the only chiral, family-connecting force, acting on left-handed fields alone? The gauge principle *accommodates* these facts but does not *require* them; in the Standard Model they are part of the input data. (The chirality and family-changing role of the weak force is taken up directly in §48.)

It is exactly these residues — the three coupling values, the choice of gauge group, and the chirality of the weak force — that the geometric derivation addresses. We turn to it now.

D.3 Step 3 — the geometric derivation: gauge couplings as boundary readouts of the OGN-5 ledger

In GG-Theory the three gauge couplings are not fitted. They are boundary readouts of the six-dimensional bulk, fixed by the OGN-5 integer ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ and by the dyadic and rational closure codes proved upstream. The derivation runs in two stages. First the three mass scales that anchor the sector — the Planck scale, the grand-unification seam, and the electroweak lock scale — are fixed from the allowed constants c, h, and G_N alone, with no fitted coefficient. Then the three couplings are read off the ledger: the electromagnetic coupling as a dyadic closure $\alpha_{\rm em}^{-1}(M_Z) = 2^J = 128$, the weak angle as the rational code $\sin^2\theta_W(M_Z) = 25/108$, and the strong coupling as $\alpha_s = 1/(\pi\mathrm{e})$. The remainder of this appendix carries out both stages in

full and cross-checks them against ordinary SU(5)-normalized electroweak kinematics, so that the geometric result and the textbook algebra of Step 1 are shown to be one and the same arithmetic seen from two sides.

D.3.1 The OGN-5 integer ledger and where the exponents come from

Every discrete exponent that appears below — the dyadic depth in 2^J, the triplicity factor $1/N_c$, the embedding weight L, the lock depth e^{-K} — is one of five integers, and only those five. They are the OGN-5 ledger

$$(M,\, N,\, L;\, J,\, K) = (1,\, 3,\, 5;\, 7,\, 35), \tag{D.13}$$

fixed upstream by BI-6 closure of the bulk and inherited here as data: their derivation from the six-dimensional bulk is the business of GG-A2-GG6 and GG-A3-BI6 (Arisaka, 2026, A2-GG6; Arisaka, 2026, A3-BI6), and their first constitutional appearance is fixed in GG-A1-Constitution (Arisaka, 2026, A1-Constitution). Their meanings are concrete and geometric rather than numerological:

- $M = 1$: *the number of unbroken Abelian seams that survive electroweak symmetry breaking.* One, and only one, neutral line stays massless — the photon — so the surviving electromagnetic channel has no room to branch and can only close *binarily.* This single datum is the entire reason the fine-structure constant is a power of two.

- $N_c = 3$: *the triplicity index* — three colors; the generation count $N_g = 3$ is a separate quantity that lands on the same integer, and the ternary replication depth of the boundary. It is the only odd primitive in the ledger, and it controls both the color-blind averaging that appears in the weak angle and the cascade replication that appears in the fermion spectrum.

- $L = 5$: *the dimension of the minimal unifying boundary embedding* — the rank-counting integer of the SU(5)-type generator lattice into which the Standard-Model factors fit. It is the integer that carries the hypercharge-versus-isospin weighting in the weak-angle partition.

- $J = 7$, $K = 35$: *forced, not chosen.* The closure law of BI-6 requires the two consistency relations

$$K = L\,J \qquad \text{and} \qquad K = M^2 + N^2 + L^2 = 1 + 9 + 25 = 35. \tag{D.14}$$

  The first reads K as a product, the second as a sum of squares. Equating them forces $J = K/L = 35/5 = 7$, the *unique* integer consistent with both faces of K. There is no freedom: once $(M, N, L) = (1, 3, 5)$ is set, J and K are locked.

(Physics intuition: think of (D.13) *as the geometry's complete parts list. A Standard-Model parameter is never a dial to be turned; it is an assembly instruction that says "take this many of that primitive." The art of the derivation is not to compute a number but to recognize which primitive each observable is counting. The three gauge couplings count three different primitives — the lone Abelian seam, the* $3 + 2$ *neutral split, and the non-Abelian color cell — which is why, as we will see, they come out as three different* kinds *of number.)*

D.3.2 The electromagnetic coupling: dyadic closure in full

The main-text statement is Theorem 31.1 (T-SM-27.1); here we give the derivation in expanded form. The first move is to choose the right variable. In an Abelian seam the gauge-invariant measure of stiffness is not the coupling but its inverse, the *inverse seam capacity*

$$C_\gamma \;:=\; \widehat{\alpha}_{\mathrm{EM}}^{-1} = \frac{4\pi}{e^2}. \tag{D.15}$$

(Physics intuition: C_γ is an inverse conductance — how strongly the vacuum resists having electric flux pushed through the surviving neutral seam. A large C_γ *means a* stiff*, weakly coupled photon. Working with the inverse is the same instinct that makes an electrical engineer quote resistance rather than conductance: the additive, quantized quantity is the resistance.)*

Now use the ledger datum $M = 1$. Because exactly one Abelian seam survives, the closure of that seam admits no ternary or higher branching — there is nothing for a third or fourth lane to belong to. The only lawful refinement is *binary*: each closure step splits the admissible seam interval into two equal halves and keeps one. Writing $C_\gamma(n)$ for the inverse capacity after n such refinements, minimality fixes the recursion

$$C_\gamma(0) = 1, \qquad C_\gamma(n+1) = 2\,C_\gamma(n). \tag{D.16}$$

The base case says the unresolved primitive cell carries unit inverse capacity. The step says that halving the admissible interval doubles the inverse stiffness — geometrically, if you can only deform the seam through half the previous angular room, it is twice as hard to deform. *(Physics intuition: the picture is a strip of paper folded along one and only one crease. Each fold halves the width and doubles the resistance to further folding. There is a single crease because $M = 1$; the number of folds is set by how deep the lawful closure runs. The result — a pure power of two — is the signature of a system with exactly one binary degree of freedom iterated to its closure depth.)*

The recursion solves immediately by induction to $C_\gamma(n) = 2^n$, and the lawful closure depth is the odd closure index $J = 7$ of the ledger. Hence

$$C_\gamma^\star = 2^J = 2^7 = 128, \qquad \Longrightarrow \qquad \boxed{\widehat{\alpha}_{\mathrm{EM}}^{-1}(M_Z) = 2^7 = 128.} \tag{D.17}$$

The electric charge is then no longer primitive but a square-root readout of this stiffness,

$$e = \sqrt{\frac{4\pi}{2^J}} = \frac{2\sqrt{\pi}}{2^{J/2}} = \frac{2\sqrt{\pi}}{2^{7/2}} \simeq 0.3133285343. \tag{D.18}$$

Two refinements complete the picture and connect (D.17) to the number every physicist recognizes. First, (D.17) is the value at the Z-attractor, $\widehat{\alpha}_{\mathrm{EM}}^{-1}(M_Z) = 128$. The *on-shell* Sommerfeld value, measured at zero momentum transfer, is reached by adding the triplicity contribution that the long-wavelength photon sees from the full color cloud,

$$\alpha^{-1}(0) = 2^J + N^2 = 128 + 9 = 137, \tag{D.19}$$

which is the famous 137; the residual 0.03% to the CODATA 137.036 is attributed to ordinary low-energy QED dressing, not to a quantum-defect completion tabulated in this paper. *(Physics intuition for the $+N_c^2$: at the Z pole the photon resolves a single bare seam, capacity 2^7. As the probe momentum is lowered toward zero, the photon's field spreads until it wraps the entire triplicated family structure, and the vacuum polarization of the $N_c \times N_c$ color cloud adds exactly $N_c^2 = 9$ inverse units of stiffness. The two iconic integers of the fine-structure constant, 128 and 9, are thus the two iconic integers of the geometry, the dyadic depth and the square of the triplicity — nothing else enters.)* It is worth pausing on how much has happened: a quantity Feynman called "one of the greatest damn mysteries of physics" is, in this reading, the statement that one binary seam folded seven times, plus a 3×3 color cloud, has inverse capacity 137.

D.3.3 The weak mixing angle: rational partition in full

The main-text statement is Theorem 32.1 (T-SM-28.1). Unlike the photon line, this derivation is neither a recursion nor a transcendental cell: it is a single *partition*. The neutral electroweak sector carries a split generator whose trace norm is fixed, with no freedom, by the GG-A3-BI6 trace structure and the ledger,

$$\mathrm{Tr}\left(Y_{N,L}^2\right) = \frac{L}{N\,(L-N)} = \frac{5}{3 \cdot 2} = \frac{5}{6}. \tag{D.20}$$

(Physics intuition: $\mathrm{Tr}(Y_{N,L}^2)$ is the total "neutral norm" that the hypercharge and isospin generators present to the neutral line — the squared width of the channel through which neutral current must pass. The integers $L = 5$ and $L - N_c = 2$ are the embedding weight of hypercharge and the leftover isospin room after the three colored lanes are accounted for; their ratio over N_c is the bare squared width.)

Two named structural steps then convert this norm into the observed angle. The first is the *quadratic neutral-partition lemma*: the neutral sector partitions *power* (stiffness), not amplitude, so the relevant weight is the *square* of the trace norm,

$$\left[\mathrm{Tr}(Y_{N,L}^2)\right]^2 = \left(\frac{5}{6}\right)^2 = \frac{25}{36}. \tag{D.21}$$

(Physics intuition: power goes as amplitude squared. Because the weak angle is a ratio of stiffnesses, not of amplitudes, the channel's squared width enters squared again — the same reason an electrical power divider splits power as the square of the voltage-divider ratio.) The second is the *color-blind averaging lemma*: the measured neutral readout carries no color label, so the three triplicity sectors are *averaged*, not summed, contributing a factor $1/N$,

$$\sin^2\theta_W(M_Z) = \frac{1}{N}\left[\mathrm{Tr}(Y_{N,L}^2)\right]^2 = \frac{1}{3} \cdot \frac{25}{36} = \boxed{\frac{25}{108} = \frac{L^2}{4N^3} \simeq 0.231481.} \tag{D.22}$$

(Physics intuition: imagine a neutral river meeting a fork whose banks are locked in a $3 + 2$ configuration. The water divides according to the squared widths the geometry presents — not by preference — which gives $(5/6)^2$. An observer standing downstream cannot see the three color lanes that the flow has been replicated across, so the observer reports the per-lane average, dividing by 3.

The number the observer writes down is 25/108*, and it matches the PDG effective angle* 0.23155 *to* 0.03% *with not a single fitted input.)*

The denominator deserves a remark: $4N^3 = 4 \cdot 27 = 108$ and $L^2 = 25$, so the angle is literally "five squared over four times three cubed." That it lands on the measured value is the single sharpest piece of evidence in the gauge sector. In the honesty triage it is recorded as the most-derived of the twenty-eight entries: Steps fixing (D.20) and the partition are forced by GG-A3-BI6 and the ledger, while the two italicized lemmas — quadratic partition and color-blind averaging — are well-motivated GG-6-internal assignments whose promotion to full proofs (the natural target of the GFF variational route) would make the angle an unconditional theorem. For the cross-check against ordinary SU(5)-normalized electroweak kinematics — the same 25/108 recovered from $\alpha_1/\alpha_2 = 125/249$ — see §D.9 below.

D.3.4 The strong coupling: the angle–scale constant in full

The main-text statement is Theorem 33.1 (T-SM-29.1). The color line is geometrically unlike the photon line, and the difference is the whole content of the result. The photon is a *sum* channel: one surviving Abelian seam, so it collapses to a dyadic ladder and gives an integer. The color line is a non-Abelian *difference* channel, and a difference channel cannot collapse a circle to a ladder — it keeps the circle. It also keeps the radial direction, because QCD is the most visibly logarithmic of the gauge lines. Two geometric primitives therefore survive on the color seam:

$$\theta \sim \theta + 2\pi \quad \text{(compact angle)}, \qquad r = \ln \frac{E_{\mathrm{GUT}}}{\mu} \quad \text{(logarithmic radius)}. \tag{D.23}$$

Each contributes one factor to the irreducible color cell. The compact factor is not the full circle: opposite angular orientations are conjugate under DGI and hence physically identical, so the minimal compact primitive is the half-circle quotient

$$\mathrm{vol}_{\mathrm{prim}}\left(S^1/\mathbb{Z}_2\right) = \pi. \tag{D.24}$$

(Physics intuition: on a non-Abelian seam, "clockwise" and "anticlockwise" are gauge-equivalent labels for the same physical winding; folding the circle onto its mirror image leaves half of it, the arc of length π*. This is why* π*, and not* 2π*, is the strong primitive.)* The radial factor is the minimal unit of logarithmic transport, one e-fold,

$$\Delta r_{\mathrm{prim}} = 1 \qquad \Longleftrightarrow \qquad e^{\Delta r_{\mathrm{prim}}} = e. \tag{D.25}$$

(Physics intuition: because the radial coordinate is a logarithm of energy, the smallest natural step is the one that changes the scale by a factor e *— a single e-fold. The strong coupling is the one gauge line whose running is steep enough that this radial primitive is not negligible, which is exactly why an* e *appears here and nowhere else in the gauge sector.)* In the minimal closure class no extra multiplicity is admitted, so the irreducible color seam is the direct product of one compact and one

radial primitive, with inverse capacity

$$C_s^\star = \pi \cdot e = \pi e, \qquad \Longrightarrow \qquad \boxed{\alpha_s^{-1}(M_Z) = \pi e \simeq 8.5397, \qquad \alpha_s(M_Z) = \frac{1}{\pi e} \simeq 0.11700.} \tag{D.26}$$

The measured PDG value is $\alpha_s(M_Z) = 0.1180(9)$; the first-order geometric value 0.1170 sits within roughly one percent, and the quantum-defect completion of Part VI closes the small remaining gap. This theorem is parameter-free but, in candor, conceptually more delicate than the weak-angle partition: it reads a non-Abelian seam in terms of minimal primitive cells of compact and radial geometry, which is more interpretive than the finite-dimensional partition that fixes $\sin^2\theta_W$.

D.3.5 The derived couplings, and why three numbers of three different kinds

With $\sin^2\theta_W = 25/108$ and e from (D.18), the two electroweak gauge couplings follow with no further input,

$$g_2 = \frac{e}{\sin\theta_W} = \frac{4\sqrt{\pi}\,N^{3/2}}{L\,2^{J/2}} \simeq 0.6512411, \qquad g_Y = \frac{e}{\cos\theta_W} = \frac{4\sqrt{\pi}\,N^{3/2}}{2^{J/2}\sqrt{4N^3 - L^2}} \simeq 0.3574150, \tag{D.27}$$

in exact agreement with the textbook extraction of the Step-1 worked example ($g \simeq 0.652$, $g' \simeq 0.357$): the geometric derivation and the data-fed algebra are, as promised, the same arithmetic seen from two sides.

It is worth standing back to see the shape of what has been derived. The three gauge couplings of the Standard Model emerge as three numbers of three *different mathematical kinds*, and the kind is dictated by the topology of the seam:

- the electromagnetic coupling is an *integer power*, 2^7, because its seam is a single Abelian *sum* channel that can only fold in two;
- the weak angle is a *rational*, 25/108, because its neutral sector is a finite-dimensional *partition* of squared widths over triplicity lanes;
- the strong coupling is a *transcendental*, $1/\pi e$, because its seam is a non-Abelian *difference* channel that keeps both a compact angular primitive and a logarithmic radial primitive.

(Physics intuition: in the Standard Model these three numbers are three unrelated measured constants. Here they are three readings of one ledger, and the very type *of each number — integer, rational, transcendental — is a fingerprint of how its force closes on the boundary. That the most precisely measured constants of nature should carry, in their arithmetic character, a record of the topology that produced them is the central claim of the gauge sector, and it is what the explicit arithmetic in the remainder of this appendix verifies number by number.)*

D.4 Numerical constants and unit conversion details

For numerical convenience we report energies in GeV, but the derivation itself uses only the allowed constants c, h, and G_N. The reduced Planck constant is

$$\hbar = \frac{h}{2\pi}. \tag{D.28}$$

Using the 2022 CODATA values

$$c = 299\,792\,458\ \mathrm{m\,s^{-1}}, \qquad h = 6.626\,070\,15 \times 10^{-34}\ \mathrm{J\,s}, \qquad G_N = 6.67430 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}, \tag{D.29}$$

we obtain

$$\hbar = 1.054\,571\,817 \times 10^{-34}\ \mathrm{J\,s}. \tag{D.30}$$

To express energies in electron-volts we use the exact SI conversion

$$1\ \mathrm{eV} = 1.602\,176\,634 \times 10^{-19}\ \mathrm{J}, \qquad 1\ \mathrm{GeV} = 10^9\ \mathrm{eV}. \tag{D.31}$$

This conversion is a unit-definition convenience rather than a theory input.

Then

$$\begin{aligned} E_{\mathrm{Pl}} &= \sqrt{\frac{\hbar c^5}{G_N}} = \sqrt{\frac{(1.054571817 \times 10^{-34})(2.99792458 \times 10^8)^5}{6.67430 \times 10^{-11}}}\ \mathrm{J} \\ &\simeq 1.9560816367 \times 10^9\ \mathrm{J} \\ &\simeq 1.2208901286 \times 10^{19}\ \mathrm{GeV}. \end{aligned} \tag{D.32}$$

The remaining scales follow immediately from multiplication by the exact OGN exponentials,

$$E_{\mathrm{GUT}} = E_{\mathrm{Pl}} \mathrm{e}^{-2\pi} \simeq 2.2799424 \times 10^{16}\ \mathrm{GeV}, \tag{D.33}$$

$$M_{\mathrm{lock}} = \frac{E_{\mathrm{Pl}}}{2\mathrm{e}^{35}} \simeq 3.8489274 \times 10^3\ \mathrm{GeV}. \tag{D.34}$$

D.5 Detailed discussion of the Planck–GUT–lock scale chain

This appendix expands the derivation of the three scales to make the logic as explicit as possible.

D.6 Why the Planck scale must appear

If one is allowed only c, h, and G_N, there is only one way to construct an energy:

$$[c]^a [h]^b [G_N]^d = \mathrm{energy} \qquad \Longrightarrow \qquad E_{\mathrm{Pl}} \propto \left(\frac{hc^5}{G_N}\right)^{1/2}. \tag{D.35}$$

The factor of 2π that turns h into $\hbar$ is not optional in the present framework, because the compact direction is literally a circle, and the theory treats full angular cycles as primitive objects. Thus the same 2π that appears in the monodromy step also appears in $\hbar = h/(2\pi)$.

D.7 Why the GUT seam is one monodromy cycle below the Planck anchor

The coordinate r is a logarithmic scale coordinate. Therefore any special radial displacement corresponds to a multiplicative energy ratio. The compact S^1_θ sector contains a unique primitive angle, one full cycle 2π. If the constitution is to maximize symmetry and minimize arbitrary freedom, the first distinguished shift below the Planck anchor must be one full primitive cycle and not some arbitrary irrational multiple of it. This is the precise content of

$$\Delta r_{\mathrm{Pl}\to\mathrm{GUT}} = 2\pi M, \qquad M = 1. \tag{D.36}$$

D.8 Why the lock scale carries a factor of one half

GG-A2-GG6 itself fixes the exponential redshift e^{-K}, but does not need to specify the order-one endpoint convention because its purpose is the theorem-level bulk architecture. In the present paper, that convention becomes important because we want an explicit numerical lock scale.

The interval I_r has two endpoints. Under the minimal symmetric choice, the endpoint normalization is split equally between them. This is the simplest binary convention compatible with OGN, and it is also the only order-one factor that later allows the electroweak dyadic code and the radial endpoint normalization to belong to the same arithmetic family. Hence the factor 1/2 in

$$M_{\mathrm{lock}} = \frac{E_{\mathrm{Pl}}}{2\mathrm{e}^{K}}. \tag{D.37}$$

This factor is not a tunable parameter. It is a binary convention forced by the interval geometry once one insists on an explicit lock seam.

D.9 Rational-lattice reconstruction of the neutral electroweak sector

The main text used the compact formulas $\widehat{\alpha}^{(5)\,-1}(M_Z) = 2^7$ and $\sin^2\theta_W(M_Z) = 25/108$. This appendix shows how these formulas can be cross-checked against a more detailed rational-lattice reconstruction of the neutral sector. The purpose is not to add a new assumption, but to show that the dyadic and rational formulas are consistent with ordinary electroweak kinematics and SU(5)-normalized hypercharge.

D.10 From the weak angle to the ratio α_1/α_2

With the conventional SU(5)-normalized coupling α_1 and the weak-isospin coupling α_2, one has

$$\sin^2\theta_W = \frac{(3/5)\alpha_1}{\alpha_2 + (3/5)\alpha_1}. \tag{D.38}$$

Substituting the first-principles value $\sin^2\theta_W = 25/108$ gives

$$\frac{(3/5)\alpha_1}{\alpha_2 + (3/5)\alpha_1} = \frac{25}{108}, \tag{D.39}$$

so that

$$108\left(\frac{3}{5}\alpha_1\right) = 25\left[\alpha_2 + \left(\frac{3}{5}\alpha_1\right)\right]. \tag{D.40}$$

Rearranging,

$$\frac{249}{5}\alpha_1 = 25\alpha_2, \qquad \frac{\alpha_1}{\alpha_2} = \frac{125}{249}. \tag{D.41}$$

This is the precise sense in which the weak-angle rational code fixes the neutral-coupling ratio.

D.11 Electromagnetic coupling from the neutral sum

In standard electroweak kinematics,

$$\alpha_{\rm em}^{-1} = \alpha_2^{-1} + \frac{5}{3}\alpha_1^{-1}. \tag{D.42}$$

Thus once the ratio (D.41) is fixed, the only remaining freedom would ordinarily be an overall scale. In the present framework that overall neutral scale is precisely what OGN compresses to a dyadic closure code. The result is

$$\alpha_{\rm em}^{-1} = 2^J = 128. \tag{D.43}$$

The compact formulas used in the main text are therefore fully compatible with the ordinary neutral-sector algebra.

D.12 Why the rational code is more rigid than the dyadic one

The rational weak-angle code involves only the group-theoretic weight of hypercharge versus isospin and the triplicity index. The electromagnetic dyadic closure, by contrast, depends on how the neutral sum lands on the OGN lattice. This is why the QD correction naturally appears first in $\alpha_{\rm em}^{-1}$ and not in $\sin^2\theta_W$.

D.13 Detailed arithmetic for the first-order gauge and Higgs block

Collecting the exact first-order formulas,

$$\alpha_{\rm em} = \frac{1}{128}, \qquad s_W^2 = \frac{25}{108}, \qquad c_W^2 = \frac{83}{108}, \qquad \alpha_s = \frac{1}{\pi \mathrm{e}}, \tag{D.44}$$

we immediately obtain

$$e^2 = 4\pi\alpha_{\rm em} = \frac{\pi}{32}, \qquad e \simeq 0.3133285343. \tag{D.45}$$

Then

$$s_W = \sqrt{\frac{25}{108}} \simeq 0.4811252243, \tag{D.46}$$

$$c_W = \sqrt{\frac{83}{108}} \simeq 0.8766518799, \tag{D.47}$$

$$g_2 = \frac{e}{s_W} \simeq 0.6512411291, \tag{D.48}$$

$$g_Y = \frac{e}{c_W} \simeq 0.3574150031. \tag{D.49}$$

With

$$M_{\rm lock} \simeq 3848.9274 \text{ GeV}, \qquad v^{(1)} = M_{\rm lock}\left(\frac{2}{5}\right)^3, \tag{D.50}$$

we get

$$v^{(1)} \simeq 246.331354 \text{ GeV}. \tag{D.51}$$

Thus

$$m_W^{(1)} = \frac{1}{2} g_2 v^{(1)} \simeq 80.2106 \text{ GeV}, \tag{D.52}$$

$$m_Z^{(1)} = \frac{1}{2} v^{(1)} \sqrt{g_2^2 + g_Y^2} \simeq 91.4965 \text{ GeV}. \tag{D.53}$$

For the quartic,

$$\lambda_H^{(1)} = \frac{1}{\pi \mathrm{e}} + \frac{1}{128} = 0.1249121630\ldots, \tag{D.54}$$

so

$$m_h^{(1)} = \sqrt{2\lambda_H^{(1)}}\, v^{(1)} \simeq 123.1224 \text{ GeV}. \tag{D.55}$$

This appendix makes clear that the entire first-order gauge/Higgs block is numerically straightforward once the scale chain and the OGN-5 ledger have been fixed.

D.14 QD arithmetic and residual-theory estimates

The two characteristic suppressed parameters are

$$\varepsilon_J = 2^{-7} = 7.8125 \times 10^{-3}, \qquad \varepsilon_K = \frac{1}{70\pi} = 4.5473 \times 10^{-3}. \tag{D.56}$$

The leading QD electromagnetic correction is

$$\delta\big(\widehat{\alpha}^{(5)\,-1}\big) = -\frac{N^2}{2^J} = -\frac{9}{128} = -0.0703125. \tag{D.57}$$

The leading QD strong correction is

$$\delta\alpha_s = \frac{1}{\pi \mathrm{e}} \frac{1}{128} \simeq 9.1484 \times 10^{-4}. \tag{D.58}$$

The VEV shift adopted in the main text is

$$\delta_v \equiv \frac{v^{\rm QD} - v^{(1)}}{v^{(1)}} \simeq -4.52 \times 10^{-4}, \tag{D.59}$$

which is comfortably smaller than ε_K, as expected for a finite seam-transport correction.

For the quartic, the adopted shift is

$$\delta\lambda_H \equiv \lambda_H^{\rm QD} - \lambda_H^{(1)} \simeq 0.00437, \tag{D.60}$$

large enough to move the Higgs mass by approximately

$$\delta m_h \simeq 2.08 \text{ GeV}. \tag{D.61}$$

This explains why the Higgs is the most sensitive of the five primary observables to next-order dressing.

D.15 Why the second-order uncertainty on the Higgs is kept conservative

In principle one could assign a much smaller residual uncertainty to the QD Higgs mass by iterating the same suppression logic once more. However, that would overstate the formal completeness of the current preprint. The present paper has reconstructed the second-order structure faithfully within the GG-6 logic, but it has not yet written a fully explicit closed two-loop Wilson-line computation in the 6D bulk. For that reason the QD uncertainty on m_h is kept at the conservative level of 0.30 GeV, which is more realistic for this present stage.

Problems and worked solutions. D.1.1. Derive the electric charge from the neutral-sector rotation: $e = g\sin\theta_W = g'\cos\theta_W$.
Solution. Step 1: the neutral gauge fields (W^3_μ, B_μ) mix into the mass eigenstates (Z_μ, A_μ) by the rotation (D.3); the photon is the combination $A_\mu = \sin\theta_W\, W^3_\mu + \cos\theta_W\, B_\mu$. Step 2: insert into the covariant derivative (D.2) and collect the coefficient of A_μ acting on a field of isospin T^3 and hypercharge Y: it is $g\sin\theta_W\, T^3 + g'\cos\theta_W\, \frac{Y}{2}$. Step 3: for this to be eQ with $Q = T^3 + Y/2$ — one charge, one coupling — the two coefficients must be equal: $e = g\sin\theta_W = g'\cos\theta_W$ (D.4), equivalently $1/e^2 = 1/g^2 + 1/g'^2$. Physics intuition: electromagnetism is not one of the Standard Model's primitive forces — it is the unbroken diagonal remnant of two of them, and its coupling is the parallel-resistor sum of theirs; the dyadic readout of the third step prices exactly this remnant.

D.1.2. Show that exact unification fixes the tree-level weak angle at $\sin^2\theta_W = 3/8$.
Solution. Step 1: embedding hypercharge in a simple group forces the normalization $g_1^2 = \frac{5}{3}g'^2$ (D.9) (the 5/3 is the trace ratio of the hypercharge generator over one fermion family). Step 2: at the unification point $g_1 = g_2 \equiv g$, so $g'^2 = \frac{3}{5}g^2$. Step 3: insert into $\sin^2\theta_W = g'^2/(g^2+g'^2) = \frac{3/5}{1+3/5} = \frac{3}{8}$ (D.10) — a pure group-theory number, no dynamics anywhere. Physics intuition: 3/8 is what the weak angle would be if the three forces were one and undressed; everything between 3/8 and the measured 0.231 is running — which is why the angle at M_Z is a probe of the entire desert between the weak scale and the seam.

D.1.3. Verify the one-loop coefficient $b_2 = -19/6$, including the Higgs' contribution.
Solution. Step 1: for SU(N_c) with n_f Dirac-fermion doublets-worth of matter and n_s complex scalar doublets, the coefficient in the convention of (D.7) is $b = -\frac{11}{3}N_c + \frac{2}{3}\cdot 2n_f\cdot\frac{1}{2} + \frac{1}{3}n_s\cdot\frac{1}{2}$ (adjoint gauge loop; Weyl-fermion and scalar matter loops). Step 2: for SU(2)$_L$: the gauge term gives $-22/3$; the fermions — four doublets per generation (three quark colors plus one lepton), three generations, i.e. twelve Weyl doublets — give $+\frac{2}{3}\cdot 12\cdot\frac{1}{2} = +4$; the single Higgs doublet gives $+\frac{1}{3}\cdot\frac{1}{2} = +\frac{1}{6}$. Step 3: total $b_2 = -\frac{22}{3} + 4 + \frac{1}{6} = -\frac{19}{6}$ (D.8); the same counting gives $b_3 = -11 + 4 = -7$ (no scalar) and, with the $\frac{5}{3}$ normalization, $b_1 = +\frac{41}{10}$. Physics intuition: every particle in the theory votes on how

each force runs — gauge bosons screen-in (asymptotic freedom), matter screens out — and the Higgs' lonely +1/6 in b_2 is the fingerprint of the scalar sector on the gauge flow: remove it and the near-miss of D.1.4 shifts visibly.

D.1.4. Quantify the classic near-miss: where do the couplings actually cross?
Solution. Step 1: with the measured inverse couplings at M_Z ($\alpha_1^{-1} \approx 59.0$, $\alpha_2^{-1} \approx 29.6$, $\alpha_3^{-1} \approx 8.4$) run by (D.7), two couplings cross where $\alpha_i^{-1} - \alpha_j^{-1}$ is consumed by $(b_i - b_j)\ln(\mu/M_Z)/2\pi$. Step 2: the 1–2 crossing: $\ln(\mu/M_Z) = 2\pi\,(59.0 - 29.6)/(41/10 + 19/6) \approx 25.4$, i.e. $\mu_{12} \approx 1.0 \times 10^{13}$ GeV. Step 3: the 2–3 crossing lands at $\mu_{23} \approx 1 \times 10^{17}$ GeV — four orders of magnitude away: the Standard Model *almost* unifies, and measurably does not. Physics intuition: the near-miss is data, not aesthetics — whatever completes the Standard Model must either supply new thresholds that bend the lines together (the supersymmetric bet) or, as the third step of this chapter argues, stop asking the three couplings to meet dynamically at all and derive each from the seam geometry instead.

D.3.1. Evaluate the dyadic electromagnetic code against both measured faces of α.
Solution. Step 1: the closure recursion (D.16) gives $C_\gamma(n) = 2^n$, and the lawful depth is the ledger's odd closure index $J = 7$: $\widehat{\alpha}^{-1}(M_Z) = 2^7 = 128$ (D.17), against the measured running value 127.930(8): agreement to 0.055%. Step 2: the on-shell Sommerfeld face adds the color-cloud triplicity N_c^2: $\alpha^{-1}(0) = 2^J + N^2 = 128 + 9 = 137$ (D.19), against 137.036: agreement to 0.026%. Step 3: both faces from two ledger integers, with the *difference* between them (the running from 0 to M_Z) carried by standard QED — the code prices the endpoints, the textbook prices the path. Physics intuition: the most famous pure number in physics is read here as a power of two plus a square — and the reading is falsifiable, since neither face has a dial to absorb a mismatch; the QD refinement that closes the residual 0.055% at M_Z is priced in the chapter's second-order section; the 0.026% at $q^2 = 0$ is attributed to ordinary QED dressing, not to a QD completion tabulated here.

D.3.2. Verify the rational weak angle $\sin^2\theta_W = 25/108$ and its pull.
Solution. Step 1: the quadratic hypercharge lemma gives $[\mathrm{Tr}(Y_{N,L}^2)]^2 = (5/6)^2 = 25/36$ (D.20); the color-blind neutral readout *averages* over the $N_c = 3$ triplicity sectors, contributing $1/N_c$. Step 2: $\sin^2\theta_W(M_Z) = \frac{1}{3}\cdot\frac{25}{36} = \frac{25}{108} = \frac{L^2}{4N^3} = 0.231481$ (D.22). Step 3: against the measured effective leptonic angle 0.23155, the pull is 0.03% — and the number is a ratio of ledger integers, $L^2/4N^3$, with nothing continuous anywhere. Physics intuition: compare D.1.2 — the textbook's 3/8 lives at the seam and needs the desert's running to reach the data, while the geometric 25/108 is stated *at* M_Z directly: the two readings bracket the same physics from opposite ends, which is exactly how the chapter reconciles them.

D.3.3. Evaluate the strong-coupling closure $\alpha_s^{-1}(M_Z) = \pi e$.
Solution. Step 1: the color seam admits no integer multiplicity; its inverse capacity is the product of the one compact and the one radial primitive (D.26): $\alpha_s^{-1} = \pi e = 8.5397$. Step 2: hence $\alpha_s(M_Z) = 1/\pi e = 0.11710$, against the world average 0.1180 ± 0.0009. Step 3: the pull is 0.9σ — one standard deviation, from two transcendental constants and no dial (and F.10.1 explains why exactly these two constants are the only ones available). Physics intuition: where the abelian sector counts (powers of two), the confining sector *measures* (volumes of the compact and radial primitives)

— the appearance of πe here and $1/\pi$ in the cosmological inventory is one geometry pricing different sectors with the same two lengths.

D.3.4. Verify the ledger's double identity $K = LJ = M^2 + N^2 + L^2$.
Solution. Step 1: from the ledger (D.13), $(M, N, L; J, K) = (1, 3, 5; 7, 35)$: compute $LJ = 5 \times 7 = 35$. Step 2: compute $M^2 + N^2 + L^2 = 1 + 9 + 25 = 35$. Step 3: both equal K — the lock depth is simultaneously the product of the embedding weight and the closure index *and* the sum of the squared multiplicities: one integer, two independent readings, which is the kind of overdetermination that makes a five-integer ledger rigid rather than adjustable (change any entry and both identities break at once). Physics intuition: the ledger is not a list of five free integers but a single interlocked object — the same rigidity that C3's black-hole module leans on when it borrows K for the species-sum scale and that the cosmology paper spends as $e^{-K/2}$ in the baryon asymmetry.

Take-away. *Standard result:* three gauge groups, three measured couplings, one measured angle, and one-loop running that nearly unifies. *What it assumed:* the groups and the couplings themselves — four experimental inputs and one unexplained near-miss. *The geometric reading:* the gauge pattern and the boundary couplings descend from the bulk, the weak angle becomes a geometric ratio, and the two-loop Wilson-line computation is named as the open refinement, with its uncertainty carried conservatively.

E Electroweak symmetry breaking and the Higgs

What this appendix teaches. Electroweak symmetry breaking as a graduate student meets it — the Higgs mechanism, the W and Z masses, the massless photon, the custodial structure — with its free numbers stated plainly; and the geometric reading in which the electroweak scale is set by a Wilson-line selector, that scale an exponentially suppressed, triply filtered shadow of the Planck scale, and the hierarchy problem dissolved by a change in what sets the scale rather than solved by a cancellation — for the selector; the doublet's own stability is O_{11}.

Read alongside: Schwartz (2014) for the modern treatment; Weinberg (1995–1996) at depth; P. W. Higgs (1964) and Englert & Brout (1964) for the originals.

This is the second sector appendix of the course. Its subject is the mechanism that gives the W and Z their masses and leaves the photon massless — electroweak symmetry breaking — together with the scalar that drives it, the Higgs. We again follow the three-step plan: the textbook mechanism as a graduate student meets it; the two free numbers it must take from experiment, and the naturalness puzzle they raise; and finally the geometric derivation, in which the Higgs vacuum value and the quartic coupling descend from a Wilson-line selector, the layering hypothesis and the integer ledger, with the hierarchy problem dissolving into a statement about exponential scale suppression. The textbook development of electroweak symmetry breaking and the Higgs mechanism is given in Schwartz (2014) and Weinberg (1995–1996).

E.1 Step 1 — electroweak symmetry breaking as a graduate student meets it

Why a scalar is needed at all. An unbroken gauge theory has only massless gauge bosons: a mass term $\frac{1}{2}m^2 W_\mu W^\mu$ is not gauge invariant. Yet the weak bosons are heavy, $m_W \simeq 80$ GeV

and $m_Z \simeq 91$ GeV, while the photon is exactly massless. The Standard Model reconciles this by *spontaneous* breaking: the gauge symmetry is intact in the Lagrangian but not in the vacuum. *(Physics intuition: nothing in the laws picks a direction, but the ground state does — like a pencil balanced on its tip that must fall some way. The laws keep their symmetry; the vacuum quietly spends it.)*

The Higgs field and the Mexican-hat potential. One introduces a complex scalar doublet of $\mathrm{SU}(2)_L$ with hypercharge $Y = 1$,

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad V(\Phi) = \mu^2\, \Phi^\dagger \Phi + \lambda\, (\Phi^\dagger \Phi)^2. \tag{E.1}$$

If $\mu^2 < 0$ and $\lambda > 0$ the potential is the famous Mexican hat: the origin is a local maximum and the minima form a circle at

$$\langle \Phi^\dagger \Phi \rangle = \frac{v^2}{2}, \qquad v = \sqrt{\frac{-\mu^2}{\lambda}}. \tag{E.2}$$

Choosing the unitary gauge $\langle \Phi \rangle = \frac{1}{\sqrt{2}}(0,\, v)^{\mathsf{T}}$ breaks $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{\mathrm{em}}$. *(Physics intuition: the field rolls off the central bump into the circular trough. Which point of the trough it picks is irrelevant — they are gauge-equivalent — but that it picks one at all is what breaks the symmetry and sets the scale v.)*

Eaten Goldstones and gauge-boson masses. Of the four real components of Φ, three are massless Goldstone modes; by the Higgs mechanism they become the longitudinal polarizations of $W^\pm$ and Z. Expanding the scalar kinetic term $|D_\mu \Phi|^2$ about the vacuum gives the masses directly,

$$m_W = \tfrac{1}{2}\, g\, v, \qquad m_Z = \tfrac{1}{2}\, v \sqrt{g^2 + g'^2}, \qquad m_\gamma = 0, \tag{E.3}$$

with the photon protected by the unbroken $\mathrm{U}(1)_{\mathrm{em}}$. The single surviving scalar fluctuation h is the physical Higgs, with mass

$$m_h^2 = 2\lambda v^2. \tag{E.4}$$

(Physics intuition: the gauge bosons grow heavy by swallowing the would-be Goldstone modes — a massless gauge boson has two polarizations, a massive one has three, and the extra longitudinal state is exactly the eaten Goldstone. The photon eats nothing and stays massless.)

Custodial symmetry and $\rho = 1$. The doublet structure leaves a residual "custodial" SU(2) that forces the tree-level relation

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = 1, \tag{E.5}$$

verified experimentally to the per-mille level in the Standard Model. In the present construction the relation holds for the doublet itself but is violated at tree level by the lock-scale Kaluza–Klein tower, $\rho - 1 = \alpha T = 1.3 \times 10^{-2}$ (O_{22}, §34.6), which is the paper's standing exposure rather than

a confirmation. In the textbook reading it is one of the sharpest confirmations that the order parameter is a doublet.

Worked example (STANDARD TEXTBOOK PEDAGOGY). The vacuum value is fixed by the Fermi constant, measured from muon decay as $G_F = 1.1664 \times 10^{-5}$ GeV^{-2}. Since $G_F/\sqrt{2} = g^2/(8m_W^2) = 1/(2v^2)$,

$$v = \left(\sqrt{2}\, G_F\right)^{-1/2} = \left(1.6495 \times 10^{-5}\right)^{-1/2} \simeq 246.2 \text{ GeV}. \tag{E.6}$$

With the measured Higgs mass $m_h = 125$ GeV, pinned down by the 2012 ATLAS and CMS discovery of the boson (ATLAS Collaboration, 2012; CMS Collaboration, 2012), the quartic follows from (E.4),

$$\lambda = \frac{m_h^2}{2v^2} = \frac{125^2}{2 \cdot 246^2} \simeq 0.129. \tag{E.7}$$

The machinery again works perfectly — *given* G_F (hence v) and m_h (hence λ) from experiment. Where the two numbers v and λ come from, and why v sits so far below the Planck scale, is exactly what the Standard Model cannot say.

E.2 Step 2 — what electroweak symmetry breaking assumes but cannot explain

The mechanism of Step 1 is exact and experimentally vindicated, but it leaves three questions open, and each is sharper than the gauge-coupling questions of the previous appendix.

First, *the two scalar parameters are pure inputs.* The vacuum value v (or equivalently μ^2) and the quartic λ are two more of the twenty-eight free numbers. Nothing fixes $v \simeq 246$ GeV rather than any other scale, and nothing fixes $\lambda \simeq 0.13$. Indeed the sign $\mu^2 < 0$ that triggers breaking is simply *posited*: the Standard Model does not explain why the vacuum breaks the symmetry at all, only what follows if it does.

Second, and most famously, *the hierarchy and naturalness problem.* The Higgs mass-squared receives quantum corrections that scale quadratically with whatever heavy scale Λ the theory is embedded in,

$$\delta m_h^2 \sim \frac{\Lambda^2}{16\pi^2} \times (\text{couplings}). \tag{E.8}$$

If Λ is the Planck or GUT scale, δm_h^2 is some thirty orders of magnitude larger than the observed m_h^2; keeping the physical Higgs light then requires the bare parameter to cancel the correction to one part in 10^{34}. The electroweak scale is, in this sense, unnaturally small, and no symmetry of the Standard Model protects it. *(Physics intuition: a scalar mass is like a microphone left next to a loudspeaker — it picks up every high-energy mode it couples to. Fermion masses are protected by chiral symmetry and gauge-boson masses by gauge symmetry, but a lone scalar has no such shield, so its mass is dragged up to the highest scale in the problem unless something cancels it by hand.)*

Third, *vacuum metastability.* Run to high energy, the measured λ drifts toward zero and turns slightly negative near 10^{10}–10^{12} GeV, placing our vacuum on the knife-edge between stability and decay. That the Standard-Model parameters should land precisely on this edge is an unexplained coincidence in the bottom-up picture.

These three — the two inputs, the unnatural smallness of v, and the metastable λ — are what the geometric derivation must address. It does so by refusing to treat the Higgs as a primitive scalar at all.

E.3 Step 3 — the geometric derivation: the electroweak scale as a holonomy shadow

In GG-Theory the Higgs is not a field with a freely chosen potential. Its branch is set by a local chart on a Wilson-line holonomy — the order parameter that Hosotani showed to be the natural electroweak variable on a multiply connected space (Y. Hosotani, 1983; K. G. Wilson, 1974), promoted to the electroweak order parameter of the cohort in GG-A6-GRIP (Arisaka, 2026, A6-GRIP) — and its vacuum value and quartic are read off the same integer ledger $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ that fixed the gauge couplings. We give the derivation first in its essential geometric form (the order parameter, the lock scale, the branch angle, the quartic) and then in explicit arithmetic, cross-checked against the gauge-sector numbers of Appendix D.3.

E.3.1 The order parameter is a Wilson-line holonomy

GG-A6-GRIP proves that the only admissible global order parameter for electroweak breaking along the compact GG-6 direction is the conjugacy class of the Wilson line, and any local Higgs field is a *chart* on that holonomy orbit — but the doublet it charts is required by the fermion mass sector and selected as the minimal $\mathbf{5}_H$, the $(\mathbf{1}, \mathbf{2})_{+1/2}$ of the bulk $\mathbf{5}_H$, and not a mode of the connection (§23.3). What the holonomy removes is the freedom in the scalar's potential, which is inherited from the geometry the angle charts; what it does not remove is the scalar. *(Physics intuition: in ordinary field theory the Higgs is an actor who may be handed any script — any μ^2 and λ. Here the Higgs is the shadow of a rod threaded once around a compact circle; you cannot paint the shadow independently of the rod. The potential the shadow obeys is whatever the rod's geometry dictates, and the geometry is already fixed.)* Because the order parameter is an angle, the boundary-visible scalar amplitude is a trigonometric projection of the selected holonomy branch,

$$v = M_{\text{lock}} \sin \theta_H^\star, \tag{E.9}$$

where $\theta_H^\star$ is the stable branch selected by the GRIP cascade DGI $\Rightarrow$ GSSB $\Rightarrow$ GA of GG-A6-GRIP, and M_{lock} is the first visible electroweak scale.

E.3.2 The lock scale: where the electroweak scale comes from

The first boundary scale available to the electroweak packet is not a free parameter but the *lock scale*, the Planck scale pulled down by the scale profile e^{-K},

$$M_{\text{lock}} = \tfrac{1}{2}\, M_{\text{Pl}}\, e^{-K}, \qquad K = 35. \tag{E.10}$$

The exponent $K = 35$ is the ledger integer (D.14), the same one that satisfies $K = LJ = M^2+N^2+L^2$; the factor $\frac{1}{2}$ is the binary endpoint convention of the radial interval (Appendix D.5), not a tunable

coefficient. Numerically

$$M_{\text{lock}} = \frac{1.2209 \times 10^{19}\ \text{GeV}}{2\,e^{35}} \simeq 3.849 \times 10^{3}\ \text{GeV}. \tag{E.11}$$

(Physics intuition: the scale profile e^{-K} is an exponential elevator. The Planck scale is the penthouse, 10^{19} GeV; riding down $K = 35$ e-folds of the scale profile drops one to the TeV lobby. The enormous ratio between the Planck and electroweak scales is therefore not a fine-tuning but a count*: thirty-five e-folds, fixed by the same integer that closes the bulk. This is the first half of the answer to the hierarchy problem — the big scale and the small scale are separated by an integer number of scale e-folds, not by an accident.)*

E.3.3 The branch angle: the repeated doublet fraction

The selected stable holonomy branch is not arbitrary. At each triplicity layer of the reduction, only the electroweak doublet — two directions out of the unified L-block — survives into the visible branch, contributing one factor $2/L$. The fraction $2/L$ is derived once, as the doublet's survival fraction. That the *same* fraction reappears at each of the N layers is asserted, not derived: it is the content of the triplicity-layering lemma L2 (Appendix P), which is not proved here, and it is the assumption on which the electroweak scale rests (main-text Theorem 34.1, Step 3). Granting it, after N layers,

$$\sin\theta_H^{\star} = \left(\frac{2}{L}\right)^{N} = \left(\frac{2}{5}\right)^{3} = \frac{8}{125} = 0.064. \tag{E.12}$$

(Physics intuition: the holonomy twist is filtered through three successive doublet gates, each passing the fraction $2/5$ of the available block. Three gates in series pass $(2/5)^3$, a little over six percent. That there are three such gates, and that each passes the same fraction, is the assumption; the fraction itself is not.)

E.3.4 The Higgs vacuum value

Combining the projection (E.9), the lock scale (E.10), and the branch angle (E.12) gives the Wilson–Hosotani vev (main-text Theorem 34.1, T-SM-30.1),

$$\boxed{v = M_{\text{lock}} \left(\frac{2}{L}\right)^{N} = \tfrac{1}{2}\, M_{\text{Pl}}\, e^{-K} \left(\frac{2}{L}\right)^{N} \simeq 246.33\ \text{GeV}.} \tag{E.13}$$

Every symbol on the right is fixed: M_{Pl} by c, h, G_N; $K = 35$, $L = 5$, $N = 3$ by the ledger. There is no continuous input — and one structural assertion, the repetition of the doublet fraction over the N layers (lemma L2). *(Physics intuition: picture a lighthouse beam wound around a spiral staircase. The bulk knows the whole spiral angle, but the boundary observer sees only the horizontal shadow the beam casts at a selected height. The lock scale fixes the height; the factor $(2/L)^N$ is how much of the spiral projects into the visible doublet channel. The Higgs vacuum value is the horizontal shadow of a selected geometric twist — nothing adjustable, though the three-fold repetition of the filter is assumed rather than proved.)*

E.3.5 The quartic coupling and the Higgs mass

The quartic is the second and final scalar number, and it too is no longer free once hypothesis **H-L2** is granted: the quartic self-stiffening of the selected branch is the neutral fraction of the doublet raised to the triplicity index. MSMF forbids a new continuous coefficient, so the quartic must be built from the discrete packet already in hand; under H-L2 the suppression is dyadic, and the only relevant depth is the triplicity depth N_c. One factor $\frac{1}{2}$ per layer gives (main-text Theorem 35.1, T-SM-31.1)

$$\boxed{\lambda_H = \left(\frac{1}{2}\right)^N = 2^{-N} = \frac{1}{8}.} \tag{E.14}$$

(Physics intuition: the quartic measures how stiffly the branch resists being pushed away from its minimum — the curvature of the trough floor. "One half per triplicity layer" is the same dyadic logic that folded the photon seam in the gauge appendix, here applied to the scalar's self-stiffening: three layers, three halvings, $1/8$*. The scalar sector and the Abelian gauge sector share one arithmetic of halving.)* The physical Higgs mass then follows with no further input,

$$m_h = \sqrt{2\lambda_H}\, v = \sqrt{\tfrac{1}{4}}\, v = \frac{v}{2} \simeq 123.1 \text{ GeV}, \tag{E.15}$$

the small remaining gap to the measured 125.10 GeV being closed by the quantum-defect completion of Part VI.

E.3.6 Explicit arithmetic and the descendant masses

The explicit numbers are collected in the gauge-and-Higgs arithmetic block of the previous appendix; we record the chain here for completeness. With $M_{\text{lock}} \simeq 3848.93$ GeV (Eq. D.34) and $\sin\theta_H^\star = (2/5)^3$,

$$v^{(1)} = M_{\text{lock}} \left(\tfrac{2}{5}\right)^3 \simeq 246.33 \text{ GeV} \quad \text{(Eq. D.51)}, \tag{E.16}$$

and with the gauge couplings $g_2 \simeq 0.6512$, $g_Y \simeq 0.3574$ of Eq. (D.27),

$$m_W^{(1)} = \tfrac{1}{2} g_2 v^{(1)} \simeq 80.21 \text{ GeV} \quad \text{(Eq. D.52)}, \tag{E.17}$$

$$m_Z^{(1)} = \tfrac{1}{2} v^{(1)} \sqrt{g_2^2 + g_Y^2} \simeq 91.50 \text{ GeV} \quad \text{(Eq. D.53)}, \tag{E.18}$$

$$m_h^{(1)} = \sqrt{2\lambda_H^{(1)}}\, v^{(1)} \simeq 123.12 \text{ GeV} \quad \text{(Eq. D.55)}, \tag{E.19}$$

and the Fermi constant $G_F = 1/(\sqrt{2}\, v^2)$ becomes a descendant rather than an input. Thus the entire electroweak mass spectrum — v, m_W, m_Z, m_h, G_F — flows from the two ledger readings v and λ_H, which in turn flow from c, h, G_N and the integers $(N, L, K) = (3, 5, 35)$.

E.3.7 Honesty triage and the dissolution of the hierarchy problem

In the codebook (§8) the vev v is recorded as **Derived (cond.)**: every factor in (E.13) is fixed, granting the triplicity-layering lemma L2. The quartic λ_H is recorded as **Derived (cond.)** on lemma L6 — the same hypothesis **H-L2**, in its second instance: the dyadic value $2^{-N} = 1/8$ is the

neutral fraction of the doublet raised to the triplicity index, and promoting it to an unconditional grade requires a QD transport computed from the ledger before comparison, named under L6; the printed λ_H^{QD} is the datum read backwards, not that transport.

It is worth stating plainly what has happened to the naturalness problem. In the bottom-up picture the smallness of v relative to the Planck scale is a fine-tuning of one part in 10^{34}. Here it is two compositions of fixed structure: an exponential scale profile e^{-K} with $K = 35$, and a triple doublet filter $(2/L)^N$ with $N = 3, L = 5$. Neither is adjustable, and neither is tuned; both are counts. The quadratic sensitivity (E.8) that powers the hierarchy problem presupposes a primitive scalar whose mass can be dragged to the cutoff. What is protected here by the topology of the compact direction — in the same way a gauge-boson mass is protected by gauge symmetry — is the *selector*, the holonomy whose stationary point fixes where the scale sits; the doublet itself is a bulk scalar (§23.3), and its radiative stability is the little hierarchy of O_{11}, not dissolved here. The *large* hierarchy, in this reading, is not solved by a cancellation but *dissolved* by a change in what sets the scale. The electroweak scale is small because it is an exponentially suppressed, triply filtered shadow of the Planck scale — and the scale depth and the filter depth are both integers in the ledger.

Problems and worked solutions. **E.1.1. Extract the vacuum value from the Fermi constant.**
Solution. Step 1: at energies far below m_W, W exchange in muon decay collapses to the contact interaction with strength $G_F/\sqrt{2} = g^2/8m_W^2$. Step 2: insert $m_W = gv/2$ (E.3): the coupling cancels, $G_F/\sqrt{2} = 1/2v^2$. Step 3: invert with the measured $G_F = 1.1664 \times 10^{-5}\,\mathrm{GeV}^{-2}$: $v = (\sqrt{2}\,G_F)^{-1/2} = 246.2$ GeV (E.6). Physics intuition: the weak scale was measured in 1934 — in the lifetime of the muon's ancestor decays — half a century before the W was seen: v is the rare fundamental scale extracted from a tabletop lifetime, because the contact strength remembers the vev even when the energy cannot resolve the boson.

E.1.2. Prove $\rho = 1$ at tree level and state what it tests.
Solution. Step 1: from the doublet's kinetic term, $m_W^2 = \frac{1}{4}g^2v^2$ and $m_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2$. Step 2: form the ratio (E.5): $\rho = m_W^2/(m_Z^2\cos^2\theta_W) = g^2/[(g^2+g'^2)\cdot g^2/(g^2+g'^2)] = 1$ identically — the couplings and the vev cancel. Step 3: the cancellation is a consequence of the custodial SU(2) of the *doublet* representation: a scalar triplet with a vev v_T shifts ρ by $O(v_T^2/v^2)$, so the measured $\rho \approx 1.0004$ confines any triplet to the few-GeV level. Physics intuition: $\rho = 1$ is the Standard Model's fingerprint of *which* representation broke the symmetry — the data are not merely consistent with a doublet, they nearly exclude everything else; any completion (this chapter's selected doublet included) must reproduce a doublet's custodial structure; this one does for the doublet alone; the tree-level correction to T from the Kaluza–Klein tower is O_{22}'s.

E.1.3. Extract the quartic coupling from the Higgs discovery.
Solution. Step 1: the potential (E.1) gives $m_h^2 = 2\lambda v^2$ (E.4). Step 2: invert with the 2012 discovery value: $\lambda = m_h^2/2v^2 = 125^2/(2\cdot 246^2) = 0.129$ (E.7). Step 3: note what happened: the last parameter of the minimal Standard Model was measured — and it landed at a small, unexplained number, close to (but, at this order, not equal to) simple fractions. Physics intuition: λ is the Higgs talking to itself; its measured smallness is what makes the electroweak vacuum famously near-critical — and E.3.2 reads the number as a dyadic depth, 2^{-N}, rather than a dial.

E.1.4. Derive the unitarity ceiling that guaranteed the LHC would find something.
Solution. Step 1: without a Higgs, the amplitude for longitudinal W_LW_L scattering grows like G_FE^2 — the gauge cancellations leave the growth the scalar was born to cancel. Step 2: partial-wave unitarity ($|a_0| \le \frac{1}{2}$) then fails at $\sqrt{s} \sim \left[8\pi\sqrt{2}/(3G_F)\right]^{1/2}$. Step 3: numerically $\left[8\pi\sqrt{2}/(3\times 1.1664\times 10^{-5})\right]^{1/2} \approx$ 1.0 TeV — by that energy, *either* a Higgs appears *or* the weak interaction becomes strong and new resonances appear: no third option. Physics intuition: this inequality is why the LHC was a no-lose experiment — the machine was built to reach the energy at which the Standard Model must show its completion; at 125 GeV the scalar arrived with room to spare, and the quartic stayed weakly coupled (E.1.3).

E.3.1. Evaluate the geometric vev formula.
Solution. Step 1: the chain (E.13): $v = M_{\text{lock}}\,(2/L)^N$ with the lock scale $M_{\text{lock}} = \frac{1}{2}M_{\text{Pl}}e^{-K} =$ 3.849 TeV (E.11). Step 2: $(2/5)^3 = 0.064$; $v = 3849 \times 0.064 = 246.3$ GeV. Step 3: against the Fermi-constant extraction of E.1.1 (246.22), the ledger reproduces the weak scale at the per-mille level from $(M_{\text{Pl}}; N, L, K)$ and nothing else. Physics intuition: the electroweak scale is here the end of a pipeline — $M_{\text{Pl}} \to e^{-K} \to$ three branch filters of $2/L$ each — every stage an integer of the ledger: E.3.3 turns this into the hierarchy statement, and the collider falsifier at the first stage (3.85 TeV) is standing.

E.3.2. Run the dyadic quartic to the Higgs mass, and state the register exactly.
Solution. Step 1: the only admissible dimensionless suppression is dyadic and the relevant depth is the triplicity: $\lambda_H = 2^{-N} = \frac{1}{8}$ (E.14). Step 2: then $m_h = \sqrt{2\lambda_H}\,v = v/2 = 123.1$ GeV at first order (E.15) — compare the measured 125.10: the first-order value lands within 1.6%. Step 3: the QD completion (the chapter's second-order section) supplies the remaining dressing and carries, deliberately, the conservative 0.30 GeV theory uncertainty — the register is: structure derived, final digit QD-completed, uncertainty stated. Physics intuition: $m_h = v/2$ at leading order is the cleanest slogan in the codebook — the Higgs weighs half its own vacuum — and the 1/8 behind it is three halvings, one per triplicity layer: the same arithmetic that put N^2 on top of 2^J in D.3.1.

E.3.3. State the hierarchy as an identity of integers.
Solution. Step 1: measured, $v/M_{\text{Pl}} = 246.33/1.2209 \times 10^{19} = 2.018 \times 10^{-17}$ — the infamous seventeen orders. Step 2: the ledger's formula: $\frac{1}{2}e^{-35}(2/5)^3 = 2.018 \times 10^{-17}$ — the same number, digit for digit, from $K = 35$, $L = 5$, $N = 3$. Step 3: the hierarchy "problem" is thereby restated: not *why is a continuous parameter tuned small*, but *why these integers* — a question the ledger answers upstream, and a question that cannot be un-asked by radiative corrections because (E.3.4) nothing continuous is being protected. Physics intuition: exponential hierarchies from order-one integers are cheap — e^{-35} does in one factor what fine-tuning buys with seventeen digits; the entire naturalness anxiety was priced on the assumption that the small number was an input.

E.3.4. Why does the electroweak scale have no quadratic divergence to cancel — and what that does not cover?
Solution. Step 1: the ordinary complaint (E.8): a primitive scalar's mass-squared drags $\Lambda^2/16\pi^2$ from every loop. Step 2: here the variable that fixes the scale is a Wilson-line *phase* θ_H, compact and periodic (F.10): no local counterterm θ_H^2 exists (it is not gauge invariant — only $\cos\theta_H$ structures

are), so the effective potential (F.12) is finite, and its stationary point is set by the compactification scale, not the cutoff. Step 3: the hierarchy then enters multiplicatively (F.14) — filters compose by products, not subtractions — which is why no cancellation, and no partner spectrum to enforce one, is required to set the scale. Step 4, the limit: the doublet whose vacuum value lands at that point is a bulk scalar, not the phase (§23.3); its own loops at the lock scale are the little hierarchy, registered at O_{11}, and this problem does not address them. Physics intuition: you cannot quadratically renormalize an angle — periodicity is the symmetry that scalar-mass counterterms cannot break, and the Hosotani mechanism is that protection made dynamical; F.14.1 works the potential.

Take-away. *Standard result:* the Higgs mechanism gives the W and Z their masses, keeps the photon massless, and fits the data — with the scale and the quartic taken from experiment. *What it assumed:* a primitive scalar with a chosen potential, and no answer to why its scale is small. *The geometric reading:* the scale is selected by a Wilson line and is $M_{\rm Pl}$ filtered through integer depths; the large hierarchy is dissolved, not solved. The doublet is a required, selected scalar, and its little hierarchy is O_{11}'s.

F Operational export, radial flow, and Wilson–Hosotani geometry

What this appendix teaches. The connective tissue of the course: how the completed gauge/Higgs, fermion-mass, CKM, and PMNS blocks feed the rest of the paper — the operational export map, the radial flow, and the Wilson–Hosotani geometry that the sector chapters consume. A reference chapter, to be consulted as the course proceeds rather than worked through in one sitting.

F.1 How the gauge/Higgs, fermion-mass, CKM, and PMNS/Majorana derivation feeds the rest of GG-A9-SM

The main text already gave the roadmap of the full paper. Here we state explicitly how the completed 28-parameter block becomes the input for the deep-origin synthesis of the Standard Model.

F.2 CKM structure (completed in the present version)

The quark-mixing problem has been solved at the working level. The diagonal up-type and down-type masses fixed by Section 49.2 were promoted to full family-link textures in Section 52.4, and their left-handed misalignment produced the four CKM parameters without introducing any new fitted flavor constant. This completed quark block is now an input, not a future task.

F.3 PMNS and Majorana structure (completed in the present version)

The lepton side has now been completed at the same working level. The charged-lepton dyadic block and the Majorana neutrino block were promoted in Section 54.3 to a full family-projector misalignment problem, and that misalignment produced the three PMNS angles, the leptonic Dirac phase, and the two physical Majorana phases without introducing any new empirical mixing constant.

F.4 Final CP angle

Only one CP-related parameter remains outside the already-derived CKM and PMNS/Majorana counting: the strong-CP-like final angle. Because every ordinary Yukawa mass and every ordinary flavor quantity has now been fixed, that angle is isolated as a genuinely topological or BI-6-level question rather than a generic flavor ambiguity.

F.5 Why the remaining problem is dramatically narrower than the present one

The present paper had to solve the scales, the gauge couplings, the Higgs sector, the diagonal masses, the CKM quark-misalignment block, and the PMNS/Majorana lepton-misalignment block all at once. The remaining work is much narrower: it does not need to discover new normalization scales, new mass codebooks, or new family matrices. It needs only to identify the remaining BI-6/topological CP datum of already-fixed sectors.

F.6 Detailed derivation of the radial renormalization group from the sixth dimension

This appendix expands Section 22 and makes explicit why the GG-6 statement

$$r = \ln\left(\frac{E_{\mathrm{GUT}}}{\mu}\right)$$

is not a decorative analogy but the exact coordinate statement that turns bulk transport into boundary running. The purpose of the appendix is not to introduce any new assumption. It is to show that once the GG-6 metric and the GDOS projection postulates are accepted, a radial renormalization law follows with almost no freedom left (Arisaka, 2026, A2-GG6; Arisaka, 2026, A5-GDOS).

F.7 Redshift and the operational meaning of the radial coordinate

The GG-6 metric is written in the form

$$ds_6^2 = e^{-2A(r)} g_{\mu\nu}(x) dx^\mu dx^\nu + dr^2 + R_\theta^2\, d\theta^2. \tag{F.1}$$

A local process at radial position r with proper energy scale μ_{loc} is seen by the four-dimensional boundary observer with a redshifted energy

$$\mu(r) = \mu_{\mathrm{loc}}\, e^{-A(r)}. \tag{F.2}$$

Choosing the UV anchor at the GUT seam means normalizing the local reference scale to $\mu_{\mathrm{loc}} = E_{\mathrm{GUT}}$ at the top of the throat. In the linearized throat approximation used throughout the GG-6 construction, $A(r) = kr$. Measuring r in units of k^{-1} gives

$$\mu(r) = E_{\mathrm{GUT}} e^{-r}, \qquad r = \ln\left(\frac{E_{\mathrm{GUT}}}{\mu}\right). \tag{F.3}$$

The first equality says that every equal radial step multiplies the local energy by the same factor. The second says that the radial coordinate is literally the logarithmic RG scale.

This point is conceptually important. In ordinary four-dimensional field theory one introduces the running scale μ to reorganize perturbation theory. In GG-6 the same scale appears geometrically before perturbation theory is ever discussed. The renormalization variable is therefore not an external bookkeeping convention imposed on the theory after the fact. It is already present in the bulk geometry.

F.8 From bulk shell integration to the differential beta law

Consider a gauge-field sector inside the composite connection of GG-6. The six-dimensional kinetic term is schematically

$$S_{\text{gauge}} = -\frac{1}{4g_6^2}\int d^4x\, dr\, d\theta\, \sqrt{-G}\, G^{MP}G^{NQ}F_{MN}F_{PQ}. \tag{F.4}$$

To obtain a boundary-effective action, one integrates over the compact angle and then over radial shells. Let the shell $[r, r+dr]$ be thin enough that the couplings and scale profile vary only infinitesimally across it. The shell contributes a correction to the four-dimensional inverse coupling of the form

$$d\left(\frac{1}{g_4^2}\right) = \Pi(r)\, dr, \tag{F.5}$$

where $\Pi(r)$ is the shell polarization density, i.e. the integrated effect of the allowed GG-6 degrees of freedom on that gauge channel.

Equation (F.5) is the geometric origin of the RG law. Passing from g_4 to a generic coupling g_i, and using $d\ln\mu = -dr$, one obtains

$$\mu\frac{dg_i}{d\mu} = \beta_i(g) \qquad \Longleftrightarrow \qquad \frac{dg_i}{dr} = -\beta_i(g). \tag{F.6}$$

This is precisely the radial beta equation quoted in the main text. The minus sign is not optional: moving deeper into the throat means moving toward lower energy.

At one loop, when the shell response is linear in the quadratic Casimir data, one recovers the familiar form

$$\frac{d}{dr}\,\alpha_i^{-1}(r) = \frac{b_i}{2\pi}, \qquad \alpha_i^{-1}(r) = \alpha_i^{-1}(0) + \frac{b_i}{2\pi}\,r. \tag{F.7}$$

The important difference from an ordinary textbook derivation is that the coefficients are not the whole story. In the present framework, the shell data must still respect BI-6, orbifold parity, OGN-5 normalization, and the GDOS projection discipline. The continuum running is therefore only the transport skeleton. The seam values themselves remain discretely constrained.

F.9 Stationary seams as evaluation surfaces of the integrated flow

The main text used language such as the lock seam and the neutral seam. It is helpful to state their RG meaning explicitly. Let

$$\alpha_i^{-1}(r_*) = \alpha_i^{-1}(0) + \int_0^{r_*} \frac{b_i(r)}{2\pi}\, dr + \Delta_i^{\mathrm{seam}}. \tag{F.8}$$

The integral term is the continuum radial transport. The term Δ_i^{seam} collects the finite matching conditions imposed by the particular stationary seam at which the observable is read out. The first-order predictions of the present paper should be understood as the leading discrete representatives of (F.8). The QD corrections are then finite next-order refinements of the seam matching, not a change in the underlying geometry.

This viewpoint explains why the running picture and the integer-coded formulas fit together rather than compete. The RG part transports information through the throat. The OGN/BI-6 part decides which channels are actually admissible at the measurement seam.

F.10 Why only two transcendental numbers survive the transport

No hidden empirical numbers are used in the manuscript. At first sight one might wonder why the final formulas still involve π and e. The radial derivation explains this. The only two irreducible geometric sources of transcendental structure are

1. the compact angular measure on S^1_θ, which introduces 2π;
2. the exponential scale/redshift law, which introduces e^{-r}.

There is no independent geometric origin in the current preprint system for any other transcendental constant. Therefore the appearance of π and e in the gauge/Higgs sector is not a numerical convenience. It is the direct shadow of the only two non-algebraic structures present in the minimal GG-6 stage.

F.11 A practical summary for later sections of GG-A9-SM

Later parts of GG-A9-SM will need the same logic in the Yukawa and neutrino sectors. The operational recipe is the same in every case:

1. write the admissible six-dimensional operator in the anomaly-free GG-6 bulk;
2. transport it radially using (F.6);
3. evaluate it on the appropriate GDOS stationary seam;
4. impose OGN-5 and BI-6 selection rules to eliminate non-constitutional freedom.

Thus the radial RG is not merely one ingredient among many. It is the common transport law that every later parameter derivation must share.

F.12 Hierarchy and naturalness in GG-6 revisited in full detail

On the withdrawn identification. This subsection and those that follow it through the closure relation (F.24), together with the worked problems F.14.1 and F.3.2, were written on the reading that the Higgs doublet is the boundary image of the Wilson-line phase; that reading is withdrawn in §23.3. What survives, read on the selector: the holonomy angle is compact, its effective potential is finite and forced, and its stationary point sets the electroweak scale with no dial. What does not: the doublet is the $(\mathbf{1},\mathbf{2})$ of the bulk $\mathbf{5}_H$, not the phase, its quartic rests on hypothesis H-L2 (lemma L6), and its own radiative stability is O_{11}'s. Equations (F.11), (F.13) and (F.22) below are the withdrawn reading's and are kept as its record.

The main text stated that the hierarchy and naturalness problems are not merely improved numerically but reformulated conceptually. Because this claim is central to the whole GG-A9-SM program, it deserves a slower and more explicit argument.

F.13 Why the ordinary 4D hierarchy problem appears severe

In a purely four-dimensional description with a fundamental local Higgs scalar, one writes schematically

$$m_H^2(\Lambda) = m_{H,0}^2 + c_1\Lambda^2 + c_2 m^2 \ln\!\left(\frac{\Lambda}{\mu}\right) + \cdots . \tag{F.9}$$

If Λ is taken near the Planck or GUT scale, then obtaining a weak-scale physical Higgs mass seems to require a delicate cancellation between quantities of wildly different magnitude. This is the standard form of the naturalness complaint.

The important GG-6 observation is that (F.9) uses the wrong primitive variable for the UV problem. It assumes from the outset that the Higgs is a local scalar field whose mass term is fundamental. For the selector that is the wrong variable; for the doublet, which this paper now reads as a bulk scalar with a mass term of its own, the question is O_{11}'s — the reading below, on which the preprints did not identify the Higgs that way, is the withdrawn one.

F.14 The selector's ultraviolet variable is a Wilson-line phase (withdrawn as the doublet's)

In the present framework the primitive compact degree of freedom is the holonomy angle

$$\Theta_H = g_6 \oint_{S^1_\theta} A_\theta \, d\theta, \qquad \Theta_H \sim \Theta_H + 2\pi. \tag{F.10}$$

The periodic identification is essential. It says that the fundamental object is compact and gauge-geometric, not an unconstrained local scalar coordinate. On the withdrawn reading, only after projection to the four-dimensional boundary did one write a local field variable h near a selected vacuum,

$$h = f_H\,(\Theta_H - \Theta_0) \qquad \text{(withdrawn; see §23.3)}, \tag{F.11}$$

where Θ_0 is the stationary point selected by the finite Hosotani potential and f_H is the appropriate normalization constant for the boundary mode.

Once the ultraviolet degree of freedom is the compact phase (F.10), the usual intuition behind an arbitrary local counterterm $\Lambda^2 h^2$ becomes misleading. Local UV-sensitive pieces can renormalize vacuum energy and other phase-independent quantities, but the curvature of the periodic potential at a given minimum is a genuinely nonlocal observable tied to transport around the compact direction.

F.15 Why the Hosotani potential is finite in the Higgs direction

The most useful schematic form of the effective potential is

$$V_{\rm eff}(\Theta_H; r) = \sum_{n=1}^{\infty} C_n(r) \cos(n\Theta_H). \tag{F.12}$$

Each Fourier coefficient $C_n(r)$ is produced by matter and gauge transport around the compact direction. The crucial structural point, already familiar from gauge–Higgs unification, is that divergent local pieces are independent of Θ_H, whereas the Θ_H-dependent terms are finite because they probe the global holonomy around the circle.

On that reading the physical Higgs mass was not the coefficient of a primitive quadratically divergent local operator but the curvature of a finite periodic potential at its minimum,

$$m_h^2 = \frac{1}{f_H^2} \left. \frac{\partial^2 V_{\rm eff}}{\partial \Theta_H^2} \right|_{\Theta_H = \Theta_0} \qquad \text{(withdrawn; see §23.3).} \tag{F.13}$$

Likewise the quartic self-interaction is controlled by the higher derivatives of the same finite function. For the *selector* the naturalness problem is therefore not solved by claiming that radiative corrections vanish. It is solved by recognizing that what radiatively corrects is a finite global quantity rather than a free local mass term; for the doublet, which is not the phase, the question is O_{11}'s.

F.16 The hierarchy becomes multiplicative rather than subtractive

The hierarchy itself is already visible before any loop calculation. The lock scale is

$$M_{\rm lock} = \frac{E_{\rm Pl}}{2e^{35}}, \tag{F.14}$$

and the leading electroweak VEV is

$$v^{(1)} = M_{\rm lock} \left(\frac{2}{5}\right)^3. \tag{F.15}$$

Thus the electroweak scale is obtained by successive multiplicative projections from the Planck anchor. There is no stage in the calculation where two enormous unrelated numbers are subtracted to many decimal places in order to recover a small residue. The scale chain is geometric and factorized.

The significance of the fixed integer $K = 35$ can be stated cleanly. If one were to treat K as a continuous dial, then

$$\frac{\partial \ln M_{\rm lock}}{\partial K} = -1. \tag{F.16}$$

But in the GG-6 constitution K is not a dial. It is fixed by OGN-5. Equation (F.16) therefore measures sensitivity to a variation that the theory does not allow. From the constitutional point of view there is nothing to tune.

F.17 What remains after the conceptual reformulation

The previous paragraphs do not mean that every quantitative question disappears. Finite seam-matching corrections remain. The QD layer precisely represents those effects. In particular, the VEV and Higgs mass still receive controlled next-order shifts. What disappears, for the selector, is the special ultraviolet instability that comes from treating the scale-setting variable as a primitive pointlike scalar with an arbitrary bare mass parameter; for the doublet it does not, and that is O_{11}.

This distinction matters for the later parameter program. Once the gauge/Higgs background is finite and geometrically fixed, the remaining fermion and flavor sectors inherit a stable electroweak normalization rather than an ad hoc one. That is why hierarchy and naturalness had to be addressed before the final topological completion and the deep-origin synthesis.

F.18 Wilson–Hosotani reduction and the emergence of the 4D Higgs field

This appendix gives a slower derivation of the statement — withdrawn as the doublet's, retained for the selector — that the electroweak selector is a boundary image of the compact gauge connection rather than an elementary four-dimensional scalar added by hand; the doublet it selects is the bulk $\mathbf{5}_H$'s (§23.3).

F.19 Orbifold parities and the separation of gauge components

The GG-6 construction uses an orbifold-type boundary condition with parity matrix

$$P = \mathrm{diag}(+1, +1, +1, -1, -1). \tag{F.17}$$

The parity action distinguishes the ordinary four-dimensional gauge components from the compact component according to

$$A_\mu(x, -\theta) = P\, A_\mu(x, \theta)\, P^{-1}, \qquad A_\theta(x, -\theta) = -P\, A_\theta(x, \theta)\, P^{-1}. \tag{F.18}$$

The even zero modes of A_μ generate the surviving low-energy gauge symmetry, while selected components of A_θ in the broken directions remain as scalar boundary modes. This is the geometric origin of the electroweak selector in the GG-6 language (Arisaka, 2026, A2-GG6; Arisaka, 2026, A5-GDOS); the doublet is not among these modes.

F.20 Gauge-invariant order parameter: the Wilson line

Because A_θ itself is gauge dependent, the physically meaningful order parameter is the compact holonomy

$$W_\theta = \mathcal{P}\exp\left(ig_6\oint_{S^1_\theta} A_\theta\, d\theta\right) = \exp(i\Theta_H) \quad \text{in the abelianized notation.} \tag{F.19}$$

This quantity is globally defined, periodic, and directly sensitive to the compact geometry. The vacuum chooses a particular equivalence class of W_θ, or equivalently of Θ_H, once the finite effective potential is generated.

The physical intuition is straightforward. A local scalar takes a value at a point. The Wilson-line selector instead measures the integrated gauge connection around the compact direction. It is therefore a global quantity. That is exactly why the electroweak selector fits naturally into the GDOS viewpoint, where observable data arise from projected global structure rather than from an isolated microscopic point value.

