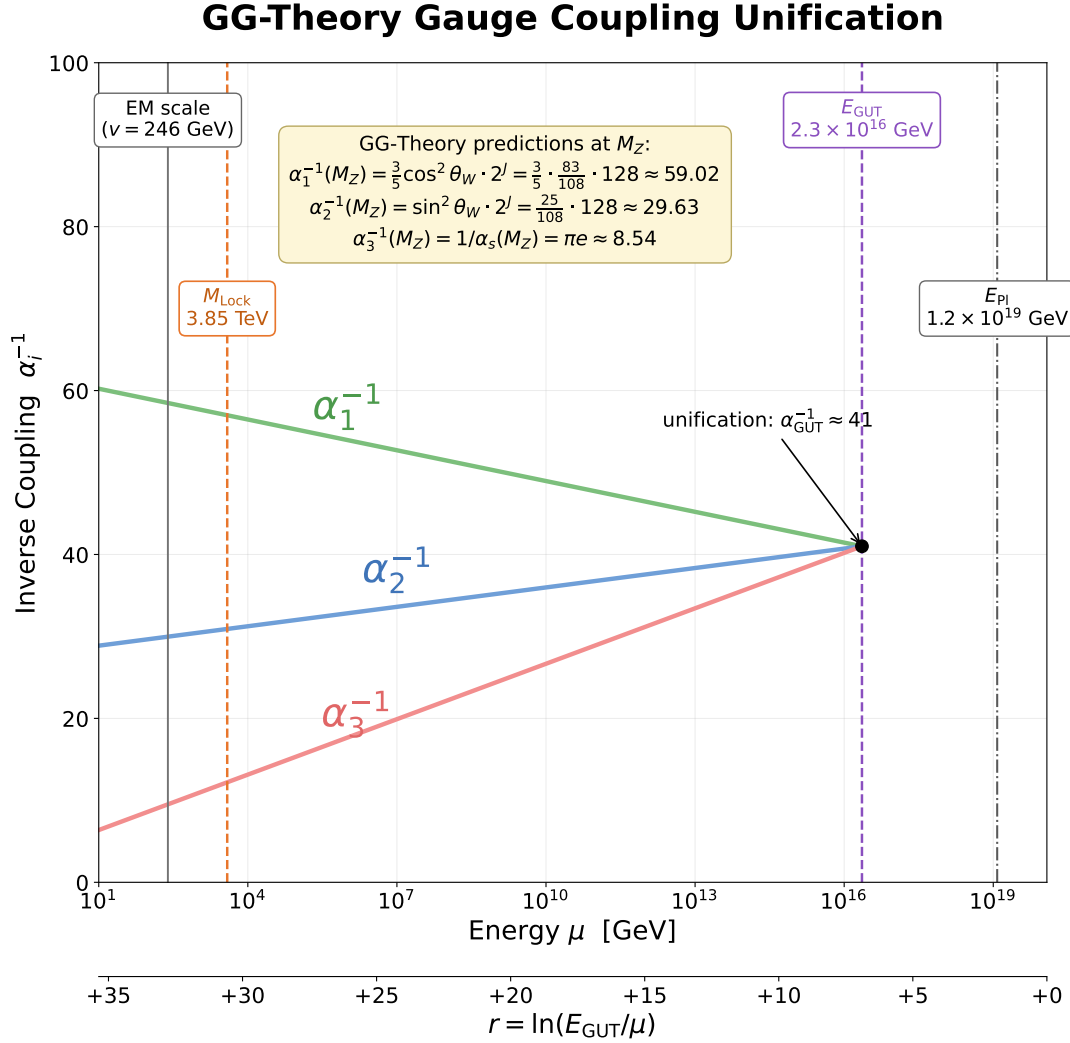




$$\alpha_1^{-1}(M_Z) = \frac{3}{5} \cos^2 \theta_W \cdot 2^J = \frac{3}{5} \cdot \frac{83}{108} \cdot 128 = 59.02, \quad \alpha_2^{-1}(M_Z) = \sin^2 \theta_W \cdot 2^J = \frac{25}{108} \cdot 128 = 29.63,$$

$$\alpha_3^{-1}(M_Z) = 1/\alpha_s(M_Z) = \pi e = 8.54; \quad \alpha_{\text{GUT}}^{-1} \approx 41 \text{ at } E_{\text{GUT}} = M_{\text{Pl}} e^{-2\pi} \approx 2.3 \times 10^{16} \text{ GeV}$$

Figure 3: GG-Theory gauge-coupling unification across the 6D bulk. Inverse couplings α_1^{-1} (green, hypercharge in SU(5) normalization), α_2^{-1} (blue, SU(2)_L), and α_3^{-1} (coral, SU(3)_C) versus the renormalization-group scale μ and the radial coordinate $r = \ln(E_{\text{GUT}}/\mu)$, with $r = 0$ at the unification point. The boundary values at M_Z are pinned by the GG-Constitution and the OGN-5 ledger with the three closed forms displayed above — no continuous fitted parameter intervenes anywhere between M_Z and E_{GUT} . Four constitutional scales are marked: the EM scale $v = 246$ GeV, the KK lock-cluster mass $M_{\text{lock}} = \frac{1}{2} M_{\text{Pl}} e^{-35} \approx 3.85$ TeV, the unification scale $E_{\text{GUT}} = M_{\text{Pl}} e^{-2\pi} \approx 2.3 \times 10^{16}$ GeV, and the Planck scale $E_{\text{Pl}} \approx 1.2 \times 10^{19}$ GeV. Below M_{lock} the running uses the standard one-loop SM beta functions; above M_{lock} the KK lock cluster opens and the three couplings converge to $\alpha_{\text{GUT}}^{-1} \approx 41$ at exactly E_{GUT} (black dot), where the SM gauge-coupling description ends and the bulk geometry takes over. This three-line meeting is the structural claim of the 6D ultraviolet completion of the Standard Model; the segment above M_{lock} is drawn as that claim, not computed, and the lock-cluster spectrum is its empirical content. Full discussion: Figure 13 in Part IV.