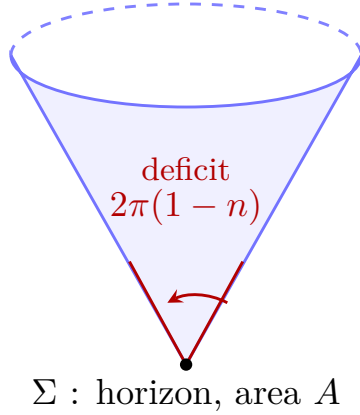


Where the $1/4$ Comes From, and Why Nothing Was Fitted to Get It

Euclidean time τ (replica cover $\mathcal{M}_n$)



$$\int_{\mathcal{M}_n} \sqrt{g} R \supset 4\pi(1-n) A$$

$$\Rightarrow S_{\text{BH}} = \frac{A}{4G_N}, \quad \frac{1}{4} = \frac{4\pi}{16\pi}$$

The number that is usually quoted and rarely derived. On the replica cover the Euclidean-time circle is pinched at the horizon with deficit angle $2\pi(1-n)$; the curvature concentrated on the horizon contributes $4\pi(1-n)A$ to $\int \sqrt{g} R$, and fed through the induced Einstein–Hilbert action the replica formula returns $S_{\text{BH}} = A/4G_N$. The coefficient is the ratio of two normalizations: $1/4 = 4\pi/16\pi$. *Neither number knows anything about black holes.* The 4π is fixed by the geometry of spheres, the 16π by demanding Newton’s law in the weak-field limit, and both were set by consistency requirements decades before anyone computed a horizon entropy. No free parameter enters, no regularization choice survives, and supersymmetry is never used. In full: Figure 9, p. 70.