F.21 Expansion of the effective action around the selected vacuum

Let $\Theta_H = \Theta_0 + \delta\Theta$ be a stationary vacuum. Expanding the finite effective action around this point gives

$$V_{\text{eff}}(\Theta_H) = V_0 + \frac{1}{2}V_2\,(\delta\Theta)^2 + \frac{1}{3!}V_3\,(\delta\Theta)^3 + \frac{1}{4!}V_4\,(\delta\Theta)^4 + \cdots, \tag{F.20}$$

where $V_n \equiv \partial^n_{\Theta_H} V_{\text{eff}}|_{\Theta_0}$. In a CP-even vacuum the odd derivatives vanish or are strongly suppressed, and the quadratic and quartic terms dominate the low-energy Higgs dynamics.

Introducing the canonically normalized boundary field $h = f_H\delta\Theta$, one obtains

$$V_{\text{eff}}(h) = V_0 + \frac{1}{2}\left(\frac{V_2}{f_H^2}\right)h^2 + \frac{1}{4!}\left(\frac{V_4}{f_H^4}\right)h^4 + \cdots. \tag{F.21}$$

On the withdrawn reading the physical Higgs mass and quartic were not independent objects:

$$m_h^2 = \frac{V_2}{f_H^2}, \qquad \lambda_{H,\text{eff}} \propto \frac{V_4}{f_H^4} \qquad \text{(withdrawn; see §23.3).} \tag{F.22}$$

The proportionality constant in the quartic depends on the precise low-energy normalization convention. The structural message of that reading was that mass and quartic are successive derivatives of the same finite holonomy potential; on the present reading the two are the doublet's, $m_h^2 = 2\lambda_H v^2$ with λ_H on H-L2.

F.22 Why the doublet survives while the color triplet does not

The Arisaka-GG preprints emphasize that a $\mathbf{5}$ of $SU(5)$ decomposes as

$$\mathbf{5}_H \to (\mathbf{3},\mathbf{1})_{-1/3} \oplus (\mathbf{1},\mathbf{2})_{+1/2}. \tag{F.23}$$

In a conventional SU(5) setting the coexistence of the triplet and doublet is the source of the notorious doublet–triplet splitting problem. In the GG-6 construction the split is done by the intrinsic parity $\eta_5 = -1$ of the $\mathbf{5}_H$ (§23.3), a discrete choice named there: the parity acting on the $\mathbf{5}$ by the gauge rule alone would keep the triplet. The low-energy Higgs doublet is the $(\mathbf{1}, \mathbf{2})_{+1/2}$ that survives it, while the color-triplet partner is projected out of the zero-mode spectrum or pushed to the high seam and hence does not appear as a light boundary scalar.

This is one of the clearest examples of the phrase "geometry replaces tuning." The theory does not ask why an arbitrary scalar triplet mass happened to become huge while an arbitrary scalar doublet mass remained small. It organizes the spectrum so that only the doublet has the correct boundary parity and seam access to appear in the low-energy theory.

F.23 Connection to the first-order quartic closure used in the main text

The main text used the leading closure

$$\lambda_H^{(1)} = \alpha_s + \alpha_{\mathrm{em}}. \tag{F.24}$$

This relation should not be read as an arbitrary numerical guess. It is the lowest-order boundary image, on the withdrawn reading, of the statement that the curvature of the finite holonomy potential is fed by the two most stable seam channels in the present sector: the color channel and the electric channel; on the present reading it is the operative one-loop closure the comparison tables use, and the quartic's basis is H-L2. The QD uplift is read from the datum and carried as representative (§43.9).

F.24 Operational extraction of couplings and masses in GDOS/GFF

The manuscript uses GDOS and GFF = GEP + CEP as the foundation of both calculation and observation. This appendix makes that operational statement more explicit.

F.25 Why a prediction is not complete until an observation rule is specified

A theory number becomes physically meaningful only after one states how it will be read out on the boundary. In GDOS this point is elevated to a principle: observation is an open-system projection from the six-dimensional bulk to the four-dimensional effective boundary. Therefore a coupling or a mass is never a free-floating symbol. It is a response coefficient of the projected stationary state.

Let ρ denote the effective GDOS state variable and let $F_{\mathsf{GFF}}[\rho]$ be the corresponding free-energy functional. Introducing a set of sources J_a for boundary observables $\mathcal{O}^a$, one writes the formal generating functional

$$Z[J] = \int \mathcal{D}\rho \, \exp\left[-F_{\mathsf{GFF}}[\rho] + \int J_a \mathcal{O}^a\right]. \tag{F.25}$$

Then the expectation values and connected responses are generated by

$$\langle \mathcal{O}^a \rangle = \frac{\delta \ln Z}{\delta J_a}, \qquad G^{ab}_{\mathrm{conn}} = \frac{\delta^2 \ln Z}{\delta J_a \delta J_b}. \tag{F.26}$$

The effective action obtained by Legendre transform packages the inverse responses. Pole masses are extracted from the zeros of the appropriate inverse propagator, while running couplings are extracted from properly normalized vertices. This is the GDOS/GFF language behind the ordinary phenomenological phrases "mass" and "coupling."

F.26 Why the comparison basis of the main text is the correct one

The theory side is forbidden from using measured Standard-Model observables as inputs. But once a theoretical value is derived, it must still be compared with a quantity defined in the same operational scheme used experimentally. That is why the main text compares primary predictions against $\widehat{\alpha}^{(5)}(M_Z)$, $\sin^2\theta^{\ell}_{\mathrm{eff}}$, $\alpha_s(M_Z)$, v, and m_h, and treats the pole masses m_W and m_Z as secondary checks.

This is not a compromise with the "no empirical input" rule. The rule forbids using these quantities to determine the prediction. It does not forbid asking whether the finished prediction agrees with them.

F.27 Meaning of the theory error bars used in the tables

The uncertainties quoted in Section 44 are not statistical errors in the experimental sense. They are truncation estimates attached to the theoretical construction. More precisely, they quantify the expected size of terms omitted when one stops at a given layer of the radial/seam expansion.

For the first-order predictions, the theory uncertainty estimates the combined size of the leading omitted seam-matching and QD corrections. For the QD-completed predictions, the uncertainty estimates the next omitted layer beyond QD. This is why the second-order error bars are much smaller but not zero. There is still residual structure not yet written in closed form.

F.28 How GFF organizes vacuum selection and measurement simultaneously

The language GFF = GEP + CEP is particularly useful because it unifies two roles that are often split apart in ordinary presentations. The same functional that determines the preferred stationary state also determines the response of that state to small external probes. In this sense GFF is both a selection principle and a metrological principle. That dual role is precisely what makes it the right foundational language for a parameter-derivation paper.

Problems and worked solutions. F.10.1. Why do exactly two transcendental numbers survive the radial transport?

Solution. Step 1: the radial renormalization group of this chapter transports couplings along one non-compact direction (the exponential redshift of (F.2) supplies e) around one compact direction (the circle's period supplies π). Step 2: every lawful readout at a stationary seam is built from

integrals of these two primitives (F.7); no other transcendental has a geometric source in the arena — there is nothing else to integrate over. Step 3: hence the codebook's striking texture: rationals and dyadics from counting (D.3.1, D.3.2, E.3.2), and π's and e's from measuring (D.3.3's πe; the $e^{-\pi}$ tolls of the quark ladder; the e^{-K} of the lock scale) — and *nothing else* anywhere in the twenty-eight parameters. Physics intuition: a theory's constants betray its arena — circles pay in π, exponential flows pay in e, discrete branchings pay in integers; the absence of any other species of number in the codebook is itself a structural prediction, and one a single rogue constant would falsify.

F.14.1. Work the Hosotani protection: from periodic phase to finite Higgs curvature. *Solution.* Step 1: the order parameter is the gauge-invariant Wilson line (F.19); its scalar is the phase θ_H, periodic by construction (F.10). Step 2: gauge invariance forbids any polynomial potential in θ_H; the effective potential generated at one loop is necessarily a Fourier series, $V_{\text{eff}} \propto \cos\theta_H + \dots$ (F.12), with finite coefficients — the would-be quadratic divergence has no invariant operator to inhabit. Step 3: expanding around the selected minimum (F.20), on the withdrawn reading the 4D Higgs emerged (F.11) with mass-squared given by the finite curvature (F.13); on the present reading the finite potential selects the branch that sets the doublet's scale — the input E.3.4 consumed. Physics intuition: the deepest protection mechanisms in physics are all topological bookkeeping — here, the impossibility of writing a divergent counterterm for an angle; the selector is light for the same class of reason the axion is light in the strong-CP chapter (App. O) — a compact variable whose potential must respect its circle. The doublet is not an angle, and its lightness is O_{11}'s question.

F.3.1. Show that the radial coordinate is not a new dimension but a resolution label. *Solution.* Step 1: in the export picture the radial variable orders boundary observables by the scale at which they are read, and a shift in it is a change of resolution, not a translation in space. Step 2: the test is whether anything propagates in it — a genuine dimension supports a wave equation and a conserved momentum, and the radial variable supports neither, since the flow is first order and dissipative in it. Step 3: consequence — there is no Kaluza–Klein tower associated with the radial direction, which is why the only tower in this framework's collider phenomenology is the compact one of GG-P1-KK. Physics intuition: the distinction is the same one that separates the renormalization-group scale from a real dimension. Both are labeled by a positive number and both organize physics hierarchically, but only one of them has a momentum conjugate to it, and a student who keeps that test in mind will not be misled by the geometric language.

F.3.2. Show that the Wilson–Hosotani phase is periodic, and say what the periodicity buys. *Solution.* Step 1: the Hosotani phase is the holonomy of a gauge field around a compact direction, so it is defined modulo 2π and physically equivalent configurations differ by that shift. Step 2: hence its effective potential must be a periodic function — generated only at loop level, since a tree-level potential for a holonomy is forbidden by higher-dimensional gauge invariance. Step 3: consequence — the scalar mass generated is finite and calculable rather than quadratically divergent, which is the mechanism's whole point. Physics intuition: periodicity is a symmetry, and symmetries protect. A compact variable cannot have an arbitrary potential, so its excitation cannot have an arbitrary mass; this is exactly the same argument that keeps an axion light, and it is why the electroweak

scale in this framework is a selected number rather than a tuned one; the doublet's mass, $2\lambda_H v^2$ on H-L2, is not protected by this argument (O_{11}).

F.3.3. Check the export at a scale where it should fail, and confirm that it does.
Solution. Step 1: the export delivers four-dimensional effective operators valid below the compactification scale; above it, the tower is not decoupled and the four-dimensional description must break. Step 2: the diagnostic is the growth of the effective description's amplitudes — once a would-be four-dimensional amplitude violates its unitarity bound, the omitted states are the ones restoring it. Step 3: for this framework the scale is the lock, $M_{\rm lock} = 3.85$ TeV, which is precisely why GG-P1-KK exists and why F-SM-1 is written against a collider rather than against a low-energy observable. Physics intuition: an effective theory that never fails is not an effective theory — it is either exact or untestable. Being able to say where a description must break, and to name the experiment that will look there, is the difference between a framework and a fit, and the exercise of finding your own theory's ceiling is one every student should perform on whatever they are working on.

F.3.4. Trace one exported coefficient back to its ledger integer.
Solution. Step 1: pick the neutrino operator coefficient of App. J: $m_\star = \pi v^2/E_{\rm GUT}$. Step 2: follow it back — v from the Wilson–Hosotani vev of this chapter, $E_{\rm GUT} = E_{\rm Pl}e^{-2\pi}$ from the gauge chapter's seam, and the prefactor π from a single traversal of the compact direction S^1_θ. Step 3: so a meV-scale neutrino mass is three upstream quantities and no new input, which is the claim the export chapter exists to support. Physics intuition: this is the exercise to repeat for every derived number in the paper, and it is the most useful critical habit a reader can develop here. Ask of each: which upstream quantities does it use, and were any of them fixed *after* the downstream number was known? If the answer to the second question is ever yes, the derivation is a fit; the point of the export chapter is to make the dependency graph explicit enough that the question can be asked at all.

G The assembled Lagrangian: the course walk

What this appendix teaches. The complete Standard-Model Lagrangian, assembled from scratch as a constitutional export — the same eight-stage chain audited at results register in Part X, here walked at course pace: the bulk action — earned brace by brace, not recorded — and the forced integer ledger, one bulk spinor becoming a chiral family with every hypercharge computed, the group splitting and the covariant derivative, the five term groups exported one by one, the operators the bulk does *not* send, and the assembled equation read line by line. Every manipulation is shown, and every step carries its physics intuition.

Read alongside: Peskin & Schroeder (1995) for gauge theory and the Standard-Model Lagrangian in textbook form; Schwartz (2014) for the electroweak sector; Georgi & Glashow (1974) for the unified family packet; Y. Hosotani (1983) for the holonomy order parameter.

This chapter is the course's *assembly interlude*. It is not one of the sector appendices — it derives no new parameter — and it deliberately interrupts the sequence at the halfway point, after the gauge chapter and the Higgs chapter have built the bosonic half of the theory and before the fermion chapters build the other half. The reason for the interruption is pedagogical and, we

would argue, psychological. Every reader of this course learned the subject in the same way: the Standard-Model Lagrangian was written on the board, complete, on the first day, and everything afterwards was consequence. The single most disorienting claim of GG-Theory is that the object on the board is not a starting point at all. Part X of the main text states that claim at results register; this chapter is where the student *earns* it, by assembling the Lagrangian with their own hands, one exported structure at a time, with nothing pulled from a hat. The two treatments share one governing sentence, and it bears repeating before the first step: *the visible four-dimensional Lagrangian is exported from the six-dimensional bulk, not assembled by listing all gauge invariants. (Physics intuition: think of the boundary theory as the shadow cast by a rigid six-dimensional object. One does not build a shadow by choosing its parts; one builds the object, shines the light, and reads the shadow off. This chapter builds the object and reads.)*

G.1 How to read this chapter

The walk has eight stages, numbered S-1 to S-8 as in the main text, plus one interpolated station the main text compresses to a paragraph: S-1a, where the bulk action itself is earned rather than cited. Each stage below is worked from scratch: the claim, the computation with no skipped manipulations, the register of the result (**Derived**, **Derived (cond.)** on a named lemma, or **Asserted**), and the physics intuition. Where a stage consumes a result already derived in full elsewhere in the course — the dyadic gauge readings of the gauge chapter, the lock-scale chain of the Higgs chapter, the Wilson-line geometry of the export chapter just concluded — we work the assembly step here and cite the chapter that owns the ingredient, so that nothing is derived twice and nothing is asserted without an address. The chapter closes with the assembled equation, eight problems with complete worked solutions, and the take-away box. A reader with limited time can read §G.11 first and work backwards; a reader with no time at all should at least sit with the assembled equation until the strangeness registers: *there is no term in it anyone chose.*

G.2 S-1 — the inputs on the table, and the bulk action

What is on the table. Before the first computation, the complete list of what the derivation is allowed to touch — the admissible inputs of the standard of derivation (§4) — is short enough to memorize: the four axioms and seven postulates of the GG-Constitution; the six-dimensional bulk $M_4 \times I_r \times S^1_\theta$ (four-dimensional spacetime, a radial interval that acts as an energy ladder, and one compact circle); the five dimensional constants; the two transcendentals native to the compact directions (e for the exponential redshift along the radial interval, π for the angle of the circle); and the five integers of the OGN-5 ledger, forced in S-1 below. *Not* on the table: any four-dimensional Lagrangian, any choice of gauge group, any fermion representation, any scalar potential, and any of the twenty-eight numbers. *(Physics intuition: the honest way to keep score in any derivation is to lock the pantry before cooking. Everything served below must be traceable to this shelf, and the shelf contains no continuous dial anywhere — the strongest statement on it is a pair of closure identities among five integers.)*

The bulk action, term by term. The bulk dynamics is Einstein–Yang–Mills–Dirac theory with flux on the six-dimensional bulk, written schematically as

$$S_6 \;=\; \int_{M_6} \sqrt{-G}\,\Big[\underbrace{\tfrac{1}{2\kappa_6^2}\,(R_6 - 2\Lambda_6)}_{\text{geometry}} \;-\; \underbrace{\tfrac{1}{4}\,\mathcal{F}^{A}_{MN}\mathcal{F}^{A\,MN}}_{\mathrm{SU}(5)\times\mathrm{U}(1)_B\times\mathrm{U}(1)_{Qi}} \;+\; \underbrace{i\,\overline{\Psi}\,\Gamma^M D_M \Psi}_{\text{one bulk spinor per family}} \;-\; \underbrace{\tfrac{1}{12}\,H_{MNP}H^{MNP}}_{\text{flux sector}}\Big], \tag{G.1}$$

plus the Green–Schwarz-type terms that the anomaly bookkeeping of GG-A3-BI6 (Arisaka, 2026, A3-BI6) requires. Read the four braces as the complete cast of characters of the *gauge and gravitational* sector: the metric, the unified gauge field, the matter spinor, and the flux whose closure law is about to do all the work. (The bulk cosmological term Λ_6 rides with the geometry brace; its boundary shadow is not a tuned number but an export — the lock-scale seesaw of the codebook.)

One field is required and selected rather than forced, and this paper says so. The electroweak doublet is the $(\mathbf{1},\mathbf{2})_{+1/2}$ of a bulk $\mathbf{5}_H$ (§23.3), and (G.1) as written does not carry it: an earlier reading identified the doublet with a component of the connection already in the action, and that reading is withdrawn. The scalar multiplet must therefore be added to the bulk content, and it is *admitted*, not derived — the ledger fixes the couplings and the scales, and it does not fix that there is exactly one Higgs five-plet. The consequence for the paper's headline is stated once, here, and carried in the codebook: the twenty-eight parameters are derived with no adjustable continuous input, *given* the boundary field content, and that content is forced by the cohort's theorems *up to the Higgs multiplet*, which is an assumption of minimality rather than a theorem. It is the same assumption classic SU(5) makes, made explicit here rather than left silent.

Equation (G.1) is not an ansatz, and this chapter does not ask the reader to accept it on citation. Its remaining field content is forced by theorems of the cohort — the stage and the warped metric by T-GG6-0/1/2 of GG-A2-GG6 (Arisaka, 2026, A2-GG6), the gauge group by T-GG6-3, the flux sector and its Green–Schwarz completion by the anomaly closure of GG-A3-BI6 — its canonical form is recorded in GG-A2-GG6 ahead of the BRST-consistency construction there, and the uniqueness of its dynamics — *why these terms and no others* — is earned brace by brace in S-1a below, with the register of every claim stated. The machinery that then spends it is the nine-part descent theorem T-GDOS-9 of GG-A5-GDOS (Arisaka, 2026, A5-GDOS), which carries the action from the bulk to the boundary. *(Physics intuition: nothing in Eq. (G.1) is exotic — every term is the six-dimensional version of machinery a graduate student already trusts. The novelty of the program is not in the ingredients; it is in the claim that this one fixed dish, with no seasoning allowed, exports the entire Standard Model.)*

The closure that forces the ledger. The flux sector is not free: its Bianchi identity is modified by the anomaly inflow, $\mathrm{d}H_3 = X_4$, and demanding that this BI-6 closure be globally consistent on the bulk is a Diophantine condition on the discrete data. GG-A3-BI6 proves that the condition admits exactly one solution, the OGN-5 ledger, and the two identities that lock it can be checked by hand:

$$\begin{gathered}(M,N,L;J,K) = (1,3,5;7,35),\\ K \;=\; L\,J \;=\; 5\times 7 \;=\; 35 \;=\; 1+9+25 \;=\; M^2+N^2+L^2.\end{gathered} \tag{G.2}$$

Problem G.2.1 works the uniqueness scan: among small integer triples, no other (M, N, L) produces an integer, odd closure index J from the same pair of relations. At the end of S-1 the theory carries five integers and zero continuous freedom, and every subsequent stage spends from this account only. *(Physics intuition: the ledger is the theory's DNA — five integers that are not settings but survival conditions. Change any one and the closure dies at once, the way changing one gear tooth stops a clock rather than making it run at a different speed. That is what separates an integer ledger from a parameter set: parameters bend, ledgers break.)*

G.3 S-1a — why this action and no others: the bulk action earned

The claim. Stage S-1 put Eq. (G.1) on the table with its provenance named; this station proves the sharper statement the walk actually needs:

> *On the bulk forced by T-GG6-0/1/2 of GG-A2-GG6, with the gauge group forced by T-GG6-3 and the matter restricted to the minimal anomaly-admissible chiral sector, the constitutional clauses LC (locality–causality), POS-2 (a single variational principle), and MSMF (read at lowest derivative order) admit exactly one local action functional: Eq. (G.1), completed by the Green–Schwarz terms that BI-6 closure forces.*

The proof is a brace-by-brace uniqueness scan, and its honesty matters as much as its conclusion: two of the five steps close on constitutional theorems, two close on elementary classification arguments, and one stands on a named lemma. The registers are stated as we go and collected in the table at the end. *(Physics intuition: the reader has seen this move before, in S-1's locked pantry — but a locked pantry only helps if the recipe is also forced. This station shows the recipe book has one page.)*

Brace 1 — geometry. What scalar densities can the metric alone supply? LC demands a local functional; at zero derivatives the only invariant is the volume element itself — the cosmological term — and at two derivatives the classification is classical: the Ricci scalar R_6 is the *only* diffeomorphism-invariant density, so the Einstein–Hilbert term is not a choice but the entire two-derivative shelf. The honest caveat lives one order up. In six dimensions Lovelock's classification admits one further term with second-order field equations — the Gauss–Bonnet density $R^2 - 4R_{MN}R^{MN} + R_{MNPQ}R^{MNPQ}$ — which in four dimensions integrates to a topological invariant but in $D = 6$ carries dynamics. It is removed not by a uniqueness theorem but by MSMF's lowest-derivative-order reading: a four-derivative term is structure the two-derivative theory does not need, and MSMF forbids freedom that buys nothing. Its boundary image, like that of every higher-curvature correction, is an irrelevant operator diluted along the radial throat — the geometrized Wilsonian statement of T-GDOS-9.9 (Arisaka, 2026, A5-GDOS). Register: **D** at lowest derivative order, with the Lovelock caveat stated; Problem G.3.1 works the scan by hand. *(Physics intuition: gravity's kinetic term is the loneliest object in field theory — ask for general covariance and two derivatives, and the market stocks exactly one item.)*

Brace 2 — gauge. Given the group, the kinetic term is again the whole shelf: $\mathcal{F}_{MN}\mathcal{F}^{MN}$ is the unique gauge-invariant scalar density at two derivatives. The parity-odd invariants a six-dimensional

connection can also supply — the Chern-class densities of $\mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F}$ type — are not free additions here: they belong to the topological sector, and BI-6 closure prices every topological coefficient it admits (brace 4). What the classification does *not* settle is the group itself, and the register must say so: T-GG6-3 of GG-A2-GG6 forces $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ from GGEP + MSMF + gauge closure, but that theorem carries two conditional clauses stated inside it — the requirement that the boundary contain the Standard-Model gauge content, and the empirically-preferred four-component class — and this walk inherits them rather than hiding them. Register: form **D**; group **D*** on T-GG6-3's stated clauses. *(Physics intuition: the gauge shelf is as bare as gravity's — one item — but the label on the jar is the real content. The classification fixes what a gauge field can do; the Constitution fixes which gauge field exists. Keeping those two claims at their two different grades is what honest bookkeeping looks like.)*

Brace 3 — matter. The Dirac term $i\,\overline{\Psi}\,\Gamma^M D_M \Psi$ is the unique LC-admissible first-order kinetic term for a spinor — a classification even shorter than the bosonic ones. The substantive question is not the term but the *cast*: why one bulk spinor per family and nothing else? Here the walk names its conditional plainly. Anomaly admissibility (the factorization theorem T-GG6-5 of GG-A2-GG6, with its global completion in GG-A3-BI6) constrains the chiral sector to representations whose anomaly polynomial factorizes against X_4; the *minimal* such sector is the one bulk spinor whose zero modes are the $\overline{\mathbf{5}} \oplus \mathbf{10}$ packet of S-2. That minimality — no second packet, no exotics, no vector-like passengers — is MSMF again, and its precise statement is a lemma this course names rather than proves: the **Minimal-Matter Lemma**, that the family packet is the unique minimal anomaly-admissible chiral sector on the forced bulk (evidence base: the benchmark-spectrum analysis of GG-A3-BI6). Replication $N_g = 3$ is the pincer of GG-P2-N3, already carried at **D*** in S-2. Register: form **D**; content **D*** on the Minimal-Matter Lemma. *(Physics intuition: the Dirac term is the cheapest sentence in the action, and the cast list is the expensive one. Saying "one spinor, nothing else" is a strong minimality claim — strong enough that this course prints the lemma's name beside it rather than pretending it has the proof.)*

Brace 4 — flux, and the terms with no dial. The two-form sector is the one place the action is *added to*, and the addition is forced rather than chosen: consistency of the chiral sector requires the Chern–Simons-improved field strength — the H_3 whose Bianchi identity reads $\mathrm{d}H_3 = X_4$ — its kinetic term $H_{MNP}H^{MNP}$ is the unique two-derivative invariant, and the Green–Schwarz coupling with the coefficient vector $(1, -1, -\frac{2}{5}, -\frac{2}{5})$ is a theorem (T-GG6-4 of GG-A2-GG6; structural and global closure in GG-A3-BI6 (Arisaka, 2026, A3-BI6)). Nothing in this brace is graded below **D**: the flux sector is where the Constitution shows its teeth, because here even the topological coefficients — the ones a four-dimensional θ-angle would leave free — are fixed by closure. Register: **D**. *(Physics intuition: in four dimensions the θ-angle is the loose thread of the Standard Model, a topological coefficient nothing fixes. Here the analogous coefficients sit inside a closure identity and are priced by it: the flux sector is where "no free dials" stops being a slogan and becomes arithmetic.)*

Brace 5 — the scalar, and what the export admits. An earlier version of this walk argued that the action contains no fundamental scalar and that the cast list of Eq. (G.1) is therefore closed:

the symmetry-breaking order parameter of S-6 being the Wilson-line holonomy, a fundamental scalar alongside it would be redundant continuous freedom of the kind MSMF forbids. *That argument is withdrawn*, and with it the register it carried. It rested on the identification of the electroweak doublet with a component of the connection, and §23.3 shows on the paper's own group theory that the identification cannot hold: the coset of the adjoint is colored.

What the export actually admits is one bulk $\mathbf{5}_H$, whose $(\mathbf{1},\mathbf{2})_{+1/2}$ is the electroweak doublet and whose survival fraction $2/L$ is the factor Theorem 34.1 reads. MSMF still does real work here, and it is worth saying exactly how much: it forbids a *tuned* scalar mass, it forbids a second scalar sector, and it selects the minimal multiplet that the SU(5) embedding admits. It also derives that a scalar multiplet is present at all: the BI-6-forced fermions need a mass term, and §23.3 shows the minimal representation that supplies one is the $\mathbf{5}_H$. Register: **required and MSMF-selected**, not forced by a theorem — the same status classic SU(5) gives its $\mathbf{5}_H$, stated here rather than left silent. *(Physics intuition: the textbook's scalar, with its hand-tuned $\mu^2 < 0$, is not replaced by a hole in the action. It is replaced by a multiplet the geometry admits and whose every parameter the geometry then fixes — which is a smaller claim than the one this brace used to make, and a true one.)*

Table 80: The assembled action, brace by brace: the form of each term, the content it carries, and the register it belongs to. The table is the bookkeeping behind the claim that the Lagrangian is exported rather than postulated, and it is arranged so that the absence of a term is as visible as the presence of one. The textbook's scalar potential appears only through its two numbers, v on L2 and λ_H on L6, which is exactly the point of the paragraph above it.

Brace	Form of the term	Content it carries	Register
geometry	EH unique at two derivatives	Gauss–Bonnet removed at lowest order	**D** (Lovelock caveat stated)
gauge	$\mathcal{F}^2$ unique at two derivatives	group from T-GG6-3 of GG-A2-GG6	form **D**; group **D***
spinor	Dirac unique at first order	minimal packet; $N_g = 3$ (GG-P2-N3)	form **D**; content **D***
flux + GS	forced by BI-6 closure	coefficients $(1, -1, -\frac{2}{5}, -\frac{2}{5})$	**D**
$\mathbf{5}_H$ (required scalar)	required by the fermion mass sector; selected as minimal	doublet–triplet split by an intrinsic parity (discrete choice); holonomy is the selector	**selected** (representation) / **D*** (potential)

The provenance of the action, collected.

What promotion would look like. The two named conditionals of this station are research targets with precise statements, not soft spots hidden in prose: a proof of the Minimal-Matter Lemma promotes brace 3 to **D**, and a constitutional derivation of the lowest-derivative-order reading — or a boundary observable sensitive to a Gauss–Bonnet remnant — settles brace 1; brace 5 is a required field in a selected representation, and stays so. Nothing downstream of this station

upgrades a grade, and the assembled equation at the end of the walk is exactly as strong as this page plus the descent behind it. Problems G.3.1 and G.3.2 work the two halves — the operator scan, and the machine that spends the action — by hand. *(Physics intuition: a derivation is only as strong as its weakest named link, and naming the links is the point. The two lemmas of this station are not embarrassments to be smoothed over; they are the shortest list anyone has yet attached to the question "why this Lagrangian?".)*

G.4 S-2 — one bulk spinor becomes a chiral family

The reduction, worked. Take the bulk Dirac term of Eq. (G.1) and expand the spinor in modes of the compact directions,

$$\Psi(x,r,\theta) \;=\; \sum_n \Big[\,\psi_L^{(n)}(x)\,f_n(r)\,e^{in\theta} \;+\; \psi_R^{(n)}(x)\,g_n(r)\,e^{in\theta}\Big], \tag{G.3}$$

where L/R are the four-dimensional chiralities. On the circle, single- valuedness quantizes n; on the interval, the variational principle forces a boundary condition, and the consistent choice sets one chirality to zero at the ends, $\psi_R\big|_{\partial I_r} = 0$. The massless sector is then read off: the would-be right-handed zero mode needs a constant profile g_0, which the Dirichlet condition kills; the left-handed zero mode survives with its profile f_0 intact. The four-dimensional boundary spectrum is therefore *chiral* — purely left-handed Weyl — before any dynamics has been discussed (§7 carries the full statement). *(Physics intuition: chirality here is a filter, not a choice. The interval acts like a polarizer: both polarizations of the bulk field arrive, one is absorbed at the walls, and only the other is transmitted to the boundary world. The famous left-handedness of the weak interaction starts life as a boundary condition.)*

The packet, decomposed with every hypercharge computed. The surviving zero modes arrive in complete multiplets of the unified SU(5), one $\overline{\mathbf{5}} \oplus \mathbf{10}$ per family, and the entire quantum-number table of the Standard Model is now a *computation* rather than a list. Fix the hypercharge generator inside SU(5) acting on the fundamental $\mathbf{5}$ as

$$Y \;=\; \mathrm{diag}\Big(-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3},+\tfrac{1}{2},+\tfrac{1}{2}\Big), \qquad \mathrm{Tr}\,Y = 3(-\tfrac{1}{3}) + 2(+\tfrac{1}{2}) = 0, \tag{G.4}$$

traceless as a generator of a simple group must be — and that tracelessness is already the whole mystery of charge quantization: hypercharge cannot be rescaled per multiplet, because it is one matrix. Then the $\overline{\mathbf{5}}$ (conjugate, so Y flips sign) contains the color antitriplet with $Y = +\frac{1}{3}$ — the d^c — and the weak doublet with $Y = -\frac{1}{2}$ — the lepton doublet $L = (\nu, e)$. The $\mathbf{10}$ is the antisymmetric pair of $\mathbf{5}$ indices, and its hypercharges are simply *sums of pairs* of the entries in Eq. (G.4):

$$\underbrace{-\tfrac{1}{3}-\tfrac{1}{3} = -\tfrac{2}{3}}_{\text{two color indices: } u^c\ (\overline{\mathbf{3}},\mathbf{1})} \qquad \underbrace{-\tfrac{1}{3}+\tfrac{1}{2} = +\tfrac{1}{6}}_{\text{one color, one weak: } Q\ (\mathbf{3},\mathbf{2})} \qquad \underbrace{+\tfrac{1}{2}+\tfrac{1}{2} = +1}_{\text{two weak indices: } e^c\ (\mathbf{1},\mathbf{1})} \;. \tag{G.5}$$

Fifteen Weyl fields, five irreducible blocks, every hypercharge — the strange fractions $\frac{1}{6}, -\frac{2}{3}, \frac{1}{3}, -\frac{1}{2}, 1$ that the textbook must postulate — read off one traceless matrix and its pair sums. The electric

charges follow from $Q = T_3 + Y$: for the quark doublet, $Q(u) = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$ and $Q(d) = -\frac{1}{2} + \frac{1}{6} = -\frac{1}{3}$; for the leptons, $Q(\nu) = \frac{1}{2} - \frac{1}{2} = 0$ and $Q(e) = -\frac{1}{2} - \frac{1}{2} = -1$; for the singlets, $Q(u^c) = -\frac{2}{3}$, $Q(d^c) = +\frac{1}{3}$, $Q(e^c) = +1$. Problem G.4.1 completes the classic corollary: the tracelessness of Q on the $\overline{\mathbf{5}}$ forces $|q_e| = |q_p|$ *exactly*, which is why atoms are neutral to better than one part in 10^{21}. *(Physics intuition: the textbook presents the hypercharge table the way a phone book presents numbers — facts without reasons. Here the table is the multiplication table of one matrix. When a student first computes $-\frac{1}{3} + \frac{1}{2} = \frac{1}{6}$ and recognizes the quark doublet's hypercharge, the fifteen-field table stops being a list to memorize and becomes a pattern with one seed.)*

Anomaly cancellation for free, and three families. The textbook must verify, after choosing its representations, that the gauge anomalies cancel — four independent sums over each family that have no right to work and do. Here they cannot fail: each family is one bulk packet, and the packet's consistency is the BI-6 closure already paid for in S-1. Problem G.4.2 works all four sums explicitly — $[\mathrm{SU}(3)]^2\,\mathrm{U}(1)_Y$, $[\mathrm{SU}(2)]^2\,\mathrm{U}(1)_Y$, $[\mathrm{U}(1)_Y]^3$, and gravitational — and each vanishes term by term using only the hypercharges just computed. Replication is the one entry of this stage carried at conditional register: the family count is the two-sided pincer of GG-P2-N3 (Arisaka, 2026, P2-N3) (§21), $N_g \geq 3$ from the MSMF CP lower bound and $N_g \leq 3$ from the Bogomolnyi/BI-6 action cap — **Derived (cond.)**, as flagged there. *(Physics intuition: anomaly cancellation is the Standard Model's most impressive conspiracy — fractions conspiring across quarks and leptons to sum to zero four separate ways. Conspiracies dissolve when you find the committee: the quarks and leptons sit in one room upstairs, in a single bulk multiplet, and the four sums are just that room's bookkeeping viewed four ways.)*

G.5 S-3 — the splitting, and the covariant derivative assembled

The splitting. Boundary conditions and the holonomy structure on the compact directions reduce the bulk gauge symmetry to the subgroup that survives on the boundary,

$$\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi} \;\longrightarrow\; \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y, \tag{G.6}$$

with the mechanism worked in the fundamental-architecture section (§16) and the Wilson-line geometry in the chapter just concluded. The essential point for the assembly is what the splitting does *not* touch: the hypercharge generator (G.4) is part of the surviving algebra, so every quantum number computed in S-2 descends unchanged. *(Physics intuition: breaking a symmetry by boundary conditions is like slicing a crystal along a plane — the bulk lattice fixes which faces are possible, and the exposed face keeps the lattice's spacings. The Standard-Model group is the exposed face of the unified crystal, and the hypercharges are its inherited spacings.)*

The 5/3, computed in four lines. The unified origin fixes the relative normalization of hypercharge, and with it the tree-level weak angle — the number the gauge chapter's problems already met from the running side (D.1.2). Compute both traces over one family using the table of

S-2. For hypercharge:

$$\mathrm{Tr}\, Y^2 = \underbrace{6\cdot\tfrac{1}{36}}_{Q} + \underbrace{3\cdot\tfrac{4}{9}}_{u^c} + \underbrace{3\cdot\tfrac{1}{9}}_{d^c} + \underbrace{2\cdot\tfrac{1}{4}}_{L} + \underbrace{1}_{e^c} = \tfrac{1}{6}+\tfrac{4}{3}+\tfrac{1}{3}+\tfrac{1}{2}+1 = \tfrac{10}{3}, \tag{G.7}$$

while the same trace of T_3^2 receives $\frac{1}{4}$ from each of the eight doublet states (six quark, two lepton): $\mathrm{Tr}\, T_3^2 = 8\cdot\frac{1}{4} = 2$. The ratio is $\mathrm{Tr}\, Y^2/\,\mathrm{Tr}\, T_3^2 = \frac{10/3}{2} = \frac{5}{3}$ — the normalization $g_1^2 = \frac{5}{3}g'^2$ of the gauge chapter (D.9) — and at the unification point it gives $\sin^2\theta_W = \frac{3}{8}$ (D.10). Problem G.5.1 extends the computation to $\mathrm{Tr}\, Q^2 = \frac{16}{3}$ and recovers 3/8 directly as $\mathrm{Tr}\, T_3^2/\,\mathrm{Tr}\, Q^2$. *(Physics intuition: the 5/3 is not a convention — it is the statement that hypercharge and weak isospin are two rulers cut from the same block of wood, so their relative markings are fixed. Any theory in which Y is its own free abelian charge can rescale it at will; a theory in which Y is five entries of one traceless matrix cannot.)*

The covariant derivative. Everything a gauge theory does to matter is packaged in one operator, and that operator is now fully determined:

$$D_\mu \;=\; \partial_\mu \;-\; i\,g_s\,G_\mu^a T^a \;-\; i\,g\,W_\mu^i \tau^i \;-\; i\,g'\,\tfrac{Y}{2}\,B_\mu, \tag{G.8}$$

with the three factors from the splitting (G.6), every Y from the pair sums (G.5), and every T^a, τ^i from the surviving algebra. No entry of Eq. (G.8) was chosen at any point of the chain. *(Physics intuition: in the textbook, D_μ is where the physicist's choices are stored — the group, the charges, the couplings all typed into one line. Here the same line is a receipt: every symbol on it was paid for upstream, and the three couplings, the only continuous entries, are about to be read off the seam rather than measured in.)*

G.6 S-4 — term group I: the gauge kinetic terms

Integrating the compact directions in the bulk gauge kinetic term of Eq. (G.1) produces the boundary kinetic terms for the three surviving factors,

$$\mathcal{L}_{\text{gauge}} = -\tfrac{1}{4}\,G^a_{\mu\nu}G^{a\,\mu\nu} - \tfrac{1}{4}\,W^i_{\mu\nu}W^{i\,\mu\nu} - \tfrac{1}{4}\,B_{\mu\nu}B^{\mu\nu}, \qquad \frac{1}{g_4^2} \;=\; \frac{V_2}{g_6^2}, \tag{G.9}$$

where V_2 is the compact volume: the six-dimensional coupling and the compact geometry set the four-dimensional couplings, exactly as the inverse-square law of the force chapter descends from flux conservation. The *structure* of this term group is therefore pure export. Its three *coefficients* are the gauge chapter's business, and the course has already derived them as seam registrations: $\widehat{\alpha}^{-1}(M_Z) = 2^J = 128$ (D.17), the rational weak angle $\sin^2\theta_W = 25/108$ (D.22), and the strong closure $\alpha_s^{-1}(M_Z) = \pi e$ (D.26), each with its stated pull and QD refinement. *(Physics intuition: a coupling constant, in this picture, is a measured property of a geometric seam — like the pitch of a taut string, set by length and tension rather than dialed. The gauge chapter measured the three pitches; this chapter only has to hang the strings in the right frame.)*

G.7 S-5 — term group II: the fermion kinetic terms

The matter half of the kinetic sector is now a single sum with no freedom in it. Three families times fifteen Weyl fields gives forty-five two-component spinors, each coupled through the one covariant derivative:

$$\mathcal{L}_{\text{fermion}} = \sum_{\psi \in 3\times(Q,\,u^c,\,d^c,\,L,\,e^c)} \overline{\psi}\; i\gamma^\mu D_\mu\, \psi. \tag{G.10}$$

Count the freedom: the spectrum was computed (S-2), the charges were computed (S-2), the operator was computed (S-3), and the family number is the pincer theorem. There is nothing left to decide. The one feature that startles every student — that the $\mathrm{SU}(2)_L$ factor sees only left-handed fields, maximal parity violation, the 1957 shock — is the polarizer of S-2 read dynamically: the boundary condition transmitted one chirality, so the gate force can only couple to what arrived. *(Physics intuition: forty-five fields sounds baroque until one remembers they are one bulk field, viewed through a compactification. An orchestra of overtones is not a complicated instrument; it is one string, heard properly.)*

G.8 S-6 — term group III: the Higgs sector, with no dial

This is the stage where the export departs furthest from the board, and the chapter slows down accordingly. The textbook writes two terms by hand: a kinetic term for a postulated doublet, and the potential $V = \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$ with $\mu^2 < 0$ inserted to force the vacuum. The export replaces all four decisions (the field, the kinetic term, the shape, the sign) with geometry already on the table.

The field. The would-be Higgs is required by the fermion mass sector and selected as the minimal $\mathbf{5}_H$, and the export says where it enters: the doublet is the $(\mathbf{1},\mathbf{2})_{+1/2}$ inside a bulk $\mathbf{5}_H$, minimal by MSMF and not forced by a theorem. An earlier reading took its quantum numbers from the coset directions of the splitting, and §23.3 withdraws that reading: on the paper's own group theory the coset of the adjoint is colored. What the compact circle supplies is not the field but the *selector* — the Wilson-line phase $\theta_H \sim \oint A_\theta$ (F.19), whose branch fixes where the doublet's vacuum value lands. *(Physics intuition: the geometry does not manufacture the scalar; it tells the scalar where to sit. Something was added to the universe in 1964 on this reading — what the export removes is the freedom in its parameters, not the field.)*

The kinetic term. The selector's kinetic term is a gauge-field component's, a slice of the bulk Yang–Mills kinetic term — the mixed $F_{\mu\theta}F^{\mu\theta}$ components of Eq. (G.1) — not a separate postulate; the doublet's $|D_\mu\Phi|^2$ is the boundary image of the $\mathbf{5}_H$'s own bulk kinetic term, as §90.6 states. *(Physics intuition: one term of Eq. (G.1), cut along two different planes, yields both the gauge kinetic terms of S-4 and the selector's kinetic term of S-6. For the selector the bulk action is being spent, not supplemented; the doublet's kinetic term is the $\mathbf{5}_H$'s own.)*

The shape of the potential, forced. Here is the argument in full, because it is the dissolution of the naturalness problem and it fits in a paragraph. A Wilson-line phase is compact: a large gauge

transformation — one that winds once around S^1_θ — shifts

$$\theta_H \longrightarrow \theta_H + 2\pi, \tag{G.11}$$

and is exactly invisible to physics (F.10). Every admissible term in the effective potential must therefore be periodic in θ_H. But a polynomial — in particular the mass term $\mu^2\,\theta_H^2$ that the textbook writes and the quadratic divergence that renormalization wants to feed it — is not periodic: under Eq. (G.11) it shifts, so gauge invariance forbids it *as an operator*. What survives is the Fourier series generated at one loop by the fields running around the circle, $V_{\text{eff}} \propto \cos\theta_H + \dots$ (F.12), with finite coefficients set by the compactification scale — the Hosotani mechanism (Y. Hosotani, 1983; K. G. Wilson, 1974). For the selector, the textbook's hand-tuned $\mu^2 < 0$ becomes a *branch-selection condition*, $\partial_{\theta_H} V_{\text{eff}} = 0$ with $\partial^2_{\theta_H} V_{\text{eff}} > 0$: the vacuum sits where the forced potential has its stable minimum, and there is no dial in the mechanism anywhere. Problem G.8.1 works the periodicity argument as an exercise and states its consequence plainly: *you cannot quadratically renormalize an angle.* The hierarchy problem is not solved here — it is dissolved, because the operator that carries the problem does not exist on the export list for the selector; the doublet's is O_{11}'s. *(Physics intuition: an angle has no length to drag. Every naturalness anxiety of the past fifty years was an anxiety about a length — a scalar mass pulled toward the cutoff. The export's* selector *is a phase on a circle, and a phase can be pushed around the circle but never stretched; the divergence has no invariant to live in.)*

The two readings. The Higgs chapter derived the two numbers this sector owes the codebook, and the assembly imports them: the vacuum value from the lock-scale chain, $v = M_{\text{lock}}\,(2/L)^N = 3849 \times (2/5)^3$ GeV ≈ 246.3 GeV (E.13), against the Fermi-constant extraction 246.2 GeV; and the quartic from the dyadic reading $\lambda_H = 2^{-N} = \frac{1}{8}$ (E.14), giving $m_h = v/2 = 123.1$ GeV at first order (E.15), with the QD completion carrying the stated conservative theory uncertainty. Registers as graded there: the vev **Derived (cond.)** on lemma L2; the quartic **Derived (cond.)** on lemma L6, both on hypothesis H-L2. *(Physics intuition: the electroweak scale, in this assembly, is the end of a pipeline — M_{Pl}, one exponential lock e^{-K}, three doublet filters of $2/L$ — every stage an integer of the ledger. The number* 246 *stops being a measurement to explain and becomes an address in the geometry.)*

G.9 S-7 — term group IV: the Yukawa block

The last interacting term group couples the two halves just built:

$$\mathcal{L}_{\text{Yuk}} = -\,\overline{Q}_L\,Y_u\,\widetilde{\Phi}\,u_R - \,\overline{Q}_L\,Y_d\,\Phi\,d_R - \,\overline{L}_L\,Y_e\,\Phi\,e_R \;+\; \text{h.c.} \tag{G.12}$$

Two separate claims must be kept distinct here, because they carry different registers. The *form* of Eq. (G.12) is forced: given the chiral packets of S-2 and the doublet-and-selector sector of S-6, these are the only boundary-admissible couplings between them (read subject to O_{24}) — **Derived**. The *content* of the matrices Y_f — in the textbook, fifty-four free real numbers feeding twelve masses and eight mixing parameters plus phases — is the subject the course takes up immediately after

this interlude, and the assembly records only its skeleton: the top quark sits at the zero-winding anchor with $y_t = 1$ at first order (H.7); the quark ladder descends by Bogomolnyi tolls $e^{-\pi k}$ with small rational crowd factors (H.6); the colorless leptons carry dyadic–rational codes (H.8); the CKM and PMNS matrices are the misalignment holonomies between family projectors — the gates of the cascade — with the Cabibbo angle carrying the pentagonal phase; and the neutrino sector enters through the geometric seesaw $m_\star = \pi v^2/E_{\rm GUT}$ (H.9), with no free right-handed mass scale anywhere. Per-entry registers are graded in Table 7; the cascade-depth assignments are **Derived (cond.)** on the named cascade lemma. *(Physics intuition: in the textbook the Yukawa sector is a wall of arbitrary numbers — the least explained corner of physics. The assembly reframes the wall before the next chapters climb it: the numbers are not couplings that were set, they are distances on a landscape — windings, tolls, and misalignment angles — and the chapters that follow will pace out each one.)*

G.10 S-8 — what the bulk does not send

The textbook's construction rule — write every renormalizable term the symmetries allow — generates three famous embarrassments, and the course must be honest that removing them by hand is part of what "learning the Standard Model" has always meant. The export rule replaces the listing rule, and each embarrassment becomes a theorem.

The θ-term is not exported. The listing rule admits $\theta\,\frac{g_s^2}{32\pi^2}G\widetilde{G}$, and experiment then demands $\bar{\theta} < 10^{-10}$ with no protecting symmetry — a tuning of one part in ten billion, accepted silently in every course. The export generates no coefficient for the topological density (the descent of the admissible bulk action contains none), and the first-order quark determinant is real because the family-link texture is Hermitian (T-SM-47.1), so $\bar{\theta}^{(1)} \in \{0, \pi\}$ with the sign on lemma L9, with the constitutionally minimal residue $\sin(2\pi/L)\,2^{-J}e^{-K} \simeq 4.7 \times 10^{-18}$ (O.4) — seven orders below the bound, **Derived (cond.)** on lemma L9. The strong-CP chapter of this course works the full reduction. *(Physics intuition: the deepest solution to a fine-tuning problem is to discover there was never a knob. The neutron's electric dipole moment is silent not because someone turned the dial to zero with superhuman care, but because the instrument was shipped without that dial.)*

The proton-decay operators are not exported. Classic SU(5) died by the listing rule: its heavy bosons generate $\Delta B = 1$ operators, Super-Kamiokande looked, and the proton does not decay. Here every $\Delta B = \pm 1, \pm 2$ operator carries non-zero $\mathrm{U}(1)_B$ charge, and the exact global baryon number residual to the boundary-terminated $\mathrm{U}(1)_B$ forbids them to all orders (Theorem 86.1, conditional on its stated hypotheses). The force-law chapter of this course carries the full treatment. *(Physics intuition: the 1974 unification put quarks and leptons in one multiplet and paid for the elegance with the proton's life. The six-dimensional version keeps the multiplet but banishes the dangerous mediators to the bulk, and gauges baryon number on the way down — the elegance survives, the proton does too.)*

The selector's scalar-mass dial is not exportable. Worked in S-6: no gauge-invariant polynomial in a compact phase exists, so the μ^2 term — the single most tuned object in the textbook tradition — is absent as a matter of admissibility, not choice — for the selector; the doublet's own mass term is O_{11}'s (§23.3).

The honest overshoot. Finally, the export is slightly *larger* than the textbook Standard Model: the boundary retains the terminated $\mathrm{U}(1)_B$ and the $\mathrm{U}(1)_{Qi}$ structure, protected remnants of the bulk gauge content with observable consequences (the fifth-force absence among the falsification channels). The assembly records this as a feature: a derivation that exactly reproduced the textbook operator list, no more and no less, would be suspicious; this one overshoots by two protected structures and stakes falsifiable claims on both. *(Physics intuition: a tailor who delivers a suit that fits perfectly plus two extra buttons you did not order has probably actually measured you.)*

G.11 The assembled equation, read line by line

Collecting Eqs. (G.9), (G.10), the Wilson-line sector of S-6, (G.12), and the $\mathbb{Z}_2$ of S-8:

$$\begin{aligned}\mathcal{L}_{\mathrm{SM}} = {} & \underbrace{-\tfrac{1}{4}\,G^a_{\mu\nu}G^{a\,\mu\nu} - \tfrac{1}{4}\,W^i_{\mu\nu}W^{i\,\mu\nu} - \tfrac{1}{4}\,B_{\mu\nu}B^{\mu\nu}}_{\substack{\text{S-4: exported structure;}\\ \text{couplings = seam registrations}}} + \underbrace{\sum_{\psi}\overline{\psi}\,i\gamma^\mu D_\mu\psi}_{\substack{\text{S-2, S-3, S-5: one bulk spinor,}\\ \text{45 chiral zero modes}}} \\ & + \underbrace{|D_\mu\Phi|^2 \;-\; V_{\mathrm{eff}}(\theta_H)}_{\substack{\text{S-6: Wilson line;}\\ \text{forced Fourier potential}}} - \underbrace{\left(\overline{Q}_L Y_u \widetilde{\Phi} u_R + \overline{Q}_L Y_d \Phi d_R + \overline{L}_L Y_e \Phi e_R + \mathrm{h.c.}\right)}_{\substack{\text{S-7: forced form;}\\ \text{cascade content}}} \\ & + \underbrace{0 \cdot \frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\widetilde{G}^{a\,\mu\nu}}_{\substack{\text{S-8: } \bar{\theta}^{(1)}\in\{0,\pi\}\text{, zero branch;}\\ \text{sign on L9}}}\end{aligned} \tag{G.13}$$

This is the equation from the first day of the reader's education, symbol for symbol — compare Peskin & Schroeder (1995) or Schwartz (2014) — with one line no textbook writes: the θ-term displayed with its coefficient, zero on the zero branch, because here the first-order angle is a $\mathbb{Z}_2$ by Theorem 79.1 and the branch is the neutron bound's, with the sign lemma L9's (§90.8). The reading protocol is the point of the whole interlude. Read each brace and ask the textbook question "who chose this?" — and observe that the answer, five times over, is *no one*: the structures were exported (S-1 to S-6), the coefficients were registered at the seam or graded in the codebook (Table 7), and the absences were derived to a sign (S-8). The main-text audit of Part X tabulates all of this element by element (its six-silent-axioms and master provenance tables); Problem G.11.1 asks the reader to reconstruct that mapping from memory, which is the honest test of whether the interlude has done its job. *(Physics intuition: the deepest change this equation undergoes in the present framework is grammatical — every verb turns passive. The group* was chosen *becomes the group* survived the splitting*; the charges* were assigned *becomes the charges* were computed*; the potential* was written

becomes the potential was generated*; the θ-term* was tuned away *becomes the θ-term* was never sent. *Physics has spent fifty years as the subject of these verbs; the export makes it the object.)*

One boundary statement, repeated from the main text because a course must not oversell: the framework of quantum field theory itself is imported, not derived; the boundary Lorentz structure is constitutional input; every **Derived (cond.)** above stands on its named lemma, cataloged in the promotion-roadmap chapter (Appendix P); and "complete" means the complete term content at stated grades — six **Derived**, fourteen **Derived (cond.)**, eight **Asserted** — never "every decimal derived." The assembly is exactly as strong as the descent behind it, no stronger.

Problems and worked solutions. G.2.1. Verify the ledger's double identity and run the uniqueness scan over small candidates.

Solution. Step 1: for $(M, N, L) = (1, 3, 5)$, compute both closure readings of K: the lock product $K = LJ$ requires $J = K/L$ to be an odd integer, and the sphere sum gives $K = M^2 + N^2 + L^2 = 1 + 9 + 25 = 35$, whence $J = 35/5 = 7$: odd, integer, and the ledger closes with $(1, 3, 5; 7, 35)$ (G.2). Step 2: scan the neighboring triples: $(1, 2, 3)$ gives $K = 14$, $J = 14/3$ — not an integer; $(1, 3, 4)$ gives $26/4 = 6.5$ — fails; $(1, 2, 5)$ gives $30/5 = 6$ — integer but even, and the closure index must be odd; $(2, 3, 5)$ gives $38/5$ — fails; $(1, 4, 5)$ gives $42/5$ — fails; $(3, 4, 5)$ gives $50/5 = 10$ — even again; $(1, 3, 7)$ gives $59/7$ — fails. Step 3: within the scanned window only $(1, 3, 5)$ satisfies both relations with an odd integer J — the full uniqueness statement, over all admissible triples with the complete BI-6 conditions, is the theorem of GG-A3-BI6. Physics intuition: this is what "the integers are forced" feels like in the fingers — every neighboring tuple dies in one line of arithmetic, not in a fit; the ledger sits at the intersection of two integer curves that barely deign to meet at all.

G.3.1. Run the two-derivative operator scan of S-1a: show the shelf holds exactly the terms of the bulk action, and name what removes the one Lovelock intruder.

Solution. Step 1: the gravity shelf. At zero derivatives the only diffeomorphism-invariant density is $\sqrt{-G}$ itself — the cosmological term. At one derivative there is nothing: the first derivatives of the metric vanish at any point in normal coordinates, so no scalar survives. At two derivatives every invariant is linear in the Riemann tensor, and its only full contraction is the Ricci scalar R_6 — the Einstein–Hilbert term, alone on the shelf. Step 2: the gauge and spinor shelves. For a connection, the lowest gauge-covariant object is $\mathcal{F}_{MN}$ and the lowest invariant scalar is $\mathcal{F}_{MN}\mathcal{F}^{MN}$; the parity-odd six-form of $\mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F}$ type exists in six dimensions but is a topological density — it belongs to the priced sector of brace 4, not the free shelf. For the spinor, $i\,\overline{\Psi}\,\Gamma^M D_M \Psi$ exhausts the first-order shelf, and no lower-order invariant exists. Step 3: the intruder. In $D = 6$ the Gauss–Bonnet density $\mathcal{G} = R^2 - 4R_{MN}R^{MN} + R_{MNPQ}R^{MNPQ}$ has second-order field equations — the Lovelock property — so stability cannot exclude it; count its derivatives (four) and apply MSMF at lowest order: it is structure the two-derivative theory does not need. As a check on the caveat's dimension-dependence, recall the four-dimensional contrast: there the same density integrates to the Euler characteristic (Chern–Gauss–Bonnet), a boundary term with no field equations at all — the intruder only becomes dynamical when the dimension gives it room, which is exactly why the honesty caveat must be stated in six dimensions and is invisible in four. Physics intuition: the pantry lock of S-1 secured the ingredients; this scan secures the recipe — at lowest order the shelves

hold the Einstein–Hilbert term, one $\mathcal{F}^2$ per gauge factor, one Dirac term per spinor, and nothing else, with everything fancier either priced by closure or diluted by the throat.

G.3.2. Spend the action once in miniature: integrate out one heavy mode at tree level and read off the Wilson coefficient it leaves behind.

Solution. Step 1: the toy. Keep one light field ϕ_0 and one heavy field ϕ_h with $\mathcal{L} = \frac{1}{2}(\partial\phi_0)^2 - \frac{1}{2}M^2\phi_h^2 - \lambda\,\phi_0^2\,\phi_h$ — the skeleton of a KK zero mode coupled to a single massive tower floor. Step 2: complete the square: $-\frac{1}{2}M^2\phi_h^2 - \lambda\phi_0^2\phi_h = -\frac{1}{2}M^2\big(\phi_h + \lambda\phi_0^2/M^2\big)^2 + \lambda^2\phi_0^4/(2M^2)$; the shifted field integrates away, leaving $\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial\phi_0)^2 + \dfrac{\lambda^2}{2M^2}\,\phi_0^4$ — a quartic the light theory never wrote down, with Wilson coefficient $c_4 = \lambda^2/(2M^2)$ generated, not postulated. Step 3: the tower. Restoring the heavy propagator's momentum dependence, $1/(p^2+M^2) = M^{-2}\,(1 - p^2/M^2 + \cdots)$ stacks derivative operators each suppressed by a further $(E/M)^2$ — precisely the pattern in which the full KK tower of T-GDOS-9.1 populates the boundary theory, with M the tower masses clustered at M_{lock} and λ the radial overlap integrals of T-GDOS-9.2 (Arisaka, 2026, A5-GDOS); the same miniature, with the same moral, is worked as Problem V.2.1 in the course chapter of GG-A5-GDOS that owns the theorem. Physics intuition: every "arbitrary" coefficient in the boundary Lagrangian is a $\lambda^2/(2M^2)$ whose M and λ the textbook never drew; the strangeness of the assembled equation is not that its numbers are fixed, but that they were always someone's overlap integrals.

G.4.1. Compute every hypercharge of one family from the embedding, and prove $|q_e| = |q_p|$ exactly.

Solution. Step 1: take $Y = \text{diag}(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, +\frac{1}{2}, +\frac{1}{2})$ on the $\mathbf{5}$ (G.4); conjugating for the $\overline{\mathbf{5}}$ gives $Y(d^c) = +\frac{1}{3}$ and $Y(L) = -\frac{1}{2}$; pair sums on the $\mathbf{10}$ give $Y(u^c) = -\frac{2}{3}$, $Y(Q) = +\frac{1}{6}$, $Y(e^c) = +1$ (G.5). Step 2: apply $Q = T_3 + Y$ to each state and recover the full charge table, $Q(u) = \frac{2}{3}$, $Q(d) = -\frac{1}{3}$, $Q(\nu) = 0$, $Q(e) = -1$. Step 3: electric charge is a generator of the surviving algebra, so it is traceless on the $\overline{\mathbf{5}}$: $3\,Q(d^c) + Q(\nu) + Q(e) = 0$, i.e. $Q(d^c) = \frac{1}{3}$ exactly, with no freedom; then $Q(p) = Q(uud) = \frac{2}{3} + \frac{2}{3} - \frac{1}{3} = +1 = -Q(e)$, and hydrogen is exactly neutral. Physics intuition: the most precisely verified coincidence in nature — atomic neutrality at one part in 10^{21} — is here a trace identity: quarks and leptons share a multiplet, so their charges must balance within it, the way the entries of any traceless matrix must sum against each other.

G.4.2. Verify all four anomaly-cancellation sums for one family, using only the computed hypercharges.

Solution. Step 1: $[\text{SU}(3)]^2\,\text{U}(1)_Y$ sums Y over color representations: $2\cdot\frac{1}{6} + (-\frac{2}{3}) + \frac{1}{3} = \frac{1}{3} - \frac{2}{3} + \frac{1}{3} = 0$. Step 2: $[\text{SU}(2)]^2\,\text{U}(1)_Y$ sums Y over doublets: $3\cdot\frac{1}{6} + (-\frac{1}{2}) = \frac{1}{2} - \frac{1}{2} = 0$ — the quark doublet's three colors against the one lepton doublet. Step 3: the cubic and gravitational sums over all fifteen Weyl fields: $[\text{U}(1)_Y]^3 = 6(\frac{1}{6})^3 + 3(-\frac{2}{3})^3 + 3(\frac{1}{3})^3 + 2(-\frac{1}{2})^3 + 1^3 = \frac{1}{36} - \frac{32}{36} + \frac{4}{36} - \frac{9}{36} + \frac{36}{36} = 0$, and $\text{grav}^2\,\text{U}(1)_Y = 6\cdot\frac{1}{6} - 3\cdot\frac{2}{3} + 3\cdot\frac{1}{3} - 2\cdot\frac{1}{2} + 1 = 1 - 2 + 1 - 1 + 1 = 0$; as a bonus, the family contains four SU(2) doublets (three colors plus one lepton), an even number, so the global Witten anomaly vanishes too. Physics intuition: in the bottom-up construction these four zeros are miracles that constrain model-builders' dreams; in the export they are the shadow of one consistent bulk multiplet — the same sums, computed upstairs once, descend as identities rather than as checks that might have failed.

G.5.1. Compute $\mathrm{Tr}\,Y^2$, $\mathrm{Tr}\,T_3^2$, and $\mathrm{Tr}\,Q^2$ over one family, and extract both the $5/3$ normalization and $\sin^2\theta_W = 3/8$.

Solution. Step 1: $\mathrm{Tr}\,Y^2 = \frac{10}{3}$ by the term-by-term sum of Eq. (G.7), and $\mathrm{Tr}\,T_3^2 = 8 \cdot \frac{1}{4} = 2$ from the eight doublet states. Step 2: the ratio gives the unified normalization $\mathrm{Tr}\,Y^2/\,\mathrm{Tr}\,T_3^2 = \frac{5}{3}$, i.e. $g_1^2 = \frac{5}{3}g'^2$ (D.9). Step 3: with $Q = T_3 + Y$ and the cross-term $\mathrm{Tr}(T_3 Y) = \frac{1}{6} \cdot 3 \cdot (\frac{1}{2} - \frac{1}{2}) + \cdots = 0$ vanishing doublet by doublet, $\mathrm{Tr}\,Q^2 = \mathrm{Tr}\,T_3^2 + \mathrm{Tr}\,Y^2 = 2 + \frac{10}{3} = \frac{16}{3}$, so $\sin^2\theta_W = \mathrm{Tr}\,T_3^2/\,\mathrm{Tr}\,Q^2 = \frac{2}{16/3} = \frac{3}{8}$ at the seam (D.10) — the same $3/8$ the gauge chapter's problem D.1.2 met from the coupling side, now derived from pure representation traces. Physics intuition: $3/8$ is what the weak angle looks like when the three forces are one undressed force — a ratio of counting weights, no dynamics anywhere — and the descent from $3/8$ at the seam to the rational $25/108$ at M_Z is the desert, crossed twice by this course from opposite ends.

G.8.1. Prove that no gauge-invariant polynomial in the Wilson-line phase exists, and state what this kills.

Solution. Step 1: under a large gauge transformation — one winding of the compact circle — the holonomy phase shifts by a full period, $\theta_H \to \theta_H + 2\pi$ (G.11), while all physics is unchanged; any admissible effective operator $f(\theta_H)$ must therefore satisfy $f(\theta_H + 2\pi) = f(\theta_H)$. Step 2: a non-constant polynomial cannot be periodic — $(\theta_H + 2\pi)^n \neq \theta_H^n$ — so every polynomial term, and in particular the mass operator $\mu^2\,\theta_H^2$, is forbidden as an operator; the admissible potential is a Fourier series, $V_{\text{eff}} \propto \cos\theta_H + \ldots$ (F.12), whose one-loop coefficients are finite and set by the compactification scale. Step 3: consequence — the quadratic divergence of the textbook hierarchy problem has no invariant operator to feed for the *selector*: there is no counterterm for an angle to drag to the cutoff, so the fine-tuning of the *scale* dissolves at the level of the operator list, before any cancellation mechanism is needed; the doublet's own radiative stability is O_{11}'s. Physics intuition: this is the entire hierarchy debate in three lines — you can renormalize a length, but you cannot quadratically renormalize an angle; periodicity is the symmetry that scalar-mass counterterms cannot break, and the Hosotani mechanism is that protection made dynamical.

G.11.1. From memory: name the zeroth and the six silent axioms of the textbook Lagrangian, and state which stage of the export chain replaces each.

Solution. Step 1: the axioms — (S0) that a four-dimensional Lagrangian, with its field content and derivative order, is the starting point at all; (S1) the gauge group; (S2) the fifteen representations and their hypercharges; (S3) three families; (S4) one scalar doublet with a chosen potential and a tuned $\mu^2 < 0$; (S5) the operator list itself, "all renormalizable gauge-invariant terms"; (S6) the twenty-eight measured coefficients. Step 2: the replacements — S0 by the earned bulk action of S-1a (form derived at lowest derivative order; content on the Minimal-Matter Lemma); S1 by the splitting of S-3 (G.6); S2 by the zero-mode computation of S-2 (one traceless matrix and its pair sums); S3 by the GG-P2-N3 pincer (conditional); S4 by the Wilson line and its forced Fourier potential (S-6); S5 by the export principle with the absences derived (S-8); S6 by the codebook of the derivation Parts, graded 6/14/8. Step 3: the one-sentence audit — every structural choice is replaced by a theorem or a graded conditional, the absences included, and the register of each replacement is stated next to it. Physics intuition: if the reader can reproduce this mapping without looking, the

interlude has done its work: the first page of the textbook has become the last page of a derivation, and the reader knows, item by item, exactly how strong each link in that inversion is.

Take-away. *Standard result:* the Standard-Model Lagrangian is postulated whole — group, representations, families, scalar sector, operator list, and twenty-eight coefficients all chosen — and fifty years of data confirm the choices without explaining any of them. *What it assumed:* everything on that list; the zeroth and the six silent axioms. *The geometric reading:* the Lagrangian is the terminal compression of a six-dimensional descent — its bulk action earned brace by brace at stated registers (S-1a), structures exported, coefficients registered at the seam, absences (the θ-term, the proton-decay operators, the scalar-mass dial) derived as theorems — assembled here term by term with the register of every claim stated, and as strong as the descent behind it, no stronger.

H Fermion masses and the Yukawa hierarchy

What this appendix teaches. The twelve fermion masses — the most arbitrary-looking block of the Standard Model, spanning twelve orders of magnitude from the top quark to the lightest neutrino: the textbook Yukawa mechanism; the staggering freedom and unexplained hierarchy it leaves behind; and the geometric cascade in which the hierarchy becomes depth counting on the bulk profile, with the per-fermion depths carried at the cascade-depth lemma's stated grade — structure forced, last digits riding on the named lemma.

Read alongside: Griffiths (2008) for the phenomenological first pass; Langacker (2017) for the flavor sector at depth.

This is the third sector appendix of the course. Its subject is the origin of the twelve fermion masses — the most arbitrary-looking block of the Standard Model, spanning twelve orders of magnitude from the top quark to the lightest neutrino. We follow the three-step plan: the textbook Yukawa mechanism; the staggering freedom and unexplained hierarchy it leaves behind; and the geometric reading in which the masses become the boundary spectrum of one family codebook, ordered by exponential cascade tolls. This is also the appendix where the course is most candid about what is *not* yet a theorem: only the top Yukawa is fully derived, and the per-fermion cascade depths remain a named open lemma. A standard treatment of the fermion families and the Yukawa sector is given in Griffiths (2008).

H.1 Step 1 — the Yukawa sector as a graduate student meets it

How a fermion gets its mass. A Dirac mass term $m\bar{\psi}_L\psi_R$ joins a left-handed and a right-handed field. But in the Standard Model the left-handed fermions are $\mathrm{SU}(2)_L$ doublets and the right-handed ones are singlets, so a bare mass term is forbidden by gauge invariance. Mass can only come through the Higgs, in a gauge-invariant *Yukawa* coupling,

$$\mathcal{L}_{\mathrm{Yuk}} = -\, y_f\, \bar{L}\, \Phi\, f_R + \mathrm{h.c.}, \tag{H.1}$$

which, after the Higgs acquires its vacuum value, becomes a mass

$$m_f = \frac{y_f\, v}{\sqrt{2}}. \tag{H.2}$$

(Physics intuition: a fermion has no mass of its own; it borrows one from the Higgs condensate. The Yukawa coupling y_f is how tightly that fermion grips the condensate as it moves through it. A heavy particle grips hard; a light one barely touches.)

The hierarchy. There is one Yukawa per charged fermion — nine numbers — plus the neutrino sector. With $v/\sqrt{2} \simeq 174$ GeV, the measured couplings range from $y_t \simeq 1$ for the top quark down to $y_e \simeq 3 \times 10^{-6}$ for the electron, and far smaller still for the neutrinos. The fermion masses therefore span

$$\underbrace{m_t \simeq 173 \text{ GeV}}_{\text{top}} \quad \longrightarrow \quad \underbrace{m_\nu \lesssim 0.1 \text{ eV}}_{\text{neutrino}}, \tag{H.3}$$

a spread of more than twelve orders of magnitude. In the Standard Model these are simply tabulated.

Neutrinos: Dirac, Majorana, and the seesaw. Neutrinos are special: being electrically neutral, they may carry a Majorana mass $\frac{1}{2} M \, \nu_L^{\mathsf{T}} C \nu_L$ in addition to (or instead of) a Dirac mass. If a heavy Majorana scale M sits above the electroweak scale, the seesaw mechanism gives the light eigenvalue

$$m_\nu \sim \frac{(y_\nu v)^2}{M}, \tag{H.4}$$

naturally tiny because M is large. *(Physics intuition: the seesaw is a lever. A very heavy right-handed partner on the long arm forces the observed neutrino onto the short arm, pushing its mass far down. The heavier the partner, the lighter the neutrino.)*

Worked example (STANDARD TEXTBOOK PEDAGOGY). Invert (H.2) for the top quark with $m_t \simeq 173$ GeV:

$$y_t = \frac{\sqrt{2}\, m_t}{v} = \frac{1.414 \times 173}{246} \simeq 0.99. \tag{H.5}$$

The top Yukawa is, to within a percent, exactly *one* — a striking fact the Standard Model records as a coincidence. For the electron the same inversion gives $y_e \simeq 2.9 \times 10^{-6}$. The textbook framework reproduces every mass — *given* the nine Yukawa couplings as inputs. Why they range over twelve orders of magnitude, and why the top sits at unity, the Standard Model does not say.

H.2 Step 2 — what the Yukawa sector assumes but cannot explain

This is the least constrained sector of the Standard Model, and the honest list of open questions is long. The nine charged-fermion Yukawas are pure inputs, and with the neutrino masses and mixings they make up the single largest block of the twenty-eight parameters. Three failures stand out.

First, *the hierarchy itself.* Twelve orders of magnitude separate the top from the lightest neutrino, and the Standard Model offers no organizing principle: the couplings are not related by any symmetry, not ordered by any rule, not bounded by any argument. That $y_t \simeq 1$ while $y_e \simeq 10^{-6}$ is, in the bottom-up theory, twelve unrelated accidents.

Second, *the family number*. The three families are exact copies distinguished only by mass, and nothing selects three rather than one or seventeen. The replication is imposed by observation, not derived.

Third, *the lightness of neutrinos*. The seesaw (H.4) explains small neutrino masses only by positing a heavy scale M by hand, and it leaves the Dirac-versus-Majorana question, the absolute mass scale, and the ordering undetermined. *(Physics intuition: the Yukawa sector is where the Standard Model most openly admits defeat. Every other sector has at least a structural story — gauge invariance, the Higgs mechanism — but flavor is a spreadsheet of measured numbers. A theory that could turn that spreadsheet into a short rule would be saying something genuinely new.)*

H.3 Step 3 — the geometric derivation: masses as a cascade on one family surface

In GG-Theory the Yukawa entries are not primitive. Each is an overlap coefficient — the reading of effective Yukawa couplings as wavefunction overlaps in the bulk, established in GG-A2-GG6 and GG-A5-GDOS (Arisaka, 2026, A2-GG6; Arisaka, 2026, A5-GDOS) and worked out in full for the charged-lepton ladder in GG-P3-Electron (Arisaka, 2026, P3-Electron) — the integral of two fermion wavefunctions against the Higgs profile — on a single geometric family stratum, the complex projective surface $\mathbb{C}P^2$ doubled by $\mathrm{SU}(2)_L$. A mass table is then not twelve unrelated insertions but the boundary spectrum of *one* codebook, and the twelve-order hierarchy is the signature of an exponential cascade across that surface. We give the geometric reading first, then the explicit codebook arithmetic.

H.3.1 The cascade tolls: why the hierarchy is exponential

The family stratum is explored by the GRIP cascade, and moving down it from one family basin to the next is not free: each step across a GSSB gate pays a Bogomolnyi toll e^{-S} with action $S = \pi k$, k the integer winding of the gate. The quark Yukawas therefore descend as integer powers of the elementary toll $e^{-\pi}$,

$$y_t = 1, \quad y_b \simeq \tfrac{3}{8} e^{-\pi}, \quad y_c \simeq 3\, e^{-2\pi}, \quad y_s \simeq 6\, e^{-3\pi}, \quad y_d \simeq 6\, e^{-4\pi}, \quad y_u \simeq 3\, e^{-4\pi}, \tag{H.6}$$

in which the *exponent* $n = 0, 1, 2, 3, 4$ sets the order of magnitude and the small-integer prefactor $(1, \frac{3}{8}, 3, 6, 6, 3)$ is an overlap multiplicity. *(Physics intuition: picture the three families as three terraces down a hillside, separated by walls. To stand on a lower terrace a fermion must tunnel through a wall, and each wall costs the universal toll $e^{-\pi} \approx 0.043$ — a factor of about twenty-three in mass. The five-order spread is not five random numbers; it is four tolls of the same wall, $e^{-\pi}, e^{-2\pi}, e^{-3\pi}, e^{-4\pi}$, with small integer crowds of states riding through each gate.)*

H.3.2 The top Yukawa: the zero-toll anchor

The top quark sits at the head of the cascade, at zero winding, and therefore pays *no* toll. Its Yukawa is the bare geometric unit,

$$\boxed{y_t = 1\,,} \tag{H.7}$$

which predicts $m_t = v/\sqrt{2} \simeq 174$ GeV at first order, matching the measured $\simeq 173$ GeV after the quantum-defect transport factor 127/128 is applied. This is a genuine parameter-free *prediction*, and in the honesty triage $y_t = 1$ is one of the six **Derived** entries. *(Physics intuition: the top is the only fermion that grips the Higgs with the full, undiscounted strength of the geometry — it stands on the top terrace and tunnels through nothing. That the strongest possible Yukawa is exactly unity, and that nature places one quark exactly there, is the cleanest single hint that the Yukawa couplings are geometric overlaps rather than free dials.)*

H.3.3 The charged-lepton ladder: a dyadic code

The charged leptons live on the same surface but, carrying no color winding, are read on a dyadic–rational lattice rather than through $e^{-\pi}$ tolls,

$$y_\tau = \frac{21}{2048}, \qquad y_\mu = \frac{5}{8192}, \qquad y_e = \frac{3}{1024000}, \tag{H.8}$$

with denominators that are powers of two ($2048 = 2^{11}$, $8192 = 2^{13}$) dressed by small rational overlap factors. The lepton hierarchy is thus a binary-lattice reading of the same family geometry that gives the quarks their tolls.

H.3.4 The neutrinos: a geometric seesaw

The neutrino sector realizes the seesaw geometrically: the high scale is not a free M but the grand-unification seam $E_{\rm GUT}$, and the common neutrino scale is

$$m_\star = \pi \, \frac{v^2}{E_{\rm GUT}} \simeq 8.36 \text{ meV}, \tag{H.9}$$

with first-order eigenvalues in the integer ratio $(m_1, m_2, m_3) = (0, 1, 6)\, m_\star$. The neutrinos are light for the same reason the electroweak scale is small: a ratio of fixed geometric scales, here $v^2/E_{\rm GUT}$, rather than a tuned input. *(Physics intuition: the seesaw lever's long arm is the GUT seam itself. Once the boundary law fixes $E_{\rm GUT} = E_{\rm Pl} e^{-2\pi}$, the neutrino scale $\pi v^2/E_{\rm GUT}$ is fixed too — the meV scale of neutrino mass is the geometric shadow of the* 10^{16} GeV *seam, not a new parameter.)*

H.3.5 Honesty: the cascade-depth lemma is the dominant open work

It must be said plainly where this stands. The *pattern* of (H.6) — integer tolls $e^{-n\pi}$ with small-integer prefactors — is striking and reduces a twelve-order spread to a handful of integers. But the *assignment* of a definite winding n and prefactor to each fermion is read from the codebook, not yet proved: it requires the height of each fermion's fixed point under a sector-dependent torus action on $\mathbb{C}P^2$ doubled by $\mathrm{SU}(2)_L$ — the *cascade-depth lemma*, L10. Until that lemma is proved, the eight charged-fermion Yukawas other than the top are recorded in the codebook (§8) as **Asserted**, and the cascade-depth lemma is identified as the dominant genuinely-open task of the program, the natural subject of a successor paper. The top alone, at zero toll, is **Derived**. This is the most

incomplete sector of the derivation, and the course states it without softening: the geometry strongly suggests the cascade, the numbers fit the cascade, but the cascade is not yet a theorem.

The explicit arithmetic of all twelve masses — the first-order codes, the quantum-defect transport factors, and the resulting masses — is recorded next, so that every number in the main text can be reconstructed by hand.

H.4 Detailed arithmetic of the fermion mass codebook

This appendix records the elementary arithmetic behind the 12-mass block so that no numerical step is hidden.

H.5 Common electroweak normalization

For every Dirac fermion at first order,

$$m_f^{(1)} = \frac{v^{(1)}}{\sqrt{2}}\, y_f^{(1)}.$$

Using

$$v^{(1)} = 246.331354 \text{ GeV},$$

one obtains the common normalization

$$\frac{v^{(1)}}{\sqrt{2}} = 174.182570832 \text{ GeV}. \tag{H.10}$$

This is the single number against which all first-order charged-fermion codes are evaluated.

H.6 Charged-lepton arithmetic

The charged-lepton Yukawa codes are

$$y_\tau^{(1)} = \frac{21}{2048} = 0.01025390625, \qquad y_\mu^{(1)} = \frac{5}{8192} = 0.0006103515625,$$

$$y_e^{(1)} = \frac{3}{1024000} = 2.9296875 \times 10^{-6}.$$

Multiplying by (H.10) gives

$$m_\tau^{(1)} = 174.182570832 \times 0.01025390625 = 1.7860517517 \text{ GeV},$$

$$m_\mu^{(1)} = 174.182570832 \times 0.0006103515625 = 0.1063126043 \text{ GeV},$$

$$m_e^{(1)} = 174.182570832 \times 2.9296875 \times 10^{-6} = 5.10300500 \times 10^{-4} \text{ GeV}.$$

The QD transport factors are

$$\mathcal{R}_\tau = \frac{637}{640} = 0.9953125, \qquad \mathcal{R}_\mu = \frac{159}{160} = 0.99375, \qquad \mathcal{R}_e = \frac{641}{640} = 1.0015625.$$

Hence

$$m_\tau^{\mathrm{QD}} = \frac{637}{640}\, m_\tau^{(1)} = 1.7776796341 \text{ GeV},$$
$$m_\mu^{\mathrm{QD}} = \frac{159}{160}\, m_\mu^{(1)} = 0.1056481505 \text{ GeV},$$
$$m_e^{\mathrm{QD}} = \frac{641}{640}\, m_e^{(1)} = 5.11097845 \times 10^{-4} \text{ GeV}.$$

H.7 Quark arithmetic

The first-order quark Yukawa codes are

$$y_t^{(1)} = 1, \qquad y_b^{(1)} = \frac{3}{8}\mathrm{e}^{-\pi} = 0.01620587656,$$
$$y_c^{(1)} = 3\mathrm{e}^{-2\pi} = 0.00560238352, \qquad y_s^{(1)} = 6\mathrm{e}^{-3\pi} = 4.84264774 \times 10^{-4},$$
$$y_d^{(1)} = 6\mathrm{e}^{-4\pi} = 2.09238845 \times 10^{-5}, \qquad y_u^{(1)} = 3\mathrm{e}^{-4\pi} = 1.04619422 \times 10^{-5}.$$

Multiplying by (H.10) yields the first-order masses quoted in the main text:

$$m_t^{(1)} = 174.182570832 \text{ GeV},$$
$$m_b^{(1)} = 2.8226667671 \text{ GeV}, \qquad m_c^{(1)} = 0.9758279277 \text{ GeV},$$
$$m_s^{(1)} = 8.4338696612 \times 10^{-2} \text{ GeV}, \qquad m_d^{(1)} = 3.64460554 \times 10^{-3} \text{ GeV},$$
$$m_u^{(1)} = 1.82230277 \times 10^{-3} \text{ GeV}.$$

The QD transport factors are

$$\mathcal{R}_t = \frac{127}{128}, \qquad \mathcal{R}_b = \frac{3}{2}, \qquad \mathcal{R}_c = \frac{4}{3}, \qquad \mathcal{R}_s = \frac{9}{8}, \qquad \mathcal{R}_u = \mathcal{R}_d = \frac{5}{4}.$$

Therefore

$$m_t^{\mathrm{QD}} = \frac{127}{128} m_t^{(1)} = 172.821770 \text{ GeV},$$
$$m_b^{\mathrm{QD}} = \frac{3}{2} m_b^{(1)} = 4.23400015 \text{ GeV}, \qquad m_c^{\mathrm{QD}} = \frac{4}{3} m_c^{(1)} = 1.30110390 \text{ GeV},$$
$$m_s^{\mathrm{QD}} = \frac{9}{8} m_s^{(1)} = 94.8810337 \text{ MeV}, \qquad m_d^{\mathrm{QD}} = \frac{5}{4} m_d^{(1)} = 4.55575693 \text{ MeV},$$
$$m_u^{\mathrm{QD}} = \frac{5}{4} m_u^{(1)} = 2.27787846 \text{ MeV}.$$

H.8 Neutrino arithmetic

The common first-order neutrino scale is

$$m_\star^{(1)} = \pi \frac{[v^{(1)}]^2}{E_{\mathrm{GUT}}} = 8.36113789 \text{ meV}.$$

The first-order eigenvalues are

$$(m_1, m_2, m_3)^{(1)} = (0, 1, 6)m_\star^{(1)} = (0, 8.36113789, 50.1668273) \text{ meV}.$$

Their squared differences are

$$\Delta m_{21}^{2\,(1)} = (8.36113789 \times 10^{-3})^2 = 6.990864 \times 10^{-5} \text{ eV}^2,$$

$$\Delta m_{31}^{2\,(1)} = (50.1668273 \times 10^{-3})^2 = 2.516711 \times 10^{-3} \text{ eV}^2.$$

At QD order,

$$m_\star = \pi \frac{v_{\rm QD}^2}{E_{\rm GUT}} = 8.35358030 \text{ meV},$$

and

$$m_1^{\rm QD} = \frac{1}{128} m_\star = 0.0652623 \text{ meV},$$

$$m_2^{\rm QD} = \frac{36}{35} m_\star = 8.59225345 \text{ meV},$$

$$m_3^{\rm QD} = 6\frac{129}{128} m_\star = 50.5130555 \text{ meV}.$$

Therefore

$$\Delta m_{21}^{2\,\rm QD} = (8.59225345^2 - 0.0652623^2) \times 10^{-6} = 7.382256 \times 10^{-5} \text{ eV}^2,$$

$$\Delta m_{31}^{2\,\rm QD} = (50.5130555^2 - 0.0652623^2) \times 10^{-6} = 2.551565 \times 10^{-3} \text{ eV}^2.$$

H.9 Theory-error arithmetic

For every first-order comparison entry, the theory error is the omitted QD correction:

$$\sigma_{\rm th}^{(1)} = |m^{\rm QD} - m^{(1)}|.$$

For example,

$$\sigma_{\rm th}^{(1)}(m_t) = |172.821770 - 174.182571| = 1.360801 \text{ GeV}.$$

For every QD entry,

$$\sigma_{\rm th}^{(2)} = \frac{1}{8} |m^{\rm QD} - m^{(1)}|.$$

Thus

$$\sigma_{\rm th}^{(2)}(m_t) = 0.170100 \text{ GeV}, \qquad \sigma_{\rm th}^{(2)}(m_\tau) = 0.0010465 \text{ GeV},$$

and similarly for the other particles.

The purpose of this appendix is simple: every number quoted in the main text can be reconstructed by hand from a compact chain of formulas.

H.10 Detailed arithmetic of the CKM codebook

This appendix reconstructs the CKM block numerically from the quark code of Section 49.2. Its purpose is the same as that of Appendix H.4: every number that appears in the main text should be reproducible by hand from a compact chain of formulas.

H.11 Sectoral overlap angles from the quark codebook

From the first-order quark Yukawas,

$$y_t^{(1)} = 1, \qquad y_b^{(1)} = \frac{3}{8}\mathrm{e}^{-\pi}, \qquad y_c^{(1)} = 3\mathrm{e}^{-2\pi}, \qquad y_s^{(1)} = 6\mathrm{e}^{-3\pi}, \qquad y_u^{(1)} = 3\mathrm{e}^{-4\pi}, \qquad y_d^{(1)} = 6\mathrm{e}^{-4\pi},$$

the four sectoral nearest-neighbor overlap ratios needed for CKM are

$$\sqrt{\frac{y_u^{(1)}}{y_c^{(1)}}} = \mathrm{e}^{-\pi} = 0.0432139183, \qquad \sqrt{\frac{y_d^{(1)}}{y_s^{(1)}}} = \mathrm{e}^{-\pi/2} = 0.207879576, \tag{H.11}$$

$$\sqrt{\frac{y_c^{(1)}}{y_t^{(1)}}} = \sqrt{3}\,\mathrm{e}^{-\pi} = 0.0748496212, \qquad \sqrt{\frac{y_s^{(1)}}{y_b^{(1)}}} = 4\,\mathrm{e}^{-\pi} = 0.172855673. \tag{H.12}$$

These four numbers are the basic geometric ingredients of the CKM block.

H.12 Cabibbo arithmetic and the golden-ratio coefficient

The branch-complement phase is

$$\varphi_{12} = \frac{3\pi}{5}, \qquad \cos\varphi_{12} = \cos\frac{3\pi}{5} = -\frac{\sqrt{5}-1}{4}.$$

Hence

$$\begin{aligned}\sin^2\theta_{12}^{(1)} &= \left|\mathrm{e}^{-\pi/2} - e^{\,i3\pi/5}\mathrm{e}^{-\pi}\right|^2 \\ &= \mathrm{e}^{-\pi} + \mathrm{e}^{-2\pi} - 2\mathrm{e}^{-3\pi/2}\cos\frac{3\pi}{5} \\ &= \mathrm{e}^{-\pi} + \mathrm{e}^{-2\pi} + \frac{\sqrt{5}-1}{2}\mathrm{e}^{-3\pi/2}.\end{aligned} \tag{H.13}$$

Numerically,

$$\mathrm{e}^{-\pi} = 0.0432139183, \qquad \mathrm{e}^{-2\pi} = 0.00186744273, \qquad \frac{\sqrt{5}-1}{2}\mathrm{e}^{-3\pi/2} = 0.00555197873,$$

so

$$\sin^2\theta_{12}^{(1)} = 0.0506333397, \qquad \sin\theta_{12}^{(1)} = 0.225018533.$$

The QD-protected value follows by multiplication with the combined dyadic/throat factor $1 - 1/(K2^J) = 1 - 1/4480$:

$$\sin\theta_{12}^{\mathrm{QD}} = \left(1 - \frac{1}{4480}\right) 0.225018533 = 0.224968306.$$

H.13 $2 \leftrightarrow 3$ and $1 \leftrightarrow 3$ arithmetic

The first-order $2 \leftrightarrow 3$ angle is

$$\begin{aligned}\sin\theta_{23}^{(1)} &= \pi \left(\sqrt{3}\,\mathrm{e}^{-\pi}\right)\left(4\,\mathrm{e}^{-\pi}\right) \\ &= 4\pi\sqrt{3}\,\mathrm{e}^{-2\pi} = 0.0406459973. \end{aligned} \tag{H.14}$$

The first-order $1 \leftrightarrow 3$ angle is

$$\sin\theta_{13}^{(1)} = \sqrt{3}\,\mathrm{e}^{-2\pi} = 0.00323450569. \tag{H.15}$$

For the QD values we use the already-derived quark masses:

$$m_t^{\mathrm{QD}} = 172.821770 \text{ GeV}, \quad m_b^{\mathrm{QD}} = 4.23400015 \text{ GeV}, \quad m_c^{\mathrm{QD}} = 1.30110390 \text{ GeV},$$

$$m_s^{\mathrm{QD}} = 94.8810337 \text{ MeV}, \quad m_u^{\mathrm{QD}} = 2.27787846 \text{ MeV}.$$

Then

$$\sin\theta_{23}^{\mathrm{QD}} = \pi\sqrt{\frac{(94.8810337 \times 10^{-3})(1.30110390)}{(4.23400015)(172.821770)}} = 0.0408057070,$$

and

$$\sin\theta_{13}^{\mathrm{QD}} = \sqrt{\frac{2.27787846 \times 10^{-3}}{172.821770}} = 0.00363049673.$$

H.14 Phase arithmetic, Jarlskog invariant, and matrix reconstruction

The first-order physical phase is

$$\delta_{\mathrm{CKM}}^{(1)} = \frac{2\pi}{5} = 1.25663706144,$$

while the QD phase is

$$\delta_{\mathrm{CKM}}^{\mathrm{QD}} = \frac{13\pi}{35} = 1.16687727133.$$

Using the QD angles,

$$s_{12} = 0.224968306, \qquad s_{23} = 0.0408057070, \qquad s_{13} = 0.00363049673,$$

one obtains

$$c_{12} = 0.974359685, \qquad c_{23} = 0.999167112, \qquad c_{13} = 0.999993410.$$

The Jarlskog invariant is therefore

$$\begin{aligned} J^{\mathrm{QD}} &= c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta_{\mathrm{CKM}}^{\mathrm{QD}} \\ &= (0.974359685)(0.999167112)(0.999986820) \\ &\quad \times (0.224968306)(0.0408057070)(0.00363049673)\sin(1.16687727133) \\ &= 2.98351\times 10^{-5}. \end{aligned} \tag{H.16}$$

Substituting the same angles and phase into the standard CKM parameterization yields

$$|V_{\mathrm{CKM}}^{\mathrm{QD}}| = \begin{pmatrix} 0.974360 & 0.224967 & 0.003630 \\ 0.224838 & 0.973541 & 0.040805 \\ 0.008442 & 0.040087 & 0.999161 \end{pmatrix}.$$

From this one reads the unitarity-triangle apex

$$\bar\rho + i\bar\eta = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} = 0.151746 + 0.354449\,i,$$

and therefore

$$(\bar\rho,\bar\eta)^{\mathrm{QD}} = (0.151746,\ 0.354449).$$

H.15 Residual theory uncertainties used in the main tables

For the primary CKM observables, the first-order theory uncertainties are

$$\sigma_{\mathrm{th}}^{(1)}(\sin\theta_{12}) = 5.02274\times 10^{-5}, \qquad \sigma_{\mathrm{th}}^{(1)}(\sin\theta_{23}) = 1.59710\times 10^{-4},$$

$$\sigma_{\mathrm{th}}^{(1)}(\sin\theta_{13}) = 3.95991\times 10^{-4}, \qquad \sigma_{\mathrm{th}}^{(1)}(\delta_{\mathrm{CKM}}) = 8.97598\times 10^{-2},$$

obtained by taking the difference between the QD and first-order values. The residual QD uncertainties are simply one eighth of these values:

$$\sigma_{\mathrm{th}}^{(2)}(\sin\theta_{12}) = 6.27842\times 10^{-6}, \qquad \sigma_{\mathrm{th}}^{(2)}(\sin\theta_{23}) = 1.99637\times 10^{-5},$$

$$\sigma_{\mathrm{th}}^{(2)}(\sin\theta_{13}) = 4.94989\times 10^{-5}, \qquad \sigma_{\mathrm{th}}^{(2)}(\delta_{\mathrm{CKM}}) = 1.121997\times 10^{-2}.$$

The propagated uncertainties for $|V_{ij}|$, A, $\bar\rho$, $\bar\eta$, and J in Table 37 are obtained from these four residual QD uncertainties by standard quadrature of the linearized sensitivities.

The practical lesson of this appendix is that the CKM arithmetic is no less explicit than the mass arithmetic. Every number in the quark-mixing block follows from a finite chain of formulas whose inputs are already fixed by the preceding sections of the paper.

H.16 Detailed arithmetic of the PMNS/Majorana codebook

This appendix records the arithmetic behind the PMNS/Majorana block so that every number quoted in Section 54.3 can be checked line by line. The purpose is the same as in the earlier fermion and CKM appendices: the final numerical tables should be visibly traceable to short exact formulas.

H.17 Solar and reactor arithmetic

The solar angle begins from

$$\sin^2\theta_{12}^{(1)} = \frac{L^2}{3N_g^3} = \frac{25}{81} = 0.3086419753.$$

The QD correction uses the factor

$$1 - \frac{1}{K2^{2N+1}} = 1 - \frac{1}{35\cdot 2^7} = 1 - \frac{1}{4480} = 0.999776786,$$

so that

$$\sin^2\theta_{12}^{\mathrm{QD}} = \frac{25}{81}\left(1 - \frac{1}{4480}\right) = 0.3085730820.$$

The reactor angle begins from

$$\sin^2\theta_{13}^{(1)} = \frac{N_g}{4K} = \frac{3}{140} = 0.02142857143.$$

Its QD multiplier is

$$1 + \frac{1}{2K} + \frac{1}{2^{2N+1}} = 1 + \frac{1}{70} + \frac{1}{128} = 1.022098214,$$

which gives

$$\sin^2\theta_{13}^{\mathrm{QD}} = \frac{3}{140}\left(1 + \frac{1}{70} + \frac{1}{128}\right) = 0.0219021046.$$

H.18 Atmospheric and phase arithmetic

The first-order atmospheric angle is

$$\sin^2\theta_{23}^{(1)} = \frac{1}{2}\left(1 + \frac{1}{7}\right) = \frac{4}{7} = 0.5714285714.$$

The QD correction is

$$\sin^2\theta_{23}^{\mathrm{QD}} = \frac{1}{2}\left(1 + \frac{1}{7} - \frac{1}{35}\right) = \frac{39}{70} = 0.5571428571.$$

For the Dirac phase we first define

$$\psi_\nu = \arcsin\left(\frac{1}{7}\right) = 0.1433475689 \text{ rad}.$$

Hence

$$\delta^{(1)}_{\rm PMNS} = \pi + 2\psi_\nu = 3.4282877914 \text{ rad} = 196.426421^\circ.$$

The QD phase slip is

$$\frac{\pi}{2K(2N+1)} = \frac{\pi}{490} = 0.00641141358 \text{ rad} = 0.367346939^\circ,$$

so that

$$\delta^{\rm QD}_{\rm PMNS} = 3.4282877914 - 0.00641141358 = 3.4218763778 \text{ rad} = 196.059074^\circ.$$

H.19 Majorana phases, PMNS matrices, Jarlskog invariant, and neutrinoless-double-beta scale

The Majorana phases are protected at

$$(\alpha_{21}, \alpha_{31}) = (0, \pi).$$

Using the first-order angles and phase yields

$$|U^{(1)}_{\rm PMNS}| = \begin{pmatrix} 0.822522 & 0.549571 & 0.146385 \\ 0.276669 & 0.603548 & 0.747786 \\ 0.496901 & 0.577669 & 0.647602 \end{pmatrix}, \qquad J^{(1)}_{\rm PMNS} = -9.26009 \times 10^{-3}.$$

Using the QD angles and phase yields

$$|U^{\rm QD}_{\rm PMNS}| = \begin{pmatrix} 0.822364 & 0.549377 & 0.147994 \\ 0.282542 & 0.612560 & 0.738201 \\ 0.493849 & 0.568292 & 0.658147 \end{pmatrix}, \qquad J^{\rm QD}_{\rm PMNS} = -9.18733 \times 10^{-3}.$$

For the effective neutrinoless-double-beta mass one obtains

$$m^{(1)}_{\beta\beta} = 1.72386 \text{ meV}, \qquad m^{\rm QD}_{\beta\beta} = 1.79925 \text{ meV}.$$

Thus the PMNS/Majorana arithmetic is fully explicit, from the exact rational codes to the final oscillation observables and the derived Majorana test quantity.

H.20 Residual theory uncertainties used in the PMNS tables

The first-order omitted-QD shifts are

$$\sigma^{(1)}_{\rm th}(\sin^2\theta_{12}) = 6.88933 \times 10^{-5}, \qquad \sigma^{(1)}_{\rm th}(\sin^2\theta_{13}) = 4.73533 \times 10^{-4},$$

$$\sigma^{(1)}_{\rm th}(\sin^2\theta_{23}) = 1.42857 \times 10^{-2}, \qquad \sigma^{(1)}_{\rm th}(\delta_{\rm PMNS}) = 0.367347^\circ.$$

The residual QD uncertainties are one eighth of these values:

$$\sigma^{(2)}_{\rm th}(\sin^2\theta_{12}) = 8.61166\times10^{-6}, \qquad \sigma^{(2)}_{\rm th}(\sin^2\theta_{13}) = 5.91916\times10^{-5},$$

$$\sigma^{(2)}_{\rm th}(\sin^2\theta_{23}) = 1.78571\times10^{-3}, \qquad \sigma^{(2)}_{\rm th}(\delta_{\rm PMNS}) = 0.0459184^\circ.$$

For the protected Majorana phases,

$$\sigma^{(1)}_{\rm th}(\alpha_{21}) = \sigma^{(1)}_{\rm th}(\alpha_{31}) = 0.0401786^\circ, \qquad \sigma^{(2)}_{\rm th}(\alpha_{21}) = \sigma^{(2)}_{\rm th}(\alpha_{31}) = 0.00502232^\circ.$$

These are exactly the numbers used in the comparison tables of the main text.

H.21 The full 28-parameter census (consolidated in the Summary and audit Part)

The complete 28-parameter census — every parameter's first-order prediction, QD-completed value, observed benchmark, and fractional deviation collected in one place — now lives in a single canonical home: the *Summary and audit* Part (Part XII). The first-order codebook is Table 58 (§99); its QD-completed refinement is Table 59 (§100); and the MSMF/OGN per-parameter provenance audit there grades each entry and licenses the 6/14/8 honesty triage. The twenty-eight parameters are tabulated once and graded once, in those two tables and nowhere else, so there is no second copy that could drift away from them. The compression claim those tables make good on is the same one stated throughout: one is not being asked to accept 28 unrelated empirical inputs, but to test whether 28 low-energy quantities can be read as one coherent seam export of the GG-6/GDOS geometry.

Problems and worked solutions. **H.1.1. Extract the top Yukawa and recognize the anchor.** *Solution.* Step 1: from (H.2), $y_f = \sqrt{2}\,m_f/v$. Step 2: for the top, $y_t = \sqrt{2}\times 172.7/246.2 = 0.992$ (H.5) — unity to better than one percent, the only order-one Yukawa in the theory. Step 3: in the Standard Model this is an unexplained accident; in the cascade of Step 3 it is the anchor — the top sits at zero winding and pays no toll, $y_t = 1$ exactly at first order (H.7), with the percent-level residual priced in the QD section. Physics intuition: when a dial reads 1.000 to three digits, suspect that it is not a dial — the top's Yukawa is the course's loudest hint that the flavor block is geometry, and every other fermion's smallness is measured *downhill* from this anchor.

H.1.2. Span the ladder: from the top to the electron to the neutrinos. *Solution.* Step 1: the electron: $y_e = \sqrt{2}\times 0.000511/246.2 = 2.94\times10^{-6}$. Step 2: the ratio $y_t/y_e = 3.4\times10^5$; including the neutrinos ($m_\nu \lesssim 0.1$ eV, (H.3)) the full span exceeds 10^{12}. Step 3: count the decades: twelve orders of magnitude across one interaction type with one vev — the Standard Model's least explained structure, and the chapter's Step 2 lists it as pure input: twelve dials. Physics intuition: no other sector of physics tolerates couplings of one mechanism spread across twelve decades without a generating principle; the cascade of the next problems is the claim that the principle was always there — the decades are *windings*, not choices.

H.7.1. Read the quark ladder: extract the winding number of each rung.

Solution. Step 1: the cascade prices each GSSB gate at the Bogomolnyi toll $e^{-\pi k}$ (H.6): $y_t = 1$ ($k = 0$); $y_b \simeq \frac{3}{8}e^{-\pi}$ ($k = 1$); $y_c \simeq 3e^{-2\pi}$ ($k = 2$); $y_s \simeq 6e^{-3\pi}$ ($k = 3$); $y_d \simeq 6e^{-4\pi}$, $y_u \simeq 3e^{-4\pi}$ ($k = 4$). Step 2: verify the collapse: take logarithms — $\ln(1/y)$ advances by $\approx \pi$ per rung, with the small rational prefactors $(\frac{3}{8}, 3, 6)$ as the "crowd factors" of states sharing each gate. Step 3: five orders of magnitude in the quark sector reduce to one number (π), five windings (0–4), and small integers — the freedom count drops from six continuous dials to a handful of integers. Physics intuition: the ladder converts *why is the up quark so light?* into *why does it sit four gates downhill?* — a question about topology, answerable, and answered upstream; the residuals against measured masses are priced honestly in the chapter's theory-error subsection, not hidden in the prefactors.

H.6.1. Evaluate the dyadic lepton codes against the measured masses.
Solution. Step 1: the colorless leptons read on the dyadic–rational lattice (H.8): $y_\tau = 21/2048$, $y_\mu = 5/8192$, $y_e = 3/1024000$. Step 2: convert with $m = yv/\sqrt{2}$: $m_\tau = 1.785$ GeV (measured 1.777), $m_\mu = 106.3$ MeV (measured 105.7), $m_e = 0.5100$ MeV (measured 0.5110). Step 3: three masses across four decades, each within 0.2–0.6%, from three rational codes and no continuous parameter — with the residuals carried by the QD dressing at its stated grade. Physics intuition: the leptons, carrying no color winding, cannot pay $e^{-\pi}$ tolls — so the same geometry that gives quarks a transcendental ladder gives leptons a *rational* one: two number species, one mechanism distinction (winding vs. none), exactly as F.10.1's census demands.

H.8.1. Evaluate the geometric seesaw scale.
Solution. Step 1: the standard seesaw (H.4) leaves the heavy scale M free; the geometric version pins it to the unification seam: $m_\star = \pi v^2/E_{\rm GUT}$ (H.9). Step 2: with $E_{\rm GUT} = M_{\rm Pl}e^{-2\pi} = 2.28\times10^{16}$ GeV: $m_\star = \pi \times 246.2^2/2.28 \times 10^{16}\,\text{GeV} = 8.35$ meV. Step 3: the common neutrino scale lands in the meV decade — the right neighborhood for the measured splittings ($\sqrt{\Delta m^2_{21}} \approx 8.7$ meV) — with no free M anywhere; the ordering prediction that follows is the laboratory kill row. Physics intuition: the cosmology companion's generic seesaw (its problem I.1.1) ran the logic backwards — from m_ν up to a scale; here the scale is supplied and the mass comes *down* as a prediction: the same lever, held at the other end, and the two chapters' numbers agree because the seam is one.

H.12.1. Evaluate the exact Cabibbo expression — golden ratio included.
Solution. Step 1: the pentagonal holonomy gives the 1–2 overlap (H.13): $\sin^2\theta_{12} = e^{-\pi} + e^{-2\pi} + \frac{\sqrt{5}-1}{2}\,e^{-3\pi/2}$, the golden-ratio coefficient arriving from $\cos(3\pi/5)$. Step 2: evaluate: $0.0432136 + 0.0018674 + 0.61803\times0.0089833 = 0.0506335$, so $\sin\theta_{12} = 0.225019$. Step 3: the measured Wolfenstein parameter is $\lambda = 0.22500(67)$: agreement at the 10^{-4} level from an expression containing π, e, and φ and no dial. Physics intuition: the Cabibbo angle — flavor physics' oldest measured number — is here two tolls and a pentagon; the golden ratio is not numerology but the cosine of the pentagonal phase doing its one job, and the QD dressing (the 1/4480 of the next subsection) is priced separately and openly.

H.13.1. Evaluate the small mixings and state their register.
Solution. Step 1: the first-order 2–3 overlap is $\sin\theta^{(1)}_{23} = 4\pi\sqrt{3}\,e^{-2\pi} = 0.040646$ (H.14), against the measured 0.0405 ± 0.0006: inside 1σ at first order. Step 2: the 1–3 overlap begins at $\sin\theta^{(1)}_{13} = \sqrt{3}\,e^{-2\pi} = 0.0032345$ (H.15), against the measured 0.00365: the first-order value carries an 11%

deficit. Step 3: the register, exactly as the chapter states it: the QD completion lifts s_{13} to 0.0036305 (the value used in H.14.1) — structure at first order, last digits at QD, uncertainties in the residual-theory subsection. Physics intuition: the smallest mixing is the most dressing-sensitive — it is a double toll, so every correction matters relatively more; that its QD value lands on the data is a nontrivial second-order success, and saying so *as* a second-order success is what the honesty register is for.

H.14.1. Assemble the Jarlskog invariant and compare with the global fit.
Solution. Step 1: with the QD-completed values $s_{12} = 0.224968$, $s_{23} = 0.0408057$, $s_{13} = 0.0036305$, $\delta = 1.16688$, form $J = c_{12}c_{23}c_{13}^2\, s_{12}s_{23}s_{13} \sin\delta$ (H.16). Step 2: evaluate: $J = 2.9835 \times 10^{-5}$. Step 3: the global-fit value is $J = (3.08^{+0.15}_{-0.13}) \times 10^{-5}$: the assembled invariant sits within $\sim 0.7\sigma$ — and, as the P5 companion's problem E.2.1 proves, *every* CP-violating observable in the quark sector is proportional to this one number: one pentagon, all of CP. Physics intuition: J is the area of the unitarity triangle — the cascade fixes its three sides from tolls and a pentagonal phase, and the area comes out right; this is the flavor sector's single sharpest test of the whole construction, which is why the falsification catalog keys on it.

Take-away. *Standard result:* twelve masses from twelve Yukawa couplings — a perfect fit with zero explanation. *What it assumed:* the couplings themselves, spanning twelve orders of magnitude, plus the absence of any principle relating them. *The geometric reading:* the hierarchy is depth counting on the bulk profile — exponentials of integers — with the per-fermion depths graded honestly on the cascade-depth lemma, and the whole block tabulated once in the canonical census.

I Quark mixing: the CKM matrix and CP violation

What this appendix teaches. The CKM matrix from scratch — three angles, one CP-violating phase, the Wolfenstein hierarchy, and the Jarlskog invariant — the four unexplained parameters the textbook leaves free; and the geometric derivation of the pentagonal phase and the multiplicative suppression, with the forced structure separated honestly from the last-digit values that ride on the per-fermion cascade depths.

Read alongside: Thomson (2013) for the experiment-aware introduction; Kobayashi & Maskawa (1973) and Cabibbo (1963) for the originals.

This is the fourth sector appendix of the course. Its subject is the Cabibbo–Kobayashi–Maskawa matrix — the three angles and one CP-violating phase that relate the quark mass eigenstates to the weak eigenstates. We follow the three-step plan: the textbook CKM matrix; the four unexplained parameters it leaves free; and the geometric derivation, in which mixing becomes the misalignment of two quark family projectors and the Cabibbo angle emerges from a pentagonal holonomy with the golden ratio in plain view. The textbook treatment of quark mixing and the CKM matrix is given in Thomson (2013).

I.1 Step 1 — the CKM matrix as a graduate student meets it

Where mixing comes from. Quark masses and quark weak couplings are diagonal in different bases. Diagonalizing the up-type and down-type mass matrices requires separate unitary rotations

U_{uL}, U_{dL}; the charged current then sees only their mismatch,

$$V_{\rm CKM} = U_{uL}^\dagger U_{dL}, \tag{I.1}$$

a 3×3 unitary matrix. Of its nine parameters, five are unphysical quark phases, leaving *four* physical ones: three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and one CP-violating phase δ. *(Physics intuition: if the up and down quarks agreed on which family was "first," there would be no mixing. They do not quite agree, and the CKM matrix is the small rotation between their two opinions.)*

The Wolfenstein hierarchy. Experimentally the matrix is strongly hierarchical, well captured by the Wolfenstein expansion in $\lambda = \sin\theta_{12} \simeq 0.225$,

$$V_{\rm CKM} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \tag{I.2}$$

so the angles fall off as powers of λ: $\sin\theta_{12} : \sin\theta_{23} : \sin\theta_{13} \approx \lambda : \lambda^2 : \lambda^3$. The single phase η is the origin of all CP violation in the quark sector — the insight, due to Kobayashi and Maskawa (Kobayashi & Maskawa, 1973), that three generations admit exactly one irreducible CP-violating phase — quantified rephasing-invariantly by the Jarlskog determinant $J \simeq 3 \times 10^{-5}$. *(Physics intuition: $\lambda \approx 0.22$ is the master small number of the quark world; nearest-neighbor mixing is order λ, next-neighbor order λ^2, and the corner element order λ^3. One number organizes the whole matrix.)*

Worked example (STANDARD TEXTBOOK PEDAGOGY). The Cabibbo angle is read directly from strangeness-changing decays: $|V_{us}| = \sin\theta_{12} \simeq 0.2243$, whence $\lambda \simeq 0.225$. The next element gives $A = |V_{cb}|/\lambda^2 \simeq 0.041/0.0506 \simeq 0.81$. With these two numbers the entire matrix (I.2) is reconstructed — given λ, A, ρ, η from data. Why $\lambda \simeq 0.225$, and why there is a CP phase at all, the Standard Model does not say.

I.2 Step 2 — what the CKM matrix assumes but cannot explain

The four physical CKM parameters are four more free inputs, and they raise three questions the Standard Model cannot answer. First, *the hierarchy*: why is mixing small and ordered in powers of λ, rather than either absent ($V = \mathbf{1}$) or maximal? Nothing forbids large quark mixing. Second, *the origin of CP violation*: the single phase δ is inserted by hand; the Standard Model guarantees only that three families *permit* one irreducible phase, not that it is nonzero or of any particular size. Third, *why CP violation is small but not tiny*: $J \sim 10^{-5}$ is a specific number with no structural account. *(Physics intuition: the CKM matrix is the quark sector's second spreadsheet, after the masses. It is more constrained than the masses — it is nearly the identity — but the smallness of its off-diagonal entries, and the lone complex phase that makes matter and antimatter behave differently, are simply measured, not understood.)*

I.3 Step 3 — the geometric derivation: mixing as projector misalignment

In GG-Theory the CKM matrix is not an independent input. By (I.1) it is the misalignment between the up-type and down-type quark family projectors — the off-diagonal transport that survives after the common family rotations are quotiented out — on the *same* stabilized quark packet whose diagonal masses were derived in the previous appendix. We give the geometric core and then point to the explicit arithmetic.

I.3.1 The Cabibbo angle from pentagonal holonomy

The $1 \leftrightarrow 2$ (Cabibbo) channel, named for the rotation Cabibbo introduced in 1963 (Cabibbo, 1963), is the least suppressed. The full projector-misalignment construction is given in the main text (§52); here we follow the Cabibbo channel in detail. The up-type and down-type nearest-neighbor overlap angles are read straight off the quark mass codebook (H.6),

$$\theta_{12}^{(u)} = \sqrt{\frac{y_u}{y_c}} = e^{-\pi}, \qquad \theta_{12}^{(d)} = \sqrt{\frac{y_d}{y_s}} = e^{-\pi/2}. \tag{I.3}$$

If these two amplitudes carried no relative phase, their difference would miss the measured Cabibbo angle. The geometry supplies the phase. Because the only compact holonomy direction in the flavor sector is the single circle S^1_θ, and the Higgs doublet keeps the fraction $2/L$ of the SU(5) block, the complementary branch phase is forced to the pentagonal value

$$\varphi_{12} = \pi\left(1 - \frac{2}{L}\right) = \frac{3\pi}{5}. \tag{I.4}$$

The first-order Cabibbo angle is then the modulus of the difference of the two nearest-neighbor amplitudes,

$$\boxed{\sin\theta_{12}^{(1)} = \left| e^{-\pi/2} - e^{\,i3\pi/5}\, e^{-\pi} \right| = 0.225019,} \tag{I.5}$$

and using $\cos(3\pi/5) = -(\sqrt{5}-1)/4$ it can be written in closed form as

$$\sin^2\theta_{12}^{(1)} = e^{-\pi} + e^{-2\pi} + \frac{\sqrt{5}-1}{2}\, e^{-3\pi/2}. \tag{I.6}$$

This is within 0.01% of the measured 0.2250 before any sharpening, and it is the thirteenth benchmarked closed form of the codebook. *(Physics intuition: the up and down nearest-neighbor links lie on complementary arcs of one pentagon — the $L = 5$ circle. The Higgs keeps two-fifths of the block, so the flavor-complement phase is the remaining three-fifths of a half-circle, $3\pi/5$. The golden ratio in (I.6) is not numerology; it is the trigonometry of a pentagon, the literal signature of $L = 5$ showing up in the quark mixing angle. The Cabibbo angle is the interference of two known tunneling amplitudes across a pentagonal gate.)*

I.3.2 The smaller angles and the CP phase

The $2\leftrightarrow3$ angle is much smaller because the left-handed doublet is shared between the up and down sectors: a common $2\leftrightarrow3$ rotation is unobservable and cancels in $U_{uL}^{\dagger}U_{dL}$, so what survives is only the *bilinear* mismatch of the two sectoral overlaps — multiplicative, not additive, hence the extra power of λ. The deep $1\leftrightarrow3$ channel is suppressed further still.

The CP phase is the sharpest structural point. There is *no second flavor torus* on which an independent quark CP angle could float: the same single circle S^1_θ that supplies the pentagonal phase $3\pi/5$ supplies the rephasing step $\pi/5$, and the physical CP phase is the topological-projection remainder left after the unphysical quark phases are removed. CP violation is therefore not an inserted parameter but a forced consequence of the pentagonal holonomy — it is nonzero precisely because $3\pi/5$ is not a multiple of π. *(Physics intuition: matter and antimatter differ in the quark sector for a geometric reason — the flavor pentagon is chiral. A phase of $3\pi/5$ cannot be rotated away to 0 or π, so a complex residue always remains, and that residue is CP violation. The same pentagon that sets the size of the Cabibbo angle sets the existence of CP violation.)*

I.3.3 Explicit arithmetic and honesty

The full sectoral-overlap reconstruction of the $2\leftrightarrow3$ and $1\leftrightarrow3$ angles, the Dirac phase, and the precision comparison against the measured matrix are recorded in the CKM codebook arithmetic of Appendix H.10, and the underlying minimal family-link algebra $V_{\rm CKM}=U_{uL}^{\dagger}U_{dL}$ is set out in full in the main text (§52). In the honesty triage the Cabibbo angle stands with the best-derived quantities — a parameter-free closed form matching data to 0.01% — while the smaller angles and the Dirac phase inherit their precision from the same quark cascade depths (H.6), and so share the status of the cascade-depth lemma discussed in the previous appendix: the *structure* (pentagonal phase, multiplicative suppression, topological CP remainder) is forced, while the last-digit values ride on the per-fermion depths that the angular-cascade-depth lemma would close.

Problems and worked solutions. I.1.1. Count the physical parameters of the CKM matrix, and locate the one that violates CP.

Solution. Step 1: a 3×3 unitary matrix carries nine real parameters; three are the angles of an orthogonal rotation, so six are phases. Step 2: the six quark fields can each absorb a phase, but one overall common phase does nothing to (I.1), so $2N-1=5$ phases are removable. Step 3: what survives is $6-5=1$ physical phase beside the three angles — four parameters, exactly as the chapter states; the general-N_g counting (ledger-chapter problem M.11.1) shows this single phase is a three-family novelty. Physics intuition: CP violation lives in the one corner of the matrix that no amount of quark rephasing can reach — a residue of complexity that exists only because the family space is three-dimensional; two families would have none, and the 1973 Kobayashi–Maskawa argument ran this very counting backwards to predict a third generation nobody had seen.

I.1.2. Reconstruct the Wolfenstein parameters, reading A from the *derived* angle.

Solution. Step 1: the expansion (I.2) organizes the matrix in powers of $\lambda=\sin\theta_{12}\simeq0.225$, with $\sin\theta_{23}=A\lambda^2$. Step 2: instead of the measured $|V_{cb}|$, insert the codebook's own derived

$\sin\theta_{23} = 4\pi\sqrt{3}\,e^{-2\pi} = 0.040646$ (H.14): $A = 0.040646/0.225^2 = 0.803$, against the global-fit $A \simeq 0.81$ — the derived second angle lands the Wolfenstein slope within one percent. Step 3: the corner elements follow as $|V_{ub}| \sim A\lambda^3\sqrt{\rho^2+\eta^2} \approx 0.0036$ and $|V_{td}| \sim A\lambda^3\sqrt{(1-\rho)^2+\eta^2} \approx 0.0084$ with the global-fit apex — the whole hierarchical matrix hangs on numbers the codebook supplies. Physics intuition: in the textbook, λ and A are two more measured dials; here λ is a pentagonal interference (I.3.1) and A rides the double-toll $e^{-2\pi}$ — the Wolfenstein hierarchy is the cascade's toll schedule written in matrix form.

I.3.1. Work the Cabibbo interference to six digits, both routes.
Solution. Step 1: the two nearest-neighbor amplitudes (I.3) are $e^{-\pi/2} = 0.2078796$ (down-type) and $e^{-\pi} = 0.0432139$ (up-type), and the pentagonal phase (I.4) carries $\cos(3\pi/5) = -0.3090170$. Step 2: the modulus (I.5): $\sin^2\theta_{12} = e^{-\pi} + e^{-2\pi} - 2e^{-3\pi/2}\cos(3\pi/5) = 0.0432139 + 0.0018674 + 0.0055520 = 0.0506333$, so $\sin\theta_{12} = 0.225018$. Step 3: the closed form (I.6) reproduces it exactly, because $-2\cos(3\pi/5) = (\sqrt{5}-1)/2 = 0.618034$ — the golden ratio conjugate — so the cross term is $\frac{\sqrt{5}-1}{2}e^{-3\pi/2} = 0.618034 \times 0.0089833 = 0.0055520$; against the measured 0.22500(7) the closed form sits at +0.01%. Physics intuition: the oldest measured number of flavor physics is here two tunneling amplitudes interfering across a pentagonal gate — and the golden ratio is not decoration but the literal trigonometry of $L = 5$: work problem I.3.2 to see what happens to the agreement if the pentagon is traded for a square or a hexagon.

I.3.2. Falsify the wrong polygons: recompute the Cabibbo angle with $L = 4$ and $L = 6$.
Solution. Step 1: the branch phase (I.4) is $\varphi_{12} = \pi(1 - 2/L)$; for $L = 4$ (square) it is $\pi/2$, killing the cross term: $\sin\theta_{12} = \sqrt{e^{-\pi} + e^{-2\pi}} = 0.21232$, off the measurement by −5.6%. Step 2: for $L = 6$ (hexagon) $\varphi_{12} = 2\pi/3$, $\cos = -\frac{1}{2}$: $\sin\theta_{12} = \sqrt{0.0432139 + 0.0018674 + 0.0089833} = 0.23252$, off by +3.3%. Step 3: only the pentagon, $L = 5$, lands at +0.01% — the agreement is not generic to the construction but specific to the ledger's unified block, and a measured Cabibbo angle a percent away would have falsified $L = 5$ outright. Physics intuition: this is what separates a derivation from a fit — there is no dial to turn; the polygon is fixed upstream by the same integer that sets the SU(5) spine and the 2/5 doublet fraction, and the Cabibbo angle is its trigonometric fingerprint at the fourth decimal place.

I.3.3. Explain why θ_{23} is quadratically small and why CP violation is compulsory.
Solution. Step 1: the left-handed doublet is shared between the up and down sectors, so a *common* $2 \leftrightarrow 3$ family rotation cancels in $V = U_{uL}^\dagger U_{dL}$ (I.1); only the bilinear mismatch of the two sectoral overlaps survives, hence $\sin\theta_{23} \propto e^{-2\pi}$ — one toll from each sector, multiplicative, not additive — and the deep $1 \leftrightarrow 3$ channel is suppressed further still. Step 2: the CP phase cannot be dialed away: the flavor sector owns exactly one compact circle, its rephasing step is $\pi/5$, and the branch phase $3\pi/5$ is not a multiple of π, so a complex remainder survives every allowed rephasing. Step 3: consequence — $J \neq 0$ is *forced*, and its size is assembled from the same tolls in the fermion chapter's problem H.14.1; the companion paper's problem E.2.1 (GG-P5-CPT) then shows every CP-violating quark observable is proportional to this one J. Physics intuition: matter differs from antimatter, on this reading, because a pentagon is chiral — $3\pi/5$ can face neither 0 nor π — so the universe's CP violation is not an optional decoration but a topological remainder; and the hierarchy $\lambda : \lambda^2 : \lambda^3$ is simply one, then two, then three tolls paid.

I.2.1. Show that the CKM phase convention is unphysical, and find what is not.
Solution. Step 1: rephasing the six quark fields, $q \to e^{i\phi_q} q$, changes individual V_{ij} by phases while leaving all observables fixed. Step 2: therefore no single matrix element's phase is physical; what is physical is any rephasing-invariant combination, the simplest being the modulus $|V_{ij}|$ and the quartet $V_{ud}V_{cs}V_{us}^*V_{cd}^*$, whose imaginary part is the Jarlskog invariant. Step 3: hence a statement like "V_{ub} has phase $-\gamma$" is convention-dependent, while "$J = 3 \times 10^{-5}$" is not. Physics intuition: whenever a quantity in a gauge theory can be changed by a field redefinition, it is bookkeeping. The discipline of asking "is this rephasing-invariant?" before quoting a phase is the same discipline that makes the Majorana counting of problem K.1.4 tractable, and it is why the Jarlskog invariant, not δ, is the right thing to derive.

I.2.2. Estimate how much of the CKM matrix is actually independent data.
Solution. Step 1: the matrix has nine complex entries, eighteen real numbers, but unitarity imposes nine real conditions and rephasing removes five phases: four physical parameters remain. Step 2: yet the Particle Data Group quotes ten independent $|V_{ij}|$ measurements plus several angle and side determinations of the unitarity triangle — far more measurements than parameters. Step 3: the excess is the content of the unitarity tests, and their agreement at the percent level is the strongest evidence that there is no fourth family and no significant new flavor physics below the multi-TeV scale. Physics intuition: an over-determined system is where a theory earns its keep. Four numbers predicting fifteen measurements is a much stronger statement than four numbers fitting four, and this is the right frame for judging the codebook of this paper too — count the outputs against the inputs, not the successes against the failures.

I.2.3. Ask why the quark and lepton sectors must be compared, not treated separately.
Solution. Step 1: both mixing matrices arise the same way — as the misalignment $U_L^\dagger U'_L$ of two family bases (I.1). Step 2: yet one is nearly the identity and the other nearly democratic, and no Standard-Model input distinguishes the two cases: the gauge structure, the number of families and the mechanism are identical. Step 3: therefore the contrast is a *single* datum about family structure, not two independent ones, and any framework that explains one without the other has explained half a fact. Physics intuition: this is the most important thing the quark chapter has to say to the lepton chapters, and it is the reason they sit consecutively in this course. "CKM small, PMNS large" should be read as one sentence with a contrast in it, and a theory should be judged on whether it produces the contrast from one construction — which is exactly the claim App. J.3.3 makes and the claim a reader should test hardest.

Take-away. *Standard result:* the CKM matrix — three angles, one phase — fits every flavor observable. *What it assumed:* all four numbers, and the origin of CP violation itself. *The geometric reading:* the pentagonal phase and the multiplicative suppression are forced structure; the last-digit values await the same cascade-depth lemma as the masses — one lemma, two sectors, stated in one register.

J Neutrino masses and PMNS mixing

What this appendix teaches. The lepton sector that inverts the quark story — tiny masses, large mixing angles. Step 1 develops the oscillation formalism from scratch (including the matter effect, the three-flavor CP-odd term, and the three independent absolute-mass probes) and derives the seesaw in all three of its types. Step 2 lists ten quantities the Standard Model must take from experiment. Step 3 gives the geometric reading, in which one reused scale sets the meV floor, a dyadic-against-democratic projector misalignment forces the large angles as ratios of small integers, and the ordering becomes a near-term kill row — with the one place the ledger visibly strains, the solar splitting, identified and quantified rather than averaged into a ratio.

Read alongside: Giunti & Kim (2007) Ch. 7–8 and Zuber (2020) for the formalism and the experiments; Langacker (2017) Ch. 8 for the seesaw; Fukuda et al. (1998) and Ahmad et al. (2002) for the discovery papers; Thomson (2013) Ch. 13 for the cleanest short derivation of the oscillation probability.

This is the fifth sector appendix of the course, and the first of three consecutive chapters on the lepton sector — this one on masses and mixing, the next (App. K) on the Majorana question and the CP phases, the third (App. L) on what the same structure does to the early universe. Its subject inverts the quark story of the previous two appendices in every particular: the masses are not merely hierarchical but *absolutely* tiny, six orders of magnitude below the electron; and the mixing angles are not small but large, two of them within a factor of two of maximal. We follow the three-step plan.

J.1 Step 1 — neutrino masses and lepton mixing as a graduate student meets it

J.1.1 Why the Standard Model predicts a massless neutrino

It is worth being precise about this, because "the Standard Model does not accommodate neutrino mass" is often said and is not quite what is true. The statement is structural, and it has three steps.

Step (a): the field content. The minimal Standard Model contains the left-handed doublet $L = (\nu_L, e_L)^{\mathsf{T}}$ and the right-handed singlet e_R, and *no* ν_R. A Dirac mass term $m\,\bar{\nu}_L \nu_R$ therefore has nothing to pair with.

Step (b): the gauge quantum numbers. Could one write a Majorana mass $\frac{1}{2} m\, \overline{\nu_L^c}\, \nu_L$ instead? The bilinear $\nu_L \nu_L$ sits in LL, which carries weak isospin $\mathbf{2} \otimes \mathbf{2} = \mathbf{3} \oplus \mathbf{1}$ and hypercharge $2Y_L = -1$. It is not a gauge singlet, so no dimension-three mass term is gauge invariant.

Step (c): what is allowed, and at what dimension. Contract LL with two Higgs doublets. The unique gauge-invariant, Lorentz-scalar operator built from Standard-Model fields alone that gives the neutrino a mass is

$$\mathcal{O}_5 = \frac{c_5}{\Lambda}\,(L^{\mathsf{T}}\epsilon H)(L^{\mathsf{T}}\epsilon H) + \text{h.c.}, \tag{J.1}$$

of *dimension five* — the celebrated Weinberg operator (Weinberg, 1979). After electroweak breaking it delivers $m_\nu = c_5 v^2/\Lambda$.

Three consequences follow immediately and are worth stating separately, because each is a fact about the Standard Model rather than about any model beyond it. First, $\mathcal{O}_5$ is the *only* dimension-five operator in the whole Standard Model: neutrino mass is the leading signature of *anything* above the electroweak scale, whatever that thing is. Second, $\mathcal{O}_5$ violates lepton number by two units, so a nonzero neutrino mass of this origin makes the neutrino its own antiparticle. Third, the suppression is *automatic*: a mass generated at dimension five is smaller than an ordinary Yukawa mass by v/Λ, which is why the neutrino is light without anybody tuning anything. *(Physics intuition: the Standard Model does not forbid neutrino mass — it* postpones *it. Every other fermion mass is allowed at dimension four and therefore arrives at full strength; the neutrino's is allowed only at dimension five and therefore arrives suppressed by one power of a high scale. The neutrino is light not because its coupling is small but because its mass operator is the first one that has to reach outside the theory to exist at all. Its smallness is a measurement of how far outside.)*

J.1.2 Oscillations, derived

The two-flavor calculation, with no steps skipped. Suppose two flavor states are related to two mass states by a rotation,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}. \tag{J.2}$$

A source produces $|\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$. Each mass eigenstate propagates with its own phase $e^{-i(E_i t - p_i L)}$. For ultrarelativistic neutrinos of common momentum $p \simeq E$,

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p}, \tag{J.3}$$

so over a distance $L \simeq t$ the two states accumulate a *relative* phase

$$\Delta\phi = (E_2 - E_1)t - (p_2 - p_1)L \;\simeq\; \frac{\Delta m^2_{21} L}{2E}, \qquad \Delta m^2_{21} \equiv m_2^2 - m_1^2. \tag{J.4}$$

Projecting the evolved state back onto $\langle\nu_\mu|$ and squaring, the cross term gives

$$\boxed{P(\nu_e \to \nu_\mu) = \sin^2 2\theta\, \sin^2\!\left(\frac{\Delta m^2_{21} L}{4E}\right).} \tag{J.5}$$

Two features are the whole subject in miniature. The *amplitude* of the oscillation is set by the mixing angle and the *frequency* by the mass-squared difference: an oscillation experiment measures θ and Δm^2, and *cannot* measure m_1 or m_2 separately, because the absolute phase common to both states cancels in (J.4).

The conversion constant, derived. Restoring $\hbar$ and c: with $1\ \text{km} = 5.0677 \times 10^{18}\ \text{GeV}^{-1}$ and $1\ \text{eV}^2 = 10^{-18}\ \text{GeV}^2$,

$$\frac{\Delta m^2 L}{4E} = \frac{5.0677}{4}\,\frac{\Delta m^2[\text{eV}^2]\ L[\text{km}]}{E[\text{GeV}]} = 1.2669\,\frac{\Delta m^2 L}{E}, \tag{J.6}$$

so the first oscillation maximum sits at $L_{\max} = (\pi/2)/1.2669 \times E/\Delta m^2 = 1.240\,E/\Delta m^2$ in these units, and the full oscillation length is $L_{\text{osc}} = 2.480\,E/\Delta m^2$. *(Physics intuition: memorize* 1.27. *Every neutrino baseline ever built — a reactor experiment at a kilometer, a solar experiment at an astronomical unit, an accelerator at several hundred kilometers — is this one number solved for L with a different Δm^2 in the denominator. The experiments look nothing alike and are all the same experiment.)*

The caveat the plane-wave derivation hides. Equation (J.5) was obtained by giving the two mass states a common momentum, which is not consistent: they have different masses and are produced by a process with definite energy–momentum bookkeeping. The honest derivation uses wave packets (Giunti & Kim, 2007), and it returns (J.5) multiplied by a coherence factor that stays near one provided the packet separation $\Delta v\, L$ remains small compared to the packet width. For every terrestrial experiment this is satisfied by many orders of magnitude; for supernova and solar neutrinos it is not, and the oscillating term averages to $\tfrac{1}{2}$. A student should know that the standard formula is a limit, and know which limit.

Three flavors. With three families the same computation gives, for $\Delta_{ij} \equiv \Delta m^2_{ij} L/4E$,

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \text{Re}\Big(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\Big) \sin^2\Delta_{ij} + 2\sum_{i>j} \text{Im}\Big(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\Big) \sin 2\Delta_{ij}. \tag{J.7}$$

The last term is odd under $\alpha \leftrightarrow \beta$ and changes sign between neutrinos and antineutrinos: it is the CP-violating term, and it collapses to a single rephasing-invariant,

$$P(\nu_\alpha \to \nu_\beta) - P(\bar\nu_\alpha \to \bar\nu_\beta) = 16\, J_{\text{PMNS}} \sin\Delta_{21} \sin\Delta_{31} \sin\Delta_{32}, \tag{J.8}$$

with the leptonic Jarlskog invariant

$$J_{\text{PMNS}} = c_{12}s_{12}\, c_{23}s_{23}\, c^2_{13}s_{13}\, \sin\delta_{\text{PMNS}}. \tag{J.9}$$

(Physics intuition: CP violation in oscillations needs all three *splittings to be active at once — the product of three sines in (J.8) vanishes if any two mass states are degenerate or if any angle is zero. That is why the discovery of a nonzero θ_{13} in 2012 was the event that made leptonic CP violation an experimental program rather than a hope: it lifted the last factor off the floor.)*

J.1.3 Matter: the MSW effect, and why it matters twice

Neutrinos crossing matter forward-scatter off electrons. All three flavors scatter through Z exchange, which contributes a common phase and drops out; only ν_e additionally scatters through

W exchange off the electrons themselves. That difference is a potential (Mikheyev & Smirnov, 1985; Wolfenstein, 1978)

$$V_{\rm CC} = \sqrt{2}\, G_F n_e, \tag{J.10}$$

which enters the effective Hamiltonian exactly like a mass-squared term,

$$H_{\rm eff} = \frac{1}{4E}\begin{pmatrix} -\Delta m^2 \cos 2\theta + 2EV_{\rm CC} & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & \Delta m^2 \cos 2\theta - 2EV_{\rm CC} \end{pmatrix}. \tag{J.11}$$

The in-medium mixing angle is maximal — *whatever the vacuum angle* — when the diagonal vanishes:

$$2E\, V_{\rm CC} = \Delta m^2 \cos 2\theta \quad \Longleftrightarrow \quad n_e^{\rm res} = \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2}\, G_F E}. \tag{J.12}$$

Numerically, for the solar parameters and a 10 MeV ^{8}B neutrino, $n_e^{\rm res} = 1.14 \times 10^{25}$ cm^{-3} $\simeq 19\, N_A$ cm^{-3}, comfortably below the solar core density $n_e \simeq 5.8 \times 10^{25}$ cm^{-3}. The high-energy solar neutrinos are therefore *born above the resonance*, and if the density profile varies slowly enough for the evolution to be adiabatic they emerge as the pure heavier state ν_2, giving a survival probability $P_{ee} \to \sin^2\theta_{12} \simeq 0.31$ — against the vacuum-averaged $1 - \frac{1}{2}\sin^2 2\theta_{12} = 0.57$ that the low-energy *pp* neutrinos show. Both are observed, and the transition between them across the ^{8}B spectrum is the signature of the effect.

The MSW effect matters twice. It is how the solar solution was *identified*, and — because $V_{\rm CC}$ flips sign for antineutrinos and its interference with Δm^2_{31} depends on the *sign* of that splitting — it is also the handle by which a long-baseline accelerator experiment determines the mass ordering. *(Physics intuition: matter is a flavor-dependent index of refraction for neutrinos, and it is flavor-dependent for the mundane reason that ordinary matter contains electrons and not muons. That accident of chemistry is what lets an experiment on Earth read the sign of a mass-squared difference — and it is also the reason a long-baseline CP measurement must disentangle a genuine CP asymmetry from the matter-induced fake one, which is the single hardest systematic in the field.)*

J.1.4 What each experiment actually measures

The current determinations, which this course scores its predictions against throughout, are the global fit (Esteban et al., 2025) — cross-checked against the independent global analysis of Capozzi et al. (2025), which uses a different treatment of the atmospheric and solar data and agrees on every parameter except the atmospheric octant — together with the two measurements that most sharpen it: JUNO's direct determination of the solar parameters (JUNO Collaboration, 2026) and the T2K–NOvA joint fit (T2K and NOvA Collaborations, 2025). The reader should note two live disagreements rather than a settled table. The atmospheric angle is reported in the *lower* octant by the global fit and the *upper* octant by the accelerator joint fit, a split treated at length in §55.5; and the ordering is preferred normal at only 2.2–2.3σ after JUNO (Esteban et al., 2026). A student scoring any lepton-sector prediction should score it against both octants and say which.

Table 81: The measurement landscape. Each row is an oscillation channel with its characteristic L/E; the phase column is (J.6) evaluated at the quoted baseline and energy, and shows that every one of these experiments was built to sit near the first oscillation maximum of the splitting it targets — $\pi/2 = 1.571$. This is the single most useful table in the subject: it explains why the baselines are what they are.

Channel	L	E	Splitting	Phase	Fixes
Reactor, short (Daya Bay, RENO)	1.65 km	4 MeV	Δm^2_{31}	1.31	θ_{13}
Reactor, medium (JUNO)	52.5 km	4 MeV	Δm^2_{21}	1.25	θ_{12}, Δm^2_{21}, ordering
Reactor, long (KamLAND)	180 km	4 MeV	Δm^2_{21}	4.29	Δm^2_{21} (past first max)
Accelerator (T2K)	295 km	0.6 GeV	Δm^2_{31}	1.56	θ_{23}, δ_{PMNS}
Accelerator (NOvA)	810 km	2 GeV	Δm^2_{31}	1.29	θ_{23}, ordering
Accelerator (DUNE)	1300 km	2.5 GeV	Δm^2_{31}	1.65	δ_{PMNS}, ordering
Atmospheric (Super-K, IceCube)	10–10^4 km	1 GeV	Δm^2_{31}	spectrum	θ_{23}, octant
Solar (^{8}B, *pp*)	1 AU	0.1–15 MeV	Δm^2_{21} (MSW)	averaged	θ_{12}, sign of Δm^2_{21}

J.1.5 The absolute scale: three independent probes

Oscillations are blind to the absolute scale. Three quite different experiments are not, and they measure three *different* combinations — which is why all three are needed and why a discrepancy between them would be informative rather than merely awkward.

1. **Kinematic (single β decay).** The endpoint of the tritium spectrum is displaced by the incoherent sum $m_\beta^2 = \sum_i |U_{ei}|^2 m_i^2$. This is model-independent — it uses only energy conservation — and therefore the most robust. KATRIN reports $m_\beta < 0.45$ eV (KATRIN Collaboration, 2025).

2. **Cosmological.** Massive neutrinos free-stream and suppress structure below their free-streaming scale, so a galaxy survey plus the CMB constrains $\Sigma m_\nu = \sum_i m_i$. This is the most *sensitive* probe by two orders of magnitude and the most model-dependent: DESI DR2 gives $\Sigma m_\nu < 64.2$ meV in ΛCDM but < 163 meV once the dark-energy equation of state is allowed to vary (DESI Collaboration, 2025). The tightening is recent and worth registering: the DR1 analysis (DESI Collaboration, 2024) gave < 72 meV on the same assumptions, so this bound has moved by more than ten percent in one data release — which is the right frame for judging how much headroom any prediction pressed against it actually has.

3. **Neutrinoless double beta decay.** The rate is controlled by the coherent, phase-sensitive combination $m_{\beta\beta} = \left|\sum_i U_{ei}^2 m_i\right|$, and is nonzero only if the neutrino is Majorana. Treated in full in App. K.

(Physics intuition: the three probes weight the same three masses with a square, a first power, and a first power with phases. A theory that predicts the spectrum predicts all three, and they are not independent tests of it — but they are *independent tests of each other's systematics, which for a*

quantity this small is the more useful service. If cosmology reported a sum incompatible with a measured $m_{\beta\beta}$, the news would be about cosmology.)

J.1.6 The seesaw, in its three types

Where does (J.1) come from? Integrating out a heavy state produces it, and at tree level there are exactly three ways (Mohapatra & Pal, 2004): the heavy state can be a fermion singlet, a scalar triplet, or a fermion triplet.

Type I: a heavy fermion singlet. Add N_R with a Dirac Yukawa and a Majorana mass (Gell-Mann et al., 1979; Minkowski, 1977; Mohapatra & Senjanović, 1980; Yanagida, 1979). In the (ν_L, N_R^c) basis the mass matrix is

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^{\mathsf{T}} & M_N \end{pmatrix}, \qquad m_D = Y_\nu\, v. \tag{J.13}$$

A convention worth pinning, because it is a factor of two. Textbooks divide here on whether the Dirac mass is $Y_\nu v$ or $Y_\nu v/\sqrt{2}$, and the two differ in m_ν by a factor of two. *This paper uses $m_D = Y_\nu v$ with $v = 246.33$ GeV throughout*, which is the convention under which the effective washout mass is $\widetilde{m} = {Y_\nu}^2 v^2/M_N$ and under which the lock-scale matching of §60 is an identity with no factor; in the standard normalization $m_D = Y^{\mathrm{L}} v/\sqrt{2}$ it is the same identity with $Y^{\mathrm{L}} = \sqrt{2}\, Y_\nu$ (§60). Note that the Davidson–Ibarra bound below is conventionally written with $\bar{v} = 174$ GeV: that is a property of *that* formula, not a second convention for m_D, and the two do not interfere. A student comparing numbers across papers should check this first — more disagreements in the seesaw literature come from it than from physics. Block-diagonalizing for $M_N \gg m_D$ gives the two eigenvalues

$$m_\nu \simeq -m_D M_N^{-1} m_D^{\mathsf{T}}, \qquad m_{\mathrm{heavy}} \simeq M_N, \tag{J.14}$$

the light state suppressed by exactly the ratio the heavy state is enhanced by — which is the image the name records.

Type II: a heavy scalar triplet. A triplet Δ with hypercharge 1 couples as $f\, L^{\mathsf{T}} \Delta L$ and acquires an induced vev $v_\Delta \simeq \mu v^2/M_\Delta^2$, giving $m_\nu = 2 f v_\Delta$.

Type III: a heavy fermion triplet. Identical in structure to type I with the singlet replaced by an SU(2) triplet Σ; the light mass is again $m_\nu \sim v^2/M_\Sigma$.

The parametrization that shows what is free. For type I with three heavy states, Casas & Ibarra (2001) showed that the most general Dirac Yukawa reproducing a given light spectrum and mixing matrix is

$$Y_\nu = \frac{1}{v}\, \sqrt{\widehat{M}_N}\; R\; \sqrt{\widehat{m}_\nu}\; U_{\mathrm{PMNS}}^{\dagger}, \tag{J.15}$$

with R an arbitrary *complex* orthogonal matrix. This equation is the honest statement of the sector's freedom, and every student should internalize it: the low-energy data fix $\widehat{m}_\nu$ and U and say

nothing whatever about $\widehat{M}_N$ or R. The three heavy masses and the three complex angles of R — nine real parameters — are entirely unconstrained by oscillation experiments. It is this freedom that App. L will find is exactly what a leptogenesis prediction needs and cannot get.

Worked example (STANDARD TEXTBOOK PEDAGOGY). Take the atmospheric splitting and assume $m_1 \approx 0$, so that $m_3 \approx \sqrt{|\Delta m^2_{31}|} = \sqrt{2.511 \times 10^{-3}} = 50.1$ meV. Insert this in (J.14) with a Dirac Yukawa of order the top's, $Y_\nu \sim 1$, so $m_D = Y_\nu v = 246$ GeV in the convention pinned above:

$$M_N \sim \frac{m_D^2}{m_\nu} = \frac{(246.33\ \text{GeV})^2}{5.01 \times 10^{-11}\ \text{GeV}} = 1.2 \times 10^{15}\ \text{GeV}, \tag{J.16}$$

within an order of magnitude of the grand-unification scale — a coincidence the Standard Model records and cannot explain. Note carefully what was assumed to get it: $Y_\nu \sim 1$, for no reason except that the top quark's is. Choose $Y_\nu \sim 10^{-6}$ instead, and the same neutrino mass comes from $M_N \sim 1$ TeV — which is, within a factor of a few, exactly where App. L will find the geometry putting it, with the Yukawa derived rather than chosen. *The seesaw fixes only the combination m_D^2/M_N; it does not fix the scale, and any textbook statement that "the seesaw points at grand unification" is a statement about the assumed Yukawa, not about the data.* This will matter a great deal in App. L.

J.2 Step 2 — what the lepton sector assumes but cannot explain

The lepton sector is the least constrained part of the Standard Model. Where the quark sector left four numbers free, this one leaves ten, and adds two structural puzzles that no choice of parameters resolves.

1. **The absolute mass scale.** Free. Equivalently, Λ in (J.1) is free.
2. **The operator coefficient c_5.** Free, and degenerate with Λ — only the ratio is observable.
3. **The two mass-squared splittings.** Two independent measured inputs. Nothing explains why their ratio is ~ 33 rather than ~ 1 or $\sim 10^6$.
4. **The mass ordering.** Not predicted; the sign of Δm^2_{31} is an experimental question.
5. **Dirac or Majorana.** Not predicted. The Standard Model extended by ν_R gives Dirac; extended by (J.1) gives Majorana; nothing selects.
6. **The three mixing angles.** Three measured inputs.
7. **The Dirac phase.** One measured input, currently determined only to a broad interval.
8. **The two Majorana phases.** Two inputs, unmeasured and unconstrained by any experiment yet performed.
9. **The heavy sector.** $\widehat{M}_N$ and R in (J.15): nine further real parameters, invisible to low-energy data.

10. **The number of light species.** Three, from the Z width and cosmology — observed, not derived.

And the two structural puzzles, which are the more important half of the list because no amount of data will resolve them:

Puzzle A: why are the lepton angles large when the quark angles are small? This is the *opposite* of the quark puzzle, in the same theory, with the same gauge structure and the same number of families. Both facts are recorded and neither is explained.

Puzzle B: why is the neutrino mass scale six orders of magnitude below the electron's? The seesaw explains how a small mass can arise without a small coupling, but — as (J.16) showed — it converts one unexplained number into two, and fixes only their ratio.

(Physics intuition: the quark mixing matrix is nearly the identity; the lepton one is nearly democratic. The Standard Model has no reason for either, and certainly no reason for the striking contrast between them. A theory that produced "CKM small, PMNS large" from one *construction would be explaining a headline fact of particle physics — and it would be doing so in the one sector where the data are still arriving, which is the only interesting place for a theory to make a claim.)*

J.3 Step 3 — the geometric derivation: a Majorana floor and a democratic misalignment

In GG-Theory both the smallness of the neutrino masses and the largeness of the lepton angles come from one construction, with no new scale and no right-handed neutrino sector. Every item in Step 2's list of ten is addressed; two of them are addressed *less well than the others*, and this section identifies which and by how much rather than quoting an average.

J.3.1 The neutrino floor from the minimal Majorana channel

BI-6 forbids any new intermediate mass scale and any second compact loop, so the neutrino mass must come from the one constitutionally admitted operator — the Weinberg dimension-five $(LH)(LH)$ of (J.1) — reusing the already-derived grand-unification seam $E_{\rm GUT}$ as its scale. This is the whole of the answer to Step 2's items 1 and 2: Λ is not free because it is not new. The common neutrino scale is therefore fixed,

$$m_\star = \pi\,\frac{v^2}{E_{\rm GUT}} = \frac{\pi \times (246.33\ \text{GeV})^2}{2.280 \times 10^{16}\ \text{GeV}} = 8.36114\ \text{meV}, \tag{J.17}$$

the same expression recorded in the codebook arithmetic (Eq. H.9), with first-order eigenvalues in the integer ratio $(m_1, m_2, m_3) = (0, 1, 6)\, m_\star$.

Two structural predictions follow with no further input. First, the Majorana ledger is *rank two*, so there are exactly two independent mass-squared splittings and the absolute m_1 enters only through the operator coefficient — precisely the observed pattern, and the answer to item 3's first half. Second, the cascade-depth assignment places the ground state (the GGA of the Majorana

stratum) at ν_1, forcing the *normal* ordering $m_1 < m_2 < m_3$ — item 4 — and an established inverted ordering would falsify the assignment outright.

Where the ledger strains, stated with the number. The integer ratio $36 = (2N)^2$ is the framework's answer to item 3's second half, and it should be scored honestly. Against Esteban et al. (2025) the measured ratio is $\Delta m^2_{31}/\Delta m^2_{21} = 33.35$, so the first-order 36 is +7.96% high; the QD-completed spectrum gives 34.56, or +3.65%. But the ratio is the wrong quantity to look at, because it averages two very different agreements:

$$\Delta m^{2\,(1)}_{31} = 2.5167 \times 10^{-3}\ \text{eV}^2 \quad (+0.23\%), \qquad \Delta m^{2\,(1)}_{21} = 6.9909 \times 10^{-5}\ \text{eV}^2 \quad (-7.16\%). \tag{J.18}$$

The atmospheric splitting is right to two parts in a thousand and essentially all of the discrepancy sits in the solar one. The QD layer moves the solar splitting to −1.96% and the atmospheric to +1.62%, which is an improvement in the ratio bought partly by degrading the quantity that was already right. A student should read this as the framework's clearest live tension in the lepton sector: the integer 6 in $(0, 1, 6)$ is doing very well and the integer 1 is doing less well, and with JUNO now measuring Δm^2_{21} to 1.6% (JUNO Collaboration, 2026) this is a discrepancy that is no longer hiding inside the error bar. *(Physics intuition: the neutrinos are light for the same reason the electroweak scale is small — a ratio of fixed geometric scales, here v^2/E_{GUT}, not a tuned seesaw mass. The meV scale of neutrino mass is the geometric shadow of the* 10^{16} GeV *seam, and the two most extreme mass scales ever measured become one ratio. And because the Majorana ledger has rank two, nature can hand the oscillation experiments only two beat frequencies — the absence of a third is a prediction, not a gap.)*

J.3.2 The forced triplet: why (D, S, A) and not some other basis

Before the angles, the basis. The neutrino block is a rank-two Majorana packet on a three-dimensional family space, and three facts fix its natural basis with no freedom left.

Rank two singles out a vector plus a plane. A rank-two symmetric form on a three-dimensional space has a one-dimensional kernel; the decomposition is therefore $\mathbf{3} = \mathbf{1} \oplus \mathbf{2}$, a distinguished direction and its orthogonal plane. *Permutation symmetry picks the vector.* The Majorana channel is generated by the single operator (J.1) acting identically on all three families, so the distinguished direction must be invariant under family permutations: it is the democratic vector. *The charged-lepton hierarchy splits the plane.* The remaining two-dimensional space is resolved by the only structure that distinguishes families, the dyadic charged-lepton ladder, into the $\mu-\tau$-even and $\mu-\tau$-odd combinations. The result is forced:

$$|D\rangle = \tfrac{1}{\sqrt{3}}(1,1,1)^{\mathsf{T}}, \quad |S\rangle = \tfrac{1}{\sqrt{6}}(2,-1,-1)^{\mathsf{T}}, \quad |A\rangle = \tfrac{1}{\sqrt{2}}(0,1,-1)^{\mathsf{T}}. \tag{J.19}$$

(Physics intuition: this is the same argument that gives the democratic, symmetric and antisymmetric combinations in any three-body problem with an interaction that does not know which body is which — and the point is that the neutrino sector's operator genuinely does not know, because it is one operator, while the charged-lepton sector emphatically does, because it is three very different Yukawas.

The PMNS matrix is the angle between a sector that cannot tell the families apart and a sector that can barely do anything else.)

J.3.3 Large angles from a dyadic-versus-democratic misalignment

Reading the misalignment of the dyadic charged-lepton basis against (J.19) gives the three angles as closed-form rationals,

$$\boxed{\sin^2\theta_{12} = \frac{L^2}{3N_g^3} = \frac{25}{81}, \qquad \sin^2\theta_{23} = \frac{1}{2}\Big(1+\frac{1}{J}\Big) = \frac{4}{7}, \qquad \sin^2\theta_{13} = \frac{N_g}{4K} = \frac{3}{140},} \tag{J.20}$$

each built from the ledger integers $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ and no others. Each deserves a sentence of its own geometry, because the three are structurally different and a student who reads them as three fitted fractions has missed the chapter.

The solar angle: a democratic overlap, depressed. A pure democratic overlap gives exactly $1/3$. The ledger gives

$$\frac{25}{81} = \frac{1}{3}\Big(1 - \frac{2}{27}\Big) = 0.308642, \tag{J.21}$$

a 7.4% depression of the democratic value, arising because the charged-lepton plane is not exactly orthogonal to $|D\rangle$. This number now decides between hypotheses: JUNO measures 0.3092 ± 0.0087 (JUNO Collaboration, 2026), so the pure democratic $1/3$ sits 2.8σ high and the depressed value sits 0.07σ away. The measurement has become precise enough to *see* the $2/27$, and it sees it. Tri-bimaximal models, which predict the undepressed $1/3$, predict the value the data have moved off.

The reactor angle: the one residue of sharpness. The factor of four in $N_g/4K$ is two factors of $1/\sqrt{2}$ — the two independent $\mu-\tau$ projections through which the dyadic comb reaches the democratic sector — and $K = 35$ is the full closure depth. Small because the reach is second order; nonzero because the comb is not orthogonal to the packet.

The atmospheric angle: a tilt about maximal, not about zero. This is the structurally most interesting of the three and the one most often misread. The ledger does not predict a small angle plus a correction; it predicts a *traceless tilt about maximal mixing*,

$$\sin^2\theta_{23} = \frac{1}{2}\Big(1+\frac{1}{J}\Big), \qquad J = 7, \tag{J.22}$$

so the deviation from $1/2$ is $1/(2J) = 1/14$ and its *sign* is forced by which direction the charged-lepton hierarchy distinguishes — upward, into the upper octant. Maximal mixing is the symmetric point and J is what breaks it. Consequently the octant is not a detail of this prediction, it is the prediction: $4/7 = 0.5714$ is inside the accelerator joint fit's $0.56^{+0.03}_{-0.05}$ and more than four sigma from the global fit's $0.470^{+0.017}_{-0.014}$. §55.5 treats the split, and F-SM-3e registers its resolution as a kill criterion.

(Physics intuition: "CKM small, PMNS large" is not two constitutions but one. On the quark side both sectors are hierarchical, so their family bases nearly agree and the mixing is small. On the lepton side a sharp dyadic comb is laid over a near-uniform democratic background, and two combs of such different teeth can only mesh at large angles. The small reactor angle is the one residue of the charged-lepton sharpness that survives into the democratic sector — and this single picture answers Puzzle A of Step 2, which is the reason the chapter exists.)

J.3.4 Phases, and the honest status of the lepton sector

The Dirac phase is fixed by the same compact-angle holonomy logic as the quark sector rather than by a new continuous parameter, and the two Majorana phases are parity data of the rank-two ledger rather than dials. Both are derived in the next chapter (App. K), where they are the subject rather than a postscript; the full projector construction and the precision comparison against the global fit are in the main text (§50.3, §54), and the explicit mass arithmetic is in the codebook arithmetic of the fermion-mass appendix (§H.3.4).

Scoring Step 2's ten items honestly: items 1, 2, 4, 5, 6, 7, 8 and 10 are structural outputs of the construction, and both puzzles are answered. Item 9 — the nine parameters of the heavy sector — is answered by the lock-scale construction of App. L, where it becomes the chapter's central claim and also its most conditional one. Item 3 is answered in its first half (*why two*) and answered with a visible strain in its second (*why 33*), quantified in (J.18). And the angle values inherit whatever residual freedom the charged-lepton dyadic codes carry — the same cascade-depth lemma that governs the charged fermions, item 13 of the Part's open list.

Problems and worked solutions. **J.1.1. Derive the oscillation probability, and the constant 1.27.**

Solution. Step 1: expand the dispersion relation (J.3) for $m \ll p$ and form the relative phase (J.4): only the *difference* of m_i^2 survives, because the common p cancels between the two mass states. Step 2: project and square — the interference term carries $\sin^2 2\theta$ from the two rotations and $\sin^2(\Delta m^2 L/4E)$ from the phase, giving (J.5). Step 3: restore units. With $1\,\text{km} = 5.0677 \times 10^{18}\ \text{GeV}^{-1}$ and $1\ \text{eV}^2 = 10^{-18}\ \text{GeV}^2$ the phase is $(5.0677/4)\,\Delta m^2 L/E = 1.2669\,\Delta m^2[\text{eV}^2]L[\text{km}]/E[\text{GeV}]$. Physics intuition: the entire subject is one interferometer formula and one conversion constant; every baseline in Table 81 is 1.27 solved for L. Memorize the constant and you can reconstruct the design of any neutrino experiment ever built from its target splitting alone.

J.1.2. Design the experiments: check that every baseline sits at its first maximum.

Solution. Step 1: the first maximum is at phase $\pi/2 = 1.571$. Step 2: evaluate (J.6) for T2K ($L = 295$ km, $E = 0.6$ GeV, $\Delta m^2_{31} = 2.511 \times 10^{-3}$): $1.2669 \times 2.511 \times 10^{-3} \times 295/0.6 = 1.564$ — within half a percent of $\pi/2$. For Daya Bay (1.65 km, 4 MeV, same splitting): 1.31. For JUNO (52.5 km, 4 MeV, $\Delta m^2_{21} = 7.53 \times 10^{-5}$): 1.25. For DUNE (1300 km, 2.5 GeV): 1.65. Step 3: every entry in Table 81 lands between 1.25 and 1.65 except KamLAND at 4.29 — and KamLAND is the exception that proves the rule, since it was a reactor experiment sited at an existing cavern rather than built to a baseline, and reads Δm^2_{21} from the *spectral* distortion of several oscillations rather

than from a single dip. Physics intuition: a neutrino experiment is a tuned instrument, and the tuning is visible in one arithmetic line. When a proposal quotes a baseline, this is the calculation that tells you what it is really for — and, for DUNE at 1.65, that it is deliberately placed just *past* the maximum, where the matter effect separates the orderings most cleanly.

J.1.3. Invert the seesaw — and then show the inversion proves less than it appears to. *Solution.* Step 1: the Weinberg operator gives $m_\nu \sim v^2/\Lambda$; invert, $\Lambda \sim v^2/m_\nu$. Step 2: with $m_\nu = 0.05$ eV and $v = 246$ GeV, $\Lambda \sim (246)^2/(5\times10^{-11})$ GeV $= 1.2\times10^{15}$ GeV — the grand-unification decade. Step 3: now repeat the argument with the type-I mass (J.14), $M_N = m_D^2/m_\nu$, and notice that $m_D = Y_\nu v$ contains a Yukawa nobody has measured. With $Y_\nu \sim 1$ one gets 1.2×10^{15} GeV; with $Y_\nu \sim 10^{-6}$ one gets 1 TeV; both give the same neutrino mass. Physics intuition: this is the most commonly overstated inference in the field. The data fix m_D^2/M_N and nothing else, so "neutrino mass points at grand unification" is a statement about an *assumed* Yukawa of order one — assumed because the top quark's is. A framework that derives Y_ν instead of assuming it can put the heavy scale anywhere the geometry says, and App. L puts it at 3.85 TeV.

J.1.4. Count the physical phases of the lepton sector, Dirac versus Majorana, at general n.

Solution. Step 1: an $n\times n$ unitary matrix has n^2 real parameters, of which $n(n-1)/2$ are angles and $n(n+1)/2$ are phases. Step 2: if the neutrinos are Dirac, both sectors absorb phases and $2n-1$ are removable, leaving $n(n+1)/2-(2n-1) = (n-1)(n-2)/2$ physical phases: zero for $n=2$, one for $n=3$. If the neutrinos are Majorana, $\nu \to e^{i\phi}\nu$ would change the Majorana mass term and is forbidden, so only the n charged-lepton phases are available and $n(n+1)/2-n = n(n-1)/2$ survive: one for $n=2$, *three* for $n=3$ — one Dirac plus two Majorana. Step 3: the two extra phases are invisible in oscillations, which depend only on the rephasing-invariant products $U_{\alpha i}U^*_{\beta i}$, and surface only in lepton-number-violating processes. Physics intuition: a Majorana particle cannot hide a phase in its own redefinition because it *is* its own antiparticle; the phases that quarks shrug off become physical bookkeeping. Counting them is the cleanest way to see that "Dirac or Majorana?" is not metaphysics but two more measurable numbers — and the next chapter measures them.

J.1.5. Derive the solar MSW resonance density, and show the Sun is dense enough. *Solution.* Step 1: the charged-current potential (J.10) is $V_{\rm CC} = \sqrt{2}G_F n_e$, and the in-medium mixing is maximal when it cancels the vacuum diagonal: $n_e^{\rm res} = \Delta m^2\cos 2\theta/(2\sqrt{2}G_F E)$. Step 2: with $\Delta m^2_{21} = 7.53\times10^{-5}$ eV^2, $\cos 2\theta_{12} = 1-2(0.308) = 0.384$, $E = 10$ MeV and $G_F = 1.166\times10^{-5}$ GeV^{-2}: $n_e^{\rm res} = 8.6\times10^{-17}$ GeV$^3 = 1.14\times10^{25}$ cm$^{-3} = 19\,N_A$ cm^{-3}. Step 3: the solar core has $\rho \simeq 150$ g cm^{-3} and $Y_e \simeq 0.64$, so $n_e \simeq 5.8\times10^{25}$ cm^{-3} — a factor five *above* resonance. A 10 MeV ^{8}B neutrino is therefore born on the far side of the resonance and, if the profile is adiabatic, exits as pure ν_2 with $P_{ee} \to \sin^2\theta_{12} = 0.31$; the *pp* neutrinos at 0.3 MeV never reach resonance and average to $1-\frac{1}{2}\sin^2 2\theta_{12} = 0.57$. Physics intuition: both numbers are measured, and the factor-of-two step between them across the solar spectrum is the fingerprint. The Sun works as a neutrino spectrometer because its density happens to bracket the resonance for the energies it emits — had the core been ten times less dense, the solar neutrino problem would have been a flat deficit with no structure and far harder to diagnose.

J.1.6. Show that CP violation in oscillations requires all three splittings and all three angles.

Solution. Step 1: the CP-odd difference (J.8) is $16 J_{\rm PMNS} \sin\Delta_{21} \sin\Delta_{31} \sin\Delta_{32}$, and $J_{\rm PMNS}$ itself (J.9) is $c_{12}s_{12}c_{23}s_{23}c_{13}^2 s_{13}\sin\delta$. Step 2: the product vanishes if any one of the three angles is 0 or $\pi/2$, if δ is 0 or π, or if any two masses are degenerate (which kills one of the three sines). Step 3: historically this is why θ_{13} mattered so much: before 2012 it was consistent with zero, and a zero would have closed the leptonic CP program entirely. Its measured value $\sin^2\theta_{13} \simeq 0.0225$ is small but decisively nonzero, and it is the smallest factor in the product — the leptonic CP asymmetry is small *because of the reactor angle*, not because of the phase. Physics intuition: CP violation is an area, and an area needs all three sides. A degenerate pair collapses the triangle onto a line; a vanishing angle collapses it onto a point. Nature gave us a triangle with one short side, which is why DUNE needs a megawatt beam and a decade.

J.1.7. Read the three absolute-mass probes as three different weightings.

Solution. Step 1: write the three combinations: $m_\beta^2 = \sum_i |U_{ei}|^2 m_i^2$ (incoherent, squares), $\Sigma m_\nu = \sum_i m_i$ (unweighted, first power), and $m_{\beta\beta} = |\sum_i U_{ei}^2 m_i|$ (coherent, first power, with phases). Step 2: with $|U_{e1}|^2 = c_{12}^2 c_{13}^2$, $|U_{e2}|^2 = s_{12}^2 c_{13}^2$, $|U_{e3}|^2 = s_{13}^2$ and the derived first-order spectrum (0, 8.36114, 50.1668) meV: $m_\beta = \sqrt{0.3020 \times 69.909 + 0.02143 \times 2516.7} = 8.66$ meV, while $\Sigma m_\nu = 58.53$ meV and (next chapter) $m_{\beta\beta} = 1.72$ meV. Step 3: the same spectrum gives three numbers spanning a factor of thirty-four, and only the middle one is within reach of a current experiment — 8.66 meV is fifty times below KATRIN's 450 meV limit and below even its ultimate design sensitivity. Physics intuition: the ordering of the three numbers is itself a prediction. The coherent combination is the smallest because the phases interfere destructively; the kinematic one is small because squaring a spectrum dominated by its top eigenvalue and then weighting that eigenvalue by $s_{13}^2 = 0.021$ throws most of it away. A framework that gets the spectrum right must get all three, and the one that will decide is the sum.

J.1.8. Locate the Casas–Ibarra freedom, and count it.

Solution. Step 1: (J.15) writes the Dirac Yukawa as $\sqrt{\widehat{M}_N} R \sqrt{\widehat{m}_\nu} U^\dagger$ with R complex orthogonal. Step 2: count. A complex orthogonal 3×3 matrix has three complex angles, so six real parameters; the three heavy masses add three more; nine in total, *none* of which appears in any oscillation observable, because $m_\nu = -m_D M_N^{-1} m_D^{\mathsf{T}}$ is invariant under $R \to R'$ by construction. Step 3: consequence — any statement of the form "the measured PMNS phase implies the sign of the baryon asymmetry" is false in the general type-I seesaw, because R can supply an arbitrary extra phase. Physics intuition: Casas–Ibarra is the field's honesty device. It makes visible, in one equation, exactly which parameters low-energy experiments can never touch — and therefore exactly what a theory must supply from elsewhere if it wants to predict leptogenesis rather than accommodate it. Hold this count of nine; App. L returns every one of them.

J.2.1. State what a global fit is fitting, and why the octant disagreement is not a mistake.

Solution. Step 1: a global fit minimizes a χ^2 over six oscillation parameters against several hundred data points from experiments with different L/E, different channels, and different systematics. Step 2: the atmospheric octant is determined mainly by two competing handles — the ν_μ disappearance

amplitude, which depends on $\sin^2 2\theta_{23}$ and so is *octant-blind*, and the ν_e appearance rate, which depends on $\sin^2\theta_{23}$ and is not, but is entangled with $\delta_{\rm PMNS}$ and the ordering. Step 3: the global fit (Esteban et al., 2025) weights the atmospheric data heavily and lands at 0.470; the accelerator joint fit (T2K and NOvA Collaborations, 2025) weights appearance and lands at 0.56. Both are correct analyses of different data. Physics intuition: this is what a genuine experimental tension looks like from inside, and a student should resist the urge to average. The disagreement is *information*: it says the octant is being determined by a sub-leading effect that two datasets see differently, and it will be settled by a measurement (Hyper-K, DUNE) rather than by a weighted mean. A prediction that lands in one octant should say so and wait.

J.2.2. Show that the seesaw explains the smallness but not the scale.

Solution. Step 1: the seesaw formula (J.14) contains two unknowns, m_D and M_N, and the data determine one combination. Step 2: therefore, for any M_N in the range $[10^2, 10^{16}]$ GeV there is a Yukawa $Y_\nu = \sqrt{2m_\nu M_N}/v$ that reproduces the measured mass: fourteen orders of magnitude of heavy scale are equally consistent with all oscillation data. Step 3: the constraint that does bite is not from the neutrino sector at all — it is that thermal leptogenesis in the standard picture requires $M_N \gtrsim 10^9$ GeV (next chapter), and even that is a bound on a *mechanism*, not on the seesaw. Physics intuition: the seesaw is a mechanism, not a prediction. It converts "why is the neutrino light?" into "what is the heavy scale, and what is the Yukawa?" — two questions where there was one. That is real progress, because the new questions are about structure rather than about a small number; but a course should not let a student leave believing the seesaw predicts 10^{15} GeV.

J.2.3. Estimate what would have to be true for the mass ordering to be inverted.

Solution. Step 1: inverted ordering means $m_3 < m_1 < m_2$, so the two nearly degenerate states are the heavy pair and $\Sigma m_\nu \gtrsim 2\sqrt{|\Delta m^2_{31}|} = 100$ meV. Step 2: DESI DR2 in ΛCDM gives $\Sigma m_\nu < 64.2$ meV (DESI Collaboration, 2025), which is already *below* the inverted-ordering minimum — cosmology disfavors inverted ordering independently of any oscillation data. Step 3: the caveat that keeps this honest: the same data allow $\Sigma m_\nu < 163$ meV once w_0 and w_a float, and the inverted minimum survives comfortably there. So the correct statement is that inverted ordering is disfavored *conditional on* Λ*CDM*. Physics intuition: this is the cleanest example in contemporary physics of a particle-physics question being settled by cosmology — and of why the settlement is provisional. A student should be able to say, in one sentence, which assumption is carrying the conclusion; here it is the dark-energy equation of state, which has nothing to do with neutrinos and everything to do with the answer.

J.3.1. Evaluate the neutrino floor from the reused seam.

Solution. Step 1: the one admissible operator reuses $E_{\rm GUT} = E_{\rm Pl}\, e^{-2\pi} = 2.280 \times 10^{16}$ GeV — no new scale — so the floor (J.17) is $m_\star = \pi v^2/E_{\rm GUT}$. Step 2: with the first-order $v = 246.33$ GeV (O.11): $m_\star = \pi \times 60678/2.280 \times 10^{16}$ GeV $= 8.36114$ meV. Step 3: compare the fermion chapter's problem H.8.1 and the cosmology companion's problem I.1.1 (GG-A10-Cosmos): the same lever read from three directions — here the scale is *supplied* and the mass descends as a prediction, with no free right-handed mass anywhere in the construction, which is the concrete content of the answer to problem J.1.3. Physics intuition: the meV decade of neutrino physics is the geometric shadow of

the 10^{16}-GeV seam — the two most extreme mass scales ever measured are one ratio, and the π out front is the same compact circle that runs the whole flavor sector.

J.3.2. Score the rank-two ledger honestly: the two splittings separately, not their ratio. *Solution.* Step 1: the first-order eigenvalues $(m_1, m_2, m_3) = (0, 1, 6)\, m_\star$ give $\Delta m^2_{21} = m^2_\star = 6.9909 \times 10^{-5}$ eV^2 and $\Delta m^2_{31} = 36\, m^2_\star = 2.5167 \times 10^{-3}$ eV^2. Step 2: score each against Esteban et al. (2025) (7.53×10^{-5} and 2.511×10^{-3}): the atmospheric splitting is $+0.23\%$, the solar one -7.16%. The ratio 36 against the measured 33.35 is $+7.96\%$, and the QD-completed spectrum gives 34.56, or $+3.65\%$. Step 3: the honest reading is that quoting the *ratio* hides the structure — one integer of $(0, 1, 6)$ is performing to two parts in a thousand and the other is off by seven percent, and it is the solar one. With JUNO now measuring Δm^2_{21} to 1.6% (JUNO Collaboration, 2026) the gap is 4.2σ against JUNO alone and no longer inside anybody's error bar. Physics intuition: a framework that reports an average when its two components disagree is hiding from its own data. The right response is to name the strained integer, which is the 1 in $(0, 1, 6)$, and to say what would close it — the QD layer already moves it two-thirds of the way, so the remaining gap is a question about the QD completion of the Majorana channel and not about the channel itself.

J.3.3. Test the ordering prediction against the one number that could kill it. *Solution.* Step 1: the cascade-depth assignment puts the GGA at ν_1, forcing $m_1 < m_2 < m_3$. Step 2: the current evidence is 2.2–2.3σ for normal after JUNO (Esteban et al., 2026), and JUNO alone reaches 3σ around 2032 by a method independent of matter effects. Step 3: this is a genuine one-sided kill row — confirmation adds little, since normal is already preferred, but an established inverted ordering falsifies the depth assignment outright and with it the whole cascade reading of the neutrino block. Physics intuition: the most valuable predictions are the ones that can only lose. This one has no free parameter to absorb a surprise: the ground state of a cascade is its lightest member, and if nature puts the lightest state at the top of the spectrum then the object being described is not a cascade.

J.3.4. Derive the (D, S, A) triplet rather than adopting it. *Solution.* Step 1: a rank-two symmetric form on a three-dimensional family space has a one-dimensional kernel, so the decomposition is $\mathbf{3} = \mathbf{1} \oplus \mathbf{2}$ — one distinguished direction and a plane. Step 2: the Majorana channel is generated by the *single* operator (J.1) acting identically on all three families, so the distinguished direction is permutation-invariant: $|D\rangle = (1, 1, 1)^{\mathsf{T}}/\sqrt{3}$. Step 3: the residual plane is resolved by the only family-distinguishing structure available, the dyadic charged-lepton ladder, into $\mu{-}\tau$-even $|S\rangle$ and $\mu{-}\tau$-odd $|A\rangle$; nothing is chosen at any step. Physics intuition: the basis is not an ansatz, it is a consequence of the rank and of the fact that one operator cannot tell three identical families apart. This is the whole reason the lepton angles come out large: the neutrino sector's natural basis is maximally spread over the families while the charged sector's is maximally concentrated, and the PMNS matrix is the angle between "spread" and "concentrated".

J.3.5. Score the closed-form angles against the global fit — and against the octant split. *Solution.* Step 1: the misalignment (J.20) gives $\sin^2\theta_{12} = 25/81 = 0.30864$ against JUNO's 0.3092 ± 0.0087: 0.07σ. Step 2: $\sin^2\theta_{13} = 3/140 = 0.021429$ against the reactor $0.02248^{+0.00055}_{-0.00059}$:

1.8σ low. And $\sin^2\theta_{23} = 4/7 = 0.57143$ scored *twice* — inside the accelerator joint fit's $0.56^{+0.03}_{-0.05}$, and more than four sigma above the global fit's $0.470^{+0.017}_{-0.014}$. Step 3: all three are ratios of the ledger integers $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ with no others available, and the famous contrast "CKM small, PMNS large" becomes one statement: hierarchical-against-hierarchical meshes finely (chapter I), hierarchical-against-democratic can only mesh coarsely. Physics intuition: when three angles that dominated two decades of experimental effort come out as fractions a student can write on one line, the right response is not belief but the falsification channels — and the atmospheric row is currently the sharpest of them, because the two datasets disagree and the prediction sits squarely in one of the two camps.

J.3.6. Show that the solar angle now discriminates between hypotheses.

Solution. Step 1: the pure democratic overlap gives $\sin^2\theta_{12} = 1/3 = 0.33333$, which is what tri-bimaximal mixing predicts. Step 2: JUNO's direct measurement is 0.3092 ± 0.0087, so the democratic value is 2.8σ high. The ledger's $25/81 = 0.30864 = \frac{1}{3}(1 - 2/27)$ is 0.07σ away. Step 3: the discriminating quantity is the 7.4% depression $2/27$, and it is now resolved by experiment — the measurement crossed the threshold of being able to see it somewhere around the 2020s and JUNO has made it decisive. Physics intuition: for twenty years $\sin^2\theta_{12} \simeq 1/3$ was the most-quoted approximate relation in flavor physics, and a whole model-building literature was built on making it exact. The data have moved off it. That is the best kind of experimental result — one that kills a beautiful approximation — and any framework claiming this sector must now predict the *deviation*, not the value.

J.3.7. Show why the atmospheric angle is a tilt about maximal, and why the sign is not free.

Solution. Step 1: write the prediction in the form (J.22), $\sin^2\theta_{23} = \frac{1}{2}(1 + 1/J)$: the central value is *maximal* mixing and the ledger supplies a traceless tilt of size $1/(2J) = 1/14 = 0.0714$. Step 2: the sign is fixed by which direction the charged-lepton hierarchy distinguishes, so the prediction is $0.5 + 0.0714 = 0.5714$ and *not* $0.5 - 0.0714 = 0.4286$ — the two would be experimentally indistinguishable in $\sin^2 2\theta_{23}$ and are very distinguishable in appearance. Step 3: therefore the octant is the prediction, and F-SM-3e registers it: a global determination settling in the lower octant at 5σ kills the assignment, and the same measurement in the upper octant confirms it. Physics intuition: most flavor predictions are about a magnitude; this one is about a *sign*, and signs are much harder to accommodate after the fact. Note also what the form says structurally — maximal mixing is the symmetric point, μ–τ exchange is the symmetry, and $J = 7$ is what breaks it. That is a rather different statement from "θ_{23} happens to be near 45°".

J.3.8. Compute Σm_ν and locate it under the cosmological bound.

Solution. Step 1: the first-order spectrum sums to $\Sigma m_\nu{}^{(1)} = 0 + 8.36114 + 50.1668 = 58.528$ meV; the QD-completed spectrum gives 59.171 meV. Step 2: against DESI DR2's ΛCDM bound $\Sigma m_\nu < 64.2$ meV (DESI Collaboration, 2025) the prediction sits at 92% of the bound, with 5.0 meV of headroom. Step 3: this is F-SM-3c, and it is the channel most likely to fire first: the next cosmological data release either pushes the bound below 59 meV and kills the spectrum, or measures a nonzero Σm_ν and tests it directly. Physics intuition: the prediction is not merely consistent with the bound, it is *pressed against* it, which is the most dangerous place a prediction can sit and the

most informative. There is no version of this framework in which Σm_ν is comfortably small — the spectrum is nearly hierarchical and its sum is nearly the atmospheric mass, so the number is what it is, and the experiment either finds it in the next few years or removes it.

J.3.9. Compute m_β and explain why β decay will not see it.
Solution. Step 1: $m_\beta^2 = \sum_i |U_{ei}|^2 m_i^2$ with $|U_{e2}|^2 = s_{12}^2 c_{13}^2 = 0.30203$ and $|U_{e3}|^2 = s_{13}^2 = 0.021429$. Step 2: with the derived spectrum, $m_\beta^2 = 0.30203 \times 69.909 + 0.021429 \times 2516.7 = 21.11 + 53.93 = 75.04$ meV^2, so $m_\beta = 8.66$ meV. Step 3: KATRIN's current limit is 450 meV (KATRIN Collaboration, 2025) and its design sensitivity is about 200 meV — a factor of twenty-three above the prediction. No planned kinematic experiment reaches it. Physics intuition: it is worth being explicit that a prediction can be unfalsifiable in one channel and sharply falsifiable in another. The same spectrum that hides from KATRIN by a factor of twenty-three sits at 92% of the DESI bound. A student choosing a thesis should read this pair of numbers as a map of where the decisive measurements are — and they are in cosmology and in double beta decay, not at the tritium endpoint.

J.3.10. Audit the chapter: which of Step 2's ten items are actually closed?
Solution. Step 1: list the closures. Items 1 and 2 (the scale and the operator coefficient) close together, because $\Lambda = E_{\rm GUT}$ is reused rather than introduced. Item 3 closes in its first half — rank two forces exactly two splittings — and strains in its second, at -7.16% on the solar splitting (J.18). Items 4 (ordering), 5 (Majorana), 6 (three angles), 7 (Dirac phase), 8 (Majorana phases) and 10 (three families, imported from GG-P2-N3) are structural outputs. Step 2: item 9, the nine Casas–Ibarra parameters of problem J.1.8, is answered in App. L — and answered *conditionally*, on a hypothesis whose crux is open. Step 3: so the honest tally is eight closed, one closed with a stated strain, one closed conditionally; and both structural puzzles answered. Physics intuition: the point of running this audit at the end of the chapter is that it is very easy, in a framework with many derived quantities, to let the successes carry the failures. Writing the tally down forces the strained solar splitting and the conditional heavy sector to appear next to the eight clean closures, at the same size, which is where a reader deciding what to believe needs them.

Take-away. *Standard result:* oscillations, two mass splittings, three large angles, and a sector the Standard Model must leave massless at dimension four. *What it assumed:* ten quantities, plus the nine invisible parameters of the heavy sector, plus both structural puzzles. *The geometric reading:* one reused scale sets the meV floor; a rank-two Majorana ledger forces exactly two splittings and the normal ordering; a dyadic-against-democratic misalignment forces the large angles as ratios of the ledger integers; the atmospheric angle is a tilt about maximal whose *sign* is the prediction. *Where it strains:* the solar splitting, at -7.2%, named and quantified rather than averaged into a ratio.

K Majorana neutrinos, leptonic CP violation, and neutrinoless double beta decay

What this appendix teaches. The three questions the previous chapter deferred. Step 1 builds the spinor technology from the Lorentz algebra — Weyl, Dirac and Majorana fields, and why only a *neutral* fermion may be its own antiparticle — then derives the seesaw exactly, counts the physical phases at general generation number, sets out CP violation in oscillations with its degeneracies, and obtains the $0\nu\beta\beta$ amplitude with the propagator step that produces $m_{\beta\beta}$. Step 2 lists eight quantities left open. Step 3 derives all three phases as *parity data of a rank-two ledger* rather than as dials, works $m_{\beta\beta}$ term by term, and reports the uncomfortable consequence: the derived parity puts the prediction near the *destructive* end of its own accessible envelope.

Read alongside: Bilenky (2018) for the phase counting; Mohapatra & Pal (2004) Ch. 5–7 for the seesaw variants and $0\nu\beta\beta$; Srednicki (2007) Ch. 33–36 for two-component spinor technology; Giunti & Kim (2007) Ch. 6 for the charge-conjugation bookkeeping done carefully.

This is the sixth sector appendix and the second of the three lepton chapters. It takes up the three questions the previous chapter posed and deferred: *is the neutrino its own antiparticle*, *what are the three leptonic CP phases*, and *what would it take to measure them*. The three are one question, because the only laboratory access to the two Majorana phases runs through a process that exists only if the neutrino is Majorana.

K.1 Step 1 — Majorana fermions and CP violation as a graduate student meets them

K.1.1 Weyl, Dirac, Majorana: three ways to be a spin-half field

The Lorentz algebra $\mathfrak{so}(3,1) \simeq \mathfrak{su}(2)\oplus\mathfrak{su}(2)$ has two inequivalent two-dimensional representations, $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$ — the left- and right-handed *Weyl* spinors ψ_L, ψ_R. A Dirac field is the pair $\Psi = (\psi_L, \psi_R)^{\mathsf{T}}$, four complex components, and its mass term $m\bar{\Psi}\Psi = m(\bar{\psi}_L\psi_R + \text{h.c.})$ couples the two.

The Majorana observation is that one does not need two Weyl fields to write a mass. Charge conjugation

$$\Psi^c \equiv C\bar{\Psi}^{\mathsf{T}}, \qquad C = i\gamma^2\gamma^0, \tag{K.1}$$

maps a left-handed field to a *right-handed* one: $(\psi_L)^c$ transforms in $(0,\frac{1}{2})$. So ψ_L and $(\psi_L)^c$ are exactly the two ingredients a mass term needs, and one may write

$$\mathcal{L}_M = -\tfrac{1}{2}\, m\, \overline{(\psi_L)^c}\, \psi_L + \text{h.c.} = -\tfrac{1}{2}\, m\, \psi_L^{\mathsf{T}} C \psi_L + \text{h.c.} \tag{K.2}$$

A field satisfying $\Psi = \Psi^c$ is a *Majorana* field. It has two, not four, independent complex components — the particle and the antiparticle are the same state, distinguished only by helicity. *(Physics intuition: a Dirac fermion is a fermion with a conserved arrow — something that distinguishes it from its antiparticle, and that something is always a charge. Remove the charge and the arrow has nothing to point along; the particle and antiparticle become two helicity states of one object. The*

factor of $\frac{1}{2}$ in (K.2) is the same $\frac{1}{2}$ that appears in front of any real field's mass term, and for the same reason: you are double-counting if you do not put it there.)

The one-sentence reason this is a neutrino question. (K.2) takes *two* ψ_L's, so it carries *twice* whatever charge ψ_L carries. It is therefore forbidden for every charged fermion by electromagnetic gauge invariance alone, and permitted for exactly one Standard-Model field: the neutrino. The Dirac-or-Majorana question exists for the neutrino and for nothing else in the observed particle content — which is why an answer would be a structural fact and not a detail.

K.1.2 The seesaw, diagonalized exactly

Take one generation for clarity. With ν_L and a singlet N_R, the Lagrangian permits a Dirac mass m_D and, since N_R is a total gauge singlet, a Majorana mass M for N_R — but *not* for ν_L, by the argument above. In the basis $(\nu_L,\, N_R^c)$ this is

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}. \tag{K.3}$$

The eigenvalues are exactly

$$\lambda_\pm = \frac{M}{2}\left(1 \pm \sqrt{1 + \frac{4m_D^2}{M^2}}\right), \tag{K.4}$$

and for $m_D \ll M$ these expand to

$$\lambda_- \simeq -\frac{m_D^2}{M}, \qquad \lambda_+ \simeq M + \frac{m_D^2}{M}, \qquad \tan 2\vartheta = \frac{2m_D}{M} \;\Rightarrow\; \vartheta \simeq \frac{m_D}{M}. \tag{K.5}$$

Both eigenstates are Majorana — the mass matrix (K.3) is symmetric, and a symmetric mass matrix for neutral fields has Majorana eigenstates. That is the essential structural point and it is easy to lose: the light neutrino is Majorana not because somebody added a Majorana mass for it, but because the heavy singlet's Majorana mass propagates down. The minus sign in λ_- is absorbed into the Majorana phase, which is the first hint that these phases are bookkeeping about *signs* rather than free angles. *(Physics intuition: the determinant of (K.3) is $-m_D^2$, fixed; so the product of the two eigenvalues is fixed. Push one up and the other must come down by exactly the same factor. That is the entire mechanism, and it is why the name is right: a seesaw has a fixed fulcrum, and the fulcrum here is m_D, not M.)*

K.1.3 Counting the physical phases

An $n \times n$ unitary matrix has n^2 real parameters: $n(n-1)/2$ angles and $n(n+1)/2$ phases. How many phases survive rephasing depends on whether the neutrino mass term permits $\nu \to e^{i\phi}\nu$:

$$N_{\text{phase}}^{\text{Dirac}} = \frac{(n-1)(n-2)}{2}, \qquad N_{\text{phase}}^{\text{Majorana}} = \frac{n(n-1)}{2}. \tag{K.6}$$

For $n = 3$: one phase if Dirac, *three* if Majorana — one Dirac phase δ_{PMNS} and two Majorana phases α_{21}, α_{31}, entering as

$$U = \tilde{U}\,\mathrm{diag}\Big(1,\, e^{i\alpha_{21}/2},\, e^{i\alpha_{31}/2}\Big)\,. \tag{K.7}$$

The half-angles are not a convention chosen for elegance: the mass term (K.2) is quadratic in the field, so a phase ϕ on the field appears as 2ϕ in the mass. Writing $\alpha/2$ in the matrix makes α the physical, mass-term phase — and it is α, not $\alpha/2$, that Step 3 derives. *(Physics intuition: the two extra phases are exactly the two relative signs — more generally, the two relative phases — among the three Majorana mass eigenvalues, which a Dirac theory would have rotated away into the fields. They are invisible in oscillations because an oscillation probability depends on $U_{\alpha i}U^*_{\beta i}$, in which the diagonal phase matrix cancels identically. Any process that is to see them must be quadratic in U without the conjugate — and that means lepton-number violating.)*

K.1.4 CP violation in oscillations, and the three degeneracies

The Dirac phase is measurable in oscillations through the asymmetry (J.8). In practice the measurement is hard for three reasons, and a student should know all three because they are what the next decade of experiments is designed around.

The matter fake. The charged-current potential (J.10) has opposite sign for antineutrinos, so matter generates an asymmetry that has nothing to do with CP. At DUNE's 1300 km the matter effect is comparable to the CP effect, and the two are separated by their different energy dependence, not by a single measurement.

The octant degeneracy. The appearance rate depends on $\sin^2\theta_{23}\sin^2 2\theta_{13}$, so an upper-octant θ_{23} with one δ_{PMNS} can mimic a lower-octant θ_{23} with another. This is the same octant question that §55.5 finds the datasets disagreeing about.

The ordering degeneracy. Flipping the sign of Δm^2_{31} changes the matter term, and can be partially compensated by a shift in δ_{PMNS}.

The current status reflects all three: the T2K–NOvA joint fit (T2K and NOvA Collaborations, 2025) gives $\delta_{\text{PMNS}} = 203.4^{+63.0}_{-37.8}$ degrees for normal ordering and excludes $\delta_{\text{PMNS}} = +\pi/2$ at 3σ — but notably *not* $3\pi/2$, the value that maximizes CP violation with the opposite sign.

K.1.5 Neutrinoless double beta decay

The process and the amplitude. Ordinary double beta decay, $(A, Z) \to (A, Z+2) + 2e^- + 2\bar{\nu}_e$, is a second-order weak process observed in about a dozen isotopes. Its neutrinoless partner

$$(A, Z) \to (A, Z+2) + 2e^- \tag{K.8}$$

violates lepton number by two units and is forbidden unless the neutrino is Majorana. The amplitude joins the two vertices with a neutrino propagator; because the exchanged neutrino is virtual with momentum of order the nuclear Fermi scale $|p| \sim 100$ MeV $\gg m_i$, the propagator's *mass* term is

what survives the chirality projection,

$$\frac{\not{p} + m_i}{p^2 - m_i^2} \xrightarrow{\text{chirality}} \frac{m_i}{p^2}, \tag{K.9}$$

and each mass eigenstate contributes with weight U_{ei}^2 — the square, *not* $|U_{ei}|^2$, because no conjugate appears. Summing, the whole particle-physics content is one number:

$$\boxed{m_{\beta\beta} = \left| \sum_i U_{ei}^2 \, m_i \right| = \left| \, m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i(\alpha_{31} - 2\delta_{\text{PMNS}})} \right|.} \tag{K.10}$$

The rate is

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu} \left| M^{0\nu} \right|^2 \frac{{m_{\beta\beta}}^2}{m_e^2}, \tag{K.11}$$

with $G^{0\nu}$ an exactly calculable phase-space factor and $M^{0\nu}$ a nuclear matrix element whose value differs between many-body methods by a factor of two to three — the dominant theory uncertainty in the entire field. *(Physics intuition: the reason $m_{\beta\beta}$ is a coherent sum with phases, while the β-decay observable m_β is an incoherent sum of squares, is that here the three mass eigenstates are exchanged as* virtual *particles inside one amplitude and there interfere, while in β decay they are produced as* real *particles in distinguishable final states and there do not. Interference is exactly what makes the Majorana phases observable — and exactly what allows the answer to be small through cancellation.)*

The two bands, and the funnel. Plotting (K.10) against the lightest mass with the phases scanned gives two allowed bands. For *inverted* ordering the two dominant states are nearly degenerate and their contributions cannot fully cancel, so there is a floor: $m_{\beta\beta} \gtrsim 19$ meV, computed in problem K.1.7. For *normal* ordering there is no floor — the band passes through zero in a narrow "funnel" near a lightest mass of a few meV, where the three terms can cancel exactly. *This asymmetry is the field's strategic fact:* a tonne-scale experiment covers the inverted band completely and therefore either finds a signal or excludes inverted ordering, whereas the normal band is only partially covered and a null result there proves nothing.

Current limits. The strongest are $T_{1/2}^{0\nu} > 3.8 \times 10^{26}$ yr in ^{136}Xe (KamLAND-Zen Collaboration, 2025), $> 1.9 \times 10^{26}$ yr in ^{76}Ge (LEGEND Collaboration, 2026) and $> 3.5 \times 10^{25}$ yr in ^{130}Te (CUORE Collaboration, 2025), corresponding to $m_{\beta\beta}$ limits in the few tens of meV with the nuclear-matrix-element spread dominating. The next generation, LEGEND-1000 and CUPID, targets 9–21 meV.

The black-box theorem, and its honest size. Schechter & Valle (1982) proved that *any* mechanism producing (K.8) generates a Majorana neutrino mass, by closing the process's own diagram into a self-energy. So an observation of $0\nu\beta\beta$ proves the neutrino is Majorana whatever the underlying mechanism. The theorem is qualitative and must be quoted as such: the mass so generated is of order 10^{-24} eV, thirteen orders of magnitude below the observed scale, so the

theorem establishes *that* the neutrino is Majorana and says nothing about *how much* of the observed rate comes from light-neutrino exchange.

Worked example (STANDARD TEXTBOOK PEDAGOGY). The rate (K.11) scales as ${m_{\beta\beta}}^2$, so *half-life sensitivity* scales as ${m_{\beta\beta}}^{-2}$. In the background-limited regime a half-life reach grows only as $\sqrt{Mt/(b\,\Delta E)}$ — the square root of exposure over background. Chaining the two: improving the $m_{\beta\beta}$ reach by a factor f costs f^2 in half-life and f^4 in exposure. From the next generation's 9 meV to a hypothetical 1.8 meV is $f = 5$, hence 25 in half-life and **625** in exposure. That single arithmetic line is the reason a meV-scale $m_{\beta\beta}$ is not an incremental goal, and Step 3 will show it is exactly where this framework puts its prediction.

K.2 Step 2 — what the Majorana sector assumes but cannot explain

1. **Whether the neutrino is Majorana at all.** Not predicted; both completions are consistent.
2. **The Dirac phase $\delta_{\rm PMNS}$.** A free parameter, currently constrained only to a broad interval.
3. **The two Majorana phases.** Free, and *unconstrained by any measurement ever made.*
4. **The value of $m_{\beta\beta}$.** With the phases free, even a perfectly known spectrum leaves $m_{\beta\beta}$ uncertain by the full width of its envelope — typically a factor of two to three.
5. **Whether light-neutrino exchange dominates the rate.** Other lepton-number-violating mechanisms contribute; the black-box theorem does not separate them.
6. **The nuclear matrix elements.** A factor two to three of theory uncertainty, unrelated to particle physics and currently irreducible.
7. **Why leptonic CP violation should be large when the quark one is small.** Recorded, not explained.
8. **Whether any of this connects to the baryon asymmetry.** As problem J.1.8 showed, the Casas–Ibarra matrix R severs the connection in the general seesaw.

(Physics intuition: items 2, 3 and 7 are the same item seen three ways. The Standard Model's answer to "why is there a CP phase?" is "because three families permit one" — a statement about what is allowed*. Nothing anywhere in the framework says what the phase* is*, and for the two Majorana phases nothing even says they are not both zero. A theory that derived all three would be making its strongest possible claim in its least constrained sector.)*

K.3 Step 3 — the geometric derivation: phases as parity data

K.3.1 Why the parities are data and not dials

The framework's answer to Step 2's items 1, 2 and 3 rests on a single structural fact established in the previous chapter: the Majorana ledger is *rank two*. A rank-two symmetric form on the three-family space has a one-dimensional kernel, and the relative phases among its eigenvalues are

therefore not continuous parameters at all — they are the *signs* that a rank-two form is allowed to carry, which is to say a discrete parity assignment. The framework derives

$$\boxed{(\alpha_{21},\, \alpha_{31}) = (0,\, \pi).} \tag{K.12}$$

Two things follow at once. The neutrino is Majorana — item 1 — because the operator (J.1) is the only admitted mass channel and it is lepton-number violating by construction. And the phases are not tunable: (K.12) is a choice between a finite number of sign assignments, not a point in a two-torus, so the framework cannot adjust $m_{\beta\beta}$ to taste.

The half-angle, and why a parity of π is not the same as a phase of π. Because the Majorana phases enter (K.7) halved, the parity $\alpha_{31} = \pi$ appears in the mixing matrix as

$$e^{i\alpha_{31}/2} = e^{i\pi/2} = i. \tag{K.13}$$

A parity of π in the mass term is a factor of i — a *quarter* turn — in the amplitude. In $m_{\beta\beta}$, which is quadratic in U, the i squares back to -1 and flips the sign of the third term; in any amplitude *linear* in U it would instead rotate by ninety degrees. The leptogenesis chapter uses precisely this, and §64 is where the sign it sets is cashed. *(Physics intuition: this is the single most useful thing to understand about Majorana phases. They are not angles you dial; they are square roots of signs. A relative minus sign between two Majorana masses — the only thing a rank-two ledger is free to have — appears in the mixing matrix as a factor of i, and whether you see a sign flip or a quarter turn depends entirely on whether your observable is quadratic or linear in U. That is why the same parity makes $m_{\beta\beta}$ smaller and makes the CP asymmetry of leptogenesis nonzero.)*

K.3.2 The Dirac phase, derived

The Dirac phase is fixed by the same compact-angle holonomy logic that fixes the quark phase, read on the lepton sector's reference channel. The result is the closed form

$$\delta_{\mathrm{PMNS}} = \pi + 2\arcsin\frac{1}{J} = \pi + 2\arcsin\frac{1}{7} = 196.426421^\circ, \tag{K.14}$$

with the QD-completed value 196.059074°. Three features deserve comment. The base value is π, *not* zero, because the conventional parameterization refers the phase to a channel entering with a negative real coefficient; converting to a positive real axis is a rotation by half a turn. It is worth being explicit that this is *not* the Majorana parity $\alpha_{31} = \pi$ reappearing: the two are both π, and they may well share a geometric origin in the parity-flipped reference channel, but they are distinct observables and K.1.5 shows that only the first is visible to oscillation. The residual is $2\arcsin(1/J)$ and not $\arcsin(1/J)$, because the phase is read on a channel traversed twice, which is the same doubling that put the half-angle in (K.13). And $J = 7$ is the ledger's own integer, the same one that tilts the atmospheric angle in (J.22) — so the atmospheric angle and the Dirac phase are two readings of one number, and cannot be adjusted independently.

Against data: the T2K–NOvA joint fit gives $203.4^{+63.0}_{-37.8}$ degrees (T2K and NOvA Collaborations, 2025), so 196.4° sits well inside the measured interval — but the interval is 100 degrees wide, and this is a prediction awaiting an experiment rather than one that has passed a test. F-SM-3d registers it: a 5σ determination of $\delta_{\rm PMNS}$ outside $[186°, 207°]$ kills it, and DUNE and Hyper-K are the facilities.

K.3.3 $m_{\beta\beta}$, worked

Rather than quote the result, work it; the way the three terms combine is the content. At first order $m_1 = 0$, so the first term of (K.10) vanishes identically and the answer is a two-term interference. With $s^2_{12} = 25/81$, $s^2_{13} = 3/140$ and hence $c^2_{13} = 137/140$, and with $\alpha_{21} = 0$ so that the second term is real and positive:

$$m_2 s^2_{12} c^2_{13} = 8.36114 \times \tfrac{25}{81} \times \tfrac{137}{140} = +2.52530 \text{ meV}. \tag{K.15}$$

The third term carries the phase $\alpha_{31} - 2\delta_{\rm PMNS} = \pi - 2(196.426421°)$ and the modulus $m_3 s^2_{13} = 50.1668 \times 3/140 = 1.07500$ meV, giving

$$m_3 s^2_{13} e^{i(\alpha_{31} - 2\delta_{\rm PMNS})} = -0.90307 + 0.58317\, i \text{ meV}. \tag{K.16}$$

The real parts oppose: +2.52530 against −0.90307. Adding and taking the modulus,

$$m_{\beta\beta}{}^{(1)} = |\, 2.52530 - 0.90307 + 0.58317\, i \,| = 1.72386 \text{ meV}, \tag{K.17}$$

and at QD order the first term is no longer zero ($m_1^{\rm QD} = 0.0653$ meV contributes +0.04416 meV), giving $m_{\beta\beta}{}^{\rm QD} = 1.79925$ meV.

K.3.4 The finding: the derived parity sits near the worst case

The cancellation in (K.17) is not incidental — it is most of the answer, and it is caused by the derived parity. Two comparisons on the *same* derived spectrum and angles make its size concrete.

- With the other admissible parity, $\alpha_{31} = 0$, the third term's real part changes sign and adds: $m_{\beta\beta}$ would be 3.62 meV, twice as large.
- Scanning both Majorana phases freely over the full circle — which is what an experiment analyzing a generic normal-ordering spectrum must allow — gives the envelope $m_{\beta\beta} \in [1.44, 3.74]$ meV.

The derived value 1.80 meV therefore lands in the *lowest quarter* of the accessible range, near the maximally destructive end. **This framework's Majorana prediction is harder to confirm than a generic one**, and the difficulty is a direct consequence of the parity the framework derives rather than of any choice made to avoid a constraint. The cost is paid for the parity being *derived*: a framework free to choose α_{31} would place its prediction at the top of the envelope and claim a better chance of confirmation, about a number it had fitted.

Two consequences. The prediction is one-sided in the sense of F-SM-3b — a confirmed $0\nu\beta\beta$ line anywhere in the tonne-scale window of 9–21 meV kills it, while a null result there proves nothing —

and by the exposure arithmetic of the Step 1 worked example, reaching 1.8 meV needs 625 times the exposure of an experiment sensitive to 9 meV. This is not a prediction any planned experiment will confirm. It is a prediction that a positive result in the approved program would *refute*, and it is offered on that basis.

K.3.5 The two unitarity triangles, at true relative area

The Jarlskog invariant is twice the area of the unitarity triangle, so the two sectors can be compared directly. This paper derives both:

$$\frac{|J_{\mathrm{PMNS}}|}{|J_{\mathrm{CKM}}|} = \frac{9.187 \times 10^{-3}}{2.984 \times 10^{-5}} = 308 \qquad \text{(QD order; 338 at first order).} \tag{K.18}$$

The leptonic unitarity triangle has about three hundred times the area of the quark one, and because both numbers are outputs of the same ledger the ratio is a prediction rather than an observation. The quark side compares two hierarchical Dirac sectors and gets a thin triangle; the lepton side compares a dyadic charged-lepton sector against a nearly democratic Majorana one and gets a fat one. *Same constitution, different projector geometry* — and three hundred is how much different.

The caveat matters and is stated wherever this ratio is used: three hundred is *not* ten orders of magnitude, so this is not why the lepton sector can pay for baryogenesis when the quark sector cannot. That is the $\Delta L = 2$ door and the out-of-equilibrium lock crossing, and the next chapter finds that the *unflavored* invariant — the one J_{PMNS} belongs to — drops out of the matter asymmetry entirely.

K.3.6 Honest status

Items 1, 2, 3, 4 and 7 of Step 2 are answered: the neutrino is Majorana by construction, all three phases are derived, and $m_{\beta\beta}$ follows with no envelope left. Item 8 is answered in the next chapter, and answered conditionally. Items 5 and 6 are *not* answered and are not the framework's to answer — whether light-neutrino exchange dominates, and what the nuclear matrix elements are, are questions for the lepton-number-violation and nuclear-structure communities respectively, and the second of them is currently the larger uncertainty on any $m_{\beta\beta}$ limit quoted anywhere.

Problems and worked solutions. K.1.1. Show that charge conjugation turns a left-handed Weyl field into a right-handed one, and count the degrees of freedom.
Solution. Step 1: under the Lorentz algebra $\psi_L \in (\frac{1}{2}, 0)$; the operation (K.1) involves complex conjugation, which exchanges the two $\mathfrak{su}(2)$ factors, so $(\psi_L)^c \in (0, \frac{1}{2})$. Step 2: therefore $\overline{(\psi_L)^c}\psi_L$ is a Lorentz scalar built from *one* Weyl field, and (K.2) is a legitimate mass term. Step 3: count. A Dirac field is two Weyl fields, four complex components, describing particle and antiparticle each with two helicities. A Majorana field is one Weyl field, two complex components, and the "antiparticle" is the opposite helicity state of the same object. Physics intuition: mass, for a fermion, is whatever couples left to right. Nature offers two ways to find a right-handed partner for ψ_L — import one (ψ_R, Dirac) or manufacture one by conjugation ($(\psi_L)^c$, Majorana) — and the second

is free. It is available only when there is no charge to conjugate, which is why this is a neutrino question and only a neutrino question.

K.1.2. Prove that no charged fermion can have a Majorana mass.
Solution. Step 1: the term (K.2) is $\psi_L^{\mathsf{T}} C \psi_L$, bilinear in ψ_L with *no* conjugate. Step 2: under a $\mathrm{U}(1)_{\mathrm{EM}}$ rotation $\psi_L \to e^{iq\theta}\psi_L$, the term picks up $e^{2iq\theta}$, so it is invariant only for $q = 0$. Step 3: hence the electron, muon, tau and every quark are excluded by electromagnetic gauge invariance alone, and among Standard-Model fermions only the neutrino survives — and even it survives only at dimension five (J.1), because ν_L sits in a doublet carrying weak isospin and hypercharge. Physics intuition: "is it its own antiparticle?" is really "does it carry any conserved charge at all?" A photon is its own antiparticle for the same reason. The neutrino is the only *fermion* that could be, and whether it actually is depends on whether $B - L$ — the last global symmetry that could still forbid it — is a symmetry of nature or an accident of the Standard Model's field content.

K.1.3. Diagonalize the seesaw exactly, then check the expansion.
Solution. Step 1: the eigenvalues of (K.3) solve $\lambda^2 - M\lambda - m_D^2 = 0$, giving (K.4). Step 2: expand for $m_D \ll M$ using $\sqrt{1+x} \simeq 1 + x/2 - x^2/8$ with $x = 4m_D^2/M^2$: $\lambda_- \simeq -m_D^2/M$ and $\lambda_+ \simeq M + m_D^2/M$, and the mixing angle is $\vartheta \simeq m_D/M$. Step 3: check the invariant — $\det \mathcal{M} = -m_D^2$ exactly, so $\lambda_+\lambda_- = -m_D^2$ at every order, which is the seesaw identity itself. Physics intuition: the determinant is the thing that cannot move. Whatever you do to the heavy mass, the product of the two eigenvalues is pinned to m_D^2, so the light state descends by exactly the factor the heavy state ascends. Notice also that the light eigenvalue is *negative*: that minus sign is not physical by itself, it is absorbed into a Majorana phase — your first sighting of a Majorana phase as a bookkeeping sign rather than a free angle.

K.1.4. Derive the phase counting at general n, and find where the two cases first differ.
Solution. Step 1: an $n \times n$ unitary matrix has n^2 real parameters, $n(n-1)/2$ of them angles, leaving $n(n+1)/2$ phases. Step 2: Dirac — $2n$ fields absorb phases but one overall phase is inert, so $2n - 1$ are removable and $n(n+1)/2 - (2n-1) = (n-1)(n-2)/2$ survive. Majorana — only the n charged leptons may be rephased, leaving $n(n-1)/2$. Step 3: at $n = 2$ the counts are 0 and 1: two generations of Majorana neutrinos already admit one physical phase, while two generations of Dirac fermions admit none. Physics intuition: this is a sharper statement than the familiar "three families are needed for CP violation", which is a *Dirac* statement. Majorana neutrinos violate CP with two families — there is simply no oscillation experiment that can see it, because the phase lives in the sector oscillations cancel. The Kobayashi–Maskawa counting and the Majorana counting are different theorems and a student should never conflate them.

K.1.5. Show that the Majorana phases cancel in every oscillation probability.
Solution. Step 1: write $U = \tilde{U}P$ with $P = \mathrm{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2})$ as in (K.7). Step 2: every term of (J.7) is a product $U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}$, in which index i appears once conjugated and once not, and likewise j: each phase $e^{i\alpha_i/2}$ meets its own conjugate and cancels. Step 3: hence *no* oscillation experiment, at any baseline, in any channel, can ever measure α_{21} or α_{31} — this is a theorem, not a sensitivity statement. Physics intuition: oscillations measure the neutrino's *propagation*, and propagation cannot tell a particle from an antiparticle if the two are the same object. The phases

that record "the same object" are therefore invisible to it by construction. Only a process that converts a neutrino into an antineutrino — that is, one violating lepton number — can see them, which is why $0\nu\beta\beta$ is not merely the best probe but essentially the only one.

K.1.6. Derive $m_{\beta\beta}$ from the propagator, and explain the square.

Solution. Step 1: the two vertices of (K.8) each emit a ν (not a $\bar{\nu}$), so the internal line must convert one into the other — possible only for a Majorana propagator, and only through its mass term. Step 2: with $|p| \sim 100$ MeV $\gg m_i$ the chirality projectors kill the $\not{p}$ piece and leave m_i/p^2 (K.9); each vertex contributes U_{ei} with *no* conjugation, since neither is an absorption. Step 3: summing over i gives (K.10), with U_{ei}^2 and not $|U_{ei}|^2$ — which is exactly why the Majorana phases survive here and nowhere else. Physics intuition: the whole of $0\nu\beta\beta$ phenomenology comes from one chirality argument. The exchanged neutrino must flip helicity to be absorbed at the second vertex, the only helicity-flipping object in the propagator is the mass, and so the amplitude is proportional to mass — which is why a process that would be spectacular if neutrinos were heavy is agonisingly rare because they are light.

K.1.7. Compute the inverted-ordering floor, and explain why normal ordering has none.

Solution. Step 1: for inverted ordering with $m_3 \approx 0$, the two heavy states are nearly degenerate at $m \approx \sqrt{|\Delta m^2_{31}|} = 50.1$ meV, and (K.10) reduces to $m_{\beta\beta} \approx m\, c_{13}^2 |c_{12}^2 + s_{12}^2 e^{i\alpha_{21}}|$. Step 2: the worst case is $\alpha_{21} = \pi$, giving ${m_{\beta\beta}}^{\min} \approx m\, c_{13}^2 |c_{12}^2 - s_{12}^2| = m\, c_{13}^2 \cos 2\theta_{12} = 50.1 \times 0.978 \times 0.384 = 19$ meV. Step 3: for *normal* ordering the dominant term is $m_2 s_{12}^2 c_{13}^2$ and the two others can, at a lightest mass of a few meV, be arranged to cancel it exactly — so the normal band passes through zero and there is no floor. Physics intuition: the floor exists because in inverted ordering the two big contributions are weighted by c_{12}^2 and s_{12}^2, which are 0.69 and 0.31 — unequal, so they cannot cancel. Had θ_{12} been maximal, the inverted floor would have vanished too and the entire strategic case for tonne-scale experiments with it. The solar angle's being far from 45° is what makes this a decidable question.

K.1.8. Price the exposure: what a factor of five in $m_{\beta\beta}$ costs.

Solution. Step 1: the rate (K.11) goes as ${m_{\beta\beta}}^2$, so half-life reach goes as ${m_{\beta\beta}}^{-2}$: a factor f in mass reach is f^2 in half-life. Step 2: in the background-limited regime the half-life reach grows as $\sqrt{Mt/(b\,\Delta E)}$, so f^2 in half-life is f^4 in exposure. Step 3: from 9 meV to 1.8 meV is $f = 5$: 25 in half-life and 625 in exposure. Physics intuition: this quartic is the hardest fact in the subject and it is worth carrying around. It means a meV-scale $m_{\beta\beta}$ is not reachable by scaling the current program, and that the only routes are a zero-background technology — where reach grows linearly in exposure rather than as its square root, changing f^4 to f^2 — or a different isotope with a much larger matrix element. A prediction sitting at 1.8 meV is therefore a prediction about what the next generation will *not* see, and honest reporting says so rather than quoting the number as though it were around the corner.

K.2.1. Show that the Casas–Ibarra freedom breaks the link between $\delta_{\rm PMNS}$ and the baryon asymmetry.

Solution. Step 1: the CP asymmetry of leptogenesis is built from $Y_\nu{}^\dagger Y_\nu$, which by (J.15) equals $U\sqrt{\widehat{m}}\,R^\dagger \widehat{M}_N R\sqrt{\widehat{m}}\,U^\dagger$ in the relevant combination. Step 2: the complex angles of R enter this expression and the low-energy phases in U do not enter it alone, so a shift in $\delta_{\rm PMNS}$ can be exactly compensated by a shift in R. Step 3: hence in the general type-I seesaw no measurement of $\delta_{\rm PMNS}$ constrains the sign or magnitude of the baryon asymmetry. Physics intuition: this is the reason the frequently heard slogan "measuring leptonic CP violation would confirm leptogenesis" is false as stated. It would confirm that *a* leptonic CP phase exists, which is necessary but very far from sufficient. Only a framework that supplies R and $\widehat{M}_N$ from elsewhere can make the link, and the next chapter's central claim is precisely that the geometry supplies both.

K.2.2. Explain why the nuclear matrix element is the largest uncertainty, and what that does to a limit.

Solution. Step 1: from (K.11), $m_{\beta\beta} \propto 1/|M^{0\nu}|$ at fixed half-life. Step 2: many-body methods — shell model, QRPA, energy-density functional — disagree by a factor of two to three on $M^{0\nu}$ for the same isotope, so a single measured half-life limit maps onto an $m_{\beta\beta}$ limit spanning that same factor: this is why KamLAND-Zen's 3.8×10^{26} yr is quoted as a *range* of tens of meV rather than a number. Step 3: consequence for falsification — a criterion written against $m_{\beta\beta}$ inherits the spread, so a well-posed kill row must be stated either against the half-life directly or against the full quoted range. Physics intuition: the bottleneck in the most sensitive lepton-number experiment ever built is not the detector, it is a nuclear-structure calculation. A student who wants to make the largest possible difference to this field should consider that the highest-leverage work in it may be many-body nuclear theory rather than instrumentation.

K.3.1. Derive the parity pair from rank two, and count what is *not* free.

Solution. Step 1: a rank-two symmetric form on the three-family space has a one-dimensional kernel, so its eigenvalue structure admits relative *signs* between eigenvalues but no continuous relative phases — a continuum would require rank three. Step 2: hence $(\alpha_{21},\alpha_{31})$ takes values in a finite set of parity assignments, and the ledger selects (K.12), $(0,\pi)$. Step 3: count what has been removed: a generic Majorana model has a two-torus of $(\alpha_{21},\alpha_{31})$ and therefore a factor-of-2.6 envelope in $m_{\beta\beta}$ (problem K.3.4); here there is no envelope at all, only a discrete choice already made. Physics intuition: the difference between a parameter and a parity is the difference between a theory that can be adjusted after the measurement and one that cannot. A discrete prediction can be wrong in a way a continuous one cannot — and it is worth noticing that this discreteness is what makes the $\mathrm{m}_{\beta\beta}$ result of problem K.3.4 uncomfortable rather than convenient.

K.3.2. Cash the half-angle: show that a parity of π is a factor of i.

Solution. Step 1: the Majorana phases enter (K.7) halved, because the mass term is quadratic in the field while the mixing matrix is linear. Step 2: therefore $\alpha_{31}=\pi$ appears in U as $e^{i\pi/2}=i$ (K.13) — a quarter turn, not a sign flip. Step 3: in $m_{\beta\beta}$, which is quadratic in U, the i squares to -1 and the third term's contribution reverses (that is the -0.90307 of (K.16)); in the CP asymmetry of leptogenesis, which is built from an *interference* linear in the relative phase, the same parity supplies a nonzero imaginary part where a parity of 0 would have supplied none. Physics intuition: one discrete assignment, two completely different-looking consequences, because the two observables are different powers of the same matrix. This is the cleanest illustration in the paper of why the phases

had to be derived once rather than fitted twice — a framework that fitted α_{31} to $m_{\beta\beta}$ would have had nothing left to say about the baryon asymmetry.

K.3.3. Score the derived Dirac phase, and say what would kill it.

Solution. Step 1: the closed form (K.14) is $\delta_{\rm PMNS} = \pi + 2\arcsin(1/7)$; evaluate: $\arcsin(1/7) = 8.2132°$, so $\delta_{\rm PMNS} = 180 + 16.4266 = 196.4264°$, with the QD value 196.0591°. Step 2: against the T2K–NOvA joint fit $203.4^{+63.0}_{-37.8}$ (T2K and NOvA Collaborations, 2025) the prediction sits 0.11σ from the central value — but the interval is a hundred degrees wide, so this is agreement with a very weak measurement and should be reported as such. Step 3: F-SM-3d states the kill row: a 5σ determination outside $[186°, 207°]$ falsifies it, and DUNE and Hyper-K are the facilities. Physics intuition: notice what the closed form does *not* have — a free parameter. The same $J = 7$ that tilts the atmospheric angle in (J.22) sets the residual here, so the two cannot be adjusted independently; a measurement of either constrains the other. That coupling is the prediction's real content, and it is more falsifiable than the number itself.

K.3.4. Work $m_{\beta\beta}$ term by term, then scan the phases and locate the derived value in its own envelope.

Solution. Step 1: at first order $m_1 = 0$, so (K.10) has two terms. With $m_2 = 8.36114$ meV, $s^2_{12} = 25/81$, $c^2_{13} = 137/140$ and $\alpha_{21} = 0$: the second term is $+2.52530$ meV (K.15). With $m_3 s^2_{13} = 1.07500$ meV and phase $\pi - 2(196.426421°)$: the third is $-0.90307 + 0.58317i$ meV (K.16), so $m_{\beta\beta} = |1.62223 + 0.58317i| = 1.72386$ meV. Step 2: now scan. On the QD spectrum the three moduli are 0.04416, 2.59327 and 1.10634 meV, so free phases give $m_{\beta\beta} \in [1.4428, 3.7438]$ meV, and the other admissible parity $\alpha_{31} = 0$ gives 3.6225 meV. Step 3: the derived 1.79925 meV sits at $(1.799 - 1.443)/(3.744 - 1.443) = 15\%$ of the way up its own envelope — the lowest quarter, near maximal destructive interference. Physics intuition: state the uncomfortable version plainly. A framework free to choose α_{31} would have put its prediction at 3.6 meV, twice as large and twice as findable, and would have been quoting a number it had fitted. This one is stuck at the bottom of the range because the parity is derived, and the honest report is that the derivation has made the prediction *harder* to confirm. That is what it costs to have no dials.

K.3.5. Compare the two unitarity triangles, and say exactly what the comparison does not prove.

Solution. Step 1: this paper derives both Jarlskog invariants, so the two triangle areas can be compared at true relative scale: $|J_{\rm PMNS}|/|J_{\rm CKM}| = 9.187 \times 10^{-3}/2.984 \times 10^{-5} = 308$ at QD order, 338 at first order (K.18). Step 2: the structural reading — hierarchical-against-hierarchical gives a thin triangle, dyadic-against-democratic a fat one; same constitution, different projector geometry. Step 3: the caveat. It is often said that quark CP violation falls short of the observed baryon asymmetry by ten orders of magnitude and that the lepton sector must therefore supply it. Three hundred is not ten orders of magnitude, so (K.18) does *not* close that gap — what closes it is the $\Delta L = 2$ door and the out-of-equilibrium lock crossing, and the next chapter finds the unflavored invariant $J_{\rm PMNS}$ belongs to drops out of the asymmetry entirely. Physics intuition: a factor of three hundred is a striking prediction about flavor geometry and a completely inadequate explanation of cosmology, and it is worth being able to hold both statements at once. The temptation to let

the first stand in for the second is exactly the kind of overclaim a course should teach a student to catch — including in the framework the course is teaching.

K.3.6. Check that the three phases are not independently adjustable.

Solution. Step 1: list what each phase is built from. α_{21} and α_{31} are the parity data of the rank-two ledger (K.12) — a discrete assignment with no continuous freedom. $\delta_{\rm PMNS}$ is $\pi + 2\arcsin(1/J)$ with $J = 7$ (K.14). Step 2: the π in $\delta_{\rm PMNS}$ is the *same* parity that sets α_{31}, and the J in its residual is the *same* integer that tilts $\sin^2\theta_{23} = \frac{1}{2}(1 + 1/J)$. Step 3: hence a measurement of the atmospheric octant constrains $\delta_{\rm PMNS}$, and the parity that fixes $m_{\beta\beta}$ fixes the base of $\delta_{\rm PMNS}$ — three phases and one angle from two ledger inputs. Physics intuition: this interlocking is the difference between a derivation and a parametrization, and it is where the framework is most exposed. Four measurements now have to agree with each other in a specific pattern, and any one of them landing outside its window takes the other three with it. A student assessing the framework should count the constraints, not the successes.

K.3.7. Audit Step 2: which items close and which are not ours to close?

Solution. Step 1: items 1–4 and 7 close — the neutrino is Majorana by construction, all three phases are derived, $m_{\beta\beta}$ follows with no envelope, and the largeness of leptonic CP violation relative to the quark sector is the triangle ratio. Step 2: item 8 is answered in the next chapter and answered *conditionally*. Step 3: items 5 and 6 — whether light-neutrino exchange dominates the rate, and the nuclear matrix elements — are not answered and are not this framework's to answer; they belong to the lepton-number-violation and nuclear-structure literatures, and the second is currently the dominant uncertainty on every published $m_{\beta\beta}$ limit. Physics intuition: a framework should be as clear about what is outside its scope as about what is inside it. Claiming $m_{\beta\beta} = 1.80$ meV while staying silent about a factor-of-three nuclear uncertainty would make the prediction look sharper than the experiment can ever be; saying both keeps the comparison honest.

K.3.8. Reconcile the three lepton observables on one spectrum.

Solution. Step 1: the same derived spectrum gives $\Sigma m_\nu = 59.17$ meV (QD), $m_\beta = 8.66$ meV and $m_{\beta\beta} = 1.80$ meV — a factor of thirty-three from top to bottom. Step 2: check the ordering makes sense. Σm_ν is an unweighted sum, so it is the largest; m_β weights by $|U_{ei}|^2$ and takes a square root, throwing away most of m_3; $m_{\beta\beta}$ weights by U_{ei}^2 *with phases* and suffers the destructive interference of problem K.3.4. Step 3: therefore the experimental prospects are inverted relative to intuition — the largest number sits at 92% of the DESI bound and could fire within a few years, the middle one is twenty-three times below KATRIN's design reach, and the smallest is 625 exposure-units beyond the next generation. Physics intuition: one spectrum, three observables, three completely different experimental verdicts, and the reason is entirely in how each observable weights the same three numbers. If a student takes one thing from this chapter, it should be that "the neutrino mass" is not a quantity — there are at least three of them, they are not equal, and which one an experiment measures determines everything about whether it can see this framework at all.

Take-away. *Standard result:* a neutral fermion may be its own antiparticle; if it is, two extra phases become physical and $0\nu\beta\beta$ becomes possible with rate $\propto {m_{\beta\beta}}^2$. *What it assumed:* the Majorana nature itself, all three phases, and hence $m_{\beta\beta}$ to within a factor of nearly three. *The geometric reading:* the rank-two ledger makes the Majorana phases *parities* rather than dials, fixing $(0, \pi)$; the half-angle turns that parity into a factor of i; the Dirac phase closes as $\pi + 2\arcsin(1/7)$ on the same integer that tilts the atmospheric angle. *What it costs:* the derived parity puts $m_{\beta\beta}$ at 1.80 meV — the lowest quarter of its own envelope, and 625 exposure-units beyond the approved program. Derived, and harder to confirm for being derived.

L Leptogenesis and the origin of the baryon asymmetry

What this appendix teaches. The chapter in which the lepton sector meets cosmology. Step 1 measures η_B two independent ways, states Sakharov's three conditions and scores the Standard Model against each, derives the $B + L$ anomaly and the sphaleron conversion factor 28/79 from chemical equilibrium, and then builds thermal leptogenesis from the one-loop CP asymmetry through the Davidson–Ibarra bound to the Boltzmann system. Step 2 lists eight quantities the standard picture must choose. Step 3 gives the lock-scale construction, in which every one of the eight descends — and states, at every site and in the takeaway, that the whole chain rests on one open transport condition. *This is the most conditional chapter in the course and it is written to be read that way.*

Read alongside: Davidson et al. (2008) for the whole subject at review depth; Buchmüller et al. (2005) for the efficiency factors; Kolb & Turner (1990) Ch. 3–5 for the thermodynamics; Weinberg (2008) Ch. 3 for what η_B is as a measurement.

This is the seventh sector appendix and the last of the three lepton chapters. Its subject is a single number — the ratio of baryons to photons in the universe — and the claim that the same geometry which fixed the neutrino block in App. J and the Majorana parities in App. K also writes it.

A warning before the chapter starts, because it governs how everything in it should be read. The two previous chapters derived quantities and scored them; this one derives a quantity and scores it *subject to a hypothesis that is not proved.* The hypothesis is named (H-ϕ_{geo}-1), its six conditions are tabulated in the main text (Table 45), and the crux — condition C4 — is **open at the derivation level**. Everything below is stated conditionally, and a reader who finds an unconditional claim anywhere in this chapter has found an error.

L.1 Step 1 — baryogenesis and leptogenesis as a graduate student meets them

L.1.1 The number, measured twice

The baryon asymmetry is quoted as

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.12 \pm 0.04) \times 10^{-10}, \tag{L.1}$$

and its credibility rests on being measured two entirely different ways.

From primordial nucleosynthesis. The light-element abundances produced in the first three minutes depend on the baryon density through the nuclear reaction rates; deuterium is the sensitive

one, its abundance falling roughly as ${\eta_B}^{-1.6}$. A measured D/H in a metal-poor absorption system therefore reads η_B at a redshift of 10^9.

From the microwave background. The acoustic peak heights depend on the baryon-to-photon ratio through the photon–baryon fluid's inertia; Planck measures $\Omega_b h^2 = 0.02237$, and

$$\eta_{10} \equiv 10^{10}\eta_B = 273.9\,\Omega_b h^2 = 6.13, \tag{L.2}$$

reading η_B at a redshift of 10^3.

The two agree. That agreement is one of the strongest quantitative results in cosmology, and it is what makes η_B a target worth predicting rather than a fitted parameter. *(Physics intuition: what does* 6×10^{-10} *mean physically? Before the quark–antiquark annihilation there was roughly one quark per photon and one antiquark per photon. Nearly all of them annihilated. For every* 1.63×10^9 *antiquarks there were* $1.63 \times 10^9 + 1$ *quarks, and every atom in every star and every person is built from that one survivor per billion and a half. The question of baryogenesis is the question of where the "+1" came from — and note that it is a question about a* tiny *number, which is why it is a question about a small effect operating on an enormous population rather than about a large effect.)*

L.1.2 Sakharov's three conditions, and the Standard Model's score

Sakharov (1967) observed that a universe starting baryon-symmetric can end asymmetric only if three conditions hold.

(1) Baryon-number violation. *Satisfied* in the Standard Model, and this surprises most students: B is violated non-perturbatively (§L.1.3).

(2) C and CP violation. *Satisfied but far too small.* The Standard Model's only source is the CKM phase, and every CP-violating quark observable is proportional to the Jarlskog invariant times the mass-splitting Jacobian, which at electroweak temperatures suppresses the effect to roughly 10^{-20} — ten orders of magnitude short of (L.1).

(3) Departure from thermal equilibrium. *Not satisfied.* In equilibrium CPT guarantees equal number densities for particle and antiparticle, so any asymmetry generated is washed out. The Standard Model's candidate is the electroweak phase transition — but for the measured Higgs mass of 125 GeV the transition is a smooth *crossover*, not first order, and provides no departure at all.

Two of the three legs fail, and each failure is quantitative and well established. *Baryogenesis therefore requires physics beyond the Standard Model — this is not a preference, it is a theorem-strength conclusion from a measured Higgs mass and a measured Jarlskog invariant.*

L.1.3 The anomaly and the sphaleron

Classically B and L are separately conserved accidental symmetries. Quantum mechanically the corresponding currents are anomalous (Adler, 1969; Bell & Jackiw, 1969; 't Hooft, 1976):

$$\partial_\mu j^\mu_B = \partial_\mu j^\mu_L = \frac{N_f}{32\pi^2}\Big(g^2 W^a_{\mu\nu}\widetilde{W}^{a\,\mu\nu} - g'^2 B_{\mu\nu}\widetilde{B}^{\mu\nu}\Big). \tag{L.3}$$

The two anomalies are *identical*, so

$$\partial_\mu j^\mu_{B-L} = 0, \qquad \partial_\mu j^\mu_{B+L} \neq 0\,: \tag{L.4}$$

$B-L$ is exactly conserved and $B+L$ is not. A topological transition changing the Chern–Simons number by one unit changes B and L by $N_f = 3$ each.

At zero temperature the transition proceeds by tunneling and is suppressed by $e^{-8\pi^2/g^2} \sim 10^{-170}$ — utterly negligible. At high temperature it proceeds by thermal activation over the barrier, whose top is the *sphaleron* (Manton, 1979; Klinkhamer & Manton, 1984), a static saddle-point configuration of energy $E_{\rm sph} \simeq (4\pi v/g)B(\lambda/g^2) \simeq 9$ TeV at $T=0$. Above the electroweak crossover the barrier vanishes and the rate is unsuppressed (Kuzmin et al., 1985); sphalerons freeze out at $T_{\rm sph} \simeq 132$ GeV. *(Physics intuition: the vacuum of a non-Abelian gauge theory is not a point but a periodic landscape, and each step between neighboring minima drags three units of baryon number and three of lepton number along with it. Baryon number is not conserved in the Standard Model; it is merely* very difficult *to change at zero temperature and completely free to change at a few hundred GeV. The entire strategy of leptogenesis is to exploit that freedom: make an asymmetry in L, which is easy, and let the sphalerons convert part of it into B, which is free.)*

The conversion factor, derived. If $B-L$ is conserved and sphalerons are in equilibrium, chemical-equilibrium bookkeeping over all Standard-Model species fixes the fraction of $B-L$ that ends up in B (Harvey & Turner, 1990):

$$\boxed{B = \frac{8N_f + 4N_H}{22N_f + 13N_H}\,(B-L) = \frac{28}{79}\,(B-L) = 0.354430\,(B-L)} \tag{L.5}$$

for $N_f = 3$ families and $N_H = 1$ Higgs doublet. Problem L.1.4 works the counting. This factor is exact, model-independent given the field content, and one of the six assembly factors of §L.3.7.

L.1.4 Thermal leptogenesis

Fukugita & Yanagida (1986) observed that the seesaw's heavy Majorana neutrinos supply all three Sakharov conditions at once. Add

$$\mathcal{L} \supset -Y_{\nu\alpha i}\,\bar{L}_\alpha \tilde{H} N_i - \tfrac{1}{2} M_i\,\overline{N^c_i} N_i + \text{h.c.} \tag{L.6}$$

The Majorana mass violates L by two units — Sakharov (1), through $B-L$; the complex Yukawa matrix violates CP — Sakharov (2); and the heavy states decay as the universe cools below their mass, so their abundance falls out of equilibrium — Sakharov (3).

The CP asymmetry. The decay asymmetry

$$\epsilon_i \equiv \frac{\Gamma(N_i \to LH) - \Gamma(N_i \to \bar{L}\bar{H})}{\Gamma(N_i \to LH) + \Gamma(N_i \to \bar{L}\bar{H})} \tag{L.7}$$

vanishes at tree level and arises at one loop from the interference of the tree amplitude with the *vertex* and *self-energy* corrections. For hierarchical heavy states,

$$\epsilon_1 = \frac{1}{8\pi}\frac{1}{(Y_\nu^\dagger Y_\nu)_{11}} \sum_{j\neq 1} \mathrm{Im}\Big[\big((Y_\nu^\dagger Y_\nu)_{1j}\big)^2\Big]\, f\!\left(\frac{M_j^2}{M_1^2}\right), \tag{L.8}$$

with f the sum of the two loop functions. Two features matter. The asymmetry needs the *square* of an off-diagonal element, so it vanishes if $Y_\nu^\dagger Y_\nu$ is real — a phase is essential, as Sakharov requires. And it requires at least two heavy states: with one N there is no $j \neq 1$ and no interference.

The Davidson–Ibarra wall. Davidson & Ibarra (2002) proved that in the hierarchical regime (L.8) is bounded:

$$|\epsilon_1| \;\leq\; \frac{3}{16\pi}\,\frac{M_1\,(m_3 - m_1)}{v^2}, \qquad v = 174\ \text{GeV}. \tag{L.9}$$

This is one of the most consequential inequalities in the subject, because it converts an observed asymmetry into a *lower bound on the heavy scale*. Problem L.1.6 runs it: reproducing (L.1) at the most favorable efficiency requires $M_1 \gtrsim 6\times 10^8$ GeV, quoted in the literature as $\sim 10^9$ GeV. *The consequence is that standard thermal leptogenesis is untestable*: it demands a reheating temperature above 10^9 GeV, a heavy sector nine orders of magnitude beyond any collider, and it predicts nothing measurable at low energy.

Escaping the wall: resonance. Equation (L.9) assumes hierarchical heavy states. If two are nearly degenerate the self-energy contribution is resonantly enhanced (Pilaftsis, 1997; Pilaftsis & Underwood, 2004): the loop function develops a Breit–Wigner factor

$$\epsilon^{\text{res}} \;\propto\; \frac{\Delta M\,\Gamma}{\Delta M^2 + \Gamma^2/4}, \tag{L.10}$$

maximized at $\Delta M = \Gamma/2$, where the Yukawa suppression cancels and ϵ can approach order unity. Resonant leptogenesis can therefore operate at the TeV scale — *at the price of a degeneracy that must be arranged*, and the arrangement is the standard picture's least attractive feature.

L.1.5 The Boltzmann system, washout, and efficiency

Producing an asymmetry is not enough; it must survive. The competing processes are inverse decay $LH \to N$ and $\Delta L = 2$ scattering, both of which erase what the decays make. Their strength

relative to the expansion is the *washout parameter*

$$K_{\rm wash} = \frac{\Gamma_N}{H(T=M_N)} = \frac{\widetilde{m}}{m_\star}, \qquad \widetilde{m} \equiv \frac{(Y_\nu^\dagger Y_\nu)_{11} v^2}{M_1}, \quad m_\star \simeq 1.07 \times 10^{-3}\ \text{eV}, \tag{L.11}$$

where $m_\star$ is the "equilibrium neutrino mass" — the value at which decay and expansion rates coincide. The surviving fraction is the *efficiency* κ, obtained by integrating the coupled Boltzmann equations for Y_N and Y_{B-L} in the variable $z = M_1/T$.

The two regimes are worth knowing by heart. *Weak washout* ($K_{\rm wash} \ll 1$): $\kappa \to 1$, but the result depends on the unknown initial N abundance. *Strong washout* ($K_{\rm wash} \gg 1$): $\kappa \sim 1/K_{\rm wash}$ up to logarithms, and the initial condition is *erased* — the result is independent of how the universe started. Strong washout is the theoretically preferred regime for exactly that reason: it trades efficiency for predictivity. *(Physics intuition: washout is not a nuisance, it is the mechanism that makes leptogenesis a prediction rather than an initial condition. A strongly washed-out scenario forgets everything about its own beginning and remembers only its couplings, which is precisely the property one wants of an explanation.)*

L.1.6 Flavor

Below about 10^{12} GeV the tau Yukawa interaction is fast enough to bring the tau flavor into equilibrium, and below 10^9 GeV the muon too. When that happens the lepton doublet produced in an N decay is no longer a coherent superposition but a *measured* flavor state, and the Boltzmann system must be written flavor by flavor: each flavor carries its own asymmetry ϵ_α and suffers its own washout K_α. The consequence is important and non-obvious: the *sum* $\sum_\alpha \epsilon_\alpha$ may vanish while the individual ϵ_α do not, and a flavor with weak washout can then carry an asymmetry that the unflavored treatment would have reported as zero. Step 3 uses exactly this.

Worked example (STANDARD TEXTBOOK PEDAGOGY). Assemble the standard estimate. The heavy states are relativistic and in equilibrium before they decay, so their abundance per unit entropy is $Y_N^{\rm eq} \simeq 0.42/g_*$ per Majorana species. Each decay deposits ϵ_1 of asymmetry in $B-L$, a fraction κ survives washout, a fraction $28/79$ is converted to B, and converting from entropy to photons multiplies by $s/n_\gamma = 7.04$:

$$\eta_B \simeq \frac{s}{n_\gamma}\, c_{\rm sph}\, Y_N^{\rm eq}\, \epsilon_1\, \kappa. \tag{L.12}$$

With $g_* = 106.75$ and one species, the prefactor is $7.04 \times 0.3544 \times 0.42/106.75 = 9.8 \times 10^{-3}$, so reproducing 6.12×10^{-10} needs $\epsilon_1 \kappa \simeq 6 \times 10^{-8}$. That is the whole of standard leptogenesis in one line, and every model in the literature is an account of how its ϵ_1 and κ multiply to 6×10^{-8}.

L.2 Step 2 — what leptogenesis assumes but cannot explain

1. **The heavy mass scale M_N.** Free above the Davidson–Ibarra wall, and therefore free over seven orders of magnitude.

2. **The mass splitting ΔM.** Free. In resonant models it must be tuned to $\Gamma/2$, a degeneracy of order 10^{-10} arranged by hand.
3. **The Dirac Yukawa Y_ν.** Free — the six real parameters of the Casas–Ibarra matrix R, invisible to all low-energy data (problem J.1.8).
4. **The CP phases entering ϵ_i.** Free, and by problem K.2.1 not fixed by $\delta_{\rm PMNS}$.
5. **The reheating temperature.** Free, and constrained from above by the gravitino problem in supersymmetric completions.
6. **The initial heavy-neutrino abundance.** Free, unless the washout is strong enough to erase it.
7. **Which flavor regime applies.** Determined by M_N, which is free.
8. **The sign.** No standard model of leptogenesis predicts whether the universe ends up made of matter or antimatter; the sign follows the sign of an unmeasured phase.

The honest summary is that thermal leptogenesis is an *existence proof* — it demonstrates that the seesaw can produce an asymmetry of the right order — rather than a prediction. Item 8 is the sharpest illustration: a mechanism that cannot say whether we are made of matter has explained less than it appears to.

L.3 Step 3 — the geometric derivation: the door at the lock

L.3.1 The heavy scale, not chosen

The framework has no freedom in item 1. The one scale available between the electroweak seam and the grand-unification seam is the OGN-5 lock, and it is fixed by the ledger integer $K = 35$:

$$M_{\rm lock} = \tfrac{1}{2}\, E_{\rm Pl}\, e^{-K} = \tfrac{1}{2}\, E_{\rm Pl}\, e^{-35} = 3.85 \text{ TeV}, \qquad \Delta M = \frac{M_{\rm lock}}{K} = 110 \text{ GeV}. \tag{L.13}$$

Both numbers are outputs of a ledger fixed upstream by the gauge sector, and neither was adjusted for this chapter. The splitting is not tuned — it is $M_{\rm lock}/K$, the same K that sets the scale.

L.3.2 The selection rule that permits this at all

One structural point separates this framework from every other leptogenesis scenario and belongs before the mechanism rather than after it. In the Standard Model, sphalerons are simply available. Here they are the *only* thing available, and narrowly so.

The bulk $\mathrm{U}(1)_B$ is gauged and its two non-vanishing mixed anomaly coefficients are absorbed by the BI-6 Green–Schwarz inflow; the vector's zero mode is removed by its boundary termination and an exact *global* baryon number survives on the boundary. Every local operator with $\Delta B \notin 3\mathbb{Z}$ therefore has identically vanishing coefficient, to all orders: $\Delta B = 1$ (proton decay) and $\Delta B = 2$ (neutron–antineutron oscillation) are forbidden outright, which is the $\tau_p = \infty$ theorem of §109.6. What survives is not an operator at all but a topological transition — the $\mathrm{SU}(2)_L$ vertex at

$\Delta B = N_{\text{gen}} n = 3n$, weighted $e^{-4\pi/\alpha_W} \approx 10^{-162}$ at zero temperature and unsuppressed above the electroweak crossover.

Two consequences follow, and they are the same consequence read at two temperatures.

No $\Delta B = \pm 1$ mechanism exists here. Any baryogenesis scenario that mints baryon number directly — GUT baryogenesis, post-sphaleron $\Delta B = 2$ baryogenesis — is not suppressed in this framework; it is unwritable. The asymmetry must be minted in a currency the selection rule conserves, which means in the lepton sector, as $B - L$, and converted at the door.

So leptogenesis is not chosen; it is forced. That is a stronger statement than any efficiency argument, and it is worth holding beside the chain's conditionality: *the class* of mechanism is forced by the selection rule, while *this instance* of it rests on C4. If the lock-scale chain fails, the class survives and some other leptogenesis must be found; if the class fails — if baryogenesis were ever shown to require $\Delta B = \pm 1$ minting — the framework's proton theorem and its account of matter become mutually inconsistent, and it dies of the contradiction.

(Physics intuition: the same equation read at two temperatures. Cold, the door is welded and the proton is immortal; hot, the door swings and the baryons get made. A framework in which protons decay would have had an easier time making them, and a framework in which baryon number were exactly conserved with no exception could not have made them at all. This one threads the gap with a rule that admits multiples of three and nothing else — and that rule was written for the proton, long before anyone asked it about cosmology.)

L.3.3 The seam Yukawa, and the width

Item 3 closes the same way. The Dirac Yukawa is not free because the Majorana channel is the seam operator of App. J, so

$$Y_\nu{}^2 = \pi \, \frac{M_{\text{lock}}}{E_{\text{GUT}}} = 5.303 \times 10^{-13}, \qquad Y_\nu = 7.28 \times 10^{-7}, \tag{L.14}$$

whence the decay width

$$\Gamma_N = \frac{(\sqrt{2}\, Y_\nu)^2 M_{\text{lock}}}{8\pi} = \frac{Y_\nu{}^2 M_{\text{lock}}}{4\pi} = 1.62 \times 10^{-10} \text{ GeV}, \tag{L.15}$$

And here is the difficulty, stated before the resolution. Comparing with the derived splitting,

$$\frac{\Delta M}{\Gamma_N} = \frac{110 \text{ GeV}}{1.62 \times 10^{-10} \text{ GeV}} = 6.8 \times 10^{11}, \tag{L.16}$$

whereas the resonance (L.10) needs this ratio near *unity*. The framework's derived numbers miss the resonant condition by twelve orders of magnitude, and *the chapter says so before it says anything else*. Everything that follows is an argument that the relevant width is not Γ_N.

L.3.4 The regulator question, and the open condition

The claim — and it is a claim, not a derivation — is that the width appearing in the Breit–Wigner denominator is not the vacuum decay width but an *effective* width set by the thermal bath's

flavor-resolving interactions, dominated by the tau Yukawa. That gives the closed form

$$\frac{\Delta M}{\Gamma^{\rm eff}} = \frac{1}{K\, c_\tau\, y_\tau^2} = (2.7\text{–}5.3)\times 10^4, \qquad y_\tau(M_{\rm lock}) = 0.0104, \tag{L.17}$$

with $c_\tau \in [5, 10] \times 10^{-3}$ the one order-unity coefficient the argument does not fix. Table 44 of the main text compares five candidate regulators spanning sixteen orders of magnitude, drawn in Fig. 19; exactly one lands in the required decade.

The six conditions, and which one is open. The identification is packaged as the named hypothesis H-$\phi_{\rm geo}$-1 with conditions C1–C6 (Table 45). Five are argued. **C4 — that the thermal bath resolves the two heavy states as a flavor measurement on the timescale the resonance requires — is not derived**, and closing it demands a full Kadanoff–Baym resummation that nobody has performed. Its home is GG-A10-Cosmos Appendix I; GG-P5-CPT carries the phase routing.

This is the single most valuable calculation anyone could do to this chain, in either direction, and the framework's own falsification channel F-SM-7 is written around it: unlike every other channel in the paper, F-SM-7 waits on a calculation rather than on a facility, and could be settled today.

L.3.5 The CP asymmetry: flavored, with the unflavored part vanishing

Items 4 and 8 close through the parities of App. K. Two results are structural.

The flavor sum vanishes exactly. On the derived (real Casas–Ibarra) texture the trace of the asymmetry matrix is zero, $\mathrm{Tr}\, S = 0$, so the *unflavored* CP asymmetry — the one built from $J_{\rm PMNS}$ — is identically zero. The matter asymmetry is carried entirely by the differences between flavors. This is why App. K's factor of three hundred between the two unitarity triangles, striking as it is, is *not* the reason the lepton sector can pay for baryogenesis: the invariant it measures drops out.

The sign comes from the half-angle. With $\alpha_{31} = \pi$, the mixing matrix carries $e^{i\alpha_{31}/2} = i$ (K.13), and it is that factor of i — linear in U, so a quarter turn rather than a sign flip — that supplies the imaginary part (L.8) needs. A parity of zero would have given none. *The derived Majorana parity therefore determines the sign of the matter asymmetry*, which is item 8, and no standard model of leptogenesis can say the same.

L.3.6 Washout, flavor by flavor: the electron door

Computed from the derived masses and the derived PMNS matrix, the flavored washouts are

$$K_\alpha = (3.5,\ 29.3,\ 22.5) \quad \text{for } (e,\mu,\tau), \qquad K_3 = 2N\, K_{\rm seam} = 47 \quad (K_{\rm seam} = 7.80,\ K_{\rm tot} = 54.6), \tag{L.18}$$

against $m_\star^{\rm wash} = 1.07 \times 10^{-3}$ eV. The electron flavor is washed out an order of magnitude less than the other two: it is the *door* through which the asymmetry escapes, and its weakness is a consequence of the derived reactor angle being small, not an assumption. The efficiency the chain adopts is $\kappa = 1.0 \times 10^{-2}$; the Boltzmann integration returns η_B directly and not κ, and at the adopted $\epsilon^{\rm res}$ its throughput implies $\kappa \approx 1.2 \times 10^{-3}$, which is what E measures.

L.3.7 The six factors, and the assembly

Every factor in (L.12) is now either derived, standard, adopted, or postulated — the fourth, $\epsilon^{\rm res}\kappa = e^{-K/2}$, is the postulated identity of §71:

$$\eta_B = \underbrace{7.04}_{s/n_\gamma} \times \underbrace{\frac{28}{79}}_{c_{\rm sph}} \times \underbrace{\frac{0.83}{g_*}}_{Y_N^{\rm eq},\ \text{two sourcing states}} \times \underbrace{e^{-K/2}}_{\epsilon^{\rm res}\kappa} = 4.89 \times 10^{-10}, \qquad g_* = 106.75, \tag{L.19}$$

against 6.12×10^{-10} observed — low by 20%.

L.3.8 The throughput identity, and the exponent that is postulated

The fourth factor is the claim of the chapter. The Boltzmann integration of §68 returns a throughput a factor 8.4 below it ($E = 10.6$ below observation); the identity is asserted, not produced:

$$\epsilon^{\rm res}\,\kappa \;\simeq\; e^{-K/2} = e^{-17.5} = 2.511 \times 10^{-8}, \tag{L.20}$$

so the entire dynamical content of the chain compresses to *half the ledger integer in the exponent.* That compression is elegant and it is **postulated, not derived**: the exponent $K/2$ is read off the result rather than obtained from the transport equations, and it is exactly the statement that condition C4 would have to establish. A reader who wants one sentence for what this chapter owes, it is this one.

L.3.9 The error bar, at three levels

A factor-three band is honest only if it is derived. The main text (§72) states it at three levels and this chapter quotes them without softening:

- **Level A — the ledger identity.** ${\eta_B}^{\rm A} = 4.89 \times 10^{-10} \times 3^{\pm 1}$, band $(1.6\text{–}15) \times 10^{-10}$. Span of nine; the exponent is postulated.
- **Level B — the Boltzmann computation.** ${\eta_B}^{\rm B} = 0.58^{+0.35}_{-0.22} \times 10^{-10}$, band width $\times 2.6$. Sits a factor $E = 10.6$ $[6.6, 17.0]$ *below* observation.
- **Level C — the completed inventory.** Folding in four standard transport effects omitted from Level B, all pointing up: ${\eta_B}^{\rm C} = 1.9^{+1.4}_{-0.8} \times 10^{-10}$. The observation sits 2.2σ above the median on a logarithmic scale, just above the 95% band: a central deficit of a factor 3.2.

The three numbers answer three different questions, and the gap between Level A and Level C — about a factor of two and a half — is the measure of how much of the chain is still owed. *All three levels are conditional on C4.*

L.3.10 What this chapter owes

Stated plainly, and repeated because it is the point. This chapter does not own the finite-temperature transport argument or the conditions C1–C6 — their home is GG-A10-Cosmos Ap-

pendix I. It does not own the Boltzmann integration or the phase routing. Condition C4 is open there, and no statement in this chapter is stronger than that condition.

What the chapter *does* establish, subject to C4, is that all eight of Step 2's free quantities descend from a ledger fixed by the gauge sector: the scale, the splitting, the Yukawa, the phases, the flavor regime, the efficiency, and the sign. Item 5 (reheating) and item 6 (initial abundance) are removed by the strong-washout regime the derived couplings put the chain in. That is a great deal to get from one integer — which is exactly why the open condition matters as much as it does.

Problems and worked solutions. L.1.1. Convert the CMB baryon density into η_B, and check it against BBN.

Solution. Step 1: the standard conversion is $\eta_{10} = 273.9\,\Omega_b h^2$ (L.2), which comes from $n_B = \Omega_b \rho_c / m_N$ divided by $n_\gamma = (2\zeta(3)/\pi^2) T_0^3$ at $T_0 = 2.725$ K. Step 2: with Planck's $\Omega_b h^2 = 0.02237$, $\eta_{10} = 273.9 \times 0.02237 = 6.13$. Step 3: the deuterium abundance in metal-poor systems independently gives $\eta_{10} \simeq 6.1$, from physics at a redshift of 10^9 rather than 10^3. Physics intuition: two measurements separated by six decades of cosmic expansion, using completely different physics — nuclear reaction rates in one case, acoustic oscillations of a photon–baryon fluid in the other — agree to better than a percent. That agreement is why η_B is a number worth trying to predict. It is also the reason a prediction cannot hide in the error bar: the observational uncertainty is 0.7%.

L.1.2. Count the survivors.

Solution. Step 1: at $T \gg 1$ GeV quarks and antiquarks are both relativistic and roughly as abundant as photons, so $n_q \simeq n_{\bar{q}} \simeq n_\gamma$. Step 2: the asymmetry is $(n_q - n_{\bar{q}})/n_\gamma = \eta_B = 6.12 \times 10^{-10}$, so the fractional excess is one part in $1/6.12 \times 10^{-10} = 1.63 \times 10^9$. Step 3: after annihilation only the excess survives, so every baryon in the present universe is one of those survivors. Physics intuition: baryogenesis is not the problem of making matter — there was plenty — it is the problem of making matter and antimatter *slightly* unequal. A mechanism that produced an asymmetry of order one would be as badly wrong as one that produced none, and this is worth remembering when assessing claims: the target is a small number, and small numbers are easy to produce by accident and hard to produce for a reason.

L.1.3. Score the Standard Model against Sakharov's three conditions.

Solution. Step 1: baryon-number violation — satisfied, by the anomaly (L.3) and unsuppressed sphaleron transitions above the electroweak crossover. Step 2: CP violation — present, but the CKM contribution is suppressed by the Jarlskog invariant times a Jacobian of quark mass-squared differences over T^{12}, giving of order 10^{-20} against the required 10^{-10}. Step 3: departure from equilibrium — absent, because at $m_H = 125$ GeV the electroweak transition is a crossover rather than first order. Physics intuition: two failures, both quantitative and both settled by measurements (the Jarlskog invariant, the Higgs mass). This is one of the very few places where "the Standard Model is incomplete" is not a philosophical preference but a conclusion from data — and note it became a conclusion only in 2012, when the Higgs mass was measured and the transition was thereby shown to be a crossover.

L.1.4. Derive the sphaleron conversion factor $28/79$**.**

Solution. Step 1: above the electroweak crossover, assign a chemical potential to each species and impose equilibrium for every fast reaction — gauge interactions, Yukawa interactions, and the sphaleron process itself, which enforces $\sum_i (3\mu_{q_i} + \mu_{\ell_i}) = 0$. Step 2: impose also hypercharge neutrality of the plasma, and solve the linear system for B in terms of the conserved $B - L$. Step 3: the general result is $B = (8N_f + 4N_H)/(22N_f + 13N_H)(B - L)$; at $N_f = 3$, $N_H = 1$ this is $28/79 = 0.354430$ (L.5). Physics intuition: the factor is not 1 because sphalerons redistribute rather than convert — they push the plasma toward the equilibrium partition of a conserved $B-L$ between baryons and leptons, and the partition is fixed by how many species of each there are to share it among. The appearance of N_H is worth noting: a two-Higgs-doublet model gives 8/23 instead, so this "standard" factor is a statement about field content.

L.1.5. Show that $B - L$ is anomaly-free and explain why leptogenesis needs it to be.
Solution. Step 1: the anomalies of j^μ_B and j^μ_L in (L.3) are *identical*, being proportional to the same N_f and the same field strengths. Step 2: therefore $\partial_\mu j^\mu_{B-L} = 0$ exactly, while $\partial_\mu j^\mu_{B+L} \neq 0$. Step 3: consequence — an asymmetry stored in $B + L$ would be erased by the very sphalerons that are supposed to convert it, whereas one stored in $B - L$ is immune and survives to be partitioned by (L.5). Physics intuition: this is the hinge of the whole subject. Leptogenesis works because L violation feeds the *conserved* combination, so the sphalerons act as a redistribution channel rather than a drain. Any mechanism that made an asymmetry in $B + L$ alone before the electroweak crossover would have nothing left to show for it, and this is exactly why several early baryogenesis proposals failed.

L.1.6. Run the Davidson–Ibarra bound and get the 10^9 GeV wall.
Solution. Step 1: from the assembly (L.12) the required asymmetry is $\epsilon_1 \simeq \eta_B/(9.8 \times 10^{-3}\kappa)$. Take the theoretical maximum efficiency $\kappa \to 1$: then $\epsilon_1 \simeq 6.12 \times 10^{-10}/9.8 \times 10^{-3} = 6.2 \times 10^{-8}$. Step 2: invert (L.9) for M_1 with $m_3 = 50$ meV and $v = 174$ GeV: $M_1 \geq 16\pi\epsilon_1 v^2/(3m_3) = 50.27 \times 6.2 \times 10^{-8} \times 30276/(1.5 \times 10^{-10}) = 6 \times 10^8$ GeV. Step 3: quoted in the literature as $M_1 \gtrsim 10^9$ GeV, and it forces a reheating temperature of the same order. Physics intuition: this single inequality is why standard thermal leptogenesis is beautiful and untestable. It puts the mechanism nine orders of magnitude beyond any conceivable experiment and leaves no low-energy signature, so the theory can be neither confirmed nor refuted — which is the condition a framework should be most suspicious of in itself, and the reason the resonant escape route (L.10) attracts so much attention.

L.1.7. Show why the resonance needs $\Delta M \sim \Gamma/2$, and why that is uncomfortable.
Solution. Step 1: the self-energy contribution carries the Breit–Wigner factor (L.10), $\Delta M\Gamma/(\Delta M^2 + \Gamma^2/4)$. Step 2: maximize — set the derivative to zero and find $\Delta M = \Gamma/2$, where the factor equals 1 and the Yukawa suppression of ϵ cancels entirely. Step 3: for a TeV-scale N_g with a seesaw-sized Yukawa, $\Gamma/M \sim 10^{-13}$, so the required degeneracy is one part in 10^{13} — arranged by hand in every standard resonant model. Physics intuition: resonant leptogenesis trades one implausibility for another. It escapes the 10^9 GeV wall and becomes testable, but it does so by demanding a degeneracy nobody can motivate. A framework that *derived* the splitting would be making a genuinely different claim — which is what Step 3 attempts, and why the twelve-order mismatch of (L.16) has to be confronted rather than buried.

L.1.8. Contrast weak and strong washout, and say which is preferable and why.
Solution. Step 1: $K_{\rm wash} = \widetilde{m}/m_\star$ (L.11) compares the decay rate to the expansion rate at $T = M_N$. Step 2: for $K_{\rm wash} \ll 1$ the efficiency $\kappa \to 1$ but the result depends on the unknown initial abundance of N; for $K_{\rm wash} \gg 1$, $\kappa \sim 1/K_{\rm wash}$ and any pre-existing asymmetry *and* the initial abundance are both erased. Step 3: strong washout is preferable despite being less efficient, because it makes the answer a function of couplings alone. Physics intuition: this is a general lesson about early-universe model-building and worth carrying beyond this chapter. A process that erases its own initial conditions converts a cosmological accident into a particle-physics prediction; one that does not requires you to specify what happened before, which is usually a way of hiding a free parameter in a place nobody will check.

L.1.9. Explain how a vanishing flavor sum can still give a nonzero asymmetry.
Solution. Step 1: below 10^{12} GeV the tau Yukawa equilibrates and the lepton doublet from an N_g decay is measured into flavor states; each flavor then has its own ϵ_α and its own K_α. Step 2: suppose $\sum_\alpha \epsilon_\alpha = 0$. The surviving asymmetry is $\sum_\alpha \epsilon_\alpha \kappa(K_\alpha)$, and since κ differs between flavors, this is *not* zero. Step 3: the physical picture — the decays deposit equal and opposite asymmetries in different flavors, and washout destroys them at different rates, leaving a residue. Physics intuition: an unflavored calculation of such a scenario would report exactly zero and conclude the mechanism fails. This is one of the clearest cases in cosmology of a "more careful treatment" changing an answer from nothing to something, and it is the reason the flavored Boltzmann system became standard after 2006. Step 3 of this chapter is built on it.

L.1.10. Assemble the standard estimate and find what $\epsilon\kappa$ must be.
Solution. Step 1: the chain is $\eta_B = (s/n_\gamma) c_{\rm sph} Y_N^{\rm eq} \epsilon_1 \kappa$ (L.12), with $s/n_\gamma = 7.04$, $c_{\rm sph} = 28/79$ and $Y_N^{\rm eq} = 0.42/g_*$ per species. Step 2: with $g_* = 106.75$ the prefactor is $7.04 \times 0.3544 \times 0.42/106.75 = 9.8 \times 10^{-3}$. Step 3: matching 6.12×10^{-10} needs $\epsilon_1\kappa = 6.2 \times 10^{-8}$. Physics intuition: every leptogenesis paper ever written is, at bottom, an account of how its ϵ and its κ multiply to 6×10^{-8}. Knowing this one line lets a reader evaluate any such paper in a minute: find its ϵ, find its κ, multiply, and see whether the claimed agreement is a calculation or a choice. Apply the test to Step 3 of this chapter as well.

L.2.1. Show that low-energy CP measurements do not test leptogenesis in the general seesaw.
Solution. Step 1: ϵ_1 (L.8) is built from $\mathrm{Im}[((Y_\nu^\dagger Y_\nu)_{1j})^2]$, and by Casas–Ibarra (J.15) the combination $Y_\nu^\dagger Y_\nu$ depends on R as well as on U. Step 2: the six real parameters of R can supply an arbitrary phase, so ϵ_1 can be given any value, including zero and either sign, at fixed $\delta_{\rm PMNS}$. Step 3: hence measuring $\delta_{\rm PMNS}$ constrains leptogenesis only in models that fix R by some other principle. Physics intuition: it is worth being precise about a claim one hears often. Measuring leptonic CP violation would establish that the lepton sector *has* a CP phase, which is a necessary ingredient; it would not establish that this phase is the one that made the baryons. A framework wanting to claim the link must derive R, and the derivation is then the thing to scrutinize.

L.2.2. Ask what a mechanism that cannot predict the sign has explained.

Solution. Step 1: in the standard picture the sign of η_B follows the sign of an unmeasured phase in R, so both signs are equally available. Step 2: therefore the mechanism predicts $|\eta_B|$ of the right order and nothing about whether we are made of matter or antimatter. Step 3: since "the universe is made of matter" is one of the two facts to be explained — the other being the magnitude — an existence proof that reproduces the magnitude for either sign has answered half the question. Physics intuition: this is a useful discipline to apply generally. Ask of any mechanism not only "does it give the right size?" but "how many of the observed features does it *not* get to choose?" The magnitude is one number and the sign is one bit, and a framework that fixes both from structure is making a claim twice as strong as one that fixes only the first.

L.3.1. Derive the lock scale and its splitting from the ledger.

Solution. Step 1: the OGN-5 closure integer is $K = LJ = 5 \times 7 = 35$, fixed by the gauge sector upstream and not by anything in this chapter. Step 2: the lock sits at $M_{\rm lock} = \frac{1}{2} E_{\rm Pl} e^{-K}$; with $E_{\rm Pl} = 1.2209 \times 10^{19}$ GeV and $e^{-35} = 6.305 \times 10^{-16}$ this is 3.85 TeV (L.13). Step 3: the splitting is $\Delta M = M_{\rm lock}/K = 3849/35 = 110$ GeV — the same integer appearing twice, so the degeneracy is not an independent choice. Physics intuition: contrast this with problem L.1.7, where a resonant model had to arrange $\Delta M/M \sim 10^{-13}$ by hand. Here $\Delta M/M = 1/K = 1/35$, which is not small at all. That is either the chapter's most interesting feature or its central difficulty, and problem L.3.2 shows it is both.

L.3.2. Confront the twelve-order mismatch before accepting the resolution.

Solution. Step 1: the derived Yukawa (L.14) gives ${Y_\nu}^2 = \pi M_{\rm lock}/E_{\rm GUT} = 5.303 \times 10^{-13}$, hence the vacuum width $\Gamma_N = (\sqrt{2}\, Y_\nu)^2 M_{\rm lock}/8\pi = 1.62 \times 10^{-10}$ GeV (L.15). Step 2: compare to the derived splitting: $\Delta M/\Gamma_N = 110/1.62 \times 10^{-10} = 6.8 \times 10^{11}$, while the resonance (L.10) wants order unity. *The framework's own derived numbers miss the resonant condition by nearly twelve orders of magnitude.* Step 3: the proposed resolution is that the relevant width is not the vacuum width but a thermal effective width (L.17), $\Delta M/\Gamma^{\rm eff} = 1/(K c_\tau y_\tau^2) = (2.7\text{–}5.3) \times 10^4$ — eight orders closer, and in the decade the chain needs. Physics intuition: a student should sit with the mismatch before accepting the fix. A twelve-order-of-magnitude gap closed by identifying a different width is exactly the kind of move that deserves the most scrutiny in the paper, and the framework's own assessment agrees: condition C4, which is what would justify the identification, is marked open, and F-SM-7 is written so that a single Kadanoff–Baym calculation settles it either way.

L.3.3. Locate the one regulator that works among five candidates, and price the coincidence.

Solution. Step 1: the required window is $\Delta M/\Gamma \sim 10^4$–$10^5$, from the throughput the assembly needs. Step 2: Table 44 of the main text lists five candidate widths — the N widths on the S-matrix prescription, the flavor-measurement rate $c_\tau y_\tau^2 T$, the oscillation rate ΔM itself, gauge damping $g^2 T$, and the Hubble rate — spanning sixteen orders of magnitude in $\Delta M/\Gamma^{\rm eff}$ (Fig. 19). Exactly one, the flavor-measurement width, lands in the decade. Step 3: this is presented in the paper as a *selection*, not a derivation, and the distinction matters — five candidates spread over sixteen orders sit about three decades apart, so one of them falling inside the required decade is a coincidence of roughly the odds of a fair die, not a derivation. Physics intuition: the honest reading is that the regulator identification is a hypothesis with a testable consequence, not a calculation with a result.

What makes it more than numerology is that the same identification fixes the flavor structure and the sign independently, and those are separate checks; what would make it a derivation is C4.

L.3.4. Show that the unflavored asymmetry vanishes, and why that is a feature.
Solution. Step 1: on the derived texture the Casas–Ibarra matrix is real, so the asymmetry matrix S has $\mathrm{Tr}\, S = 0$ exactly. Step 2: hence the unflavored CP asymmetry — the one proportional to the invariant J_{PMNS} — is identically zero, and the entire matter asymmetry is carried by *differences* between the flavored ϵ_α acting through different washouts (problem L.1.9). Step 3: the consequence for the previous chapter is sharp — the factor of three hundred between the two unitarity triangles (K.18), striking as it is, is not why the lepton sector can pay for baryogenesis, because the invariant it measures drops out of the answer. Physics intuition: this is a good example of a framework's own result cutting against a rhetorically attractive claim. It would have been very convenient to point at $J_{\mathrm{PMNS}}/J_{\mathrm{CKM}} = 308$ and say "there is the missing CP violation"; the calculation says that quantity contributes exactly nothing, and the paper says so in both chapters.

L.3.5. Cash the half-angle for the sign of the universe.
Solution. Step 1: the derived parity is $\alpha_{31} = \pi$, which enters the mixing matrix halved as $e^{i\pi/2} = i$ (K.13). Step 2: ϵ_1 (L.8) needs the imaginary part of a product of Yukawa combinations; the factor of i — linear in U here, unlike in $m_{\beta\beta}$ where it squared — supplies exactly that imaginary part, and its sign fixes whether the surviving asymmetry is in leptons or antileptons. Step 3: hence the derived parity determines the sign of η_B, answering item 8 of Step 2, which no standard leptogenesis model does. Physics intuition: this is the single most striking structural claim in the chapter. One discrete parity, forced by the rank of the Majorana ledger, fixes $m_{\beta\beta} = 1.80$ meV in the previous chapter and the sign of the matter content of the universe in this one — the same bit read through two different powers of the same matrix. It is also, of course, conditional on C4 like everything else here, and the right attitude is that this is what makes C4 worth someone's year.

L.3.6. Find the electron door.
Solution. Step 1: the flavored washouts computed from the derived masses and derived PMNS matrix are $K_\alpha = (3.5,\ 29.3,\ 22.5)$ for (e, μ, τ) (L.18), against the unflavored third-channel $K_3 = 47$ and the total $K_{\mathrm{tot}} = 54.6$. Step 2: the electron flavor is washed out roughly an order of magnitude less than the other two, so its asymmetry survives with an order of magnitude better efficiency: the electron is the door. Step 3: trace the cause — K_e is small because the electron row of the PMNS matrix is dominated by the small reactor angle, so the derived $\sin^2\theta_{13} = 3/140$ of App. J is what opens the door. Physics intuition: a reactor experiment measuring a mixing angle at a kilometer baseline, and the survival of the matter asymmetry at 10^{-11} seconds, are here the same number. Whether or not the chain holds up, that is the kind of connection a unification claim is supposed to make, and it is worth noting it is not an adjustable one: θ_{13} was measured before this was computed.

L.3.7. Assemble η_B from the six factors and check the arithmetic.
Solution. Step 1: list the six — $s/n_\gamma = 7.04$; $c_{\mathrm{sph}} = 28/79 = 0.354430$; $Y_N^{\mathrm{eq}} = 0.83/g_*$ for the two sourcing states of the triplet; $g_* = 106.75$; and $\epsilon^{\mathrm{res}}\kappa = e^{-K/2}$. Step 2: multiply. $7.04 \times 0.354430 = 2.4952$; $\times 0.833 = 2.078$; $/106.75 = 1.947\times10^{-2}$; $\times e^{-17.5} = 1.947\times10^{-2}\times2.511\times10^{-8} = 4.89\times10^{-10}$

(L.19). Step 3: against 6.12×10^{-10} this is low by 20.1%, or −20%. Physics intuition: four of the six factors are textbook thermodynamics that any leptogenesis calculation uses; the fifth is the field content; the entire content of the framework's claim is the sixth, $\epsilon^{\rm res}\kappa = e^{-K/2}$. When assessing this result, that is the only factor to argue about — and the next problem is about why it is the one that is not derived.

L.3.8. Identify precisely what is postulated.
Solution. Step 1: the throughput identity (L.20) states $\epsilon^{\rm res}\kappa \simeq e^{-K/2} = e^{-17.5} = 2.511 \times 10^{-8}$, compressing all the dynamics into half the ledger integer. Step 2: the exponent $K/2$ is *read off the result* rather than obtained from the transport equations — it is not a consequence of anything derived in the chapter. Step 3: what would derive it is condition C4 of H-$\phi_{\rm geo}$-1, namely that the thermal bath resolves the two heavy states as a flavor measurement on the required timescale; establishing it needs a full Kadanoff–Baym resummation that has not been done. Physics intuition: a student wanting one sentence for what this chain owes should take this one. Everything else — the scale, the splitting, the Yukawa, the washouts, the sign — is computed; the compression into $e^{-K/2}$ is asserted. A framework that quietly let this look like the rest of the chapter would be misrepresenting itself, which is why C4 is named in the table, in the takeaway, in the falsification channel, and here.

L.3.9. Read the three error-bar levels without averaging them.
Solution. Step 1: Level A takes the ledger identity at face value: $4.89 \times 10^{-10} \times 3^{\pm 1}$, band (1.6–15) $\times 10^{-10}$, a span of nine. Step 2: Level B integrates the Boltzmann system and scans the four remaining freedoms: $0.58^{+0.35}_{-0.22} \times 10^{-10}$, band width ×2.6 — a factor $E = 10.6$ $[6.6, 17.0]$ *below* observation. Level C folds in four standard transport effects Level B omits, all pointing up: $1.9^{+1.4}_{-0.8} \times 10^{-10}$, with the observation 2.2σ above the median on a logarithmic scale. Step 3: they answer three different questions — what the identity gives, what the declared dynamics produces, and what the completed inventory would produce — and the factor of two and a half between A and C measures what is still owed. Physics intuition: resist averaging them. The instinct to collapse three numbers into one is exactly wrong here, because their *spread* is the informative quantity: it says the chain's arithmetic and the chain's dynamics currently disagree by a factor of a few, which is a much more useful statement than any single number would be.

L.3.10. Audit Step 2: eight items, and the one condition they all hang on.
Solution. Step 1: items 1 (scale), 2 (splitting), 3 (Yukawa), 4 (phases), 7 (flavor regime) and 8 (sign) descend from the ledger; items 5 (reheating) and 6 (initial abundance) are removed because the derived couplings put the chain in strong washout, which erases both. Step 2: so all eight close — *subject to* the regulator identification, whose condition C4 is open. Step 3: the correct summary statement is therefore conditional and should always be written that way: *given* that the thermal bath supplies the flavor-resolving regulator, the baryon asymmetry descends from the same integer $K = 35$ that fixes the lock scale and the neutrino floor. Physics intuition: eight free parameters replaced by one integer is a very large claim, and one open analytic condition is a very small-looking price — which is exactly why it must be stated every time. The useful question for a student is not whether the claim is impressive but what would settle it, and here the answer is unusually concrete:

one finite-temperature calculation, no new data, and F-SM-7 is written so that it decides the matter in either direction.

Take-away. *Standard result:* sphalerons convert a lepton asymmetry into a baryon one at the exact rate 28/79; the seesaw supplies all three Sakharov conditions; and the Davidson–Ibarra bound then puts the mechanism above 10^9 GeV, where it is untestable. *What it assumed:* eight quantities, including the sign of the universe. *The geometric reading:* the lock scale $M_{\rm lock} = \frac{1}{2} E_{\rm Pl} e^{-K} = 3.85$ TeV, its splitting $M_{\rm lock}/K$, the seam Yukawa, the flavored washouts and the sign all descend from $K = 35$; the unflavored asymmetry vanishes identically; the electron flavor is the door, opened by the derived reactor angle. *What it owes:* the compression $\epsilon^{\rm res}\kappa \simeq e^{-K/2}$ is **postulated**, and condition C4 — the finite-temperature transport step — is **open**. Every number in this chapter is conditional on it, and F-SM-7 is written so that one calculation settles it.

M Constitutional ledgers and Standard-Model architecture

What this appendix teaches. The audit chapter: the constitutional ledger of exactly what the visible derivation consumed — which preprints, which restricted inputs, which kill criteria — and the architecture cards that compare the constitutional reading with its stabilized boundary projection. The course's provenance record: to be checked rather than worked.

M.1 Constitutional ledger of the present gauge/Higgs, fermion-mass, CKM, PMNS/Majorana, and topological-CP derivation

Because the visible derivation relies only on the written Arisaka-GG preprints and on the restricted input set, it is useful to close the appendix sequence with a compact ledger of exactly what the present paper does. Table 82 lists each major stage of the argument, what it is allowed to consume, and what it is forbidden to borrow from observation.

Table 82: Constitutional ledger for the present gauge/Higgs, fermion-mass, CKM, PMNS/Majorana, and topological-CP derivation. Each stage of the derivation is one row, and each row names what may be used, what comes out, and what would be a shortcut at that particular stage. The last column changes from row to row, which is the point: an input that is legitimate at a later stage can be circular at an earlier one, and a ledger naming only a single global list of forbidden moves would miss exactly that.

Stage	Allowed inputs	Outputs produced	Forbidden shortcut at this stage
Constitutional setup	GG-A1-Constitution axioms/postulates and GG-A5-GDOS observation rules	Meaning of derivation, admissible inputs, role of GDOS/GFF	Smuggling in measured SM parameters as if they were theoretical primitives

Stage	**Allowed inputs**	**Outputs produced**	**Forbidden shortcut at this stage**
GG-6 stage selection	GG-A2-GG6 geometry, BI-6 discipline, OGN-5 ledger	Minimal bulk $M_4 \times I_r \times S^1_\theta$, composite connection, throat depth	Adding extra dimensions, supersymmetry, or string states not present in the written preprints
Scale chain	c, h, G_N, π, e, and OGN-5 integers	$E_{\rm Pl}$, $E_{\rm GUT}$, $M_{\rm lock}$	Using G_F, m_Z, or any electroweak datum to set the weak scale
GDOS/GFF layer	GG-A5-GDOS open-system dynamics and GFF functional	Meaning of stationary seams, vacuum selection, measurement map	Treating observables as external empirical labels without projection theory
Radial RG layer	Warped metric and logarithmic radial coordinate	Transport equation $\partial_r g_i = -\beta_i$ and seam evaluation logic	Importing textbook beta functions as free phenomenological inputs divorced from GG-6 geometry
Wilson–Hosotani layer	Compact holonomy and orbifold parity structure	Geometric Higgs origin, finite holonomy potential, naturalness reformulation	Introducing an elementary 4D scalar with an arbitrary bare mass parameter
Gauge/Higgs first order	Same restricted constants plus OGN/BI-6 rational structure	Leading formulas for $\widehat{\alpha}^{(5)\,-1}(M_Z)$, $\sin^2\theta_W$, $\alpha_s(M_Z)$, v, and m_h	Fitting first-order coefficients directly to benchmark central values
Gauge/Higgs QD	Finite residual seam corrections within the same framework	Second-order sharpened gauge/Higgs predictions and theory truncation errors	Replacing the second-order layer by ad hoc numerical regression
Charged-fermion overlap block	Yukawa overlap theorem from GG-A2-GG6/3, fixed v, OGN dyadic/winding codebook	First-order charged-lepton and quark Yukawas and masses	Inserting empirical Yukawa matrices or family-specific fit constants
Neutrino Majorana block	Allowed $(LH)(LH)$ operator, fixed $E_{\rm GUT}$, fixed v, OGN family projector	First-order and QD neutrino masses and oscillation scales	Choosing an arbitrary seesaw scale or measured neutrino mass as an input

Stage	**Allowed inputs**	**Outputs produced**	**Forbidden shortcut at this stage**
Fermion QD transport	Same dyadic and rational closure logic as above	QD-completed fermion masses and truncation errors	Independently fitting each fermion mass with a separate floating correction factor
CKM block	Fixed quark masses, family-link algebra, compact-angle phase logic, and the same OGN/BI-6 ledger	First-order and QD CKM angles, phase, matrix elements, and Jarlskog invariant	Importing measured CKM angles or a generic fitted unitary flavor matrix
PMNS/Majorana block	Fixed charged-lepton codebook, fixed Majorana neutrino block, family projector algebra, compact-angle phase logic, and the same OGN/BI-6 ledger	First-order and QD PMNS angles, leptonic Dirac phase, Majorana phases, full matrix, and $m_{\beta\beta}$	Importing measured PMNS angles/phases or a generic fitted lepton mixing matrix
Comparison stage	Only now: benchmark observations in the matching scheme	Dev and Pull tables, assessment of agreement, identification of residual work	Feeding the comparison numbers back into the derivation as hidden inputs

The value of this ledger is methodological. It clarifies why the present manuscript is a genuine first-principles construction in the sense adopted here. Every stage is allowed to use only the written constitutional language, the minimal conversion constants, the geometric transcendentals, and the OGN-5 integers. No stage is allowed to consume a measured Standard-Model number in order to make the subsequent algebra work.

This point also explains why the comparison tables are placed late in the paper. In an empirical fit paper, the comparison table often acts as an input filter: one varies unknown constants until the table looks good. In the present paper the comparison table is a post-derivational auditor.

M.2 Explicit SU(5) decomposition, hypercharge embedding, and charge assignments

This appendix supports the new phenomenological prologue by collecting the basic SU(5) decomposition formulae used implicitly in the main text. None of the formulae are novel by themselves; the point is to display them in the notation of GG-A9-SM so that the reader can see exactly how the historical Georgi–Glashow structure is being lifted into GG-6.

M.3 Adjoint decomposition

The adjoint of SU(5) has dimension 24. Under the Standard-Model subgroup it decomposes as

$$\mathbf{24} \to (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\bar{\mathbf{3}},\mathbf{2})_{5/6}. \tag{M.1}$$

The interpretation is immediate. The octet gives the gluons, the triplet gives the weak bosons, the singlet gives the hypercharge boson, and the mixed-color weak doublets correspond to the heavy X and Y gauge bosons.

M.4 Matter-family decomposition

A single visible family is organized into

$$\bar{\mathbf{5}} \oplus \mathbf{10}, \tag{M.2}$$

with the decompositions

$$\bar{\mathbf{5}} \to (\bar{\mathbf{3}},\mathbf{1})_{1/3} \oplus (\mathbf{1},\mathbf{2})_{-1/2}, \tag{M.3}$$

$$\mathbf{10} \to (\mathbf{3},\mathbf{2})_{1/6} \oplus (\bar{\mathbf{3}},\mathbf{1})_{-2/3} \oplus (\mathbf{1},\mathbf{1})_1. \tag{M.4}$$

These are precisely the familiar quark and lepton assignments of one family.

M.5 Hypercharge generator and electric charge

Inside SU(5), the conventional hypercharge generator is represented by

$$Y = \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right), \tag{M.5}$$

with the trace condition $\mathrm{Tr}\, Y = 0$. Electric charge is then

$$Q = T_3 + Y. \tag{M.6}$$

One immediately obtains the standard charge pattern shown in Table 83.

Table 83: Electric charges from the SU(5) embedding. Each row gives a field, its representation with the weak-isospin component where that matters, the electric charge that follows, and the sector the field belongs to. The charges are outputs here and not assignments: they are read off the embedding, and the fractional values in the quark rows follow from the hypercharge normalization rather than from a separate postulate.

State	SM representation	Q	Interpretation
Left quark doublet upper component	$(\mathbf{3},\mathbf{2})_{1/6}$ with $T_3=+1/2$	$+2/3$	Up-type quark
Left quark doublet lower component	$(\mathbf{3},\mathbf{2})_{1/6}$ with $T_3=-1/2$	$-1/3$	Down-type quark
Right up antiquark field	$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	$-2/3$ on conjugate field	Up-type singlet sector
Right charged antilepton field	$(\mathbf{1},\mathbf{1})_1$	$+1$ on conjugate field	Charged-lepton singlet sector
Lepton doublet upper component	$(\mathbf{1},\mathbf{2})_{-1/2}$ with $T_3=+1/2$	0	Neutrino
Lepton doublet lower component	$(\mathbf{1},\mathbf{2})_{-1/2}$ with $T_3=-1/2$	-1	Charged lepton

The table is elementary, but it is valuable because it shows why charge quantization is already explained once the unified embedding is fixed.

M.6 Baryon number in the GG-6 extension

The historical SU(5) model does not by itself make local baryon number a fundamental gauge symmetry. GG-6 extends the visible gauge sector by local $\mathrm{U}(1)_B$. At the seam this gives the familiar assignments

$$B(q)=\frac{1}{3}, \qquad B(\ell)=0. \tag{M.7}$$

The importance of writing Eq. (M.7) explicitly is that it clarifies why baryon bookkeeping is not treated in GG-A9-SM as an accidental approximate global symmetry. It belongs to the constitutional visible gauge structure.

M.7 Spin, family counting, and why three generations are minimal under MSMF

The new main-text sections argued that three generations are the minimal nontrivial family number compatible with hierarchical mixing and CP violation. This appendix records the underlying logic more explicitly.

M.8 Spin counting from a 6D spinor to a 4D massive fermion

A six-dimensional Dirac spinor carries more components than a four-dimensional Dirac spinor, but the boundary reduction and chiral projection remove unwanted light-sector components. Schematically,

$$\Psi_{6D} \longrightarrow \psi_{L,4D} \oplus \psi_{R,4D} \oplus \text{heavy mirror/KK components.} \tag{M.8}$$

After electroweak mass generation, a massive 4D fermion supports two physical spin projections,

$$s_z = \pm\frac{1}{2}. \tag{M.9}$$

This is the precise technical meaning of the phrase “half spin, spin up/down.

M.9 Flavor counting versus state counting

For one generation the visible matter content contains four flavor types: up-type quark, down-type quark, charged lepton, and neutrino. With three generations the flavor content therefore becomes twelve. State counting, however, is richer because quarks carry color and all massive fermions carry two spin projections.

Table 84 keeps these notions separate.

Table 84: Why “six quarks and six leptons” refers to flavor counting rather than full state counting. The distinction matters because the two counts are used interchangeably in the literature and differ by the color and spin multiplicities. The table separates them line by line, so that the twelve of “six quarks and six leptons” can be traced to flavor counting and the larger numbers that appear in thermodynamic and anomaly arguments can be traced to state counting. Nothing here is derived; it is bookkeeping made explicit so that later counts cannot be misread.

Object counted	**Per family**	**For three families**	**Comment**
Quark flavors	2	6	Up-type and down-type across three generations
Lepton flavors	2	6	Charged lepton and neutrino across three generations
Quark color states	2×3	6×3	Color multiplies state count but not flavor count
Spin states for massive fermions	factor of 2	factor of 2	Spin doubles physical state multiplicity for each massive species

M.10 Why one generation is too poor

With one generation there is no nontrivial flavor mixing. The family space is one-dimensional, so any would-be mixing matrix is trivial. This immediately fails to account for the observed pattern of quark and lepton misalignment.

M.11 Why two generations are still insufficient

With two generations one can have one real mixing angle, but no irreducible Kobayashi–Maskawa phase. Put differently, all apparent complex phases can be removed by rephasing, so there is no physical Jarlskog invariant. In that sense two generations are the first nontrivial mixing case but not the first full flavor case.

M.12 Why three generations are the first fully nontrivial case

With three generations the family space is large enough to support

(i) nontrivial hierarchical masses,

(ii) nontrivial quark and lepton misalignment,

(iii) a physical CP-odd invariant,

(iv) and, in the neutrino case, genuinely distinct Majorana-phase structure.

This is the real meaning of "minimal but nontrivial" in the present paper. MSMF does not merely say "choose the smallest number because it is small." It says "choose the smallest number that can actually carry the observed structure." For flavor, that number is three.

M.13 Relation to the later codebook construction

The later sections of GG-A9-SM make this argument constructive by writing down minimal family projectors and overlap codebooks. The present appendix does not repeat all of that arithmetic. Its role is to record the logical reason why the family number should already be expected to be three before the detailed codebook is displayed.

M.14 Packet cards for the unified and Standard-Model seam descriptions

The main text introduced both a unified packet and a Standard-Model seam packet. This appendix presents them in compact "packet card" form for quick reference. The purpose is practical: it lets the reader see in one place how the abstract SIP formalism is instantiated in the particle-physics context.

M.15 Unified visible packet card

M.16 Standard-Model seam packet card

The comparison between the two cards makes the logic of the manuscript explicit. The second is not independent of the first. It is its stabilized boundary projection.

Problems and worked solutions. M.3.1. Decompose the adjoint 24 and read off the leptoquark charges.

Solution. Step 1: count the decomposition (M.1): $(8,1)_0 \oplus (1,3)_0 \oplus (1,1)_0 \oplus (3,2)_{-5/6} \oplus (\bar{3},2)_{+5/6}$ gives $8+3+1+6+6=24$ states — the gluons, the weak triplet, the hypercharge boson, and twelve leptoquark directions X, Y. Step 2: the leptoquark hypercharge is a pair difference of the generator (M.5): a color row against a weak row gives $Y = -\frac{1}{3} - \frac{1}{2} = -\frac{5}{6}$; then $Q = T_3 + Y$ (M.6) yields $Q(X) = -\frac{4}{3}$ and $Q(Y) = -\frac{1}{3}$. Step 3: these twelve are exactly the directions the boundary conditions remove from the four-dimensional gauge sector — bulk-only in GG-6 — which is why proton decay by X, Y exchange never reaches the boundary (chapter L). Physics intuition: the

Table 85: The unified visible packet before branch selection, in the compact card form used through this appendix. Each field of the abstract formalism is instantiated for the particle-physics case, so that the correspondence can be read in one place rather than reconstructed from the general definitions. The kill criteria are the operative row: they say what would falsify the instantiation, and they are stated before the branch is chosen rather than after it.

Field	**Unified visible packet before branch selection**
State manifold	Unified gauge connection, compact-angle holonomy, radial transport variables, matter multiplets, anomaly-compatible boundary data
Ports	Boundary conditions, bulk-to-boundary projection channels, UV/IR matching interfaces, admissible preparation/readout channels
Generator	Warped-bulk transport plus projected open-system evolution
GFF ledger	Unified gauge + gravity + matter + corrective scalar ledger
Observables	Holonomy class, anomaly checks, surviving zero modes, projected response functions
Kill criteria	Anomaly failure, wrong surviving algebra, failure of positivity/projection, proton-instability operators

Table 86: The Standard-Model seam packet after branch selection, in the same card form and with the same fields as the unified packet above. The two cards are meant to be read together and compared field by field. What changes across the branch selection is as informative as what survives it, and the fields that survive unchanged are the ones on which the selection has no purchase at all.

Field	**Standard-Model seam packet after branch selection**
State manifold	Running gauge couplings, Wilson-line/Higgs order parameter, family projectors, renormalized masses and mixings
Ports	Scattering channels, decay channels, oscillation baselines, detector readout and reconstruction pipelines
Generator	Effective seam RG transport plus low-energy interaction dynamics
GFF ledger	Scalar budget governing branch stability and observable extraction
Observables	$\widehat{\alpha}^{(5)\,-1}$, $\sin^2\theta_W$, α_s, v, m_h, fermion masses, CKM, PMNS, Majorana phases
Kill criteria	Wrong gauge pattern, wrong stability class, catastrophic failure of predicted observables beyond theory error, inconsistency with proton stability

adjoint is the unified force's full keyboard; the boundary gets to play only the $8+3+1$ white keys, and the twelve black keys — the ones whose chords turn quarks into leptons — stay locked in the bulk.

M.6.1. Audit the anomalies of gauged baryon number, and find the one that bites. *Solution.* Step 1: with the assignments (M.7) ($B = \frac{1}{3}$ on quark fields, 0 on leptons), three sums vanish identically per family: $[\mathrm{SU}(3)]^2\,\mathrm{U}(1)_B$: $2(\frac{1}{3}) - \frac{1}{3} - \frac{1}{3} = 0$; $\mathrm{grav}^2\,\mathrm{U}(1)_B$: $6(\frac{1}{3}) - 3(\frac{1}{3}) - 3(\frac{1}{3}) = 0$; $[\mathrm{U}(1)_B]^3$: $\frac{6}{27} - \frac{3}{27} - \frac{3}{27} = 0$. Step 2: two do not: $[\mathrm{SU}(2)]^2\,\mathrm{U}(1)_B$ sums B over doublets — three colored quark doublets at $\frac{1}{3}$ against a lepton doublet at 0 — giving $1 \neq 0$; and the mixed $\mathrm{U}(1)_B\,[\mathrm{U}(1)_Y]^2$ sum gives $-\frac{1}{2} \neq 0$. Step 3: this is why baryon number in the Standard Model is only an *accidental* symmetry, and why gauging it — as GG-6 does — is impossible in four dimensions alone: the surviving anomalies must be canceled by the BI-6 Green–Schwarz inflow, which is precisely the load-bearing hypothesis (H3) of the proton-stability theorem (Theorem 86.1). Physics intuition: the proton's guardian symmetry is anomalous on the boundary and can only be rescued from the bulk — so proton stability, in this framework, is not a low-energy accident but a six-dimensional bookkeeping identity; the assembly interlude's problem G.3.2 is the same audit run for hypercharge, where all four sums close on their own.

M.11.1. Prove that two generations admit no CP violation: the general-N_g counting. *Solution.* Step 1: an $N \times N$ unitary mixing matrix carries N^2 parameters, of which $N(N-1)/2$ are angles and the rest phases; $2N-1$ phases are removable by quark rephasing. Step 2: physical phases number $\frac{N^2+N}{2} - (2N-1) = \frac{(N_g-1)(N_g-2)}{2}$: for $N_g = 1$, no mixing at all; for $N_g = 2$, one angle (Cabibbo's 1963 rotation) and *zero* phases — every complex entry can be rephased away, so no Jarlskog invariant exists; for $N = 3$, three angles and one phase — the first N with room for CP. Step 3: this is leg one of the MSMF minimality argument of this chapter: three generations are not decoration but the smallest family space that can carry hierarchical masses, nontrivial misalignment, *and* a CP-odd invariant — and the closed two-sided theorem lives in GG-P2-N3 (§21). Physics intuition: nature needed CP violation to make more matter than antimatter, and CP violation needed three families to exist at all — the counting on this page, run forwards, is the anthropic-sounding fact made mechanical: a two-family universe would have been perfectly symmetric, and perfectly empty.

M.2.1. Show that the ledger integers are not independently adjustable. *Solution.* Step 1: the ledger is $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ with the closure relation $K = LJ$. Step 2: so K is not a fifth input — fixing L and J fixes it, and $K = 35$ then appears in the reactor angle $N_g/4K$, in the lock scale $\frac{1}{2}E_{\mathrm{Pl}}e^{-K}$, and in the throughput exponent $e^{-K/2}$. Step 3: consequently a change to L or J propagates into the lepton mixing, the collider scale and the baryon asymmetry simultaneously; they cannot be tuned one at a time. Physics intuition: this is where the framework is most exposed and most testable. Five integers, only four of them independent, feeding twenty-eight parameters — so the falsification channels are not independent either, and a single decisive failure in one sector cannot be repaired locally. A reader assessing the program should look for this interlocking rather than for individual agreements.

M.2.2. Count the inputs against the outputs.

Solution. Step 1: the ledger supplies four independent integers plus the constitutional structure. Step 2: the codebook delivers twenty-eight Standard-Model parameters, and Stage-5 material adds the baryon asymmetry, the two Majorana phases and $m_{\beta\beta}$ — conditionally in the first case. Step 3: the ratio of outputs to inputs is therefore of order seven to one, and it is that ratio, not the precision of any single entry, that is the claim. Physics intuition: a framework with as many parameters as observables explains nothing however well it fits, and one with far fewer is falsifiable even if some entries are imprecise. When judging any unification proposal, count first and admire second — and apply the count honestly, which here means noting that the conditional entries should be weighted less than the derived ones.

M.2.3. Locate where the architecture would break if N_g were four.
Solution. Step 1: $N_g = 3$ is imported from GG-P2-N3 rather than derived here, on three independent legs. Step 2: trace the consequences of $N_g = 4$: the cascade-closure bound $N_g(N_g - 1)/2 \leq N_g$ fails at $N_g = 4$; the reactor angle $N_g/4K$ would move by a third; the family projector geometry on $\mathbb{C}P^{N_g-1}$ would change dimension; and the Majorana ledger's rank-two structure, which gives exactly two splittings, would admit a third. Step 3: so a fourth family is not an extension of this framework but a refutation of it, and the observation of one falsifies the whole chain. Physics intuition: the useful question about any structural input is not "is it true?" but "what does the theory look like if it is false?" Here the answer is that there is no theory left, which is a much stronger statement than an adjustable parameter would allow — and it is why the $N = 3$ derivation is carried by a dedicated paper rather than assumed in passing.

M.2.4. Distinguish a ledger from a fit by a test a reader can apply.
Solution. Step 1: state the test — for each derived quantity, ask whether any ledger input was chosen *after* that quantity was known, and whether the same input is used elsewhere with no freedom to differ. Step 2: apply it to the reactor angle $\sin^2\theta_{13} = N_g/4K$: N_g comes from the family count and K from the closure LJ, both fixed by the gauge sector, and both reused in the lock scale and the throughput — so the angle has no private dial. Step 3: apply it to a place where the answer is less comfortable: the Weinberg-operator coefficient of App. J: its coefficient 2π is conditional on lemma L8, and the ratio m_3/m_2 it multiplies is recorded in the provenance table as **Derived (selected)** rather than **Derived** precisely because the projector ratio $2N$ was selected against the measured ratio by the MSMF/OGN audit, with its look-elsewhere factor reported. Physics intuition: the test is worth applying to every row of the codebook, and the honest outcome is a mixture rather than a clean sweep. A framework's credibility rests less on how many rows pass than on whether its own provenance table reports the failures — which is what the **(selected)** and **(cond.)** markers exist to do.

N Force laws, anomaly cancellation, and proton stability

What this appendix teaches. Three structural facts the Standard Model presents as facts and the geometry reads as consequences: the inverse-square form of the classical force laws (from boundary Green's functions), the family-by-family cancellation of gauge anomalies (inherited from the bulk identity rather than imposed), and the stability of the proton (a gauged baryon symmetry rather than an accident) — with the Super-Kamiokande bound as the worked example.

Read alongside: Halzen & Martin (1984) for the cross-section and decay estimates; Adler (1969) and Bell & Jackiw (1969) for the anomaly originals.

This appendix collects three structural features that the Standard Model presents as facts and GG-Theory reads as consequences of the boundary geometry: the inverse-square form of the classical force laws, the family-by-family cancellation of gauge anomalies, and the stability of the proton. We follow the three-step plan, with the explicit Green's-function and $U(1)_B$ derivations carried in the detailed subsections that follow this framing. The classical force laws and proton phenomenology are developed in Halzen & Martin (1984).

N.1 Step 1 — force laws, anomalies, and proton decay as a graduate student meets them

The inverse-square laws. Both Newtonian gravity and the Coulomb force fall off as $1/r^2$. This is not an independent postulate but a consequence of a massless mediator in three spatial dimensions: the static potential is the Green's function of the Laplacian, $\nabla^2\phi = -\rho$, whose solution $\phi \propto 1/r$ gives a force $\propto 1/r^2$. The explicit derivation from the three-dimensional Gauss law is given in the subsection below. *(Physics intuition: field lines from a point source spread over a sphere whose area grows as r^2, so their density — the field strength — falls as $1/r^2$. The inverse-square law is the geometry of three-dimensional space, not a separate force rule.)*

Anomaly cancellation. A chiral gauge theory is consistent only if its gauge anomalies cancel. In the Standard Model the triangle anomalies, identified by Adler and by Bell and Jackiw (Adler, 1969; Bell & Jackiw, 1969) and shown to cancel family-by-family in the Standard Model by Bouchiat, Iliopoulos, and Meyer (Bouchiat et al., 1972) — $[SU(3)]^2U(1)$, $[SU(2)]^2U(1)$, $[U(1)]^3$, and the mixed gravitational $[\text{grav}]^2U(1)$ — all cancel *within each generation*, a delicate conspiracy between the quark and lepton hypercharges that also forces electric-charge quantization (why $|q_e| = |q_p|$ to 10^{-21}). *(Physics intuition: anomaly cancellation is the quantum consistency tax. Each fermion contributes to a potential breakdown of gauge invariance at the loop level, and only because the quark and lepton charges add up to exactly zero in every channel does the theory survive. That the charges of utterly different particles sum to zero is one of the deepest hints that a family is a single object.)*

Proton decay in grand unification. Grand unified theories place quarks and leptons in common multiplets, so they generally contain heavy X, Y gauge bosons that mediate baryon-number-violating decays such as $p \to e^+\pi^0$. Minimal SU(5) (Georgi & Glashow, 1974) predicts a lifetime $\tau_p \sim 10^{30}$–10^{31} yr, already excluded by the Super-Kamiokande bound $\tau_p > 1.6 \times 10^{34}$ yr. *(Physics intuition:*

unify quarks with leptons and you open a door between them — a quark can turn into a lepton — and the proton can fall apart. The non-observation of proton decay is therefore a sharp constraint on how, and whether, the forces unify.)

Worked example (STANDARD TEXTBOOK PEDAGOGY). Dimensionally, $\tau_p \sim M_X^4/(\alpha_{\rm GUT}^2 m_p^5)$. With $M_X \sim 10^{15}$ GeV and $\alpha_{\rm GUT} \sim 1/40$ this gives $\tau_p \sim 10^{31}$ yr — below the experimental bound, which is exactly why minimal SU(5) is ruled out and why the proton's stability needs an explanation.

N.2 Step 2 — what these structural facts assume but cannot explain

The Standard Model uses all three facts without grounding them. The inverse-square law is inherited from working in three spatial dimensions, which is itself unexplained. Anomaly cancellation *works*, but the theory does not say why the fermion charges take the precise values that make it work — it is imposed as a consistency condition, not derived. And the proton's stability rests on a coincidence: baryon number B and lepton number L are only *accidental* global symmetries of the renormalizable Standard-Model Lagrangian, not gauged and not fundamental, so nothing protects the proton once the theory is embedded in anything larger. Generic unification breaks B and predicts a decay that is not seen. *(Physics intuition: the proton is stable in the Standard Model only by an accident of which operators happen to be renormalizable. Lean on the theory from above — unify it — and the accident evaporates. A framework that made baryon number exact rather than accidental would protect the proton on purpose.)*

N.3 Step 3 — the geometric derivation: gauged baryon number and boundary force laws

In GG-Theory all three facts descend from the boundary geometry. The inverse-square laws are the boundary Green's functions of the seam Laplacian, derived explicitly from the three-dimensional Gauss law in the next subsection. Anomaly freedom is not an imposed condition but is inherited from upstream: BI-6 (GG-A3-BI6) proves the parent bundle anomaly-free before any low-energy descent, so the family-by-family cancellation is a theorem of the legal parent theory rather than a numerical accident. Most decisively, baryon number is *gauged*: the GG-6 extension carries a local $\mathrm{U}(1)_B$ that makes B an exact rather than accidental symmetry, excluding the X, Y-type operators that would let the proton decay. The proton is therefore not merely long-lived but *absolutely stable* in the minimal construction — a sharp, falsifiable difference from ordinary four-dimensional grand unification. The detailed mechanism, and its contrast with minimal 4D unification, is set out in the proton-stability subsection below. *(Physics intuition: where ordinary unification opens a door between quarks and leptons and then must explain why the proton does not walk through it, the geometric construction never builds the door. Local baryon number is a gauged wall, not an accidental fence, so the proton is stable for the same kind of reason the photon is massless — a symmetry protects it.)*

N.4 Green's functions, Gauss laws, and the inverse-square form of Newton and Coulomb

This appendix gives the compact mathematical reason why both Newton's and Coulomb's laws take inverse-square form in three spatial dimensions.

N.5 Green's function of the 3D Laplacian

Let $G(\mathbf{x})$ solve

$$\nabla^2 G(\mathbf{x}) = -\delta^{(3)}(\mathbf{x}). \tag{N.1}$$

The rotationally symmetric solution is

$$G(r) = \frac{1}{4\pi r}. \tag{N.2}$$

Therefore any static massless field whose source equation reduces to a Poisson problem in three dimensions produces a $1/r$ potential and a $1/r^2$ force.

N.6 Application to Newtonian gravity

For a point mass M,

$$\Phi_N(\mathbf{x}) = -4\pi G_N M\, G(r) = -\frac{G_N M}{r}. \tag{N.3}$$

Differentiating gives

$$\mathbf{F}_N = -m\nabla\Phi_N = -G_N \frac{mM}{r^2}\,\hat{\mathbf{r}}. \tag{N.4}$$

N.7 Application to electrostatics

For a point charge q,

$$\phi(\mathbf{x}) = \frac{q}{\varepsilon_0} G(r) = \frac{q}{4\pi\varepsilon_0 r}, \tag{N.5}$$

so the electric field is

$$\mathbf{E} = -\nabla\phi = \frac{q}{4\pi\varepsilon_0 r^2}\,\hat{\mathbf{r}}, \tag{N.6}$$

and the force on a second charge q' is

$$\mathbf{F}_C = q'\mathbf{E} = \frac{qq'}{4\pi\varepsilon_0 r^2}\,\hat{\mathbf{r}}. \tag{N.7}$$

N.8 Why the GG-6 viewpoint adds more than this elementary derivation

The Green's-function derivation explains the inverse-square form, but not why the relevant massless fields should exist or why one should have both a geometric and a gauge sector in the first place. The GG-6 framework adds exactly that missing layer. It says that the two long-range sectors are boundary exports of one composite geometry–gauge constitution. The familiar inverse-square laws are therefore not merely historical laws; they are the infrared trace of a deeper unified structure.

N.9 Phenomenological comparison between historical unification language and the GG-6 seam language

This appendix supports the professional phenomenological tone of the revised manuscript by making explicit how the present paper relates to familiar historical languages of unification. The intent is not polemical. The intent is to help a reader trained in ordinary gauge-theory phenomenology locate exactly what is standard, what is reinterpreted, and what is genuinely new in GG-6/GDOS.

N.10 Why this comparison is useful

A phenomenological paper is easier to evaluate when the reader can quickly answer three questions:

(i) Which features are simply inherited from the successful Standard-Model and historical GUT traditions?

(ii) Which features are retained but geometrically reinterpreted?

(iii) Which features are genuinely additional structures introduced by the new framework?

The present appendix answers those questions in a compact but explicit way.

N.11 The historical baseline

The baseline traditional story is familiar. One starts from a 4D gauge theory, asks whether the gauge couplings suggest unification at high energy, and then studies simple groups such as SU(5) or larger groups as candidate UV symmetries. The Higgs sector is usually introduced by scalar multiplets, proton stability is a strong empirical constraint, and flavor is often left largely as a separate problem.

This story contains genuine achievements. It correctly emphasizes gauge symmetry, representation theory, anomaly cancellation, RG running, and symmetry breaking. The present paper does not discard any of those achievements. Instead it asks whether they can be embedded in a more constrained constitutional and open-system setting.

N.12 What remains standard in GG-6

Several core ingredients remain completely standard at the phenomenological level.

(1) The low-energy gauge group visible in laboratory physics is still the Standard-Model group before electroweak breaking and its familiar remnants after breaking.

(2) Representation theory still matters. Quarks and leptons are still read in a language inherited from unified group embeddings.

(3) Renormalization-group running still matters. The present paper does not reject RG thinking; it geometrizes it.

(4) CKM and PMNS matrices remain the correct low-energy observable language for flavor mixing.

(5) Benchmark comparison to precision data still uses the ordinary observable basis collected by the PDG and neutrino global analyses.

These points are worth stating because they show that GG-6 is not attempting to replace successful phenomenology by unfamiliar jargon. It is attempting to give a deeper origin to structures phenomenology already knows are real.

N.13 What is reinterpreted rather than discarded

The clearest examples of reinterpretation are the following.

Unification scale. In ordinary language, the unification scale is where couplings approximately meet. In GG-6, the relevant ultraviolet seam is tied to the six-dimensional stage and to the scale chain $E_{\mathrm{Pl}} \to E_{\mathrm{GUT}} \to M_{\mathrm{lock}}$. The observable meeting of couplings is thus a seam consequence rather than the sole definition of the UV scale.

Higgs mechanism. In ordinary language, the Higgs is a scalar doublet with a symmetry-breaking potential. In GG-6, this language survives only as a low-energy shadow. The fundamental variable is the Wilson-line holonomy and the selected vacuum branch.

Yukawa sector. In ordinary language, Yukawa matrices are phenomenological inputs. In GG-6 they are effective seam images of overlaps and family projectors.

Mass. In ordinary language, mass may be introduced sector by sector. In GG-6, mass is recast as a geometric spectral gap and an GFF-curvature quantity.

Observable. In ordinary phenomenology, an observable is often treated as a renormalized number quoted in a review table. In GDOS it is a registered packet export under a lawful boundary map.

N.14 What is genuinely additional in GG-6/GDOS

The following structures are not merely reinterpretations of standard jargon; they are genuine additions.

(1) The constitutional layer of four axioms and seven postulates.

(2) The six-dimensional bulk as the minimal admissible stage.

(3) The BI-6 anomaly-cancellation structure with OGN-fixed coefficients.

(4) Local $\mathrm{U}(1)_B$ and local $\mathrm{U}(1)_{Qi}$ as part of the visible unified group.

(5) The OGGEP/GDOS observational layer that turns the UV theory into an explicit open-system boundary theory.

(6) The SIP/GFF/DGI/GSSB/GA/Qi-4 runtime vocabulary used to organize branch selection and measurement.

These are the distinctive conceptual features that the present paper asks the reader to evaluate.

N.15 A compact comparison table

Table 87: Phenomenological comparison between historical unification language and the GG-6/GDOS reinterpretation. The left column is stated in the language its own authors used, so that the comparison is between positions rather than between caricatures. The table is a reading aid for the Parts that follow and not an argument against the historical language: on several rows the two columns describe the same physics, and the difference is in what each treats as primitive.

Phenomenological issue	**Historical unification language**	**GG-6/GDOS language**
Unified gauge structure	Simple 4D group such as SU(5)	Composite 6D geometry–gauge structure with $\mathrm{SU}(5)\times\mathrm{U}(1)_B\times\mathrm{U}(1)_{Qi}$ on a bulk
Scale running	4D RG flow of effective field theory	Radial transport in $r=\ln(E_{\mathrm{GUT}}/\mu)$ plus seam evaluation
Electroweak breaking	Scalar potential and VEV	Wilson–Hosotani holonomy selection and lock-seam export
Flavor masses	Yukawa inputs	Overlap/projector codebook with QD completion
CKM/PMNS	Observable unitary matrices	Observable matrices derived from family-projector misalignment
Proton stability	Strong empirical constraint, often problematic in minimal GUTs	Local $\mathrm{U}(1)_B$ and operator-selection discipline
Measurement theory	Usually implicit or external to the field theory	Explicit OGGEP/GDOS boundary map
Open-system language	Rare in particle phenomenology	Built in from AX-3 onward
Hierarchy/naturalness	Bare-scalar sensitivity problem	Finite holonomy potential and multiplicative throat hierarchy
Status of parameters	Often free inputs of the effective theory	Seam observables of a constrained packet

N.16 Why the comparison supports the present paper's ambitions

The comparison table helps clarify the ambition level of GG-A9-SM. The paper is not content to say that a few Standard-Model numbers happen to be reproduced by a creative formula. Its stronger claim is that once the phenomenological target is re-embedded in a more constrained geometric and operational framework, a large part of the Standard-Model parameter problem becomes derivable rather than merely fittable.

Whether one ultimately agrees with every detailed formula in the later sections, this is the proper standard by which the paper should be judged. It is a paper about the origin of the parameter structure, not just about numerical coincidence.

N.17 Final interpretive remark

A useful way to summarize the appendix is the following. Historical unification language already knew that gauge symmetry and representation theory are powerful. GG-6/GDOS asks whether gauge symmetry, geometry, anomaly freedom, scale, measurement, and open-system dynamics can all be treated as one non-separable chain. The present revised manuscript is an attempt to show what happens when that question is answered in the affirmative and then carried all the way through the phenomenology of couplings, masses, and mixings.

N.18 Proton stability, local $\mathrm{U}(1)_B$, and why the GG-6 Standard-Model foundation differs from minimal 4D unification

A recurring phenomenological objection to simple grand unification is proton instability. The objection is entirely reasonable: any credible UV completion of the Standard Model must respect the extraordinary empirical longevity of the proton. Because the foundation sections of the present paper emphasize historical SU(5) as the correct conceptual starting point, it is useful to state explicitly why the GG-6 realization differs in its treatment of baryon number and proton decay.

N.19 Why proton stability is a foundational rather than secondary issue

Proton stability is not merely another observable among many. It is one of the sharpest filters on theories that unify quarks and leptons. If a candidate unification scheme predicts rapid baryon-number violation, then it fails before one even reaches the level of detailed flavor fitting. This is why the present paper treats local $\mathrm{U}(1)_B$ as part of the visible unified structure rather than as an optional afterthought.

In other words, proton stability belongs to the Standard-Model foundation section for the same reason anomaly freedom does: it is part of the admissibility filter that any later parameter derivation must inherit.

N.20 The usual tension in minimal 4D unification

In minimal 4D SU(5)-type unification, the same unified structure that puts quarks and leptons into common multiplets also tends to introduce heavy bosons or effective operators that mediate

baryon-number violation. The tension is famous: the unification idea is elegant, but proton decay becomes a powerful empirical threat.

The present paper does not need to rehearse every historical model-building route around this tension. It is enough to emphasize the conceptual point: a unified group alone does not automatically guarantee sufficiently strong protection of the proton. Additional structure is required.

N.21 Why GG-6 treats baryon number differently

GG-6 provides that additional structure by elevating baryon number to a local $U(1)_B$ factor in the visible unified sector. At the conceptual level this changes the problem from "how can we suppress dangerous baryon-violating operators well enough?" to "which operators are constitutionally admissible once local baryon number is part of the gauge structure?"

That is a major shift. It means proton stability is not merely an accidental numerical consequence of a high mass scale. It is tied to the admissible operator content of the theory.

N.22 Why this matters for a parameter paper

One might wonder why an appendix on proton stability belongs in a paper devoted to Standard-Model parameters. The answer is that the parameter paper is not being written inside an arbitrary effective theory. It is being written inside a very specific ultraviolet framework. The credibility of the entire parameter construction depends on whether that framework respects the strongest known structural constraints on the visible world. Proton stability is one of those constraints.

This appendix therefore supports the main text in a simple way: it reminds the reader that the same framework used to derive couplings, masses, and mixings also addresses one of the most severe historical problems of unified model building.

N.23 Compact comparison

Table 88: Proton stability in minimal historical four-dimensional unification and in the geometric foundation this paper uses, question by question. The comparison is included because proton decay is the constraint that ended the first generation of unified models, and a framework that did not address it would be incomplete however well it did elsewhere. The table states the contrast; the quantitative treatment is delegated to the companion paper that owns it.

Question	Minimal historical 4D unification	GG-6 foundation used in this paper
Status of baryon number	Often accidental or approximate at low energy	Local $U(1)_B$ in the visible unified structure
Proton-decay issue	Potentially severe phenomenological tension	Reframed as an operator-admissibility question under local baryon number
Role in parameter derivation	Often treated as a separate model-building constraint	Built into the foundation inherited by the parameter derivation

The practical moral is straightforward. In the present paper, proton stability is not disconnected from the rest of the phenomenology. It is one of the reasons the GG-6 route to the Standard Model is being taken seriously in the first place.

Problems and worked solutions. N.4.1. Derive both inverse-square laws from one Green's function.

Solution. Step 1: the static potential of a point source obeys $\nabla^2 G = \delta^3(\mathbf{r})$, whose solution (N.2) is $G = -1/4\pi r$ — fixed entirely by the dimensionality of space, via Gauss: the flux of ∇G through a sphere is constant, and the sphere's area grows as $4\pi r^2$. Step 2: feed the two static sectors the same function: gravity's potential $\phi = -G_N M/r$ gives $F = G_N Mm/r^2$ (N.4); electrostatics' $\phi = q/4\pi\epsilon_0 r$ gives Coulomb's $F = q_1 q_2/4\pi\epsilon_0 r^2$ (N.7). Step 3: in D spatial dimensions the same argument gives $F \propto r^{-(D-1)}$ — so the observed r^{-2}, tested from micrometers to solar-system scales, is a live measurement of $D = 3$: the boundary force laws are a standing check that the *visible* world has exactly the dimensionality the seam assigns it, with the extra directions compactified out of the flux count. Physics intuition: an inverse-square law is not a law about forces but a law about real estate — flux shared over the surface of a growing sphere; two interactions as different as gravity and electricity obey the same power because they rent area in the same three-dimensional space.

N.4.2. Compute the 10^{39} and say what kind of explanation could reach it.

Solution. Step 1: form the dimensionless ratio of the electric to gravitational force between a proton and an electron (any separation — both fall as r^{-2}): $e^2/4\pi\epsilon_0 \div G_N m_p m_e$. Step 2: numerically, $\alpha\hbar c = 2.31 \times 10^{-28}$ N m^2 against $G_N m_p m_e = 1.02 \times 10^{-67}$ N m^2: ratio 2.3×10^{39}. Step 3: no perturbative mechanism generates 10^{39} from order-one inputs, but exponentials of modest integers do it effortlessly — $e^{91} \simeq 3 \times 10^{39}$ — and the paper's worked instance is the hierarchy identity $v/E_{\rm Pl} = \frac{1}{2}e^{-35}(2/5)^3$ (assembly-interlude register: integers of the ledger, not tuned decimals); gravity is feeble because it alone couples through the full exponential depth of the radial dimension. Physics intuition: the most extreme number in everyday physics — the reason a toy magnet beats the whole planet's gravity — is, on this reading, a *length* in the fifth dimension measured in e-folds; hierarchy problems dissolve when ratios stop being dials and become distances.

N.6.1. Kill classic SU(5) by arithmetic, and state why the export is immune.

Solution. Step 1: four-dimensional X, Y exchange generates the $\Delta B = 1$ operator $qqql/M_X^2$, giving the standard estimate $\Gamma_p \sim \alpha_G^2 m_p^5/M_X^4$. Step 2: with $\alpha_G \simeq 1/25$ and $M_X \simeq 10^{15}$ GeV: $\Gamma_p \simeq (1/625)(0.938)^5/10^{60}$ GeV $= 1.2 \times 10^{-63}$ GeV, i.e. $\tau_p = \hbar/\Gamma_p \simeq 1.8 \times 10^{31}$ yr — three orders of magnitude below the Super-Kamiokande bound $\tau(p \to e^+\pi^0) > 2.4 \times 10^{34}$ yr: the 1974 theory is excluded by a wide margin, not a technicality. Step 3: the export escapes at the operator level, not by raising M_X: the X, Y are bulk-only (M.3.1), so no four-dimensional exchange exists, and every $\Delta B \neq 0$ Wilson coefficient vanishes under the exact global baryon number of Theorem 86.1 — conditional on its stated hypotheses, with (H3) the anomaly-inflow audit of M.6.1. Physics intuition: classic unification put the quark and the lepton in one multiplet and paid with the proton's life; the six-dimensional version keeps the marriage but moves the matchmakers out of reach — the difference between suppressing an operator and never writing it is the difference between 10^{31} years and forever.

N.6.2. Show that even the anomaly cannot kill a proton: $\Delta B \in 3\mathbb{Z}$.
Solution. Step 1: the one exact non-conservation of baryon number is the electroweak anomaly, $\partial_\mu J^\mu_B = N_{\rm gen}(g^2/32\pi^2)\,\mathrm{tr}\,W\widetilde{W}$ — each family contributes once, so any topological transition changes B by $N_{\rm gen}\,n = 3n$. Step 2: a proton decay is $\Delta B = -1$; but $-1 \notin 3\mathbb{Z}$, so no anomalous process, however violent, can remove a single nucleon — the minimal event costs three baryons at once (and three leptons with it, preserving $B - L$). Step 3: the rate is in any case tunneling-suppressed by the instanton action $S = 8\pi^2/g^2 \approx 187$ — utterly negligible today — but the $3\mathbb{Z}$ selection rule is the structural point: it ties proton safety to the same $N_g = 3$ that the family pincer fixed. Physics intuition: even the one legal loophole in baryon conservation is quantized in family units — the anomaly can reshuffle the cosmic books only three baryons at a time, so the generation count literally stands guard over every atom; $N_g = 3$ is not flavor trivia, it is nuclear security.

N.6.3. Defend the terminated $\mathrm{U}(1)_B$ with the massive-photon analogy.
Solution. Step 1: the objection — "your $\mathrm{U}(1)_B$ has no light gauge boson — its vector tower starts at the lock scale — so baryon number is broken and the proton should decay." Step 2: separate the two things a gauge symmetry does: the *local* redundancy (its massless mode is removed — the vector's zero mode is killed by the boundary condition, and the lightest Z'_B sits at $M_{\rm lock}$) and the *global* charge (exact — every operator still carries its B, and $\Delta B \neq 0$ coefficients vanish identically). Step 3: the controlled analogy: give the photon a mass and electric charge is still exactly conserved — gapping or removing the force does not touch the bookkeeping; hence the prediction is "exact global B, residual to a boundary-terminated $\mathrm{U}(1)_B$," with the massive Z'_B tested through the lock cluster and the force's absence through the fifth-force channel of the falsification catalog. Physics intuition: symmetries have a body and a soul — the gauge boson is the body, the conservation law the soul; the termination removes the body's long range entirely and leaves the soul untouched, and the proton answers to the soul.

Take-away. *Standard result:* inverse-square laws, anomaly-free generations, and a proton stable to $> 10^{34}$ yr. *What it assumed:* anomaly cancellation imposed as a consistency check, and baryon number protected only by accident. *The geometric reading:* Green's functions on the boundary give the force laws; anomaly freedom is inherited from the bulk identity; and a gauged $U(1)_B$ makes proton stability absolute — a prediction, not a tolerance.

O The strong-CP problem, confinement, and the mass-energy dictionary

What this appendix teaches. The protected corners of the Standard Model — $\bar\theta$ and the neutron-EDM bound; color confinement and the flux tube; and the mass-energy dictionary that keeps one ontology for mass across sectors: the textbook statements, the two famous non-explanations (why $\bar\theta$ is small, why quarks never appear alone), and the geometric readings — the protected topological zero, and mass as entries of one seam ledger.

Read alongside: Peskin & Schroeder (1995) for the instanton and θ-vacuum background; Peccei & Quinn (1977) for the Peccei–Quinn mechanism, whose relaxation the cohort realizes geometrically through the Kaxion of GG-C1-Kaxion rather than through an added global symmetry.

This appendix takes up two of the protected corners of the Standard Model — the strong-CP angle and color confinement — together with the unified language for mass that the framework uses throughout. Each is treated in the three-step course form: the textbook statement, what it leaves unexplained, and the geometric reading. We begin with the twenty-eighth and last free parameter, the strong-CP angle. The instanton and topological background to this section is developed in Peskin & Schroeder (1995).

O.1 Step 1 — the strong-CP problem as a graduate student meets it

The $\bar{\theta}$ term. Quantum chromodynamics admits a gauge-invariant, renormalizable term that violates CP,

$$\mathcal{L}_\theta = \theta \, \frac{g_s^2}{32\pi^2} \, G^a_{\mu\nu} \widetilde{G}^{a\,\mu\nu}, \tag{O.1}$$

a total derivative that nonetheless contributes through topologically nontrivial gauge configurations (instantons). Chiral rotations of the quark fields shift θ by the phase of the quark mass determinant, so the physical, basis-independent quantity is

$$\bar{\theta} = \theta_{\rm QCD} + \arg\det(M_u M_d). \tag{O.2}$$

(Physics intuition: $\bar{\theta}$ measures how much the strong vacuum distinguishes left from right. There is no symmetry reason for it to vanish, and on general grounds a dimensionless angle should be of order one.)

The experimental bound. A nonzero $\bar{\theta}$ would give the neutron an electric dipole moment $d_n \sim \bar{\theta} \times (2\text{–}4) \times 10^{-16}\ e\,\mathrm{cm}$, the chiral estimate (Baluni, 1979; Crewther et al., 1979); this paper adopts 2.4×10^{-16} throughout, and the conclusion below is unchanged anywhere in that range. The measured bound (Abel et al., 2020) $d_n < 1.8 \times 10^{-26}\ e\,\mathrm{cm}$ forces

$$\bar{\theta} \lesssim 7.5 \times 10^{-11}. \tag{O.3}$$

The strong-CP problem is the question of why this angle is ten orders of magnitude below its natural value. The leading proposed answer is the Peccei–Quinn axion (Peccei & Quinn, 1977; Weinberg, 1978; Wilczek, 1978), taken up within the cohort as the cold-dark-matter Kaxion of GG-C1-Kaxion (Arisaka, 2026, C1-Kaxion): a new symmetry that promotes $\bar{\theta}$ to a dynamical field relaxing to zero, predicting a light pseudoscalar that has gone undiscovered through more than forty years of searches. *(Physics intuition: the neutron is a sensitive CP antenna. If the strong vacuum were even slightly chiral, the neutron would carry a measurable dipole; that it does not, to one part in 10^{10}, is the puzzle.)*

O.2 Step 2 — what the strong-CP sector assumes but cannot explain

In the Standard Model $\bar{\theta}$ is simply the twenty-eighth free parameter, tuned to zero by hand to one part in 10^{10} with no protecting symmetry. The axion mechanism would explain the smallness,

but at the cost of a new global symmetry of uncertain quality and a new particle that remains unobserved. The Standard Model, on its own, offers no reason for $\bar{\theta}$ to be small.

O.3 Step 3 — the geometric derivation: a protected topological readout

In GG-Theory $\bar{\theta}$ is not tuned, and its smallness needs no Peccei–Quinn global symmetry; the cohort's Kaxion relaxes the observable angle to the CKM floor $\sim 10^{-17}$ (GG-C1-Kaxion), and the derivation below estimates the bare angle from which that relaxation starts. Both terms of (O.2) are fixed at first order for structural reasons — the first to zero, the second to 0 or π — and the infrared region suppresses any residue to far below detection. The first term vanishes because the visible four-dimensional Lagrangian is *exported* from the six-dimensional bulk, not assembled by listing all gauge invariants: the topological density $G\widetilde{G}$ can appear only if the bulk descent generates a coefficient for it, and GG-A2-GG6/GG-A3-BI6 introduce no such coefficient, so $\theta^{(1)}_{\text{top}} = 0$. The second term is 0 or π because the family-link texture is Hermitian (Theorem 79.1); the older argument — that in the first-order quark codebook the singular values are positive and the physical phases live only in the misalignment matrices $L_u^\dagger L_d$, giving $\arg\det U_u + \arg\det U_d = 0$ — is incomplete as stated and is retained only as the physical picture. Hence $\bar{\theta}^{(1)} \in \{0, \pi\}$, the zero branch selected by the neutron bound and the sign lemma L9's. The smallest constitutionally natural residue is the triple product

$$\varepsilon^{\text{QD}}_{\text{topo}} = \underbrace{\sin(2\pi/L)}_{\text{compact-phase decoration}} \quad \underbrace{2^{-J}}_{\text{minimal dyadic leakage}} \quad \underbrace{e^{-K}}_{\text{deep topological suppression}} \simeq 4.7 \times 10^{-18}, \qquad \text{(O.4)}$$

some seven orders of magnitude below the experimental bound (O.3); the explicit reduction is recorded in Appendix O.4. *(Physics intuition: strong-CP is solved here not by adding a particle but by the* absence of a knob. *The bulk simply does not hand the boundary a θ-term, and the first-order quark determinant is real, so the angle starts at* 0 *or* π *with the sign the one open item; the smallest thing that can move it off the branch is a deep topological leakage through the throat, and that is suppressed by the same* e^{-K} *scale and* 2^{-J} *dyadic factors that appear everywhere else — down to* 10^{-18}*, a minimum the ledger admits and not a bound on shallower windings (§79). The neutron antenna stays silent because there is nothing to drive it.)* In the honesty triage $\bar{\theta}$ is recorded as **Derived (cond.)**: Theorem 79.1 fixes the first-order angle to $\{0, \pi\}$, and the sign and the leakage beyond first order are not yet proved end-to-end (the named lemma L9).

O.4 Explicit topological-phase reduction for the strong-CP sector

This appendix records the logic of the strong-CP reduction in a more algebraic form. The main text emphasized the physical combination $\bar{\theta} = \theta_{\text{top}} + \arg\det(M_u M_d)$. Two claims were used: first, there is no independent first-order seam θ_{top} term; second, the determinant of the first-order quark blocks is real, so its phase is 0 or π.

The first claim is a statement about admissible descent. The visible seam Lagrangian is not written by listing all 4D gauge-invariant terms. It is exported from the 6D bulk. Therefore the 4D topological density $G\widetilde{G}$ can appear only if the descent of the admissible bulk action generates

a free coefficient multiplying it. The written GG-A2-GG6 and GG-A3-BI6 papers do not introduce such a coefficient. Hence $\theta_{\rm top}^{(1)} = 0$.

The second claim may be expressed in terms of the polar decomposition. Write

$$M_f = H_f U_f, \qquad H_f = (M_f M_f^\dagger)^{1/2}, \qquad U_f \in U(3). \tag{O.5}$$

Then

$$\arg\det M_f = \arg\det U_f. \tag{O.6}$$

In the present first-order codebook the singular values are all positive and — this is the older reading, incomplete as stated, kept as the physical picture — the nontrivial phases enter only in relative misalignment matrices such as $L_u^\dagger L_d$, not in an independent determinant phase of U_f. What Theorem 79.1 proves is the weaker statement $\arg\det(M_u M_d)^{(1)} \in \{0, \pi\}$, the sign being lemma L9's gap (a′).

The QD estimate may be derived by the same three-factor logic used in the main text:

$$\varepsilon_{\rm topo}^{\rm QD} = \underbrace{\sin(2\pi/L)}_{\text{compact-phase decoration}} \quad \underbrace{2^{-J}}_{\text{minimal dyadic leakage}} \quad \underbrace{e^{-K}}_{\text{deep topological suppression}}, \tag{O.7}$$

which for $(L, J, K) = (5, 7, 35)$ gives 4.68×10^{-18}. The value is not claimed to be the final output of a completed nonperturbative topological computation. It is the smallest constitutionally natural scale available to the finished parameter paper.

O.5 Arithmetic of the derived electroweak constants

The main text derived the familiar electroweak constants from the primary ledger. Here the arithmetic is recorded explicitly.

From the primary gauge/Higgs block one has

$$\alpha_{\rm em}^{-1} = 128, \qquad \sin^2\theta_W = \frac{25}{108}.$$

Therefore

$$\alpha_2^{-1} = \alpha_{\rm em}^{-1}\sin^2\theta_W = 128 \times \frac{25}{108} = \frac{800}{27} = 29.6296296, \tag{O.8}$$

and

$$\alpha_Y^{-1} = \alpha_{\rm em}^{-1} - \alpha_2^{-1} = 128 - \frac{800}{27} = \frac{2656}{27} = 98.3703704. \tag{O.9}$$

Hence

$$g_2 = \sqrt{4\pi\alpha_2} = 0.651241, \qquad g_Y = \sqrt{4\pi\alpha_Y} = 0.357415, \qquad e = \sqrt{4\pi\alpha_{\rm em}} = 0.313329. \tag{O.10}$$

For the weak scale,

$$v = \frac{E_{\rm Pl}}{2e^{35}}\left(\frac{2}{5}\right)^3 = 246.331354 \text{ GeV} \quad \text{(first order)}, \tag{O.11}$$

which gives

$$G_F^{(1)} = \frac{1}{\sqrt{2}\,v^2} = 1.16532111 \times 10^{-5}\ \text{GeV}^{-2}. \tag{O.12}$$

Likewise

$$m_W^{(1)} = \frac{1}{2} g_2 v = 80.2106\ \text{GeV}, \qquad m_Z^{(1)} = \frac{1}{2} v\sqrt{g_2^2 + g_Y^2} = 91.4965\ \text{GeV}. \tag{O.13}$$

The same steps can be repeated with the QD values

$$\alpha_{\text{em}}^{-1} = 127.9296875, \qquad v^{\text{QD}} = 246.220000\ \text{GeV},$$

leading to

$$g_2^{\text{QD}} = 0.651420, \qquad g_Y^{\text{QD}} = 0.357513, \qquad G_F^{\text{QD}} = 1.16637539 \times 10^{-5}\ \text{GeV}^{-2},$$

$$m_W^{\text{QD}} = 80.3700\ \text{GeV}, \qquad m_Z^{\text{QD}} = 91.1900\ \text{GeV}.$$

These are the numbers quoted in the main text.

O.6 Step 1 — confinement and the mass-energy of matter as a graduate student meets it

Asymptotic freedom and dimensional transmutation. The strong coupling weakens at high energy and grows at low energy (Appendix D.3.4). As one runs downward, α_s becomes large near a scale at which perturbation theory breaks down, and quarks and gluons are confined into color-singlet hadrons. That scale is generated from the dimensionless coupling by *dimensional transmutation*,

$$\Lambda_{\text{QCD}} = \mu\, \exp\!\left(-\frac{8\pi^2}{b_0\, g_3^2(\mu)}\right) \simeq 200\ \text{MeV}, \tag{O.14}$$

a mass scale conjured from a pure number. *(Physics intuition: QCD has no mass in its Lagrangian for the light quarks, yet it produces the proton. The trick is that the running coupling hits strong coupling at a definite energy, and that energy — where the force can no longer be ignored — is itself the scale of hadron masses.)*

Most mass is binding energy. The proton weighs $\simeq 938$ MeV, but its three valence quarks carry only a few MeV of current mass. More than 98% of the proton's mass — and hence of all visible matter — is the energy of the confined gluon field, $E = mc^2$ made concrete. *(Physics intuition: you are made almost entirely of binding energy. The quark masses contribute a percent; the rest is the stored energy of the strong field holding the quarks together. Mass, here, is mostly trapped motion and field.)*

Worked example (STANDARD TEXTBOOK PEDAGOGY). Running $\alpha_s(M_Z) = 0.118$ down with one-loop b_0 for QCD gives Λ_{QCD} of order a few hundred MeV — the observed hadronic scale — confirming that the proton mass is fixed by where the coupling runs strong, not by the Higgs Yukawa of the light quarks.

O.7 Step 2 — what confinement assumes but cannot explain

Two things are taken for granted. First, confinement itself is not analytically proven: that Yang–Mills theory has a mass gap and confines is one of the Clay Millennium Problems, established only numerically on the lattice, via Wilson's lattice formulation of gauge theory (K. G. Wilson, 1974). Second, the value of $\Lambda_{\rm QCD}$ relative to the electroweak or Planck scale is, in the bottom-up theory, just where the measured coupling happens to run strong — an emergent input with no deeper account. *(Physics intuition: the Standard Model can compute how the strong coupling runs, but it cannot say why it runs strong at* 200 MeV *rather than at some other scale, nor prove from first principles that quarks must be permanently jailed.)*

O.8 Step 3 — the geometric derivation: confinement as a scale window

In GG-Theory the running of the couplings is radial transport in the sixth dimension, and the confinement scale is set by the geometry rather than left as an emergent accident. The remaining subsections record the explicit formulas: the radial renormalization-group law, the dimensional-transmutation scale read as a *scale window*, the role of local $\mathrm{U}(1)_B$ in protecting the baryon sector, and the unified mass-energy dictionary in which every sector's mass — weak-boson, Higgs, fermion, baryon, gravitational — is one and the same seam spectral gap in the GFF ledger. *(Physics intuition: the strong coupling runs because moving in energy is moving radially through the infrared region, and the place where it runs strong — the confinement scale — is a fixed window of that throat. The proton's mass is then a geometric quantity: the spectral gap of the strong seam, the same ledger entry that, in other sectors, reads out as the W mass or the Higgs mass.)*

O.9 Running couplings, dimensional transmutation, and the baryonic confinement window

The main text tied radial transport to renormalization-group flow. This appendix records a few useful formulas explicitly.

For a generic gauge factor,

$$\mu\frac{dg_i}{d\mu} = \beta_i(g_i) = -\frac{b_i}{16\pi^2}g_i^3 + \cdots , \tag{O.15}$$

which is equivalent to

$$\frac{d}{dr}\frac{1}{g_i^2} = \frac{b_i}{8\pi^2} + \cdots , \qquad r = \ln\!\left(\frac{E_{\rm GUT}}{\mu}\right). \tag{O.16}$$

Thus the sign of b_i determines whether the seam coupling strengthens or weakens as one moves to larger r.

For QED the beta function is positive in α, so the electric charge is screened in the infrared and unscreened in the ultraviolet. For QCD the sign flips because of non-Abelian self-interactions, and one obtains the RG-invariant scale

$$\Lambda_{\rm QCD} = \mu\exp\!\left(-\frac{8\pi^2}{b_0 g_3^2(\mu)}\right) \quad \text{(one loop)}. \tag{O.17}$$

This is dimensional transmutation: the strong interaction generates a mass scale from a dimensionless coupling.

The additional GG-6 point is the role of local $U(1)_B$. Confinement by itself requires asymptotic color singlets. Local baryon number then organizes which singlets are baryonic and excludes operators that would violate the baryonic label. One may therefore speak of a *baryonic confinement window*: the regime in which QCD has already forced color neutralization while local $U(1)_B$ still protects the baryon sector from unified-theory decay operators.

O.10 The unified mass-energy dictionary

The mass-origin manuscripts repeatedly emphasize that "mass" is a sector-dependent word. This appendix summarizes the dictionary used in the present paper.

Table 89: Dictionary from sector-specific mass language to the unified GG-6/GFF language. Each row takes a mass statement as its sector normally makes it, restates it in the unified language, and gives the formula carrying the restatement. The fourth column is what makes the restatements more than verbal: the same geometric object — a selected branch, a counting fraction, an overlap depth — appears in the formula for a boson mass and for a fermion mass, and the dictionary is what makes that visible.

Sector language	Standard statement	GG-6/GFF reinterpretation	Representative formula
Weak boson mass	Higgs VEV gives W and Z masses	The doublet's vacuum value, its scale set by the Wilson-line selector (the holonomy's own gap is ultraviolet, §34.6)	$m_W = gv/2$
Higgs mass	Curvature of scalar potential	$2\lambda_H v^2$ with v from the selector and λ_H on H-L2	$m_h^2 = 2\lambda_H v^2$
Fermion mass	Yukawa coupling times VEV	Overlap integral of left, Higgs, and right internal profiles	cf. §85
Baryon mass	QCD energy plus anomaly	GFF split $E_{\mathrm{GEP}} + E_{\mathrm{CEP}}$	cf. §85
Gravitational mass-energy	Source of curvature	Same seam action varied with respect to $g_{\mu\nu}$	$T_{\mu\nu} = -2\delta S_{\mathrm{eff}}/\delta g^{\mu\nu}$

The dictionary matters because it prevents an easy conceptual mistake. Different sectors use different calculational machinery, but the present paper does not treat them as different ontologies. They are different modes of one seam ledger.

Problems and worked solutions. O.1.1. Turn the neutron's silence into a bound on $\bar{\theta}$.

Solution. Step 1: a nonzero $\bar{\theta}$ (O.2) induces a neutron electric dipole moment of the standard estimated size $d_n \sim 2.4 \times 10^{-16}\,\bar{\theta}\,e\,\mathrm{cm}$ (the chiral-loop scale m_q/m_n^2 with order-one hadronic factors). Step 2: the experimental bound $|d_n| < 1.8 \times 10^{-26}\ e\,\mathrm{cm}$ then gives $|\bar{\theta}| < 1.8 \times 10^{-26}/2.4 \times 10^{-16} = 7.5 \times 10^{-11}$ — the bound quoted as (O.3). Step 3: state the puzzle exactly: $\bar{\theta}$ is a compact angle, any value in $[0, 2\pi)$ equally lawful in the textbook, and the flavor sector demonstrates (chapter I) that the theory has no aversion to large phases — yet this one angle sits below 10^{-10}. Physics intuition: the neutron is a CP antenna — a charge-neutral object whose charge *layout* would tilt if the strong force distinguished past from future mirror-wise; fifty years of listening have heard nothing, and the textbook's only comment is that someone must have set the dial with eleven-digit care.

O.4.1. Evaluate the protected residue and place it against the antenna.

Solution. Step 1: with the export generating no θ-term and the first-order determinant real (O.4.2; Theorem 79.1), the constitutionally minimal leakage is the triple product (O.4): the compact-phase decoration $\sin(2\pi/L)$, the minimal dyadic depth 2^{-J}, and the topological lock e^{-K}. Step 2: evaluate with the ledger: $\sin(2\pi/5) = 0.95106$, $2^{-7} = 0.0078125$, $e^{-35} = 6.305 \times 10^{-16}$: product $= 4.7 \times 10^{-18}$. Step 3: that is seven orders of magnitude below the 7.5×10^{-11} of O.1.1 — silent to any foreseeable neutron experiment — and it is built from the same three integers (L, J, K) that price the Cabibbo pentagon, the fine-structure dyad, and the weak scale. Physics intuition: the strong-CP problem is solved here by poverty, not fine-tuning — the theory simply cannot afford a θ-term; the largest CP leak this estimate constructs is a pentagon phase filtered through seven halvings and the full thirty-five-fold lock, and that number was spent elsewhere in the codebook long before the neutron asked.

O.4.2. Work the first-order determinant sign.

Solution. Step 1: the physical angle is $\bar{\theta} = \theta_{\mathrm{top}} + \arg\det(M_u M_d)$ (O.2); polar-decompose each mass block (O.5): $M_f = H_f U_f$ with H_f positive. Step 2: at first order the codebook's singular values (tolls and codes) are real and positive, so $\arg\det H_f = 0$, and the unitary factors are the eigenvector rotations of a Hermitian texture (Theorem 79.1), whose determinants are real, so their product over the up and down blocks is a sign (O.6)–(O.7), leaving $\arg\det(M_u M_d)^{(1)} \in \{0, \pi\}$. Step 3: with $\theta_{\mathrm{top}}^{(1)} = 0$ from the export (no coefficient generated), $\bar{\theta}^{(1)} \in \{0, \pi\}$ exactly, a $\mathbb{Z}_2$ and not an angle — and the honesty register holds: the sign, and the leakage at all orders, are the named lemma L9's two gaps, so the entry is **Derived (cond.)**, not oversold. Physics intuition: the same pentagonal phase that *must* appear in the CKM matrix (I.3.3) is exactly the kind that *cannot* appear in $\bar{\theta}$ — misalignment phases live between the blocks and leave the determinant real; CP violation flows where the geometry channels it, into flavor, and not where the textbook feared it, into the vacuum.

O.5.1. Chain the derived electroweak constants from two ledger readings.

Solution. Step 1: from $\alpha^{-1} = 128$ and $\sin^2\theta_W = 25/108$ alone: $\alpha_2^{-1} = 128 \times \frac{25}{108} = \frac{800}{27} = 29.630$ (O.8) and $\alpha_Y^{-1} = 128 - \frac{800}{27} = \frac{2656}{27} = 98.370$ (O.9) — note both are exact rationals. Step 2: couplings (O.10): $g_2 = \sqrt{4\pi/29.630} = 0.65124$, $g_Y = 0.35742$; with $v^{(1)} = 246.331$ GeV (O.11): $m_W^{(1)} = \frac{1}{2} g_2 v = 80.211$ GeV, $m_Z^{(1)} = \frac{1}{2} v\sqrt{g_2^2 + g_Y^2} = 91.497$ GeV, and $G_F^{(1)} = 1/\sqrt{2}\,v^2 = 1.16532 \times 10^{-5}$ GeV^{-2} (O.12). Step 3: the QD pass (O.13) sharpens to 80.370 and 91.190 against the measured 80.37 and 91.19 — the entire electroweak constant sheet, G_F included, descending from two ledger fractions

and the lock scale. Physics intuition: this problem is the codebook's compression made tactile — five constants every student memorizes (α, θ_W, G_F, m_W, m_Z) reduce to *two* readings and arithmetic; the textbook measures five dials, the ledger prints one page.

O.6.1. Weigh a person in quark masses.
Solution. Step 1: the proton's valence content carries current masses $2m_u + m_d \simeq 2(2.2) + 4.7 \simeq 9.0$ MeV against $m_p = 938.3$ MeV: the Higgs-given share is 0.96%. Step 2: scale up: of a 70-kg human, roughly 0.7 kg is Yukawa mass; the remaining 69-plus kilograms are confined gluon field and quark kinetic energy — $E = mc^2$ running in reverse, energy read as weight. Step 3: the dictionary entry (Table 89): baryon mass is the GFF split $E_{\mathrm{GEP}} + E_{\mathrm{CEP}}$ — so the Higgs chapter explains the *origin* of mass for the elementary fermions, while the mass you feel on a scale is overwhelmingly the strong seam's stored energy; both are entries of one ledger, not two ontologies. Physics intuition: "the Higgs gives everything mass" is the most popular wrong sentence in physics outreach — the Higgs prices the quarks' tolls at the percent level, and confinement pays the other ninety-nine; you weigh what your protons' fields are doing, not what their parts are.

O.9.1. Work dimensional transmutation honestly, from the derived coupling.
Solution. Step 1: the one-loop RG-invariant scale (O.17) is $\Lambda = \mu \exp[-8\pi^2/(b_0 g_3^2)]$; take $\mu = m_Z$, the five-flavor $b_0 = 23/3$, and the derived closure $\alpha_s(M_Z) = 1/\pi e = 0.11710$ (D.3.3): the exponent is $-8\pi^2/(\frac{23}{3} \times 4\pi \times 0.11710) = -7.00$, so $\Lambda \simeq 91.19\, e^{-7.00} \simeq 83$ MeV. Step 2: state the register plainly: strict one loop lands the *decade* (order 10^2 MeV, an e^{-7} suppression from m_Z); the standard multi-loop, threshold-matched value is $\Lambda^{(5)}_{\overline{\mathrm{MS}}} \simeq 210$ MeV, and the chapter's Step-1 quote of "$\simeq 200$ MeV" lives at that standard. Step 3: sensitivity — because Λ depends exponentially on $1/\alpha_s$, the 0.9σ difference between $1/\pi e$ and the world average moves Λ by tens of percent: the confinement scale is the codebook's most leveraged output, which is exactly why the radial-transport reading matters — the window, not the last digit, is the derived object. Physics intuition: QCD is the only sector that manufactures a mass scale out of pure arithmetic — a dimensionless coupling and an exponential — and the geometry reads that exponential as literal distance down the radial dimension: the proton mass marks how far below the seam the strong coupling's transport runs out of room.

O.10.1. From memory: one word "mass," five sectors, one ledger.
Solution. Step 1: name the five sector meanings (Table 89): weak-boson mass = the doublet's vacuum value, its scale the selector's; Higgs mass = $2\lambda_H v^2$ on H-L2; fermion mass = overlap of internal profiles (tolls and codes); baryon mass = confined field energy, the GFF split; gravitational mass-energy = the same seam action varied against the metric. Step 2: identify the common structure: every entry is a property of *one* seam ledger read by different instruments — no sector owns a private ontology of mass, which is why the paper may add masses across sectors (a person = Yukawa tolls + confinement energy + binding corrections) without category error. Step 3: the one-sentence audit: the Higgs sector prices the elementary tolls (O.5.1), confinement prices the composite bulk (O.6.1), and gravity weighs the total — three prices, one currency. Physics intuition: the deepest service of the dictionary is negative — it forbids the question "but what is mass *really*?" from having five different answers; in this framework mass is always the same thing, stored seam energy, and the five formulas are five doors into one room.

Take-away. *Standard result:* $\bar{\theta} < 10^{-10}$ from the neutron EDM, confinement from the lattice, and mass from several bookkeeping systems. *What it assumed:* the smallness of $\bar{\theta}$ unexplained (or bought with an axion), confinement demonstrated but not derived, and the mass ontologies left separate. *The geometric reading:* $\bar{\theta}$ is a protected topological zero of the ledger; confinement is read geometrically; and mass has one ontology — the seam ledger — across every sector of the course.

P Promotion roadmap for the named structural lemmas

The MSMF/OGN per-parameter audit (§101.2) promotes ten of the eighteen entries a conservative reading leaves Asserted and identifies fourteen Standard-Model parameters as Derived (cond.) modulo a single named structural lemma each. This appendix lists in one place the eleven named parameter-closing lemmas: L1–L6 and L12 on the conditional electroweak and PMNS entries (L12 the leptonic Dirac-phase assignment lemma of Row 22), L7–L9 on the quark codebook, the Weinberg-operator coefficient and the strong-CP sign, and L10 the cascade-depth lemma covering the eight remaining **Asserted** Yukawas (the Higgs vev's binary-endpoint sub-lemma is folded into L1). Each lemma carries statement, parameter(s) closed, dependencies on upstream papers, a one-line physics intuition, expected proof difficulty, and successor-paper assignment. Each is a finite, well-posed mathematical claim; each, if proved, converts one or more codebook entries from **Asserted** or **Derived (cond.)** to fully **Derived** end-to-end. Alongside these eleven parameter-closing lemmas we register one *structural* lemma, L11 (the gate-operator lemma of §48.6): it closes no codebook entry, but, if proved, it would establish the identification of the weak interaction as the unique gate force of the family landscape and — through the gate/three-basin co-entailment — the joint existence of the weak interaction and the three generations. It is listed after L10 in Table 90, and L12 follows it.

The roadmap is summarized in Table 90, then unpacked lemma by lemma below.

Table 90: Promotion roadmap for the eleven parameter-closing lemmas L1–L10 and L12, and the one structural lemma L11. Each of L1–L10 and L12, if proved, would close one or more codebook entries by upgrading their status from **Asserted** or **Derived (cond.)** to fully **Derived**. "Difficulty" rates the expected effort in person-months, banded as Easy (~ 1), Medium (~ 6), Hard (~ 18), Very-hard (~ 36).

#	Lemma	Closes parameter(s)	Dependencies	Difficulty	Successor paper
L1	Binary-recursion lemma	$\alpha^{-1}(M_Z) = 2^J$ (Row 1)	GG-A2-GG6 T-GG6-6; GG-A4-Euclid §31	Easy	GG-A6-GRIP (extension) or dedicated note
L2	Triplicity-layering lemma	$v = M_{\rm lock}(2/L)^N$ (Row 4); $\lambda_H = 2^{-N}$ (Row 5, via L6)	GG-A2-GG6 T-GG6-6; Wilson–Hosotani descent of GG-A6-GRIP T4.1	Medium	GG-A6-GRIP (extension)
L3	Democratic-weight assignment lemma	$\sin^2\theta_{12}^{\rm PMNS} = L^2/3N_g^3 = 25/81$ (Row 19)	GDOS/GFF projector overlap on the family stratum	Medium	GG-A6-GRIP (extension) or dedicated PMNS paper

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#	Lemma	Closes parameter(s)	Dependencies	Difficulty	Successor paper
L4	Reactor-overlap assignment lemma	$\sin^2\theta_{13}^{\mathrm{PMNS}} = N_g/4K = 3/140$ (Row 20)	GDOS/GFF reactor-overlap on the family stratum	Hard	Dedicated PMNS-derivation paper
L5	Rank-2 atmospheric-deviation lemma	$\sin^2\theta_{23}^{\mathrm{PMNS}} = (J{+}1)/(2J) = 4/7$ (Row 21)	Rank-2 family-projector stratum + cost-tie discrimination on $\{4/7, 5/9\}$	Medium	Dedicated PMNS-derivation paper
L6	Quartic lemma (H-L2, second instance)	$\lambda_H = 2^{-N}$ (Row 5; Derived (cond.))	neutral fraction of the doublet to the power N under H-L2; a ledger-computed QD transport	As L2; medium for the transport	With L2, or a dedicated Higgs-sector note
L7	Quark-codebook overlap lemma	Four CKM mixing parameters (Rows 15–18)	GG-A6-GRIP §28 angular-cascade decoration on the quark up/down family projectors; pentagonal-holonomy interference	Hard	Dedicated CKM-derivation paper (jointly with the cascade-depth paper of L10)
L8	Weinberg-operator coefficient lemma	The neutrino mass scale and the solar splitting (Rows 25–26; Row 27's scale is Row 25's, its grade its own coefficient's)	A2-GG-6 T6 ($K = 35$); Wilson–Hosotani descent of GG-A6-GRIP T4.1; Majorana operator $(LH)(LH)/E_{\mathrm{GUT}}$	Medium	GG-A6-GRIP (extension) or dedicated neutrino-mass-scale paper
L9	Quark-determinant sign-and-leakage lemma	Strong-CP angle $\bar\theta$ (Row 28)	A2-GG-6 (no first-order θ-term on the GG-6 seam); GG-A6-GRIP §28 quark-determinant phase reading	Hard	Dedicated strong-CP paper or jointly with L7
L10	Cascade-depth lemma: each fermion's depth is the height of its fixed point under a sector-dependent torus action on the doubled stratum	Eight charged-fermion Yukawas: m_e, m_μ, m_τ (Rows 6–8); m_u, m_d, m_c, m_s, m_b (Rows 9–12, 14)	Torus actions on $\mathbb{C}P^2$ doubled by $\mathrm{SU}(2)_L$ and the heights of their fixed points; GG-A6-GRIP's family-weights theorem (the dilatation charge); the inflow tolls πk as Chern numbers; the leptons placed first	Very-hard	Dedicated cascade-depth paper (provisional title: "Arisaka-GG-A9X-Yukawa cascade depths")
L12	Leptonic Dirac-phase assignment lemma	$\delta_{\mathrm{CP}}^{\mathrm{PMNS}} = \pi + 2\arcsin(1/(2N+1))$ (Row 22)	The leak operator of (54.6) on the rank-2 stratum; L3–L5; $\alpha_{31} = \pi$ (Row 24)	Medium	Dedicated PMNS-derivation paper (with L3–L5)

Structural lemma — closes a structural result, not a codebook parameter

continued on next page

#	Lemma	Closes parameter(s)	Dependencies	Difficulty	Successor paper
L11	Gate-operator lemma	*Structural*: weak-as-gate-force; weak ⇔ three generations (§48)	Morse transport between adjacent critical points on $\mathbb{C}P^{N_g-1}$; GSSB/GRIP-Out; GDOS open system; $N_g = 3$ closure (Thm 21.1)	Hard	Dedicated flavor-architecture note, jointly with GG-A11-Origin

Lemma L1: the binary-recursion lemma

Statement. On the seven-step binary-recursion stratum that A2-GG-6 T6 identifies with the Diophantine ledger position $J = 7$, the inverse fine-structure constant at the electroweak seam is forced by binary recursion to take the value $\alpha^{-1}(M_Z) = 2^J = 128$, with no remaining freedom in the dyadic step structure or in the numeric base.

Parameters closed. Row 1 of Table 7 ($\alpha^{-1}(M_Z) = 2^J = 128$), currently **Derived (cond.)** modulo this A9-local lemma; promotion would make Row 1 fully **Derived** end-to-end.

Dependencies. The lemma uses the Diophantine ledger position $J = 7$ from A2-GG-6 T6 (Arisaka, 2026, A2-GG6) and the dyadic-recursion structure of GG-A4-Euclid §31 (Arisaka, 2026, A4-Euclid). No new physics input is required.

Physics intuition. Each dyadic step halves the residual coupling-renormalization envelope, and the seven-step binary recursion is exactly the depth that the OGN-5 ledger position $J = 7$ admits. After all, 128 is what one gets by halving from 1 down through seven layers; the lemma proves what the QED running already suggests numerically.

Expected difficulty. Easy. The binary-recursion calculation is essentially a numerical coincidence elevated to a structural statement; the recursion endpoint $J = 7$ is itself already proved upstream. The lemma reduces to "write down the proof of an observation".

Successor paper. Most naturally an extension of GG-A6-GRIP, or a short dedicated note (10–15 pp).

Lemma L2 (with L6): the triplicity-layering lemma, in sharp form

Statement. Let $f_D = \dim(\mathbf{1},\mathbf{2})_{+1/2}/\dim \mathbf{5}_H = 2/L$ be the doublet fraction of the Higgs multiplet and $f_0 = 1/2$ the neutral fraction of the doublet — two numbers of representation theory. *Hypothesis* (**H-L2**): on the electroweak seam the branch amplitude selected by the descent of GG-A6-GRIP T4.1

is f_D raised to the triplicity index, and the quartic self-stiffening of the selected branch is f_0 raised to the same — one factor per triplicity layer, N_c layers. Under **H-L2**, $\sin\theta_H^\star = (2/L)^N$ (Row 4) and $\lambda_H = 2^{-N}$ (Row 5). No mechanism is supplied for the exponent: the hypothesis is the statement that the selection factorizes over the N_c triplicity layers, and it is what this lemma is.

What this replaces. The withdrawn reading's lemmas L2 and L6 stated the same rule twice — "one factor of $2/L$ per triplicity layer" and "one factor of $1/2$ per triplicity layer" — each as a freestanding assertion. They are one lemma: both are the statement that the electroweak branch selection factorizes over the N_c triplicity layers, and they differ only in which fraction — the doublet's of the multiplet, or the neutral's of the doublet — is being raised to N. The two fractions are representation theory; **H-L2** is the content, and it is a single claim about the GG-A6-GRIP descent that can be checked against T4.1 directly.

Why selection does not substitute for it. The MSMF/OGN audit of §101.2, run on $v/M_{\rm lock} = 0.06397$, finds $(2/L)^N = 0.06400$ within 0.05% — and finds it at cost 6, where the admissible monomials are dense enough that a random target is matched to 0.1% some 84% of the time, with a cost-2 competitor $1/(L\pi)$ within 0.5%. The form is not a numerological coincidence to be graded by look-elsewhere; it is a structural claim, and it is graded on its lemma.

Status. **H-L2** is the assumption on which the electroweak scale and the Higgs quartic rest. It is named here in one sentence so that it can be proved or refuted against GG-A6-GRIP in one place.

Expected difficulty. Medium.

Successor paper. An extension of GG-A6-GRIP.

Lemma L3: the democratic-weight assignment lemma

Statement. On the family-projector stratum, the PMNS solar mixing angle is forced by the GDOS/GFF democratic-weight projector overlap to take the structural value $\sin^2\theta_{12}^{\rm PMNS} = L^2/3N_g^3 = 25/81$ (QD-completed to 0.30857).

Parameters closed. Row 19 of Table 7 ($\sin^2\theta_{12}^{\rm PMNS} = L^2/3N_g^3 = 25/81$), currently **Derived (cond.)** modulo the present lemma.

Dependencies. The lemma uses the GDOS/GFF democratic-weight projector overlap on the family-projector stratum of GG-A6-GRIP (Arisaka, 2026, A6-GRIP).

Physics intuition. Roughly speaking, the solar mixing weight is the democratic projector overlap $L^2/3N_g^3$ on the family stratum — the same L^2/N_g^3 skeleton — read on the family stratum, where the integer is the generation count N_g, rather than on the gauge stratum, where it is the color index N_c — that fixes the weak angle $\sin^2\theta_W = L^2/4N^3$, with the family denominator $3N_g^3$ in place of the gauge $4N_g^3$. The PMNS solar angle inherits the family-stratum overlap geometry.

Expected difficulty. Medium. The proof reduces to showing that the family-stratum projector overlap forces the democratic weight $L^2/3N_g^3 = 25/81$ rather than a neighboring rational.

Successor paper. Either an extension of GG-A6-GRIP or a dedicated PMNS-derivation paper (provisional title: "Arisaka-GG-A9Y: PMNS angles from the rank-stratified $\mathbb{C}P^2$").

Lemma L4: the reactor-overlap assignment lemma

Statement. On the family-projector stratum, the PMNS reactor mixing angle is forced by the GDOS/GFF reactor-overlap to take the structural value $\sin^2\theta_{13}^{\mathrm{PMNS}} = N_g/4K = 3/140$ (QD-completed to 0.021902).

Parameters closed. Row 20 of Table 7 ($\sin^2\theta_{13}^{\mathrm{PMNS}} = N_g/4K = 3/140$), currently **Derived (cond.)** modulo the present lemma.

Dependencies. The lemma requires a Hopkins–Singer-style differential-cohomology descent calculation on the rank-3 family-projector stratum, building on the QD-completion descent of §43.2 of the present paper but applied to the family-projector $\mathbb{C}P^2$ rather than to the energy stratum.

Physics intuition. The reactor overlap on the family stratum is the $N_g/4K$ amplitude-squared readout — the rank-3 family projector feeding one power of N_g over the master lock $K = LJ$, with the dyadic 1/4. It is the structural reactor form of §54, not a bounded-cost selection.

Expected difficulty. Hard. The calculation must show that the family-stratum reactor overlap forces $N_g/4K = 3/140$ rather than a neighboring rational.

Successor paper. Most naturally a dedicated PMNS-derivation paper.

Lemma L5: the rank-2 atmospheric-deviation lemma

Statement. On the rank-2 family-projector stratum, the PMNS atmospheric mixing angle is forced by the structural rule "atmospheric angle deviates from $\pi/4$ by $1/(2J)$" to take the value $\sin^2\theta_{23}^{\mathrm{PMNS}} = (J{+}1)/(2J) = 4/7$ at the structural-precision target. The structural-naming criterion (OGN admissibility 3) breaks the cost-3 tie with the equally-cheap candidate 5/9 in favor of 4/7.

Parameters closed. Row 21 of Table 7 ($\sin^2\theta_{23}^{\mathrm{PMNS}} = 4/7$), currently **Derived (cond., MSMF/OGN)** modulo the present lemma.

Dependencies. The lemma requires identifying the rank-2 family-projector stratum with the atmospheric mass-eigenstate doublet and verifying the deviation $1/(2J)$ from the maximally-mixed configuration $\pi/4$.

Physics intuition. Maximal mixing $\sin^2\theta_{23} = 1/2$ is the symmetric reading of a rank-2 doublet; the empirical deviation by $1/(2J)$ is the small structural correction that the rank-2 family-projector adds. After all, two of the three families share the atmospheric block, and the deviation depends on the third — encoded by the Diophantine atom $J = 7$.

Expected difficulty. Medium. The structural-naming step that breaks the cost tie is the cleaner part; the geometric identification of the atmospheric doublet with the rank-2 stratum requires care.

Successor paper. Most naturally a dedicated PMNS-derivation paper, jointly with L4.

Lemma L6: the quartic lemma (H-L2, second instance)

Statement. Lemma L6 is the second instance of the triplicity-layering hypothesis **H-L2** stated under Lemma L2 above: with $f_0 = 1/2$ the neutral fraction of the doublet, $(f_0)^N = 2^{-N}$, and under **H-L2** the quartic self-stiffening of the selected branch is $\lambda_H = 2^{-N} = 1/8$. The base of the suppression is the neutral fraction of one doublet, $1/2$, and the depth N is the layering; that 2 is also the smallest admissible element of $\mathcal{S}_\Sigma$ greater than one is a consistency remark under MSMF, not a second basis. It is listed separately from L2 so that Row 5 can be graded on it by name; it closes with L2 and not otherwise.

Parameters closed. Row 5 of Table 7 ($\lambda_H = 1/8$), recorded as **Derived (cond.)** on the present lemma: the cost-minimum reading is weak (cost-3, look-elsewhere $p \approx 1$), and the GG-A6-GRIP GFF quartic-curvature reading that once carried the status is withdrawn with the coset reading (§36.5). A second route exists and is not yet written: a QD transport computed from the ledger *before* comparison and landing near 0.129 would carry Row 5 to the **Derived (predicted)** grade of Row 13 on the same logic (y_t and v fixed independently, the mass then landing). The printed $\lambda_H^{\rm QD} = 0.12928$ of (43.10) does not qualify: it is $m_h^2/2v^2$ at 125.20 GeV read from the datum, the representative value of a transport not yet written, and a closed form found afterwards for it would be a post-hoc rational (Row 18's precedent), not a prediction. The forward transport is the calculation this lemma names.

Dependencies. The doublet's neutral fraction (representation theory); **H-L2**; the GG-A6-GRIP descent (T4.1) that L2 rests on.

Expected difficulty. As L2 for the layering; *medium* for the closed-form transport.

Successor paper. Jointly with L2; a dedicated Higgs-sector calculation for the transport.

Lemma L7: the quark-codebook overlap lemma

Statement. On the up-type and down-type quark family-projector strata of $\mathbb{C}P^2$, the four CKM mixing parameters ($\theta_{12}^{\rm CKM}$ Cabibbo, $\theta_{13}^{\rm CKM}$, $\theta_{23}^{\rm CKM}$, $\delta_{\rm CP}^{\rm CKM}$) are forced by the misalignment of the two

quark family projectors and the pentagonal-holonomy interference structure of GG-A6-GRIP §28 to take the values derived in §52 of the present paper.

Parameters closed. Rows 15–18 of Table 7 (the four CKM parameters), currently **Derived (cond.)** on the quark codebook of #9–14; promotion would make all four fully **Derived** end-to-end.

Dependencies. GG-A6-GRIP §28 angular-cascade decoration on the up/down family projectors; the pentagonal-holonomy interference identified in §52; the cascade-depth assignments to be supplied by L10. L7 therefore depends on L10 in part: the CKM angles inherit per-quark depth assignments and cannot be fully closed until L10 closes.

Physics intuition. Two slightly misaligned family projectors interfere at five overlap points (the pentagonal-holonomy interference of §52); the CKM angles read out the geometry of the misalignment. Indeed, the small Cabibbo angle is the smallest pentagonal-holonomy step, and the suppressed $\theta_{13}^{\mathrm{CKM}}$ is the deepest one.

Expected difficulty. Hard. The lemma is jointly conditional on L10 (per-quark cascade depths); once L10 closes, the misalignment calculation is tractable.

Successor paper. Dedicated CKM-derivation paper, jointly with the cascade-depth paper of L10 (provisional title "Arisaka-GG-A9Z: CKM mixing from quark-codebook overlaps").

Lemma L8: the Weinberg-operator coefficient lemma

Statement. On the constitutional Majorana channel $(LH)(LH)/E_{\mathrm{GUT}}$, the operator coefficient 2π — which, once the two Higgs insertions ($v/\sqrt{2}$ each) halve it, pins the absolute neutrino mass scale $m_* = \pi v^2/E_{\mathrm{GUT}}$ — is read off the canonical S^1_θ traversal of A2-GG-6 as the natural dimensionless prefactor — plausible, and not proved by an A1–A7 theorem, which is why Row 25 is D* — of any boundary observable that wraps the compact direction once. The rank-2 projector ratio $2N = 6$ in $\Delta m^2_{31}/\Delta m^2_{21}$ is *selected*, not forced: it is the unique cost-2 integer-ledger monomial whose QD-completed squared ratio lies within 5% of the measured 33.35 (Row 27, graded **Derived (selected)** on the coefficient it adds, look-elsewhere $p \approx 0.21$). This lemma would close the scale m_*; the coefficient's selection stands on its own row.

Parameters closed. Rows 25–27 of Table 7 (the absolute Majorana mass scale m_* and the two squared-mass splittings Δm^2_{21}, Δm^2_{31}): Rows 25 and 26 are **Derived (cond.)** on the Weinberg-operator coefficient choice; Row 27 multiplies the same scale by its selected coefficient $2N$ and is graded on that coefficient. Promotion would make Rows 25 and 26 fully **Derived** end-to-end; Row 27's grade does not depend on it.

Dependencies. A2-GG-6 T6 for the lock scale E_{GUT} via $K = 35$; Wilson–Hosotani descent of GG-A6-GRIP T4.1 for v; the canonical S^1_θ traversal of the compact direction for the 2π coefficient.

Physics intuition. Any boundary observable that crosses the compact S^1_θ once picks up one factor of the circumference 2π; the neutrino Majorana scale is exactly such an observable. Roughly speaking, the operator carries a factor 2π because it wraps the compact circle once, no more, no less; the two Higgs insertions ($v/\sqrt{2}$ each) then halve it, giving $m_* = \pi v^2/E_{\rm GUT}$.

Expected difficulty. Medium. The 2π coefficient is structurally suggested by the S^1_θ traversal; the rigorous proof requires showing that no half-traversal is admissible; the coefficient $2N$ is Row 27's and not this lemma's.

Successor paper. An extension of GG-A6-GRIP, or a dedicated neutrino-mass-scale paper.

Lemma L9: the quark-determinant sign-and-leakage lemma

Statement. On the GG-6 boundary, the strong-CP angle $\bar\theta = \theta_{\rm QCD} + \arg\det(M_u M_d)$ is 0 or π at first order by Theorem 79.1, on two structural facts: (i) A2-GG-6 admits no independent first-order θ-term on the GG-6 seam (the seam topology forbids it); (ii) the family-link texture is Hermitian, so $\det(M_u M_d)$ is real in the descent basis in which the texture is written, and $\bar\theta$ — the basis-independent combination — is therefore 0 or π. What this lemma must supply is the sign — that the number of negative eigenvalues across the two sectors is even at the link strengths that reproduce the codebook masses — and the exclusion of shallow-winding leakage beyond first order. The smallest natural quantum-defect leakage the ledger can build is $\bar\theta_{\rm bare} \simeq 4.68 \times 10^{-18}$ — a minimum the ledger admits, not an upper limit on it (§79).

Two named gaps. *(a′) The sign.* The basis identification that was gap (a) is closed by Theorem 79.1 — the family-link texture is Hermitian, so $\bar\theta^{(1)} \in \{0, \pi\}$ in every basis — and is replaced by a sharper gap: which of the two. At the leading-order link strengths the texture's parity is even for every nonvanishing up-sector loop phase (item (3): $\det K_d = -1$ for all phases, $\det K_u = 2(\cos\Phi_u - 1) < 0$ for $\Phi_u \neq 0$), so the texture as printed selects the zero branch; whether that parity survives the κ_{ij} that reproduce the codebook masses is O_{23}. *(b) The shallow-winding leakage.* The first-order statement is a $\mathbb{Z}_2$, not a bound; the paper's estimate of the residual leakage is the *smallest* product the ledger admits, while the Bogomolnyi weights $e^{-n\pi}$ of §49.3 supply larger CP-odd candidates, the shallowest six orders above the neutron limit (§79). Excluding those in the topological channel is part of what this lemma must do.

Parameters closed. Row 28 of Table 7 ($\bar\theta$), currently **Derived (cond.)** on the sign (gap (a′)) and the shallow-winding leakage (gap (b)); promotion would make Row 28 fully **Derived** end-to-end.

Dependencies. A2-GG-6 for the absence of a first-order θ-term on the GG-6 seam; the family-link texture of §52 (Theorem 79.1) for the reality of the determinant; the link strengths κ_{ij} that reproduce the codebook masses (O_{23}) for the sign; GG-A6-GRIP §28 and the per-quark cascade-depth assignments of L10 for the leakage arithmetic.

Physics intuition. The strong-CP angle is the relative phase between the QCD vacuum and the quark-mass-matrix determinant. In GG-6 the first one is forbidden by topology and the second is real by Hermiticity, so the first-order angle is a $\mathbb{Z}_2$ rather than a continuous number; what remains is which of the two values, and π is excluded empirically. The celebrated strong-CP problem becomes, on the 6D bulk, a sign to be fixed rather than an angle to be tuned.

Expected difficulty. Hard. The sign arithmetic is jointly conditional on L10 (per-quark depths) and O_{23}; the topological exclusion of the first-order θ-term is plausible from A2-GG-6 but a fully rigorous proof requires identifying the relevant cohomology class.

Successor paper. Dedicated strong-CP paper, or jointly with L7.

Lemma L10: the cascade-depth lemma

Statement. On the family-projector stratum $\mathbb{C}P^2$ doubled by $\mathrm{SU}(2)_L$, each charged fermion's depth is the *height of its fixed point* under a sector-dependent torus action on the doubled stratum, with the weights of that action derived, on a stratum whose leptons the lemma must first place. The depths this codebook uses — $n_{y_t} = 0$, $n_{y_b} = 1$, $n_{y_c} = 2$, $n_{y_s} = 3$, $n_{y_u} = n_{y_d} = 4$ for the quarks, and the dyadic seam depths of the leptons — are what the lemma must return, not what it assumes. Two things about the statement are now fixed. First, the depth is not the dimension of a cohomology class, which an earlier statement of this lemma said it was: read as $\mathbb{C}P^2 \times \mathbb{C}P^1$ the doubled stratum has six torus-fixed points and an equivariant cohomology of rank six, so three of the nine readouts have no class to occupy before any question of degree arises, and for the six quarks the half-degrees $\{0, 1, 1, 2, 2, 3\}$ are not the codebook's $\{0, 1, 2, 3, 4, 4\}$ under any bijection; read as $\mathbb{C}P^2 \times \mathbb{C}P^2$ the nine half-degrees $\{0, 1, 1, 2, 2, 2, 3, 3, 4\}$ are not the lemma's multiset either. What distinguishes depths is the height of the fixed point, the value of the moment-map component there. Second, on the family stratum GG-A6-GRIP has now derived which height its own action assigns: the boundary dilatation charge, $\Delta_+(\ell) = 2 + \sqrt{4 + \ell^2}$ at the index-$2(\ell - 1)$ point, with gap ratio $(\sqrt{13} - \sqrt{8})/(\sqrt{8} - \sqrt{5}) = 1.31$; the toll πk of the gates is the inflow charge, a Chern number, and not a height (Arisaka, 2026, A6-GRIP). A single height cannot serve all three charged sectors (§101.2), so the action must differ sector by sector, and the isospin factor must enter the family weights themselves.

Parameters closed. All eight non-top charged-fermion Yukawa entries (rows 6–12, 14 of Table 7), currently **Asserted**. This is the dominant single open task in the A9 closure program: *one calculation closes eight parameters*, and through the dependencies of L7 and L9, indirectly closes five more.

Dependencies. The torus actions on $\mathbb{C}P^2$ doubled by $\mathrm{SU}(2)_L$ and the heights of their fixed points; GG-A6-GRIP's transport proposition and its family-weights theorem, which fix the height of the family action as the dilatation charge (Arisaka, 2026, A6-GRIP); the inflow tolls $S_{[2k]\to[2(k-1)]} = \pi k$ as the gates' Chern numbers, distinct from the heights; and the placement of the leptons, whose

skeleton this codebook reads as dyadic rather than exponential. The chiral doubling by $\mathrm{SU}(2)_L$ is structurally important, and the count it must supply is nine distinct readouts (eight charged-fermion Yukawas and the cascade top) from a stratum whose fixed points a product of a family action and an isospin action does not provide in the required pattern.

Physics intuition. The chiral doubling is what makes a fermion *chiral* — left-handed and right-handed components live on geometrically distinct sub-strata — and the height of a fermion's fixed point is how far down the landscape it sits: the Yukawa is the equilibrium weight e^{-S} of that point, with S the height difference from the top. The spread of five and a half orders of magnitude across the charged-fermion Yukawas is then the reading of nine heights, not nine fitted numbers — provided the actions that assign the heights are derived, which is the lemma's whole content. Its prefactors are a separate matter and remain ungraded: no selection on the ledger alphabet can raise them (§101.2), and no rate prefactor is claimed.

Expected difficulty. Very-hard. This is the deepest of the eleven parameter-closing lemmas and the dominant remaining piece of upstream work for the A9 codebook. The expected effort is comparable to a standalone preprint ($\sim$ 100–150 pp), conducted as a dedicated successor paper to the foundation block. A derivation of the sector actions and of the heights of their fixed points that returned the eight depths would close one of the longest-standing problems in particle physics — the absence of a structural rationale for the Yukawa hierarchy — by replacing eight assertions with one geometric statement.

Successor paper. Provisional title "Arisaka-GG-A9X: Yukawa cascade depths from the heights of the doubled family stratum"; estimated completion 2027.

Lemma L11: the gate-operator lemma (structural)

Statement. On the family stratum $\mathbb{C}P^{N_g-1}$, a gauge connection that parallel-transports the family projector between adjacent critical points of the GFF Morse height function is necessarily (i) off-diagonal in the critical-point basis, (ii) chiral — it carries the orientation of the downhill gradient flow — and (iii) Higgs/Stueckelberg-massive, its range set by the saddle width; and within the boundary gauge group the unique factor admitting all three properties is $\mathrm{SU}(2)_L$.

What it would establish. Unlike L1–L10 and L12, this lemma closes no codebook *parameter*; it closes a *structural* result. A proof would upgrade the identification of the weak interaction as the unique gate force of the family landscape (§48) from a structural reading to a theorem, and — through the gate/three-basin co-entailment and the toll-budget closure $\frac{1}{2}N(N-1)=3$ — would yield a first-principles derivation of the *joint* existence of the weak interaction and the three generations.

Dependencies. The Morse basin–gate–toll structure on $\mathbb{C}P^{N_g-1}$ and the Bogomolnyi spectrum $S_{[2k]\to[2(k-1)]}=\pi k$ of GG-A6-GRIP §28 (Arisaka, 2026, A6-GRIP); the GSSB/GRIP-Out reading of

electroweak symmetry breaking; the GDOS open-system relaxation law; and the $N = 3$ cascade closure (Theorem 21.1). It is the flavor-side complement of the four-force-uniqueness theorem (Theorem 95.1) of GG-A11-Origin.

Expected difficulty. Hard. The statement is a clean equivariant/Morse-theoretic claim about transport operators on $\mathbb{C}P^{N_g-1}$, but tying "off-diagonal + chiral + massive transport operator" uniquely to $\mathrm{SU}(2)_L$ requires a careful representation-theoretic step. If proved, it answers two of the oldest questions in particle physics — why the weak interaction is special, and why there are three generations — in one stroke.

Successor paper. Naturally developed jointly with the four-force-uniqueness program of GG-A11-Origin; provisional home a dedicated flavor-architecture note in the 2027 timeframe.

Lemma L12: the leptonic Dirac-phase assignment lemma

Statement. On the rank-2 Majorana family stratum, the leptonic Dirac phase in the standard parametrization is the doubled minimal neutrino phase on the reference channel, $\delta_{\mathrm{CP}}^{\mathrm{PMNS}} = \pi + 2\phi_\nu$ with $\phi_\nu = \arcsin(1/(2N+1)) = \arcsin(1/7)$ (§54): the leak operator $e^{-i\phi_\nu}T_{eA} + e^{+i\phi_\nu}T_{Ae}$ of (54.6) carries the only physical phase of the block, a rephasing invariant traverses it twice, and the base π is the reference-channel convention of the standard parametrization, not a geometric datum. What is to be proved is the assignment: that no admixture of the second leak channel enters the invariant at first order, so that the physical phase is $2\phi_\nu$ with coefficient exactly two and not a sum over channels, and that the $3\pi/2$ branch of the subdominant degeneracy is excluded structurally and not by the empirical preference recorded at Row 22.

Parameters closed. Row 22 of Table 7 ($\delta_{\mathrm{CP}}^{\mathrm{PMNS}} = \pi + 2\arcsin(1/7) = 196.4°$, QD-completed 196.06°), currently **Derived (cond.)** on the present lemma; promotion would make Row 22 fully **Derived** end-to-end, and with it the last of the four PMNS entries.

Dependencies. The three PMNS angles on L3–L5 (the leak operator's normalization is Row 21's $1/(2N+1)$); the Majorana parity $\alpha_{31} = \pi$ of Row 24 (the phase is a Dirac phase only once the parity is fixed); the reference-channel convention of §54.

Physics intuition. A rephasing-invariant quantity must go through the leak and come back, so it collects the phase twice; the atmospheric tilt and the Dirac phase are the same integer $J = 7$ read twice, and the lemma is the statement that nothing else is read.

Expected difficulty. Medium. The doubling is a two-line argument on the printed operator; the exclusion of the second channel and of the $3\pi/2$ branch requires the rank-2 block's invariant to be computed rather than read off.

Successor paper. With L3–L5, in the dedicated PMNS-derivation paper.

Programmatic summary of the roadmap

The eleven lemmas L1–L10 and L12 represent the full open structural-naming agenda for the GG-A9-SM codebook as it stands at the present. Seven of the eleven (L1, L2, L3, L5, L6, L8, L12) are tractable in person-month timescales (Easy or Medium) and could be closed in successor papers within 2026–2027. Three (L4, L7, L9) are Hard and are the natural subject of dedicated extension papers. The tenth (L10) is the dominant single open task, requires a dedicated equivariant-cohomology calculation, and is the natural subject of a separate preprint in the 2027 timeframe; through the dependencies of L7 and L9, L10 also indirectly closes the four CKM parameters and the strong-CP angle. The structural lemma L11 (the gate-operator lemma) stands outside this parameter-closure arithmetic: it closes no codebook entry, but its proof would establish the weak-as-gate-force result and the joint weak/three-generation derivation of §48.

The closure arithmetic stages as follows. Closing the seven tractable lemmas L1, L2, L3, L5, L6, L8, L12 promotes Rows 1, 4, 5, 19, 21, 22, 25 and 26 — eight rows — to fully **Derived**, raising the count from 6 to $6 + 8 = 14$ **Derived** end-to-end. Closing the three Hard lemmas L4, L7, L9 (with L7, L9 themselves dependent on L10) promotes Rows 20, 15–18 and 28 — six rows — taking the count to $14 + 6 = 20$ **Derived**, with L10 still outstanding and the eight Yukawas still **Asserted**. Closing L10 then promotes the eight Yukawas, taking the count to $20 + 8 = 28$ **Derived**. The phase measurement to be supplied by T2K-II / NOvA-II / DUNE / Hyper-Kamiokande over 2027–2032 is then a test of Row 22, not an input to it.

Remark 45 (The shape of the closure program)**.** Indeed, the roadmap above makes the closure program finite, named, and dated: seven tractable lemmas, three hard ones and one very hard one, and the 28-parameter codebook is closed. That is what a structural framework should produce as a successor agenda: a numbered list with dependencies, difficulties and successor papers. Whether each lemma admits a clean proof is the content of that work; the achievement here is the reduction of twenty-eight free parameters to eleven named lemmas with a test each.

Part XVI
Common Objections and Responses

The strongest objections a particle theorist, cosmologist, mathematician, philosopher, or string theorist is most likely to raise on first reading — answered, with pointers into the paper

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A consequence paper that claims to derive the twenty-eight free parameters of the Standard Model from a five-integer ledger will be met — rightly — with skepticism from every quarter. This appendix collects eighty-three objections, grouped by the background of the reader most likely to raise each, together with an extended response. The objections are not strawmen; they are the questions the authors have actually been asked, sharpened. Where the honest answer is "not yet," we say so, and point to the codebook status (Table 7) and the promotion roadmap (§P) that record exactly which claims are theorems, which are conditional, and which are asserted. After all, the test of a framework is not that it deflects every question, but that it knows which questions it has answered and which it has not.

Q Methodology, numerology, and the honesty triage

Objection 1 (O-SM-1) (the parameter-counting skeptic)
"You advertise a derivation of twenty-eight parameters, but your own triage marks only six as Derived and eight as Asserted. Isn't '28 derived' an overstatement?"

Response. It would be, and that is precisely why the paper does not make that claim without the triage. The headline result is structural: all twenty-eight parameters are placed on a single integer ledger and assigned a closed form, so that none remains a freely floating real number typed into a Lagrangian by hand. The honesty triage (Table 7) then grades how strong each placement is: six **Derived**, fourteen **Derived (conditional)** — forced once a single named lemma is supplied — and eight **Asserted**. The correct one-sentence summary is therefore not "twenty-eight derived" but "twenty-eight reduced to a five-integer ledger plus eleven named lemmas," with the strength of each reduction stated explicitly. We regard refusing to inflate the count as a feature, not a concession.

Objection 2 (O-SM-2) (the look-elsewhere statistician)
"With π, e, and a handful of small integers one can approximate almost any number. How is the codebook not curve-fitting with an enormous look-elsewhere effect?"

Response. This is the single most important methodological objection, and the paper takes it seriously enough to quantify it rather than wave it away. The MSMF/OGN selection principle is

a complexity-bounded Diophantine search, and a search has a look-elsewhere cost. For each parameter we report that cost. Where the candidate space is genuinely sparse the selection is strong: $\alpha_s^{-1}(M_Z) = \pi e$ is the unique cost-2 monomial on $\{\pi, e\}$ within 1% of the datum, look-elsewhere $p \approx 0.27$. Where it is dense we say so and decline to claim strength: $\lambda_H = 2^{-N}$ is admissible but the cost-≤ 3 band is saturated ($p \approx 1$), so we do *not* rest its status on the cost-minimum at all (Objection 7). The discipline is the opposite of curve-fitting: a curve-fit hides its degrees of freedom, whereas the audit publishes them parameter by parameter in §101.2. The decisive program-level statistic is the *global* trials factor over the full alphabet evaluated against all twenty-eight targets at once (O-SM-73); we regard that single number, not the favorable per-parameter costs, as what should ultimately make or break the look-elsewhere charge. Two further points, developed in §102.1, complete the reply and are the heart of the matter: the alphabet is not a per-parameter toolbox but *one closed ledger* reused coherently across all twenty-eight observables (the same $N = 3$ in $\sin^2\theta_W$, $\alpha^{-1}(0)$, and λ_H at once), and the *form* of each entry is forced by the stratum it inhabits rather than scanned — so this is the descent of propositions from the GG-Constitution, in the lineage of Euclid's *Elements*, not a search. We do not claim the look-elsewhere effect is zero; we claim it is forced down to a bounded residual that the global trials factor quantifies.

Objection 3 (O-SM-3) (the careful Diophantine analyst)
"Were the ledger integers $(M, N, L; J, K) = (1, 3, 5; 7, 35)$ chosen before or after the data were known?"

Response. Before, and not by us. $M = 1$ is the dimensional-minimality output of A2-GG-6 (Arisaka, 2026, A2-GG6); $L = 5$ is Georgi–Glashow's 1974 identification of SU(5) as the smallest simple unifying group (Georgi & Glashow, 1974); $N_g = 3$ is established by the independent three-leg pincer of GG-P2-N3 (Arisaka, 2026, P2-N3) — a CP lower bound $N_g \geq 3$ and a GDOS/GRIP stability-budget upper bound $N_g \leq 3$ (the latter the capacity theorem T-N3-6, conditional on the coupling clause and the integrality theorem L-INT (residue R-INT)) — while the SU(5) trace-ratio identity serves only as a downstream consistency check; and $(J, K) = (7, 35)$ is then fixed by the dual-lock identity $K = LJ = M^2 + N^2 + L^2 = 35$, whose uniqueness over small integers is checked explicitly in A2-GG-6 §10. Each integer is determined by a structural theorem that pre-dates, logically and historically, the parameter fits of the present paper. The codebook formulas are downstream of the integers, not the reverse.

Objection 4 (O-SM-4) (the precision-physics critic)
"Four-significant-figure agreement appears only after a 'QD completion.' Isn't the QD layer where the fitting actually happens?"

Response. The QD completion is constrained, not free. Its rationals carry denominators forced by Hopkins–Singer descent integrality on the relevant stratum (for the leptons, $640 = 2^J L$) (Hopkins & Singer, 2005), and integer numerators drawn from the same OGN-5 ledger by cascade depth. There are no continuous knobs: a skeptic can verify that no alternative small-integer numerator on the same lattice reproduces all three lepton ratios at once. The honest statement is that the leading-order ledger value and the QD-completed value are two predictions of

different precision, both reported, with the completion adding structure (a named integrality theorem) rather than adjustable parameters. If the QD layer contained hidden freedom, the same machinery would not also have to reproduce the quark and mixing sectors with the *same* ledger integers, which it does. A final clarification, in the light of the renormalization analysis of Arisaka (2026, A5-GDOS) (Theorem T9.9): the QD layer is the *scale-flow* (Tier-2) completion of the first-order value — the part the radial geometry legitimately organizes — so where its pull is small the four-figure agreement is honest. Where its pull is large (α_s, m_H, the δ_{CP} shift) it reaches into the scale-invariant content the geometry cannot derive, and there it is an honest bounded *selection*, carried in the same global trials accounting (§102.1), not a loop derivation.

Objection 5 (O-SM-5) (the philosopher of science)
"If each value can be assigned a status after the fact, is the triage falsifiable, or is it flexible bookkeeping that always survives?"

Response. The triage is falsifiable because the status labels carry commitments that data can break. A **Derived** entry asserts a parameter-free value; a measured deviation beyond the stated tolerance falsifies it directly (e.g. $\sin^2\theta_W = 25/108$ at $> 0.5\%$, or any fourth chiral family). A **Derived (conditional)** entry names a specific mathematical lemma whose failure to be provable would demote it. An **Asserted** entry makes no theorem claim and so cannot be "rescued" by relabeling — it is simply open. What the triage forbids is the move the objection fears: silently upgrading an entry without either a new theorem or a passing datum. The λ_H reclassification of the present version is the test case — it moved only because an explicit curvature calculation with a passing vev cross-check was carried out, and it carries its remaining open lemma in the open (Objection 7).

R The individual derivations

Objection 6 (O-SM-6) (the electroweak theorist)
"$\sin^2\theta_W = 25/108$ is your one genuine theorem, yet you now concede it rests on two unproven lemmas. So is anything derived end-to-end?"

Response. The honest answer is: one parameter is derived end-to-end *modulo two named structural steps*, and we label it exactly that way (Theorem 32.1). The upstream half is airtight: the trace ratio $\mathrm{Tr}(B_0^2)/\mathrm{Tr}(Y_0^2) = 2/5$ is elementary SU(5) representation theory, giving $\mathrm{Tr}(\hat{Y}^2_{\mathrm{split}}) = 5/6$. The assembly $\sin^2\theta_W = (1/N)[\mathrm{Tr}(\hat{Y}^2)]^2 = 25/108$ then rests on two physically-motivated but GG-6-internal steps — the quadratic neutral-partition lemma (the squaring) and the color-blind averaging lemma (the factor $1/N$). We name them rather than bury them. "Nothing derived end-to-end" overstates the deficit: the value uses *no empirical input*, reproduces the measured angle, and its two open steps are ordinary structural assignments, not hidden fits. It is the most-derived of the twenty-eight, and we say precisely how derived it is.

Objection 7 (O-SM-7) (the Higgs/gauge-Higgs-unification specialist)
"You relabeled λ_H from a 'selection' to a 'curvature' after the cost-minimum argument failed. Isn't that moving the goalposts?"

Response. The relabeling followed a calculation, not a rhetorical retreat, and it was made only after the cost-minimum was found wanting. The status moved from "selected (degenerate)" to **Derived (cond.)** on lemma L6 — the triplicity-layering hypothesis **H-L2**, under which the quartic is the neutral fraction of the doublet raised to the triplicity index, 2^{-N}, the same hypothesis the vacuum value v rests on. The curvature reading that stood between those two — λ_H as the quartic curvature of the holonomy potential, with its closed form, its vev cross-check and its natural one-loop size — is withdrawn with the coset reading (§36.5) and is recorded in §35.5 as withdrawn. The goalpost did not move; a hypothesis with a name and a grade replaced a cost-minimum that carried none, and the one route to an unconditional grade — a QD transport computed from the ledger before comparison — is stated openly under lemma L6, with the caution that the printed λ_H^{QD} is the datum read backwards and does not qualify.

Objection 8 (O-SM-8) (the precision critic, on λ_H)
"If the curvature calculation only gives an order of magnitude, why call λ_H derived at all?"

Response. Because "derived" in the triage means "fixed by a structural object, not a free real number," and a representation-theoretic fraction raised to the triplicity index is exactly such an object. The curvature reading that once supplied the microscopic backing — a fourth derivative of a potential whose shape is fixed by the geometry — is withdrawn with the coset reading (§36.5), and the grade now rests on lemma L6 alone: the triplicity-layering hypothesis **H-L2**, under which the quartic self-stiffening of the selected branch is the neutral fraction of the doublet raised to the triplicity index, 2^{-N}. The counting is a theorem; the identification is the hypothesis, stated as Step 3 of Thm 35.1 and not proved there, and λ_H is labeled "Derived (cond.)" on lemma L6 accordingly. The residual is small and quantified: 1/8 sits 3.2% below the measured 0.1291, and the QD-completed value 0.12928 is representative, read from the datum, not a transport that closes it. Calling it "derived modulo one lemma" is the accurate description; calling it either "fully derived" or "merely asserted" would be the dishonest options.

Objection 9 (O-SM-9) (the GUT theorist)
"Textbook SU(5) gives $\sin^2\theta_W = 3/8$ at unification, evolved down by running. You quote $25/108 \approx 0.231$ at M_Z with no running. How can both be defensible?"

Response. They describe different objects. The 3/8 is the group-theoretic ratio of gauge generators at the unification scale, which must then be run down through the desert; the 25/108 is the value of the boundary observable at M_Z, read directly off the split-generator partition of the electroweak attractor. In GG-Theory the radial coordinate *is* the renormalization-group axis, so "running" and "descent along the throat" are the same operation, and the boundary readout already incorporates it. The non-trivial claim is that the geometry delivers the low-energy value without a fitted threshold between M_Z and E_{GUT}. That is a falsifiable statement: a measured

$\sin^2\theta_W$ departing from 25/108 at the half-percent level would break it, independently of any GUT-scale bookkeeping.

Objection 10 (O-SM-10) (the flavor physicist)
"The mass hierarchy via $e^{-n\pi}$ looks like Froggatt–Nielsen with the Froggatt–Nielsen charge renamed 'cascade depth.' What is new?"

Response. The resemblance is real and we welcome it, but the exponent is fixed where Froggatt–Nielsen leaves it free. In Froggatt–Nielsen the suppression is ϵ^q with both the small parameter ϵ and the charges q chosen to fit. Here the suppression is the Bogomolnyi instanton action $S_{[2k]\to[2(k-1)]} = \pi k$ on the family-projector moduli $\mathbb{C}P^{N_g-1}$ (Arisaka, 2026, P2-N3; Witten, 1982): the base $e^{-\pi}$ is not adjustable and the integer depths are the only freedom. That residual freedom is exactly what the triage flags — the eight charged-fermion depths are the eight **Asserted** entries (Objection 11). So the honest claim is narrow and testable: GG-6 fixes the *base* of the hierarchy and the doubling structure, and isolates the per-fermion integers as the open problem, rather than fitting a continuous ϵ.

Objection 11 (O-SM-11) (the flavor physicist, continued)
"All eight of your Asserted parameters are the charged-fermion cascade depths — precisely the hard part of flavor. Haven't you simply not solved flavor?"

Response. Correct, and the paper states it plainly: the charged-fermion Yukawa sector is where the framework is weakest, and the eight depths $n_X \in \{0,1,2,3,4\}$ remain **Asserted**. What the paper claims is more limited and, we think, still substantial: the spectrum lives on a fixed cascade with a fixed base, the up/down interleaving is structural, and the entire residual ignorance is compressed into one well-posed lemma — the heights of the fixed points of sector-dependent torus actions on the doubled family stratum, which would fix each depth as a height rather than as a fitted integer (the cascade-depth lemma, $L10$). "Flavor is unsolved" and "flavor is reduced to one named calculation" are both true; the second is the contribution. We do not claim the first is the second.

Objection 12 (O-SM-12) (the mixing-matrix critic)
"You now mark the PMNS Dirac phase 'Derived (conditional)' at 196°, yet it is barely measured. Isn't deriving an as-yet-unmeasured number an unfalsifiable bet?"

Response. It is the opposite of unfalsifiable. The structural rule fixes $\delta_{\rm CP}^{\rm PMNS} = \pi + 2\arcsin(1/7) = 196°$ from the same odd integer $J = 2N+1 = 7$ that orders the atmospheric angle, *before* the measurement — a forward commitment, not a fit. DUNE and Hyper-Kamiokande will measure $\delta_{\rm CP}$ to $\sim \pm 15°$ over 2027–2032; a central value far from 196° falsifies the assignment (falsifier F-Origin-6 of GG-A11-Origin). The phase is **Derived (conditional)**, not **Asserted**, because the structural rule selects a unique value: a flat-monomial cost search does surface a subdominant near-degeneracy $\{\pi, 3\pi/2\}$, but the derivation breaks it by tying the phase to the same $J = 7$ that fixes the atmospheric sector. It is 'conditional' only in that it rests on one named lemma — the leptonic Dirac-phase assignment lemma — exactly as the three

PMNS angles do. A derived value whose experimental test is named, dated, and pending is the strongest form of empirical exposure, not the weakest.

Objection 13 (O-SM-13) (the strong-CP critic)
"How is $\bar{\theta}$ handled — is the strong-CP angle derived, or quietly set to zero?"

Response. It is confined by structure rather than tuned, and it is labeled conditional, not derived end-to-end. At first order the *bare* $\bar{\theta}$ is 0 or π: the exported Lagrangian carries no θ-coefficient, and the family-link texture is Hermitian, so the quark determinant is real (Theorem 79.1). The zero branch is the one the neutron bound selects; that the texture's sign delivers it is lemma L9's gap (a′), registered at O_{23} — a named sign lemma rather than a Peccei–Quinn field inserted by hand (Peccei & Quinn, 1977). The associated light-axion phenomenology — the Kaxion — is a genuine prediction, but it is deferred to the companion paper C1-Kaxion (Arisaka, 2026, C1-Kaxion) rather than claimed here. Within A9 the honest statement is that the *bare* angle's smallness is structural and conditional on one lemma; the observable angle is the Kaxion-relaxed minimum at the CKM floor $\sim 10^{-17}$, and the experimental exposure is two-fold: the neutron electric dipole moment falsifies the zero branch at any level (F-SM-4), and the companion's laboratory line tests the relaxation.

S Comparison with other approaches

Objection 14 (O-SM-14) (the string theorist)
"String theory offers a UV-complete quantum gravity. GG-6 posits a six-dimensional bulk without proving UV completeness. Isn't this just an effective model dressed as a first-principles theory?"

Response. GG-6 makes a different trade than string theory and is explicit about it. String theory secures UV completeness at the cost of an unconstrained moduli landscape ($\sim 10^{500}$ vacua), which is exactly the unforced continuous freedom that the parsimony axiom AX-4 forbids. GG-6 instead fixes the bulk uniquely — six dimensions, one composite group, one tensor sector, one five-integer ledger — and pays for it by not yet exhibiting a string-theoretic UV completion. The two programs are not in zero-sum competition: GG-6 uses string-theoretic results as inputs (the Green–Schwarz mechanism appears as the four-component BI-6 statement (Arisaka, 2026, A3-BI6; M. Green & J. Schwarz, 1984)) while rejecting the open moduli space. Whether the unique GG-6 bulk admits a string realization is an open and, to us, interesting question — but the predictive content of the codebook does not wait on its answer.

Objection 15 (O-SM-15) (the supersymmetry advocate)
"Without supersymmetry, what stabilizes the electroweak hierarchy, and why is the weak scale not dragged up to E_{GUT}?"

Response. The weak scale is not a tuned residual here; it is a derived geometric quantity, $v = M_{\mathrm{lock}}(2/L)^N$, set by the lock scale and the scale/triplicity factor (Theorem 34.1). The

little-hierarchy smallness $\xi = (v/f)^2 \approx (2/L)^{2N}$ is therefore geometric rather than fine-tuned — the same scale profile that fixes the vev supplies the suppression that, in composite-Higgs language, is usually put in by hand. Supersymmetry is not invoked, and the Coleman–Mandula constraints that motivate it are respected because the extra structure is Abelian gauge symmetry, not a spacetime-symmetry extension. The cost is that the Higgs-mass closure rests on the same hypothesis H-L2 as λ_H (Objection 8); we do not claim a complete hierarchy proof, but we do replace "tune the weak scale" with "read it off the geometry."

Objection 16 (O-SM-16) (the neutrino-anarchy advocate)
"Large, seemingly random PMNS mixing is well described by anarchy with no structure. Why prefer specific rationals over the simpler anarchic null hypothesis?"

Response. Because anarchy and GG-6 make different, testable commitments, and the difference is becoming measurable. Anarchy predicts distributions, not values, and is comfortable with essentially any mixing pattern; GG-6 predicts specific structural forms for the PMNS angles (conditional on named lemmas) and, crucially, a definite reactor and atmospheric structure rather than a flat prior. The discriminator is precision: as θ_{13}, θ_{23}, and eventually δ_{CP} tighten, anarchy can only ever be "consistent," whereas the GG-6 forms can be falsified by a measured value outside their tolerance. We do not claim anarchy is excluded today; we claim GG-6 stakes a sharper position and is therefore the more useful hypothesis to test.

Objection 17 (O-SM-17) (the lepton-spectrum specialist)
"Does GG-6 reproduce or explain the Koide relation, which already ties the charged-lepton masses with a single empirical formula?"

Response. The charged-lepton masses in GG-6 come from the cascade plus the QD-completion rationals on the lepton stratum, and the Koide combination is a derived consequence of that spectrum rather than an independent input. The framework's claim is stronger than Koide in ambition — it assigns each mass a ledger value, not merely a relation among the three — but weaker in present rigor for the depths (Objection 11). We regard the Koide relation as a useful consistency check: any structural lepton spectrum must reproduce its near-saturation, and the GG-6 spectrum does, without the relation having been imposed. We do not claim to have a first-principles proof of Koide saturation; we note its compatibility and flag the deeper explanation as part of the same open cascade-depth problem.

Objection 18 (O-SM-18) (the renormalization-group critic)
"You reinterpret the fifth dimension as the renormalization-group axis rather than a Randall–Sundrum infrared region. Isn't that an *ad hoc* relabeling to dodge Kaluza–Klein graviton bounds?"

Response. It is a substantive identification with consequences, not a cosmetic relabeling, and it is shared across the cohort. Treating the radial coordinate as the RG scale, $r = \ln(E_{\mathrm{GUT}}/\mu)$, means the boundary couplings are RG flows rather than modes of a literal extra dimension, so the spectrum of light Randall–Sundrum gravitons that precision and collider bounds constrain is simply not predicted. What *is* predicted is the lock-cluster structure at $M_{\mathrm{lock}} \approx 3.85$ TeV

(Objection 19). The reframing is testable precisely because it changes what should and should not appear: no tower of sub-TeV KK gravitons, but a specific lock-cluster signature. A relabeling with no observational consequence would be idle; this one reorganizes the predictions.

T Phenomenology, falsifiability, and proton decay

Objection 19 (O-SM-19) (the LHC phenomenologist)
"Your lock cluster at ≈ 3.85 TeV — why has the LHC not seen it, and what exactly would falsify it?"

Response. The detailed lock-cluster phenomenology — the resonance pattern, the branching ratios, and the collider reach — is deferred to the dedicated preprint P1-KK (Arisaka, 2026, P1-KK), and A9 imports only the scale. At 3.85 TeV the states sit near or beyond current sensitivity for the relevant channels, so non-observation to date is expected, not anomalous. The falsifier is sharp and pre-declared: the cluster predicts a specific pattern of nearly-degenerate states at that scale; the absence of any such structure within the predicted window at HL-LHC energies, or a clean resonance at a markedly different scale, would break the lock mechanism. We state the scale here so that the prediction is on the record before the data arrive, and point to P1-KK for the line shapes.

Objection 20 (O-SM-20) (the proton-decay veteran)
"SU(5) is famous for predicting proton decay that Super-Kamiokande has not seen. You assert protons are 'absolutely stable.' Isn't that equally exposed — falsified the moment a single decay is seen?"

Response. Yes, and we embrace that exposure: proton stability is a primary falsifier of GG-6, stated as such. The mechanism is specific. GG-6 does not stop at SU(5); the anomaly structure BI-6 forces an additional gauged $\mathrm{U}(1)_B$ (Arisaka, 2026, A3-BI6). The dangerous dimension-six X, Y operators that drive decay in ordinary SU(5) carry net baryon number and are therefore forbidden by exact baryon-number gauge invariance, even though the $\mathrm{U}(1)_B$ boson is itself heavy. The prediction is thus the opposite of minimal SU(5): not a lifetime near 10^{31} years, but absolute stability — the (H2) descent step this paper once flagged open is now discharged by GG-P6-Proton (the status correction of §109.6), with the standing conditions those recorded there. A single confirmed $p \to e^+\pi^0$ event would falsify GG-6 as decisively as continued non-observation falsified minimal SU(5). We regard turning SU(5)'s most famous failure into a sharp, opposite prediction as a strength, and we welcome the next-generation searches.

Objection 21 (O-SM-21) (the gauge-theory specialist)
"A gauged $\mathrm{U}(1)_B$ whose vector is removed from the light spectrum by a boundary condition still leaves only *global* baryon number. Doesn't the massive tower — or the termination itself — reintroduce baryon-number violation?"

Response. No, and the distinction is the crux. The Dirichlet condition that terminates $\mathrm{U}(1)_B$ acts on the vector's mode functions, not on the symmetry: the boundary action retains exact

$\mathrm{U}(1)_B$ invariance, so every operator with $\Delta B \neq 0$ has identically vanishing Wilson coefficient, to all orders. The massive Kaluza–Klein states at M_{lock} couple through the conserved baryon current with B-symmetric vertices and source no net $\Delta B \neq 0$ process; nothing is eaten at the zero mode, because the zero mode is simply not in the spectrum, and what the massive levels do among themselves is the ordinary Kaluza–Klein pairing of $A_{B,\mu}$ with $A_{B,r}$, which carries no ΔB. What is absent is the local redundancy's massless mode; what survives is an *exact global* baryon number, which is all that proton stability requires. This is why the prediction is "exact global baryon number, residual to a boundary-terminated $\mathrm{U}(1)_B$" rather than "unbroken local $\mathrm{U}(1)_B$." The dangerous operators are forbidden by the surviving symmetry regardless of where the vector tower sits, exactly as a massive photon would still conserve electric charge.

Objection 22 (O-SM-22) (the gauge-coupling-unification critic)
"Non-supersymmetric SU(5) **famously fails to unify the three couplings. What makes the GG-6 unification work without superpartners?"**

Response. The claim is precise and falsifiable: the three couplings meet at $E_{\mathrm{GUT}} \approx 2.28 \times 10^{16}$ GeV with no fitted parameter inserted between M_Z and E_{GUT}. The mechanism is not added matter tuned to fix the slopes but the geometric identification of the radial/RG axis together with the lock-scale threshold at $M_{\mathrm{lock}} \approx 3.85$ TeV, which modifies the running in a structurally-fixed way rather than a fitted one. The honest test is the lock-cluster spectrum and the threshold evolution it induces: one-loop Standard-Model running alone does not meet the three couplings at E_{GUT} (it gives 37.4, 46.3, 45.5), the convergence drawn above M_{lock} is the framework's structural claim and not yet a computed evolution, and a computed evolution with the lock cluster open that failed to meet within the predicted geometry — given the measured low-energy inputs — would falsify the construction. We do not claim ordinary non-SUSY SU(5) unifies; we claim the GG-6 runtime does, and we expose that claim to the data.

Objection 23 (O-SM-23) (the four-generation hunter)
"How firmly is $N_g = 3$ established? Could a fourth, heavier family be hiding?"

Response. $N_g = 3$ is among the firmest results in the chain, and a fourth chiral family is a clean falsifier. The family number is fixed three independent ways that must agree: the SU(5) trace-ratio identity, the anomaly bookkeeping of BI-6, and the topological three-leg pincer proved in GG-P2-N3 (Arisaka, 2026, P2-N3). It is not a fitted multiplicity but a forced output of the same ledger that fixes everything else — indeed $N = 3$ enters $\sin^2\theta_W$, $\lambda_H = 2^{-N}$, and the vev. A discovered fourth chiral generation would not merely add a row to the codebook; it would break the ledger and falsify the framework. The triage records $N_g = 3$ as derived, and we regard the absence of a fourth family as a prediction rather than an accident.

Objection 24 (O-SM-24) (the cosmology critic)
"Deriving the vev from M_{lock} and the scale profile seems to commit you to cosmological outputs — dark matter, the cosmological constant. Don't those expose the whole edifice to cosmological falsification?"

Response. They do, and we accept the exposure while keeping the bookkeeping clean. The cosmological inventory — Ω_m, Ω_Λ, the dark sector, the equation of state — is derived in the companion paper A10-Cosmos (Arisaka, 2026, A10-Cosmos), not in A9, precisely so that each claim has a single owner. The relevant point for A9 is consistency: the same lock scale and ledger that fix the electroweak sector also feed the cosmological predictions, so a cosmological falsification (say, a dark-matter mass or an Ω value outside the predicted structure) would propagate back and threaten the shared inputs. Far from insulating the particle codebook, the shared ledger means the cohort stands or falls together — which is why we keep the cross-paper coherence map explicit.

U Foundations, scope, and the meta-question

Objection 25 (O-SM-25) (the quantum-field-theory critic)
"Is the boundary Standard Model you derive an internally consistent quantum field theory, or only a pattern of numbers that happens to match?"

Response. It is the ordinary Standard Model as a quantum field theory, with the parameters supplied by the ledger rather than by measurement. GG-6 does not modify the renormalizable $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ dynamics on the boundary; it derives the values of the couplings, masses, and mixings that the Standard Model leaves free. Anomaly cancellation, unitarity, and renormalizability are inherited from the established structure (and, upstream, are tied to the bulk anomaly theorem BI-6). So the answer is both/and: it is a genuine QFT, and the contribution is the parameter pattern. The novelty is not a new low-energy dynamics but a derivation of the constants the low-energy dynamics needs.

Objection 26 (O-SM-26) (the reproducibility critic)
"Can an independent reader regenerate every number from scratch, without your code and without tuned constants?"

Response. That is a design goal of the paper, and the means are provided. Every codebook entry is a closed-form expression in the ledger integers and the named anchors; Table 7 gives the formula, the status, and the owning theorem for each, and the fermion-arithmetic appendix walks the explicit numbers. There are no continuous fitted constants to reproduce — only the integers $(1, 3, 5; 7, 35)$, the structural anchors, and arithmetic. A reader can recompute $\alpha^{-1} = 2^J + N^2$, $\sin^2\theta_W = 25/108$, $v = M_{\text{lock}}(2/L)^N$, and the cascade spectrum by hand. We regard "checkable with a pencil" as a higher standard than "checkable with our code," and the codebook is written to meet it.

Objection 27 (O-SM-27) (the foundational skeptic)
"You take $\{c, G_N, \hbar; k_B, Q_B\}$ as anchors. Why should the Standard Model need a *energy–information* constant Q_B at all?"

Response. Q_B does no work in the particle-physics codebook, and we are explicit that the twenty-eight Standard-Model parameters are fixed by the ledger plus the four standard constants $\{c, G_N, \hbar, k_B\}$. Q_B enters only as the boundary anchor of the optional information sector $\mathrm{U}(1)_{Qi}$, which is adopted on empirical grounds elsewhere in the cohort (GG-A11-Origin, GG-A7-Qi4) and is not required for any number in this paper. We list it among the anchors for cohort-wide uniformity, not because the electroweak or flavor sectors depend on it. A reader who rejects the $\mathrm{U}(1)_{Qi}$ sector entirely loses nothing in the Standard-Model derivation; the codebook is untouched.

Objection 28 (O-SM-28) (the Occam critic)
"Is a twenty-two-paper program that derives twenty-eight numbers actually *simpler*, by any reasonable measure, than the Standard Model it claims to explain?"

Response. By the measure that matters for parsimony — the count of unexplained continuous inputs — yes. The Standard Model takes twenty-eight real numbers as free inputs; GG-6 replaces them with five integers, a handful of standard constants, and a list of named lemmas. The page count of the program is large because it shows its work, not because the underlying structure is complicated: the generative content is the five-integer ledger, which is far smaller in information than twenty-eight independent reals. Length of exposition and economy of assumptions are different axes. The honest comparison is integers-and-lemmas versus twenty-eight reals, and on that comparison the framework is the more economical hypothesis — with the lemmas as the explicit IOUs.

Objection 29 (O-SM-29) (the predictive-power critic)
"What does GG-6 predict that the Standard Model does not, and which could be tested soon?"

Response. Several sharp, near-term exposures, each pre-declared. Absolute proton stability rather than SU(5)-scale decay (Objection 20). No fourth chiral family (Objection 23). $\sin^2\theta_W = 25/108$ at the half-percent level with no fitted threshold (Objection 9). A lock cluster near 3.85 TeV with a specific pattern rather than a generic resonance (Objection 19). A Kaxion at ~ 8 μeV in the companion axion paper. The Standard Model predicts none of these — it accommodates the measured parameters but forecasts no particular value. Each GG-6 item can move from "consistent" to "falsified" with foreseeable data, which is the property the objection rightly demands. We collect them in the predictions-and-falsifiability section so that the framework's neck is visibly on the block.

Objection 30 (O-SM-30) (the closing meta-question)
"Suppose several of your Asserted entries are never derived. What survives, and why is the paper still worth publishing?"

Response. Even in that pessimistic case, the durable contribution is the reframing: the twenty-eight Standard-Model parameters are organized onto a single five-integer ledger, with a published, parameter-by-parameter honesty triage that says exactly how strong each placement is and names the precise mathematical step that would settle each open one. The strong empirical hits — the integer 137, $\sin^2\theta_W = 25/108$, the 246 GeV vev, the gauge-coupling meeting, the four-significant-figure codebook — stand on their own as falsifiable claims regardless of the fate of the cascade-depth lemmas. And the open problems are not vague gestures but a numbered roadmap (§P). A framework that reduces twenty-eight free reals to a short list of integers plus an explicit, falsifiable to-do list has changed the question, even if it has not closed every line of it. That, in our view, is what a foundational paper should do: not pretend to finish the subject, but make the remaining questions sharp.

V Sharper methodology and statistics

Objection 31 (O-SM-31) (the two-readings critic)
"You report *two* numbers for most parameters — a 'first-order' ledger value and a 'QD-completed' value. With two shots per parameter, can you ever miss?"

Response. The two readings are a single prediction at two stated precisions, not two independent shots, and the rule for which applies is fixed in advance, not chosen after seeing the datum. The first-order value is the leading ledger monomial; the QD completion is the *same* value corrected by Hopkins–Singer integrality rationals whose denominators are forced by the stratum (Objection 4). A reader is owed both because they answer different questions — "what does the bare ledger give?" and "what does the ledger give once the named integrality theorem is applied?" — and the honesty triage (Table 7) reports the status of the completed value, never the more permissive of the two. The discipline that forbids the hedge is that the QD rationals are the *same* integers across the lepton, quark, and mixing sectors; one cannot tune them for one parameter without breaking another.

Objection 32 (O-SM-32) (the Bayesian)
"Your 'complexity cost' assigns the integer ledger essentially zero prior penalty. A skeptic would assign the entire six-dimensional bulk, the composite group, and the tensor sector a large penalty. Isn't the whole parsimony claim prior-dependent?"

Response. It is prior-dependent, and we state the prior rather than smuggle it. The complexity measure is explicitly the MSMF/OGN ℓ^1 cost on the integer ledger (§C cites the selection principle, Theorem 101.1), and the look-elsewhere numbers of Objection 2 are computed *within* that prior. A skeptic is free to assign the bulk machinery a large structural penalty; the point we defend is narrower and prior-independent — that *given* the constitutional bulk, the residual

continuous freedom in the parameter sector is zero, so the Bayesian model-complexity debate is about the one-time cost of the framework, not about per-parameter fitting. The honest division is: the bulk is an expensive but fixed prior, paid once; the codebook adds no further adjustable cost. We argue the first is worth it only because the second is genuinely free.

Objection 33 (O-SM-33) (the version-history auditor)
"Across versions you have *reclassified* entries — λ_H moved from 'selected' to 'curvature'. Each reclassification is a hidden degree of freedom. How many revisions before the triage is allowed to stabilize?"

Response. Reclassifications are permitted only in the direction that data and proof allow, and each one is logged with its trigger, so the version history is an audit trail rather than a hidden knob. λ_H moved because an explicit curvature calculation with a passing vev cross-check was performed (Objection 7), not because a fit improved; the triage's own rule (Objection 5) forbids upgrading an entry without a new theorem or a passing datum. A genuine degree of freedom would let a value be re-tuned to track a shifting measurement; the triage instead pins the value and lets only the *status label* change, monotonically, as proofs arrive. If a future revision had to *downgrade* a Derived entry, that too would be logged — and would itself be the falsification the objection is right to demand.

Objection 34 (O-SM-34) (the marketing skeptic)
"'Twenty-eight to zero' is the slogan, but your own triage has fourteen Conditional and eight Asserted — twenty-two of twenty-eight are not derived end-to-end. Isn't '28-to-zero' overselling?"

Response. '28-to-zero' is a statement about *continuous freedom*, not about completed proofs, and on that axis it is exact: not one of the twenty-eight is a free real number once the ledger is fixed. The fourteen Conditional and eight Asserted entries are not free parameters — each is pinned to a ledger value and an owning lemma — so the count of *adjustable reals* is genuinely zero even though the count of *fully proved* entries is six. We agree the slogan can mislead if read as "twenty-eight proved", which is exactly why the paper never states it without the triage (Objection 1) and why the accurate phrasing is "twenty-eight reduced to five integers plus named lemmas plus one measurement." The zero refers to knobs, not to theorems; both numbers are reported.

Objection 35 (O-SM-35) (the correlated-evidence statistician)
"The integer $N = 3$ enters $\sin^2\theta_W$, $\lambda_H = 2^{-N}$, and the vev. These are not independent confirmations — they share one input, so your tally of 'hits' overstates the evidence."

Response. Correct that they are correlated, and the paper neither hides nor double-counts the correlation — it is the point. $N = 3$ is a single forced integer (Theorem 21.1); the fact that it then appears in the weak angle, the quartic, and the vev is not three independent coincidences but *one* structural commitment with three exposed consequences. That makes the evidence *more* dangerous to the framework, not less: a single measured failure in any of the three would impugn the shared $N = 3$ and hence the others, so the correlation concentrates the falsification

risk rather than inflating a success count. We do not claim three independent confirmations of $N = 3$; we claim one input with three falsifiable outputs, which is the honest and the more exposed statement.

W Sharper technical derivations

Objection 36 (O-SM-36) (the QED-precision critic)
"$\alpha^{-1}(0) = 2^J + N^2 = 128 + 9 = 137$, but the measured value is $137.035999\ldots$. You call the 0.026% residual 'higher-order QED'. Isn't that hand-waving — why should the structural value be exactly the integer part?"

Response. The structural object is the bare OGN normalization $\alpha^{-1} = 2^J + N^2$, an integer because it counts a dyadic seam depth plus a triplicity square (Theorem 39.1); the measured 137.036 is that bare value *dressed* by the ordinary low-energy QED vacuum polarization, which is computed in the Standard Model, not invented here. The claim is therefore falsifiable and specific: the integer part is structural and parameter-free, and the residual must match the known QED correction — not an adjustable offset. If the measured integer part of $\alpha^{-1}(0)$ were ever revised away from 137, the prediction breaks (Objection 29). "Higher-order QED" is not a fudge but a named, independently-calculable quantity; the test is that it has the right sign and magnitude, which it does.

Objection 37 (O-SM-37) (the integrality skeptic)
"Your QD denominators differ by sector — $640 = 2^J L$ for leptons, a different rational for quarks. If Hopkins–Singer integrality really forced them, why are they not the same everywhere?"

Response. Because the denominator is the integrality modulus of the *specific stratum* a sector lives on, and the lepton and quark family-projectors sit on different strata, so different moduli are not a freedom but a prediction of the descent (Objection 4). $640 = 2^J L$ is the lepton-stratum modulus; the quark strata carry their own forced denominators from the same Hopkins–Singer mechanism (Hopkins & Singer, 2005) applied to a different cohomology. The skeptic's test is the right one and we invite it: the denominators are not chosen to fit each sector but are read off the stratum's integral structure, and *no* alternative small-integer numerator on the resulting lattice reproduces all the ratios in a sector simultaneously. Same mechanism, stratum-dependent modulus — that is what an integrality theorem looks like, not a per-sector knob.

Objection 38 (O-SM-38) (the top-quark specialist)
"The top sits at cascade depth zero, i.e. unsuppressed, so it lands near the vev 'for free'. Isn't a near-vev top mass imposed by the depth-zero assignment rather than predicted?"

Response. The depth-zero *assignment* is the one non-trivial claim, and it is exposed, not free. Given depth zero the top mass is $\sim v$ with no suppression — that part is arithmetic — but *why*

the top is the unique unsuppressed family member is the structural content: it is the apex of the family cascade, the only fermion whose overlap integral is order one. That assignment is among the eight **Asserted** cascade depths (Objection 11), so the paper does *not* claim it as derived; it claims that *if* the cascade-depth lemma assigns the top depth zero — the natural reading of the apex — then $m_t \approx v$ follows. A measured top mass far from the vev scale would already be in tension; that it sits where the unsuppressed slot predicts is a consistency the framework gets right but does not yet prove.

Objection 39 (O-SM-39) (the gauge-boson-mass critic)
"m_W, m_Z, and m_H all follow from the vev and the gauge couplings. Counting them as three separate 'theorems' (T-SM-33.2, 33.3, 33.4) inflates the score — they are not independent."

Response. They are not independent, and the paper's logic depends on their *not* being independent — that is the closure claim, not a padded tally. Once the vev (Theorem 34.1) and the gauge couplings are fixed by the ledger, m_W, m_Z, and m_H are *forced* by the ordinary tree-level relations $m_W = \frac{1}{2}gv$, $m_Z = m_W/\cos\theta_W$, $m_H^2 = 2\lambda_H v^2$; we state them as separate theorems because each is a separate *checkable* number against the PDG, not because each is a separate assumption. The honest reading is the opposite of inflation: three masses are predicted from *zero* additional inputs beyond v, the couplings, and λ_H, so the three agreements are one structural success exposed three times — and a single mass off its predicted value would break the shared inputs (cf. Objection 35).

Objection 40 (O-SM-40) (the seam-monomial critic)
"Each parameter is a monomial in π, e, and the ledger integers. What fixes *which* monomial — the exponents — for a given parameter, beyond searching until one lands near the datum?"

Response. The exponents are fixed by the physical role of the constant in the seam expression, and the MSMF/OGN principle then forbids importing any exponent not already demanded by that role (Theorem 101.1). π enters only as the angular holonomy of the compact S^1_θ (so it appears with the cascade/instanton action πk), and e only as the base of the radial redshift along the RG axis (so it appears in scale/overlap factors); a monomial that used π where a redshift belongs, or e where a holonomy belongs, is inadmissible regardless of how well it fits. That is why the look-elsewhere counts of Objection 2 are small for the sparse sectors: the admissible monomial space is constrained by the geometry of where each constant can appear, not by a free exponent search. Where the geometry does *not* pin the exponent, the entry is Asserted and we say so.

Objection 41 (O-SM-41) (the neutrino-mass-scale critic)
"The absolute neutrino mass scale and the two Majorana phases are essentially unconstrained in your scheme. In the sector where data are weakest, the framework predicts nothing testable. Isn't that telling?"

Response. It is telling, and the triage tells it: the neutrino sector is, with the charged-fermion depths, where the framework is weakest, and the absolute scale and Majorana phases carry different grades that this response had not kept up with. The codebook and the MSMF/OGN promotion audit both grade $\alpha_{21} = 0$ **Derived (convention)** and $\alpha_{31} = \pi$ **Derived (structural)** — a discrete opposite-parity datum of the rank-two block, not a fit — and the absolute scale $m_\star$ **Derived (cond.)**. None of the three is Asserted; the Asserted class is enumerated exhaustively as the eight charged-fermion Yukawas, which excludes the Majorana sector by construction. What the scheme does fix conditionally is the *pattern* — the PMNS angles as projector misalignments on the lepton stratum (Objection 12) and a normal-ordering preference — while the absolute scale awaits the same cascade-depth lemma that governs the charged sector. We make two honest statements rather than one dishonest one: the framework is not silent (it forecasts mixing structure and ordering, falsifiable as precision improves, Objection 16), but it is genuinely open on the absolute scale and Majorana phases, and a measured neutrinoless-double-beta rate (Objection 49) bears directly on the latter. Weakest-sector honesty is exactly what the triage exists to enforce.

X Dependence on the rest of the cohort

Objection 42 (O-SM-42) (the import-auditor)
"Strip out what A9 imports — the ledger from A2, $N = 3$ from P2, the cascade from A6, the boundary law from A5 — and what theorem is *A9's own*?"

Response. A9's own content is the boundary parameter map: the theorems that take the imported ledger and bulk and *compute the Standard-Model constants* from them. The weak angle (Theorem 32.1), the electroweak attractor coordinates (Theorem 30.1), the vev (Theorem 34.1), the boundary α^{-1} (Theorem 31.1), the mass theorems (T-SM-33.x), the conditional proton-stability theorem (Theorem 86.1), the four-force uniqueness (Theorem 95.1), and the MSMF/OGN selection on the boundary (Theorem 101.1) are stated and proved here, not imported. A9 is explicitly a *consequence* paper — its job is to carry the upstream structure to the 28 numbers — so heavy importing is by design, and the cross-paper coherence map names every borrowed result. The contribution is the map from a fixed ledger to the measured constants; that map is A9's.

Objection 43 (O-SM-43) (the conditional-inheritance critic)
"A6's GRIP cascade carries open problems M9/M10/M11. Your mass derivations use that cascade. Don't your 'Derived (conditional)' entries inherit *A6's* conditionals on top of your own — conditional on conditionals?"

Response. Yes, and the triage is built to make that inheritance explicit rather than to launder it. An A9 entry labeled **Derived (conditional)** on a cascade lemma is, transitively, conditional on whatever that cascade rests on upstream — including A6's M9 (the MSMF norm), M10 (the BI-6 budget that fixes $N_g \leq 3$), and M11 (the compensator monodromy) (Arisaka, 2026, A6-GRIP). We regard this as a feature of an honest cohort: the conditionals do not multiply silently because each is *named* and traceable through the coherence map, so a reader can see that, e.g., the $N_g \leq 3$ half of the family count ultimately rests on M10. The alternative — presenting A9's results as unconditional while the upstream remains open — is exactly the dishonesty the triage forbids. A9 is no more secure than its imports, and it says so.

Objection 44 (O-SM-44) (the parameter-count auditor)
"A3 speaks of *twenty-six* Standard-Model parameters, A9 of *twenty-eight*. Which is it, and does the discrepancy signal loose bookkeeping?"

Response. Both are right under stated conventions, and the two extra are the neutrino Majorana phases. The count is nineteen non-neutrino parameters (nine charged-fermion masses, four CKM, three gauge couplings, two Higgs, and $\bar{\theta}$) plus the neutrino sector. With *Dirac* neutrinos that sector adds seven (three masses, three angles, one phase) for twenty-six; with *Majorana* neutrinos it adds nine (the same plus two Majorana phases) for twenty-eight. A3's earlier statement used the Dirac count; A9, which treats the neutrinos as Majorana, uses twenty-eight. This is a convention difference, not a bookkeeping error, and we will harmonize the wording so both papers quote "twenty-six (Dirac) / twenty-eight (Majorana)" to remove the apparent conflict. The codebook itself is unaffected: the two Majorana phases are graded **Derived** — α_{21} by convention, α_{31} structurally — so the two extra parameters are counted but not fitted (Objection 41).

Objection 45 (O-SM-45) (the modular-flavor theorist)
"Modular flavor symmetry also generates masses and mixings from a single modular parameter τ. Why is the GG-6 codebook not just another point in that crowded landscape of flavor models?"

Response. Because the GG-6 flavor structure has *no* continuous modulus to vary, which is exactly the freedom modular models retain. In modular flavor symmetry the complex modulus τ is a continuous input fixed by a vacuum-selection assumption; vary τ and the spectrum moves, so the framework spans a family of models. In GG-6 the analog of τ is frozen — the family-projector geometry is $\mathbb{C}P^{N_g-1}$ with $N_g = 3$ forced (Theorem 21.1) and the cascade base $e^{-\pi}$ fixed (Objection 10) — so there is a single point, not a landscape, and the residual freedom is integer cascade depths, not a continuous parameter. The two programs share the ambition of structuring flavor; they differ on whether a continuous modulus survives. We claim none does, which is a sharper and more falsifiable position than a modular family.

Y Further phenomenology and existing limits

Objection 46 (O-SM-46) (the resonance-search critic)
"The LHC has already excluded many TeV-scale resonances up to 4–5 TeV in dilepton and dijet channels. Is a lock cluster at 3.85 TeV not already in tension with existing limits?"

Response. Not for the cluster's actual quantum numbers, and the distinction is why the detailed analysis lives in P1-KK (Arisaka, 2026, P1-KK) rather than being hand-waved here. The strongest existing limits near 4 TeV are for narrow, strongly-produced, leptonically-decaying Z'-like states; the lock cluster is a set of nearly-degenerate composite resonances whose production and decay pattern is *not* that of a canonical sequential Z', so the published exclusions do not map directly onto it (Objection 19). A9 commits only to the scale, $M_{\text{lock}} \approx 3.85$ TeV; whether the specific cluster is already constrained is a line-shape-and-channel question answered in P1-KK against the actual searches. The honest statement is that the scale is on the record and the channel-by-channel confrontation with HL-LHC limits is the companion paper's burden — and a clean exclusion of the predicted pattern in its predicted channels would falsify it.

Objection 47 (O-SM-47) (the muon-g-2 critic)
"Where does GG-6 stand on the muon anomalous magnetic moment? Does it predict the measured anomaly, accommodate it, or stay silent?"

Response. On a_μ the boundary theory is the ordinary Standard Model (Objection 25), so at the level of A9 the prediction is the Standard-Model value, with no new light states inserted to manufacture an anomaly. That is a genuine exposure rather than a dodge: the framework forbids the unforced extra sectors that beyond-Standard-Model models routinely invoke to fit a_μ, so if the experimental anomaly survives the current lattice-versus-data-driven debate on the hadronic vacuum polarization and demands genuinely new physics at the relevant scale, that would be evidence *against* the minimal GG-6 boundary content. We do not claim a GG-6 contribution to a_μ; we note that the framework's rigidity means it cannot quietly absorb such an anomaly, which is the kind of vulnerability the predictive-power objection (Objection 29) rightly wants stated.

Objection 48 (O-SM-48) (the FCNC critic)
"You introduce a gauged $\mathrm{U}(1)_B$ and an optional $\mathrm{U}(1)_{Qi}$. New neutral gauge bosons generically induce flavor-changing neutral currents tightly bounded by kaon and B-meson data. Are yours safe?"

Response. They are, because both extra Abelian currents are *flavor-universal* at the boundary, which is precisely the property that suppresses dangerous FCNC. $\mathrm{U}(1)_B$ couples to baryon number, a flavor-diagonal charge identical for all three generations, so it generates no tree-level flavor-changing vertex; the same holds for the information charge of $\mathrm{U}(1)_{Qi}$. FCNC constraints bite on *non-universal* Z' couplings, which the GG-6 charges are not. The boson masses are additionally large (the $\mathrm{U}(1)_B$ vector tower begins at the lock scale, Objection 21), further

suppressing any loop-level effect. So the kaon and B bounds that exclude generic Z' models do not constrain these currents at leading order; a measured flavor-changing signal attributable to a universal current would, conversely, be a surprise the framework would have to answer.

Objection 49 (O-SM-49) (the double-beta-decay critic)
"If the neutrinos are Majorana, neutrinoless double beta decay should occur. Does GG-6 predict a rate, and is it within reach of the next-generation experiments?"

Response. The framework commits to the *Majorana character* — hence to the existence of the process — but not yet to a sharp rate, because the rate depends on the absolute mass scale and the Majorana phases, which are graded **Derived** but not yet *measured* — which is the constraint that matters here, since a derived-but-unmeasured phase still leaves the rate to be predicted rather than checked (Objection 41). So the honest prediction is qualitative-but-real: $0\nu\beta\beta$ must occur at some level, and a definitive demonstration that neutrinos are *Dirac* (e.g. a robust null result combined with a cosmological mass detection forcing the Dirac interpretation) would contradict the Majorana assignment and count against the scheme. A measured effective Majorana mass would, conversely, feed back and help fix the currently-open phases, turning an Asserted entry conditional. We neither claim a rate the structure does not yet give nor hide that the sector's openness is exactly what a $0\nu\beta\beta$ measurement would help close.

Z Rigor, scope, and standing

Objection 50 (O-SM-50) (the dynamical-systems critic)
"Your gauge and Higgs values are read off 'attractor coordinates' of the boundary RG flow. Is the attractor structure — existence, basin, uniqueness — actually proved, or assumed because it gives the right numbers?"

Response. The attractor-coordinate theorem (Theorem 30.1) establishes the structure it uses, and where the proof is partial the entries that depend on it are labeled conditional, not derived. What is established is that the boundary Wilson–Hosotani / RG flow possesses fixed points whose coordinates are the split-generator partitions delivering $\sin^2\theta_W$ and the gauge couplings; what is *not* claimed in full generality is global uniqueness of the basin for every sector, which is among the named lemmas behind the **Derived (conditional)** entries. So the answer is calibrated: the attractor existence and the relevant coordinates are theorems, the global basin-uniqueness is a stated open step, and no value rests on an unproved attractor while being advertised as fully derived. The objection's worry — "assumed because it fits" — is exactly what the conditional label is there to prevent.

Objection 51 (O-SM-51) (the boundary-emergence critic)
"A5/GDOS gives a CPTP boundary projection. Is the boundary theory *proved* to be the Standard Model, or is 'the boundary is the Standard Model' the thing you assume and then decorate with numbers?"

Response. The boundary gauge group and chiral content are forced upstream, not assumed here — the scalar representation included, which the Yukawa sector requires and MSMF selects as the minimal representation closing it (§23.3): the four-factor bulk group and its anomaly-free chiral spectrum come from BI-6 (Arisaka, 2026, A3-BI6) and the GDOS projection (Arisaka, 2026, A5-GDOS), and the Standard-Model $SU(3) \times SU(2) \times U(1)$ is what survives the projection of that fixed bulk. A9 does not posit "the boundary is the SM" and fit constants to it; it inherits a boundary theory whose group, fermion representations, and anomaly structure are already determined (the scalar representation is required by the Yukawa sector and selected as minimal), and then computes the *parameters* that the inherited theory leaves free. The genuinely open part is not the identity of the boundary theory but the rigor of some parameter lemmas (the Conditional and Asserted entries). So "which theory" is upstream and forced; "which constants" is A9's, and exposed.

Objection 52 (O-SM-52) (the tuned-constant skeptic)
"The $\mathcal{H}_1+\mathcal{H}_2$ reduction carries an explicit $c_R = 1$. An $\mathcal{O}(1)$ constant that happens to equal exactly one looks tuned. Is it derived, or set to the convenient value?"

Response. $c_R = 1$ is a normalization forced by the reduction, not a coefficient chosen to land a number, and it is flagged as one of the named steps rather than slipped in. In the $\mathcal{H}_1+\mathcal{H}_2$ reduction imported from GG-P2-N3 (Arisaka, 2026, P2-N3), c_R is the ratio of two integrally-normalized cohomology pairings on the same stratum, so it is a ratio of integers that the construction sets to unity by the matching of the two legs — not a continuous dial. The honest exposure is that the *value* $c_R = 1$ depends on the same descent integrality (Objection 37) that fixes the QD denominators; were that integrality to fail, c_R would not be free to drift to a fitted value but the reduction itself would break. So the proper worry is not "why exactly one?" — the construction answers that — but "is the integrality airtight?", which is the open lemma the triage already names.

Objection 53 (O-SM-53) (the peer-review realist)
"The entire cohort is single-author, unrefereed preprints. Why should the community accept twenty-eight derivations, none of which has cleared peer review?"

Response. It should not *accept* them on authority — it should *check* them, which is why the paper is written to be checkable without trusting the author. Every codebook entry is a closed-form expression in named integers with its owning theorem and status given (Table 7, Objection 26), so a critic can recompute $\alpha^{-1} = 2^J + N^2$, $\sin^2\theta_W = 25/108$, and the vev with a pencil and confront each against the PDG independently of any code or claim of authority. Peer review is the right next step and we welcome it; the preprint's job is to put the structure, the triage, and the falsifiers on the record in a form that makes review *possible* and the claims

independently falsifiable. That the record is new is a true statement about its standing; it is not an argument about correctness, which the explicit arithmetic settles either way.

Objection 54 (O-SM-54) (the single-point-of-failure critic)
"Name the one result whose collapse would do the most damage. If you had to bet against GG-6, where is the framework most likely to break first?"

Response. The most load-bearing single result is the integer ledger $(1, 3, 5; 7, 35)$ with $N = 3$: it enters the weak angle, the quartic, the vev, the fine-structure value, and the cascade base, so its failure would propagate through the whole codebook (Objection 35). The most likely *near-term* break, by contrast, is empirical and sharp — a measured $\sin^2\theta_W$ off 25/108 at the half-percent level (Objection 9), a confirmed proton decay (Objection 20), or a fourth chiral family (Objection 23) — any of which would falsify the framework without waiting on the open lemmas. We would bet against the framework, if at all, at the *charged-fermion cascade depths* (Objection 11): they are the weakest, the most Asserted, and the place where "reduced to a named calculation" is most likely to turn out to be "reduced to a calculation that does not close." Naming that as the fragile point, rather than the headline numbers, is the honest answer.

Objection 55 (O-SM-55) (the scheme-coherence critic)
"Your flagship $\sin^2\theta_W = 25/108$ is labeled the $\overline{\text{MS}}$ angle at M_Z, yet you compare it to 0.23155, which is the *effective leptonic* angle. Against the $\overline{\text{MS}}$ value $0.23122(4)$ it is a $+6.5\sigma$ miss. Which observable do you actually predict?"

Response. Fair, and we have stopped letting the label and the value drift apart. The geometric output $L^2/(4N^3) = 25/108 = 0.23148$ is a prediction of the *effective leptonic* mixing angle $\sin^2\theta^\ell_{\text{eff}}$, which it matches to 0.03%. Against the $\overline{\text{MS}}$ angle $0.23122(4)$ the same number is a $+6.5\sigma$ tension — and the marquee caption now states exactly that, in the open, rather than quoting the convenient value under the inconvenient label. A reader is entitled to judge the closed form as a statement about the effective angle; the $\overline{\text{MS}}$ comparison is reported honestly alongside, not hidden.

Objection 56 (O-SM-56) (the pull-tautology critic)
"Your first-order theory error σ_{th} is *defined* as the gap between the first-order value and the quantum-defect-completed value. That forces $\text{Pull} \approx \pm 1$ on every row by construction. The pull column carries no information."

Response. Correct, and we no longer ask that column to do evidential work. The first-order σ_{th} is the estimated size of the omitted quantum-defect correction, so by construction the first-order pull sits near unity; that is a statement about the size of the next term, not about agreement with data. The honest measures are the two we now report alongside it: the relative deviation $\text{Dev} = (\text{Pred} - \text{Obs})/|\text{Obs}|$, and the experimental-only significance $(\text{Pred} - \text{Obs})/\sigma_{\text{obs}}$, which is frankly large for the most precisely measured constants because those rows are post-dictions of long-known inputs. We would rather show that plainly than dress a manufactured pull as a fit quality.

Objection 57 (O-SM-57) (the transcendental-mixing critic)
"$\alpha_s^{-1}(M_Z) = \pi e$ multiplies an angular constant by the base of the natural logarithm. Mixing π and e in one coupling is the textbook fingerprint of numerology."

Response. The two transcendentals are not thrown together; they label two different primitives of the color-difference channel — π the angular (holonomy) primitive and e the radial (renormalization-group) primitive — which is exactly why this channel, unlike the electromagnetic one, retains both. That said, we do not over-sell it: πe is reported as an MSMF/OGN cost-2 *selection* within 1% of the datum, look-elsewhere $p \approx 0.27$, with the empirical value openly acknowledged to enter the filter. It is listed as a selection, not as a derived theorem, and the decisive test is the global trials figure (O-SM-2, O-SM-73), not the cleanliness of the form.

Objection 58 (O-SM-58) (the bulk-construction critic)
"You have dropped the words 'holographic' and 'warped.' Good — but then what *is* your six-dimensional bulk? There is no bulk action, no boundary CFT, no central charge, no compactification manifold. Without those it is not a higher-dimensional theory at all."

Response. We agree the prestige vocabulary was misleading and have removed it. The 6D bulk is not advanced as a string background or an AdS/CFT dual, and we claim no central charge or GKP–Witten dictionary. It is defined *axiomatically*: AX-3 (OGGEP) posits a lawful projection from a six-dimensional bulk — four spacetime dimensions, a radial direction identified with the renormalization-group scale $r = \ln(E_{\mathrm{GUT}}/\mu)$, and a compact phase circle — onto the observed boundary. GG-Theory is an axiomatic–deductive program (§1.1, "the deductive structure in three stages"), to be judged on whether its theorems follow from its axioms and reproduce the Standard Model, not on whether it reproduces a string construction it never claimed. The honest cost of that stance is that the bulk's status is postulated, not derived from a UV-complete theory — which is precisely what the axiom asserts.

Objection 59 (O-SM-59) (the landscape-inversion critic)
"You scorn the string landscape for leaving the vacuum undetermined. But string theory at least *constructs* its 10^{500} vacua; you construct none and simply *assert* uniqueness. Asserting uniqueness is the easy half."

Response. A fair correction, and we have softened the contrast accordingly. GG-Theory does not build a moduli space and count its minima; it fixes its inputs by axiom — the OGN-5 integer ledger — and derives the rest. The honest comparison is therefore not "one vacuum versus 10^{500}" but "an axiomatic–deductive program versus a vacuum-counting construction." The burden that falls on us in return is real: the uniqueness of the $(1, 3, 5; 7, 35)$ ledger must be argued, not assumed, and we do so in A2-GG-6 and GG-P2-N3 — with the one genuinely open link, the BI-6 budget bound behind $N_g \leq 3$, flagged as a named conjecture rather than a theorem. We claim the deductive route is cleaner; we do not claim it is finished.

Objection 60 (O-SM-60) (the swampland critic)
"A serious higher-dimensional proposal must confront the swampland program — the Weak Gravity, distance, and de Sitter conjectures. You engage none of them."

Response. We separate the conjectures rather than plead a blanket exemption. The distance and de Sitter conjectures police low-energy effective theories descending from a gravitational UV completion of the usual string type, and GG-Theory does not present itself as such a compactification, so in their standard form they do not directly bind it; its own consistency conditions are different in kind — the BI-6 anomaly closure of GG-A3-BI6 and the positivity and finite-resolution requirements of GDOS (GG-A5-GDOS) are the constraints an admissible boundary law must satisfy — and a genuine swampland engagement awaits an independent gravitational completion of the bulk. The Weak Gravity Conjecture is different: it is argued to bind *any* effective theory with dynamical gravity and a gauge field, so we engage it on its own terms rather than plead exemption. GG-6 carries two abelian factors coupled to dynamical gravity — the gauged baryon number $\mathrm{U}(1)_B$ and the information charge $\mathrm{U}(1)_{Qi}$. $\mathrm{U}(1)_B$'s vector has no massless mode — its zero mode is removed by a boundary condition (Objection 21) — and the sharp electric form of the conjecture — a state with $m \leq g\,q\,M_{\mathrm{Pl}}$ — is formulated for *massless* U(1)s; its status for a terminated U(1) is itself unsettled, while the magnetic form bounds only the cutoff, $\Lambda \lesssim g_B M_{\mathrm{Pl}}$, which the sub-Planckian bulk scale and the derived coupling ($g_B = 0.565$, GG-A3-BI6 T-BI6-6.4) satisfy with orders of magnitude to spare. For $\mathrm{U}(1)_{Qi}$ the required light charged state is, honestly, an open consistency check we have not discharged rather than a box we claim to tick. The honest position is therefore narrower than "the swampland does not apply" and sharper than silence: the magnetic Weak Gravity bound is met, the electric bound is either inapplicable (no massless mode; the electric form is formulated for massless U(1)s) or an explicitly open check ($\mathrm{U}(1)_{Qi}$), and a full swampland analysis with the charged spectrum written out is named future work.

Objection 61 (O-SM-61) (the $N_g = 3$ circularity critic)
"Your $N = 3$ rests on a trace ratio $2/5 = (L - N)/L$ — but that ratio is computed in the 5 of SU(5) using *three* color entries. You put $N_c = 3$ in to get $2/5$ and read $N = 3$ back out."

Response. The objection correctly identifies a circularity *in the trace-ratio presentation*, and that presentation is not where the derivation lives. The actual argument is the three-leg pincer of GG-P2-N3: a lower bound $N_g \geq 3$ from the requirement of a physical CP-violating phase (independent of any color count), and an upper bound $N_g \leq 3$ from a GDOS/GRIP stability-budget theorem (the cascade cannot fund a fourth family), the latter the capacity theorem T-N3-6, conditional on the coupling clause and the integrality theorem L-INT (residue R-INT). The two bounds meet at three. The SU(5) trace-ratio identity is a downstream consistency check that the same integer reappears, not the proof; and $N = 3$ is graded in the codebook as a *conditional* derivation, not an unconditional theorem, precisely so that the open budget bound is visible.

Objection 62 (O-SM-62) (the QD-completion critic)
"Four-significant-figure agreement only appears after a 'quantum-defect completion' that multiplies each first-order value by a bespoke rational (637/640, 159/160, 641/640, ...). That is a second fit with one knob per observable."

Response. The concern is exactly right to raise, and it has a sharp test attached. We claim the QD denominators are not free: they are the Hopkins–Singer descent-integrality coefficients of the boundary X_4, with $2^J L = 640$ the controlling integer, so the admissible numerators form a fixed small set rather than a continuum. If that claim is correct, the QD layer is a computed completion, not a fit; if it is wrong, the four-figure agreement is manufactured. We regard the decisive evidence as the explicit enumeration showing that no alternative small-integer numerator reproduces all three lepton ratios simultaneously, and we agree that this enumeration — not the agreement itself — is what should persuade. The first-order value remains the bare prediction, reported separately and with its own honest deviation.

Objection 63 (O-SM-63) (the blind-prediction critic)
"Every marquee number — α, $\sin^2\theta_W$, α_s, v, m_H, y_t — was measured decades ago. Where is a single blind prediction of a not-yet-measured quantity, with an error bar?"

Response. We accept the distinction and now separate it explicitly. The gauge–Higgs and charged-lepton closed forms are post-dictions of long-known constants; their value is internal consistency, not foresight. The genuinely forward, presently-testable stakes are few and we name them as such: the neutrino mass sum $\Sigma m_\nu \approx 59$ meV (the near-term cosmological squeeze, DESI-DR2 + CMB ∼2026–2028 — possibly the first channel to fire), the leptonic Dirac phase $\delta_{\mathrm{PMNS}} \approx 196^\circ$ (DUNE/JUNO/Hyper-K), and absolute proton stability (a null; the (H2) step it once rested on is discharged by GG-P6-Proton, §109.6). Two further quantitative falsifiers — the Kaluza–Klein lock near 3.85 TeV and the Kaxion near 8.1 μeV — carry no cross-section or coupling *in this paper* and are deferred to GG-P1-KK and GG-C1-Kaxion. That is a thin set of live predictions, and we present it as thin rather than padding it with the post-dictions.

Objection 64 (O-SM-64) (the flavor-hierarchy critic)
"The fermion mass hierarchy spanning five orders of magnitude is *the* flavor puzzle. Your own table marks all eight non-top charged-fermion Yukawas 'Asserted, chosen post-hoc.' So you have not solved flavor at all."

Response. Correct, and we say so without hedging. GG-Theory fixes the *base* of the hierarchy — the top Yukawa reference and the geometric cascade scale — but the eight remaining charged-fermion Yukawas are an explicit discrete ansatz, graded **Asserted** in the codebook with the per-particle exponents and prefactors flagged as post-hoc. The abstract and summary now state this in the 6/14/8 triage rather than implying the masses are an output of geometry. What GG-Theory offers in the flavor sector is a structural template (a geometric-series cascade of mass ratios) with the rungs not yet derived; the honest headline is "the base, not the rungs." Closing the cascade-depth lemma that would *predict* each exponent is named as the principal open problem of the sector.

Objection 65 (O-SM-65) (the hierarchy-tuning critic)
"$v = \frac{1}{2} M_{\rm Pl}\, e^{-K} (2/L)^N$ with $K = 35$ has $dv/dK = -v$: one unit of your 'derived' integer moves the electroweak scale by a factor e. You have relocated the hierarchy problem into the assertion that K is exactly 35."

Response. The sensitivity is real and we now state it in the body. But the point cuts the other way once it is read carefully: K is a *discrete* integer of the OGN-5 ledger, not a continuous dial, so the hierarchy is carried by an integer-valued exponent rather than by tuning a small real number. There are no continuous dimensionless free parameters in the construction at all; the only dimensionful input is Newton's constant G_N, the energy↔spacetime conversion factor of general relativity, from which both $M_{\rm Pl}$ and $E_{\rm GUT}$ descend. Whether $K = 35$ is itself *forced* (as the dual-lock identity $K = LJ = M^2 + N^2 + L^2$ claims) is the proper locus of the question, and we put the burden there — not on a fine-tuned coefficient, of which there is none.

Objection 66 (O-SM-66) (the Planck-mass critic)
"Your $v \approx 246$ GeV uses the non-reduced Planck mass. With the reduced $M_{\rm Pl}$ standard in every effective theory it becomes ≈ 49 GeV — a factor of five. Which is it, and why?"

Response. The convention is now fixed once, cohort-wide: $M_{\rm lock}$ and the vev chain use the non-reduced Planck mass $M_{\rm Pl} = (\hbar c/G_N)^{1/2}$. This is not opportunism but a consequence of the single-input structure — G_N is the lone dimensionful constant, and both $M_{\rm Pl}$ and the unification scale are defined from it through the geometric scale chain, with the factor of $\sqrt{8\pi}$ absorbed into the once-stated definition rather than re-inserted per equation. We agree that a floating convention would be a factor-of-five escape hatch; that is exactly why the definition is stated explicitly at the single-input paragraph and used uniformly thereafter.

Objection 67 (O-SM-67) (the oblique-parameter critic)
"A gauge–Higgs-unification model with a holonomy Higgs and Kaluza–Klein towers generically shifts the oblique parameters S and T, which are measured to ± 0.1. You predict neither."

Response. We now do, and the answer is not comfortable. §34.6 computes the electroweak Kaluza–Klein tower on the canonical metric and evaluates its tree-level contribution to T against a Higgs registered at the infrared end: $\sum_{n\geq 1} \xi_n^2/m_n^2 = 17.0\, e^{2K}/k^2$ with $\xi_n^2 \to 2K = 70$, giving $T = 1.7$ with the first level at $M_{\rm lock}$, 4.2 on the free-field placement at 2.46 TeV, and 0.55 on the bulk profile $h \propto e^{2r}$ — which is a floor: the whole-tower sum has a closed form in the Higgs scaling dimension, increasing from $T = 0$ at the unitarity bound $\Delta_H = 1$ to the brane value, so no single-exponential profile that protects the hierarchy gives less than 0.55 at the lock. The global fit allows $T = 0.00 \pm 0.06$ with $U = 0$. S is suppressed relative to T by the ultraviolet-to-infrared amplitude ratio $\xi^2_{\rm UV}/\xi^2_{\rm IR} \simeq 5 \times 10^{-4}$ for endpoint-localized light fermions, which is why the comparison is made on T alone and why that is a conclusion rather than an omission. The bulk group $\mathrm{SU}(5) \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{Qi}$ contains no $\mathrm{SU}(2)_R$ — SU(5) has rank four, and the two Abelian factors are not an SU(2), so no custodial structure is available to soften it. This is Open Problem O_{22}, it is registered at kill grade, the collider companion carries the same

registration beside its own falsifier (its O-KK-68), and the four-significant-figure electroweak agreements of Part VI are quoted in the absence of this tower — both cannot stand. Which is interesting, and is not proof.

Objection 68 (O-SM-68) (the anomaly-polynomial critic)
"You call BI-6 'anomaly closure,' but what GG-A9-SM actually uses is a trace-ratio $|a/c| = |a/d| = 5/2$ — a constraint on U(1) charges, silent on the non-abelian SU(5) anomalies. That is not Green–Schwarz factorization."

Response. Agreed on the division of labor, and the text now marks it. Within GG-A9-SM we consume only the trace-ratio constraint from BI-6; we do not re-derive anomaly cancellation here. The full factorizing six-dimensional anomaly polynomial $I_8 = \frac{1}{2}X_4^2$, the modified Bianchi identity, and the flux quantization are established in GG-A3-BI6 (Arisaka, 2026, A3-BI6), where they are presented honestly as an *existence* construction rather than a uniqueness proof. So the correct reading is: GG-A9-SM inherits a checked anomaly structure and applies one of its consequences; the claim that the observed chiral spectrum is the *unique* anomaly-free choice is not made, and the gap is A3's to close.

Objection 69 (O-SM-69) (the abstract-versus-triage critic)
"Your front matter says the twenty-eight parameters are 'outputs of geometry, observables of one projection.' Your own codebook says six Derived, fourteen conditional, eight Asserted. Those cannot both be the headline."

Response. The two are reconciled by keeping apart two claims that are easy to run together. "The parameters are observables of the projection" is an *ontological* statement about what they are — it holds whether or not this paper succeeds in computing each one. "Six Derived, fourteen conditional, eight Asserted" is the *computational* status of those attempts. The executive summary and the marquee now carry the 6/14/8 triage at the point of the strong claim, with the explicit sentence that one parameter ($\sin^2\theta_W$) is derived end-to-end modulo two named lemmas and eight charged-fermion Yukawas remain asserted. The vision is intact; the bookkeeping is honest; and a reader of the front matter alone is no longer misled about what was achieved.

Objection 70 (O-SM-70) (the conditional-lemma critic)
"The fourteen 'Derived (conditional)' entries are where the fitting hides: each is 'forced once a single named lemma is supplied,' and several lemmas look reverse-engineered to land on the answer."

Response. This is the right place to press, and the honest answer distinguishes two kinds of named lemma. Some are genuinely upstream — fixed by GG-A3-BI6, GG-A6-GRIP, or GG-A5-GDOS before any Standard-Model number is consulted — and for those the conditional grade is conservative. Others are *A9-local structural assignments* (the squaring step and the color-blind average behind $\sin^2\theta_W$ are the clearest examples) that we do not claim to have proved from the upstream variational principle; the codebook says so in each comment. The proper standard is a held-out test: a lemma fixed by structure before the observable is known is a derivation, one

chosen to match is a fit, and we mark which is which rather than blurring them. Promoting the A9-local lemmas to theorems of the GFF variational route is named as open work.

Objection 71 (O-SM-71) (the axiom-shield skeptic)
"Calling OGGEP and MSMF 'axioms' looks like a shield: anything you cannot derive you can simply postulate. Why is that not a way of immunizing the theory against criticism?"

Response. Because declaring something an axiom *raises* the bar on everything downstream rather than lowering it. Once OGGEP and MSMF are fixed, every later step must be a genuine deduction: blind to the data, well-defined in scheme and scale, and free of circularity — and the paper is built to expose exactly those failures where they occur. The honesty triage publishes which outputs are theorems and which are not; the MSMF/OGN audit publishes the look-elsewhere cost of each selection; the conditional grades flag every undischarged lemma. An axiom that quietly consulted the data to fix a downstream number would violate MSMF itself, and we treat such cases as defects to be marked, not licenses. The axioms constrain us; they do not excuse us. The legitimate criticism is not "you postulated axioms" but "did your deductions honor them" — and that is the criticism the apparatus invites.

Objection 72 (O-SM-72) (the top-Yukawa critic)
"$y_t = 1$ is quoted as a marquee hit, but the running top Yukawa is ≈ 0.93–0.94 at the scales where it is measured. At m_t that is many standard deviations from one."

Response. The headline now reads "$y_t = 1$ at the matching scale," which is where the cascade-top reference is defined; we no longer present it as an unqualified equality. The physical, testable statement is $m_t = v/\sqrt{2}$, a parameter-free relation that lands within about one percent of the measured pole mass — itself a post-diction of a quantity known since 1995, and labeled as such. The running of y_t down to m_t is ordinary Standard-Model physics and is not in dispute; the only correction we have made is to attach the scale to the marquee statement so that "$y_t = 1$" cannot be read as a claim at m_t.

Objection 73 (O-SM-73) (the Eddington critic)
"$\alpha^{-1}(0) = 2^7 + 3^2 = 137$ is Eddington redux. With a small alphabet of integers and transcendentals one reaches most $O(1)$ targets. How is this different?"

Response. The difference Eddington never supplied is a quantified look-elsewhere accounting, and that is exactly the test we accept as decisive. Per parameter we already publish the trials cost (sparse where the selection is strong, $p \approx 1$ where it is not, and we decline to claim strength in the latter case). The honest, program-level number is the *global* trials factor over the full declared alphabet and operation set, evaluated against all twenty-eight targets at once; we regard that single figure — not the cleanliness of $2^7 + 3^2$ — as what should make or break the numerological charge. If the global p is large, the program fails by its own stated standard, and we would report it as such. We will not pretend the per-parameter picture already settles the matter: a rough back-of-the-envelope pass over the declared alphabet suggests that, of the dozen-or-so closures that look individually clean, an order-half could plausibly be reached by

chance — so the favorable single-parameter costs are suggestive, not probative, and only the global figure, computed once over all twenty-eight targets together, can discharge the charge. Committing to that test, rather than admiring individual coincidences, is the line between this and Eddington. The structural complement to this statistical answer — why the codebook is *forced* rather than searched, one closed ledger reused across all twenty-eight observables in the lineage of Euclid's *Elements* — is set out in §102.1.

Objection 74 (O-SM-74) (the single-input auditor)
"You advertise 'one fixed input,' but the starting point is a six-dimensional bulk *plus* a five-integer ledger *plus* five dimensional constants. That is not one input."

Response. "One input" means one continuous dimensionful free parameter, and on that precise reading the count holds. Of the five dimensional constants, c, $\hbar$, and k_B are unit conventions and Q_B is a fixed conversion ratio, leaving Newton's constant G_N as the single dimensionful scale — and there are no continuous *dimensionless* free parameters at all. The OGN-5 ledger is discrete, and its values are argued to be forced (conditionally, in the case of $N_g \leq 3$), not chosen; the bulk itself is posited by AX-3 rather than fitted. So the accurate phrasing, which the text now uses, is "one continuous dimensionful input (G_N), a discrete integer ledger argued to be unique, and no tunable dimensionless numbers" — not the looser "one input," which we agree overstates the economy if read literally.

AA The limits of geometrization and the standing of the forcing argument

Objection 75 (O-SM-75) (the renormalization critic)
"You geometrize the renormalization group as a radial fifth dimension $r = \ln(E_{\mathrm{GUT}}/\mu)$. If so, you should be able to shortcut the brutal high-order loop calculations — the muon $g-2$ at five loops, say. Can you?"

Response. No, and the reason is structural rather than a matter of effort; it is set out in full in the companion Arisaka (2026, A5-GDOS) (Theorem T9.9). The radial coordinate is the variable conjugate to scale, so it geometrizes exactly the *scale-dependent* (logarithmic, RG-resummable) content of an amplitude — the running couplings, the mass-ratio logs — and is, by construction, blind to the *scale-invariant* finite coefficients in which the labor of $g-2$ actually resides. Schwinger's $\alpha/2\pi$ and the higher coefficients, carrying $\zeta(3)$, $\pi^2 \ln 2$, and worse, are pure numbers with no scale dependence, and no geometry of the scale direction can manufacture them. So GG-Theory does not replace the Feynman expansion. What it does instead is compute the *inputs* that expansion takes as given — the value of α, the mass ratios — a different and complementary task. Perturbation theory computes the corrections to nature's constants; GG-Theory computes the constants.

Objection 76 (O-SM-76) (the hadronic / lattice critic)
"Grant the perturbative periods. But the strong-coupling regime is where holography helps. Surely there GG-6 can derive the hadronic contributions — vacuum polarization, light-by-light — that dominate the $g-2$ uncertainty?"

Response. Here the honest answer is 'structure and order of magnitude, but not precision,' analyzed in Arisaka (2026, A5-GDOS) (T9.9). The hadronic contribution is an irreducible *dynamical* correlator of the boundary QCD — the full spectral function $R(s)$ — not an input and not a scale-logarithm; deriving it means solving QCD. A holographic geometrization, GG-6's radial dimension included, genuinely helps at strong coupling, but only in the controlled limit of large N and large 't Hooft coupling, whereas real QCD has $N = 3$ and coupling of order one, where the corrections are themselves of order one. The GG-6 bulk, being parameter-free, could in principle yield a structural, fitted-dial-free estimate — a genuine advantage over tuned AdS/QCD — but at the $\sim$ 10–30% level, not the sub-percent the muon anomaly demands. That the data-driven and lattice determinations presently disagree is itself evidence that no geometric shortcut exists. Consistent with this paper's discipline, we make *no* claim to predict the hadronic $g-2$.

Objection 77 (O-SM-77) (the reverse-engineering critic)
"Your strongest move is 'the form is forced by the stratum' — $\alpha_s^{-1} = \pi e$ because the color channel carries both the angular and radial primitives. But was that assignment rule fixed *before* you knew the answer, or reverse-engineered to land on πe? If the latter, the forcing is circular."

Response. This is the sharpest form of the numerology charge, and it presses in the right place. The defense is provenance, not rhetoric, and it is honest about its own limits. The assignment rules that carry the most weight — the dyadic seam-depth giving 2^J, the sector/color ratio giving $\sin^2\theta_W = L^2/(4N^3)$, the Bogomolnyi instanton action πk giving the Yukawa cascade — are fixed by constructions (the GG-Constitution, BI-6, the GFF Morse cascade) that exist independently of the numerical targets and are applied identically in sectors with very different data. Where an assignment is *not* independently forced — and $\alpha_s^{-1} = \pi e$ is exactly such a case, a complexity *selection* on $\{\pi, e\}$ in which the empirical value enters the filter — we say so and grade it a selection, not a theorem (the six/fourteen/eight triage; §102.1). So the answer divides cleanly: the load-bearing forms are pre-registered by geometry, the selected forms are flagged as such, and the residual circularity of the selected set is precisely what the global trials factor exists to bound. We do not claim every form is forced; we claim the forced ones are forced and the selected ones are labeled.

Objection 78 (O-SM-78) (the promissory-note critic)
"Your entire anti-numerology defense rests on a 'global trials factor' you have not computed. You have written pages explaining why I should withhold judgment until a number appears that does not yet exist. Why should I?"

Response. The charge is fair, and we will not dodge it: the global trials factor is the decisive computation, it is not yet done, and until it is, the cohort's closures are an honest *ansatz*,

not a proof. What we claim in the meantime is bounded and, we think, defensible — that the construction is *forced or rule-selected* rather than freely searched (§102.1), that the per-parameter costs are already published (§101.2), and that the single number which would settle the matter is *named and accepted as decisive*, the precise commitment Eddington never made. A reader is entitled to withhold final judgment until that number is reported; what the reader is not entitled to do, having seen the over-constraint and the published costs, is assert that the freedom is *hidden*. The computation belongs to a dedicated statistics note, and we regard producing it as the most important single task ahead of the program — precisely because, by our own stated standard, a large global p would sink it.

Objection 79 (O-SM-79) (the Euclid critic)
"You liken the codebook to Euclid's *Elements*. But Euclid *proved* every proposition. You have eight Asserted entries and fourteen conditional ones. A triage is not the *Elements*."

Response. Correct, and the analogy is meant narrowly: it concerns the *method*, not the state of completion. The Euclidean point is that the parameters are presented as *propositions descending from a fixed constitution*, not as expressions assembled from a free toolbox — which is what separates a deductive architecture from a fit, and which holds whether a given proposition is yet fully proved or not. The triage is the honest ledger of *how far* each descent has been carried: six proved end-to-end, fourteen forced modulo one named lemma each, eight still asserted. We would rather publish that ledger than claim a completeness we have not reached — which is itself the Euclidean virtue, since the *Elements* is admired not for asserting its propositions but for proving them and stating which postulates each rests on. The analogy commits us to finishing the proofs, not to having finished them.

Objection 80 (O-SM-80) (the self-consistency critic)
"Suppose the geometry is internally self-consistent. So what? A self-consistent mathematical structure can describe nothing in nature. Consistency is not correctness, and it is not uniqueness."

Response. Entirely agreed, and we are careful not to conflate the three. Internal consistency is necessary but not sufficient: it earns the framework a fair hearing, not assent. *Correctness* is decided empirically, and the program stakes it on falsifiable forward commitments — the 3.85 TeV lock cluster, the 8.1 μeV Kaxion, $\delta_{\mathrm{CP}}^{\mathrm{PMNS}} = 196°$, $\sum m_\nu \approx 59$ meV — any of which can break it within the decade. *Uniqueness* — that this is *the* geometry rather than *a* geometry — is the weakest of the three claims and is carried explicitly as a conjecture, the spectrum-level uniqueness theorem being open. So the standing we assert is precise: a self-consistent geometry with zero continuous freedom and a dated falsification program, not a proof of nature. The objection names the right hierarchy, and the paper's own triage and falsifier table are built to respect it.

Objection 81 (O-SM-81) (the discrete-symmetry critic)
"Your CP story presupposes the standard C, P, T apparatus — unitary this, antiunitary that — imported wholesale from quantum field theory. You derive parameters while assuming the symmetry table."

Response. The table is now derived, not assumed (T-SM-46.1). In the real presentation the three discrete operations are orientation reversals of the three real factors of the bulk — fiber, space, clock — and their unitary/antiunitary character follows: reversal of the fiber commutes with the complex structure because the signed charge reverses with the generator; reversal of the clock anticommutes with it; hence a symmetry is antiunitary exactly when it contains T. CPT is total orientation reversal and a symmetry of any real texture; CP fails exactly when the texture is chiral on the circle, which is what the lattice angle $2\pi/5$ measures. The apparatus and the parameters come from the same geometry.

Objection 82 (O-SM-82) (the flavor-provenance critic)
"A 'derived' CKM phase is meaningless: phases are convention. You can put δ anywhere you like by rephasing, so deriving $2\pi/5$ derives nothing."

Response. Precisely because phases are convention, the paper derives the invariant, not the convention (T-SM-46.2). Rephasing is per-family dial re-zeroing; a spanning-tree gauge nulls every removable angle; what survives are loop holonomies — for three families, one — and the quoted $\delta^{(1)} = 2\pi/5$ is the standard-parameterization name of that single closed-loop angle, whose convention-free content is the Jarlskog invariant the paper checks against PDG. The claim is exactly as rephasing-invariant as an Aharonov–Bohm phase is gauge-invariant, and for the same holonomy reason.

Objection 83 (O-SM-83) (the neutrino-nature critic)
"Your neutrino block bets the house on Majorana neutrinos. If the light neutrinos turn out Dirac, the floors, the phases, and the downstream baryogenesis all collapse — and Majorana-ness was your choice, not your derivation."

Response. The direction of the bet is a theorem and the exposure is stated. T-SM-44.1 proves the Majorana channel is the unique $\Delta L = 2$ structure the fiber admits — open only at weight zero, where the reality condition *is* the Majorana condition — and the LHN portal is the unique fiber-neutral door; a Dirac-only sector would require leaving a constitutionally permitted, uniquely minimal term unused, an unforced suppression the framework's own economy clause forbids. So the framework does not merely prefer Majorana: it stakes itself on it, and the falsifier is the standard one — an established Dirac nature for the light neutrinos (e.g. a decisive null program in neutrinoless double beta decay at the relevant masses) retires the neutrino floors as derived and the leptogenesis module with them. The bet is forced, sized, and on the table.

Part XVII
References

Cited works, each with a relevance note linking it to the present paper

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Unlike a conventional reference list, every entry below carries a short *Relevance* note: a sentence or two saying what the cited work contributes and where in the present paper it bears. The aim is to turn the bibliography from a bare list of sources into a navigable companion — a reader can move through the references and see, at a glance, which theorem, table, or section each one underwrites. The notes are written for two readers at once. For the newcomer, they trace the intellectual lineage of the parameter derivation: from the foundational work on gauge theory, grand unification, the chiral anomaly, and the renormalization group, through to the twenty-two-paper GG-Theory cohort whose Standard-Model results the present paper completes. For the specialist, they make the paper's debts explicit and checkable — each note names the specific result it supports, so that no citation is merely decorative.

The references are organized in two groups — first the GG-Theory papers by the author (Section A below), then all other works (Section B), each in alphabetical order by first author. The cohort papers (GG-A1-Constitution–GG-A11-Origin, with the C- and P-series) are cited under the canonical keys fixed by the cross-paper coherence map of the GG-Theory program.

A. Papers by Katsushi Arisaka (the GG-Theory cohort)

Arisaka, K.• (2026). *The GG-Constitution: Four Axioms and Seven Postulates of the GG-Theory Program, From Which GG-6, BI-6, OGN-5, and GDOS Descend as Theorems*. Preprint, available at https://preprints.arisaka-gg.org/a1-constitution/.
doi:10.5281/zenodo.22063867.

Relevance. The present manuscript. It serves as the citeable constitutional foundation for the subsequent technical GG-6 and GDOS papers. It fixes the vocabulary (axiom/postulate/theorem), the operational meaning of measurement (OGGEP), and the parsimony discipline (MSMF/OGN) that constrains all later derivations. First cited in §1.

Arisaka, K.• (2026). *GG-Theory with GG-6: Geometry–Gauge Locking at the 6D Bulk (GG-6) at the Heart of Grand Geometric Theory (GG-Theory)*. Preprint, available at https://preprints.arisaka-gg.org/a2-gg6/.
doi:10.5281/zenodo.22064298.

Relevance. This companion paper develops the concrete six-dimensional geometric stage suggested by the axioms (especially GGEP and anomaly inflow requirements). In the present constitutional paper we cite it whenever the 6D bulk, the four-component BI-6 anomaly polynomial, the OGN-5

integer ledger, or the $\mathrm{Spin}(1,5)\times\mathrm{SU}(5)\times\mathrm{U}(1)_B\times\mathrm{U}(1)_{Qi}$ composite gauge group is invoked. First cited in §1.

Arisaka, K.• (2026). *Anomaly Freedom of BI-6: The Modified Bianchi Identity (BI-6) for the Six-Force, Six-Dimensional Bulk (GG-6)*. Preprint, available at https://preprints.arisaka-gg.org/a3-bi6/.
doi:10.5281/zenodo.22064300.
Relevance. This companion paper supplies the rigorous Hopkins-Singer differential-cohomology proof that the four-component anomaly polynomial $X_4=\mathrm{tr}(R\wedge R)-\mathrm{tr}(F_{\mathrm{SU}(5)}\wedge F_{\mathrm{SU}(5)})-\frac{2}{5}F_B\wedge F_B-\frac{2}{5}F_{Qi}\wedge F_{Qi}$ is exact, establishing $|a/c|=|a/d|=5/2$ as the geometry-gauge trace-ratio lock that POS-AF demands. First cited in §1.

Arisaka, K.• (2026). *From Euclid to GG-6: The Realization of Euclid's Elements in Six Dimensions — GU-5, OGN-5, and a Parameter-Free Reconstruction of Modern Physics*. Preprint, available at https://preprints.arisaka-gg.org/a4-euclid/.
doi:10.5281/zenodo.22064302.
Relevance. The historical companion to the GG6 Trio. Pedagogical commentary on the long chain from Euclid through Newton, Riemann, Maxwell, and Einstein to the GG-6 stage. We cite this paper whenever the historical positioning of the 6D bulk against its predecessors is invoked. First cited in §2.1.

Arisaka, K.• (2026). *GDOS: Geometric Dynamics of Open Systems — 6D GG-6 Bulk Projected to 4D Boundary for Quantum Measurement, General Relativity, and QFT*. Preprint, available at https://preprints.arisaka-gg.org/a5-gdos/.
doi:10.5281/zenodo.22064304.
Relevance. This companion paper develops the open-system "runtime" implied by OGGEP and the postulates (positivity, RG-compatibility, falsifiability). We cite it whenever the boundary law $\mathcal{L}_{\mathsf{GDOS}}$, the Hilbert-space embedding, the Born rule, the Schrödinger limit, the modified TOV equation, or the BRST/Slavnov-Taylor identities are invoked, since their proofs live there as the eleven GDOS Principal Theorems T1–T11. First cited in §1.

Arisaka, K.• (2026). *The GRIP on GDOS — GRIP: the Geometric Engine of Symmetry Breaking, from the Big Bang to Mind*. Preprint, available at https://preprints.arisaka-gg.org/a6-grip/.
doi:10.5281/zenodo.22064306.
Relevance. This companion paper formalizes the four Geometric-Invariance pillars descending from OGGEP: the Total Geometric-Gauge object TGG, the Covariant Conservation Identity CCI, the Geometric Free-Energy Functional GFF, and the Gauge-Runtime Invariance Pipeline GRIP (DGI → GSSB → GA). We cite it when the geometric-side consequences of the GG-Constitution are invoked. First cited in §1.

Arisaka, K.• (2026). *The Qi-4 Pillars — Qi-QUA, Qi-EP, Qi-CON, Qi-HAL: The Quantum-Information Backbone of GDOS*. Preprint, available at https://preprints.arisaka-gg.org/a7-qi4/.

doi:10.5281/zenodo.22064308.

Relevance. This companion paper formalizes the four Quantum-Information pillars descending from OGGEP: Qi-QUA (qua-state preparation), Qi-EP (quantum equivalence principle), Qi-CON (decoherence/contraction), and Qi-HAL (holographic alphabet). We cite it when the quantum-information-side consequences of the GG-Constitution are invoked. First cited in §7.1.

Arisaka, K.• (2026). *The Standard Cosmology as Theorem of GG-Theory: The Cosmic Inventory as Observations of One Six-Dimensional Geometry*. Preprint, available at https://preprints.arisaka-gg.org/a10-cosmos/.
doi:10.5281/zenodo.22064314.

Relevance. This companion paper derives the cosmological parameters $\Omega_m = 1/\pi$, $\Omega_\Lambda = 1 - 1/\pi$, $\Omega_b = 1/(2\pi^2)$, $\Omega_{\rm DM} = 1/\pi - 1/(2\pi^2)$ from the GG-Constitution and the 6D bulk. We cite it whenever the constitution's cosmological-prediction column is invoked. First cited in §81.

Arisaka, K.• (2026). *The Geometric Inevitability of Cosmos, Life, and Mind: The Self-Referential Closure of the GG-Theory Program*. Preprint, available at https://preprints.arisaka-gg.org/a11-origin/.
doi:10.5281/zenodo.22064319. No result of the present paper depends on it.

Relevance. The capstone of the program. Defines the Ground Geometric Attractor (GGA) and proves T-GGA-as-Ground-State, the ground-state refinement on which §94's answer to "why is every electron the same?" now rests; its eighteen-layer cascade is the recursive extension of the seven-layer survey given in that section. First cited in §94.1.

Arisaka, K.• (2026). *The GG-6 Lock Cluster: A Non-Spatial Kaluza–Klein Resonance near 3.85 TeV as the Standing LHC Falsifier of GG-Theory*. Preprint, available at https://preprints.arisaka-gg.org/p1-kk/.
doi:10.5281/zenodo.22064321.

Relevance. The first paper of the Predictions Branch. Specific phenomenology of the lock cluster at $M_{\rm lock} = (1/2)\, M_{\rm Pl}\, e^{-35} \approx 3.85\,\text{TeV}$, with explicit LHC and future-collider falsifiers. Any verified collider deviation from the predicted Kaluza-Klein resonance pattern at 5σ falsifies T4 + T6 jointly. First cited in §81.1.

Arisaka, K.• (2026). *Why Three Generations: A Three-Leg Derivation of $N_g = 3$ from BI-6, MSMF, and GDOS Morse Stability*. Preprint, available at https://preprints.arisaka-gg.org/p2-n3/.
doi:10.5281/zenodo.22064323.

Relevance. The second paper of the Predictions Branch. The dedicated proof that the fermion family number $N_g = 3$ is uniquely forced within GG-Theory by three logically independent legs: (L1) the MSMF lower bound $N_g \geq 3$ from Kobayashi–Maskawa CP-violation combinatorics; (L2) the GDOS Morse upper bound $N_g \leq 3$ from family-projector stability on $\mathbb{C}P^{N_g-1}$ via the cascade-closure constraint $S_{\rm tot}(N_g) = \pi N_g(N_g-1)/2 \leq S_{\rm BI}^{\rm max} = 3\pi$; (L3) the constructive codebook self-consistency check from the present paper (GG-A9-SM), which reproduces all twelve fermion masses (nine charged, three neutrino), at the few-per-cent level and the gauge and charged-lepton entries to four significant figures, uniquely at $N = 3$. Its capacity architecture dissolves the

would-be soft steps: the bulk-only derivation of the descended class is closed in the negative (its rigidity lemma), the transport geometry is supplied at the Born theorem's own grade by its adiabatic theorems T-AD-1–T-AD-3, strict Morse-positivity is off the critical path (holding as a theorem at the admissible value), and the integrality statement L-INT is a derived theorem with a single named topological residue R-INT — the same appendix chain proving the no-go for the naive formulation, the literal scale-profile metric pullback of $\mathrm{tr}\, R \wedge R$ vanishes identically — so that the coefficient is fixed by the Fubini–Study normalization of its Appendix F conventions, the Berry connection being constructed rather than awaited. The instanton budget $S_{\mathrm{BI}}^{\max} = 3\pi$ is the capacity theorem T-N3-6 there (extensive form πN; the residue R-INT is discharged on the benchmark background in that paper's Appendix H). Any discovery of a fourth standard family, a precision deviation in m_τ/m_μ from the cascade rational 16.800 by more than 0.5%, or a theoretical inconsistency between BI-6 and GDOS Morse-positivity would falsify the chain. Cited in the present paper (GG-A9-SM) as the canonical owner of the $N = 3$ theorem; GG-A9-SM imports the result without re-derivation. First cited in §7.5.

Arisaka, K.˙ (2026). *The Electron as the Ground State of Nature: The Six Oldest Questions about the Electron, Closed and Fenced within GG-Theory*. Preprint, available at https://preprints.arisaka-gg.org/p3-electron/.
doi:10.5281/zenodo.22064325.
Relevance. The third paper of the Predictions Branch. The dedicated review-article treatment of the six oldest electron questions — identity (with the values y_e, m_e, e, spin-$\frac{1}{2}$ derived or imported from the present paper's codebook), absolute stability (no exit gate from the GGA basin), the excited copies (importing GG-P2-N3 with its fences), the positron (the conjugate readout of the same basin, with the origin of the twin imported at theorem grade from GG-P5-CPT), pointlikeness (structure entirely along the non-spatial directions; readout locality derived from AX-1 of A1; the geometric pointlikeness theorem, with $F \equiv 1$ as its laboratory-facing corollary, held within the declared axiom system), and field response derived as composite geodesic motion on the AX-2 connection — Newton and Coulomb in parallel from one bulk, reproducing A5's Part VI — with the two-sector open-system reading (the phase sector moves the electron, the scale sector transmutes it; radiation is the dissipative channel of motion) — assembled as one theorem-complex about ground-state-ness, with claim-status ledger and pre-registered kill criteria. Consumes the present section's geometric-origin-of-identity reading and the lepton cascade-depth assignment without re-derivation. First cited in §94.2.

Arisaka, K.˙ (2026). *The Real Geometry Beneath the Complex Numbers of Physics: Positive Frequency is Holomorphy — from Cardano's Cubics to Twistor Space, via the Sixth Dimension of GG-Theory*. Preprint, available at https://preprints.arisaka-gg.org/p4-real/.
doi:10.5281/zenodo.22064327.
Relevance. The origin-of-i member paper of the Foundations Branch: the standalone, full-length development of the real presentation this paper uses in T-SM-44.1, T-SM-46.1, and T-SM-46.2 — the two real sources of the derived complex structure, the holomorphy bridge at its Hardy, tube,

and twistor levels, and the open items stated as open. For the present paper it supplies the origin of the complex phase structure in which the ledger's valued phases live. First cited in §2.1.

Arisaka, K.• (2026). *The Mirrors of Matter: Why Every Particle Has a Twin, Why Nature's Mirrors Break, and Why the Universe Is Made of Matter*. Preprint, available at https://preprints.arisaka-gg.org/p5-cpt/.
doi:10.5281/zenodo.22064329.
Relevance. The CPT member paper of the Foundations Branch: antiparticles from the handshake of the circle and the clock, C/P/T as orientation reversals of the three real bulk factors, CPT at quasi-free grade from the Wick-quartet analytic fact, the chiral-pentagon CP phases, and the matter–antimatter asymmetry through the weight-zero door. It is the GG-A9-SM–GG-A10-Cosmos interface of the cohort map: the ledger phases this paper derives — the CKM and PMNS Dirac phases and the Majorana parities, each at the grade recorded in the codebook — reach the baryon-asymmetry chain of GG-A10-Cosmos through it. First cited in §2.1.

Arisaka, K.• (2026). *The Proton as the Ground Geometric Attractor of Matter: The Eleven Questions of the Proton, Closed and Fenced within GG-Theory*. Preprint, available at https://preprints.arisaka-gg.org/p6-proton/.
doi:10.5281/zenodo.22064331.
Relevance. The Predictions-Branch member that runs the ground-attractor method of GG-P3-Electron on the baryonic stratum, and the one companion this paper is in debt to rather than the other way round. GG-A9-SM states proton stability as a theorem conditional on the (H2) baryon-descent step and defers that step to a successor; GG-P6-Proton discharges it, with a lock theorem that realizes the descended gauged $\mathrm{U}(1)_B$ and makes exact global baryon number a consequence rather than an assumption. The selection rule this paper cites in the leptogenesis appendix — the one that makes every $\Delta B = \pm 1$ channel unwritable — is proved there. Nothing in the twenty-eight-parameter derivation depends on it, and the proton-stability statement of §109.6 is unchanged by it; what changes is that a step this paper left open is now closed elsewhere in the cohort. First cited in §2.1.

Arisaka, K.• (2026). *The Fifth Force in GG-6: One Prediction, $\eta = 0$ at Any Range and Precision, and the Instrument That Tests It (MADO)*. Preprint, in preparation; the fifth-force companion of the Predictions branch.
Relevance. Carries the cohort's single fifth-force prediction — absence, $\eta = 0$ at any range and any precision — that replaces the (g_B, m_B) window this paper's falsification section formerly listed, and names the instrument that tests it. First cited in §109.7.

Arisaka, K.• (2026). *The Kaxion: Cold Dark Matter as the Non-Spatial Compact-Angle Pseudoscalar of GG-6*. Preprint, available at https://preprints.arisaka-gg.org/c1-kaxion/.
doi:10.5281/zenodo.22064333.
Relevance. The cosmology phenomenology companion to GG-A10-Cosmos. The Kaxion — the boundary dual of the BI-6 Green–Schwarz tensor — produces a single dark-matter line at 8.1 μeV (1.95 GHz) through one calibrated scale, with explicit haloscope falsifiers: the line at the DFSZ-line

coupling inside the 1.59–2.13 GHz window, and the derived-conditional count of its exposure ledger. Exclusion of the window at the derived coupling falsifies the prediction and, upstream, the constitutional clauses AX-4 MSMF and POS-6 OGN. First cited in §81.

Arisaka, K.• (planned). *The Geometric Inevitability of Cosmos, Life, and Mind — A GG-Theory Foundation Book*. Capstone volume of the GG-Theory program, in preparation.
Relevance. The planned capstone volume that synthesizes the cohort papers into a single treatment. GG-A9-SM cites it as the natural successor work for the higher-layer uniqueness theorems of the structure-formation cascade (hadrons, atoms, molecules, single-cell life, and the consciousness layer) beyond the lowest layer closed at theorem level here. First cited in §94.3.

Arisaka, K. (2026). *Quantum Mechanics Derived from GG-Theory as Projection from 6D Bulk to 4D Boundary*, Arisaka-GG-A8-QM.
Preprint, available at https://preprints.arisaka-gg.org/a8-qm/.
doi:10.5281/zenodo.22064310.
Relevance. The quantum-mechanics stratum of the Consequence Trio; recasts textbook QM via GDOS and OGGEP. First cited in §2.1.

Arisaka, K. (2026). *The Standard Model as Theorem of GG-Theory: Its 28 Parameters as Observations of One Six-Dimensional Geometry*, Arisaka-GG-A9-SM.
Preprint, available at https://preprints.arisaka-gg.org/a9-sm/.
doi:10.5281/zenodo.22064312.
Relevance. The present paper; listed for completeness as the GG-A9-SM node of the cohort map. First cited in §2.1.

Arisaka, K. (2026). *The Hubble Tension in the Light of GG-Theory: The Hubble Constant as One Reading of a Derived Expansion History $H(t)$*, Arisaka-GG-C2-Hubble.
Preprint, available at https://preprints.arisaka-gg.org/c2-hubble/.
doi:10.5281/zenodo.22064336.
Relevance. Cosmology and Astrophysics Branch: reads the Hubble tension as one six-dimensional geometric corridor. First cited in §2.1.

Arisaka, K. (2026). *Black-Hole Entropy under GG-Theory: The GDOS Reading of the Horizon as an OGGEP Port — with the Exact Bekenstein–Hawking $A/4G_N$ Derived from Induced Gravity*, Arisaka-GG-C3-BH.
Preprint, available at https://preprints.arisaka-gg.org/c3-bh/.
doi:10.5281/zenodo.22064338.
Relevance. Cosmology and Astrophysics Branch: black-hole entropy with the horizon as an OGGEP port. First cited in §2.1.

Arisaka, K. (2026). *Symmetry, Conservation, and Geometry: Seven Steps from Noether, Bianchi, to Hopkins–Singer, and the Ledger the Open-System Turn Pays Out*, Arisaka-GG-W1-Symmetry.
Preprint, available at https://preprints.arisaka-gg.org/w1-symmetry/.
doi:10.5281/zenodo.22064340.

Relevance. The cohort's symmetry review. Its ladder ends at protection without symmetry, which is the register in which §82's tier-2 argument is stated, and its reading of symmetry breaking as the one-way *reading* of holonomy data is the sentence §36.2 uses to relabel the Higgs mechanism honestly. First cited in §36.2.

Arisaka, K. (2026). *The Circle That Remembers, the Interval That Forgets: GG-Theory and the Sixteen Problems of Particle Physics and Cosmology*, Arisaka-GG-W2-Holonomy.
Preprint, available at https://preprints.arisaka-gg.org/w2-holonomy/.
doi:10.5281/zenodo.22064342.
Relevance. The treatise on the fork this sector's account runs on — holonomy requires closed loops, and an interval has none — proving the group law for circle transport and the semigroup law for interval transport, and carrying the stage-custody correspondence that §36.3 applies to the electroweak sheet. Its readability theorem is the source of the Elitzur reading. First cited in §36.3.

B. Other references

Abachi, S. et al. (D0 Collaboration). (1995). *Observation of the top quark.* Physical Review Letters, *74*, 2632–2637. https://doi.org/10.1103/PhysRevLett.74.2632
Relevance. The D0 co-discovery of the top quark at the Tevatron, confirming the heaviest Standard-Model fermion at the mass anticipated by precision electroweak fits. Cited in the introduction as part of the framework's record of prediction confirmed. First cited in §1.

Abe, F. et al. (CDF Collaboration). (1995). *Observation of top quark production in $\bar{p}p$ collisions with the Collider Detector at Fermilab.* Physical Review Letters, *74*, 2626–2631. https://doi.org/10.1103/PhysRevLett.74.2626
Relevance. The CDF discovery of the top quark at ~ 173 GeV, the value indirectly predicted beforehand by electroweak radiative corrections. Cited in the introduction as a triumph of Standard-Model predictivity. First cited in §1.

Adler, S. L. (1969). *Axial-vector vertex in spinor electrodynamics.* Physical Review, *177*, 2426–2438. https://doi.org/10.1103/PhysRev.177.2426
Relevance. With Bell and Jackiw, the discovery of the chiral (triangle) anomaly whose family-by-family cancellation constrains the Standard-Model charge assignments. Cited in the introduction's account of the theory's internal consistency. First cited in §1.

ADMX Collaboration (C. Bartram et al.). (2025). *Search for axion dark matter from 1.1 to 1.3 GHz with ADMX.* Phys. Rev. Lett. arXiv:2504.07279.
Relevance. DFSZ-sensitive microwave-cavity haloscope; its Generation-2 band (0.6–2 GHz) reaches the Kaxion line at 1.95 GHz. First cited in §109.3.

Ahmad, Q. R. et al. (SNO Collaboration). (2002). *Direct evidence for neutrino flavor transformation from neutral-current interactions in the Sudbury Neutrino Observatory*. Physical Review Letters, *89*, 011301. https://doi.org/10.1103/PhysRevLett.89.011301
Relevance. SNO's neutral-current measurement established solar-neutrino flavor transformation, completing (with Super-Kamiokande) the proof that neutrinos are massive. Cited in the introduction as the empirical origin of the neutrino sector. First cited in §1.

Arnison, G. et al. (UA1 Collaboration). (1983). *Experimental observation of isolated large transverse energy electrons with associated missing energy at* $\sqrt{s} = 540$ *GeV*. Physics Letters B, *122*, 103–116. https://doi.org/10.1016/0370-2693(83)91177-2
Relevance. The UA1 discovery of the W boson at CERN, confirming the electroweak prediction of massive charged mediators. Cited in the introduction as a triumph of Standard-Model predictivity. First cited in §1.

ATLAS Collaboration (Aad, G. et al.). (2012). *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*. Physics Letters B, *716*, 1–29. https://doi.org/10.1016/j.physletb.2012.08.020
Relevance. The ATLAS observation of the ~ 125 GeV Higgs boson, confirming electroweak symmetry breaking. Cited in the introduction and in the discussion of the Higgs sector. First cited in §1.

Barger, V., D. Marfatia and K. Whisnant. (2012). *The Physics of Neutrinos*. Princeton University Press.
Relevance. The shortest complete route from a graduate quantum-field-theory background to the current neutrino literature. Recommended in the course's textbook review as the concise survey; reviewed there for what it covers and where it stops. First cited in §XV.

Bell, J. S., & Jackiw, R. (1969). *A PCAC puzzle:* $\pi^0 \to \gamma\gamma$ *in the* σ*-model*. Il Nuovo Cimento A, *60*, 47–61. https://doi.org/10.1007/BF02823296
Relevance. With Adler, the discovery of the chiral anomaly. Cited in the introduction in the discussion of anomaly cancellation as an internal-consistency constraint. First cited in §1.

Bethe, H. A. (1947). *The electromagnetic shift of energy levels*. Physical Review, *72*, 339–341. https://doi.org/10.1103/PhysRev.72.339
Relevance. Bethe's 1947 non-relativistic estimate of the Lamb shift was the first successful renormalized QED calculation. We cite Bethe in the historical opening of the IR Abelian transport theorem as the founding moment of low-energy QED transport, extended by Schwinger and Tomonaga to the full $g-2$ program. First cited in §38.4.

Bilenky, S. M. (2018). *Introduction to the Physics of Massive and Mixed Neutrinos* (2nd ed.). Lecture Notes in Physics 947, Springer.
Relevance. The careful general-n counting of physical angles and phases for Dirac and Majorana neutrinos, and the proof that the Majorana phases cancel identically in every oscillation probability. The counting of (K.6) and problems K.1.4 and K.1.5 follow it. First cited in §K.

Bouchiat, C., Iliopoulos, J., & Meyer, P. (1972). *An anomaly-free version of Weinberg's model.* Physics Letters B, *38*, 519–523. https://doi.org/10.1016/0370-2693(72)90532-1
Relevance. Showed that the electroweak gauge anomalies cancel between quarks and leptons precisely when each family is complete, locking the quark and lepton charge assignments together. Cited in the introduction's account of internal consistency. First cited in §1.

W. Buchmüller, P. Di Bari, and M. Plümacher. (2005). *Leptogenesis for pedestrians.* Annals of Physics, *315*, 305–351. hep-ph/0401240. (See also Nuclear Physics B, *643*, 367–390 (2002), Erratum *793*, 362 (2008).)
Relevance. The field-defining review, and the source of the strong-washout efficiency fit (68.4) used in Part VIII. First cited in §68.

Cabrer, J. A., von Gersdorff, G., & Quirós, M. (2011). *Suppressing electroweak precision observables in 5D warped models.* Journal of High Energy Physics, *2011*(05), 083. arXiv:1103.1388.
Relevance. The published soft-wall route to a warped model inside the oblique bounds: a deformed infrared end with a Higgs profile fixed by the background and a strongly renormalized light mode, reaching Kaluza–Klein scales above a TeV. Cited in §34.6 as the escape this paper's throat deformation does not reproduce, and as the route whose applicability to the framework's bulk scalar is a question for the constitution. First cited in §34.6.

Carena, M., Pontón, E., Tait, T. M. P., & Wagner, C. E. M. (2003). *Opaque branes in warped backgrounds.* Physical Review D, *67*, 096006. arXiv:hep-ph/0212307.
Relevance. Brane-localized gauge kinetic terms in a warped background: a kinetic term on the infrared end makes the first Kaluza–Klein gauge mode light and repels the tower from the boundary, lowering its oblique contribution. Cited in §34.6 for the fourth escape from Open Problem O_{22}, the seam term GG-A3-BI6 registers as its Open Problem O15. First cited in §34.6.

J. A. Casas and A. Ibarra. (2001). *Oscillating neutrinos and $\mu \to e\gamma$.* Nuclear Physics B, *618*, 171–204. hep-ph/0103065.
Relevance. The parameterization in which the neutrino Yukawa matrix is decomposed into light masses, the PMNS matrix and an orthogonal matrix R. It is what makes the unflavored CP invariant manifestly U_{PMNS}-independent, and hence what forces the chain of Part VIII to be flavored. First cited in §63.

Cabibbo, N. (1963). *Unitary symmetry and leptonic decays.* Physical Review Letters, *10*, 531–533. https://doi.org/10.1103/PhysRevLett.10.531
Relevance. Cabibbo's 1963 introduction of the rotation angle that mixes strange and non-strange decays is the original quark-mixing matrix. We cite Cabibbo in the historical opening of the CKM section as the inventor of the structural object that Kobayashi–Maskawa later extended to three families with one CP phase. First cited in §52.

CMS Collaboration (Chatrchyan, S. et al.). (2012). *Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC.* Physics Letters B, *716*, 30–61. https://doi.org/10.1016/j.physletb.2012.08.021

Relevance. The CMS observation of the ~ 125 GeV Higgs boson, simultaneous with ATLAS. Cited in the introduction and in the discussion of the Higgs sector. First cited in §1.

Cheng, T.-P. and L.-F. Li. (1984). *Gauge Theory of Elementary Particle Physics*. Oxford University Press.
Relevance. A classic graduate text on the gauge structure of the Standard Model. Recommended in the technical-appendix graduate course for in-depth study of the gauge sector, the SU(5) embedding, and anomaly cancellation. First cited in §D.1.

CMS Collaboration (A. M. Sirunyan et al.). (2019). *Search for resonant $t\bar{t}$ production in proton–proton collisions at $\sqrt{s} = 13$ TeV*. JHEP, *04*, 031. arXiv:1810.05905.
Relevance. The $t\bar{t}$ resonance search whose Randall–Sundrum Kaluza–Klein-gluon exclusion reaches 4.55 TeV. Cited in F-SM-1 to explain why that limit, set on a broad template with $\Gamma/M \sim 15$–30% and BR$(t\bar{t})$ near unity, does not transfer to the narrow lock-cluster member. First cited in §109.2.

CMS Collaboration (A. M. Sirunyan et al.). (2020). *Search for high mass dijet resonances with a new background prediction method in pp collisions at $\sqrt{s} = 13$ TeV*. J. High Energy Phys., *05*, 033. arXiv:1911.03947.
Relevance. Run-2 dijet exclusions (leptophobic Z' to ~ 4 TeV; color-octet scalars to ~ 3.7 TeV) bracketing the predicted 3.85 TeV cluster. First cited in §109.2.

ACT Collaboration (E. Calabrese et al.). (2025). *The Atacama Cosmology Telescope: DR6 constraints on extended cosmological models*. JCAP, *11*, 063. arXiv:2503.14454.
Relevance. Independent CMB constraint on the neutrino mass sum, $\sum m_\nu < 0.089$ eV (95%, ΛCDM), quoted beside the DESI bound in F-SM-3 so that the model dependence of the squeeze is visible rather than hidden. First cited in §109.4.

F. Capozzi, E. Lisi, A. Marrone, and A. Palazzo. (2025). *Neutrino masses and mixing: entering the era of subpercent precision*. Physical Review D, *111*, 093006. arXiv:2503.07752.
Relevance. The independent Bari global fit, corroborating the ordering preference and the subpercent determination of the atmospheric splitting; cited so that the octant discussion does not rest on a single group's analysis. First cited in §55.5.

CUORE Collaboration (D. Q. Adams et al.). (2025). *With or without ν? Hunting for the seed of the matter-antimatter asymmetry*. Science, *390*, 1029. arXiv:2404.04453.
Relevance. The ^{130}Te $0\nu\beta\beta$ limit, $T^{0\nu}_{1/2} > 3.5 \times 10^{25}$ yr, completing the three-isotope picture against which the predicted $m_{\beta\beta} \simeq 1.8$ meV is scored. First cited in §109.4.

Coleman, S. R. and Weinberg, E. (1973). *Radiative corrections as the origin of spontaneous symmetry breaking*. Physical Review D, *7*, 1888–1910. https://doi.org/10.1103/PhysRevD.7.1888
Relevance. Coleman and Weinberg's 1973 paper showed that radiative corrections can generate spontaneous symmetry breaking even when the tree-level potential has no such feature, fixing the dimensionless quartic coupling at one-loop order. We cite Coleman–Weinberg in the historical

opening of the minimal quartic theorem as the inventors of the radiative-quartic mechanism that the geometric-attractor picture replaces with a tree-level branch normal form. First cited in §34.1.

Davoudiasl, H., Hewett, J. L., & Rizzo, T. G. (2003). *Brane-localized kinetic terms in the Randall–Sundrum model*. Physical Review D, *68*, 045002. arXiv:hep-ph/0212279.
Relevance. The companion analysis of brane kinetic terms for the gauge fields and the graviton in the Randall–Sundrum background, spectrum and precision-electroweak consequences included; with Carena et al. (2003) the reference for the price list of the seam-term escape in §34.6. First cited in §34.6.

S. Davidson and A. Ibarra. (2002). *A lower bound on the right-handed neutrino mass from leptogenesis*. Physics Letters B, *535*, 25–32. hep-ph/0202239.
Relevance. The bound (61.1) that forces standard hierarchical thermal leptogenesis above 10^9 GeV, and therefore the obstacle that the lock-scale mechanism of Part VIII must evade. First cited in §61.

S. Davidson, E. Nardi and Y. Nir. (2008). *Leptogenesis*. Physics Reports, *466*, 105–177. arXiv:0802.2962.
Relevance. The definitive review of the subject: the Boltzmann system, the flavor effects, the bound structure, and the efficiency parametrizations. Where the leptogenesis chapter of this course departs from the standard treatment it departs from this reference, and a reader should be able to locate exactly where. First cited in §XV.

DESI Collaboration (A. G. Adame et al.). (2024). *DESI 2024 VI: Cosmological constraints from the measurements of baryon acoustic oscillations*. arXiv:2404.03002.
Relevance. DESI + CMB neutrino-mass bound $\sum m_\nu < 0.072$ eV, pressing on the predicted $\simeq 59$ meV. First cited in §109.4.

DESI Collaboration (M. Abdul-Karim et al.). (2025). *Constraints on neutrino physics from DESI DR2 BAO and DR1 full shape*. Physical Review D, *112*, 083513.
Relevance. The current tightest cosmological bound on the neutrino mass sum: < 0.0642 eV (95%, ΛCDM), < 0.053 eV in the Feldman–Cousins variant, < 0.163 eV under w_0w_aCDM. Because this paper's F-SM-6 pins $w = -1$ two-sidedly, the ΛCDM row is the one that applies, and the predicted 59.17 meV sits eight percent beneath it. First cited in §109.4.

Dodelson, S. and F. Schmidt. (2020). *Modern Cosmology* (2nd ed.). Academic Press.
Relevance. The modern data pipeline from a CMB power spectrum or galaxy survey to a parameter constraint — the background a student needs to read the Σm_ν bounds of §J.1.5 critically rather than as quoted numbers. First cited in §XV.

J. Ellis and K. Sakurai. (2016). *Search for sphalerons in proton–proton collisions*. Journal of High Energy Physics, *04*, 086. arXiv:1601.03654.
Relevance. The quantitative collider phenomenology of sphaleron-induced $\Delta B = \Delta L = 3$ events — final states, rates under stated suppression assumptions, and LHC constraints. With Tye &

Wong (2015) it brackets the contested near-threshold question on which the FCC-hh rate of §76 rides. First cited in §76.

Englert, F. and Brout, R. (1964). *Broken symmetry and the mass of gauge vector mesons.* Physical Review Letters, *13*, 321–323. https://doi.org/10.1103/PhysRevLett.13.321
Relevance. Englert and Brout's 1964 paper, simultaneous with Higgs, gave the first complete relativistic gauge-invariant mechanism for vector-boson mass generation by spontaneous symmetry breaking. Cited in the historical opening of the Wilson–Hosotani Higgs-vev theorem and the minimal quartic theorem as one of the two co-original sources. First cited in §34.

I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler, and T. Schwetz. (2024). *NuFit-6.0: updated global analysis of three-flavor neutrino oscillations.* J. High Energy Phys., *12*, 216. arXiv:2410.05380.
Relevance. Global three-flavor oscillation fit; normal ordering favored, with a $\delta_{\rm CP}$ band consistent with the predicted 196°. First cited in §109.4.

I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler, J. P. Pinheiro, and T. Schwetz. (2025). *NuFIT 6.1: three-neutrino fit based on data available in November 2025.* Release at `nu-fit.org`; methodology as in NuFit-6.0, JHEP *12*, 216.
Relevance. The global three-flavor fit used as the primary oscillation benchmark throughout this paper. Its atmospheric best fit lies in the lower octant, against this paper's derived upper-octant value and against the accelerator joint fit; the disagreement is the subject of §55.5. First cited in §55.1.

I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler, J. P. Pinheiro, and T. Schwetz. (2026). *Lessons from the first JUNO results.* JHEP, *04*, 089. arXiv:2601.09791.
Relevance. The post-JUNO global analysis: $\Delta\chi^2(\mathrm{IO}-\mathrm{NO}) = 4.6$ without atmospheric data and 9.4 with, roughly doubling the discrimination and placing normal ordering — this paper's prediction — at about 2.2–2.3σ. First cited in §109.4.

Fermi, E. (1934). *Versuch einer Theorie der β-Strahlen. I.* Zeitschrift für Physik, *88*, 161–177. https://doi.org/10.1007/BF01351864
Relevance. Fermi's 1934 four-fermion theory of beta decay introduced the dimensionful coupling G_F that now defines the electroweak scale. We cite Fermi in the historical opening of the boson-mass and Fermi-constant theorem package as the originator of the structural constant whose value the present paper derives from the OGN-5 ledger and the scale chain. First cited in §37.4.

Fukugita, M. and T. Yanagida. (2003). *Physics of Neutrinos and Applications to Astrophysics.* Springer.
Relevance. The bridge from the neutrino mass matrix to cosmology, written by the authors of the leptogenesis proposal itself; the natural book-length companion to App. L. Reviewed in the course's textbook survey, where its stopping point — the heavy-neutrino scale and the Dirac Yukawa remain model inputs — is stated. First cited in §XV.

M. Fukugita and T. Yanagida. (1986). *Baryogenesis without grand unification.* Physics Letters B, *174*, 45–47.

Relevance. The paper that proposed leptogenesis: make a lepton asymmetry in heavy-neutrino decay and let the electroweak sphalerons convert it. Part VIII is a realization of this mechanism in which the heavy scale is an output rather than an input. First cited in §57.

Fritzsch, H. and Gell-Mann, M. (1972). *Current algebra: Quarks and what else?.* In *Proceedings of the XVI International Conference on High Energy Physics.* Chicago/Batavia.

Relevance. Fritzsch and Gell-Mann's 1972 conference paper proposed three color charges (the original color–gauge identification) as the resolution of the Δ^{++} statistics paradox. We cite Fritzsch–Gell-Mann in the historical opening of the strong angle–scale theorem as the inventors of the color-gauge identification that this paper now derives geometrically. First cited in §32.3.

Fukuda, Y. et al. (Super-Kamiokande Collaboration). (1998). *Evidence for oscillation of atmospheric neutrinos.* Physical Review Letters, *81*, 1562–1567. https://doi.org/10.1103/PhysRevLett.81.1562

Relevance. The Super-Kamiokande discovery of atmospheric-neutrino oscillation, the first proof that neutrinos have mass and the one empirical fact that forced the Standard Model to grow. Cited in the introduction as the origin of the neutrino sector. First cited in §1.

Gell-Mann, M. (1962). *Symmetries of baryons and mesons.* Physical Review, *125*, 1067–1084. https://doi.org/10.1103/PhysRev.125.1067

Relevance. Gell-Mann's 1962 paper introduced the $SU(3)$ flavor symmetry now called the Eightfold Way, the structural antecedent of $SU(3)_c$ as the color-confinement gauge group. We cite Gell-Mann in the historical opening of the strong angle–scale theorem as the originator of the structural identification of the strong sector with an $SU(3)$ algebra. First cited in §32.3.

Georgi, H. (1999). *Lie Algebras in Particle Physics* (2nd ed.). Perseus Books.

Relevance. Representation theory as a working tool: the SU(3) chapters, the tensor methods, and the embedding of the Standard-Model group in SU(5) are the technical backing for the branching arguments of the ledger chapter. First cited in §XV.

Georgi, H., & Glashow, S. L. (1974). *Unity of all elementary-particle forces.* Physical Review Letters, *32*, 438–441.

Relevance. The original SU(5) grand-unified theory paper. Georgi and Glashow embed $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ into a single simple group whose adjoint **24** contains the Standard-Model gauge bosons plus heavy X, Y bosons mediating proton decay; the matter content of one family fits into $\bar{\mathbf{5}} \oplus \mathbf{10}$. We cite Georgi–Glashow 1974 in §6 as the historical stepping-stone whose non-Abelian spine GG-Theory retains and embeds inside the 6D bulk. First cited in §6.

M. Gell-Mann, P. Ramond, and R. Slansky. (1979). *Complex spinors and unified theories.* In *Supergravity* (Stony Brook Workshop), Conf. Proc. C790927, 315–321. arXiv:1306.4669.

Relevance. One of the three independent 1977–1980 statements of the seesaw mechanism. Cited beside Minkowski and Yanagida in Part VIII as the historical origin of the heavy-Majorana route that the constitutional door of §58 arrives at without freedom. First cited in §58.

J. A. Harvey and M. S. Turner. (1990). *Cosmological baryon and lepton number in the presence of electroweak fermion number violation.* Physical Review D, *42*, 3344–3349.
Relevance. The sphaleron transfer coefficient $B = (28/79)(B - L)$ of (69.1). This factor is inherited unchanged by the present framework and is flagged as textbook rather than derived. First cited in §69.

Giunti, C. and C. W. Kim. (2007). *Fundamentals of Neutrino Physics and Astrophysics.* Oxford University Press.
Relevance. The standard reference for the wave-packet treatment of oscillations — why the plane-wave derivation of (J.5) returns the right answer, under what coherence conditions, and what happens when they fail — and for the Majorana charge-conjugation bookkeeping. The caveat stated in §J.1.2 is theirs. First cited in §J.

Glashow, S. L. (1961). https://doi.org/10.1016/0029-5582(61)90469-2
Relevance. Glashow's gauge unification of electromagnetic and weak interactions initiated the electroweak structure completed by Weinberg and Salam. We cite these as the historical backbone for POS-1 (Gauge Invariance) and for the claim that symmetry principles organize viable theories. First cited in §5.

Glashow, S. L., Iliopoulos, J., & Maiani, L. (1970). *Weak interactions with lepton-hadron symmetry.* Physical Review D, *2*, 1285–1292. https://doi.org/10.1103/PhysRevD.2.1285
Relevance. The GIM mechanism, which suppressed flavor-changing neutral currents by requiring a fourth quark and so predicted charm before its 1974 discovery. Cited in the introduction as an instance of internal consistency forcing a prediction. First cited in §1.

M. Green and J. Schwarz. (1984). *Anomaly cancellations in supersymmetric D=10 gauge theory and superstring theory.* Phys. Lett. B, *149*, 117.
Relevance. The 1984 Green–Schwarz paper proved that 10-dimensional anomalies in superstring theory cancel for the gauge groups SO(32) and $E_8 \times E_8$, launching the first superstring revolution. The mathematical mechanism — a higher-form B_2 tensor field with modified Bianchi identity $dH_3 = X_4$ that absorbs the chiral anomaly via inflow — is exactly what GG-Theory adapts to six dimensions as BI-6 (Theorem 4 of GG-A2-GG6). We cite it in Part III of the present paper as the historical origin of the Green–Schwarz mechanism, with explicit acknowledgment that the technical machinery is shared while the constitutional principles differ (10D vs. 6D, SUSY vs. non-SUSY, landscape vs. unique vacuum). First cited in §11.1.

Gross, D. J., & Wilczek, F. (1973). https://doi.org/10.1103/PhysRevLett.30.1343
Relevance. Asymptotic freedom is a key structural fact about non-Abelian gauge theories and underlies the consistency of QCD. We cite it to emphasize that "viability filters" (renormalizability, RG behavior) are part of what makes gauge theories physically admissible. First cited in §32.3.

Griffiths, D. J. (2008). *Introduction to Elementary Particles* (2nd ed.). Wiley-VCH.
Relevance. An accessible upper-undergraduate/early-graduate introduction to particle physics. Recommended in the technical-appendix graduate course for a gentle first pass, in particular on the fermion families and the Yukawa spectrum. First cited in §H.1.

Gross, D. J. and Wilczek, F. (1973). *Ultraviolet behavior of non-Abelian gauge theories*. Physical Review Letters, *30*, 1343–1346. https://doi.org/10.1103/PhysRevLett.30.1343
Relevance. Companion to GrossWilczek1973: Gross and Wilczek's 1973 letter is the inaugural statement of asymptotic freedom for non-Abelian gauge theories, sharing the 2004 Nobel Prize with Politzer's independent derivation. We cite Gross 1973 in the historical opening of the strong angle–scale theorem as the founding moment of the perturbative QCD high-energy regime that GG-Theory now derives from the infrared region. First cited in §32.3.

Halzen, F. and A. D. Martin. (1984). *Quarks and Leptons: An Introductory Course in Modern Particle Physics*. John Wiley & Sons.
Relevance. A classic phenomenology textbook. Recommended in the technical-appendix graduate course as the companion for the cross-section, decay, and force-law estimates used in the worked examples, in particular on the classical force laws and proton phenomenology. First cited in §N.1.

P. W. Higgs. (1964). *Broken symmetries and the masses of gauge bosons*. Phys. Rev. Lett., *13*, 508.
Relevance. Foundational paper on gauge-boson mass generation. GG-6 reproduces the mass relations from holonomy. First cited in §34.

Hirzebruch, F. (1995). *Topological Methods in Algebraic Geometry*. Berlin: Springer. Classics in Mathematics. Reprint of the 3rd ed.
Relevance. Hirzebruch's classic monograph develops the theory of characteristic classes, Pontryagin classes, and the L-genus that bears his name. We cite Hirzebruch for the signature theorem $\sigma(M) = \int_M L_k(p_1, \ldots, p_k)$ used in Step 2 of the parity route (T-N3-8 of GG-P2-N3) reducing strict GDOS Morse-positivity to a parity statement on the complex dimension of the family-projector moduli. First cited in §43.11.

Hopkins, M. J., & Singer, I. M. Quadratic functions in geometry, topology, and M-theory. *Journal of Differential Geometry* **70** (2005) 329–452. arXiv:math/0211216.
Relevance. The differential-cohomology framework of the cohort: the quadratic refinement technology behind self-dual field quantization and the global consistency of the GG-6 tensor sector, and the setting of the integrality theorem L-INT that Arisaka (2026, P2-N3) names as the single open statement of its $N = 3$ chain. First cited in §43.11.

S. Elitzur. (1975). *Impossibility of spontaneously breaking local symmetries*. Phys. Rev. D, *12*, 3978.
Relevance. The theorem that a local gauge symmetry cannot break spontaneously — conceded in a footnote by most treatments of the Standard Model and then set aside. On the holonomy reading it becomes a result rather than an embarrassment: what acquires a value is the gauge-invariant Wilson-line modulus, and nothing breaks. First cited in §36.2.

H. Hatanaka, T. Inami & C. S. Lim. (1998). *The gauge hierarchy problem and higher-dimensional gauge theories*. Mod. Phys. Lett. A, *13*, 2601.
Relevance. Establishes the property on which this paper's hierarchy argument rests: the effective potential of a Wilson-line phase is finite because no local counterterm for it exists. The tier-2

protection of §82 is this result, read on the cohort's anatomy; it is inherited, not claimed as new. First cited in §107.0.6.

N. S. Manton. (1979). *A new six-dimensional approach to the Weinberg–Salam model.* Nucl. Phys. B, *158*, 141.
Relevance. The first construction obtaining a Higgs-like scalar from the extra-dimensional components of a gauge field, and in six dimensions — the ancestor of a reading this paper once held and withdraws in §23.3: the doublet is not a connection component here, the selector is. First cited in §107.0.6.

Y. Hosotani. (1983). *Dynamical mass generation by compact extra dimensions.* Phys. Lett. B, *126*, 309.
Relevance. The Hosotani mechanism is the field-theory foundation of the GG-6 electroweak branch selector. First cited in §34.

Y. Hosotani. (1989). *Dynamics of non-integrable phases and gauge symmetry breaking.* Ann. Phys., *190*, 233.
Relevance. The developed dynamics of the non-integrable phase — the systematic treatment of how the holonomy's effective potential selects a vacuum, which is the mechanism Theorem 34.1 closes within the ledger. First cited in §107.0.6.

Huang, G.-y., & Zhou, S. (2021). *Precise values of running quark and lepton masses in the Standard Model.* Physical Review D, *103*, 016010. arXiv:2009.04851.
Relevance. The $\overline{\text{MS}}$ running quark masses at M_Z (its Table 2, full Standard Model) used for the sector log-gap ratios of §101.2, where the three sectors are read at one scale. First cited in §101.2.

Hyper-Kamiokande Collaboration (K. Abe et al.). (2018). *Hyper-Kamiokande Design Report.* arXiv:1805.04163.
Relevance. Next-generation proton-decay reach $\sim 10^{35}$ yr ($p \to e^+\pi^0$); data-taking from 2027. First cited in §109.6.

R. Jost. (1957). *Eine Bemerkung zum CPT-Theorem.* Helvetica Physica Acta, *30*, 409–416.
Relevance. The CPT theorem in its axiomatic form, reaching interacting Wightman field theories through analytic continuation to the complexified Lorentz group. It is cited here for the relation the cohort actually claims — *complementarity*: Jost's scope exceeds the geometric account's, which is quasi-free with the interacting case conditional, while the geometric account supplies the anatomy Jost's proof does not (what C, P and T individually are, and why antiunitarity traces to the time factor). See Arisaka, 2026, P5-CPT for the statement of that division. First cited in §95.

JUNO Collaboration (A. Abusleme et al.). (2026). *Measurement of reactor neutrino oscillation with the first JUNO data.* Nature, *654*, 343–348.
Relevance. The world's most precise determination of the solar pair, from 59.1 days of running: $\sin^2\theta_{12} = 0.3092 \pm 0.0087$ and $\Delta m^2_{21} = (7.50 \pm 0.12) \times 10^{-5}$ eV^2. The derived closed form $L^2/3N_g^3 = 25/81$ sits 0.07σ from it. First cited in §55.1.

KamLAND-Zen Collaboration (S. Abe et al.). (2025). *Search for Majorana neutrinos with the complete KamLAND-Zen dataset*. Physical Review Letters, *135*, 262501. arXiv:2406.11438.
Relevance. $0\nu\beta\beta$ bound $T^{0\nu}_{1/2} > 3.8 \times 10^{26}$ yr ($m_{\beta\beta} \lesssim$ 28–122 meV), probing the predicted Majorana structure. First cited in §109.4.

KATRIN Collaboration (M. Aker et al.). (2025). *Direct neutrino-mass measurement based on 259 days of KATRIN data*. Science, *388*, 180.
Relevance. Direct kinematic bound $m_\nu < 0.45$ eV, consistent with the predicted light spectrum. First cited in §109.4.

W.-Y. Keung and G. Senjanović. (1983). *Majorana neutrinos and the production of the right-handed charged gauge boson*. Physical Review Letters, *50*, 1427–1430.
Relevance. The canonical collider signature of a TeV-scale Majorana neutrino: gauge-portal production followed by same-sign dileptons, exhibiting $\Delta L = 2$ in a detector. Cited in §76 for the channel this framework's leptophobic cluster *closes* — and that the open mirror-portal question of §76.3 could reopen at a future machine. First cited in §76.

F. R. Klinkhamer and N. S. Manton. (1984). *A saddle-point solution in the Weinberg–Salam theory*. Physical Review D, *30*, 2212–2220.
Relevance. The sphaleron itself: the static saddle-point configuration at the top of the barrier between topologically distinct vacua, of energy $\simeq 9$ TeV, whose thermal activation makes $B + L$ violation unsuppressed above the electroweak crossover. First cited in §L.1.3.

V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov. (1985). *On anomalous electroweak baryon-number non-conservation in the early universe*. Physics Letters B, *155*, 36–42.
Relevance. The electroweak sphaleron: unsuppressed $B + L$ violation above the electroweak scale, conserving $B - L$. It is the converter without which the lepton asymmetry of Part VIII would never become a baryon asymmetry, and it is compatible with this framework's exact proton stability because it moves baryon number in units of three. First cited in §69.

Kobayashi, M. and Maskawa, T. (1973). *CP-violation in the renormalizable theory of weak interaction*. Progress of Theoretical Physics, *49*, 652–657. https://doi.org/10.1143/PTP.49.652
Relevance. Kobayashi and Maskawa's 1973 extension of Cabibbo to three quark generations (predicting a third family before its discovery) and the resulting one-phase CP violation define the modern 3×3 CKM matrix. Cited in the historical opening of the CKM section as the inventors of the four-parameter structure that the present paper derives as projector misalignment. First cited in §7.5.

Kolb, E. W. and M. S. Turner. (1990). *The Early Universe*. Addison-Wesley.
Relevance. The machinery of a species in and out of equilibrium: entropy density, the effective degrees of freedom g_*, freeze-out by rate-against-Hubble comparison, and the Boltzmann equation in FRW coordinates. Every one of the six assembly factors of (L.19) is a Kolb–Turner quantity. First cited in §XV.

Langacker, P. (2017). *The Standard Model and Beyond* (2nd ed.). CRC Press.
Relevance. A graduate text presenting the Standard Model as an effective theory together with its extensions. Recommended in the technical-appendix graduate course for in-depth study of the neutrino sector and lepton mixing. First cited in §J.1.

LEGEND Collaboration. (2026). *Search for neutrinoless double-beta decay with LEGEND-200.* Physical Review Letters, *136*, 022701. arXiv:2505.10440. Program description: arXiv:2501.10046.
Relevance. ^{76}Ge program. The quoted $T^{0\nu}_{1/2} > 1.9 \times 10^{26}$ yr is the *combined* LEGEND-200, GERDA and Majorana Demonstrator limit and is not LEGEND-200's alone; LEGEND-1000 projects $m_{\beta\beta}$ of 9–21 meV in the 2030s. First cited in §109.4.

S. P. Mikheyev and A. Yu. Smirnov. (1985). *Resonance amplification of oscillations in matter and spectroscopy of solar neutrinos.* Soviet Journal of Nuclear Physics, *42*, 913–917.
Relevance. With Wolfenstein (1978), the resonant matter effect: the observation that the in-medium mixing angle becomes maximal at a critical electron density (J.12) regardless of the vacuum angle, which is how the solar solution was identified and how a long-baseline experiment reads the sign of Δm^2_{31}. First cited in §J.1.3.

P. Minkowski. (1977). $\mu \to e\gamma$ *at a rate of one out of* 10^9 *muon decays?* Physics Letters B, *67*, 421–428.
Relevance. The first statement of the seesaw mechanism. See the note at Gell-Mann et al., 1979. First cited in §58.

Mohapatra, R. N. and P. B. Pal. (2004). *Massive Neutrinos in Physics and Astrophysics* (3rd ed.). World Scientific.
Relevance. The model-building menu — type I, type II and type III seesaws, left–right symmetric completions, radiative mass generation — set out so that the common structure and the differences are both visible. The three-type classification of §J.1.6 follows it. First cited in §J.1.6.

R. N. Mohapatra and G. Senjanović. (1980). *Neutrino mass and spontaneous parity nonconservation.* Physical Review Letters, *44*, 912–915.
Relevance. The seesaw embedded in a left–right symmetric theory, and the standard reference for the connection between Majorana neutrino mass and parity violation. First cited in §58.

Majorana, E. (1937). *Teoria simmetrica dell'elettrone e del positrone.* Il Nuovo Cimento, *14*, 171–184. https://doi.org/10.1007/BF02961314
Relevance. Majorana's 1937 paper introduced the self-conjugate fermion that bears his name and the additional CP phases now called Majorana phases. We cite Majorana in the historical opening of the neutrino-mass section as the inventor of the channel through which the constitutional Majorana mass enters the GG-Theory neutrino sector. First cited in §49.19.

Maki, Z., Nakagawa, M., and Sakata, S. (1962). *Remarks on the unified model of elementary particles.* Progress of Theoretical Physics, *28*, 870–880. https://doi.org/10.1143/PTP.28.870
Relevance. Maki, Nakagawa, and Sakata's 1962 paper introduced the unitary mixing matrix that bears their initials, generalizing Pontecorvo's two-flavor oscillation to a multi-flavor structure.

We cite MNS in the historical opening of the PMNS section as the original PMNS-matrix authors; the modern Dirac-plus-Majorana phase structure is the projector-misalignment image of their construction. First cited in §54.

MICROSCOPE Collaboration (P. Touboul et al.). (2022). *MICROSCOPE mission: final results of the test of the equivalence principle*. Phys. Rev. Lett., *129*, 121102. arXiv:2209.15487.
Relevance. Final weak-equivalence-principle test $\eta(\mathrm{Ti},\mathrm{Pt}) = [-1.5 \pm 2.3 \pm 1.5] \times 10^{-15}$, bounding a long-range baryonic fifth force. First cited in §109.7.

Milnor, J. (1963). *Morse Theory*. Princeton, NJ: Princeton University Press. Annals of Mathematics Studies.
Relevance. Milnor's classic 1963 monograph established Morse theory as a tool for studying the topology of smooth manifolds via critical points of smooth real-valued functions. We cite Milnor for the perfect-Morse-function structure of the Fubini–Study height function on $\mathbb{C}P^{N_g-1}$, which is the structural backbone of Leg L2 of the three-leg $N_g = 3$ derivation. First cited in §21.5.

Workman, R. L. et al. (Particle Data Group). (2022). *Review of Particle Physics*. Progress of Theoretical and Experimental Physics, *2022*, 083C01. https://doi.org/10.1093/ptep/ptac097
Relevance. The standard compilation of measured particle masses, couplings, and mixing parameters. Cited as the source of the empirical benchmark values that the present paper's 28-parameter codebook is audited against. First cited in §1.

Navas, S. et al. (Particle Data Group). (2024). *Review of Particle Physics*. Physical Review D, *110*, 030001. https://doi.org/10.1103/PhysRevD.110.030001
Relevance. The current edition of the compilation; cited for the global electroweak fit's oblique parameters, $T = 0.00 \pm 0.06$ with $U = 0$, against which the lock-scale electroweak tower is priced in §34.6. First cited there.

Pauli, W. (1930). *Letter to the Tübingen conference, December 4, 1930 ("Liebe Radioaktive Damen und Herren")*. Reprinted in W. Pauli, Wissenschaftlicher Briefwechsel, Band II.
Relevance. Pauli's 1930 letter postulating the neutrino to rescue energy conservation in beta decay is the originating moment of the neutrino concept. We cite Pauli in the historical opening of the neutrino-mass section as the inventor of the particle whose mass-generation mechanism this paper now derives geometrically. First cited in §49.19.

Abel, C. et al. (nEDM Collaboration). (2020). *Measurement of the permanent electric dipole moment of the neutron*. Physical Review Letters, *124*, 081803. https://doi.org/10.1103/PhysRevLett.124.081803
Relevance. The current best direct bound on the neutron electric dipole moment, $|d_n| < 1.8 \times 10^{-26}\, e\,\mathrm{cm}$ at 90% C.L. It is the measurement that converts the strong-CP angle into an experimental statement, and every $\bar{\theta}$ bound quoted in this paper rests on it. First cited in Appendix O.1.

Baluni, V. (1979). *CP-nonconserving effects in quantum chromodynamics*. Physical Review D, *19*, 2227–2230. https://doi.org/10.1103/PhysRevD.19.2227

Relevance. With Crewther et al., the original computation of the neutron electric dipole moment induced by a nonzero $\bar{\theta}$. Cited for the coefficient relating d_n to $\bar{\theta}$.

Crewther, R. J., Di Vecchia, P., Veneziano, G. and Witten, E. (1979). *Chiral estimate of the electric dipole moment of the neutron in quantum chromodynamics*. Physics Letters B, *88*, 123–127. https://doi.org/10.1016/0370-2693(79)90128-X
Relevance. The chiral estimate $d_n \simeq 2.4 \times 10^{-16}\,\bar{\theta}\; e\,\mathrm{cm}$ used throughout the strong-CP treatment of this paper. It is the coefficient that turns the nEDM bound into the bound on $\bar{\theta}$.

Peccei, R. D. and Quinn, H. R. (1977). *CP conservation in the presence of pseudoparticles*. Physical Review Letters, *38*, 1440–1443. https://doi.org/10.1103/PhysRevLett.38.1440
Relevance. Peccei and Quinn's 1977 introduction of the global $U(1)_{PQ}$ symmetry whose spontaneous breaking dynamically suppresses the strong-CP $\bar{\theta}$ angle is the original framework in which the smallness of $\bar{\theta}$ becomes a derived rather than fine-tuned statement. We cite Peccei–Quinn in the historical opening of the strong-CP section; GG-Theory replaces the global PQ symmetry with a topological projection that fixes the *bare* angle structurally, the *observable* angle being relaxed by the bulk's own tensor remnant, the Kaxion, rather than by an added field. First cited in §79.

Perlmutter, S. et al. (Supernova Cosmology Project). (1999). *Measurements of* Ω *and* Λ *from 42 high-redshift supernovae*. The Astrophysical Journal, *517*, 565–586. https://doi.org/10.1086/307221
Relevance. With Riess et al., the discovery of cosmic acceleration and hence of dark energy. Cited in the introduction in the statement of the dark-energy and cosmological-constant problem. First cited in §1.

Vafa, C. and Witten, E. (1984). *Parity conservation in quantum chromodynamics*. Physical Review Letters, *53*, 535–536. https://doi.org/10.1103/PhysRevLett.53.535
Relevance. The theorem that the vacuum energy of a vector-like gauge theory is minimized at vanishing θ. It certifies the minimum that the companion's relaxation reaches; it does not drive the relaxation, and this paper carries it in that order, following GG-C1-Kaxion. First cited in §79.

Peskin, M. E. and D. V. Schroeder. (1995). *An Introduction to Quantum Field Theory*. Addison-Wesley (Westview Press).
Relevance. The standard graduate text on quantum field theory and the gauge sector of the Standard Model. Recommended as a primary reference for the technical-appendix graduate course, in particular on gauge theories, renormalization-group running, and the instanton/topological background of the strong-CP sector. First cited in §D.1.

A. Pilaftsis. (1997). *Resonant CP violation induced by particle mixing in transition amplitudes*. Nuclear Physics B, *504*, 61–107. hep-ph/9702393.
Relevance. The origin of the resonant enhancement (61.3) on which the lock-scale mechanism depends: quasi-degenerate heavy states cancel the Yukawa suppression of the CP asymmetry. First cited in §61.

A. Pilaftsis and T. E. J. Underwood. (2004). *Resonant leptogenesis*. Nuclear Physics B, *692*, 303–345. hep-ph/0309342. (See also Physical Review D, *72*, 113001 (2005).)

Relevance. The systematic treatment of leptogenesis in the resonant regime, and the reference against which the regulator question of §66 is posed. In the general treatment the quasi-degeneracy is assumed; here it is the ledger's own level spacing M_{lock}/K. First cited in §61.

Planck Collaboration (N. Aghanim et al.). (2020). *Planck 2018 results. VI. Cosmological parameters*. Astron. Astrophys., *641*, A6. arXiv:1807.06209.

Relevance. CMB cosmological parameters (the observed Ω's, z_*, and $H_0^{\mathrm{CMB}} \approx 67$) compared with the derived inventory of GG-A10-Cosmos. First cited in §111.1.

Politzer, H. D. (1973).

Relevance. Along with Gross–Wilczek, this established asymptotic freedom. It reinforces the centrality of RG structure and motivates scale covariance as a constitutional requirement. First cited in §32.3.

Pontecorvo, B. (1957). *Mesonium and antimesonium*. Soviet Physics JETP, *6*, 429–431.

Relevance. Pontecorvo's 1957 paper introduced neutrino oscillations as a meson–antimeson analog. We cite Pontecorvo in the historical opening of the PMNS section as the inventor of lepton-flavor mixing, which Maki–Nakagawa–Sakata 1962 cast in the modern projector form. First cited in §49.19.

Riess, A. G. et al. (High-z Supernova Search Team). (1998). *Observational evidence from supernovae for an accelerating universe and a cosmological constant*. The Astronomical Journal, *116*, 1009–1038. https://doi.org/10.1086/300499

Relevance. With Perlmutter et al., the discovery of the accelerating expansion of the universe. Cited in the introduction in the statement of the dark-energy and cosmological-constant problem. First cited in §1.

Rubin, V. C., & Ford, W. K. (1970). *Rotation of the Andromeda nebula from a spectroscopic survey of emission regions*. The Astrophysical Journal, *159*, 379–403. https://doi.org/10.1086/150317

Relevance. The flat galactic rotation curves that, with Zwicky's cluster dynamics, established the gravitational case for dark matter. Cited in the introduction in the statement of the dark-matter problem. First cited in §1.

Sakharov, A. D. (1967). *Violation of CP invariance, C asymmetry, and baryon asymmetry of the universe*. JETP Letters, *5*, 24–27.

Relevance. Sakharov's three conditions for dynamically generating the matter–antimatter asymmetry (baryon-number violation, C and CP violation, departure from equilibrium). Cited in the introduction in the statement of the baryon-asymmetry problem. First cited in §1.

Salam, A. (1968).

Relevance. Salam's electroweak unification is part of the Standard Model foundation. It is cited here to emphasize that modern physics already depends on strong "admissibility rules" (gauge

invariance and renormalizability) even if not presented as an explicit constitution. First cited in §5.

J. Schechter and J. W. F. Valle. (1982). *Neutrinoless double beta decay in $SU(2)\times U(1)$ theories.* Physical Review D, *25*, 2951–2954.
Relevance. The black-box theorem: any mechanism producing $0\nu\beta\beta$ generates a Majorana neutrino mass, so an observation establishes the Majorana nature whatever the underlying operator. Quoted in §K.1.5 together with its honest magnitude — the mass so generated is of order 10^{-24} eV, so the theorem is qualitative. First cited in §K.1.5.

Schwartz, M. D. (2014). *Quantum Field Theory and the Standard Model.* Cambridge University Press.
Relevance. A modern graduate text covering quantum field theory through the full Standard Model. Recommended as a primary reference for the technical-appendix graduate course, in particular on the Higgs mechanism and electroweak symmetry breaking. First cited in §E.1.

Schwinger, J. (1948). *On quantum-electrodynamics and the magnetic moment of the electron.* Physical Review, *73*, 416–417. https://doi.org/10.1103/PhysRev.73.416
Relevance. Schwinger's 1948 calculation $a_e = \alpha/(2\pi)$ for the electron anomalous magnetic moment is the first technical instance of an Abelian IR transport coefficient. We cite Schwinger as the originator of the structural object that the IR Abelian transport theorem in this paper generalizes. First cited in §38.4.

Sommerfeld, A. (1916). *Zur Quantentheorie der Spektrallinien.* Annalen der Physik, *356*(17), 1–94. https://doi.org/10.1002/andp.19163561702
Relevance. Sommerfeld's 1916 paper introduced the fine-structure constant α as the dimensionless coupling controlling the relativistic correction to the hydrogen spectrum. We cite Sommerfeld in the historical opening of the electromagnetic dyadic-closure theorem (T) as the originator of α as a derivable structural constant rather than an empirical fit. First cited in §31.

G. 't Hooft. (1976). *Symmetry breaking through Bell–Jackiw anomalies.* Physical Review Letters, *37*, 8–11.
Relevance. The observation that the anomaly (L.3) makes baryon and lepton number non-conserved in the Standard Model through instanton transitions, with $B - L$ exactly preserved — the structural fact on which leptogenesis rests. First cited in §L.1.3.

't Hooft, G., & Veltman, M. (1972). *Regularization and renormalization of gauge fields.* Nuclear Physics B, *44*, 189–213. https://doi.org/10.1016/0550-3213(72)90279-9
Relevance. The proof that spontaneously broken non-Abelian gauge theories are renormalizable, which turned the electroweak synthesis into a predictive quantum field theory. Cited in the introduction's account of how the Standard Model was assembled. First cited in §1.

Srednicki, M. (2007). *Quantum Field Theory.* Cambridge University Press.

Relevance. Two-component spinor technology developed from the outset, which is the cleanest preparation for the Majorana material of §K.1.1, where the distinction between ψ and ψ^c must be handled without apology. First cited in §XV.

Super-Kamiokande Collaboration (A. Takenaka et al.). (2020). *Search for proton decay via $p \to e^+\pi^0$ and $p \to \mu^+\pi^0$ with an enlarged fiducial volume in Super-Kamiokande I–IV*. Phys. Rev. D, *102*, 112011. arXiv:2010.16098.
Relevance. Current 90% C.L. bound $\tau/B(p \to e^+\pi^0) > 2.4 \times 10^{34}$ yr against which exact stability is tested. First cited in §109.6.

Super-Kamiokande Collaboration (K. Abe et al.). (2021). *Neutron–antineutron oscillation search using a 0.37 megaton-years exposure of Super-Kamiokande*. Phys. Rev. D, *103*, 012008. arXiv:2012.02607.
Relevance. Current $\Delta B = 2$ bound $\tau_{n\bar{n}} > 4.7 \times 10^8$ s, forbidden exactly in GG-6. First cited in §109.6.

T2K and NOvA Collaborations (K. Abe, M. A. Acero et al.). (2025). *Joint neutrino oscillation analysis from the T2K and NOvA experiments*. Nature, *646*, 818–824. arXiv:2510.19888.
Relevance. The direct accelerator determination of the Dirac phase, adopted here as the δ_{PMNS} benchmark: for normal ordering, best fit 203.4° with 1σ interval $[165.6°, 266.4°]$, containing the derived 196.06°. It also places $\sin^2\theta_{23}$ in the upper octant, against the global fit. It excludes $\delta = +\pi/2$ at 3σ but not $3\pi/2$. First cited in §55.1.

S.-H. H. Tye and S. S. C. Wong. (2015). *Bloch wave function for the periodic sphaleron potential and unsuppressed baryon and lepton number violating processes*. Physical Review D, *92*, 045005. arXiv:1505.03690.
Relevance. The proposal that the exponential suppression of sphaleron-induced $\Delta B = \Delta L = 3$ processes may lift near the barrier energy $E_{\text{sph}} \simeq 9$ TeV, making the transition observable at a 100 TeV collider. Contested (see Ellis & Sakurai, 2016); this paper takes no side and cites both. First cited in §76.

Thomson, M. (2013). *Modern Particle Physics*. Cambridge University Press.
Relevance. A modern, experiment-aware graduate textbook. Recommended in the technical-appendix graduate course for a gentle first pass, in particular on quark mixing and the CKM matrix. First cited in §I.1.

Weinberg, S. (1967). https://doi.org/10.1103/PhysRevLett.19.1264
Relevance. Weinberg's paper, together with Glashow and Salam, established the electroweak gauge theory. We cite it as part of the historical backbone of gauge invariance and spontaneous symmetry breaking, which GG-Theory aims to generalize to open systems via GDOS. First cited in §5.

Weinberg, S. (1978). *A new light boson?*. Physical Review Letters, *40*, 223–226. https://doi.org/10.1103/PhysRevLett.40.223

Relevance. With Wilczek, the recognition that a spontaneously broken Peccei–Quinn symmetry implies a light pseudoscalar, the axion. Cited in the introduction in the statement of the strong-CP problem. First cited in §1.

S. Weinberg. (1979). *Baryon- and lepton-nonconserving processes*. Physical Review Letters, *43*, 1566–1570.
Relevance. The dimension-five operator $(LH)(LH)/\Lambda$: the unique lowest-dimension addition to the Standard Model that gives neutrinos mass, and the operator the constitutional door of §58 arrives at as the only legal $\Delta L = 2$ structure. First cited in §58.

Weinberg, S. (1989). *The cosmological constant problem*. Reviews of Modern Physics, *61*, 1–23. https://doi.org/10.1103/RevModPhys.61.1
Relevance. The canonical statement of the cosmological-constant problem and the $\sim 10^{120}$ discrepancy between the natural and observed vacuum-energy scales. Cited in the introduction in the statement of the dark-energy problem. First cited in §1.

Weinberg, S. (2008). *Cosmology*. Oxford University Press.
Relevance. The derivation of the cosmological observables — acoustic peaks, baryon density, light-element abundances — from first principles, so that a student knows precisely what η_B *is* as a measured quantity before being asked to predict it. The two independent determinations of §L.1.1 are set out there. First cited in §XV.

Weinberg, S. (1995–1996). *The Quantum Theory of Fields*, Vols. I–II. Cambridge University Press.
Relevance. The authoritative advanced treatment of quantum field theory and the electroweak Standard Model. Recommended in the technical-appendix graduate course for in-depth study of the electroweak sector and its effective-operator structure. First cited in §E.1.

Wilczek, F. (1978). *Problem of strong P and T invariance in the presence of instantons*. Physical Review Letters, *40*, 279–282. https://doi.org/10.1103/PhysRevLett.40.279
Relevance. With Weinberg, the identification of the axion as the pseudo-Goldstone boson of a broken Peccei–Quinn symmetry. Cited in the introduction in the statement of the strong-CP problem. First cited in §1.

K. G. Wilson. (1974). *Confinement of quarks*. Phys. Rev. D, *10*, 2445.
Relevance. Wilson loops as gauge-invariant observables. The holonomy language central to GG-6. First cited in §34.

Witten, E. (1982). *Supersymmetry and Morse theory*. Journal of Differential Geometry, *17*(4), 661–692. https://doi.org/10.4310/jdg/1214437492
Relevance. Witten's 1982 paper introduced supersymmetric quantum mechanics on a Riemannian manifold and showed that the Witten index of the deformed exterior derivative computes the Euler characteristic via Morse-theoretic critical-point counting. We cite Witten for the Bogomolnyi-bound saturation of the gradient flow on $\mathbb{C}P^{N_g-1}$, which gives the explicit instanton action $S_{[2k]\to[2(k-1)]} = \pi k$ used in the cascade-closure constraint of Leg L2. First cited in §21.5.

L. Wolfenstein. (1978). *Neutrino oscillations in matter*. Physical Review D, *17*, 2369–2374.
Relevance. The charged-current matter potential (J.10), $V_{\mathrm{CC}} = \sqrt{2} G_F n_e$, arising because ordinary matter contains electrons and not muons. It is the origin of both the solar solution and the long-baseline handle on the mass ordering, and of the matter-induced fake CP asymmetry that is the field's hardest systematic. First cited in §J.1.3.

Xing, Z.-z., Zhang, H., & Zhou, S. (2012). *Updated values of running quark and lepton masses*. Physical Review D, *86*, 013013. arXiv:1112.3112.
Relevance. The $\overline{\mathrm{MS}}$ charged-lepton masses at M_Z (its Table II) and the second quark compilation used for the sector log-gap ratios of §101.2; the spread between its quark ratios and those of Huang & Zhou (2021) is the uncertainty quoted there. First cited in §101.2.

T. Yanagida. (1979). *Horizontal gauge symmetry and masses of neutrinos*. In *Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe* (KEK, Tsukuba), eds. O. Sawada and A. Sugamoto, Conf. Proc. C7902131, 95–99.
Relevance. The third independent statement of the seesaw mechanism. The 1979 KEK proceedings is the reference intended here; the companion paper Progress of Theoretical Physics *64*, 1103 (1980) is a distinct work and is not the one cited. First cited in §58.

Yukawa, H. (1935). *On the interaction of elementary particles. I*. Proceedings of the Physico-Mathematical Society of Japan, *17*, 48–57.
Relevance. Yukawa's 1935 paper postulated the exchange-particle origin of the nucleon-nucleon force and introduced the dimensionless coupling that bears his name. We cite Yukawa in the historical opening of the charged-lepton mass section as the originator of the dimensionless fermion-mass coupling later identified with m_f/v. First cited in §47.

Zuber, K. (2020). *Neutrino Physics* (3rd ed.). CRC Press.
Relevance. The experiments in detail and up to date — what each detector measures, its systematics, and why the error bars are the size they are. Recommended in §XV as the book to read before writing a falsification criterion against a neutrino observable. First cited in §J.

Zwicky, F. (1933). *Die Rotverschiebung von extragalaktischen Nebeln*. Helvetica Physica Acta, *6*, 110–127.
Relevance. Zwicky's measurement of the velocity dispersion of the Coma cluster, the first dynamical evidence for unseen (dark) mass. Cited in the introduction in the statement of the dark-matter problem. First cited in §1.